Computer algebra independent integration tests

6-Hyperbolic-functions/6.5-Hyperbolic-secant/6.5.3-Hyperbolic-secant-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [201]. This is test number [179].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (201)	% 0.00 (0)
Mathematica	% 95.52 (192)	% 4.48 (9)
Maple	% 69.65 (140)	% 30.35 (61)
Maxima	% 44.78 (90)	% 55.22 (111)
Fricas	% 70.65 (142)	% 29.35 (59)
Sympy	% 4.48 (9)	% 95.52 (192)
Giac	% 56.72 (114)	% 43.28 (87)
Mupad	% 46.77 (94)	% 53.23 (107)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

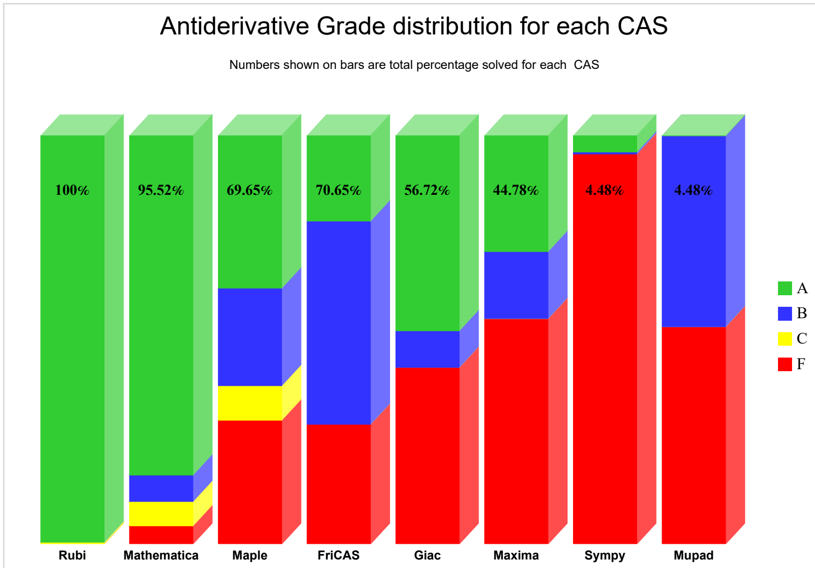
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

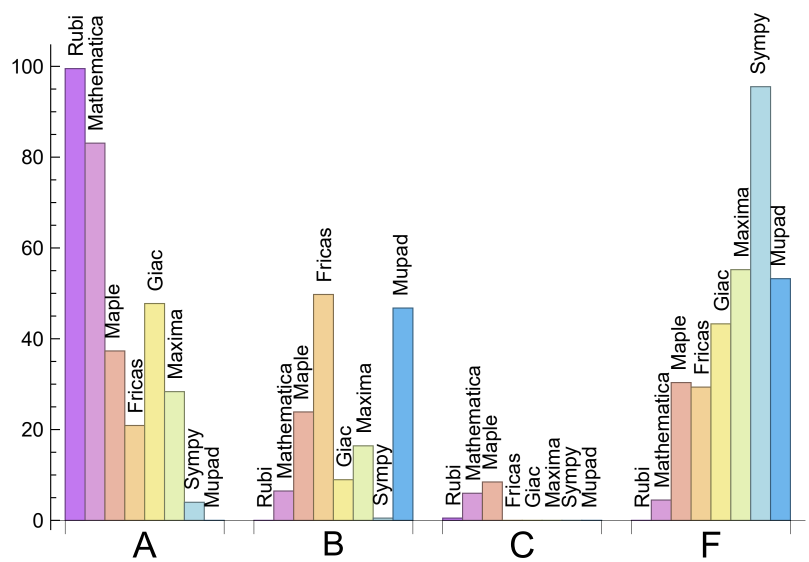
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.50	0.00	0.50	0.00
Mathematica	83.08	6.47	5.97	4.48
Maple	37.31	23.88	8.46	30.35
Maxima	28.36	16.42	0.00	55.22
Fricas	20.90	49.75	0.00	29.35
Sympy	3.98	0.50	0.00	95.52
Giac	47.76	8.96	0.00	43.28
Mupad	0.00	46.77	0.00	53.23

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	44.44 %	55.56 %	0.00 %
Maple	61	100.00 %	0.00 %	0.00 %
Maxima	111	81.98 %	0.00 %	18.02 %
Fricas	59	86.44 %	13.56 %	0.00 %
Sympy	192	94.79 %	5.21 %	0.00 %
Giac	87	77.01 %	14.94 %	8.05 %
Mupad	107	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

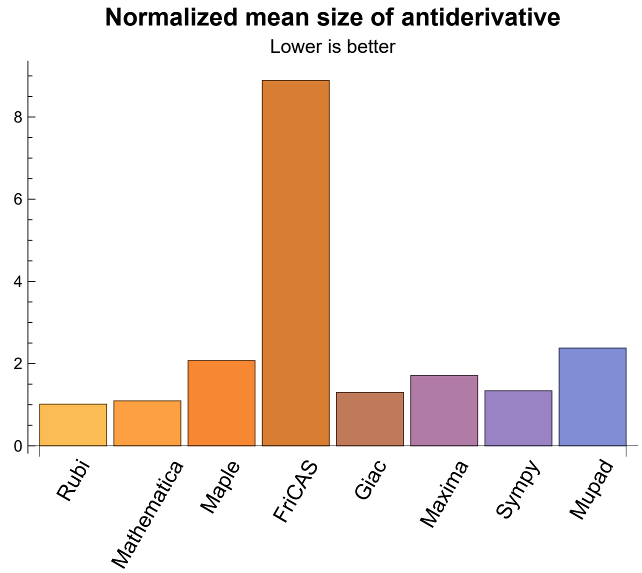
1.3 Performance

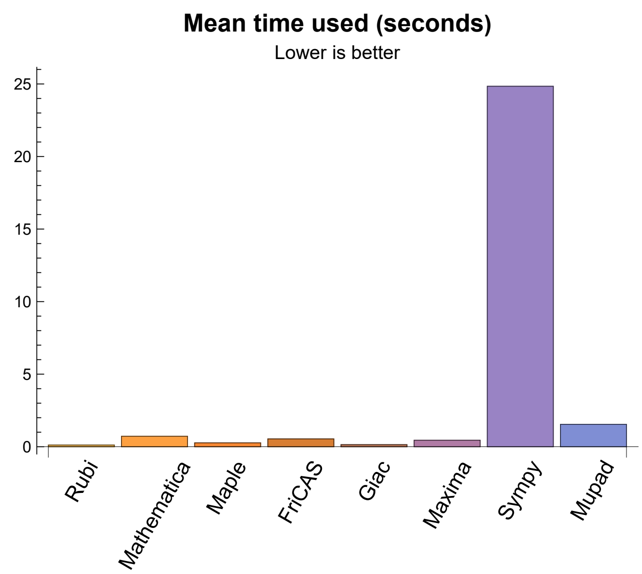
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.11	93.02	1.01	66.00	1.00
Mathematica	0.72	84.65	1.09	58.00	1.00
Maple	0.27	129.71	2.07	100.00	1.50
Maxima	0.45	88.78	1.71	62.00	1.56
Fricas	0.54	791.25	8.89	267.00	6.46
Sympy	24.84	46.11	1.34	41.00	1.09
Giac	0.14	74.34	1.30	54.50	1.19
Mupad	1.54	154.05	2.38	75.50	2.14

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {186}

Mathematica {183, 184, 185}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

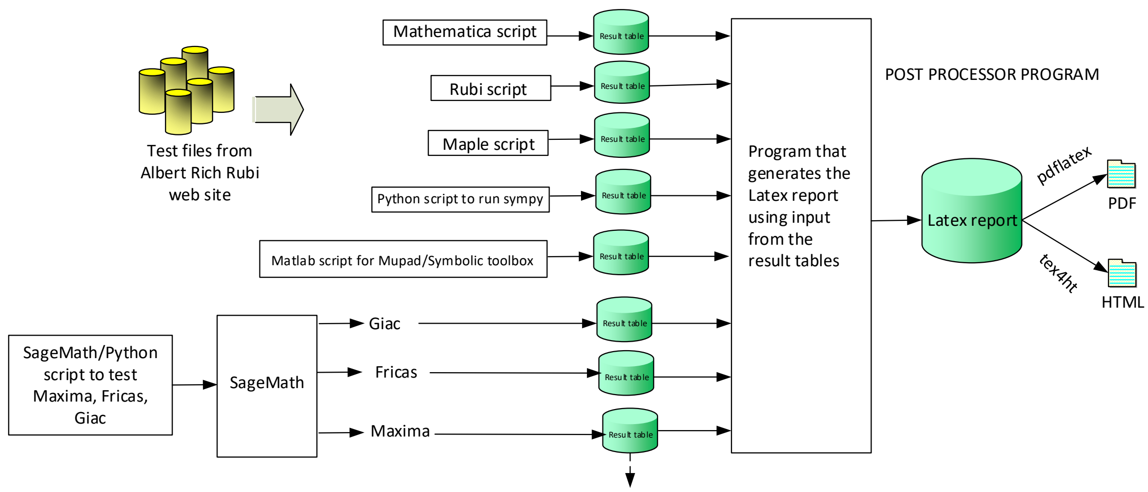
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade: { }

C grade: { 186 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 140, 142, 143, 144, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 164, 165, 167, 169, 171, 173, 175, 178, 179, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade: { 27, 85, 86, 129, 132, 135, 136, 137, 145, 146, 185, 187, 188 }

C grade: { 158, 160, 162, 166, 168, 170, 172, 174, 176, 177, 180, 181 }

F grade: { 130, 131, 138, 139, 141, 147, 148, 149, 150 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 45, 46, 47, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 104, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 127, 135, 144, 151, 152, 153, 154, 155, 156, 157, 159, 161, 167, 169, 171, 173, 177, 178, 191, 192, 193, 194, 195, 197, 200 }

B grade: { 9, 11, 12, 13, 14, 19, 28, 29, 30, 31, 35, 36, 37, 38, 48, 49, 50, 51, 52, 53, 54, 60, 61, 62, 68, 69, 70, 71, 92, 93, 95, 96, 97, 103, 105, 106, 107, 113, 114, 115, 116, 117, 118, 164, 196, 198, 199, 201 }

C grade: { 24, 25, 26, 27, 32, 33, 34, 158, 160, 162, 166, 168, 170, 172, 174, 176, 186 }

F grade: { 15, 16, 17, 18, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 163, 165, 175, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190 }

2.1.4 Maxima

A grade: { 1, 2, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 49, 50, 51, 52, 54, 56, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 88, 89, 90, 107, 108, 109, 110, 111, 117, 119, 120, 122, 152, 153, 154, 155, 156, 157, 159, 171, 179, 191, 192 }

B grade: { 3, 4, 5, 6, 7, 8, 24, 25, 45, 46, 47, 53, 55, 57, 58, 59, 61, 63, 87, 103, 104, 105, 106, 112, 113, 115, 124, 151, 167, 186, 187, 188, 194 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 60, 62, 65, 67, 78, 79, 80, 81, 82, 83, 84, 85, 86, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 114, 116, 118, 121, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 193, 195, 196, 197, 198, 199, 200, 201 }

2.1.5 FriCAS

A grade: { 1, 27, 28, 29, 30, 31, 34, 52, 53, 54, 64, 70, 71, 72, 73, 76, 77, 90, 91, 99, 100, 107, 108, 110, 118, 119, 120, 154, 155, 156, 157, 159, 161, 165, 167, 169, 173, 175, 177, 179, 181, 191 }

B grade: { 2, 3, 4, 5, 6, 7, 8, 24, 25, 26, 32, 33, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 109, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 135, 136, 142, 143, 144, 151, 152, 153, 163, 171, 186, 187, 188, 189, 190, 192, 193, 194, 195 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 94, 129, 130, 131, 132, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 158, 160, 162, 164, 166, 168, 170, 172, 174, 176, 178, 180, 182, 183, 184, 185, 196, 197, 198, 199, 200, 201 }

2.1.6 Sympy

A grade: { 28, 29, 30, 35, 36, 37, 38, 119 }

B grade: { 108 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

2.1.7 Giac

A grade: { 1, 2, 4, 6, 7, 8, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 151, 152, 153, 154, 155, 156, 157, 187, 188, 191, 192, 194, 195 }

B grade: { 3, 5, 26, 58, 59, 66, 79, 80, 83, 85, 86, 106, 113, 115, 117, 124, 186, 193 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 81, 82, 84, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 196, 197, 198, 199, 200, 201 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 28, 35, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 135, 144, 151, 152, 153, 159, 167, 171, 179, 186, 187, 188, 191, 192, 193, 194, 195 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 49, 50, 51, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 196, 197, 198, 199, 200, 201 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	19	0	12	23
normalized size	1	1.00	1.00	1.09	1.00	1.73	0.00	1.09	2.09
time (sec)	N/A	0.005	0.002	0.016	0.361	0.433	0.000	0.129	0.076

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	41	0	18	18
normalized size	1	1.00	1.00	1.10	1.80	4.10	0.00	1.80	1.80
time (sec)	N/A	0.010	0.004	0.228	0.306	0.833	0.000	0.135	0.079

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	65	267	0	76	81
normalized size	1	1.00	1.00	0.88	1.91	7.85	0.00	2.24	2.38
time (sec)	N/A	0.016	0.010	0.243	1.127	1.129	0.000	0.139	0.083

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	90	164	0	31	31
normalized size	1	1.00	1.00	0.88	3.46	6.31	0.00	1.19	1.19
time (sec)	N/A	0.012	0.006	0.246	0.323	1.355	0.000	0.110	0.062

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	47	50	112	812	0	102	189
normalized size	1	1.00	0.85	0.91	2.04	14.76	0.00	1.85	3.44
time (sec)	N/A	0.028	0.042	0.288	0.568	1.024	0.000	0.115	1.305

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	205	344	0	42	42
normalized size	1	1.00	1.00	0.80	5.00	8.39	0.00	1.02	1.02
time (sec)	N/A	0.015	0.011	0.252	0.318	2.853	0.000	0.114	1.354

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	49	116	0	18	30
normalized size	1	1.00	1.00	0.89	2.58	6.11	0.00	0.95	1.58
time (sec)	N/A	0.010	0.004	0.234	0.315	0.593	0.000	0.112	0.096

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	27	137	280	0	30	30
normalized size	1	1.00	1.00	0.77	3.91	8.00	0.00	0.86	0.86
time (sec)	N/A	0.014	0.004	0.276	2.080	0.473	0.000	0.136	1.518

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	217	0	0	0	0	-1
normalized size	1	1.00	0.77	3.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.078	0.508	0.000	0.658	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	103	0	0	0	0	-1
normalized size	1	1.00	0.79	1.66	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.046	0.572	0.000	0.562	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	0	0	0	-1
normalized size	1	1.00	1.00	3.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.029	0.400	0.000	0.479	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	0	0	0	-1
normalized size	1	1.00	1.00	3.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.037	0.411	0.000	1.138	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	53	174	0	0	0	0	-1
normalized size	1	1.00	0.80	2.64	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.047	0.579	0.000	1.719	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	59	188	0	0	0	0	-1
normalized size	1	1.00	0.89	2.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.071	0.524	0.000	0.530	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	68	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.203	0.319	0.000	1.112	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	56	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.073	0.281	0.000	0.397	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.040	0.289	0.000	0.395	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.021	0.380	0.000	0.391	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	244	0	0	0	0	-1
normalized size	1	1.00	1.00	5.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.029	0.367	0.000	0.393	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.068	0.269	0.000	0.440	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.085	0.297	0.000	0.416	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	70	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.132	0.313	0.000	0.451	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.061	0.443	0.000	0.430	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	81	230	156	1604	0	124	-1
normalized size	1	1.00	0.90	2.56	1.73	17.82	0.00	1.38	-0.01
time (sec)	N/A	0.030	0.106	0.493	0.477	0.424	0.000	0.118	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	208	112	812	0	102	-1
normalized size	1	1.00	0.85	3.20	1.72	12.49	0.00	1.57	-0.02
time (sec)	N/A	0.022	0.123	0.423	0.421	0.444	0.000	0.139	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	183	65	267	0	76	-1
normalized size	1	1.00	1.15	4.58	1.62	6.68	0.00	1.90	-0.02
time (sec)	N/A	0.017	0.042	0.434	0.421	0.392	0.000	0.128	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	29	130	11	19	0	12	-1
normalized size	1	1.00	2.64	11.82	1.00	1.73	0.00	1.09	-0.09
time (sec)	N/A	0.011	0.017	0.433	0.316	0.381	0.000	0.131	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	97	26	10	29	23	53
normalized size	1	1.00	1.00	4.41	1.18	0.45	1.32	1.05	2.41
time (sec)	N/A	0.016	0.026	0.414	0.323	0.381	17.320	0.130	0.155

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	201	54	32	54	48	-1
normalized size	1	1.00	0.86	3.94	1.06	0.63	1.06	0.94	-0.02
time (sec)	N/A	0.020	0.071	0.436	0.326	0.388	18.847	0.115	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	47	305	82	66	80	70	-1
normalized size	1	1.00	0.62	4.01	1.08	0.87	1.05	0.92	-0.01
time (sec)	N/A	0.027	0.084	0.414	0.326	0.456	38.199	0.120	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	57	409	100	108	0	92	-1
normalized size	1	1.00	0.56	4.05	0.99	1.07	0.00	0.91	-0.01
time (sec)	N/A	0.035	0.139	0.418	0.325	0.494	0.000	0.134	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	127	72	1082	0	65	-1
normalized size	1	1.00	0.65	1.95	1.11	16.65	0.00	1.00	-0.02
time (sec)	N/A	0.034	0.036	0.243	0.484	0.494	0.000	0.125	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	29	106	39	310	0	48	-1
normalized size	1	1.00	0.63	2.30	0.85	6.74	0.00	1.04	-0.02
time (sec)	N/A	0.024	0.020	0.206	1.448	0.458	0.000	0.134	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	21	72	8	145	0	8	-1
normalized size	1	1.00	0.84	2.88	0.32	5.80	0.00	0.32	-0.04
time (sec)	N/A	0.016	0.006	0.222	0.482	0.442	0.000	0.126	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	17	79	15	14	33
normalized size	1	1.00	1.00	4.46	1.31	6.08	1.15	1.08	2.54
time (sec)	N/A	0.029	0.006	0.211	0.450	0.436	0.594	0.110	0.120

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	130	35	277	37	29	-1
normalized size	1	1.00	0.75	3.61	0.97	7.69	1.03	0.81	-0.03
time (sec)	N/A	0.020	0.022	0.189	0.438	0.629	1.252	0.135	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	196	53	580	60	41	-1
normalized size	1	1.00	0.65	3.56	0.96	10.55	1.09	0.75	-0.02
time (sec)	N/A	0.029	0.039	0.194	0.420	0.580	10.170	0.116	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	262	71	970	80	53	-1
normalized size	1	1.00	0.57	3.54	0.96	13.11	1.08	0.72	-0.01
time (sec)	N/A	0.040	0.052	0.195	0.428	0.738	136.543	0.132	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.103	0.255	0.000	0.500	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	47	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.039	0.191	0.000	0.715	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.019	0.215	0.000	1.044	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.042	0.221	0.000	0.525	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	47	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.094	0.188	0.000	1.589	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.098	0.188	0.000	0.411	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	54	72	620	2804	0	51	498
normalized size	1	1.00	0.33	0.44	3.80	17.20	0.00	0.31	3.06
time (sec)	N/A	0.045	0.171	0.255	0.465	0.532	0.000	0.118	1.453

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	42	60	322	1475	0	39	356
normalized size	1	1.00	0.36	0.51	2.75	12.61	0.00	0.33	3.04
time (sec)	N/A	0.035	0.096	0.201	0.435	0.473	0.000	0.132	1.374

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	46	120	516	0	27	46
normalized size	1	1.00	0.49	0.75	1.97	8.46	0.00	0.44	0.75
time (sec)	N/A	0.023	0.058	0.196	0.437	0.428	0.000	0.114	1.344

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	13	81	0	13	71
normalized size	1	1.00	1.00	1.93	0.87	5.40	0.00	0.87	4.73
time (sec)	N/A	0.017	0.006	0.218	0.436	0.430	0.000	0.108	0.058

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	23	89	30	253	0	28	-1
normalized size	1	1.00	0.64	2.47	0.83	7.03	0.00	0.78	-0.03
time (sec)	N/A	0.016	0.023	0.233	0.438	0.441	0.000	0.112	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	65	1141	0	52	-1
normalized size	1	1.00	0.44	2.67	0.76	13.27	0.00	0.60	-0.01
time (sec)	N/A	0.036	0.037	0.207	0.466	0.447	0.000	0.131	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	103	2600	0	76	-1
normalized size	1	1.00	0.42	2.74	0.78	19.70	0.00	0.58	-0.01
time (sec)	N/A	0.056	0.079	0.213	0.494	0.462	0.000	0.135	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	28	130	54	36	0	42	59
normalized size	1	1.00	0.64	2.95	1.23	0.82	0.00	0.95	1.34
time (sec)	N/A	0.139	0.104	0.140	0.317	0.413	0.000	0.112	1.483

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	67	46	30	0	37	53
normalized size	1	1.00	1.00	2.91	2.00	1.30	0.00	1.61	2.30
time (sec)	N/A	0.122	0.048	0.115	0.316	0.382	0.000	0.130	1.359

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	16	78	42	14	0	28	41
normalized size	1	1.00	0.59	2.89	1.56	0.52	0.00	1.04	1.52
time (sec)	N/A	0.101	0.067	0.107	0.313	0.398	0.000	0.110	1.341

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	27	35	50	0	32	15
normalized size	1	1.00	0.94	1.59	2.06	2.94	0.00	1.88	0.88
time (sec)	N/A	0.073	0.020	0.100	0.317	0.399	0.000	0.133	0.068

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	44	23	48	103	0	52	51
normalized size	1	1.00	1.33	0.70	1.45	3.12	0.00	1.58	1.55
time (sec)	N/A	0.098	0.055	0.129	0.318	0.391	0.000	0.128	1.439

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	90	71	0	31	91
normalized size	1	1.00	1.09	1.00	3.91	3.09	0.00	1.35	3.96
time (sec)	N/A	0.139	0.041	0.148	0.314	0.377	0.000	0.112	1.352

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	59	45	99	630	0	90	121
normalized size	1	1.00	1.28	0.98	2.15	13.70	0.00	1.96	2.63
time (sec)	N/A	0.195	0.220	0.150	0.316	0.398	0.000	0.114	1.351

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	39	39	292	219	0	59	236
normalized size	1	1.00	1.15	1.15	8.59	6.44	0.00	1.74	6.94
time (sec)	N/A	0.146	0.064	0.154	0.317	0.386	0.000	0.114	1.376

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	219	488	0	1812	0	197	275
normalized size	1	1.00	1.66	3.70	0.00	13.73	0.00	1.49	2.08
time (sec)	N/A	0.370	0.736	0.124	0.000	0.445	0.000	0.138	2.006

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	66	361	128	490	0	87	123
normalized size	1	1.00	1.08	5.92	2.10	8.03	0.00	1.43	2.02
time (sec)	N/A	0.180	0.135	0.119	0.320	0.406	0.000	0.137	1.602

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	213	0	536	0	100	173
normalized size	1	1.00	0.93	2.60	0.00	6.54	0.00	1.22	2.11
time (sec)	N/A	0.212	0.195	0.118	0.000	0.432	0.000	0.137	1.670

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	46	78	0	34	20
normalized size	1	1.00	0.95	1.55	2.30	3.90	0.00	1.70	1.00
time (sec)	N/A	0.088	0.011	0.102	0.314	0.399	0.000	0.117	1.350

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	37	48	59	58	0	65	148
normalized size	1	1.00	0.70	0.91	1.11	1.09	0.00	1.23	2.79
time (sec)	N/A	0.118	0.074	0.135	0.318	0.408	0.000	0.115	1.744

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	77	0	452	0	64	151
normalized size	1	1.00	1.14	1.17	0.00	6.85	0.00	0.97	2.29
time (sec)	N/A	0.133	0.261	0.156	0.000	0.420	0.000	0.120	1.561

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	86	82	148	828	0	174	255
normalized size	1	1.00	1.01	0.96	1.74	9.74	0.00	2.05	3.00
time (sec)	N/A	0.238	0.347	0.180	0.336	0.449	0.000	0.140	1.831

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	156	154	0	2340	0	149	295
normalized size	1	1.00	1.41	1.39	0.00	21.08	0.00	1.34	2.66
time (sec)	N/A	0.304	0.598	0.161	0.000	0.452	0.000	0.143	1.747

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	139	80	139	0	86	88
normalized size	1	1.00	0.94	2.07	1.19	2.07	0.00	1.28	1.31
time (sec)	N/A	0.096	0.089	0.142	0.320	0.394	0.000	0.111	1.447

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	111	66	100	0	70	70
normalized size	1	1.00	0.98	2.06	1.22	1.85	0.00	1.30	1.30
time (sec)	N/A	0.086	0.077	0.148	0.319	0.385	0.000	0.114	1.358

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	45	87	56	70	0	51	52
normalized size	1	1.00	1.10	2.12	1.37	1.71	0.00	1.24	1.27
time (sec)	N/A	0.080	0.051	0.138	0.317	0.382	0.000	0.111	1.360

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	32	59	41	47	0	35	34
normalized size	1	1.00	1.23	2.27	1.58	1.81	0.00	1.35	1.31
time (sec)	N/A	0.057	0.063	0.133	0.309	0.384	0.000	0.113	1.312

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	12	14	0	11	11
normalized size	1	1.00	0.91	0.82	1.09	1.27	0.00	1.00	1.00
time (sec)	N/A	0.024	0.008	0.073	0.329	0.368	0.000	0.136	1.311

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	21	23	29	0	20	31
normalized size	1	1.00	1.10	1.05	1.15	1.45	0.00	1.00	1.55
time (sec)	N/A	0.066	0.029	0.082	0.448	0.387	0.000	0.133	1.305

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	45	39	45	127	0	36	58
normalized size	1	1.00	1.73	1.50	1.73	4.88	0.00	1.38	2.23
time (sec)	N/A	0.101	0.086	0.090	0.416	0.376	0.000	0.135	1.318

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	51	61	73	325	0	48	73
normalized size	1	1.00	1.13	1.36	1.62	7.22	0.00	1.07	1.62
time (sec)	N/A	0.085	0.087	0.112	0.413	0.384	0.000	0.118	1.345

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	58	58	33	48	0	29	24
normalized size	1	1.00	2.00	2.00	1.14	1.66	0.00	1.00	0.83
time (sec)	N/A	0.015	0.143	0.227	0.312	0.381	0.000	0.117	1.299

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	59	60	35	50	0	29	24
normalized size	1	1.00	1.97	2.00	1.17	1.67	0.00	0.97	0.80
time (sec)	N/A	0.015	0.146	0.227	0.315	0.379	0.000	0.135	1.263

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	0	0	924	0	151	-1
normalized size	1	1.00	1.01	0.00	0.00	9.43	0.00	1.54	-0.01
time (sec)	N/A	0.120	0.338	0.512	0.000	0.440	0.000	0.322	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	75	0	0	697	0	118	-1
normalized size	1	1.00	1.14	0.00	0.00	10.56	0.00	1.79	-0.02
time (sec)	N/A	0.042	0.200	0.470	0.000	0.412	0.000	0.251	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	60	0	0	637	0	83	-1
normalized size	1	1.00	1.62	0.00	0.00	17.22	0.00	2.24	-0.03
time (sec)	N/A	0.019	0.092	0.605	0.000	0.407	0.000	0.215	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	118	0	0	868	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	10.21	0.00	0.00	-0.01
time (sec)	N/A	0.074	1.206	0.473	0.000	0.444	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	1190	0	0	-1
normalized size	1	1.00	1.55	0.00	0.00	10.44	0.00	0.00	-0.01
time (sec)	N/A	0.128	4.774	0.442	0.000	0.443	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	70	0	0	642	0	101	-1
normalized size	1	1.00	1.84	0.00	0.00	16.89	0.00	2.66	-0.03
time (sec)	N/A	0.023	2.383	0.582	0.000	0.402	0.000	0.212	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	871	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	10.01	0.00	0.00	-0.01
time (sec)	N/A	0.079	2.232	0.464	0.000	0.421	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	39	0	0	233	0	52	-1
normalized size	1	1.00	2.05	0.00	0.00	12.26	0.00	2.74	-0.05
time (sec)	N/A	0.017	0.042	0.327	0.000	0.382	0.000	0.144	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	51	0	0	235	0	69	-1
normalized size	1	1.00	2.43	0.00	0.00	11.19	0.00	3.29	-0.05
time (sec)	N/A	0.019	0.565	0.316	0.000	0.389	0.000	0.131	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	78	121	211	1028	0	141	233
normalized size	1	1.00	0.73	1.13	1.97	9.61	0.00	1.32	2.18
time (sec)	N/A	0.124	0.253	0.430	0.924	0.413	0.000	0.127	1.409

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	80	114	521	0	92	165
normalized size	1	1.00	0.75	1.10	1.56	7.14	0.00	1.26	2.26
time (sec)	N/A	0.052	0.135	0.339	0.503	0.403	0.000	0.118	1.398

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	42	41	157	0	43	70
normalized size	1	1.00	0.97	1.27	1.24	4.76	0.00	1.30	2.12
time (sec)	N/A	0.028	0.066	0.281	0.307	0.416	0.000	0.138	0.106

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	26	0	17	38
normalized size	1	1.00	1.00	1.06	1.00	1.62	0.00	1.06	2.38
time (sec)	N/A	0.009	0.002	0.017	1.185	0.400	0.000	0.112	1.299

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	88	0	270	0	56	131
normalized size	1	1.00	1.02	1.49	0.00	4.58	0.00	0.95	2.22
time (sec)	N/A	0.055	0.111	0.209	0.000	0.413	0.000	0.120	0.399

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	203	221	0	1207	0	134	296
normalized size	1	1.00	1.86	2.03	0.00	11.07	0.00	1.23	2.72
time (sec)	N/A	0.159	0.411	0.199	0.000	0.425	0.000	0.121	1.849

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	205	660	0	4125	0	261	-1
normalized size	1	1.00	1.18	3.82	0.00	23.84	0.00	1.51	-0.01
time (sec)	N/A	0.308	0.735	0.270	0.000	0.502	0.000	0.137	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	2.468	0.596	0.000	2.252	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	126	406	0	2402	0	182	251
normalized size	1	1.00	0.86	2.78	0.00	16.45	0.00	1.25	1.72
time (sec)	N/A	0.657	0.283	0.157	0.000	0.475	0.000	0.122	1.851

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	264	0	1562	0	133	209
normalized size	1	1.00	0.88	2.36	0.00	13.95	0.00	1.19	1.87
time (sec)	N/A	0.423	0.167	0.148	0.000	0.458	0.000	0.140	1.713

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	174	0	860	0	92	167
normalized size	1	1.00	0.92	2.05	0.00	10.12	0.00	1.08	1.96
time (sec)	N/A	0.265	0.129	0.150	0.000	0.437	0.000	0.137	1.579

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	94	0	430	0	62	139
normalized size	1	1.00	0.92	1.52	0.00	6.94	0.00	1.00	2.24
time (sec)	N/A	0.092	0.118	0.138	0.000	0.430	0.000	0.113	1.481

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	165	0	32	43
normalized size	1	1.00	0.98	0.86	0.00	3.93	0.00	0.76	1.02
time (sec)	N/A	0.055	0.026	0.078	0.000	0.408	0.000	0.129	0.116

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	219	0	45	286
normalized size	1	1.00	1.00	0.94	0.00	4.06	0.00	0.83	5.30
time (sec)	N/A	0.100	0.053	0.090	0.000	0.442	0.000	0.119	4.006

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	73	0	504	0	61	294
normalized size	1	1.00	0.98	1.14	0.00	7.88	0.00	0.95	4.59
time (sec)	N/A	0.140	0.109	0.095	0.000	0.449	0.000	0.120	3.881

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	146	0	1444	0	89	476
normalized size	1	1.00	0.94	1.68	0.00	16.60	0.00	1.02	5.47
time (sec)	N/A	0.242	0.226	0.095	0.000	0.527	0.000	0.117	5.079

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	117	93	686	0	69	143
normalized size	1	1.00	1.25	2.44	1.94	14.29	0.00	1.44	2.98
time (sec)	N/A	0.097	0.121	0.156	0.541	0.427	0.000	0.119	1.461

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	34	74	437	0	61	96
normalized size	1	1.00	1.06	0.94	2.06	12.14	0.00	1.69	2.67
time (sec)	N/A	0.059	0.074	0.129	0.469	0.404	0.000	0.118	1.429

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	41	75	51	210	0	42	67
normalized size	1	1.00	1.32	2.42	1.65	6.77	0.00	1.35	2.16
time (sec)	N/A	0.070	0.059	0.141	0.513	0.398	0.000	0.115	1.438

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	10	54	33	85	0	35	33
normalized size	1	1.00	0.71	3.86	2.36	6.07	0.00	2.50	2.36
time (sec)	N/A	0.048	0.036	0.129	0.706	0.402	0.000	0.124	1.357

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	35	16	14	0	14	25
normalized size	1	1.00	1.07	2.50	1.14	1.00	0.00	1.00	1.79
time (sec)	N/A	0.046	0.030	0.102	0.532	0.386	0.000	0.129	1.321

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	19	18	16	19	17	14
normalized size	1	1.00	1.33	2.11	2.00	1.78	2.11	1.89	1.56
time (sec)	N/A	0.027	0.007	0.104	0.342	0.421	0.176	0.115	1.305

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	47	52	136	0	56	65
normalized size	1	1.00	1.10	1.18	1.30	3.40	0.00	1.40	1.62
time (sec)	N/A	0.058	0.053	0.158	0.487	0.393	0.000	0.116	1.368

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	56	47	46	0	40	94
normalized size	1	1.00	0.87	1.47	1.24	1.21	0.00	1.05	2.47
time (sec)	N/A	0.089	0.077	0.159	0.364	0.397	0.000	0.130	1.352

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	69	108	773	0	94	160
normalized size	1	1.00	0.97	1.01	1.59	11.37	0.00	1.38	2.35
time (sec)	N/A	0.086	0.186	0.161	0.478	0.407	0.000	0.132	1.429

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	69	78	105	151	0	64	264
normalized size	1	1.00	1.25	1.42	1.91	2.75	0.00	1.16	4.80
time (sec)	N/A	0.120	0.105	0.166	0.352	0.384	0.000	0.135	1.535

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	132	415	332	4077	0	267	316
normalized size	1	1.00	1.09	3.43	2.74	33.69	0.00	2.21	2.61
time (sec)	N/A	0.149	0.336	0.154	0.584	0.512	0.000	0.138	1.988

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	185	575	0	4914	0	250	1001
normalized size	1	1.00	0.99	3.07	0.00	26.28	0.00	1.34	5.35
time (sec)	N/A	0.293	0.616	0.153	0.000	0.746	0.000	0.142	8.505

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	233	164	1280	0	152	155
normalized size	1	1.00	1.18	3.24	2.28	17.78	0.00	2.11	2.15
time (sec)	N/A	0.097	0.185	0.147	0.486	0.445	0.000	0.136	1.797

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	113	248	0	1254	0	111	700
normalized size	1	1.00	1.20	2.64	0.00	13.34	0.00	1.18	7.45
time (sec)	N/A	0.319	0.419	0.141	0.000	0.513	0.000	0.141	7.263

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	107	67	200	0	73	260
normalized size	1	1.00	1.06	3.06	1.91	5.71	0.00	2.09	7.43
time (sec)	N/A	0.075	0.087	0.127	0.438	0.407	0.000	0.115	1.598

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	113	0	193	0	52	273
normalized size	1	1.00	1.00	1.82	0.00	3.11	0.00	0.84	4.40
time (sec)	N/A	0.171	0.086	0.117	0.000	0.446	0.000	0.146	3.921

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	26	27	41	19	23
normalized size	1	1.00	0.58	1.11	1.37	1.42	2.16	1.00	1.21
time (sec)	N/A	0.032	0.019	0.100	0.311	0.429	0.458	0.130	0.109

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	44	78	67	81	0	67	271
normalized size	1	1.00	0.67	1.18	1.02	1.23	0.00	1.02	4.11
time (sec)	N/A	0.106	0.090	0.155	0.471	0.428	0.000	0.120	1.721

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	81	104	0	646	0	82	383
normalized size	1	1.00	0.71	0.91	0.00	5.67	0.00	0.72	3.36
time (sec)	N/A	0.205	0.349	0.175	0.000	0.421	0.000	0.132	1.667

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	112	119	164	1222	0	193	339
normalized size	1	1.00	0.99	1.05	1.45	10.81	0.00	1.71	3.00
time (sec)	N/A	0.190	0.323	0.164	0.489	0.466	0.000	0.129	2.218

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	166	179	0	3530	0	190	713
normalized size	1	1.00	0.80	0.86	0.00	17.05	0.00	0.92	3.44
time (sec)	N/A	0.329	0.778	0.175	0.000	0.487	0.000	0.142	1.833

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	167	215	366	5181	0	380	623
normalized size	1	1.00	0.94	1.21	2.06	29.11	0.00	2.13	3.50
time (sec)	N/A	0.320	1.033	0.174	0.366	0.609	0.000	0.143	2.746

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	160	0	0	4363	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	25.82	0.00	0.00	-0.01
time (sec)	N/A	0.194	5.125	0.644	0.000	1.104	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	108	0	0	1589	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	15.89	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.989	0.523	0.000	1.070	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	90	43	0	605	0	0	47
normalized size	1	1.00	1.76	0.84	0.00	11.86	0.00	0.00	0.92
time (sec)	N/A	0.054	0.152	0.108	0.000	1.036	0.000	0.000	1.692

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	195	0	0	8620	0	0	-1
normalized size	1	1.00	1.84	0.00	0.00	81.32	0.00	0.00	-0.01
time (sec)	N/A	0.175	1.828	0.565	0.000	0.974	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	518	0	0	0	0	0	-1
normalized size	1	1.00	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	20.686	0.609	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	180.002	0.494	0.000	0.631	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	7.855	0.520	0.000	2.245	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	539	0	0	0	0	0	-1
normalized size	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	18.227	0.549	0.000	1.269	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	167	0	0	2813	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	19.01	0.00	0.00	-0.01
time (sec)	N/A	0.165	4.412	0.684	0.000	1.085	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	111	0	0	925	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	11.71	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.637	0.653	0.000	1.041	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	73	26	0	558	0	0	27
normalized size	1	1.00	2.35	0.84	0.00	18.00	0.00	0.00	0.87
time (sec)	N/A	0.047	0.140	0.094	0.000	1.023	0.000	0.000	1.635

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	226	0	0	8908	0	0	-1
normalized size	1	1.00	2.13	0.00	0.00	84.04	0.00	0.00	-0.01
time (sec)	N/A	0.148	3.680	0.580	0.000	1.305	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	902	0	0	0	0	0	-1
normalized size	1	1.00	3.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	7.396	0.654	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.791	180.001	0.638	0.000	0.667	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	180.001	0.491	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.634	0.013	0.000	3.433	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.436	91.352	0.589	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	155	0	0	3745	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	25.30	0.00	0.00	-0.01
time (sec)	N/A	0.191	3.170	0.660	0.000	2.682	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	103	0	0	1107	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	12.58	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.673	0.560	0.000	1.239	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	79	46	0	917	0	0	50
normalized size	1	1.00	1.46	0.85	0.00	16.98	0.00	0.00	0.93
time (sec)	N/A	0.062	0.246	0.093	0.000	1.057	0.000	0.000	1.770

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	904	0	0	0	0	0	-1
normalized size	1	1.00	6.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	7.367	0.569	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	996	0	0	0	0	0	-1
normalized size	1	1.00	3.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	7.612	0.696	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.366	180.001	0.612	0.000	7.754	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	180.001	0.471	0.000	9.076	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	86.669	0.461	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.983	110.553	0.565	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	84	91	386	589	0	64	405
normalized size	1	1.00	0.44	0.48	2.02	3.08	0.00	0.34	2.12
time (sec)	N/A	0.282	0.091	0.750	0.320	0.774	0.000	0.135	0.167

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	72	80	209	315	0	51	91
normalized size	1	1.00	0.51	0.57	1.48	2.23	0.00	0.36	0.65
time (sec)	N/A	0.167	0.069	0.683	0.320	0.504	0.000	0.134	1.429

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	44	69	84	120	0	38	78
normalized size	1	1.00	0.79	1.23	1.50	2.14	0.00	0.68	1.39
time (sec)	N/A	0.113	0.059	0.677	0.321	0.609	0.000	0.133	0.138

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	66	21	42	0	20	-1
normalized size	1	1.00	0.95	1.50	0.48	0.95	0.00	0.45	-0.02
time (sec)	N/A	0.087	0.040	0.719	0.407	0.420	0.000	0.131	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	106	29	66	0	33	-1
normalized size	1	1.00	0.65	1.43	0.39	0.89	0.00	0.45	-0.01
time (sec)	N/A	0.113	0.053	0.830	0.320	0.402	0.000	0.135	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	78	216	74	126	0	82	-1
normalized size	1	1.00	0.48	1.33	0.46	0.78	0.00	0.51	-0.01
time (sec)	N/A	0.155	0.067	0.842	0.317	0.411	0.000	0.113	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	326	112	218	0	110	-1
normalized size	1	1.00	0.42	1.30	0.45	0.87	0.00	0.44	-0.00
time (sec)	N/A	0.196	0.106	0.747	0.321	0.434	0.000	0.121	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	77	130	0	0	0	0	-1
normalized size	1	1.00	0.71	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.178	0.240	0.000	0.415	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	39	30	48	0	0	42
normalized size	1	1.00	1.57	1.39	1.07	1.71	0.00	0.00	1.50
time (sec)	N/A	0.042	0.047	0.205	0.447	0.410	0.000	0.000	1.471

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	65	134	0	0	0	0	-1
normalized size	1	1.00	0.32	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.115	0.232	0.000	0.427	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	97	0	90	0	0	-1
normalized size	1	1.00	1.15	1.45	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.146	0.263	0.000	0.405	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	58	114	0	0	0	0	-1
normalized size	1	1.00	0.67	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.104	0.209	0.000	0.406	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	77	0	0	100	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	1.69	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.096	0.196	0.000	0.405	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	167	0	0	0	0	-1
normalized size	1	1.00	1.00	4.64	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.029	0.058	0.488	0.000	0.412	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	55	0	0	57	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	1.42	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.122	0.202	0.000	0.427	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	59	134	0	0	0	0	-1
normalized size	1	1.00	0.43	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.122	0.200	0.000	0.414	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	33	38	42	37	0	0	58
normalized size	1	1.00	1.43	1.65	1.83	1.61	0.00	0.00	2.52
time (sec)	N/A	0.040	0.040	0.191	0.406	0.433	0.000	0.000	1.352

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	65	117	0	0	0	0	-1
normalized size	1	1.00	0.81	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.101	0.200	0.000	0.430	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	98	121	0	109	0	0	-1
normalized size	1	1.00	0.80	0.99	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.194	0.245	0.000	0.441	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	77	138	0	0	0	0	-1
normalized size	1	1.00	0.55	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.184	0.200	0.000	0.442	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	47	30	56	0	0	42
normalized size	1	1.00	1.57	1.68	1.07	2.00	0.00	0.00	1.50
time (sec)	N/A	0.043	0.053	0.203	0.454	0.420	0.000	0.000	1.453

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	65	147	0	0	0	0	-1
normalized size	1	1.00	0.26	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.119	0.211	0.000	0.432	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	90	113	0	101	0	0	-1
normalized size	1	1.00	0.98	1.23	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.172	0.282	0.000	0.437	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	61	129	0	0	0	0	-1
normalized size	1	1.00	0.55	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.112	0.215	0.000	0.429	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	109	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.168	0.194	0.000	0.418	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	65	159	0	0	0	0	-1
normalized size	1	1.00	0.30	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.116	0.115	0.208	0.000	0.482	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	64	131	0	106	0	0	-1
normalized size	1	1.00	0.70	1.42	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.090	0.238	0.000	0.427	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	45	127	0	0	0	0	-1
normalized size	1	1.00	0.80	2.27	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.108	0.617	0.000	0.432	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	0	39	28	0	0	28
normalized size	1	1.00	1.28	0.00	1.56	1.12	0.00	0.00	1.12
time (sec)	N/A	0.040	0.035	0.170	0.401	0.415	0.000	0.000	1.334

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	65	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.112	0.165	0.000	0.440	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	0	0	93	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	1.41	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.111	0.181	0.000	0.436	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.151	0.163	0.000	0.424	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	126	0	0	0	0	0	-1
normalized size	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	5.553	1.692	0.000	0.407	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	101	0	0	0	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.903	1.999	0.000	0.425	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	192	0	0	0	0	0	-1
normalized size	1	1.00	2.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	13.701	1.827	0.000	0.439	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	B	B	F	B	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	139	29	509	96	189	0	215	66
normalized size	1	3.48	0.72	12.72	2.40	4.72	0.00	5.38	1.65
time (sec)	N/A	0.135	0.319	1.065	0.612	0.430	0.000	1.219	1.400

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	62	0	74	48	0	38	49
normalized size	1	1.00	2.48	0.00	2.96	1.92	0.00	1.52	1.96
time (sec)	N/A	0.039	0.129	0.495	0.338	0.412	0.000	0.131	1.531

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	64	0	49	49	0	37	36
normalized size	1	1.00	2.56	0.00	1.96	1.96	0.00	1.48	1.44
time (sec)	N/A	0.046	0.108	0.696	0.344	0.401	0.000	0.130	1.449

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	0	0	474	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	5.33	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.772	0.627	0.000	0.445	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	0	0	538	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	8.28	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.877	0.615	0.000	0.446	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	34	0	27	41
normalized size	1	1.00	1.00	1.05	1.00	1.79	0.00	1.42	2.16
time (sec)	N/A	0.016	0.049	0.023	0.313	0.423	0.000	0.118	1.407

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	28	70	0	28	24
normalized size	1	1.00	1.00	1.06	1.56	3.89	0.00	1.56	1.33
time (sec)	N/A	0.028	0.060	0.291	0.344	0.415	0.000	0.145	1.328

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	51	0	452	0	115	139
normalized size	1	1.00	1.00	0.93	0.00	8.22	0.00	2.09	2.53
time (sec)	N/A	0.040	0.059	0.301	0.000	0.428	0.000	0.155	1.403

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	91	272	0	47	55
normalized size	1	1.00	1.00	0.86	2.17	6.48	0.00	1.12	1.31
time (sec)	N/A	0.034	0.049	0.293	0.349	0.435	0.000	0.150	1.341

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	75	84	0	1326	0	152	314
normalized size	1	1.00	0.84	0.94	0.00	14.90	0.00	1.71	3.53
time (sec)	N/A	0.057	0.094	0.321	0.000	0.448	0.000	0.166	1.348

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	295	0	0	0	0	-1
normalized size	1	1.00	0.76	3.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.167	0.727	0.000	0.429	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	72	141	0	0	0	0	-1
normalized size	1	1.00	0.77	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.093	0.641	0.000	0.441	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	0	0	0	-1
normalized size	1	1.00	1.00	3.16	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.069	0.542	0.000	0.431	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	0	0	0	-1
normalized size	1	1.00	1.00	3.16	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.076	0.512	0.000	0.420	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	76	237	0	0	0	0	-1
normalized size	1	1.00	0.78	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.114	0.675	0.000	0.428	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	256	0	0	0	0	-1
normalized size	1	1.00	0.90	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.130	0.674	0.000	0.435	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [177] had the largest ratio of [.6364]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	2	1	1.00	8	0.125
5	A	3	2	1.00	8	0.250
6	A	2	1	1.00	8	0.125
7	A	2	1	1.00	6	0.167
8	A	2	1	1.00	6	0.167
9	A	3	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	2	2	1.00	10	0.200
12	A	2	2	1.00	10	0.200
13	A	3	3	1.00	10	0.300
14	A	3	3	1.00	10	0.300
15	A	4	3	1.00	12	0.250
16	A	3	3	1.00	12	0.250
17	A	3	3	1.00	12	0.250
18	A	2	2	1.00	12	0.167
19	A	2	2	1.00	12	0.167
20	A	3	3	1.00	12	0.250
21	A	3	3	1.00	12	0.250
22	A	4	3	1.00	12	0.250
23	A	2	2	1.00	10	0.200
24	A	5	3	1.00	12	0.250
25	A	4	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	3	1.00	12	0.250
27	A	2	2	1.00	12	0.167
28	A	2	2	1.00	12	0.167
29	A	3	3	1.00	12	0.250
30	A	4	3	1.00	12	0.250
31	A	5	3	1.00	12	0.250
32	A	5	4	1.00	10	0.400
33	A	4	4	1.00	10	0.400
34	A	3	3	1.00	10	0.300
35	A	2	2	1.00	10	0.200
36	A	3	3	1.00	10	0.300
37	A	4	3	1.00	10	0.300
38	A	5	3	1.00	10	0.300
39	A	7	4	1.00	10	0.400
40	A	5	4	1.00	10	0.400
41	A	4	4	1.00	10	0.400
42	A	4	4	1.00	10	0.400
43	A	5	4	1.00	10	0.400
44	A	7	4	1.00	10	0.400
45	A	3	2	1.00	10	0.200
46	A	3	2	1.00	10	0.200
47	A	3	2	1.00	10	0.200
48	A	3	3	1.00	10	0.300
49	A	3	3	1.00	10	0.300
50	A	5	3	1.00	10	0.300
51	A	7	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	7	7	1.00	13	0.538
53	A	6	5	1.00	13	0.385
54	A	5	5	1.00	13	0.385
55	A	5	4	1.00	11	0.364
56	A	6	6	1.00	11	0.546
57	A	6	5	1.00	13	0.385
58	A	7	7	1.00	13	0.538
59	A	7	6	1.00	13	0.462
60	A	6	5	1.00	13	0.385
61	A	5	4	1.00	13	0.308
62	A	5	5	1.00	13	0.385
63	A	5	4	1.00	11	0.364
64	A	4	3	1.00	11	0.273
65	A	5	5	1.00	13	0.385
66	A	6	5	1.00	13	0.385
67	A	6	5	1.00	13	0.385
68	A	7	5	1.00	13	0.385
69	A	6	5	1.00	13	0.385
70	A	5	5	1.00	13	0.385
71	A	4	4	1.00	11	0.364
72	A	1	1	1.00	11	0.091
73	A	3	3	1.00	13	0.231
74	A	4	4	1.00	13	0.308
75	A	6	6	1.00	13	0.462
76	A	2	2	1.00	12	0.167
77	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	5	5	1.00	14	0.357
79	A	4	4	1.00	14	0.286
80	A	2	2	1.00	14	0.143
81	A	5	4	1.00	14	0.286
82	A	6	5	1.00	14	0.357
83	A	2	2	1.00	15	0.133
84	A	5	4	1.00	15	0.267
85	A	2	2	1.00	10	0.200
86	A	2	2	1.00	10	0.200
87	A	6	5	1.00	12	0.417
88	A	5	4	1.00	12	0.333
89	A	4	4	1.00	12	0.333
90	A	2	1	1.00	10	0.100
91	A	3	3	1.00	12	0.250
92	A	5	5	1.00	12	0.417
93	A	6	6	1.00	12	0.500
94	A	1	1	1.00	14	0.071
95	A	8	6	1.00	13	0.462
96	A	7	6	1.00	13	0.462
97	A	6	6	1.00	13	0.462
98	A	5	5	1.00	11	0.454
99	A	3	3	1.00	11	0.273
100	A	5	5	1.00	13	0.385
101	A	6	6	1.00	13	0.462
102	A	7	7	1.00	13	0.538
103	A	5	3	1.00	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
104	A	3	2	1.00	13	0.154
105	A	4	3	1.00	13	0.231
106	A	3	2	1.00	13	0.154
107	A	3	2	1.00	13	0.154
108	A	2	2	1.00	11	0.182
109	A	3	2	1.00	11	0.182
110	A	4	3	1.00	13	0.231
111	A	3	2	1.00	13	0.154
112	A	5	3	1.00	13	0.231
113	A	3	2	1.00	13	0.154
114	A	15	8	1.00	13	0.615
115	A	3	2	1.00	13	0.154
116	A	6	6	1.00	13	0.462
117	A	3	2	1.00	13	0.154
118	A	7	7	1.00	13	0.538
119	A	4	4	1.00	11	0.364
120	A	3	2	1.00	11	0.182
121	A	9	8	1.00	13	0.615
122	A	3	2	1.00	13	0.154
123	A	15	8	1.00	13	0.615
124	A	3	2	1.00	13	0.154
125	A	5	4	1.00	23	0.174
126	A	5	4	1.00	23	0.174
127	A	4	4	1.00	21	0.190
128	A	7	5	1.00	21	0.238
129	A	13	9	1.00	23	0.391

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	7	7	1.00	23	0.304
131	A	1	1	1.00	14	0.071
132	A	5	4	1.00	23	0.174
133	A	5	4	1.00	23	0.174
134	A	5	4	1.00	23	0.174
135	A	3	3	1.00	21	0.143
136	A	7	5	1.00	21	0.238
137	A	11	6	1.00	23	0.261
138	A	11	8	1.00	23	0.348
139	A	6	6	1.00	23	0.261
140	A	1	1	1.00	14	0.071
141	A	9	8	1.00	23	0.348
142	A	5	4	1.00	23	0.174
143	A	5	4	1.00	23	0.174
144	A	4	4	1.00	21	0.190
145	A	7	4	1.00	21	0.190
146	A	11	5	1.00	23	0.217
147	A	17	11	1.00	23	0.478
148	A	7	7	1.00	23	0.304
149	A	6	6	1.00	14	0.429
150	A	14	11	1.00	23	0.478
151	A	6	5	1.00	25	0.200
152	A	6	5	1.00	25	0.200
153	A	4	4	1.00	25	0.160
154	A	4	4	1.00	25	0.160
155	A	5	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	6	5	1.00	25	0.200
157	A	6	5	1.00	25	0.200
158	A	6	6	1.00	15	0.400
159	A	3	3	1.00	15	0.200
160	A	8	8	1.00	15	0.533
161	A	6	6	1.00	15	0.400
162	A	5	5	1.00	13	0.385
163	A	6	6	1.00	11	0.546
164	A	3	2	1.00	15	0.133
165	A	5	5	1.00	15	0.333
166	A	6	6	1.00	15	0.400
167	A	3	3	1.00	15	0.200
168	A	5	5	1.00	15	0.333
169	A	8	7	1.00	15	0.467
170	A	7	6	1.00	15	0.400
171	A	3	3	1.00	15	0.200
172	A	9	8	1.00	15	0.533
173	A	7	6	1.00	15	0.400
174	A	6	5	1.00	15	0.333
175	A	7	6	1.00	15	0.400
176	A	8	7	1.00	13	0.538
177	A	7	7	1.00	11	0.636
178	A	4	3	1.00	15	0.200
179	A	3	3	1.00	15	0.200
180	A	5	5	1.00	15	0.333
181	A	6	6	1.00	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	4	1.00	11	0.364
183	A	4	4	1.00	13	0.308
184	A	4	4	1.00	13	0.308
185	A	4	4	1.00	13	0.308
186	C	9	4	3.48	44	0.091
187	A	3	3	1.00	15	0.200
188	A	4	4	1.00	15	0.267
189	A	3	3	1.00	20	0.150
190	A	3	3	1.00	21	0.143
191	A	2	1	1.00	15	0.067
192	A	3	2	1.00	17	0.118
193	A	3	2	1.00	17	0.118
194	A	3	1	1.00	17	0.059
195	A	4	2	1.00	17	0.118
196	A	4	3	1.00	19	0.158
197	A	4	3	1.00	19	0.158
198	A	3	2	1.00	19	0.105
199	A	3	2	1.00	19	0.105
200	A	4	3	1.00	19	0.158
201	A	4	3	1.00	19	0.158

Chapter 3

Listing of integrals

3.1 $\int \operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

```
[Out] arctan(sinh(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}(a+bx)\,dx = \frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x],x]
[Out] ArcTan[Sinh[a + b*x]]/b
```

fricas [A] time = 0.43, size = 19, normalized size = 1.73

$$\frac{2 \arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="fricas")
[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

giac [A] time = 0.13, size = 12, normalized size = 1.09

$$\frac{2 \arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="giac")
[Out] 2*arctan(e^(b*x + a))/b
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$\frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a),x)
[Out] arctan(sinh(b*x+a))/b
```

maxima [A] time = 0.36, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x, algorithm="maxima")
[Out] arctan(sinh(b*x + a))/b
```

mupad [B] time = 0.08, size = 23, normalized size = 2.09

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x),x)
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a),x)
[Out] Integral(sech(a + b*x), x)
```

3.2 $\int \operatorname{sech}^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tanh(a+bx)}{b}$$

[Out] tanh(b*x+a)/b

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3767, 8}

$$\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + b*x]^2,x]

[Out] Tanh[a + b*x]/b

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int 1\,dx, x, -i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]/b
```

fricas [B] time = 0.83, size = 41, normalized size = 4.10

$$-\frac{2}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 + b)
```

giac [A] time = 0.14, size = 18, normalized size = 1.80

$$-\frac{2}{b\left(e^{\left(2\,bx+2\,a\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*(e^(2*b*x + 2*a) + 1))
```

maple [A] time = 0.23, size = 11, normalized size = 1.10

$$\frac{\tanh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2,x)
```

```
[Out] tanh(b*x+a)/b
```

maxima [A] time = 0.31, size = 18, normalized size = 1.80

$$\frac{2}{b\left(e^{\left(-2\,bx-2\,a\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/(b*(e^(-2*b*x - 2*a) + 1))
```

mupad [B] time = 0.08, size = 18, normalized size = 1.80

$$-\frac{2}{b\,\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^2,x)
[Out] -2/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{2}{\left (a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2,x)
[Out] Integral(sech(a + b*x)**2, x)
```

3.3 $\int \operatorname{sech}^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[Out] 1/2*arctan(sinh(b*x+a))/b+1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3770}

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^3,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^3(a+bx)\,dx &= \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{\tan^{-1}(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^3,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

fricas [B] time = 1.13, size = 267, normalized size = 7.85

$$\frac{\cosh(bx+a)^3 + 3\,\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\,\cosh(bx+a)\sinh(bx+a\right.}{b\cosh(bx+a)^4 + 4\,b\cosh(bx+a)\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.14, size = 76, normalized size = 2.24

$$\frac{\pi + \frac{4\left(e^{(bx+a)} - e^{(-bx-a)}\right)}{\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4} + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.24, size = 30, normalized size = 0.88

$$\frac{\mathrm{sech}(bx+a)\tanh(bx+a)}{2b} + \frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3,x)
```

```
[Out] 1/2*sech(b*x+a)*tanh(b*x+a)/b+arctan(exp(b*x+a))/b
```

maxima [B] time = 1.13, size = 65, normalized size = 1.91

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

mupad [B] time = 0.08, size = 81, normalized size = 2.38

$$\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}+\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*
x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3,x)
```

```
[Out] Integral(sech(a + b*x)**3, x)
```

3.4 $\int \text{sech}^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^4,x]
```

```
[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^4(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^4,x]
```

[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)

fricas [B] time = 1.36, size = 164, normalized size = 6.31

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{5}+5\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}+b\sinh\left(bx+a\right)^{5}+3\,b\cosh\left(bx+a\right)^{3}+\left(10\,b\cosh\left(bx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b*x+a)^4,x, algorithm="fricas")

[Out] -8/3*(2*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + (10*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a))*sinh(b*x + a)^2 + 4*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))

giac [A] time = 0.11, size = 31, normalized size = 1.19

$$-\frac{4\left(3\,e^{(2\,bx+2\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b*x+a)^4,x, algorithm="giac")

[Out] -4/3*(3*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)

maple [A] time = 0.25, size = 23, normalized size = 0.88

$$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^{2}}{3}\right)\tanh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(b*x+a)^4,x)

[Out] 1/b*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a)

maxima [B] time = 0.32, size = 90, normalized size = 3.46

$$\frac{4\,e^{(-2\,bx-2\,a)}}{b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}+\frac{4}{3\,b\left(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1)) + 4/3/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1))
```

mupad [B] time = 0.06, size = 31, normalized size = 1.19

$$-\frac{4\left(3\,e^{2\,a+2\,b\,x}+1\right)}{3\,b\,\left(e^{2\,a+2\,b\,x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^4,x)
```

```
[Out] -(4*(3*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^4{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**4,x)
```

```
[Out] Integral(sech(a + b*x)**4, x)
```

3.5 $\int \operatorname{sech}^5(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^3(a+bx)}{4b}+\frac{3\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[Out] 3/8*arctan(sinh(b*x+a))/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+1/4*sech(b*x+a)^3*t
anh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3770}

$$\frac{3\tan^{-1}(\sinh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^3(a+bx)}{4b}+\frac{3\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) + (
Sech[a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^5(a+bx)\,dx &= \frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{3}{4}\int \operatorname{sech}^3(a+bx)\,dx\\ &= \frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}+\frac{3}{8}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{3\tan^{-1}(\sinh(a+bx))}{8b}+\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.85

$$\frac{3\tan^{-1}(\sinh(a+bx))+2\tanh(a+bx)\text{sech}^3(a+bx)+3\tanh(a+bx)\text{sech}(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]] + 3*Sech[a + b*x]*Tanh[a + b*x] + 2*Sech[a + b*x]^
3*Tanh[a + b*x])/(8*b)
```

fricas [B] time = 1.02, size = 812, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

giac [B] time = 0.11, size = 102, normalized size = 1.85

$$\frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{3}+20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{2}+4\right)^{2}} + 6\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-
b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x +
 2*a) - 1)*e^(-b*x - a)))/b
```

maple [A] time = 0.29, size = 50, normalized size = 0.91

$$\frac{\operatorname{sech}(bx+a)^{3}\tanh(bx+a)}{4b} + \frac{3\operatorname{sech}(bx+a)\tanh(bx+a)}{8b} + \frac{3\arctan\left(e^{bx+a}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^5,x)
```

```
[Out] 1/4*sech(b*x+a)^3*tanh(b*x+a)/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+3/4*arctan(ex
p(b*x+a))/b
```

maxima [B] time = 0.57, size = 112, normalized size = 2.04

$$-\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{3\,e^{(-bx-a)} + 11\,e^{(-3\,bx-3\,a)} - 11\,e^{(-5\,bx-5\,a)} - 3\,e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

mupad [B] time = 1.31, size = 189, normalized size = 3.44

$$\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} + \frac{e^{a+b\,x}}{2\,b\,\left(2\,e^{2\,a+2\,b\,x} + e^{4\,a+4\,b\,x} + 1\right)} - \frac{2\,e^{a+b\,x}}{b\,\left(3\,e^{2\,a+2\,b\,x} + 3\,e^{4\,a+4\,b\,x} + e^{6\,a+6\,b\,x} + 1\right)} - \frac{}{b\,\left(4\,e^{2\,a+2\,b\,x} + 6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^5,x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + exp(a + b*x)/(2
*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*ex
p(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*exp(3*a +
 3*b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) +
 exp(8*a + 8*b*x) + 1)) + (3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**5,x)
```

```
[Out] Integral(sech(a + b*x)**5, x)
```

3.6 $\int \operatorname{sech}^6(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

```
[Out] tanh(b*x+a)/b-2/3*tanh(b*x+a)^3/b+1/5*tanh(b*x+a)^5/b
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3767}

$$\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^6,x]
```

```
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(a+bx)\,dx &= \frac{i\,\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^6,x]
```

[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)

fricas [B] time = 2.85, size = 344, normalized size = 8.39

$$-\frac{}{15\left(b\cosh\left(bx+a\right)^{8}+8\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{7}+b\sinh\left(bx+a\right)^{8}+5\,b\cosh\left(bx+a\right)^{6}+\left(28\,b\cosh\left(bx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 + 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
b*x + a)^8 + 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 + 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 + 25*b*cosh(b*x + a)^3 + 10*b*cosh(
b*x + a))*sinh(b*x + a)^3 + 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 +
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 + 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

giac [A] time = 0.11, size = 42, normalized size = 1.02

$$-\frac{16\left(10\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b\left(e^{(2\,bx+2\,a)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1
)^5)
```

maple [A] time = 0.25, size = 33, normalized size = 0.80

$$\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(bx+a)^{4}}{5}+\frac{4\mathrm{sech}(bx+a)^{2}}{15}\right)\tanh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^6,x)
```

```
[Out] 1/b*(8/15+1/5*sech(b*x+a)^4+4/15*sech(b*x+a)^2)*tanh(b*x+a)
```

maxima [B] time = 0.32, size = 205, normalized size = 5.00

$$\frac{16\,e^{(-2\,bx-2\,a)}}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1\right)}+\frac{}{3\,b\left(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 32/3*e^(-4*
b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*
a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15/(b*(5*e^(-2*b*x
- 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e
^(-10*b*x - 10*a) + 1))
```

mupad [B] time = 1.35, size = 42, normalized size = 1.02

$$-\frac{16\,\left(5\,\mathrm{e}^{2\,a+2\,b\,x}+10\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}{15\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^6,x)
```

```
[Out] -(16*(5*exp(2*a + 2*b*x) + 10*exp(4*a + 4*b*x) + 1))/(15*b*(exp(2*a + 2*b*x
) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{6}\left(a + bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**6,x)
```

```
[Out] Integral(sech(a + b*x)**6, x)
```

3.7 $\int \operatorname{sech}^4(7x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)$$

```
[Out] 1/7*tanh(7*x)-1/21*tanh(7*x)^3
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767}

$$\frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[7*x]^4,x]
```

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(7x)\,dx &= \frac{1}{7}i\,\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(7x)\right)\\ &= \frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{7}\tanh(7x)-\frac{1}{21}\tanh^3(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[7*x]^4,x]
```

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

fricas [B] time = 0.59, size = 116, normalized size = 6.11

$$-\frac{8\,(2\,\cosh(7\,x) + }{21\,\left(\cosh(7\,x)^5 + 5\,\cosh(7\,x)\sinh(7\,x)^4 + \sinh(7\,x)^5 + \left(10\,\cosh(7\,x)^2 + 3\right)\sinh(7\,x)^3 + 3\,\cosh(7\,x)^3 + \left(10\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="fricas")
```

```
[Out] -8/21*(2*cosh(7*x) + sinh(7*x))/(cosh(7*x)^5 + 5*cosh(7*x)*sinh(7*x)^4 + si
nh(7*x)^5 + (10*cosh(7*x)^2 + 3)*sinh(7*x)^3 + 3*cosh(7*x)^3 + (10*cosh(7*x
)^3 + 9*cosh(7*x))*sinh(7*x)^2 + (5*cosh(7*x)^4 + 9*cosh(7*x)^2 + 2)*sinh(7
*x) + 4*cosh(7*x))
```

giac [A] time = 0.11, size = 18, normalized size = 0.95

$$-\frac{4\left(3\,e^{(14\,x)} + 1\right)}{21\left(e^{(14\,x)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="giac")
```

```
[Out] -4/21*(3*e^(14*x) + 1)/(e^(14*x) + 1)^3
```

maple [A] time = 0.23, size = 17, normalized size = 0.89

$$\frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(7x)^2}{3}\right)\tanh(7x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(7*x)^4,x)
```

```
[Out] 1/7*(2/3+1/3*sech(7*x)^2)*tanh(7*x)
```

maxima [B] time = 0.31, size = 49, normalized size = 2.58

$$\frac{4\,e^{(-14\,x)}}{7\left(3\,e^{(-14\,x)} + 3\,e^{(-28\,x)} + e^{(-42\,x)} + 1\right)} + \frac{4}{21\left(3\,e^{(-14\,x)} + 3\,e^{(-28\,x)} + e^{(-42\,x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)^4,x, algorithm="maxima")
```

```
[Out] 4/7*e^(-14*x)/(3*e^(-14*x) + 3*e^(-28*x) + e^(-42*x) + 1) + 4/21/(3*e^(-14*
x) + 3*e^(-28*x) + e^(-42*x) + 1)
```

mupad [B] time = 0.10, size = 30, normalized size = 1.58

$$-\frac{2\left(3\,\mathrm{e}^{14\,x}-3\,\mathrm{e}^{28\,x}-\mathrm{e}^{42\,x}+1\right)}{21\,{\left(\mathrm{e}^{14\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(7*x)^4,x)
[Out] -(2*(3*exp(14*x) - 3*exp(28*x) - exp(42*x) + 1))/(21*(exp(14*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^4(7x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(7*x)**4,x)
[Out] Integral(sech(7*x)**4, x)
```

3.8 $\int \operatorname{sech}^6(\pi x)\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^5(\pi x)}{5\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh(\pi x)}{\pi}$$

```
[Out] tanh(Pi*x)/Pi-2/3*tanh(Pi*x)^3/Pi+1/5*tanh(Pi*x)^5/Pi
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767}

$$\frac{\tanh^5(\pi x)}{5\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[Pi*x]^6,x]
```

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(\pi x)\,dx&=\frac{i\operatorname{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,-i\tanh(\pi x)\right)}{\pi}\\&=\frac{\tanh(\pi x)}{\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh^5(\pi x)}{5\pi}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{\tanh^5(\pi x)}{5\pi}-\frac{2\tanh^3(\pi x)}{3\pi}+\frac{\tanh(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[Pi*x]^6,x]
```

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

fricas [B] time = 0.47, size = 280, normalized size = 8.00

$$-\frac{}{15\left(5\,\pi + \pi\cosh\left(\pi x\right)^{8} + 8\,\pi\cosh\left(\pi x\right)\sinh\left(\pi x\right)^{7} + \pi\sinh\left(\pi x\right)^{8} + 5\,\pi\cosh\left(\pi x\right)^{6} + \left(5\,\pi + 28\,\pi\cosh\left(\pi x\right)^{2}\right)\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(pi*x)^2 + 18*cosh(pi*x)*sinh(pi*x) + 11*sinh(pi*x)^2 + 5)/(
5*pi + pi*cosh(pi*x)^8 + 8*pi*cosh(pi*x)*sinh(pi*x)^7 + pi*sinh(pi*x)^8 + 5
*pi*cosh(pi*x)^6 + (5*pi + 28*pi*cosh(pi*x)^2)*sinh(pi*x)^6 + 2*(28*pi*cosh
(pi*x)^3 + 15*pi*cosh(pi*x))*sinh(pi*x)^5 + 10*pi*cosh(pi*x)^4 + 5*(2*pi +
14*pi*cosh(pi*x)^4 + 15*pi*cosh(pi*x)^2)*sinh(pi*x)^4 + 4*(14*pi*cosh(pi*x)
^5 + 25*pi*cosh(pi*x)^3 + 10*pi*cosh(pi*x))*sinh(pi*x)^3 + 11*pi*cosh(pi*x)
^2 + (11*pi + 28*pi*cosh(pi*x)^6 + 75*pi*cosh(pi*x)^4 + 60*pi*cosh(pi*x)^2)
*sinh(pi*x)^2 + 2*(4*pi*cosh(pi*x)^7 + 15*pi*cosh(pi*x)^5 + 20*pi*cosh(pi*x
)^3 + 9*pi*cosh(pi*x))*sinh(pi*x))
```

giac [A] time = 0.14, size = 30, normalized size = 0.86

$$-\frac{16\left(10\,e^{(4\,\pi x)} + 5\,e^{(2\,\pi x)} + 1\right)}{15\,\pi\left(e^{(2\,\pi x)} + 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*pi*x) + 5*e^(2*pi*x) + 1)/(pi*(e^(2*pi*x) + 1)^5)
```

maple [A] time = 0.28, size = 27, normalized size = 0.77

$$\frac{\left(\frac{8}{15} + \frac{\mathrm{sech}(\pi x)^{4}}{5} + \frac{4\mathrm{sech}(\pi x)^{2}}{15}\right)\tanh\left(\pi x\right)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(Pi*x)^6,x)
```

```
[Out] 1/Pi*(8/15+1/5*sech(Pi*x)^4+4/15*sech(Pi*x)^2)*tanh(Pi*x)
```

maxima [B] time = 2.08, size = 137, normalized size = 3.91

$$\frac{16\,e^{(-2\,\pi x)}}{3\,\pi\left(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1\right)} + \frac{32\,e^{(-4\,\pi x)}}{3\,\pi\left(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)^6,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*pi*x)/(pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e
^(-8*pi*x) + e^(-10*pi*x) + 1)) + 32/3*e^(-4*pi*x)/(pi*(5*e^(-2*pi*x) + 10*
e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-10*pi*x) + 1)) + 16/15/(
pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-1
0*pi*x) + 1))
```

mupad [B] time = 1.52, size = 30, normalized size = 0.86

$$-\frac{16\,\left(5\,{\mathrm{e}}^{2\,\Pi\,x}+10\,{\mathrm{e}}^{4\,\Pi\,x}+1\right)}{15\,\Pi\,{\left({\mathrm{e}}^{2\,\Pi\,x}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(Pi*x)^6,x)
```

```
[Out] -(16*(5*exp(2*Pi*x) + 10*exp(4*Pi*x) + 1))/(15*Pi*(exp(2*Pi*x) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{6}{\left(\pi x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(pi*x)**6,x)
```

```
[Out] Integral(sech(pi*x)**6, x)
```

3.9 $\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

```
[Out] 2/3*sech(b*x+a)^(3/2)*sinh(b*x+a)/b-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3768, 3771, 2641}

$$\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sech[a + b*x]^(3/2)*Sinh[a + b*x])/(3*b)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b} + \frac{1}{3}\int \sqrt{\operatorname{sech}(a+bx)}\,dx$$
$$= \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b} + \frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$
$$= -\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}$$

Mathematica [A] time = 0.08, size = 51, normalized size = 0.77

$$\frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\left(\sinh(a+bx) - i\cosh^{\frac{3}{2}}(a+bx)F\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(5/2),x]
```

```
[Out] (2*Sech[a + b*x]^(3/2)*((-I)*Cosh[a + b*x]^(3/2)*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[a + b*x]))/(3*b)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{sech}(bx+a)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(5/2), x)
```

maple [B] time = 0.51, size = 217, normalized size = 3.29

$$\frac{2\left(2\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{-\left(\sin\right.}\right.}{3\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(5/2),x)
```

```
[Out] 2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1
/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/
2*a),2^(1/2))+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2)*((2*cosh(1/2*b*x
+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(3/2)/sinh(1/2*b*x+1/2*a)/
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x))^(5/2),x)
```

```
[Out] int((1/cosh(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(5/2),x)
[Out] Integral(sech(a + b*x)**(5/2), x)
```

3.10 $\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] 2*sinh(b*x+a)*sech(b*x+a)^(1/2)/b+2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/
2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sec
h(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3768, 3771, 2639}

$$\frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b}+\frac{2i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]
])/b + (2*Sqrt[Sech[a + b*x]]*Sinh[a + b*x])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \frac{2\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)}{b} - \int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx$$
$$= \frac{2\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)}{b} - \left(\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx$$
$$= \frac{2i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b} + \frac{2\sqrt{\operatorname{sech}(a+bx)}\,\sinh(a+bx)}{b}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 0.79

$$\frac{2\sqrt{\operatorname{sech}(a+bx)}\left(\sinh(a+bx)+i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sech[a + b*x]]*(I*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]
 + Sinh[a + b*x]))/b
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{sech}(bx+a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(3/2), x)
```

maple [A] time = 0.57, size = 103, normalized size = 1.66

$$\frac{2\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}+4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sinh^2\left(\frac{bx}{2}+\right.\right.}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(3/2),x)
```

```
[Out] 2*((-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2)
)*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^
2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cosh(a+b\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x))^(3/2),x)
```

```
[Out] int((1/cosh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(sech(a + b*x)**(3/2), x)
```

3.11 $\int \sqrt{\operatorname{sech}(a+bx)}\,dx$

Optimal. Leaf size=40

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2
*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\operatorname{sech}(a+bx)}\,dx &= \left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.00

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\operatorname{sech}(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\operatorname{sech}(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sech(b*x + a)), x)
```

maple [B] time = 0.40, size = 135, normalized size = 3.38

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\ \text{EllipticF}\left(\cosh\right.}{\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\ \sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^(1/2),x)
```

```
[Out] 2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4
```

```
+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sinh(1
/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}\left(bx + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sech(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{\cosh\left(a + b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x))^(1/2),x)
[Out] int((1/cosh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**(1/2),x)
[Out] Integral(sqrt(sech(a + b*x)), x)
```

3.12 $\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx$

Optimal. Leaf size=40

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2
*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3771, 2639}

$$-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx &= \left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/(b*Sqrt[Cosh[a + b*x]]*Sqrt[Sech[a +
 b*x]])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\operatorname{sech}\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(sech(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

maple [B] time = 0.41, size = 135, normalized size = 3.38

$$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\ \text{EllipticE}\left(\text{co}\right.}{\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\ \sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(1/2),x)
```

```
[Out] -2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*x+
```

```
1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(
1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cosh(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x))^(1/2),x)
[Out] int(1/(1/cosh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(1/2),x)
[Out] Integral(1/sqrt(sech(a + b*x)), x)
```

3.13 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=66

$$\frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}-\frac{2i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

```
[Out] 2/3*sinh(b*x+a)/b/sech(b*x+a)^(1/2)-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3769, 3771, 2641}

$$\frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}-\frac{2i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(3*b*Sqrt[Sech[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} + \frac{1}{3}\int \sqrt{\operatorname{sech}(a+bx)}\,dx$$
$$= \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} + \frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$
$$= -\frac{2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.80

$$\frac{\sqrt{\operatorname{sech}(a+bx)}\left(\sinh(2(a+bx)) - 2i\sqrt{\cosh(a+bx)}\,F\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Sech[a + b*x]]*((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[2*(a + b*x)]))/(3*b)
```

fricas [F] time = 1.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^(-3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

maple [B] time = 0.58, size = 174, normalized size = 2.64

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-6\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\cdots\right.}\right.}{3\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(3/2),x)
```

```
[Out] 2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*b
*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh
(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1/
2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/
2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{1}{\cosh\left(a+b\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x))^(3/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(sech(a + b*x)**(-3/2), x)
```

3.14 $\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=66

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{6i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

```
[Out] 2/5*sinh(b*x+a)/b/sech(b*x+a)^(3/2)-6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3769, 3771, 2639}

$$\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{6i\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*x]^(-5/2),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(5*b*Sech[a + b*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx\\
&= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}+\frac{1}{5}\left(3\sqrt{\cosh(a+bx)}\,\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx\\
&= -\frac{6i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{5b}+\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.89

$$
\frac{\sqrt{\operatorname{sech}(a+bx)}\left(\sinh(a+bx)+\sinh(3(a+bx))-12i\sqrt{\cosh(a+bx)}\,E\left(\frac{1}{2}i(a+bx)\middle|2\right)\right)}{10b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*x]^(-5/2),x]
[Out] (Sqrt[Sech[a + b*x]]*((-12*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x)
, 2] + Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(10*b)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sech(b*x + a)^(-5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

maple [B] time = 0.52, size = 188, normalized size = 2.85

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\ \left(8\left(\cosh^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-16\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right.\right.}{5\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\ \sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(b*x+a)^(5/2),x)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*b
*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/2*
b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x))^(5/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(b*x+a)**(5/2),x)
```

```
[Out] Integral(sech(a + b*x)**(-5/2), x)
```

3.15 $\int (b\,\mathbf{sech}(c+dx))^{7/2}\,dx$

Optimal. Leaf size=102

$$\frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}+\frac{6b^3\sinh(c+dx)\sqrt{b\operatorname{sech}(c+dx)}}{5d}+\frac{2b\sinh(c+dx)(b\operatorname{sech}(c+dx))^{5/2}}{5d}$$

```
[Out] 2/5*b*(b*sech(d*x+c))^(5/2)*sinh(d*x+c)/d+6/5*I*b^4*(cosh(1/2*d*x+1/2*c)^2)
^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/d/cosh(
d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)+6/5*b^3*sinh(d*x+c)*(b*sech(d*x+c))^(1/2
)/d
```

Rubi [A] time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2639}

$$\frac{6b^3\sinh(c+dx)\sqrt{b\operatorname{sech}(c+dx)}}{5d}+\frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}+\frac{2b\sinh(c+dx)(b\operatorname{sech}(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (((6*I)/5)*b^4*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b
*Sech[c + d*x]]) + (6*b^3*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/(5*d) + (2*b
*(b*Sech[c + d*x])^(5/2)*Sinh[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} + \frac{1}{5}\left(3b^2\right)\int (b\text{sech}(c+dx))^{3/2}\,dx$$
$$= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} - \frac{1}{5}\left(3b^4\right)\int \frac{}{\sqrt{b}}$$
$$= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} - \frac{\left(3b^4\right)\int \sqrt{c}}{5\sqrt{\cosh(c+d}}$$
$$= \frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{6b^3\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+}{}$$

Mathematica [A] time = 0.20, size = 68, normalized size = 0.67

$$\frac{b^2(b\text{sech}(c+dx))^{3/2}\left(3\sinh(2(c+dx))+2\tanh(c+dx)+6i\cosh^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (b^2*(b*Sech[c + d*x])^(3/2)*((6*I)*Cosh[c + d*x]^(3/2)*EllipticE[(I/2)*(c
+ d*x), 2] + 3*Sinh[2*(c + d*x)] + 2*Tanh[c + d*x]))/(5*d)
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)}\,b^3\,\text{sech}\,(dx+c)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))*b^3*sech(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \left(b \,\mathrm{sech}\left(dx + c\right)\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(7/2),x)
```

```
[Out] int((b*sech(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \,\mathrm{sech}\left(dx + c\right)\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(7/2),x)
```

```
[Out] int((b/cosh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.16 $\int (b\operatorname{sech}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=74

$$\frac{2b\sinh(c+dx)(b\operatorname{sech}(c+dx))^{3/2}}{3d}-\frac{2ib^2\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3d}$$

```
[Out] 2/3*b*(b*sech(d*x+c))^(3/2)*sinh(d*x+c)/d-2/3*I*b^2*(cosh(1/2*d*x+1/2*c)^2)
^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*
x+c)^(1/2)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2641}

$$\frac{2b\sinh(c+dx)(b\operatorname{sech}(c+dx))^{3/2}}{3d}-\frac{2ib^2\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*b^2*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Se
ch[c + d*x]])/d + (2*b*(b*Sech[c + d*x])^(3/2)*Sinh[c + d*x])/(3*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \frac{2b(b\text{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d} + \frac{1}{3}b^2\int \sqrt{b\text{sech}(c+dx)}\,dx$$
$$= \frac{2b(b\text{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d} + \frac{1}{3}\left(b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{}{\sqrt{\cosh}}$$
$$= -\frac{2ib^2\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{3d} + \frac{2b(b\text{sech}(c+dx))^{3/2}\sinh(c}{3d}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.76

$$\frac{2b^2\sqrt{b\text{sech}(c+dx)}\left(\tanh(c+dx) - i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(5/2),x]
[Out] (2*b^2*Sqrt[b*Sech[c + d*x]]*((-I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c +
 d*x), 2] + Tanh[c + d*x]))/(3*d)
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)}\,b^2\,\text{sech}\,(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c))*b^2*sech(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{sech}\,(dx+c))^{\frac{5}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (b\,\text{sech}\,(dx+c))^{\frac{5}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(5/2),x)
```

```
[Out] int((b*sech(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cosh(c + d\,x)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(5/2),x)
```

```
[Out] int((b/cosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(c + dx))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(5/2), x)
```

3.17 $\int (b\,\mathbf{sech}(c+dx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b\sinh(c+dx)\sqrt{b\mathrm{sech}(c+dx)}}{d}+\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\mathrm{sech}(c+dx)}}$$

```
[Out] 2*I*b^2*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(
1/2*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)+2*b*sinh(
d*x+c)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2639}

$$\frac{2b\sinh(c+dx)\sqrt{b\mathrm{sech}(c+dx)}}{d}+\frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\mathrm{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*b^2*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sec
h[c + d*x]]) + (2*b*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{d} - b^2\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx$$
$$= \frac{2b\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{d} - \frac{b^2\int \sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\,\sqrt{b\text{sech}(c+dx)}}$$
$$= \frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\,\sqrt{b\text{sech}(c+dx)}} + \frac{2b\sqrt{b\text{sech}(c+dx)}\,\sinh(c+dx)}{d}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.74

$$\frac{2b\sqrt{b\text{sech}(c+dx)}\left(\sinh(c+dx)+i\sqrt{\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sech[c + d*x]]*(I*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x)
, 2] + Sinh[c + d*x]))/d
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)}\,b\,\text{sech}\,(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))*b*sech(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (b\,\text{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(3/2),x)
```

```
[Out] int((b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx+c))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cosh(c+d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int((b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(c+dx))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(3/2), x)
```

3.18 $\int \sqrt{b\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=42

$$-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2
*d*x+1/2*c),2^(1/2))*cosh(d*x+c)^(1/2)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\,\mathrm{sech}(c+dx)}\,dx &= \left(\sqrt{\cosh(c+dx)}\sqrt{b\,\mathrm{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,dx\\ &= -\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$-\frac{2i\sqrt{\cosh(c+dx)}F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\,\text{sech}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int\sqrt{b\,\text{sech}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^(1/2),x)
```

```
[Out] int((b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\,\text{sech}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^(1/2),x)
[Out] int((b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*sech(c + d*x)), x)
```

3.19 $\int \frac{1}{\sqrt{b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2
*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\operatorname{sech}(c+dx)}}\,dx &= \frac{\int \sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

fricas [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)}}{b\,\text{sech}\,(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b*sech(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

maple [B] time = 0.37, size = 244, normalized size = 5.81

$$\frac{\sqrt{2}}{d\sqrt{\frac{b\,e^{dx+c}}{1+e^{2dx+2c}}}}+\frac{\left(-\frac{2\left(b\,e^{2dx+2c}+b\right)}{b\sqrt{e^{dx+c}\left(b\,e^{2dx+2c}+b\right)}}+\frac{i\sqrt{-i\left(e^{dx+c}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{dx+c}-i\right)}\,\sqrt{i e^{dx+c}}\,\left(-2i\,\text{EllipticE}\left(\sqrt{-i\left(e^{dx+c}+i\right)},\frac{\sqrt{2}}{2}\right)+i\,\text{EllipticF}\left(\sqrt{-i\left(e^{dx+c}\right.}\right.\right.}{\sqrt{e^{3dx+3c}b+b\,e^{dx+c}}}\right.}{d\sqrt{\frac{b\,e^{dx+c}}{1+e^{2dx+2c}}}\,\left(1+e^{2dx+2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2+1))^(1/2)+1/d*(-2*(b*exp(d*x+c)^2+b
)/b/(exp(d*x+c)*(b*exp(d*x+c)^2+b))^(1/2)+I*(-I*(exp(d*x+c)+I))^(1/2)*2^(1/
2)*(I*(exp(d*x+c)-I))^(1/2)*(I*exp(d*x+c))^(1/2)/(exp(d*x+c)^3*b+b*exp(d*x+
c))^(1/2)*(-2*I*EllipticE((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))+I*Elliptic
F((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c
)^2+1))^(1/2)*(b*exp(d*x+c)*(exp(d*x+c)^2+1))^(1/2)/(exp(d*x+c)^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\mathrm{sech}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sech(c + d*x)), x)
```

3.20 $\int \frac{1}{(b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\,\mathrm{sech}(c+dx)}}-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{3b^2d}$$

```
[Out] 2/3*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(1/2)-2/3*I*(cosh(1/2*d*x+1/2*c)^2)^(1/
2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*x+c)
^(1/2)*(b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2641}

$$\frac{2\sinh(c+dx)}{3bd\sqrt{b\,\mathrm{sech}(c+dx)}}-\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c
 + d*x]])/(b^2*d) + (2*Sinh[c + d*x])/(3*b*d*Sqrt[b*Sech[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}} + \frac{\int \sqrt{b\text{sech}(c+dx)}\,dx}{3b^2}$$
$$= \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}} + \frac{\left(\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,dx}{3b^2}$$
$$= -\frac{2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d} + \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.83

$$\frac{\text{sech}^2(c+dx)\left(\sinh(2(c+dx)) - 2i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{3d(b\text{sech}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Sech[c + d*x]^2*((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]
+ Sinh[2*(c + d*x)]))/(3*d*(b*Sech[c + d*x])^(3/2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)}}{b^2\,\text{sech}\,(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b^2*sech(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(-3/2), x)
```

3.21 $\int \frac{1}{(b\mathbf{sech}(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}-\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

```
[Out] 2/5*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(3/2)-6/5*I*(cosh(1/2*d*x+1/2*c)^2)^(1/
2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cosh(
d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2639}

$$\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}-\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(c + d*x), 2])/(b^2*d*Sqrt[Cosh[c + d*x]]*Sqrt[
b*Sech[c + d*x]]) + (2*Sinh[c + d*x])/(5*b*d*(b*Sech[c + d*x])^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx}{5b^2}$$
$$= \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}+\frac{3\int \sqrt{\cosh(c+dx)}\,dx}{5b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$
$$= -\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}$$

Mathematica [A] time = 0.09, size = 64, normalized size = 0.84

$$\frac{\sqrt{b\text{sech}(c+dx)}\left(\sinh(c+dx)+\sinh(3(c+dx))-12i\sqrt{\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{10b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-5/2),x]
```

```
[Out] (Sqrt[b*Sech[c + d*x]]*((-12*I)*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*
x), 2] + Sinh[c + d*x] + Sinh[3*(c + d*x)]))/(10*b^3*d)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)}}{b^3\,\text{sech}\,(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b^3*sech(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(5/2),x)
[Out] int(1/(b*sech(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\cosh(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(5/2),x)
[Out] int(1/(b/cosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(5/2),x)
[Out] Integral((b*sech(c + d*x))**(-5/2), x)
```

3.22 $\int \frac{1}{(b\,\mathbf{sech}(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{10i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{21b^4d}+\frac{10\sinh(c+dx)}{21b^3d\sqrt{b\,\mathrm{sech}(c+dx)}}+\frac{2\sinh(c+dx)}{7bd(b\,\mathrm{sech}(c+dx))^{5/2}}$$

```
[Out] 2/7*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(5/2)+10/21*sinh(d*x+c)/b^3/d/(b*sech(d
*x+c))^(1/2)-10/21*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*Elli
pticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*x+c)^(1/2)*(b*sech(d*x+c))^(1/2
)/b^4/d
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2641}

$$\frac{10\sinh(c+dx)}{21b^3d\sqrt{b\,\mathrm{sech}(c+dx)}}-\frac{10i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\,\mathrm{sech}(c+dx)}}{21b^4d}+\frac{2\sinh(c+dx)}{7bd(b\,\mathrm{sech}(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (((-10*I)/21)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech
[c + d*x]])/(b^4*d) + (2*Sinh[c + d*x])/(7*b*d*(b*Sech[c + d*x])^(5/2)) + (
10*Sinh[c + d*x])/(21*b^3*d*Sqrt[b*Sech[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

EqQ[n^2, 1/4]

Rubi steps

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{5\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx}{7b^2}$$
$$= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}} + \frac{5\int \sqrt{b\text{sech}(c+dx)}\,dx}{21b^4}$$
$$= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}} + \frac{\left(5\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right.}{21b^4}$$
$$= -\frac{10i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\sqrt{b\text{sech}(c+dx)}}{21b^4d} + \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{}{21}$$

Mathematica [A] time = 0.13, size = 70, normalized size = 0.67

$$\frac{\sqrt{b\text{sech}(c+dx)}\left(26\sinh(2(c+dx))+3\sinh(4(c+dx))-40i\sqrt{\cosh(c+dx)}\,F\left(\frac{1}{2}i(c+dx)\middle|2\right)\right)}{84b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Sech[c + d*x]]*((-40*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*
x), 2] + 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^4*d)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)}}{b^4\,\text{sech}\,(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c))/(b^4*sech(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sech(d*x+c))^(7/2),x)
[Out] int(1/(b*sech(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\cosh(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cosh(c + d*x))^(7/2),x)
[Out] int(1/(b/cosh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\mathrm{sech}\,(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sech(d*x+c))**(7/2),x)
[Out] Integral((b*sech(c + d*x))**(-7/2), x)
```

3.23 $\int (b\,\mathrm{sech}(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b\sinh(c+dx)(b\,\mathrm{sech}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

```
[Out] -b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cosh(d*x+c)^2)*(b*sech(d*x+c))^(-
1+n)*sinh(d*x+c)/d/(1-n)/(-sinh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3772, 2643}

$$-\frac{b\sinh(c+dx)(b\,\mathrm{sech}(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sech[c + d*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cosh[c + d*x]^2]*(b*Sech[
c + d*x])^(-1 + n)*Sinh[c + d*x])/(d*(1 - n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int (b\text{sech}(c+dx))^n\,dx = \left(\frac{\cosh(c+dx)}{b}\right)^n (b\text{sech}(c+dx))^n \int \left(\frac{\cosh(c+dx)}{b}\right)^{-n} dx$$
$$= -\frac{\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cosh^2(c+dx)\right)(b\text{sech}(c+dx))^n\sinh(c+dx)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.80

$$-\frac{\sqrt{\tanh^2(c+dx)}\coth(c+dx)(b\text{sech}(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\text{sech}^2(c+dx)\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sech[c + d*x])^n,x]
```

```
[Out] -((Coth[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sech[c + d*x]^2]*(b
*Sech[c + d*x])^n*Sqrt[Tanh[c + d*x]^2])/(d*n))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((b\,\text{sech}\,(dx+c))^n\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sech(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{sech}\,(dx+c))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c))^n, x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int (b\,\text{sech}\,(dx+c))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sech(d*x+c))^n,x)
```

```
[Out] int((b*sech(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cosh(c + d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cosh(c + d*x))^n,x)
```

```
[Out] int((b/cosh(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sech(d*x+c))**n,x)
```

```
[Out] Integral((b*sech(c + d*x))**n, x)
```

3.24 $\int \text{sech}^2(a+bx)^{7/2}\,dx$

Optimal. Leaf size=90

$$\frac{5\sin^{-1}(\tanh(a+bx))}{16b}+\frac{\tanh(a+bx)\text{sech}^2(a+bx)^{5/2}}{6b}+\frac{5\tanh(a+bx)\text{sech}^2(a+bx)^{3/2}}{24b}+\frac{5\tanh(a+bx)\sqrt{\text{sech}^2(a}}{16b}$$

```
[Out] 5/16*arcsin(tanh(b*x+a))/b+5/24*(sech(b*x+a)^2)^(3/2)*tanh(b*x+a)/b+1/6*(se
ch(b*x+a)^2)^(5/2)*tanh(b*x+a)/b+5/16*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4122, 195, 216}

$$\frac{5\sin^{-1}(\tanh(a+bx))}{16b}+\frac{\tanh(a+bx)\text{sech}^2(a+bx)^{5/2}}{6b}+\frac{5\tanh(a+bx)\text{sech}^2(a+bx)^{3/2}}{24b}+\frac{5\tanh(a+bx)\sqrt{\text{sech}^2(a}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (5*ArcSin[Tanh[a + b*x]])/(16*b) + (5*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/
(16*b) + (5*(Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(24*b) + ((Sech[a + b*x]
^2)^(5/2)*Tanh[a + b*x])/(6*b)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^{5/2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\,\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\,dx, x, \tanh(a+bx)\right)}{6b}$$
$$= \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\,\text{Subst}\left(\int \sqrt{1-x^2}$$
$$= \frac{5\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{16b} + \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\operatorname{sech}^2(a+bx)^{5/2}}{6b}$$
$$= \frac{5\sin^{-1}(\tanh(a+bx))}{16b} + \frac{5\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{16b} + \frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+}{24b}$$

Mathematica [A] time = 0.11, size = 81, normalized size = 0.90

$$\frac{\cosh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}\left(15\tan^{-1}(\sinh(a+bx)) + 8\tanh(a+bx)\operatorname{sech}^5(a+bx) + 10\tanh(a+bx)\operatorname{sech}^3(a\right.}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2]*(15*ArcTan[Sinh[a + b*x]] + 15*Sech[a
+ b*x]*Tanh[a + b*x] + 10*Sech[a + b*x]^3*Tanh[a + b*x] + 8*Sech[a + b*x]^5
*Tanh[a + b*x]))/(48*b)
```

fricas [B] time = 0.42, size = 1604, normalized size = 17.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/24*(15*cosh(b*x + a)^11 + 165*cosh(b*x + a)*sinh(b*x + a)^10 + 15*sinh(b*
x + a)^11 + 5*(165*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^9 + 85*cosh(b*x + a)
^9 + 45*(55*cosh(b*x + a)^3 + 17*cosh(b*x + a))*sinh(b*x + a)^8 + 18*(275*c
osh(b*x + a)^4 + 170*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^7 + 198*cosh(b*x +
 a)^7 + 42*(165*cosh(b*x + a)^5 + 170*cosh(b*x + a)^3 + 33*cosh(b*x + a))*s
```

```
inh(b*x + a)^6 + 18*(385*cosh(b*x + a)^6 + 595*cosh(b*x + a)^4 + 231*cosh(b
*x + a)^2 - 11)*sinh(b*x + a)^5 - 198*cosh(b*x + a)^5 + 90*(55*cosh(b*x + a
)^7 + 119*cosh(b*x + a)^5 + 77*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x
 + a)^4 + 5*(495*cosh(b*x + a)^8 + 1428*cosh(b*x + a)^6 + 1386*cosh(b*x + a
)^4 - 396*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^3 - 85*cosh(b*x + a)^3 + 3*(2
75*cosh(b*x + a)^9 + 1020*cosh(b*x + a)^7 + 1386*cosh(b*x + a)^5 - 660*cosh
(b*x + a)^3 - 85*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(b*x + a)^12 + 12
*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 6*(11*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^10 + 6*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + 3*cos
h(b*x + a))*sinh(b*x + a)^9 + 15*(33*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^8 + 15*cosh(b*x + a)^8 + 24*(33*cosh(b*x + a)^5 + 30*cosh
(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 3
15*cosh(b*x + a)^4 + 105*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^6 + 20*cosh(b*x
 + a)^6 + 24*(33*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 + 35*cosh(b*x + a)^3
+ 5*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(33*cosh(b*x + a)^8 + 84*cosh(b*x +
 a)^6 + 70*cosh(b*x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*c
osh(b*x + a)^4 + 20*(11*cosh(b*x + a)^9 + 36*cosh(b*x + a)^7 + 42*cosh(b*x
+ a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(11*cosh
(b*x + a)^10 + 45*cosh(b*x + a)^8 + 70*cosh(b*x + a)^6 + 50*cosh(b*x + a)^4
 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 + 12*(cosh(b
*x + a)^11 + 5*cosh(b*x + a)^9 + 10*cosh(b*x + a)^7 + 10*cosh(b*x + a)^5 +
5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a)
+ sinh(b*x + a)) + 3*(55*cosh(b*x + a)^10 + 255*cosh(b*x + a)^8 + 462*cosh(
b*x + a)^6 - 330*cosh(b*x + a)^4 - 85*cosh(b*x + a)^2 - 5)*sinh(b*x + a) -
15*cosh(b*x + a))/(b*cosh(b*x + a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11
 + b*sinh(b*x + a)^12 + 6*b*cosh(b*x + a)^10 + 6*(11*b*cosh(b*x + a)^2 + b)
*sinh(b*x + a)^10 + 20*(11*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x
+ a)^9 + 15*b*cosh(b*x + a)^8 + 15*(33*b*cosh(b*x + a)^4 + 18*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a)^8 + 24*(33*b*cosh(b*x + a)^5 + 30*b*cosh(b*x + a)^3
 + 5*b*cosh(b*x + a))*sinh(b*x + a)^7 + 20*b*cosh(b*x + a)^6 + 4*(231*b*cos
h(b*x + a)^6 + 315*b*cosh(b*x + a)^4 + 105*b*cosh(b*x + a)^2 + 5*b)*sinh(b*
x + a)^6 + 24*(33*b*cosh(b*x + a)^7 + 63*b*cosh(b*x + a)^5 + 35*b*cosh(b*x
+ a)^3 + 5*b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*b*cosh(b*x + a)^4 + 15*(33
*b*cosh(b*x + a)^8 + 84*b*cosh(b*x + a)^6 + 70*b*cosh(b*x + a)^4 + 20*b*cos
h(b*x + a)^2 + b)*sinh(b*x + a)^4 + 20*(11*b*cosh(b*x + a)^9 + 36*b*cosh(b*
x + a)^7 + 42*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))
*sinh(b*x + a)^3 + 6*b*cosh(b*x + a)^2 + 6*(11*b*cosh(b*x + a)^10 + 45*b*co
sh(b*x + a)^8 + 70*b*cosh(b*x + a)^6 + 50*b*cosh(b*x + a)^4 + 15*b*cosh(b*x
 + a)^2 + b)*sinh(b*x + a)^2 + 12*(b*cosh(b*x + a)^11 + 5*b*cosh(b*x + a)^9
 + 10*b*cosh(b*x + a)^7 + 10*b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)^3 + b*co
sh(b*x + a))*sinh(b*x + a) + b)
```

giac [A] time = 0.12, size = 124, normalized size = 1.38

$$\frac{15\,\pi + \frac{4\left(15\left(e^{(bx+a)}-e^{(-bx-a)}\right)^5+160\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+528\,e^{(bx+a)}-528\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^3} + 30\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/96*(15*pi + 4*(15*(e^(b*x + a) - e^(-b*x - a))^5 + 160*(e^(b*x + a) - e^(
-b*x - a))^3 + 528*e^(b*x + a) - 528*e^(-b*x - a))/((e^(b*x + a) - e^(-b*x
- a))^2 + 4)^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [C] time = 0.49, size = 230, normalized size = 2.56

$$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\left(15\,e^{10bx+10a}+85\,e^{8bx+8a}+198\,e^{6bx+6a}-198\,e^{4bx+4a}-85\,e^{2bx+2a}-15\right)}{24\left(1+e^{2bx+2a}\right)^5 b}+\frac{5i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2\cdots}}{\left(1+e^{2\cdots}\right.}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(7/2),x)
```

```
[Out] 1/24/(1+exp(2*b*x+2*a))^5*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(15
*exp(10*b*x+10*a)+85*exp(8*b*x+8*a)+198*exp(6*b*x+6*a)-198*exp(4*b*x+4*a)-8
5*exp(2*b*x+2*a)-15)/b+5/16*I*ln(exp(b*x)+I*exp(-a))/b*(1/(1+exp(2*b*x+2*a)
)^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)-5/16*I*ln(exp(b*x)
-I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*
a))*exp(-b*x-a)
```

maxima [B] time = 0.48, size = 156, normalized size = 1.73

$$-\frac{5\,\arctan\left(e^{(-bx-a)}\right)}{8\,b}+\frac{15\,e^{(-bx-a)}+85\,e^{(-3\,bx-3\,a)}+198\,e^{(-5\,bx-5\,a)}-198\,e^{(-7\,bx-7\,a)}-85\,e^{(-9\,bx-9\,a)}-15\,e^{(-11\,bx-\cdots}}{24\,b\left(6\,e^{(-2\,bx-2\,a)}+15\,e^{(-4\,bx-4\,a)}+20\,e^{(-6\,bx-6\,a)}+15\,e^{(-8\,bx-8\,a)}+6\,e^{(-10\,bx-10\,a)}+e^{(-12\,bx-1\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -5/8*arctan(e^(-b*x - a))/b + 1/24*(15*e^(-b*x - a) + 85*e^(-3*b*x - 3*a) +
 198*e^(-5*b*x - 5*a) - 198*e^(-7*b*x - 7*a) - 85*e^(-9*b*x - 9*a) - 15*e^(
-11*b*x - 11*a))/(b*(6*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) + 20*e^(-6*b*
```

```
x - 6*a) + 15*e^(-8*b*x - 8*a) + 6*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a)
+ 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cosh (a+b\,x)^2}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(7/2),x)
[Out] int((1/cosh(a + b*x)^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(7/2),x)
[Out] Timed out
```

3.25 $\int \operatorname{sech}^2(a+bx)^{5/2}\,dx$

Optimal. Leaf size=65

$$\frac{3\sin^{-1}(\tanh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)^{3/2}}{4b}+\frac{3\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{8b}$$

```
[Out] 3/8*arcsin(tanh(b*x+a))/b+1/4*(sech(b*x+a)^2)^(3/2)*tanh(b*x+a)/b+3/8*(sech
(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4122, 195, 216}

$$\frac{3\sin^{-1}(\tanh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)^{3/2}}{4b}+\frac{3\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Tanh[a + b*x]])/(8*b) + (3*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(
8*b) + ((Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(4*b)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} + \frac{3\,\text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{4b}$$
$$= \frac{3\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx\right.}{8b}$$
$$= \frac{3\sin^{-1}(\tanh(a+bx))}{8b} + \frac{3\sqrt{\operatorname{sech}^2(a+bx)}\,\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+b}{4b}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.85

$$\frac{\operatorname{sech}^2(a+bx)^{3/2}\left(3\sinh(2(a+bx)) + 4\tanh(a+bx) + 6\cosh^3(a+bx)\tan^{-1}(\sinh(a+bx))\right)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] ((Sech[a + b*x]^2)^(3/2)*(6*ArcTan[Sinh[a + b*x]]*Cosh[a + b*x]^3 + 3*Sinh[
2*(a + b*x)] + 4*Tanh[a + b*x]))/(16*b)
```

fricas [B] time = 0.44, size = 812, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
```

```
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

giac [A] time = 0.14, size = 102, normalized size = 1.57

$$\frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{3}+20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^{2}+4\right)^{2}} + 6\,\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-
b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x +
 2*a) - 1)*e^(-b*x - a)))/b
```

maple [C] time = 0.42, size = 208, normalized size = 3.20

$$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,\left(3\,e^{6bx+6a}+11\,e^{4bx+4a}-11\,e^{2bx+2a}-3\right)}{4\left(1+e^{2bx+2a}\right)^{3}b}+\frac{3i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}\,\left(1+e^{2bx+2a}\right)e^{-bx-a}}{8b}-\frac{3i\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(5/2),x)
```

```
[Out] 1/4/(1+exp(2*b*x+2*a))^3*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(3*e
xp(6*b*x+6*a)+11*exp(4*b*x+4*a)-11*exp(2*b*x+2*a)-3)/b+3/8*I*ln(exp(b*x)+I*
exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))
*exp(-b*x-a)-3/8*I*ln(exp(b*x)-I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b
*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)
```

maxima [B] time = 0.42, size = 112, normalized size = 1.72

$$-\frac{3\,\arctan\left(e^{(-bx-a)}\right)}{4\,b}+\frac{3\,e^{(-bx-a)}+11\,e^{(-3\,bx-3\,a)}-11\,e^{(-5\,bx-5\,a)}-3\,e^{(-7\,bx-7\,a)}}{4\,b\left(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{{\cosh\left(a+b\,x\right)}^2}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(5/2),x)
```

```
[Out] int((1/cosh(a + b*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\operatorname{sech}^{2}{\left(a+bx \right)}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(5/2),x)
```

```
[Out] Integral((sech(a + b*x)**2)**(5/2), x)
```

3.26 $\int \operatorname{sech}^2(a+bx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{\sin^{-1}(\tanh(a+bx))}{2b}+\frac{\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{2b}$$

```
[Out] 1/2*arcsin(tanh(b*x+a))/b+1/2*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4122, 195, 216}

$$\frac{\sin^{-1}(\tanh(a+bx))}{2b}+\frac{\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(3/2),x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/(2*b) + (Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(2*b)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \mathrm{sech}^2(a+bx)^{3/2}\,dx = \frac{\mathrm{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\sqrt{\mathrm{sech}^2(a+bx)}\,\tanh(a+bx)}{2b} + \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{2b}$$
$$= \frac{\sin^{-1}(\tanh(a+bx))}{2b} + \frac{\sqrt{\mathrm{sech}^2(a+bx)}\,\tanh(a+bx)}{2b}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.15

$$\frac{\mathrm{sech}(a+bx)\left(\tan^{-1}(\sinh(a+bx))+\tanh(a+bx)\mathrm{sech}(a+bx)\right)}{2b\sqrt{\mathrm{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(3/2),x]
```

```
[Out] (Sech[a + b*x]*(ArcTan[Sinh[a + b*x]] + Sech[a + b*x]*Tanh[a + b*x]))/(2*b*
Sqrt[Sech[a + b*x]^2])
```

fricas [B] time = 0.39, size = 267, normalized size = 6.68

$$\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(bx+a)\right.}{b\cosh(bx+a)^4 + 4b\cosh(bx+a)s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

giac [B] time = 0.13, size = 76, normalized size = 1.90

$$\frac{\pi + \frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

maple [C] time = 0.43, size = 183, normalized size = 4.58

$$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\left(e^{2bx+2a}-1\right)}{\left(1+e^{2bx+2a}\right)b}+\frac{i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\left(1+e^{2bx+2a}\right)e^{-bx-a}}{2b}-\frac{i\ln\left(e^{bx}-ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\left(\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(3/2),x)
```

```
[Out] 1/(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b
*x+2*a)-1)/b+1/2*I*ln(exp(b*x)+I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b
*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)-1/2*I*ln(exp(b*x)-I*exp(-a))/
b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x
-a)
```

maxima [A] time = 0.42, size = 65, normalized size = 1.62

$$-\frac{\arctan\left(e^{(-bx-a)}\right)}{b}+\frac{e^{(-bx-a)}-e^{(-3\,bx-3\,a)}}{b\left(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{{\cosh\left(a+b\,x\right)}^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(3/2),x)
```

```
[Out] int((1/cosh(a + b*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\operatorname{sech}^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((sech(a + b*x)**2)**(3/2), x)
```

3.27 $\int \sqrt{\text{sech}^2(a+bx)}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(\tanh(a+bx))}{b}$$

```
[Out] arcsin(tanh(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4122, 216}

$$\frac{\sin^{-1}(\tanh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/b
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\text{sech}^2(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\sin^{-1}(\tanh(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.02, size = 29, normalized size = 2.64

$$\frac{\cosh(a+bx)\sqrt{\text{sech}^2(a+bx)}\,\tan^{-1}(\sinh(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] (ArcTan[Sinh[a + b*x]]*Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2])/b
```

fricas [A] time = 0.38, size = 19, normalized size = 1.73

$$\frac{2 \arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

giac [A] time = 0.13, size = 12, normalized size = 1.09

$$\frac{2 \arctan\left(e^{(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(b*x + a))/b
```

maple [C] time = 0.43, size = 130, normalized size = 11.82

$$\frac{i\ln\left(\mathrm{e}^{bx}+i\mathrm{e}^{-a}\right)\sqrt{\frac{\mathrm{e}^{2bx+2a}}{\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}}\,\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{-bx-a}}{b}-\frac{i\ln\left(\mathrm{e}^{bx}-i\mathrm{e}^{-a}\right)\sqrt{\frac{\mathrm{e}^{2bx+2a}}{\left(1+\mathrm{e}^{2bx+2a}\right)^{2}}}\,\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{-bx-a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sech(b*x+a)^2)^(1/2),x)
```

```
[Out] I*ln(exp(b*x)+I*exp(-a))/b*(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2
*b*x+2*a))^(1/2)*exp(-b*x-a)-I*ln(exp(b*x)-I*exp(-a))/b*(1+exp(2*b*x+2*a))*
(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-b*x-a)
```

maxima [A] time = 0.32, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sinh(b*x + a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \sqrt{\frac{1}{\cosh\left(a+b\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*x)^2)^(1/2),x)
```

```
[Out] int((1/cosh(a + b*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{sech}^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sech(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sech(a + b*x)**2), x)
```

3.28 $\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[Out] tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4122, 191}

$$\frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[a + b*x]^2],x]
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

fricas [A] time = 0.38, size = 10, normalized size = 0.45

$$\frac{\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] sinh(b*x + a)/b
```

giac [A] time = 0.13, size = 23, normalized size = 1.05

$$\frac{e^{(bx+a)}-e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(e^(b*x + a) - e^(-b*x - a))/b
```

maple [B] time = 0.41, size = 97, normalized size = 4.41

$$\frac{e^{2bx+2a}}{2b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}-\frac{1}{2b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(1/2),x)
[Out] 1/2/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(
2*b*x+2*a)-1/2/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))
^(1/2)
```

maxima [A] time = 0.32, size = 26, normalized size = 1.18

$$\frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

mupad [B] time = 0.15, size = 53, normalized size = 2.41

$$\frac{\mathrm{e}^{-2\,a-2\,b\,x}\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)\,\sqrt{\frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^2}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x)^2)^(1/2),x)
```

```
[Out] (exp(- 2*a - 2*b*x)*(exp(4*a + 4*b*x) - 1)*((4*exp(2*a + 2*b*x))/(exp(2*a +
 2*b*x) + 1)^2)^(1/2))/(4*b)
```

sympy [A] time = 17.32, size = 29, normalized size = 1.32

$$\begin{cases} \frac{\tanh{\left(a+bx \right)}}{b\sqrt{\operatorname{sech}^{2}{\left(a+bx \right)}}} & \text{for}\: b \neq 0 \\ \frac{x}{\sqrt{\operatorname{sech}^{2}{\left(a \right)}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(1/2),x)
```

```
[Out] Piecewise((tanh(a + b*x)/(b*sqrt(sech(a + b*x)**2)), Ne(b, 0)), (x/sqrt(sec
h(a)**2), True))
```

3.29 $\int \frac{1}{\mathbf{sech}^2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}$$

```
[Out] 1/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+2/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4122, 192, 191}

$$\frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-3/2),x]
```

```
[Out] Tanh[a + b*x]/(3*b*(Sech[a + b*x]^2)^(3/2)) + (2*Tanh[a + b*x])/(3*b*Sqrt[S
ech[a + b*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{3b}$$

$$= \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Mathematica [**A**] time = 0.07, size = 44, normalized size = 0.86

$$\frac{\tanh^3(a+bx) + 3\tanh(a+bx)\operatorname{sech}^2(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-3/2),x]
```

```
[Out] (3*Sech[a + b*x]^2*Tanh[a + b*x] + Tanh[a + b*x]^3)/(3*b*(Sech[a + b*x]^2)^
(3/2))
```

fricas [**A**] time = 0.39, size = 32, normalized size = 0.63

$$\frac{\sinh(bx+a)^3 + 3\left(\cosh(bx+a)^2 + 3\right)\sinh(bx+a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

giac [**A**] time = 0.11, size = 48, normalized size = 0.94

$$-\frac{\left(9\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)} - e^{(3\,bx+3\,a)} - 9\,e^{(bx+a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - 9*e^(b*
x + a))/b
```

maple [B] time = 0.44, size = 201, normalized size = 3.94

$$\frac{e^{4bx+4a}}{24b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}+\frac{3\,e^{2bx+2a}}{8b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}-\frac{3}{8b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}-\frac{e^{-2b}}{24b\left(1+e^{2bx+2a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(3/2),x)
```

```
[Out] 1/24/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp
(4*b*x+4*a)+3/8/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a)
)^(1/2)*exp(2*b*x+2*a)-3/8/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp
(2*b*x+2*a))^(1/2)-1/24/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*
b*x+2*a))^(1/2)*exp(-2*b*x-2*a)
```

maxima [A] time = 0.33, size = 54, normalized size = 1.06

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}-\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x)^2)^(3/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x)^2)^(3/2), x)
```

sympy [A] time = 18.85, size = 54, normalized size = 1.06

$$\begin{cases} -\frac{2\tanh^{3}(a+bx)}{3b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{3}{2}}} + \frac{\tanh(a+bx)}{b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\operatorname{sech}^{2}(a)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Piecewise((-2*tanh(a + b*x)**3/(3*b*(sech(a + b*x)**2)**(3/2)) + tanh(a + b
*x)/(b*(sech(a + b*x)**2)**(3/2)), Ne(b, 0)), (x/(sech(a)**2)**(3/2), True)
)
```

3.30 $\int \frac{1}{\text{sech}^2(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{8\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}+\frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}}+\frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}}$$

```
[Out] 1/5*tanh(b*x+a)/b/(sech(b*x+a)^2)^(5/2)+4/15*tanh(b*x+a)/b/(sech(b*x+a)^2)^
(3/2)+8/15*tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4122, 192, 191}

$$\frac{8\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}+\frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}}+\frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-5/2),x]
```

```
[Out] Tanh[a + b*x]/(5*b*(Sech[a + b*x]^2)^(5/2)) + (4*Tanh[a + b*x])/(15*b*(Sech
[a + b*x]^2)^(3/2)) + (8*Tanh[a + b*x])/(15*b*Sqrt[Sech[a + b*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\text{sech}^2(a+bx)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{5b}$$

$$= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}} + \frac{8\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{15b}$$

$$= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}} + \frac{8\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}$$

Mathematica [A] time = 0.08, size = 47, normalized size = 0.62

$$\frac{\left(3\sinh^4(a+bx)+10\sinh^2(a+bx)+15\right)\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-5/2),x]
```

```
[Out] ((15 + 10*Sinh[a + b*x]^2 + 3*Sinh[a + b*x]^4)*Tanh[a + b*x])/(15*b*Sqrt[Se
ch[a + b*x]^2])
```

fricas [A] time = 0.46, size = 66, normalized size = 0.87

$$\frac{3\,\sinh\left(bx+a\right)^5 + 5\left(6\,\cosh\left(bx+a\right)^2+5\right)\sinh\left(bx+a\right)^3 + 15\left(\cosh\left(bx+a\right)^4 + 5\,\cosh\left(bx+a\right)^2+10\right)\sinh\left(bx\right.}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

giac [A] time = 0.12, size = 70, normalized size = 0.92

$$-\frac{\left(150\,e^{(4\,bx+4\,a)}+25\,e^{(2\,bx+2\,a)}+3\right)e^{(-5\,bx-5\,a)}-3\,e^{(5\,bx+5\,a)}-25\,e^{(3\,bx+3\,a)}-150\,e^{(bx+a)}}{480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*b*x + 4*a) + 25*e^(2*b*x + 2*a) + 3)*e^(-5*b*x - 5*a) - 3
*e^(5*b*x + 5*a) - 25*e^(3*b*x + 3*a) - 150*e^(b*x + a))/b
```

maple [B] time = 0.41, size = 305, normalized size = 4.01

$$\frac{e^{6bx+6a}}{160b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{5\,e^{4bx+4a}}{96b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{5\,e^{2bx+2a}}{16b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}-\frac{}{16b\left(1+e^{2b\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(5/2),x)
```

```
[Out] 1/160/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*ex
p(6*b*x+6*a)+5/96/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*
a))^(1/2)*exp(4*b*x+4*a)+5/16/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*
exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-5/16/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*
b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)-5/96/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x
+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-1/160/b/(1+exp(2*b*x+2*a))/(
1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-4*b*x-4*a)
```

maxima [A] time = 0.33, size = 82, normalized size = 1.08

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}+\frac{5\,e^{(3\,bx+3\,a)}}{96\,b}+\frac{5\,e^{(bx+a)}}{16\,b}-\frac{5\,e^{(-bx-a)}}{16\,b}-\frac{5\,e^{(-3\,bx-3\,a)}}{96\,b}-\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x)^2)^(5/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x)^2)^(5/2), x)
```

sympy [A] time = 38.20, size = 80, normalized size = 1.05

$$\begin{cases} \frac{8\tanh^5(a+bx)}{15b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} - \frac{4\tanh^3(a+bx)}{3b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} + \frac{\tanh(a+bx)}{b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\operatorname{sech}^2(a)\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(5/2),x)
```

```
[Out] Piecewise((8*tanh(a + b*x)**5/(15*b*(sech(a + b*x)**2)**(5/2)) - 4*tanh(a +
 b*x)**3/(3*b*(sech(a + b*x)**2)**(5/2)) + tanh(a + b*x)/(b*(sech(a + b*x)*
*2)**(5/2)), Ne(b, 0)), (x/(sech(a)**2)**(5/2), True))
```

3.31 $\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}} \, dx$

Optimal. Leaf size=101

$$\frac{16\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}}+\frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}}$$

```
[Out] 1/7*tanh(b*x+a)/b/(sech(b*x+a)^2)^(7/2)+6/35*tanh(b*x+a)/b/(sech(b*x+a)^2)^
(5/2)+8/35*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+16/35*tanh(b*x+a)/b/(sech(b*
x+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4122, 192, 191}

$$\frac{16\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}+\frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}}+\frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}}+\frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] Tanh[a + b*x]/(7*b*(Sech[a + b*x]^2)^(7/2)) + (6*Tanh[a + b*x])/(35*b*(Sech
[a + b*x]^2)^(5/2)) + (8*Tanh[a + b*x])/(35*b*(Sech[a + b*x]^2)^(3/2)) + (1
6*Tanh[a + b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{9/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{7b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{24\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{35b}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{16\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3}}\right.}{}$$

$$= \frac{\tanh(a+bx)}{7b\operatorname{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{16\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx}}$$

Mathematica [A] time = 0.14, size = 57, normalized size = 0.56

$$\frac{\left(5\sinh^6(a+bx)+21\sinh^4(a+bx)+35\sinh^2(a+bx)+35\right)\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] ((35 + 35*Sinh[a + b*x]^2 + 21*Sinh[a + b*x]^4 + 5*Sinh[a + b*x]^6)*Tanh[a
+ b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

fricas [A] time = 0.49, size = 108, normalized size = 1.07

$$\frac{5\sinh(bx+a)^7+7\left(15\cosh(bx+a)^2+7\right)\sinh(bx+a)^5+35\left(5\cosh(bx+a)^4+14\cosh(bx+a)^2+7\right)\sinh}{2240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2240*(5*sinh(b*x + a)^7 + 7*(15*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^5 + 35
*(5*cosh(b*x + a)^4 + 14*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 35*(cosh(b*
x + a)^6 + 7*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 35)*sinh(b*x + a))/b
```

giac [A] time = 0.13, size = 92, normalized size = 0.91

$$-\frac{\left(1225\,e^{(6\,bx+6\,a)}+245\,e^{(4\,bx+4\,a)}+49\,e^{(2\,bx+2\,a)}+5\right)e^{(-7\,bx-7\,a)}-5\,e^{(7\,bx+7\,a)}-49\,e^{(5\,bx+5\,a)}-245\,e^{(3\,bx+3\,a)}-1225\,e}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*b*x + 6*a) + 245*e^(4*b*x + 4*a) + 49*e^(2*b*x + 2*a) +
 5)*e^(-7*b*x - 7*a) - 5*e^(7*b*x + 7*a) - 49*e^(5*b*x + 5*a) - 245*e^(3*b*
x + 3*a) - 1225*e^(b*x + a))/b
```

maple [B] time = 0.42, size = 409, normalized size = 4.05

$$\frac{e^{8bx+8a}}{896b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}+\frac{7\,e^{6bx+6a}}{640b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}+\frac{7\,e^{4bx+4a}}{128b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^{2}}}}+\frac{}{128b\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sech(b*x+a)^2)^(7/2),x)
```

```
[Out] 1/896/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*ex
p(8*b*x+8*a)+7/640/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2
*a))^(1/2)*exp(6*b*x+6*a)+7/128/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^
2*exp(2*b*x+2*a))^(1/2)*exp(4*b*x+4*a)+35/128/b/(1+exp(2*b*x+2*a))/(1/(1+ex
p(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-35/128/b/(1+exp(2*b*x+
2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)-7/128/b/(1+exp(2*b*x+2*
a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-7/640/b/(
1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-4*b*x-
4*a)-1/896/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/
2)*exp(-6*b*x-6*a)
```

maxima [A] time = 0.32, size = 100, normalized size = 0.99

$$\frac{\left(49\,e^{(-2\,bx-2\,a)}+245\,e^{(-4\,bx-4\,a)}+1225\,e^{(-6\,bx-6\,a)}+5\right)e^{(7\,bx+7\,a)}}{4480\,b}-\frac{1225\,e^{(-bx-a)}+245\,e^{(-3\,bx-3\,a)}+49\,e^{(-5\,bx-5\,a)}+5}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4480*(49*e^(-2*b*x - 2*a) + 245*e^(-4*b*x - 4*a) + 1225*e^(-6*b*x - 6*a)
+ 5)*e^(7*b*x + 7*a)/b - 1/4480*(1225*e^(-b*x - a) + 245*e^(-3*b*x - 3*a) +
 49*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(a + b*x)^2)^(7/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x)^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sech(b*x+a)**2)**(7/2),x)
```

```
[Out] Timed out
```

3.32 $\int \left(a\mathrm{sech}^2(x)\right)^{5/2} dx$

Optimal. Leaf size=65

$$\frac{3}{8}a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\mathrm{sech}^2(x)}}\right)+\frac{3}{8}a^2\tanh(x)\sqrt{a\mathrm{sech}^2(x)}+\frac{1}{4}a\tanh(x)\left(a\mathrm{sech}^2(x)\right)^{3/2}$$

```
[Out] 3/8*a^(5/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+1/4*a*(a*sech(x)^2)
^(3/2)*tanh(x)+3/8*a^2*(a*sech(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4122, 195, 217, 203}

$$\frac{3}{8}a^2\tanh(x)\sqrt{a\mathrm{sech}^2(x)}+\frac{3}{8}a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\mathrm{sech}^2(x)}}\right)+\frac{1}{4}a\tanh(x)\left(a\mathrm{sech}^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/8 + (3*a^2*Sqrt[a*S
ech[x]^2]*Tanh[x])/8 + (a*(a*Sech[x]^2)^(3/2)*Tanh[x])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{sech}^2(x)\right)^{5/2} dx &= a\,\text{Subst}\left(\int \left(a-ax^2\right)^{3/2} dx, x, \tanh(x)\right)\\ &= \frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x)+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{a-ax^2}\,dx, x, \tanh(x)\right)\\ &= \frac{3}{8}a^2\sqrt{a\operatorname{sech}^2(x)}\,\tanh(x)+\frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x)+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x\right.\\ &= \frac{3}{8}a^2\sqrt{a\operatorname{sech}^2(x)}\,\tanh(x)+\frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x)+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x,\right.\\ &= \frac{3}{8}a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right)+\frac{3}{8}a^2\sqrt{a\operatorname{sech}^2(x)}\,\tanh(x)+\frac{1}{4}a\left(a\operatorname{sech}^2(x)\right)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.65

$$\frac{1}{8}\cosh(x)\left(a\operatorname{sech}^2(x)\right)^{5/2}\left(2\sinh(x)+3\sinh(x)\cosh^2(x)+6\cosh^4(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (Cosh[x]*(a*Sech[x]^2)^(5/2)*(6*ArcTan[Tanh[x/2]]*Cosh[x]^4 + 2*Sinh[x] + 3
*Cosh[x]^2*Sinh[x]))/8
```

fricas [B] time = 0.49, size = 1082, normalized size = 16.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cosh(x)^7 + 3*(a^2*e^(2*x) + a^2)*sinh(x)^7 + 11*a^2*cosh(x)^5 +
 21*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^6 + (63*a^2*cosh(x)^2 + 11*
```

```
a^2 + (63*a^2*cosh(x)^2 + 11*a^2)*e^(2*x))*sinh(x)^5 - 11*a^2*cosh(x)^3 + 5
*(21*a^2*cosh(x)^3 + 11*a^2*cosh(x) + (21*a^2*cosh(x)^3 + 11*a^2*cosh(x))*e
^(2*x))*sinh(x)^4 + (105*a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2 + (105*
a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 - 3*a^2*cosh
(x) + (63*a^2*cosh(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) + (63*a^2*cosh
(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 + 3*(a^2*cos
h(x)^8 + (a^2*e^(2*x) + a^2)*sinh(x)^8 + 4*a^2*cosh(x)^6 + 8*(a^2*cosh(x)*e
^(2*x) + a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 + a^2 + (7*a^2*cosh(x)
^2 + a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 + 3*a^2
*cosh(x) + (7*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 2*(35*a^2
*cosh(x)^4 + 30*a^2*cosh(x)^2 + 3*a^2 + (35*a^2*cosh(x)^4 + 30*a^2*cosh(x)^
2 + 3*a^2)*e^(2*x))*sinh(x)^4 + 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 + 10*a
^2*cosh(x)^3 + 3*a^2*cosh(x) + (7*a^2*cosh(x)^5 + 10*a^2*cosh(x)^3 + 3*a^2*
cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2
*cosh(x)^2 + a^2 + (7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 +
a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^8 + 4*a^2*cosh(x)^6 + 6*a^2*co
sh(x)^4 + 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^7 + 3*a^2*cosh(x)
^5 + 3*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^7 + 3*a^2*cosh(x)^5 + 3*a
^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (
3*a^2*cosh(x)^7 + 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2
*x) + (21*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2 + (21
*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2)*e^(2*x))*sinh
(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*s
inh(x)^8 + 4*(7*cosh(x)^2 + 1)*e^x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*co
sh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cos
h(x)^4 + 9*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*co
sh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4
*cosh(x)^2 + 1)*e^x)
```

giac [A] time = 0.13, size = 65, normalized size = 1.00

$$\frac{1}{16}\left(3\,\pi-\frac{4\left(3\left(e^{(-x)}-e^{x}\right)^{3}+20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)^{2}}+6\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*pi - 4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2
+ 4)^2 + 6*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a^(5/2)
```

maple [C] time = 0.24, size = 127, normalized size = 1.95

$$\frac{a^{2}\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(3\,\mathrm{e}^{6x}+11\,\mathrm{e}^{4x}-11\,\mathrm{e}^{2x}-3\right)}{4\left(1+\mathrm{e}^{2x}\right)^{3}}+\frac{3ia^{2}\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\ln\left(\mathrm{e}^{x}+i\right)}{8}-\frac{3ia^{2}\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\ln}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(5/2),x)
```

```
[Out] 1/4*a^2/(1+exp(2*x))^3*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*(3*exp(6*x)+11*exp
(4*x)-11*exp(2*x)-3)+3/8*I*a^2*exp(-x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x)
)^2)^(1/2)*ln(exp(x)+I)-3/8*I*a^2*exp(-x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2
*x))^2)^(1/2)*ln(exp(x)-I)
```

maxima [A] time = 0.48, size = 72, normalized size = 1.11

$$\frac{3}{4}\,a^{\frac{5}{2}}\arctan\left(e^{x}\right)+\frac{3\,a^{\frac{5}{2}}e^{(7\,x)}+11\,a^{\frac{5}{2}}e^{(5\,x)}-11\,a^{\frac{5}{2}}e^{(3\,x)}-3\,a^{\frac{5}{2}}e^{x}}{4\left(e^{(8\,x)}+4\,e^{(6\,x)}+6\,e^{(4\,x)}+4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/4*a^(5/2)*arctan(e^x) + 1/4*(3*a^(5/2)*e^(7*x) + 11*a^(5/2)*e^(5*x) - 11*
a^(5/2)*e^(3*x) - 3*a^(5/2)*e^x)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*
x) + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{a}{\cosh(x)^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^2)^(5/2),x)
```

```
[Out] int((a/cosh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{sech}^{2}(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(5/2),x)
[Out] Integral((a*sech(x)**2)**(5/2), x)
```

3.33 $\int \left(a\,\text{sech}^2(x)\right)^{3/2} dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\ \tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{1}{2}a\tanh(x)\sqrt{a\text{sech}^2(x)}$$

```
[Out] 1/2*a^(3/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+1/2*a*(a*sech(x)^2)
^(1/2)*tanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4122, 195, 217, 203}

$$\frac{1}{2}a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\ \tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{1}{2}a\tanh(x)\sqrt{a\text{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/2 + (a*Sqrt[a*Sech[x]
^2]*Tanh[x])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\operatorname{sech}^2(x)\right)^{3/2} dx &= a\,\text{Subst}\left(\int \sqrt{a-ax^2}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}a\sqrt{a\operatorname{sech}^2(x)}\,\tanh(x)+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x, \tanh(x)\right)\\ &= \frac{1}{2}a\sqrt{a\operatorname{sech}^2(x)}\,\tanh(x)+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right)\\ &= \frac{1}{2}a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right)+\frac{1}{2}a\sqrt{a\operatorname{sech}^2(x)}\,\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.63

$$\frac{1}{2}a\sqrt{a\operatorname{sech}^2(x)}\left(\tanh(x)+2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(3/2),x]
[Out] (a*Sqrt[a*Sech[x]^2]*(2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x]))/2
```

fricas [B] time = 0.46, size = 310, normalized size = 6.74

$$\frac{\left(a\cosh(x)^3+\left(ae^{(2x)}+a\right)\sinh(x)^3+3\left(a\cosh(x)e^{(2x)}+a\cosh(x)\right)\sinh(x)^2+\left(a\cosh(x)^4+\left(ae^{(2x)}+a\right)\sinh(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="fricas")
[Out] (a*cosh(x)^3 + (a*e^(2*x) + a)*sinh(x)^3 + 3*(a*cosh(x)*e^(2*x) + a*cosh(x)
)*sinh(x)^2 + (a*cosh(x)^4 + (a*e^(2*x) + a)*sinh(x)^4 + 4*(a*cosh(x)*e^(2*
```

```
x) + a*cosh(x))*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^4 + 2*a*cosh(x)^2 + a)*e^(2*x)
+ 4*(a*cosh(x)^3 + a*cosh(x) + (a*cosh(x)^3 + a*cosh(x))*e^(2*x))*sinh(x) +
 a)*arctan(cosh(x) + sinh(x)) - a*cosh(x) + (a*cosh(x)^3 - a*cosh(x))*e^(2*
x) + (3*a*cosh(x)^2 + (3*a*cosh(x)^2 - a)*e^(2*x) - a)*sinh(x))*sqrt(a/(e^(
4*x) + 2*e^(2*x) + 1))*e^x/(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*
cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(
x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

giac [A] time = 0.13, size = 48, normalized size = 1.04

$$\frac{1}{4}\left(\pi - \frac{4\left(e^{(-x)} - e^{x}\right)}{\left(e^{(-x)} - e^{x}\right)^{2} + 4} + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*(e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 2*arctan(1/2*(e^(2*x) -
 1)*e^(-x)))*a^(3/2)
```

maple [C] time = 0.21, size = 106, normalized size = 2.30

$$\frac{a\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\left(\mathrm{e}^{2x}-1\right)}{1+\mathrm{e}^{2x}} + \frac{ia\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\ln\left(\mathrm{e}^{x}+i\right)}{2} - \frac{ia\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\ln\left(\mathrm{e}^{x}-i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(3/2),x)
```

```
[Out] a/(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*(exp(2*x)-1)+1/2*I*a*exp(-
x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*ln(exp(x)+I)-1/2*I*a*exp(
-x)*(1+exp(2*x))*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*ln(exp(x)-I)
```

maxima [A] time = 1.45, size = 39, normalized size = 0.85

$$a^{\frac{3}{2}}\arctan\left(e^{x}\right) + \frac{a^{\frac{3}{2}}e^{(3x)} - a^{\frac{3}{2}}e^{x}}{e^{(4x)} + 2\,e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a^(3/2)*arctan(e^x) + (a^(3/2)*e^(3*x) - a^(3/2)*e^x)/(e^(4*x) + 2*e^(2*x)
+ 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{a}{\cosh(x)^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^2)^(3/2),x)
```

```
[Out] int((a/cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \operatorname{sech}^2(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**2)**(3/2), x)
```

3.34 $\int \sqrt{a\text{sech}^2(x)}\,dx$

Optimal. Leaf size=25

$$\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$

[Out] arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))*a^(1/2)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 217, 203}

$$\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*Sech[x]^2],x]

[Out] Sqrt[a]*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a\text{sech}^2(x)}\,dx = a\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x, \tanh(x)\right)$$
$$= a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$
$$= \sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.84

$$2\cosh(x)\sqrt{a\text{sech}^2(x)}\,\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^2],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[a*Sech[x]^2]
```

fricas [A] time = 0.44, size = 145, normalized size = 5.80

$$\left[\sqrt{-a}\,\log\left(\frac{2\,a\cosh(x)e^x\sinh(x) + ae^x\sinh(x)^2 + 2\left(\cosh(x)e^{(2\,x)} + \left(e^{(2\,x)}+1\right)\sinh(x) + \cosh(x)\right)\sqrt{-a}\,\sqrt{\frac{a}{e^{(4\,x)}+2}}}{2\,\cosh(x)e^x\sinh(x) + e^x\sinh(x)^2 + \left(\cosh(x)^2+1\right)e^x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cosh(x)*e^x*sinh(x) + a*e^x*sinh(x)^2 + 2*(cosh(x)*e^(2*
x) + (e^(2*x) + 1)*sinh(x) + cosh(x))*sqrt(-a)*sqrt(a/(e^(4*x) + 2*e^(2*x)
+ 1))*e^x + (a*cosh(x)^2 - a)*e^x)/(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 +
 (cosh(x)^2 + 1)*e^x)), 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*a
rctan(cosh(x) + sinh(x))]
```

giac [A] time = 0.13, size = 8, normalized size = 0.32

$$2\,\sqrt{a}\,\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*arctan(e^x)
```

maple [C] time = 0.22, size = 72, normalized size = 2.88

$$i\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln\left(\mathrm{e}^{x}+i\right)-i\sqrt{\frac{a\,\mathrm{e}^{2x}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\ln\left(\mathrm{e}^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^2)^(1/2),x)
```

```
[Out] I*(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(2*x))*ln(exp(x)+I)-I*(a*
exp(2*x)/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(2*x))*ln(exp(x)-I)
```

maxima [A] time = 0.48, size = 8, normalized size = 0.32

$$2\sqrt{a}\,\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*arctan(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{a}{\cosh(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^2)^(1/2),x)
```

```
[Out] int((a/cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**2), x)
```

3.35 $\int \frac{1}{\sqrt{a\,\mathbf{sech}^2(x)}}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}$$

```
[Out] tanh(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4122, 191}

$$\frac{\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[a*Sech[x]^2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\,\mathrm{sech}^2(x)}}\,dx &= a\,\mathrm{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tanh(x)}{\sqrt{a\mathrm{sech}^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^2],x]
[Out] Tanh[x]/Sqrt[a*Sech[x]^2]
```

fricas [B] time = 0.44, size = 79, normalized size = 6.08

$$\frac{\left(\left(e^{(2x)}+1\right)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2-1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{a}{e^{(4x)}+2e^{(2x)}+1}}\,e^x}{2\left(a\cosh(x)e^x+ae^x\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 - 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

giac [A] time = 0.11, size = 14, normalized size = 1.08

$$-\frac{e^{(-x)}-e^x}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(e^(-x) - e^x)/sqrt(a)
```

maple [B] time = 0.21, size = 58, normalized size = 4.46

$$\frac{e^{2x}}{2\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}\,\left(1+e^{2x}\right)}-\frac{1}{2\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(1/2),x)
```

```
[Out] 1/2/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(2*x)-1/2/(1+exp(2*x)
)/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)
```

maxima [A] time = 0.45, size = 17, normalized size = 1.31

$$-\frac{e^{(-x)}}{2\sqrt{a}}+\frac{e^{x}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*e^(-x)/sqrt(a) + 1/2*e^x/sqrt(a)
```

mupad [B] time = 0.12, size = 33, normalized size = 2.54

$$-\frac{\left(\frac{e^{-2x}}{2}-\frac{e^{2x}}{2}\right)\sqrt{\frac{1}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^2)^(1/2),x)
[Out] -((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 + exp(x)/2)^2)^(1/2))/(2*a^(1/2)
)
```

sympy [A] time = 0.59, size = 15, normalized size = 1.15

$$\frac{\tanh(x)}{\sqrt{a}\sqrt{\operatorname{sech}^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(1/2),x)
[Out] tanh(x)/(sqrt(a)*sqrt(sech(x)**2))
```

3.36 $\int \frac{1}{\left(a\,\mathbf{sech}^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh(x)}{3a\sqrt{a\operatorname{sech}^2(x)}}+\frac{\tanh(x)}{3\left(a\operatorname{sech}^2(x)\right)^{3/2}}$$

```
[Out] 1/3*tanh(x)/(a*sech(x)^2)^(3/2)+2/3*tanh(x)/a/(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{2\tanh(x)}{3a\sqrt{a\operatorname{sech}^2(x)}}+\frac{\tanh(x)}{3\left(a\operatorname{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-3/2),x]
```

```
[Out] Tanh[x]/(3*(a*Sech[x]^2)^(3/2)) + (2*Tanh[x])/(3*a*Sqrt[a*Sech[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = a\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{5/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{3\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{3\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{2\tanh(x)}{3a\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.75

$$\frac{(9\sinh(x)+\sinh(3x))\text{sech}^3(x)}{12\left(a\text{sech}^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-3/2),x]
```

```
[Out] (Sech[x]^3*(9*Sinh[x] + Sinh[3*x]))/(12*(a*Sech[x]^2)^(3/2))
```

fricas [B] time = 0.63, size = 277, normalized size = 7.69

$$\frac{\left(\left(e^{(2x)}+1\right)\sinh(x)^6+\cosh(x)^6+6\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)^5+3\left(5\cosh(x)^2+\left(5\cosh(x)^2+3\right)e^{(2x)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(cosh(x)*e^(2*x) + cosh(x))*s
inh(x)^5 + 3*(5*cosh(x)^2 + (5*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^4 + 9*co
sh(x)^4 + 4*(5*cosh(x)^3 + (5*cosh(x)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*s
inh(x)^3 + 3*(5*cosh(x)^4 + 18*cosh(x)^2 + (5*cosh(x)^4 + 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^6 + 9*cosh(x)^4 - 9*cosh(x
)^2 - 1)*e^(2*x) + 6*(cosh(x)^5 + 6*cosh(x)^3 + (cosh(x)^5 + 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

giac [A] time = 0.13, size = 29, normalized size = 0.81

$$-\frac{\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}}{24\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a^(3/2)
```

maple [B] time = 0.19, size = 130, normalized size = 3.61

$$\frac{e^{4x}}{24a\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}}+\frac{3\,e^{2x}}{8a\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}}-\frac{3}{8\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\,\left(1+e^{2x}\right)a}-\frac{e^{-2x}}{24a\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(3/2),x)
```

```
[Out] 1/24/a*exp(4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+3/8/a*exp(2*
x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)-3/8/(a*exp(2*x)/(1+exp(2*
x))^2)^(1/2)/(1+exp(2*x))/a-1/24/a*exp(-2*x)/(1+exp(2*x))/(a*exp(2*x)/(1+ex
p(2*x))^2)^(1/2)
```

maxima [A] time = 0.44, size = 35, normalized size = 0.97

$$\frac{e^{(3\,x)}}{24\,a^{\frac{3}{2}}}-\frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}}-\frac{e^{(-3\,x)}}{24\,a^{\frac{3}{2}}}+\frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*x)/a^(3/2) - 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^x
/a^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^{2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a/cosh(x)^2)^(3/2), x)
```

sympy [A] time = 1.25, size = 37, normalized size = 1.03

$$-\frac{2\tanh^{3}(x)}{3a^{\frac{3}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{3}{2}}}+\frac{\tanh(x)}{a^{\frac{3}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(3/2),x)
```

```
[Out] -2*tanh(x)**3/(3*a**(3/2)*(sech(x)**2)**(3/2)) + tanh(x)/(a**(3/2)*(sech(x)
**2)**(3/2))
```

3.37 $\int \frac{1}{\left(a\mathbf{sech}^2(x)\right)^{5/2}} dx$

Optimal. Leaf size=55

$$\frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}+\frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}}+\frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}}$$

```
[Out] 1/5*tanh(x)/(a*sech(x)^2)^(5/2)+4/15*tanh(x)/a/(a*sech(x)^2)^(3/2)+8/15*tan
h(x)/a^2/(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}+\frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}}+\frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-5/2),x]
```

```
[Out] Tanh[x]/(5*(a*Sech[x]^2)^(5/2)) + (4*Tanh[x])/(15*a*(a*Sech[x]^2)^(3/2)) +
(8*Tanh[x])/(15*a^2*Sqrt[a*Sech[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{5/2}}\,dx &= a\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{7/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{5\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{5\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{8\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx, x, \tanh(x)\right)}{15a}\\
&= \frac{\tanh(x)}{5\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{8\tanh(x)}{15a^2\sqrt{a\operatorname{sech}^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.65

$$
\frac{(150\sinh(x)+25\sinh(3x)+3\sinh(5x))\cosh(x)\sqrt{a\operatorname{sech}^2(x)}}{240a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^2]*(150*Sinh[x] + 25*Sinh[3*x] + 3*Sinh[5*x]))/(240
*a^3)
```

fricas [B] time = 0.58, size = 580, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) + 1)*sinh(x)^10 + 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) + co
sh(x))*sinh(x)^9 + 5*(27*cosh(x)^2 + (27*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x
)^8 + 25*cosh(x)^8 + 40*(9*cosh(x)^3 + (9*cosh(x)^3 + 5*cosh(x))*e^(2*x) +
5*cosh(x))*sinh(x)^7 + 10*(63*cosh(x)^4 + 70*cosh(x)^2 + (63*cosh(x)^4 + 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 + 150*cosh(x)^6 + 4*(189*cosh(x)^5
 + 350*cosh(x)^3 + (189*cosh(x)^5 + 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 + 10*(63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 +
 (63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 - 15)*e^(2*x) - 15)*sinh(x)^
```

```
4 - 150*cosh(x)^4 + 40*(9*cosh(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 + (9*cosh
(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sin
h(x)^3 + 5*(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 +
(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 - 25*cosh(x)^2 + (3*cosh(x)^10 + 25*cosh(x)^8 + 150*cosh(x)^
6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 3)*e^(2*x) + 10*(3*cosh(x)^9 + 20*cosh(x
)^7 + 90*cosh(x)^5 - 60*cosh(x)^3 + (3*cosh(x)^9 + 20*cosh(x)^7 + 90*cosh(x
)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
^(4*x) + 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

giac [A] time = 0.12, size = 41, normalized size = 0.75

$$-\frac{\left(150\,e^{(4\,x)}+25\,e^{(2\,x)}+3\right)e^{(-5\,x)}-3\,e^{(5\,x)}-25\,e^{(3\,x)}-150\,e^{x}}{480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*x) + 25*e^(2*x) + 3)*e^(-5*x) - 3*e^(5*x) - 25*e^(3*x) -
150*e^x)/a^(5/2)
```

maple [B] time = 0.19, size = 196, normalized size = 3.56

$$\frac{e^{6x}}{160a^{2}\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}}+\frac{5\,e^{4x}}{96a^{2}\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}}+\frac{5\,e^{2x}}{16a^{2}\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}}-\frac{5}{16\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^{2}}}\left(1+e^{2x}\right)a^{2}}-\frac{}{96a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(5/2),x)
```

```
[Out] 1/160/a^2*exp(6*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+5/96/a^2*
exp(4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+5/16/a^2*exp(2*x)/(
1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)-5/16/(a*exp(2*x)/(1+exp(2*x))
^2)^(1/2)/(1+exp(2*x))/a^2-5/96/a^2*exp(-2*x)/(1+exp(2*x))/(a*exp(2*x)/(1+e
xp(2*x))^2)^(1/2)-1/160/a^2*exp(-4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))
^2)^(1/2)
```

maxima [A] time = 0.42, size = 53, normalized size = 0.96

$$\frac{e^{(5\,x)}}{160\,a^{\frac{5}{2}}}+\frac{5\,e^{(3\,x)}}{96\,a^{\frac{5}{2}}}-\frac{5\,e^{(-x)}}{16\,a^{\frac{5}{2}}}-\frac{5\,e^{(-3\,x)}}{96\,a^{\frac{5}{2}}}-\frac{e^{(-5\,x)}}{160\,a^{\frac{5}{2}}}+\frac{5\,e^{x}}{16\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) - 5/96*e
^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) + 5/16*e^x/a^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a/cosh(x)^2)^(5/2), x)
```

sympy [A] time = 10.17, size = 60, normalized size = 1.09

$$\frac{8\tanh^5(x)}{15a^{\frac{5}{2}}\left(\operatorname{sech}^2(x)\right)^{\frac{5}{2}}} - \frac{4\tanh^3(x)}{3a^{\frac{5}{2}}\left(\operatorname{sech}^2(x)\right)^{\frac{5}{2}}} + \frac{\tanh(x)}{a^{\frac{5}{2}}\left(\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(5/2),x)
```

```
[Out] 8*tanh(x)**5/(15*a**(5/2)*(sech(x)**2)**(5/2)) - 4*tanh(x)**3/(3*a**(5/2)*(
sech(x)**2)**(5/2)) + tanh(x)/(a**(5/2)*(sech(x)**2)**(5/2))
```

3.38 $\int \frac{1}{\left(a\,\mathrm{sech}^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{16\tanh(x)}{35a^3\sqrt{a\,\mathrm{sech}^2(x)}}+\frac{8\tanh(x)}{35a^2\left(a\,\mathrm{sech}^2(x)\right)^{3/2}}+\frac{6\tanh(x)}{35a\left(a\,\mathrm{sech}^2(x)\right)^{5/2}}+\frac{\tanh(x)}{7\left(a\,\mathrm{sech}^2(x)\right)^{7/2}}$$

```
[Out] 1/7*tanh(x)/(a*sech(x)^2)^(7/2)+6/35*tanh(x)/a/(a*sech(x)^2)^(5/2)+8/35*tan
h(x)/a^2/(a*sech(x)^2)^(3/2)+16/35*tanh(x)/a^3/(a*sech(x)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4122, 192, 191}

$$\frac{16\tanh(x)}{35a^3\sqrt{a\,\mathrm{sech}^2(x)}}+\frac{8\tanh(x)}{35a^2\left(a\,\mathrm{sech}^2(x)\right)^{3/2}}+\frac{6\tanh(x)}{35a\left(a\,\mathrm{sech}^2(x)\right)^{5/2}}+\frac{\tanh(x)}{7\left(a\,\mathrm{sech}^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] Tanh[x]/(7*(a*Sech[x]^2)^(7/2)) + (6*Tanh[x])/(35*a*(a*Sech[x]^2)^(5/2)) +
(8*Tanh[x])/(35*a^2*(a*Sech[x]^2)^(3/2)) + (16*Tanh[x])/(35*a^3*Sqrt[a*Sech
[x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = a\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{9/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{7/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{24\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{5/2}}\,dx, x, \tanh(x)\right)}{35a}$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{16\,\text{Subst}\left(\int \frac{1}{\left(a-ax^2\right)^{3/2}}\,dx\right.}{35a^2}$$
$$= \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{16\tanh(x)}{35a^3\sqrt{a\operatorname{sech}^2(x)}}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.57

$$\frac{(1225\sinh(x)+245\sinh(3x)+49\sinh(5x)+5\sinh(7x))\cosh(x)\sqrt{a\operatorname{sech}^2(x)}}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] (Cosh[x]*Sqrt[a*Sech[x]^2]*(1225*Sinh[x] + 245*Sinh[3*x] + 49*Sinh[5*x] + 5
*Sinh[7*x]))/(2240*a^4)
```

fricas [B] time = 0.74, size = 970, normalized size = 13.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) + 1)*sinh(x)^14 + 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) + c
osh(x))*sinh(x)^13 + 7*(65*cosh(x)^2 + (65*cosh(x)^2 + 7)*e^(2*x) + 7)*sinh
```

```
(x)^12 + 49*cosh(x)^12 + 28*(65*cosh(x)^3 + (65*cosh(x)^3 + 21*cosh(x))*e^(
2*x) + 21*cosh(x))*sinh(x)^11 + 7*(715*cosh(x)^4 + 462*cosh(x)^2 + (715*cos
h(x)^4 + 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 + 245*cosh(x)^10 + 70
*(143*cosh(x)^5 + 154*cosh(x)^3 + (143*cosh(x)^5 + 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 + 35*(429*cosh(x)^6 + 693*cosh(x)^4 + 3
15*cosh(x)^2 + (429*cosh(x)^6 + 693*cosh(x)^4 + 315*cosh(x)^2 + 35)*e^(2*x)
 + 35)*sinh(x)^8 + 1225*cosh(x)^8 + 8*(2145*cosh(x)^7 + 4851*cosh(x)^5 + 36
75*cosh(x)^3 + (2145*cosh(x)^7 + 4851*cosh(x)^5 + 3675*cosh(x)^3 + 1225*cos
h(x))*e^(2*x) + 1225*cosh(x))*sinh(x)^7 + 7*(2145*cosh(x)^8 + 6468*cosh(x)^
6 + 7350*cosh(x)^4 + 4900*cosh(x)^2 + (2145*cosh(x)^8 + 6468*cosh(x)^6 + 73
50*cosh(x)^4 + 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 - 1225*cosh(x
)^6 + 14*(715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3
+ (715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 + 35*(143*cosh(x)^10 + 693*cosh(x)
^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 + (143*cosh(x)^10 + 69
3*cosh(x)^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 - 7)*e^(2*x)
- 7)*sinh(x)^4 - 245*cosh(x)^4 + 140*(13*cosh(x)^11 + 77*cosh(x)^9 + 210*co
sh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 + (13*cosh(x)^11 + 77*cosh(x)^9 + 2
10*cosh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 - 7*cosh(x))*e^(2*x) - 7*cosh(
x))*sinh(x)^3 + 7*(65*cosh(x)^12 + 462*cosh(x)^10 + 1575*cosh(x)^8 + 4900*c
osh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 + (65*cosh(x)^12 + 462*cosh(x)^10
 + 1575*cosh(x)^8 + 4900*cosh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 - 49*cosh(x)^2 + (5*cosh(x)^14 + 49*cosh(x)^12 + 245*c
osh(x)^10 + 1225*cosh(x)^8 - 1225*cosh(x)^6 - 245*cosh(x)^4 - 49*cosh(x)^2
- 5)*e^(2*x) + 14*(5*cosh(x)^13 + 42*cosh(x)^11 + 175*cosh(x)^9 + 700*cosh(
x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 + (5*cosh(x)^13 + 42*cosh(x)^11 + 175*c
osh(x)^9 + 700*cosh(x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

giac [A] time = 0.13, size = 53, normalized size = 0.72

$$-\frac{\left(1225\,e^{(6\,x)} + 245\,e^{(4\,x)} + 49\,e^{(2\,x)} + 5\right)e^{(-7\,x)} - 5\,e^{(7\,x)} - 49\,e^{(5\,x)} - 245\,e^{(3\,x)} - 1225\,e^{x}}{4480\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*x) + 245*e^(4*x) + 49*e^(2*x) + 5)*e^(-7*x) - 5*e^(7*x)
 - 49*e^(5*x) - 245*e^(3*x) - 1225*e^x)/a^(7/2)
```

maple [B] time = 0.20, size = 262, normalized size = 3.54

$$\frac{e^{8x}}{896a^3\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}}+\frac{7\,e^{6x}}{640a^3\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}}+\frac{7\,e^{4x}}{128a^3\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}}+\frac{35\,e^{2x}}{128a^3\left(1+e^{2x}\right)\sqrt{\frac{a\,e^{2x}}{\left(1+e^{2x}\right)^2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^2)^(7/2),x)
```

```
[Out] 1/896/a^3*exp(8*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+7/640/a^3
*exp(6*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+7/128/a^3*exp(4*x)
/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)+35/128/a^3*exp(2*x)/(1+exp(
2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)-35/128/(a*exp(2*x)/(1+exp(2*x))^2)^
(1/2)/(1+exp(2*x))/a^3-7/128/a^3*exp(-2*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(
2*x))^2)^(1/2)-7/640/a^3*exp(-4*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)
^(1/2)-1/896/a^3*exp(-6*x)/(1+exp(2*x))/(a*exp(2*x)/(1+exp(2*x))^2)^(1/2)
```

maxima [A] time = 0.43, size = 71, normalized size = 0.96

$$\frac{e^{(7\,x)}}{896\,a^{\frac{7}{2}}}+\frac{7\,e^{(5\,x)}}{640\,a^{\frac{7}{2}}}+\frac{7\,e^{(3\,x)}}{128\,a^{\frac{7}{2}}}-\frac{35\,e^{(-x)}}{128\,a^{\frac{7}{2}}}-\frac{7\,e^{(-3\,x)}}{128\,a^{\frac{7}{2}}}-\frac{7\,e^{(-5\,x)}}{640\,a^{\frac{7}{2}}}-\frac{e^{(-7\,x)}}{896\,a^{\frac{7}{2}}}+\frac{35\,e^{x}}{128\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) + 7/128*e^(3*x)/a^(7/2) - 35/
128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) - 7/640*e^(-5*x)/a^(7/2) - 1/89
6*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^2)^(7/2),x)
```

```
[Out] int(1/(a/cosh(x)^2)^(7/2), x)
```

sympy [A] time = 136.54, size = 80, normalized size = 1.08

$$-\frac{16\tanh^{7}(x)}{35a^{\frac{7}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{7}{2}}}+\frac{8\tanh^{5}(x)}{5a^{\frac{7}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{7}{2}}}-\frac{2\tanh^{3}(x)}{a^{\frac{7}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{7}{2}}}+\frac{\tanh(x)}{a^{\frac{7}{2}}\left(\operatorname{sech}^{2}(x)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**2)**(7/2),x)
```

```
[Out] -16*tanh(x)**7/(35*a**(7/2)*(sech(x)**2)**(7/2)) + 8*tanh(x)**5/(5*a**(7/2)
*(sech(x)**2)**(7/2)) - 2*tanh(x)**3/(a**(7/2)*(sech(x)**2)**(7/2)) + tanh(
x)/(a**(7/2)*(sech(x)**2)**(7/2))
```

3.39 $\int \left(a\,\text{sech}^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=121

$$\frac{154}{585}a^2\tanh(x)\sqrt{a\,\text{sech}^3(x)}+\frac{2}{13}a^2\tanh(x)\text{sech}^4(x)\sqrt{a\,\text{sech}^3(x)}+\frac{22}{117}a^2\tanh(x)\text{sech}^2(x)\sqrt{a\,\text{sech}^3(x)}+\frac{154}{195}ia^2\,\text{c}$$

```
[Out] 154/195*I*a^2*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*s
inh(1/2*x),2^(1/2))*(a*sech(x)^3)^(1/2)+154/195*a^2*cosh(x)*sinh(x)*(a*sech
(x)^3)^(1/2)+154/585*a^2*(a*sech(x)^3)^(1/2)*tanh(x)+22/117*a^2*sech(x)^2*(
a*sech(x)^3)^(1/2)*tanh(x)+2/13*a^2*sech(x)^4*(a*sech(x)^3)^(1/2)*tanh(x)
```

Rubi [A] time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3768, 3771, 2639}

$$\frac{2}{13}a^2\tanh(x)\text{sech}^4(x)\sqrt{a\,\text{sech}^3(x)}+\frac{22}{117}a^2\tanh(x)\text{sech}^2(x)\sqrt{a\,\text{sech}^3(x)}+\frac{154}{585}a^2\tanh(x)\sqrt{a\,\text{sech}^3(x)}+\frac{154}{195}ia^2\,\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(5/2),x]
```

```
[Out] ((154*I)/195)*a^2*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (
154*a^2*Cosh[x]*Sqrt[a*Sech[x]^3]*Sinh[x])/195 + (154*a^2*Sqrt[a*Sech[x]^3]
*Tanh[x])/585 + (22*a^2*Sech[x]^2*Sqrt[a*Sech[x]^3]*Tanh[x])/117 + (2*a^2*S
ech[x]^4*Sqrt[a*Sech[x]^3]*Tanh[x])/13
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{15}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(11a^2\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{11}{2}}(x)\,dx}{13\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(77a^2\sqrt{a\text{sech}\right.}{11}$$

$$= \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sec}}$$

$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sec}}$$

$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sec}}$$

$$= \frac{154}{195}ia^2\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.52

$$\frac{2}{585}a\text{sech}(x)\left(a\text{sech}^3(x)\right)^{3/2}\left(45\tanh(x) + 231i\cosh^{\frac{11}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 231\sinh(x)\cosh^5(x) + 77\sinh(x)\cosh^3(x)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(5/2),x]
```

```
[Out] (2*a*Sech[x]*(a*Sech[x]^3)^(3/2)*((231*I)*Cosh[x]^(11/2)*EllipticE[(I/2)*x,
 2] + 55*Cosh[x]*Sinh[x] + 77*Cosh[x]^3*Sinh[x] + 231*Cosh[x]^5*Sinh[x] + 4
5*Tanh[x]))/585
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\,\text{sech}(x)^3}\,a^2\,\text{sech}(x)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3)*a^2*sech(x)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\text{sech}(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(a\text{sech}(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(5/2),x)
```

```
[Out] int((a*sech(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\text{sech}(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{a}{\cosh(x)^3}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^3)^(5/2),x)
```

```
[Out] int((a/cosh(x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{sech}^3(x)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(5/2), x)
```

3.40 $\int \left(a\,\textbf{sech}^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{10}{21}a\sinh(x)\sqrt{a\text{sech}^3(x)}+\frac{2}{7}a\tanh(x)\text{sech}(x)\sqrt{a\text{sech}^3(x)}-\frac{10}{21}ia\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)}$$

```
[Out] -10/21*I*a*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh
(1/2*x),2^(1/2))*(a*sech(x)^3)^(1/2)+10/21*a*sinh(x)*(a*sech(x)^3)^(1/2)+2/
7*a*sech(x)*(a*sech(x)^3)^(1/2)*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3768, 3771, 2641}

$$\frac{10}{21}a\sinh(x)\sqrt{a\text{sech}^3(x)}+\frac{2}{7}a\tanh(x)\text{sech}(x)\sqrt{a\text{sech}^3(x)}-\frac{10}{21}ia\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(3/2),x]
```

```
[Out] ((-10*I)/21)*a*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (10*
a*Sqrt[a*Sech[x]^3]*Sinh[x])/21 + (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*Tanh[x])/7
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{9}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{5}{2}}(x)\,dx}{7\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int \sqrt{\text{sech}(}}{21\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\,\tanh(x) + \frac{1}{21}\left(5a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x}\right.$$

$$= -\frac{10}{21}ia\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\,\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.68

$$\frac{2}{21}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\left(3\tanh(x) - 5i\cosh^{\frac{5}{2}}(x)F\left(\frac{ix}{2}\middle|2\right) + 5\sinh(x)\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(3/2),x]
```

```
[Out] (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2]
+ 5*Cosh[x]*Sinh[x] + 3*Tanh[x]))/21
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\,\text{sech}(x)^3}\,a\,\text{sech}(x)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sech(x)^3)*a*sech(x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \left(a\mathrm{sech}(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(3/2),x)
[Out] int((a*sech(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{a}{{\mathrm{cosh}\left(x\right)}^3}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^3)^(3/2),x)
[Out] int((a/cosh(x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{sech}^{3}(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(3/2), x)
```

3.41 $\int \sqrt{a\operatorname{sech}^3(x)}\,dx$

Optimal. Leaf size=46

$$2\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^3(x)}+2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{sech}^3(x)}$$

```
[Out] 2*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x)
,2^(1/2))*(a*sech(x)^3)^(1/2)+2*cosh(x)*sinh(x)*(a*sech(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3768, 3771, 2639}

$$2\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^3(x)}+2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{sech}^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^3],x]
```

```
[Out] (2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + 2*Cosh[x]*Sqr
t[a*Sech[x]^3]*Sinh[x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \frac{\sqrt{a\text{sech}^3(x)}\int \text{sech}^{\frac{3}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$
$$= 2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x) - \frac{\sqrt{a\text{sech}^3(x)}\int \frac{1}{\sqrt{\text{sech}(x)}}\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$
$$= 2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x) - \left(\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}\right)\int \sqrt{\cosh(x)}\,dx$$
$$= 2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + 2\cosh(x)\sqrt{a\text{sech}^3(x)}\,\sinh(x)$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.78

$$2\cosh(x)\sqrt{a\text{sech}^3(x)}\left(\sinh(x) + i\sqrt{\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^3],x]
```

```
[Out] 2*Cosh[x]*Sqrt[a*Sech[x]^3]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x
])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\,\text{sech}(x)^3}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sech(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sech(x)^3), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a\mathrm{sech}(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^3)^(1/2),x)
[Out] int((a*sech(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sech(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{a}{\cosh(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^3)^(1/2),x)
[Out] int((a/cosh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}^3\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**3), x)
```

3.42 $\int \frac{1}{\sqrt{a \operatorname{sech}^3(x)}} \, dx$

Optimal. Leaf size=48

$$\frac{2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}$$

```
[Out] -2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))/c
osh(x)^(3/2)/(a*sech(x)^3)^(1/2)+2/3*tanh(x)/(a*sech(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3769, 3771, 2641}

$$\frac{2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^3],x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*x, 2])/(Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) + (2*T
anh[x])/(3*Sqrt[a*Sech[x]^3])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\mathrm{sech}^3(x)}}\,dx &= \frac{\mathrm{sech}^{\frac{3}{2}}(x)\int \frac{1}{\mathrm{sech}^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\mathrm{sech}^3(x)}}\\ &= \frac{2\tanh(x)}{3\sqrt{a\mathrm{sech}^3(x)}}+\frac{\mathrm{sech}^{\frac{3}{2}}(x)\int \sqrt{\mathrm{sech}(x)}\,dx}{3\sqrt{a\mathrm{sech}^3(x)}}\\ &= \frac{2\tanh(x)}{3\sqrt{a\mathrm{sech}^3(x)}}+\frac{\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\mathrm{sech}^3(x)}}\\ &= -\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\mathrm{sech}^3(x)}}+\frac{2\tanh(x)}{3\sqrt{a\mathrm{sech}^3(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.79

$$\frac{2\tanh(x)-\frac{2iF\left(\frac{ix}{2}\middle|2\right)}{\cosh^{\frac{3}{2}}(x)}}{3\sqrt{a\mathrm{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^3],x]
```

```
[Out] (((-2*I)*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + 2*Tanh[x])/(3*Sqrt[a*Sech[x
]^3])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a\,\mathrm{sech}(x)^3}}{a\,\mathrm{sech}(x)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sech(x)^3)/(a*sech(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\operatorname{sech}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\operatorname{sech}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(1/2),x)
[Out] int(1/(a*sech(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\operatorname{sech}(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{a}{\cosh(x)^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a/cosh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \operatorname{sech}^{3}(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(x)**3), x)
```

3.43 $\int \frac{1}{\left(a\,\mathrm{sech}^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{14\sinh(x)}{45a\sqrt{a\,\mathrm{sech}^3(x)}}-\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{2\sinh(x)\cosh^2(x)}{9a\sqrt{a\,\mathrm{sech}^3(x)}}$$

```
[Out] -14/15*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))
/a/cosh(x)^(3/2)/(a*sech(x)^3)^(1/2)+14/45*sinh(x)/a/(a*sech(x)^3)^(1/2)+2/
9*cosh(x)^2*sinh(x)/a/(a*sech(x)^3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3769, 3771, 2639}

$$\frac{14\sinh(x)}{45a\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{2\sinh(x)\cosh^2(x)}{9a\sqrt{a\,\mathrm{sech}^3(x)}}-\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\,\mathrm{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(-3/2),x]
```

```
[Out] (((-14*I)/15)*EllipticE[(I/2)*x, 2])/(a*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) +
(14*Sinh[x])/(45*a*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^2*Sinh[x])/(9*a*Sqrt[a*S
ech[x]^3])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx &= \frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\text{sech}^3(x)}}\\
&= \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}} + \frac{\left(7\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\text{sech}^{\frac{5}{2}}(x)}\,dx}{9a\sqrt{a\text{sech}^3(x)}}\\
&= \frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}} + \frac{\left(7\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\text{sech}(x)}}\,dx}{15a\sqrt{a\text{sech}^3(x)}}\\
&= \frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}} + \frac{7\int \sqrt{\cosh(x)}\,dx}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}\\
&= -\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} + \frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 47, normalized size = 0.61

$$\frac{33\sinh(x) + 5\sinh(3x) - \frac{84iE\left(\frac{ix}{2}\middle|2\right)}{\cosh^{\frac{3}{2}}(x)}}{90a\sqrt{a\text{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(-3/2),x]
```

```
[Out] (((-84*I)*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + 33*Sinh[x] + 5*Sinh[3*x])/
(90*a*Sqrt[a*Sech[x]^3])
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\,\text{sech}(x)^3}}{a^2\,\text{sech}(x)^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sech(x)^3)/(a^2*sech(x)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\text{sech}(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(-3/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\text{sech}(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(3/2),x)
[Out] int(1/(a*sech(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\text{sech}(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="maxima")
```

[Out] integrate((a*sech(x)^3)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a/cosh(x)^3)^(3/2),x)

[Out] int(1/(a/cosh(x)^3)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\mathrm{sech}^3\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sech(x)**3)**(3/2),x)

[Out] Integral((a*sech(x)**3)**(-3/2), x)

3.44 $\int \frac{1}{\left(a\,\mathrm{sech}^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{26\tanh(x)}{77a^2\sqrt{a\,\mathrm{sech}^3(x)}}-\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{2\sinh(x)\cosh^5(x)}{15a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{26\sinh(x)\cosh^3(x)}{165a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\,\mathrm{sech}^3(x)}}$$

```
[Out] -26/77*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))
/a^2/cosh(x)^(3/2)/(a*sech(x)^3)^(1/2)+78/385*cosh(x)*sinh(x)/a^2/(a*sech(x
)^3)^(1/2)+26/165*cosh(x)^3*sinh(x)/a^2/(a*sech(x)^3)^(1/2)+2/15*cosh(x)^5*
sinh(x)/a^2/(a*sech(x)^3)^(1/2)+26/77*tanh(x)/a^2/(a*sech(x)^3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4123, 3769, 3771, 2641}

$$\frac{26\tanh(x)}{77a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{2\sinh(x)\cosh^5(x)}{15a^2\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{26\sinh(x)\cosh^3(x)}{165a^2\sqrt{a\,\mathrm{sech}^3(x)}}-\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\,\mathrm{sech}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\,\mathrm{sech}^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^3)^(-5/2),x]
```

```
[Out] (((-26*I)/77)*EllipticF[(I/2)*x, 2])/(a^2*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3])
+ (78*Cosh[x]*Sinh[x])/(385*a^2*Sqrt[a*Sech[x]^3]) + (26*Cosh[x]^3*Sinh[x])
/(165*a^2*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^5*Sinh[x])/(15*a^2*Sqrt[a*Sech[x]
^3]) + (26*Tanh[x])/(77*a^2*Sqrt[a*Sech[x]^3])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \frac{\text{sech}^{\frac{3}{2}}(x)\int \frac{1}{\text{sech}^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{\left(13\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\text{sech}^{\frac{11}{2}}(x)}\,dx}{15a^2\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{\left(39\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\text{sech}^{\frac{7}{2}}(x)}\,dx}{55a^2\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{\left(39\text{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\text{sech}^{\frac{3}{2}}(}}{77a^2\sqrt{a\text{sech}^3(x)}}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\tanh(x)}{77a^2\sqrt{a\text{sech}^3(x)}} +$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\tanh(x)}{77a^2\sqrt{a\text{sech}^3(x)}} + $$

$$= -\frac{26iF\left(\frac{ix}{2}\middle|2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\,\text{s}}{15a^2\sqrt{a\text{sec}}}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.52

$$\frac{\cosh(x)\sqrt{a\text{sech}^3(x)}\left(19122\sinh(2x)+4406\sinh(4x)+826\sinh(6x)+77\sinh(8x)-24960i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right.\right.}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^3)^(-5/2),x]
[Out] (Cosh[x]*Sqrt[a*Sech[x]^3]*((-24960*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]
+ 19122*Sinh[2*x] + 4406*Sinh[4*x] + 826*Sinh[6*x] + 77*Sinh[8*x]))/(73920*
a^3)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\,\text{sech}(x)^3}}{a^3\,\text{sech}(x)^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sech(x)^3)/(a^3*sech(x)^9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\text{sech}(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\text{sech}(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^3)^(5/2),x)
[Out] int(1/(a*sech(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\mathrm{sech}(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^3)^(5/2),x)
[Out] int(1/(a/cosh(x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\mathrm{sech}^3\,(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**3)**(5/2),x)
[Out] Integral((a*sech(x)**3)**(-5/2), x)
```

3.45 $\int \left(a\text{sech}^4(x)\right)^{7/2} \, dx$

Optimal. Leaf size=163

$$a^3 \sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}+\frac{1}{13}a^3\sinh^2(x)\tanh^{11}(x)\sqrt{a\text{sech}^4(x)}-\frac{6}{11}a^3\sinh^2(x)\tanh^9(x)\sqrt{a\text{sech}^4(x)}+\frac{5}{3}a$$

```
[Out] a^3*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-2*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)
*tanh(x)+3*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3-20/7*a^3*sinh(x)^2*(
a*sech(x)^4)^(1/2)*tanh(x)^5+5/3*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^
7-6/11*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^9+1/13*a^3*sinh(x)^2*(a*se
ch(x)^4)^(1/2)*tanh(x)^11
```

Rubi [A] time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4123, 3767}

$$a^3 \sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}+\frac{1}{13}a^3\sinh^2(x)\tanh^{11}(x)\sqrt{a\text{sech}^4(x)}-\frac{6}{11}a^3\sinh^2(x)\tanh^9(x)\sqrt{a\text{sech}^4(x)}+\frac{5}{3}a$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] a^3*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - 2*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*T
anh[x] + 3*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3 - (20*a^3*Sqrt[a*Sech[
x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (5*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^7
)/3 - (6*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^9)/11 + (a^3*Sqrt[a*Sech[x
]^4]*Sinh[x]^2*Tanh[x]^11)/13
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx = \left(a^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^{14}(x)\,dx$$
$$= \left(ia^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)dx, x,\right.$$
$$= a^3\cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x) - 2a^3\sqrt{a\text{sech}^4(x)}\,\sinh^2(x)\tanh(x) + 3a^3\sqrt{a\text{sech}^4(x)}\,\sin$$

Mathematica [A] time = 0.17, size = 54, normalized size = 0.33

$$\frac{\sinh(x)\cosh(x)(2380\cosh(2x)+1093\cosh(4x)+378\cosh(6x)+92\cosh(8x)+14\cosh(10x)+\cosh(12x)+20}{6006}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] (Cosh[x]*(2048 + 2380*Cosh[2*x] + 1093*Cosh[4*x] + 378*Cosh[6*x] + 92*Cosh[
8*x] + 14*Cosh[10*x] + Cosh[12*x])*(a*Sech[x]^4)^(7/2)*Sinh[x])/6006
```

fricas [B] time = 0.53, size = 2804, normalized size = 17.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) +
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) + 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 + a^3 + (88*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(88*a^3*cosh(x)^2 + a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (88*a^3*co
sh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 + 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 + 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(11
88*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 + 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 + 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) + 2*(1188*a^3*cosh(x)^5 + 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3 + (5544*a^3*c
osh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(5544*a^
3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^
```

6 + 572*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^
3*cosh(x))*e^(4*x) + 2*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
+ 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) + 2*(32670*a^3*cosh(x)^8 + 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(3630*a^3
*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 + 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3
 + (8712*a^3*cosh(x)^10 + 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(8712*a^3*cosh(x)^10 + 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 + a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) + 2*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x) + (792*a^3*cosh(x)^11 + 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a
^3*cosh(x))*e^(4*x) + 2*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 + 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 + 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 +
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 + 483*co
sh(x)^4 + 63*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 + 483*
cosh(x)^5 + 105*cosh(x)^3 + 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 + 12236*cosh(x)^6 + 3990*cosh(x)^4 + 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 + 15732*cosh(x)^7 + 7182*cosh(x)^5 + 1140*cosh
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 + 66861*cosh
(x)^8 + 40698*cosh(x)^6 + 9690*cosh(x)^4 + 765*cosh(x)^2 + 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 + 81719*cosh(x)^9 + 63954*cosh(x)^7 + 21318*c
osh(x)^5 + 2805*cosh(x)^3 + 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 + 490314*cosh(x)^10 + 479655*cosh(x)^8 + 213180*cosh(x)^6 + 42075*c
osh(x)^4 + 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
+ 4056234*cosh(x)^11 + 4849845*cosh(x)^9 + 2771340*cosh(x)^7 + 765765*cosh(
x)^5 + 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh
(x)^14 + 676039*cosh(x)^12 + 969969*cosh(x)^10 + 692835*cosh(x)^8 + 255255*
cosh(x)^6 + 45045*cosh(x)^4 + 3003*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^12 + 208

```
*(37145*cosh(x)^15 + 156009*cosh(x)^13 + 264537*cosh(x)^11 + 230945*cosh(x)
^9 + 109395*cosh(x)^7 + 27027*cosh(x)^5 + 3003*cosh(x)^3 + 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 + 178296*cosh(x)^14 + 352716*cosh(x)
^12 + 369512*cosh(x)^10 + 218790*cosh(x)^8 + 72072*cosh(x)^6 + 12012*cosh(x
)^4 + 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 + 59432
*cosh(x)^15 + 135660*cosh(x)^13 + 167960*cosh(x)^11 + 121550*cosh(x)^9 + 51
480*cosh(x)^7 + 12012*cosh(x)^5 + 1320*cosh(x)^3 + 45*cosh(x))*e^(2*x)*sinh
(x)^9 + 143*(10925*cosh(x)^18 + 66861*cosh(x)^16 + 174420*cosh(x)^14 + 2519
40*cosh(x)^12 + 218790*cosh(x)^10 + 115830*cosh(x)^8 + 36036*cosh(x)^6 + 59
40*cosh(x)^4 + 405*cosh(x)^2 + 5)*e^(2*x)*sinh(x)^8 + 1144*(575*cosh(x)^19
+ 3933*cosh(x)^17 + 11628*cosh(x)^15 + 19380*cosh(x)^13 + 19890*cosh(x)^11
+ 12870*cosh(x)^9 + 5148*cosh(x)^7 + 1188*cosh(x)^5 + 135*cosh(x)^3 + 5*cos
h(x))*e^(2*x)*sinh(x)^7 + 286*(805*cosh(x)^20 + 6118*cosh(x)^18 + 20349*cos
h(x)^16 + 38760*cosh(x)^14 + 46410*cosh(x)^12 + 36036*cosh(x)^10 + 18018*co
sh(x)^8 + 5544*cosh(x)^6 + 945*cosh(x)^4 + 70*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 572*(115*cosh(x)^21 + 966*cosh(x)^19 + 3591*cosh(x)^17 + 7752*cosh(x)
^15 + 10710*cosh(x)^13 + 9828*cosh(x)^11 + 6006*cosh(x)^9 + 2376*cosh(x)^7
+ 567*cosh(x)^5 + 70*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^5 + 26*(575*cos
h(x)^22 + 5313*cosh(x)^20 + 21945*cosh(x)^18 + 53295*cosh(x)^16 + 84150*cos
h(x)^14 + 90090*cosh(x)^12 + 66066*cosh(x)^10 + 32670*cosh(x)^8 + 10395*cos
h(x)^6 + 1925*cosh(x)^4 + 165*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^4 + 104*(25*co
sh(x)^23 + 253*cosh(x)^21 + 1155*cosh(x)^19 + 3135*cosh(x)^17 + 5610*cosh(x
)^15 + 6930*cosh(x)^13 + 6006*cosh(x)^11 + 3630*cosh(x)^9 + 1485*cosh(x)^7
+ 385*cosh(x)^5 + 55*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 13*(25*cosh
(x)^24 + 276*cosh(x)^22 + 1386*cosh(x)^20 + 4180*cosh(x)^18 + 8415*cosh(x)^
16 + 11880*cosh(x)^14 + 12012*cosh(x)^12 + 8712*cosh(x)^10 + 4455*cosh(x)^8
 + 1540*cosh(x)^6 + 330*cosh(x)^4 + 36*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 2
6*(cosh(x)^25 + 12*cosh(x)^23 + 66*cosh(x)^21 + 220*cosh(x)^19 + 495*cosh(x
)^17 + 792*cosh(x)^15 + 924*cosh(x)^13 + 792*cosh(x)^11 + 495*cosh(x)^9 + 2
20*cosh(x)^7 + 66*cosh(x)^5 + 12*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (co
sh(x)^26 + 13*cosh(x)^24 + 78*cosh(x)^22 + 286*cosh(x)^20 + 715*cosh(x)^18
+ 1287*cosh(x)^16 + 1716*cosh(x)^14 + 1716*cosh(x)^12 + 1287*cosh(x)^10 + 7
15*cosh(x)^8 + 286*cosh(x)^6 + 78*cosh(x)^4 + 13*cosh(x)^2 + 1)*e^(2*x))
```

giac [A] time = 0.12, size = 51, normalized size = 0.31

$$-\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)}+1287\,e^{(10\,x)}+715\,e^{(8\,x)}+286\,e^{(6\,x)}+78\,e^{(4\,x)}+13\,e^{(2\,x)}+1\right)}{3003\left(e^{(2\,x)}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) + 1287*e^(10*x) + 715*e^(8*x) + 286*e^(6*
x) + 78*e^(4*x) + 13*e^(2*x) + 1)/(e^(2*x) + 1)^13
```

maple [A] time = 0.26, size = 72, normalized size = 0.44

$$-\frac{2048 a^{3} e^{-2 x} \sqrt{\frac{a\,e^{4 x}}{\left(1+e^{2 x}\right)^{4}}}\, \left(1716\,e^{12 x}+1287\,e^{10 x}+715\,e^{8 x}+286\,e^{6 x}+78\,e^{4 x}+13\,e^{2 x}+1\right)}{3003 \left(1+e^{2 x}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(7/2),x)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(1+exp(2*x))^11*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*
(1716*exp(12*x)+1287*exp(10*x)+715*exp(8*x)+286*exp(6*x)+78*exp(4*x)+13*exp
(2*x)+1)
```

maxima [B] time = 0.46, size = 620, normalized size = 3.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] 2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e
^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x)
 + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x)
+ 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) +
 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(
-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-
26*x) + 1) + 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-
6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 12
87*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x)
+ e^(-26*x) + 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 2
86*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*
x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(
-24*x) + e^(-26*x) + 1) + 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) + 78*e^(-4*
x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e
^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) +
13*e^(-24*x) + e^(-26*x) + 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) + 78*
e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) +
1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22
*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) + 78*e
^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1
716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*
x) + 13*e^(-24*x) + e^(-26*x) + 1)
```

mupad [B] time = 1.45, size = 498, normalized size = 3.06

$$\frac{1536\,a^3\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{\left(\mathrm{e}^{2\,x}+1\right)^8\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}-\frac{2048\,a^3\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}{7\left(\mathrm{e}^{2\,x}+1\right)^7\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(7/2),x)
```

```
[Out] (1536*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*e
xp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6*x
))) - (2048*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) + 2*exp(4*x) +
 exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6
*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^9*(exp(2*x) + 2*e
xp(4*x) + exp(6*x))) + (4096*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(
2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^10*(exp(2*x
) + 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)
*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(11*(exp(2*x) + 1)^
11*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*a^3*(a/(exp(-x)/2 + exp(x)/2
)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x
) + 1)^12*(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a/(exp(-x)/2 + e
xp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(1
3*(exp(2*x) + 1)^13*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(7/2),x)
```

```
[Out] Timed out
```

3.46 $\int \left(a\,\mathbf{sech}^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=117

$$a^2\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{9}a^2\sinh^2(x)\tanh^7(x)\sqrt{a\,\text{sech}^4(x)}-\frac{4}{7}a^2\sinh^2(x)\tanh^5(x)\sqrt{a\,\text{sech}^4(x)}+\frac{6}{5}a^2\,\text{si}$$

```
[Out] a^2*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-4/3*a^2*sinh(x)^2*(a*sech(x)^4)^(1/
2)*tanh(x)+6/5*a^2*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3-4/7*a^2*sinh(x)^
2*(a*sech(x)^4)^(1/2)*tanh(x)^5+1/9*a^2*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(
x)^7
```

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4123, 3767}

$$a^2\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{9}a^2\sinh^2(x)\tanh^7(x)\sqrt{a\,\text{sech}^4(x)}-\frac{4}{7}a^2\sinh^2(x)\tanh^5(x)\sqrt{a\,\text{sech}^4(x)}+\frac{6}{5}a^2\,\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] a^2*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (4*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*
Tanh[x])/3 + (6*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5 - (4*a^2*Sqrt[
a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x]^7)/9
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\operatorname{sech}^4(x)\right)^{5/2} dx = \left(a^2\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\int \operatorname{sech}^{10}(x)\,dx$$
$$= \left(ia^2\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right) dx, x, -i\tanh(x)\right)$$
$$= a^2\cosh(x)\sqrt{a\operatorname{sech}^4(x)}\,\sinh(x) - \frac{4}{3}a^2\sqrt{a\operatorname{sech}^4(x)}\,\sinh^2(x)\tanh(x) + \frac{6}{5}a^2\sqrt{a\operatorname{sech}^4(x)}\,\sin$$

Mathematica [A] time = 0.10, size = 42, normalized size = 0.36

$$\frac{1}{315}\sinh(x)\cosh(x)(130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x)+128)\left(a\operatorname{sech}^4(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] (Cosh[x]*(128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*(a
*Sech[x]^4)^(5/2)*Sinh[x])/315
```

fricas [B] time = 0.47, size = 1475, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) + 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 + 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) + 2*a^2*cosh(x)*e^(2*x)
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 + a^2 + (42*a^2*cosh(x)^2
+ a^2)*e^(4*x) + 2*(42*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 + a^2*cosh(x) + (14*a^2*cosh(x)^3 + a^2*cosh
(x))*e^(4*x) + 2*(14*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 + 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) + 2*(245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 + 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) + 2*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 +
a^2 + (392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^(4
*x) + 2*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 + 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(126*a^2*cosh(x)^8 + 84*a^2*cos
```

```
h(x)^6 + 36*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x) + (56*a^2*cosh(x)
^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(56*a^2*
cosh(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 + 1)*e^(2
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 + 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 + 10*c
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 + 195*cosh(x)^4
+ 39*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 + 273*cosh(x)^5
 + 91*cosh(x)^3 + 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 + 4004
*cosh(x)^6 + 2002*cosh(x)^4 + 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 + 25740*cosh(x)^7 + 18018*cosh(x)^5 + 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 + 6435*cosh(x)^8 + 6006*cosh
(x)^6 + 2310*cosh(x)^4 + 315*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 + 715*cosh(x)^9 + 858*cosh(x)^7 + 462*cosh(x)^5 + 105*cosh(x)^3 +
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 + 858*cosh(x)^10 + 1287*c
osh(x)^8 + 924*cosh(x)^6 + 315*cosh(x)^4 + 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 + 78*cosh(x)^11 + 143*cosh(x)^9 + 132*cosh(x)^7 +
63*cosh(x)^5 + 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 + 455*cosh(x)^12 + 1001*cosh(x)^10 + 1155*cosh(x)^8 + 735*cosh(x)^6 + 245
*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 + 105*
cosh(x)^13 + 273*cosh(x)^11 + 385*cosh(x)^9 + 315*cosh(x)^7 + 147*cosh(x)^5
 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 + 120*cos
h(x)^14 + 364*cosh(x)^12 + 616*cosh(x)^10 + 630*cosh(x)^8 + 392*cosh(x)^6 +
 140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 + 8*c
osh(x)^15 + 28*cosh(x)^13 + 56*cosh(x)^11 + 70*cosh(x)^9 + 56*cosh(x)^7 + 2
8*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 + 9*cosh
(x)^16 + 36*cosh(x)^14 + 84*cosh(x)^12 + 126*cosh(x)^10 + 126*cosh(x)^8 + 8
4*cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))
```

giac [A] time = 0.13, size = 39, normalized size = 0.33

$$-\frac{256\, a^{\frac{5}{2}} \left(126\, e^{(8\,x)} + 84\, e^{(6\,x)} + 36\, e^{(4\,x)} + 9\, e^{(2\,x)} + 1\right)}{315\, \left(e^{(2\,x)} + 1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(e
^(2*x) + 1)^9
```

maple [A] time = 0.20, size = 60, normalized size = 0.51

$$-\frac{256a^{2}e^{-2x}\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(126\,e^{8x}+84\,e^{6x}+36\,e^{4x}+9\,e^{2x}+1\right)}{315\left(1+e^{2x}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(5/2),x)
```

```
[Out] -256/315*a^2*exp(-2*x)/(1+exp(2*x))^7*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*(12
6*exp(8*x)+84*exp(6*x)+36*exp(4*x)+9*exp(2*x)+1)
```

maxima [B] time = 0.43, size = 322, normalized size = 2.75

$$\frac{256\,a^{\frac{5}{2}}e^{(-2\,x)}}{35\left(9\,e^{(-2\,x)}+36\,e^{(-4\,x)}+84\,e^{(-6\,x)}+126\,e^{(-8\,x)}+126\,e^{(-10\,x)}+84\,e^{(-12\,x)}+36\,e^{(-14\,x)}+9\,e^{(-16\,x)}+e^{(-18\,x)}+1\right)}+\frac{}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) + 126*e^(-8
*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) + e^(-18*x)
 + 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) +
126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) +
e^(-18*x) + 1) + 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^
(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(
-16*x) + e^(-18*x) + 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1) + 256/315*a^(5/2)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1)
```

mupad [B] time = 1.37, size = 356, normalized size = 3.04

$$\frac{256\,a^{2}\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^{4}}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{3\left(e^{2\,x}+1\right)^{6}\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}-\frac{128\,a^{2}\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^{4}}}\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{5\left(e^{2\,x}+1\right)^{5}\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}-\frac{768}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(5/2),x)
```

```
[Out] (256*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*ex
p(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^6*(exp(2*x) + 2*exp(4*x) + exp(6*
x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*x) +
 exp(6*x))) - (768*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*e
xp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) + 2*exp
(4*x) + exp(6*x))) + (64*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x)
 + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(2*x) + 2
*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp
(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) + 1)^9*(exp(2
*x) + 2*exp(4*x) + exp(6*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\operatorname{sech}^{4}(x)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(5/2), x)
```

3.47 $\int \left(a\,\text{sech}^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$a\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{5}a\sinh^2(x)\tanh^3(x)\sqrt{a\,\text{sech}^4(x)}-\frac{2}{3}a\sinh^2(x)\tanh(x)\sqrt{a\,\text{sech}^4(x)}$$

```
[Out] a*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-2/3*a*sinh(x)^2*(a*sech(x)^4)^(1/2)*t
anh(x)+1/5*a*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4123, 3767}

$$a\sinh(x)\cosh(x)\sqrt{a\,\text{sech}^4(x)}+\frac{1}{5}a\sinh^2(x)\tanh^3(x)\sqrt{a\,\text{sech}^4(x)}-\frac{2}{3}a\sinh^2(x)\tanh(x)\sqrt{a\,\text{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(3/2),x]
```

```
[Out] a*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (2*a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x])/3 + (a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\text{sech}^4(x)\right)^{3/2} dx = \left(a\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^6(x)\,dx$$
$$= \left(ia\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -i\tanh(x)\right)$$
$$= a\cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x) - \frac{2}{3}a\sqrt{a\text{sech}^4(x)}\,\sinh^2(x)\tanh(x) + \frac{1}{5}a\sqrt{a\text{sech}^4(x)}\,\text{sinh}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 0.49

$$\frac{1}{15}\sinh(x)\cosh(x)(6\cosh(2x)+\cosh(4x)+8)\left(a\text{sech}^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(3/2),x]
[Out] (Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*(a*Sech[x]^4)^(3/2)*Sinh[x])/15
```

fricas [B] time = 0.43, size = 516, normalized size = 8.46

$$-\frac{16\left(10\,a\cosh(x)^4+10\left(ae\right.\right.}{15\left(10\,\cosh(x)e^{(2x)}\sinh(x)^9+e^{(2x)}\sinh(x)^{10}+5\left(9\,\cosh(x)^2+1\right)e^{(2x)}\sinh(x)^8+40\left(3\,\cosh(x)^3+\cosh(x)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="fricas")
[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 + 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 + a)*e^(4*x) + 2*(12*a*cosh(x)^2 +
 a)*e^(2*x) + a)*sinh(x)^2 + (10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(4*x) +
 2*(10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 + a*cos
h(x) + (4*a*cosh(x)^3 + a*cosh(x))*e^(4*x) + 2*(4*a*cosh(x)^3 + a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 + 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 + 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 + 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cos
h(x)^4 + 15*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^
5 + 5*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^
6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 + 4*
```

```
cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
^10 + 5*cosh(x)^8 + 10*cosh(x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))
```

giac [A] time = 0.11, size = 27, normalized size = 0.44

$$-\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\,\left(e^{(2\,x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

maple [A] time = 0.20, size = 46, normalized size = 0.75

$$-\frac{16a\,\mathrm{e}^{-2x}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^{4}}}\,\left(10\,\mathrm{e}^{4x}+5\,\mathrm{e}^{2x}+1\right)}{15\left(1+\mathrm{e}^{2x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(3/2),x)
```

```
[Out] -16/15*a*exp(-2*x)/(1+exp(2*x))^3*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*(10*exp
(4*x)+5*exp(2*x)+1)
```

maxima [B] time = 0.44, size = 120, normalized size = 1.97

$$\frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}+\frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x)
+ e^(-10*x) + 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(
-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) + 16/15*a^(3/2)/(5*e^(-2*x) + 10*e^(-4*
x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

mupad [B] time = 1.34, size = 46, normalized size = 0.75

$$-\frac{4\,a\,\mathrm{e}^{-2\,x}\sqrt{\frac{a}{{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}}\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\,{\left(\mathrm{e}^{2\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(3/2),x)
```

```
[Out] -(4*a*exp(-2*x)*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*x
) + 1))/(15*(exp(2*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}^4\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(3/2), x)
```

3.48 $\int \sqrt{a \operatorname{sech}^4(x)}\, dx$

Optimal. Leaf size=15

$$\sinh(x)\cosh(x)\sqrt{a \operatorname{sech}^4(x)}$$

```
[Out] cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4123, 3767, 8}

$$\sinh(x)\cosh(x)\sqrt{a \operatorname{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sech[x]^4],x]
```

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\text{sech}^4(x)}\,dx &= \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^2(x)\,dx\\ &= \left(i\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}(\int 1\,dx, x, -i\tanh(x))\\ &= \cosh(x)\sqrt{a\text{sech}^4(x)}\,\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sech[x]^4],x]
```

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

fricas [B] time = 0.43, size = 81, normalized size = 5.40

$$-\frac{2\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 + 1)*e^(2*x))
```

giac [A] time = 0.11, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{a}}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)/(e^(2*x) + 1)
```

maple [B] time = 0.22, size = 29, normalized size = 1.93

$$-2\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^{4}}}\,\mathrm{e}^{-2x}\left(1+\mathrm{e}^{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sech(x)^4)^(1/2),x)
[Out] -2*(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))
```

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{2\sqrt{a}}{e^{(-2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)/(e^(-2*x) + 1)
```

mupad [B] time = 0.06, size = 71, normalized size = 4.73

$$-\frac{\sqrt{a}\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}}\,\left(2\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+2\,\mathrm{e}^{6\,x}+\frac{\mathrm{e}^{8\,x}}{2}+\frac{1}{2}\right)}{\left(\mathrm{e}^{2\,x}+1\right)\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cosh(x)^4)^(1/2),x)
[Out] -(a^(1/2)*(1/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(2*exp(2*x) + 3*exp(4*x) + 2*e
xp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) + 1)*(exp(2*x) + 2*exp(4*x) + exp(6
*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}^4\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sech(x)**4)**(1/2),x)
[Out] Integral(sqrt(a*sech(x)**4), x)
```

3.49 $\int \frac{1}{\sqrt{a\operatorname{sech}^4(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

```
[Out] 1/2*x*sech(x)^2/(a*sech(x)^4)^(1/2)+1/2*tanh(x)/(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4123, 2635, 8}

$$\frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + Tanh[x]/(2*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\text{sech}^2(x)\int \cosh^2(x)\,dx}{\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{\tanh(x)}{2\sqrt{a\text{sech}^4(x)}} + \frac{\text{sech}^2(x)\int 1\,dx}{2\sqrt{a\text{sech}^4(x)}}$$
$$= \frac{x\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}} + \frac{\tanh(x)}{2\sqrt{a\text{sech}^4(x)}}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.64

$$\frac{\tanh(x) + x\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Sech[x]^2 + Tanh[x])/(2*Sqrt[a*Sech[x]^4])
```

fricas [B] time = 0.44, size = 253, normalized size = 7.03

$$\frac{\left(\left(e^{(4\,x)} + 2\,e^{(2\,x)} + 1\right)\sinh(x)^4 + \cosh(x)^4 + 4\left(\cosh(x)e^{(4\,x)} + 2\,\cosh(x)e^{(2\,x)} + \cosh(x)\right)\sinh(x)^3 + 4\,x\cosh(x)^2 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 + 2*x)*e^(4*x) + 2*(3*cosh(x)^2 + 2*x)*e^(2*x) + 2*x)*sinh(x)^
2 + (cosh(x)^4 + 4*x*cosh(x)^2 - 1)*e^(4*x) + 2*(cosh(x)^4 + 4*x*cosh(x)^2
- 1)*e^(2*x) + 4*(cosh(x)^3 + 2*x*cosh(x) + (cosh(x)^3 + 2*x*cosh(x))*e^(4*
x) + 2*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*
e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

giac [A] time = 0.11, size = 28, normalized size = 0.78

$$-\frac{\left(2\,e^{(2\,x)} + 1\right)e^{(-2\,x)} - 4\,x - e^{(2\,x)}}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/sqrt(a)
```

maple [B] time = 0.23, size = 89, normalized size = 2.47

$$\frac{e^{2x}x}{2\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(1+e^{2x}\right)^{2}}+\frac{e^{4x}}{8\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}\,\left(1+e^{2x}\right)^{2}}-\frac{1}{8\left(1+e^{2x}\right)^{2}\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(1/2),x)
```

```
[Out] 1/2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x+1/8/(a*exp(
4*x)/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(4*x)-1/8/(1+exp(2*x))^2/(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)
```

maxima [A] time = 0.44, size = 30, normalized size = 0.83

$$-\frac{\left(\sqrt{a}\,e^{(-4\,x)}-\sqrt{a}\right)e^{(2\,x)}}{8\,a}+\frac{x}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x)/a + 1/2*x/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^4)^(1/2),x)
```

```
[Out] int(1/(a/cosh(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}^{4}\,(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(x)**4), x)
```

3.50 $\int \frac{1}{\left(a\mathbf{sech}^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{5x\mathrm{sech}^2(x)}{16a\sqrt{a\mathrm{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\mathrm{sech}^4(x)}}+\frac{\sinh(x)\cosh^3(x)}{6a\sqrt{a\mathrm{sech}^4(x)}}+\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\mathrm{sech}^4(x)}}$$

```
[Out] 5/16*x*sech(x)^2/a/(a*sech(x)^4)^(1/2)+5/24*cosh(x)*sinh(x)/a/(a*sech(x)^4)
^(1/2)+1/6*cosh(x)^3*sinh(x)/a/(a*sech(x)^4)^(1/2)+5/16*tanh(x)/a/(a*sech(x
)^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4123, 2635, 8}

$$\frac{5x\mathrm{sech}^2(x)}{16a\sqrt{a\mathrm{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\mathrm{sech}^4(x)}}+\frac{\sinh(x)\cosh^3(x)}{6a\sqrt{a\mathrm{sech}^4(x)}}+\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\mathrm{sech}^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(-3/2),x]
```

```
[Out] (5*x*Sech[x]^2)/(16*a*Sqrt[a*Sech[x]^4]) + (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a
*Sech[x]^4]) + (Cosh[x]^3*Sinh[x])/(6*a*Sqrt[a*Sech[x]^4]) + (5*Tanh[x])/(1
6*a*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
```

```
& !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx = \frac{\operatorname{sech}^2(x)\int \cosh^6(x)\,dx}{a\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(5\operatorname{sech}^2(x)\right)\int \cosh^4(x)\,dx}{6a\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(5\operatorname{sech}^2(x)\right)\int \cosh^2(x)\,dx}{8a\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(5\operatorname{sech}^2(x)\right)\int 1\,dx}{16a\sqrt{a\operatorname{sech}^4(x)}}$$
$$= \frac{5x\operatorname{sech}^2(x)}{16a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.44

$$\frac{(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x))\operatorname{sech}^6(x)}{192\left(a\operatorname{sech}^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(-3/2),x]
[Out] (Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Sech[x]
^4)^(3/2))
```

fricas [B] time = 0.45, size = 1141, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="fricas")
[Out] 1/384*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
```

```
x)^2 + 3)*e^(4*x) + 2*(22*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^10 + 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 + 9*cosh(x))*e^(4*x) + 2*(22*cosh(x
)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 + 9*cosh
(x)^2 + (11*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(4*x) + 2*(11*cosh(x)^4 + 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 + 15*co
sh(x)^3 + (11*cosh(x)^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) + 2*(11*cosh(x)
^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 + 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 + 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 +
315*cosh(x)^4 + 210*cosh(x)^2 + 20*x)*e^(4*x) + 2*(154*cosh(x)^6 + 315*cosh
(x)^4 + 210*cosh(x)^2 + 20*x)*e^(2*x) + 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
+ 63*cosh(x)^5 + 70*cosh(x)^3 + 20*x*cosh(x) + (22*cosh(x)^7 + 63*cosh(x)^5
 + 70*cosh(x)^3 + 20*x*cosh(x))*e^(4*x) + 2*(22*cosh(x)^7 + 63*cosh(x)^5 +
70*cosh(x)^3 + 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 + 42*cos
h(x)^6 + 70*cosh(x)^4 + 40*x*cosh(x)^2 + (11*cosh(x)^8 + 42*cosh(x)^6 + 70*
cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(4*x) + 2*(11*cosh(x)^8 + 42*cosh(x)^6 +
70*cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
20*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 + (11*cos
h(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
) + 2*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 + 135*cosh(x)^8 +
420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 + 135*cosh(
x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 - 3)*e^(4*x) + 2*(22*
cosh(x)^10 + 135*cosh(x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2
 - 3)*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^12 + 9*cosh(x)^10 + 4
5*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4 - 9*cosh(x)^2 - 1)*e^(4*x) + 2
*(cosh(x)^12 + 9*cosh(x)^10 + 45*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4
 - 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(x)^7
 + 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(
x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x))*e^(4*x) + 2*(2*cosh(x)^1
1 + 15*cosh(x)^9 + 60*cosh(x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x
))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x
) + 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

giac [A] time = 0.13, size = 52, normalized size = 0.60

$$-\frac{\left(110\,e^{(6\,x)}+45\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)e^{(-6\,x)}-120\,x-e^{(6\,x)}-9\,e^{(4\,x)}-45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6*
x) - 9*e^(4*x) - 45*e^(2*x))/a^(3/2)
```

maple [B] time = 0.21, size = 230, normalized size = 2.67

$$\frac{5\,\mathrm{e}^{2x}x}{16a\left(1+\mathrm{e}^{2x}\right)^{2}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^{4}}}}+\frac{\mathrm{e}^{8x}}{384a\left(1+\mathrm{e}^{2x}\right)^{2}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^{4}}}}+\frac{3\,\mathrm{e}^{6x}}{128a\left(1+\mathrm{e}^{2x}\right)^{2}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^{4}}}}+\frac{15\,\mathrm{e}^{4x}}{128a\left(1+\mathrm{e}^{2x}\right)^{2}\sqrt{\frac{a\,\mathrm{e}^{4x}}{\left(1+\mathrm{e}^{2x}\right)^{4}}}}-\frac{}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(3/2),x)
```

```
[Out] 5/16/a*exp(2*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*x+1/384/a*
exp(8*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+3/128/a*exp(6*x)/
(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+15/128/a*exp(4*x)/(1+exp(2
*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)-15/128/(a*exp(4*x)/(1+exp(2*x))^4)
^(1/2)/(1+exp(2*x))^2/a-3/128/a*exp(-2*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp
(2*x))^4)^(1/2)-1/384/a*exp(-4*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4
)^(1/2)
```

maxima [A] time = 0.47, size = 65, normalized size = 0.76

$$\frac{\left(9\,\sqrt{a}\,e^{(-2\,x)}+45\,\sqrt{a}\,e^{(-4\,x)}-45\,\sqrt{a}\,e^{(-8\,x)}-9\,\sqrt{a}\,e^{(-10\,x)}-\sqrt{a}\,e^{(-12\,x)}+\sqrt{a}\right)e^{(6\,x)}}{384\,a^{2}}+\frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/384*(9*sqrt(a)*e^(-2*x) + 45*sqrt(a)*e^(-4*x) - 45*sqrt(a)*e^(-8*x) - 9*s
qrt(a)*e^(-10*x) - sqrt(a)*e^(-12*x) + sqrt(a))*e^(6*x)/a^2 + 5/16*x/a^(3/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\mathrm{cosh}\left(x\right)}^4}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^4)^(3/2),x)
```

```
[Out] int(1/(a/cosh(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \operatorname{sech}^4(x)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(-3/2), x)
```

3.51 $\int \frac{1}{\left(a\,\mathbf{sech}^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{63x\mathrm{sech}^2(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{\sinh(x)\cosh^7(x)}{10a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{9\sinh(x)\cosh^5(x)}{80a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{21\sinh(x)\cosh^3(x)}{160a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{21\sinh(x}{128a^2\sqrt{a}}$$

```
[Out] 63/256*x*sech(x)^2/a^2/(a*sech(x)^4)^(1/2)+21/128*cosh(x)*sinh(x)/a^2/(a*se
ch(x)^4)^(1/2)+21/160*cosh(x)^3*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+9/80*cosh(x
)^5*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+1/10*cosh(x)^7*sinh(x)/a^2/(a*sech(x)^4
)^(1/2)+63/256*tanh(x)/a^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4123, 2635, 8}

$$\frac{63x\mathrm{sech}^2(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{\sinh(x)\cosh^7(x)}{10a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{9\sinh(x)\cosh^5(x)}{80a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{21\sinh(x)\cosh^3(x)}{160a^2\sqrt{a\mathrm{sech}^4(x)}}+\frac{21\sinh(x}{128a^2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sech[x]^4)^(-5/2),x]
```

```
[Out] (63*x*Sech[x]^2)/(256*a^2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]*Sinh[x])/(128*a^
2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]^3*Sinh[x])/(160*a^2*Sqrt[a*Sech[x]^4]) +
 (9*Cosh[x]^5*Sinh[x])/(80*a^2*Sqrt[a*Sech[x]^4]) + (Cosh[x]^7*Sinh[x])/(10
*a^2*Sqrt[a*Sech[x]^4]) + (63*Tanh[x])/(256*a^2*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
```

```
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{5/2}}\,dx = \frac{\operatorname{sech}^2(x)\int \cosh^{10}(x)\,dx}{a^2\sqrt{a\operatorname{sech}^4(x)}}$$

$$= \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(9\operatorname{sech}^2(x)\right)\int \cosh^8(x)\,dx}{10a^2\sqrt{a\operatorname{sech}^4(x)}}$$

$$= \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(63\operatorname{sech}^2(x)\right)\int \cosh^6(x)\,dx}{80a^2\sqrt{a\operatorname{sech}^4(x)}}$$

$$= \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(21\operatorname{sech}^2(x)\right)\int \cosh^4(}{32a^2\sqrt{a\operatorname{sech}^4(x)}}$$

$$= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} +$$

$$= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} +$$

$$= \frac{63x\operatorname{sech}^2(x)}{256a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} +$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.42

$$\frac{(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) + 25\sinh(8x) + 2\sinh(10x))\cosh^2(x)\sqrt{a\operatorname{sech}^4(x)}}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sech[x]^4)^(-5/2),x]
```

```
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] + 150
*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

fricas [B] time = 0.46, size = 2600, normalized size = 19.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
*cosh(x)^2 + 5)*e^(4*x) + 2*(76*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x)^18 + 25
*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 + 15*cosh(x))*e^(4*x) + 2*(7
6*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 + 255*cosh(x)^2 + (646*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(4*x) + 2*(646
*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
+ 48*(646*cosh(x)^5 + 425*cosh(x)^3 + (646*cosh(x)^5 + 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) + 2*(646*cosh(x)^5 + 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(4*x) + 2*(129
2*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^14
 + 600*cosh(x)^14 + 120*(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 +
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(4*x) + 2*
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(2*x) + 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4
 + 910*cosh(x)^2 + (4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4 + 910*
cosh(x)^2 + 35)*e^(4*x) + 2*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)
^4 + 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + (4199*co
sh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) + 2*(4199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 + 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x
)^6 + 300300*cosh(x)^4 + 69300*cosh(x)^2 + 2520*x)*e^(4*x) + 2*(184756*cosh
(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^4 + 69300*cos
h(x)^2 + 2520*x)*e^(2*x) + 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 + 6077
5*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 2520*x*
cosh(x) + (16796*cosh(x)^11 + 60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cos
h(x)^5 + 23100*cosh(x)^3 + 2520*x*cosh(x))*e^(4*x) + 2*(16796*cosh(x)^11 +
60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 + 36465*cosh(x)^10 +
64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 + 36465*cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 346
50*cosh(x)^4 + 7560*x*cosh(x)^2 - 70)*e^(4*x) + 2*(8398*cosh(x)^12 + 36465*
cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*cosh(x)^5 + 2
520*x*cosh(x)^3 + (646*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580
*cosh(x)^7 + 6930*cosh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) + 2*(6
```

```
46*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*co
sh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 + 7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8
+ 32340*cosh(x)^6 + 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 +
7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17
640*x*cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(4*x) + 2*(1292*cosh(x)^14 + 7735*c
osh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17640*x*
cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^6 - 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 +
 69300*cosh(x)^7 + 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 +
8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7 + 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(4*x) + 2*(1292*cosh(x)^1
5 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7
+ 52920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(2*x) - 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*
cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cos
h(x)^2 + (323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*cosh(x
)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cosh(x)^2
 - 5)*e^(4*x) + 2*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 200
20*cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
+ 170*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040
*x*cosh(x)^7 - 980*cosh(x)^5 - 100*cosh(x)^3 + (19*cosh(x)^17 + 170*cosh(x)
^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040*x*cosh(x)^7
- 980*cosh(x)^5 - 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) + 2*(19*cosh(x)^17 + 1
70*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040*x*
cosh(x)^7 - 980*cosh(x)^5 - 100*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))
*sinh(x)^3 + 5*(76*cosh(x)^18 + 765*cosh(x)^16 + 3600*cosh(x)^14 + 10920*co
sh(x)^12 + 27720*cosh(x)^10 + 45360*x*cosh(x)^8 - 11760*cosh(x)^6 - 1800*co
sh(x)^4 - 180*cosh(x)^2 + (76*cosh(x)^18 + 765*cosh(x)^16 + 3600*cosh(x)^14
 + 10920*cosh(x)^12 + 27720*cosh(x)^10 + 45360*x*cosh(x)^8 - 11760*cosh(x)^
6 - 1800*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(4*x) + 2*(76*cosh(x)^18 + 765*co
sh(x)^16 + 3600*cosh(x)^14 + 10920*cosh(x)^12 + 27720*cosh(x)^10 + 45360*x*
cosh(x)^8 - 11760*cosh(x)^6 - 1800*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(2*x) -
 5)*sinh(x)^2 - 25*cosh(x)^2 + (2*cosh(x)^20 + 25*cosh(x)^18 + 150*cosh(x)^
16 + 600*cosh(x)^14 + 2100*cosh(x)^12 + 5040*x*cosh(x)^10 - 2100*cosh(x)^8
- 600*cosh(x)^6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 2)*e^(4*x) + 2*(2*cosh(x)^
20 + 25*cosh(x)^18 + 150*cosh(x)^16 + 600*cosh(x)^14 + 2100*cosh(x)^12 + 50
40*x*cosh(x)^10 - 2100*cosh(x)^8 - 600*cosh(x)^6 - 150*cosh(x)^4 - 25*cosh(
x)^2 - 2)*e^(2*x) + 10*(4*cosh(x)^19 + 45*cosh(x)^17 + 240*cosh(x)^15 + 840
*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 - 1680*cosh(x)^7 - 360*cos
h(x)^5 - 60*cosh(x)^3 + (4*cosh(x)^19 + 45*cosh(x)^17 + 240*cosh(x)^15 + 84
0*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 - 1680*cosh(x)^7 - 360*co
sh(x)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(4*x) + 2*(4*cosh(x)^19 + 45*cosh(x)^
17 + 240*cosh(x)^15 + 840*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 -
```

```
 1680*cosh(x)^7 - 360*cosh(x)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cos
h(x))*sinh(x) - 2)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1)
)*e^(2*x)/(a^3*cosh(x)^10*e^(2*x) + 10*a^3*cosh(x)^9*e^(2*x)*sinh(x) + 45*a
^3*cosh(x)^8*e^(2*x)*sinh(x)^2 + 120*a^3*cosh(x)^7*e^(2*x)*sinh(x)^3 + 210*
a^3*cosh(x)^6*e^(2*x)*sinh(x)^4 + 252*a^3*cosh(x)^5*e^(2*x)*sinh(x)^5 + 210
*a^3*cosh(x)^4*e^(2*x)*sinh(x)^6 + 120*a^3*cosh(x)^3*e^(2*x)*sinh(x)^7 + 45
*a^3*cosh(x)^2*e^(2*x)*sinh(x)^8 + 10*a^3*cosh(x)*e^(2*x)*sinh(x)^9 + a^3*e
^(2*x)*sinh(x)^10)
```

giac [A] time = 0.13, size = 76, normalized size = 0.58

$$-\frac{\left(5754\,e^{(10\,x)}+2100\,e^{(8\,x)}+600\,e^{(6\,x)}+150\,e^{(4\,x)}+25\,e^{(2\,x)}+2\right)e^{(-10\,x)}-5040\,x-2\,e^{(10\,x)}-25\,e^{(8\,x)}-150\,e^{(6\,x)}-\cdots}{20480\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*((5754*e^(10*x) + 2100*e^(8*x) + 600*e^(6*x) + 150*e^(4*x) + 25*e^
(2*x) + 2)*e^(-10*x) - 5040*x - 2*e^(10*x) - 25*e^(8*x) - 150*e^(6*x) - 600
*e^(4*x) - 2100*e^(2*x))/a^(5/2)
```

maple [B] time = 0.21, size = 362, normalized size = 2.74

$$\frac{63\,e^{2x}x}{256a^2\left(1+e^{2x}\right)^2\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}}+\frac{e^{12x}}{10240a^2\left(1+e^{2x}\right)^2\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}}+\frac{5\,e^{10x}}{4096a^2\left(1+e^{2x}\right)^2\sqrt{\frac{a\,e^{4x}}{\left(1+e^{2x}\right)^4}}}+\frac{15\,e^{8x}}{2048a^2\left(1+e^{2x}\right)^2\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sech(x)^4)^(5/2),x)
```

```
[Out] 63/256/a^2*exp(2*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)*x+1/10
240/a^2*exp(12*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+5/4096/a
^2*exp(10*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+15/2048/a^2*e
xp(8*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+15/512/a^2*exp(6*x
)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)+105/1024/a^2*exp(4*x)/(1
+exp(2*x))^2/(a*exp(4*x)/(1+exp(2*x))^4)^(1/2)-105/1024/(a*exp(4*x)/(1+exp(
2*x))^4)^(1/2)/(1+exp(2*x))^2/a^2-15/512/a^2*exp(-2*x)/(1+exp(2*x))^2/(a*ex
p(4*x)/(1+exp(2*x))^4)^(1/2)-15/2048/a^2*exp(-4*x)/(1+exp(2*x))^2/(a*exp(4*
x)/(1+exp(2*x))^4)^(1/2)-5/4096/a^2*exp(-6*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1
+exp(2*x))^4)^(1/2)-1/10240/a^2*exp(-8*x)/(1+exp(2*x))^2/(a*exp(4*x)/(1+exp
(2*x))^4)^(1/2)
```

maxima [A] time = 0.49, size = 103, normalized size = 0.78

$$\frac{\left(25\,\sqrt{a}\,e^{(-2\,x)}+150\,\sqrt{a}\,e^{(-4\,x)}+600\,\sqrt{a}\,e^{(-6\,x)}+2100\,\sqrt{a}\,e^{(-8\,x)}-2100\,\sqrt{a}\,e^{(-12\,x)}-600\,\sqrt{a}\,e^{(-14\,x)}-150\,\sqrt{a}\,e^{(-16}}{20480\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20480*(25*sqrt(a)*e^(-2*x) + 150*sqrt(a)*e^(-4*x) + 600*sqrt(a)*e^(-6*x)
+ 2100*sqrt(a)*e^(-8*x) - 2100*sqrt(a)*e^(-12*x) - 600*sqrt(a)*e^(-14*x) -
150*sqrt(a)*e^(-16*x) - 25*sqrt(a)*e^(-18*x) - 2*sqrt(a)*e^(-20*x) + 2*sqrt
(a))*e^(10*x)/a^3 + 63/256*x/a^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cosh(x)^4}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cosh(x)^4)^(5/2),x)
```

```
[Out] int(1/(a/cosh(x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\operatorname{sech}^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sech(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(-5/2), x)
```

3.52 $\int \frac{\sinh^4(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=44

$$-\frac{x}{8a}-\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\sinh(x)\cosh(x)}{8a}$$

```
[Out] -1/8*x/a-1/8*cosh(x)*sinh(x)/a+1/4*cosh(x)^3*sinh(x)/a-1/3*sinh(x)^3/a
```

Rubi [A] time = 0.14, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$-\frac{x}{8a}-\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\sinh(x)\cosh(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] -x/(8*a) - (Cosh[x]*Sinh[x])/(8*a) + (Cosh[x]^3*Sinh[x])/(4*a) - Sinh[x]^3/
(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^4(x)}{-a-a\cosh(x)}\,dx\\
&= -\frac{\int \cosh(x)\sinh^2(x)\,dx}{a}+\frac{\int \cosh^2(x)\sinh^2(x)\,dx}{a}\\
&= \frac{\cosh^3(x)\sinh(x)}{4a}-\frac{i\,\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{a}-\frac{\int \cosh^2(x)\,dx}{4a}\\
&= -\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}-\frac{\int 1\,dx}{8a}\\
&= -\frac{x}{8a}-\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 28, normalized size = 0.64

$$\frac{24\sinh(x)-8\sinh(3x)+3(\sinh(4x)-4x)}{96a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (24*Sinh[x] - 8*Sinh[3*x] + 3*(-4*x + Sinh[4*x]))/(96*a)
```

fricas [A] time = 0.41, size = 36, normalized size = 0.82

$$\frac{(3\cosh(x)-2)\sinh(x)^3+3\left(\cosh(x)^3-2\cosh(x)^2+2\right)\sinh(x)-3x}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((3*cosh(x) - 2)*sinh(x)^3 + 3*(cosh(x)^3 - 2*cosh(x)^2 + 2)*sinh(x) -
 3*x)/a
```

giac [A] time = 0.11, size = 42, normalized size = 0.95

$$-\frac{\left(24e^{(3x)}-8e^{x}+3\right)e^{(-4x)}+24x-3e^{(4x)}+8e^{(3x)}-24e^{x}}{192a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/192*((24*e^(3*x) - 8*e^x + 3)*e^(-4*x) + 24*x - 3*e^(4*x) + 8*e^(3*x) -
24*e^x)/a
```

maple [B] time = 0.14, size = 130, normalized size = 2.95

$$\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{5}{6a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{7}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a}-\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+a*sech(x)),x)
```

```
[Out] 1/4/a/(tanh(1/2*x)-1)^4+5/6/a/(tanh(1/2*x)-1)^3+7/8/a/(tanh(1/2*x)-1)^2+1/8
/a/(tanh(1/2*x)-1)+1/8/a*ln(tanh(1/2*x)-1)-1/4/a/(tanh(1/2*x)+1)^4+5/6/a/(t
anh(1/2*x)+1)^3-7/8/a/(tanh(1/2*x)+1)^2+1/8/a/(tanh(1/2*x)+1)-1/8/a*ln(tanh
(1/2*x)+1)
```

maxima [A] time = 0.32, size = 54, normalized size = 1.23

$$-\frac{\left(8e^{(-x)}-24e^{(-3x)}-3\right)e^{(4x)}}{192a}-\frac{x}{8a}-\frac{24e^{(-x)}-8e^{(-3x)}+3e^{(-4x)}}{192a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*e^(-x) - 24*e^(-3*x) - 3)*e^(4*x)/a - 1/8*x/a - 1/192*(24*e^(-x)
- 8*e^(-3*x) + 3*e^(-4*x))/a
```

mupad [B] time = 1.48, size = 59, normalized size = 1.34

$$\frac{e^{-3x}}{24a} - \frac{e^{-x}}{8a} - \frac{e^{3x}}{24a} - \frac{e^{-4x}}{64a} + \frac{e^{4x}}{64a} - \frac{x}{8a} + \frac{e^{x}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) - exp(-x)/(8*a) - exp(3*x)/(24*a) - exp(-4*x)/(64*a) + exp
(4*x)/(64*a) - x/(8*a) + exp(x)/(8*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sinh^4(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**4/(sech(x) + 1), x)/a
```

3.53 $\int \frac{\sinh^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=23

$$\frac{\cosh^3(x)}{3a}-\frac{\sinh^2(x)}{2a}$$

```
[Out] 1/3*cosh(x)^3/a-1/2*sinh(x)^2/a
```

Rubi [A] time = 0.12, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2835, 2564, 30, 2565}

$$\frac{\cosh^3(x)}{3a}-\frac{\sinh^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] Cosh[x]^3/(3*a) - Sinh[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
```

```
2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^3(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\int \cosh(x)\sinh(x)\,dx}{a}+\frac{\int \cosh^2(x)\sinh(x)\,dx}{a} \\
&= \frac{\text{Subst}(\int x\,dx, x, i\sinh(x))}{a}+\frac{\text{Subst}\left(\int x^2\,dx, x, \cosh(x)\right)}{a} \\
&= \frac{\cosh^3(x)}{3a}-\frac{\sinh^2(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 23, normalized size = 1.00

$$\frac{3\cosh(x)-3\cosh(2x)+\cosh(3x)-7}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + a*Sech[x]),x]
[Out] (-7 + 3*Cosh[x] - 3*Cosh[2*x] + Cosh[3*x])/(12*a)
```

fricas [A] time = 0.38, size = 30, normalized size = 1.30

$$\frac{\cosh(x)^3+3\,(\cosh(x)-1)\sinh(x)^2-3\,\cosh(x)^2+3\,\cosh(x)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/12*(cosh(x)^3 + 3*(cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 3*cosh(x))/a
```

giac [A] time = 0.13, size = 37, normalized size = 1.61

$$\frac{\left(3\,e^{(2\,x)}-3\,e^{x}+1\right)e^{(-3\,x)}+e^{(3\,x)}-3\,e^{(2\,x)}+3\,e^{x}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*((3*e^(2*x) - 3*e^x + 1)*e^(-3*x) + e^(3*x) - 3*e^(2*x) + 3*e^x)/a
```

maple [B] time = 0.12, size = 67, normalized size = 2.91

$$\frac{-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{8}{8\tanh\left(\frac{x}{2}\right)+8}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+a*sech(x)),x)
```

```
[Out] 8/a*(-1/24/(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)^2-1/8/(tanh(1/2*x)-1)+1/24
/(tanh(1/2*x)+1)^3-1/8/(tanh(1/2*x)+1)^2+1/8/(tanh(1/2*x)+1))
```

maxima [B] time = 0.32, size = 46, normalized size = 2.00

$$-\frac{\left(3\,e^{(-x)}-3\,e^{(-2\,x)}-1\right)e^{(3\,x)}}{24\,a}+\frac{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x)/a + 1/24*(3*e^(-x) - 3*e^(-2*x) +
 e^(-3*x))/a
```

mupad [B] time = 1.36, size = 53, normalized size = 2.30

$$\frac{e^{-x}}{8\,a}-\frac{e^{-2\,x}}{8\,a}-\frac{e^{2\,x}}{8\,a}+\frac{e^{-3\,x}}{24\,a}+\frac{e^{3\,x}}{24\,a}+\frac{e^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + a/cosh(x)),x)
```

```
[Out] exp(-x)/(8*a) - exp(-2*x)/(8*a) - exp(2*x)/(8*a) + exp(-3*x)/(24*a) + exp(3
*x)/(24*a) + exp(x)/(8*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sinh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**3/(sech(x) + 1), x)/a
```

3.54 $\int \frac{\sinh^2(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] 1/2*x/a-sinh(x)/a+1/2*cosh(x)*sinh(x)/a
```

Rubi [A] time = 0.10, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2637, 2635, 8}

$$\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/(2*a) - Sinh[x]/a + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^2(x)}{-a-a\cosh(x)}\,dx\\ &= -\frac{\int \cosh(x)\,dx}{a}+\frac{\int \cosh^2(x)\,dx}{a}\\ &= -\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.07, size = 16, normalized size = 0.59

$$\frac{x+\sinh(x)(\cosh(x)-2)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + a*Sech[x]),x]
[Out] (x + (-2 + Cosh[x])*Sinh[x])/(2*a)
```

fricas [A] time = 0.40, size = 14, normalized size = 0.52

$$\frac{(\cosh(x)-2)\sinh(x)+x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/2*((cosh(x) - 2)*sinh(x) + x)/a
```

giac [A] time = 0.11, size = 28, normalized size = 1.04

$$\frac{(4\,e^x-1)e^{(-2\,x)}+4\,x+e^{(2\,x)}-4\,e^x}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/8*((4*e^x - 1)*e^(-2*x) + 4*x + e^(2*x) - 4*e^x)/a
```

maple [B] time = 0.11, size = 78, normalized size = 2.89

$$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+a*sech(x)),x)
```

```
[Out] 1/2/a/(tanh(1/2*x)-1)^2+3/2/a/(tanh(1/2*x)-1)-1/2/a*ln(tanh(1/2*x)-1)-1/2/a
/(tanh(1/2*x)+1)^2+3/2/a/(tanh(1/2*x)+1)+1/2/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 42, normalized size = 1.56

$$-\frac{\left(4\,e^{(-x)}-1\right)e^{(2\,x)}}{8\,a}+\frac{x}{2\,a}+\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*e^(-x) - 1)*e^(2*x)/a + 1/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a
```

mupad [B] time = 1.34, size = 41, normalized size = 1.52

$$\frac{e^{-x}}{2\,a}-\frac{e^{-2\,x}}{8\,a}+\frac{e^{2\,x}}{8\,a}+\frac{x}{2\,a}-\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + a/cosh(x)),x)
```

```
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + x/(2*a) - exp(x)/(2*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sinh^{2}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(sinh(x)**2/(sech(x) + 1), x)/a
```

3.55 $\int \frac{\sinh(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\cosh(x)}{a} - \frac{\log(\cosh(x)+1)}{a}$$

[Out] cosh(x)/a-ln(1+cosh(x))/a

Rubi [A] time = 0.07, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3872, 2833, 12, 43}

$$\frac{\cosh(x)}{a} - \frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(a + a*Sech[x]),x]

[Out] Cosh[x]/a - Log[1 + Cosh[x]]/a

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{a+a\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh(x)}{-a-a\cosh(x)}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cosh(x)\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cosh(x)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)\,dx, x, -a\cosh(x)\right)}{a^2}\\
&= \frac{\cosh(x)}{a}-\frac{\log(1+\cosh(x))}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.94

$$\frac{\cosh(x)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x] - 2*Log[Cosh[x/2]])/a
```

fricas [B] time = 0.40, size = 50, normalized size = 2.94

$$\frac{2\,x\cosh(x)+\cosh(x)^2-4\,(\cosh(x)+\sinh(x))\log\left(\cosh(x)+\sinh(x)+1\right)+2\,(x+\cosh(x))\sinh(x)+\sinh(x)^2}{2\,(a\cosh(x)+a\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*cosh(x) + cosh(x)^2 - 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 2*(x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(a*cosh(x) + a*sinh(x))
```

giac [A] time = 0.13, size = 32, normalized size = 1.88

$$\frac{x}{a}+\frac{e^{(-x)}}{2\,a}+\frac{e^x}{2\,a}-\frac{2\,\log\left(e^x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="giac")
[Out] x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^x + 1)/a
```

maple [A] time = 0.10, size = 27, normalized size = 1.59

$$-\frac{\ln(1+\mathrm{sech}(x))}{a}+\frac{1}{a\,\mathrm{sech}(x)}+\frac{\ln(\mathrm{sech}(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+a*sech(x)),x)
[Out] -1/a*ln(1+sech(x))+1/a/sech(x)+1/a*ln(sech(x))
```

maxima [B] time = 0.32, size = 35, normalized size = 2.06

$$-\frac{x}{a}+\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}-\frac{2\,\log\left(e^{(-x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="maxima")
[Out] -x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^(-x) + 1)/a
```

mupad [B] time = 0.07, size = 15, normalized size = 0.88

$$-\frac{\ln(\cosh(x)+1)-\cosh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + a/cosh(x)),x)
[Out] -(log(cosh(x) + 1) - cosh(x))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sinh(x)}{\mathrm{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*sech(x)),x)
[Out] Integral(sinh(x)/(sech(x) + 1), x)/a
```

3.56 $\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\operatorname{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

```
[Out] -1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

Rubi [A] time = 0.10, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3872, 2706, 2606, 30, 2611, 3770}

$$\frac{\operatorname{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + a*Sech[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/(2*a) - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)}{-a-a\cosh(x)}\,dx \\
&= \frac{\int \coth^2(x)\operatorname{csch}(x)\,dx}{a} - \frac{\int \coth(x)\operatorname{csch}^2(x)\,dx}{a} \\
&= -\frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{\int \operatorname{csch}(x)\,dx}{2a} - \frac{\operatorname{Subst}(\int x\,dx, x, -i\operatorname{csch}(x))}{a} \\
&= -\frac{\tanh^{-1}(\cosh(x))}{2a} - \frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{\operatorname{csch}^2(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 1.33

$$-\frac{\operatorname{sech}(x)\left(2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+1\right)}{2a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + a*Sech[x]),x]
```

```
[Out] -1/2*((1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))*Sech[x])/(a*(1
+ Sech[x]))
```

fricas [B] time = 0.39, size = 103, normalized size = 3.12

$$-\frac{\left(\cosh(x)^2+2\left(\cosh(x)+1\right)\sinh(x)+\sinh(x)^2+2\,\cosh(x)+1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(\cosh(x)^2+2\right.}{2\left(a\cosh(x)^2+a\sinh(x)^2+2\,a\cosh(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.13, size = 52, normalized size = 1.58

$$-\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{4\,a}+\frac{e^{(-x)}+e^{x}-2}{4\,a\left(e^{(-x)}+e^{x}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x - 2)/(a*(e^(-x) + e^x + 2))
```

maple [A] time = 0.13, size = 23, normalized size = 0.70

$$\frac{\tanh^{2}\left(\frac{x}{2}\right)}{4a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+a*sech(x)),x)
```

```
[Out] 1/4/a*tanh(1/2*x)^2+1/2/a*ln(tanh(1/2*x))
```

maxima [A] time = 0.32, size = 48, normalized size = 1.45

$$-\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

mupad [B] time = 1.44, size = 51, normalized size = 1.55

$$\frac{1}{a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)}-\frac{1}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + a/cosh(x))),x)
```

```
[Out] 1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)^
(1/2))/a)/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)/(sech(x) + 1), x)/a
```

3.57 $\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)} dx$

Optimal. Leaf size=23

$$\frac{\operatorname{csch}^3(x)}{3a} - \frac{\coth^3(x)}{3a}$$

[Out] -1/3*coth(x)^3/a+1/3*csch(x)^3/a

Rubi [A] time = 0.14, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2839, 2606, 30, 2607}

$$\frac{\operatorname{csch}^3(x)}{3a} - \frac{\coth^3(x)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^2/(a + a*Sech[x]),x]

[Out] -Coth[x]^3/(3*a) + Csch[x]^3/(3*a)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}(x)}{-a-a\cosh(x)}\,dx\\ &= \frac{\int \coth^2(x)\operatorname{csch}^2(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^3(x)\,dx}{a}\\ &= -\frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)}{a}-\frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\ &= -\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 1.09

$$-\frac{(2\cosh(x)+\cosh(2x)+3)\operatorname{csch}(x)}{6a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + a*Sech[x]),x]
```

```
[Out] -1/6*((3 + 2*Cosh[x] + Cosh[2*x])*Csch[x])/(a*(1 + Cosh[x]))
```

fricas [B] time = 0.38, size = 71, normalized size = 3.09

$$-\frac{4\,(2\,\cosh(x)+\sinh(x)+1)}{3\,\left(a\cosh(x)^3+a\sinh(x)^3+2\,a\cosh(x)^2+(3\,a\cosh(x)+2\,a)\sinh(x)^2-a\cosh(x)+\left(3\,a\cosh(x)^2+4\,a\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + sinh(x) + 1)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 +
 (3*a*cosh(x) + 2*a)*sinh(x)^2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) +
 a)*sinh(x) - 2*a)
```

giac [A] time = 0.11, size = 31, normalized size = 1.35

$$-\frac{1}{2\,a(e^{x}-1)}+\frac{3\,e^{(2\,x)}+1}{6\,a(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] -1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 1)/(a*(e^x + 1)^3)
```

maple [A] time = 0.15, size = 23, normalized size = 1.00

$$\frac{-\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+a*sech(x)),x)
[Out] 1/4/a*(-1/3*tanh(1/2*x)^3-1/tanh(1/2*x))
```

maxima [B] time = 0.31, size = 90, normalized size = 3.91

$$-\frac{4\,e^{(-x)}}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}-\frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a}-\frac{2}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] -4/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a*
e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) -
a*e^(-4*x) + a)
```

mupad [B] time = 1.35, size = 91, normalized size = 3.96

$$\frac{\frac{e^{2\,x}}{6\,a}+\frac{1}{6\,a}-\frac{e^{x}}{3\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{\frac{1}{6\,a}-\frac{e^{x}}{6\,a}}{e^{2\,x}+2\,e^{x}+1}-\frac{1}{2\,a\,\left(e^{x}-1\right)}+\frac{1}{6\,a\,\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + a/cosh(x))),x)
```

```
[Out] (exp(2*x)/(6*a) + 1/(6*a) - exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp(x)
 + 1) - (1/(6*a) - exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(x)
 - 1)) + 1/(6*a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**2/(sech(x) + 1), x)/a
```

3.58 $\int \frac{\mathbf{csch}^3(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=46

$$\frac{\mathrm{csch}^4(x)}{4a}+\frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\mathrm{csch}^3(x)}{4a}-\frac{\coth(x)\mathrm{csch}(x)}{8a}$$

```
[Out] 1/8*arctanh(cosh(x))/a-1/8*coth(x)*csch(x)/a-1/4*coth(x)*csch(x)^3/a+1/4*cs
ch(x)^4/a
```

Rubi [A] time = 0.19, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$\frac{\mathrm{csch}^4(x)}{4a}+\frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\mathrm{csch}^3(x)}{4a}-\frac{\coth(x)\mathrm{csch}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + a*Sech[x]),x]
```

```
[Out] ArcTanh[Cosh[x]]/(8*a) - (Coth[x]*Csch[x])/(8*a) - (Coth[x]*Csch[x]^3)/(4*a
) + Csch[x]^4/(4*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{csch}^3(x)}{a+a\,\text{sech}(x)}\,dx &= -\int \frac{\coth(x)\text{csch}^2(x)}{-a-a\cosh(x)}\,dx\\
&= \frac{\int \coth^2(x)\text{csch}^3(x)\,dx}{a}-\frac{\int \coth(x)\text{csch}^4(x)\,dx}{a}\\
&= -\frac{\coth(x)\text{csch}^3(x)}{4a}+\frac{\int \text{csch}^3(x)\,dx}{4a}+\frac{\text{Subst}\left(\int x^3\,dx,x,-i\text{csch}(x)\right)}{a}\\
&= -\frac{\coth(x)\text{csch}(x)}{8a}-\frac{\coth(x)\text{csch}^3(x)}{4a}+\frac{\text{csch}^4(x)}{4a}-\frac{\int \text{csch}(x)\,dx}{8a}\\
&= \frac{\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth(x)\text{csch}(x)}{8a}-\frac{\coth(x)\text{csch}^3(x)}{4a}+\frac{\text{csch}^4(x)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 59, normalized size = 1.28

$$\frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(-2\operatorname{csch}^2\left(\frac{x}{2}\right)+\operatorname{sech}^4\left(\frac{x}{2}\right)-4\log\left(\sinh\left(\frac{x}{2}\right)\right)+4\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)}{16(a\operatorname{sech}(x)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*(-2*Csch[x/2]^2 + 4*Log[Cosh[x/2]] - 4*Log[Sinh[x/2]] + Sech[x
/2]^4)*Sech[x])/(16*(a + a*Sech[x]))
```

fricas [B] time = 0.40, size = 630, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(x)^5 + 2*(5*cosh(x) + 2)*sinh(x)^4 + 2*sinh(x)^5 + 4*cosh(x)^4
 + 4*(5*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x)^3 + 20*cosh(x)^3 + 4*(5*cosh(x)^
3 + 6*cosh(x)^2 + 15*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 - (cosh(x)^6 + 2*
(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*co
sh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) -
 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 -
12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*co
sh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) +
 sinh(x) + 1) + (cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*co
sh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh
(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4
+ 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3
*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)
 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(5*cosh(x)^4 + 8*cosh(x)^3
 + 30*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 2*cosh(x))/(a*cosh(x)^6 + a*sinh
(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4 + (15*a
*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*cosh(x)^3
 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*cosh(x)^4
 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*a*cosh(
x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^2 - a*c
osh(x) + a)*sinh(x) + a)
```

giac [B] time = 0.11, size = 90, normalized size = 1.96

$$\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}-\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}+\frac{e^{(-x)}+e^{x}-6}{16\,a\left(e^{(-x)}+e^{x}-2\right)}-\frac{3\left(e^{(-x)}+e^{x}\right)^{2}+12\,e^{(-x)}+12\,e^{x}-4}{32\,a\left(e^{(-x)}+e^{x}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/16*log(e^(-x) + e^x + 2)/a - 1/16*log(e^(-x) + e^x - 2)/a + 1/16*(e^(-x)
+ e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(3*(e^(-x) + e^x)^2 + 12*e^(-x) +
12*e^x - 4)/(a*(e^(-x) + e^x + 2)^2)
```

maple [A] time = 0.15, size = 45, normalized size = 0.98

$$\frac{\tanh^4\left(\frac{x}{2}\right)}{32a}-\frac{\tanh^2\left(\frac{x}{2}\right)}{16a}-\frac{1}{16a\tanh\left(\frac{x}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+a*sech(x)),x)
```

```
[Out] 1/32/a*tanh(1/2*x)^4-1/16/a*tanh(1/2*x)^2-1/16/a/tanh(1/2*x)^2-1/8/a*ln(tan
h(1/2*x))
```

maxima [B] time = 0.32, size = 99, normalized size = 2.15

$$-\frac{e^{(-x)}+2\,e^{(-2\,x)}+10\,e^{(-3\,x)}+2\,e^{(-4\,x)}+e^{(-5\,x)}}{4\left(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a\right)}+\frac{\log\left(e^{(-x)}+1\right)}{8\,a}-\frac{\log\left(e^{(-x)}-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(e^(-x) + 2*e^(-2*x) + 10*e^(-3*x) + 2*e^(-4*x) + e^(-5*x))/(2*a*e^(-x
) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) + a)
 + 1/8*log(e^(-x) + 1)/a - 1/8*log(e^(-x) - 1)/a
```

mupad [B] time = 1.35, size = 121, normalized size = 2.63

$$\frac{1}{2\,a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)}+\frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{x}-1\right)}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + a/cosh(x))),x)
```

```
[Out] 1/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(
2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) -
```

```
1)) + atan((exp(x)*(-a^2)^(1/2))/a)/(4*(-a^2)^(1/2)) - 1/(a*(3*exp(2*x) +
exp(3*x) + 3*exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**3/(sech(x) + 1), x)/a
```

3.59 $\int \frac{\mathbf{csch}^4(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=34

$$-\frac{\coth^5(x)}{5a}+\frac{\coth^3(x)}{3a}+\frac{\text{csch}^5(x)}{5a}$$

```
[Out] 1/3*coth(x)^3/a-1/5*coth(x)^5/a+1/5*csch(x)^5/a
```

Rubi [A] time = 0.15, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3872, 2839, 2606, 30, 2607, 14}

$$-\frac{\coth^5(x)}{5a}+\frac{\coth^3(x)}{3a}+\frac{\text{csch}^5(x)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + a*Sech[x]),x]
```

```
[Out] Coth[x]^3/(3*a) - Coth[x]^5/(5*a) + Csch[x]^5/(5*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^3(x)}{-a-a\cosh(x)}\,dx\\
&= \frac{\int \coth^2(x)\operatorname{csch}^4(x)\,dx}{a}-\frac{\int \coth(x)\operatorname{csch}^5(x)\,dx}{a}\\
&= \frac{i\,\text{Subst}\left(\int x^4\,dx, x, -i\operatorname{csch}(x)\right)}{a}+\frac{i\,\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, i\coth(x)\right)}{a}\\
&= \frac{\operatorname{csch}^5(x)}{5a}+\frac{i\,\text{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, i\coth(x)\right)}{a}\\
&= \frac{\coth^3(x)}{3a}-\frac{\coth^5(x)}{5a}+\frac{\operatorname{csch}^5(x)}{5a}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 1.15

$$\frac{(-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x)-15)\operatorname{csch}^3(x)}{60a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + a*Sech[x]),x]
```

```
[Out] ((-15 - 6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(60*a
*(1 + Cosh[x]))
```

fricas [B] time = 0.39, size = 219, normalized size = 6.44

$$-\frac{}{15\left(a\cosh(x)^6 + a\sinh(x)^6 + 2\,a\cosh(x)^5 + 2\,(3\,a\cosh(x) + a)\sinh(x)^5 - 2\,a\cosh(x)^4 + \left(15\,a\cosh(x)^2 + 10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -8/15*(7*cosh(x)^2 + 4*(4*cosh(x) + 1)*sinh(x) + 7*sinh(x)^2 + 2*cosh(x) +
1)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)
^5 - 2*a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - 2*a)*sinh(x)^4 - 6*a*
cosh(x)^3 + 2*(10*a*cosh(x)^3 + 10*a*cosh(x)^2 - 4*a*cosh(x) - 3*a)*sinh(x)
^3 - a*cosh(x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 12*a*cosh(x)^2 - 18*a
*cosh(x) - a)*sinh(x)^2 + 4*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 -
4*a*cosh(x)^3 - 9*a*cosh(x)^2 + a*cosh(x) + 4*a)*sinh(x) + 2*a)
```

giac [B] time = 0.11, size = 59, normalized size = 1.74

$$\frac{3\,e^{(2\,x)} - 12\,e^x + 5}{24\,a(e^x - 1)^3} - \frac{15\,e^{(4\,x)} + 60\,e^{(3\,x)} + 10\,e^{(2\,x)} + 20\,e^x + 7}{120\,a(e^x + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) - 12*e^x + 5)/(a*(e^x - 1)^3) - 1/120*(15*e^(4*x) + 60*e^(3
*x) + 10*e^(2*x) + 20*e^x + 7)/(a*(e^x + 1)^5)
```

maple [A] time = 0.15, size = 39, normalized size = 1.15

$$\frac{-\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5} + \frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3} - \frac{1}{3\tanh\left(\frac{x}{2}\right)^3} + \frac{2}{\tanh\left(\frac{x}{2}\right)}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+a*sech(x)),x)
```

```
[Out] 1/16/a*(-1/5*tanh(1/2*x)^5+2/3*tanh(1/2*x)^3-1/3/tanh(1/2*x)^3+2/tanh(1/2*x
))
```

maxima [B] time = 0.32, size = 292, normalized size = 8.59

$$\frac{8\,e^{(-x)}}{15\left(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a\right)} - \frac{}{15\left(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 8/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*
e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 8/15*e^(-2*x)/(2*a*e^(-x) - 2*a
*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e
^(-8*x) + a) - 8/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a
*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 4*e^(-4*x)/(2*a
*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e
^(-7*x) - a*e^(-8*x) + a) + 4/15/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x)
+ 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

mupad [B] time = 1.38, size = 236, normalized size = 6.94

$$\frac{1}{6\,a\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}-\frac{\frac{3\,e^{2\,x}}{40\,a}+\frac{e^{3\,x}}{40\,a}+\frac{1}{40\,a}-\frac{e^{x}}{8\,a}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}-\frac{\frac{e^{2\,x}}{40\,a}-\frac{1}{24\,a}+\frac{e^{x}}{20\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{\frac{e^{3\,x}}{10\,a}-\frac{e^{2\,x}}{4\,a}+\frac{e^{4\,x}}{40\,a}+\frac{1}{40\,a}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + a/cosh(x))),x)
```

```
[Out] 1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(40*a) + exp
(3*x)/(40*a) + 1/(40*a) - exp(x)/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*x)
 + 4*exp(x) + 1) - (exp(2*x)/(40*a) - 1/(24*a) + exp(x)/(20*a))/(3*exp(2*x)
 + exp(3*x) + 3*exp(x) + 1) - (exp(3*x)/(10*a) - exp(2*x)/(4*a) + exp(4*x)/
(40*a) + 1/(40*a) + exp(x)/(10*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x)
+ exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(8*a*(ex
p(x) - 1)) - 1/(20*a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{4}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**4/(sech(x) + 1), x)/a
```

3.60 $\int \frac{\sinh^4(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=132

$$-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5}-\frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2}+\frac{\sinh(x)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)c\cdots\right.}{8a^4}$$

```
[Out] 1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctan((a-b)
^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^5+1/8*(8*b*(a^2-b^2)-a*(3*a^2-4*b^2)*cosh
(x))*sinh(x)/a^4-1/12*(4*b-3*a*cosh(x))*sinh(x)^3/a^2
```

Rubi [A] time = 0.37, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2865, 2735, 2659, 205}

$$\frac{x\left(-12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{\sinh(x)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)}{8a^4}-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\cdots}{\sqrt{a+b}}\right.}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^5 + ((8*b*(a^2 - b^2) - a*(
3*a^2 - 4*b^2)*Cosh[x])*Sinh[x])/(8*a^4) - ((4*b - 3*a*Cosh[x])*Sinh[x]^3)/
(12*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\operatorname{sech}(x)}\,dx = -\int \frac{\cosh(x)\sinh^4(x)}{-b-a\cosh(x)}\,dx$$

$$= -\frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2} + \frac{\int \frac{\left(-ab+\left(3a^2-4b^2\right)\cosh(x)\right)\sinh^2(x)}{-b-a\cosh(x)}\,dx}{4a^2}$$

$$= \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2} - \frac{\int \frac{-ab\left(5a^2-4b^2\right.}{}}{}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x}{12a^2}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cosh(x)\right)\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x}{12a^2}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a\right.\right.}{}$$

Mathematica [A] time = 0.74, size = 219, normalized size = 1.66

$$\frac{36a^4x + 3a^4\sinh(4x) - 8a^3b\sinh(3x) - 144a^2b^2x + 24ab\left(5a^2 - 4b^2\right)\sinh(x) - 24a^2\left(a^2 - b^2\right)\sinh(2x) + \frac{192b^5}{\cdots}}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (36*a^4*x - 144*a^2*b^2*x + 96*b^4*x + (192*a^4*b*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (384*a^2*b^3*ArcTan[((-a + b)*Tanh[x
/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (192*b^5*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 24*a*b*(5*a^2 - 4*b^2)*Sinh[x] - 24*
a^2*(a^2 - b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

fricas [B] time = 0.45, size = 1812, normalized size = 13.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*co
sh(x) - a^3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)
^2 - 14*a^3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b
^2 + 8*b^4)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh
(x)^3 - 7*a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*
sinh(x)^5 + 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 -
180*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a
^3*b - 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)
^3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a
^4 - a^2*b^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5
*a^3*b - 4*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*
(7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^
4 - 2*a^2*b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b -
 4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 - 192*((a^2*
b - b^3)*cosh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*co
sh(x)^2*sinh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(
x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x
) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
 + b)*sinh(x) + a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^
2*b^2)*cosh(x)^5 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*
```

```
b - 4*a*b^3)*cosh(x)^4 + a^3*b - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 -
 a^2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^
5*cosh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4), 1/192*(3*
a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cosh(x) - a^
3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^2 - 14*a^
3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b^2 + 8*b^4
)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(x)^3 - 7*
a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^5
+ 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 - 180*(a^4 -
 a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a^3*b - 4*a
*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)^3 + 8*(21
*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a^4 - a^2*b
^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5*a^3*b - 4
*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(7*a^4*cos
h(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^4 - 2*a^2*
b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b - 4*a*b^3)*
cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 + 384*((a^2*b - b^3)*c
osh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*cosh(x)^2*si
nh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(x)^4)*sqrt
(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + 8*(3*a^4
*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^2*b^2)*cosh(x)^5 + 12*(3*a^4 -
 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*b - 4*a*b^3)*cosh(x)^4 + a^3*b
 - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x))/(a
^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*cosh(x)^2*sinh(x)^2 + 4*a^5*
cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)]
```

giac [A] time = 0.14, size = 197, normalized size = 1.49

$$\frac{3\,a^3e^{(4\,x)} - 8\,a^2be^{(3\,x)} - 24\,a^3e^{(2\,x)} + 24\,ab^2e^{(2\,x)} + 120\,a^2be^x - 96\,b^3e^x}{192\,a^4} + \frac{\left(3\,a^4 - 12\,a^2b^2 + 8\,b^4\right)x}{8\,a^5} + \frac{\left(8\,a^3be^x - 3\,a^4 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) - 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
+ 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x/a^5
+ 1/192*(8*a^3*b*e^x - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*e^(3*x) + 24*(a^4 - a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - 2*(a^4*b - 2*a^2*b^3 + b^5)*arctan((a*e^x +
 b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5)
```

maple [B] time = 0.12, size = 488, normalized size = 3.70

$$\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a}-\frac{1}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{3}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a}+\frac{1}{8a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3}{8a\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sech(x)),x)
```

```
[Out] 3/8/a*ln(tanh(1/2*x)+1)-1/8/a/(tanh(1/2*x)-1)^2-3/8/a/(tanh(1/2*x)-1)-3/8/a
*ln(tanh(1/2*x)-1)+1/8/a/(tanh(1/2*x)+1)^2-3/8/a/(tanh(1/2*x)+1)-2*b/a/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+4*b^3/a^3/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2*b^5/a^5/((
a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/4/a/(tanh
(1/2*x)-1)^4+1/2/a/(tanh(1/2*x)-1)^3-1/4/a/(tanh(1/2*x)+1)^4+1/2/a/(tanh(1/
2*x)+1)^3+1/a^5*ln(tanh(1/2*x)+1)*b^4-1/a^2/(tanh(1/2*x)+1)*b+1/2/a^3/(tanh
(1/2*x)+1)*b^2+1/a^4/(tanh(1/2*x)+1)*b^3+1/3/a^2/(tanh(1/2*x)+1)^3*b+1/2/a^
3/(tanh(1/2*x)-1)*b^2+1/a^4/(tanh(1/2*x)-1)*b^3-1/a^2/(tanh(1/2*x)-1)*b+1/2
/a^2/(tanh(1/2*x)-1)^2*b+1/2/a^3/(tanh(1/2*x)-1)^2*b^2+3/2/a^3*ln(tanh(1/2*
x)-1)*b^2-1/a^5*ln(tanh(1/2*x)-1)*b^4+1/3/a^2/(tanh(1/2*x)-1)^3*b-1/2/a^2/(
tanh(1/2*x)+1)^2*b-1/2/a^3/(tanh(1/2*x)+1)^2*b^2-3/2/a^3*ln(tanh(1/2*x)+1)*
b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 2.01, size = 275, normalized size = 2.08

$$\frac{e^{4x}}{64a}-\frac{e^{-4x}}{64a}+\frac{x\left(3a^4-12a^2b^2+8b^4\right)}{8a^5}-\frac{e^{-x}\left(5a^2b-4b^3\right)}{8a^4}+\frac{e^{-2x}\left(a^2-b^2\right)}{8a^3}-\frac{e^{2x}\left(a^2-b^2\right)}{8a^3}+\frac{b\,e^{-3x}}{24a^2}-\frac{b\,e^{3x}}{24a^2}+\frac{e^{x}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b/cosh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 - 12*a^2*b^2))/(8*a^
5) - (exp(-x)*(5*a^2*b - 4*b^3))/(8*a^4) + (exp(-2*x)*(a^2 - b^2))/(8*a^3)
- (exp(2*x)*(a^2 - b^2))/(8*a^3) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(2
4*a^2) + (exp(x)*(5*a^2*b - 4*b^3))/(8*a^4) + (b*log((2*exp(x)*(a^4*b + b^5
 - 2*a^2*b^3))/a^6 - (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/a^6)*
(a + b)^(3/2)*(b - a)^(3/2))/a^5 - (b*log((2*exp(x)*(a^4*b + b^5 - 2*a^2*b^
3))/a^6 + (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/a^6)*(a + b)^(3/
2)*(b - a)^(3/2))/a^5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**4/(a + b*sech(x)), x)
```

3.61 $\int \frac{\sinh^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{b\cosh^2(x)}{2a^2}+\frac{b\left(a^2-b^2\right)\log(a\cosh(x)+b)}{a^4}-\frac{\left(a^2-b^2\right)\cosh(x)}{a^3}+\frac{\cosh^3(x)}{3a}$$

```
[Out] -(a^2-b^2)*cosh(x)/a^3-1/2*b*cosh(x)^2/a^2+1/3*cosh(x)^3/a+b*(a^2-b^2)*ln(b
+a*cosh(x))/a^4
```

Rubi [A] time = 0.18, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3872, 2837, 12, 772}

$$-\frac{\left(a^2-b^2\right)\cosh(x)}{a^3}+\frac{b\left(a^2-b^2\right)\log(a\cosh(x)+b)}{a^4}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -(((a^2 - b^2)*Cosh[x])/a^3) - (b*Cosh[x]^2)/(2*a^2) + Cosh[x]^3/(3*a) + (b
*(a^2 - b^2)*Log[b + a*Cosh[x]])/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{a+b\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^3(x)}{-b-a\cosh(x)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a^3}\\
&= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)dx, x, -a\cosh(x)\right)}{a^4}\\
&= -\frac{\left(a^2-b^2\right)\cosh(x)}{a^3}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}+\frac{b\left(a^2-b^2\right)\log(b+a\cosh(x))}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 66, normalized size = 1.08

$$\frac{\left(12ab^2-9a^3\right)\cosh(x)+a^3\cosh(3x)-3a^2b\cosh(2x)+12a^2b\log(a\cosh(x)+b)-12b^3\log(a\cosh(x)+b)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] - 3*a^2*b*Cosh[2*x] + a^3*Cosh[3*x] + 12*a^2*b
*Log[b + a*Cosh[x]] - 12*b^3*Log[b + a*Cosh[x]])/(12*a^4)
```

fricas [B] time = 0.41, size = 490, normalized size = 8.03

$$\frac{a^3\cosh(x)^6+a^3\sinh(x)^6-3\,a^2b\cosh(x)^5+3\left(2\,a^3\cosh(x)-a^2b\right)\sinh(x)^5-24\left(a^2b-b^3\right)x\cosh(x)^3-3\left(3\,a^3 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
- a^2*b)*sinh(x)^5 - 24*(a^2*b - b^3)*x*cosh(x)^3 - 3*(3*a^3 - 4*a*b^2)*cos
h(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) - 3*a^3 + 4*a*b^2)*sinh(x)^4
- 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 - 12*(a^2*b -
b^3)*x - 6*(3*a^3 - 4*a*b^2)*cosh(x))*sinh(x)^3 + a^3 - 3*(3*a^3 - 4*a*b^2)
*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 + 4*a*b^2 - 24
*(a^2*b - b^3)*x*cosh(x) - 6*(3*a^3 - 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 24*((
a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b - b^3
)*cosh(x)*sinh(x)^2 + (a^2*b - b^3)*sinh(x)^3)*log(2*(a*cosh(x) + b)/(cosh(
x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 - 24*(a^2*b - b^3)*
x*cosh(x)^2 - 4*(3*a^3 - 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 - 4*a*b^2)*c
osh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*s
inh(x)^2 + a^4*sinh(x)^3)
```

giac [A] time = 0.14, size = 87, normalized size = 1.43

$$\frac{a^2\left(e^{(-x)}+e^x\right)^3-3\,ab\left(e^{(-x)}+e^x\right)^2-12\,a^2\left(e^{(-x)}+e^x\right)+12\,b^2\left(e^{(-x)}+e^x\right)}{24\,a^3}+\frac{\left(a^2b-b^3\right)\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(-x) + e^x)^3 - 3*a*b*(e^(-x) + e^x)^2 - 12*a^2*(e^(-x) + e^x)
 + 12*b^2*(e^(-x) + e^x))/a^3 + (a^2*b - b^3)*log(abs(a*(e^(-x) + e^x) + 2*
b))/a^4
```

maple [B] time = 0.12, size = 361, normalized size = 5.92

$$-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{b^3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^4}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{}{2a\left(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sech(x)),x)
```

```
[Out] -b/a^2*ln(tanh(1/2*x)-1)+b^3/a^4*ln(tanh(1/2*x)-1)-1/3/a/(tanh(1/2*x)-1)^3-
1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*b+1/2/a/(tanh(1/2*x)-1)-1
/2/a^2/(tanh(1/2*x)-1)*b-1/a^3/(tanh(1/2*x)-1)*b^2-1/2/a/(tanh(1/2*x)+1)+1/
2/a^2/(tanh(1/2*x)+1)*b+1/a^3/(tanh(1/2*x)+1)*b^2-b/a^2*ln(tanh(1/2*x)+1)+b
^3/a^4*ln(tanh(1/2*x)+1)-1/2/a/(tanh(1/2*x)+1)^2-1/2/a^2/(tanh(1/2*x)+1)^2*
b+1/3/a/(tanh(1/2*x)+1)^3+b/a/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
-b^2/a^2/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-b^3/a^3/(a-b)*ln(a*t
anh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+b^4/a^4/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2
*x)^2*b+a+b)
```

maxima [B] time = 0.32, size = 128, normalized size = 2.10

$$-\frac{\left(3\,abe^{(-x)}-a^2+3\left(3\,a^2-4\,b^2\right)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3}-\frac{3\,abe^{(-2\,x)}-a^2e^{(-3\,x)}+3\left(3\,a^2-4\,b^2\right)e^{(-x)}}{24\,a^3}+\frac{\left(a^2b-b^3\right)x}{a^4}+\frac{\left(a^2b-b^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*
(3*a*b*e^(-2*x) - a^2*e^(-3*x) + 3*(3*a^2 - 4*b^2)*e^(-x))/a^3 + (a^2*b - b
^3)*x/a^4 + (a^2*b - b^3)*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^4
```

mupad [B] time = 1.60, size = 123, normalized size = 2.02

$$\frac{e^{-3\,x}}{24\,a}+\frac{e^{3\,x}}{24\,a}-\frac{x\,\left(a^2\,b-b^3\right)}{a^4}-\frac{e^x\,\left(3\,a^2-4\,b^2\right)}{8\,a^3}-\frac{b\,e^{-2\,x}}{8\,a^2}-\frac{b\,e^{2\,x}}{8\,a^2}+\frac{\ln\left(a+2\,b\,e^x+a\,e^{2\,x}\right)\,\left(a^2\,b-b^3\right)}{a^4}-\frac{e^{-x}\,\left(3\,a^2-4\,b^2\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) + exp(3*x)/(24*a) - (x*(a^2*b - b^3))/a^4 - (exp(x)*(3*a^2
 - 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) + (log(a
+ 2*b*exp(x) + a*exp(2*x))*(a^2*b - b^3))/a^4 - (exp(-x)*(3*a^2 - 4*b^2))/(
8*a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{a+b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*sech(x)), x)
```

3.62 $\int \frac{\sinh^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=82

$$\frac{2b\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}-\frac{x\left(a^2-2b^2\right)}{2a^3}$$

```
[Out] -1/2*(a^2-2*b^2)*x/a^3-1/2*(2*b-a*cosh(x))*sinh(x)/a^2+2*b*arctan((a-b)^(1/
2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^3
```

Rubi [A] time = 0.21, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2865, 2735, 2659, 205}

$$-\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{2b\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] -((a^2 - 2*b^2)*x)/(2*a^3) + (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(Sqrt[a -
b]*Tanh[x/2])/Sqrt[a + b]])/a^3 - ((2*b - a*Cosh[x])*Sinh[x])/(2*a^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh^2(x)}{-b-a\cosh(x)}\,dx\\
&= -\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}+\frac{\int \frac{-ab+\left(a^2-2b^2\right)\cosh(x)}{-b-a\cosh(x)}\,dx}{2a^2}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}-\frac{\left(b\left(a^2-b^2\right)\right)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}-\frac{\left(2b\left(a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx,x,\tanh\right.}{a^3}\\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3}+\frac{2\sqrt{a-b}\,b\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 76, normalized size = 0.93

$$\frac{-8b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)-2a^2x+a^2\sinh(2x)-4ab\sinh(x)+4b^2x}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sech[x]),x]
```

[Out] (-2*a^2*x + 4*b^2*x - 8*b*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[
a^2 - b^2]] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)

fricas [B] time = 0.43, size = 536, normalized size = 6.54

$$\left[\frac{a^2\cosh(x)^4 + a^2\sinh(x)^4 - 4\,ab\cosh(x)^3 - 4\left(a^2 - 2\,b^2\right)x\cosh(x)^2 + 4\left(a^2\cosh(x) - ab\right)\sinh(x)^3 + 4\,ab\cosh}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="fricas")

[Out] [1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*c
osh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(
x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*
cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*si
nh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2
*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a
*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^2 + 2
*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2), 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 -
 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*cosh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh
(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2
)*x)*sinh(x)^2 - 16*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(
a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - a^2 + 4*(
a^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))
/(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)]

giac [A] time = 0.14, size = 100, normalized size = 1.22

$$\frac{ae^{(2\,x)} - 4\,be^x}{8\,a^2} - \frac{\left(a^2 - 2\,b^2\right)x}{2\,a^3} + \frac{\left(4\,abe^x - a^2\right)e^{(-2\,x)}}{8\,a^3} + \frac{2\left(a^2b - b^3\right)\arctan\left(\frac{ae^x + b}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="giac")

[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x -
a^2)*e^(-2*x)/a^3 + 2*(a^2*b - b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sq
rt(a^2 - b^2)*a^3)

maple [B] time = 0.12, size = 213, normalized size = 2.60

$$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b}{a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b^{2}}{a^{3}}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sech(x)),x)
```

```
[Out] 1/2/a/(tanh(1/2*x)-1)^2+1/2/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b+1/2/a
*ln(tanh(1/2*x)-1)-1/a^3*ln(tanh(1/2*x)-1)*b^2-1/2/a/(tanh(1/2*x)+1)^2+1/2/
a/(tanh(1/2*x)+1)+1/a^2/(tanh(1/2*x)+1)*b-1/2/a*ln(tanh(1/2*x)+1)+1/a^3*ln(
tanh(1/2*x)+1)*b^2+2*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b
)*(a-b))^(1/2))-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+
b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.67, size = 173, normalized size = 2.11

$$\frac{e^{2x}}{8a}-\frac{e^{-2x}}{8a}-\frac{b\,e^{x}}{2a^{2}}+\frac{b\,e^{-x}}{2a^{2}}-\frac{x\left(a^{2}-2b^{2}\right)}{2a^{3}}+\frac{b\ln\left(-\frac{2b\,e^{x}\left(a^{2}-b^{2}\right)}{a^{4}}-\frac{2b\sqrt{a+b}\,(a+b\,e^{x})\sqrt{b-a}}{a^{4}}\right)\sqrt{a+b}\,\sqrt{b-a}}{a^{3}}-\frac{b\ln\left(\frac{2b\sqrt{a+b}\,(\right.}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2)
 - (x*(a^2 - 2*b^2))/(2*a^3) + (b*log(- (2*b*exp(x)*(a^2 - b^2))/a^4 - (2*b
*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4)*(a + b)^(1/2)*(b - a)^(1/
2))/a^3 - (b*log((2*b*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4 - (2*
b*exp(x)*(a^2 - b^2))/a^4)*(a + b)^(1/2)*(b - a)^(1/2))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*sech(x)), x)
```

3.63 $\int \frac{\sinh(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\cosh(x)}{a}-\frac{b\log(a\cosh(x)+b)}{a^2}$$

[Out] cosh(x)/a-b*ln(b+a*cosh(x))/a^2

Rubi [A] time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3872, 2833, 12, 43}

$$\frac{\cosh(x)}{a}-\frac{b\log(a\cosh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(a + b*Sech[x]),x]

[Out] Cosh[x]/a - (b*Log[b + a*Cosh[x]])/a^2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx &= -\int \frac{\cosh(x)\sinh(x)}{-b-a\cosh(x)}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a}\\ &= -\frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(1-\frac{b}{b-x}\right)\,dx, x, -a\cosh(x)\right)}{a^2}\\ &= \frac{\cosh(x)}{a}-\frac{b\log(b+a\cosh(x))}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{a\cosh(x)-b\log(a\cosh(x)+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sech[x]),x]
[Out] (a*Cosh[x] - b*Log[b + a*Cosh[x]])/a^2
```

fricas [B] time = 0.40, size = 78, normalized size = 3.90

$$\frac{2\,bx\cosh(x)+a\cosh(x)^2+a\sinh(x)^2-2\,(b\cosh(x)+b\sinh(x))\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)+2\,(bx+a\cosh(x))\sinh}{2\left(a^2\cosh(x)+a^2\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="fricas")
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) +
a)/(a^2*cosh(x) + a^2*sinh(x))
```

giac [A] time = 0.12, size = 34, normalized size = 1.70

$$\frac{e^{(-x)}+e^x}{2\,a}-\frac{b\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="giac")
[Out] 1/2*(e^(-x) + e^x)/a - b*log(abs(a*(e^(-x) + e^x) + 2*b))/a^2
```

maple [A] time = 0.10, size = 31, normalized size = 1.55

$$-\frac{b\ln\left(a+b\,\mathrm{sech}(x)\right)}{a^2}+\frac{1}{a\,\mathrm{sech}(x)}+\frac{b\ln\left(\mathrm{sech}(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sech(x)),x)
[Out] -1/a^2*b*ln(a+b*sech(x))+1/a/sech(x)+1/a^2*b*ln(sech(x))
```

maxima [B] time = 0.31, size = 46, normalized size = 2.30

$$-\frac{bx}{a^2}+\frac{e^{(-x)}}{2\,a}+\frac{e^x}{2\,a}-\frac{b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="maxima")
[Out] -b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a - b*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^
2
```

mupad [B] time = 1.35, size = 20, normalized size = 1.00

$$\frac{\cosh(x)}{a}-\frac{b\,\ln\left(b+a\cosh(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b/cosh(x)),x)
[Out] cosh(x)/a - (b*log(b + a*cosh(x)))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(x\right)}{a+b\,\mathrm{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sech(x)),x)
[Out] Integral(sinh(x)/(a + b*sech(x)), x)
```

3.64 $\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=53

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] 1/2*ln(1-cosh(x))/(a+b)-1/2*ln(1+cosh(x))/(a-b)+b*ln(b+a*cosh(x))/(a^2-b^2)
```

Rubi [A] time = 0.12, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3872, 2721, 801}

$$\frac{b\log(a\cosh(x)+b)}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[b + a*
Cosh[x]])/(a^2 - b^2)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = -\int \frac{\coth(x)}{-b-a\cosh(x)}\,dx$$
$$= \operatorname{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cosh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)} - \frac{b}{(a-b)(a+b)(b-x)} + \frac{1}{2(a+b)(a+x)}\right)dx, x, -a\cosh(x)\right)$$
$$= \frac{\log(1-\cosh(x))}{2(a+b)} - \frac{\log(1+\cosh(x))}{2(a-b)} + \frac{b\log(b+a\cosh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 0.70

$$\frac{b\log(a\cosh(x)+b) + a\log\left(\tanh\left(\frac{x}{2}\right)\right) - b\log(\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sech[x]),x]
```

```
[Out] (b*Log[b + a*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]])/(a^2 - b^2)
```

fricas [A] time = 0.41, size = 58, normalized size = 1.09

$$\frac{b\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right) - (a+b)\log\left(\cosh(x)+\sinh(x)+1\right) + (a-b)\log\left(\cosh(x)+\sinh(x)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] (b*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh(
x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

giac [A] time = 0.12, size = 65, normalized size = 1.23

$$\frac{ab\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^3-ab^2} - \frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)} + \frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) - 1/2*log(e^(-x) + e^x +
 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

maple [A] time = 0.14, size = 48, normalized size = 0.91

$$\frac{b\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b\right)}{(a+b)(a-b)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sech(x)),x)
```

```
[Out] b/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+1/(a+b)*ln(tanh(1/2*x
))
```

maxima [A] time = 0.32, size = 59, normalized size = 1.11

$$\frac{b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^2-b^2}-\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
log(e^(-x) - 1)/(a + b)
```

mupad [B] time = 1.74, size = 148, normalized size = 2.79

$$\frac{\ln\left(128\,a\,b-32\,a^2-128\,b^2+32\,a^2\,e^x+128\,b^2\,e^x-128\,a\,b\,e^x\right)}{a+b}-\frac{\ln\left(-128\,a\,b-32\,a^2-128\,b^2-32\,a^2\,e^x-128\,b^2\right.}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b/cosh(x))),x)
```

```
[Out] log(128*a*b - 32*a^2 - 128*b^2 + 32*a^2*exp(x) + 128*b^2*exp(x) - 128*a*b*e
xp(x))/(a + b) - log(- 128*a*b - 32*a^2 - 128*b^2 - 32*a^2*exp(x) - 128*b^2
*exp(x) - 128*a*b*exp(x))/(a - b) + (b*log(16*a*b^2 - 4*a^3*exp(2*x) - 4*a^
3 + 32*b^3*exp(x) - 8*a^2*b*exp(x) + 16*a*b^2*exp(2*x)))/(a^2 - b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sech(x)),x)
[Out] Integral(csch(x)/(a + b*sech(x)), x)
```

3.65 $\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)} dx$

Optimal. Leaf size=66

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2ab\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] 2*a*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a+b)^(3/2)/(a-b)^(3/2)+(
b-a*cosh(x))*csch(x)/(a^2-b^2)
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2866, 12, 2659, 205}

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2ab\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*a*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}(x)}{-b-a\cosh(x)}\,dx\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{\int \frac{ab}{-b-a\cosh(x)}\,dx}{a^2-b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{(ab)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^2-b^2}\\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{(2ab)\operatorname{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\
&= \frac{2ab\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 75, normalized size = 1.14

$$\frac{1}{2}\left(-\frac{4ab\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{\tanh\left(\frac{x}{2}\right)}{b-a}-\frac{\coth\left(\frac{x}{2}\right)}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((-4*a*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
Coth[x/2]/(a + b) + Tanh[x/2]/(-a + b))/2
```

fricas [B] time = 0.42, size = 452, normalized size = 6.85

$$\left[\frac{2\,a^3 - 2\,ab^2 - \left(ab\cosh(x)^2 + 2\,ab\cosh(x)\sinh(x) + ab\sinh(x)^2 - ab\right)\sqrt{-a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2\,ab\,c}{a\cosh}\right.}{a^4 - 2\,a^2b^2 + b^4 - \left(a^4 - 2\,a^2b^2 + b^4\right)\cosh(x)^2 - 2\left(a^4 - 2\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
- a*b)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
- a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) + a)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(
a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a
*b^2 + (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - (a^2*b - b
^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*
b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)^2)]
```

giac [A] time = 0.12, size = 64, normalized size = 0.97

$$\frac{2\,ab\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{\frac{3}{2}}} + \frac{2\,(be^x - a)}{\left(a^2-b^2\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*a*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*(b*e^x - a)
/((a^2 - b^2)*(e^(2*x) - 1))
```

maple [A] time = 0.16, size = 77, normalized size = 1.17

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2\,(a-b)} - \frac{1}{2\,(a+b)\tanh\left(\frac{x}{2}\right)} + \frac{2ab\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sech(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a+b)/(a-b)*a*b/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.56, size = 151, normalized size = 2.29

$$\frac{a\,b\,\ln\left(-\frac{2\,b\,e^x}{a^2-b^2}-\frac{2\,b\,(a+b\,e^x)}{(a+b)^{3/2}\,(b-a)^{3/2}}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1}-\frac{a\,b\,\ln\left(\frac{2\,b\,(a+b\,e^x)}{(a+b)^{3/2}\,(b-a)^{3/2}}-\frac{2\,b\,e^x}{a^2-b^2}\right)}{(a+b)^{3/2}\,(b-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b/cosh(x))),x)
```

```
[Out] (a*b*log(- (2*b*exp(x))/(a^2 - b^2) - (2*b*(a + b*exp(x)))/((a + b)^(3/2)*(
b - a)^(3/2))))/((a + b)^(3/2)*(b - a)^(3/2)) - ((2*a)/(a^2 - b^2) - (2*b*e
xp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (a*b*log((2*b*(a + b*exp(x)))/((a + b)
^(3/2)*(b - a)^(3/2)) - (2*b*exp(x))/(a^2 - b^2)))/((a + b)^(3/2)*(b - a)^(
3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*sech(x)), x)
```

3.66 $\int \frac{\mathbf{csch}^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{a^2b\log(a\cosh(x)+b)}{(a^2-b^2)^2}+\frac{\text{csch}^2(x)(b-a\cosh(x))}{2(a^2-b^2)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(\cosh(x)+1)}{4(a-b)^2}$$

```
[Out] 1/2*(b-a*cosh(x))*csch(x)^2/(a^2-b^2)-1/4*a*ln(1-cosh(x))/(a+b)^2+1/4*a*ln(
1+cosh(x))/(a-b)^2-a^2*b*ln(b+a*cosh(x))/(a^2-b^2)^2
```

Rubi [A] time = 0.24, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2837, 12, 823, 801}

$$-\frac{a^2b\log(a\cosh(x)+b)}{(a^2-b^2)^2}+\frac{\text{csch}^2(x)(b-a\cosh(x))}{2(a^2-b^2)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - (a*Log[1 - Cosh[x]])/(4*(a +
b)^2) + (a*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^2*b*Log[b + a*Cosh[x]])/(a^
2 - b^2)^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx &= -\int \frac{\coth(x)\operatorname{csch}^2(x)}{-b-a\cosh(x)}\,dx \\
&= -\left(a^3\operatorname{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cosh(x)\right)\right) \\
&= -\left(a^2\operatorname{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cosh(x)\right)\right) \\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{\operatorname{Subst}\left(\int \frac{a^2b+a^2x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cosh(x)\right)}{2\left(a^2-b^2\right)} \\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{\operatorname{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)(a-x)} - \frac{2a^2b}{(a-b)(a+b)(b-x)} + \frac{a(a-b)}{2(a+b)(a+x)}\right)\,dx, x, -a\cosh\right.}{2\left(a^2-b^2\right)} \\
&= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{a\log(1-\cosh(x))}{4(a+b)^2} + \frac{a\log(1+\cosh(x))}{4(a-b)^2} - \frac{a^2b\log(b+a\cosh(x}{\left(a^2-b^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 86, normalized size = 1.01

$$\frac{1}{8}\left(-\frac{4a\left(\left(a^2+b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)-2ab\log(\sinh(x))+2ab\log(a\cosh(x)+b)\right)}{(a-b)^2(a+b)^2}-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b}-\frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-(Csch[x/2]^2/(a + b)) - (4*a*(2*a*b*Log[b + a*Cosh[x]] - 2*a*b*Log[Sinh[x
]] + (a^2 + b^2)*Log[Tanh[x/2]]))/((a - b)^2*(a + b)^2) - Sech[x/2]^2/(a -
b))/8
```

fricas [B] time = 0.45, size = 828, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b^
3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 + 2*
(a^3 - a*b^2)*cosh(x) + 2*(a^2*b*cosh(x)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^
2*b*sinh(x)^4 - 2*a^2*b*cosh(x)^2 + a^2*b + 2*(3*a^2*b*cosh(x)^2 - a^2*b)*s
inh(x)^2 + 4*(a^2*b*cosh(x)^3 - a^2*b*cosh(x))*sinh(x))*log(2*(a*cosh(x) +
b)/(cosh(x) - sinh(x))) - ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 +
 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b +
 a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x)
 + sinh(x) + 1) + ((a^3 - 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 - 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^3 + (a^3 - 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 - 2*a^2*b
 + a*b^2 - 2*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 - 2*a^2*b + a*b^2 -
 3*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 2*a^2*b + a*b^2
)*cosh(x)^3 - (a^3 - 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(
x) - 1) + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh
(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4
)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
 b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*co
sh(x)^3 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 174, normalized size = 2.05

$$-\frac{a^3b\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{a\log\left(e^{(-x)}+e^x+2\right)}{4\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(e^{(-x)}+e^x-2\right)}{4\left(a^2+2\,ab+b^2\right)}-\frac{a^2b\left(e^{(-x)}+e^x\right)^2+2\,a^3\left(e^{(-x)}+e^x\right)}{2\left(a^4-2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="giac")

[Out] -a^3*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*a*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*a*log(e^(-x) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^2*b*(e^(-x) + e^x)^2 + 2*a^3*(e^(-x) + e^x) - 2*a*b^2*(e^(-x) + e^x) - 8*a^2*b + 4*b^3)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e^x)^2 - 4))

maple [A] time = 0.18, size = 82, normalized size = 0.96

$$\frac{\tanh^2\left(\frac{x}{2}\right)}{8a-8b}-\frac{a^2 b\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b\right)}{(a+b)^2(a-b)^2}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(x)^3/(a+b*sech(x)),x)

[Out] 1/8*tanh(1/2*x)^2/(a-b)-a^2*b/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-1/8/(a+b)/tanh(1/2*x)^2-1/2*a/(a+b)^2*ln(tanh(1/2*x))

maxima [A] time = 0.34, size = 148, normalized size = 1.74

$$-\frac{a^2 b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^4-2\,a^2b^2+b^4}+\frac{a\log\left(e^{(-x)}+1\right)}{2\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(e^{(-x)}-1\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{ae^{(-x)}-2\,be^{(-2\,x)}+ae^{(-3\,x)}}{a^2-b^2-2\left(a^2-b^2\right)e^{(-2\,x)}+\left(a^2-b^2\right)e^{(-}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="maxima")

[Out] -a^2*b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*a*log(e^(-x) - 1)/(a^2 + 2*a*b + b^2) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))

mupad [B] time = 1.83, size = 255, normalized size = 3.00

$$\frac{\frac{2\left(a^2\,b-b^3\right)}{\left(a^2-b^2\right)^2}+\frac{e^x\left(a\,b^2-a^3\right)}{\left(a^2-b^2\right)^2}}{e^{2\,x}-1}+\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{a\,\ln\left(e^x-1\right)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{a\,\ln\left(e^x+1\right)}{2\,a^2-4\,a\,b+2\,b^2}-\frac{a^2\,b\,\ln\left(a^6\,e^{2\,x}+a^6+a^2\,b^4-14\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(x)^3*(a + b/cosh(x))),x)

```
[Out] ((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(e
xp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) - 2
*exp(2*x) + 1) - (a*log(exp(x) - 1))/(4*a*b + 2*a^2 + 2*b^2) + (a*log(exp(x
) + 1))/(2*a^2 - 4*a*b + 2*b^2) - (a^2*b*log(a^6*exp(2*x) + a^6 + a^2*b^4 -
 14*a^4*b^2 + a^2*b^4*exp(2*x) - 14*a^4*b^2*exp(2*x) + 2*a*b^5*exp(x) + 2*a
^5*b*exp(x) - 28*a^3*b^3*exp(x)))/(a^4 + b^4 - 2*a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*sech(x)), x)
```

3.67 $\int \frac{\textbf{csch}^4(x)}{a+b\textbf{sech}(x)}\,dx$

Optimal. Leaf size=111

$$-\frac{2a^3b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\operatorname{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{3\left(a^2-b^2\right)^2}$$

```
[Out] -2*a^3*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a+b)^(5/2)/(a-b)^(5/2
)-1/3*(3*a^2*b-a*(2*a^2+b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+1/3*(b-a*cosh(x))
*csch(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.30, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3872, 2866, 12, 2659, 205}

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\operatorname{csch}(x)\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)}{3\left(a^2-b^2\right)^2}-\frac{2a^3b\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (-2*a^3*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a +
b)^(5/2)) - ((3*a^2*b - a*(2*a^2 + b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2)
 + ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx = -\int \frac{\coth(x)\operatorname{csch}^3(x)}{-b-a\cosh(x)}\,dx$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} - \frac{\int \frac{\left(ab-2a^2\cosh(x)\right)\operatorname{csch}^2(x)}{-b-a\cosh(x)}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\int \frac{3a^3b}{-b-a\cosh(x)}\,dx}{3\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(a^3b\right)\int \frac{1}{-b-a\cosh(x)}\,d}{\left(a^2-b^2\right)^2}$$

$$= -\frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(2a^3b\right)\operatorname{Subst}\left(\int \frac{}{-a-}\right.}{\left(a^2\right.}$$

$$= -\frac{2a^3b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csc}}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.60, size = 156, normalized size = 1.41

$$\frac{\operatorname{sech}(x)(a\cosh(x)+b)\left(\frac{48a^3b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{2b\tanh\left(\frac{x}{2}\right)}{(a-b)^2}+\frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2}+\frac{2(4a+b)\coth\left(\frac{x}{2}\right)}{(a+b)^2}-\frac{\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right)}{2(a+b)}+\frac{8\sinh^4\left(\frac{x}{2}\right)\operatorname{csc}}{a-b}\right.}{24(a+b\operatorname{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]*((48*a^3*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + (2*(4*a + b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3
*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])
/(a - b)^2 - (2*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[x]))
```

fricas [B] time = 0.45, size = 2340, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(6*(a^4*b - a^2*b^3)*cosh(x)^5 + 6*(a^4*b - a^2*b^3)*sinh(x)^5 - 4*a^
5 + 2*a^3*b^2 + 2*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^
4 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^3 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)
^2 + 6*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2
+ 12*(a^5 - a^3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*c
osh(x)^2 - (5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 + 3*(a^3*b*cosh
(x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3
*a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a
^3*b*cosh(x)^3 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*
b*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a
^3*b*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2
+ 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2
 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(
x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 6*(a^4*b - a^2*b^3)*cosh(x) + 6*(a^4
*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^
3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*
sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*
```

```
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*
sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 -
 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), -2/3*(3
*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 - 2*a^5 + a^3*
b^2 + a*b^4 - 3*(a^3*b^2 - a*b^4)*cosh(x)^4 - 3*(a^3*b^2 - a*b^4 - 5*(a^4*b
 - a^2*b^3)*cosh(x))*sinh(x)^4 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^3
- 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 + 6*(a^3*
b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 6*(a^5 - a^3*b^2)*cosh(x)^2 + 6*(a^5 - a^
3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*cosh(x)^2 - (5*
a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(x)^6 + 6*a^3*
b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3*a^3*b*cosh(x)
^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a^3*b*cosh(x)^3
 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*b*cosh(x)^2 +
a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a^3*b*cosh(x))*
sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2
)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(a^4*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)
*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(
x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*
a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^
5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.14, size = 149, normalized size = 1.34

$$-\frac{2\,a^3b\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}-\frac{2\left(3\,a^2be^{(5\,x)}-3\,ab^2e^{(4\,x)}-10\,a^2be^{(3\,x)}+4\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,a^2be^x-2\,a^3-ab\right.}{3\left(a^4-2\,a^2b^2+b^4\right)\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(
a^2 - b^2)) - 2/3*(3*a^2*b*e^(5*x) - 3*a*b^2*e^(4*x) - 10*a^2*b*e^(3*x) + 4
```

```
*b^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a^2*b*e^x - 2*a^3 - a*b^2)/((a^4 - 2*a^2*b
^2 + b^4)*(e^(2*x) - 1)^3)
```

maple [A] time = 0.16, size = 154, normalized size = 1.39

$$-\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24\left(a-b\right)^2}+\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{24\left(a-b\right)^2}+\frac{3a\tanh\left(\frac{x}{2}\right)}{8\left(a-b\right)^2}-\frac{\tanh\left(\frac{x}{2}\right)b}{8\left(a-b\right)^2}-\frac{2a^3b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a-b\right)^2\left(a+b\right)^2\sqrt{\left(a+b\right)\left(a-b\right)}}-\frac{1}{24\left(a+b\right)\tanh\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sech(x)),x)
```

```
[Out] -1/24/(a-b)^2*a*tanh(1/2*x)^3+1/24/(a-b)^2*tanh(1/2*x)^3*b+3/8/(a-b)^2*a*ta
nh(1/2*x)-1/8/(a-b)^2*tanh(1/2*x)*b-2/(a-b)^2/(a+b)^2*a^3*b/((a+b)*(a-b))^(
1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3
+3/8/(a+b)^2/tanh(1/2*x)*a+1/8/(a+b)^2/tanh(1/2*x)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.75, size = 295, normalized size = 2.66

$$\frac{\frac{4\left(a\,b^2-a^3\right)}{\left(a^2-b^2\right)^2}+\frac{8\,e^x\left(a^2\,b-b^3\right)}{3\left(a^2-b^2\right)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{8\,a}{3\left(a^2-b^2\right)}-\frac{8\,b\,e^x}{3\left(a^2-b^2\right)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}+\frac{\frac{2\,a\,b^2}{\left(a^2-b^2\right)^2}-\frac{2\,a^2\,b\,e^x}{\left(a^2-b^2\right)^2}}{e^{2\,x}-1}+\frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,e^x}{\left(a^2-b^2\right)^2}-\frac{2\,a^2\,b\,\left(a+b\,e^x\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}-\frac{a^3\,b\,\ln\Big(2 \ldots}{(a \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b/cosh(x))),x)
```

```
[Out] ((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^
2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*
(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*a*b^2)/(a^2 -
b^2)^2 - (2*a^2*b*exp(x))/(a^2 - b^2)^2)/(exp(2*x) - 1) + (a^3*b*log((2*a^2
*b*exp(x))/(a^2 - b^2)^2 - (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b - a)^
```

```
(5/2))))/((a + b)^(5/2)*(b - a)^(5/2)) - (a^3*b*log((2*a^2*b*exp(x))/(a^2 -
 b^2)^2 + (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b - a)^(5/2)))))/((a + b)
^(5/2)*(b - a)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*sech(x)), x)
```

3.68 $\int \frac{\cosh^4(x)}{a+a\textbf{sech}(x)}\,dx$

Optimal. Leaf size=67

$$\frac{15x}{8a}-\frac{4\sinh^3(x)}{3a}-\frac{4\sinh(x)}{a}+\frac{5\sinh(x)\cosh^3(x)}{4a}+\frac{15\sinh(x)\cosh(x)}{8a}-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}$$

```
[Out] 15/8*x/a-4*sinh(x)/a+15/8*cosh(x)*sinh(x)/a+5/4*cosh(x)^3*sinh(x)/a-cosh(x)
^3*sinh(x)/(a+a*sech(x))-4/3*sinh(x)^3/a
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2635, 8, 2633}

$$\frac{15x}{8a}-\frac{4\sinh^3(x)}{3a}-\frac{4\sinh(x)}{a}+\frac{5\sinh(x)\cosh^3(x)}{4a}+\frac{15\sinh(x)\cosh(x)}{8a}-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sinh[x])/a + (15*Cosh[x]*Sinh[x])/(8*a) + (5*Cosh[x]^3*Si
nh[x])/(4*a) - (Cosh[x]^3*Sinh[x])/(a + a*Sech[x]) - (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx = -\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{\int \cosh^4(x)(-5a+4a\text{sech}(x))\,dx}{a^2}$$
$$= -\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{4\int \cosh^3(x)\,dx}{a} + \frac{5\int \cosh^4(x)\,dx}{a}$$
$$= \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{(4i)\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -i\sinh(x)\right)}{a} + \frac{15\int$$
$$= -\frac{4\sinh(x)}{a} + \frac{15\cosh(x)\sinh(x)}{8a} + \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{4\sinh^3(x)}{3a}$$
$$= \frac{15x}{8a} - \frac{4\sinh(x)}{a} + \frac{15\cosh(x)\sinh(x)}{8a} + \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{4\sinh}{3a}$$

Mathematica [A] time = 0.09, size = 63, normalized size = 0.94

$$\frac{\text{sech}\left(\frac{x}{2}\right)\left(-360\sinh\left(\frac{x}{2}\right)-120\sinh\left(\frac{3x}{2}\right)+40\sinh\left(\frac{5x}{2}\right)-5\sinh\left(\frac{7x}{2}\right)+3\sinh\left(\frac{9x}{2}\right)+360x\cosh\left(\frac{x}{2}\right)\right)}{192a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Sech[x/2]*(360*x*Cosh[x/2] - 360*Sinh[x/2] - 120*Sinh[(3*x)/2] + 40*Sinh[(
5*x)/2] - 5*Sinh[(7*x)/2] + 3*Sinh[(9*x)/2]))/(192*a)
```

fricas [B] time = 0.39, size = 139, normalized size = 2.07

$$\frac{3\cosh(x)^5 + (15\cosh(x) - 8)\sinh(x)^4 + 3\sinh(x)^5 - 8\cosh(x)^4 + \left(30\cosh(x)^2 - 8\cosh(x) + 35\right)\sinh(x)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*cosh(x)^5 + (15*cosh(x) - 8)*sinh(x)^4 + 3*sinh(x)^5 - 8*cosh(x)^4
 + (30*cosh(x)^2 - 8*cosh(x) + 35)*sinh(x)^3 + 45*cosh(x)^3 + (30*cosh(x)^3
 - 48*cosh(x)^2 + 135*cosh(x) - 160)*sinh(x)^2 + 24*(15*x - 2)*cosh(x) - 16
0*cosh(x)^2 + (15*cosh(x)^4 - 8*cosh(x)^3 + 105*cosh(x)^2 + 360*x - 160*cos
h(x) - 288)*sinh(x) + 360*x + 552)/(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.11, size = 86, normalized size = 1.28

$$\frac{15\,x}{8\,a}+\frac{\left(552\,e^{(4\,x)}+120\,e^{(3\,x)}-40\,e^{(2\,x)}+5\,e^{x}-3\right)e^{(-4\,x)}}{192\,a(e^{x}+1)}+\frac{3\,a^{3}e^{(4\,x)}-8\,a^{3}e^{(3\,x)}+48\,a^{3}e^{(2\,x)}-168\,a^{3}e^{x}}{192\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 15/8*x/a + 1/192*(552*e^(4*x) + 120*e^(3*x) - 40*e^(2*x) + 5*e^x - 3)*e^(-4
*x)/(a*(e^x + 1)) + 1/192*(3*a^3*e^(4*x) - 8*a^3*e^(3*x) + 48*a^3*e^(2*x) -
 168*a^3*e^x)/a^4
```

maple [B] time = 0.14, size = 139, normalized size = 2.07

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a}+\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5}{6a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{15}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{25}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{15\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)+1/4/a/(tanh(1/2*x)-1)^4+5/6/a/(tanh(1/2*x)-1)^3+15/8/a/(ta
nh(1/2*x)-1)^2+25/8/a/(tanh(1/2*x)-1)-15/8/a*ln(tanh(1/2*x)-1)-1/4/a/(tanh(
1/2*x)+1)^4+5/6/a/(tanh(1/2*x)+1)^3-15/8/a/(tanh(1/2*x)+1)^2+25/8/a/(tanh(1
/2*x)+1)+15/8/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.32, size = 80, normalized size = 1.19

$$\frac{15\,x}{8\,a}+\frac{168\,e^{(-x)}-48\,e^{(-2\,x)}+8\,e^{(-3\,x)}-3\,e^{(-4\,x)}}{192\,a}-\frac{5\,e^{(-x)}-40\,e^{(-2\,x)}+120\,e^{(-3\,x)}+552\,e^{(-4\,x)}-3}{192\left(ae^{(-4\,x)}+ae^{(-5\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 15/8*x/a + 1/192*(168*e^(-x) - 48*e^(-2*x) + 8*e^(-3*x) - 3*e^(-4*x))/a - 1
/192*(5*e^(-x) - 40*e^(-2*x) + 120*e^(-3*x) + 552*e^(-4*x) - 3)/(a*e^(-4*x)
 + a*e^(-5*x))
```

mupad [B] time = 1.45, size = 88, normalized size = 1.31

$$\frac{7\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{4\,a}+\frac{\mathrm{e}^{2\,x}}{4\,a}+\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}+\frac{15\,x}{8\,a}+\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{7\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] (7*exp(-x))/(8*a) - exp(-2*x)/(4*a) + exp(2*x)/(4*a) + exp(-3*x)/(24*a) - e
xp(3*x)/(24*a) - exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (15*x)/(8*a) + 2/(a*(
exp(x) + 1)) - (7*exp(x))/(8*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh^{4}{\left(x \right)}}{\operatorname{sech}{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**4/(sech(x) + 1), x)/a
```

3.69 $\int \frac{\cosh^3(x)}{a+a\text{sech}(x)} \, dx$

Optimal. Leaf size=54

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}$$

```
[Out] -3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*sech(x))+
4/3*sinh(x)^3/a
```

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2633, 2635, 8}

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh^2(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[
x])/(a + a*Sech[x]) + (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{\int \cosh^3(x)(-4a+3a\text{sech}(x))\,dx}{a^2}\\ &= -\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{3\int \cosh^2(x)\,dx}{a}+\frac{4\int \cosh^3(x)\,dx}{a}\\ &= -\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}+\frac{(4i)\,\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}-\frac{3\int}{2}\\ &= -\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\text{sech}(x)}+\frac{4\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 0.98

$$\frac{\text{sech}\left(\frac{x}{2}\right)\left(45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)-36x\cosh\left(\frac{x}{2}\right)\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*x
)/2] + Sinh[(7*x)/2]))/(24*a)
```

fricas [B] time = 0.39, size = 100, normalized size = 1.85

$$\frac{\cosh(x)^4+(4\cosh(x)-1)\sinh(x)^3+\sinh(x)^4-3\cosh(x)^3+\left(6\cosh(x)^2-9\cosh(x)+20\right)\sinh(x)^2-3\,(12}{24\,(a\cosh(x)+a\,\text{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6*
cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)^2
 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x - 69
)/(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.11, size = 70, normalized size = 1.30

$$-\frac{3\,x}{2\,a}-\frac{\left(69\,e^{(3\,x)}+18\,e^{(2\,x)}-2\,e^{x}+1\right)e^{(-3\,x)}}{24\,a(e^{x}+1)}+\frac{a^{2}e^{(3\,x)}-3\,a^{2}e^{(2\,x)}+21\,a^{2}e^{x}}{24\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1)
) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

maple [B] time = 0.15, size = 111, normalized size = 2.06

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/3/a/(tanh(1/2*x)-1)^3-1/a/(tanh(1/2*x)-1)^2-5/2/a/(tanh(1
/2*x)-1)+3/2/a*ln(tanh(1/2*x)-1)-1/3/a/(tanh(1/2*x)+1)^3+1/a/(tanh(1/2*x)+1
)^2-5/2/a/(tanh(1/2*x)+1)-3/2/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.32, size = 66, normalized size = 1.22

$$-\frac{3\,x}{2\,a}-\frac{21\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}}{24\,a}-\frac{2\,e^{(-x)}-18\,e^{(-2\,x)}-69\,e^{(-3\,x)}-1}{24\left(ae^{(-3\,x)}+ae^{(-4\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 18
*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

mupad [B] time = 1.36, size = 70, normalized size = 1.30

$$\frac{\mathrm{e}^{-2\,x}}{8\,a}-\frac{7\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{2\,x}}{8\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}+\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{3\,x}{2\,a}-\frac{2}{a\,\left(\mathrm{e}^{x}+1\right)}+\frac{7\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + a/cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + e
xp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**3/(sech(x) + 1), x)/a
```

3.70 $\int \frac{\cosh^2(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=41

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

```
[Out] 3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)*sinh(x)/(a+a*sech(x))
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3819, 3787, 2635, 8, 2637}

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]*Sinh[x])
/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a+a\text{sech}(x)}\,dx &= -\frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{\int \cosh^2(x)(-3a+2a\text{sech}(x))\,dx}{a^2}\\ &= -\frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{2\int \cosh(x)\,dx}{a}+\frac{3\int \cosh^2(x)\,dx}{a}\\ &= -\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)}+\frac{3\int 1\,dx}{2a}\\ &= \frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 1.10

$$\frac{\text{sech}\left(\frac{x}{2}\right)\left(-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)+12x\cosh\left(\frac{x}{2}\right)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2]
))/(8*a)
```

fricas [A] time = 0.38, size = 70, normalized size = 1.71

$$\frac{\cosh(x)^3+(3\cosh(x)-4)\sinh(x)^2+\sinh(x)^3+(12x-1)\cosh(x)-4\cosh(x)^2+\left(3\cosh(x)^2+12x-4\cos\right.}{8\left(a\cosh(x)+a\sinh(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x)
 - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20)/
(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.11, size = 51, normalized size = 1.24

$$\frac{3x}{2a} + \frac{\left(20e^{(2x)} + 3e^x - 1\right)e^{(-2x)}}{8a(e^x + 1)} + \frac{ae^{(2x)} - 4ae^x}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(2
*x) - 4*a*e^x)/a^2
```

maple [B] time = 0.14, size = 87, normalized size = 2.12

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a} + \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a} - \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*sech(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)+1/2/a/(tanh(1/2*x)-1)^2+3/2/a/(tanh(1/2*x)-1)-3/2/a*ln(tan
h(1/2*x)-1)-1/2/a/(tanh(1/2*x)+1)^2+3/2/a/(tanh(1/2*x)+1)+3/2/a*ln(tanh(1/2
*x)+1)
```

maxima [A] time = 0.32, size = 56, normalized size = 1.37

$$\frac{3x}{2a} + \frac{4e^{(-x)} - e^{(-2x)}}{8a} - \frac{3e^{(-x)} + 20e^{(-2x)} - 1}{8\left(ae^{(-2x)} + ae^{(-3x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(a
*e^(-2*x) + a*e^(-3*x))
```

mupad [B] time = 1.36, size = 52, normalized size = 1.27

$$\frac{e^{-x}}{2a} - \frac{e^{-2x}}{8a} + \frac{e^{2x}}{8a} + \frac{3x}{2a} + \frac{2}{a\,(e^x + 1)} - \frac{e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + a/cosh(x)),x)
```

```
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp(
x) + 1)) - exp(x)/(2*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**2/(sech(x) + 1), x)/a
```

3.71 $\int \frac{\cosh(x)}{a+a\,\textbf{sech}(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a\operatorname{sech}(x)+a}$$

```
[Out] -x/a+2*sinh(x)/a-sinh(x)/(a+a*sech(x))
```

Rubi [A] time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3819, 3787, 2637, 8}

$$-\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a\operatorname{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Sech[x]),x]
```

```
[Out] -(x/a) + (2*Sinh[x])/a - Sinh[x]/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = -\frac{\sinh(x)}{a + a\text{sech}(x)} - \frac{\int \cosh(x)(-2a + a\text{sech}(x))\,dx}{a^2}$$
$$= -\frac{\sinh(x)}{a + a\text{sech}(x)} - \frac{\int 1\,dx}{a} + \frac{2\int \cosh(x)\,dx}{a}$$
$$= -\frac{x}{a} + \frac{2\sinh(x)}{a} - \frac{\sinh(x)}{a + a\text{sech}(x)}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 1.23

$$\frac{-2x + 3\tanh\left(\frac{x}{2}\right) + \sinh\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Sech[x]),x]
```

```
[Out] (-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

fricas [A] time = 0.38, size = 47, normalized size = 1.81

$$-\frac{2\,x\cosh(x) - \cosh(x)^2 + 2\,(x - \cosh(x) - 1)\sinh(x) - \sinh(x)^2 + 2\,x + 5}{2\,(a\cosh(x) + a\sinh(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 + 2
*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.11, size = 35, normalized size = 1.35

$$-\frac{x}{a} - \frac{(5\,e^x + 1)e^{(-x)}}{2\,a(e^x + 1)} + \frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

maple [B] time = 0.13, size = 59, normalized size = 2.27

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/a/(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)-1)-1/a/(tanh(1/2*x)+
1)-1/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 41, normalized size = 1.58

$$-\frac{x}{a}+\frac{5\,e^{(-x)}+1}{2\left(ae^{(-x)}+ae^{(-2\,x)}\right)}-\frac{e^{(-x)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

mupad [B] time = 1.31, size = 34, normalized size = 1.31

$$\frac{e^x}{2\,a}-\frac{x}{a}-\frac{2}{a\,\left(e^x+1\right)}-\frac{e^{-x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a/cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cosh(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)/(sech(x) + 1), x)/a
```

3.72 $\int \frac{\mathbf{sech}(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{a\text{sech}(x)+a}$$

```
[Out] tanh(x)/(a+a*sech(x))
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3794}

$$\frac{\tanh(x)}{a\text{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Sech[x]),x]
```

```
[Out] Tanh[x]/(a + a*Sech[x])
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\text{sech}(x)}{a+a\text{sech}(x)}\,dx=\frac{\tanh(x)}{a+a\text{sech}(x)}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 0.91

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Sech[x]),x]
```

```
[Out] Tanh[x/2]/a
```

fricas [A] time = 0.37, size = 14, normalized size = 1.27

$$-\frac{2}{a\cosh(x) + a\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -2/(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.14, size = 11, normalized size = 1.00

$$-\frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -2/(a*(e^x + 1))
```

maple [A] time = 0.07, size = 9, normalized size = 0.82

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)
```

maxima [A] time = 0.33, size = 12, normalized size = 1.09

$$\frac{2}{ae^{(-x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/(a*e^(-x) + a)
```

mupad [B] time = 1.31, size = 11, normalized size = 1.00

$$-\frac{2}{a\,\left(\mathrm{e}^x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + a/cosh(x))),x)
```

```
[Out] -2/(a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)/(sech(x) + 1), x)/a
```

3.73 $\int \frac{\mathbf{sech}^2(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

[Out] arctan(sinh(x))/a-tanh(x)/(a+a*sech(x))

Rubi [A] time = 0.07, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3789, 3770, 3794}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(a + a*Sech[x]),x]

[Out] ArcTan[Sinh[x]]/a - Tanh[x]/(a + a*Sech[x])

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \operatorname{sech}(x)\,dx}{a} - \int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.10

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + a*Sech[x]),x]
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

fricas [A] time = 0.39, size = 29, normalized size = 1.45

$$\frac{2\left((\cosh(x)+\sinh(x)+1)\arctan\left(\cosh(x)+\sinh(x)\right)+1\right)}{a\cosh(x)+a\sinh(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] 2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*si
nh(x) + a)
```

giac [A] time = 0.13, size = 20, normalized size = 1.00

$$\frac{2\arctan\left(e^x\right)}{a}+\frac{2}{a(e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] 2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

maple [A] time = 0.08, size = 21, normalized size = 1.05

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+a*sech(x)),x)
[Out] -1/a*tanh(1/2*x)+2/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.45, size = 23, normalized size = 1.15

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}-\frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

mupad [B] time = 1.30, size = 31, normalized size = 1.55

$$\frac{2}{a\left(e^x+1\right)}+\frac{2\,\mathrm{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + a/cosh(x))),x)
[Out] 2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\operatorname{sech}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+a*sech(x)),x)
[Out] Integral(sech(x)**2/(sech(x) + 1), x)/a
```

3.74 $\int \frac{\textbf{sech}^3(x)}{a+a\textbf{sech}(x)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}+\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

```
[Out] -arctan(sinh(x))/a+tanh(x)/a+tanh(x)/(a+a*sech(x))
```

Rubi [A] time = 0.10, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3790, 3789, 3770, 3794}

$$\frac{\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}+\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Sech[x]),x]
```

```
[Out] -(ArcTan[Sinh[x]]/a) + Tanh[x]/a + Tanh[x]/(a + a*Sech[x])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx &= \frac{\tanh(x)}{a}-\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx\\ &= \frac{\tanh(x)}{a}-\frac{\int \operatorname{sech}(x)\,dx}{a}+\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx\\ &= -\frac{\tan^{-1}(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a+a\operatorname{sech}(x)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 1.73

$$\frac{2\cosh\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(\tanh(x)-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)\right)}{a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (2*Cosh[x/2]*Sech[x]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x]
)))/(a*(1 + Sech[x]))
```

fricas [B] time = 0.38, size = 127, normalized size = 4.88

$$-\frac{2\left(\left(\cosh(x)^3+(3\cosh(x)+1)\sinh(x)^2+\sinh(x)^3+\cosh(x)^2+\left(3\cosh(x)^2+2\cosh(x)+1\right)\sinh(x)+\cosh\right.\right.}{a\cosh(x)^3+a\sinh(x)^3+a\cosh(x)^2+(3a\cosh(x)+a)\sinh(x)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*cos
h(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) +
cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x)^3
 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (3
*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.13, size = 36, normalized size = 1.38

$$-\frac{2\arctan\left(e^x\right)}{a}-\frac{2\left(e^{(2x)}+e^x+2\right)}{a\left(e^{(3x)}+e^{(2x)}+e^x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

maple [A] time = 0.09, size = 39, normalized size = 1.50

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}+\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*sech(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)+2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.42, size = 45, normalized size = 1.73

$$\frac{2\left(e^{(-x)}+e^{(-2x)}+2\right)}{ae^{(-x)}+ae^{(-2x)}+ae^{(-3x)}+a}+\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arct
an(e^(-x))/a
```

mupad [B] time = 1.32, size = 58, normalized size = 2.23

$$-\frac{\frac{2e^{2x}}{a}+\frac{4}{a}+\frac{2e^{x}}{a}}{e^{2x}+e^{3x}+e^{x}+1}-\frac{2\,\mathrm{atan}\left(\frac{e^{x}\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + a/cosh(x))),x)
```

```
[Out] - ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
- (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\operatorname{sech}^{3}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**3/(sech(x) + 1), x)/a
```

3.75 $\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=45

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\operatorname{sech}^2(x)}{a\operatorname{sech}(x)+a}+\frac{3\tanh(x)\operatorname{sech}(x)}{2a}$$

```
[Out] 3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)/(
a+a*sech(x))
```

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3818, 3787, 3767, 8, 3768, 3770}

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\operatorname{sech}^2(x)}{a\operatorname{sech}(x)+a}+\frac{3\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]^2*Tanh[x])/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{\int \operatorname{sech}^2(x)(2a-3a\operatorname{sech}(x))\,dx}{a^2}\\ &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{2\int \operatorname{sech}^2(x)\,dx}{a}+\frac{3\int \operatorname{sech}^3(x)\,dx}{a}\\ &= \frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}-\frac{(2i)\operatorname{Subst}(\int 1\,dx,x,-i\tanh(x))}{a}+\frac{3\int \operatorname{sech}(x)\,d}{2a}\\ &= \frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 1.13

$$\frac{\cosh\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(\cosh\left(\frac{x}{2}\right)\left(6\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)(\operatorname{sech}(x)-2)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]*Sech[x]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + S
ech[x])*Tanh[x])))/(a*(1 + Sech[x]))
```

fricas [B] time = 0.38, size = 325, normalized size = 7.22

$$\frac{3\cosh(x)^4+3(4\cosh(x)+1)\sinh(x)^3+3\sinh(x)^4+3\cosh(x)^3+\left(18\cosh(x)^2+9\cosh(x)+5\right)\sinh(x)^2 -}{a\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (1
8*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*sin
h(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^3
+ 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 2
*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)*sinh
(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh(x)^3
+ 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5 + a*sin
h(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3 + 2*(5*a
*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*cosh(x)^3
+ 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*cosh(x)^4 +
 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.12, size = 48, normalized size = 1.07

$$\frac{3 \arctan\left(e^{x}\right)}{a} + \frac{e^{(3x)} + 2\,e^{(2x)} - e^{x} + 2}{a\left(e^{(2x)} + 1\right)^{2}} + \frac{2}{a\left(e^{x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/(
a*(e^x + 1))
```

maple [A] time = 0.11, size = 61, normalized size = 1.36

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a} - \frac{3\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}} - \frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}} + \frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-3/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-1/a/(tanh(1/2*x)^2+1
)^2*tanh(1/2*x)+3/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.41, size = 73, normalized size = 1.62

$$-\frac{e^{(-x)} + 5\,e^{(-2x)} + 3\,e^{(-3x)} + 3\,e^{(-4x)} + 4}{ae^{(-x)} + 2\,ae^{(-2x)} + 2\,ae^{(-3x)} + ae^{(-4x)} + ae^{(-5x)} + a} - \frac{3 \arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2*
x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

mupad [B] time = 1.35, size = 73, normalized size = 1.62

$$\frac{2}{a\,\left({\mathrm{e}}^x+1\right)}+\frac{\frac{2}{a}+\frac{{\mathrm{e}}^x}{a}}{{\mathrm{e}}^{2\,x}+1}+\frac{3\,\mathrm{atan}\left(\frac{{\mathrm{e}}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}-\frac{2\,{\mathrm{e}}^x}{a\,\left(2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + a/cosh(x))),x)
```

```
[Out] 2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2
)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^{4}{\left(x \right)}}{\operatorname{sech}{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**4/(sech(x) + 1), x)/a
```

3.76 $\int \frac{1}{a+a\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a\,\mathrm{sech}(c+dx)+a)}$$

[Out] x/a-tanh(d*x+c)/d/(a+a*sech(d*x+c))

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a\,\mathrm{sech}(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sech[c + d*x])^(-1),x]

[Out] x/a - Tanh[c + d*x]/(d*(a + a*Sech[c + d*x]))

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\,\mathrm{sech}(c+dx)}\,dx &= -\frac{\tanh(c+dx)}{d(a+a\,\mathrm{sech}(c+dx))}+\frac{\int a\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\tanh(c+dx)}{d(a+a\,\mathrm{sech}(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.14, size = 58, normalized size = 2.00

$$\frac{\mathrm{sech}\left(\frac{c}{2}\right)\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\left(dx\cosh\left(c+\frac{dx}{2}\right)-2\sinh\left(\frac{dx}{2}\right)+dx\cosh\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(-1),x]
```

```
[Out] (Sech[c/2]*Sech[(c + d*x)/2]*(d*x*Cosh[(d*x)/2] + d*x*Cosh[c + (d*x)/2] - 2
*Sinh[(d*x)/2]))/(2*a*d)
```

fricas [A] time = 0.38, size = 48, normalized size = 1.66

$$\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)+dx+2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) + d*x + 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) + a*d)
```

giac [A] time = 0.12, size = 29, normalized size = 1.00

$$\frac{\frac{dx+c}{a}+\frac{2}{a\left(e^{(dx+c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a + 2/(a*(e^(d*x + c) + 1)))/d
```

maple [A] time = 0.23, size = 58, normalized size = 2.00

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c)),x)
```

```
[Out] -1/d/a*tanh(1/2*d*x+1/2*c)-1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/
2*d*x+1/2*c)+1)
```

maxima [A] time = 0.31, size = 33, normalized size = 1.14

$$\frac{dx+c}{ad}-\frac{2}{\left(ae^{(-dx-c)}+a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + a)*d)
```

mupad [B] time = 1.30, size = 24, normalized size = 0.83

$$\frac{x}{a}+\frac{2}{a\,d\,\left({\mathrm{e}}^{c+d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cosh(c + d*x)),x)
```

```
[Out] x/a + 2/(a*d*(exp(c + d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\operatorname{sech}\left(c+dx\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c)),x)
```

```
[Out] Integral(1/(sech(c + d*x) + 1), x)/a
```

3.77 $\int \frac{1}{a-a\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=30

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\operatorname{sech}(c+dx))}$$

```
[Out] x/a-tanh(d*x+c)/d/(a-a*sech(d*x+c))
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3777, 8}

$$\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\operatorname{sech}(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - Tanh[c + d*x]/(d*(a - a*Sech[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-a\operatorname{sech}(c+dx)}\,dx&=-\frac{\tanh(c+dx)}{d(a-a\operatorname{sech}(c+dx))}+\frac{\int a\,dx}{a^2}\\&=\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\operatorname{sech}(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.15, size = 59, normalized size = 1.97

$$\frac{\operatorname{csch}\left(\frac{c}{2}\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)\left(dx\cosh\left(c+\frac{dx}{2}\right)+2\sinh\left(\frac{dx}{2}\right)-dx\cosh\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sech[c + d*x])^(-1),x]
```

```
[Out] (Csch[c/2]*Csch[(c + d*x)/2]*(-(d*x*Cosh[(d*x)/2]) + d*x*Cosh[c + (d*x)/2]
+ 2*Sinh[(d*x)/2]))/(2*a*d)
```

fricas [A] time = 0.38, size = 50, normalized size = 1.67

$$\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)-dx-2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) - d*x - 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) - a*d)
```

giac [A] time = 0.13, size = 29, normalized size = 0.97

$$\frac{\frac{dx+c}{a}-\frac{2}{a\left(e^{(dx+c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - 2/(a*(e^(d*x + c) - 1)))/d
```

maple [A] time = 0.23, size = 60, normalized size = 2.00

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}-\frac{1}{da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sech(d*x+c)),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/a/tanh
(1/2*d*x+1/2*c)
```

maxima [A] time = 0.31, size = 35, normalized size = 1.17

$$\frac{dx+c}{ad}+\frac{2}{\left(ae^{(-dx-c)}-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(a*d) + 2/((a*e^(-d*x - c) - a)*d)
```

mupad [B] time = 1.26, size = 24, normalized size = 0.80

$$\frac{x}{a}-\frac{2}{a\,d\,\left({\mathrm{e}}^{c+d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cosh(c + d*x)),x)
```

```
[Out] x/a - 2/(a*d*(exp(c + d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\operatorname{sech}{\left(c + d x \right)} - 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c)),x)
```

```
[Out] -Integral(1/(sech(c + d*x) - 1), x)/a
```

3.78 $\int (a+a\,\text{sech}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a\,\text{sech}(c+dx)+a}}+\frac{2a^2\tanh(c+dx)\sqrt{a\,\text{sech}(c+dx)+a}}{3d}$$

```
[Out] 2*a^(5/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+14/3*a^3*t
anh(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)+2/3*a^2*(a+a*sech(d*x+c))^(1/2)*tanh(d
*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3775, 3915, 3774, 203, 3792}

$$\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a\,\text{sech}(c+dx)+a}}+\frac{2a^2\tanh(c+dx)\sqrt{a\,\text{sech}(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(14*a^3*Tanh[c + d*x])/(3*d*Sqrt[a + a*Sech[c + d*x]]) + (2*a^2*Sqrt[a + a*
Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
```

```
Q[2*n]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\operatorname{sech}(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\,\tanh(c+dx)}{3d}+\frac{1}{3}(2a)\int\sqrt{a+a\operatorname{sech}(c+dx)}\left(\frac{3a}{2}+\frac{7}{2}as\right.$$

$$= \frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\,\tanh(c+dx)}{3d}+a^2\int\sqrt{a+a\operatorname{sech}(c+dx)}\,dx+\frac{1}{3}\left(7a^2\right)$$

$$= \frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\operatorname{sech}(c+dx)}}+\frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\,\tanh(c+dx)}{3d}+\frac{\left(2ia^3\right)\text{Subst}\Big(}{}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\operatorname{sech}(c+dx)}}+\frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx}}{3d}$$

Mathematica [A] time = 0.34, size = 99, normalized size = 1.01

$$\frac{a^2\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\operatorname{sech}(c+dx)\sqrt{a(\operatorname{sech}(c+dx)+1)}\left(-6\sinh\left(\frac{1}{2}(c+dx)\right)+8\sinh\left(\frac{3}{2}(c+dx)\right)+3\sqrt{2}\,\sinh^{-1}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sech[(c + d*x)/2]*Sech[c + d*x]*Sqrt[a*(1 + Sech[c + d*x])]*(3*Sqrt[2]
*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Cosh[c + d*x]^(3/2) - 6*Sinh[(c + d*x)/
2] + 8*Sinh[(3*(c + d*x))/2]))/(3*d)
```

fricas [B] time = 0.44, size = 924, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(
d*x + c)^2 + a^2)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x
 + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 +
 (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5
 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x
 + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15
*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^
4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*
x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(
d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c)
+ 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(
d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a)*log((a*cosh(d*x
 + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh
(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*co
sh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d
*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(
d*x + c))) + 8*(4*a^2*cosh(d*x + c)^3 + 4*a^2*sinh(d*x + c)^3 - 3*a^2*cosh(
d*x + c)^2 + 3*a^2*cosh(d*x + c) + 3*(4*a^2*cosh(d*x + c) - a^2)*sinh(d*x +
 c)^2 - 4*a^2 + 3*(4*a^2*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c) + a^2)*sinh(
d*x + c))*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 + 1)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*si
nh(d*x + c)^2 + d)
```

giac [A] time = 0.32, size = 151, normalized size = 1.54

$$\frac{\frac{6\,a^{3}\arctan\left(-\frac{\sqrt{a}\,e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-3\,a^{\frac{5}{2}}\log\left(\left|-\sqrt{a}\,e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)-\frac{4\left(4\,a^{4}-\left(3\,a^{4}e^{c}+\left(4\,a^{4}e^{(dx+3\,c)}-3\,a^{4}e^{(2\,c)}\right)e^{(dx)}\right)e^{\cdots}\right)}{\left(ae^{(2\,dx+2\,c)}+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*a^3*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt
(-a))/sqrt(-a) - 3*a^(5/2)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x +
 2*c) + a))) - 4*(4*a^4 - (3*a^4*e^c + (4*a^4*e^(d*x + 3*c) - 3*a^4*e^(2*c)
)*e^(d*x))*e^(d*x))/(a*e^(2*d*x + 2*c) + a)^(3/2))/d
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \left(a + a\,\mathrm{sech}\left(dx + c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(5/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}\left(dx + c\right) + a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cosh\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cosh(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,\mathrm{sech}\left(c + dx\right) + a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(5/2), x)
```

3.79 $\int (a + a\,\mathbf{sech}(c + dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\text{sech}(c+dx)+a}}$$

```
[Out] 2*a^(3/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+2*a^2*tanh
(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3775, 21, 3774, 203}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a\text{sech}(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(2*a^2*Tanh[c + d*x])/(d*Sqrt[a + a*Sech[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rubi steps

$$\begin{aligned}\int (a+a\operatorname{sech}(c+dx))^{3/2}\,dx &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}}+(2a)\int \frac{\frac{a}{2}+\frac{1}{2}a\operatorname{sech}(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx\\ &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}}+a\int \sqrt{a+a\operatorname{sech}(c+dx)}\,dx\\ &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}}+\frac{\left(2ia^2\right)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{ia\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d}\\ &= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d}+\frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\operatorname{sech}(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.20, size = 75, normalized size = 1.14

$$\frac{a\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\operatorname{sech}(c+dx)+1)}\left(2\sinh\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(3/2),x]
[Out] (a*Sech[(c + d*x)/2]*Sqrt[a*(1 + Sech[c + d*x])]*(Sqrt[2]*ArcSinh[Sqrt[2]*S
inh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]] + 2*Sinh[(c + d*x)/2]))/d
```

fricas [B] time = 0.41, size = 697, normalized size = 10.56

$$a^{\frac{3}{2}}\log\left(-\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4-3a\cosh(dx+c)^3+(4a\cosh(dx+c)-3a)\sinh(dx+c)^3+5a\cosh(dx+c)^2+\left(6a\cosh(dx+c)^2-9a\cosh(dx+c)+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^(3/2)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + a^(3/2)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))) + 4*(a*cosh(d*x
 + c) + a*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

giac [B] time = 0.25, size = 118, normalized size = 1.79

$$\frac{\frac{2\,a^{2}\arctan\left(-\frac{\sqrt{a}\,e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-a^{\frac{3}{2}}\log\left(\left|-\sqrt{a}\,e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)+\frac{2\left(a^{2}e^{(dx+c)}-a^{2}\right)}{\sqrt{ae^{(2\,dx+2\,c)}+a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] (2*a^2*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a)
)/sqrt(-a) - a^(3/2)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c)
+ a))) + 2*(a^2*e^(d*x + c) - a^2)/sqrt(a*e^(2*d*x + 2*c) + a))/d
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int (a + a\,\mathrm{sech}\,(dx + c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(3/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,\mathrm{sech}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sech(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+\frac{a}{\cosh\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cosh(c + d*x))^(3/2),x)
[Out] int((a + a/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,\mathrm{sech}\,(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(3/2),x)
[Out] Integral((a*sech(c + d*x) + a)**(3/2), x)
```

3.80 $\int \sqrt{a + a\,\mathrm{sech}(c + dx)}\, dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a + a\mathrm{sech}(c + dx)}\, dx &= \frac{(2ia)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\, dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\mathrm{sech}(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a+a\mathrm{sech}(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 1.62

$$\frac{\sqrt{2}\sinh^{-1}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\operatorname{sech}(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]]*Sech[(c + d
*x)/2]*Sqrt[a*(1 + Sech[c + d*x])])/d
```

fricas [B] time = 0.41, size = 637, normalized size = 17.22

$$\sqrt{a}\log\left(-\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4-3a\cosh(dx+c)^3+(4a\cosh(dx+c)-3a)\sinh(dx+c)^3+5a\cosh(dx+c)^2+\left(6a\cosh(dx+c)^2-9a\cosh(dx+c)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + sqrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

giac [B] time = 0.21, size = 83, normalized size = 2.24

$$\frac{\frac{2a\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2dx+2c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\sqrt{a}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2dx+2c)}+a}\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))/
sqrt(-a) - sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c) +
a))))/d
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \sqrt{a + a\,\mathrm{sech}\,(dx + c)}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sech(d*x+c))^(1/2),x)
```

```
[Out] int((a+a*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}\,(dx + c) + a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a + \frac{a}{\cosh\left(c + d\,x\right)}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\,\mathrm{sech}\,(c + dx) + a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(c + d*x) + a), x)
```

3.81 $\int \frac{1}{\sqrt{a+a\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/
2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx = \frac{\int \sqrt{a+a\operatorname{sech}(c+dx)}\,dx}{a} - \int \frac{\operatorname{sech}(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx$$
$$= \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d} - \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx}}\right.}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 1.21, size = 118, normalized size = 1.39

$$\frac{\left(e^{c+dx}+1\right)\left(\sqrt{2}\sinh^{-1}\left(e^{c+dx}\right)-2\tanh^{-1}\left(\frac{e^{c+dx}-1}{\sqrt{2}\sqrt{e^{2(c+dx)}+1}}\right)-\sqrt{2}\tanh^{-1}\left(\sqrt{e^{2(c+dx)}+1}\right)\right)}{\sqrt{2}\,d\sqrt{e^{2(c+dx)}+1}\sqrt{a(\operatorname{sech}(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] ((1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(-1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] - Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a*(1 + Sech[c + d*x
])])
```

fricas [B] time = 0.44, size = 868, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) - 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x +
```

```
c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) - 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)) + sqrt(a)*log(-(a*cosh(d*x +
 c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)
*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*
x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*si
nh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 -
 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
 c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh
(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2
- 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a
/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) -
 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh
(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*
sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(
a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + a\,\mathrm{sech}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+a*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}\,(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{a}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + a/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}\,(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(c + d*x) + a), x)
```

3.82 $\int \frac{1}{(a+a\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a\mathrm{sech}(c+dx)+a)^{3/2}}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan
h(1/2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2
)-1/2*tanh(d*x+c)/d/(a+a*sech(d*x+c))^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3777, 3920, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\mathrm{sech}(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a\mathrm{sech}(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(a^(3/2)*d)
- (5*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - Tanh[c + d*x]/(2*d*(a + a*Sech[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
```

```
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\operatorname{sech}(c+dx))^{3/2}}\,dx &= -\frac{\tanh(c+dx)}{2d(a+a\operatorname{sech}(c+dx))^{3/2}}-\frac{\int \frac{-2a+\frac{1}{2}a\operatorname{sech}(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx}{2a^2}\\
&= -\frac{\tanh(c+dx)}{2d(a+a\operatorname{sech}(c+dx))^{3/2}}+\frac{\int \sqrt{a+a\operatorname{sech}(c+dx)}\,dx}{a^2}-\frac{5\int \frac{\operatorname{sech}(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\,dx}{4a}\\
&= -\frac{\tanh(c+dx)}{2d(a+a\operatorname{sech}(c+dx))^{3/2}}+\frac{(2i)\operatorname{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{ia\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{ad}-\frac{(5i)\operatorname{Sub}
\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tanh(c+dx)}{2d(a+a\operatorname{sech}(c+dx))
\end{aligned}$$

Mathematica [A] time = 4.77, size = 177, normalized size = 1.55

$$\frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\operatorname{sech}(c+dx)\left(4\left(e^{c+dx}+1\right)\sinh^{-1}\left(e^{c+dx}\right)+5\sqrt{2}\left(e^{c+dx}+1\right)\tanh^{-1}\left(\frac{1-e^{c+dx}}{\sqrt{2}\sqrt{e^{2(c+dx)}+1}}\right)-4\left(e^{c+dx}+\right.}{2d\sqrt{e^{2(c+dx)}+1}\,(a(\operatorname{sech}(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^2*Sech[c + d*x]*(4*(1 + E^(c + d*x))*ArcSinh[E^(c + d*x)
] + 5*Sqrt[2]*(1 + E^(c + d*x))*ArcTanh[(1 - E^(c + d*x))/(Sqrt[2]*Sqrt[1 +
 E^(2*(c + d*x))])] - 4*(1 + E^(c + d*x))*ArcTanh[Sqrt[1 + E^(2*(c + d*x))]
] - 2*Sqrt[1 + E^(2*(c + d*x))]*Tanh[(c + d*x)/2]))/(2*d*Sqrt[1 + E^(2*(c +
 d*x))]*(a*(1 + Sech[c + d*x]))^(3/2))
```

fricas [B] time = 0.44, size = 1190, normalized size = 10.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(3*a*cosh(d*x + c)^2 + 3*a
*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(
d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cos
h(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*a*cosh(
d*x + c) + 2*(3*a*cosh(d*x + c) - a)*sinh(d*x + c) + 3*a)/(cosh(d*x + c)^2
+ 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) +
 1)) + 4*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x
+ c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 +
5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(
d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sin
h(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) +
 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x
 + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*co
sh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c)
+ 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c)
+ (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*si
nh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 4*(cosh(d*x + c)^2 + 2*(
cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*s
qrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*c
osh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*
cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*s
qrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/
(cosh(d*x + c) + sinh(d*x + c))) - 4*(cosh(d*x + c)^3 + (3*cosh(d*x + c) -
1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2
 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(a^2*d*cos
```

```
h(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a
^2*d*cosh(d*x + c) + a^2*d)*sinh(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + a\,\mathrm{sech}\left(dx + c\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+a*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,\mathrm{sech}\left(dx + c\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{a}{\mathrm{cosh}\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \operatorname{sech}(c + dx) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(-3/2), x)
```

3.83 $\int \sqrt{a - a\,\mathbf{sech}(c + dx)}\, dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}$$

[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))*a^(1/2)/d

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - a*Sech[c + d*x]],x]

[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/d

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a - a\text{sech}(c + dx)}\, dx &= -\frac{(2ia)\,\text{Subst}\left(\int \frac{1}{a+x^2}\, dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}\\
&= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}
\end{aligned}$$

Mathematica [A] time = 2.38, size = 70, normalized size = 1.84

$$\frac{\sqrt{e^{2(c+dx)}+1}\,\sqrt{a-a\operatorname{sech}(c+dx)}\left(\sinh^{-1}\left(e^{c+dx}\right)+\tanh^{-1}\left(\sqrt{e^{2(c+dx)}+1}\right)\right)}{d\left(e^{c+dx}-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[1 + E^(2*(c + d*x))]*(ArcSinh[E^(c + d*x)] + ArcTanh[Sqrt[1 + E^(2*(c
 + d*x))]])*Sqrt[a - a*Sech[c + d*x]])/(d*(-1 + E^(c + d*x)))
```

fricas [B] time = 0.40, size = 642, normalized size = 16.89

$$\sqrt{a}\log\left(\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4+3\,a\cosh(dx+c)^3+(4\,a\cosh(dx+c)+3\,a)\sinh(dx+c)^3+5\,a\cosh(dx+c)^2+\left(6\,a\cosh(dx+c)^2+9\,a\cosh(dx+c)+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log((a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)
^3 + (4*a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a
*cosh(d*x + c)^2 + 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c
)^5 + (5*cosh(d*x + c) + 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x
+ c)^4 + (10*cosh(d*x + c)^2 + 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*co
sh(d*x + c)^3 + (10*cosh(d*x + c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c)
 + 7)*sinh(d*x + c)^2 + 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*
x + c)^3 + 15*cosh(d*x + c)^2 + 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cos
h(d*x + c) + 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)) + 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9
*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d
*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)
^2 + sinh(d*x + c)^3)) + sqrt(a)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x +
c) - a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

giac [B] time = 0.21, size = 101, normalized size = 2.66

$$-\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}\,e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)\operatorname{sgn}\left(e^{(dx+c)}-1\right)}{\sqrt{-a}}+\sqrt{a}\,\log\left(\left|-\sqrt{a}\,e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)\operatorname{sgn}\left(e^{(dx+c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))
*sgn(e^(d*x + c) - 1)/sqrt(-a) + sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqr
t(a*e^(2*d*x + 2*c) + a)))*sgn(e^(d*x + c) - 1))/d
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \sqrt{a - a\,\mathrm{sech}\,(dx + c)}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sech(d*x+c))^(1/2),x)
```

```
[Out] int((a-a*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\,\mathrm{sech}\,(dx + c) + a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a - \frac{a}{\cosh\left(c + d\,x\right)}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a - a/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\,\mathrm{sech}\,(c + dx) + a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sech(c + d*x) + a), x)
```

3.84 $\int \frac{1}{\sqrt{a-a\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/
2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a-a*sech(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\mathrm{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\text{sech}(c+dx)}}\,dx = \frac{\int \sqrt{a-a\text{sech}(c+dx)}\,dx}{a} + \int \frac{\text{sech}(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\,dx$$
$$= -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d} + \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 2.23, size = 118, normalized size = 1.36

$$\frac{\left(e^{c+dx}-1\right)\left(\sqrt{2}\sinh^{-1}\left(e^{c+dx}\right)-2\tanh^{-1}\left(\frac{e^{c+dx}+1}{\sqrt{2}\sqrt{e^{2(c+dx)}+1}}\right)+\sqrt{2}\tanh^{-1}\left(\sqrt{e^{2(c+dx)}+1}\right)\right)}{\sqrt{2}\,d\sqrt{e^{2(c+dx)}+1}\sqrt{a-a\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] ((-1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] + Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a - a*Sech[c + d*x]
])
```

fricas [B] time = 0.42, size = 871, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x +
```

```
 c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) + 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 2*cosh(d*x + c) + 1)) + sqrt(a)*log((a*cosh(d*x +
c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + 3*a)*
sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 9*a*cosh(d*x
 + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) + 3)*sin
h(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 +
12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) + 7)*sinh(d*x + c)^2 + 7*cosh(
d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 +
 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) + 4)*sqrt(a)*sqrt(a/
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) +
4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9*a*cosh(d*x + c)^2 + 10*a*cosh(
d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*s
inh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(a
)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh
(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x + c) - a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(exp
(d*x+c)-1)]Warning, replacing 0 by ` u`, a substitution variable should per
haps be purged.Warning, replacing 0 by ` u`, a substitution variable should
 perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable sh
ould perhaps be purged.Error: Bad Argument Type
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - a\,\mathrm{sech}\left(dx + c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a-a*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\operatorname{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-a*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-\frac{a}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - a/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\operatorname{sech}\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sech(c + d*x) + a), x)
```

3.85 $\int \sqrt{3+3\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=19

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\operatorname{sech}(x)+1}}\right)$$

```
[Out] 2*arctanh(tanh(x)/(1+sech(x))^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3774, 203}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{\operatorname{sech}(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 + Sech[x]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+3\operatorname{sech}(x)}\,dx &= 6i\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3i\tanh(x)}{\sqrt{3+3\operatorname{sech}(x)}}\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1+\operatorname{sech}(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.04, size = 39, normalized size = 2.05

$$\sqrt{6}\sinh^{-1}\left(\sqrt{2}\sinh\left(\frac{x}{2}\right)\right)\sqrt{\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{\operatorname{sech}(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 3*Sech[x]],x]
```

```
[Out] Sqrt[6]*ArcSinh[Sqrt[2]*Sinh[x/2]]*Sqrt[Cosh[x]]*Sech[x/2]*Sqrt[1 + Sech[x]
]
```

fricas [B] time = 0.38, size = 233, normalized size = 12.26

$$\frac{1}{2}\sqrt{3}\log\left(-\frac{\cosh(x)^4 + (4\cosh(x) - 3)\sinh(x)^3 + \sinh(x)^4 - 3\cosh(x)^3 + \left(6\cosh(x)^2 - 9\cosh(x) + 5\right)\sinh(x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(-(cosh(x)^4 + (4*cosh(x) - 3)*sinh(x)^3 + sinh(x)^4 - 3*cos
h(x)^3 + (6*cosh(x)^2 - 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(
cosh(x) - 1)*sinh(x)^2 + sinh(x)^3 - 3*cosh(x)^2 + (3*cosh(x)^2 - 6*cosh(x)
 + 4)*sinh(x) + 4*cosh(x) - 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x
)^2 + (4*cosh(x)^3 - 9*cosh(x)^2 + 10*cosh(x) - 4)*sinh(x) - 4*cosh(x) + 4)
/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2
*sqrt(3)*log((sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
+ 1) + cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

giac [B] time = 0.14, size = 52, normalized size = 2.74

$$-\sqrt{3}\left(\log\left(\sqrt{e^{(2x)} + 1} - e^{x} + 1\right) + \log\left(\sqrt{e^{(2x)} + 1} - e^{x}\right) - \log\left(-\sqrt{e^{(2x)} + 1} + e^{x} + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1) + log(sqrt(e^(2*x) + 1) - e^x) -
 log(-sqrt(e^(2*x) + 1) + e^x + 1))
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \sqrt{3 + 3\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+3*sech(x))^(1/2),x)
```

```
[Out] int((3+3*sech(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3\,\operatorname{sech}(x)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(3*sech(x) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{\frac{3}{\cosh(x)}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3/cosh(x) + 3)^(1/2),x)
[Out] int((3/cosh(x) + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3}\int \sqrt{\operatorname{sech}{\left(x \right)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+3*sech(x))**(1/2),x)
[Out] sqrt(3)*Integral(sqrt(sech(x) + 1), x)
```

3.86 $\int \sqrt{3-3\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=21

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\operatorname{sech}(x)}}\right)$$

```
[Out] 2*arctanh(tanh(x)/(1-sech(x))^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3774, 203}

$$2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\operatorname{sech}(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 - Sech[x]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-3\operatorname{sech}(x)}\,dx &= -\left(6i\,\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\tanh(x)}{\sqrt{3-3\operatorname{sech}(x)}}\right)\right)\\ &= 2\sqrt{3}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{1-\operatorname{sech}(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.57, size = 51, normalized size = 2.43

$$\frac{\sqrt{3}\sqrt{e^{2x}+1}\sqrt{1-\operatorname{sech}(x)}\left(\sinh^{-1}(e^x)+\tanh^{-1}\left(\sqrt{e^{2x}+1}\right)\right)}{e^x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 3*Sech[x]],x]
```

```
[Out] (Sqrt[3]*Sqrt[1 + E^(2*x)]*(ArcSinh[E^x] + ArcTanh[Sqrt[1 + E^(2*x)]])*Sqrt
[1 - Sech[x]])/(-1 + E^x)
```

fricas [B] time = 0.39, size = 235, normalized size = 11.19

$$\frac{1}{2}\sqrt{3}\log\left(\frac{\cosh(x)^4 + (4\cosh(x) + 3)\sinh(x)^3 + \sinh(x)^4 + 3\cosh(x)^3 + \left(6\cosh(x)^2 + 9\cosh(x) + 5\right)\sinh(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log((cosh(x)^4 + (4*cosh(x) + 3)*sinh(x)^3 + sinh(x)^4 + 3*cosh
(x)^3 + (6*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(c
osh(x) + 1)*sinh(x)^2 + sinh(x)^3 + 3*cosh(x)^2 + (3*cosh(x)^2 + 6*cosh(x)
+ 4)*sinh(x) + 4*cosh(x) + 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x)
^2 + (4*cosh(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 4)*sinh(x) + 4*cosh(x) + 4)/
(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2*
sqrt(3)*log(-(sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
- 1) + cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

giac [B] time = 0.13, size = 69, normalized size = 3.29

$$\sqrt{3}\left(\log\left(\sqrt{e^{(2x)} + 1} - e^x + 1\right)\mathrm{sgn}\left(e^x - 1\right) - \log\left(\sqrt{e^{(2x)} + 1} - e^x\right)\mathrm{sgn}\left(e^x - 1\right) - \log\left(-\sqrt{e^{(2x)} + 1} + e^x + 1\right)\mathrm{sgn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1)*sgn(e^x - 1) - log(sqrt(e^(2*x) +
 1) - e^x)*sgn(e^x - 1) - log(-sqrt(e^(2*x) + 1) + e^x + 1)*sgn(e^x - 1))
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \sqrt{3 - 3\,\mathrm{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-3*sech(x))^(1/2),x)
```

```
[Out] int((3-3*sech(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3 \operatorname{sech}(x) + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-3*sech(x) + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{3 - \frac{3}{\cosh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 3/cosh(x))^(1/2),x)
[Out] int((3 - 3/cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\sqrt{3} \int \sqrt{1 - \operatorname{sech}{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-3*sech(x))**(1/2),x)
[Out] sqrt(3)*Integral(sqrt(1 - sech(x)), x)
```

3.87 $\int (a + b\,\mathbf{sech}(c + dx))^4\, dx$

Optimal. Leaf size=107

$$a^4x+\frac{b^2\left(17a^2+2b^2\right)\tanh(c+dx)}{3d}+\frac{2ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{d}+\frac{4ab^3\tanh(c+dx)\text{sech}(c+dx)}{3d}+\frac{b^2\tan$$

```
[Out] a^4*x+2*a*b*(2*a^2+b^2)*arctan(sinh(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tanh(d
*x+c)/d+4/3*a*b^3*sech(d*x+c)*tanh(d*x+c)/d+1/3*b^2*(a+b*sech(d*x+c))^2*tan
h(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3782, 4048, 3770, 3767, 8}

$$\frac{b^2\left(17a^2+2b^2\right)\tanh(c+dx)}{3d}+\frac{2ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{d}+a^4x+\frac{4ab^3\tanh(c+dx)\text{sech}(c+dx)}{3d}+\frac{b^2\tan$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^4,x]
```

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tanh[c + d*x])/(3*d) + (4*a*b^3*Sech[c + d*x]*Tanh[c + d*x])/(3*d) + (b^
2*(a + b*Sech[c + d*x])^2*Tanh[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
```

```
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\text{sech}(c+dx))^4\,dx = \frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d}+\frac{1}{3}\int (a+b\text{sech}(c+dx))\left(3a^3+b\left(9a^2+2b^2\right)\right.$$
$$= \frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d}+\frac{1}{6}\int \left(6a^4\right.$$
$$= a^4x+\frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d}+\left(2ab\right.$$
$$= a^4x+\frac{2ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{d}+\frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a}{}$$
$$= a^4x+\frac{2ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\left(17a^2+2b^2\right)\tanh(c+dx)}{3d}+\frac{4ab^3\text{sec}}{}$$

Mathematica [A] time = 0.25, size = 78, normalized size = 0.73

$$\frac{3a^4dx+6ab\left(2a^2+b^2\right)\tan^{-1}(\sinh(c+dx))+3b^2\tanh(c+dx)\left(6a^2+2ab\text{sech}(c+dx)+b^2\right)-b^4\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^4,x]
```

```
[Out] (3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + 3*b^2*(6*a^2 + b^2
 + 2*a*b*Sech[c + d*x])*Tanh[c + d*x] - b^4*Tanh[c + d*x]^3)/(3*d)
```

fricas [B] time = 0.41, size = 1028, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 + 12*a*b^3*cosh(
d*x + c)^5 + 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) + 2*a*b^3)*sinh(d*x + c
)^5 - 12*a*b^3*cosh(d*x + c) + 9*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^4 + 3*
(15*a^4*d*x*cosh(d*x + c)^2 + 3*a^4*d*x + 20*a*b^3*cosh(d*x + c) - 12*a^2*b
^2)*sinh(d*x + c)^4 - 36*a^2*b^2 - 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 +
10*a*b^3*cosh(d*x + c)^2 + 3*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x
*cosh(d*x + c)^4 + 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x - 24*a^2*b^2 - 4*b^
4 + 18*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((2*a^3*
b + a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c)*sinh(d*x + c
)^5 + (2*a^3*b + a*b^3)*sinh(d*x + c)^6 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)
^4 + 3*(2*a^3*b + a*b^3 + 5*(2*a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 3*(2*a^3*b
 + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c
)^2 + 3*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^4 + 2*a^3*b + a*b^3 + 6*(2*a^3*b
 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b + a*b^3)*cosh(d*x
+ c)^5 + 2*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + (2*a^3*b + a*b^3)*cosh(d*x +
 c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(3*a^4*d*x*co
sh(d*x + c)^5 + 10*a*b^3*cosh(d*x + c)^4 - 2*a*b^3 + 6*(a^4*d*x - 4*a^2*b^2
)*cosh(d*x + c)^3 + (3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x
 + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4
 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d
*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)
^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

giac [A] time = 0.13, size = 141, normalized size = 1.32

$$\frac{3\,(dx+c)a^4 + 12\left(2\,a^3b + ab^3\right)\arctan\left(e^{(dx+c)}\right) + \frac{4\left(3\,ab^3e^{(5\,dx+5\,c)} - 9\,a^2b^2e^{(4\,dx+4\,c)} - 18\,a^2b^2e^{(2\,dx+2\,c)} - 3\,b^4e^{(2\,dx+2\,c)} - 3\,ab^3e^{(dx+c)} - 9 \cdots\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 12*(2*a^3*b + a*b^3)*arctan(e^(d*x + c)) + 4*(3*a*b^
3*e^(5*d*x + 5*c) - 9*a^2*b^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c)
- 3*b^4*e^(2*d*x + 2*c) - 3*a*b^3*e^(d*x + c) - 9*a^2*b^2 - b^4)/(e^(2*d*x
+ 2*c) + 1)^3)/d
```

maple [A] time = 0.43, size = 121, normalized size = 1.13

$$a^4x+\frac{a^4c}{d}+\frac{8a^3b\arctan\left(e^{dx+c}\right)}{d}+\frac{6a^2b^2\tanh(dx+c)}{d}+\frac{2a\,b^3\text{sech}(dx+c)\tanh(dx+c)}{d}+\frac{4a\,b^3\arctan\left(e^{dx+c}\right)}{d}+\frac{2b^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+8/d*a^3*b*arctan(exp(d*x+c))+6/d*a^2*b^2*tanh(d*x+c)+2*a*b^
3*sech(d*x+c)*tanh(d*x+c)/d+4/d*a*b^3*arctan(exp(d*x+c))+2/3/d*b^4*tanh(d*x
+c)+1/3/d*b^4*tanh(d*x+c)*sech(d*x+c)^2
```

maxima [B] time = 0.92, size = 211, normalized size = 1.97

$$a^4x-4\,ab^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)+\frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4*a*b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))
/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 4/3*b^4*(3*e^(-2*d*x -
2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) +
 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) +
4*a^3*b*arctan(sinh(d*x + c))/d + 12*a^2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.41, size = 233, normalized size = 2.18

$$a^4\,x-\frac{\frac{12\,a^2\,b^2}{d}-\frac{4\,a\,b^3\,e^{c+d\,x}}{d}}{e^{2\,c+2\,d\,x}+1}-\frac{\frac{4\,b^4}{d}+\frac{8\,a\,b^3\,e^{c+d\,x}}{d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+\frac{8\,b^4}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{4\,\text{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a\,b^3\right.}{d\,\sqrt{4\,a^6\,b^2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^4,x)
```

```
[Out] a^4*x - ((12*a^2*b^2)/d - (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1)
- ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*
d*x) + 1) + (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(d^2)^(1/2) + 2*a^3*b*(d^
2)^(1/2)))/(d*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 + 4*a^4*b^
4 + 4*a^6*b^2)^(1/2))/(d^2)^(1/2)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{sech}\,(c + dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**4, x)
```

3.88 $\int (a+b\,\text{sech}(c+dx))^3\,dx$

Optimal. Leaf size=73

$$a^3x+\frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))}{2d}$$

```
[Out] a^3*x+1/2*b*(6*a^2+b^2)*arctan(sinh(d*x+c))/d+5/2*a*b^2*tanh(d*x+c)/d+1/2*b
^2*(a+b*sech(d*x+c))*tanh(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3782, 3770, 3767, 8}

$$\frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d}+a^3x+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^3,x]
```

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*d) + (5*a*b^2*Tanh[c + d
*x])/(2*d) + (b^2*(a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\text{sech}(c+dx))^3\,dx = \frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d}+\frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\text{sech}(c+dx)+5ab\right.$$

$$= a^3x+\frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d}+\frac{1}{2}\left(5ab^2\right)\int \text{sech}^2(c+dx)\,dx+\frac{1}{2}\left(b\left(6\right.\right.$$

$$= a^3x+\frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d}+\frac{(5}{}$$

$$= a^3x+\frac{b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2(a+b\text{sech}(c+dx}{2d}$$

Mathematica [A] time = 0.13, size = 55, normalized size = 0.75

$$\frac{2a^3dx+b\left(6a^2+b^2\right)\tan^{-1}(\sinh(c+dx))+b^2\tanh(c+dx)(6a+b\text{sech}(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^3,x]
```

```
[Out] (2*a^3*d*x + b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + b^2*(6*a + b*Sech[c +
d*x])*Tanh[c + d*x])/(2*d)
```

fricas [B] time = 0.40, size = 521, normalized size = 7.14

$$\frac{a^3dx\cosh(dx+c)^4+a^3dx\sinh(dx+c)^4+b^3\cosh(dx+c)^3+a^3dx-b^3\cosh(dx+c)+\left(4a^3dx\cosh(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (a^3*d*x*cosh(d*x + c)^4 + a^3*d*x*sinh(d*x + c)^4 + b^3*cosh(d*x + c)^3 +
a^3*d*x - b^3*cosh(d*x + c) + (4*a^3*d*x*cosh(d*x + c) + b^3)*sinh(d*x + c)
^3 - 6*a*b^2 + 2*(a^3*d*x - 3*a*b^2)*cosh(d*x + c)^2 + (6*a^3*d*x*cosh(d*x
+ c)^2 + 2*a^3*d*x + 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh(d*x + c)^2 + ((6*a
^2*b + b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)
^3 + (6*a^2*b + b^3)*sinh(d*x + c)^4 + 6*a^2*b + b^3 + 2*(6*a^2*b + b^3)*co
sh(d*x + c)^2 + 2*(6*a^2*b + b^3 + 3*(6*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(
```

```
d*x + c)^2 + 4*((6*a^2*b + b^3)*cosh(d*x + c)^3 + (6*a^2*b + b^3)*cosh(d*x
+ c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (4*a^3*d*x*cos
h(d*x + c)^3 + 3*b^3*cosh(d*x + c)^2 - b^3 + 4*(a^3*d*x - 3*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^
3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [A] time = 0.12, size = 92, normalized size = 1.26

$$\frac{(dx+c)a^3 + \left(6\,a^2b + b^3\right)\arctan\left(e^{(dx+c)}\right) + \frac{b^3e^{(3\,dx+3\,c)} - 6\,ab^2e^{(2\,dx+2\,c)} - b^3e^{(dx+c)} - 6\,ab^2}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^3 + (6*a^2*b + b^3)*arctan(e^(d*x + c)) + (b^3*e^(3*d*x + 3*c)
 - 6*a*b^2*e^(2*d*x + 2*c) - b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c) +
1)^2)/d
```

maple [A] time = 0.34, size = 80, normalized size = 1.10

$$a^3x + \frac{a^3c}{d} + \frac{6a^2b\arctan\left(e^{dx+c}\right)}{d} + \frac{3a\,b^2\tanh\left(dx+c\right)}{d} + \frac{b^3\mathrm{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{2d} + \frac{b^3\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+6/d*a^2*b*arctan(exp(d*x+c))+3*a*b^2*tanh(d*x+c)/d+1/2/d*b^
3*sech(d*x+c)*tanh(d*x+c)+1/d*b^3*arctan(exp(d*x+c))
```

maxima [A] time = 0.50, size = 114, normalized size = 1.56

$$a^3x - b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) + \frac{3\,a^2b\arctan\left(\sinh\left(dx+c\right)\right)}{d} + \frac{6\,ab^2}{d\left(e^{(-2\,dx-2\,c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*
(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*arctan(sinh(d*x + c
))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 1.40, size = 165, normalized size = 2.26

$$a^3\,x-\frac{\frac{6\,a\,b^2}{d}-\frac{b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{2\,c+2\,d\,x}+1}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(b^3\,\sqrt{d^2}+6\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}\right)\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}{\sqrt{d^2}}-\frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^3,x)
```

```
[Out] a^3*x - ((6*a*b^2)/d - (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1) + (atan
((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 6*a^2*b*(d^2)^(1/2)))/(d*(b^6 + 12*a^2
*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 + 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(d^2)^(1/
2) - (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{sech}\,(c + dx))^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**3, x)
```

3.89 $\int (a + b\,\mathbf{sech}(c + dx))^2\,dx$

Optimal. Leaf size=33

$$a^2x + \frac{2ab\tan^{-1}(\sinh(c + dx))}{d} + \frac{b^2\tanh(c + dx)}{d}$$

```
[Out] a^2*x+2*a*b*arctan(sinh(d*x+c))/d+b^2*tanh(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$a^2x + \frac{2ab\tan^{-1}(\sinh(c + dx))}{d} + \frac{b^2\tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*Tanh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\text{sech}(c+dx))^2\,dx = a^2x + (2ab)\int \text{sech}(c+dx)\,dx + b^2\int \text{sech}^2(c+dx)\,dx$$
$$= a^2x + \frac{2ab\tan^{-1}(\sinh(c+dx))}{d} + \frac{\left(ib^2\right)\text{Subst}(\int 1\,dx, x, -i\tanh(c+dx))}{d}$$
$$= a^2x + \frac{2ab\tan^{-1}(\sinh(c+dx))}{d} + \frac{b^2\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.07, size = 32, normalized size = 0.97

$$\frac{a\left(adx+2b\tan^{-1}(\sinh(c+dx))\right)+b^2\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^2,x]
```

```
[Out] (a*(a*d*x + 2*b*ArcTan[Sinh[c + d*x]]) + b^2*Tanh[c + d*x])/d
```

fricas [B] time = 0.42, size = 157, normalized size = 4.76

$$\frac{a^2dx\cosh(dx+c)^2 + 2\,a^2dx\cosh(dx+c)\sinh(dx+c) + a^2dx\sinh(dx+c)^2 + a^2dx - 2\,b^2 + 4\left(ab\cosh(dx+\right.}{d\cosh(dx+c)^2 + 2\,d\cosh(dx+c)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 + a^2*d*x - 2*b^2 + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b)*arctan(cosh(d*x + c) + sin
h(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(
d*x + c)^2 + d)
```

giac [A] time = 0.14, size = 43, normalized size = 1.30

$$\frac{(dx+c)a^2 + 4\,ab\arctan\left(e^{(dx+c)}\right) - \frac{2\,b^2}{e^{(2\,dx+2\,c)}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 4*a*b*arctan(e^(d*x + c)) - 2*b^2/(e^(2*d*x + 2*c) + 1))/d
```

maple [A]   time = 0.28, size = 42, normalized size = 1.27

$$a^2x + \frac{b^2\tanh(dx+c)}{d} + \frac{4ab\arctan\left(e^{dx+c}\right)}{d} + \frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^2,x)
```

```
[Out] a^2*x+b^2*tanh(d*x+c)/d+4/d*a*b*arctan(exp(d*x+c))+1/d*a^2*c
```

maxima [A]   time = 0.31, size = 41, normalized size = 1.24

$$a^2x + \frac{2\,ab\arctan(\sinh(dx+c))}{d} + \frac{2\,b^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*arctan(sinh(d*x + c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B]   time = 0.11, size = 70, normalized size = 2.12

$$a^2\,x - \frac{2\,b^2}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} + \frac{4\,\mathrm{atan}\left(\frac{a\,b\,e^{d\,x}\,e^c\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^2,x)
```

```
[Out] a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) + (4*atan((a*b*exp(d*x)*exp(c)*(
d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{sech}\,(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**2, x)
```

3.90 $\int (a+b\,\mathrm{sech}(c+dx))\,dx$

Optimal. Leaf size=16

$$ax+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

```
[Out] a*x+b*arctan(sinh(d*x+c))/d
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3770}

$$ax+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sech[c + d*x],x]
```

```
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\,\mathrm{sech}(c+dx))\,dx &= ax+b\int \mathrm{sech}(c+dx)\,dx\\ &= ax+\frac{b\tan^{-1}(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$ax+\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sech[c + d*x],x]
```

```
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

fricas [A] time = 0.40, size = 26, normalized size = 1.62

$$\frac{adx + 2\,b\arctan\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="fricas")
[Out] (a*d*x + 2*b*arctan(cosh(d*x + c) + sinh(d*x + c)))/d
```

giac [A] time = 0.11, size = 17, normalized size = 1.06

$$ax + \frac{2\,b\arctan\left(e^{(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="giac")
[Out] a*x + 2*b*arctan(e^(d*x + c))/d
```

maple [A] time = 0.02, size = 17, normalized size = 1.06

$$ax + \frac{b\arctan\left(\sinh\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sech(d*x+c),x)
[Out] a*x+b*arctan(sinh(d*x+c))/d
```

maxima [A] time = 1.18, size = 16, normalized size = 1.00

$$ax + \frac{b\arctan\left(\sinh\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="maxima")
[Out] a*x + b*arctan(sinh(d*x + c))/d
```

mupad [B] time = 1.30, size = 38, normalized size = 2.38

$$a\,x + \frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/cosh(c + d*x),x)
```

```
[Out] a*x + (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))
/(d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{sech}\,(c + dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sech(d*x+c),x)
```

```
[Out] Integral(a + b*sech(c + d*x), x)
```

3.91 $\int \frac{1}{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] x/a-2*b*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/d/(a-b)^(1/2)
/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3783, 2659, 208}

$$\frac{x}{a}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx = \frac{x}{a} - \frac{\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a}$$
$$= \frac{x}{a} + \frac{(2i)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}$$
$$= \frac{x}{a} - \frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.11, size = 60, normalized size = 1.02

$$\frac{\frac{2b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}} + \frac{c}{d} + x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-1),x]
```

```
[Out] (c/d + x + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt
[a^2 - b^2]*d))/a
```

fricas [A] time = 0.41, size = 270, normalized size = 4.58

$$\left[\frac{\left(a^2-b^2\right)dx - \sqrt{-a^2+b^2}\,b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)-a^2+2\,b^2+2\left(a^2\cosh(dx+c)+ab\right)\sinh(dx+c)+2\,\sqrt{-a^2+b^2}}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sinh(dx+c)}\right.}{\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] [((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(
d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b
))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d
*x + c) + b)*sinh(d*x + c) + a)))/((a^3 - a*b^2)*d), ((a^2 - b^2)*d*x + 2*s
```

```
qrt(a^2 - b^2)*b*arctan(-(a*cosh(d*x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 -
 b^2)))/((a^3 - a*b^2)*d)]
```

giac [A] time = 0.12, size = 56, normalized size = 0.95

$$-\frac{\frac{2\,b\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*b*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a) - (d*
x + c)/a)/d
```

maple [A] time = 0.21, size = 88, normalized size = 1.49

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{da\sqrt{(a+b)\,(a-b)}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c)),x)
```

```
[Out] -1/d/a*ln(tanh(1/2*d*x+1/2*c)-1)-2/d*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
anh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/a*ln(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.40, size = 131, normalized size = 2.22

$$\frac{x}{a}+\frac{b\,\ln\left(\frac{2\,b\,\mathrm{e}^{c+d\,x}}{a^2}-\frac{2\,b\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b\,\ln\left(\frac{2\,b\,\mathrm{e}^{c+d\,x}}{a^2}+\frac{2\,b\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x)),x)
```

```
[Out] x/a + (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a + b*exp(c + d*x)))/(a^2*(a +
b)^(1/2)*(b - a)^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2)) - (b*log((2*b*e
xp(c + d*x))/a^2 + (2*b*(a + b*exp(c + d*x)))/(a^2*(a + b)^(1/2)*(b - a)^(1
/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{sech}(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*sech(c + d*x)), x)
```

3.92 $\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))}+\frac{x}{a^2}$$

```
[Out] x/a^2-2*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c)
)
```

Rubi [A] time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 3919, 3831, 2659, 208}

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*(a + b*Sech[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
```

```
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx = \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} - \frac{\int \frac{-a^2+b^2+ab\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} - \frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))} + \frac{\left(2i\left(2a^2-b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x\right.}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{x}{a^2} - \frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))}$$

Mathematica [A] time = 0.41, size = 203, normalized size = 1.86

$$\frac{b\left(\left(a^2-b^2\right)^{3/2}(c+dx)+ab\sqrt{a^2-b^2}\,\sinh(c+dx)+\left(4a^2b-2b^3\right)\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)+a\cosh(c+dx)\left(\left(a^2\right.\right.}{a^2d(a-b)(a+b)\sqrt{a^2-b^2}\,(a\cosh(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-2),x]
```

```
[Out] (a*((a^2 - b^2)^(3/2)*(c + d*x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(
c + d*x)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x] + b*((a^2 - b^2)^(3/2)*(c + d*
x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x]))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]
*d*(b + a*Cosh[c + d*x]))
```

fricas [B] time = 0.42, size = 1207, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x
 + (2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)
*sinh(d*x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 +
 (2*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*
cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x
 + c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(a^2*b^3 -
b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^
5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*s
inh(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5
*b^2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(
d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d
*cosh(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)), -(2*a^3*b
^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^5 - 2*a^3
*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x - 2*(2*a^
3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)*sinh(d*
x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 + (2*a^3*
b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cosh(d*
x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 - b^2)) + 2*(a^2*b^3 - b^5 - (a^4*b
```

```
- 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2
 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*sinh(d*x + c))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^
4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c) + (a
^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c
) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))]
```

giac [A] time = 0.12, size = 134, normalized size = 1.23

$$-\frac{\frac{2\left(2\,a^{2}b-b^{3}\right)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\left(a^{4}-a^{2}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{2\left(b^{3}e^{(dx+c)}+ab^{2}\right)}{\left(a^{4}-a^{2}b^{2}\right)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}+a\right)}-\frac{dx+c}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(2*a^2*b - b^3)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/((a^4 - a^2
*b^2)*sqrt(a^2 - b^2)) + 2*(b^3*e^(d*x + c) + a*b^2)/((a^4 - a^2*b^2)*(a*e^
(2*d*x + 2*c) + 2*b*e^(d*x + c) + a)) - (d*x + c)/a^2)/d
```

maple [B] time = 0.20, size = 221, normalized size = 2.03

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{2}}+\frac{2b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(a^{2}-b^{2}\right)\left(\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{4b\arctan\left(\frac{(a-b)\tan}{\sqrt{(a+}}\right.}{d\,(a+b)\,(a-b)\,\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*ln(tanh(1/2*d*x+1/2*c)-1)+2/d/a*b^2/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/
(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)-4/d*b/(a+b)/(a-b)/((a
+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d/
a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))+1/d/a^2*ln(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.85, size = 296, normalized size = 2.72

$$\frac{\frac{2\,b^2}{d\,\left(a\,b^2-a^3\right)}+\frac{2\,b^3\,{\mathrm{e}}^{c+d\,x}}{a\,d\,\left(a\,b^2-a^3\right)}}{a+2\,b\,{\mathrm{e}}^{c+d\,x}+a\,{\mathrm{e}}^{2\,c+2\,d\,x}}+\frac{x}{a^2}+\frac{b\,\ln\left(\frac{2\,{\mathrm{e}}^{c+d\,x}\,\left(2\,a^2\,b-b^3\right)}{a^3\,\left(a^2-b^2\right)}-\frac{2\,b\,\left(2\,a^2-b^2\right)\,\left(a+b\,{\mathrm{e}}^{c+d\,x}\right)}{a^3\,{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}\right)\,\left(2\,a^2-b^2\right)}{a^2\,d\,{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}-\frac{b\,\ln\left(\frac{2\,{\mathrm{e}}^{c+d\,x}\,\left(2\,a^2\,b-b^3\right)}{a^3\,\left(a^2-b^2\right)}+\frac{2 \cdots}{\cdots}\right.}{a^2\,d\,(a+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^2,x)
```

```
[Out] ((2*b^2)/(d*(a*b^2 - a^3)) + (2*b^3*exp(c + d*x))/(a*d*(a*b^2 - a^3)))/(a +
 2*b*exp(c + d*x) + a*exp(2*c + 2*d*x)) + x/a^2 + (b*log((2*exp(c + d*x)*(2
*a^2*b - b^3))/(a^3*(a^2 - b^2)) - (2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x)))
/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b
- a)^(3/2)) - (b*log((2*exp(c + d*x)*(2*a^2*b - b^3))/(a^3*(a^2 - b^2)) + (
2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x)))/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*
(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{sech}\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-2), x)
```

3.93 $\int \frac{1}{(a+b\,\mathbf{sech}(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$\frac{x}{a^3}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\operatorname{sech}(c+dx))}+\frac{b^2\tanh(c+dx)}{2ad\left(a^2-b^2\right)(a+b\operatorname{sech}(c+dx))^2}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\ \cdots}{\cdots}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/\cdots}}$$

```
[Out] x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a
+b*sech(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tanh(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*
sech(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3785, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\operatorname{sech}(c+dx))}+\frac{b^2\tanh(c+\cdots}{2ad\left(a^2-b^2\right)(a+b\operatorname{se}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tanh[c + d*x])/
(2*a*(a^2 - b^2)*d*(a + b*Sech[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tanh[c +
 d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sech[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} - \frac{\int \frac{-2\left(a^2-b^2\right)+2ab\operatorname{sech}(c+dx)-b^2\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} + \frac{\int \frac{2\left(a^2\right.}{}}{}$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} -$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} -$$

$$= \frac{x}{a^3} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\operatorname{sech}(c+dx))} +$$

$$= \frac{x}{a^3} - \frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\operatorname{sech}(c}$$

Mathematica [A] time = 0.74, size = 205, normalized size = 1.18

$$\frac{\operatorname{sech}^3(c+dx)(a\cosh(c+dx)+b)\left(\frac{3ab^2\left(2a^2-b^2\right)\sinh(c+dx)(a\cosh(c+dx)+b)}{(a-b)^2(a+b)^2} + \frac{2b\left(6a^4-5a^2b^2+2b^4\right)(a\cosh(c+dx)+b)^2\tan^{-1}\left(\frac{(b-a)\tan}{\sqrt{a}}\right.}{\left(a^2-b^2\right)^{5/2}}\right.}{2a^3d(a+b\operatorname{sech}(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] ((b + a*Cosh[c + d*x])*Sech[c + d*x]^3*(2*(c + d*x)*(b + a*Cosh[c + d*x])^2
 + (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqr
t[a^2 - b^2]]*(b + a*Cosh[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sinh[c +
d*x])/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cosh[c + d*x])*Sin
h[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sech[c + d*x])^3)
```

fricas [B] time = 0.50, size = 4125, normalized size = 23.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(12*a^6*b^2 - 18*a^4*b^4 + 6*a^2*b^6 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
 - a^2*b^6)*d*x*cosh(d*x + c)^4 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)
*d*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a
^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 - 11*a^3*b^
5 + 4*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) -
 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - 2*(a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^
6 + 6*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x
 + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^
2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4
 + 5*a^2*b^6 - 2*b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a
^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x +
 c)^4 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 -
5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 +
 (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^
6*b + 7*a^4*b^3 - 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b
^3 - 8*a^2*b^5 + 4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^
2 + 6*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*
(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4
+ 2*a*b^6 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + 3*(6*a^5*b^
2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5
 + 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*cosh(d*x
+ c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a
*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x
+ c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(17*a^5*b^3 - 25*a^3
*b^5 + 8*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x +
c) + 2*(17*a^5*b^3 - 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
- a^2*b^6)*d*x*cosh(d*x + c)^3 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*
d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6
*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)
)*sinh(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^
4 + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b
- 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c)^3 + 2*(a^11 - a^9*b^2 -
3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 - 3*a^9*b^2
```

```
 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
 a^4*b^7)*d)*sinh(d*x + c)^3 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c) + 2*(3*(a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x
 + c)^2 + 6*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + (a
^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^
11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4
- a^5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c)^2 + (a^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d
*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c
)), -(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^
2*b^6)*d*x*cosh(d*x + c)^4 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*si
nh(d*x + c)^4 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7
 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) - 4*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - (a^8 - 3*a^6*b^2 + 3
*a^4*b^4 - a^2*b^6)*d*x + (6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 2*(
a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)^2 + (6*a^
6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a
^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*
b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - (6*a^6*b - 5*a^4*b^3
+ 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^4 + (6*a^6*b
- 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b
^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^6*b - 5*a^4
*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^6*b + 7*a^4*b^3 -
 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 +
4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2 + 6*(6*a^5*b^2
- 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(6*a^5*b^2 - 5*a^
3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^
6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + 3*(6*a^5*b^2 - 5*a^3*b^4 + 2
*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 + 4*b^7)*cosh(d*
x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cosh(d*x + c) + a*sinh(d*
x + c) + b)/sqrt(a^2 - b^2)) + (17*a^5*b^3 - 25*a^3*b^5 + 8*a*b^7 - 4*(a^7*
b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c) + (17*a^5*b^3 - 25*a^
3*b^5 + 8*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x +
c)^3 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x + 3*(7*a^5*b^3 - 11*a^
3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x +
 c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 2*(a^8 - a^6*b^2 -
3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^11 -
3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^4 + (a^11 - 3*a^9*b^2 + 3*
a^7*b^4 - a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
a^4*b^7)*d*cosh(d*x + c)^3 + 2*(a^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*
a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*
cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c)
^3 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + 2*(3*(a
```

```
^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^2 + 6*(a^10*b - 3*a^
8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + (a^11 - a^9*b^2 - 3*a^7*b^4
+ 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^11 - 3*a^9*b^2 + 3*a^7*b^4
 - a^5*b^6)*d + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)
^3 + 3*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c)^2 + (a^11
 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c) + (a^10*b -
 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c))]
```

giac [A] time = 0.14, size = 261, normalized size = 1.51

$$-\frac{\frac{\left(6\,a^4b-5\,a^2b^3+2\,b^5\right)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}+\frac{7\,a^3b^3e^{(3\,dx+3\,c)}-4\,ab^5e^{(3\,dx+3\,c)}+6\,a^4b^2e^{(2\,dx+2\,c)}+9\,a^2b^4e^{(2\,dx+2\,c)}-6\,b^6e^{(2\,dx+2\,c)}+17\,a^3b^3e^{(dx+c)}-8\,ab^5e^{(dx}}{\left(a^7-2\,a^5b^2+a^3b^4\right)\left(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*a^4*b - 5*a^2*b^3 + 2*b^5)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (7*a^3*b^3*e^(3*d*x + 3*c)
 - 4*a*b^5*e^(3*d*x + 3*c) + 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x
 + 2*c) - 6*b^6*e^(2*d*x + 2*c) + 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x +
 c) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x + 2*c
) + 2*b*e^(d*x + c) + a)^2) - (d*x + c)/a^3)/d
```

maple [B] time = 0.27, size = 660, normalized size = 3.82

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3}+\frac{6b^2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)^2(a-b)\left(a^2+2ab+b^2\right)}+\frac{}{da\left(\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*ln(tanh(1/2*d*x+1/2*c)-1)+6/d*b^2/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/d/a*b
^3/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^3-2/d/a^2*b^4/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d
*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+6/d*b^2/(t
anh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)-1/d/a*b^3/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)
^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-2/d/a^2*b^4/(tanh(1/2
*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tanh(1
```

```
/2*d*x+1/2*c)-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*
tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+5/d/a*b^3/(a^4-2*a^2*b^2+b^4)/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d/a
^3*b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))+1/d/a^3*ln(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-3), x)
```

3.94 $\int \frac{1}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sec}}{a-}}}{ad}$$

Mathematica [A] time = 2.47, size = 168, normalized size = 1.58

$$\frac{2b\tanh\left(\frac{1}{2}(c+dx)\right)\sqrt{a\cosh(c+dx)+b}\sqrt{\frac{b(\operatorname{sech}(c+dx)+1)}{b-a}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right)|\frac{a+b}{a-b}\right)}{\sqrt{a}\,d\sqrt{a+b}\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(\operatorname{sech}(c+dx)-1)}{a+b}}\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

fricas [F] time = 2.25, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{\sqrt{b\operatorname{sech}(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

3.95 $\int \frac{\cosh^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=146

$$-\frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)\cosh^2(x)}{3a^2}-\frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4}+\frac{\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{8a^3}+\frac{x\left(3a^4+4\right.}{8}$$

```
[Out] 1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-1/3*b*(2*a^2+3*b^2)*sinh(x)/a^4+1/8*(3*a^
2+4*b^2)*cosh(x)*sinh(x)/a^3-1/3*b*cosh(x)^2*sinh(x)/a^2+1/4*cosh(x)^3*sinh
(x)/a-2*b^5*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^5/(a-b)^(1/2)/(a+
b)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3853, 4104, 3919, 3831, 2659, 205}

$$\frac{x\left(4a^2b^2+3a^4+8b^4\right)}{8a^5}-\frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4}-\frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}+\frac{\left(3a^2+4b^2\right)\sinh(x)\cosh(x)}{8a^3}-\frac{b\sinh(x}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]) - (b*(2*a^2 + 3*b^2)*Sinh[
x])/(3*a^4) + ((3*a^2 + 4*b^2)*Cosh[x]*Sinh[x])/(8*a^3) - (b*Cosh[x]^2*Sinh
[x])/(3*a^2) + (Cosh[x]^3*Sinh[x])/(4*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh^3(x)\sinh(x)}{4a} + \frac{\int \frac{\cosh^3(x)\left(-4b+3a\text{sech}(x)+3b\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{4a}$$

$$= -\frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{\int \frac{\cosh^2(x)\left(-3\left(3a^2+4b^2\right)-ab\text{sech}(x)+8b^2\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{12a^2}$$

$$= \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} + \int \frac{\cosh(x)\left(-8b\left(2a^2+\right.\right.}{}$$

$$= -\frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\text{s}}{4a}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(}{3}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(}{3}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(}{3}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\,\sqrt{a+b}} - \frac{b\left(2a^2+3b^2\right)\sinh(x)}{3a^4} + \frac{\left(3a^2+4b^2\right)\text{c}}{8}$$

Mathematica [A] time = 0.28, size = 126, normalized size = 0.86

$$\frac{3a^4\sinh(4x)-8a^3b\sinh(3x)-24ab\left(3a^2+4b^2\right)\sinh(x)+24a^2\left(a^2+b^2\right)\sinh(2x)+\frac{192b^5\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+12}{96a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x + (192*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Sinh[x] + 24*a^2*(
a^2 + b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

fricas [B] time = 0.47, size = 2402, normalized size = 16.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 - 8*(a^5*
b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh(x))*si
nh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(a^6 - a
^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6 + 3*a^4
*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*a^5*b +
 a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*(a^6 -
a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh
(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3*b^3)*c
osh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4
- 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 24*(3*a^
5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b^5 + 21
*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^6 - a^2
*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x) - 30*(
3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^4)*cosh
(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^5*b - a
^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b^2 + 4*
a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 +
 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 - 192*(b^5*cosh(x)^4 +
4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)
^3 + b^5*sinh(x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2
*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
+ 2*(a*cosh(x) + b)*sinh(x) + a)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6
 - a^4*b^2)*cosh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 1
8*(a^6 - a^2*b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*co
sh(x)^3 - 15*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3
 - 4*a*b^5)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2
)*cosh(x)^4 + 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(
x)^2*sinh(x)^2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh
(x)^4), 1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 -
8*(a^5*b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh
(x))*sinh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(
a^6 - a^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6
+ 3*a^4*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*
a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*
(a^6 - a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^
4)*cosh(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3
*b^3)*cosh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a
^2*b^4 - 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 2
4*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b
^5 + 21*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^
```

```
6 - a^2*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)
 - 30*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^
4)*cosh(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^
5*b - a^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b
^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh
(x)^3 + 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 + 384*(b^5*cosh(
x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*
sinh(x)^3 + b^5*sinh(x)^4)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6 - a^4*b^2)*c
osh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 18*(a^6 - a^2*
b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^3 - 15*
(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3 - 4*a*b^5)*c
osh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2)*cosh(x)^4 +
 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(x)^2*sinh(x)^
2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh(x)^4)]
```

giac [A] time = 0.12, size = 182, normalized size = 1.25

$$-\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{3\,a^3e^{(4\,x)}-8\,a^2be^{(3\,x)}+24\,a^3e^{(2\,x)}+24\,ab^2e^{(2\,x)}-72\,a^2be^x-96\,b^3e^x}{192\,a^4}+\frac{\left(3\,a^4+4\,a^2b^2+8\right.}{8\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5) + 1/192*(3
*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x) - 72*a^2
*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x/a^5 + 1/192*(8
*a^3*b*e^x - 3*a^4 + 24*(3*a^3*b + 4*a*b^3)*e^(3*x) - 24*(a^4 + a^2*b^2)*e^
(2*x))*e^(-4*x)/a^5
```

maple [B] time = 0.16, size = 406, normalized size = 2.78

$$\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a}+\frac{7}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{5}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a}-\frac{7}{8a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{5}{8a\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sech(x)),x)
```

```
[Out] 3/8/a*ln(tanh(1/2*x)+1)+7/8/a/(tanh(1/2*x)-1)^2+5/8/a/(tanh(1/2*x)-1)-3/8/a
*ln(tanh(1/2*x)-1)-7/8/a/(tanh(1/2*x)+1)^2+5/8/a/(tanh(1/2*x)+1)-2*b^5/a^5/
((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/4/a/(ta
nh(1/2*x)-1)^4+1/2/a/(tanh(1/2*x)-1)^3+1/2/a/(tanh(1/2*x)+1)^3+1/a^4/(tanh(
```

```
1/2*x)-1)*b^3-1/a^5*ln(tanh(1/2*x)-1)*b^4+1/3/a^2/(tanh(1/2*x)-1)^3*b-1/2/a
^3/(tanh(1/2*x)+1)^2*b^2+1/a^4/(tanh(1/2*x)+1)*b^3-1/4/a/(tanh(1/2*x)+1)^4+
1/a^5*ln(tanh(1/2*x)+1)*b^4+1/a^2/(tanh(1/2*x)+1)*b+1/2/a^3/(tanh(1/2*x)+1)
*b^2+1/3/a^2/(tanh(1/2*x)+1)^3*b+1/2/a^3/(tanh(1/2*x)-1)*b^2+1/a^2/(tanh(1/
2*x)-1)*b+1/2/a^2/(tanh(1/2*x)-1)^2*b+1/2/a^3/(tanh(1/2*x)-1)^2*b^2-1/2/a^3
*ln(tanh(1/2*x)-1)*b^2-1/2/a^2/(tanh(1/2*x)+1)^2*b+1/2/a^3*ln(tanh(1/2*x)+1
)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.85, size = 251, normalized size = 1.72

$$\frac{e^{4x}}{64a}-\frac{e^{-4x}}{64a}+\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{8a^5}-\frac{e^{-2x}\left(a^2+b^2\right)}{8a^3}+\frac{e^{2x}\left(a^2+b^2\right)}{8a^3}+\frac{e^{-x}\left(3a^2b+4b^3\right)}{8a^4}+\frac{b\,e^{-3x}}{24a^2}-\frac{b\,e^{3x}}{24a^2}-\frac{e^{x}\left(3a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b/cosh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 4*a^2*b^2))/(8*a^5
) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) + (exp
(-x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(24
*a^2) - (exp(x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b^5*log((2*b^5*exp(x))/a^6 -
(2*b^5*(a + b*exp(x)))/(a^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b)^(1/
2)*(b - a)^(1/2)) - (b^5*log((2*b^5*exp(x))/a^6 + (2*b^5*(a + b*exp(x)))/(a
^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*sech(x)), x)
```

3.96 $\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx$

Optimal. Leaf size=112

$$\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)\cosh(x)}{2a^2}-\frac{bx\left(a^2+2b^2\right)}{2a^4}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}+\frac{\sinh(x)\cosh^2(x)}{3a}$$

```
[Out] -1/2*b*(a^2+2*b^2)*x/a^4+1/3*(2*a^2+3*b^2)*sinh(x)/a^3-1/2*b*cosh(x)*sinh(x
)/a^2+1/3*cosh(x)^2*sinh(x)/a+2*b^4*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1
/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3853, 4104, 3919, 3831, 2659, 205}

$$-\frac{bx\left(a^2+2b^2\right)}{2a^4}+\frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3}+\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)\cosh(x)}{2a^2}+\frac{\sinh(x)\cosh^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -(b*(a^2 + 2*b^2)*x)/(2*a^4) + (2*b^4*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
 + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Sinh[x])/(3*a^3) -
 (b*Cosh[x]*Sinh[x])/(2*a^2) + (Cosh[x]^2*Sinh[x])/(3*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{\int \frac{\cosh^2(x)\left(-3b+2a\text{sech}(x)+2b\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{3a}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} - \frac{\int \frac{\cosh(x)\left(-2\left(2a^2+3b^2\right)-ab\text{sech}(x)+3b^2\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{6a^2}$$

$$= \frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{\int \frac{-3b\left(a^2+2b^2\right)-3ab^2\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{6a^3}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{b^4\int \frac{\text{s}}{a+}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{b^3\int \frac{}{1+}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{\left(2b^3\right)\text{S}}{}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}} + \frac{\left(2a^2+3b^2\right)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} +$$

Mathematica **[A]** time = 0.17, size = 99, normalized size = 0.88

$$\frac{a^3\sinh(3x) - 6bx\left(a^2+2b^2\right) + 3a\left(3a^2+4b^2\right)\sinh(x) - \frac{24b^4\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 3a^2b\sinh(2x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*x - (24*b^4*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sinh[x] - 3*a^2*b*Sinh[2*x] + a^3*
Sinh[3*x])/(12*a^4)
```

fricas **[B]** time = 0.46, size = 1562, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(x)^6 - 3*(a^4*b - a
^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^2)*cosh(x))*sinh(x)
^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh(x)^3 + 3*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4 + 5*(a^5 - a^3*
b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 - a^3*
b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4*b + a^2*b^3 - 2*b^
5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^3*b^
2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 - 5*(a^5 - a^3*b^2)*c
osh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b + a^2*b^3 - 2*b^5)*x*
cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 - 24*(b^4*cosh
(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*
sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 +
 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*
sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*
sinh(x) + a)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^
5 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
 2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 -
a^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a
^4*b^2)*sinh(x)^3), 1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(
x)^6 - 3*(a^4*b - a^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^
2)*cosh(x))*sinh(x)^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh
(x)^3 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*
b^4 + 5*(a^5 - a^3*b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4
+ 2*(10*(a^5 - a^3*b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4
*b + a^2*b^3 - 2*b^5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3
- 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 -
5*(a^5 - a^3*b^2)*cosh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b +
a^2*b^3 - 2*b^5)*x*cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(
x)^2 - 48*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^
2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt
(a^2 - b^2)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^5
 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 + a
^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a
^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a^
4*b^2)*sinh(x)^3)]
```

giac [A] time = 0.14, size = 133, normalized size = 1.19

$$\frac{2\,b^4\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^4}+\frac{a^2e^{(3\,x)}-3\,abe^{(2\,x)}+9\,a^2e^x+12\,b^2e^x}{24\,a^3}-\frac{\left(a^2b+2\,b^3\right)x}{2\,a^4}+\frac{\left(3\,a^2be^x-a^3-3\left(3\,a^3+4\,ab^2\right)e^{(2\,x)}\right)e}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^2
*e^(3*x) - 3*a*b*e^(2*x) + 9*a^2*e^x + 12*b^2*e^x)/a^3 - 1/2*(a^2*b + 2*b^3
)*x/a^4 + 1/24*(3*a^2*b*e^x - a^3 - 3*(3*a^3 + 4*a*b^2)*e^(2*x))*e^(-3*x)/a
^4
```

maple [B] time = 0.15, size = 264, normalized size = 2.36

$$-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b}{2a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{b^{2}}{a^{3}\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sech(x)),x)
```

```
[Out] -1/3/a/(tanh(1/2*x)-1)^3-1/2/a/(tanh(1/2*x)-1)^2-1/2/a^2/(tanh(1/2*x)-1)^2*
b-1/a/(tanh(1/2*x)-1)-1/2/a^2/(tanh(1/2*x)-1)*b-1/a^3/(tanh(1/2*x)-1)*b^2+1
/2*b/a^2*ln(tanh(1/2*x)-1)+b^3/a^4*ln(tanh(1/2*x)-1)+2*b^4/a^4/((a+b)*(a-b)
)^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/3/a/(tanh(1/2*x)+1)
^3+1/2/a/(tanh(1/2*x)+1)^2+1/2/a^2/(tanh(1/2*x)+1)^2*b-1/a/(tanh(1/2*x)+1)-
1/2/a^2/(tanh(1/2*x)+1)*b-1/a^3/(tanh(1/2*x)+1)*b^2-1/2*b/a^2*ln(tanh(1/2*x
)+1)-b^3/a^4*ln(tanh(1/2*x)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.71, size = 209, normalized size = 1.87

$$\frac{e^{3x}}{24a}-\frac{e^{-3x}}{24a}-\frac{x\left(a^{2}b+2b^{3}\right)}{2a^{4}}+\frac{e^{x}\left(3a^{2}+4b^{2}\right)}{8a^{3}}+\frac{b\,e^{-2x}}{8a^{2}}-\frac{b\,e^{2x}}{8a^{2}}-\frac{e^{-x}\left(3a^{2}+4b^{2}\right)}{8a^{3}}+\frac{b^{4}\ln\left(-\frac{2b^{4}e^{x}}{a^{5}}-\frac{2b^{4}\left(a+b\,e^{x}\right)}{a^{5}\sqrt{a+b}\sqrt{b-a}}\right)}{a^{4}\sqrt{a+b}\sqrt{b-a}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) - (x*(a^2*b + 2*b^3))/(2*a^4) + (exp(x)*
(3*a^2 + 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (
exp(-x)*(3*a^2 + 4*b^2))/(8*a^3) + (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^4*(
a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))))/(a^4*(a + b)^(1/2)*(b -
a)^(1/2)) - (b^4*log((2*b^4*(a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)^(1/2
)) - (2*b^4*exp(x))/a^5))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*sech(x)), x)
```

3.97 $\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)}{a^2}+\frac{x\left(a^2+2b^2\right)}{2a^3}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] 1/2*(a^2+2*b^2)*x/a^3-b*sinh(x)/a^2+1/2*cosh(x)*sinh(x)/a-2*b^3*arctan((a-b
)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3853, 4104, 3919, 3831, 2659, 205}

$$\frac{x\left(a^2+2b^2\right)}{2a^3}-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)}{a^2}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Sinh[x])/a^2 + (Cosh[x]*Sinh[x])/(2
*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int \frac{\cosh(x)\left(-2b+a\text{sech}(x)+b\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{2a}$$
$$= -\frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{\int \frac{-a^2-2b^2-ab\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{2a^2}$$
$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^3\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{a^3}$$
$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^3}$$
$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3}$$
$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}$$

Mathematica [A] time = 0.13, size = 78, normalized size = 0.92

$$\frac{\frac{8b^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 2a^2x + a^2\sinh(2x) - 4ab\sinh(x) + 4b^2x}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*a^2*x + 4*b^2*x + (8*b^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

fricas [B] time = 0.44, size = 860, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*sinh(x)^4 - a^4 + a^2*b^2
 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*
```

```
(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 - a^2*b^2)*
cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x
)^2 - 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(-a^2 +
 b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*
(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b
))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a
)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cosh(x)
^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)*sin
h(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) + (a^5
 - a^3*b^2)*sinh(x)^2), 1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*si
nh(x)^4 - a^4 + a^2*b^2 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b
- a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3
+ 2*(3*(a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b -
 a*b^3)*cosh(x))*sinh(x)^2 + 16*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^
3*sinh(x)^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 -
 b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cos
h(x)^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)
*sinh(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) +
(a^5 - a^3*b^2)*sinh(x)^2)]
```

giac [A] time = 0.14, size = 92, normalized size = 1.08

$$-\frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{ae^{(2\,x)}-4\,be^x}{8\,a^2}+\frac{\left(a^2+2\,b^2\right)x}{2\,a^3}+\frac{\left(4\,abe^x-a^2\right)e^{(-2\,x)}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*e
^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x - a^2)*e^(
-2*x)/a^3
```

maple [B] time = 0.15, size = 174, normalized size = 2.05

$$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b}{a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)b^2}{a^3}-\frac{2b^3\arctan\left(\right.}{a^3\sqrt{(a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sech(x)),x)
```

```
[Out] 1/2/a/(tanh(1/2*x)-1)^2+1/2/a/(tanh(1/2*x)-1)+1/a^2/(tanh(1/2*x)-1)*b-1/2/a
*ln(tanh(1/2*x)-1)-1/a^3*ln(tanh(1/2*x)-1)*b^2-2*b^3/a^3/((a+b)*(a-b))^(1/2
```

```
)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2/a/(tanh(1/2*x)+1)^2+1/2
/a/(tanh(1/2*x)+1)+1/a^2/(tanh(1/2*x)+1)*b+1/2/a*ln(tanh(1/2*x)+1)+1/a^3*ln
(tanh(1/2*x)+1)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.58, size = 167, normalized size = 1.96

$$\frac{e^{2x}}{8a}-\frac{e^{-2x}}{8a}-\frac{b\,e^x}{2a^2}+\frac{b\,e^{-x}}{2a^2}+\frac{x\left(a^2+2b^2\right)}{2a^3}+\frac{b^3\,\ln\left(\frac{2b^3\,e^x}{a^4}-\frac{2b^3\,(a+b\,e^x)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b^3\,\ln\left(\frac{2b^3\,e^x}{a^4}+\frac{2b^3\,(a+b\,e^x)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2)
 + (x*(a^2 + 2*b^2))/(2*a^3) + (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a + b*
exp(x)))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^(1/
2)) - (b^3*log((2*b^3*exp(x))/a^4 + (2*b^3*(a + b*exp(x)))/(a^4*(a + b)^(1/
2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{a+b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*sech(x)), x)
```

3.98 $\int \frac{\cosh(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sinh(x)}{a}$$

```
[Out] -b*x/a^2+sinh(x)/a+2*b^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^2/(a
-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3853, 12, 3783, 2659, 205}

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqr
t[a - b]*Sqrt[a + b]) + Sinh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(x)}{a+b\,\mathrm{sech}(x)}\,dx &= \frac{\sinh(x)}{a}-\frac{\int \frac{b}{a+b\,\mathrm{sech}(x)}\,dx}{a}\\
&= \frac{\sinh(x)}{a}-\frac{b\int \frac{1}{a+b\,\mathrm{sech}(x)}\,dx}{a}\\
&= -\frac{bx}{a^2}+\frac{\sinh(x)}{a}+\frac{b\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^2}\\
&= -\frac{bx}{a^2}+\frac{\sinh(x)}{a}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2}+\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\sinh(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 57, normalized size = 0.92

$$
\frac{b\left(-\frac{2b\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-x\right)+a\sinh(x)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] (b*(-x - (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
) + a*Sinh[x])/a^2
```

fricas [B] time = 0.43, size = 430, normalized size = 6.94

$$\left[-\frac{a^3 - ab^2 + 2\left(a^2b - b^3\right)x\cosh(x) - \left(a^3 - ab^2\right)\cosh(x)^2 - \left(a^3 - ab^2\right)\sinh(x)^2 + 2\left(b^2\cosh(x) + b^2\sinh(x)\right)\sqrt{-a}}{2\left(\left(a^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) - (a^3 - a*b^2)*cosh(x)^2 -
(a^3 - a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*lo
g((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*co
sh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 2*(
(a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*cosh(x)
+ (a^4 - a^2*b^2)*sinh(x)), -1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) -
 (a^3 - a*b^2)*cosh(x)^2 - (a^3 - a*b^2)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*s
inh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)
) + 2*((a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*c
osh(x) + (a^4 - a^2*b^2)*sinh(x))]
```

giac [A] time = 0.11, size = 62, normalized size = 1.00

$$\frac{2\,b^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2} - \frac{bx}{a^2} - \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) - b*x/a^2 -
 1/2*e^(-x)/a + 1/2*e^x/a
```

maple [A] time = 0.14, size = 94, normalized size = 1.52

$$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{2b^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)\,(a-b)}}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sech(x)),x)
```

```
[Out] -1/a/(tanh(1/2*x)-1)+b/a^2*ln(tanh(1/2*x)-1)+2*b^2/a^2/((a+b)*(a-b))^(1/2)*
arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/a/(tanh(1/2*x)+1)-b/a^2*ln(
tanh(1/2*x)+1)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 1.48, size = 139, normalized size = 2.24

$$\frac{e^x}{2\,a}-\frac{e^{-x}}{2\,a}-\frac{b\,x}{a^2}+\frac{b^2\,\ln\left(-\frac{2\,b^2\,e^x}{a^3}-\frac{2\,b^2\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b^2\,\ln\left(\frac{2\,b^2\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{2\,b^2\,e^x}{a^3}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b/cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - exp(-x)/(2*a) - (b*x)/a^2 + (b^2*log(- (2*b^2*exp(x))/a^3 -
(2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a^2*(a + b)^(1/
2)*(b - a)^(1/2)) - (b^2*log((2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b -
 a)^(1/2)) - (2*b^2*exp(x))/a^3))/(a^2*(a + b)^(1/2)*(b - a)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{a+b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*sech(x)), x)
```

3.99 $\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx$

Optimal. Leaf size=42

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] 2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3831, 2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sech[x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b}$$

$$=\frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.98

$$-\frac{2\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sech[x]),x]
```

```
[Out] (-2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

fricas [A] time = 0.41, size = 165, normalized size = 3.93

$$\left[-\frac{\sqrt{-a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{-a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)}{a^2-b^2},-\frac{2\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) +
a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b
)*sinh(x) + a))/(a^2 - b^2), -2*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^
2 - b^2))/sqrt(a^2 - b^2)]
```

giac [A] time = 0.13, size = 32, normalized size = 0.76

$$\frac{2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

maple [A] time = 0.08, size = 36, normalized size = 0.86

$$\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sech(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.12, size = 43, normalized size = 1.02

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b/cosh(x))),x)
```

```
[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*exp(x))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^(1
/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*sech(x)), x)
```

3.100 $\int \frac{\text{sech}^2(x)}{a+b\,\text{sech}(x)}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}(\sinh(x))}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] arctan(sinh(x))/b-2*a*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(
1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3789, 3770, 3831, 2659, 205}

$$\frac{\tan^{-1}(\sinh(x))}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/b - (2*a*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b*Sqrt[a + b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{b} - \frac{a\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b} \\
&= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{a\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^2} \\
&= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= \frac{\tan^{-1}(\sinh(x))}{b} - \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$\frac{2\left(\frac{a\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] + (a*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2]))/b
```

fricas [A] time = 0.44, size = 219, normalized size = 4.06

$$\left[-\frac{\sqrt{-a^2+b^2}\,a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{-a^2+b^2}\,(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)-2\left(a^2\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) + a)) - 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)
, 2*(sqrt(a^2 - b^2)*a*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2))
 + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)]
```

giac [A] time = 0.12, size = 45, normalized size = 0.83

$$-\frac{2\,a\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b}+\frac{2\,\arctan\left(e^x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b) + 2*arctan(e^x
)/b
```

maple [A] time = 0.09, size = 51, normalized size = 0.94

$$-\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)\,(a-b)}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sech(x)),x)
```

```
[Out] -2*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2/
b*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.01, size = 286, normalized size = 5.30

$$\frac{a\ln\left(64\,a\,b^4-64\,a^3\,b^2+128\,b^5\,e^x-64\,a\,b^3\,\sqrt{b^2-a^2}+32\,a^3\,b\,\sqrt{b^2-a^2}+32\,a^4\,b\,e^x-128\,b^4\,e^x\,\sqrt{b^2-a^2}-160\right.}{b\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b/cosh(x))),x)
```

```
[Out] (a*log(64*a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) - 64*a*b^3*(b^2 - a^2)^(1/2)
+ 32*a^3*b*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) - 128*b^4*exp(x)*(b^2 - a^2)
^(1/2) - 160*a^2*b^3*exp(x) + 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(b^2
 - a^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b - (a*log(64*
a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) + 64*a*b^3*(b^2 - a^2)^(1/2) - 32*a^3*b
*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) + 128*b^4*exp(x)*(b^2 - a^2)^(1/2) - 1
60*a^2*b^3*exp(x) - 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(b^2 - a^2)^(1
/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*sech(x)), x)
```

3.101 $\int \frac{\text{sech}^3(x)}{a+b\text{sech}(x)}\,dx$

Optimal. Leaf size=64

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}$$

```
[Out] -a*arctan(sinh(x))/b^2+2*a^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^
2/(a-b)^(1/2)/(a+b)^(1/2)+tanh(x)/b
```

Rubi [A] time = 0.14, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3790, 3789, 3770, 3831, 2659, 205}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -((a*ArcTan[Sinh[x]])/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]) + Tanh[x]/b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx &= \frac{\tanh(x)}{b}-\frac{a\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx}{b}\\
&= \frac{\tanh(x)}{b}-\frac{a\int \operatorname{sech}(x)\,dx}{b^2}+\frac{a^2\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b^2}\\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}+\frac{a^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^3}\\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= -\frac{a\tan^{-1}(\sinh(x))}{b^2}+\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}+\frac{\tanh(x)}{b}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 63, normalized size = 0.98

$$\frac{-\frac{2a^2\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+b\tanh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-2*a*ArcTan[Tanh[x/2]] - (2*a^2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + b*Tanh[x])/b^2
```

fricas [B] time = 0.45, size = 504, normalized size = 7.88

$$\left[-\frac{2\,a^2b - 2\,b^3 + \left(a^2\cosh(x)^2 + 2\,a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 + a^2\right)\sqrt{-a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2\,ab\cos}{a\cosh(}\right.}{a^2b^2 - b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(2*a^2*b - 2*b^3 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
 + a^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 -
 a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(x) + sinh(x)
))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^4)*cosh(x)*s
inh(x) + (a^2*b^2 - b^4)*sinh(x)^2), -2*(a^2*b - b^3 + (a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x
) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^
2 + 2*(a^3 - a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(
x) + sinh(x)))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^
4)*cosh(x)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2)]
```

giac [A] time = 0.12, size = 61, normalized size = 0.95

$$\frac{2\,a^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2} - \frac{2\,a\arctan\left(e^x\right)}{b^2} - \frac{2}{b\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^2) - 2*a*arcta
n(e^x)/b^2 - 2/(b*(e^(2*x) + 1))
```

maple [A] time = 0.10, size = 73, normalized size = 1.14

$$\frac{2a^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)\,(a-b)}} + \frac{2\tanh\left(\frac{x}{2}\right)}{b\left(\tanh^2\left(\frac{x}{2}\right)+1\right)} - \frac{2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sech(x)),x)
```

```
[Out] 2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
+2/b*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/b^2*a*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.88, size = 294, normalized size = 4.59

$$\frac{a^2\ln\Big(64\,a^3\,b-64\,a\,b^3+32\,a^3\,\sqrt{b^2-a^2}-32\,a^4\,\mathrm{e}^x-128\,b^4\,\mathrm{e}^x-64\,a\,b^2\,\sqrt{b^2-a^2}-128\,b^3\,\mathrm{e}^x\,\sqrt{b^2-a^2}+160\,a^2}{b^2\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b/cosh(x))),x)
```

```
[Out] (a*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/b^2 - 2/(b + b*exp(
2*x)) + (a^2*log(64*a^3*b - 64*a*b^3 + 32*a^3*(b^2 - a^2)^(1/2) - 32*a^4*ex
p(x) - 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128*b^3*exp(x)*(b^2 -
a^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2 - a^2)^(1/2)))/(b^2*
(b^2 - a^2)^(1/2)) - (a^2*log(64*a*b^3 - 64*a^3*b + 32*a^3*(b^2 - a^2)^(1/2
) + 32*a^4*exp(x) + 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128*b^3*e
xp(x)*(b^2 - a^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2 - a^2)^
(1/2)))/(b^2*(b^2 - a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}(x)}{a+b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*sech(x)), x)
```

3.102 $\int \frac{\text{sech}^4(x)}{a+b\text{sech}(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

```
[Out] 1/2*(2*a^2+b^2)*arctan(sinh(x))/b^3-2*a^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a
+b)^(1/2))/b^3/(a-b)^(1/2)/(a+b)^(1/2)-a*tanh(x)/b^2+1/2*sech(x)*tanh(x)/b
```

Rubi [A] time = 0.24, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3851, 4082, 3998, 3770, 3831, 2659, 205}

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Tanh[x])/b^2 + (Sech[
x]*Tanh[x])/(2*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\operatorname{sech}(x)\tanh(x)}{2b} + \frac{\int \frac{\operatorname{sech}(x)\left(a+b\operatorname{sech}(x)-2a\operatorname{sech}^2(x)\right)}{a+b\operatorname{sech}(x)}\,dx}{2b}$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} + \frac{\int \frac{\operatorname{sech}(x)\left(ab+\left(2a^2+b^2\right)\operatorname{sech}(x)\right)}{a+b\operatorname{sech}(x)}\,dx}{2b^2}$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{a^3\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b^3} + \frac{\left(2a^2+b^2\right)\int \operatorname{sech}(x)\,dx}{2b^3}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^4}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx\right.}{b^4}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b}$$

Mathematica [**A**] time = 0.23, size = 82, normalized size = 0.94

$$\frac{2\left(2a^2+b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{4a^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + b\tanh(x)(b\operatorname{sech}(x)-2a)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*ArcTan[Tanh[x/2]] + (4*a^3*ArcTan[((-a + b)*Tanh[x/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + b*(-2*a + b*Sech[x])*Tanh[x])/(2*b^3)
```

fricas [**B**] time = 0.53, size = 1444, normalized size = 16.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3
 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cos
```

```
h(x))*sinh(x)^2 - (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4
+ 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh
(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sin
h(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sq
rt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*
b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + ((2*a^4 - a^2*b^2 - b^4)*cosh
(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2*a^4 - a^2*b^2 - b^
4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2
+ 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2
 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b^2 - b^4)*cosh(x))*
sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cosh(x) - (a^2*b^2 - b
^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*sinh(x))/(a^2
*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^
3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 -
b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3
 + (a^2*b^3 - b^5)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*
cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^
3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 2*(a^3*cosh(x)^4 + 4
*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*c
osh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + ((2*a^4 -
a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2
*a^4 - a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^
2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*c
osh(x)^2)*sinh(x)^2 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b
^2 - b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cos
h(x) - (a^2*b^2 - b^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cos
h(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^
5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x
)^2 + 2*(a^2*b^3 - b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b
^3 - b^5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))]
```

giac [A] time = 0.12, size = 89, normalized size = 1.02

$$-\frac{2\,a^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{\left(2\,a^2+b^2\right)\arctan\left(e^x\right)}{b^3}+\frac{be^{(3\,x)}+2\,ae^{(2\,x)}-be^x+2\,a}{b^2\left(e^{(2\,x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^3) + (2*a^2 +
 b^2)*arctan(e^x)/b^3 + (b*e^(3*x) + 2*a*e^(2*x) - b*e^x + 2*a)/(b^2*(e^(2*
x) + 1)^2)
```

maple [A] time = 0.10, size = 146, normalized size = 1.68

$$-\frac{2a^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}}-\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)a}{b^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tanh^3\left(\frac{x}{2}\right)}{b\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{2\tanh\left(\frac{x}{2}\right)a}{b^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{\tanh\left(\frac{x}{2}\right)}{b\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+\cdots 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sech(x)),x)
```

```
[Out] -2/b^3*a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)-2/b^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a-1/b/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^3-2/b^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a+1/b/(tanh(1/2*x)^2+1)^2*tanh
(1/2*x)+2/b^3*arctan(tanh(1/2*x))*a^2+1/b*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.08, size = 476, normalized size = 5.47

$$\frac{e^x}{b+b\,e^{2x}}-\frac{2\,e^x}{b+2\,b\,e^{2x}+b\,e^{4x}}+\frac{2\,a}{b^2\,e^{2x}+b^2}-\frac{\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i}{2\,b}-\frac{a^2\,\left(\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1\cdots\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b/cosh(x))),x)
```

```
[Out] exp(x)/(b + b*exp(2*x)) - (2*exp(x))/(b + 2*b*exp(2*x) + b*exp(4*x)) + (2*a
)/(b^2*exp(2*x) + b^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2*b
) - (a^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/b^3 - (a^3*log(16*a
*b^5 - 48*a^5*b - 24*a^5*(b^2 - a^2)^(1/2) + 32*a^3*b^3 + 24*a^6*exp(x) + 3
2*b^6*exp(x) + 16*a*b^4*(b^2 - a^2)^(1/2) + 40*a^3*b^2*(b^2 - a^2)^(1/2) +
32*b^5*exp(x)*(b^2 - a^2)^(1/2) + 56*a^2*b^4*exp(x) - 112*a^4*b^2*exp(x) +
72*a^2*b^3*exp(x)*(b^2 - a^2)^(1/2) - 72*a^4*b*exp(x)*(b^2 - a^2)^(1/2)))/(
b^3*(b^2 - a^2)^(1/2)) + (a^3*log(16*a*b^5 - 48*a^5*b + 24*a^5*(b^2 - a^2)^
```

```
(1/2) + 32*a^3*b^3 + 24*a^6*exp(x) + 32*b^6*exp(x) - 16*a*b^4*(b^2 - a^2)^(
1/2) - 40*a^3*b^2*(b^2 - a^2)^(1/2) - 32*b^5*exp(x)*(b^2 - a^2)^(1/2) + 56*
a^2*b^4*exp(x) - 112*a^4*b^2*exp(x) - 72*a^2*b^3*exp(x)*(b^2 - a^2)^(1/2) +
 72*a^4*b*exp(x)*(b^2 - a^2)^(1/2)))/(b^3*(b^2 - a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*sech(x)), x)
```

3.103 $\int \frac{\tanh^6(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{a}-\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)(4-3\mathrm{sech}(x))}{12a}-\frac{\tanh(x)(8-3\mathrm{sech}(x))}{8a}$$

```
[Out] x/a-3/8*arctan(sinh(x))/a-1/8*(8-3*sech(x))*tanh(x)/a-1/12*(4-3*sech(x))*ta
nh(x)^3/a
```

Rubi [A] time = 0.10, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$\frac{x}{a}-\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)(4-3\mathrm{sech}(x))}{12a}-\frac{\tanh(x)(8-3\mathrm{sech}(x))}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*a) - ((8 - 3*Sech[x])*Tanh[x])/(8*a) - ((4 - 3
*Sech[x])*Tanh[x]^3)/(12*a)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^6(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\int(-a+a\operatorname{sech}(x))\tanh^4(x)\,dx}{a^2}$$
$$= -\frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a} - \frac{\int(-4a+3a\operatorname{sech}(x))\tanh^2(x)\,dx}{4a^2}$$
$$= -\frac{(8-3\operatorname{sech}(x))\tanh(x)}{8a} - \frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a} - \frac{\int(-8a+3a\operatorname{sech}(x))\,dx}{8a^2}$$
$$= \frac{x}{a} - \frac{(8-3\operatorname{sech}(x))\tanh(x)}{8a} - \frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a} - \frac{3\int\operatorname{sech}(x)\,dx}{8a}$$
$$= \frac{x}{a} - \frac{3\tan^{-1}(\sinh(x))}{8a} - \frac{(8-3\operatorname{sech}(x))\tanh(x)}{8a} - \frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a}$$

Mathematica [A] time = 0.12, size = 60, normalized size = 1.25

$$\frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(6\left(4x-3\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\tanh(x)\left(-6\operatorname{sech}^3(x)+8\operatorname{sech}^2(x)+15\operatorname{sech}(x)-32\right)\right)}{12a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*Sech[x]*(6*(4*x - 3*ArcTan[Tanh[x/2]]) + (-32 + 15*Sech[x] + 8
*Sech[x]^2 - 6*Sech[x]^3)*Tanh[x]))/(12*a*(1 + Sech[x]))
```

fricas [B] time = 0.43, size = 686, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*x*cosh(x)^8 + 12*x*sinh(x)^8 + 3*(32*x*cosh(x) + 5)*sinh(x)^7 + 48
*(x + 1)*cosh(x)^6 + 15*cosh(x)^7 + 3*(112*x*cosh(x)^2 + 16*x + 35*cosh(x)
+ 16)*sinh(x)^6 + 3*(224*x*cosh(x)^3 + 96*(x + 1)*cosh(x) + 105*cosh(x)^2 -
 3)*sinh(x)^5 + 24*(3*x + 4)*cosh(x)^4 - 9*cosh(x)^5 + 3*(280*x*cosh(x)^4 +
 240*(x + 1)*cosh(x)^2 + 175*cosh(x)^3 + 24*x - 15*cosh(x) + 32)*sinh(x)^4
+ 3*(224*x*cosh(x)^5 + 320*(x + 1)*cosh(x)^3 + 175*cosh(x)^4 + 32*(3*x + 4)
*cosh(x) - 30*cosh(x)^2 + 3)*sinh(x)^3 + 16*(3*x + 5)*cosh(x)^2 + 9*cosh(x)
^3 + (336*x*cosh(x)^6 + 720*(x + 1)*cosh(x)^4 + 315*cosh(x)^5 + 144*(3*x +
4)*cosh(x)^2 - 90*cosh(x)^3 + 48*x + 27*cosh(x) + 80)*sinh(x)^2 - 9*(cosh(x
)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*c
```

```
osh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cos
h(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^
2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(
x) + 1)*arctan(cosh(x) + sinh(x)) + (96*x*cosh(x)^7 + 288*(x + 1)*cosh(x)^5
 + 105*cosh(x)^6 + 96*(3*x + 4)*cosh(x)^3 - 45*cosh(x)^4 + 32*(3*x + 5)*cos
h(x) + 27*cosh(x)^2 - 15)*sinh(x) + 12*x - 15*cosh(x) + 32)/(a*cosh(x)^8 +
8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4*(7*a*cosh(x)^2 + a)
*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5 + 6*a*cosh(x)^4 + 2*
(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(7*a*cosh(x)^5 + 10*a
*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4*(7*a*cosh(x)^6 + 15
*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh(x)^7 + 3*a*cosh(x)^
5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

giac [A] time = 0.12, size = 69, normalized size = 1.44

$$\frac{x}{a} - \frac{3 \arctan\left(e^{x}\right)}{4\,a} + \frac{15\,e^{(7\,x)} + 48\,e^{(6\,x)} - 9\,e^{(5\,x)} + 96\,e^{(4\,x)} + 9\,e^{(3\,x)} + 80\,e^{(2\,x)} - 15\,e^{x} + 32}{12\,a\left(e^{(2\,x)} + 1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 3/4*arctan(e^x)/a + 1/12*(15*e^(7*x) + 48*e^(6*x) - 9*e^(5*x) + 96*e^
(4*x) + 9*e^(3*x) + 80*e^(2*x) - 15*e^x + 32)/(a*(e^(2*x) + 1)^4)
```

maple [B] time = 0.16, size = 117, normalized size = 2.44

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{11\left(\tanh^{7}\left(\frac{x}{2}\right)\right)}{4a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{137\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{12a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{71\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{12a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{5}{4a\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+a*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-11/4/a/(tanh(1/2*x)^2+1)^4*tan
h(1/2*x)^7-137/12/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5-71/12/a/(tanh(1/2*x)^
2+1)^4*tanh(1/2*x)^3-5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)-3/4/a*arctan(tan
h(1/2*x))
```

maxima [B] time = 0.54, size = 93, normalized size = 1.94

$$\frac{x}{a}+\frac{15\,e^{(-x)} - 80\,e^{(-2\,x)} - 9\,e^{(-3\,x)} - 96\,e^{(-4\,x)} + 9\,e^{(-5\,x)} - 48\,e^{(-6\,x)} - 15\,e^{(-7\,x)} - 32}{12\left(4\,ae^{(-2\,x)} + 6\,ae^{(-4\,x)} + 4\,ae^{(-6\,x)} + ae^{(-8\,x)} + a\right)}+\frac{3 \arctan\left(e^{(-x)}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 1/12*(15*e^(-x) - 80*e^(-2*x) - 9*e^(-3*x) - 96*e^(-4*x) + 9*e^(-5*x)
 - 48*e^(-6*x) - 15*e^(-7*x) - 32)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6
*x) + a*e^(-8*x) + a) + 3/4*arctan(e^(-x))/a
```

mupad [B] time = 1.46, size = 143, normalized size = 2.98

$$\frac{\frac{8}{3a}+\frac{6\,e^x}{a}}{3\,e^{2x}+3\,e^{4x}+e^{6x}+1}-\frac{\frac{4}{a}+\frac{9\,e^x}{2a}}{2\,e^{2x}+e^{4x}+1}+\frac{x}{a}+\frac{\frac{4}{a}+\frac{5\,e^x}{4a}}{e^{2x}+1}-\frac{3\,\mathrm{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{4\sqrt{a^2}}-\frac{4\,e^x}{a\left(4\,e^{2x}+6\,e^{4x}+4\,e^{6x}+e^{8x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + a/cosh(x)),x)
```

```
[Out] (8/(3*a) + (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (4/a +
(9*exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + x/a + (4/a + (5*exp(x))/(4*
a))/(exp(2*x) + 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2)) - (4*
exp(x))/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^6(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**6/(sech(x) + 1), x)/a
```

3.104 $\int \frac{\tanh^5(x)}{a+a\textbf{sech}(x)} dx$

Optimal. Leaf size=36

$$-\frac{\text{sech}^3(x)}{3a}+\frac{\text{sech}^2(x)}{2a}+\frac{\text{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

```
[Out] ln(cosh(x))/a+sech(x)/a+1/2*sech(x)^2/a-1/3*sech(x)^3/a
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 75}

$$-\frac{\text{sech}^3(x)}{3a}+\frac{\text{sech}^2(x)}{2a}+\frac{\text{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + a*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Sech[x]/a + Sech[x]^2/(2*a) - Sech[x]^3/(3*a)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx, x, \cosh(x)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^3}{x^3}-\frac{a^3}{x^2}+\frac{a^3}{x}\right)dx, x, \cosh(x)\right)}{a^4}$$
$$= \frac{\log(\cosh(x))}{a}+\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^2(x)}{2a}-\frac{\operatorname{sech}^3(x)}{3a}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 1.06

$$\frac{\operatorname{sech}^3(x)(6\cosh(2x)+3\cosh(3x)\log(\cosh(x))+\cosh(x)(9\log(\cosh(x))+6)+2)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + a*Sech[x]),x]
```

```
[Out] ((2 + 6*Cosh[2*x] + 3*Cosh[3*x]*Log[Cosh[x]] + Cosh[x]*(6 + 9*Log[Cosh[x]])
)*Sech[x]^3)/(12*a)
```

fricas [B] time = 0.40, size = 437, normalized size = 12.14

$$-\frac{3\,x\cosh(x)^6+3\,x\sinh(x)^6+6\,(3\,x\cosh(x)-1)\sinh(x)^5+3\,(3\,x-2)\cosh(x)^4-6\,\cosh(x)^5+3\,\left(15\,x\cosh(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*x*cosh(x)^6 + 3*x*sinh(x)^6 + 6*(3*x*cosh(x) - 1)*sinh(x)^5 + 3*(3*
x - 2)*cosh(x)^4 - 6*cosh(x)^5 + 3*(15*x*cosh(x)^2 + 3*x - 10*cosh(x) - 2)*
sinh(x)^4 + 4*(15*x*cosh(x)^3 + 3*(3*x - 2)*cosh(x) - 15*cosh(x)^2 - 1)*sin
h(x)^3 + 3*(3*x - 2)*cosh(x)^2 - 4*cosh(x)^3 + 3*(15*x*cosh(x)^4 + 6*(3*x -
 2)*cosh(x)^2 - 20*cosh(x)^3 + 3*x - 4*cosh(x) - 2)*sinh(x)^2 - 3*(cosh(x)^
6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cos
h(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)
^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sin
h(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*x*cosh(x)^5 + 2*(3*x -
2)*cosh(x)^3 - 5*cosh(x)^4 + (3*x - 2)*cosh(x) - 2*cosh(x)^2 - 1)*sinh(x) +
 3*x - 6*cosh(x))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*
cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x
```

```
))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x
)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

giac [A] time = 0.12, size = 61, normalized size = 1.69

$$\frac{\log\left(e^{(-x)} + e^{x}\right)}{a} - \frac{11\left(e^{(-x)} + e^{x}\right)^{3} - 12\left(e^{(-x)} + e^{x}\right)^{2} - 12\,e^{(-x)} - 12\,e^{x} + 16}{6\,a\left(e^{(-x)} + e^{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - 1/6*(11*(e^(-x) + e^x)^3 - 12*(e^(-x) + e^x)^2 - 12*e
^(-x) - 12*e^x + 16)/(a*(e^(-x) + e^x)^3)
```

maple [A] time = 0.13, size = 34, normalized size = 0.94

$$-\frac{\mathrm{sech}(x)^{3}}{3a} + \frac{\mathrm{sech}(x)^{2}}{2a} + \frac{\mathrm{sech}(x)}{a} - \frac{\ln\left(\mathrm{sech}(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+a*sech(x)),x)
```

```
[Out] -1/3*sech(x)^3/a+1/2*sech(x)^2/a+sech(x)/a-1/a*ln(sech(x))
```

maxima [B] time = 0.47, size = 74, normalized size = 2.06

$$\frac{x}{a} + \frac{2\left(3\,e^{(-x)} + 3\,e^{(-2\,x)} + 2\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 3\,e^{(-5\,x)}\right)}{3\left(3\,ae^{(-2\,x)} + 3\,ae^{(-4\,x)} + ae^{(-6\,x)} + a\right)} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2/3*(3*e^(-x) + 3*e^(-2*x) + 2*e^(-3*x) + 3*e^(-4*x) + 3*e^(-5*x))/(3
*a*e^(-2*x) + 3*a*e^(-4*x) + a*e^(-6*x) + a) + log(e^(-2*x) + 1)/a
```

mupad [B] time = 1.43, size = 96, normalized size = 2.67

$$\frac{\ln\left(\mathrm{e}^{2\,x} + 1\right)}{a} - \frac{\frac{2}{a} + \frac{8\,\mathrm{e}^{x}}{3\,a}}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1} - \frac{x}{a} + \frac{\frac{2}{a} + \frac{2\,\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x} + 1} + \frac{8\,\mathrm{e}^{x}}{3\,a\,\left(3\,\mathrm{e}^{2\,x} + 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - (2/a + (8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1)
- x/a + (2/a + (2*exp(x))/a)/(exp(2*x) + 1) + (8*exp(x))/(3*a*(3*exp(2*x) +
 3*exp(4*x) + exp(6*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^5(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**5/(sech(x) + 1), x)/a
```

3.105 $\int \frac{\tanh^4(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)(2-\operatorname{sech}(x))}{2a}$$

```
[Out] x/a-1/2*arctan(sinh(x))/a-1/2*(2-sech(x))*tanh(x)/a
```

Rubi **[A]** time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3881, 3770}

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)(2-\operatorname{sech}(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/(2*a) - ((2 - Sech[x])*Tanh[x])/(2*a)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3881

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(e*(e*Cot[c + d*x])^(m - 1)*(a*m + b*(m - 1)*Csc[c
 + d*x]))/(d*m*(m - 1)), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a*m
 + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1
]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^4(x)}{a+a\operatorname{sech}(x)}\,dx &= -\frac{\int(-a+a\operatorname{sech}(x))\tanh^2(x)\,dx}{a^2}\\
&= -\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}-\frac{\int(-2a+a\operatorname{sech}(x))\,dx}{2a^2}\\
&= \frac{x}{a}-\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}-\frac{\int\operatorname{sech}(x)\,dx}{2a}\\
&= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 41, normalized size = 1.32

$$
\frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(2\left(x-\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\tanh(x)(\operatorname{sech}(x)-2)\right)}{a(\operatorname{sech}(x)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*Sech[x]*(2*(x - ArcTan[Tanh[x/2]]) + (-2 + Sech[x])*Tanh[x]))/
(a*(1 + Sech[x]))
```

fricas [B] time = 0.40, size = 210, normalized size = 6.77

$$
x\cosh(x)^4+x\sinh(x)^4+(4\,x\cosh(x)+1)\sinh(x)^3+2\,(x+1)\cosh(x)^2+\cosh(x)^3+\left(6\,x\cosh(x)^2+2\,x+3\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 + (4*x*cosh(x) + 1)*sinh(x)^3 + 2*(x + 1)*cosh(x
)^2 + cosh(x)^3 + (6*x*cosh(x)^2 + 2*x + 3*cosh(x) + 2)*sinh(x)^2 - (cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) +
 (4*x*cosh(x)^3 + 4*(x + 1)*cosh(x) + 3*cosh(x)^2 - 1)*sinh(x) + x - cosh(x
) + 2)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

giac [A] time = 0.11, size = 42, normalized size = 1.35

$$
\frac{x}{a}-\frac{\arctan\left(e^x\right)}{a}+\frac{e^{(3\,x)}+2\,e^{(2\,x)}-e^x+2}{a\left(e^{(2\,x)}+1\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2)
```

maple [B] time = 0.14, size = 75, normalized size = 2.42

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{3\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-3/a/(tanh(1/2*x)^2+1)^2*tanh(1
/2*x)^3-1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)-1/a*arctan(tanh(1/2*x))
```

maxima [B] time = 0.51, size = 51, normalized size = 1.65

$$\frac{x}{a}+\frac{e^{(-x)}-2\,e^{(-2\,x)}-e^{(-3\,x)}-2}{2\,ae^{(-2\,x)}+ae^{(-4\,x)}+a}+\frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + (e^(-x) - 2*e^(-2*x) - e^(-3*x) - 2)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
+ arctan(e^(-x))/a
```

mupad [B] time = 1.44, size = 67, normalized size = 2.16

$$\frac{x}{a}+\frac{\frac{2}{a}+\frac{\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+1}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}-\frac{2\,\mathrm{e}^{x}}{a\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] x/a + (2/a + exp(x)/a)/(exp(2*x) + 1) - atan((exp(x)*(a^2)^(1/2))/a)/(a^2)^
(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{4}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**4/(sech(x) + 1), x)/a
```

3.106 $\int \frac{\tanh^3(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=14

$$\frac{\operatorname{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

[Out] ln(cosh(x))/a+sech(x)/a

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 43}

$$\frac{\operatorname{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3/(a + a*Sech[x]),x]

[Out] Log[Cosh[x]]/a + Sech[x]/a

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+a\mathrm{sech}(x)}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{a-ax}{x^2}\,dx,x,\cosh(x)\right)}{a^2}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)dx,x,\cosh(x)\right)}{a^2}$$
$$= \frac{\log(\cosh(x))}{a}+\frac{\mathrm{sech}(x)}{a}$$

Mathematica [A] time = 0.04, size = 10, normalized size = 0.71

$$\frac{\mathrm{sech}(x)+\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (Log[Cosh[x]] + Sech[x])/a
```

fricas [B] time = 0.40, size = 85, normalized size = 6.07

$$-\frac{x\cosh(x)^2+x\sinh(x)^2-\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+2\,(x\cosh(x)-}{a\cosh(x)^2+2\,a\cosh(x)\sinh(x)+a\sinh(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(x*cosh(x) - 1)*sinh(x) + x - 2*c
osh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

giac [B] time = 0.12, size = 35, normalized size = 2.50

$$\frac{\log\left(e^{(-x)}+e^x\right)}{a}-\frac{e^{(-x)}+e^x-2}{a\left(e^{(-x)}+e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - (e^(-x) + e^x - 2)/(a*(e^(-x) + e^x))
```

maple [B] time = 0.13, size = 54, normalized size = 3.86

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{2}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+a*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+2/a/(tanh(1/2*x)^2+1)+1/a*ln(t
anh(1/2*x)^2+1)
```

maxima [B] time = 0.71, size = 33, normalized size = 2.36

$$\frac{x}{a}+\frac{2\,e^{(-x)}}{ae^{(-2\,x)}+a}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(a*e^(-2*x) + a) + log(e^(-2*x) + 1)/a
```

mupad [B] time = 1.36, size = 33, normalized size = 2.36

$$\frac{\ln\left(e^{2\,x}+1\right)}{a}-\frac{x}{a}+\frac{2\,e^{x}}{a\,\left(e^{2\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - x/a + (2*exp(x))/(a*(exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{3}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**3/(sech(x) + 1), x)/a
```

3.107 $\int \frac{\tanh^2(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=14

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{a}$$

```
[Out] x/a-arctan(sinh(x))/a
```

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3888, 3770}

$$\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/a
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{a+a\,\mathrm{sech}(x)}\,dx &= -\frac{\int (-a+a\,\mathrm{sech}(x))\,dx}{a^2}\\ &= \frac{x}{a}-\frac{\int \mathrm{sech}(x)\,dx}{a}\\ &= \frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.07

$$\frac{x-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + a*Sech[x]),x]
[Out] (x - 2*ArcTan[Tanh[x/2]])/a
```

fricas [A] time = 0.39, size = 14, normalized size = 1.00

$$\frac{x-2\arctan\left(\cosh(x)+\sinh(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] (x - 2*arctan(cosh(x) + sinh(x)))/a
```

giac [A] time = 0.13, size = 14, normalized size = 1.00

$$\frac{x}{a}-\frac{2\arctan\left(e^{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] x/a - 2*arctan(e^x)/a
```

maple [B] time = 0.10, size = 35, normalized size = 2.50

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+a*sech(x)),x)
[Out] -1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-2/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.53, size = 16, normalized size = 1.14

$$\frac{x}{a}+\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*arctan(e^(-x))/a
```

mupad [B] time = 1.32, size = 25, normalized size = 1.79

$$\frac{x}{a}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + a/cosh(x)),x)
```

```
[Out] x/a - (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**2/(sech(x) + 1), x)/a
```

3.108 $\int \frac{\tanh(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(\cosh(x)+1)}{a}$$

```
[Out] ln(1+cosh(x))/a
```

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 31}

$$\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + a*Sech[x]),x]
```

```
[Out] Log[1 + Cosh[x]]/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m - 1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+a\operatorname{sech}(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cosh(x)\right)\\ &= \frac{\log(1+\cosh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.33

$$\frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + a*Sech[x]),x]
[Out] (2*Log[Cosh[x/2]])/a
```

fricas [A] time = 0.42, size = 16, normalized size = 1.78

$$-\frac{x-2\,\log\left(\cosh(x)+\sinh(x)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="fricas")
[Out] -(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

giac [A] time = 0.11, size = 17, normalized size = 1.89

$$-\frac{x}{a}+\frac{2\,\log\left(e^{x}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="giac")
[Out] -x/a + 2*log(e^x + 1)/a
```

maple [A] time = 0.10, size = 19, normalized size = 2.11

$$\frac{\ln\left(1+\mathrm{sech}(x)\right)}{a}-\frac{\ln\left(\mathrm{sech}(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+a*sech(x)),x)
[Out] 1/a*ln(1+sech(x))-1/a*ln(sech(x))
```

maxima [A] time = 0.34, size = 18, normalized size = 2.00

$$\frac{x}{a}+\frac{2\,\log\left(e^{(-x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 2*log(e^(-x) + 1)/a
```

mupad [B] time = 1.31, size = 14, normalized size = 1.56

$$-\frac{x-2\,\ln\left(\mathrm{e}^x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + a/cosh(x)),x)
[Out] -(x - 2*log(exp(x) + 1))/a
```

sympy [B] time = 0.18, size = 19, normalized size = 2.11

$$\frac{x}{a}-\frac{\log\left(\tanh\left(x\right)+1\right)}{a}+\frac{\log\left(\operatorname{sech}\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*sech(x)),x)
[Out] x/a - log(tanh(x) + 1)/a + log(sech(x) + 1)/a
```

3.109 $\int \frac{\coth(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=40

$$\frac{1}{2a(\cosh(x)+1)}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(\cosh(x)+1)}{4a}$$

```
[Out] 1/2/a/(1+cosh(x))+1/4*ln(1-cosh(x))/a+3/4*ln(1+cosh(x))/a
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3879, 88}

$$\frac{1}{2a(\cosh(x)+1)}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(\cosh(x)+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + a*Sech[x]),x]
```

```
[Out] 1/(2*a*(1 + Cosh[x])) + Log[1 - Cosh[x]]/(4*a) + (3*Log[1 + Cosh[x]])/(4*a)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\coth(x)}{a+a\text{sech}(x)}\,dx = -\left(a^2\,\text{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx, x, \cosh(x)\right)\right)$$
$$= -\left(a^2\,\text{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)}+\frac{1}{2a^3(1+x)^2}-\frac{3}{4a^3(1+x)}\right)dx, x, \cosh(x)\right)\right)$$
$$= \frac{1}{2a(1+\cosh(x))}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(1+\cosh(x))}{4a}$$

Mathematica [**A**] time = 0.05, size = 44, normalized size = 1.10

$$\frac{\text{sech}(x)\left(2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\sinh\left(\frac{x}{2}\right)\right)+3\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)+1\right)}{2a(\text{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + a*Sech[x]),x]
```

```
[Out] ((1 + 2*Cosh[x/2]^2*(3*Log[Cosh[x/2]] + Log[Sinh[x/2]]))*Sech[x])/(2*a*(1 +
 Sech[x]))
```

fricas [**B**] time = 0.39, size = 136, normalized size = 3.40

$$-\frac{2\,x\cosh(x)^2+2\,x\sinh(x)^2+2\,(2\,x-1)\cosh(x)-3\left(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+2\,\cosh(x\right.}{2\left(a\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x)^2 + 2*x*sinh(x)^2 + 2*(2*x - 1)*cosh(x) - 3*(cosh(x)^2 +
2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x)
+ 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) - 1) + 2*(2*x*cosh(x) + 2*x - 1)*sinh(x) + 2*x)/(a*cosh
(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

giac [**A**] time = 0.12, size = 56, normalized size = 1.40

$$\frac{3\,\log\left(e^{(-x)}+e^x+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{4\,a}-\frac{3\,e^{(-x)}+3\,e^x+2}{4\,a\left(e^{(-x)}+e^x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a - 1/4*(3*e^(-x) +
 3*e^x + 2)/(a*(e^(-x) + e^x + 2))
```

maple [A] time = 0.16, size = 47, normalized size = 1.18

$$-\frac{\tanh^{2}\left(\frac{x}{2}\right)}{4a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+a*sech(x)),x)
```

```
[Out] -1/4/a*tanh(1/2*x)^2-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+1/2/a*ln(t
anh(1/2*x))
```

maxima [A] time = 0.49, size = 52, normalized size = 1.30

$$\frac{x}{a}+\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}+\frac{3\,\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) + 3/2*log(e^(-x) + 1)/a + 1/2*lo
g(e^(-x) - 1)/a
```

mupad [B] time = 1.37, size = 65, normalized size = 1.62

$$\frac{\ln\left(\mathrm{e}^{2\,x}-1\right)}{a}-\frac{x}{a}-\frac{1}{a+2\,a\,\mathrm{e}^{x}+a\,\mathrm{e}^{2\,x}}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}+\frac{1}{a+a\,\mathrm{e}^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) - 1)/a - x/a - 1/(a + 2*a*exp(x) + a*exp(2*x)) + atan((exp(x)*
(-a^2)^(1/2))/a)/(-a^2)^(1/2) + 1/(a + a*exp(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)/(sech(x) + 1), x)/a
```

3.110 $\int \frac{\coth^2(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}$$

```
[Out] x/a-1/3*coth(x)*(3-2*sech(x))/a-1/3*coth(x)^3*(1-sech(x))/a
```

Rubi [A] time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$\frac{x}{a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/a - (Coth[x]*(3 - 2*Sech[x]))/(3*a) - (Coth[x]^3*(1 - Sech[x]))/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a + a\mathrm{sech}(x)}\,dx = -\frac{\int \coth^4(x)(-a + a\mathrm{sech}(x))\,dx}{a^2}$$
$$= -\frac{\coth^3(x)(1 - \mathrm{sech}(x))}{3a} + \frac{\int \coth^2(x)(3a - 2a\mathrm{sech}(x))\,dx}{3a^2}$$
$$= -\frac{\coth(x)(3 - 2\mathrm{sech}(x))}{3a} - \frac{\coth^3(x)(1 - \mathrm{sech}(x))}{3a} - \frac{\int -3a\,dx}{3a^2}$$
$$= \frac{x}{a} - \frac{\coth(x)(3 - 2\mathrm{sech}(x))}{3a} - \frac{\coth^3(x)(1 - \mathrm{sech}(x))}{3a}$$

Mathematica [A] time = 0.08, size = 33, normalized size = 0.87

$$\frac{6x - 4\tanh(x) - 4\coth(x) - 2\mathrm{csch}(x) + 6x\mathrm{sech}(x)}{6a\mathrm{sech}(x) + 6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (6*x - 4*Coth[x] - 2*Csch[x] + 6*x*Sech[x] - 4*Tanh[x])/(6*a + 6*a*Sech[x])
```

fricas [A] time = 0.40, size = 46, normalized size = 1.21

$$-\frac{2\,\cosh(x)^2 - ((3\,x + 4)\cosh(x) + 3\,x + 4)\sinh(x) + 2\,\sinh(x)^2 + \cosh(x)}{3\,(a\cosh(x) + a)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cosh(x)^2 - ((3*x + 4)*cosh(x) + 3*x + 4)*sinh(x) + 2*sinh(x)^2 + c
osh(x))/((a*cosh(x) + a)*sinh(x))
```

giac [A] time = 0.13, size = 40, normalized size = 1.05

$$\frac{x}{a} - \frac{1}{2\,a(e^x - 1)} + \frac{15\,e^{(2\,x)} + 24\,e^x + 13}{6\,a(e^x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/2/(a*(e^x - 1)) + 1/6*(15*e^(2*x) + 24*e^x + 13)/(a*(e^x + 1)^3)
```

maple [A] time = 0.16, size = 56, normalized size = 1.47

$$-\frac{\tanh^{3}\left(\frac{x}{2}\right)}{12a}-\frac{\tanh\left(\frac{x}{2}\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{1}{4a\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+a*sech(x)),x)
```

```
[Out] -1/12/a*tanh(1/2*x)^3-1/a*tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2
*x)+1)-1/4/a/tanh(1/2*x)
```

maxima [A] time = 0.36, size = 47, normalized size = 1.24

$$\frac{x}{a}-\frac{2\left(5e^{(-x)}-3e^{(-3x)}+4\right)}{3\left(2ae^{(-x)}-2ae^{(-3x)}-ae^{(-4x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/3*(5*e^(-x) - 3*e^(-3*x) + 4)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*
x) + a)
```

mupad [B] time = 1.35, size = 94, normalized size = 2.47

$$\frac{\frac{5e^{2x}}{6a}+\frac{5}{6a}+\frac{e^{x}}{a}}{3e^{2x}+e^{3x}+3e^{x}+1}+\frac{\frac{1}{2a}+\frac{5e^{x}}{6a}}{e^{2x}+2e^{x}+1}+\frac{x}{a}-\frac{1}{2a\,(e^{x}-1)}+\frac{5}{6a\,(e^{x}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + a/cosh(x)),x)
```

```
[Out] ((5*exp(2*x))/(6*a) + 5/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x)
 + 1) + (1/(2*a) + (5*exp(x))/(6*a))/(exp(2*x) + 2*exp(x) + 1) + x/a - 1/(2
*a*(exp(x) - 1)) + 5/(6*a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\coth^{2}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**2/(sech(x) + 1), x)/a
```

3.111 $\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal. Leaf size=68

$$\frac{1}{8a(1-\cosh(x))}+\frac{3}{4a(\cosh(x)+1)}-\frac{1}{8a(\cosh(x)+1)^2}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(\cosh(x)+1)}{16a}$$

```
[Out] 1/8/a/(1-cosh(x))-1/8/a/(1+cosh(x))^2+3/4/a/(1+cosh(x))+5/16*ln(1-cosh(x))/
a+11/16*ln(1+cosh(x))/a
```

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3879, 88}

$$\frac{1}{8a(1-\cosh(x))}+\frac{3}{4a(\cosh(x)+1)}-\frac{1}{8a(\cosh(x)+1)^2}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(\cosh(x)+1)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + a*Sech[x]),x]
```

```
[Out] 1/(8*a*(1 - Cosh[x])) - 1/(8*a*(1 + Cosh[x])^2) + 3/(4*a*(1 + Cosh[x])) + (
5*Log[1 - Cosh[x]])/(16*a) + (11*Log[1 + Cosh[x]])/(16*a)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3879

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[((a - b*x)^((m -
1)/2)*(a + b*x)^((m - 1)/2 + n))/x^(m + n), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx = a^4\,\text{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx, x, \cosh(x)\right)$$
$$= a^4\,\text{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2}+\frac{5}{16a^5(-1+x)}+\frac{1}{4a^5(1+x)^3}-\frac{3}{4a^5(1+x)^2}+\frac{11}{16a^5(1+x)}\right)\right.$$
$$= \frac{1}{8a(1-\cosh(x))}-\frac{1}{8a(1+\cosh(x))^2}+\frac{3}{4a(1+\cosh(x))}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(1\cdot}{1}$$

Mathematica [A]   time = 0.19, size = 66, normalized size = 0.97

$$\frac{\operatorname{sech}(x)\left(-2\coth^2\left(\frac{x}{2}\right)-\operatorname{sech}^2\left(\frac{x}{2}\right)+4\cosh^2\left(\frac{x}{2}\right)\left(5\log\left(\sinh\left(\frac{x}{2}\right)\right)+11\log\left(\cosh\left(\frac{x}{2}\right)\right)\right)+12\right)}{16a(\operatorname{sech}(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + a*Sech[x]),x]
```

```
[Out] ((12 - 2*Coth[x/2]^2 + 4*Cosh[x/2]^2*(11*Log[Cosh[x/2]] + 5*Log[Sinh[x/2]])
 - Sech[x/2]^2)*Sech[x])/(16*a*(1 + Sech[x]))
```

fricas [B]   time = 0.41, size = 773, normalized size = 11.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*x*cosh(x)^6 + 8*x*sinh(x)^6 + 2*(8*x - 5)*cosh(x)^5 + 2*(24*x*cosh(
x) + 8*x - 5)*sinh(x)^5 - 4*(2*x - 3)*cosh(x)^4 + 2*(60*x*cosh(x)^2 + 5*(8*
x - 5)*cosh(x) - 4*x + 6)*sinh(x)^4 - 4*(8*x - 7)*cosh(x)^3 + 4*(40*x*cosh(
x)^3 + 5*(8*x - 5)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) - 8*x + 7)*sinh(x)^3 - 4
*(2*x - 3)*cosh(x)^2 + 4*(30*x*cosh(x)^4 + 5*(8*x - 5)*cosh(x)^3 - 6*(2*x -
 3)*cosh(x)^2 - 3*(8*x - 7)*cosh(x) - 2*x + 3)*sinh(x)^2 + 2*(8*x - 5)*cosh
(x) - 11*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5
 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 +
 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*co
sh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x
)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*co
sh(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^6 + 2*(3*cosh(x) + 1)*si
nh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)
^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*
cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*si
```

```
nh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x
)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*
(24*x*cosh(x)^5 + 5*(8*x - 5)*cosh(x)^4 - 8*(2*x - 3)*cosh(x)^3 - 6*(8*x -
7)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) + 8*x - 5)*sinh(x) + 8*x)/(a*cosh(x)^6 +
 a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4
+ (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*co
sh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*co
sh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*
a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^
2 - a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.13, size = 94, normalized size = 1.38

$$\frac{11\,\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}+\frac{5\,\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}-\frac{5\,e^{(-x)}+5\,e^{x}-6}{16\,a\left(e^{(-x)}+e^{x}-2\right)}-\frac{33\,\left(e^{(-x)}+e^{x}\right)^{2}+84\,e^{(-x)}+84\,e^{x}+52}{32\,a\left(e^{(-x)}+e^{x}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 11/16*log(e^(-x) + e^x + 2)/a + 5/16*log(e^(-x) + e^x - 2)/a - 1/16*(5*e^(-
x) + 5*e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(33*(e^(-x) + e^x)^2 + 84*e^(
-x) + 84*e^x + 52)/(a*(e^(-x) + e^x + 2)^2)
```

maple [A] time = 0.16, size = 69, normalized size = 1.01

$$-\frac{\tanh^{4}\left(\frac{x}{2}\right)}{32a}-\frac{5\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{16a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{1}{16a\tanh\left(\frac{x}{2}\right)^{2}}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+a*sech(x)),x)
```

```
[Out] -1/32/a*tanh(1/2*x)^4-5/16/a*tanh(1/2*x)^2-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tan
h(1/2*x)+1)-1/16/a/tanh(1/2*x)^2+5/8/a*ln(tanh(1/2*x))
```

maxima [A] time = 0.48, size = 108, normalized size = 1.59

$$\frac{x}{a}+\frac{5\,e^{(-x)}-6\,e^{(-2\,x)}-14\,e^{(-3\,x)}-6\,e^{(-4\,x)}+5\,e^{(-5\,x)}}{4\left(2\,ae^{(-x)}-ae^{(-2\,x)}-4\,ae^{(-3\,x)}-ae^{(-4\,x)}+2\,ae^{(-5\,x)}+ae^{(-6\,x)}+a\right)}+\frac{11\,\log\left(e^{(-x)}+1\right)}{8\,a}+\frac{5\,\log\left(e^{(-x)}-1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 1/4*(5*e^(-x) - 6*e^(-2*x) - 14*e^(-3*x) - 6*e^(-4*x) + 5*e^(-5*x))/(
2*a*e^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-
6*x) + a) + 11/8*log(e^(-x) + 1)/a + 5/8*log(e^(-x) - 1)/a
```

mupad [B] time = 1.43, size = 160, normalized size = 2.35

$$\frac{\ln\left(9\,e^{2\,x}-9\right)}{a}-\frac{x}{a}-\frac{1}{2\,\left(a+4\,a\,e^{x}+6\,a\,e^{2\,x}+4\,a\,e^{3\,x}+a\,e^{4\,x}\right)}+\frac{1}{a+3\,a\,e^{x}+3\,a\,e^{2\,x}+a\,e^{3\,x}}-\frac{1}{4\,\left(a-2\,a\,e^{x}+a\,e^{2\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + a/cosh(x)),x)
```

```
[Out] log(9*exp(2*x) - 9)/a - x/a - 1/(2*(a + 4*a*exp(x) + 6*a*exp(2*x) + 4*a*exp
(3*x) + a*exp(4*x))) + 1/(a + 3*a*exp(x) + 3*a*exp(2*x) + a*exp(3*x)) - 1/(
4*(a - 2*a*exp(x) + a*exp(2*x))) - 2/(a + 2*a*exp(x) + a*exp(2*x)) + (3*ata
n((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) + 3/(2*(a + a*exp(x))) + 1/(4*
(a - a*exp(x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^{3}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**3/(sech(x) + 1), x)/a
```

3.112 $\int \frac{\coth^4(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=55

$$\frac{x}{a}-\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}-\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a}-\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a}$$

```
[Out] x/a-1/15*coth(x)*(15-8*sech(x))/a-1/15*coth(x)^3*(5-4*sech(x))/a-1/5*coth(x
)^5*(1-sech(x))/a
```

Rubi [A] time = 0.12, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3888, 3882, 8}

$$\frac{x}{a}-\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}-\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a}-\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - (Coth[x]*(15 - 8*Sech[x]))/(15*a) - (Coth[x]^3*(5 - 4*Sech[x]))/(15*a
) - (Coth[x]^5*(1 - Sech[x]))/(5*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3882

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[((e*Cot[c + d*x])^(m + 1)*(a + b*Csc[c + d*x]))/(d
*e*(m + 1)), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(m
+ 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[
m, -1]
```

Rule 3888

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a + a\text{sech}(x)}\,dx = -\frac{\int \coth^6(x)(-a + a\text{sech}(x))\,dx}{a^2}$$
$$= -\frac{\coth^5(x)(1 - \text{sech}(x))}{5a} + \frac{\int \coth^4(x)(5a - 4a\text{sech}(x))\,dx}{5a^2}$$
$$= -\frac{\coth^3(x)(5 - 4\text{sech}(x))}{15a} - \frac{\coth^5(x)(1 - \text{sech}(x))}{5a} - \frac{\int \coth^2(x)(-15a + 8a\text{sech}(x))\,dx}{15a^2}$$
$$= -\frac{\coth(x)(15 - 8\text{sech}(x))}{15a} - \frac{\coth^3(x)(5 - 4\text{sech}(x))}{15a} - \frac{\coth^5(x)(1 - \text{sech}(x))}{5a} + \frac{\int 15a\,dx}{15a^2}$$
$$= \frac{x}{a} - \frac{\coth(x)(15 - 8\text{sech}(x))}{15a} - \frac{\coth^3(x)(5 - 4\text{sech}(x))}{15a} - \frac{\coth^5(x)(1 - \text{sech}(x))}{5a}$$

Mathematica [A] time = 0.10, size = 69, normalized size = 1.25

$$\frac{\text{csch}^3(x)\text{sech}(x)(-90x\sinh(x) - 30x\sinh(2x) + 30x\sinh(3x) + 15x\sinh(4x) + 8\cosh(x) + 16\cosh(2x) - 16\text{ c}}{120a(\text{sech}(x) + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Csch[x]^3*Sech[x]*(-25 + 8*Cosh[x] + 16*Cosh[2*x] - 16*Cosh[3*x] - 23*Cosh
[4*x] - 90*x*Sinh[x] - 30*x*Sinh[2*x] + 30*x*Sinh[3*x] + 15*x*Sinh[4*x]))/(
120*a*(1 + Sech[x]))
```

fricas [B] time = 0.38, size = 151, normalized size = 2.75

$$-\frac{23\,\cosh(x)^4 - 2\,(2\,(15\,x + 23)\cosh(x) + 15\,x + 23)\sinh(x)^3 + 23\,\sinh(x)^4 + 16\,\cosh(x)^3 + 2\left(69\,\cosh(x)^2 +\right.}{30\left((2\,a\cosh(x) + a)\,\text{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/30*(23*cosh(x)^4 - 2*(2*(15*x + 23)*cosh(x) + 15*x + 23)*sinh(x)^3 + 23*
sinh(x)^4 + 16*cosh(x)^3 + 2*(69*cosh(x)^2 + 24*cosh(x) - 8)*sinh(x)^2 - 16
*cosh(x)^2 - 2*(2*(15*x + 23)*cosh(x)^3 + 3*(15*x + 23)*cosh(x)^2 - 2*(15*x
 + 23)*cosh(x) - 45*x - 69)*sinh(x) - 8*cosh(x) + 25)/((2*a*cosh(x) + a)*si
nh(x)^3 + (2*a*cosh(x)^3 + 3*a*cosh(x)^2 - 2*a*cosh(x) - 3*a)*sinh(x))
```

giac [A] time = 0.13, size = 64, normalized size = 1.16

$$\frac{x}{a} - \frac{21\,e^{(2\,x)} - 36\,e^{x} + 19}{24\,a(e^{x} - 1)^3} + \frac{115\,e^{(4\,x)} + 380\,e^{(3\,x)} + 530\,e^{(2\,x)} + 340\,e^{x} + 91}{40\,a(e^{x} + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/24*(21*e^(2*x) - 36*e^x + 19)/(a*(e^x - 1)^3) + 1/40*(115*e^(4*x) +
 380*e^(3*x) + 530*e^(2*x) + 340*e^x + 91)/(a*(e^x + 1)^5)
```

maple [A] time = 0.17, size = 78, normalized size = 1.42

$$-\frac{\tanh^5\left(\frac{x}{2}\right)}{80a}-\frac{\tanh^3\left(\frac{x}{2}\right)}{8a}-\frac{\tanh\left(\frac{x}{2}\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{1}{48a\tanh\left(\frac{x}{2}\right)^3}-\frac{3}{8a\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+a*sech(x)),x)
```

```
[Out] -1/80/a*tanh(1/2*x)^5-1/8/a*tanh(1/2*x)^3-1/a*tanh(1/2*x)-1/a*ln(tanh(1/2*x
)-1)+1/a*ln(tanh(1/2*x)+1)-1/48/a/tanh(1/2*x)^3-3/8/a/tanh(1/2*x)
```

maxima [B] time = 0.35, size = 105, normalized size = 1.91

$$\frac{x}{a}-\frac{2\left(31\,e^{(-x)}-31\,e^{(-2\,x)}-73\,e^{(-3\,x)}+25\,e^{(-4\,x)}+65\,e^{(-5\,x)}+15\,e^{(-6\,x)}-15\,e^{(-7\,x)}+23\right)}{15\left(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/15*(31*e^(-x) - 31*e^(-2*x) - 73*e^(-3*x) + 25*e^(-4*x) + 65*e^(-5*
x) + 15*e^(-6*x) - 15*e^(-7*x) + 23)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3
*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

mupad [B] time = 1.53, size = 264, normalized size = 4.80

$$\frac{\frac{9\,e^{2\,x}}{4\,a}+\frac{3\,e^{3\,x}}{2\,a}+\frac{23\,e^{4\,x}}{40\,a}+\frac{23}{40\,a}+\frac{3\,e^{x}}{2\,a}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^{x}+1}+\frac{\frac{9\,e^{2\,x}}{8\,a}+\frac{23\,e^{3\,x}}{40\,a}+\frac{3}{8\,a}+\frac{9\,e^{x}}{8\,a}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}+\frac{\frac{23\,e^{2\,x}}{40\,a}+\frac{3}{8\,a}+\frac{3\,e^{x}}{4\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}+\frac{\frac{3}{8\,a}+\frac{23\,e^{x}}{40\,a}}{e^{2\,x}+2\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + a/cosh(x)),x)
```

```
[Out] ((9*exp(2*x))/(4*a) + (3*exp(3*x))/(2*a) + (23*exp(4*x))/(40*a) + 23/(40*a)
 + (3*exp(x))/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5
*exp(x) + 1) + ((9*exp(2*x))/(8*a) + (23*exp(3*x))/(40*a) + 3/(8*a) + (9*ex
p(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) + ((23*exp
```

```
(2*x))/(40*a) + 3/(8*a) + (3*exp(x))/(4*a))/(3*exp(2*x) + exp(3*x) + 3*exp(
x) + 1) + (3/(8*a) + (23*exp(x))/(40*a))/(exp(2*x) + 2*exp(x) + 1) + 1/(6*a
*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)
) + x/a - 7/(8*a*(exp(x) - 1)) + 23/(40*a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**4/(sech(x) + 1), x)/a
```

3.113 $\int \frac{\tanh^7(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=121

$$-\frac{\left(a^2-b^2\right)^3\log(a+b\operatorname{sech}(x))}{ab^6}-\frac{a\left(a^2-3b^2\right)\operatorname{sech}^2(x)}{2b^4}+\frac{\left(a^2-3b^2\right)\operatorname{sech}^3(x)}{3b^3}+\frac{\left(a^4-3a^2b^2+3b^4\right)\operatorname{sech}(x)}{b^5}-\frac{a\operatorname{sech}^4}{4b^2}$$

```
[Out] ln(cosh(x))/a-(a^2-b^2)^3*ln(a+b*sech(x))/a/b^6+(a^4-3*a^2*b^2+3*b^4)*sech(
x)/b^5-1/2*a*(a^2-3*b^2)*sech(x)^2/b^4+1/3*(a^2-3*b^2)*sech(x)^3/b^3-1/4*a*
sech(x)^4/b^2+1/5*sech(x)^5/b
```

Rubi [A] time = 0.15, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\frac{\left(a^2-3b^2\right)\operatorname{sech}^3(x)}{3b^3}-\frac{a\left(a^2-3b^2\right)\operatorname{sech}^2(x)}{2b^4}+\frac{\left(-3a^2b^2+a^4+3b^4\right)\operatorname{sech}(x)}{b^5}-\frac{\left(a^2-b^2\right)^3\log(a+b\operatorname{sech}(x))}{ab^6}-\frac{a\operatorname{sech}^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a - ((a^2 - b^2)^3*Log[a + b*Sech[x]])/(a*b^6) + ((a^4 - 3*a^2
*b^2 + 3*b^4)*Sech[x])/b^5 - (a*(a^2 - 3*b^2)*Sech[x]^2)/(2*b^4) + ((a^2 -
3*b^2)*Sech[x]^3)/(3*b^3) - (a*Sech[x]^4)/(4*b^2) + Sech[x]^5/(5*b)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{x(a+x)}\,dx, x, b\text{sech}(x)\right)}{b^6}$$

$$= -\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2\left(-a^2+b^2\right)}{a^4}\right)+\frac{b^6}{ax}+a\left(a^2-3b^2\right)x-\left(a^2-3b^2\right)x^2+ax^3-x^4+\frac{\left(a^2-b^2\right.}{a(a+x}\right.\right.}{b^6}$$

$$= \frac{\log(\cosh(x))}{a} - \frac{\left(a^2-b^2\right)^3\log(a+b\text{sech}(x))}{ab^6} + \frac{\left(a^4-3a^2b^2+3b^4\right)\text{sech}(x)}{b^5} - \frac{a\left(a^2-3b^2\right.}{2b}$$

Mathematica [A] time = 0.34, size = 132, normalized size = 1.09

$$\frac{-30ab^2\left(a^2-3b^2\right)\text{sech}^2(x) - \frac{60\left(a^2-b^2\right)^3\log(a\cosh(x)+b)}{a} + 20b^3\left(a^2-3b^2\right)\text{sech}^3(x) + 60b\left(a^4-3a^2b^2+3b^4\right)\text{sech}(x}{60b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] (60*a*(a^4 - 3*a^2*b^2 + 3*b^4)*Log[Cosh[x]] - (60*(a^2 - b^2)^3*Log[b + a*
Cosh[x]])/a + 60*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sech[x] - 30*a*b^2*(a^2 - 3*b^
2)*Sech[x]^2 + 20*b^3*(a^2 - 3*b^2)*Sech[x]^3 - 15*a*b^4*Sech[x]^4 + 12*b^5
*Sech[x]^5)/(60*b^6)
```

fricas [B] time = 0.51, size = 4077, normalized size = 33.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 - 30*(a^5*b - 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) - a^5*b + 3*a^3*b^3 - 3*a*b^5)*sin
h(x)^9 + 15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 + 5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4 - 18*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 - 27*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x))*sinh(x)^7
+ 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 - 21*a^2*b^4 - 252*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 + 42*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 - 28*(3*
```

a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 - 4*(45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 - 45*a^5*b + 115*a^3*b^3 - 99*a*b^5 - 945*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 + 210*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^2 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^5 + 30*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 + 15*b^6*x + 9*a^4*b^2 - 21*a^2*b^4 - 378*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 105*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^2 - 2*(45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 - 63*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^6 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 + 21*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^3 - (45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^3 + 15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 - 216*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x + 84*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 - 18*a^2*b^4 - 168*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4 - 8*(45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 + 36*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^2 - 30*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x) + 15*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^10 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^7 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 4*(63*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 10*(21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 5*(9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 28*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 4*

```
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - 15*
((a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
)*cosh(x)*sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 + 5*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 9*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 +
21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 4*(63*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^5 + 10*(a^6 - 3*a^4*b^2 +
3*a^2*b^4)*cosh(x)^4 + 10*(21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + a^6
 - 3*a^4*b^2 + 3*a^2*b^4 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 15*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4)*cosh(x)^7 + 7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 5*(a^
6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x
))*sinh(x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 5*(9*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4)*cosh(x)^8 + 28*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 +
 a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 +
 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 - 3*a^4*b
^2 + 3*a^2*b^4)*cosh(x)^9 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7 + 6*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*co
sh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*cosh(x)/(co
sh(x) - sinh(x))) + 10*(15*b^6*x*cosh(x)^9 - 27*(a^5*b - 3*a^3*b^3 + 3*a*b^
5)*cosh(x)^8 + 12*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^7 - 28*(3*a^5*b
 - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^6 - 3*a^5*b + 9*a^3*b^3 - 9*a*b^5 + 18*(5*b
^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^5 - 2*(45*a^5*b - 115*a^3*b^3 + 99*a*
b^5)*cosh(x)^4 + 12*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^3 - 12*(3*a^5
*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*c
osh(x))*sinh(x))/(a*b^6*cosh(x)^10 + 10*a*b^6*cosh(x)*sinh(x)^9 + a*b^6*sin
h(x)^10 + 5*a*b^6*cosh(x)^8 + 10*a*b^6*cosh(x)^6 + 10*a*b^6*cosh(x)^4 + 5*a
*b^6*cosh(x)^2 + 5*(9*a*b^6*cosh(x)^2 + a*b^6)*sinh(x)^8 + 40*(3*a*b^6*cosh
(x)^3 + a*b^6*cosh(x))*sinh(x)^7 + a*b^6 + 10*(21*a*b^6*cosh(x)^4 + 14*a*b^
6*cosh(x)^2 + a*b^6)*sinh(x)^6 + 4*(63*a*b^6*cosh(x)^5 + 70*a*b^6*cosh(x)^3
 + 15*a*b^6*cosh(x))*sinh(x)^5 + 10*(21*a*b^6*cosh(x)^6 + 35*a*b^6*cosh(x)^
4 + 15*a*b^6*cosh(x)^2 + a*b^6)*sinh(x)^4 + 40*(3*a*b^6*cosh(x)^7 + 7*a*b^6
*cosh(x)^5 + 5*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x)^3 + 5*(9*a*b^6*cosh
(x)^8 + 28*a*b^6*cosh(x)^6 + 30*a*b^6*cosh(x)^4 + 12*a*b^6*cosh(x)^2 + a*b^
6)*sinh(x)^2 + 10*(a*b^6*cosh(x)^9 + 4*a*b^6*cosh(x)^7 + 6*a*b^6*cosh(x)^5
+ 4*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 267, normalized size = 2.21

$$\frac{\left(a^{5}-3\,a^{3}b^{2}+3\,ab^{4}\right)\log\left(e^{(-x)}+e^{x}\right)}{b^{6}}-\frac{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{ab^{6}}-\frac{137\,a^{5}\left(e^{(-x)}+e^{x}\right)^{5}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] (a^5 - 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + e^x)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b^6) - 1/60*(137*a^5*(e^(-
x) + e^x)^5 - 411*a^3*b^2*(e^(-x) + e^x)^5 + 411*a*b^4*(e^(-x) + e^x)^5 - 1
20*a^4*b*(e^(-x) + e^x)^4 + 360*a^2*b^3*(e^(-x) + e^x)^4 - 360*b^5*(e^(-x)
+ e^x)^4 + 120*a^3*b^2*(e^(-x) + e^x)^3 - 360*a*b^4*(e^(-x) + e^x)^3 - 160*
a^2*b^3*(e^(-x) + e^x)^2 + 480*b^5*(e^(-x) + e^x)^2 + 240*a*b^4*(e^(-x) + e
^x) - 384*b^5)/(b^6*(e^(-x) + e^x)^5)
```

maple [B] time = 0.15, size = 415, normalized size = 3.43

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{a^{5}\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}{b^{6}}+\frac{3a^{3}\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^7/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-a^5/b^6*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+3*a^
3/b^4*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-3*a/b^2*ln(a*tanh(1/2*x)^2-ta
nh(1/2*x)^2*b+a+b)+1/a*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-1/a*ln(tanh(
1/2*x)+1)+2/b^5/(tanh(1/2*x)^2+1)*a^4+2/b^4/(tanh(1/2*x)^2+1)*a^3-4/b^3/(ta
nh(1/2*x)^2+1)*a^2-4/b^2/(tanh(1/2*x)^2+1)*a+2/b/(tanh(1/2*x)^2+1)+1/b^6*ln
(tanh(1/2*x)^2+1)*a^5-3/b^4*ln(tanh(1/2*x)^2+1)*a^3+3/b^2*ln(tanh(1/2*x)^2+
1)*a-2/b^4/(tanh(1/2*x)^2+1)^2*a^3-4/b^3/(tanh(1/2*x)^2+1)^2*a^2+4/b/(tanh(
1/2*x)^2+1)^2+32/5/b/(tanh(1/2*x)^2+1)^5+8/3/b^3/(tanh(1/2*x)^2+1)^3*a^2+8/
b^2/(tanh(1/2*x)^2+1)^3*a+8/b/(tanh(1/2*x)^2+1)^3-4/b^2/(tanh(1/2*x)^2+1)^4
*a-16/b/(tanh(1/2*x)^2+1)^4
```

maxima [B] time = 0.58, size = 332, normalized size = 2.74

$$\frac{2\left(15\left(a^{4}-3\,a^{2}b^{2}+3\,b^{4}\right)e^{(-x)}-15\left(a^{3}b-3\,ab^{3}\right)e^{(-2\,x)}+20\left(3\,a^{4}-8\,a^{2}b^{2}+6\,b^{4}\right)e^{(-3\,x)}-15\left(3\,a^{3}b-7\,ab^{3}\right)e^{(-4\,x)}+}{15\left(5\,b^{5}e^{(-2\,x)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b - 3*a*b^3)*e^(-2*x) +
 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-3*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 - 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-6
*x) + 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-7*x) - 15*(a^3*b - 3*a*b^3)*e^(-8*
x) + 15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) + 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) + 5*b^5*e^(-8*x) + b^5*e^(-10*x) + b^5) + x/a + (a^5 -
 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^6)
```

mupad [B] time = 1.99, size = 316, normalized size = 2.61

$$\frac{\frac{8\,a}{b^2}-\frac{8\,{\mathrm{e}}^x\,\left(5\,a^2-27\,b^2\right)}{15\,b^3}}{3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1}-\frac{\frac{4\,a}{b^2}+\frac{64\,{\mathrm{e}}^x}{5\,b}}{4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1}+\frac{\frac{8\,{\mathrm{e}}^x\,\left(a^2-3\,b^2\right)}{3\,b^3}+\frac{2\,\left(a^4-5\,a^2\,b^2\right)}{a\,b^4}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{x}{a}+\frac{\frac{2\,{\mathrm{e}}^x\,\left(a^4-3\,a^2\,b^2+3\,b^4\right)}{b^5}-\frac{2\,\left(a^4 \cdots\right.}{\cdots}}{{\mathrm{e}}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^7/(a + b/cosh(x)),x)
```

```
[Out] ((8*a)/b^2 - (8*exp(x)*(5*a^2 - 27*b^2))/(15*b^3))/(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1) - ((4*a)/b^2 + (64*exp(x))/(5*b))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) + ((8*exp(x)*(a^2 - 3*b^2))/(3*b^3) + (2*(a^4
 - 5*a^2*b^2))/(a*b^4))/(2*exp(2*x) + exp(4*x) + 1) - x/a + ((2*exp(x)*(a^4
 + 3*b^4 - 3*a^2*b^2))/b^5 - (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*x) + 1)
+ (32*exp(x))/(5*b*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + e
xp(10*x) + 1)) + (log(exp(2*x) + 1)*(3*a*b^4 + a^5 - 3*a^3*b^2))/b^6 - (log
(a + 2*b*exp(x) + a*exp(2*x))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(a*b^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^7(x)}{a+b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**7/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**7/(a + b*sech(x)), x)
```

3.114 $\int \frac{\tanh^6(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=187

$$\frac{a\left(a^2-3b^2\right)\tanh(x)}{b^4}-\frac{\left(a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{\left(a^2-3b^2\right)\tanh(x)\text{sech}(x)}{2b^3}-\frac{\left(a^4-3a^2b^2+3b^4\right)\tan^{-1}(\sinh(x))}{b^5}-$$

```
[Out] x/a-3/8*arctan(sinh(x))/b-1/2*(a^2-3*b^2)*arctan(sinh(x))/b^3-(a^4-3*a^2*b^
2+3*b^4)*arctan(sinh(x))/b^5+2*(a-b)^(5/2)*(a+b)^(5/2)*arctan((a-b)^(1/2)*t
anh(1/2*x)/(a+b)^(1/2))/a/b^5+a*tanh(x)/b^2+a*(a^2-3*b^2)*tanh(x)/b^4-3/8*s
ech(x)*tanh(x)/b-1/2*(a^2-3*b^2)*sech(x)*tanh(x)/b^3-1/4*sech(x)^3*tanh(x)/
b-1/3*a*tanh(x)^3/b^2
```

Rubi [A] time = 0.29, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3898, 2897, 2659, 205, 3770, 3767, 8, 3768}

$$\frac{a\left(a^2-3b^2\right)\tanh(x)}{b^4}-\frac{\left(a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{\left(-3a^2b^2+a^4+3b^4\right)\tan^{-1}(\sinh(x))}{b^5}-\frac{\left(a^2-3b^2\right)\tanh(x)\text{sech}(x}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*b) - ((a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) -
 ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTan[Sinh[x]])/b^5 + (2*(a - b)^(5/2)*(a + b)
^(5/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^5) + (a*Tanh[x])/b
^2 + (a*(a^2 - 3*b^2)*Tanh[x])/b^4 - (3*Sech[x]*Tanh[x])/(8*b) - ((a^2 - 3*
b^2)*Sech[x]*Tanh[x])/(2*b^3) - (Sech[x]^3*Tanh[x])/(4*b) - (a*Tanh[x]^3)/(
3*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\sinh(x)\tanh^5(x)}{b+a\cosh(x)}\,dx$$

$$= -\int \left(-\frac{1}{a} - \frac{(a^2-b^2)^3}{ab^5(b+a\cosh(x))} + \frac{(a^4-3a^2b^2+3b^4)\,\text{sech}(x)}{b^5} + \frac{(-a^3+3ab^2)\,\text{sech}^2(x)}{b^4} + \ldots$$

$$= \frac{x}{a} + \frac{a\int \text{sech}^4(x)\,dx}{b^2} - \frac{\int \text{sech}^5(x)\,dx}{b} + \frac{\left(a(a^2-3b^2)\right)\int \text{sech}^2(x)\,dx}{b^4} - \frac{(a^2-3b^2)\int \text{sech}^3(\ldots}{b^3}$$

$$= \frac{x}{a} - \frac{(a^4-3a^2b^2+3b^4)\tan^{-1}(\sinh(x))}{b^5} - \frac{(a^2-3b^2)\,\text{sech}(x)\tanh(x)}{2b^3} - \frac{\text{sech}^3(x)\tanh(x)}{4b} + \ldots$$

$$= \frac{x}{a} - \frac{(a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\tan^{-1}(\sinh(x))}{b^5} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\ldots}{ab\ldots}$$

$$= \frac{x}{a} - \frac{3\tan^{-1}(\sinh(x))}{8b} - \frac{(a^2-3b^2)\tan^{-1}(\sinh(x))}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\tan^{-1}(\sinh(x))}{b^5} + \ldots$$

Mathematica [A] time = 0.62, size = 185, normalized size = 0.99

$$\frac{\frac{48\left(b^5x\sqrt{a^2-b^2}-2(a^2-b^2)^3\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)}{a\sqrt{a^2-b^2}} - 12\left(8a^4-20a^2b^2+15b^4\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + b\tanh(x)\text{sech}^3(x)\left(12a^3\cos\ldots\right.}{48b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] (-12*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Tanh[x/2]] + (48*(b^5*Sqrt[a^2 -
b^2]*x - 2*(a^2 - b^2)^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]))/(a*
Sqrt[a^2 - b^2]) + b*(-12*a^2*b + 15*b^3 + 4*a*(9*a^2 - 17*b^2)*Cosh[x] + 3
*b*(-4*a^2 + 9*b^2)*Cosh[2*x] + 12*a^3*Cosh[3*x] - 28*a*b^2*Cosh[3*x])*Sech
[x]^3*Tanh[x])/(48*b^5)
```

fricas [B] time = 0.75, size = 4914, normalized size = 26.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2 - 9*a*b^4)*co
sh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^7 + 24*(2*b^5*
x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 16*b^5*x - 8*a^
4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6 + 12*b^5*x -
3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*a^3*b^2 + a*b^
4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^2 + 48*(2*b^5*x - a^4*b + 3*a^2*b^3)*c
osh(x))*sinh(x)^5 - 24*a^4*b + 56*a^2*b^3 + 24*(3*b^5*x - 3*a^4*b + 7*a^2*b
^3)*cosh(x)^4 + 3*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b + 56*a^2*b^3 -
 35*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^3 + 120*(2*b^5*x - a^4*b + 3*a^2*b^3)*cos
h(x)^2 - 5*(4*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 3*(4*a^3*b^2 - a*b^4)*c
osh(x)^3 + 3*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 - a*b^4 - 35*(4*a^3*b^2 - 9*a
*b^4)*cosh(x)^4 + 160*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^3 - 10*(4*a^3*b
^2 - a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x))*sinh(x)
^3 + 8*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x)^2 + (336*b^5*x*cosh(x)^6 +
48*b^5*x - 63*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^5 - 72*a^4*b + 152*a^2*b^3 + 36
0*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^4 - 30*(4*a^3*b^2 - a*b^4)*cosh(x)^
3 + 144*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^2 + 12*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^8 + 4*(a^4 -
 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4
+ 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2
 + b^4)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)
^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-
a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*co
sh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*((8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^8 + 8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5
- 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^6 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3
*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 +
15*a*b^4 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 - 60*a^3
*b^2 + 45*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(8*a^5
- 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 20*a^3*b^2 +
15*a*b^4)*cosh(x)^5 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(8*a^5 - 20*a^3*b^2 + 15
*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5
```

- 20*a^3*b^2 + 15*a*b^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 20*a^3*b
^2 + 15*a*b^4)*cosh(x)^7 + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(4*a^3*b^2 - 9*a*b^4)*cosh
(x) + (96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^6 + 144*(2*b^5
*x - a^4*b + 3*a^2*b^3)*cosh(x)^5 + 12*a^3*b^2 - 27*a*b^4 - 15*(4*a^3*b^2 -
a*b^4)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^3 + 9*(4*a^3
*b^2 - a*b^4)*cosh(x)^2 + 16*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x))*sinh
(x))/(a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)^8 + 4*a*b
^5*cosh(x)^6 + 6*a*b^5*cosh(x)^4 + 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5*cosh(x)^2
+ a*b^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(
x)^5 + 2*(35*a*b^5*cosh(x)^4 + 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh(x)^4 + 8*
(7*a*b^5*cosh(x)^5 + 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^3 + 4*(7
*a*b^5*cosh(x)^6 + 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 + a*b^5)*sinh(x)^
2 + 8*(a*b^5*cosh(x)^7 + 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 + a*b^5*cosh
(x))*sinh(x)), 1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2
- 9*a*b^4)*cosh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^
7 + 24*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 1
6*b^5*x - 8*a^4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6
+ 12*b^5*x - 3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*
a^3*b^2 + a*b^4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^2 + 48*(2*b^5*x - a^4*b
+ 3*a^2*b^3)*cosh(x))*sinh(x)^5 - 24*a^4*b + 56*a^2*b^3 + 24*(3*b^5*x - 3*a
^4*b + 7*a^2*b^3)*cosh(x)^4 + 3*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b
+ 56*a^2*b^3 - 35*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^3 + 120*(2*b^5*x - a^4*b +
3*a^2*b^3)*cosh(x)^2 - 5*(4*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 3*(4*a^3*
b^2 - a*b^4)*cosh(x)^3 + 3*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 - a*b^4 - 35*(4
*a^3*b^2 - 9*a*b^4)*cosh(x)^4 + 160*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^3
- 10*(4*a^3*b^2 - a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*co
sh(x))*sinh(x)^3 + 8*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x)^2 + (336*b^5*
x*cosh(x)^6 + 48*b^5*x - 63*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^5 - 72*a^4*b + 15
2*a^2*b^3 + 360*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^4 - 30*(4*a^3*b^2 - a
*b^4)*cosh(x)^3 + 144*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*
b^2 - a*b^4)*cosh(x))*sinh(x)^2 - 24*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8
*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^8 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*c
osh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^
2*b^2 + 3*b^4 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a
^2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2
+ b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 -
2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b
^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(

```
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a
^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x)
+ a*sinh(x) + b)/sqrt(a^2 - b^2)) - 3*((8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^8 + 8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 - 20*a
^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6
+ 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^
5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 + 15*a*b
^4 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 - 60*a^3*b^2 +
 45*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(8*a^5 - 20*a
^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b
^4)*cosh(x)^5 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 - 2
0*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^2 + 4*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5 - 20*a
^3*b^2 + 15*a*b^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 20*a^3*b^2 + 1
5*a*b^4)*cosh(x)^7 + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(
x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(4*a^3*b^2 - 9*a*b^4)*cosh(x) +
(96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^6 + 144*(2*b^5*x - a
^4*b + 3*a^2*b^3)*cosh(x)^5 + 12*a^3*b^2 - 27*a*b^4 - 15*(4*a^3*b^2 - a*b^4
)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^3 + 9*(4*a^3*b^2 -
 a*b^4)*cosh(x)^2 + 16*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x))*sinh(x))/(
a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)^8 + 4*a*b^5*cos
h(x)^6 + 6*a*b^5*cosh(x)^4 + 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5*cosh(x)^2 + a*b
^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^5 +
 2*(35*a*b^5*cosh(x)^4 + 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh(x)^4 + 8*(7*a*b
^5*cosh(x)^5 + 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^3 + 4*(7*a*b^5
*cosh(x)^6 + 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 + a*b^5)*sinh(x)^2 + 8*
(a*b^5*cosh(x)^7 + 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 + a*b^5*cosh(x))*s
inh(x))]
```

giac [A] time = 0.14, size = 250, normalized size = 1.34

$$\frac{x}{a} - \frac{\left(8\,a^4 - 20\,a^2b^2 + 15\,b^4\right)\arctan\left(e^x\right)}{4\,b^5} + \frac{2\left(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6\right)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,ab^5} - \frac{12\,a^2be^{(7\,x)} - 27\,b^3e^{(7\,x)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/4*(8*a^4 - 20*a^2*b^2 + 15*b^4)*arctan(e^x)/b^5 + 2*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a
*b^5) - 1/12*(12*a^2*b*e^(7*x) - 27*b^3*e^(7*x) + 24*a^3*e^(6*x) - 72*a*b^2
```

```
*e^(6*x) + 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) + 72*a^3*e^(4*x) - 168*a*b^2*e^
(4*x) - 12*a^2*b*e^(3*x) + 3*b^3*e^(3*x) + 72*a^3*e^(2*x) - 152*a*b^2*e^(2*
x) - 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 - 56*a*b^2)/(b^4*(e^(2*x) + 1)^4)
```

maple [B] time = 0.15, size = 575, normalized size = 3.07

$$\frac{6\left(\tanh^5\left(\frac{x}{2}\right)\right)a^3}{b^4\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}+\frac{2a^5\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^5\sqrt{(a+b)(a-b)}}+\frac{\left(\tanh^7\left(\frac{x}{2}\right)\right)a^2}{b^3\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}-\frac{44\left(\tanh^3\left(\frac{x}{2}\right)\right)a}{3b^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}+\frac{2\tanh\left(\frac{x}{2}\right)a^3}{b^4\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*sech(x)),x)
```

```
[Out] -1/b^3/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^2+6/b^4/(tanh(1/2*x)^2+1)^4*tanh
(1/2*x)^3*a^3+2/b^4/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^3+1/b^3/(tanh(1/2*x
)^2+1)^4*tanh(1/2*x)^7*a^2+2*a^5/b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(
1/2*x)/((a+b)*(a-b))^(1/2))+1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-2*b
/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-44/3/b
^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a+2/b^4/(tanh(1/2*x)^2+1)^4*tanh(1/2*x
)*a^3-4/b^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a-1/b^3/(tanh(1/2*x)^2+1)^4*tan
h(1/2*x)*a^2-4/b^2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a+6/b^4/(tanh(1/2*x)^2
+1)^4*tanh(1/2*x)^5*a^3+1/b^3/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^2-44/3/b^
2/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a+6*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b
)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-6/b^3*a^3/((a+b)*(a-b))^(1/2)*arctan((a-
b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+5/b^3*arctan(tanh(1/2*x))*a^2-2/b^5*arc
tan(tanh(1/2*x))*a^4-7/4/b/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7-15/4/b/(tanh(1
/2*x)^2+1)^4*tanh(1/2*x)^5+15/4/b/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3+7/4/b/(
tanh(1/2*x)^2+1)^4*tanh(1/2*x)-15/4/b*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.50, size = 1001, normalized size = 5.35

$$\frac{\frac{8a}{3b^2}+\frac{6e^x}{b}}{3e^{2x}+3e^{4x}+e^{6x}+1}-\frac{\frac{e^x(4a^2-9b^2)}{4b^3}+\frac{2(a^4-3a^2b^2)}{ab^4}}{e^{2x}+1}-\frac{\frac{4a}{b^2}-\frac{e^x(4a^2-13b^2)}{2b^3}}{2e^{2x}+e^{4x}+1}+\frac{x}{a}+\frac{\ln(e^x-i)\left(a^4\,8i-a^2b^2\,20i+b^4\,15i\right)}{8b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + b/cosh(x)),x)
```

```
[Out] ((8*a)/(3*b^2) + (6*exp(x))/b)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (
(exp(x)*(4*a^2 - 9*b^2))/(4*b^3) + (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*x)
 + 1) - ((4*a)/b^2 - (exp(x)*(4*a^2 - 13*b^2))/(2*b^3))/(2*exp(2*x) + exp(4
*x) + 1) + x/a + (log(exp(x) - 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5
) - (log(exp(x) + 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) - (4*exp(x)
)/(b*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) + (log(((-(a +
b)^5*(a - b)^5)^(1/2)*((128*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4*b^8 -
3160*a^6*b^6 + 2240*a^8*b^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 192*a^11*b
*exp(x) + 3075*a^3*b^9*exp(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5*exp(x) -
 1216*a^9*b^3*exp(x))/(2*a^6*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(a^2
- b^2)*(16*a*b^4 + 16*a^5 - 32*a^3*b^2 + 32*b^5*exp(x) + 28*a^4*b*exp(x) -
57*a^2*b^3*exp(x)))/(a^6*b^2) + (32*(-(a + b)^5*(a - b)^5)^(1/2)*(3*a*b^2 -
 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b^3)))/(a*b^5)))/(a*b^5) - ((
a^2 - b^2)^3*(8*a^4 + 15*b^4 - 20*a^2*b^2)*(30*a*b^4 + 16*a^5 - 40*a^3*b^2
+ 52*b^5*exp(x) + 28*a^4*b*exp(x) - 71*a^2*b^3*exp(x)))/(2*a^6*b^12))*(-(a
+ b)^5*(a - b)^5)^(1/2))/(a*b^5) - (log(- ((-(a + b)^5*(a - b)^5)^(1/2)*((1
28*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4*b^8 - 3160*a^6*b^6 + 2240*a^8*b
^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 192*a^11*b*exp(x) + 3075*a^3*b^9*ex
p(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5*exp(x) - 1216*a^9*b^3*exp(x))/(2*
a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(a^2 - b^2)*(16*a*b^4 + 16*a^5
 - 32*a^3*b^2 + 32*b^5*exp(x) + 28*a^4*b*exp(x) - 57*a^2*b^3*exp(x)))/(a^6*
b^2) - (32*(-(a + b)^5*(a - b)^5)^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3
*a^2*b*exp(x)))/(a^6*b^3)))/(a*b^5)))/(a*b^5) - ((a^2 - b^2)^3*(8*a^4 + 15*
b^4 - 20*a^2*b^2)*(30*a*b^4 + 16*a^5 - 40*a^3*b^2 + 52*b^5*exp(x) + 28*a^4*
b*exp(x) - 71*a^2*b^3*exp(x)))/(2*a^6*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/
(a*b^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**6/(a + b*sech(x)), x)
```

3.115 $\int \frac{\tanh^5(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal. Leaf size=72

$$\frac{(a^2-b^2)^2\log(a+b\operatorname{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\operatorname{sech}(x)}{b^3}+\frac{a\operatorname{sech}^2(x)}{2b^2}+\frac{\log(\cosh(x))}{a}-\frac{\operatorname{sech}^3(x)}{3b}$$

```
[Out] ln(cosh(x))/a+(a^2-b^2)^2*ln(a+b*sech(x))/a/b^4-(a^2-2*b^2)*sech(x)/b^3+1/2
*a*sech(x)^2/b^2-1/3*sech(x)^3/b
```

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{(a^2-2b^2)\operatorname{sech}(x)}{b^3}+\frac{(a^2-b^2)^2\log(a+b\operatorname{sech}(x))}{ab^4}+\frac{a\operatorname{sech}^2(x)}{2b^2}+\frac{\log(\cosh(x))}{a}-\frac{\operatorname{sech}^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((a^2 - b^2)^2*Log[a + b*Sech[x]])/(a*b^4) - ((a^2 - 2*b^2
)*Sech[x])/b^3 + (a*Sech[x]^2)/(2*b^2) - Sech[x]^3/(3*b)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx, x, b\text{sech}(x)\right)}{b^4}$$

$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)dx, x, b\text{sech}(x)\right)}{b^4}$$

$$= \frac{\log(\cosh(x))}{a}+\frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\text{sech}(x)}{b^3}+\frac{a\text{sech}^2(x)}{2b^2}-\frac{\text{sech}^3(x}{3b}$$

Mathematica [A] time = 0.19, size = 85, normalized size = 1.18

$$\frac{3a^2b^2\text{sech}^2(x)-6ab(a^2-2b^2)\text{sech}(x)-6a^2(a^2-2b^2)\log(\cosh(x))+6(a^2-b^2)^2\log(a\cosh(x)+b)-2ab^3\text{se}}{6ab^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]),x]
```

```
[Out] (-6*a^2*(a^2 - 2*b^2)*Log[Cosh[x]] + 6*(a^2 - b^2)^2*Log[b + a*Cosh[x]] - 6
*a*b*(a^2 - 2*b^2)*Sech[x] + 3*a^2*b^2*Sech[x]^2 - 2*a*b^3*Sech[x]^3)/(6*a*
b^4)
```

fricas [B] time = 0.45, size = 1280, normalized size = 17.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b - 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b - 2*a*b^3)*sinh(x)^5 + 3*b^4*x + 3*(3*b^4*x -
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 + 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b - 2*a*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^3*b - 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 + 3*a^3*b - 4*a*b^3 + 15*(a^3*b - 2*a*b^3)*cosh(x)^2 + 3*
(3*b^4*x - 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x - 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x - 2*a^2*b^2 + 20*(a^3*b - 2*a*b^3)*cosh
(x)^3 + 6*(3*b^4*x - 2*a^2*b^2)*cosh(x)^2 + 4*(3*a^3*b - 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b - 2*a*b^3)*cosh(x) - 3*((a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2 + b^4)
*sinh(x)^6 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + b^4
```

```
 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 4*(5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 - 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 - 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + 5*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 + a^4 - 2*a^2*b^2 + 4*(5*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
 6*((a^4 - 2*a^2*b^2)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + (a^4 - 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b - 2*a*b^3)*cosh(x)^4 + a^3*b - 2*a*b^3 + 2*(3*b^4*x -
2*a^2*b^2)*cosh(x)^3 + 2*(3*a^3*b - 4*a*b^3)*cosh(x)^2 + (3*b^4*x - 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
sinh(x)^6 + 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 + a*b^4 + 3*(5*a*b^4*cosh
(x)^2 + a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 + 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 + 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 + 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 152, normalized size = 2.11

$$-\frac{\left(a^{3}-2\,ab^{2}\right)\log\left(e^{(-x)}+e^{x}\right)}{b^{4}}+\frac{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{ab^{4}}+\frac{11\,a^{3}\left(e^{(-x)}+e^{x}\right)^{3}-22\,ab^{2}\left(e^{(-x)}+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -(a^3 - 2*a*b^2)*log(e^(-x) + e^x)/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(a*
(e^(-x) + e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) + e^x)^3 - 22*a*b^2*(e
^(-x) + e^x)^3 - 12*a^2*b*(e^(-x) + e^x)^2 + 24*b^3*(e^(-x) + e^x)^2 + 12*a
*b^2*(e^(-x) + e^x) - 16*b^3)/(b^4*(e^(-x) + e^x)^3)
```

maple [B] time = 0.15, size = 233, normalized size = 3.24

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{a^{3}\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}{b^{4}}-\frac{2a\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right.}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+a^3/b^4*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-2*a/
b^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+1/a*ln(a*tanh(1/2*x)^2-tanh(1/2
*x)^2*b+a+b)-1/a*ln(tanh(1/2*x)+1)-2/b^3/(tanh(1/2*x)^2+1)*a^2-2/b^2/(tanh(
1/2*x)^2+1)*a+2/b/(tanh(1/2*x)^2+1)+2/b^2/(tanh(1/2*x)^2+1)^2*a+4/b/(tanh(1
/2*x)^2+1)^2-1/b^4*ln(tanh(1/2*x)^2+1)*a^3+2/b^2*ln(tanh(1/2*x)^2+1)*a-8/3/
b/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.49, size = 164, normalized size = 2.28

$$\frac{2\left(3\,abe^{(-2\,x)}+3\,abe^{(-4\,x)}-3\left(a^2-2\,b^2\right)e^{(-x)}-2\left(3\,a^2-4\,b^2\right)e^{(-3\,x)}-3\left(a^2-2\,b^2\right)e^{(-5\,x)}\right)}{3\left(3\,b^3e^{(-2\,x)}+3\,b^3e^{(-4\,x)}+b^3e^{(-6\,x)}+b^3\right)}+\frac{x}{a}-\frac{\left(a^3-2\,ab^2\right)\log\left(e^{(-\right.}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] 2/3*(3*a*b*e^(-2*x) + 3*a*b*e^(-4*x) - 3*(a^2 - 2*b^2)*e^(-x) - 2*(3*a^2 -
4*b^2)*e^(-3*x) - 3*(a^2 - 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) + 3*b^3*e^(-4*x
) + b^3*e^(-6*x) + b^3) + x/a - (a^3 - 2*a*b^2)*log(e^(-2*x) + 1)/b^4 + (a^
4 - 2*a^2*b^2 + b^4)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^4)
```

mupad [B] time = 1.80, size = 155, normalized size = 2.15

$$\frac{\frac{2\,a}{b^2}-\frac{2\,e^x\left(a^2-2\,b^2\right)}{b^3}}{e^{2\,x}+1}-\frac{x}{a}-\frac{\frac{2\,a}{b^2}+\frac{8\,e^x}{3\,b}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\ln\left(e^{2\,x}+1\right)\left(2\,a\,b^2-a^3\right)}{b^4}+\frac{8\,e^x}{3\,b\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)}+\frac{\ln\left(a+2\,b\,e^x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + b/cosh(x)),x)
```

```
[Out] ((2*a)/b^2 - (2*exp(x)*(a^2 - 2*b^2))/b^3)/(exp(2*x) + 1) - x/a - ((2*a)/b^
2 + (8*exp(x))/(3*b))/(2*exp(2*x) + exp(4*x) + 1) + (log(exp(2*x) + 1)*(2*a
*b^2 - a^3))/b^4 + (8*exp(x))/(3*b*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
) + (log(a + 2*b*exp(x) + a*exp(2*x))*(a^4 + b^4 - 2*a^2*b^2))/(a*b^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{a+b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**5/(a + b*sech(x)), x)
```

3.116 $\int \frac{\tanh^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=94

$$\frac{\left(2a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3}-\frac{a\tanh(x)}{b^2}+\frac{x}{a}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

```
[Out] x/a+1/2*(2*a^2-3*b^2)*arctan(sinh(x))/b^3-2*(a-b)^(3/2)*(a+b)^(3/2)*arctan(
(a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/b^3-a*tanh(x)/b^2+1/2*sech(x)*tanh(x
)/b
```

Rubi [A] time = 0.32, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3898, 2893, 3057, 2659, 205, 3770}

$$\frac{\left(2a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3}-\frac{a\tanh(x)}{b^2}-\frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3}+\frac{x}{a}+\frac{\tanh(x)\text{sech}(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*(a - b)^(3/2)*(a + b)^
(3/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^3) - (a*Tanh[x])/b^
2 + (Sech[x]*Tanh[x])/(2*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
```

```
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3057

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(C*x)/(b*d), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\sinh(x)\tanh^3(x)}{b+a\cosh(x)}\,dx$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{\int \frac{\left(-2a^2+3b^2-ab\cosh(x)-2b^2\cosh^2(x)\right)\text{sech}(x)}{b+a\cosh(x)}\,dx}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{\left(a^2-b^2\right)^2 \int \frac{1}{b+a\cosh(x)}\,dx}{ab^3} - \frac{\left(-2a^2+3b^2\right)\int \text{sech}(x)\,d}{2b^3}$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{\left(2\left(a^2-b^2\right)^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{x}{a} + \frac{\left(2a^2-3b^2\right)\tan^{-1}(\sinh(x))}{2b^3} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3} - \frac{a\tanh(x)}{b^2} +$$

Mathematica [A] time = 0.42, size = 113, normalized size = 1.20

$$\frac{\text{sech}^2(x)(a\cosh(x)+b)\left(2\cosh(x)\left(a\left(2a^2-3b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)+b^3x\right)+\right.}{2ab^3(a+b\text{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]^2*(2*(b^3*x + a*(2*a^2 - 3*b^2)*ArcTan[Tanh[x/2]]
+ 2*(a^2 - b^2)^(3/2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])*Cosh[x]
 + a*b*(-2*a*Sinh[x] + b*Tanh[x])))/(2*a*b^3*(a + b*Sech[x]))
```

fricas [B] time = 0.51, size = 1254, normalized size = 13.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh(
x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cosh
(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^2
 - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*s
inh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2
)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*s
```

```
inh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*
cosh(x)*sinh(x)^3 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^
3 - 3*a*b^2)*cosh(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*s
inh(x))*arctan(cosh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2
- a*b^2 + 4*(b^3*x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*co
sh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*
cosh(x)^2 + a*b^3)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))
, (b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh
(x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cos
h(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^
2 + 2*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2
)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 -
b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x)
)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b
^2)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*cosh(x)*sinh(x)^3
 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^3 - 3*a*b^2)*cosh
(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*(
(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*sinh(x))*arctan(co
sh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2 - a*b^2 + 4*(b^3*
x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 +
 a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 + a*b^3
)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))]
```

giac [A] time = 0.14, size = 111, normalized size = 1.18

$$\frac{x}{a}+\frac{\left(2\,a^{2}-3\,b^{2}\right)\arctan\left(e^{x}\right)}{b^{3}}-\frac{2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\arctan\left(\frac{ae^{x}+b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,ab^{3}}+\frac{be^{(3\,x)}+2\,ae^{(2\,x)}-be^{x}+2\,a}{b^{2}\left(e^{(2\,x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a + (2*a^2 - 3*b^2)*arctan(e^x)/b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((a
*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*b^3) + (b*e^(3*x) + 2*a*e^(2*
x) - b*e^x + 2*a)/(b^2*(e^(2*x) + 1)^2)
```

maple [B] time = 0.14, size = 248, normalized size = 2.64

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{2a^{3}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{3}\sqrt{(a+b)\,(a-b)}}+\frac{4a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)\,(a-b)}}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)\,(a-b)}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-2/b^3*a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*
x)/((a+b)*(a-b))^(1/2))+4*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/
((a+b)*(a-b))^(1/2))-2*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a
+b)*(a-b))^(1/2))+1/a*ln(tanh(1/2*x)+1)-2/b^2/(tanh(1/2*x)^2+1)^2*tanh(1/2*
x)^3*a-1/b/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-2/b^2/(tanh(1/2*x)^2+1)^2*tanh
(1/2*x)*a+1/b/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)+2/b^3*arctan(tanh(1/2*x))*a^2
-3/b*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.26, size = 700, normalized size = 7.45

$$\frac{\frac{2a}{b^2}+\frac{e^x}{b}}{e^{2x}+1}+\frac{x}{a}-\frac{\ln\left(e^x-i\right)\left(a^2\,2i-b^2\,3i\right)}{2b^3}+\frac{\ln\left(e^x+1i\right)\left(a^2\,2i-b^2\,3i\right)}{2b^3}-\frac{2e^x}{b\left(2e^{2x}+e^{4x}+1\right)}+\ln\left(\left(\frac{64a^8+96e^x a^7 b-288a^6 b^2-}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b/cosh(x)),x)
```

```
[Out] ((2*a)/b^2 + exp(x)/b)/(exp(2*x) + 1) + x/a - (log(exp(x) - 1i)*(a^2*2i - b
^2*3i))/(2*b^3) + (log(exp(x) + 1i)*(a^2*2i - b^2*3i))/(2*b^3) - (2*exp(x))
/(b*(2*exp(2*x) + exp(4*x) + 1)) + (log((((64*a^8 + 32*b^8 - 272*a^2*b^6 +
456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) + 96*a^7*b*exp(x) + 600*a^3*b^
5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) - (((16*(a^2 - b^2)*(4*a*b^2 - 4*a
```

```
^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 + (32*(-(a + b)^3*(a - b)^3)^(1/2)
*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b))*(-(a + b)^3*(a
 - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (8*(a^2 -
b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 + 10*b^3*exp(x) - 7*a^2*b*exp(x)))/
(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (log(- (((64*a^8 + 32*b^
8 - 272*a^2*b^6 + 456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) + 96*a^7*b*e
xp(x) + 600*a^3*b^5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) + (((16*(a^2 - b
^2)*(4*a*b^2 - 4*a^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 - (32*(-(a + b)^
3*(a - b)^3)^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*
b))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(a
*b^3) - (8*(a^2 - b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 + 10*b^3*exp(x) -
 7*a^2*b*exp(x)))/(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sech(x)), x)
```

3.117 $\int \frac{\tanh^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=35

$$\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\,\mathrm{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}+\frac{\mathrm{sech}(x)}{b}$$

```
[Out] ln(cosh(x))/a+(1-a^2/b^2)*ln(a+b*sech(x))/a+sech(x)/b
```

Rubi [A] time = 0.08, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\,\mathrm{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}+\frac{\mathrm{sech}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((1 - a^2/b^2)*Log[a + b*Sech[x]])/a + Sech[x]/b
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\operatorname{sech}(x)\right)}{b^2}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)dx, x, b\operatorname{sech}(x)\right)}{b^2}$$
$$= \frac{\log(\cosh(x))}{a} + \frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\operatorname{sech}(x))}{a} + \frac{\operatorname{sech}(x)}{b}$$

Mathematica [A] time = 0.09, size = 37, normalized size = 1.06

$$\frac{(b^2-a^2)\log(a\cosh(x)+b)+a^2\log(\cosh(x))+ab\operatorname{sech}(x)}{ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (a^2*Log[Cosh[x]] + (-a^2 + b^2)*Log[b + a*Cosh[x]] + a*b*Sech[x])/(a*b^2)
```

fricas [B] time = 0.41, size = 200, normalized size = 5.71

$$-\frac{b^2x\cosh(x)^2+b^2x\sinh(x)^2+b^2x-2\,ab\cosh(x)+\left((a^2-b^2)\cosh(x)^2+2\left(a^2-b^2\right)\cosh(x)\sinh(x)+\left(a^2-b^2\right.\right.}{ab^2\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 + b^2*x - 2*a*b*cosh(x) + ((a^2 - b^2)*
cosh(x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2 + a^2 - b
^2)*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a^2*cosh(x)^2 + 2*a^2*cos
h(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*
(b^2*x*cosh(x) - a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) +
 a*b^2*sinh(x)^2 + a*b^2)
```

giac [B] time = 0.12, size = 73, normalized size = 2.09

$$\frac{a\log\left(e^{(-x)}+e^x\right)}{b^2} - \frac{(a^2-b^2)\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{ab^2} - \frac{a\left(e^{(-x)}+e^x\right)-2\,b}{b^2\left(e^{(-x)}+e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*log(e^(-x) + e^x)/b^2 - (a^2 - b^2)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b
^2) - (a*(e^(-x) + e^x) - 2*b)/(b^2*(e^(-x) + e^x))
```

maple [B] time = 0.13, size = 107, normalized size = 3.06

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{a\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}{b^{2}}+\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+a+b\right)}{a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-a/b^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)+1/a*ln
(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-1/a*ln(tanh(1/2*x)+1)+2/b/(tanh(1/2*x
)^2+1)+1/b^2*ln(tanh(1/2*x)^2+1)*a
```

maxima [A] time = 0.44, size = 67, normalized size = 1.91

$$\frac{x}{a}+\frac{2\,e^{(-x)}}{be^{(-2\,x)}+b}+\frac{a\log\left(e^{(-2\,x)}+1\right)}{b^{2}}-\frac{\left(a^{2}-b^{2}\right)\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2*e^(-x)/(b*e^(-2*x) + b) + a*log(e^(-2*x) + 1)/b^2 - (a^2 - b^2)*log
(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^2)
```

mupad [B] time = 1.60, size = 260, normalized size = 7.43

$$\frac{2\,\mathrm{e}^{x}}{b+b\,\mathrm{e}^{2\,x}}-\frac{x}{a}+\frac{\ln\left(16\,a^{5}\,\mathrm{e}^{2\,x}+4\,a\,b^{4}+16\,a^{5}-16\,a^{3}\,b^{2}+8\,b^{5}\,\mathrm{e}^{x}-16\,a^{3}\,b^{2}\,\mathrm{e}^{2\,x}+32\,a^{4}\,b\,\mathrm{e}^{x}+4\,a\,b^{4}\,\mathrm{e}^{2\,x}-32\,a^{2}\,b^{3}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] (2*exp(x))/(b + b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) + 4*a*b^4 + 16*a^5
- 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a*b
^4*exp(2*x) - 32*a^2*b^3*exp(x))/a - (a*log(16*a^5*exp(2*x) + 4*a*b^4 + 16*
a^5 - 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4
*a*b^4*exp(2*x) - 32*a^2*b^3*exp(x)))/b^2 + (a*log(16*a^6*exp(2*x) - 4*b^6*
exp(2*x) + 16*a^6 - 4*b^6 + 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(2*x) -
 32*a^4*b^2*exp(2*x)))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\operatorname{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sech(x)), x)
```

3.118 $\int \frac{\tanh^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}+\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}$$

```
[Out] x/a-arctan(sinh(x))/b+2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(
1/2)*(a+b)^(1/2)/a/b
```

Rubi [A] time = 0.17, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3894, 4051, 3770, 3919, 3831, 2659, 205}

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}+\frac{x}{a}-\frac{\tan^{-1}(\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/b + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4051

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx &= -\int \frac{-1+\text{sech}^2(x)}{a+b\text{sech}(x)}\,dx\\
&= -\frac{\int \text{sech}(x)\,dx}{b} - \frac{\int \frac{-b-a\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{b}\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \frac{\left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b}\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{a} - \frac{\tan^{-1}(\sinh(x))}{b} + \frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 62, normalized size = 1.00

$$
\frac{-2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right) - 2a\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + bx}{ab}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (b*x - 2*a*ArcTan[Tanh[x/2]] - 2*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2]
)/Sqrt[a^2 - b^2]])/(a*b)
```

fricas [A] time = 0.45, size = 193, normalized size = 3.11

$$
\left[\frac{bx - 2\,a\arctan\left(\cosh(x)+\sinh(x)\right) + \sqrt{-a^2+b^2}\,\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2\,ab\cosh(x) - a^2 + 2\,b^2 + 2\,\left(a^2\cosh(x) + ab\right)\sinh(x}{a\cosh(x)^2 + a\sinh(x)^2 + 2\,b\cosh(x) + 2\,(a\cosh(x) +}\right.}{ab}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b*x - 2*a*arctan(cosh(x) + sinh(x)) + sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2
 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)))/(a*b), (b*x - 2*a*ar
ctan(cosh(x) + sinh(x)) - 2*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x)
+ b)/sqrt(a^2 - b^2)))/(a*b)]
```

giac [A] time = 0.15, size = 52, normalized size = 0.84

$$\frac{x}{a} - \frac{2\,\arctan\left(e^{x}\right)}{b} + \frac{2\,\sqrt{a^{2} - b^{2}}\,\arctan\left(\frac{ae^{x} + b}{\sqrt{a^{2} - b^{2}}}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 2*arctan(e^x)/b + 2*sqrt(a^2 - b^2)*arctan((a*e^x + b)/sqrt(a^2 - b^2
))/(a*b)
```

maple [B] time = 0.12, size = 113, normalized size = 1.82

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)\,(a-b)}}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)\,(a-b)}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)+2*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/(
(a+b)*(a-b))^(1/2))-2*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+
b)*(a-b))^(1/2))+1/a*ln(tanh(1/2*x)+1)-2/b*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.92, size = 273, normalized size = 4.40

$$\frac{\ln(e^x - i)\,1i - \ln(e^x + 1i)\,1i}{b} + \frac{\ln\left(2\,a\,b^3 - 2\,a^3\,b + a^3\,\sqrt{b^2 - a^2} + a^4\,e^x + 4\,b^4\,e^x - 2\,a\,b^2\,\sqrt{b^2 - a^2} - 4\,b^3\,e^x\,\sqrt{b^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b + (log(2*a*b^3 - 2*a^3*b + a^
3*(b^2 - a^2)^(1/2) + a^4*exp(x) + 4*b^4*exp(x) - 2*a*b^2*(b^2 - a^2)^(1/2)
 - 4*b^3*exp(x)*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) + 3*a^2*b*exp(x)*(b^2
- a^2)^(1/2))*(b^2 - a^2)^(1/2) - log(2*a*b^3 - 2*a^3*b - a^3*(b^2 - a^2)^(
1/2) + a^4*exp(x) + 4*b^4*exp(x) + 2*a*b^2*(b^2 - a^2)^(1/2) + 4*b^3*exp(x)
*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) - 3*a^2*b*exp(x)*(b^2 - a^2)^(1/2))*(
b^2 - a^2)^(1/2) + b*x)/(a*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)), x)
```

3.119 $\int \frac{\tanh(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(a+b\operatorname{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}$$

```
[Out] ln(cosh(x))/a+ln(a+b*sech(x))/a
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3885, 36, 29, 31}

$$\frac{\log(a+b\operatorname{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[a + b*Sech[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{a + b\text{sech}(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\text{sech}(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\text{sech}(x)\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\text{sech}(x)\right)}{a}$$
$$= \frac{\log(\cosh(x))}{a} + \frac{\log(a + b\text{sech}(x))}{a}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 0.58

$$\frac{\log(a\cosh(x) + b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]),x]
[Out] Log[b + a*Cosh[x]]/a
```

fricas [A] time = 0.43, size = 27, normalized size = 1.42

$$-\frac{x - \log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="fricas")
[Out] -(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

giac [A] time = 0.13, size = 19, normalized size = 1.00

$$\frac{\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="giac")
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

maple [A] time = 0.10, size = 21, normalized size = 1.11

$$\frac{\ln\left(a + b\,\text{sech}(x)\right)}{a} - \frac{\ln\left(\text{sech}(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sech(x)),x)
```

```
[Out] ln(a+b*sech(x))/a-1/a*ln(sech(x))
```

maxima [A] time = 0.31, size = 26, normalized size = 1.37

$$\frac{x}{a} + \frac{\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

mupad [B] time = 0.11, size = 23, normalized size = 1.21

$$-\frac{x - \ln\left(a + 2\,b\,e^{x} + a\,e^{2\,x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b/cosh(x)),x)
```

```
[Out] -(x - log(a + 2*b*exp(x) + a*exp(2*x)))/a
```

sympy [A] time = 0.46, size = 41, normalized size = 2.16

$$\begin{cases} \frac{\tilde{\infty}}{\operatorname{sech}(x)} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{1}{b \operatorname{sech}(x)} & \text{for}\: a = 0 \\ \frac{x - \log(\tanh(x) + 1)}{a} & \text{for}\: b = 0 \\ \frac{x}{a} + \frac{\log\left(\frac{a}{b} + \operatorname{sech}(x)\right)}{a} - \frac{\log(\tanh(x) + 1)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sech(x)),x)
```

```
[Out] Piecewise((zoo/sech(x), Eq(a, 0) & Eq(b, 0)), (1/(b*sech(x)), Eq(a, 0)), ((
x - log(tanh(x) + 1))/a, Eq(b, 0)), (x/a + log(a/b + sech(x))/a - log(tanh(
x) + 1)/a, True))
```

3.120 $\int \frac{\coth(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=66

$$-\frac{b^2\log(a+b\operatorname{sech}(x))}{a\left(a^2-b^2\right)}+\frac{\log(1-\operatorname{sech}(x))}{2(a+b)}+\frac{\log(\operatorname{sech}(x)+1)}{2(a-b)}+\frac{\log(\cosh(x))}{a}$$

```
[Out] ln(cosh(x))/a+1/2*ln(1-sech(x))/(a+b)+1/2*ln(1+sech(x))/(a-b)-b^2*ln(a+b*se
ch(x))/a/(a^2-b^2)
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3885, 894}

$$-\frac{b^2\log(a+b\operatorname{sech}(x))}{a\left(a^2-b^2\right)}+\frac{\log(1-\operatorname{sech}(x))}{2(a+b)}+\frac{\log(\operatorname{sech}(x)+1)}{2(a-b)}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[1 - Sech[x]]/(2*(a + b)) + Log[1 + Sech[x]]/(2*(a - b)
) - (b^2*Log[a + b*Sech[x]])/(a*(a^2 - b^2))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx = -\left(b^2\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)}\,dx, x, b\text{sech}(x)\right)\right)$$
$$= -\left(b^2\,\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)}+\frac{1}{ab^2x}+\frac{1}{a(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)b^2(b+x)}\right) d$$
$$= \frac{\log(\cosh(x))}{a}+\frac{\log(1-\text{sech}(x))}{2(a+b)}+\frac{\log(1+\text{sech}(x))}{2(a-b)}-\frac{b^2\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)}$$

Mathematica [**A**] time = 0.09, size = 44, normalized size = 0.67

$$-\frac{a^2(-\log(\sinh(x)))+b^2\log(a\cosh(x)+b)+ab\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sech[x]),x]
```

```
[Out] -((b^2*Log[b + a*Cosh[x]] - a^2*Log[Sinh[x]] + a*b*Log[Tanh[x/2]])/(a^3 - a
*b^2))
```

fricas [**A**] time = 0.43, size = 81, normalized size = 1.23

$$-\frac{b^2\log\left(\frac{2(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)+\left(a^2-b^2\right)x-\left(a^2+ab\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(a^2-ab\right)\log\left(\cosh(x)+\sinh(x)-\right.}{a^3-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + (a^2 - b^2)*x - (a^2 + a
*b)*log(cosh(x) + sinh(x) + 1) - (a^2 - a*b)*log(cosh(x) + sinh(x) - 1))/(a
^3 - a*b^2)
```

giac [**A**] time = 0.12, size = 67, normalized size = 1.02

$$-\frac{b^2\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^3-ab^2}+\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) + 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

maple [A] time = 0.16, size = 78, normalized size = 1.18

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{b^2\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b\right)}{a\left(a+b\right)\left(a-b\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sech(x)),x)
```

```
[Out] -1/a*ln(tanh(1/2*x)-1)-b^2/a/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b
+a+b)-1/a*ln(tanh(1/2*x)+1)+1/(a+b)*ln(tanh(1/2*x))
```

maxima [A] time = 0.47, size = 67, normalized size = 1.02

$$-\frac{b^2\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^3-ab^2}+\frac{x}{a}+\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b^2*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^3 - a*b^2) + x/a + log(e^(-x) + 1)
/(a - b) + log(e^(-x) - 1)/(a + b)
```

mupad [B] time = 1.72, size = 271, normalized size = 4.11

$$\frac{\ln\left(64\,a\,b^4+32\,a^4\,b+32\,b^5+96\,a^2\,b^3+64\,a^3\,b^2+32\,b^5\,e^x+64\,a\,b^4\,e^x+32\,a^4\,b\,e^x+96\,a^2\,b^3\,e^x+64\,a^3\,b^2\,e^x\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b/cosh(x)),x)
```

```
[Out] log(64*a*b^4 + 32*a^4*b + 32*b^5 + 96*a^2*b^3 + 64*a^3*b^2 + 32*b^5*exp(x)
+ 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(x) + 64*a^3*b^2*exp(x)
)/(a - b) - x/a + log(64*a*b^4 - 32*a^4*b - 32*b^5 - 96*a^2*b^3 + 64*a^3*b^
2 + 32*b^5*exp(x) - 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(x) -
 64*a^3*b^2*exp(x))/(a + b) + (b^2*log(4*a^5*exp(2*x) + 4*a*b^4 + 4*a^5 + 4
*a^3*b^2 + 8*b^5*exp(x) + 4*a^3*b^2*exp(2*x) + 8*a^4*b*exp(x) + 4*a*b^4*exp
(2*x) + 8*a^2*b^3*exp(x)))/(a*b^2 - a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(x\right)}{a+b\,\mathrm{sech}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*sech(x)), x)
```

3.121 $\int \frac{\coth^2(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=114

$$-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{ax}{a^2-b^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] a*x/(a^2-b^2)-b^2*x/a/(a^2-b^2)+2*b^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^
(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)-a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

Rubi [A] time = 0.20, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3898, 2902, 2606, 8, 3473, 2735, 2659, 205}

$$-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{ax}{a^2-b^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b^2*x)/(a*(a^2 - b^2)) + (2*b^3*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)) - (a*Coth[x])/(a^2 -
 b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx$$
$$= \frac{a\int \coth^2(x)\,dx}{a^2-b^2} - \frac{b\int \coth(x)\text{csch}(x)\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{a^2-b^2}$$
$$= -\frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\coth(x)}{a^2-b^2} + \frac{a\int 1\,dx}{a^2-b^2} + \frac{(ib)\,\text{Subst}(\int 1\,dx, x, -i\text{csch}(x))}{a^2-b^2} + \frac{b^3\int \frac{1}{b+a\cosh(x)}\,d}{a\left(a^2-b^2\right)}$$
$$= \frac{ax}{a^2-b^2} - \frac{b^2x}{a\left(a^2-b^2\right)} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\text{csch}(x)}{a^2-b^2} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+b-(-a+b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right.\right.}{a\left(a^2-b^2\right)}$$
$$= \frac{ax}{a^2-b^2} - \frac{b^2x}{a\left(a^2-b^2\right)} + \frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\text{csch}(x)}{a^2-b^2}$$

Mathematica [A] time = 0.35, size = 81, normalized size = 0.71

$$\frac{\frac{2b^3\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a^2x - a^2\coth(x) + ab\text{csch}(x) - b^2x}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (a^2*x - b^2*x + (2*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] - a^2*Coth[x] + a*b*Csch[x])/(a^3 - a*b^2)
```

fricas [B] time = 0.42, size = 646, normalized size = 5.67

$$\left[\frac{2\,a^4 - 2\,a^2b^2 - \left(a^4 - 2\,a^2b^2 + b^4\right)x\cosh(x)^2 - \left(a^4 - 2\,a^2b^2 + b^4\right)x\sinh(x)^2 - \left(b^3\cosh(x)^2 + 2\,b^3\cosh(x)\sinh(\right.}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^
2 + b^4)*x*sinh(x)^2 - (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)
```

```
^2 - b^3)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*co
sh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cos
h(x) + b)*sinh(x) + a)) + (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cos
h(x) - 2*(a^3*b - a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5
- 2*a^3*b^2 + a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*
b^2 + a*b^4)*cosh(x)*sinh(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2), (2*a^4
 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^2 + b^4
)*x*sinh(x)^2 + 2*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 -
b^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) +
 (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(a^3*b - a*b^3 +
 (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^
5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)*sinh
(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2)]
```

giac [A] time = 0.13, size = 82, normalized size = 0.72

$$\frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^3-ab^2\right)\sqrt{a^2-b^2}}+\frac{x}{a}+\frac{2\,(be^x-a)}{\left(a^2-b^2\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^3 - a*b^2)*sqrt(a^2 - b^2)) +
 x/a + 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

maple [A] time = 0.18, size = 104, normalized size = 0.91

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2\,(a-b)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)\,a\,(a+b)\,\sqrt{(a+b)\,(a-b)}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{1}{2\,(a+b)\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sech(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)+2/(a-b)/a/(a+b)*b^3/((a+b)*(a-
b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a*ln(tanh(1/2*x)+
1)-1/2/(a+b)/tanh(1/2*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.67, size = 383, normalized size = 3.36

$$\frac{x}{a}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,b^3}{a^3\,\left(a\,b^2-a^3\right)\,\left(a^2-b^2\right)\,\sqrt{b^6}}-\frac{2\,\left(a\,b^3\,\sqrt{b^6}-a^3\,b\,\sqrt{b^6}\right)}{a^2\,b^2\,\left(a\,b^2-a^3\right)\,\sqrt{a^2\,\left(a^2-b^2\right)^3}\,\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}\right)+\frac{\cdots}{a^2\,b^2\,\left(a\,b^2-a^3\right)\cdots}\cdots\right.\right.\right.}{\sqrt{a^8-3\,a^6\,b^2+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b/cosh(x)),x)
```

```
[Out] x/a - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*at
an((exp(x)*((2*b^3)/(a^3*(a*b^2 - a^3)*(a^2 - b^2)*(b^6)^(1/2)) - (2*(a*b^3
*(b^6)^(1/2) - a^3*b*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^2)^
3)^(1/2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2))) + (2*(a^4*(b^6)^(1
/2) - a^2*b^2*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^2)^3)^(1/2
)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2)))*((a^4*(a^8 - a^2*b^6 + 3*
a^4*b^4 - 3*a^6*b^2)^(1/2))/2 - (a^2*b^2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6
*b^2)^(1/2))/2))*(b^6)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{a+b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sech(x)), x)
```

3.122 $\int \frac{\coth^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=113

$$\frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2}-\frac{1}{4(a+b)(1-\text{sech}(x))}-\frac{1}{4(a-b)(\text{sech}(x)+1)}+\frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(\text{s}}{4(a-b}$$

```
[Out] ln(cosh(x))/a+1/4*(2*a+3*b)*ln(1-sech(x))/(a+b)^2+1/4*(2*a-3*b)*ln(1+sech(x
))/(a-b)^2+b^4*ln(a+b*sech(x))/a/(a^2-b^2)^2-1/4/(a+b)/(1-sech(x))-1/4/(a-b
)/(1+sech(x))
```

Rubi [A] time = 0.19, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$\frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2}-\frac{1}{4(a+b)(1-\text{sech}(x))}-\frac{1}{4(a-b)(\text{sech}(x)+1)}+\frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(\text{s}}{4(a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((2*a + 3*b)*Log[1 - Sech[x]])/(4*(a + b)^2) + ((2*a - 3*b
)*Log[1 + Sech[x]])/(4*(a - b)^2) + (b^4*Log[a + b*Sech[x]])/(a*(a^2 - b^2)
^2) - 1/(4*(a + b)*(1 - Sech[x])) - 1/(4*(a - b)*(1 + Sech[x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = -\left(b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\text{sech}(x)\right)\right)$$
$$= -\left(b^4\,\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2} + \frac{2a+3b}{4b^4(a+b)^2(b-x)} + \frac{1}{ab^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+\right.\right.\right.$$
$$= \frac{\log(\cosh(x))}{a} + \frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2} + \frac{(2a-3b)\log(1+\text{sech}(x))}{4(a-b)^2} + \frac{b^4\log(a+b\text{se}}{a\left(a^2-b^2\right.}$$

Mathematica [A] time = 0.32, size = 112, normalized size = 0.99

$$\frac{4a\left(2a\left(a^2-2b^2\right)\log(\sinh(x)) + b\left(3b^2-a^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right) + 8b^4\log(a\cosh(x)+b) - a(a-b)^2(a+b)\text{csch}^2\left(\right.}{8a(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-(a*(a - b)^2*(a + b)*Csch[x/2]^2) + 8*b^4*Log[b + a*Cosh[x]] + 4*a*(2*a*(
a^2 - 2*b^2)*Log[Sinh[x]] + b*(-a^2 + 3*b^2)*Log[Tanh[x/2]]) + a*(a - b)*(a
 + b)^2*Sech[x/2]^2)/(8*a*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.47, size = 1222, normalized size = 10.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*x*s
inh(x)^4 - 2*(a^3*b - a*b^3)*cosh(x)^3 - 2*(a^3*b - a*b^3 - 4*(a^4 - 2*a^2*
b^2 + b^4)*x*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4
)*x)*cosh(x)^2 + 2*(2*a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)
^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*x - 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 + 2
*(a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(b^4*cosh(x)^4 +
 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 - 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4
*cosh(x)^2 - b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - b^4*cosh(x))*sinh(x))*log(
2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - ((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*
b^3)*cosh(x)^4 + 4*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)*sinh(x)^3
+ (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*sinh(x)^4 + 2*a^4 + a^3*b - 4*a^2*b
^2 - 3*a*b^3 - 2*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^2 - 2*(2*a^4
```

```
 + a^3*b - 4*a^2*b^2 - 3*a*b^3 - 3*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*co
sh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^3 - (
2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x
) + 1) - ((2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^4 + 4*(2*a^4 - a^3*
b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)*sinh(x)^3 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3
*a*b^3)*sinh(x)^4 + 2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - 2*(2*a^4 - a^3*b
- 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2 - 2*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 -
 3*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 -
 a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^3 - (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*
b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*(a^4 - 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 - a^3*b + a*b^3 - 3*(a^3*b - a*b^3)*cosh(x)^2 + 4*(a^4 -
a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a
*b^4 + (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 - 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^4 - 2*(a^5 - 2*a^3*b^2
+ a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^5 - 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^3 - (a^5 - 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

giac [A] time = 0.13, size = 193, normalized size = 1.71

$$\frac{b^4 \log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\left(a^2-2\,ab+b^2\right)}+\frac{(2\,a+3\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\left(a^2+2\,ab+b^2\right)}-\frac{a^3\left(e^{(-x)}+e^x\right)^2-2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*(2*a -
 3*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + 3*b)*log(e^(-x
) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a*b^2*(e^(
-x) + e^x)^2 - 2*a^2*b*(e^(-x) + e^x) + 2*b^3*(e^(-x) + e^x) + 4*a*b^2)/((a
^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e^x)^2 - 4))
```

maple [A] time = 0.16, size = 119, normalized size = 1.05

$$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8\,(a-b)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{b^4\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b\right)}{(a-b)^2\,(a+b)^2\,a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{1}{8\,(a+b)\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sech(x)),x)
```

```
[Out] -1/8*tanh(1/2*x)^2/(a-b)-1/a*ln(tanh(1/2*x)-1)+1/(a-b)^2*b^4/(a+b)^2/a*ln(a
*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-1/a*ln(tanh(1/2*x)+1)-1/8/(a+b)/tanh(1/
2*x)^2+1/(a+b)^2*ln(tanh(1/2*x))*a+3/2/(a+b)^2*ln(tanh(1/2*x))*b
```

maxima [A] time = 0.49, size = 164, normalized size = 1.45

$$\frac{b^4 \log\left(2\, be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^5 - 2\,a^3b^2 + ab^4} + \frac{(2\,a - 3\,b)\log\left(e^{(-x)} + 1\right)}{2\left(a^2 - 2\,ab + b^2\right)} + \frac{(2\,a + 3\,b)\log\left(e^{(-x)} - 1\right)}{2\left(a^2 + 2\,ab + b^2\right)} + \frac{be^{(-x)} - 2\,ae^{(-2\,x)} +}{a^2 - b^2 - 2\left(a^2 - b^2\right)e^{(-2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + 1/2*(2*a -
 3*b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + 3*b)*log(e^(-x) - 1)
/(a^2 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 -
2*(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x)) + x/a
```

mupad [B] time = 2.22, size = 339, normalized size = 3.00

$$\frac{\ln\left(e^x - 1\right)\,(2\,a + 3\,b)}{2\,a^2 + 4\,a\,b + 2\,b^2} - \frac{x}{a} - \frac{\frac{2\,a}{a^2 - b^2} - \frac{2\,b\,e^x}{a^2 - b^2}}{e^{4\,x} - 2\,e^{2\,x} + 1} - \frac{\frac{2\left(a^4 - a^2\,b^2\right)}{a\left(a^2 - b^2\right)^2} - \frac{e^x\left(a^2\,b - b^3\right)}{\left(a^2 - b^2\right)^2}}{e^{2\,x} - 1} + \frac{\ln\left(e^x + 1\right)\,(2\,a - 3\,b)}{2\,a^2 - 4\,a\,b + 2\,b^2} + \frac{b^4\,\ln\left(4\,a^9\,e^{2\,x} + 4\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1)*(2*a + 3*b))/(4*a*b + 2*a^2 + 2*b^2) - x/a - ((2*a)/(a^2 -
 b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(a^4 -
a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(exp(2*
x) - 1) + (log(exp(x) + 1)*(2*a - 3*b))/(2*a^2 - 4*a*b + 2*b^2) + (b^4*log(
4*a^9*exp(2*x) + 4*a*b^8 + 4*a^9 + 7*a^3*b^6 + 14*a^5*b^4 - 17*a^7*b^2 + 8*
b^9*exp(x) + 7*a^3*b^6*exp(2*x) + 14*a^5*b^4*exp(2*x) - 17*a^7*b^2*exp(2*x)
 + 8*a^8*b*exp(x) + 4*a*b^8*exp(2*x) + 14*a^2*b^7*exp(x) + 28*a^4*b^5*exp(x
) - 34*a^6*b^3*exp(x)))/(a*b^4 + a^5 - 2*a^3*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sech(x)), x)
```

3.123 $\int \frac{\coth^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=207

$$-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}+\frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{b^4x}{a\left(a^2-b^2\right)^2}-\frac{b^3\operatorname{csch}(x)}{\left(a^2-b^2\right)^2}-\frac{2b^5}{a}$$

```
[Out] -a*b^2*x/(a^2-b^2)^2+b^4*x/a/(a^2-b^2)^2+a*x/(a^2-b^2)-2*b^5*arctan((a-b)^(
1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)^(5/2)/(a+b)^(5/2)+a*b^2*coth(x)/(a^2-
b^2)^2-a*coth(x)/(a^2-b^2)-1/3*a*coth(x)^3/(a^2-b^2)-b^3*csch(x)/(a^2-b^2)^
2+b*csch(x)/(a^2-b^2)+1/3*b*csch(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.33, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3898, 2902, 2606, 3473, 8, 2735, 2659, 205}

$$\frac{b^4x}{a\left(a^2-b^2\right)^2}-\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}+\frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\operatorname{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}-\frac{2b^5\,\mathrm{t}}{a(}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sech[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a +
 b)^(5/2)) + (a*b^2*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x])/(a^2 - b^2) - (a*C
oth[x]^3)/(3*(a^2 - b^2)) - (b^3*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2
- b^2) + (b*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2902

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(a*d^2)/(a^2
 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[(
b*d)/(a^2 - b^2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[(a^2*d^2)/(g^2*(a^2 - b^2)), Int[((g*Cos[e + f*x])^(p + 2)*(d*Sin[e +
 f*x])^(n - 2))/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3898

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[(Cos[c + d*x]^m*(b + a*Sin[c + d*x])^n)/Sin[c + d*x]^(
m + n), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\cosh(x)\coth^4(x)}{b+a\cosh(x)}\,dx$$
$$= \frac{a\int \coth^4(x)\,dx}{a^2-b^2} - \frac{b\int \coth^3(x)\text{csch}(x)\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx}{a^2-b^2}$$
$$= -\frac{a\coth^3(x)}{3(a^2-b^2)} - \frac{(ab^2)\int \coth^2(x)\,dx}{(a^2-b^2)^2} + \frac{b^3\int \coth(x)\text{csch}(x)\,dx}{(a^2-b^2)^2} + \frac{b^4\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{(a^2-b^2)^2} + \frac{a\int \text{co}}{a^2}$$
$$= \frac{b^4x}{a(a^2-b^2)^2} + \frac{ab^2\coth(x)}{(a^2-b^2)^2} - \frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3(a^2-b^2)} + \frac{b\text{csch}(x)}{a^2-b^2} + \frac{b\text{csch}^3(x)}{3(a^2-b^2)} - \frac{(ab^2)\int 1}{(a^2-b^2}$$
$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} + \frac{ab^2\coth(x)}{(a^2-b^2)^2} - \frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3(a^2-b^2)} - \frac{b^3\text{csch}(x)}{(a^2-b^2)^2}$$
$$= -\frac{ab^2x}{(a^2-b^2)^2} + \frac{b^4x}{a(a^2-b^2)^2} + \frac{ax}{a^2-b^2} - \frac{2b^5\tan^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}} + \frac{ab^2\coth(x)}{(a^2-b^2)^2} - \frac{a\coth(x)}{a^2-b^2}$$

Mathematica [A] time = 0.78, size = 166, normalized size = 0.80

$$\frac{\text{sech}(x)(a\cosh(x)+b)\left(\frac{48b^5\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a(a^2-b^2)^{5/2}} + \frac{22b\tanh\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{16a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{2(8a+11b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} - \frac{\sinh(x)\text{csch}^4\left(\frac{x}{2}\right)}{2(a+b)} + \frac{8\sinh^4\left(\frac{x}{2}\right)}{a-}\right.}{24(a+b\text{sech}(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]*((24*x)/a + (48*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) - (2*(8*a + 11*b)*Coth[x/2])/(a + b)^
2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) -
 (16*a*Tanh[x/2])/(a - b)^2 + (22*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[
x]))
```

fricas [B] time = 0.49, size = 3530, normalized size = 17.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="fricas")

[Out] [-1/3*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 15*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 - 6*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 - 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*cosh(x)^2 - b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 - 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 + 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^2 + (4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*sinh(x)^5 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sinh(x)^6 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^4 - 4*(5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^2 - 6*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^5 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)), -1/3*(3*(a^6

```
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^5*b - 3*a^3*b^
3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b
^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(4*a^6 - 10*a^4
*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 + 3*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos
h(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 - 4*(a^5*b - 5*
a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 15
*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^
4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 3*(4*a^6
- 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)
^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 - 6*(4*a^6 -
10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2
 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5
)*cosh(x))*sinh(x)^2 + 6*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sin
h(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*cosh(x)^2 - b^5
)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh
(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 - 2*b^5*cosh(x)
^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b -
 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*
cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 + 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c
osh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^2 + (4*a^6
 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x
))*sinh(x))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*cosh(x)^6 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*s
inh(x)^5 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sinh(x)^6 + 3*(a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6 - 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^4 - 4*(5*(a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*cosh(x))*sinh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*co
sh(x)^2 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + 5*(a^7 - 3*a^5*b^2 + 3*a
^3*b^4 - a*b^6)*cosh(x)^4 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)
^2)*sinh(x)^2 - 6*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^5 - 2*(a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.14, size = 190, normalized size = 0.92

$$-\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\left(a^5-2\,a^3b^2+ab^4\right)\sqrt{a^2-b^2}}+\frac{x}{a}+\frac{2\left(3\,a^2be^{(5\,x)}-6\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+9\,ab^2e^{(4\,x)}-2\,a^2be^{(3\,x)}+8\,b^3e^{(3\,x)}+6\,a^3e^{(}\right.}{3\left(a^4-2\,a^2b^2+b^4\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt(
a^2 - b^2)) + x/a + 2/3*(3*a^2*b*e^(5*x) - 6*b^3*e^(5*x) - 6*a^3*e^(4*x) +
9*a*b^2*e^(4*x) - 2*a^2*b*e^(3*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) - 12*a*b^
2*e^(2*x) + 3*a^2*b*e^x - 6*b^3*e^x - 4*a^3 + 7*a*b^2)/((a^4 - 2*a^2*b^2 +
b^4)*(e^(2*x) - 1)^3)
```

maple [A] time = 0.18, size = 179, normalized size = 0.86

$$-\frac{a\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24\left(a-b\right)^{2}}+\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)b}{24\left(a-b\right)^{2}}-\frac{5a\tanh\left(\frac{x}{2}\right)}{8\left(a-b\right)^{2}}+\frac{7\tanh\left(\frac{x}{2}\right)b}{8\left(a-b\right)^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{2b^{5}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a-b\right)^{2}\left(a+b\right)^{2}a\sqrt{\left(a+b\right)\left(a-b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sech(x)),x)
```

```
[Out] -1/24/(a-b)^2*a*tanh(1/2*x)^3+1/24/(a-b)^2*tanh(1/2*x)^3*b-5/8/(a-b)^2*a*ta
nh(1/2*x)+7/8/(a-b)^2*tanh(1/2*x)*b-1/a*ln(tanh(1/2*x)-1)-2/(a-b)^2/(a+b)^2
/a*b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/
a*ln(tanh(1/2*x)+1)-1/24/(a+b)/tanh(1/2*x)^3-5/8/(a+b)^2/tanh(1/2*x)*a-7/8/
(a+b)^2/tanh(1/2*x)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.83, size = 713, normalized size = 3.44

$$\frac{x}{a}-\frac{\frac{8\,a}{3\,\left(a^2-b^2\right)}-\frac{8\,b\,{\mathrm{e}}^x}{3\,\left(a^2-b^2\right)}}{3\,{\mathrm{e}}^{2\,x}-3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}-1}-\frac{\frac{2\,\left(2\,a^4-3\,a^2\,b^2\right)}{a\,{\left(a^2-b^2\right)}^2}-\frac{2\,{\mathrm{e}}^x\,\left(a^2\,b-2\,b^3\right)}{{\left(a^2-b^2\right)}^2}}{{\mathrm{e}}^{2\,x}-1}-\frac{\frac{4\,\left(a^4-a^2\,b^2\right)}{a\,{\left(a^2-b^2\right)}^2}-\frac{8\,{\mathrm{e}}^x\,\left(a^2\,b-b^3\right)}{3\,{\left(a^2-b^2\right)}^2}}{{\mathrm{e}}^{4\,x}-2\,{\mathrm{e}}^{2\,x}+1}-\frac{2\,\mathrm{atan}\left(\left({\mathrm{e}}^x\,\left(\frac{2\,b^5}{a^3\,{\left(a^2-b^2\right)}^2\,\sqrt{b^{10}}\,\left(a^5\right.}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b/cosh(x)),x)
```

```
[Out] x/a - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*(a^2 - b^2)))/(3*exp(2*x) -
3*exp(4*x) + exp(6*x) - 1) - ((2*(2*a^4 - 3*a^2*b^2))/(a*(a^2 - b^2)^2) - (
2*exp(x)*(a^2*b - 2*b^3))/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((4*(a^4 - a^2*b^
2))/(a*(a^2 - b^2)^2) - (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^2))/(exp(4*
x) - 2*exp(2*x) + 1) - (2*atan((exp(x)*((2*b^5)/(a^3*(a^2 - b^2)^2*(b^10)^(
1/2)*(a*b^4 + a^5 - 2*a^3*b^2)) + (2*(a*b^5*(b^10)^(1/2) - 2*a^3*b^3*(b^10)
^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*b^4 + a
^5 - 2*a^3*b^2)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*
a^10*b^2)^(1/2))) + (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2) - 2*a^4*b^2
*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*b^4 + a^5 - 2*a^3*b^2
)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2
)))*((a^6*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b
^2)^(1/2))/2 + (a^2*b^4*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*
b^4 - 5*a^10*b^2)^(1/2))/2 - a^4*b^2*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*
b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1/2))/(a^12 - a^2*b^10 + 5*a
^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a + b\operatorname{sech}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*sech(x)), x)
```

3.124 $\int \frac{\coth^5(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal. Leaf size=178

$$\frac{\left(8a^2+21ab+15b^2\right)\log(1-\operatorname{sech}(x))}{16(a+b)^3}+\frac{\left(8a^2-21ab+15b^2\right)\log(\operatorname{sech}(x)+1)}{16(a-b)^3}-\frac{b^6\log(a+b\operatorname{sech}(x))}{a\left(a^2-b^2\right)^3}-\frac{5a}{16(a+b)^2}$$

```
[Out] ln(cosh(x))/a+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sech(x))/(a+b)^3+1/16*(8*a^2-
21*a*b+15*b^2)*ln(1+sech(x))/(a-b)^3-b^6*ln(a+b*sech(x))/a/(a^2-b^2)^3-1/16
/(a+b)/(1-sech(x))^2+1/16*(-5*a-7*b)/(a+b)^2/(1-sech(x))-1/16/(a-b)/(1+sech
(x))^2+1/16*(-5*a+7*b)/(a-b)^2/(1+sech(x))
```

Rubi [A] time = 0.32, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3885, 894}

$$-\frac{b^6\log(a+b\operatorname{sech}(x))}{a\left(a^2-b^2\right)^3}+\frac{\left(8a^2+21ab+15b^2\right)\log(1-\operatorname{sech}(x))}{16(a+b)^3}+\frac{\left(8a^2-21ab+15b^2\right)\log(\operatorname{sech}(x)+1)}{16(a-b)^3}-\frac{5}{16(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sech[x]])/(16*(a + b)^3
) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sech[x]])/(16*(a - b)^3) - (b^6*Log[
a + b*Sech[x]])/(a*(a^2 - b^2)^3) - 1/(16*(a + b)*(1 - Sech[x])^2) - (5*a +
 7*b)/(16*(a + b)^2*(1 - Sech[x])) - 1/(16*(a - b)*(1 + Sech[x])^2) - (5*a
- 7*b)/(16*(a - b)^2*(1 + Sech[x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx = -\left(b^6\,\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\text{sech}(x)\right)\right)$$
$$= -\left(b^6\,\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3} + \frac{5a+7b}{16b^5(a+b)^2(b-x)^2} + \frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)} + \frac{1}{ab^6x} + \right.\right.\right.$$
$$= \frac{\log(\cosh(x))}{a} + \frac{\left(8a^2+21ab+15b^2\right)\log(1-\text{sech}(x))}{16(a+b)^3} + \frac{\left(8a^2-21ab+15b^2\right)\log(1+\text{sech}}{16(a-b)^3}$$

Mathematica [A] time = 1.03, size = 167, normalized size = 0.94

$$\frac{1}{64}\left(-\frac{8\left(a\left(b\left(3a^4-10a^2b^2+15b^4\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)-8a\left(a^4-3a^2b^2+3b^4\right)\log(\sinh(x))\right)+8b^6\log(a\cosh(x)+b\right.}{a(a-b)^3(a+b)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Sech[x]),x]
```

```
[Out] ((-2*(7*a + 9*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) - (8*(8*b^6*L
og[b + a*Cosh[x]] + a*(-8*a*(a^4 - 3*a^2*b^2 + 3*b^4)*Log[Sinh[x]] + b*(3*a
^4 - 10*a^2*b^2 + 15*b^4)*Log[Tanh[x/2]])))/(a*(a - b)^3*(a + b)^3) + (2*(7
*a - 9*b)*Sech[x/2]^2)/(a - b)^2 - Sech[x/2]^4/(a - b))/64
```

fricas [B] time = 0.61, size = 5181, normalized size = 29.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^8 + 8*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*x*sinh(x)^8 - 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(
x)^7 - 2*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5 - 32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*x*cosh(x))*sinh(x)^7 + 16*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^6 + 2*(16*a^6 - 40*a^4*b^2 + 24*a^2
*b^4 + 112*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 - 16*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*x - 7*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x))*si
nh(x)^6 - 2*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^5 - 2*(3*a^5*b - 2*a^3*b^
3 - a*b^5 - 224*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 + 21*(5*a^5
```

```
*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^2 - 48*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 -
2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^5 - 16*(2*a^6 - 6
*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 -
 2*(16*a^6 - 48*a^4*b^2 + 32*a^2*b^4 - 280*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*x*cosh(x)^4 + 35*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^3 - 120*(2*a^
6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x
)^2 - 24*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 5*(3*a^5*b - 2*a^3*b^3 - a
*b^5)*cosh(x))*sinh(x)^4 - 2*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^3 + 2*(2
24*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 - 3*a^5*b + 2*a^3*b^3 +
a*b^5 - 35*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 160*(2*a^6 - 5*a^4*
b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^3 - 10*(
3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^2 - 32*(2*a^6 - 6*a^4*b^2 + 4*a^2*b^4
- 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 16*(2*a^6 -
 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2
 + 2*(112*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 16*a^6 - 40*a^4
*b^2 + 24*a^2*b^4 - 21*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^5 + 120*(2*
a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh
(x)^4 - 10*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^3 - 48*(2*a^6 - 6*a^4*b^2
+ 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2 - 16*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 3*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x))
*sinh(x)^2 + 8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 2*(5*a^5*b - 14*a^3*
b^3 + 9*a*b^5)*cosh(x) + 8*(b^6*cosh(x)^8 + 8*b^6*cosh(x)*sinh(x)^7 + b^6*s
inh(x)^8 - 4*b^6*cosh(x)^6 + 6*b^6*cosh(x)^4 - 4*b^6*cosh(x)^2 + 4*(7*b^6*c
osh(x)^2 - b^6)*sinh(x)^6 + b^6 + 8*(7*b^6*cosh(x)^3 - 3*b^6*cosh(x))*sinh(
x)^5 + 2*(35*b^6*cosh(x)^4 - 30*b^6*cosh(x)^2 + 3*b^6)*sinh(x)^4 + 8*(7*b^6
*cosh(x)^5 - 10*b^6*cosh(x)^3 + 3*b^6*cosh(x))*sinh(x)^3 + 4*(7*b^6*cosh(x)
^6 - 15*b^6*cosh(x)^4 + 9*b^6*cosh(x)^2 - b^6)*sinh(x)^2 + 8*(b^6*cosh(x)^7
 - 3*b^6*cosh(x)^5 + 3*b^6*cosh(x)^3 - b^6*cosh(x))*sinh(x))*log(2*(a*cosh(
x) + b)/(cosh(x) - sinh(x))) - ((8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3
+ 24*a^2*b^4 + 15*a*b^5)*cosh(x)^8 + 8*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a
^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)*sinh(x)^7 + (8*a^6 + 3*a^5*b - 24*a
^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*sinh(x)^8 - 4*(8*a^6 + 3*a^5*b
 - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^6 - 4*(8*a^6 +
3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 - 7*(8*a^6 + 3*a^
5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^6
 + 8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5 + 8*(7
*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x
)^3 - 3*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)
*cosh(x))*sinh(x)^5 + 6*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2
*b^4 + 15*a*b^5)*cosh(x)^4 + 2*(24*a^6 + 9*a^5*b - 72*a^4*b^2 - 30*a^3*b^3
+ 72*a^2*b^4 + 45*a*b^5 + 35*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 2
4*a^2*b^4 + 15*a*b^5)*cosh(x)^4 - 30*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3
*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^6 + 3*a^5*b
- 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^5 - 10*(8*a^6 +
3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^3 + 3*(8
```

```
*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x))*
sinh(x)^3 - 4*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*
a*b^5)*cosh(x)^2 + 4*(7*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2
*b^4 + 15*a*b^5)*cosh(x)^6 - 8*a^6 - 3*a^5*b + 24*a^4*b^2 + 10*a^3*b^3 - 24
*a^2*b^4 - 15*a*b^5 - 15*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^
2*b^4 + 15*a*b^5)*cosh(x)^4 + 9*(8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3
+ 24*a^2*b^4 + 15*a*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^6 + 3*a^5*b - 24*a^
4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^7 - 3*(8*a^6 + 3*a^5*b
- 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^5 + 3*(8*a^6 + 3
*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x)^3 - (8*a^
6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15*a*b^5)*cosh(x))*sin
h(x))*log(cosh(x) + sinh(x) + 1) - ((8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*
b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^8 + 8*(8*a^6 - 3*a^5*b - 24*a^4*b^2 +
10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)*sinh(x)^7 + (8*a^6 - 3*a^5*b -
24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*sinh(x)^8 - 4*(8*a^6 - 3*a
^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^6 - 4*(8*a^
6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5 - 7*(8*a^6 -
3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^2)*sinh(
x)^6 + 8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5 +
8*(7*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*co
sh(x)^3 - 3*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*
b^5)*cosh(x))*sinh(x)^5 + 6*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24
*a^2*b^4 - 15*a*b^5)*cosh(x)^4 + 2*(24*a^6 - 9*a^5*b - 72*a^4*b^2 + 30*a^3*
b^3 + 72*a^2*b^4 - 45*a*b^5 + 35*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3
 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^4 - 30*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10
*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^6 - 3*a^
5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^5 - 10*(8*a^
6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^3 +
3*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(
x))*sinh(x)^3 - 4*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 -
 15*a*b^5)*cosh(x)^2 + 4*(7*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24
*a^2*b^4 - 15*a*b^5)*cosh(x)^6 - 8*a^6 + 3*a^5*b + 24*a^4*b^2 - 10*a^3*b^3
- 24*a^2*b^4 + 15*a*b^5 - 15*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 2
4*a^2*b^4 - 15*a*b^5)*cosh(x)^4 + 9*(8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*
b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^6 - 3*a^5*b - 2
4*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^7 - 3*(8*a^6 - 3*a^
5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^5 + 3*(8*a^6
 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x)^3 - (
8*a^6 - 3*a^5*b - 24*a^4*b^2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cosh(x))
*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(32*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*x*cosh(x)^7 - 7*(5*a^5*b - 14*a^3*b^3 + 9*a*b^5)*cosh(x)^6 - 5*a^5*b
+ 14*a^3*b^3 - 9*a*b^5 + 48*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^5 - 5*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh
(x)^4 - 32*(2*a^6 - 6*a^4*b^2 + 4*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*x)*cosh(x)^3 - 3*(3*a^5*b - 2*a^3*b^3 - a*b^5)*cosh(x)^2 + 16*(2*a^6
```

```
 - 5*a^4*b^2 + 3*a^2*b^4 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)
)*sinh(x))/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^8 + 8*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*sinh(x)^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4
 - a*b^6)*sinh(x)^8 + a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 4*(a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 - 4*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6
 - 7*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*cosh(x))*sinh(x)^5 + 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos
h(x)^4 + 2*(3*a^7 - 9*a^5*b^2 + 9*a^3*b^4 - 3*a*b^6 + 35*(a^7 - 3*a^5*b^2 +
 3*a^3*b^4 - a*b^6)*cosh(x)^4 - 30*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*co
sh(x)^2)*sinh(x)^4 + 8*(7*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^5 -
 10*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 + 3*(a^7 - 3*a^5*b^2 +
3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^3 - 4*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*
b^6)*cosh(x)^2 - 4*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 7*(a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 + 15*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
*cosh(x)^4 - 9*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^2 +
 8*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^7 - 3*(a^7 - 3*a^5*b^2 +
3*a^3*b^4 - a*b^6)*cosh(x)^5 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh
(x)^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x))
```

giac [B] time = 0.14, size = 380, normalized size = 2.13

$$-\frac{b^6\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}+\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(e^{(-x)}+e^x+2\right)}{16\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(e^{(-x)}+e^x\cdots\right.}{16\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -b^6*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/16*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*
a*b^2 - b^3) + 1/16*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) + e^x - 2)/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) - 1/4*(3*a^5*(e^(-x) + e^x)^4 - 9*a^3*b^2*(e^(-x)
+ e^x)^4 + 9*a*b^4*(e^(-x) + e^x)^4 - 5*a^4*b*(e^(-x) + e^x)^3 + 14*a^2*b^3
*(e^(-x) + e^x)^3 - 9*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 32*a^
3*b^2*(e^(-x) + e^x)^2 - 48*a*b^4*(e^(-x) + e^x)^2 + 12*a^4*b*(e^(-x) + e^x
) - 40*a^2*b^3*(e^(-x) + e^x) + 28*b^5*(e^(-x) + e^x) - 16*a^3*b^2 + 64*a*b
^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

maple [A] time = 0.17, size = 215, normalized size = 1.21

$$-\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)a}{64\,(a-b)^2}+\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)b}{64\,(a-b)^2}-\frac{3\left(\tanh^2\left(\frac{x}{2}\right)\right)a}{16\,(a-b)^2}+\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)b}{4\,(a-b)^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{b^6\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh\cdots\right.\right.}{(a-b)^3\,(a+b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*sech(x)),x)
```

```
[Out] -1/64/(a-b)^2*tanh(1/2*x)^4*a+1/64/(a-b)^2*tanh(1/2*x)^4*b-3/16/(a-b)^2*tan
h(1/2*x)^2*a+1/4/(a-b)^2*tanh(1/2*x)^2*b-1/a*ln(tanh(1/2*x)-1)-1/(a-b)^3*b^
6/(a+b)^3/a*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)-1/a*ln(tanh(1/2*x)+1)-1
/64/(a+b)/tanh(1/2*x)^4-3/16/(a+b)^2/tanh(1/2*x)^2*a-1/4/(a+b)^2/tanh(1/2*x
)^2*b+1/(a+b)^3*ln(tanh(1/2*x))*a^2+21/8/(a+b)^3*ln(tanh(1/2*x))*a*b+15/8/(
a+b)^3*ln(tanh(1/2*x))*b^2
```

maxima [B] time = 0.37, size = 366, normalized size = 2.06

$$-\frac{b^6\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}+\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(e^{(-x)}+1\right)}{8\left(a^3-3\,a^2b+3\,ab^2-b^3\right)}+\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(e^{(-x)}-1\right)}{8\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{(5\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b^6*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/8*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + 1/8*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*
b^2 + b^3) + 1/4*((5*a^2*b - 9*b^3)*e^(-x) - 8*(2*a^3 - 3*a*b^2)*e^(-2*x) +
 (3*a^2*b + b^3)*e^(-3*x) + 16*(a^3 - 2*a*b^2)*e^(-4*x) + (3*a^2*b + b^3)*e
^(-5*x) - 8*(2*a^3 - 3*a*b^2)*e^(-6*x) + (5*a^2*b - 9*b^3)*e^(-7*x))/(a^4 -
 2*a^2*b^2 + b^4 - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2
+ b^4)*e^(-4*x) - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b
^4)*e^(-8*x)) + x/a
```

mupad [B] time = 2.75, size = 623, normalized size = 3.50

$$\frac{\ln\left(e^x-1\right)\left(8\,a^2+21\,a\,b+15\,b^2\right)}{8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3}-\frac{\frac{2\left(4\,a^4-5\,a^2\,b^2\right)}{a\left(a^2-b^2\right)^2}-\frac{e^x\left(9\,a^2\,b-13\,b^3\right)}{2\left(a^2-b^2\right)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{2\left(2\,a^6-5\,a^4\,b^2+3\,a^2\,b^4\right)}{a\left(a^2-b^2\right)^3}-\frac{e^x\left(5\,a^4\,b-14\,a^2\,b^3+9\,b^5\right)}{4\left(a^2-b^2\right)^3}}{e^{2\,x}-1}-\frac{\frac{8\left(a^4-a^2\cdots\right.}{a\left(a^2-b\cdots\right.}\cdots}{3\,e^{2\,x}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1)*(21*a*b + 8*a^2 + 15*b^2))/(24*a*b^2 + 24*a^2*b + 8*a^3 +
8*b^3) - ((2*(4*a^4 - 5*a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(9*a^2*b - 13
*b^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(2*a^6 + 3*a^2*
b^4 - 5*a^4*b^2))/(a*(a^2 - b^2)^3) - (exp(x)*(5*a^4*b + 9*b^5 - 14*a^2*b^3
))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) - ((8*(a^4 - a^2*b^2))/(a*(a^2 - b^2)^
2) - (6*exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(3*exp(2*x) - 3*exp(4*x) + exp
```

```
(6*x) - 1) - x/a - ((4*a)/(a^2 - b^2) - (4*b*exp(x))/(a^2 - b^2))/(6*exp(4*
x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) + (log(exp(x) + 1)*(8*a^2 - 21
*a*b + 15*b^2))/(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) + (b^6*log(64*a^13*ex
p(2*x) + 64*a*b^12 + 64*a^13 + 159*a^3*b^10 + 492*a^5*b^8 - 1214*a^7*b^6 +
1020*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) + 159*a^3*b^10*exp(2*x) + 492
*a^5*b^8*exp(2*x) - 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*x) - 393*a^1
1*b^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) + 318*a^2*b^11*exp(
x) + 984*a^4*b^9*exp(x) - 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*exp(x) - 786*a
^10*b^3*exp(x)))/(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{a + b\,\mathrm{sech}\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*sech(x)),x)
```

```
[Out] Integral(coth(x)**5/(a + b*sech(x)), x)
```

3.125 $\int \sqrt{a+b\,\mathbf{sech}(c+dx)}\ \tanh^5(c+dx)\,dx$

Optimal. Leaf size=169

$$-\frac{2\left(3a^2-2b^2\right)(a+b\operatorname{sech}(c+dx))^{5/2}}{5b^4d}+\frac{2a\left(a^2-2b^2\right)(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^4d}-\frac{2(a+b\operatorname{sech}(c+dx))^{9/2}}{9b^4d}+\frac{6a(a+b\operatorname{se}}{7}$$

```
[Out] 2/3*a*(a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d-2/5*(3*a^2-2*b^2)*(a+b*sech
(d*x+c))^(5/2)/b^4/d+6/7*a*(a+b*sech(d*x+c))^(7/2)/b^4/d-2/9*(a+b*sech(d*x+
c))^(9/2)/b^4/d+2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b
*sech(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$-\frac{2\left(3a^2-2b^2\right)(a+b\operatorname{sech}(c+dx))^{5/2}}{5b^4d}+\frac{2a\left(a^2-2b^2\right)(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^4d}-\frac{2(a+b\operatorname{sech}(c+dx))^{9/2}}{9b^4d}+\frac{6a(a+b\operatorname{se}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d + (2*a*(a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(3/2))/(3*b^4*d)
 - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) + (6*a*(a + b*
Sech[c + d*x])^(7/2))/(7*b^4*d) - (2*(a + b*Sech[c + d*x])^(9/2))/(9*b^4*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\,\tanh^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\left(b^2-x^2\right)^2}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^4 d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2\left(-a^2+b^2+2ax^2-x^4\right)^2}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4 d}$$

$$= -\frac{2\,\text{Subst}\left(\int \left(b^4-a\left(a^2-2b^2\right)x^2+\left(3a^2-2b^2\right)x^4-3ax^6+x^8+\frac{a}{-a}\right.\right.}{b^4 d}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2a\left(a^2-2b^2\right)(a+b\text{sech}(c+dx))^{3/2}}{3b^4 d}-\frac{2(}{}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2a\left(a^2-2b\right.}{}$$

Mathematica [A] time = 5.12, size = 160, normalized size = 0.95

$$\frac{2\sqrt{a+b\text{sech}(c+dx)}\left(\frac{16a^4}{b^4}+\left(\frac{42a}{b}-\frac{8a^3}{b^3}\right)\text{sech}(c+dx)+\left(\frac{6a^2}{b^2}+126\right)\text{sech}^2(c+dx)-\frac{84a^2}{b^2}-\frac{5a\text{sech}^3(c+dx)}{b}+\frac{315\sqrt{a}}{}\right.}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a + b*Sech[c + d*x]]*(-315 + (16*a^4)/b^4 - (84*a^2)/b^2 + (315*Arc
Tanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh[c + d*x]]
```

```
)/Sqrt[b + a*Cosh[c + d*x]] + ((-8*a^3)/b^3 + (42*a)/b)*Sech[c + d*x] + (12
6 + (6*a^2)/b^2)*Sech[c + d*x]^2 - (5*a*Sech[c + d*x]^3)/b - 35*Sech[c + d*
x]^4))/(315*d)
```

fricas [B] time = 1.10, size = 4363, normalized size = 25.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="fricas")

[Out] [1/630*(315*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^
4*sinh(d*x + c)^8 + 4*b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4
*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*
x + c)^4 + 30*b^4*cosh(d*x + c)^2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4
*cosh(d*x + c)^5 + 10*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(7*b^4*cosh(d*x + c)^6 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x
+ c)^2 + b^4)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c
)^5 + 3*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log
(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 +
4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^
2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) +
 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x +
 c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*c
osh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x +
 c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (
4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*((16*a^4 - 84*a^2*b^2 - 315*b^4)*c
osh(d*x + c)^8 + (16*a^4 - 84*a^2*b^2 - 315*b^4)*sinh(d*x + c)^8 - 4*(4*a^3
*b - 21*a*b^3)*cosh(d*x + c)^7 - 4*(4*a^3*b - 21*a*b^3 - 2*(16*a^4 - 84*a^2
*b^2 - 315*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(16*a^4 - 78*a^2*b^2 - 1
89*b^4)*cosh(d*x + c)^6 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4 + 7*(16*a^4 - 84
*a^2*b^2 - 315*b^4)*cosh(d*x + c)^2 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c))
*sinh(d*x + c)^6 - 4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^5 - 4*(12*a^3*b -
53*a*b^3 - 14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^3 + 21*(4*a^3*b
 - 21*a*b^3)*cosh(d*x + c)^2 - 6*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^4 +
 2*(35*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^4 + 48*a^4 - 228*a^2*b
^2 - 721*b^4 - 70*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^3 + 30*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c)^2 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^4 + 16*a^4 - 84*a^2*b^2 - 315*b^4 - 4*(12*a^3*b - 53*a*b^3)*c
osh(d*x + c)^3 + 4*(14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^5 - 35
```

```
*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 53*a*b^3 + 20*(16*a^4 -
78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^3 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x +
 c)^2 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^2 + 4*(7*(16*a^4 - 84*a^2*
b^2 - 315*b^4)*cosh(d*x + c)^6 - 21*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^5 +
15*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^4 + 16*a^4 - 78*a^2*b^2 -
189*b^4 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^3 + 3*(48*a^4 - 228*a^2*b^
2 - 721*b^4)*cosh(d*x + c)^2 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(
d*x + c)^2 - 4*(4*a^3*b - 21*a*b^3)*cosh(d*x + c) + 4*(2*(16*a^4 - 84*a^2*b
^2 - 315*b^4)*cosh(d*x + c)^7 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^6 + 6*
(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^5 - 5*(12*a^3*b - 53*a*b^3)*c
osh(d*x + c)^4 - 4*a^3*b + 21*a*b^3 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*co
sh(d*x + c)^3 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)))/(b^4*d*cosh(d*x + c)^8 + 8*b^4*d*cosh(d*x + c)*sinh(d*x + c)
^7 + b^4*d*sinh(d*x + c)^8 + 4*b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c
)^4 + 4*b^4*d*cosh(d*x + c)^2 + 4*(7*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*
x + c)^6 + 8*(7*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c
)^5 + b^4*d + 2*(35*b^4*d*cosh(d*x + c)^4 + 30*b^4*d*cosh(d*x + c)^2 + 3*b^
4*d)*sinh(d*x + c)^4 + 8*(7*b^4*d*cosh(d*x + c)^5 + 10*b^4*d*cosh(d*x + c)^
3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*d*cosh(d*x + c)^6 + 1
5*b^4*d*cosh(d*x + c)^4 + 9*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2
+ 8*(b^4*d*cosh(d*x + c)^7 + 3*b^4*d*cosh(d*x + c)^5 + 3*b^4*d*cosh(d*x + c
)^3 + b^4*d*cosh(d*x + c))*sinh(d*x + c)), -1/315*(315*(b^4*cosh(d*x + c)^8
 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 + 4*b^4*cosh(d
*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*
x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*x + c)^4 + 30*b^4*cosh(d*x + c)^
2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*b^4*cosh(d
*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*cosh(d*x + c)^6
 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 +
8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c)^5 + 3*b^4*cosh(d*x + c)^3 + b^
4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh
(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*
sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c))) - 2*((16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^8 + (1
6*a^4 - 84*a^2*b^2 - 315*b^4)*sinh(d*x + c)^8 - 4*(4*a^3*b - 21*a*b^3)*cosh
(d*x + c)^7 - 4*(4*a^3*b - 21*a*b^3 - 2*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cos
h(d*x + c))*sinh(d*x + c)^7 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x +
c)^6 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4 + 7*(16*a^4 - 84*a^2*b^2 - 315*b^4)
*cosh(d*x + c)^2 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 -
4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^5 - 4*(12*a^3*b - 53*a*b^3 - 14*(16*a
^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^3 + 21*(4*a^3*b - 21*a*b^3)*cosh(d
*x + c)^2 - 6*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c))*sinh(d*x + c)^
```

```
5 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^4 + 2*(35*(16*a^4 - 84
*a^2*b^2 - 315*b^4)*cosh(d*x + c)^4 + 48*a^4 - 228*a^2*b^2 - 721*b^4 - 70*(
4*a^3*b - 21*a*b^3)*cosh(d*x + c)^3 + 30*(16*a^4 - 78*a^2*b^2 - 189*b^4)*co
sh(d*x + c)^2 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 1
6*a^4 - 84*a^2*b^2 - 315*b^4 - 4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^3 + 4*
(14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^5 - 35*(4*a^3*b - 21*a*b^
3)*cosh(d*x + c)^4 - 12*a^3*b + 53*a*b^3 + 20*(16*a^4 - 78*a^2*b^2 - 189*b^
4)*cosh(d*x + c)^3 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(48*a^4 -
 228*a^2*b^2 - 721*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^4 - 78*a^2
*b^2 - 189*b^4)*cosh(d*x + c)^2 + 4*(7*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh
(d*x + c)^6 - 21*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^5 + 15*(16*a^4 - 78*a^2
*b^2 - 189*b^4)*cosh(d*x + c)^4 + 16*a^4 - 78*a^2*b^2 - 189*b^4 - 10*(12*a^
3*b - 53*a*b^3)*cosh(d*x + c)^3 + 3*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d
*x + c)^2 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(4*a
^3*b - 21*a*b^3)*cosh(d*x + c) + 4*(2*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(
d*x + c)^7 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^6 + 6*(16*a^4 - 78*a^2*b^
2 - 189*b^4)*cosh(d*x + c)^5 - 5*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^4 - 4*
a^3*b + 21*a*b^3 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^3 - 3*(
12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(16*a^4 - 78*a^2*b^2 - 189*b^4)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(b^4
*d*cosh(d*x + c)^8 + 8*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*d*sinh(d*x
 + c)^8 + 4*b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(
d*x + c)^2 + 4*(7*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^6 + 8*(7*b^4
*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^5 + b^4*d + 2*(35
*b^4*d*cosh(d*x + c)^4 + 30*b^4*d*cosh(d*x + c)^2 + 3*b^4*d)*sinh(d*x + c)^
4 + 8*(7*b^4*d*cosh(d*x + c)^5 + 10*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*b^4*d*cosh(d*x + c)^6 + 15*b^4*d*cosh(d*x +
c)^4 + 9*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 + 8*(b^4*d*cosh(d*x
 + c)^7 + 3*b^4*d*cosh(d*x + c)^5 + 3*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*
x + c))*sinh(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(dx + c) + a} \tanh(dx + c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}(dx + c)} \left(\tanh^5(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(dx+c)+a}\,\tanh(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(c+d\,x)^5\,\sqrt{a+\frac{b}{\cosh(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b \operatorname{sech}(c+dx)}\,\tanh^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**5,x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**5, x)
```

3.126 $\int \sqrt{a+b\,\mathbf{sech}(c+dx)}\,\tanh^3(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}$$

```
[Out] -2/3*a*(a+b*sech(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sech(d*x+c))^(5/2)/b^2/d+2*ar
ctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sech(d*x+c))^(1/2)/
d
```

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 207}

$$\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^2d}-\frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d - (2*a*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*S
ech[c + d*x])^(5/2))/(5*b^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\,\tanh^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}\,(b^2-x^2)}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^2d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$

$$= -\frac{2\,\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d}$$

$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\text{sech}(c}{5b^2d}$$

$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{2a(a+b\text{sec}}{3}$$

Mathematica [A] time = 0.99, size = 108, normalized size = 1.08

$$\frac{2\sqrt{a+b\text{sech}(c+dx)}\left(-\frac{2a^2}{b^2}+\frac{a\text{sech}(c+dx)}{b}+\frac{15\sqrt{a\cosh(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)+b}}+3\text{sech}^2(c+dx)-15\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a + b*Sech[c + d*x]]*(-15 - (2*a^2)/b^2 + (15*ArcTanh[Sqrt[b + a*Co
sh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh[c + d*x]])/Sqrt[b + a*Cosh[
c + d*x]] + (a*Sech[c + d*x])/b + 3*Sech[c + d*x]^2))/(15*d)
```

fricas [B] time = 1.07, size = 1589, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*s
inh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*
cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cos
h(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a
*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c
)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(
d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
 + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x
+ c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*(2*a*b*cosh(d*x
+ c)^3 - (2*a^2 + 15*b^2)*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^
4 + 2*(a*b - 2*(2*a^2 + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh
(d*x + c) - 2*(2*a^2 + 9*b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*
(2*a^2 + 15*b^2)*cosh(d*x + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 -
 15*b^2 + 2*(3*a*b*cosh(d*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a
*b - 2*(2*a^2 + 9*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c)
+ b)/cosh(d*x + c)))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*
x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b
^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 +
b^2*d*cosh(d*x + c))*sinh(d*x + c)), -1/15*(15*(b^2*cosh(d*x + c)^4 + 4*b^2
*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^
2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x
 + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c
)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*
x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*
x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*
x + c) + a*b)*sinh(d*x + c))) - 2*(2*a*b*cosh(d*x + c)^3 - (2*a^2 + 15*b^2)
*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^4 + 2*(a*b - 2*(2*a^2 + 1
5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh(d*x + c) - 2*(2*a^2 + 9*
b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*(2*a^2 + 15*b^2)*cosh(d*x
 + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 - 15*b^2 + 2*(3*a*b*cosh(d
*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a*b - 2*(2*a^2 + 9*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(b^2
```

```
*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x
 + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^
2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh
(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{sech}\,(dx+c)}\,\left(\tanh^3(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\,(c+d\,x)^3\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}(c + dx)}\,\tanh^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**3, x)
```

3.127 $\int \sqrt{a+b\operatorname{sech}(c+dx)}\,\tanh(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sech(d*x+c))^(1
/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 50, 63, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d}\\ &= -\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{d}-\frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d}\\ &= -\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{d}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.15, size = 90, normalized size = 1.76

$$-\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}\left(\sqrt{a\cosh(c+dx)+b}-\sqrt{a\cosh(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)\right)}{d\sqrt{a\cosh(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

```
[Out] (-2*(-(ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[a*Cosh
[c + d*x]]) + Sqrt[b + a*Cosh[c + d*x]])*Sqrt[a + b*Sech[c + d*x]])/(d*Sqrt
[b + a*Cosh[c + d*x]])
```

fricas [B] time = 1.04, size = 605, normalized size = 11.86

$$\left[\frac{\sqrt{a}\log\left(-\frac{2a^2\cosh(dx+c)^4+2a^2\sinh(dx+c)^4+4ab\cosh(dx+c)^3+4\left(2a^2\cosh(dx+c)+ab\right)\sinh(dx+c)^3+4ab\cosh(dx+c)+\left(4a^2+b^2\right)\cosh(dx+c)^2}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*c
osh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh
(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*
b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)
^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d
*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
+ 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(
d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh
(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x +
 c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*sqrt((a*cosh(d*x
+ c) + b)/cosh(d*x + c)))/d, -(sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(
d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*s
qrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)
*sinh(d*x + c))) + 2*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\;\tanh\,(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

maple [A] time = 0.11, size = 43, normalized size = 0.84

$$-\frac{2\sqrt{a+b\,\mathrm{sech}\,(dx+c)}\;-2\sqrt{a}\;\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(dx+c)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x)
```

```
[Out] -1/d*(2*(a+b*sech(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a
^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}\left(dx + c\right) + a} \tanh\left(dx + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

mupad [B] time = 1.69, size = 47, normalized size = 0.92

$$\frac{2\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] (2*a^(1/2)*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/d - (2*(a + b/cosh(c
 + d*x))^(1/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}\left(c + dx\right)} \tanh\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x), x)
```

3.128 $\int \coth(c+dx)\sqrt{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))*(a+b)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 898, 1287, 206, 207}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a + b]])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
```

```
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \coth(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx = -\frac{b^2\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d}$$
$$= -\frac{(2b^2)\operatorname{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d}$$
$$= -\frac{(2b^2)\operatorname{Subst}\left(\int \left(-\frac{a}{b^2(a-x^2)}+\frac{a+b}{2b^2(a+b-x^2)}+\frac{-a+b}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{a+b\operatorname{se}}\right.}{d}$$
$$= \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d} + \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{-a+b+}\right.}{}$$
$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+}}{}$$

Mathematica [A] time = 1.83, size = 195, normalized size = 1.84

$$\frac{\sqrt{a\cosh(c+dx)}\sqrt{a+b\operatorname{sech}(c+dx)}\left(2\sqrt{a}\log\left(\sqrt{a\cosh(c+dx)+b}+\sqrt{a\cosh(c+dx)}\right)-\sqrt{-a-b}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-a}}\right.\right.}{\sqrt{a}\,d\sqrt{a\cosh(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[a*Cosh[c + d*x]]*(-(Sqrt[-a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c +
d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])]) - Sqrt[a - b]*ArcTanh[(Sqrt[a
]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c + d*x]])] + 2*Sqrt[
a]*Log[Sqrt[a*Cosh[c + d*x]] + Sqrt[b + a*Cosh[c + d*x]]])*Sqrt[a + b*Sech[
c + d*x]])/(Sqrt[a]*d*Sqrt[b + a*Cosh[c + d*x]])
```

fricas [B] time = 0.97, size = 8620, normalized size = 81.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*
a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b -
 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 -
8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2
 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2
 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x
+ c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x +
 c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*
cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a -
b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*s
inh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x +
c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b +
 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) +
 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c
)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*
x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(
d*x + c) + 1)) + sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 +
(8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 +
 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh
(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh
(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b
)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)
*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x +
c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2
*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x
+ c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*
cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^
```

```
2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh
(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(c
osh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 +
 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c
) - 4*cosh(d*x + c) + 1)) + 2*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*s
inh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh
(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2
*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*
a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*c
osh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c
)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cos
h(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) +
 a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x +
 c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d
*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
)))/d, -1/4*(4*sqrt(-a)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*
x + c) + b)*sinh(d*x + c) + a)) - sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*c
osh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*
cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 -
8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x
+ c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*c
osh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a
 - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2
*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^
2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*
b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x
 + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(
d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*
cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c)
+ 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) - sqrt(a + b)*log(-((8*a^2 + 8*a
*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*
b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(
3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b
+ 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b
)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*
(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2
+ 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x +
c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x +
```

c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*
sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c)
 + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x +
c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)
)/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^
4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x
+ c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cos
h(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)))/d, -1/4*(2*sqrt(-a +
 b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a - b
)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a
 - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - sqrt(a + b)*log(-((8*a
^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 +
 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)
^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6
*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*(
(2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3
 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x
 + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sin
h(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cos
h(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt
(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(
d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cos
h(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(
d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d
*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*s
inh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2
 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) - 2*sqrt(a)*l
og(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3
+ 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*
a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c)
 + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x
 + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a
*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x
 + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b +
 (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)))/d, -1/4*(4*sqrt(-a)*arctan((cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sq
rt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c
)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*sqr
t(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2

```
*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2
*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - sqrt(a + b)*log(
-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x +
c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*
x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b
^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2
 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x
+ c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*co
sh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a +
b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3
*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b
)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)
*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^
2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))
*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c)
+ 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x
+ c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)))/d, 1/4
*(2*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
))/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x +
c) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) + sqrt(a - b
)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(
d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2
- 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*c
osh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b
 + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b
 + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cos
h(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a -
 b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) +
2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)
^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2
*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b -
3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b
- 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x
 + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x
 + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cos
h(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) +
 2*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh
(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*
x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*c
osh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4
+ a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x
```

```
+ c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2
*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x
 + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*
x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)
^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/d, -1/4*(4*sqrt(-a)*arc
tan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*
sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) +
 a)) - 2*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*
x + c))/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d
*x + c) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - sqrt(
a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*
sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8
*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b
^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 -
8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 -
8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*
b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(
2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x +
c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x
 + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c
) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a
*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4
*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cos
h(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*
x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cos
h(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 +
3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) +
1)))/d, -1/2*(sqrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c))/((2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) -
 sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/
((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
+ 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - sqrt(a)*log(-
(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*
(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2
+ b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4
*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c
)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cos
h(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c
)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*co
```

```
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh
(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*
a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2)))/d, -1/2*(2*sqrt(-a)*arctan((cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((
a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2
+ b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + sqrt(-a +
 b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a - b
)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a
 - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - sqrt(-a - b)*arctan(2*
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqr
t(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a + b)*cosh(d*x + c
)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a + b)*cosh(d*x
 + c) + b)*sinh(d*x + c) + 2*a + b)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(dx+c)+a}\, \coth(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \coth(dx+c)\sqrt{a+b \operatorname{sech}(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(dx+c)+a}\, \coth(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(c+d\,x)\,\sqrt{a+\frac{b}{\cosh(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\,\coth(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x), x)
```

3.129 $\int \coth^3(c+dx)\sqrt{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=217

$$-\frac{\coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{2d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{se}}}{\sqrt{}}\right)}{d\sqrt{a-b}}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-a*arctanh((a+b*sech(d*
x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+3/4*b*arctanh((a+b*sech(d*x+c))^(1/2
)/(a-b)^(1/2))/d/(a-b)^(1/2)-a*arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))
/d/(a+b)^(1/2)-3/4*b*arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(
1/2)-1/2*coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.33, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3885, 898, 1315, 1178, 12, 1093, 206, 1170, 207}

$$-\frac{\coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{2d}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\text{se}}}{\sqrt{}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (a*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) + (3*b*ArcTanh[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) - (a*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) - (3*b*ArcTanh[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Coth[c + d*x]^2*Sqrt[a + b*Sech
[c + d*x]])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^
2 - b*d*e + a*e^2), Int[((f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1))/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
```

```
p, -1] && GtQ[m, 0]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \coth^3(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx = -\frac{b^4\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{x\left(b^2-x^2\right)^2}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d}$$

$$= -\frac{\left(2b^4\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{-a^2+b^2+ax^2}{\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d} - \frac{\left(2ab^2\right)\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\operatorname{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\operatorname{sech}(c+dx))+(a+b\operatorname{sech}(c+dx))^2\right)} - \frac{\left(2ab^2\right)\cdots}{\cdots}$$

$$= \frac{b^2\sqrt{a+b\operatorname{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\operatorname{sech}(c+dx))+(a+b\operatorname{sech}(c+dx))^2\right)} - \frac{a\operatorname{Subs}\cdots}{\cdots}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \frac{a\tanh^{-1}\left(\cdots\right)}{\sqrt{\cdots}}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{3b\tanh^{-1}\cdots}{4\cdots}$$

Mathematica [B] time = 20.69, size = 518, normalized size = 2.39

$$\frac{\sqrt{a+b\operatorname{sech}(c+dx)}\left(\frac{8\sqrt{-a\cosh(c+dx)}\tan^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{-a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)+b}} - \frac{2\sqrt{a}\sqrt{-a\cosh(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{a\cosh(c+dx)+b}}{\sqrt{a-b}\sqrt{-a\cosh(c+dx)}}\right)}{\sqrt{a-b}\sqrt{a\cosh(c+dx)+b}} - \frac{2\sqrt{a}\sqrt{-a\cosh(c+dx}\cdots}{\sqrt{a+b}\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (((8*ArcTan[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[-(a*Cosh[c + d*x])]]*Sqrt[-(a*Co
sh[c + d*x])])/Sqrt[b + a*Cosh[c + d*x]] - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[
b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cosh[c + d*x])])]*Sqrt[-(a*Cosh
[c + d*x])])/(Sqrt[a - b]*Sqrt[b + a*Cosh[c + d*x]]) - (2*Sqrt[a]*ArcTan[(S
qrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cosh[c + d*x])])]*S
qrt[-(a*Cosh[c + d*x])])/(Sqrt[a + b]*Sqrt[b + a*Cosh[c + d*x]]) + (3*b*Arc
Tan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]]
)]*Sqrt[a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[-a - b]*Sqrt[b + a*Cosh[c + d*x]])
- ((2*a - 3*b)*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqr
t[a*Cosh[c + d*x]])]*Sqrt[a*Cosh[c + d*x]])/(Sqrt[a]*Sqrt[a - b]*Sqrt[b + a
*Cosh[c + d*x]]) - (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(
Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])]*Sqrt[a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqr
t[b + a*Cosh[c + d*x]]) - 2*Coth[c + d*x]^2)*Sqrt[a + b*Sech[c + d*x]])/(4*
d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}\left(dx + c\right) + a} \coth\left(dx + c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \left(\coth^{3}\left(dx + c\right)\right) \sqrt{a + b \,\operatorname{sech}\left(dx + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{sech}(dx + c) + a} \coth(dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \coth(c + d\,x)^3 \sqrt{a + \frac{b}{\cosh(c + d\,x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{sech}(c + dx)} \coth^3(c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**3, x)
```

3.130 $\int \sqrt{a+b\,\mathbf{sech}(c+dx)}\,\tanh^2(c+dx)\,dx$

Optimal. Leaf size=344

$$-\frac{2a(a-b)\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2\tanh(c+dx)}{}$$

[Out] -2/3*a*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^2/d-2/3*(a+2*b)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/d-2/3*(a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)/d

Rubi [A] time = 0.39, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4057, 4058, 3921, 3784, 3832, 4004}

$$-\frac{2a(a-b)\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\mathrm{sech}(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}-\frac{2\tanh(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a +
 2*b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
h[c + d*x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b)
)])/d - (2*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4057

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)
/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*
A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\text{sech}(c+dx)}\,\tanh^2(c+dx)\,dx = -\int \sqrt{a+b\text{sech}(c+dx)}\,\left(-1+\text{sech}^2(c+dx)\right)\,dx$$
$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}\,\tanh(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a}{2} - b\text{sech}(c+dx) + \dots}{\sqrt{a+b\text{sech}(c\dots}}$$
$$= -\frac{2\sqrt{a+b\text{sech}(c+dx)}\,\tanh(c+dx)}{3d} - \frac{2}{3}\int \frac{-\frac{3a}{2} + \left(-\frac{a}{2}-b\right)\text{sech}(c\dots}{\sqrt{a+b\text{sech}(c+dx\dots}}$$
$$= -\frac{2a(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\dots}{\dots}}\dots}{3b^2d}$$
$$= -\frac{2a(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\dots}{\dots}}\dots}{3b^2d}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
[Out] $Aborted
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{sech}\,(dx + c)}\,\left(\tanh^2(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{sech}\,(dx + c) + a}\,\tanh(dx + c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh(c + d\,x)^2\,\sqrt{a + \frac{b}{\cosh(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\mathrm{sech}\,(c + dx)}\,\tanh^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**2,x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**2, x)
```

3.131 $\int \sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\mathrm{sech}(c+dx))}{a+b\mathrm{sech}(c+dx)}}\sqrt{\frac{b(\mathrm{sech}(c+dx)+1)}{a+b\mathrm{sech}(c+dx)}}\,(a+b\mathrm{sech}(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\mathrm{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b),((a-b)
/(a+b))^(1/2))*(a+b*sech(d*x+c))*(-b*(1-sech(d*x+c))/(a+b*sech(d*x+c)))^(1/
2)*(b*(1+sech(d*x+c))/(a+b*sech(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3780}

$$\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\mathrm{sech}(c+dx))}{a+b\mathrm{sech}(c+dx)}}\sqrt{\frac{b(\mathrm{sech}(c+dx)+1)}{a+b\mathrm{sech}(c+dx)}}\,(a+b\mathrm{sech}(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\mathrm{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sech[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sech[c +
d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Sech[c +
 d*x]))/(Sqrt[a + b]*d)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int\sqrt{a+b\mathrm{sech}(c+dx)}\,dx=\frac{2\coth(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\mathrm{sech}(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\mathrm{sech}(c+dx))}{a+b\mathrm{sech}(c+dx)}}\sqrt{\frac{b(1+\mathrm{sech}(c+dx)}{a+b\mathrm{sech}(c+dx}}}{\sqrt{a+b}\,d}$$

Mathematica [F] time = 7.86, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\text{sech}(c + dx)}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]],x]
[Out] Integrate[Sqrt[a + b*Sech[c + d*x]], x]
```

fricas [F] time = 2.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{sech}\left(dx + c\right) + a}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{sech}\left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\,\text{sech}\left(dx + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sech(d*x+c))^(1/2),x)
[Out] int((a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{sech}\left(dx + c\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{sech}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x)), x)
```

3.132 $\int \coth^2(c+dx)\sqrt{a+b\,\mathbf{sech}(c+dx)}\,dx$

Optimal. Leaf size=246

$$-\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right.\right.}{d}$$

```
[Out] coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^
(1/2)/d+2*coth(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b
),((a-b)/(a+b))^(1/2))*(a+b*sech(d*x+c))*(-b*(1-sech(d*x+c))/(a+b*sech(d*x+
c)))^(1/2)*(b*(1+sech(d*x+c))/(a+b*sech(d*x+c)))^(1/2)/d/(a+b)^(1/2)-coth(d
*x+c)*(a+b*sech(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3896, 3780, 3875, 3832}

$$-\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sech[c + d*x]))/(a - b))])/d - (Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]]
)/d + (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*
Sech[c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sec
h[c + d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Se
ch[c + d*x]))/(Sqrt[a + b]*d)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rubi steps

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\int \left(-\sqrt{a+b\text{sech}(c+dx)} - \text{csch}^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\right)dx$$

$$= \int \sqrt{a+b\text{sech}(c+dx)}\,dx + \int \text{csch}^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$= -\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d} + \frac{2\coth(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\cdots\right.\right.}{\cdots}$$

$$= \frac{\sqrt{a+b}\,\coth(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\,\sqrt{\cdots}}{d}$$

Mathematica [B] time = 18.23, size = 539, normalized size = 2.19

$$\frac{\sqrt{a+b\text{sech}(c+dx)}\left(\frac{2\sqrt{b}\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}\sqrt{\frac{(a+b)(a\cosh(c+dx)+a)}{(a-b)(a-a\cosh(c+dx))}}F\left(\sin^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{b}\sqrt{a-a\cosh(c+dx)}}\right)|-\frac{2b}{a-b}\right)}{a^{3/2}\sqrt{\cosh(c+dx)-1}\sqrt{\cosh(c+dx)+1}\sqrt{\text{sech}(c+dx)}\left(-\frac{a-a\cosh(c+dx)}{a}\right)^{3/2}\sqrt{\frac{a\cosh(c+dx)+a}{a}}\sqrt{-\frac{a(a+b)\cosh(c+dx)}{b(a-a\cosh(c+dx))}}} - \frac{\cdots}{\sqrt{a}\sqrt{a\cdots}}\cdots\right.}{2d\sqrt{\text{sech}(c+dx)}\sqrt{a\cosh\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] -((Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]])/d) + (Sqrt[a + b*Sech[c + d*x]]
*((2*Sqrt[b]*(a - a*Cosh[c + d*x])^(3/2)*Sqrt[((a + b)*(a + a*Cosh[c + d*x]
))/((a - b)*(a - a*Cosh[c + d*x]))]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[b + a*Co
sh[c + d*x]])/(Sqrt[b]*Sqrt[a - a*Cosh[c + d*x]])], (-2*b)/(a - b)]*Sinh[c
+ d*x])/(a^(3/2)*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[-((a
*(a + b)*Cosh[c + d*x])/(b*(a - a*Cosh[c + d*x])))]*(-((a - a*Cosh[c + d*x]
)/a))^(3/2)*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Sech[c + d*x]]) - (4*b*(a -
a*Cosh[c + d*x])*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b + a*Cosh[c +
d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]*Sqrt[-((b*(a
+ a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a - b)))]*Sinh[c + d*x])/(Sqrt[a]*Sqr
t[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d
*x]]*Sqrt[-((a - a*Cosh[c + d*x])/a)]*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Se
ch[c + d*x]]*Sqrt[-((b*(a - a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a + b)))]))
)/(2*d*Sqrt[b + a*Cosh[c + d*x]]*Sqrt[Sech[c + d*x]])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\,\text{sech}\left(dx+c\right)+a}\,\coth\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{sech}\left(dx+c\right)+a}\,\coth\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \left(\coth^{2}\left(dx+c\right)\right)\sqrt{a+b\,\text{sech}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\operatorname{sech}(dx+c)+a}\,\coth(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \coth(c+d\,x)^2\,\sqrt{a+\frac{b}{\cosh(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\,\coth^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**2, x)
```

3.133 $\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\left(3a^2-2b^2\right)(a+b\,\mathrm{sech}(c+dx))^{3/2}}{3b^4d}+\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^4d}-\frac{2(a+b\,\mathrm{sech}(c+dx))^{7/2}}{7b^4d}+\frac{6a(a+b\,\mathrm{sec}}{5b}$$

```
[Out] -2/3*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d+6/5*a*(a+b*sech(d*x+c))^(5
/2)/b^4/d-2/7*(a+b*sech(d*x+c))^(7/2)/b^4/d+2*arctanh((a+b*sech(d*x+c))^(1/
2)/a^(1/2))/d/a^(1/2)+2*a*(a^2-2*b^2)*(a+b*sech(d*x+c))^(1/2)/b^4/d
```

Rubi [A] time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1153, 207}

$$-\frac{2\left(3a^2-2b^2\right)(a+b\,\mathrm{sech}(c+dx))^{3/2}}{3b^4d}+\frac{2a\left(a^2-2b^2\right)\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^4d}-\frac{2(a+b\,\mathrm{sech}(c+dx))^{7/2}}{7b^4d}+\frac{6a(a+b\,\mathrm{sec}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sech[c + d*x]])/(b^4*d) - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c
+ d*x])^(3/2))/(3*b^4*d) + (6*a*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) - (2
*(a + b*Sech[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(b^2-x^2)^2}{x\sqrt{a+x}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{b^4d}$$

$$= -\frac{2\operatorname{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{-a+x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^4d}$$

$$= -\frac{2\operatorname{Subst}\left(\int \left(-a^3+2ab^2+(3a^2-2b^2)x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right) dx, x, \sqrt{a+b\operatorname{sech}}\right.}{b^4d}$$

$$= \frac{2a(a^2-2b^2)\sqrt{a+b\operatorname{sech}(c+dx)}}{b^4d} - \frac{2(3a^2-2b^2)(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^4d} + \frac{6a(a+}{}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} + \frac{2a(a^2-2b^2)\sqrt{a+b\operatorname{sech}(c+dx)}}{b^4d} - \frac{2(3a^2-2b^2)(a}{3}$$

Mathematica [A] time = 4.41, size = 167, normalized size = 1.13

$$\frac{2\left(48a^4+(24a^3b-70ab^3)\operatorname{sech}(c+dx)-140a^2b^2+(70b^4-6a^2b^2)\operatorname{sech}^2(c+dx)+\frac{105b^4\sqrt{a\cosh(c+dx)+b}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{\sqrt{a\cosh(c+dx)}}\right.}{105b^4d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*(48*a^4 - 140*a^2*b^2 + (105*b^4*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[
a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] + (24*a^
3*b - 70*a*b^3)*Sech[c + d*x] + (-6*a^2*b^2 + 70*b^4)*Sech[c + d*x]^2 + 3*a
*b^3*Sech[c + d*x]^3 - 15*b^4*Sech[c + d*x]^4))/(105*b^4*d*Sqrt[a + b*Sech[
c + d*x]])
```

fricas [B] time = 1.09, size = 2813, normalized size = 19.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)*sinh(d*x + c)^5 + b^
4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh(d*x + c)^2 + 3*(5*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*b^4*cosh(d*x + c)^3 +
 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(d*x + c)^4 + 6*b^4*co
sh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x + c)^5 + 2*b^4*cosh(
d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x
 + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x
 + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sin
h(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*
x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 +
(6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*
x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b
)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*
(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh
(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2)) + 16*((12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^6 + (12*a^4 -
 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c)^5 - (12*
a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 + (36*a^4 - 87*a^2*b^2 + 15*(12*a^
4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^3
 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^3 + 5*(1
2*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 + (15*(12*a^4 -
 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87*a^2*b^2 - 10*(12*a^3*b - 35*a*b^
3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 - 24*(3*a^3*b
 - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (12*a^3*b - 35*a*b^3)*cosh(d*x
 + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^5 - 5*(12*a^3*b - 35*a*b^3)*
cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12*(12*a^4 - 29*a^2*b^2)*cosh(d*x +
 c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^2 + 6*(12*a^4 - 29*a^2*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b
```

```
^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^4*d*si
nh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4 + 3*a*b^4*d*cosh(d*x + c)^2 + a*b
^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4
*d*cosh(d*x + c)^3 + 3*a*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^4*
d*cosh(d*x + c)^4 + 6*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^2 +
6*(a*b^4*d*cosh(d*x + c)^5 + 2*a*b^4*d*cosh(d*x + c)^3 + a*b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), -1/105*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)
*sinh(d*x + c)^5 + b^4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh
(d*x + c)^2 + 3*(5*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*
b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(
d*x + c)^4 + 6*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x
 + c)^5 + 2*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a
)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cos
h(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c
) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 8*((12*a^4 - 35*a^2
*b^2)*cosh(d*x + c)^6 + (12*a^4 - 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b -
 35*a*b^3)*cosh(d*x + c)^5 - (12*a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)
*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 +
 (36*a^4 - 87*a^2*b^2 + 15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^
3*b - 35*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3
*a^3*b - 5*a*b^3)*cosh(d*x + c)^3 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 3
5*a^2*b^2)*cosh(d*x + c)^3 + 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(1
2*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2
)*cosh(d*x + c)^2 + (15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87
*a^2*b^2 - 10*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b
^2)*cosh(d*x + c)^2 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x
+ c)^5 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12
*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x +
c)^2 + 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d
*x + c) + b)/cosh(d*x + c)))/(a*b^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + a*b^4*d*sinh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4
+ 3*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^
4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4*d*cosh(d*x + c)^3 + 3*a*b^4*d*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(5*a*b^4*d*cosh(d*x + c)^4 + 6*a*b^4*d*cosh(d*x + c
)^2 + a*b^4*d)*sinh(d*x + c)^2 + 6*(a*b^4*d*cosh(d*x + c)^5 + 2*a*b^4*d*cos
h(d*x + c)^3 + a*b^4*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(dx + c\right)^5}{\sqrt{b\,\mathrm{sech}\left(dx + c\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(dx+c)}{\sqrt{a+b\,\mathrm{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^5}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(c+d\,x)^5}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/sqrt(a + b*sech(c + d*x)), x)
```

3.134 $\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2(a+b\,\mathrm{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2/3*(a+b*sech(d*x+c))^(3/2)/b^2/d+2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2)
)/d/a^(1/2)-2*a*(a+b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1153, 207}

$$\frac{2(a+b\,\mathrm{sech}(c+dx))^{3/2}}{3b^2d}-\frac{2a\sqrt{a+b\,\mathrm{sech}(c+dx)}}{b^2d}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a +
b*Sech[c + d*x]])/(b^2*d) + (2*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{b^2 d}$$

$$= -\frac{2\operatorname{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^2 d}$$

$$= -\frac{2\operatorname{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^2 d}$$

$$= -\frac{2a\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2 d} + \frac{2(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^2 d} - \frac{2\operatorname{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b}\right.}{d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{2a\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2 d} + \frac{2(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^2 d}$$

Mathematica [A] time = 0.64, size = 111, normalized size = 1.41

$$\frac{2\left(-2a^2+\frac{3b^2\sqrt{a\cosh(c+dx)+b}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)}}-ab\operatorname{sech}(c+dx)+b^2\operatorname{sech}^2(c+dx)\right)}{3b^2 d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*(-2*a^2 + (3*b^2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]
]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] - a*b*Sech[c + d*x] + b^
2*Sech[c + d*x]^2))/(3*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

fricas [B] time = 1.04, size = 925, normalized size = 11.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 + b^2)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c
)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
 c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*
cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c
) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*c
osh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^
3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a
*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 8*(a^2
*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*c
osh(d*x + c) - a*b)*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
))/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2
*d*sinh(d*x + c)^2 + a*b^2*d), -1/3*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*
x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(-a)*arctan((a*cosh(d
*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*s
inh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*c
osh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c))) + 4*(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x
 + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*cosh(d*x + c) - a*b)*sinh(d*x +
c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b^2*d*cosh(d*x + c)^2 + 2
*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^3}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(dx+c)}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^3}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(c+d\,x)^3}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

3.135 $\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3885, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]

[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [B] time = 0.14, size = 73, normalized size = 2.35

$$\frac{2\sqrt{a\cosh(c+dx)+b}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{d\sqrt{a\cosh(c+dx)}\,\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh
[c + d*x]])/(d*Sqrt[a*Cosh[c + d*x]]*Sqrt[a + b*Sech[c + d*x]])
```

fricas [B] time = 1.02, size = 558, normalized size = 18.00

$$\left[\frac{\log\left(-\frac{2\,a^2\cosh(dx+c)^4+2\,a^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^3+4\left(2\,a^2\cosh(dx+c)+ab\right)\sinh(dx+c)^3+4\,ab\cosh(dx+c)+\left(4\,a^2+b^2\right)\cosh(dx+c)^2+\left(12\,a^2\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x +
 c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c)
 + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*
x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*si
nh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3
 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*si
```

```
nh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^
2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c)
 + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2
*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))/(sqrt(a)*d), -sqrt(-a)*arctan(
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x +
c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)))/(a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(dx+c\right)}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

maple [A] time = 0.09, size = 26, normalized size = 0.84

$$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(dx+c\right)}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

mupad [B] time = 1.64, size = 27, normalized size = 0.87

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
[Out] (2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

3.136 $\int \frac{\coth(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))/d/(a+b)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3885, 898, 1170, 206, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
```

```
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx = -\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)}\,dx,x,b\,\mathrm{sech}(c+dx)\right)}{d}$$

$$= -\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)}\,dx,x,\sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \left(-\frac{1}{b^2\left(a-x^2\right)}+\frac{1}{2b^2\left(a+b-x^2\right)}-\frac{1}{2b^2\left(-a+b+x^2\right)}\right)dx,x,\sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{a+b-x^2}\,dx,x,\sqrt{a+b\,\mathrm{sech}(c+dx)}\right)}{d}+\frac{\mathrm{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx,x,\sqrt{a+b\,\mathrm{sech}\right.}{d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

Mathematica [B] time = 3.68, size = 226, normalized size = 2.13

$$\frac{\sqrt{a\cosh(c+dx)+b}\left(\frac{\frac{2\sqrt{b}\sqrt{\frac{a\cosh(c+dx)}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt{\cosh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}}-\frac{\sqrt{-a\cosh(c+dx)-b}\tanh^{-1}\left(\frac{\sqrt{-a-b}\sqrt{\cosh(c+dx)}}{\sqrt{-a\cosh(c+dx)-b}}\right)}{\sqrt{-a-b}}}{\sqrt{a\cosh(c+dx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{\cosh(c+dx)}}{\sqrt{a\cosh(c+dx)+b}}\right)}{\sqrt{a-b}}\right)}{d\sqrt{\cosh(c+dx)}\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cosh[c + d*x]]*(-(ArcTanh[(Sqrt[a - b]*Sqrt[Cosh[c + d*x]])/Sqr
t[b + a*Cosh[c + d*x]]]/Sqrt[a - b]) + (-((ArcTanh[(Sqrt[-a - b]*Sqrt[Cosh[
c + d*x]])/Sqrt[-b - a*Cosh[c + d*x]]]*Sqrt[-b - a*Cosh[c + d*x]])/Sqrt[-a
- b]) + (2*Sqrt[b]*ArcSinh[(Sqrt[a]*Sqrt[Cosh[c + d*x]])/Sqrt[b]]*Sqrt[1 +
(a*Cosh[c + d*x])/b])/Sqrt[a])/Sqrt[b + a*Cosh[c + d*x]]))/(d*Sqrt[Cosh[c +
 d*x]]*Sqrt[a + b*Sech[c + d*x]])
```

fricas [B] time = 1.31, size = 8908, normalized size = 84.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 +
 (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3
+ 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +
 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cos
h(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a -
b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b
)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x +
 c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) +
2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x
 + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)
*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a
^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cos
h(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(
cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x +
c) + 4*cosh(d*x + c) + 1)) + (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b +
 b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b +
```

3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8
*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*
b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*co
sh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a
 + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*
(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2
 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2
 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt
((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4
*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2
 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(c
osh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 -
4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)
^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*
x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) + 2*(a^2 - b^2)*sqrt(a)*l
og(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3
+ 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*
a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c)
 + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x
 + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a
*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x
 + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b +
 (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a^3 - a*b^2)*d), 1/4*(2*(a^2 - a*
b)*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
)/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c
) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) + (a^2 + a*b)
*sqrt(a - b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b +
 b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^
2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b
 + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*
a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^
4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3
 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(
d*x + c) + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*co
sh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d
*x + c) + 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) +
4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3
+ 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^
2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*s

```
inh(d*x + c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 +
 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c
)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x +
 c) + 1)) + 2*(a^2 - b^2)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(
d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x
 + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cos
h(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2
+ 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(
d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2
+ 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*
x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*
sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^
3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/
((a^3 - a*b^2)*d), -1/4*(2*(a^2 + a*b)*sqrt(-a + b)*arctan(-2*(cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a + b)*sqr
t((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a - b)*cosh(d*x + c)^2 + (2*a -
b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((2*a - b)*cosh(d*x + c) + b)*si
nh(d*x + c) + 2*a - b)) - (a^2 - a*b)*sqrt(a + b)*log(-((8*a^2 + 8*a*b + b^
2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b
^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 + 8*a*b + b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^
2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(
d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a +
b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*
(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a + b)*sinh(d*x + c)^2 +
2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 +
2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)*sqrt(a + b)*sqrt((a
*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((
8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 +
4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh
(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 4*c
osh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c)^2
+ 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x +
 c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)) - 2*(a^2 - b^2)*sqrt(a)*log(
-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4
*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2
 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) +
4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x +
c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*co
sh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x +
c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*c
osh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cos
h(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4
```

```
*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c
)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a^3 - a*b^2)*d), -1/2*((a^2 + a*b)*s
qrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(
(2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) +
 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - (a^2 - a*b)*sq
rt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2
*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2
*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - (a^2 - b^2)*sqrt
(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x +
c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c)
+ (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x
 + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sin
h(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3
+ 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sin
h(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2
 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c)
+ b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*
a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh
(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a^3 - a*b^2)*d), -1/4*(4*(a^
2 - b^2)*sqrt(-a)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a
*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c)
 + b)*sinh(d*x + c) + a)) - (a^2 + a*b)*sqrt(a - b)*log(-((8*a^2 - 8*a*b +
b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*sinh(d*x + c)^4 + 4*(4*a*b - 3
*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a^2 - 8*a*b + b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*
a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2 + 6*(4*a*b - 3*b
^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 - 4*((2*a - b)*cos
h(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^3 + 2*(2*(2*a
- b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(
3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a - b)*sinh(d*x + c)^2
+ 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2
+ 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)*sqrt(a - b)*sqrt(
(a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b - 3*b^2)*cosh(d*x + c) + 4*
((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b - 3*b^2)*cosh(d*x + c)^2
+ 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(co
sh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 4
*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c)^
2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2 + 3*cosh(d*x
 + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)) - (a^2 - a*b)*sqrt(a + b)*
log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b^2)*sinh(d*
x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2 + (8*a^2 +
8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b + 3*b^2)*cos
```

```
h(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 8*a*b +
 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 + 8*a*b +
 b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4 + 2*b*cosh(
d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*a + b
)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*
a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a
 + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b + 3*
b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a*b +
3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)*cosh(d*x +
 c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x +
 c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(
d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)))/((
a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan((cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cosh(d*x
 + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d
*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) - 2*(a^2 - a*b)*sqrt(
-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/((2*a
+ b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*((
2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)) - (a^2 + a*b)*sqrt(a
- b)*log(-((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 - 8*a*b + b^2)*si
nh(d*x + c)^4 + 4*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b - 3*b^2 + (8*a
^2 - 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 - 8*a*b + 3*b^2
)*cosh(d*x + c)^2 + 2*(3*(8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*
a*b + 3*b^2 + 6*(4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^2 - 8*
a*b + b^2 - 4*((2*a - b)*cosh(d*x + c)^4 + (2*a - b)*sinh(d*x + c)^4 + 2*b*
cosh(d*x + c)^3 + 2*(2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*(2*a - b)*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
 + 2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a - b)*cosh(d*x +
 c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c)
+ 2*a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*(4*a*b
 - 3*b^2)*cosh(d*x + c) + 4*((8*a^2 - 8*a*b + b^2)*cosh(d*x + c)^3 + 3*(4*a
*b - 3*b^2)*cosh(d*x + c)^2 + 4*a*b - 3*b^2 + (8*a^2 - 8*a*b + 3*b^2)*cosh(
d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(
d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*
cosh(d*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)
))/((a^3 - a*b^2)*d), -1/4*(4*(a^2 - b^2)*sqrt(-a)*arctan((cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*c
osh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(a^2 + a*b)*
sqrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/
```

```
((2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c)
+ 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - (a^2 - a*b)*s
qrt(a + b)*log(-((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^4 + (8*a^2 + 8*a*b + b
^2)*sinh(d*x + c)^4 + 4*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 4*(4*a*b + 3*b^2
+ (8*a^2 + 8*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^2 + 8*a*b +
 3*b^2)*cosh(d*x + c)^2 + 2*(3*(8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^2 + 8*a^
2 + 8*a*b + 3*b^2 + 6*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*a^
2 + 8*a*b + b^2 - 4*((2*a + b)*cosh(d*x + c)^4 + (2*a + b)*sinh(d*x + c)^4
+ 2*b*cosh(d*x + c)^3 + 2*(2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c)^3 +
 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 3*b*cosh(d*
x + c) + 2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(2*(2*a + b)*cosh
(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 2*(2*a + b)*cosh(d*x + c) + b)*sinh(d*x
 + c) + 2*a + b)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 4*
(4*a*b + 3*b^2)*cosh(d*x + c) + 4*((8*a^2 + 8*a*b + b^2)*cosh(d*x + c)^3 +
3*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2 + (8*a^2 + 8*a*b + 3*b^2)
*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2
*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 - 3*cosh(d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c
) + 1)))/((a^3 - a*b^2)*d), -1/2*(2*(a^2 - b^2)*sqrt(-a)*arctan((cosh(d*x +
 c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-a)*sqrt(
(a*cosh(d*x + c) + b)/cosh(d*x + c))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2
 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + (a^2 + a
*b)*sqrt(-a + b)*arctan(-2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c))/((2*a - b)*cosh(d*x + c)^2 + (2*a - b)*sinh(d*x + c)^2 + 2*b*cosh(d*x +
 c) + 2*((2*a - b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a - b)) - (a^2 - a*
b)*sqrt(-a - b)*arctan(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
)/((2*a + b)*cosh(d*x + c)^2 + (2*a + b)*sinh(d*x + c)^2 + 2*b*cosh(d*x + c
) + 2*((2*a + b)*cosh(d*x + c) + b)*sinh(d*x + c) + 2*a + b)))/((a^3 - a*b^
2)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)}{\sqrt{a+b\,\mathrm{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

3.137 $\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=262

$$-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)(1-\text{sech}(c+dx))}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a-b)(\text{sech}(c+dx)+1)}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}$$

```
[Out] 1/4*b*arctanh((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-1/4*b*arct
anh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d+2*arctanh((a+b*sech(
d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2
))/d/(a-b)^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
-1/4*(a+b*sech(d*x+c))^(1/2)/(a+b)/d/(1-sech(d*x+c))-1/4*(a+b*sech(d*x+c))^
(1/2)/(a-b)/d/(1+sech(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3885, 898, 1238, 206, 199, 207}

$$-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)(1-\text{sech}(c+dx))}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a-b)(\text{sech}(c+dx)+1)}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) + (b*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) - (b*ArcTanh[Sqrt[a + b*Sech
[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d) - Sqrt[a + b*Sech[c + d*x]]/(4*(a + b)*d
*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a - b)*d*(1 + Sech[c
+ d*x]))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{b^4\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}\left(b^2-x^2\right)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{\left(2b^4\right)\text{Subst}\left(\int \frac{1}{\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\left(2b^4\right)\text{Subst}\left(\int \left(-\frac{1}{b^4\left(a-x^2\right)} + \frac{1}{4b^3\left(a+b-x^2\right)^2} + \frac{1}{2b^4\left(a+b-x^2\right)} - \frac{1}{4b^3\left(-a+b+x^2\right)^2} - \frac{1}{2b^4\left(-a+b+x^2\right)}\right.\right.}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\text{sech}\right.}}{d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d} - \frac{}{4(}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} - $$

Mathematica [B] time = 7.40, size = 902, normalized size = 3.44

$$\frac{\sqrt{b+a\cosh(c+dx)}\,\sqrt{\text{sech}(c+dx)}\left(\frac{\left(2a^2-2b^2\right)\left(\sqrt{a}\left(\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{b+a\cosh(c+dx)}}{\sqrt{a-b}\,\sqrt{-a\cosh(c+dx)}}\right)+\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\,\sqrt{-a\cosh(c+dx)}}\right)\right)-4\sqrt{a-b}\,\sqrt{a}\right.}{\sqrt{a-b}\,\sqrt{a+b}\,\sqrt{\cosh(c+dx)-1}\,\sqrt{\cosh(c+dx)+1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cosh[c + d*x]]*((Sqrt[a]*b*(Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b
+ a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])] + Sqrt[-a - b]*Ar
cTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c + d*x]
])])*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x
]))/(Sqrt[-a - b]*Sqrt[a - b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c + d*x]
]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[Sech[c + d*x]]) - ((2*a^2 - 3*b^2)*(Sqrt[a +
 b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c
+ d*x]])] + Sqrt[a - b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a
 + b]*Sqrt[a*Cosh[c + d*x]])])*Sqrt[a*Cosh[c + d*x]]*Sqrt[(-a + a*Cosh[c +
```

```
d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Sqrt[Sech[c + d*x]])/(a^
(3/2)*Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*
x]]) + ((2*a^2 - 2*b^2)*(-4*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cosh[
c + d*x]]/Sqrt[-(a*Cosh[c + d*x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*
Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cosh[c + d*x])])] + Sqrt[a
 - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cosh
[c + d*x])])]))*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[(-a + a*Cosh[c + d*x])/(a + a
*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Cosh[2*(c + d*x)]*Sqrt[Sech[c + d*x]
])/(Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]
]*(a^2 - 2*b^2 + 4*b*(b + a*Cosh[c + d*x]) - 2*(b + a*Cosh[c + d*x])^2)))*S
qrt[Sech[c + d*x]])/(4*(a - b)*(a + b)*d*Sqrt[a + b*Sech[c + d*x]]) + ((b +
 a*Cosh[c + d*x])*(-1/2*a/(a^2 - b^2) + ((a - b*Cosh[c + d*x])*Csch[c + d*x
]^2)/(2*(-a^2 + b^2)))*Sech[c + d*x])/(d*Sqrt[a + b*Sech[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)^3}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(dx+c)}{\sqrt{a+b\,\mathrm{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(dx+c\right)^{3}}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(c+d\,x\right)^{3}}{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}\left(c+dx\right)}{\sqrt{a+b\,\mathrm{sech}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

3.138 $\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=610

$$\frac{2(a-b)\sqrt{a+b}\left(8a^2+9b^2\right)\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}+\frac{2\sqrt{a}}{}$$

```
[Out] -4*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c)
)/(a-b))^(1/2)/b^2/d+2/15*(a-b)*(8*a^2+9*b^2)*coth(d*x+c)*EllipticE((a+b*se
ch(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*
x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/d-4*coth(d*x+c)*Ell
ipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)
*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2/15*
(8*a^2-2*a*b+9*b^2)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+
sech(d*x+c))/(a-b))^(1/2)/b^3/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^
(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c
))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-8/15*a*(a+b*sech(d*x+c
))^(1/2)*tanh(d*x+c)/b^2/d+2/5*sech(d*x+c)*(a+b*sech(d*x+c))^(1/2)*tanh(d*x
+c)/b/d
```

Rubi [A] time = 0.79, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3895, 3784, 3837, 3832, 4004, 3860, 4082, 4005}

$$\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (-4*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*
(8*a^2 + 9*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sech[c + d*x]))/(a - b))])/(15*b^4*d) - (4*Sqrt[a + b]*Coth[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b
))])/(b*d) + (2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Coth[c + d*x]*EllipticF
```

```
[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(15*b
^3*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - (8*a*Sqrt[a +
b*Sech[c + d*x]]*Tanh[c + d*x])/(15*b^2*d) + (2*Sech[c + d*x]*Sqrt[a + b*Se
ch[c + d*x]]*Tanh[c + d*x])/(5*b*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3837

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[(Csc[e + f
*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
```

```
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \left(\frac{1}{\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}} + \frac{\text{sech}^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\right) dx$$
$$= -\left(2\int \frac{\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx + \int \frac{\text{sech}^4(c+dx)}{\sqrt{a+b\text{sech}(c+}}$$
$$= \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a}}}{ad}$$
$$= -\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{b^2d}$$
$$= -\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{b^2d}$$
$$= -\frac{4(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{b^2d}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
[Out] $Aborted
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh(dx+c)^4}{\sqrt{b\,\text{sech}(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^4}{\sqrt{b\,\text{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}\left(dx+c\right)}{\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(dx+c\right)^{4}}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh\left(c+d\,x\right)^4}{\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2),x)
```

```
[Out] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}\left(c+dx\right)}{\sqrt{a+b\,\mathrm{sech}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/sqrt(a + b*sech(c + d*x)), x)
```

3.139 $\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=310

$$-\frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\,\coth(c+$$

```
[Out] -2*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c)
)/(a-b))^(1/2)/b^2/d-2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*
(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))
^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+
c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.26, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3894, 4059, 3921, 3784, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\,\coth(c+$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Coth[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\int \frac{-1+\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$= -\int \frac{-1-\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx - \int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$= -\frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{b^2d}$$
$$= -\frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{b^2d}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^2}{\sqrt{b\,\text{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(dx+c)}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^2}{\sqrt{b\,\mathrm{sech}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

3.140 $\int \frac{1}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\,\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\,\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\,\sqrt{-\frac{b(1+\operatorname{sec}}{a-}}}{ad}$$

Mathematica [A] time = 0.63, size = 168, normalized size = 1.58

$$\frac{2b\tanh\left(\frac{1}{2}(c+dx)\right)\sqrt{a\cosh(c+dx)+b}\sqrt{\frac{b(\operatorname{sech}(c+dx)+1)}{b-a}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right)|\frac{a+b}{a-b}\right)}{\sqrt{a}\,d\sqrt{a+b}\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(\operatorname{sech}(c+dx)-1)}{a+b}}\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

fricas [F] time = 3.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\operatorname{sech}(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+b\operatorname{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\operatorname{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

3.141 $\int \frac{\coth^2(c+dx)}{\sqrt{a+b\,\mathbf{sech}(c+dx)}}\,dx$

Optimal. Leaf size=362

$$-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}F\left(\sin^{-}\right.}{d\sqrt{a+b}}$$

```
[Out] coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/d/(a+b
)^(1/2)-coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2
)/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2
),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*
(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-coth(d*x+c)/d/(a+b*sech(d*x+c))^(1/2)-
b^2*tanh(d*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3896, 3784, 3875, 3833, 21, 3829, 3832, 4004}

$$-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}F\left(\sin^{-}\right.}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c +
 d*x]))/(a - b))])/(Sqrt[a + b]*d) - (Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (
2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - Coth[c + d*x]/(d*Sqrt[a
 + b*Sech[c + d*x]]) - (b^2*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sech[c
 + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
```

```
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\int \left(-\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}-\frac{\text{csch}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\right)dx$$

$$=\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx+\int \frac{\text{csch}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$=\frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$

$$=\frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$

$$=\frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$

$$=\frac{2\sqrt{a+b}\,\coth(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$

$$=\frac{\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{\sqrt{a+b}\,d}-\text{co}$$

Mathematica [F] time = 91.35, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]], x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)^2}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(dx+c)}{\sqrt{a+b\,\mathrm{sech}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)^2}{\sqrt{b\,\mathrm{sech}(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth (c + d\,x)^2}{\sqrt{a + \frac{b}{\cosh(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2 (c + dx)}{\sqrt{a + b\operatorname{sech}(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

3.142 $\int \frac{\tanh^5(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}-\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}-\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*a*(a+b*sech(d*x+c))^
(3/2)/b^4/d-2/5*(a+b*sech(d*x+c))^(5/2)/b^4/d-2*(a^2-b^2)^2/a/b^4/d/(a+b*se
ch(d*x+c))^(1/2)-2*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(1/2)/b^4/d
```

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}-\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sech[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec
h[c + d*x]])/(b^4*d) + (2*a*(a + b*Sech[c + d*x])^(3/2))/(b^4*d) - (2*(a +
b*Sech[c + d*x])^(5/2))/(5*b^4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x(a+x)^{3/2}}\,dx, x, b\text{sech}(c+dx)\right)}{b^4d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{\left(-a^2+b^2+2ax^2-x^4\right)^2}{x^2\left(-a+x^2\right)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4d}$$

$$= -\frac{2\,\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{\left(a^2-b^2\right)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a\left(a-x^2\right)}\right)dx, x, \sqrt{a+b\text{sech}(c$$

$$= -\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\text{sech}(c+dx)}}{b^4d}+\frac{2a(a+b\text{sech}}{b^4c}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2\left(a^2-b^2\right)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}}-\frac{2\left(3a^2-2b^2\right)\sqrt{a+b\text{sec}}}{b^4d}$$

Mathematica [A] time = 3.17, size = 155, normalized size = 1.05

$$-\frac{2\left(16a^4-2a^2b^2\text{sech}^2(c+dx)+2ab\left(4a^2-5b^2\right)\text{sech}(c+dx)-20a^2b^2-\frac{5b^4\sqrt{a\cosh(c+dx)+b}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)}}\right.}{5ab^4d\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (-2*(16*a^4 - 20*a^2*b^2 + 5*b^4 - (5*b^4*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]
/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] +
2*a*b*(4*a^2 - 5*b^2)*Sech[c + d*x] - 2*a^2*b^2*Sech[c + d*x]^2 + a*b^3*Sec
h[c + d*x]^3))/(5*a*b^4*d*Sqrt[a + b*Sech[c + d*x]])
```

fricas [B] time = 2.68, size = 3745, normalized size = 25.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/10*(5*(a*b^4*cosh(d*x + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x +
c)^5 + 3*a*b^4*cosh(d*x + c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x +
 c)^2 + 2*b^5*cosh(d*x + c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)
^5 + a*b^4 + (15*a*b^4*cosh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*si
nh(d*x + c)^4 + 4*(5*a*b^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^
4*cosh(d*x + c) + b^5)*sinh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5
*cosh(d*x + c)^3 + 18*a*b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^
4)*sinh(d*x + c)^2 + 2*(3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6
*a*b^4*cosh(d*x + c)^3 + 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^
5)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)
^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
+ 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*c
osh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c)
 + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*co
sh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*
sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*
b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*((16*
a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^
4)*sinh(d*x + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b -
 10*a^2*b^3 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c
)^5 + 16*a^5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh
(d*x + c)^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5
*a*b^4)*cosh(d*x + c)^2 + 20*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^4 + 32*(a^4*b - a^2*b^3)*cosh(d*x + c)^3 + 4*(8*a^4*b - 8*a^2*b^3 + 5*
(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^3 + 10*(4*a^4*b - 5*a^2*b^3)*
cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^2 + (48*a^5 - 68*a^
3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^4 + 40*
(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^3 + 6*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*
cosh(d*x + c)^2 + 96*(a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(
```

```
4*a^4*b - 5*a^2*b^3)*cosh(d*x + c) + 2*(3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*c
osh(d*x + c)^5 + 8*a^4*b - 10*a^2*b^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x +
 c)^4 + 2*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^3 + 48*(a^4*b - a^
2*b^3)*cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*si
nh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^4*d*cosh(d*x
 + c)^6 + a^3*b^4*d*sinh(d*x + c)^6 + 2*a^2*b^5*d*cosh(d*x + c)^5 + 3*a^3*b
^4*d*cosh(d*x + c)^4 + 4*a^2*b^5*d*cosh(d*x + c)^3 + 3*a^3*b^4*d*cosh(d*x +
 c)^2 + 2*a^2*b^5*d*cosh(d*x + c) + a^3*b^4*d + 2*(3*a^3*b^4*d*cosh(d*x + c
) + a^2*b^5*d)*sinh(d*x + c)^5 + (15*a^3*b^4*d*cosh(d*x + c)^2 + 10*a^2*b^5
*d*cosh(d*x + c) + 3*a^3*b^4*d)*sinh(d*x + c)^4 + 4*(5*a^3*b^4*d*cosh(d*x +
 c)^3 + 5*a^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5*d
)*sinh(d*x + c)^3 + (15*a^3*b^4*d*cosh(d*x + c)^4 + 20*a^2*b^5*d*cosh(d*x +
 c)^3 + 18*a^3*b^4*d*cosh(d*x + c)^2 + 12*a^2*b^5*d*cosh(d*x + c) + 3*a^3*b
^4*d)*sinh(d*x + c)^2 + 2*(3*a^3*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^5*d*cosh(d
*x + c)^4 + 6*a^3*b^4*d*cosh(d*x + c)^3 + 6*a^2*b^5*d*cosh(d*x + c)^2 + 3*a
^3*b^4*d*cosh(d*x + c) + a^2*b^5*d)*sinh(d*x + c)), -1/5*(5*(a*b^4*cosh(d*x
 + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x + c)^5 + 3*a*b^4*cosh(d*x
+ c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x + c)^2 + 2*b^5*cosh(d*x +
 c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)^5 + a*b^4 + (15*a*b^4*c
osh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^4 + 4*(5*a*b
^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*s
inh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5*cosh(d*x + c)^3 + 18*a*
b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^2 + 2*(
3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6*a*b^4*cosh(d*x + c)^3 +
 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c))*sqrt(-
a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*c
osh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*((16*a^5 - 20*a^
3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*sinh(d*x
 + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b - 10*a^2*b^3
 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 16*a^
5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^4
 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos
h(d*x + c)^2 + 20*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 32
*(a^4*b - a^2*b^3)*cosh(d*x + c)^3 + 4*(8*a^4*b - 8*a^2*b^3 + 5*(16*a^5 - 2
0*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x +
c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (4
8*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*
a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^4 + 40*(4*a^4*b -
5*a^2*b^3)*cosh(d*x + c)^3 + 6*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x +
c)^2 + 96*(a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(4*a^4*b - 5
*a^2*b^3)*cosh(d*x + c) + 2*(3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c
)^5 + 8*a^4*b - 10*a^2*b^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^4 + 2*(
48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^3 + 48*(a^4*b - a^2*b^3)*cosh
```

```
(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)
)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^4*d*cosh(d*x + c)^6 + a
^3*b^4*d*sinh(d*x + c)^6 + 2*a^2*b^5*d*cosh(d*x + c)^5 + 3*a^3*b^4*d*cosh(d
*x + c)^4 + 4*a^2*b^5*d*cosh(d*x + c)^3 + 3*a^3*b^4*d*cosh(d*x + c)^2 + 2*a
^2*b^5*d*cosh(d*x + c) + a^3*b^4*d + 2*(3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5
*d)*sinh(d*x + c)^5 + (15*a^3*b^4*d*cosh(d*x + c)^2 + 10*a^2*b^5*d*cosh(d*x
 + c) + 3*a^3*b^4*d)*sinh(d*x + c)^4 + 4*(5*a^3*b^4*d*cosh(d*x + c)^3 + 5*a
^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5*d)*sinh(d*x
+ c)^3 + (15*a^3*b^4*d*cosh(d*x + c)^4 + 20*a^2*b^5*d*cosh(d*x + c)^3 + 18*
a^3*b^4*d*cosh(d*x + c)^2 + 12*a^2*b^5*d*cosh(d*x + c) + 3*a^3*b^4*d)*sinh(
d*x + c)^2 + 2*(3*a^3*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^5*d*cosh(d*x + c)^4 +
 6*a^3*b^4*d*cosh(d*x + c)^3 + 6*a^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*co
sh(d*x + c) + a^2*b^5*d)*sinh(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(dx+c\right)^{5}}{\left(b\operatorname{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}\left(dx+c\right)}{\left(a+b\operatorname{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(dx+c\right)^{5}}{\left(b\operatorname{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(c+d\,x\right)^5}{{\left(a+\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}{\left(c + dx \right)}}{\left(a + b \operatorname{sech}{\left(c + dx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x))**(3/2), x)
```

3.143 $\int \frac{\tanh^3(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(a
+b*sech(d*x+c))^(1/2)+2*(a+b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3885, 898, 1261, 206}

$$\frac{2\left(a^2-b^2\right)}{ab^2d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
)/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*Sqrt[a + b*Sech[c + d*x]])/(b^2*
d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{b^2 d}\\
&= -\frac{2\operatorname{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2\left(-a+x^2\right)}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^2 d}\\
&= -\frac{2\operatorname{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a\left(a-x^2\right)}\right)dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{b^2 d}\\
&= \frac{2\left(a^2-b^2\right)}{ab^2 d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2 d}+\frac{2\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+\right.}{ad}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2\left(a^2-b^2\right)}{ab^2 d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2 d}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 103, normalized size = 1.17

$$\frac{2\left(2a^2+\frac{b^2\sqrt{a\cosh(c+dx)+b}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)}}+ab\operatorname{sech}(c+dx)-b^2\right)}{ab^2 d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(2*a^2 - b^2 + (b^2*ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*
x]]]*Sqrt[b + a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]] + a*b*Sech[c + d*x]))
/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

fricas [B] time = 1.24, size = 1107, normalized size = 12.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3*cosh(d*x + c)
+ a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*
cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*
cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*
cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 +
b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x +
 c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b
*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x
+ c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b
^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2)) + 4*(2*a^2*b*cosh(d*x + c) + 2*a^3 - a*b^2 + (2
*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)*sinh(d*x + c)^2 + 2*(a^2*b
+ (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^3*b^2*d*sinh(d*x + c)^2 + 2*
a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^2*d*cosh(d*x + c) + a^2*b^3*
d)*sinh(d*x + c)), -((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3
*cosh(d*x + c) + a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(
-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
 c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 2*(2*a^2*b*cosh(d
*x + c) + 2*a^3 - a*b^2 + (2*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)
*sinh(d*x + c)^2 + 2*(a^2*b + (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^
3*b^2*d*sinh(d*x + c)^2 + 2*a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^
2*d*cosh(d*x + c) + a^2*b^3*d)*sinh(d*x + c))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^3}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(dx+c)}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^3}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

3.144 $\int \frac{\tanh(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2/a/d/(a+b*sech(d*x+c)
)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 51, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - 2/(a*d*Sqrt[a
+ b*Sech[c + d*x]])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d} \\
&= -\frac{2}{ad\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\operatorname{sech}(c+dx)\right)}{ad} \\
&= -\frac{2}{ad\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{ad} \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2}{ad\sqrt{a+b\operatorname{sech}(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 79, normalized size = 1.46

$$\frac{2\left(\frac{\sqrt{a\cosh(c+dx)+b}\,\tanh^{-1}\left(\frac{\sqrt{a\cosh(c+dx)+b}}{\sqrt{a\cosh(c+dx)}}\right)}{\sqrt{a\cosh(c+dx)}}-1\right)}{ad\sqrt{a+b\operatorname{sech}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(-1 + (ArcTanh[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[a*Cosh[c + d*x]]]*Sqrt[b +
 a*Cosh[c + d*x]])/Sqrt[a*Cosh[c + d*x]]))/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

fricas [B] time = 1.06, size = 917, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/2*((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cos h(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2* a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b) *sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (1 2*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d* x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4* a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*s inh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d *x + c)^2 + a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x + c)^2 + a^3*d*sinh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d* cosh(d*x + c) + a^2*b*d)*sinh(d*x + c)), -((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqr t(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2* a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b) /cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt((a*co sh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x + c)^2 + a^3*d*sinh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d*cosh(d*x + c) + a^2*b*d)* sinh(d*x + c))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)

maple [A] time = 0.09, size = 46, normalized size = 0.85

$$-\frac{\frac{2}{a\sqrt{a+b\,\mathrm{sech}(dx+c)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(2/a/(a+b*sech(d*x+c))^(1/2)-2/a^(3/2)*arctanh((a+b*sech(d*x+c))^(1/2)
/a^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh (dx + c)}{(b \operatorname{sech} (dx + c) + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

mupad [B] time = 1.77, size = 50, normalized size = 0.93

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d} - \frac{2}{a\,d\,\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] (2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(3/2)*d) - 2/(a*d*(a + b/
cosh(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh (c + dx)}{(a + b \operatorname{sech} (c + dx))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

3.145 $\int \frac{\coth(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3885, 898, 1287, 206}

$$\frac{2b^2}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[
a + b*Sech[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx &= -\frac{b^2\operatorname{Subst}\left(\int \frac{1}{x(a+x)^{3/2}\left(b^2-x^2\right)}\,dx, x, b\operatorname{sech}(c+dx)\right)}{d}\\
&= -\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{d}\\
&= -\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \left(\frac{1}{a\left(a^2-b^2\right)x^2}-\frac{1}{ab^2\left(a-x^2\right)}+\frac{1}{2(a-b)b^2\left(a-b-x^2\right)}+\frac{1}{2b^2(a+b)\left(a+b-x^2\right)}\right)dx, x,\right.}{d}\\
&= \frac{2b^2}{a\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{ad}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d}+
\end{aligned}$$

Mathematica [B] time = 7.37, size = 904, normalized size = 6.37

$$\frac{(b+a\cosh(c+dx))^2\left(-\frac{2b^3}{a^2\left(a^2-b^2\right)(b+a\cosh(c+dx))}-\frac{2b^2}{a^2\left(b^2-a^2\right)}\right)\operatorname{sech}^2(c+dx)}{d(a+b\operatorname{sech}(c+dx))^{3/2}}-\frac{(b+a\cosh(c+dx))^{3/2}\left(\frac{\left(a^2-b^2\right)\left(\sqrt{a}\left(\sqrt{a+\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((b + a*Cosh[c + d*x])^(3/2)*((-2*Sqrt[a]*b*(Sqrt[a - b]*ArcTan[(Sqrt[
a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])] + Sqrt[
-a - b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*Cos
h[c + d*x]])])*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Co
sh[c + d*x]))/(Sqrt[-a - b]*Sqrt[a - b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cos
h[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[Sech[c + d*x]]) - ((a^2 + b^2)*(Sq
rt[a + b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*C
osh[c + d*x]])] + Sqrt[a - b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(
Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])])*Sqrt[a*Cosh[c + d*x]]*Sqrt[(-a + a*Cos
h[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Sqrt[Sech[c + d*x]
])/(a^(3/2)*Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[
c + d*x]]) + ((a^2 - b^2)*(-4*Sqrt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cos
h[c + d*x]]/Sqrt[-(a*Cosh[c + d*x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a
]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[-(a*Cosh[c + d*x])])] + Sqrt
[a - b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Co
sh[c + d*x])])]))*Sqrt[-(a*Cosh[c + d*x])]*Sqrt[(-a + a*Cosh[c + d*x])/(a +
 a*Cosh[c + d*x])]*(a + a*Cosh[c + d*x])*Cosh[2*(c + d*x)]*Sqrt[Sech[c + d*
x]])/(Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*
x]]*(a^2 - 2*b^2 + 4*b*(b + a*Cosh[c + d*x]) - 2*(b + a*Cosh[c + d*x])^2)))
*Sech[c + d*x]^(3/2))/(a*(-a + b)*(a + b)*d*(a + b*Sech[c + d*x])^(3/2)) +
((b + a*Cosh[c + d*x])^2*((-2*b^2)/(a^2*(-a^2 + b^2)) - (2*b^3)/(a^2*(a^2 -
 b^2)*(b + a*Cosh[c + d*x])))*Sech[c + d*x]^2)/(d*(a + b*Sech[c + d*x])^(3/
2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(dx+c\right)}{\left(b\operatorname{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(dx+c)}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c+d\,x)}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

3.146 $\int \frac{\coth^3(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)^2(1-\text{sech}(c+dx))}-\frac{\sqrt{a+b\text{sech}(c+}{4d(a-b)^2(\text{sech}(c+}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/2*(2*a-3*b)*arctanh(
(a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*b*arctanh((a+b*sech(
d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d-1/4*b*arctanh((a+b*sech(d*x+c))^(1
/2)/(a+b)^(1/2))/(a+b)^(5/2)/d-1/2*(2*a+3*b)*arctanh((a+b*sech(d*x+c))^(1/2
)/(a+b)^(1/2))/(a+b)^(5/2)/d-2*b^4/a/(a^2-b^2)^2/d/(a+b*sech(d*x+c))^(1/2)-
1/4*(a+b*sech(d*x+c))^(1/2)/(a+b)^2/d/(1-sech(d*x+c))-1/4*(a+b*sech(d*x+c))
^(1/2)/(a-b)^2/d/(1+sech(d*x+c))
```

Rubi [A] time = 0.43, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3885, 898, 1335, 206, 199}

$$-\frac{2b^4}{ad\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)^2(1-\text{sech}(c+dx))}-\frac{\sqrt{a+b\text{sech}(c}{4d(a-b)^2(\text{sech}(c}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) + (b*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) - (b*ArcTa
nh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) - ((2*a + 3*
b)*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) - (2
*b^4)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) - Sqrt[a + b*Sech[c + d
*x]]/(4*(a + b)^2*d*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a
- b)^2*d*(1 + Sech[c + d*x]))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 898

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^
q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 3885

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> -Dist[(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[((b^2 - x^
2)^((m - 1)/2)*(a + x)^n)/x, x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{b^4\,\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}\left(b^2-x^2\right)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{\left(2b^4\right)\text{Subst}\left(\int \frac{1}{x^2\left(-a+x^2\right)\left(-a^2+b^2+2ax^2-x^4\right)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\left(2b^4\right)\text{Subst}\left(\int \left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4\left(a-x^2\right)} - \frac{1}{4(a-b)b^3\left(a-b-x^2\right)^2} + \frac{2a-3b}{4(a-b)^2b^4\left(a-b-x^2\right)} + \cdots\right.\right.}{d}$$

$$= -\frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad} \cdots$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} - \frac{(2a+3b)\tanh^{-1}\cdots}{2(a+\cdots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+b\text{sec}\cdots}}{\sqrt{a}\cdots}\right.}{4(a-b)^{5/2}\cdots}$$

Mathematica [B] time = 7.61, size = 996, normalized size = 3.15

$$\frac{(b+a\cosh(c+dx))^2\left(\frac{2b^5}{a^2\left(a^2-b^2\right)^2(b+a\cosh(c+dx))} + \frac{\left(-a^2+2b\cosh(c+dx)a-b^2\right)\text{csch}^2(c+dx)}{2\left(b^2-a^2\right)^2} - \frac{a^4+b^2a^2+4b^4}{2a^2\left(b^2-a^2\right)^2}\right)\text{sech}^2(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}} + \frac{(b+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cosh[c + d*x])^(3/2)*(((-(a^3*b) + 7*a*b^3)*(Sqrt[a - b]*ArcTan[(Sq
rt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[-a - b]*Sqrt[a*Cosh[c + d*x]])] + Sq
rt[-a - b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sqrt[a - b]*Sqrt[a*
Cosh[c + d*x]])])*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a
*Cosh[c + d*x]))/(Sqrt[a]*Sqrt[-a - b]*Sqrt[a - b]*Sqrt[-1 + Cosh[c + d*x]]
*Sqrt[a*Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[Sech[c + d*x]]) - ((2*a
^4 - 6*a^2*b^2 - 2*b^4)*(Sqrt[a + b]*ArcTanh[(Sqrt[a]*Sqrt[b + a*Cosh[c + d
*x]])/(Sqrt[a - b]*Sqrt[a*Cosh[c + d*x]])] + Sqrt[a - b]*ArcTanh[(Sqrt[a]*S
```

```
qrt[b + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])])*Sqrt[a*Cosh
[c + d*x]]*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c
 + d*x])*Sqrt[Sech[c + d*x]])/(a^(3/2)*Sqrt[a - b]*Sqrt[a + b]*Sqrt[-1 + Co
sh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]) + ((2*a^4 - 4*a^2*b^2 + 2*b^4)*(-4*Sq
rt[a - b]*Sqrt[a + b]*ArcTan[Sqrt[b + a*Cosh[c + d*x]]/Sqrt[-(a*Cosh[c + d*
x])]] + Sqrt[a]*(Sqrt[a + b]*ArcTan[(Sqrt[a]*Sqrt[b + a*Cosh[c + d*x]])/(Sq
rt[a - b]*Sqrt[-(a*Cosh[c + d*x])])] + Sqrt[a - b]*ArcTan[(Sqrt[a]*Sqrt[b +
 a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[-(a*Cosh[c + d*x])])])))*Sqrt[-(a*Cosh[
c + d*x])]*Sqrt[(-a + a*Cosh[c + d*x])/(a + a*Cosh[c + d*x])]*(a + a*Cosh[c
 + d*x])*Cosh[2*(c + d*x)]*Sqrt[Sech[c + d*x]])/(Sqrt[a - b]*Sqrt[a + b]*Sq
rt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*(a^2 - 2*b^2 + 4*b*(b + a*Co
sh[c + d*x]) - 2*(b + a*Cosh[c + d*x])^2)))*Sech[c + d*x]^(3/2))/(4*a*(a -
b)^2*(a + b)^2*d*(a + b*Sech[c + d*x])^(3/2)) + ((b + a*Cosh[c + d*x])^2*(-
1/2*(a^4 + a^2*b^2 + 4*b^4)/(a^2*(-a^2 + b^2)^2) + (2*b^5)/(a^2*(a^2 - b^2)
^2*(b + a*Cosh[c + d*x])) + ((-a^2 - b^2 + 2*a*b*Cosh[c + d*x])*Csch[c + d*
x]^2)/(2*(-a^2 + b^2)^2))*Sech[c + d*x]^2)/(d*(a + b*Sech[c + d*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(dx+c\right)^{3}}{\left(b\operatorname{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}\left(dx+c\right)}{\left(a+b\operatorname{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(dx+c\right)^{3}}{\left(b\operatorname{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(c+d\,x\right)^{3}}{\left(a+\frac{b}{\cosh\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}\left(c+dx\right)}{\left(a+b\operatorname{sech}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

3.147 $\int \frac{\tanh^4(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=907

$$-\frac{2\operatorname{sech}(c+dx)\tanh(c+dx)a^2}{b\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{4\tanh(c+dx)a}{\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{2\left(8a^2-5b^2\right)\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{se}}}{\sqrt{}}\right.\right.}{3b^4}$$

```
[Out] -2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
/(a+b)^(1/2)+4*a*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a
-b))^(1/2)/b^2/d/(a+b)^(1/2)-2/3*a*(8*a^2-5*b^2)*coth(d*x+c)*EllipticE((a+b
*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+
b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/d/(a+b)^(1/2)+2*coth(d*x+c)*
EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-se
ch(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)+4*
coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d/(a
+b)^(1/2)-2/3*(2*a+b)*(4*a+b)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+se
ch(d*x+c))/(a-b))^(1/2)/b^3/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sec
h(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-
sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-4*a*tanh(d
*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)+2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(
a+b*sech(d*x+c))^(1/2)-2*a^2*sech(d*x+c)*tanh(d*x+c)/b/(a^2-b^2)/d/(a+b*sec
h(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)/b^2/(a^
2-b^2)/d
```

Rubi [A] time = 1.37, antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3895, 3785, 4058, 3921, 3784, 3832, 4004, 3836, 4005, 3845, 4082}

$$-\frac{2\operatorname{sech}(c+dx)\tanh(c+dx)a^2}{b\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{4\tanh(c+dx)a}{\left(a^2-b^2\right)d\sqrt{a+b\operatorname{sech}(c+dx)}}-\frac{2\left(8a^2-5b^2\right)\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{se}}}{\sqrt{}}\right.\right.}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
```

```
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*a*Coth[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
ch[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*Sqrt[
a + b]*d) - (2*a*(8*a^2 - 5*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) +
(2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
 + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*Coth[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b*Sqrt[a + b
]*d) - (2*(2*a + b)*(4*a + b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) + (2*
Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) - (4*a*Tanh[c + d*x])/((a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*Sqrt[a + b*Sech[c + d*x]]) - (2*a^2*Sech[c + d*x]*Tanh[c + d*x])/(b*(a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sech[c
 + d*x]]*Tanh[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 3895

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int\left(\frac{1}{(a+b\text{sech}(c+dx))^{3/2}} - \frac{2\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}} + \frac{\text{sech}^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\right) d$$

$$= -\left(2\int \frac{\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx\right) + \int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx + \int \frac{\text{sech}}{(a+b\text{sec}}$$

$$= -\frac{4a\tanh(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2a^2\text{sec}}{b\left(a^2-b\right.}$$

$$= -\frac{4a\tanh(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2a^2\text{sec}}{b\left(a^2-b\right.}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
[Out] $Aborted
```

fricas [F] time = 7.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^4}{b^2\,\text{sech}\,(dx+c)^2+2\,ab\,\text{sech}\,(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^4/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^4}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(dx+c)}{(a+b\operatorname{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^4}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^4}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(c + dx)}{(a + b\operatorname{sech}(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x))**(3/2), x)
```

3.148 $\int \frac{\tanh^2(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\coth}{}$$

```
[Out] 2*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))
/(a-b))^(1/2)/a/b^2/d+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b
*(1+sech(d*x+c))/(a-b))^(1/2)/a/b/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d
*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-2*tanh(d*x+c)/a/
d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3894, 4061, 4059, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2(a-b)\sqrt{a+b}\coth}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Coth[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/
(a - b))])/(a*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) -
 (2*Tanh[c + d*x])/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
```

```
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\int \frac{-1+\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(a^2-b^2)+\frac{1}{2}(a^2-b^2)\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a(a^2-b^2)}$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}} + \frac{\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a} + \frac{2\int \frac{\frac{1}{2}(a^2-b^2)-\frac{1}{2}(a^2-b^2)\text{sech}(c\cdots}{\sqrt{a+b\text{sech}(c+dx)}}}{a(a^2-b^2)}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1\cdots}{}}}{ab^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1\cdots}{}}}{ab^2d}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] $Aborted
```

fricas [F] time = 9.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\,\text{sech}\,(dx+c)+a}\,\tanh\,(dx+c)^2}{b^2\,\text{sech}\,(dx+c)^2+2\,ab\,\text{sech}\,(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^2}{(b\,\mathrm{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(dx+c)}{(a+b\,\mathrm{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(dx+c)^2}{(b\,\mathrm{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

3.149 $\int \frac{1}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sec}}}{\sqrt{a}}\right.\right.}{a^2d}$$

```
[Out] -2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
/(a+b)^(1/2)+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b
))^(1/2)/a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(
a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b
))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tanh(d*x+c)/a/(a^2-b^
2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2\sqrt{a+b}\,\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\operatorname{sec}}}{\sqrt{a}}\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Coth[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a +
b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tanh[
c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\text{sech}(c+dx)+\frac{1}{2}b^2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\text{sech}(}{}}{}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d}$$

Mathematica [F] time = 86.67, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3/2),x]
[Out] Integrate[(a + b*Sech[c + d*x])^(-3/2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b/cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{sech}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-3/2), x)
```

3.150 $\int \frac{\coth^2(c+dx)}{(a+b\,\mathbf{sech}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=665

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{4ab^2\tanh(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)(a+b\text{sech}(c+dx))^{3/2}}+\frac{2\sqrt{a+}}{}$$

```
[Out] -coth(d*x+c)/d/(a+b*sech(d*x+c))^(3/2)+4*a*coth(d*x+c)*EllipticE((a+b*sech(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1
/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/d-(3*a-b)*coth(d*x+c
)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-
sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2
)/d-2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(
a-b))^(1/2)/a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2
)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(
a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-b^2*tanh(d*x+c)/(a^2-b^2
)/d/(a+b*sech(d*x+c))^(3/2)-4*a*b^2*tanh(d*x+c)/(a^2-b^2)^2/d/(a+b*sech(d*x
+c))^(1/2)+2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3896, 3785, 4058, 3921, 3784, 3832, 4004, 3875, 3833, 4003, 4005}

$$\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{4ab^2\tanh(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)(a+b\text{sech}(c+dx))^{3/2}}+\frac{2\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech
[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Coth[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*
Sqrt[a + b]*d) - ((3*a - b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
```

```
b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))]])/((a - b)*(a + b)^(3/2)*d) + (
2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
+ d*x]))/(a - b))]])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))]])/(a^2*d) - Coth[c + d*x]/(d*(a + b*Sech[c + d*x])^(3/2)) - (b^2*Ta
nh[c + d*x])/((a^2 - b^2)*d*(a + b*Sech[c + d*x])^(3/2)) - (4*a*b^2*Tanh[c
+ d*x])/((a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])
/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3896

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-(m/2)), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &
& ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

```
&& NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\int \left(-\frac{1}{(a+b\text{sech}(c+dx))^{3/2}} - \frac{\text{csch}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\right) dx$$

$$= \int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx + \int \frac{\text{csch}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$= -\frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}} + \frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}} + \frac{1}{2}(3b)\int \frac{\text{sec}}{(a+b\text{se}}$$

$$= -\frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}} - \frac{b^2\tanh(c+dx)}{\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^{3/2}} + \frac{2b^2\tanh(}{a\left(a^2-b^2\right)d\sqrt{a+}}$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} -$$

$$= -\frac{2\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}\,d} +$$

$$= \frac{4a\coth(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d} -$$

Mathematica [F] time = 110.55, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
[Out] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(dx+c\right)^{2}}{\left(b\,\mathrm{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}\left(dx+c\right)}{\left(a+b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(dx+c\right)^{2}}{\left(b\,\mathrm{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(c+d\,x\right)^2}{{\left(a+\frac{b}{\cosh\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)

[Out] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}{\left(c + dx \right)}}{\left(a + b \operatorname{sech}{\left(c + dx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)

[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)

3.151 $\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx$

Optimal. Leaf size=191

$$-\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}+\frac{48\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+}}{5bc\left(e^{2c(a+bx)}+1\right)^5}$$

```
[Out] 32/3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^6-1
92/5*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^5+4
8*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4-64/3
*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3
```

Rubi [A] time = 0.28, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}+\frac{48\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+}}{5bc\left(e^{2c(a+bx)}+1\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*
x)))^6) - (192*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(5*b*c*(1 + E^(
2*c*(a + b*x)))^5) + (48*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*
(1 + E^(2*c*(a + b*x)))^4) - (64*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2
])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx = \left(\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\text{sech}^7(ac+bcx)\,dx$$

$$= \frac{\left(\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{128x^7}{\left(1+x^2\right)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(128\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{x^7}{\left(1+x^2\right)^7}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{x^3}{(1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}\right)\text{Subst}\left(\int \left(-\frac{1}{(1+x)^7}+\frac{3}{(1+x)^6}-\frac{3}{(1+x)^5}+\frac{}{(1\ldots}\right.\right.}{bc}$$

$$= \frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^6} - \frac{192\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{5bc\left(1+e^{2c(a+bx)}\right)^5} \ldots$$

Mathematica [A] time = 0.09, size = 84, normalized size = 0.44

$$-\frac{16\left(6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}+1\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{15bc\left(e^{2c(a+bx)}+1\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x))
)*Cosh[c*(a + b*x)]*Sqrt[Sech[c*(a + b*x)]^2])/(15*b*c*(1 + E^(2*c*(a + b*x
)))^6)
```

fricas [B] time = 0.77, size = 589, normalized size = 3.08

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9+6\,bc\cosh\left(bcx+ac\right)^7+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)
+ 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*
sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)^7 +
 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*
x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a*c)^2 +
 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b
*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*x + a*c)^4
 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 21*b*c*cosh(
b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*x + a*c)^5 +
 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2
 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(
b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c))
```

giac [A] time = 0.14, size = 64, normalized size = 0.34

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

maple [A] time = 0.75, size = 91, normalized size = 0.48

$$-\frac{16\left(20\,\mathrm{e}^{6c(bx+a)}+15\,\mathrm{e}^{4c(bx+a)}+6\,\mathrm{e}^{2c(bx+a)}+1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}\;\mathrm{e}^{-c(bx+a)}}{15bc\left(1+\mathrm{e}^{2c(bx+a)}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] -16/15/b/c*(20*exp(6*c*(b*x+a))+15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))+1)*(
1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^5*exp
(-c*(b*x+a))
```

maxima [B] time = 0.32, size = 386, normalized size = 2.02

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{}{bc\left(e^{(12\,bcx\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
+ 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 0.17, size = 405, normalized size = 2.12

$$\frac{24\sqrt{\frac{1}{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)^{2}}}\;\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+\mathrm{e}^{4\,a\,c+4\,b\,c\,x}+1\right)}{b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}+\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)^{4}}-\frac{32\sqrt{\frac{1}{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)^{2}}}\;\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+\mathrm{e}^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}+\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)^{3}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (24*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c +
2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c +
3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^4) - (32*(1/(exp(a*c + b*c*x)/2 + exp(
- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*x) +
 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 + 1)^3) - (96*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*e
xp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(5*b*c*(exp(a*c + b*c*x) +
 exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^5) + (16*(1/(exp(a*c + b*
c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c
 + 4*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a
*c + 2*b*c*x) + 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

3.152 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=141

$$-\frac{8\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}+\frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}-\frac{4\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}$$

```
[Out] -4*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4+32/
3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3-8*co
sh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^2
```

Rubi [A] time = 0.17, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}+\frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}-\frac{4\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c*(a + b*x)
))^4) + (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3) - (8*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{5/2}\,dx &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^5(ac+bcx)\,dx\\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{32x^5}{\left(1+x^2\right)^5}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)^5}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(16\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx,x,e^{2c(a+bx)}\right)}{bc}\\
&= \frac{\left(16\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx,\right.}{bc}\\
&= -\frac{4\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4}+\frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 0.51

$$-\frac{4\left(4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{3bc\left(e^{2c(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)]*Sqrt[
Sech[c*(a + b*x)]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^4)
```

fricas [B] time = 0.50, size = 315, normalized size = 2.23

$$-\frac{}{3\left(bc\cosh(bcx+ac)^6+6\,bc\cosh(bcx+ac)\sinh(bcx+ac)^5+bc\sinh(bcx+ac)^6+4\,bc\cosh(bcx+ac)^4+(15\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

giac [A] time = 0.13, size = 51, normalized size = 0.36

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) + 1)^4)
```

maple [A] time = 0.68, size = 80, normalized size = 0.57

$$-\frac{4\left(6\,\mathrm{e}^{4c(bx+a)}+4\,\mathrm{e}^{2c(bx+a)}+1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}\;\mathrm{e}^{-c(bx+a)}}{3bc\left(1+\mathrm{e}^{2c(bx+a)}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] -4/3/b/c*(6*exp(4*c*(b*x+a))+4*exp(2*c*(b*x+a))+1)*(1/(1+exp(2*c*(b*x+a)))^
2*exp(2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^3*exp(-c*(b*x+a))
```

maxima [A] time = 0.32, size = 209, normalized size = 1.48

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 1.43, size = 91, normalized size = 0.65

$$-\frac{2\,\mathrm{e}^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1
/2)*(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*a*
c + 2*b*c*x) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.153 $\int e^{c(a+bx)}\mathbf{sech}^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=56

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

```
[Out] 2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c
*(b*x+a)))^2
```

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 264}

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^3(ac+bcx)\,dx\\ &= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{8x^3}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}\\ &= \frac{\left(8\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{x^3}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}\\ &= \frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.79

$$\frac{e^{3c(a+bx)}\sqrt{\operatorname{sech}^2(c(a+bx))}}{bce^{2c(a+bx)}+bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
[Out] (E^(3*c*(a + b*x))*Sqrt[Sech[c*(a + b*x)]^2])/(b*c + b*c*E^(2*c*(a + b*x)))
```

fricas [B] time = 0.61, size = 120, normalized size = 2.14

$$-\frac{2\,(3\,\cosh(bcx+ac)+\sinh(bcx+ac))}{bc\cosh(bcx+ac)^3+3\,bc\cosh(bcx+ac)\sinh(bcx+ac)^2+bc\sinh(bcx+ac)^3+3\,bc\cosh(bcx+ac)+\left(3\,bc\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*c*
cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

giac [A] time = 0.13, size = 38, normalized size = 0.68

$$-\frac{2\left(2 e^{(2 bcx+2 ac)}+1\right)}{bc\left(e^{(2 bcx+2 ac)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

maple [A] time = 0.68, size = 69, normalized size = 1.23

$$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}+1\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}\;\mathrm{e}^{-c(bx+a)}}{bc\left(1+\mathrm{e}^{2c(bx+a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] -2/b/c*(2*exp(2*c*(b*x+a))+1)*(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(
1/2)/(1+exp(2*c*(b*x+a)))*exp(-c*(b*x+a))
```

maxima [A] time = 0.32, size = 84, normalized size = 1.50

$$-\frac{4 e^{(2 bcx+2 ac)}}{bc\left(e^{(4 bcx+4 ac)}+2 e^{(2 bcx+2 ac)}+1\right)}-\frac{2}{bc\left(e^{(4 bcx+4 ac)}+2 e^{(2 bcx+2 ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 0.14, size = 78, normalized size = 1.39

$$-\frac{\mathrm{e}^{-a\,c-b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}}{b\,c\,\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*(1/(exp(a*c + b*c*x)/2 +
exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.154 $\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

```
[Out] cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))*(sech(b*c*x+a*c)^2)^(1/2)/b/c
```

Rubi [A] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[a*c + b*c*x]^2])/(b
*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}(ac+bcx)\,dx\\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{2x}{1+x^2}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(2\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.95

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
[Out] (Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[c*(a + b*x)]^2])/(b
*c)
```

fricas [A] time = 0.42, size = 42, normalized size = 0.95

$$\frac{\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

giac [A] time = 0.13, size = 20, normalized size = 0.45

$$\frac{\log\left(e^{(2\,bcx)} + e^{(-2\,ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

maple [A] time = 0.72, size = 66, normalized size = 1.50

$$\frac{\ln\left(e^{2bcx} + e^{-2ac}\right)\left(1 + e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^{2}}}\; e^{-c(bx+a)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] ln(exp(2*b*c*x)+exp(-2*a*c))/b/c*(1+exp(2*c*(b*x+a)))*(1/(1+exp(2*c*(b*x+a)
))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))
```

maxima [A] time = 0.41, size = 21, normalized size = 0.48

$$\frac{\log\left(e^{(2\,bcx+2\,ac)} + 1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int e^{c\,(a+b\,x)}\,\sqrt{\frac{1}{{\cosh\left(a\,c+b\,c\,x\right)}^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int \sqrt{\operatorname{sech}^2\left(ac + bcx\right)}\, e^{bcx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(sqrt(sech(a*c + b*c*x)**2)*exp(b*c*x), x)
```

3.155 $\int \frac{e^{c(a+bx)}}{\sqrt{\mathbf{sech}^2(ac+bcx)}}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/2*x*se
ch(b*c*x+a*c)/(sech(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(4*b*c*Sqrt[Sech[a*c + b*c*x]^2]) + (
x*Sech[a*c + b*c*x])/(2*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx &= \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{4bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{x\operatorname{sech}(ac+bcx)}{2\sqrt{\operatorname{sech}^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}+2bcx\right)\operatorname{sech}(c(a+bx))}{4bc\sqrt{\operatorname{sech}^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) + 2*b*c*x)*Sech[c*(a + b*x)])/(4*b*c*Sqrt[Sech[c*(a + b
*x)]^2])
```

fricas [A] time = 0.40, size = 66, normalized size = 0.89

$$\frac{(2\,bcx+1)\cosh\left(bcx+ac\right)-(2\,bcx-1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.13, size = 33, normalized size = 0.45

$$\frac{\left(2\,bcxe^{(-ac)}+e^{(2\,bcx+ac)}\right)e^{(ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*c*x*e^(-a*c) + e^(2*b*c*x + a*c))*e^(a*c)/(b*c)
```

maple [A] time = 0.83, size = 106, normalized size = 1.43

$$\frac{x\,\mathrm{e}^{c(bx+a)}}{2\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}+\frac{\mathrm{e}^{3c(bx+a)}}{4bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/2*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2
)*exp(c*(b*x+a))+1/4/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp
(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))
```

maxima [A] time = 0.32, size = 29, normalized size = 0.39

$$\frac{1}{2}\,x+\frac{a}{2\,b}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\frac{1}{\mathrm{cosh}(a\,c+b\,c\,x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\operatorname{sech}^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sech(a*c + b*c*x)**2), x)
```

3.156 $\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}} \, dx$

Optimal. Leaf size=162

$$-\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[Out] -1/16*sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+3/16*e
xp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*c*
(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+3/8*x*sech(b*c*x+a*c
)/(sech(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -Sech[a*c + b*c*x]/(16*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) + (
3*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(16*b*c*Sqrt[Sech[a*c + b*c*x]^2]) +
 (E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2]) +
 (3*x*Sech[a*c + b*c*x])/(8*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^3(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= -\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3x}{8\sqrt{\cdots}}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.48

$$\frac{\left(-e^{-2c(a+bx)}+3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{sech}^3(c(a+bx))}{16bc\operatorname{sech}^2(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((-E^(-2*c*(a + b*x)) + 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x
)*Sech[c*(a + b*x)]^3)/(16*b*c*(Sech[c*(a + b*x)]^2)^(3/2))
```

fricas [A] time = 0.41, size = 126, normalized size = 0.78

$$-\frac{\cosh\left(bcx+ac\right)^{3}+3\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}-3\sinh\left(bcx+ac\right)^{3}-6\left(2bcx+1\right)\cosh\left(bcx+ac\right)+3}{32\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*si
nh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*cosh
(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

giac [A] time = 0.11, size = 82, normalized size = 0.51

$$\frac{\left(12\,bcxe^{(-ac)}-2\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx-3\,ac)}+\left(e^{(4\,bcx+9\,ac)}+6\,e^{(2\,bcx+7\,ac)}\right)e^{(-6\,ac)}\right)e^{(ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*e^(-a*c) - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 3*a*c
) + (e^(4*b*c*x + 9*a*c) + 6*e^(2*b*c*x + 7*a*c))*e^(-6*a*c))*e^(a*c)/(b*c)
```

maple [A] time = 0.84, size = 216, normalized size = 1.33

$$\frac{3x\,e^{c(bx+a)}}{8\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^{2}}}}+\frac{e^{5c(bx+a)}}{32bc\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^{2}}}}+\frac{3\,e^{3c(bx+a)}}{16bc\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^{2}}}}-\frac{}{16bc\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 3/8*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2
)*exp(c*(b*x+a))+1/32/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*ex
p(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+3/16/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+
exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*c*(b*x+a))-1/16/b/c/(1+ex
p(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-c*(b
*x+a))
```

maxima [A] time = 0.32, size = 74, normalized size = 0.46

$$\frac{3\,(bcx+ac)}{8\,bc}+\frac{e^{(4\,bcx+4\,ac)}}{32\,bc}+\frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc}-\frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
+ 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{{\left(\frac{1}{{\mathrm{cosh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{sech}^{2}{\left(ac+bcx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(sech(a*c + b*c*x)**2)**(3/2), x)
```

3.157 $\int \frac{e^{c(a+bx)}}{\mathbf{sech}^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{e^{-4c(a+bx)}\mathrm{sech}(ac+bcx)}{128bc\sqrt{\mathrm{sech}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\mathrm{sech}(ac+bcx)}{64bc\sqrt{\mathrm{sech}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\mathrm{sech}(ac+bcx)}{32bc\sqrt{\mathrm{sech}^2(ac+bcx)}}+\frac{5e^{4c(a+bx)}\mathrm{sech}(ac+bcx)}{128bc\sqrt{\mathrm{sech}^2(ac+bcx)}}+\frac{e^{6c}}{192}$$

```
[Out] -1/128*sech(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)-5/64*
sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+5/32*exp(2*c
*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/128*exp(4*c*(b*x+
a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a))*se
ch(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/16*x*sech(b*c*x+a*c)/(sech(b*
c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-4c(a+bx)}\mathrm{sech}(ac+bcx)}{128bc\sqrt{\mathrm{sech}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\mathrm{sech}(ac+bcx)}{64bc\sqrt{\mathrm{sech}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\mathrm{sech}(ac+bcx)}{32bc\sqrt{\mathrm{sech}^2(ac+bcx)}}+\frac{5e^{4c(a+bx)}\mathrm{sech}(ac+bcx)}{128bc\sqrt{\mathrm{sech}^2(ac+bcx)}}+\frac{e^{6c}}{192}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] -Sech[a*c + b*c*x]/(128*b*c*E^(4*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) -
(5*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2])
+ (5*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2]
) + (5*E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(128*b*c*Sqrt[Sech[a*c + b*c*x]
^2]) + (E^(6*c*(a + b*x))*Sech[a*c + b*c*x])/(192*b*c*Sqrt[Sech[a*c + b*c*x
]^2]) + (5*x*Sech[a*c + b*c*x])/(16*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^5(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{\operatorname{sech}(ac+bcx)\,\text{Subst}\left(\int \frac{(1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{\operatorname{sech}(ac+bcx)\,\text{Subst}\left(\int \frac{(1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{\operatorname{sech}(ac+bcx)\,\text{Subst}\left(\int \frac{(1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= \frac{\operatorname{sech}(ac+bcx)\,\text{Subst}\left(\int \left(10+\frac{1}{x^3}+\frac{5}{x^2}+\frac{10}{x}+5x+x^2\right)dx, x, e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$
$$= -\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} +$$

Mathematica [A] time = 0.11, size = 106, normalized size = 0.42

$$\frac{\left(-\frac{1}{2}e^{-4c(a+bx)} - 5e^{-2c(a+bx)} + 10e^{2c(a+bx)} + \frac{5}{2}e^{4c(a+bx)} + \frac{1}{3}e^{6c(a+bx)} + 20bcx\right)\operatorname{sech}^5(c(a+bx))}{64bc\operatorname{sech}^2(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((-1/2*1/E^(4*c*(a + b*x)) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) + (
5*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 + 20*b*c*x)*Sech[c*(a + b*x)]^
5)/(64*b*c*(Sech[c*(a + b*x)]^2)^(5/2))
```

fricas [A] time = 0.43, size = 218, normalized size = 0.87

$$-\frac{\cosh(bcx+ac)^5 + 5\cosh(bcx+ac)\sinh(bcx+ac)^4 - 5\sinh(bcx+ac)^5 - 5\left(10\cosh(bcx+ac)^2+9\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*s
inh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 + 1
5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*sin
h(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x + a
*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*c*cos
h(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.12, size = 110, normalized size = 0.44

$$\frac{\left(120\,bcxe^{(-ac)} - 3\left(30\,e^{(4\,bcx+4\,ac)} + 10\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-4\,bcx-5\,ac)} + \left(2\,e^{(6\,bcx+20\,ac)} + 15\,e^{(4\,bcx+18\,ac)} + 60\,e^{(2\,bcx+16\,ac)}\right.}{384\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(120*b*c*x*e^(-a*c) - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a
*c) + 1)*e^(-4*b*c*x - 5*a*c) + (2*e^(6*b*c*x + 20*a*c) + 15*e^(4*b*c*x + 1
8*a*c) + 60*e^(2*b*c*x + 16*a*c))*e^(-15*a*c))*e^(a*c)/(b*c)
```

maple [A] time = 0.75, size = 326, normalized size = 1.30

$$\frac{5x\,\mathrm{e}^{c(bx+a)}}{16\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}} + \frac{\mathrm{e}^{7c(bx+a)}}{192bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}} + \frac{5\,\mathrm{e}^{5c(bx+a)}}{128bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}} + \frac{}{32bc\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 5/16*x/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/
2)*exp(c*(b*x+a))+1/192/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*
exp(2*c*(b*x+a)))^(1/2)*exp(7*c*(b*x+a))+5/128/b/c/(1+exp(2*c*(b*x+a)))/(1/
(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(5*c*(b*x+a))+5/32/b/c/(1
+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(3*
c*(b*x+a))-5/64/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*c*(b*x+a)))^2*exp(2*c*
(b*x+a)))^(1/2)*exp(-c*(b*x+a))-1/128/b/c/(1+exp(2*c*(b*x+a)))/(1/(1+exp(2*
c*(b*x+a)))^2*exp(2*c*(b*x+a)))^(1/2)*exp(-3*c*(b*x+a))
```

maxima [A] time = 0.32, size = 112, normalized size = 0.45

$$\frac{5\,(bcx+ac)}{16\,bc} + \frac{e^{(6\,bcx+6\,ac)}}{192\,bc} + \frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc} + \frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc} - \frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc} - \frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*c
*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2*a
*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{c(a+bx)}}{\left(\frac{1}{\cosh(ac+bcx)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.158 $\int \frac{x^5}{\sqrt{\text{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=108

$$\frac{2x^2}{21c^4\sqrt{\text{sech}(2\log(cx))}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{21c^5x\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\text{sech}(2\log(cx))}}$$

```
[Out] 2/21*x^2/c^4/sech(2*ln(c*x))^(1/2)+1/7*x^6/sech(2*ln(c*x))^(1/2)+1/21*(c^2+
1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arcc
ot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^5/(c^4+1/x^4)/x/s
ech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5551, 5549, 335, 277, 325, 220}

$$\frac{2x^2}{21c^4\sqrt{\text{sech}(2\log(cx))}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{21c^5x\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\text{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (2*x^2)/(21*c^4*Sqrt[Sech[2*Log[c*x]]]) + x^6/(7*Sqrt[Sech[2*Log[c*x]]]) +
(Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*
x], 1/2])/(21*c^5*(c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^6\,dx, x, cx\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{21c^7\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{2x^2}{21c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{21c^5\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [C] time = 0.18, size = 77, normalized size = 0.71

$$\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\left(\left(c^4x^4+1\right)^{3/2} - {}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)\right)}{7c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(3/2) - H
ypergeometric2F1[-1/2, 1/4, 5/4, -(c^4*x^4)]))/(7*c^6)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^5}{\sqrt{\operatorname{sech}\left(2\log(cx)\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^5/sqrt(sech(2*log(c*x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\operatorname{sech}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

maple [C] time = 0.24, size = 130, normalized size = 1.20

$$\frac{x^2\left(3c^4x^4+2\right)\sqrt{2}}{42c^4\sqrt{\frac{c^2x^2}{c^4x^4+1}}}-\frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{2}\,x}{21c^4\sqrt{ic^2}\,\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/42*x^2*(3*c^4*x^4+2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-1/21/c^4/(I*
c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*
(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\operatorname{sech}\left(2 \log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^5/(1/cosh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(sech(2*log(c*x))), x)
```

3.159 $\int \frac{x^4}{\sqrt{\textbf{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=28

$$\frac{x^5\left(c^4+\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/6*(c^4+1/x^4)*x^5/c^4/sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5551, 5549, 264}

$$\frac{x^5\left(c^4+\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] ((c^4 + x^(-4))*x^5)/(6*c^4*Sqrt[Sech[2*Log[c*x]]])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^5\,dx, x, cx\right)}{c^6\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{\left(c^4+\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 1.57

$$\frac{\left(c^4x^4+1\right)^2\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}{6c^6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[Sech[2*Log[c*x]]],x]
[Out] ((1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(6*c^6*x)
```

fricas [A] time = 0.41, size = 48, normalized size = 1.71

$$\frac{\sqrt{2}\left(c^8x^8+2\,c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{12\,c^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(2)*(c^8*x^8 + 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/(c^6*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(sech(2*log(c*x))), x)
```

maple [A] time = 0.20, size = 39, normalized size = 1.39

$$\frac{\sqrt{2}\, x \left(c^{4} x^{4}+1\right)}{12 \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}\, c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/sech(2*ln(c*x))^(1/2),x)
[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^4*x^4+1)/c^4
```

maxima [A] time = 0.45, size = 30, normalized size = 1.07

$$\frac{\left(\sqrt{2}\, c^{4} x^{4}+\sqrt{2}\right) \sqrt{c^{4} x^{4}+1}}{12\, c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] 1/12*(sqrt(2)*c^4*x^4 + sqrt(2))*sqrt(c^4*x^4 + 1)/c^5
```

mupad [B] time = 1.47, size = 42, normalized size = 1.50

$$\frac{\left(c^{4}\, x^{4}+1\right)^{2} \sqrt{\frac{2\, c^{2}\, x^{2}}{c^{4}\, x^{4}+1}}}{12\, c^{6}\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] ((c^4*x^4 + 1)^2*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(12*c^6*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\sqrt{\operatorname{sech}\left(2 \log \left(c x\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/sech(2*ln(c*x))**(1/2),x)
[Out] Integral(x**4/sqrt(sech(2*log(c*x))), x)
```

3.160 $\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=203

$$\frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{2}{5c^4x^2\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c\right.}{5c^3x\left(c^4+\right.}$$

```
[Out] 2/5/c^4/sech(2*ln(c*x))^(1/2)-2/5/c^4/(c^2+1/x^2)/x^2/sech(2*ln(c*x))^(1/2)
+1/5*x^4/sech(2*ln(c*x))^(1/2)+2/5*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)
/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/
(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)-1/5*(c^2+1/x^2
)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*
x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)/x/sech(2
*ln(c*x))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5551, 5549, 335, 277, 325, 305, 220, 1196}

$$-\frac{2}{5c^4x^2\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)|\right.}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] 2/(5*c^4*Sqrt[Sech[2*Log[c*x]]]) - 2/(5*c^4*(c^2 + x^(-2))*x^2*Sqrt[Sech[2*
Log[c*x]]]) + x^4/(5*Sqrt[Sech[2*Log[c*x]]]) + (2*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^
(-4))*x*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c
^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^(-4))*x*Sqrt[Se
ch[2*Log[c*x]]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^4}\\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^4\,dx, x, cx\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}\\
&= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{}{5c^5}\\
&= \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2}{5c^4\left(c^2+\frac{1}{x^2}\right)x^2\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2}{}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 65, normalized size = 0.32

$$\frac{\left(\frac{c^2x^2}{c^4x^4+1}\right)^{3/2}\left(c^4x^4+1\right)^{3/2}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-c^4x^4\right)}{3\sqrt{2}\,c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-1/2
, 3/4, 7/4, -(c^4*x^4)])/(3*Sqrt[2]*c^4)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(sech(2*log(c*x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(sech(2*log(c*x))), x)
```

maple [C] time = 0.23, size = 134, normalized size = 0.66

$$\frac{x^4\sqrt{2}}{10\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{i\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ic^2},i\right)-\mathrm{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}\,x}{5\sqrt{ic^2}\,\left(c^4x^4+1\right)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/10*x^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/5*I/(I*c^2)^(1/2)*(1-I*c^2*x
^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^2*(EllipticF(x*(I*c^2)^(1/2),I)
-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(sech(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] int(x^3/(1/cosh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/sech(2*ln(c*x))**(1/2),x)
[Out] Integral(x**3/sqrt(sech(2*log(c*x))), x)
```

3.161 $\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/4*x^3/sech(2*ln(c*x))^(1/2)+1/4*arctanh((1+1/c^4/x^4)^(1/2))/c^4/x/(1+1/c
^4/x^4)^(1/2)/sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5551, 5549, 266, 47, 63, 207}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}+\frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Sech[2*Log[c*x]]]) + ArcTanh[Sqrt[1 + 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^3}$$

$$= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^3\,dx, x, cx\right)}{c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [A] time = 0.15, size = 77, normalized size = 1.15

$$\frac{x\left(\sinh^{-1}\left(c^2x^2\right)+c^2x^2\sqrt{c^4x^4+1}\right)}{4\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (x*(c^2*x^2*Sqrt[1 + c^4*x^4] + ArcSinh[c^2*x^2]))/(4*Sqrt[2]*c^2*Sqrt[(c^2
*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

fricas [A] time = 0.41, size = 90, normalized size = 1.34

$$\frac{2\sqrt{2}\left(c^5x^5+cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}+\sqrt{2}\,\log\left(-2\,c^4x^4-2\left(c^5x^5+cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) + sqrt(2)*log(-
2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) - 1))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

maple [A] time = 0.26, size = 97, normalized size = 1.45

$$\frac{x^3\sqrt{2}}{8\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\sqrt{2}\,x}{8\sqrt{c^4}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,\sqrt{c^4x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] 1/8*x^3*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)/(c^4*x^4+1
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x^2/(1/cosh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(sech(2*log(c*x))), x)
```

3.162 $\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=87

$$\frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{3cx\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/3*x^2/sech(2*ln(c*x))^(1/2)-1/3*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/
cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(
c^2+1/x^2)^2)^(1/2)/c/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5551, 5549, 335, 277, 220}

$$\frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{3cx\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^
2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(3*c*(c^4 + x^(-4))*x*Sqrt[Sech[
2*Log[c*x]]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c^2} \\
&= \frac{\operatorname{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x^2\,dx, x, cx\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}} \\
&= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{3c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 58, normalized size = 0.67

$$\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-1/2,
1/4, 5/4, -(c^4*x^4)])/c^2
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{\text{sech}\left(2\log\left(cx\right)\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(sech(2*log(c*x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\text{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(sech(2*log(c*x))), x)
```

maple [C] time = 0.21, size = 114, normalized size = 1.31

$$\frac{x^2\sqrt{2}}{6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\text{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{2}\,x}{3\sqrt{ic^2}\,\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(2*ln(c*x))^(1/2),x)
```

[Out] 1/6*x^2*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/3/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(sech(2*log(c*x))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1/cosh(2*log(c*x)))^(1/2),x)

[Out] int(x/(1/cosh(2*log(c*x)))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(2*ln(c*x))**(1/2),x)

[Out] Integral(x/sqrt(sech(2*log(c*x))), x)

3.163 $\int \frac{1}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal. Leaf size=59

$$\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\,\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/2*x/sech(2*ln(c*x))^(1/2)-1/2*arccsch(c^2*x^2)/c^2/x/(1+1/c^4/x^4)^(1/2)/
sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5545, 5543, 335, 275, 277, 215}

$$\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\,\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Sech[2*Log[c*x]]]) - ArcCsch[c^2*x^2]/(2*c^2*Sqrt[1 + 1/(c^4*x^4)
]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5543

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Sech[d*(a
 + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(2\log(x))}}\,dx, x, cx\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}\,x\,dx, x, cx\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= \frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$
$$= \frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [A] time = 0.10, size = 77, normalized size = 1.31

$$\frac{x\left(2\sqrt{c^4x^4+1}-2\tanh^{-1}\left(\sqrt{c^4x^4+1}\right)\right)}{4\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (x*(2*Sqrt[1 + c^4*x^4] - 2*ArcTanh[Sqrt[1 + c^4*x^4]]))/(4*Sqrt[2]*Sqrt[(c
^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

fricas [B] time = 0.41, size = 100, normalized size = 1.69

$$\frac{\sqrt{2}\,cx\log\left(\frac{c^5x^5+2\,cx-2\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\sqrt{2}\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{8\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x
^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(
c^2*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(2 \ln \left(c x\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(2*ln(c*x))^(1/2),x)
```

```
[Out] int(1/sech(2*ln(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(2 \, \log\left(c x\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sech(2*log(c*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(1/(1/cosh(2*log(c*x)))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sech(2*log(c*x))), x)
```

3.164 $\int \frac{\sqrt{\mathrm{sech}(2\log(cx))}}{x}\,dx$

Optimal. Leaf size=36

$$-i\sqrt{\mathrm{sech}(2\log(cx))}\,\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)$$

```
[Out] -I*((1/2*c*x+1/2/c/x)^2)^(1/2)/(1/2*c*x+1/2/c/x)*EllipticF(I*(1/2*c*x-1/2/c
/x),2^(1/2))*cosh(2*ln(c*x))^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3771, 2641}

$$-i\sqrt{\mathrm{sech}(2\log(cx))}\,\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\mathrm{sech}(2\log(cx))}}{x}\,dx &= \mathrm{Subst}\left(\int \sqrt{\mathrm{sech}(2x)}\,dx, x, \log(cx)\right)\\ &= \left(\sqrt{\cosh(2\log(cx))}\,\sqrt{\mathrm{sech}(2\log(cx))}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{\cosh(2x)}}\,dx, x, \log(cx)\right)\\ &= -i\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)\sqrt{\mathrm{sech}(2\log(cx))}\end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 1.00

$$-i\sqrt{\mathrm{sech}(2\log(cx))}\,\sqrt{\cosh(2\log(cx))}\,F(i\log(cx)|2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(2*log(c*x)))/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.49, size = 167, normalized size = 4.64

$$\frac{\sqrt{\left(2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}-1\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}\,\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}\,\sqrt{-2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}+1}\ \operatorname{EllipticF}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)}{\sqrt{2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{4}+\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}\,\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x,x)
```

```
[Out] ((2*(1/2*c*x+1/2/c/x)^2-1)*(1/2*c*x-1/2/c/x)^2)^(1/2)*(-(1/2*c*x-1/2/c/x)^2
)^(1/2)*(-2*(1/2*c*x+1/2/c/x)^2+1)^(1/2)/(2*(1/2*c*x-1/2/c/x)^4+(1/2*c*x-1/
2/c/x)^2)^(1/2)*EllipticF(1/2*c*x+1/2/c/x,2^(1/2))/(1/2*c*x-1/2/c/x)/(2*(1/
2*c*x+1/2/c/x)^2-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x, x)
```

3.165 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx$

Optimal. Leaf size=40

$$-\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\,\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -1/2*c^2*x*arccsch(c^2*x^2)*(1+1/c^4/x^4)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 275, 215}

$$-\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\,\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

```
[Out] -(c^2*Sqrt[1 + 1/(c^4*x^4)]*x*ArcCsch[c^2*x^2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
```

```
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx &= c\,\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^2}\,dx, x, cx\right)\\
&= \left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^3}\,dx, x, cx\right)\\
&= -\left(\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\frac{1}{2}\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)\\
&= -\frac{1}{2}c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 1.38

$$-\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\tanh^{-1}\left(\sqrt{c^4x^4+1}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

```
[Out] -((Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*ArcTanh[Sqrt[1 + c^4*x
^4]])/x)
```

fricas [A] time = 0.43, size = 57, normalized size = 1.42

$$\frac{1}{4}\sqrt{2}\,c\log\left(\frac{c^5x^5+2\,cx-2\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*c*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c*x^5))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\ln\left(cx\right)\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^2,x)
```

```
[Out] int(sech(2*ln(c*x))^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^2,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**2, x)
```

3.166 $\int \frac{\sqrt{\mathbf{sech}(2\log(cx))}}{x^3}\,dx$

Optimal. Leaf size=137

$$-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}-\frac{1}{2}cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}\,F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)+cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\,\left(\right.$$

```
[Out] -(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)/(c^2+1/x^2)+c*(c^2+1/x^2)*x*(cos(2*arcco
t(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2
))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)-1/2*c*(c^2+1/x^2
)*x*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(
c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5551, 5549, 335, 305, 220, 1196}

$$-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}-\frac{1}{2}cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}\,F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)+cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\,\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/(c^2 + x^(-2))) + c*Sqrt[(c^4 + x
^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x*EllipticE[2*ArcCot[c*x], 1/2]*Sqr
t[Sech[2*Log[c*x]]] - (c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2
))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = c^2\,\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^3}\,dx, x, cx\right)$$
$$= \left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^4}\,dx, x, cx\right)$$
$$= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$
$$= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right) + \left(c^3\sqrt{1+\frac{1}{c^4x^4}}\right.$$
$$= -\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}} + c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xE\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\right.$$

Mathematica [C] time = 0.12, size = 59, normalized size = 0.43

$$-\frac{c^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-c^4x^4\right)}{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((c^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c^4*x^4)])/(Sqrt[1 + c^4*x^4]*Sq
rt[(c^2*x^2)/(2 + 2*c^4*x^4)]))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(2*log(c*x)))/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.20, size = 134, normalized size = 0.98

$$-\frac{\left(c^{4} x^{4}+1\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{x^{2}}+\frac{i c^{2} \sqrt{-i c^{2} x^{2}+1}\, \sqrt{i c^{2} x^{2}+1}\, \left(\mathrm{EllipticF}\left(x \sqrt{i c^{2}}, i\right)-\mathrm{EllipticE}\left(x \sqrt{i c^{2}}, i\right)\right) \sqrt{2}\, \sqrt{\frac{c^{2} x^{2}}{c^{4} x^{4}+1}}}{\sqrt{i c^{2}}\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^3,x)
[Out] -(c^4*x^4+1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)+I*c^2/(I*c^2)^(1/2)*(1
-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*(EllipticF(x*(I*c^2)^(1/2),I)-Ellipti
cE(x*(I*c^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2 \log\left(cx\right)\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(sech(2*log(c*x)))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^3,x)
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**3, x)
```

3.167 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5551, 5549, 261}

$$-\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
```

```
[Out] -((c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = c^3\operatorname{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^4}\,dx, x, cx\right)$$
$$= \left(c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^5}\,dx, x, cx\right)$$
$$= -\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 1.43

$$-\frac{c^2}{2x\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
[Out] -1/2*c^2/(x*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])
```

fricas [A] time = 0.43, size = 37, normalized size = 1.61

$$-\frac{\sqrt{2}\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/x^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.19, size = 38, normalized size = 1.65

$$-\frac{\sqrt{2}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}\,\left(c^{4}x^{4}+1\right)}{2x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^4,x)
```

```
[Out] -1/2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x^3*(c^4*x^4+1)
```

maxima [B] time = 0.41, size = 42, normalized size = 1.83

$$-\frac{1}{2}\,c^{3}\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{c^{4}x^{4}}+1}}+\frac{\sqrt{2}}{c^{4}x^{4}\sqrt{\frac{1}{c^{4}x^{4}}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*c^3*(sqrt(2)/sqrt(1/(c^4*x^4) + 1) + sqrt(2)/(c^4*x^4*sqrt(1/(c^4*x^4)
 + 1)))
```

mupad [B] time = 1.35, size = 58, normalized size = 2.52

$$-\frac{\sqrt{\frac{2\,c^{2}\,x^{2}}{c^{4}\,x^{4}+1}}}{2\,x^{3}}-\frac{c^{4}\,x\,\sqrt{\frac{2\,c^{2}\,x^{2}}{c^{4}\,x^{4}+1}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^4,x)
```

```
[Out] - ((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)/(2*x^3) - (c^4*x*((2*c^2*x^2)/(c^4*x^4
+ 1))^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**4, x)
```

3.168 $\int \frac{\sqrt{\mathbf{sech}(2\log(cx))}}{x^5}\,dx$

Optimal. Leaf size=80

$$\frac{1}{6}c^3x\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\text{sech}(2\log(cx))}\,F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}$$

```
[Out] -1/3*(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)+1/6*c^3*(c^2+1/x^2)*x*(cos(2*arccot(
c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))
*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 321, 220}

$$\frac{1}{6}c^3x\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\text{sech}(2\log(cx))}\,F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

```
[Out] -((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/3 + (c^3*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log
[c*x]]])/6
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = c^4\operatorname{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^5}\,dx, x, cx\right)$$

$$= \left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^6}\,dx, x, cx\right)$$

$$= -\left(\left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)$$

$$= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))} + \frac{1}{3}\left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int -$$

$$= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))} + \frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xF\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\sqrt{s$$

Mathematica [C] time = 0.10, size = 65, normalized size = 0.81

$$-\frac{\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-c^4x^4\right)}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*(Sqrt[2]*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4]*Hypergeometri
c2F1[-3/4, 1/2, 1/4, -(c^4*x^4)])/x^4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\mathrm{sech}\left(2\,\log\left(cx\right)\right)}}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(2*log(c*x)))/x^5, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.20, size = 117, normalized size = 1.46

$$-\frac{\left(c^4x^4+1\right)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3x^4}-\frac{c^4\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\,\mathrm{EllipticF}\left(x\sqrt{ic^2}\,,i\right)\sqrt{2}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3\sqrt{ic^2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(1/2)/x^5,x)
```

```
[Out] -1/3*(c^4*x^4+1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)-1/3*c^4/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*EllipticF(x*(I*c^2)^(1/2),I)*2^
(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\mathrm{sech}\left(2\,\log\left(cx\right)\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(2*log(c*x)))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^5,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(sech(2*log(c*x)))/x**5, x)
```

3.169 $\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=122

$$\frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{32c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{}{12\operatorname{sech}^{\frac{3}{2}}}$$

```
[Out] 1/32*x/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/16*x^5/(c^4+1/x^4)/sech(2*ln
(c*x))^(3/2)+1/12*x^9/sech(2*ln(c*x))^(3/2)-1/32*arctanh((1+1/c^4/x^4)^(1/2
))/c^12/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5551, 5549, 266, 47, 51, 63, 207}

$$\frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{32c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{}{12\operatorname{sech}^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(16*(c^4 + x^(-4))*S
ech[2*Log[c*x]]^(3/2)) + x^9/(12*Sech[2*Log[c*x]]^(3/2)) - ArcTanh[Sqrt[1 +
 1/(c^4*x^4)]]/(32*c^12*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [**A**] time = 0.19, size = 98, normalized size = 0.80

$$\frac{c^3x^3\sqrt{c^4x^4+1}\left(8c^8x^8+14c^4x^4+3\right)-3cx\sinh^{-1}\left(c^2x^2\right)}{192\sqrt{2}\,c^9\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(3 + 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSinh[c^2
*x^2])/(192*Sqrt[2]*c^9*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

fricas [A] time = 0.44, size = 109, normalized size = 0.89

$$\frac{2\sqrt{2}\left(8\,c^{13}x^{13}+22\,c^{9}x^{9}+17\,c^{5}x^{5}+3\,cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}+3\sqrt{2}\log\left(-2\,c^{4}x^{4}+2\left(c^{5}x^{5}+cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}-1\right)}{768\,c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 + 22*c^9*x^9 + 17*c^5*x^5 + 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) + 3*sqrt(2)*log(-2*c^4*x^4 + 2*(c^5*x^5 + c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) - 1))/c^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Unable
 to cancel step at 0 of 1/2/c^6*c^4*(1/2*ln(sqrt(c^4*t_nostep^4+1)-1)-1/2*l
n(sqrt(c^4*t_nostep^4+1)+1)+sqrt(c^4*t_nostep^4+1))-1/2/c^6*c^4*(-1/2*ln(sq
rt(c^4*t_nostep^4+1)-1)+1/2*ln(sqrt(c^4*t_nostep^4+1)+1)-sqrt(c^4*t_nostep^
4+1))Warning, integration of abs or sign assumes constant sign by intervals
 (correct if the argument is real):Check [abs(t_nostep)]Unable to cancel st
ep at 0 of 1/2/c^6*c^4*(1/2*ln(sqrt(c^4*t_nostep^4+1)-1)-1/2*ln(sqrt(c^4*t_
nostep^4+1)+1)+sqrt(c^4*t_nostep^4+1))-1/2/c^6*c^4*(-1/2*ln(sqrt(c^4*t_nost
ep^4+1)-1)+1/2*ln(sqrt(c^4*t_nostep^4+1)+1)-sqrt(c^4*t_nostep^4+1))Unable t
o divide, perhaps due to rounding error%%%{1,[10,4,1,0]%%%}+%%%{1,[6,0,1,0]
%%%} / %%%{1,[0,2,0,1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.24, size = 121, normalized size = 0.99

$$\frac{x^{3}\left(8c^{8}x^{8}+14c^{4}x^{4}+3\right)\sqrt{2}}{384c^{6}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}-\frac{\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}+1}\right)\sqrt{2}\,x}{128c^{6}\sqrt{c^{4}}\,\sqrt{c^{4}x^{4}+1}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/384*x^3*(8*c^8*x^8+14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sech(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^8/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**8/sech(2*log(c*x))**(3/2), x)
```

3.170 $\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=141

$$\frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{77c^5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{}{11\mathrm{sec}}$$

```
[Out] 4/77/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+6/77*x^4/(c^4+1/x^4)/sech(2*ln(c
*x))^(3/2)+1/11*x^8/sech(2*ln(c*x))^(3/2)+2/77*(c^2+1/x^2)*(cos(2*arccot(c*
x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*(
(c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^5/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2
)
```

Rubi [A] time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5551, 5549, 335, 277, 325, 220}

$$\frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{77c^5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{}{11\mathrm{sec}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + (6*x^4)/(77*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^8/(11*Sech[2*Log[c*x]]^(3/2)) + (2*Sqrt[(c^4
 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/
(77*c^5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^8}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{10}\,dx, x, cx\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^{12}}\,dx, x, \frac{1}{cx}\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{11c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x,\right.}{77c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\,l}$$

$$= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx}$$

$$= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx}$$

Mathematica [C] time = 0.18, size = 77, normalized size = 0.55

$$\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\left(\left(c^4x^4+1\right)^{5/2} - {}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)\right)}{22c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(5/2) - H
ypergeometric2F1[-3/2, 1/4, 5/4, -(c^4*x^4)]))/(22*c^8)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\operatorname{sech}\left(2\log(cx)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/sech(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\operatorname{sech}\left(2\log(cx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sech(2*log(c*x))^(3/2), x)
```

maple [C] time = 0.20, size = 138, normalized size = 0.98

$$\frac{x^2\left(7c^8x^8+13c^4x^4+4\right)\sqrt{2}}{308c^6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}-\frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ic^2}, i\right)\sqrt{2}\,x}{77c^6\sqrt{ic^2}\,\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/308*x^2*(7*c^8*x^8+13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/77/c^6/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*
EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\operatorname{sech}\left(2\log(cx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^7/sech(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^7/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**7/sech(2*log(c*x))**(3/2), x)
```

3.171 $\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=28

$$\frac{x^7\left(c^4+\frac{1}{x^4}\right)}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/10*(c^4+1/x^4)*x^7/c^4/sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5551, 5549, 264}

$$\frac{x^7\left(c^4+\frac{1}{x^4}\right)}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((c^4 + x^(-4))*x^7)/(10*c^4*Sech[2*Log[c*x]]^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7}$$
$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$= \frac{\left(c^4+\frac{1}{x^4}\right)x^7}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 1.57

$$\frac{\left(c^4x^4+1\right)^3\sqrt{\frac{c^2x^2}{2c^4x^4+2}}}{20c^8x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sech[2*Log[c*x]]^(3/2),x]
[Out] ((1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(20*c^8*x)
```

fricas [B] time = 0.42, size = 56, normalized size = 2.00

$$\frac{\sqrt{2}\left(c^{12}x^{12}+3\,c^8x^8+3\,c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{40\,c^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
[Out] 1/40*sqrt(2)*(c^12*x^12 + 3*c^8*x^8 + 3*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1))/(c^8*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Unable
 to cancel step at 0 of 1/2/c^6*c^4*(1/2*ln(sqrt(c^4*t_nostep^4+1)-1)-1/2*l
n(sqrt(c^4*t_nostep^4+1)+1)+sqrt(c^4*t_nostep^4+1))-1/2/c^6*c^4*(-1/2*ln(sq
rt(c^4*t_nostep^4+1)-1)+1/2*ln(sqrt(c^4*t_nostep^4+1)+1)-sqrt(c^4*t_nostep^
4+1))Warning, integration of abs or sign assumes constant sign by intervals
 (correct if the argument is real):Check [abs(t_nostep)]Unable to cancel st
ep at 0 of 1/2/c^6*c^4*(1/2*ln(sqrt(c^4*t_nostep^4+1)-1)-1/2*ln(sqrt(c^4*t_
nostep^4+1)+1)+sqrt(c^4*t_nostep^4+1))-1/2/c^6*c^4*(-1/2*ln(sqrt(c^4*t_nost
ep^4+1)-1)+1/2*ln(sqrt(c^4*t_nostep^4+1)+1)-sqrt(c^4*t_nostep^4+1))Unable t
o divide, perhaps due to rounding error%%%{1,[8,4,1,0]%%%}+%%%{1,[4,0,1,0]%
%%} / %%%{1,[0,2,0,1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.20, size = 47, normalized size = 1.68

$$\frac{\sqrt{2}\,x\left(c^8x^8+2c^4x^4+1\right)}{40c^6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^8*x^8+2*c^4*x^4+1)
```

maxima [A] time = 0.45, size = 30, normalized size = 1.07

$$\frac{\left(\sqrt{2}\,c^4x^4+\sqrt{2}\right)\left(c^4x^4+1\right)^{\frac{3}{2}}}{40\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/40*(sqrt(2)*c^4*x^4 + sqrt(2))*(c^4*x^4 + 1)^(3/2)/c^7
```

mupad [B] time = 1.45, size = 42, normalized size = 1.50

$$\frac{\left(c^4\,x^4+1\right)^3\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{40\,c^8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] ((c^4*x^4 + 1)^3*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(40*c^8*x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**6/sech(2*log(c*x))**(3/2), x)
```

3.172 $\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=251

$$\frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{4}{15c^4x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-$$

```
[Out] -4/15/c^4/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+4/15/c^4/(c^4+1
/x^4)/x^2/sech(2*ln(c*x))^(3/2)+2/15*x^2/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+
1/9*x^6/sech(2*ln(c*x))^(3/2)+4/15*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)
/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/
(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)-2/15*(c^2+
1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arcc
ot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)^2/x
^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.15, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5551, 5549, 335, 277, 325, 305, 220, 1196}

$$\frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{4}{15c^4x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -4/(15*c^4*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 4/(1
5*c^4*(c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + (2*x^2)/(15*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^6/(9*Sech[2*Log[c*x]]^(3/2)) + (4*Sqrt[(c^4
+ x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(
15*c^3*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (2*Sqrt[(c^4 + x^(-4)
)/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(15*c^3*(
c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log\ldots}$$

$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(c\ldots}$$

$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(c\ldots}$$

$$= -\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \ldots$$

Mathematica [C] time = 0.12, size = 65, normalized size = 0.26

$$\frac{\left(\frac{c^2x^2}{c^4x^4+1}\right)^{3/2}\left(c^4x^4+1\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-c^4x^4\right)}{6\sqrt{2}\,c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-3/2
, 3/4, 7/4, -(c^4*x^4)])/(6*Sqrt[2]*c^6)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^5}{\mathrm{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^5/sech(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\mathrm{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sech(2*log(c*x))^(3/2), x)
```

maple [C] time = 0.21, size = 147, normalized size = 0.59

$$\frac{x^4\left(5c^4x^4+11\right)\sqrt{2}}{180c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{i\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ic^2},i\right)-\mathrm{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}\,x}{15\sqrt{ic^2}\,\left(c^4x^4+1\right)c^4\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/180*x^4*(5*c^4*x^4+11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/15*I/(I*
c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^4*(Ellipti
cF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4
*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/sech(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^5/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**5/sech(2*log(c*x))**(3/2), x)
```

3.173 $\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=92

$$\frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{16c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 3/16*x/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/8*x^5/sech(2*ln(c*x))^(3/2)+3/16
*arctanh((1+1/c^4/x^4)^(1/2))/c^8/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(
3/2)
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5551, 5549, 266, 47, 63, 207}

$$\frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{16c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (3*x)/(16*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(8*Sech[2*Log[c*x]]^
(3/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(16*c^8*(1 + 1/(c^4*x^4))^(3/2)
*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^7\,dx, x, cx\right)}{c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log \ldots}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4 \ldots}}\right.}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log \ldots}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log \ldots}$$

Mathematica [A] time = 0.17, size = 90, normalized size = 0.98

$$\frac{3cx\sinh^{-1}\left(c^2x^2\right) + c^3x^3\sqrt{c^4x^4+1}\left(2c^4x^4+5\right)}{32\sqrt{2}\,c^5\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(5 + 2*c^4*x^4) + 3*c*x*ArcSinh[c^2*x^2])/(32*Sq
rt[2]*c^5*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

fricas [A] time = 0.44, size = 101, normalized size = 1.10

$$\frac{2\sqrt{2}\left(2\,c^{9}x^{9}+7\,c^{5}x^{5}+5\,cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}+3\sqrt{2}\,\log\left(-2\,c^{4}x^{4}-2\left(c^{5}x^{5}+cx\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}-1\right)}{128\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 + 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)
) + 3*sqrt(2)*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)
) - 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Unable
 to cancel step at 0 of 1/2/c^6*c^4*(1/2*ln(sqrt(c^4*t_nostep^4+1)-1)-1/2*l
n(sqrt(c^4*t_nostep^4+1)+1)+sqrt(c^4*t_nostep^4+1))-1/2/c^6*c^4*(-1/2*ln(sq
rt(c^4*t_nostep^4+1)-1)+1/2*ln(sqrt(c^4*t_nostep^4+1)+1)-sqrt(c^4*t_nostep^
4+1))Warning, integration of abs or sign assumes constant sign by intervals
 (correct if the argument is real):Check [abs(t_nostep)]Unable to cancel st
ep at 0 of 1/2/c^6*c^4*(1/2*ln(sqrt(c^4*t_nostep^4+1)-1)-1/2*ln(sqrt(c^4*t_
nostep^4+1)+1)+sqrt(c^4*t_nostep^4+1))-1/2/c^6*c^4*(-1/2*ln(sqrt(c^4*t_nost
ep^4+1)-1)+1/2*ln(sqrt(c^4*t_nostep^4+1)+1)-sqrt(c^4*t_nostep^4+1))Unable t
o divide, perhaps due to rounding error%%%{1,[6,4,1,0]%%%}+%%%{1,[2,0,1,0]%
%%} / %%%{1,[0,2,0,1]%%%} Error: Bad Argument Value
```

maple [A] time = 0.28, size = 113, normalized size = 1.23

$$\frac{x^{3}\left(2c^{4}x^{4}+5\right)\sqrt{2}}{64c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{3\ln\left(\frac{c^{4}x^{2}}{\sqrt{c^{4}}}+\sqrt{c^{4}x^{4}+1}\right)\sqrt{2}\,x}{64\sqrt{c^{4}}\,c^{2}\sqrt{c^{4}x^{4}+1}\,\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/64*x^3*(2*c^4*x^4+5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+3/64*ln(c^4*
x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4+1)^(1
/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{sech}\left(2 \log(cx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sech(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1/cosh(2*log(c*x)))^(3/2),x)
```

```
[Out] int(x^4/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}\left(2\log(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**4/sech(2*log(c*x))**(3/2), x)
```

3.174 $\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=111

$$\frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{7cx^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 2/7/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/7*x^4/sech(2*ln(c*x))^(3/2)-2/7*(c^
2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*ar
ccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c/(c^4+1/x^4)^2/x
^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 277, 220}

$$\frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{7cx^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 2/(7*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^4/(7*Sech[2*Log[c*x]]^(3/2)
) - (2*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*Arc
Cot[c*x], 1/2])/(7*c*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{\left(1+x^4\right)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\operatorname{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(c$$

$$= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(\right.}{7c\left(c^4+\frac{1}{x^4}\right)^2 x^3\operatorname{sech}^{\frac{3}{2}}(2\log(c}$$

Mathematica [C] time = 0.11, size = 61, normalized size = 0.55

$$\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-c^4x^4\right)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-3/2,
1/4, 5/4, -(c^4*x^4)])/(2*c^4)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\text{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sech(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\text{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sech(2*log(c*x))^(3/2), x)
```

maple [C] time = 0.22, size = 129, normalized size = 1.16

$$\frac{x^2\left(c^4x^4+3\right)\sqrt{2}}{28c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ic^2}, i\right)\sqrt{2}\,x}{7\sqrt{ic^2}\,\left(c^4x^4+1\right)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/28*x^2*(c^4*x^4+3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/7/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)
^(1/2),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\text{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sech(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^3/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**3/sech(2*log(c*x))**(3/2), x)
```

3.175 $\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=88

$$\frac{1}{2x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/2/(c^4+1/x^4)/x/sech(2*ln(c*x))^(3/2)+1/6*x^3/sech(2*ln(c*x))^(3/2)-1/2*a
rccsch(c^2*x^2)/c^6/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5551, 5549, 335, 275, 277, 215}

$$\frac{1}{2x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 1/(2*(c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2)) + x^3/(6*Sech[2*Log[c*x]]^(3/
2)) - ArcCsch[c^2*x^2]/(2*c^6*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^
(3/2))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{1}{2\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(c$$

$$= \frac{1}{2\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{csch}^{-1}\left(c^2x^2\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(c$$

Mathematica [A] time = 0.17, size = 88, normalized size = 1.00

$$\frac{x\left(\sqrt{c^4x^4+1}\left(c^4x^4+4\right)-3\tanh^{-1}\left(\sqrt{c^4x^4+1}\right)\right)}{12\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

[Out] (x*(Sqrt[1 + c^4*x^4]*(4 + c^4*x^4) - 3*ArcTanh[Sqrt[1 + c^4*x^4]]))/(12*Sq
rt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])

fricas [A] time = 0.42, size = 109, normalized size = 1.24

$$\frac{3\sqrt{2}\,cx\log\left(\frac{c^5x^5+2\,cx-2\,\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\sqrt{2}\left(c^8x^8+5\,c^4x^4+4\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{48\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="fricas")

[Out] 1/48*(3*sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^
4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^8*x^8 + 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c
^4*x^4 + 1)))/(c^4*x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
by intervals (correct if the argument is real):Check [abs(t_nostep)]Unable
to cancel step at 0 of 1/2/c^6*c^4*(1/2*ln(sqrt(c^4*t_nostep^4+1)-1)-1/2*l
n(sqrt(c^4*t_nostep^4+1)+1)+sqrt(c^4*t_nostep^4+1))-1/2/c^6*c^4*(-1/2*ln(sq
rt(c^4*t_nostep^4+1)-1)+1/2*ln(sqrt(c^4*t_nostep^4+1)+1)-sqrt(c^4*t_nostep^
4+1))Unable to divide, perhaps due to rounding error%%%{1,[4,4,1,0]%%%}+%%%
{1,[0,0,1,0]%%%} / %%%{1,[0,2,0,1]%%%} Error: Bad Argument Value

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{sech}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/sech(2*ln(c*x))^(3/2),x)

[Out] int(x^2/sech(2*ln(c*x))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/sech(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^2/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**2/sech(2*log(c*x))**(3/2), x)
```

3.176 $\int \frac{x}{\mathbf{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=214

$$\frac{6}{5x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{12}{5x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\right.}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -12/5/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+6/5/(c^4+1/x^4)/x^2
/sech(2*ln(c*x))^(3/2)+1/5*x^2/sech(2*ln(c*x))^(3/2)+12/5*c*(c^2+1/x^2)*(co
s(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1
/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/(c^4+1/x^4)^2/x^3/sech(2*ln(c
*x))^(3/2)-6/5*c*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x)
)*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/
2)/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5551, 5549, 335, 277, 305, 220, 1196}

$$\frac{6}{5x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{12}{5x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)F\left(2\cot^{-1}(cx)\right.}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -12/(5*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 6/(5*(c^
4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + x^2/(5*Sech[2*Log[c*x]]^(3/2)) +
(12*c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcC
ot[c*x], 1/2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (6*c*Sqrt[
(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/
2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(1+x^4\right)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\,\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log\right.}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log\right.}$$

$$= -\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{}{5\operatorname{sech}}$$

Mathematica [C] time = 0.11, size = 65, normalized size = 0.30

$$-\frac{{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-c^4x^4\right)}{2\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,\sqrt{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -1/2*Hypergeometric2F1[-3/2, -1/4, 3/4, -(c^4*x^4)]/(Sqrt[2]*c^2*Sqrt[(c^2*
x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x}{\mathrm{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x/sech(2*log(c*x))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/sech(2*log(c*x))^(3/2), x)
```

maple [C] time = 0.21, size = 159, normalized size = 0.74

$$\frac{\left(c^{8}x^{8}-4c^{4}x^{4}-5\right)\sqrt{2}}{20\left(c^{4}x^{4}+1\right)c^{2}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}+\frac{3i\sqrt{-ic^{2}x^{2}+1}\,\sqrt{ic^{2}x^{2}+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ic^{2}},i\right)-\mathrm{EllipticE}\left(x\sqrt{ic^{2}},i\right)\right)\sqrt{2}\,x}{5\sqrt{ic^{2}}\,\left(c^{4}x^{4}+1\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/20*(c^8*x^8-4*c^4*x^4-5)/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1
/2)+3/5*I/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)
*(EllipticF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2
*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x/sech(2*log(c*x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x/sech(2*log(c*x))**(3/2), x)
```

3.177 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal. Leaf size=92

$$-\frac{3}{4x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -3/4/(c^4+1/x^4)/x^3/sech(2*ln(c*x))^(3/2)+1/4*x/sech(2*ln(c*x))^(3/2)+3/4*
arctanh((1+1/c^4/x^4)^(1/2))/c^4/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3
/2)
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5545, 5543, 266, 47, 50, 63, 207}

$$-\frac{3}{4x^3\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\tanh^{-1}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(-3/2),x]
```

```
[Out] -3/(4*(c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2)) + x/(4*Sech[2*Log[c*x]]^(3
/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(4*c^4*(1 + 1/(c^4*x^4))^(3/2)*x^
3*Sech[2*Log[c*x]]^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5543

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[(Sech[d*(a
 + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c}$$

$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x\cdots}\right.}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log\cdots}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\cdots}\right.}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log\cdots}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log\cdots}$$

<u>Mathematica</u> [C] time = 0.09, size = 64, normalized size = 0.70

$$-\frac{\sqrt{c^4x^4+1}\sqrt{\frac{c^2x^2}{2c^4x^4+2}}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-c^4x^4\right)}{4c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(-3/2),x]
```

```
[Out] -1/4*(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-
3/2, -1/2, 1/2, -(c^4*x^4)])/(c^4*x^3)
```

fricas [A] time = 0.43, size = 106, normalized size = 1.15

$$\frac{3\sqrt{2}\,c^3x^3\log\left(-2\,c^4x^4-2\left(c^5x^5+cx\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)+2\sqrt{2}\left(c^8x^8-c^4x^4-2\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{32\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^
4*x^4 + 1)) - 1) + 2*sqrt(2)*(c^8*x^8 - c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(2*log(c*x))^(-3/2), x)
```

maple [A] time = 0.24, size = 131, normalized size = 1.42

$$\frac{\left(c^8x^8-c^4x^4-2\right)\sqrt{2}}{16x\left(c^4x^4+1\right)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{3c^2\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\sqrt{2}\,x}{16\sqrt{c^4}\,\sqrt{c^4x^4+1}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sech(2*ln(c*x))^(3/2),x)
```

```
[Out] 1/16*(c^8*x^8-c^4*x^4-2)/x/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1
/2)+3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(sech(2*log(c*x))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(1/(1/cosh(2*log(c*x)))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(sech(2*log(c*x))**(-3/2), x)
```

3.178 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal. Leaf size=56

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))} + i\sqrt{\operatorname{sech}(2\log(cx))}\,\sqrt{\cosh(2\log(cx))}\,E(i\log(cx)|2)$$

```
[Out] sinh(2*ln(c*x))*sech(2*ln(c*x))^(1/2)+I*((1/2*c*x+1/2/c/x)^2)^(1/2)/(1/2*c*
x+1/2/c/x)*EllipticE(I*(1/2*c*x-1/2/c/x),2^(1/2))*cosh(2*ln(c*x))^(1/2)*sec
h(2*ln(c*x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3768, 3771, 2639}

$$\sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))} + i\sqrt{\operatorname{sech}(2\log(cx))}\,\sqrt{\cosh(2\log(cx))}\,E(i\log(cx)|2)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] I*Sqrt[Cosh[2*Log[c*x]]]*EllipticE[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]] +
Sqrt[Sech[2*Log[c*x]]]*Sinh[2*Log[c*x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \operatorname{Subst}\left(\int \operatorname{sech}^{\frac{3}{2}}(2x)\,dx, x, \log(cx)\right)$$
$$= \sqrt{\operatorname{sech}(2\log(cx))}\,\sinh(2\log(cx)) - \operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(2x)}}\,dx, x, \log(cx)\right)$$
$$= \sqrt{\operatorname{sech}(2\log(cx))}\,\sinh(2\log(cx)) - \left(\sqrt{\cosh(2\log(cx))}\sqrt{\operatorname{sech}(2\log(cx))}\right)\operatorname{Subst}\left(\int \sqrt{\right.$$
$$= i\sqrt{\cosh(2\log(cx))}\,E(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))} + \sqrt{\operatorname{sech}(2\log(cx))}\,\sinh(2\log(cx))$$

Mathematica [A] time = 0.11, size = 45, normalized size = 0.80

$$\frac{\tanh(2\log(cx)) + \frac{iE(i\log(cx)|2)}{\sqrt{\cosh(2\log(cx))}}}{\sqrt{\operatorname{sech}(2\log(cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x,x]
[Out] ((I*EllipticE[I*Log[c*x], 2])/Sqrt[Cosh[2*Log[c*x]]] + Tanh[2*Log[c*x]])/Sq
rt[Sech[2*Log[c*x]]]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
[Out] integral(sech(2*log(c*x))^(3/2)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.62, size = 127, normalized size = 2.27

$$\frac{\sqrt{-2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}-1}\,\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}\,\mathrm{EllipticE}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)+2\left(\frac{cx}{2}+\frac{1}{2cx}\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}{\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x,x)
```

```
[Out] ((-2*(1/2*c*x-1/2/c/x)^2-1)^(1/2)*(-(1/2*c*x-1/2/c/x)^2)^(1/2)*EllipticE(1/
2*c*x+1/2/c/x,2^(1/2))+2*(1/2*c*x+1/2/c/x)*(1/2*c*x-1/2/c/x)^2)/(1/2*c*x-1/
2/c/x)/(2*(1/2*c*x+1/2/c/x)^2-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(2\log\left(cx\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x, x)
```

3.179 $\int \frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}x^3\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] 1/2*(c^4+1/x^4)*x^3*sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5551, 5549, 261}

$$\frac{1}{2}x^3\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] ((c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = c\,\text{Subst}\left(\int \frac{\text{sech}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx, x, cx\right)$$
$$= \left(c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^5}\,dx, x, cx\right)$$
$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^3\text{sech}^{\frac{3}{2}}(2\log(cx))$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.28

$$\sqrt{2}\,c^2x\sqrt{\frac{c^2x^2}{c^4x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]
```

fricas [A] time = 0.41, size = 28, normalized size = 1.12

$$\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2*x
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(2 \ln \left(c x\right)\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^2,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^2,x)
```

maxima [A] time = 0.40, size = 39, normalized size = 1.56

$$c\left(\frac{\sqrt{2}}{\left(\frac{1}{c^{4}x^{4}}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}}{c^{4}x^{4}\left(\frac{1}{c^{4}x^{4}}+1\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] c*(sqrt(2)/(1/(c^4*x^4) + 1)^(3/2) + sqrt(2)/(c^4*x^4*(1/(c^4*x^4) + 1)^(3/
2)))
```

mupad [B] time = 1.33, size = 28, normalized size = 1.12

$$c^{2}\,x\,\sqrt{\frac{2\,c^{2}\,x^{2}}{c^{4}\,x^{4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^2,x)
```

```
[Out] c^2*x*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}{\left(2 \log{\left(c x \right)} \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**2,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**2, x)
```

3.180 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{x^3\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{4c}$$

```
[Out] 1/2*(c^4+1/x^4)*x^2*sech(2*ln(c*x))^(3/2)-1/4*(c^4+1/x^4)*(c^2+1/x^2)*x^3*(
cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x))
,1/2*2^(1/2))*sech(2*ln(c*x))^(3/2)*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5551, 5549, 335, 288, 220}

$$\frac{1}{2}x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{x^3\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] ((c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2))/2 - ((c^4 + x^(-4))*Sqrt[(c^4 +
 x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x^3*EllipticF[2*ArcCot[c*x], 1/2]
*Sech[2*Log[c*x]]^(3/2))/(4*c)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = c^2\,\text{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx, x, cx\right)$$

$$= \left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^6}\,dx, x, cx\right)$$

$$= -\left(\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^4}{\left(1+x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3F\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\text{se}}{4c}$$

Mathematica [C] time = 0.11, size = 65, normalized size = 0.71

$$\sqrt{2}\,c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\left(\sqrt{c^4x^4+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-c^4x^4\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*(1 + Sqrt[1 + c^4*x^4]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -(c^4*x^4)])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sech(2*log(c*x))^(3/2)/x^3, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(2*ln(c*x))^(3/2)/x^3,x)
```

```
[Out] int(sech(2*ln(c*x))^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(2 \log\left(cx\right)\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^3,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**3,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**3, x)
```

3.181 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] 1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(3/2)-1/2*c^6*(1+1/c^4/x^4)^(3/2)*x^3*arc
csch(c^2*x^2)*sech(2*ln(c*x))^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5551, 5549, 335, 275, 288, 215}

$$\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))-\frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] ((c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2))/2 - (c^6*(1 + 1/(c^4*x^4))^(3/2)*
x^3*ArcCsch[c^2*x^2]*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5551

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = c^3\,\text{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right)$$

$$= \left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^7}\,dx, x, cx\right)$$

$$= -\left(\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^5}{\left(1+x^4\right)^{3/2}}\,dx, x, \frac{1}{cx}\right)\right)$$

$$= -\left(\frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

Mathematica [C] time = 0.11, size = 51, normalized size = 0.77

$$\frac{\sqrt{2}c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};c^4x^4+1\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^4,x]

[Out] (Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/2, 1 + c^4*x^4])/x

fricas [A] time = 0.44, size = 93, normalized size = 1.41

$$\frac{\sqrt{2}c^3x\log\left(\frac{c^5x^5+2\,cx-2\left(c^4x^4+1\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,c^2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")

[Out] 1/2*(sqrt(2)*c^3*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2)/x

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(2*ln(c*x))^(3/2)/x^4,x)

[Out] int(sech(2*ln(c*x))^(3/2)/x^4,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(2 \log\left(cx\right)\right)^{\frac{3}{2}}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sech(2*log(c*x))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^4,x)
```

```
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2 \log\left(cx\right)\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**4,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x**4, x)
```

3.182 $\int \text{sech}\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=63

$$\frac{2e^a x\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{bn+1}$$

```
[Out] 2*exp(a)*x*(c*x^n)^b*hypergeom([1, 1/2*(b+1/n)/b],[3/2+1/2/b/n],-exp(2*a)*(
c*x^n)^(2*b))/(b*n+1)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5545, 5547, 263, 364}

$$\frac{2e^a x\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/2
, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1+e^{-2a}x^{-2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(2e^{-a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{e^{-2a}+x^{2b}}\,dx,x,cx^n\right)}{n}$$
$$= \frac{2e^a x\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{1+bn}$$

Mathematica [A] time = 0.15, size = 64, normalized size = 1.02

$$\frac{2e^a x\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{bn+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, -
(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{sech}\left(b\log\left(cx^n\right)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

maple **[F]** time = 0.16, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n)),x)
[Out] int(sech(a+b*ln(c*x^n)),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cosh\left(a+b\ \ln\left(c\,x^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n)),x)
[Out] int(1/cosh(a + b*log(c*x^n)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n)),x)
[Out] Integral(sech(a + b*log(c*x**n)), x)
```

3.183 $\int \text{sech}^2\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=69

$$\frac{4e^{2a}x\left(cx^n\right)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{2bn+1}$$

```
[Out] 4*exp(2*a)*x*(c*x^n)^(2*b)*hypergeom([2, 1+1/2/b/n],[2+1/2/b/n],-exp(2*a)*(
c*x^n)^(2*b))/(2*b*n+1)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5545, 5547, 263, 364}

$$\frac{4e^{2a}x\left(cx^n\right)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{2bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2\left(a+b\log\left(cx^n\right)\right)dx &= \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^2(a+b\log(x))\,dx,x,cx^n\right)}{n}\\ &= \frac{\left(4e^{-2a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{\left(1+e^{-2a}x^{-2b}\right)^2}\,dx,x,cx^n\right)}{n}\\ &= \frac{\left(4e^{-2a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{\left(e^{-2a}+x^{2b}\right)^2}\,dx,x,cx^n\right)}{n}\\ &= \frac{4e^{2a}x\left(cx^n\right)^{2b}\,{}_2F_1\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right);\frac{1}{2}\left(4+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{1+2bn}\end{aligned}$$

Mathematica [A] time = 5.55, size = 126, normalized size = 1.83

$$\frac{x\left(-\frac{e^{2a}\left(cx^n\right)^{2b}\,{}_2F_1\left(1,1+\frac{1}{2bn};2+\frac{1}{2bn};-e^{2a}\left(cx^n\right)^{2b}\right)}{2bn+1}+{}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};-e^{2a}\left(cx^n\right)^{2b}\right)+\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-((E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n), 2 + 1/(2*b
*n), -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)) + Hypergeometric2F1[1, 1/(2*b*
n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Tanh[a + b*Log[c*x^n]]))/(b*
n)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^2, x)
```

maple [F] time = 1.69, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(sech(a+b*ln(c*x^n))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,x}{bc^{2\,b}ne^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+bn}+4\int\frac{1}{2\left(bc^{2\,b}ne^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+bn\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 4*integrate(1/2/(b*c^(2*b
)*n*e^(2*b*log(x^n) + 2*a) + b*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n))^2,x)
```

```
[Out] int(1/cosh(a + b*log(c*x^n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^2\left(a + b\log\left(cx^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**2, x)
```

3.184 $\int \operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=70

$$\frac{8e^{3a}x\left(cx^n\right)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

```
[Out] 8*exp(3*a)*x*(c*x^n)^(3*b)*hypergeom([3, 1/2*(3*b+1/n)/b],[5/2+1/2/b/n],-ex
p(2*a)*(c*x^n)^(2*b))/(3*b*n+1)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5545, 5547, 263, 364}

$$\frac{8e^{3a}x\left(cx^n\right)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 3*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)dx=\frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^3(a+b\log(x))\,dx,x,cx^n\right)}{n}$$

$$=\frac{\left(8e^{-3a}x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{\left(1+e^{-2a}x^{-2b}\right)^3}dx,x,cx^n\right)}{n}$$

$$=\frac{\left(8e^{-3a}x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{\left(e^{-2a}+x^{2b}\right)^3}dx,x,cx^n\right)}{n}$$

$$=\frac{8e^{3a}x\left(cx^n\right)^{3b}\,{}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{1+3bn}$$

Mathematica [A] time = 0.90, size = 101, normalized size = 1.44

$$\frac{x\left(2e^a(bn-1)\left(cx^n\right)^b\,{}_2F_1\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right);\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)+\left(bn\tanh\left(a+b\log\left(cx^n\right)\right)+1\right)\operatorname{sech}\left(a+b\log\right.\right.}{2b^2n^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(2*E^a*(-1 + b*n)*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh
[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^3, x)
```

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^n\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sech(a+b*ln(c*x^n))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$8\left(b^2c^bn^2-c^b\right)\int \frac{e^{\left(b\log\left(x^n\right)+a\right)}}{8\left(b^2c^{2\,b}n^2e^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+b^2n^2\right)}\,dx+\frac{\left(bc^{3\,b}n+c^{3\,b}\right)xe^{\left(3\,b\log\left(x^n\right)+3\,a\right)}-\left(bc^bn-c^b\right)xe^{\left(b\log\left(x^n\right)+a\right)}}{b^2c^{4\,b}n^2e^{\left(4\,b\log\left(x^n\right)+4\,a\right)}+2\,b^2c^{2\,b}n^2e^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+b^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 8*(b^2*c^b*n^2 - c^b)*integrate(1/8*e^(b*log(x^n) + a)/(b^2*c^(2*b)*n^2*e^(
2*b*log(x^n) + 2*a) + b^2*n^2), x) + ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(
x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b
*log(x^n) + 4*a) + 2*b^2*c^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n))^3,x)
```

```
[Out] int(1/cosh(a + b*log(c*x^n))^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**3, x)
```

3.185 $\int \operatorname{sech}^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=69

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

```
[Out] 16*exp(4*a)*x*(c*x^n)^(4*b)*hypergeom([4, 2+1/2/b/n],[3+1/2/b/n],-exp(2*a)*
(c*x^n)^(2*b))/(4*b*n+1)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5545, 5547, 263, 364}

$$\frac{16e^{4a}x(cx^n)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 4*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4\left(a+b\log\left(cx^n\right)\right)dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^4(a+b\log(x))\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(16e^{-4a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-4b+\frac{1}{n}}}{\left(1+e^{-2a}x^{-2b}\right)^4}\,dx,x,cx^n\right)}{n}$$
$$= \frac{\left(16e^{-4a}x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+4b+\frac{1}{n}}}{\left(e^{-2a}+x^{2b}\right)^4}\,dx,x,cx^n\right)}{n}$$
$$= \frac{16e^{4a}x\left(cx^n\right)^{4b}\,{}_2F_1\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right);\frac{1}{2}\left(6+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{1+4bn}$$

Mathematica [B] time = 13.70, size = 192, normalized size = 2.78

$$\frac{x\left(\left(8b^2n^2-2\right){}_2F_1\left(1,\frac{1}{2bn};1+\frac{1}{2bn};-e^{2a}\left(cx^n\right)^{2b}\right)+\operatorname{sech}^2\left(a+b\log\left(cx^n\right)\right)\left(\tanh\left(a+b\log\left(cx^n\right)\right)\left(\left(4b^2n^2-1\right)\mathrm{c}\right.\right.\right.}{12b^3n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(-2*E^(2*a)*(-1 + 2*b*n)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n
), 2 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + (-2 + 8*b^2*n^2)*Hypergeometr
ic2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*L
og[c*x^n]]^2*(2*b*n + (-1 + 8*b^2*n^2 + (-1 + 4*b^2*n^2)*Cosh[2*(a + b*Log[
c*x^n])])*Tanh[a + b*Log[c*x^n]])))/(12*b^3*n^3)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="giac")
[Out] integrate(sech(b*log(c*x^n) + a)^4, x)
```

maple [F] time = 1.83, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^4,x)
[Out] int(sech(a+b*ln(c*x^n))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\left(4\,b^{2}n^{2}-1\right)\int \frac{1}{48\left(b^{3}c^{2\,b}n^{3}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+b^{3}n^{3}\right)}\,dx+\frac{\left(2\,bc^{4\,b}n+c^{4\,b}\right)xe^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}-2\left(6\,b^{2}c^{2\,b}n^{2}-bc^{2\,b}n-\right.}{3\left(b^{3}c^{6\,b}n^{3}e^{\left(6\,b\log\left(x^{n}\right)+6\,a\right)}+3\,b^{3}c^{4\,b}n^{3}e^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="maxima")
[Out] 16*(4*b^2*n^2 - 1)*integrate(1/48/(b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) +
 b^3*n^3), x) + 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) - 2
*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*
b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) + 3*b^3*c^(4*b)*n^3
*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) + b^3*n^
3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*log(c*x^n))^4,x)
[Out] int(1/cosh(a + b*log(c*x^n))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^4\left(a + b\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**4,x)
[Out] Integral(sech(a + b*log(c*x**n))**4, x)
```

3.186 $\int \left(\left(1-b^2n^2\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\operatorname{sech}^3\left(a+\right.\right.$

Optimal. Leaf size=40

$$x\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)+bnx\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] x*sech(a+b*ln(c*x^n))+b*n*x*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))
```

Rubi [C] time = 0.14, antiderivative size = 139, normalized size of antiderivative = 3.48, number of steps used = 9, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5545, 5547, 263, 364}

$$\frac{16e^{3a}b^2n^2x\left(cx^n\right)^{3b}\ {}_2F_1\left(3,\frac{3b+\frac{1}{n}}{2b};\frac{1}{2}\left(5+\frac{1}{bn}\right);-e^{2a}\left(cx^n\right)^{2b}\right)}{3bn+1}+2e^ax(1-bn)\left(cx^n\right)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\left(3+\frac{1}{bn}\right);-e^{2a}\left(cx^{n}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c*x^n]]
^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*
Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, -(E^(2*a)*(c*x^
n)^(2*b))])/(1 + 3*b*n)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)+2b^2n^2\operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)\right)dx=\left(2b^2n^2\right)\int \operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)d$$

$$=\left(2b^2nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{se}\right.$$

$$=\left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{}{(1}\right.$$

$$=\left(16b^2e^{-3a}nx\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x}{(e^{-}}\right.$$

$$=2e^a(1-bn)x\left(cx^n\right)^b\ {}_2F_1\left(1,\frac{b+\frac{1}{n}}{2b};\frac{1}{2}\right.$$

Mathematica [A] time = 0.32, size = 29, normalized size = 0.72

$$x\left(bn\tanh\left(a+b\log\left(cx^n\right)\right)+1\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c
*x^n]]^3,x]
```

```
[Out] x*Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh[a + b*Log[c*x^n]])
```

fricas [B] time = 0.43, size = 189, normalized size = 4.72

$$\frac{2\left((bn+1)x\cosh\left(bn\log(x)+b\log(c)+a\right)^2+2\left(bn+1\right)x\cosh\left(bn\log(x)+\right.\right.}{\cosh\left(bn\log(x)+b\log(c)+a\right)^3+3\ \cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^2+\sinh\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="fricas")
```

```
[Out] 2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*l
og(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b*
n*log(x) + b*log(c) + a)^2 - (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)^
3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
inh(b*n*log(x) + b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a) + 3*cosh(b*n*log(x) + b*log(c) + a))
```

giac [B] time = 1.22, size = 215, normalized size = 5.38

$$\frac{2\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,bc^{b}nxx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{\cdots}{c^{4\,b}x^{4\,bn}e^{(4\,a)}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="giac")
```

```
[Out] 2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^
(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a) +
 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*
x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)*e^a/
(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

maple [C] time = 1.06, size = 509, normalized size = 12.72

$$\frac{2c^{b}\left(x^{n}\right)^{b}x\left(nb\left(x^{n}\right)^{2b}c^{2b}\mathrm{e}^{3a}\mathrm{e}^{-\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)^{3}\pi}{2}}\mathrm{e}^{\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)^{2}\mathrm{csgn}\left(ic\right)\pi}{2}}\mathrm{e}^{\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)^{2}\mathrm{csgn}\left(ix^{n}\right)\pi}{2}}\mathrm{e}^{-\frac{3ib\,\mathrm{csgn}\left(icx^{n}\right)\mathrm{csgn}\left(ic\right)\mathrm{csgn}\left(ix^{n}\right)\pi}{2}}-\mathrm{e}^{a}\mathrm{e}^{-\frac{ib\,\mathrm{csgn}\cdots}{\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^2*n^2+1)*sech(a+b*ln(c*x^n))+2*b^2*n^2*sech(a+b*ln(c*x^n))^3,x)
```

```
[Out] 2*c^b*(x^n)^b*x/(((x^n)^b)^2*(c^b)^2*exp(2*a)*exp(-I*b*csgn(I*c*x^n)^3*Pi)*
exp(I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*P
i)*exp(-I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+1)^2*(n*b*((x^n)^b)^2*(
c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^
2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3/2*I*b*cs
gn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)-exp(a)*exp(-1/2*I*b*csgn(I*c*x^n)^3*P
i)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*cs
gn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)*b*n+((x^
n)^b)^2*(c^b)^2*exp(3*a)*exp(-3/2*I*b*csgn(I*c*x^n)^3*Pi)*exp(3/2*I*b*csgn(
```

```
I*c*x^n)^2*csgn(I*c)*Pi)*exp(3/2*I*b*csgn(I*c*x^n)^2*csgn(I*x^n)*Pi)*exp(-3
/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)+exp(a)*exp(-1/2*I*b*csgn(I*c
*x^n)^3*Pi)*exp(1/2*I*b*csgn(I*c*x^n)^2*csgn(I*c)*Pi)*exp(1/2*I*b*csgn(I*c*
x^n)^2*csgn(I*x^n)*Pi)*exp(-1/2*I*b*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)*Pi)
)
```

maxima [B] time = 0.61, size = 96, normalized size = 2.40

$$\frac{2\left(\left(bc^{3b}n+c^{3b}\right)xe^{\left(3b\log(x^n)+3a\right)}-\left(bc^{b}n-c^{b}\right)xe^{\left(b\log(x^n)+a\right)}\right)}{c^{4b}e^{\left(4b\log(x^n)+4a\right)}+2c^{2b}e^{\left(2b\log(x^n)+2a\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="maxima")
```

```
[Out] 2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(
b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) + 2*c^(2*b)*e^(2*b*log(x^n
) + 2*a) + 1)
```

mupad [B] time = 1.40, size = 66, normalized size = 1.65

$$\frac{2\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^b\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-b\,n+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}{{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^2*n^2)/cosh(a + b*log(c*x^n))^3 - (b^2*n^2 - 1)/cosh(a + b*log(c*x
^n)),x)
```

```
[Out] (2*x*exp(a)*(c*x^n)^b*(exp(2*a)*(c*x^n)^(2*b) - b*n + b*n*exp(2*a)*(c*x^n)^
(2*b) + 1))/(exp(2*a)*(c*x^n)^(2*b) + 1)^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2b^{2}n^{2}\operatorname{sech}^{2}\left(a+b\log\left(cx^{n}\right)\right)-b^{2}n^{2}+1\right)\operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**2*n**2+1)*sech(a+b*ln(c*x**n))+2*b**2*n**2*sech(a+b*ln(c*x**
n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*sech(a + b*log(c*x**n))**2 - b**2*n**2 + 1)*sech(a +
b*log(c*x**n)), x)
```

3.187 $\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=25

$$\frac{2e^{-a}c^6}{\left(\frac{e^{-2a}}{x^2}+c^4\right)^2}$$

[Out] 2*c^6/exp(a)/(c^4+1/exp(2*a)/x^2)^2

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5545, 5547, 261}

$$\frac{2e^{-a}c^6}{\left(\frac{e^{-2a}}{x^2}+c^4\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]

[Out] (2*c^6)/(E^a*(c^4 + 1/(E^(2*a)*x^2))^2)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx = \frac{2\,\text{Subst}\left(\int x\operatorname{sech}^3(a+2\log(x))\,dx, x, c\sqrt{x}\right)}{c^2}$$
$$= \frac{\left(16e^{-3a}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3 x^5}\,dx, x, c\sqrt{x}\right)}{c^2}$$
$$= \frac{2c^6e^{-a}}{\left(c^4+\frac{e^{-2a}}{x^2}\right)^2}$$

Mathematica [B] time = 0.13, size = 62, normalized size = 2.48

$$-\frac{2(\cosh(a)-\sinh(a))\left(\sinh^2(a)+\cosh^2(a)-2\sinh(a)\cosh(a)+2c^4x^2\right)}{c^2\left(\sinh(a)\left(c^4x^2-1\right)+\cosh(a)\left(c^4x^2+1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (-2*(Cosh[a] - Sinh[a])*(2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((1 + c^4*x^2)*Cosh[a] + (-1 + c^4*x^2)*Sinh[a])^2)
```

fricas [B] time = 0.41, size = 48, normalized size = 1.92

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)}{c^{10}x^4e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}+c^2e^a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)/(c^10*x^4*e^(5*a) + 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

giac [A] time = 0.13, size = 38, normalized size = 1.52

$$-\frac{2\left(2\,c^4x^2e^{(2\,a)}+1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)}+1\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)*e^(-a)/((c^4*x^2*e^(2*a) + 1)^2*c^2)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a+2\ln\left(c\sqrt{x}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

```
[Out] int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

maxima [B] time = 0.34, size = 74, normalized size = 2.96

$$-\frac{2\left(\frac{2\,c^{4}x^{2}e^{(2\,a)}}{c^{8}x^{4}e^{(5\,a)}+2\,c^{4}x^{2}e^{(3\,a)}+e^{a}}+\frac{1}{c^{8}x^{4}e^{(5\,a)}+2\,c^{4}x^{2}e^{(3\,a)}+e^{a}}\right)}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a) + 1/(c^8*
x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

mupad [B] time = 1.53, size = 49, normalized size = 1.96

$$-\frac{\frac{2\,\mathrm{e}^{-a}}{c^{2}}+4\,c^{2}\,x^{2}\,\mathrm{e}^{a}}{\mathrm{e}^{4\,a}\,c^{8}\,x^{4}+2\,\mathrm{e}^{2\,a}\,c^{4}\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + 2*log(c*x^(1/2)))^3,x)
```

```
[Out] -((2*exp(-a))/c^2 + 4*c^2*x^2*exp(a))/(2*c^4*x^2*exp(2*a) + c^8*x^4*exp(4*a
) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{3}\left(a+2\log\left(c\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*ln(c*x**(1/2)))**3,x)
```

```
[Out] Integral(sech(a + 2*log(c*sqrt(x)))**3, x)
```

3.188 $\int \text{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal. Leaf size=25

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

```
[Out] 2*c^2/exp(3*a)/(exp(-2*a)+c^4/x^2)^2
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5545, 5547, 263, 261}

$$\frac{2e^{-3a}c^2}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) + c^4/x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5547

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d))
```

```
)^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx &= -\left(\left(2c^2\right)\operatorname{Subst}\left(\int \frac{\operatorname{sech}^3(a+2\log(x))}{x^3}\,dx,x,\frac{c}{\sqrt{x}}\right)\right)\\
&= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3x^9}\,dx,x,\frac{c}{\sqrt{x}}\right)\right)\\
&= -\left(\left(16c^2e^{-3a}\right)\operatorname{Subst}\left(\int \frac{x^3}{\left(e^{-2a}+x^4\right)^3}\,dx,x,\frac{c}{\sqrt{x}}\right)\right)\\
&= \frac{2c^2e^{-3a}}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}
\end{aligned}$$

Mathematica [B] time = 0.11, size = 64, normalized size = 2.56

$$-\frac{2c^6(\sinh(2a)+\cosh(2a))\left(\sinh(a)\left(c^4-2x^2\right)+\cosh(a)\left(c^4+2x^2\right)\right)}{\left(\sinh(a)\left(c^4-x^2\right)+\cosh(a)\left(c^4+x^2\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6*((c^4 + 2*x^2)*Cosh[a] + (c^4 - 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((c^4 + x^2)*Cosh[a] + (c^4 - x^2)*Sinh[a])^2
```

fricas [B] time = 0.40, size = 49, normalized size = 1.96

$$-\frac{2\left(c^{10}e^{(5a)}+2c^6x^2e^{(3a)}\right)}{c^8e^{(4a)}+2c^4x^2e^{(2a)}+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

giac [A] time = 0.13, size = 37, normalized size = 1.48

$$-\frac{2 \left(c^{10} e^{(5 a)} + 2 c^{6} x^{2} e^{(3 a)}\right)}{\left(c^{4} e^{(2 a)} + x^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) + x^2)^2
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a + 2 \ln\left(\frac{c}{\sqrt{x}}\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

```
[Out] int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

maxima [B] time = 0.34, size = 49, normalized size = 1.96

$$-\frac{2 \left(c^{10} e^{(5 a)} + 2 c^{6} x^{2} e^{(3 a)}\right)}{c^{8} e^{(4 a)} + 2 c^{4} x^{2} e^{(2 a)} + x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

mupad [B] time = 1.45, size = 36, normalized size = 1.44

$$\frac{2 c^{2} x^{4} \mathrm{e}^{a}}{\mathrm{e}^{4 a} c^{8} + 2 \mathrm{e}^{2 a} c^{4} x^{2} + x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + 2*log(c/x^(1/2)))^3,x)
```

```
[Out] (2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 + 2*c^4*x^2*exp(2*a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+2*ln(c/x**(1/2)))**3,x)
```

```
[Out] Integral(sech(a + 2*log(c/sqrt(x)))**3, x)
```

3.189 $\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=89

$$\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*exp(2*a)*(2-p)*x*(1+(c*x^n)^(2/n/(2-p))/exp(2*a))*sech(a-ln(c*x^n)/n/(2
-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5545, 5549, 261}

$$\frac{e^{2a}(2-p)x\left(cx^n\right)^{-\frac{2}{n(2-p)}}\left(e^{-2a}\left(cx^n\right)^{\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (E^(2*a)*(2 - p)*x*(1 + (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Sech[a - Log[c*x^n
]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)dx = \frac{\left(x\,(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x\,(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1+e^{-2a}\,(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}-\frac{}{n(}}\right.}{n}$$

$$= \frac{e^{2a}(2-p)x\,(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{-2a}\,(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] time = 0.77, size = 57, normalized size = 0.64

$$\frac{(p-2)x\left(e^{2a}\,(cx^n)^{\frac{2}{n(p-2)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(p-2)}\right)}{2(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((-2 + p)*x*(1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p))))*Sech[a + Log[c*x^n]/(n*(
-2 + p))]^p)/(2*(-1 + p))
```

fricas [B] time = 0.45, size = 474, normalized size = 5.33

$$\frac{\left(p-2\right)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^2+2\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(}{np-2\,n}\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] ((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*
n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p -
 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(
x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 1)))*cosh(
```

```
(a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*cosh((a*n*p -
2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh((a*n*p - 2*a*n +
 n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2
*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n
+ n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c)
)/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/
(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a + \frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a + \frac{\ln\left(c\,x^n\right)}{n\left(-2+p\right)}\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
[Out] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a + \frac{\log\left(cx^n\right)}{n\left(p-2\right)}\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cosh\left(a+\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{p}\left(a+\frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(sech(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

3.190 $\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal. Leaf size=65

$$\frac{(2-p)x\left(e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1+1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*sech(a+ln(c*x^n)/n/(2-p))^
p/(1-p)
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5545, 5549, 264}

$$\frac{(2-p)x\left(e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 + 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p)))))*Sech[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5545

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5549

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[(Sech[d*(a + b*Log[x])]^p*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)/x^(-(b*
d*p)), Int[(e*x)^m/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)dx=\frac{\left(x\left(cx^n\right)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^p\left(a-\frac{\log(x)}{n(-2+p)}\right)dx,x,cx^n\right)}{n}$$

$$=\frac{\left(x\left(cx^n\right)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1+e^{-2a}\left(cx^n\right)^{\frac{2}{n(-2+p)}}\right)^p\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}+\frac{p}{n(-2}}\right.}{n}$$

$$=\frac{(2-p)x\left(1+e^{-2a}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [**A**] time = 0.88, size = 62, normalized size = 0.95

$$\frac{e^{-2a}(p-2)x\left(e^{2a}+\left(cx^n\right)^{\frac{2}{n(p-2)}}\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{2n-np}\right)}{2(p-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((-2 + p)*x*(E^(2*a) + (c*x^n)^(2/(n*(-2 + p))))*Sech[a + Log[c*x^n]/(2*n -
 n*p)]^p)/(2*E^(2*a)*(-1 + p))
```

fricas [**B**] time = 0.45, size = 538, normalized size = 8.28

$$\left(p-2\right)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+2\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x}{np-2\,n}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] ((p - 2)*x*cosh(p*log(2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2
*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*
p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*
log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*
p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 1)))
```

```
*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*cosh(-(
a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh(-(a*n*p
- 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x)
 - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a
*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*l
og(x) - log(c))/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh(-(a*n*p - 2*a*n - n*lo
g(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) - log
(c))/(n*p - 2*n)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a - \frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

```
[Out] integrate(sech(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(a - \frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(-a + \frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

```
[Out] integrate(sech(-a + log(c*x^n)/(n*(p - 2)))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cosh\left(a-\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^{p}\left(a-\frac{\log\left(cx^{n}\right)}{n\left(p-2\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a-ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(sech(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

3.191 $\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

```
[Out] arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3770}

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.05, size = 19, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]/x,x]
```

[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)

fricas [A] time = 0.42, size = 34, normalized size = 1.79

$$\frac{2\arctan\left(\cosh\left(bn\log(x)+b\log(c)+a\right)+\sinh\left(bn\log(x)+b\log(c)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="fricas")

[Out] 2*arctan(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/(b*n)

giac [A] time = 0.12, size = 27, normalized size = 1.42

$$\frac{2\arctan\left(\frac{c^{2b}x^{bn}e^{a}}{c^{b}}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="giac")

[Out] 2*arctan(c^(2*b)*x^(b*n)*e^a/c^b)/(b*n)

maple [A] time = 0.02, size = 20, normalized size = 1.05

$$\frac{\arctan\left(\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(a+b*ln(c*x^n))/x,x)

[Out] arctan(sinh(a+b*ln(c*x^n)))/b/n

maxima [A] time = 0.31, size = 19, normalized size = 1.00

$$\frac{\arctan\left(\sinh\left(b\log\left(cx^{n}\right)+a\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="maxima")

[Out] arctan(sinh(b*log(c*x^n) + a))/(b*n)

mupad [B] time = 1.41, size = 41, normalized size = 2.16

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{b^2\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))),x)
[Out] -(2*atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(b^2*n^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))/x,x)
[Out] Integral(sech(a + b*log(c*x**n))/x, x)
```

3.192 $\int \frac{\text{sech}^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] tanh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3767, 8}

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \text{sech}^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{i\,\text{Subst}\left(\int 1\,dx,x,-i\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 18, normalized size = 1.00

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

fricas [B] time = 0.42, size = 70, normalized size = 3.89

$$-\frac{2}{bn\cosh\left(bn\log(x)+b\log(c)+a\right)^2+2\,bn\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)+bn$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n)
```

giac [A] time = 0.14, size = 28, normalized size = 1.56

$$-\frac{2}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)*b*n)
```

maple [A] time = 0.29, size = 19, normalized size = 1.06

$$\frac{\tanh\left(a+b\ln\left(c\,x^n\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^2/x,x)
```

```
[Out] tanh(a+b*ln(c*x^n))/b/n
```

maxima [A] time = 0.34, size = 28, normalized size = 1.56

$$-\frac{2}{bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

mupad [B] time = 1.33, size = 24, normalized size = 1.33

$$-\frac{2}{b\,n+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^2),x)
[Out] -2/(b*n + b*n*exp(2*a)*(c*x^n)^(2*b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**2/x,x)
[Out] Integral(sech(a + b*log(c*x**n))**2/x, x)
```

3.193 $\int \frac{\mathrm{sech}^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] 1/2*arctan(sinh(a+b*ln(c*x^n)))/b/n+1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x
^n))/b/n
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn}+\frac{\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{sech}^3(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)\tanh\left(a+b\log\left(cx^n\right)\right)}{2bn} + \frac{\operatorname{Subst}\left(\int \operatorname{sech}(a+bx)\,dx, x, \log\left(\right.\right.}{2n}$$
$$= \frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn} + \frac{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)\tanh\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{2bn} + \frac{\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

fricas [B] time = 0.43, size = 452, normalized size = 8.22

$$\frac{\cosh\left(bn\log(x)+b\log(c)+a\right)^3 + 3\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^2 + \sinh\left(bn\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] (cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c) + a)^3 + (cosh(b
*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*
log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + (3*cosh(b*n
*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*l
og(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*
```

```
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x)
+ b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

giac [B] time = 0.15, size = 115, normalized size = 2.09

$$c^{3b}\left(\frac{\arctan\left(\frac{c^{2b}x^{bn}e^{a}}{c^{b}}\right)e^{(-3a)}}{bc^{2b}c^{b}n}+\frac{\left(c^{2b}x^{3bn}e^{(2a)}-x^{bn}\right)e^{(-2a)}}{\left(c^{2b}x^{2bn}e^{(2a)}+1\right)^{2}bc^{2b}n}\right)e^{(3a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] c^(3*b)*(arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-3*a)/(b*c^(2*b)*c^b*n) + (c^(2
*b)*x^(3*b*n)*e^(2*a) - x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^
2*b*c^(2*b)*n))*e^(3*a)
```

maple [A] time = 0.30, size = 51, normalized size = 0.93

$$\frac{\mathrm{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)\tanh\left(a+b\ln\left(c\,x^{n}\right)\right)}{2bn}+\frac{\arctan\left(\mathrm{e}^{a+b\ln\left(c\,x^{n}\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] 1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))/b/n+1/b/n*arctan(exp(a+b*ln(c*x
^n)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$8\,c^{b}\int\frac{e^{\left(b\log\left(x^{n}\right)+a\right)}}{8\left(c^{2b}xe^{\left(2b\log\left(x^{n}\right)+2a\right)}+x\right)}\,dx+\frac{c^{3b}e^{\left(3b\log\left(x^{n}\right)+3a\right)}-c^{b}e^{\left(b\log\left(x^{n}\right)+a\right)}}{bc^{4b}ne^{\left(4b\log\left(x^{n}\right)+4a\right)}+2\,bc^{2b}ne^{\left(2b\log\left(x^{n}\right)+2a\right)}+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 8*c^b*integrate(1/8*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a) +
x), x) + (c^(3*b)*e^(3*b*log(x^n) + 3*a) - c^b*e^(b*log(x^n) + a))/(b*c^(4*
b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

mupad [B] time = 1.40, size = 139, normalized size = 2.53

$$\frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n+\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}}\right)}-\frac{\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(b\,n+\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}}\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{b^2\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^3),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4
*a))/(c*x^n)^(4*b))) - exp(-a)/((c*x^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^(2
*b))) - atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(b^2*n^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}{\left(a+b\log{\left(cx^{n}\right)}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**3/x, x)
```

3.194 $\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] tanh(a+b*ln(c*x^n))/b/n-1/3*tanh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3767}

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \operatorname{sech}^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{i\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 1.00

$$\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

fricas [B] time = 0.43, size = 272, normalized size = 6.48

$$-\frac{}{3\left(bn\cosh\left(bn\log(x)+b\log(c)+a\right)^{5}+5\,bn\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^{4}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/
(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

giac [A] time = 0.15, size = 47, normalized size = 1.12

$$-\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^{3}bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^3*b
*n)
```

maple [A] time = 0.29, size = 36, normalized size = 0.86

$$\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(a+b\ln(c\,x^{n}))^{2}}{3}\right)\tanh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^4/x,x)
```

[Out] 1/n/b*(2/3+1/3*sech(a+b*ln(c*x^n))^2)*tanh(a+b*ln(c*x^n))

maxima [B] time = 0.35, size = 91, normalized size = 2.17

$$-\frac{4\left(3\,c^{2\,b}e^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+1\right)}{3\left(bc^{6\,b}ne^{\left(6\,b\log\left(x^{n}\right)+6\,a\right)}+3\,bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}+3\,bc^{2\,b}ne^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="maxima")

[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)

mupad [B] time = 1.34, size = 55, normalized size = 1.31

$$\frac{4\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}\,\left({\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+3\right)}{3\,b\,n\,{\left({\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*cosh(a + b*log(c*x^n))^4),x)

[Out] (4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) + 3))/(3*b*n*(exp(2*a)*(c*x^n)^(2*b) + 1)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{4}\left(a + b \log\left(cx^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*ln(c*x**n))**4/x,x)

[Out] Integral(sech(a + b*log(c*x**n))**4/x, x)

3.195 $\int \frac{\text{sech}^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=89

$$\frac{3\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}+\frac{\tanh\left(a+b\log\left(cx^n\right)\right)\text{sech}^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\tanh\left(a+b\log\left(cx^n\right)\right)\text{sech}\left(a\right.}{8bn}$$

```
[Out] 3/8*arctan(sinh(a+b*ln(c*x^n)))/b/n+3/8*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x
^n))/b/n+1/4*sech(a+b*ln(c*x^n))^3*tanh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3768, 3770}

$$\frac{3\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}+\frac{\tanh\left(a+b\log\left(cx^n\right)\right)\text{sech}^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\tanh\left(a+b\log\left(cx^n\right)\right)\text{sech}\left(a\right.}{8bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Sech[a + b*Log[c*x^n]]*Tanh
[a + b*Log[c*x^n]])/(8*b*n) + (Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^
n]])/(4*b*n)
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\text{Subst}\left(\int \operatorname{sech}^5(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{\operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)\tanh\left(a+b\log\left(cx^n\right)\right)}{4bn} + \frac{3\,\text{Subst}\left(\int \operatorname{sech}^3(a+bx)\,dx, x,\right.}{4n}$$
$$= \frac{3\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)\tanh\left(a+b\log\left(cx^n\right)\right)}{8bn} + \frac{\operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)\tanh\left(a\right.}{4bn}$$
$$= \frac{3\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn} + \frac{3\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)\tanh\left(a+b\log\left(cx^n\right)\right.}{8bn}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.84

$$\frac{3\tan^{-1}\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)+2\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)+3\tanh\left(a+b\log\left(cx^n\right)\right)\operatorname{sec}}{8bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]] + 3*Sech[a + b*Log[c*x^n]]*Tanh[a + b*Log
[c*x^n]] + 2*Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^n]])/(8*b*n)
```

fricas [B] time = 0.45, size = 1326, normalized size = 14.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*n*log(x) + b*log(c) + a)^7 + 21*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 3*sinh(b*n*log(x) + b*log(c) + a)^7
+ (63*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*log(c) +
a)^5 + 11*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*(21*cosh(b*n*log(x) + b*log
(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c)
 + a)^4 + (105*cosh(b*n*log(x) + b*log(c) + a)^4 + 110*cosh(b*n*log(x) + b*
log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 11*cosh(b*n*log(x)
+ b*log(c) + a)^3 + (63*cosh(b*n*log(x) + b*log(c) + a)^5 + 110*cosh(b*n*lo
g(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log(x) + b
*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x
```

```
) + b*log(c) + a)^6 + 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cosh(b*n*l
og(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*cosh(b*n
*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6*cosh(b
*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*c
osh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(
b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^6 + 15*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)^2 +
 8*(cosh(b*n*log(x) + b*log(c) + a)^7 + 3*cosh(b*n*log(x) + b*log(c) + a)^5
 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*s
inh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(b*n*log(x) + b*log(c) + a)
+ sinh(b*n*log(x) + b*log(c) + a)) + (21*cosh(b*n*log(x) + b*log(c) + a)^6
+ 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(b*n*log(x) + b*log(c) + a)
^2 - 3)*sinh(b*n*log(x) + b*log(c) + a) - 3*cosh(b*n*log(x) + b*log(c) + a)
)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sinh(b*n*log(x) + b*log(c) +
a)^8 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) +
 b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b
*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(
35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*b*n*cosh(b*n*log(x) + b*log(c
) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*b*n*co
sh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*si
nh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^
6 + 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*lo
g(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*
n*log(x) + b*log(c) + a)^7 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 3*b*
n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a))
```

giac [A] time = 0.17, size = 152, normalized size = 1.71

$$\frac{1}{4}\,c^{5\,b}\left(\frac{3\,\arctan\left(\frac{c^{2\,b}x^{bn}e^{a}}{c^{b}}\right)e^{(-5\,a)}}{bc^{4\,b}c^{b}n}+\frac{\left(3\,c^{6\,b}x^{7\,bn}e^{(6\,a)}+11\,c^{4\,b}x^{5\,bn}e^{(4\,a)}-11\,c^{2\,b}x^{3\,bn}e^{(2\,a)}-3\,x^{bn}\right)e^{(-4\,a)}}{\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^{4}bc^{4\,b}n}\right)e^{(5\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/4*c^(5*b)*(3*arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-5*a)/(b*c^(4*b)*c^b*n) +
 (3*c^(6*b)*x^(7*b*n)*e^(6*a) + 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x
^(3*b*n)*e^(2*a) - 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^4*b
*c^(4*b)*n))*e^(5*a)
```

maple [A] time = 0.32, size = 84, normalized size = 0.94

$$\frac{\operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}\tanh\left(a+b\ln\left(c\,x^{n}\right)\right)}{4bn}+\frac{3\operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)\tanh\left(a+b\ln\left(c\,x^{n}\right)\right)}{8bn}+\frac{3\arctan\left(\mathrm{e}^{a+b\ln\left(c\,x^{n}\right)}\right)}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] 1/4*sech(a+b*ln(c*x^n))^3*tanh(a+b*ln(c*x^n))/b/n+3/8*sech(a+b*ln(c*x^n))*t
anh(a+b*ln(c*x^n))/b/n+3/4/b/n*arctan(exp(a+b*ln(c*x^n)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$96\,c^{b}\int\frac{e^{\left(b\log\left(x^{n}\right)+a\right)}}{128\left(c^{2\,b}xe^{\left(2\,b\log\left(x^{n}\right)+2\,a\right)}+x\right)}\,dx+\frac{3\,c^{7\,b}e^{\left(7\,b\log\left(x^{n}\right)+7\,a\right)}+11\,c^{5\,b}e^{\left(5\,b\log\left(x^{n}\right)+5\,a\right)}-11\,c^{3\,b}e^{\left(3\,b\log\left(x^{n}\right)+\dots\right.}}{4\left(bc^{8\,b}ne^{\left(8\,b\log\left(x^{n}\right)+8\,a\right)}+4\,bc^{6\,b}ne^{\left(6\,b\log\left(x^{n}\right)+6\,a\right)}+6\,bc^{4\,b}ne^{\left(4\,b\log\left(x^{n}\right)+4\,a\right)}\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 96*c^b*integrate(1/128*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a)
 + x), x) + 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) + 11*c^(5*b)*e^(5*b*log(x
^n) + 5*a) - 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) - 3*c^b*e^(b*log(x^n) + a))/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n)
```

mupad [B] time = 1.35, size = 314, normalized size = 3.53

$$\frac{2\,\mathrm{e}^{-a}}{{\left(c\,x^{n}\right)}^{b}\,\left(b\,n+\frac{3\,b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^{n}\right)}^{2\,b}}+\frac{3\,b\,n\,\mathrm{e}^{-4\,a}}{{\left(c\,x^{n}\right)}^{4\,b}}+\frac{b\,n\,\mathrm{e}^{-6\,a}}{{\left(c\,x^{n}\right)}^{6\,b}}\right)}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^{2}\,n^{2}}}{b\,n\,{\left(c\,x^{n}\right)}^{b}}\right)}{4\,\sqrt{b^{2}\,n^{2}}}-\frac{3\,\mathrm{e}^{-a}}{4\,{\left(c\,x^{n}\right)}^{b}\,\left(b\,n+\frac{b\,n\,\mathrm{e}^{-2\,a}}{{\left(c\,x^{n}\right)}^{2\,b}}\right)}+\frac{\dots}{{\left(c\,x^{n}\right)}^{3\,b}\,\left(b\,n+\frac{4\,b\,n\,\mathrm{e}^{-\dots}}{{\left(c\,x^{n}\right)}^{2\dots}}\dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^5),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n + (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp(
-4*a))/(c*x^n)^(4*b) + (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)*(
b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(b^2*n^2)^(1/2)) - (3*exp(-a))/(4*(c*x
^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^(2*b))) + (4*exp(-3*a))/((c*x^n)^(3*b)
*(b*n + (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^(4*b) +
 (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b))) - exp(-a
```

```
)/(2*(c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(c*
x^n)^(4*b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**5/x, x)
```

3.196 $\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=97

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\right.\right.}{3bn}$$

```
[Out] 2/3*sech(a+b*ln(c*x^n))^(3/2)*sinh(a+b*ln(c*x^n))/b/n-2/3*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticF(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2641}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\right.\right.}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sech[a + b*Log[c*x^n]]^(3/2)*
Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

EqQ[n^2, 1/4]

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx, x,\right.}{3n}$$
$$= \frac{2\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sec}\right.}{}$$
$$= -\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{3bn} +$$

Mathematica [A] time = 0.17, size = 74, normalized size = 0.76

$$\frac{2\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\left(\sinh\left(a+b\log\left(cx^n\right)\right)-i\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (2*Sech[a + b*Log[c*x^n]]^(3/2)*((-I)*Cosh[a + b*Log[c*x^n]]^(3/2)*Elliptic
F[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(3*b*n)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^(5/2)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.73, size = 295, normalized size = 3.04

$$\frac{2\left(2\sqrt{-\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)}\,\sqrt{-2\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right),\sqrt{2}\right)\left(\sinh^{2}\left(\frac{a}{2}+\right.\right.\right.}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(a+b*ln(c*x^n))^(5/2)/x,x)

[Out] 2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a+1/2*b*ln(c*x^n))^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2)*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(sech(b*log(c*x^n) + a)^(5/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cosh(a+b\,\ln(c\,x^{n}))}\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cosh(a + b*log(c*x^n)))^(5/2)/x,x)

```
[Out] int((1/cosh(a + b*log(c*x^n)))^(5/2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

3.197 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=93

$$\frac{2\sinh(a+b\log(cx^n))\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}+\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i\left(a+\right.\right.}{bn}$$

```
[Out] 2*sinh(a+b*ln(c*x^n))*sech(a+b*ln(c*x^n))^(1/2)/b/n+2*I*(cosh(1/2*a+1/2*b*l
n(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticE(I*sinh(1/2*a+1/2*b
*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2)/b/
n
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3768, 3771, 2639}

$$\frac{2\sinh(a+b\log(cx^n))\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}+\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\,E\left(\frac{1}{2}i\left(a+\right.\right.}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ((2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]*
Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sqrt[Sech[a + b*Log[c*x^n]]]*Sinh[
a + b*Log[c*x^n]])/(b*n)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\operatorname{Subst}\left(\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sinh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx, x, \log\right.}{n}$$
$$= \frac{2\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sinh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{se}\right.}{}$$
$$= \frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn} + \cdots$$

Mathematica [A] time = 0.09, size = 72, normalized size = 0.77

$$\frac{2\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\left(\sinh\left(a+b\log\left(cx^n\right)\right)+i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (2*Sqrt[Sech[a + b*Log[c*x^n]]]*(I*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(
I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(b*n)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sech(b*log(c*x^n) + a)^(3/2)/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.64, size = 141, normalized size = 1.52

$$\frac{2\,\mathrm{EllipticE}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right),\sqrt{2}\right)\sqrt{-2\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)-1}\,\sqrt{-\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)}+4\cosh\left(\frac{a}{2}+\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\left(\cosh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)-1}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 2/n*(EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*(-2*sinh(1/2*a+1/2*b*ln
(c*x^n))^2-1)^(1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)+2*cosh(1/2*a+1/2
*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2
*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*log(c*x^n)))^(3/2)/x,x)
```

```
[Out] int((1/cosh(a + b*log(c*x^n)))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**(3/2)/x, x)
```

3.198 $\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=58

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech
(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3771, 2641}

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, 1\right.}{n}$$
$$= -\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 1.00

$$-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sech(b*log(c*x^n) + a))/x, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.54, size = 183, normalized size = 3.16

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\ \sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\ \sqrt{-2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)}\ \sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{a}{2}+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] 2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2
+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln(c*x
^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sech(b*log(c*x^n) + a))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\cosh(a+b\,\ln(c\,x^n))}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cosh(a + b*log(c*x^n)))^(1/2)/x,x)
```

```
[Out] int((1/cosh(a + b*log(c*x^n)))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(a+b*ln(c*x**n))**(1/2)/x,x)
[Out] Integral(sqrt(sech(a + b*log(c*x**n)))/x, x)
```

3.199 $\int \frac{1}{x\sqrt{\text{sech}(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=58

$$-\frac{2i\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech
(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3771, 2639}

$$-\frac{2i\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\right)\operatorname{Subst}\left(\int \sqrt{\cosh(a+bx)}\ d\right.}{n}$$

$$= -\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

Mathematica [A] time = 0.08, size = 58, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n*Sqrt[Cosh[a + b*Log[c*
x^n]]]*Sqrt[Sech[a + b*Log[c*x^n]]])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{x\sqrt{\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

maple [B] time = 0.51, size = 183, normalized size = 3.16

$$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\sqrt{-2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.\right.}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{a}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] -2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^
2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+1/2
*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*
x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{\frac{1}{\cosh(a+b\,\ln(c\,x^n))}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)),x)
```

```
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(sech(a + b*log(c*x**n)))), x)
```

3.200 $\int \frac{1}{x\mathbf{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=97

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}$$

```
[Out] 2/3*sinh(a+b*ln(c*x^n))/b/n/sech(a+b*ln(c*x^n))^(1/2)-2/3*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticF(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2641}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(3*b*
n*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}} + \frac{\operatorname{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{3n}$$

$$= \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}} + \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right.\right.}\right.}{3n}$$

$$= -\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{3bn}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.78

$$\frac{\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\left(\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Elliptic
F[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[2*(a + b*Log[c*x^n])]))/(3*b*n)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{x\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{sech}\left(b \log\left(cx^n\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

maple [A] time = 0.68, size = 237, normalized size = 2.44

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\left(4\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-6\left(\cosh^3\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+}{3n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-sin
h(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2
)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^
n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/
sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{sech}\left(b \log\left(cx^n\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(\frac{1}{\mathrm{cosh}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)),x)
```

```
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{sech}^{\frac{3}{2}}{\left(a + b \log{\left(c x^{n} \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*sech(a + b*log(c*x**n))**(3/2)), x)
```

3.201 $\int \frac{1}{x\,\mathrm{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=97

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\mathrm{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{6i\sqrt{\mathrm{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{5bn}$$

```
[Out] 2/5*sinh(a+b*ln(c*x^n))/b/n/sech(a+b*ln(c*x^n))^(3/2)-6/5*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticE(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3769, 3771, 2639}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\mathrm{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{6i\sqrt{\mathrm{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(5*b*
n*Sech[a + b*Log[c*x^n]]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{\operatorname{Subst}\left(\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx,x,\log\left(cx^n\right)\right)}{n}$$

$$=\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{3\operatorname{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{5n}$$

$$=\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\operatorname{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(3\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right.\right.}\right.}{5n}$$

$$=-\frac{6i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{5bn}$$

Mathematica [A] time = 0.13, size = 87, normalized size = 0.90

$$\frac{\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\left(\sinh\left(a+b\log\left(cx^n\right)\right)+\sinh\left(3\left(a+b\log\left(cx^n\right)\right)\right)-12i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}i\left(\right.\right.\right.}{10bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-12*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*Ellipti
cE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]] + Sinh[3*(a + b*Lo
g[c*x^n])]))/(10*b*n)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{x\operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{sech}\left(b \log\left(cx^n\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

maple [B] time = 0.67, size = 256, normalized size = 2.64

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\left(8\left(\cosh^7\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-16\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+}{5n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10*c
osh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*c
osh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n))
,2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+s
inh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2
*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{sech}\left(b \log\left(cx^n\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)),x)
```

```
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```