Computer algebra independent integration tests

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/6.4.2-Hyperbolic-cotangent-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [224]. This is test number [175].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 81.70 (183)	% 18.30 (41)
Mathematica	% 100.00 (224)	% 0.00 (0)
Maple	% 73.21 (164)	% 26.79 (60)
Maxima	% 46.88 (105)	% 53.12 (119)
Fricas	% 79.02 (177)	% 20.98 (47)
Sympy	% 14.29 (32)	% 85.71 (192)
Giac	% 61.16 (137)	% 38.84 (87)
Mupad	% 58.48 (131)	% 41.52 (93)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

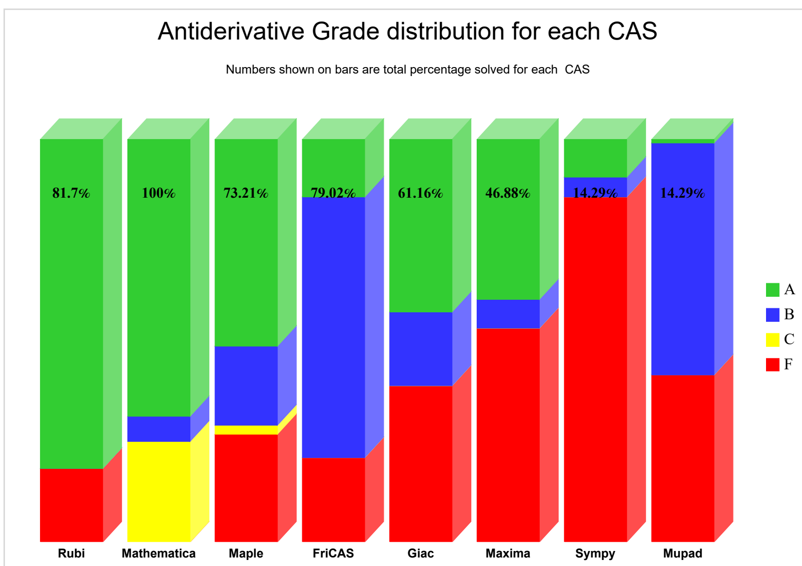
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

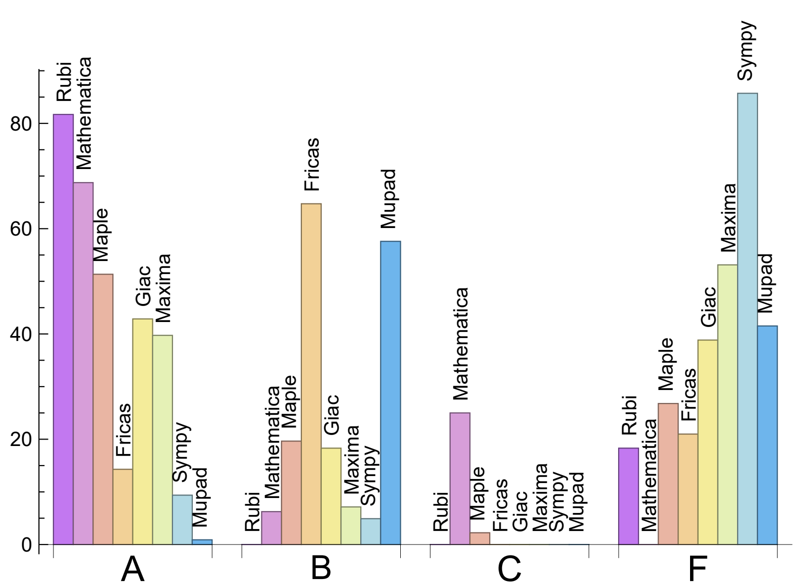
System	% A grade	% B grade	% C grade	% F grade
Rubi	81.70	0.00	0.00	18.30
Mathematica	68.75	6.25	25.00	0.00
Maple	51.34	19.64	2.23	26.79
Maxima	39.73	7.14	0.00	53.12
Fricas	14.29	64.73	0.00	20.98
Sympy	9.38	4.91	0.00	85.71
Giac	42.86	18.30	0.00	38.84
Mupad	0.89	57.59	0.00	41.52

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	41	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	60	100.00 %	0.00 %	0.00 %
Maxima	119	93.28 %	0.00 %	6.72 %
Fricas	47	74.47 %	4.26 %	21.28 %
Sympy	192	93.23 %	6.77 %	0.00 %
Giac	87	82.76 %	9.20 %	8.05 %
Mupad	93	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

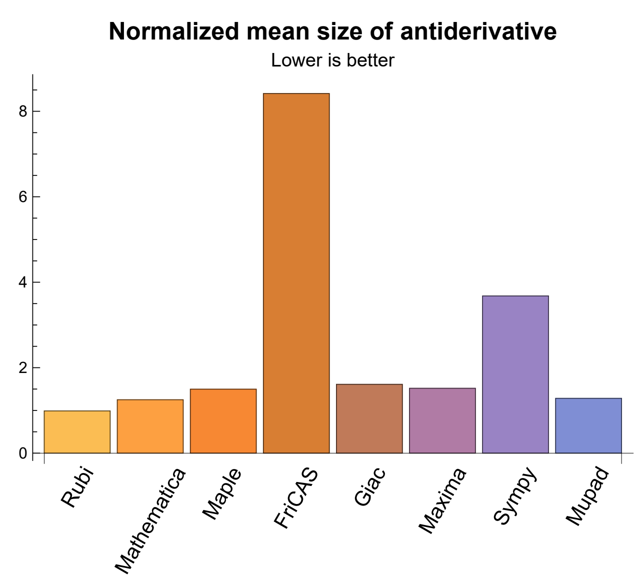
1.3 Performance

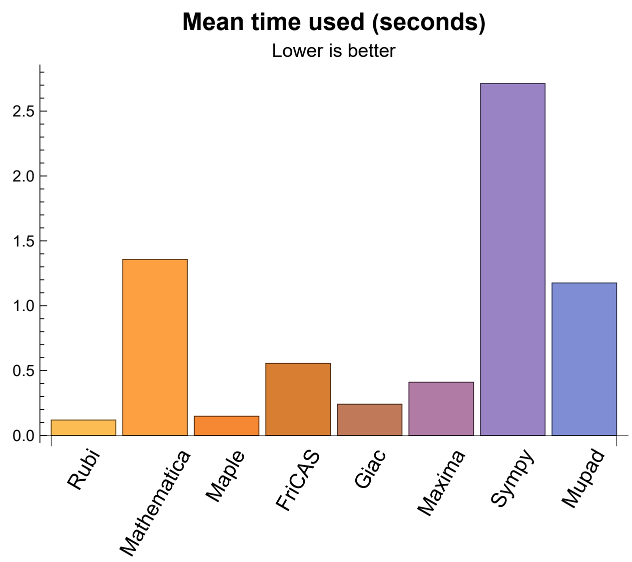
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	84.35	0.99	60.00	1.00
Mathematica	1.36	85.48	1.25	60.00	1.03
Maple	0.15	93.96	1.50	70.50	1.28
Maxima	0.41	82.33	1.52	47.00	1.12
Fricas	0.56	852.20	8.42	273.00	4.77
Sympy	2.71	192.88	3.68	98.00	2.42
Giac	0.24	97.72	1.61	66.00	1.27
Mupad	1.18	72.42	1.28	42.00	0.92

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{220, 224}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {159, 160, 161, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 196, 197, 198, 212}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

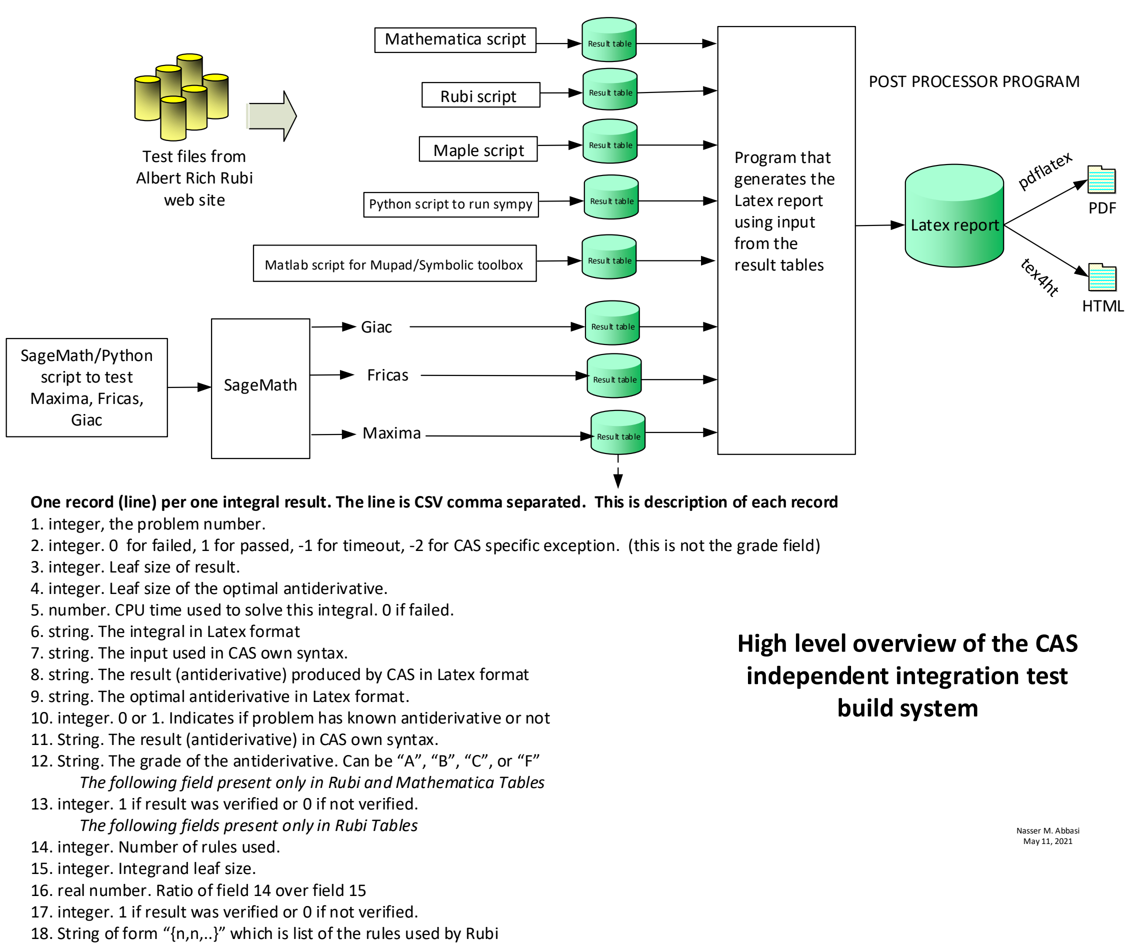
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { }

C grade: { }

F grade: { 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 10, 12, 15, 16, 17, 18, 19, 20, 21, 22, 24, 28, 29, 30, 35, 36, 37, 38, 39, 42, 43, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 155, 157, 158, 165, 166, 167, 169, 170, 171, 172, 173, 181, 184, 185, 186, 187, 189, 190, 191, 193, 194, 195, 196, 198, 199, 200, 201, 202, 205, 206, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 151, 168, 174, 175, 176, 177, 178, 179, 180, 182, 183, 197, 207, 210 }

C grade: { 6, 7, 8, 9, 11, 13, 14, 23, 25, 26, 27, 31, 32, 33, 34, 40, 41, 44, 46, 47, 48, 49, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 87, 113, 132, 133, 134, 135, 136, 137, 138, 139, 152, 154, 156, 159, 160, 161, 162, 163, 164, 188, 192, 203, 204, 212 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 29, 30, 31, 32, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 98, 100, 101, 102, 109, 115, 116, 117, 118, 119, 122, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 157, 158, 159, 160, 161, 164, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 217, 218, 219, 220, 221, 222, 223, 224 }

B grade: { 33, 34, 35, 36, 37, 38, 77, 78, 79, 80, 89, 90, 91, 92, 95, 96, 97, 99, 103, 104, 105, 106, 107, 108, 110, 111, 112, 114, 120, 121, 123, 124, 125, 126, 140, 141, 152, 154, 155, 156, 162, 181, 188, 213 }

C grade: { 163, 210, 214, 215, 216 }

F grade: { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 113, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 208, 209 }

2.1.4 Maxima

A grade: { 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 64, 65, 66, 67, 68, 69, 80, 81, 82, 85, 86, 89, 90, 91, 92, 93, 94, 97, 99, 102, 105, 106, 107, 108, 109, 110, 114, 116, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 211, 212, 213, 214, 215, 216, 220, 224 }

B grade: { 61, 62, 63, 77, 78, 79, 83, 84, 95, 96, 104, 111, 112, 191, 192, 193 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 87, 88, 98, 100, 101, 103, 113, 115, 117, 118, 120, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.1.5 FriCAS

A grade: { 9, 10, 46, 81, 85, 86, 91, 92, 93, 101, 107, 108, 118, 122, 143, 144, 145, 146, 151, 152, 153, 155, 156, 158, 163, 164, 213, 214, 218, 220, 222, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 47, 48, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 148, 149, 150, 154, 157, 159, 160, 161, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 219, 221, 223 }

C grade: { }

F grade: { 15, 16, 17, 27, 28, 39, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198 }

2.1.6 Sympy

A grade: { 34, 61, 62, 63, 64, 77, 78, 79, 80, 81, 85, 86, 144, 145, 146, 147, 148, 149, 162, 220, 224 }

B grade: { 65, 66, 67, 68, 69, 127, 128, 129, 130, 131, 155 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

2.1.7 Giac

A grade: { 18, 19, 21, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 188, 192, 211, 212, 213, 214, 215, 216, 220, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 20, 29, 30, 31, 32, 70, 71, 72, 73, 74, 75, 76, 95, 102, 104, 113, 119, 121, 132, 133, 134, 135, 136, 137, 138, 139, 150, 155, 181, 191, 193 }

C grade: { }

F grade: { 9, 10, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 87, 88, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

2.1.8 Mupad

A grade: { 220, 224 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 113, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	80	0	1574	0	379	83
normalized size	1	1.00	0.86	0.82	0.00	16.23	0.00	3.91	0.86
time (sec)	N/A	0.070	0.238	0.129	0.000	0.469	0.000	0.421	1.580

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	63	0	988	0	224	62
normalized size	1	1.00	0.87	0.81	0.00	12.67	0.00	2.87	0.79
time (sec)	N/A	0.049	0.222	0.113	0.000	0.466	0.000	0.356	1.372

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	62	0	637	0	168	61
normalized size	1	1.00	0.81	0.83	0.00	8.49	0.00	2.24	0.81
time (sec)	N/A	0.050	0.093	0.133	0.000	0.439	0.000	0.255	1.275

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	47	0	594	0	101	41
normalized size	1	1.00	0.88	0.81	0.00	10.24	0.00	1.74	0.71
time (sec)	N/A	0.036	0.041	0.128	0.000	0.436	0.000	0.191	1.215

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	0	598	0	102	38
normalized size	1	1.00	0.86	0.81	0.00	10.49	0.00	1.79	0.67
time (sec)	N/A	0.031	0.035	0.125	0.000	0.428	0.000	0.277	1.294

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	36	65	0	923	0	196	64
normalized size	1	1.00	0.46	0.83	0.00	11.83	0.00	2.51	0.82
time (sec)	N/A	0.051	0.077	0.101	0.000	0.437	0.000	0.585	1.390

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	38	64	0	1428	0	239	63
normalized size	1	1.00	0.48	0.81	0.00	18.08	0.00	3.03	0.80
time (sec)	N/A	0.050	0.072	0.095	0.000	0.457	0.000	0.607	1.491

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	38	83	0	2132	0	359	80
normalized size	1	1.00	0.38	0.83	0.00	21.32	0.00	3.59	0.80
time (sec)	N/A	0.071	0.100	0.104	0.000	0.487	0.000	0.634	1.596

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	36	209	0	292	0	0	249
normalized size	1	1.00	0.15	0.89	0.00	1.24	0.00	0.00	1.06
time (sec)	N/A	0.282	0.027	0.110	0.000	0.437	0.000	0.000	1.881

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	149	193	0	310	0	0	233
normalized size	1	1.00	0.68	0.89	0.00	1.42	0.00	0.00	1.07
time (sec)	N/A	0.294	0.186	0.108	0.000	0.431	0.000	0.000	1.499

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	38	115	0	291	0	217	146
normalized size	1	1.00	0.29	0.87	0.00	2.20	0.00	1.64	1.11
time (sec)	N/A	0.109	0.043	0.102	0.000	0.406	0.000	0.370	1.516

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	98	115	0	1598	0	216	147
normalized size	1	1.00	0.74	0.87	0.00	12.11	0.00	1.64	1.11
time (sec)	N/A	0.104	0.126	0.088	0.000	0.448	0.000	0.641	1.652

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	36	193	0	356	0	0	197
normalized size	1	1.00	0.17	0.89	0.00	1.63	0.00	0.00	0.90
time (sec)	N/A	0.235	0.031	0.090	0.000	0.458	0.000	0.000	1.386

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	36	211	0	3348	0	0	165
normalized size	1	1.00	0.15	0.89	0.00	14.07	0.00	0.00	0.69
time (sec)	N/A	0.316	0.061	0.095	0.000	0.514	0.000	0.000	1.419

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.068	0.443	0.000	0.417	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.042	0.471	0.000	0.434	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.051	0.627	0.000	0.439	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	53	97	823	0	90	-1
normalized size	1	1.00	0.92	0.87	1.59	13.49	0.00	1.48	-0.02
time (sec)	N/A	0.037	0.129	0.154	0.466	0.435	0.000	0.180	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	45	54	125	0	54	-1
normalized size	1	1.00	1.26	1.45	1.74	4.03	0.00	1.74	-0.03
time (sec)	N/A	0.020	0.047	0.156	0.468	0.423	0.000	0.122	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	34	128	0	60	30
normalized size	1	1.00	1.00	1.68	1.10	4.13	0.00	1.94	0.97
time (sec)	N/A	0.020	0.063	0.158	0.434	0.415	0.000	0.149	1.264

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	79	84	817	0	104	-1
normalized size	1	1.00	0.74	1.22	1.29	12.57	0.00	1.60	-0.02
time (sec)	N/A	0.036	0.150	0.125	0.429	0.454	0.000	0.731	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	166	0	0	1994	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	6.71	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.352	0.357	0.000	0.463	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	2037	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	7.05	0.00	0.00	-0.00
time (sec)	N/A	0.180	0.041	0.337	0.000	0.466	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	151	0	0	1618	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	6.13	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.109	0.378	0.000	0.493	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	41	0	0	8338	0	0	-1
normalized size	1	1.00	0.16	0.00	0.00	31.58	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.056	0.395	0.000	0.616	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	2066	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	7.15	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.066	0.353	0.000	0.455	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	43	0	0	0	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.157	0.362	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.042	0.627	0.000	0.666	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	82	107	0	2152	0	788	-1
normalized size	1	1.00	0.61	0.80	0.00	16.06	0.00	5.88	-0.01
time (sec)	N/A	0.060	0.531	0.203	0.000	1.815	0.000	0.650	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	63	89	0	633	0	269	-1
normalized size	1	1.00	0.61	0.86	0.00	6.09	0.00	2.59	-0.01
time (sec)	N/A	0.047	0.103	0.167	0.000	1.587	0.000	0.286	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	41	92	0	907	0	279	-1
normalized size	1	1.00	0.39	0.88	0.00	8.64	0.00	2.66	-0.01
time (sec)	N/A	0.048	0.032	0.157	0.000	1.211	0.000	0.500	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	43	106	0	3022	0	521	-1
normalized size	1	1.00	0.30	0.75	0.00	21.43	0.00	3.70	-0.01
time (sec)	N/A	0.061	0.069	0.135	0.000	1.583	0.000	2.332	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	43	145	87	1046	0	0	-1
normalized size	1	1.00	0.58	1.96	1.18	14.14	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.065	0.400	0.417	1.448	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	119	34	392	90	0	-1
normalized size	1	1.00	0.82	2.38	0.68	7.84	1.80	0.00	-0.02
time (sec)	N/A	0.024	0.033	0.396	0.418	0.657	23.764	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	192	51	148	0	0	-1
normalized size	1	1.00	1.26	6.19	1.65	4.77	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.029	0.385	0.513	0.568	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	192	32	187	0	0	-1
normalized size	1	1.00	1.00	6.19	1.03	6.03	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.031	0.414	0.827	1.534	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	119	37	287	0	0	-1
normalized size	1	1.00	0.80	2.38	0.74	5.74	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.064	0.395	1.055	0.596	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	51	149	89	1579	0	0	-1
normalized size	1	1.00	0.64	1.86	1.11	19.74	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.096	0.390	0.419	0.557	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.045	0.665	0.000	0.609	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	43	77	137	3421	0	77	-1
normalized size	1	1.00	0.39	0.70	1.25	31.10	0.00	0.70	-0.01
time (sec)	N/A	0.045	0.067	0.165	0.425	0.908	0.000	0.179	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	55	34	415	0	27	-1
normalized size	1	1.00	0.82	1.10	0.68	8.30	0.00	0.54	-0.02
time (sec)	N/A	0.023	0.035	0.157	0.416	1.097	0.000	0.155	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	59	36	422	0	32	-1
normalized size	1	1.00	0.80	1.18	0.72	8.44	0.00	0.64	-0.02
time (sec)	N/A	0.023	0.071	0.180	0.424	1.301	0.000	0.151	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	68	84	155	3473	0	99	-1
normalized size	1	1.00	0.58	0.71	1.31	29.43	0.00	0.84	-0.01
time (sec)	N/A	0.046	0.258	0.130	0.450	0.556	0.000	3.385	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	68	0	0	2864	0	0	-1
normalized size	1	1.00	0.19	0.00	0.00	8.11	0.00	0.00	-0.00
time (sec)	N/A	0.196	0.143	0.367	0.000	1.325	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	166	0	0	618	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.416	0.370	0.000	1.793	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	43	0	0	288	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.028	0.371	0.000	3.085	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	41	0	0	3316	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	11.47	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.053	0.371	0.000	0.831	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	43	0	0	1159	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	3.98	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.055	0.379	0.000	0.485	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	43	0	0	0	0	0	-1
normalized size	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.044	0.380	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.052	5.401	0.000	0.424	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.077	1.567	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.041	0.551	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.049	0.662	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.069	0.566	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.073	0.391	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.044	0.418	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.041	0.394	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.041	0.451	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.042	0.457	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.070	0.423	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	94	31	140	448	48	41	88
normalized size	1	1.00	2.29	0.76	3.41	10.93	1.17	1.00	2.15
time (sec)	N/A	0.039	0.249	0.012	0.326	0.395	1.223	0.112	1.126

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	84	25	95	273	37	35	60
normalized size	1	1.00	2.71	0.81	3.06	8.81	1.19	1.13	1.94
time (sec)	N/A	0.031	0.187	0.013	0.322	0.405	0.915	0.110	1.144

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	61	19	55	142	31	29	36
normalized size	1	1.00	2.65	0.83	2.39	6.17	1.35	1.26	1.57
time (sec)	N/A	0.021	0.155	0.011	0.316	0.391	0.682	0.112	0.041

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	19	53	22	21	20
normalized size	1	1.00	1.00	1.00	1.46	4.08	1.69	1.62	1.54
time (sec)	N/A	0.013	0.005	0.013	0.308	0.413	0.369	0.127	1.143

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	10	26	27	10	14
normalized size	1	1.00	1.12	1.50	0.62	1.62	1.69	0.62	0.88
time (sec)	N/A	0.009	0.031	0.047	0.307	0.427	0.444	0.128	0.051

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	32	16	52	88	18	16
normalized size	1	1.00	1.15	1.23	0.62	2.00	3.38	0.69	0.62
time (sec)	N/A	0.017	0.061	0.050	0.307	0.382	0.809	0.107	0.046

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	44	40	22	86	182	24	22
normalized size	1	1.00	1.22	1.11	0.61	2.39	5.06	0.67	0.61
time (sec)	N/A	0.027	0.092	0.051	0.306	0.385	1.054	0.132	0.056

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	48	28	121	299	30	28
normalized size	1	1.00	1.15	1.04	0.61	2.63	6.50	0.65	0.61
time (sec)	N/A	0.036	0.127	0.054	0.317	0.386	1.419	0.132	1.145

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	62	56	34	159	444	36	34
normalized size	1	1.00	1.11	1.00	0.61	2.84	7.93	0.64	0.61
time (sec)	N/A	0.046	0.145	0.054	0.320	0.389	1.803	0.117	1.149

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	101	43	0	438	0	160	44
normalized size	1	1.00	1.77	0.75	0.00	7.68	0.00	2.81	0.77
time (sec)	N/A	0.042	0.270	0.076	0.000	0.401	0.000	0.150	1.265

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	92	35	0	259	0	112	54
normalized size	1	1.00	2.04	0.78	0.00	5.76	0.00	2.49	1.20
time (sec)	N/A	0.032	0.159	0.063	0.000	0.407	0.000	0.150	1.195

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	69	27	0	131	0	63	26
normalized size	1	1.00	2.09	0.82	0.00	3.97	0.00	1.91	0.79
time (sec)	N/A	0.022	0.110	0.065	0.000	0.402	0.000	0.146	1.185

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	45	17	0	50	0	37	16
normalized size	1	1.00	2.14	0.81	0.00	2.38	0.00	1.76	0.76
time (sec)	N/A	0.012	0.077	0.082	0.000	0.390	0.000	0.145	1.204

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	51	27	0	85	0	66	26
normalized size	1	1.00	1.59	0.84	0.00	2.66	0.00	2.06	0.81
time (sec)	N/A	0.023	0.305	0.079	0.000	0.406	0.000	0.146	1.243

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	168	0	131	32
normalized size	1	1.00	1.76	0.71	0.00	3.43	0.00	2.67	0.65
time (sec)	N/A	0.031	0.324	0.072	0.000	0.397	0.000	0.161	1.200

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	94	43	0	266	0	179	40
normalized size	1	1.00	1.54	0.70	0.00	4.36	0.00	2.93	0.66
time (sec)	N/A	0.042	0.797	0.072	0.000	0.404	0.000	0.152	1.200

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	141	322	348	2748	325	226	244
normalized size	1	1.00	0.99	2.27	2.45	19.35	2.29	1.59	1.72
time (sec)	N/A	0.209	0.818	0.016	0.331	0.456	13.420	0.167	1.282

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	109	246	219	1396	233	153	158
normalized size	1	1.00	1.08	2.44	2.17	13.82	2.31	1.51	1.56
time (sec)	N/A	0.123	0.923	0.017	0.333	0.422	6.235	0.144	1.238

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	173	136	654	175	99	97
normalized size	1	1.00	1.25	2.51	1.97	9.48	2.54	1.43	1.41
time (sec)	N/A	0.064	0.416	0.016	0.325	0.426	2.850	0.135	0.109

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	65	116	49	205	104	57	51
normalized size	1	1.00	1.71	3.05	1.29	5.39	2.74	1.50	1.34
time (sec)	N/A	0.024	0.132	0.019	0.305	0.407	1.355	0.121	0.104

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	76	52	62	236	62	55
normalized size	1	1.00	1.28	1.52	1.04	1.24	4.72	1.24	1.10
time (sec)	N/A	0.054	0.083	0.107	0.354	0.449	2.557	0.145	1.225

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	100	101	124	426	0	130	104
normalized size	1	1.00	1.18	1.19	1.46	5.01	0.00	1.53	1.22
time (sec)	N/A	0.095	1.559	0.122	0.326	0.425	0.000	0.133	1.300

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	134	166	322	1431	0	203	195
normalized size	1	1.00	1.04	1.29	2.50	11.09	0.00	1.57	1.51
time (sec)	N/A	0.180	3.621	0.124	0.342	0.436	0.000	0.179	1.399

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	214	230	522	3698	0	303	310
normalized size	1	1.00	1.27	1.36	3.09	21.88	0.00	1.79	1.83
time (sec)	N/A	0.264	6.221	0.128	0.373	0.503	0.000	0.176	1.386

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	28	49	42	29	22
normalized size	1	1.00	1.71	1.48	0.90	1.58	1.35	0.94	0.71
time (sec)	N/A	0.044	0.043	0.110	0.311	0.406	0.811	0.139	0.045

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	53	46	29	48	42	24	22
normalized size	1	1.00	1.71	1.48	0.94	1.55	1.35	0.77	0.71
time (sec)	N/A	0.042	0.040	0.108	0.302	0.403	0.805	0.117	0.044

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	128	63	0	2231	0	0	151
normalized size	1	1.00	1.73	0.85	0.00	30.15	0.00	0.00	2.04
time (sec)	N/A	0.070	3.285	0.194	0.000	0.509	0.000	0.000	1.408

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	62	0	2307	0	0	242
normalized size	1	1.00	0.99	0.84	0.00	31.18	0.00	0.00	3.27
time (sec)	N/A	0.068	0.154	0.114	0.000	0.498	0.000	0.000	1.471

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	110	36	93	0	42	34
normalized size	1	1.00	0.70	1.83	0.60	1.55	0.00	0.70	0.57
time (sec)	N/A	0.063	0.111	0.092	0.318	0.389	0.000	0.130	1.377

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	36	80	33	60	0	31	29
normalized size	1	1.00	1.24	2.76	1.14	2.07	0.00	1.07	1.00
time (sec)	N/A	0.047	0.081	0.093	0.314	0.398	0.000	0.113	1.267

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	30	70	22	50	0	30	22
normalized size	1	1.00	0.79	1.84	0.58	1.32	0.00	0.79	0.58
time (sec)	N/A	0.052	0.054	0.092	0.306	0.395	0.000	0.143	1.245

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	40	17	25	0	19	17
normalized size	1	1.00	1.11	2.11	0.89	1.32	0.00	1.00	0.89
time (sec)	N/A	0.036	0.051	0.088	0.306	0.393	0.000	0.125	1.242

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	7	11	6	9	0	6	6
normalized size	1	1.00	0.70	1.10	0.60	0.90	0.00	0.60	0.60
time (sec)	N/A	0.022	0.004	0.012	0.302	0.429	0.000	0.114	1.197

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	18	0	12	11
normalized size	1	1.00	1.00	1.14	1.00	2.57	0.00	1.71	1.57
time (sec)	N/A	0.036	0.003	0.073	0.302	0.412	0.000	0.128	1.176

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	14	23	31	77	0	26	29
normalized size	1	1.00	1.75	2.88	3.88	9.62	0.00	3.25	3.62
time (sec)	N/A	0.040	0.038	0.088	0.304	0.389	0.000	0.128	0.079

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	41	55	0	10	16
normalized size	1	1.00	1.00	2.91	3.73	5.00	0.00	0.91	1.45
time (sec)	N/A	0.034	0.026	0.100	0.304	0.399	0.000	0.129	1.176

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	354	166	1279	0	229	143
normalized size	1	1.00	1.01	2.28	1.07	8.25	0.00	1.48	0.92
time (sec)	N/A	0.240	0.294	0.132	0.320	0.428	0.000	0.129	1.629

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	171	197	0	1859	0	163	172
normalized size	1	1.00	1.28	1.47	0.00	13.87	0.00	1.22	1.28
time (sec)	N/A	0.237	0.826	0.128	0.000	0.455	0.000	0.130	1.865

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	75	175	83	331	0	114	85
normalized size	1	1.00	0.82	1.90	0.90	3.60	0.00	1.24	0.92
time (sec)	N/A	0.142	0.170	0.126	0.329	0.406	0.000	0.119	1.469

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	93	0	431	0	72	156
normalized size	1	1.00	1.10	1.27	0.00	5.90	0.00	0.99	2.14
time (sec)	N/A	0.107	0.464	0.116	0.000	0.437	0.000	0.139	1.496

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	147	0	35	35
normalized size	1	1.00	1.21	1.03	0.00	3.87	0.00	0.92	0.92
time (sec)	N/A	0.036	0.036	0.073	0.000	0.403	0.000	0.113	0.172

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	43	0	46	51
normalized size	1	1.00	1.67	1.08	1.00	3.58	0.00	3.83	4.25
time (sec)	N/A	0.044	0.063	0.089	0.310	0.406	0.000	0.113	0.156

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	65	115	0	384	0	85	230
normalized size	1	1.00	1.14	2.02	0.00	6.74	0.00	1.49	4.04
time (sec)	N/A	0.109	0.115	0.114	0.000	0.450	0.000	0.121	1.457

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	116	110	434	0	106	88
normalized size	1	1.00	1.25	2.90	2.75	10.85	0.00	2.65	2.20
time (sec)	N/A	0.067	0.168	0.124	0.333	0.417	0.000	0.117	1.442

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	118	36	92	0	42	34
normalized size	1	1.00	0.70	1.97	0.60	1.53	0.00	0.70	0.57
time (sec)	N/A	0.067	0.091	0.099	0.335	0.407	0.000	0.116	1.422

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	34	82	27	56	0	25	23
normalized size	1	1.00	1.36	3.28	1.08	2.24	0.00	1.00	0.92
time (sec)	N/A	0.177	0.058	0.093	0.306	0.372	0.000	0.109	1.315

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	24	78	22	51	0	30	22
normalized size	1	1.00	0.63	2.05	0.58	1.34	0.00	0.79	0.58
time (sec)	N/A	0.059	0.050	0.094	0.301	0.402	0.000	0.129	0.107

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	42	11	23	0	11	11
normalized size	1	1.00	1.12	2.47	0.65	1.35	0.00	0.65	0.65
time (sec)	N/A	0.113	0.018	0.091	0.301	0.410	0.000	0.128	1.186

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	16	19	12	23	0	10	10
normalized size	1	1.00	1.60	1.90	1.20	2.30	0.00	1.00	1.00
time (sec)	N/A	0.114	0.024	0.095	0.501	0.387	0.000	0.115	1.281

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	9	36	18	78	0	27	21
normalized size	1	1.00	0.60	2.40	1.20	5.20	0.00	1.80	1.40
time (sec)	N/A	0.041	0.032	0.106	0.404	0.415	0.000	0.110	1.189

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	45	33	140	0	25	22
normalized size	1	1.00	1.00	2.25	1.65	7.00	0.00	1.25	1.10
time (sec)	N/A	0.167	0.040	0.106	0.416	0.380	0.000	0.116	1.255

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	38	75	84	0	18	18
normalized size	1	1.00	1.00	2.24	4.41	4.94	0.00	1.06	1.06
time (sec)	N/A	0.045	0.052	0.105	0.302	0.367	0.000	0.111	0.074

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	160	0	0	231	0	149	-1
normalized size	1	1.00	7.62	0.00	0.00	11.00	0.00	7.10	-0.05
time (sec)	N/A	0.046	5.271	0.378	0.000	0.435	0.000	0.130	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	144	319	154	1229	0	216	135
normalized size	1	1.00	0.98	2.17	1.05	8.36	0.00	1.47	0.92
time (sec)	N/A	0.339	0.570	0.132	0.335	0.425	0.000	0.120	1.743

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	167	200	0	1873	0	164	262
normalized size	1	1.00	1.24	1.48	0.00	13.87	0.00	1.21	1.94
time (sec)	N/A	0.251	1.483	0.131	0.000	0.444	0.000	0.141	2.044

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	146	80	334	0	104	82
normalized size	1	1.00	0.86	1.72	0.94	3.93	0.00	1.22	0.96
time (sec)	N/A	0.164	0.258	0.125	0.325	0.410	0.000	0.116	1.398

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	431	0	71	158
normalized size	1	1.00	1.10	1.28	0.00	5.99	0.00	0.99	2.19
time (sec)	N/A	0.112	0.308	0.117	0.000	0.423	0.000	0.119	1.529

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	200	0	48	164
normalized size	1	1.00	1.20	1.08	0.00	4.00	0.00	0.96	3.28
time (sec)	N/A	0.138	0.123	0.125	0.000	0.464	0.000	0.117	3.223

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	59	46	117	0	76	323
normalized size	1	1.00	0.93	2.03	1.59	4.03	0.00	2.62	11.14
time (sec)	N/A	0.055	0.097	0.135	0.420	0.437	0.000	0.134	1.584

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	187	0	856	0	102	166
normalized size	1	1.00	1.02	2.25	0.00	10.31	0.00	1.23	2.00
time (sec)	N/A	0.237	0.200	0.175	0.000	0.469	0.000	0.124	3.938

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	257	133	909	0	201	123
normalized size	1	1.00	0.86	3.25	1.68	11.51	0.00	2.54	1.56
time (sec)	N/A	0.100	0.309	0.155	0.414	0.422	0.000	0.126	1.450

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	38	41	42	41	0	26	65
normalized size	1	1.00	1.23	1.32	1.35	1.32	0.00	0.84	2.10
time (sec)	N/A	0.103	0.053	0.173	0.428	0.415	0.000	0.127	0.513

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	96	55	571	0	47	69
normalized size	1	1.00	0.93	2.23	1.28	13.28	0.00	1.09	1.60
time (sec)	N/A	0.115	0.101	0.110	0.408	0.422	0.000	0.131	1.303

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	80	43	354	0	39	35
normalized size	1	1.00	0.89	2.16	1.16	9.57	0.00	1.05	0.95
time (sec)	N/A	0.099	0.056	0.108	0.408	0.410	0.000	0.128	1.206

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	65	29	186	0	35	29
normalized size	1	1.00	0.93	2.24	1.00	6.41	0.00	1.21	1.00
time (sec)	N/A	0.074	0.049	0.097	0.421	0.437	0.000	0.117	1.216

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	47	17	73	0	17	17
normalized size	1	1.00	1.21	2.47	0.89	3.84	0.00	0.89	0.89
time (sec)	N/A	0.042	0.029	0.100	0.416	0.407	0.000	0.126	1.170

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	10	26	27	10	14
normalized size	1	1.00	1.12	1.50	0.62	1.62	1.69	0.62	0.88
time (sec)	N/A	0.008	0.027	0.049	0.307	0.408	0.445	0.127	0.002

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	24	10	26	27	10	12
normalized size	1	1.00	1.12	1.50	0.62	1.62	1.69	0.62	0.75
time (sec)	N/A	0.022	0.017	0.050	0.305	0.397	0.479	0.127	0.049

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	24	24	73	92	18	21
normalized size	1	1.00	1.21	1.26	1.26	3.84	4.84	0.95	1.11
time (sec)	N/A	0.038	0.028	0.051	0.304	0.418	0.621	0.117	0.059

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	28	38	196	160	36	21
normalized size	1	1.00	0.87	0.90	1.23	6.32	5.16	1.16	0.68
time (sec)	N/A	0.055	0.043	0.058	0.354	0.422	0.934	0.118	1.164

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	54	357	197	40	29
normalized size	1	1.00	0.89	0.86	1.46	9.65	5.32	1.08	0.78
time (sec)	N/A	0.068	0.060	0.067	0.307	0.410	1.183	0.134	0.066

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	90	35	0	259	0	135	34
normalized size	1	1.00	2.00	0.78	0.00	5.76	0.00	3.00	0.76
time (sec)	N/A	0.052	0.139	0.050	0.000	0.387	0.000	0.151	1.238

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	53	26	0	131	0	71	25
normalized size	1	1.00	1.66	0.81	0.00	4.09	0.00	2.22	0.78
time (sec)	N/A	0.038	0.122	0.061	0.000	0.397	0.000	0.159	1.197

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	97	25	0	85	0	88	24
normalized size	1	1.00	3.23	0.83	0.00	2.83	0.00	2.93	0.80
time (sec)	N/A	0.039	0.199	0.077	0.000	0.396	0.000	0.173	1.230

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	84	35	0	166	0	107	32
normalized size	1	1.00	1.71	0.71	0.00	3.39	0.00	2.18	0.65
time (sec)	N/A	0.051	0.348	0.077	0.000	0.403	0.000	0.217	1.218

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	35	0	436	0	197	34
normalized size	1	1.00	1.56	0.78	0.00	9.69	0.00	4.38	0.76
time (sec)	N/A	0.063	0.263	0.066	0.000	0.404	0.000	0.174	1.248

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	61	26	0	242	0	133	25
normalized size	1	1.00	1.79	0.76	0.00	7.12	0.00	3.91	0.74
time (sec)	N/A	0.048	0.180	0.076	0.000	0.398	0.000	0.159	1.195

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	81	35	0	189	0	88	36
normalized size	1	1.00	1.93	0.83	0.00	4.50	0.00	2.10	0.86
time (sec)	N/A	0.059	0.374	0.089	0.000	0.403	0.000	0.177	1.256

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	35	0	166	0	135	31
normalized size	1	1.00	1.76	0.71	0.00	3.39	0.00	2.76	0.63
time (sec)	N/A	0.082	0.367	0.075	0.000	0.410	0.000	0.186	1.232

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	105	283	146	1294	0	141	163
normalized size	1	1.00	1.08	2.92	1.51	13.34	0.00	1.45	1.68
time (sec)	N/A	0.525	0.358	0.151	0.414	0.453	0.000	0.143	1.612

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	88	167	94	637	0	97	111
normalized size	1	1.00	1.16	2.20	1.24	8.38	0.00	1.28	1.46
time (sec)	N/A	0.326	0.326	0.144	0.411	0.435	0.000	0.119	1.511

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	64	110	67	264	0	74	73
normalized size	1	1.00	1.07	1.83	1.12	4.40	0.00	1.23	1.22
time (sec)	N/A	0.193	0.145	0.139	0.464	0.440	0.000	0.116	1.450

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	88	50	73	0	57	58
normalized size	1	1.00	0.90	1.73	0.98	1.43	0.00	1.12	1.14
time (sec)	N/A	0.082	0.094	0.121	0.420	0.454	0.000	0.133	0.324

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	37	42	148	43	42
normalized size	1	1.00	0.74	1.41	0.95	1.08	3.79	1.10	1.08
time (sec)	N/A	0.046	0.062	0.048	0.300	0.402	0.911	0.112	0.084

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	36	43	134	43	42
normalized size	1	1.00	0.74	1.41	0.92	1.10	3.44	1.10	1.08
time (sec)	N/A	0.058	0.059	0.049	0.631	0.439	0.927	0.111	0.064

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	60	63	76	372	59	57
normalized size	1	1.00	0.78	0.95	1.00	1.21	5.90	0.94	0.90
time (sec)	N/A	0.092	0.087	0.054	0.650	0.440	1.630	0.133	1.480

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	67	82	271	636	76	74
normalized size	1	1.00	1.00	1.05	1.28	4.23	9.94	1.19	1.16
time (sec)	N/A	0.130	0.137	0.073	0.313	0.446	2.398	0.118	1.509

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	88	76	119	648	882	100	110
normalized size	1	1.00	1.16	1.00	1.57	8.53	11.61	1.32	1.45
time (sec)	N/A	0.221	0.229	0.063	0.984	0.450	3.399	0.140	1.627

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	96	169	1299	1013	143	164
normalized size	1	1.00	1.15	1.02	1.80	13.82	10.78	1.52	1.74
time (sec)	N/A	0.392	0.322	0.079	0.525	0.457	5.020	0.118	1.631

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	73	68	184	0	169	68
normalized size	1	1.00	0.91	1.35	1.26	3.41	0.00	3.13	1.26
time (sec)	N/A	0.085	0.178	0.303	0.915	0.403	0.000	0.154	1.282

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	A	A	A	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	64	24	36	28	0	24	23
normalized size	1	0.00	2.13	0.80	1.20	0.93	0.00	0.80	0.77
time (sec)	N/A	0.029	0.031	0.104	0.381	0.388	0.000	0.126	1.251

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	B	A	A	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	64	83	48	62	0	54	39
normalized size	1	0.00	1.42	1.84	1.07	1.38	0.00	1.20	0.87
time (sec)	N/A	0.021	0.242	0.148	1.074	0.402	0.000	0.138	1.213

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	26	37	36	33	0	37	25
normalized size	1	0.00	1.13	1.61	1.57	1.43	0.00	1.61	1.09
time (sec)	N/A	0.016	0.188	0.109	0.307	0.429	0.000	0.138	1.226

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	B	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	58	71	45	58	0	51	36
normalized size	1	0.00	1.45	1.78	1.12	1.45	0.00	1.28	0.90
time (sec)	N/A	0.007	0.187	0.140	0.596	0.407	0.000	0.138	1.186

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	21	26	10	18	27	21	18
normalized size	1	1.00	1.75	2.17	0.83	1.50	2.25	1.75	1.50
time (sec)	N/A	0.014	0.030	0.014	0.300	0.424	0.976	0.133	1.212

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	B	A	A	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	62	93	47	54	0	52	37
normalized size	1	0.00	1.51	2.27	1.15	1.32	0.00	1.27	0.90
time (sec)	N/A	0.022	0.172	0.126	1.218	0.418	0.000	0.135	1.205

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	27	35	30	38	0	33	25
normalized size	1	0.00	1.29	1.67	1.43	1.81	0.00	1.57	1.19
time (sec)	N/A	0.021	0.163	0.099	0.499	0.407	0.000	0.133	1.214

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	A	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	0	86	41	53	61	0	40	40
normalized size	1	0.00	1.83	0.87	1.13	1.30	0.00	0.85	0.85
time (sec)	N/A	0.066	0.110	0.096	0.328	0.389	0.000	0.109	1.247

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	A	B	F	A	B
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	154	100	66	104	0	72	60
normalized size	1	0.00	2.26	1.47	0.97	1.53	0.00	1.06	0.88
time (sec)	N/A	0.048	2.945	0.125	0.419	0.414	0.000	0.112	1.237

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	A	B	F	A	B
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	163	54	53	74	0	54	42
normalized size	1	0.00	3.98	1.32	1.29	1.80	0.00	1.32	1.02
time (sec)	N/A	0.030	3.065	0.091	0.316	0.401	0.000	0.132	1.226

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	A	B	F	A	B
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	0	153	86	60	97	0	66	54
normalized size	1	0.00	2.55	1.43	1.00	1.62	0.00	1.10	0.90
time (sec)	N/A	0.010	2.239	0.128	0.416	0.408	0.000	0.138	1.205

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	28	35	19	28	32	21	28
normalized size	1	1.00	2.00	2.50	1.36	2.00	2.29	1.50	2.00
time (sec)	N/A	0.024	0.054	0.014	0.311	0.432	7.362	0.125	1.188

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	A	A	F	A	B
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	0	153	114	69	97	0	77	60
normalized size	1	0.00	1.78	1.33	0.80	1.13	0.00	0.90	0.70
time (sec)	N/A	0.045	2.851	0.128	0.415	0.413	0.000	0.115	1.212

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	A	A	F	A	B
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	0	155	55	50	82	0	57	48
normalized size	1	0.00	2.58	0.92	0.83	1.37	0.00	0.95	0.80
time (sec)	N/A	0.046	3.250	0.086	0.332	0.396	0.000	0.138	1.230

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	0	46	0	0	0	0	0	-1
normalized size	1	0.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.102	0.108	0.000	0.405	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	77	0	0	0	0	0	-1
normalized size	1	0.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.177	0.092	0.000	0.404	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	108	0	0	0	0	0	-1
normalized size	1	0.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.233	0.149	0.000	0.399	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	259	0	0	0	0	0	-1
normalized size	1	0.00	3.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	2.058	0.270	0.000	0.407	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	126	0	0	0	0	0	-1
normalized size	1	0.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	3.230	0.114	0.000	0.407	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	0	83	0	0	0	0	0	-1
normalized size	1	0.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.432	0.138	0.000	0.409	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	125	0	0	0	0	0	-1
normalized size	1	0.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.560	0.162	0.000	0.442	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	0	142	0	0	0	0	0	-1
normalized size	1	0.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.599	0.154	0.000	0.857	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	223	0	0	0	0	0	-1
normalized size	1	0.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.947	0.153	0.000	1.749	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0	-1
normalized size	1	0.00	2.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	1.757	0.176	0.000	0.415	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0	-1
normalized size	1	0.00	2.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	1.949	0.209	0.000	0.406	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	171	0	0	0	0	0	-1
normalized size	1	0.00	2.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	2.009	0.188	0.000	0.422	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	198	0	0	0	0	0	-1
normalized size	1	0.00	3.41	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	7.164	1.252	0.000	0.403	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	207	0	0	0	0	0	-1
normalized size	1	0.00	3.34	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	4.999	1.147	0.000	0.437	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	0	193	0	0	0	0	0	-1
normalized size	1	0.00	3.57	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	6.986	1.140	0.000	0.442	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	0	198	0	0	0	0	0	-1
normalized size	1	0.00	3.81	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	8.363	1.013	0.000	0.402	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	40	56	24	76	0	74	34
normalized size	1	1.00	1.60	2.24	0.96	3.04	0.00	2.96	1.36
time (sec)	N/A	0.021	0.066	0.016	0.313	0.415	0.000	0.307	1.193

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	197	0	0	0	0	0	-1
normalized size	1	0.00	3.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	3.793	1.094	0.000	0.411	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	0	191	0	0	0	0	0	-1
normalized size	1	0.00	3.47	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	3.763	1.133	0.000	0.415	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	0	155	0	0	0	0	0	-1
normalized size	1	0.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	6.724	1.159	0.000	0.411	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	0	165	0	0	0	0	0	-1
normalized size	1	0.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	4.623	1.117	0.000	0.422	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	0	151	0	0	0	0	0	-1
normalized size	1	0.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	6.625	1.118	0.000	0.412	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	160	0	0	0	0	0	-1
normalized size	1	0.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	7.879	1.092	0.000	0.417	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	49	80	37	72	0	37	34
normalized size	1	1.00	1.75	2.86	1.32	2.57	0.00	1.32	1.21
time (sec)	N/A	0.029	0.107	0.018	0.828	0.431	0.000	0.302	1.189

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	0	158	0	0	0	0	0	-1
normalized size	1	0.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	3.630	1.125	0.000	0.416	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	156	0	0	0	0	0	-1
normalized size	1	0.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	3.593	1.152	0.000	0.535	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	52	67	330	572	0	127	95
normalized size	1	1.00	1.21	1.56	7.67	13.30	0.00	2.95	2.21
time (sec)	N/A	0.040	0.217	0.023	0.429	0.632	0.000	0.296	1.225

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	86	499	171	0	67	163
normalized size	1	1.00	0.98	1.91	11.09	3.80	0.00	1.49	3.62
time (sec)	N/A	0.037	0.118	0.021	0.453	0.512	0.000	0.314	1.208

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	88	855	1576	0	161	229
normalized size	1	1.00	1.02	1.33	12.95	23.88	0.00	2.44	3.47
time (sec)	N/A	0.057	0.289	0.021	0.500	0.423	0.000	0.336	1.199

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	158	0	0	0	0	0	-1
normalized size	1	0.00	1.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	13.477	1.582	0.000	0.528	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	312	0	0	0	0	0	-1
normalized size	1	0.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	14.881	1.560	0.000	0.575	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	306	0	600	0	0	0	0	0	-1
normalized size	1	0.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.069	16.828	1.688	0.000	0.464	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	0	387	0	0	0	0	0	-1
normalized size	1	0.00	3.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	3.973	0.281	0.000	0.403	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	174	0	0	0	0	0	-1
normalized size	1	0.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	5.420	0.260	0.000	0.431	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	93	0	626	0	0	65
normalized size	1	1.00	0.88	1.27	0.00	8.58	0.00	0.00	0.89
time (sec)	N/A	0.051	0.287	0.222	0.000	0.454	0.000	0.000	2.241

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	57	92	0	334	0	0	51
normalized size	1	1.00	0.81	1.31	0.00	4.77	0.00	0.00	0.73
time (sec)	N/A	0.050	0.149	0.127	0.000	0.473	0.000	0.000	1.860

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	72	0	305	0	0	39
normalized size	1	1.00	1.00	1.50	0.00	6.35	0.00	0.00	0.81
time (sec)	N/A	0.039	0.079	0.129	0.000	0.450	0.000	0.000	1.505

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	0	303	0	0	36
normalized size	1	1.00	1.00	0.94	0.00	6.45	0.00	0.00	0.77
time (sec)	N/A	0.040	0.115	0.137	0.000	0.490	0.000	0.000	1.644

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	93	0	625	0	0	65
normalized size	1	1.00	0.62	1.31	0.00	8.80	0.00	0.00	0.92
time (sec)	N/A	0.052	0.152	0.138	0.000	0.460	0.000	0.000	1.742

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	92	0	1104	0	0	64
normalized size	1	1.00	0.64	1.28	0.00	15.33	0.00	0.00	0.89
time (sec)	N/A	0.051	0.206	0.136	0.000	0.525	0.000	0.000	2.391

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	266	149	0	8951	0	0	-1
normalized size	1	1.00	1.97	1.10	0.00	66.30	0.00	0.00	-0.01
time (sec)	N/A	0.353	9.076	0.225	0.000	2.028	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	199	90	0	6695	0	0	-1
normalized size	1	1.00	1.90	0.86	0.00	63.76	0.00	0.00	-0.01
time (sec)	N/A	0.217	26.602	0.197	0.000	1.416	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	141	52	0	1752	0	0	-1
normalized size	1	1.00	2.43	0.90	0.00	30.21	0.00	0.00	-0.02
time (sec)	N/A	0.117	18.601	0.158	0.000	1.097	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	203	0	0	6705	0	0	-1
normalized size	1	1.00	1.92	0.00	0.00	63.25	0.00	0.00	-0.01
time (sec)	N/A	0.240	7.737	0.656	0.000	1.395	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	278	0	0	9148	0	0	-1
normalized size	1	1.00	1.52	0.00	0.00	49.99	0.00	0.00	-0.01
time (sec)	N/A	0.332	11.079	0.626	0.000	1.920	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	304	559	0	7964	0	0	-1
normalized size	1	1.00	2.30	4.23	0.00	60.33	0.00	0.00	-0.01
time (sec)	N/A	0.231	8.419	0.153	0.000	2.308	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	164	320	167	1617	0	181	-1
normalized size	1	1.00	0.51	1.00	0.52	5.07	0.00	0.57	-0.00
time (sec)	N/A	0.912	10.276	0.982	0.428	0.440	0.000	1.067	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	334	298	112	613	0	155	-1
normalized size	1	1.00	1.70	1.51	0.57	3.11	0.00	0.79	-0.01
time (sec)	N/A	0.284	3.818	0.848	0.424	0.406	0.000	0.988	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	213	56	70	0	94	-1
normalized size	1	1.00	0.61	2.57	0.67	0.84	0.00	1.13	-0.01
time (sec)	N/A	0.144	0.062	0.976	0.423	0.408	0.000	0.370	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	218	35	53	0	60	-1
normalized size	1	1.00	0.61	2.63	0.42	0.64	0.00	0.72	-0.01
time (sec)	N/A	0.194	0.126	0.931	0.427	0.392	0.000	1.329	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	104	301	90	458	0	130	-1
normalized size	1	1.00	0.54	1.56	0.47	2.37	0.00	0.67	-0.01
time (sec)	N/A	0.864	0.306	0.877	0.420	0.426	0.000	0.332	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	133	324	145	1226	0	185	-1
normalized size	1	1.00	0.43	1.04	0.47	3.94	0.00	0.59	-0.00
time (sec)	N/A	1.743	0.482	0.873	0.421	0.491	0.000	0.688	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	296	0	0	-1
normalized size	1	1.00	0.79	0.75	0.00	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.375	0.263	0.134	0.000	0.437	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	155	0	0	-1
normalized size	1	1.00	0.77	0.89	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.174	0.139	0.000	0.476	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	58	0	149	0	0	-1
normalized size	1	1.00	0.77	0.75	0.00	1.94	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.137	0.132	0.000	0.439	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	3.546	0.582	0.000	0.443	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	298	0	0	-1
normalized size	1	1.00	0.79	0.75	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.373	0.261	0.143	0.000	0.482	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	102	0	155	0	0	-1
normalized size	1	1.00	0.77	0.89	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.180	0.137	0.000	0.459	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	151	0	0	-1
normalized size	1	1.00	0.81	0.75	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.114	0.143	0.000	0.468	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	6.631	0.506	0.000	0.423	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [109] had the largest ratio of [.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	12	0.500
2	A	6	6	1.00	12	0.500
3	A	6	6	1.00	12	0.500
4	A	5	5	1.00	12	0.417
5	A	5	5	1.00	12	0.417
6	A	6	6	1.00	12	0.500
7	A	6	6	1.00	12	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	7	6	1.00	12	0.500
9	A	13	9	1.00	12	0.750
10	A	12	8	1.00	12	0.667
11	A	9	9	1.00	12	0.750
12	A	9	9	1.00	12	0.750
13	A	12	8	1.00	12	0.667
14	A	13	9	1.00	12	0.750
15	A	2	2	1.00	8	0.250
16	A	2	2	1.00	10	0.200
17	A	3	3	1.00	12	0.250
18	A	3	3	1.00	14	0.214
19	A	2	2	1.00	14	0.143
20	A	2	2	1.00	14	0.143
21	A	3	3	1.00	14	0.214
22	A	14	10	1.00	14	0.714
23	A	14	10	1.00	14	0.714
24	A	13	9	1.00	14	0.643
25	A	13	9	1.00	14	0.643
26	A	14	10	1.00	14	0.714
27	A	14	10	1.00	14	0.714
28	A	3	3	1.00	12	0.250
29	A	8	7	1.00	14	0.500
30	A	7	7	1.00	14	0.500
31	A	7	7	1.00	14	0.500
32	A	8	7	1.00	14	0.500
33	A	4	3	1.00	14	0.214
34	A	3	3	1.00	14	0.214
35	A	2	2	1.00	14	0.143
36	A	2	2	1.00	14	0.143
37	A	3	3	1.00	14	0.214
38	A	4	3	1.00	14	0.214
39	A	3	3	1.00	12	0.250
40	A	5	3	1.00	14	0.214
41	A	3	3	1.00	14	0.214
42	A	3	3	1.00	14	0.214
43	A	5	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	16	10	1.00	14	0.714
45	A	14	10	1.00	14	0.714
46	A	14	10	1.00	14	0.714
47	A	14	10	1.00	14	0.714
48	A	14	10	1.00	14	0.714
49	A	16	10	1.00	14	0.714
50	A	3	3	1.00	12	0.250
51	A	3	3	1.00	14	0.214
52	A	3	3	1.00	14	0.214
53	A	3	3	1.00	14	0.214
54	A	3	3	1.00	14	0.214
55	A	3	3	1.00	14	0.214
56	A	3	3	1.00	14	0.214
57	A	3	3	1.00	14	0.214
58	A	3	3	1.00	14	0.214
59	A	3	3	1.00	14	0.214
60	A	3	3	1.00	14	0.214
61	A	5	3	1.00	6	0.500
62	A	4	3	1.00	6	0.500
63	A	3	3	1.00	6	0.500
64	A	2	2	1.00	6	0.333
65	A	2	2	1.00	6	0.333
66	A	3	2	1.00	6	0.333
67	A	4	2	1.00	6	0.333
68	A	5	2	1.00	6	0.333
69	A	6	2	1.00	6	0.333
70	A	5	3	1.00	8	0.375
71	A	4	3	1.00	8	0.375
72	A	3	3	1.00	8	0.375
73	A	2	2	1.00	8	0.250
74	A	3	3	1.00	8	0.375
75	A	4	3	1.00	8	0.375
76	A	5	3	1.00	8	0.375
77	A	5	4	1.00	12	0.333
78	A	4	4	1.00	12	0.333
79	A	3	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	2	2	1.00	12	0.167
81	A	2	2	1.00	12	0.167
82	A	3	3	1.00	12	0.250
83	A	4	4	1.00	12	0.333
84	A	5	4	1.00	12	0.333
85	A	2	2	1.00	12	0.167
86	A	2	2	1.00	12	0.167
87	A	5	4	1.00	14	0.286
88	A	5	4	1.00	14	0.286
89	A	4	3	1.00	11	0.273
90	A	3	2	1.00	11	0.182
91	A	4	3	1.00	11	0.273
92	A	2	2	1.00	9	0.222
93	A	1	1	1.00	9	0.111
94	A	2	2	1.00	11	0.182
95	A	2	2	1.00	11	0.182
96	A	2	1	1.00	11	0.091
97	A	5	4	1.00	13	0.308
98	A	9	6	1.00	13	0.462
99	A	4	3	1.00	13	0.231
100	A	5	5	1.00	11	0.454
101	A	2	2	1.00	11	0.182
102	A	2	2	1.00	13	0.154
103	A	5	5	1.00	13	0.385
104	A	3	2	1.00	13	0.154
105	A	5	4	1.00	11	0.364
106	A	9	7	1.00	11	0.636
107	A	5	4	1.00	11	0.364
108	A	8	6	1.00	9	0.667
109	A	8	7	1.00	9	0.778
110	A	3	2	1.00	11	0.182
111	A	8	7	1.00	11	0.636
112	A	4	3	1.00	11	0.273
113	A	4	4	1.00	13	0.308
114	A	5	3	1.00	13	0.231
115	A	10	9	1.00	13	0.692

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	4	3	1.00	13	0.231
117	A	6	6	1.00	11	0.546
118	A	6	5	1.00	11	0.454
119	A	3	2	1.00	13	0.154
120	A	9	7	1.00	13	0.538
121	A	3	2	1.00	13	0.154
122	A	6	5	1.00	13	0.385
123	A	6	4	1.00	11	0.364
124	A	5	4	1.00	11	0.364
125	A	4	4	1.00	11	0.364
126	A	4	4	1.00	9	0.444
127	A	2	2	1.00	6	0.333
128	A	2	2	1.00	9	0.222
129	A	3	2	1.00	11	0.182
130	A	3	3	1.00	11	0.273
131	A	4	4	1.00	11	0.364
132	A	4	4	1.00	11	0.364
133	A	3	3	1.00	11	0.273
134	A	3	3	1.00	11	0.273
135	A	4	4	1.00	11	0.364
136	A	4	4	1.00	13	0.308
137	A	3	3	1.00	13	0.231
138	A	4	4	1.00	13	0.308
139	A	4	4	1.00	13	0.308
140	A	6	6	1.00	13	0.462
141	A	5	5	1.00	13	0.385
142	A	4	4	1.00	13	0.308
143	A	3	3	1.00	11	0.273
144	A	2	2	1.00	8	0.250
145	A	2	2	1.00	11	0.182
146	A	4	4	1.00	13	0.308
147	A	5	5	1.00	13	0.385
148	A	6	6	1.00	13	0.462
149	A	7	7	1.00	13	0.538
150	A	3	3	1.00	14	0.214
151	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	F	0	0	N/A	0	N/A
155	A	2	1	1.00	11	0.091
156	F	0	0	N/A	0	N/A
157	F	0	0	N/A	0	N/A
158	F	0	0	N/A	0	N/A
159	F	0	0	N/A	0	N/A
160	F	0	0	N/A	0	N/A
161	F	0	0	N/A	0	N/A
162	A	3	2	1.00	13	0.154
163	F	0	0	N/A	0	N/A
164	F	0	0	N/A	0	N/A
165	F	0	0	N/A	0	N/A
166	F	0	0	N/A	0	N/A
167	F	0	0	N/A	0	N/A
168	F	0	0	N/A	0	N/A
169	F	0	0	N/A	0	N/A
170	F	0	0	N/A	0	N/A
171	F	0	0	N/A	0	N/A
172	F	0	0	N/A	0	N/A
173	F	0	0	N/A	0	N/A
174	F	0	0	N/A	0	N/A
175	F	0	0	N/A	0	N/A
176	F	0	0	N/A	0	N/A
177	F	0	0	N/A	0	N/A
178	F	0	0	N/A	0	N/A
179	F	0	0	N/A	0	N/A
180	F	0	0	N/A	0	N/A
181	A	2	1	1.00	17	0.059
182	F	0	0	N/A	0	N/A
183	F	0	0	N/A	0	N/A
184	F	0	0	N/A	0	N/A
185	F	0	0	N/A	0	N/A
186	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
187	F	0	0	N/A	0	N/A
188	A	3	2	1.00	19	0.105
189	F	0	0	N/A	0	N/A
190	F	0	0	N/A	0	N/A
191	A	3	2	1.00	17	0.118
192	A	4	2	1.00	17	0.118
193	A	4	2	1.00	17	0.118
194	F	0	0	N/A	0	N/A
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	F	0	0	N/A	0	N/A
199	A	7	6	1.00	19	0.316
200	A	7	6	1.00	19	0.316
201	A	6	5	1.00	19	0.263
202	A	6	5	1.00	19	0.263
203	A	7	6	1.00	19	0.316
204	A	7	6	1.00	19	0.316
205	A	8	7	1.00	23	0.304
206	A	7	6	1.00	23	0.261
207	A	4	4	1.00	21	0.190
208	A	8	5	1.00	21	0.238
209	A	11	6	1.00	23	0.261
210	A	8	7	1.00	21	0.333
211	A	9	7	1.00	25	0.280
212	A	8	7	1.00	25	0.280
213	A	4	4	1.00	25	0.160
214	A	4	4	1.00	25	0.160
215	A	8	7	1.00	25	0.280
216	A	9	7	1.00	25	0.280
217	A	19	5	1.00	9	0.556
218	A	13	5	1.00	9	0.556
219	A	9	4	1.00	7	0.571
220	A	0	0	0.00	0	0.000
221	A	19	5	1.00	9	0.556
222	A	13	5	1.00	9	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
223	A	9	4	1.00	7	0.571
224	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int (b\coth(c+dx))^{7/2}\,dx$

Optimal. Leaf size=97

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

```
[Out] b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2/5*b*(b*coth(d*x+c))^(5/2)/d-2*b^3*(b*coth(d*x+c))^(
1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3473, 3476, 329, 212, 206, 203}

$$-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(7/2),x]
```

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Coth[c + d*x]])/d - (2*b*(b*Cot
h[c + d*x])^(5/2))/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int (b\coth(c+dx))^{7/2}\,dx &= -\frac{2b(b\coth(c+dx))^{5/2}}{5d} + b^2\int (b\coth(c+dx))^{3/2}\,dx \\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d} + b^4\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx \\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d} - \frac{b^5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(-b^2+x^2\right)}\,dx, x, b\coth(c\right.}{d} \\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d} - \frac{\left(2b^5\right)\mathrm{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(\right.}{d} \\
&= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d} + \frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+d\right.}{d} \\
&= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b^3\sqrt{b\coth(c+dx)}}{d} - 2b(b\,\mathrm{co}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 83, normalized size = 0.86

$$\frac{b^3\sqrt{b\coth(c+dx)}\left(-2\coth^{\frac{5}{2}}(c+dx) - 10\sqrt{\coth(c+dx)} + 5\tan^{-1}\left(\sqrt{\coth(c+dx)}\right) + 5\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right.\right.}{5d\sqrt{\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Coth[c + d*x]]*(5*ArcTan[Sqrt[Coth[c + d*x]]] + 5*ArcTanh[Sqrt[
Coth[c + d*x]]] - 10*Sqrt[Coth[c + d*x]] - 2*Coth[c + d*x]^(5/2)))/(5*d*Sqr
t[Coth[c + d*x]])
```

fricas [B] time = 0.47, size = 1574, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3
*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 -
 b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*(b^3*
cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4
 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x +
 c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b)
*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4
+ 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*c
osh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*b^3*cosh(d*x + c)^
4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*co
sh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 +
 4*(3*b^3*cosh(d*x + c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh
(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
 c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 -
d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)
+ d), 1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b))
+ 5*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d
*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*s
inh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*
sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + si
nh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 +
 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d
*x + c)/sinh(d*x + c)) - b) - 16*(3*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x +
 c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*cosh(d*x + c)^2 + 3*b^3
 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x +
c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh(d*x + c)/sinh(d*x +
c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 +
4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)]
```

giac [B] time = 0.42, size = 379, normalized size = 3.91

$$-\frac{10\, b^{\frac{7}{2}} \arctan\left(-\frac{\sqrt{b}\, e^{(2\,dx+2\,c)} - \sqrt{b e^{(4\,dx+4\,c)} - b}}{\sqrt{b}}\right) \operatorname{sgn}\left(e^{(2\,dx+2\,c)} - 1\right) + 5\, b^{\frac{7}{2}} \log\left(\left|-\sqrt{b}\, e^{(2\,dx+2\,c)} + \sqrt{b e^{(4\,dx+4\,c)} - b}\right|\right) \operatorname{sgn}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/10*(10*b^(7/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
 - b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) + 5*b^(7/2)*log(abs(-sqrt(b)*e^(2*
d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) - 16*(5
*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4*b^4*sgn(e^(2*d*x
 + 2*c) - 1) - 10*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3
*b^(9/2)*sgn(e^(2*d*x + 2*c) - 1) + 20*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^
```

```
(4*d*x + 4*c) - b))^2*b^5*sgn(e^(2*d*x + 2*c) - 1) - 10*(sqrt(b)*e^(2*d*x +
 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))*b^(11/2)*sgn(e^(2*d*x + 2*c) - 1) + 3*
b^6*sgn(e^(2*d*x + 2*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x +
4*c) - b) - sqrt(b))^5)/d
```

maple [A] time = 0.13, size = 80, normalized size = 0.82

$$\frac{b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b\left(b\coth\left(dx+c\right)\right)^{\frac{5}{2}}}{5d}-\frac{2b^{3}\sqrt{b\coth\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(7/2),x)
```

```
[Out] b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2/5*b*(b*coth(d*x+c))^(5/2)/d-2*b^3*(b*coth(d*x+c))^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(7/2), x)
```

mupad [B] time = 1.58, size = 83, normalized size = 0.86

$$\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,\mathrm{coth}\left(c+d\,x\right)}}{\sqrt{b}}\right)}{d}-\frac{2\,b^3\,\sqrt{b\,\mathrm{coth}\left(c+d\,x\right)}}{d}-\frac{2\,b\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{5/2}}{5\,d}-\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,\mathrm{coth}\left(c+d\,x\right)}\,1{}\mathrm{i}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(7/2),x)
```

```
[Out] (b^(7/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b^3*(b*coth(c + d*x)
)^(1/2))/d - (2*b*(b*coth(c + d*x))^(5/2))/(5*d) - (b^(7/2)*atan(((b*coth(c
 + d*x))^(1/2)*1i)/b^(1/2))*1i)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(c+dx\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(7/2), x)
```

3.2 $\int (b\coth(c+dx))^{5/2}\,dx$

Optimal. Leaf size=78

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

```
[Out] -b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3476, 329, 298, 203, 206}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(5/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Coth[c + d*x])^(3/2))/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int (b\coth(c+dx))^{5/2}\,dx &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} + b^2\int \sqrt{b\coth(c+dx)}\,dx\\
&= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} - \frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\
&= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{b+x^2}\right.}{}\\
&= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(c+dx))^{3/2}}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 68, normalized size = 0.87

$$
-\frac{(b\coth(c+dx))^{5/2}\left(2\coth^{\frac{3}{2}}(c+dx) + 3\tan^{-1}\left(\sqrt{\coth(c+dx)}\right) - 3\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{3d\coth^{\frac{5}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((b*Coth[c + d*x])^(5/2)*(3*ArcTan[Sqrt[Coth[c + d*x]]] - 3*ArcTanh[Sq
rt[Coth[c + d*x]]] + 2*Coth[c + d*x]^(3/2)))/(d*Coth[c + d*x]^(5/2))
```

fricas [B] time = 0.47, size = 988, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*si
nh(d*x + c)^2 - b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/
(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 - b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*s
inh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4))
 + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2
*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d), -1/12*(6*(b^2*cosh
(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2
)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 3*(b^2*c
osh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 -
```

```
b^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)
^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)) - b) + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x +
 c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*
x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2 - d)]
```

giac [B] time = 0.36, size = 224, normalized size = 2.87

$$\frac{6\,b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)-3\,b^{\frac{5}{2}}\log\left(\left|-\sqrt{b}\,e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)\operatorname{sgn}\left(\cdots\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b^(5/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) - 3*b^(5/2)*log(abs(-sqrt(b)*e^(2*d*x
 + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) + 8*(3*(sq
rt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*b^3*sgn(e^(2*d*x + 2
*c) - 1) + b^4*sgn(e^(2*d*x + 2*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*
e^(4*d*x + 4*c) - b) - sqrt(b))^3)/d
```

maple [A] time = 0.11, size = 63, normalized size = 0.81

$$-\frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b\left(b\coth\left(dx+c\right)\right)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(5/2),x)
```

```
[Out] -b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(5/2), x)
```

mupad [B] time = 1.37, size = 62, normalized size = 0.79

$$\frac{b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,\mathrm{coth}\left(c+d\,x\right)}}{\sqrt{b}}\right)}{d}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,\mathrm{coth}\left(c+d\,x\right)}}{\sqrt{b}}\right)}{d}-\frac{2\,b\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{3/2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(5/2),x)
```

```
[Out] (b^(5/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (b^(5/2)*atan((b*coth(
c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))^(3/2))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(5/2), x)
```

3.3 $\int (b\coth(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

```
[Out] b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3473, 3476, 329, 212, 206, 203}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Coth[c + d*x]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rubi steps

$$
\begin{aligned}
\int (b\coth(c+dx))^{3/2}\,dx &= -\frac{2b\sqrt{b\coth(c+dx)}}{d}+b^2\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx\\
&= -\frac{2b\sqrt{b\coth(c+dx)}}{d}-\frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(-b^2+x^2)}\,dx,x,b\coth(c+dx)\right)}{d}\\
&= -\frac{2b\sqrt{b\coth(c+dx)}}{d}-\frac{\left(2b^3\right)\mathrm{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{d}\\
&= -\frac{2b\sqrt{b\coth(c+dx)}}{d}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\coth(c+dx)}\right)}{d}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{b+x^2}\,d\right.}{}\\
&= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 61, normalized size = 0.81

$$\frac{(b\coth(c+dx))^{3/2}\left(-2\sqrt{\coth(c+dx)}+\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(3/2),x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[
c + d*x]])*(b*Coth[c + d*x])^(3/2))/(d*Coth[c + d*x]^(3/2))
```

fricas [B] time = 0.44, size = 637, normalized size = 8.49

$$\left[-\frac{2\sqrt{-b}\,b\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}\,b\log\left(-\frac{b\cosh(dx+c)^4+4b\cosh(dx\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt
(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/d, 1/4*(2*b^(3/2)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) + b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^
```

```
3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*b*sqrt(b*cosh(d*x + c
)/sinh(d*x + c)))/d]
```

giac [B] time = 0.26, size = 168, normalized size = 2.24

$$-\frac{\left(2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(2dx+2c)}-1\right)+\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)\operatorname{sg}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) + sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x
+ 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) - 8*b*sgn(e
^(2*d*x + 2*c) - 1)/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b)
- sqrt(b)))*b/d
```

maple [A] time = 0.13, size = 62, normalized size = 0.83

$$\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\coth\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(3/2),x)
```

```
[Out] b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(3/2), x)
```

mupad [B] time = 1.27, size = 61, normalized size = 0.81

$$\frac{b^{3/2}\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d}-\frac{2\,b\,\sqrt{b\coth\left(c+d\,x\right)}}{d}+\frac{b^{3/2}\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(3/2),x)
```

```
[Out] (b^(3/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))^
(1/2))/d + (b^(3/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(3/2), x)
```

3.4 $\int \sqrt{b \coth(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3476

Rubi steps

$$\int \sqrt{b\coth(c+dx)}\,dx = -\frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$
$$= -\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.88

$$\frac{\sqrt{b\coth(c+dx)}\left(\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)-\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\sqrt{\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ((-ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[b*Coth[
c + d*x]])/(d*Sqrt[Coth[c + d*x]])
```

fricas [B] time = 0.44, size = 594, normalized size = 10.24

$$\left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(b)*arct
an(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(b)*log(2*b*cosh(d
*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d
*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(co
sh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh
(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
 cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
- b))/d]
```

giac [B] time = 0.19, size = 101, normalized size = 1.74

$$\frac{\left(2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)-\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)\right)\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
b))/sqrt(b)) - sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x +
 4*c) - b))))*sgn(e^(2*d*x + 2*c) - 1)/d
```

maple [A] time = 0.13, size = 47, normalized size = 0.81

$$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(1/2),x)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)), x)
```

mupad [B] time = 1.22, size = 41, normalized size = 0.71

$$-\frac{\sqrt{b}\,\left(\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)-\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(1/2),x)
```

```
[Out] -(b^(1/2)*(atan((b*coth(c + d*x))^(1/2)/b^(1/2)) - atanh((b*coth(c + d*x))^
(1/2)/b^(1/2))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)), x)
```

3.5 $\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-b^2+x^2\right)}\,dx, x, b\coth(c+dx)\right)}{d} \\
&= -\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.86

$$\frac{\sqrt{\coth(c+dx)}\left(\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[Coth[c +
 d*x]])/(d*Sqrt[b*Coth[c + d*x]])
```

fricas [B] time = 0.43, size = 598, normalized size = 10.49

$$\left[-\frac{2\sqrt{-b}\,\arctan\left(\frac{\left(\cosh(dx+c)^2+2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)+\sqrt{-b}\,\log\left(-\frac{b\cosh(dx+c)^4+4\,b\cosh(d}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), 1/4*(2*sqrt(b)*a
rctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(b)*log(2*b*cos
h(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*
(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - b))/(b*d)]
```

giac [B] time = 0.28, size = 102, normalized size = 1.79

$$-\frac{\frac{2\,\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\log\left(\left|-\sqrt{b}\,e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{\sqrt{b}}}{2\,d\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqr
t(b))/sqrt(b) + log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) -
 b)))/sqrt(b))/(d*sgn(e^(2*d*x + 2*c) - 1))
```

maple [A] time = 0.12, size = 46, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(1/2),x)
```

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)), x)
```

mupad [B] time = 1.29, size = 38, normalized size = 0.67

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b\,\mathrm{coth}(c+d\,x)}}{\sqrt{b}}\right)+\mathrm{atanh}\left(\frac{\sqrt{b\,\mathrm{coth}(c+d\,x)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(1/2),x)
```

```
[Out] (atan((b*coth(c + d*x))^(1/2)/b^(1/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(1
/2)))/(b^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)), x)
```

3.6 $\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-3/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Coth[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx &= -\frac{2}{bd\sqrt{b\coth(c+dx)}}+\frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^2}\\ &= -\frac{2}{bd\sqrt{b\coth(c+dx)}}-\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{bd}\\ &= -\frac{2}{bd\sqrt{b\coth(c+dx)}}-\frac{2\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd}\\ &= -\frac{2}{bd\sqrt{b\coth(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd}-\frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x,\right.}{bd}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2}{bd\sqrt{b\coth(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.08, size = 36, normalized size = 0.46

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(c+dx)\right)}{bd\sqrt{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(b*d*Sqrt[b*Coth[c +
d*x]])
```

fricas [B] time = 0.44, size = 923, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
```

```
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*s
inh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2
- cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sq
rt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^
2*d*sinh(d*x + c)^2 + b^2*d)]
```

giac [B] time = 0.58, size = 196, normalized size = 2.51

$$\frac{\frac{(\pi+\log(|b|)+8)\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}{\sqrt{b}}+\frac{4\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}-\frac{2\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{\sqrt{b}\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}-\frac{\cdots}{\left(\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(\cdots}}\right.}}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + log(abs(b)) + 8)*sgn(e^(2*d*x + 2*c) - 1)/sqrt(b) + 4*arctan(-(s
qrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt(b))/(sqrt(b)*sgn
(e^(2*d*x + 2*c) - 1)) - 2*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d
*x + 4*c) - b)))/(sqrt(b)*sgn(e^(2*d*x + 2*c) - 1)) - 16/((sqrt(b)*e^(2*d*x
 + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))*sgn(e^(2*d*x + 2*c) - 1)))
/(b*d)
```

maple [A] time = 0.10, size = 65, normalized size = 0.83

$$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}-\frac{2}{bd\sqrt{b\coth\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(3/2),x)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-3/2), x)
```

mupad [B] time = 1.39, size = 64, normalized size = 0.82

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d}-\frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d}-\frac{2}{b\,d\,\sqrt{b\coth\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(3/2),x)
```

```
[Out] atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(3/2)*d) - atan((b*coth(c + d*x))
^(1/2)/b^(1/2))/(b^(3/2)*d) - 2/(b*d*(b*coth(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-3/2), x)
```

3.7 $\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3474, 3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d}-\frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = -\frac{2}{3bd(b\coth(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx}{b^2}$$

$$= -\frac{2}{3bd(b\coth(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-b^2+x^2\right)}\,dx, x, b\coth(c+dx)\right)}{bd}$$

$$= -\frac{2}{3bd(b\coth(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd}$$

$$= -\frac{2}{3bd(b\coth(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{b^2d} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

Mathematica [C] time = 0.07, size = 38, normalized size = 0.48

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\coth^2(c+dx)\right)}{3bd(b\coth(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[c + d*x]^2])/(3*b*d*(b*Coth[c + d*
x])^(3/2))
```

fricas [B] time = 0.46, size = 1428, normalized size = 18.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*
cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2
+ 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1
)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
```

```
c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d
*x + c))*sinh(d*x + c) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cos
h(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4 + 2*b^3*d*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*s
inh(d*x + c)^2 + 4*(b^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x +
 c)), 1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(b
)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x +
 c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c
)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
 c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*
x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqr
t(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x +
c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c
) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*
d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 + 2*b^3*d*cosh(d*x
+ c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*sinh(d*x + c)^2 + 4*(b
^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 0.61, size = 239, normalized size = 3.03

$$-\frac{\frac{\left(3\pi-3\log\left(|b|\right)-8\right)\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}{b^{\frac{5}{2}}}+\frac{12\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}+\frac{6\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)}{b^{\frac{5}{2}}\mathrm{sgn}\left(e^{(2dx+2c)}-1\right)}+\frac{16\left(3\right.}{\left(\sqrt{b}e^{(2dx+2c}\right.}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*((3*pi - 3*log(abs(b)) - 8)*sgn(e^(2*d*x + 2*c) - 1)/b^(5/2) + 12*arc
tan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt(b))/(b^(5
/2)*sgn(e^(2*d*x + 2*c) - 1)) + 6*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b
*e^(4*d*x + 4*c) - b)))/(b^(5/2)*sgn(e^(2*d*x + 2*c) - 1)) + 16*(3*(sqrt(b)
*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2 + b)/((sqrt(b)*e^(2*d*x +
 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))^3*b^2*sgn(e^(2*d*x + 2*c) -
1)))/d
```

maple [A] time = 0.10, size = 64, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}-\frac{2}{3bd\left(b\coth\left(dx+c\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(5/2),x)
```

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-5/2), x)
```

mupad [B] time = 1.49, size = 63, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d}-\frac{2}{3\,b\,d\,\left(b\coth\left(c+d\,x\right)\right)^{3/2}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(5/2),x)
```

```
[Out] atan((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d) - 2/(3*b*d*(b*coth(c + d*
x))^(3/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-5/2), x)
```

3.8 $\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+c)
)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{2}{b^3d\sqrt{b\coth(c+dx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Coth[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Coth[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = -\frac{2}{5bd(b\coth(c+dx))^{5/2}} + \frac{\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx}{b^2}$$
$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} + \frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^4}$$
$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx\right.}{b^3d}$$
$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c\right.}{b^3d}$$
$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx\right.}{b^3d}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\cot}}$$

Mathematica [C] time = 0.10, size = 38, normalized size = 0.38

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};\coth^2(c+dx)\right)}{5bd(b\coth(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, Coth[c + d*x]^2])/(5*b*d*(b*Coth[c + d
*x])^(5/2))
```

fricas [B] time = 0.49, size = 2132, normalized size = 21.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x +
c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5
*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4
+ 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x
+ c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arc
tan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqr
t(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 5*(cosh(d*x + c)^6 + 6*cosh
(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x +
```

```
c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*
x + c))*sinh(d*x + c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh
(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sin
h(d*x + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 +
 4*(15*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)
^4 + 6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d
*x + c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*
sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d
*cosh(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 3*(5*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^2 + 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), -1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x
+ c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*co
sh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3
*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x +
 c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*
(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5
*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x +
 c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*
x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*
cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*log(2*b*cosh(d*
x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cos
h(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(
d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) -
 b) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x
 + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 + 4*(1
5*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 +
6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d*x +
c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(
d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cosh
(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^4 +
 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5
*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 +
 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x + c))*
sinh(d*x + c))]
```

giac [B] time = 0.63, size = 359, normalized size = 3.59

$$\frac{\frac{\left(5\,\pi+5\,\log\left(|b|\right)+48\right)\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}{b^{\frac{7}{2}}}+\frac{20\,\arctan\left(-\frac{\sqrt{b}\,e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}-\frac{10\,\log\left(\left|-\sqrt{b}\,e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{b^{\frac{7}{2}}\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-1\right)}-\frac{32\left(5\left(\sqrt{b}\,e^{(2\cdots\right.\right.}{\cdots}}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/20*((5*pi + 5*log(abs(b)) + 48)*sgn(e^(2*d*x + 2*c) - 1)/b^(7/2) + 20*arc
tan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt(b))/(b^(7
```

```
/2)*sgn(e^(2*d*x + 2*c) - 1)) - 10*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(
b*e^(4*d*x + 4*c) - b)))/(b^(7/2)*sgn(e^(2*d*x + 2*c) - 1)) - 32*(5*(sqrt(b
)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4 + 10*(sqrt(b)*e^(2*d*x +
 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3*sqrt(b) + 20*(sqrt(b)*e^(2*d*x + 2*c
) - sqrt(b*e^(4*d*x + 4*c) - b))^2*b + 10*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b
*e^(4*d*x + 4*c) - b))*b^(3/2) + 3*b^2)/((sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*
e^(4*d*x + 4*c) - b) + sqrt(b))^5*b^3*sgn(e^(2*d*x + 2*c) - 1)))/d
```

maple [A] time = 0.10, size = 83, normalized size = 0.83

$$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}-\frac{2}{5bd\left(b\coth\left(dx+c\right)\right)^{\frac{5}{2}}}-\frac{2}{b^{3}d\sqrt{b\coth\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(7/2),x)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+c)
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-7/2), x)
```

mupad [B] time = 1.60, size = 80, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d}-\frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d}-\frac{\frac{2}{5\,b}+\frac{2\coth(c+d\,x)^2}{b}}{d\,\left(b\coth\left(c+d\,x\right)\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(7/2),x)
```

```
[Out] atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(7/2)*d) - atan((b*coth(c + d*x))
^(1/2)/b^(1/2))/(b^(7/2)*d) - (2/(5*b) + (2*coth(c + d*x)^2)/b)/(d*(b*coth(
c + d*x))^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-7/2), x)
```

3.9 $\int (b\coth(c+dx))^{4/3}\,dx$

Optimal. Leaf size=236

$$-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c\right.}{4d}$$

```
[Out] b^(4/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d-3*b*(b*coth(d*x+c))^(1/3)/
d-1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3
))/d+1/4*b^(4/3)*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(
2/3))/d-1/2*b^(4/3)*arctan(1/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))
*3^(1/2)/d+1/2*b^(4/3)*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/
2))*3^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] -(Sqrt[3]*b^(4/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*d) + (Sqrt[3]*b^(4/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))
/Sqrt[3]])/(2*d) + (b^(4/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (
3*b*(b*Coth[c + d*x])^(1/3))/d - (b^(4/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c +
 d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d) + (b^(4/3)*Log[b^(2/3) + b^(
1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (b\coth(c+dx))^{4/3}\,dx = -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + b^2\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{(3b^3)\,\text{Subst}\left(\int \frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + \frac{b^{4/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{4/3}\,\text{Su}}{}$$

$$= \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x,\right.}{4d}$$

$$= \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+}\right.}{4d}$$

$$= -\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\,\text{c}}}{}\right.}{d}$$

Mathematica [C] time = 0.03, size = 36, normalized size = 0.15

$$\frac{3b\sqrt[3]{b\coth(c+dx)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] (3*b*(b*Coth[c + d*x])^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c + 
d*x]^2]))/d
```

fricas [A] time = 0.44, size = 292, normalized size = 1.24

$$-\frac{2\sqrt{3}\,(-b)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}\,b+2\sqrt{3}\,(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) - 2\sqrt{3}\,b^{\frac{4}{3}}\arctan\left(-\frac{\sqrt{3}\,b-2\sqrt{3}\,b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right) + (-b)^{\frac{1}{3}}\,b\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*b*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(4/3)*arctan(-1/3*(s
qrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-
b)^(1/3)*b*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(4/3)*log(b^(2/3) - b^(1/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/
3)) - 2*(-b)^(1/3)*b*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
) - 2*b^(4/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*b*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Minimal poly. in rootof must be fraction fre
e Error: Bad Argument ValueMinimal poly. in rootof must be fraction free Er
ror: Bad Argument ValueMinimal poly. in rootof must be fraction free Error:
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lification only. This might return a wrong answer if simplifying 0/0!Unable
to build a single algebraic extension for simplifying.Trying rational simp

```
lification only. This might return a wrong answer if simplifying 0/0!Unable
 to build a single algebraic extension for simplifying.Trying rational simp
lification only. This might return a wrong answer if simplifying 0/0!Evalua
tion time: 1.92Done
```

maple [A] time = 0.11, size = 209, normalized size = 0.89

$$-\frac{3b\left(b\coth\left(dx+c\right)\right)^{\frac{1}{3}}}{d}-\frac{b^{\frac{4}{3}}\ln\left(\left(b\coth\left(dx+c\right)\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2d}+\frac{b^{\frac{4}{3}}\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}\left(b\coth\left(dx+c\right)\right)^{\frac{1}{3}}+\left(b\coth\left(dx+c\right)\right)^{\frac{2}{3}}\right)}{4d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(4/3),x)
```

```
[Out] -3*b*(b*coth(d*x+c))^(1/3)/d-1/2*b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)-b^(1/3)
)+1/4*b^(4/3)*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3
))/d+1/2*b^(4/3)*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^
(1/2)/d+1/2*b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4*b^(4/3)*ln(b^(2
/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/2*b^(4/3)/d*3^
(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(4/3), x)
```

mupad [B] time = 1.88, size = 249, normalized size = 1.06

$$-\frac{3\,b\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}}{d}-\frac{b^{4/3}\,\mathrm{atan}\left(\frac{{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}\,1{}\mathrm{i}}{b^{1/3}}\right)\,1{}\mathrm{i}}{d}-\frac{b^{4/3}\,\ln\left(\frac{486\,b^{37/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d^4}-\frac{486\,b^{12}\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}}{d^4}\right)\,\left(-\frac{1}{2}+\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(4/3),x)
```

```
[Out] (b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/4 - 1/4))/d^4 + (486*b^12*(b*coth(
c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(4/3)*atan(((b*coth(c
+ d*x))^(1/3)*1i)/b^(1/3))*1i)/d - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*1i)
/2 - 1/2))/d^4 - (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 -
1/2))/(2*d) - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*1i)/2 + 1/2))/d^4 - (486
*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) - (3*b*(b
*coth(c + d*x))^(1/3))/d + (b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/4 + 1/4
))/d^4 + (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 + 1/4))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(c+dx\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(4/3), x)
```

3.10 $\int (b\coth(c+dx))^{2/3}\,dx$

Optimal. Leaf size=218

$$-\frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c-\right.}{4d}$$

```
[Out] b^(2/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d-1/4*b^(2/3)*ln(b^(2/3)-b^(
1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/4*b^(2/3)*ln(b^(2/3)+
b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/2*b^(2/3)*arctan(1
/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d-1/2*b^(2/3)*arcta
n(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}+\frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c-\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])
/(2*d) - (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/
Sqrt[3]])/(2*d) + (b^(2/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (b
^(2/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2
/3)])/(4*d) + (b^(2/3)*Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*C
oth[c + d*x])^(2/3)])/(4*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (b\coth(c+dx))^{2/3}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\,\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\ldots}\right)}{d}$$

$$= \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} + \frac{b^{2/3}\,\text{S}\ldots}{\ldots}$$

$$= \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}\ldots$$

$$= \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+\ldots}}{\sqrt[3]{b}}\right)}{d}\ldots$$

Mathematica [A] time = 0.19, size = 149, normalized size = 0.68

$$\frac{(b\coth(c+dx))^{2/3}\left(-\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)+\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)+\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x])^(2/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt
[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[Co
th[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + L
og[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2/3
))
```

fricas [A] time = 0.43, size = 310, normalized size = 1.42

$$-\frac{2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+2\sqrt{3}\left(b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}\left(b^2\right)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+\left(-b^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 2*sqrt(3)*(b^2)^(1/3)*arctan(
-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/
3))/b) + (-b^2)^(1/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^
(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + (b^2)^(1/3)
*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(1/3)*log(b*(b*cosh(d*x +
 c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) - 2*(b^2)^(1/3)*log(b*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/d
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Minimal poly. in rootof must be fraction fre
e Error: Bad Argument ValueMinimal poly. in rootof must be fraction free Er
ror: Bad Argument ValueMinimal poly. in rootof must be fraction free Error:
 Bad Argument ValueMinimal poly. in rootof must be fraction free Error: Bad
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lification only. This might return a wrong answer if simplifying 0/0!Unable
 to build a single algebraic extension for simplifying.Trying rational simp
lification only. This might return a wrong answer if simplifying 0/0!Evalua
tion time: 1.67Done
```

maple [A] time = 0.11, size = 193, normalized size = 0.89

$$-\frac{b^{\frac{2}{3}}\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2d}+\frac{b^{\frac{2}{3}}\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4d}-\frac{b^{\frac{2}{3}}\arctan\left(\left(\left(1+\frac{2(b\coth\cdots}{\cdots}\right.\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(2/3),x)
```

```
[Out] -1/2*b^(2/3)/d*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*b^(2/3)*ln(b^(2/3)+b^(
1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d-1/2*b^(2/3)*arctan(1/3*
(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d+1/2*b^(2/3)/d*ln((b*
coth(d*x+c))^(1/3)+b^(1/3))-1/4*b^(2/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^
(1/3)+(b*coth(d*x+c))^(2/3))/d-1/2*b^(2/3)/d*3^(1/2)*arctan(1/3*3^(1/2)*(2*
(b*coth(d*x+c))^(1/3)/b^(1/3)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(2/3), x)
```

mupad [B] time = 1.50, size = 233, normalized size = 1.07

$$-\frac{b^{2/3}\,\mathrm{atan}\left(\frac{(b\coth(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)1\mathrm{i}}{d}-\frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{972\,b^{26/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\coth(c+d\,x))^{1/3}}{d^3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}-\frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(2/3),x)
```

```
[Out] (b^(2/3)*log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth(
c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(2/3)*log((972*b^9)/d^
3 - (972*b^(26/3)*((3^(1/2)*1i)/2 - 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*((3^
(1/2)*1i)/2 - 1/2))/(2*d) - (b^(2/3)*log((972*b^9)/d^3 - (972*b^(26/3)*((3^
(1/2)*1i)/2 + 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2))/(2
*d) - (b^(2/3)*atan(((b*coth(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d + (b^(2/3)*
log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 + 1/4)*(b*coth(c + d*x))
^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(2/3), x)
```

3.11 $\int \sqrt[3]{b \coth(c+dx)}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4d}-\frac{\sqrt{3}\sqrt[3]{b}\tan$$

```
[Out] -1/2*b^(1/3)*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(b^(4/3)+b^(
2/3)*(b*coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/d-1/2*b^(1/3)*arctan(1/3*
(b^(2/3)+2*(b*coth(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3476, 329, 275, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4d}-\frac{\sqrt{3}\sqrt[3]{b}\tan$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] -(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(
2/3))])/(2*d) - (b^(1/3)*Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)])/(2*d) + (b
^(1/3)*Log[b^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4
/3)])/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{b\coth(c+dx)}\,dx &= -\frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(3b)\,\mathrm{Subst}\left(\int \frac{x^3}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}\\
&= -\frac{(3b)\,\mathrm{Subst}\left(\int \frac{x}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}\\
&= -\frac{\sqrt[3]{b}\ \mathrm{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\ \mathrm{Subst}\left(\int \frac{-b^{2/3}+x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\,\mathrm{cot}\right.}{2d}\\
&= -\frac{\sqrt[3]{b}\ \log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\ \mathrm{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2}\right.}{4d}\\
&= -\frac{\sqrt[3]{b}\ \log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\ \log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(\right.}{4d}\\
&= -\frac{\sqrt{3}\,\sqrt[3]{b}\ \tan^{-1}\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d}-\frac{\sqrt[3]{b}\ \log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\ \log\left(b^{4/3}+\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 38, normalized size = 0.29

$$\frac{3(b\coth(c+dx))^{4/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\coth^2(c+dx)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] (3*(b*Coth[c + d*x])^(4/3)*Hypergeometric2F1[2/3, 1, 5/3, Coth[c + d*x]^2])
/(4*b*d)
```

fricas [B] time = 0.41, size = 291, normalized size = 2.20

$$-\frac{2\sqrt{3}\,(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,b-2\,\sqrt{3}\,(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}}{3\,b}\right)-2\,(-b)^{\frac{1}{3}}\log\left(-(-b)^{\frac{2}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+(-b)^{\frac{1}{3}}\log\left(\frac{(\cosh(\cdots}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b)^(1/3)*(b
*cosh(d*x + c)/sinh(d*x + c))^(2/3))/b) - 2*(-b)^(1/3)*log(-(-b)^(2/3) + (b
*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(1/3)*log(((cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)))/d
```

giac [B] time = 0.37, size = 217, normalized size = 1.64

$$-\frac{b\left(\frac{2\sqrt{3}\,|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^{2}}-\frac{|b|^{\frac{4}{3}}\log\left(\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2dx+2c)}+b\right)\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{e^{(2dx+2c)}-1}\right)}{b^{2}}+\frac{2\,|b|^{\frac{4}{3}}\log\left(\left|\left(\frac{be^{(2d\cdots}}{e^{(2dx\cdots}}\right.\right.\cdots}{b^{\cdots}}\cdots\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) +
b)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 - abs(b)^
(4/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3
) + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d
*x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 + 2*abs(b)^(4/3)*log(abs((
(b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/
d
```

maple [A] time = 0.10, size = 115, normalized size = 0.87

$$-\frac{b\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^{2}\right)^{\frac{1}{3}}\right)}{2d\left(b^{2}\right)^{\frac{1}{3}}}+\frac{b\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^{2}\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^{2}\right)^{\frac{2}{3}}\right)}{4d\left(b^{2}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\cdots}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^(1/3),x)
```

```
[Out] -1/2*b/d/(b^2)^(1/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))+1/4*b/d/(b^2)^(1
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
-1/2*b/d*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+
c))^(2/3)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(1/3), x)
```

mupad [B] time = 1.52, size = 146, normalized size = 1.11

$$\frac{(-b)^{1/3}\ln\left(81\,(-b)^{16/3}\,(b\coth(c+d\,x))^{2/3}-81\,b^6\right)}{2\,d}-\frac{(-b)^{1/3}\ln\left(-\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\coth(c+d\,x))^{2/3}}{d^4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^(1/3),x)
```

```
[Out] ((-b)^(1/3)*log(81*(-b)^(16/3)*(b*coth(c + d*x))^(2/3) - 81*b^6))/(2*d) - (
(-b)^(1/3)*log(- (81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b*c
oth(c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*log((
162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth(c + d*x))^(2/3))/d^4 - (81*b
^6)/d^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\coth(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(1/3), x)
```

3.12 $\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx$

Optimal. Leaf size=132

$$-\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d}+\frac{\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}\,d}+\frac{\sqrt{3}\tan^{-1}\left(\frac{b^{2/3}+2(b}{}\right.}{2\sqrt[3]{b}\,d}$$

```
[Out] -1/2*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/b^(1/3)/d+1/4*ln(b^(4/3)+b^(2/3)*(b*
coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/b^(1/3)/d+1/2*arctan(1/3*(b^(2/3)
+2*(b*coth(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/b^(1/3)/d
```

Rubi [A] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3476, 329, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d}+\frac{\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}\,d}+\frac{\sqrt{3}\tan^{-1}\left(\frac{b^{2/3}+2(b}{}\right.}{2\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(
2*b^(1/3)*d) - Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)]/(2*b^(1/3)*d) + Log[b
^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4/3)]/(4*b^(1
/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\left(-b^2+x^2\right)}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(3b)\,\text{Subst}\left(\int \frac{x}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}\\
&= -\frac{(3b)\,\text{Subst}\left(\int \frac{1}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} - \frac{\text{Subst}\left(\int \frac{-2b^{2/3}-x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+d\right.}{2\sqrt[3]{b}\,d}\\
&= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{4\sqrt[3]{b}\,d} + \frac{(\cdots}{\cdots}\\
&= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx)\right.}{4\sqrt[3]{b}\,d}\\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}\,d} - \frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 98, normalized size = 0.74

$$\frac{\sqrt[3]{\coth(c+dx)}\left(-2\log\left(1-\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(\coth^{\frac{4}{3}}(c+dx)+\coth^{\frac{2}{3}}(c+dx)+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\coth^{\frac{2}{3}}(c+dx}{\sqrt{3}}\right.\right.}{4d\sqrt[3]{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Coth[c + d*x]^(1/3)*(2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(2/3))/Sqrt[3]]
 - 2*Log[1 - Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(2/3) + Coth[c +
d*x]^(4/3)]))/(4*d*(b*Coth[c + d*x])^(1/3))
```

fricas [B] time = 0.45, size = 1598, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log((3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*
cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*(-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - sqrt(3)*((cosh(d*x
 + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x
 + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cos
h(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(
2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - 2
*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)
^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)/b) + 4*(3*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 3*b)/(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(-b)^(2/3)*log(-(-b)^(2/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*co
sh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*s
inh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh
(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 - 1)))/(b*d), 1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan((2*sqrt
(3)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(2/3)*sqrt(-(-b)^(1/3)/b) + sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*
x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 4*sqrt(3)*(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b))/(9*b*cosh(d*x + c
)^4 + 36*b*cosh(d*x + c)*sinh(d*x + c)^3 + 9*b*sinh(d*x + c)^4 + 14*b*cosh(
d*x + c)^2 + 2*(27*b*cosh(d*x + c)^2 + 7*b)*sinh(d*x + c)^2 + 4*(9*b*cosh(d
*x + c)^3 + 7*b*cosh(d*x + c))*sinh(d*x + c) + 9*b)) - 2*(-b)^(2/3)*log(-(-
b)^(2/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)
*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c
)/sinh(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)))/(b*d)]
```

giac [B] time = 0.64, size = 216, normalized size = 1.64

$$\frac{b\left(\frac{2\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3|b|^{\frac{2}{3}}}\right)}{b^2}+\frac{|b|^{\frac{2}{3}}\log\left(\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2dx+2c)}+b\right)\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{e^{(2dx+2c)}-1}\right)}{b^2}-\frac{2|b|^{\frac{2}{3}}\log\left(\left|\left(\frac{be^{(2dx+2c)}+\ldots}{e^{(2dx+2c)}-1}\right.\right.\right.}{b^2}\ldots\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) + b
)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 + abs(b)^(
2/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3)
 + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d*
x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 - 2*abs(b)^(2/3)*log(abs(((
b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/d
```

maple [A] time = 0.09, size = 115, normalized size = 0.87

$$-\frac{b\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{2d\left(b^2\right)^{\frac{2}{3}}}+\frac{b\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^2\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{4d\left(b^2\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\arctan\left(\ldots\right)}{2d\left(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(1/3),x)
```

```
[Out] -1/2*b/d/(b^2)^(2/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))+1/4*b/d/(b^2)^(2
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
+1/2*b/d/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+
c))^(2/3)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\coth(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-1/3), x)
```

mupad [B] time = 1.65, size = 147, normalized size = 1.11

$$\frac{\ln\left(162(-b)^{11/3}+162b^3(b\coth(c+dx))^{2/3}\right)}{2(-b)^{1/3}d}+\frac{\ln\left(\frac{81(-b)^{11/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}+\frac{162b^3(b\coth(c+dx))^{2/3}}{d^3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4(-b)^{1/3}d}-\frac{\ln\left(\frac{81\ldots}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(1/3),x)
```

```
[Out] log(162*(-b)^(11/3) + 162*b^3*(b*coth(c + d*x))^(2/3))/(2*(-b)^(1/3)*d) + (
log((81*(-b)^(11/3)*(3^(1/2)*1i - 1))/d^3 + (162*b^3*(b*coth(c + d*x))^(2/3
))/d^3)*(3^(1/2)*1i - 1))/(4*(-b)^(1/3)*d) - (log((81*(-b)^(11/3)*(3^(1/2)*
1i + 1))/d^3 - (162*b^3*(b*coth(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))/(4*
(-b)^(1/3)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b \coth(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-1/3), x)
```

3.13 $\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}-$$

```
[Out] arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d-1/4*ln(b^(2/3)-b^(1/3)*(b*
coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d+1/4*ln(b^(2/3)+b^(1/3)*
(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d-1/2*arctan(1/3*(1-2*
(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d+1/2*arctan(1/3*(1
+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d
```

Rubi [A] time = 0.24, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(
2/3)*d) + (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]
])/(2*b^(2/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) - L
og[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*
b^(2/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*
x])^(2/3)]/(4*b^(2/3)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d} \\
&= -\frac{(3b)\,\text{Subst}\left(\int \frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\cot}\right.}{b^{2/3}d} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4b^{2/3}d} + \frac{\text{Subst}\left(\int\right.}{} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d} + \\
&= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 36, normalized size = 0.17

$$
\frac{3\sqrt[3]{b\coth(c+dx)}\;{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)}{bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] (3*(b*Coth[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])
/(b*d)
```

fricas [B] time = 0.46, size = 356, normalized size = 1.63

$$2\sqrt{3}\,b\sqrt{-\left(-b^{2}\right)^{\frac{1}{3}}}\,\arctan\left(-\frac{\sqrt{3}\left(-b^{2}\right)^{\frac{1}{3}}b\sqrt{-\left(-b^{2}\right)^{\frac{1}{3}}}-2\sqrt{3}\left(-b^{2}\right)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\sqrt{-\left(-b^{2}\right)^{\frac{1}{3}}}}{3\,b^{2}}\right)+2\sqrt{3}\left(b^{2}\right)^{\frac{1}{6}}b\arctan\left(-\frac{\sqrt{3}\left(b^{2}\right)^{\frac{1}{6}}\left(\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sq
rt(-(-b^2)^(1/3)) - 2*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3)*sqrt(-(-b^2)^(1/3)))/b^2) + 2*sqrt(3)*(b^2)^(1/6)*b*arctan(-1/3*sqrt(
3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b^2) + (-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
 (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - (b^
2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2
)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(2/3)*log(b*(b*co
sh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 2*(b^2)^(2/3)*log(b*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/(b^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Minimal poly. in rootof must be fraction fre
e Error: Bad Argument ValueMinimal poly. in rootof must be fraction free Er
ror: Bad Argument ValueMinimal poly. in rootof must be fraction free Error:
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ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free Error: Bad Argument ValueMinimal poly. in rootof must be fraction free
 Error: Bad Argument ValueMinimal poly. in rootof must be fraction free Err
or: Bad Argument ValueMinimal poly. in rootof must be fraction free Error:
Bad Argument ValueMinimal poly. in rootof must be fraction free Error: Bad
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lification only. This might return a wrong answer if simplifying 0/0!Unable
 to build a single algebraic extension for simplifying.Trying rational simp
lification only. This might return a wrong answer if simplifying 0/0!Evalua
tion time: 1.62Done
```

maple [A] time = 0.09, size = 193, normalized size = 0.89

$$-\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2b^{\frac{2}{3}}d}+\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}d}+\frac{\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{2b^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(2/3),x)
```

```
[Out] -1/2/b^(2/3)/d*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*ln(b^(2/3)+b^(1/3)*(b*
coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d+1/2*arctan(1/3*(1+2*(b*
coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d+1/2/b^(2/3)/d*ln((b*
coth(d*x+c))^(1/3)+b^(1/3))-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b
*coth(d*x+c))^(2/3))/b^(2/3)/d+1/2/b^(2/3)/d*3^(1/2)*arctan(1/3*3^(1/2)*(2*
(b*coth(d*x+c))^(1/3)/b^(1/3)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\coth(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-2/3), x)
```

mupad [B] time = 1.39, size = 197, normalized size = 0.90

$$\frac{\operatorname{atanh}\left(\frac{(b\coth(c+d\,x))^{1/3}}{b^{1/3}}\right)}{b^{2/3}\,d}-\frac{\operatorname{atan}\left(\frac{b^{10/3}\,(b\coth(c+d\,x))^{1/3}\,243\mathrm{i}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}-\frac{243\,\sqrt{3}\,b^{10/3}\,(b\coth(c+d\,x))^{1/3}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)1\mathrm{i}}{2\,b^{2/3}\,d}-\frac{\operatorname{atan}\left(\frac{b^{10/3}\,(b\,\mathrm{co}}{243\,b^{11/}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(2/3),x)
```

```
[Out] atanh((b*coth(c + d*x))^(1/3)/b^(1/3))/(b^(2/3)*d) - (atan((b^(10/3)*(b*cot
h(c + d*x))^(1/3)*243i)/(3^(1/2)*b^(11/3)*243i - 243*b^(11/3)) - (243*3^(1/
2)*b^(10/3)*(b*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i - 243*b^(11/3))
)*(3^(1/2)*1i + 1)*1i)/(2*b^(2/3)*d) - (atan((b^(10/3)*(b*coth(c + d*x))^(1
/3)*243i)/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)) + (243*3^(1/2)*b^(10/3)*(b
*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)))*(3^(1/2)*1i
- 1)*1i)/(2*b^(2/3)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth\left(c + dx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-2/3), x)
```

3.14 $\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=238

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+$$

```
[Out] arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-3/b/d/(b*coth(d*x+c))^(1/3
)-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/
3)/d+1/4*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^
(4/3)/d+1/2*arctan(1/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)
/b^(4/3)/d-1/2*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1
/2)/b^(4/3)/d
```

Rubi [A] time = 0.32, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^(-4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4
/3)*d) - (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*b^(4/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d) - 3/
(b*d*(b*Coth[c + d*x])^(1/3)) - Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/
3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth
[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\int (b\coth(c+dx))^{2/3}\,dx}{b^2}$$
$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{bd}$$
$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{3\,\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{bd}$$
$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{4/3}d} + \frac{\text{Subst}\left(\int \frac{-}{b^{2/3}}\right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c\right.}{4b^{4/3}d}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\log\left(b^{2/3} - \sqrt[3]{b}\sqrt[3]{b\coth(c+dx)} + (b\,c\right.}{4b^{4/3}d}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{}{bd}$$

Mathematica [C] time = 0.06, size = 36, normalized size = 0.15

$$-\frac{3\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{bd\sqrt[3]{b\coth(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(b*d*(b*Coth[c + d*x]
)^(1/3))
```

fricas [B] time = 0.51, size = 3348, normalized size = 14.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
```

```
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*
sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x
 + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-
b)^(1/3)/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*s
qrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*
sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*
sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1
/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-
b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x +
 c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log(
(-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 +
2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2
/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*s
inh(d*x + c)^2 + b^2*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*
x + c)^2 + 6*b*cosh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))*sqrt((-b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3))
- (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b
^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cos
h(d*x + c)/sinh(d*x + c))^(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
```

```
h(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x +
 c)/sinh(d*x + c))^(1/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 12*(cosh(d*x + c)^2 + 2*cosh(d*
x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))
^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)/b)*arctan(-
1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cosh(d*x + c)/s
inh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - (cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1
/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))
^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
+ (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*co
sh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/s
inh(d*x + c))^(1/3)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) + 12*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d
*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c
) + b^2*d*sinh(d*x + c)^2 + b^2*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Minimal poly. in rootof must be fraction fre
e Error: Bad Argument ValueMinimal poly. in rootof must be fraction free Er
ror: Bad Argument ValueMinimal poly. in rootof must be fraction free Error:
 Bad Argument ValueMinimal poly. in rootof must be fraction free Error: Bad
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 Error: Bad Argument ValueMinimal poly. in rootof must be fraction free Err
or: Bad Argument ValueMinimal poly. in rootof must be fraction free Error:
Bad Argument ValueMinimal poly. in rootof must be fraction free Error: Bad
Argument ValueMinimal poly. in rootof must be fraction free Error: Bad Argu
ment ValueMinimal poly. in rootof must be fraction free Error: Bad Argument
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ueMinimal poly. in rootof must be fraction free Error: Bad Argument ValueMi
nimal poly. in rootof must be fraction free Error: Bad Argument ValueUnable
 to build a single algebraic extension for simplifying.Trying rational simp
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 to build a single algebraic extension for simplifying.Trying rational simp
lification only. This might return a wrong answer if simplifying 0/0!Unable
 to build a single algebraic extension for simplifying.Trying rational simp
lification only. This might return a wrong answer if simplifying 0/0!Evalua
tion time: 2.15Done
```

maple [A] time = 0.10, size = 211, normalized size = 0.89

$$-\frac{3}{bd\left(b\coth\left(dx+c\right)\right)^{\frac{1}{3}}}-\frac{\ln\left(\left(b\coth\left(dx+c\right)\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2b^{\frac{4}{3}}d}+\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}\left(b\coth\left(dx+c\right)\right)^{\frac{1}{3}}+\left(b\coth\left(dx+c\right)\right)^{\frac{2}{3}}\right)}{4b^{\frac{4}{3}}d}-\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c))^(4/3),x)
```

```
[Out] -3/b/d/(b*coth(d*x+c))^(1/3)-1/2/b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)-b^(1/3)
)+1/4*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/
3)/d-1/2*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^
(4/3)/d+1/2/b^(4/3)/d*ln((b*coth(d*x+c))^(1/3)+b^(1/3))-1/4*ln(b^(2/3)-b^(1
/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/3)/d-1/2/b^(4/3)/d*3^
(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1/3)-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(-4/3), x)
```

mupad [B] time = 1.42, size = 165, normalized size = 0.69

$$-\frac{3}{b\,d\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}}-\frac{\mathrm{atan}\left(\frac{{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}\,1{}\mathrm{i}}{b^{1/3}}\right)\,1{}\mathrm{i}}{b^{4/3}\,d}-\frac{\mathrm{atan}\left(\frac{b^9\,d^4\,{\left(b\,\mathrm{coth}\left(c+d\,x\right)\right)}^{1/3}\,486{}\mathrm{i}}{243\,b^{28/3}\,d^4-\sqrt{3}\,b^{28/3}\,d^4\,243{}\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,b^{4/3}\,d}+\frac{\mathrm{atan}\left(\frac{b^9}{243}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x))^(4/3),x)
```

```
[Out] (atan((b^9*d^4*(b*coth(c + d*x))^(1/3)*486i)/(243*b^(28/3)*d^4 + 3^(1/2)*b^
(28/3)*d^4*243i))*(3^(1/2)*1i - 1)*1i)/(2*b^(4/3)*d) - (atan(((b*coth(c + d
*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(4/3)*d) - (atan((b^9*d^4*(b*coth(c + d*x))^
(1/3)*486i)/(243*b^(28/3)*d^4 - 3^(1/2)*b^(28/3)*d^4*243i))*(3^(1/2)*1i + 1
)*1i)/(2*b^(4/3)*d) - 3/(b*d*(b*coth(c + d*x))^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth{\left(c + dx \right)}\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-4/3), x)
```

3.15 $\int \coth^n(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\coth^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(a+bx)\right)}{b(n+1)}$$

```
[Out] coth(b*x+a)^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(b*x+a)^2)/b/(1+
n)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3476, 364}

$$\frac{\coth^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*x]^n,x]
```

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
b*x]^2])/(b*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \coth^n(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^n}{-1+x^2}\,dx,x,\coth(a+bx)\right)}{b}\\ &= \frac{\coth^{1+n}(a+bx)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(a+bx)\right)}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 1.05

$$\frac{\coth^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\coth^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*x]^n,x]
```

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, Coth[
a + b*x]^2])/(b*(1 + n))
```

fricas [F]   time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(bx+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="fricas")
[Out] integral(coth(b*x + a)^n, x)
```

giac [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="giac")
[Out] Timed out
```

maple [F]   time = 0.44, size = 0, normalized size = 0.00

$$\int \coth^{n}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(b*x+a)^n,x)
[Out] int(coth(b*x+a)^n,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(bx+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)^n,x, algorithm="maxima")
[Out] integrate(coth(b*x + a)^n, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int {\mathrm{coth}\left(a+b\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*x)^n,x)
[Out] int(coth(a + b*x)^n, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{n}{\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)**n,x)
[Out] Integral(coth(a + b*x)**n, x)
```

3.16 $\int (b\coth(c+dx))^n\,dx$

Optimal. Leaf size=48

$$\frac{(b\coth(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(c+dx)\right)}{bd(n+1)}$$

```
[Out] (b*coth(d*x+c))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(d*x+c)^2)/b
/d/(1+n)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3476, 364}

$$\frac{(b\coth(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\coth^2(c+dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x])^n,x]
```

```
[Out] ((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[
c + d*x]^2])/(b*d*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\coth(c+dx))^n\,dx &= -\frac{b\,\text{Subst}\left(\int \frac{x^n}{-b^2+x^2}\,dx,x,b\coth(c+dx)\right)}{d}\\ &= \frac{(b\coth(c+dx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\coth^2(c+dx)\right)}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 1.06

$$\frac{\coth(c+dx)(b\coth(c+dx))^n\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\coth^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x])^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 +
 n)/2, Coth[c + d*x]^2])/(d*(1 + n))
```

fricas [F]   time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((b\coth(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c))^n, x)
```

giac [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="giac")
[Out] Timed out
```

maple [F]   time = 0.47, size = 0, normalized size = 0.00

$$\int (b\coth(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c))^n,x)
[Out] int((b*coth(d*x+c))^n,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^n, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int (b\coth(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x))^n,x)
[Out] int((b*coth(c + d*x))^n, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (b\coth(c+dx))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c))**n,x)
[Out] Integral((b*coth(c + d*x))**n, x)
```

3.17 $\int \left(b \coth^2(c+dx)\right)^n \, dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);\coth^2(c+dx)\right)}{d(2n+1)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^2)^n*hypergeom([1, 1/2+n],[3/2+n],coth(d*x+c)^2)
/d/(1+2*n)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3658, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);\coth^2(c+dx)\right)}{d(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3 +
 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\coth^2(c+dx)\right)^n\,dx &= \left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\int \coth^{2n}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{2n}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(1+2n);\frac{1}{2}(3+2n);\coth^2(c+dx)\right)}{d(1+2n)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.82

$$\frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n {}_2F_1\left(1,n+\frac{1}{2};n+\frac{3}{2};\coth^2(c+dx)\right)}{2dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, 1/2 + n, 3/2 + n,
 Coth[c + d*x]^2])/(d + 2*d*n)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\coth\left(dx+c\right)^2\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^2)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^2\left(dx+c\right)\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^n,x)
```

```
[Out] int((b*coth(d*x+c)^2)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\coth\left(c+d\,x\right)^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^n,x)
```

```
[Out] int((b*coth(c + d*x)^2)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^2(c+dx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**n, x)
```

3.18 $\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$\frac{b\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))}{d}-\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}$$

```
[Out] -1/2*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/2)/d+b*ln(sinh(d*x+c))*(b*coth(d*x+
c)^2)^(1/2)*tanh(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{b\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))}{d}-\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(3/2),x]
```

```
[Out] -(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^2])/(2*d) + (b*Sqrt[b*Coth[c + d*x]^
2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx &= \left(b\sqrt{b\coth^2(c+dx)}\,\tanh(c+dx)\right)\int \coth^3(c+dx)\,dx\\ &= -\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}+\left(b\sqrt{b\coth^2(c+dx)}\,\tanh(c+dx)\right)\int \coth(c\\ &= -\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}+\frac{b\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))\tanh(c}{d}\end{aligned}$$

Mathematica [A] time = 0.13, size = 56, normalized size = 0.92

$$-\frac{\tanh^3(c+dx)\left(b\coth^2(c+dx)\right)^{3/2}\left(\coth^2(c+dx)-2\log(\tanh(c+dx))-2\log(\cosh(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(3/2),x]
```

```
[Out] -1/2*((b*Coth[c + d*x]^2)^(3/2)*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2
*Log[Tanh[c + d*x]])*Tanh[c + d*x]^3)/d
```

fricas [B] time = 0.43, size = 823, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d*x*cosh(d*x + c)^4 - (b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x + c)^4 -
4*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sinh(d*x + c)
^3 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2 + 2*(3*b*d*x*cosh(d*x + c)^2 - b
*d*x - (3*b*d*x*cosh(d*x + c)^2 - b*d*x + b)*e^(2*d*x + 2*c) + b)*sinh(d*x
+ c)^2 - (b*d*x*cosh(d*x + c)^4 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2)*e^
(2*d*x + 2*c) - (b*cosh(d*x + c)^4 - (b*e^(2*d*x + 2*c) - b)*sinh(d*x + c)^
4 - 4*(b*cosh(d*x + c)*e^(2*d*x + 2*c) - b*cosh(d*x + c))*sinh(d*x + c)^3 -
 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - (3*b*cosh(d*x + c)^2 - b)*e
^(2*d*x + 2*c) - b)*sinh(d*x + c)^2 - (b*cosh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + b)*e^(2*d*x + 2*c) + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c) - (b*cosh
(d*x + c)^3 - b*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + b)*log(2*si
nh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b*d*x*cosh(d*x + c)^3 - (
b*d*x - b)*cosh(d*x + c) - (b*d*x*cosh(d*x + c)^3 - (b*d*x - b)*cosh(d*x +
c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x
+ 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^4 +
 (d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^4 + 4*(d*cosh(d*x + c)*e^(2*d*x + 2*
c) + d*cosh(d*x + c))*sinh(d*x + c)^3 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d
*x + c)^2 + (3*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^2
+ (d*cosh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + 4*(d*cosh
(d*x + c)^3 - d*cosh(d*x + c) + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) + d)
```

giac [A] time = 0.18, size = 90, normalized size = 1.48

$$-\frac{\left((dx+c)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)+\frac{2\,e^{(2\,dx+2\,c)}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}\right)b^{\frac{3}{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1) + 2*e^(2*d*x + 2*c)*sgn(e^(4*d*x + 4*c) - 1)/(e^(2*d*x +
 2*c) - 1)^2)*b^(3/2)/d
```

maple [A] time = 0.15, size = 53, normalized size = 0.87

$$-\frac{\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{3}{2}}\left(\coth^2(dx+c)+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)\right)}{2d\coth(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^2)^(3/2)*(coth(d*x+c)^2+ln(coth(d*x+c)-1)+ln(coth(d*x
+c)+1))/coth(d*x+c)^3
```

maxima [A] time = 0.47, size = 97, normalized size = 1.59

$$-\frac{(dx+c)b^{\frac{3}{2}}}{d}-\frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*b^(3/2)/d - b^(3/2)*log(e^(-d*x - c) + 1)/d - b^(3/2)*log(e^(-d*
x - c) - 1)/d - 2*b^(3/2)*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d
*x - 4*c) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\coth\left(c+d\,x\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(3/2),x)
```

```
[Out] int((b*coth(c + d*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^{2}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(3/2), x)
```

3.19 $\int \sqrt{b \coth^2(c+dx)}\, dx$

Optimal. Leaf size=31

$$\frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))}{d}$$

[Out] ln(sinh(d*x+c))*(b*coth(d*x+c)^2)^(1/2)*tanh(d*x+c)/d

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3658, 3475}

$$\frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Coth[c + d*x]^2],x]

[Out] (Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\coth^2(c+dx)}\, dx &= \left(\sqrt{b\coth^2(c+dx)}\,\tanh(c+dx)\right)\int \coth(c+dx)\,dx\\ &= \frac{\sqrt{b\coth^2(c+dx)}\,\log(\sinh(c+dx))\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 1.26

$$\frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\,(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Coth[c + d*x]^2],x]

[Out] (Sqrt[b*Coth[c + d*x]^2]*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c +
 d*x])/d

fricas [B] time = 0.42, size = 125, normalized size = 4.03

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*e^(2*d*x + 2*c) + d)
```

giac [A] time = 0.12, size = 54, normalized size = 1.74

$$-\frac{\left((dx+c)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)\right)\sqrt{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1))*sqrt(b)/d
```

maple [A] time = 0.16, size = 45, normalized size = 1.45

$$-\frac{\sqrt{b\left(\coth^{2}(dx+c)\right)}\,\left(\ln\left(\coth\left(dx+c\right)-1\right)+\ln\left(\coth\left(dx+c\right)+1\right)\right)}{2d\coth\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^2)^(1/2)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1))/coth(d
*x+c)
```

maxima [A] time = 0.47, size = 54, normalized size = 1.74

$$-\frac{(dx+c)\sqrt{b}}{d}-\frac{\sqrt{b}\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\sqrt{b}\log\left(e^{(-dx-c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*sqrt(b)/d - sqrt(b)*log(e^(-d*x - c) + 1)/d - sqrt(b)*log(e^(-d*
x - c) - 1)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{b\coth\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**2), x)
```

3.20 $\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx &= \frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{\sqrt{b\coth^2(c+dx)}}\\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 31, normalized size = 1.00

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

fricas [B] time = 0.41, size = 128, normalized size = 4.13

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{bde^{(2\,dx+2\,c)}+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*cosh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b*d*e^(2*d*x + 2*c) + b*
d)
```

giac [B] time = 0.15, size = 60, normalized size = 1.94

$$-\frac{\frac{dx+c}{\sqrt{b}\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(
sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

maple [A] time = 0.16, size = 52, normalized size = 1.68

$$-\frac{\coth\left(dx+c\right)\left(\ln\left(\coth\left(dx+c\right)-1\right)+\ln\left(\coth\left(dx+c\right)+1\right)-2\ln\left(\coth\left(dx+c\right)\right)\right)}{2d\sqrt{b\left(\coth^{2}\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1)-2*ln(coth(d*x+c)))/
(b*coth(d*x+c)^2)^(1/2)
```

maxima [A] time = 0.43, size = 34, normalized size = 1.10

$$-\frac{dx+c}{\sqrt{b}\,d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)/(sqrt(b)*d) - log(e^(-2*d*x - 2*c) + 1)/(sqrt(b)*d)
```

mupad [B] time = 1.26, size = 30, normalized size = 0.97

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\coth(c+d\,x)}{\sqrt{b\,\coth(c+d\,x)^2}}\right)}{\sqrt{b}\,\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(1/2),x)
```

```
[Out] atanh((b^(1/2)*coth(c + d*x))/(b*coth(c + d*x)^2)^(1/2))/(b^(1/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**2), x)
```

3.21 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/b/d/(b*coth(d*x+c)^2)^(1/2)-1/2*tanh(d*x+c)/b/d
/(b*coth(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 3475}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-3/2),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(b*d*Sqrt[b*Coth[c + d*x]^2]) - Tanh[c +
 d*x]/(2*b*d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx &= \frac{\coth(c+dx)\int \tanh^3(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}\\ &= -\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}+\frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}\\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 48, normalized size = 0.74

$$\frac{2\coth(c+dx)\log(\cosh(c+dx))-\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-3/2),x]
```

```
[Out] (2*Coth[c + d*x]*Log[Cosh[c + d*x]] - Tanh[c + d*x])/(2*b*d*Sqrt[b*Coth[c + d*x]^2])
```

fricas [B] time = 0.45, size = 817, normalized size = 12.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^4 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^4 - 4*(d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(d*x - 1)*cosh(d*x + c)^2 + 2*(3*d*x*cosh(d*x + c)^2 + d*x - (3*d*x*cosh(d*x + c)^2 + d*x - 1)*e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 + d*x - (d*x*cosh(d*x + c)^4 + 2*(d*x - 1)*cosh(d*x + c)^2 + d*x)*e^(2*d*x + 2*c) + ((e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 + 4*(cosh(d*x + c)*e^(2*d*x + 2*c) - cosh(d*x + c))*sinh(d*x + c)^3 - 2*(3*cosh(d*x + c)^2 - (3*cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) + 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + (cosh(d*x + c)^4 + 2*cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) - 4*(cosh(d*x + c)^3 - (cosh(d*x + c)^3 + cosh(d*x + c))*e^(2*d*x + 2*c) + cosh(d*x + c))*sinh(d*x + c) - 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(d*x*cosh(d*x + c)^3 + (d*x - 1)*cosh(d*x + c) - (d*x*cosh(d*x + c)^3 + (d*x - 1)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b^2*d*cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + (b^2*d*e^(2*d*x + 2*c) + b^2*d)*sinh(d*x + c)^4 + 4*(b^2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d + (3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

giac [A] time = 0.73, size = 104, normalized size = 1.60

$$-\frac{\frac{dx+c}{\sqrt{b}\operatorname{sgn}\left(e^{(4dx+4c)}-1\right)}-\frac{\log\left(e^{(2dx+2c)}+1\right)}{\sqrt{b}\operatorname{sgn}\left(e^{(4dx+4c)}-1\right)}-\frac{2e^{(2dx+2c)}}{\sqrt{b}\left(e^{(2dx+2c)}+1\right)^2\operatorname{sgn}\left(e^{(4dx+4c)}-1\right)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - 2*e^(2*d*x + 2*c)/(sqrt(b)*(e^(2*d*x + 2*c) + 1)^2*sgn(e^(4*d*x + 4*c) - 1)))/(b*d)
```

maple [A] time = 0.12, size = 79, normalized size = 1.22

$$-\frac{\coth(dx+c)\left(\ln(\coth(dx+c)-1)\left(\coth^2(dx+c)\right)+\ln(\coth(dx+c)+1)\left(\coth^2(dx+c)\right)-2\ln(\coth(d\right.}{2d\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)-1)*coth(d*x+c)^2+ln(coth(d*x+c)+1)*coth(
d*x+c)^2-2*ln(coth(d*x+c))*coth(d*x+c)^2+1)/(b*coth(d*x+c)^2)^(3/2)
```

maxima [A] time = 0.43, size = 84, normalized size = 1.29

$$-\frac{2\sqrt{b}e^{(-2\,dx-2\,c)}}{\left(2\,b^2e^{(-2\,dx-2\,c)}+b^2e^{(-4\,dx-4\,c)}+b^2\right)d}-\frac{dx+c}{b^{\frac{3}{2}}d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b)*e^(-2*d*x - 2*c)/((2*b^2*e^(-2*d*x - 2*c) + b^2*e^(-4*d*x - 4*c)
 + b^2)*d) - (d*x + c)/(b^(3/2)*d) - log(e^(-2*d*x - 2*c) + 1)/(b^(3/2)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(3/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**2)**(-3/2), x)
```

3.22 $\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=297

$$-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}-\frac{b\sqrt[3]{b\coth^2(c+dx)}\,\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth}}{}$$

```
[Out] b*arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)-3/
5*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/3)/d-1/4*b*(b*coth(d*x+c)^2)^(1/3)*ln(
1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(2/3)+1/4*b*(b*coth(d*
x+c)^2)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(2/3)
+1/2*b*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(1/3)*
3^(1/2)/d/coth(d*x+c)^(2/3)-1/2*b*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2
))*(b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)
```

Rubi [A] time = 0.28, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}-\frac{b\sqrt[3]{b\coth^2(c+dx)}\,\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{b\sqrt[3]{b\coth}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] (Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^
(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^(
1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (b*Ar
cTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3
)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^2)^(1/3))/(5*d) - (b*(b*Coth[c + d
*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[
c + d*x]^(2/3)) + (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3)
+ Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^{4/3} dx = \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int \coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x,\right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx,\right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}$$

$$= \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b}}{2d\coth^{\frac{2}{3}}(c+dx)}$$

Mathematica [A] time = 0.35, size = 166, normalized size = 0.56

$$-\frac{\left(b\coth^2(c+dx)\right)^{4/3}\left(12\coth^{\frac{5}{3}}(c+dx) - 20\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right) - 5\left(-\log\left(\coth^{\frac{2}{3}}(c+dx) - \sqrt[3]{\coth(c+}\right.\right.\right.}{20d\,\mathrm{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] -1/20*((b*Coth[c + d*x]^2)^(4/3)*(-20*ArcTanh[Coth[c + d*x]^(1/3)] + 12*Cot
h[c + d*x]^(5/3) - 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]
 - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c +
 d*x]^(2/3)])))/(d*Coth[c + d*x]^(8/3))
```

fricas [B] time = 0.46, size = 1994, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*(-b)^(1/3)*arctan(1/3*(sqrt(3)
```

```
*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*si
nh(d*x + c)^2 + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d
*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) - 10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*
sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*b^(1/3)*arctan(-1/3*
(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt
(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c
)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2/3)*((b*cosh(d*x +
 c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/
3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + b)) + 5*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d
*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cos
h(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x
+ c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
+ c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + s
inh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + 5*(b
*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)
*b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(
d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1)
)^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*s
inh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + c
osh(d*x + c))*sinh(d*x + c) + 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d
*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))
^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
+ 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 - b)*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(d
*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^{2}\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \left(b \left(\coth^2(dx + c)\right)\right)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(4/3),x)
[Out] int((b*coth(d*x+c)^2)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth(dx + c)^2\right)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b \coth(c + d\,x)^2\right)^{4/3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(4/3),x)
[Out] int((b*coth(c + d*x)^2)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth^2(c + dx)\right)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(4/3), x)
```

3.23 $\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=289

$$-\frac{3\tanh(c+dx)\left(b\coth^2(c+dx)\right)^{2/3}}{d}-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c\right.}{}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(2/3)/d/coth(d*x+c)^(4/3)-1/4*
(b*coth(d*x+c)^2)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(4/3)+1/4*(b*coth(d*x+c)^2)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(4/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^2)^(2/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(2/3)*3^(1/2)/d/coth(d*x+c)^(4/3)
-3*(b*coth(d*x+c)^2)^(2/3)*tanh(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(
2/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(2/3))/(d*Coth[c + d*x]^(4/3)) -
((b*Coth[c + d*x]^2)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^2)^(2/3)*Log[1 + Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[
c + d*x]^2)^(2/3)*Tanh[c + d*x])/d
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx = \frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}dx, x\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}dx, x\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \sqrt[3]{\text{co}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth\right.}{4d\coth^{\frac{4}{3}}(c+}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\cot\right.}{2d\coth^{\frac{4}{3}}(c+dx)}$$

Mathematica [C] time = 0.04, size = 43, normalized size = 0.15

$$\frac{3\tanh(c+dx)\left(b\coth^2(c+dx)\right)^{2/3}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] (3*(b*Coth[c + d*x]^2)^(2/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c +
d*x]^2])*Tanh[c + d*x])/d
```

fricas [B] time = 0.47, size = 2037, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh
(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x
+ c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
 + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*((b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(
3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
```

```
)^2 + b)) + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b
*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh
(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x
 + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + s
qrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + b)) + (-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(-b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 -
b*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^
2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*
cosh(d*x + c))*sinh(d*x + c) + b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d
*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(
d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d
*x + c) + 1)) + (b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*log(-((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b
*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2
 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*c
osh(d*x + c))*sinh(d*x + c) + b)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)) - 2*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(-((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh
(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*(b^2)^(1
/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*log(((b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 + 1) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x
+ c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(d*cosh(d*x + c)^2
+ 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth \left(dx+c\right)^2\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^2(dx+c)\right)\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(2/3),x)
[Out] int((b*coth(d*x+c)^2)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth(dx+c)^2\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\coth(c+d\,x)^2\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(2/3),x)
[Out] int((b*coth(c + d*x)^2)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^2(c+dx)\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(2/3), x)
```

3.24 $\int \sqrt[3]{b\coth^2(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)-1/4*
(b*coth(d*x+c)^2)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(2/3)+1/4*(b*coth(d*x+c)^2)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(2/3)+1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)-1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)
```

Rubi [A] time = 0.21, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3658, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1
/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - (
(b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(2/3)) + ((b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \frac{\sqrt[3]{b\coth^2(c+dx)}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{\sqrt[3]{b\coth^2(c+dx)}\ \text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{\sqrt[3]{b\coth^2(c+dx)}\ \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)} + \frac{\sqrt[3]{b\coth^2(c+dx)}\ \text{Subst}\ \cdots}{d\,\text{co}\cdots}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^2(c+dx)}\ \text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx \cdots\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c \cdots}\right.}{4d\coth^{\frac{2}{3}}(c+d\cdots}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^{\cdots}}}{2d\coth^{\frac{2}{3}}(c+dx)}$$

Mathematica [A] time = 0.11, size = 151, normalized size = 0.57

$$\frac{\sqrt[3]{b\coth^2(c+dx)}\left(-\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)+\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)+\cdots\right.}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sq
rt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[
Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] +
 Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2
/3))
```

fricas [B] time = 0.49, size = 1618, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3
)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 + 2*(sqrt(3)*co
sh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x +
c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(co
sh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2*sqrt(3)*b^
```

```
(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sq
rt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2
/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si
nh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (-b)^(1/3)*log(((cosh(d*x + c)^4 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 +
1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c)
)*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4
+ 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2
- 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x +
c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d
*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c
)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)) + b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x +
c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (co
sh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*(
(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c
)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)
^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))
/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b)^(1/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 -
 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c
)^2 + 1)) - 2*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth \left(dx+c\right)^{2}\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b \left(\coth^{2}\left(dx+c\right)\right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^2)^(1/3),x)
```

```
[Out] int((b*coth(d*x+c)^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\coth\left(c+d\,x\right)^2\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^2)^(1/3),x)
[Out] int((b*coth(c + d*x)^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\coth^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**2)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(1/3), x)
```

3.25 $\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx$

Optimal. Leaf size=264

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+\right.}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(2/3)/d/(b*coth(d*x+c)^2)^(1/3)-1/4*
coth(d*x+c)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c
)^2)^(1/3)+1/4*coth(d*x+c)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/
d/(b*coth(d*x+c)^2)^(1/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*c
oth(d*x+c)^(2/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(1/3)
```

Rubi [A] time = 0.17, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3658, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+\right.}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/
(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(d*(b*Coth[c + d*x]^2)^(1/3)) -
(Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4
*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c + d*x]^
(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\right.}{d\sqrt[3]{b\coth^2(c+}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth}\right.}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\mathrm{co}\right.}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx}{2d\sqrt[3]{b\coth^2(c+dx)}}$$

Mathematica [C] time = 0.06, size = 41, normalized size = 0.16

$$\frac{3\coth(c+dx)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])/(d*(b*Cot
h[c + d*x]^2)^(1/3))
```

fricas [B] time = 0.62, size = 8338, normalized size = 31.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*sqrt(3)*(cosh(d*x + c)^4 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh
(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(
d*x + c) + 1)*b^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*
x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3)*sqrt(-1/b^(2/3)) - 2*b*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + sqrt(3)*(b*cosh(d*x + c)
```

```
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x +
 c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
+ b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*sqrt(-1/b^(2/3)) + 4*(b*cosh(
d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) - (sqrt(3)*(b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*sqrt(-1/b^(2/3))
 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b
^(2/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sin
h(d*x + c)^2 - 1))^(1/3) - 3*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
 + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(
d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))) + sqrt(3
)*b*sqrt((-b)^(1/3)/b)*log(-(3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)*sinh(
d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*cosh(d*x +
c)^2 + b)*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3
 + sinh(d*x + c)^4 - 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(2*(cosh(d*x
+ c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x
+ c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh
(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) + (b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
 + b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - (b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b
*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))*s
qrt((-b)^(1/3)/b) + 4*(3*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)
 - b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)) + (-b)^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x +
c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) -
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(
1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d
*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x
+ c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^
(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4
 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(
d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - b^(2/3)*log(((cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d
*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*
sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*
x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x +
c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2
 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x +
 c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(
d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c
) + 1)) - 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*c
```

```
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 2*b^(2/3)*log(((cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) +
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b
*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^
2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)))/(b*d), -1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-1/3*(sqr
t(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1
)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x
+ c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3)*sqrt(-(-b)^(1/3)
/b))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1
)) - sqrt(3)*b*sqrt(-1/b^(2/3))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*
sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*sqrt(3)*(cosh(d*x + c)^4 + 4*cosh(d
*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(
d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d
*x + c) + 1)*b^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x
 + c)^2 + sinh(d*x + c)^2 - 1))^(2/3)*sqrt(-1/b^(2/3)) - 2*b*cosh(d*x + c)^
2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + sqrt(3)*(b*cosh(d*x + c)^
4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x +
c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
 b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*sqrt(-1/b^(2/3)) + 4*(b*cosh(d
*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) - (sqrt(3)*(b*cosh(d*x + c)^4 +
4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*sqrt(-1/b^(2/3))
+ 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*
sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^
(2/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh
(d*x + c)^2 - 1))^(1/3) - 3*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))) - (-b)^(2
/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)
^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cos
h(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c
)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 +
 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh
(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1
))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(c
osh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(
d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(
d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 +
cosh(d*x + c))*sinh(d*x + c) + 1)) + b^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh
(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh
(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c)
 + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + si
nh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(co
sh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*
x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
 + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh
(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + 2*(
-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*
x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*
```

```
sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*b^(2/3)*log(((cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c
)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3)
)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))
/(b*d), 1/4*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log(-(3*b*cosh(d*x + c)^4 + 12*b*
cosh(d*x + c)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 +
 2*(9*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^4 + 4*cosh(
d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x +
 c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(2/3)*((b*cosh(d*x + c)^2 +
 b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sq
rt(3)*(2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)
^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cos
h(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*((b*cosh(d*x +
c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3
) + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4
*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - (b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*si
nh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*
((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x +
c)^2 - 1))^(1/3))*sqrt((-b)^(1/3)/b) + 4*(3*b*cosh(d*x + c)^3 + b*cosh(d*x
+ c))*sinh(d*x + c) - b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)) - 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqrt(3)*((cosh(d*x +
 c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) - 2*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*c
osh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2
- 1))^(1/3))/((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(1/3))) + (-b)^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^
2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) +
 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cos
h(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x +
 c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*
x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(
d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x
+ c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - b^(2/3)*
log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*
x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4
*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh
(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^
2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
 (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2
 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x +
 c))*sinh(d*x + c) + 1)) - 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b
*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh
(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 2*b^(
2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
```

```
2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2
 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)))/(b*d), -1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)
*arctan(-1/3*(sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 + 1)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 2*sqrt(3)*(cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c
)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3)
*sqrt(-(-b)^(1/3)/b))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 + 1)) + 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqrt(3)*((cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(1/3) - 2*(cosh
(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh
(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1
))^(1/3))/((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*b^(1/3))) - (-b)^(2/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh
(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 +
 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)
*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)
^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x +
 c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*si
nh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2
 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) +
1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x
 + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + s
inh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c
)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + b^(2/3)*log
(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*
(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
 c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x + c)^4 + 4*co
sh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*
x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(d*x + c)^2 +
 b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 +
1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c)
)*sinh(d*x + c) + 1)) + 2*(-b)^(2/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c
)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d*x + c)^2 + 2*co
sh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*si
nh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*b^(2/3
)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)))/(b*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth \left(dx + c\right)^{2}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^{2}\left(dx+c\right)\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(1/3),x)
[Out] int(1/(b*coth(d*x+c)^2)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^{2}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(1/3),x)
[Out] int(1/(b*coth(c + d*x)^2)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\coth^{2}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(-1/3), x)
```

3.26 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=289

$$-\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+\right.}{4d\left(b\coth^2(c\right.}$$

```
[Out] -3*coth(d*x+c)/d/(b*coth(d*x+c)^2)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*
x+c)^(4/3)/d/(b*coth(d*x+c)^2)^(2/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*x+c)
^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^2)^(2/3)+1/4*coth(d*x+c)^(4/3)*l
n(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^2)^(2/3)+1/2*arct
an(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/d/(b*coth
(d*x+c)^2)^(2/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c
)^(4/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(2/3)
```

Rubi [A] time = 0.22, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+\right.}{4d\left(b\coth^2(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^2)^(2/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^2)^
(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^2)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^2)^(2/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(2
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^2)^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
```

```
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \text{cot}\ldots$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{S}\ldots}{\ldots}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,1\ldots}{\ldots}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+2\ldots}{\ldots}\right)\ldots}{2d\left(b\ldots\right.}$$

Mathematica [C] time = 0.07, size = 41, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(d*(b*C
oth[c + d*x]^2)^(2/3))
```

fricas [B] time = 0.45, size = 2066, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sin
h(d*x + c)^2 + b)*sqrt(-(-b^2)^(1/3))*arctan(1/3*(2*sqrt(3)*(-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(
-(-b^2)^(1/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^
2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
```

```
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(-b^2)^(1/3)*sqrt(-(-b^2)^(1/3
)))/(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x
 + c)^2 + b^2)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/6)*arctan(1/3*sqrt(3)*(b^2)^(1/6)*(
2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 +
sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d
*x + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/3))/(b^2*cosh(d*x + c)^2 + 2*b^2*
cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)) + (-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log((
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b^2)
^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh
(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
+ b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d
*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(
d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - (b^2)^(2/3)*(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-(
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(b^2)^
(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(
d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b
)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) +
 b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*
sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b
)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-
((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)
^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 + 1)) + 2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(((b^2)^(2/3)*(cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) + (b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^
(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
 1)) - 12*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2
+ sinh(d*x + c)^2 - 1))^(1/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x +
c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{2}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^2(dx+c)\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(2/3),x)
[Out] int(1/(b*coth(d*x+c)^2)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth(dx+c)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\coth(c+d\,x)^2\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(2/3),x)
[Out] int(1/(b*coth(c + d*x)^2)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(-2/3), x)
```

3.27 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=309

$$-\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(\right.}{4bd\sqrt[3]{b\,c}}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(2/3)/b/d/(b*coth(d*x+c)^2)^(1/3)-1/
4*coth(d*x+c)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d
*x+c)^2)^(1/3)+1/4*coth(d*x+c)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/
3))/b/d/(b*coth(d*x+c)^2)^(1/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1
/2))*coth(d*x+c)^(2/3)*3^(1/2)/b/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctan(1/3*(
1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)*3^(1/2)/b/d/(b*coth(d*x+c
)^2)^(1/3)-3/5*tanh(d*x+c)/b/d/(b*coth(d*x+c)^2)^(1/3)
```

Rubi [A] time = 0.18, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx}\right.}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^2)^(-4/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/
(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Arc
Tanh[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(b*d*(b*Coth[c + d*x]^2)^(1/
3)) - (Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c
 + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) - (
3*Tanh[c + d*x])/(5*b*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right.}{bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}} + \cdots$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\cdots}{\cdots}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}\right.}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c\cdots}{2bd\sqrt[3]{b\coth^2(c+dx)}}$$

Mathematica [C] time = 0.16, size = 43, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{5}{6},1;\frac{1}{6};\coth^2(c+dx)\right)}{5d\left(b\coth^2(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-5/6, 1, 1/6, Coth[c + d*x]^2])/(5*d*(b
*Coth[c + d*x]^2)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{2}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^{2}\left(dx+c\right)\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^2)^(4/3),x)
[Out] int(1/(b*coth(d*x+c)^2)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{2}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^{2}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^2)^(4/3),x)
[Out] int(1/(b*coth(c + d*x)^2)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^{2}\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**2)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(-4/3), x)
```

3.28 $\int \left(b\coth^3(c+dx)\right)^n\,dx$

Optimal. Leaf size=55

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};\coth^2(c+dx)\right)}{d(3n+1)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^3)^n*hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],coth(d
*x+c)^2)/d/(1+3*n)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3658, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};\coth^2(c+dx)\right)}{d(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\coth^3(c+dx)\right)^n\,dx &= \left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\int \coth^{3n}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\operatorname{Subst}\left(\int \frac{x^{3n}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(1+3n);\frac{3(1+n)}{2};\coth^2(c+dx)\right)}{d(1+3n)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.96

$$\frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};\coth^2(c+dx)\right)}{3dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d + 3*d*n)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\coth\left(dx+c\right)^3\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^3)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^3\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^3\left(dx+c\right)\right)\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^n,x)
```

```
[Out] int((b*coth(d*x+c)^3)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^3\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\coth\left(c+d\,x\right)^3\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^n,x)
```

```
[Out] int((b*coth(c + d*x)^3)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^3(c+dx)\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**n, x)
```

3.29 $\int \left(b\coth^3(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=134

$$-\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}-\frac{b\sqrt{b\coth^3(c+dx)}\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{b\sqrt{b\coth}\cdots}{\cdots}$$

```
[Out] -2/3*b*(b*coth(d*x+c)^3)^(1/2)/d-b*arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c)
^3)^(1/2)/d/coth(d*x+c)^(3/2)+b*arctanh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3
)^(1/2)/d/coth(d*x+c)^(3/2)-2/7*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3658, 3473, 3476, 329, 298, 203, 206}

$$-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}-\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{b\sqrt{b\coth^3(c+dx)}\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{b\sqrt{b\coth}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Coth[c + d*x]^3])/(3*d) - (b*ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[
b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) + (b*ArcTanh[Sqrt[Coth[c + d*x]
]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) - (2*b*Coth[c + d*x]^2*
Sqrt[b*Coth[c + d*x]^3])/(7*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx = \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int \coth^{\frac{9}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int \coth^{\frac{5}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\coth^3(c+d\right.}}{\mathrm{coth}}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\coth^3(c+d\right.}}{}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} - \frac{\left(2b\sqrt{b\coth^3(c+\right.}}{}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\coth^3(c+d\right.}}{}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{b\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{b\tanh^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.53, size = 82, normalized size = 0.61

$$-\frac{\left(b\coth^3(c+dx)\right)^{3/2}\left(6\coth^{\frac{7}{2}}(c+dx)+14\coth^{\frac{3}{2}}(c+dx)+21\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)-21\tanh^{-1}\left(\sqrt{\coth(c\right.\right.}{21d\coth^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(3/2),x]
```

```
[Out] -1/21*((b*Coth[c + d*x]^3)^(3/2)*(21*ArcTan[Sqrt[Coth[c + d*x]]] - 21*ArcTa
nh[Sqrt[Coth[c + d*x]]] + 14*Coth[c + d*x]^(3/2) + 6*Coth[c + d*x]^(7/2)))/
(d*Coth[c + d*x]^(9/2))
```

fricas [B] time = 1.82, size = 2152, normalized size = 16.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(
d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c
)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cos
h(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x +
 c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(
d*x + c) - b)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh
(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2
1*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^
6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(
5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c
)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6
*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)
- b)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) +
 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cos
h(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*b*cos
h(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d*x + c)^6 + b
*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(25*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (75*b
*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(15*b*cosh(
d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 5*b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*s
inh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x
+ c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c) - d), -1/84*(42*(b*cosh(d*x + c)^6 + 6*b*c
osh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*
(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4
 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*co
sh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*arctan(sqrt(b)*
sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*
sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 21*(b*cosh(d*x + c)^6 + 6*b*cosh(
d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b
*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d
*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d
*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*log(2*b*cosh(d*x +
c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*
x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x
+ c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh
(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b)
+ 16*(5*b*cosh(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d
*x + c)^6 + b*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4
```

```
+ 4*(25*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x +
 c)^2 + (75*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 +
2*(15*b*cosh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
 c) + 5*b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*co
sh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(
5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4
- 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cos
h(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)]
```

giac [B] time = 0.65, size = 788, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/42*(42*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
- b))/sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c)
- 1)*sgn(e^(4*d*x + 4*c) - 1) - 21*sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c)
 + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) +
3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) + 16*(21*(sqrt(b)*e^(2*d*x
+ 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^6*b*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x
+ 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) - 42*(sqrt(b)*e^(2
*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^5*b^(3/2)*sgn(e^(6*d*x + 6*c) -
3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) + 119*(
sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4*b^2*sgn(e^(6*d*x +
 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)
 - 56*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3*b^(5/2)*sgn
(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x
+ 4*c) - 1) + 63*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*
b^3*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^
(4*d*x + 4*c) - 1) - 14*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))*b^(7/2)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
1)*sgn(e^(4*d*x + 4*c) - 1) + 5*b^4*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c)
 + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c
) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b))^7)*b/d
```

maple [A] time = 0.20, size = 107, normalized size = 0.80

$$\frac{\left(b\left(\coth^{3}\left(dx+c\right)\right)\right)^{\frac{3}{2}}\left(21b^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)-21b^{\frac{7}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)-6\left(b\coth\left(dx+c\right)\right)^{\frac{7}{2}}-14b^{2}\left(\right.\right.}{21d\coth\left(dx+c\right)^{3}\left(b\coth\left(dx+c\right)\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(3/2),x)
```

```
[Out] 1/21/d*(b*coth(d*x+c)^3)^(3/2)*(21*b^(7/2)*arctanh((b*coth(d*x+c))^(1/2)/b^
(1/2))-21*b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))-6*(b*coth(d*x+c))^(
7/2)-14*b^2*(b*coth(d*x+c))^(3/2))/coth(d*x+c)^3/(b*coth(d*x+c))^(3/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\coth\left(c+d\,x\right)^3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(3/2),x)
[Out] int((b*coth(c + d*x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^3\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**3)**(3/2), x)
```

3.30 $\int \sqrt{b \coth^3(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{2\tanh(c+dx)\sqrt{b\coth^3(c+dx)}}{d}+\frac{\sqrt{b\coth^3(c+dx)}\,\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\coth^3(c+dx)}\,\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}$$

```
[Out] arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)+arcta
nh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)-2*(b*coth
(d*x+c)^3)^(1/2)*tanh(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3658, 3473, 3476, 329, 212, 206, 203}

$$\frac{\sqrt{b\coth^3(c+dx)}\,\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\coth^3(c+dx)}\,\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}-\frac{2\tanh(c+dx)\sqrt{b\coth^3(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2
)) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x
]^(3/2)) - (2*Sqrt[b*Coth[c + d*x]^3]*Tanh[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \frac{\sqrt{b\coth^3(c+dx)}\int \coth^{\frac{3}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt{b\coth^3(c+dx)}\int \frac{1}{\sqrt{\coth(c+dx)}}\,dx}{\coth^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} - \frac{\sqrt{b\coth^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, \coth$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} - \frac{\left(2\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth$$

$$= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} + \frac{\sqrt{b\coth^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(c+$$

$$= \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx}}{d\coth^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.61

$$\frac{\sqrt{b\coth^3(c+dx)}\left(-2\sqrt{\coth(c+dx)}+\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)\right)}{d\coth^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[
c + d*x]])*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2))
```

fricas [B] time = 1.59, size = 633, normalized size = 6.09

$$\left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4b\cosh(d}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*sqrt(b*cosh(d*x + c)
/sinh(d*x + c)))/d, 1/4*(2*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh
(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) + sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh
(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d
*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh
(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - c
osh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/d]
```

giac [B] time = 0.29, size = 269, normalized size = 2.59

$$-\frac{2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\operatorname{sgn}\left(e^{(4dx+4c)}-1\right)+\sqrt{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
 1)*sgn(e^(4*d*x + 4*c) - 1) + sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + s
qrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^
(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) - 8*b*sgn(e^(6*d*x + 6*c) - 3*e
^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)/(sqrt(b)*e
^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b)))/d
```

maple [A] time = 0.17, size = 89, normalized size = 0.86

$$-\frac{\sqrt{b\left(\coth^{3}(dx+c)\right)}\left(2\sqrt{b\coth(dx+c)}-\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)-\sqrt{b}\,\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\right)}{d\coth(dx+c)\sqrt{b\coth(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(1/2),x)
```

```
[Out] -1/d*(b*coth(d*x+c)^3)^(1/2)*(2*(b*coth(d*x+c))^(1/2)-b^(1/2)*arctanh((b*co
th(d*x+c))^(1/2)/b^(1/2))-b^(1/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2)))/co
th(d*x+c)/(b*coth(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \coth(dx+c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b \coth(c+d\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \coth^3(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**3), x)
```

3.31 $\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

```
[Out] -2*coth(d*x+c)/d/(b*coth(d*x+c)^3)^(1/2)-arctan(coth(d*x+c)^(1/2))*coth(d*x
+c)^(3/2)/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c)^
(3/2)/d/(b*coth(d*x+c)^3)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3658, 3474, 3476, 329, 298, 203, 206}

$$-\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (-2*Coth[c + d*x])/(d*Sqrt[b*Coth[c + d*x]^3]) - (ArcTan[Sqrt[Coth[c + d*x]
]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c
+ d*x]]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx &= \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{3}{2}}(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}\\
&= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\int \sqrt{\coth(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}\\
&= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d\sqrt{b\coth^3(c+dx)}}\\
&= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}\\
&= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}-\coth^{\frac{3}{2}}(c\\
&= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}-\frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}}+\frac{\tanh^{-1}\left(\sqrt{\coth(c+d}\right.}{d\sqrt{b\coth^3}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.39

$$
-\frac{2\coth(c+dx)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(c+dx)\right)}{d\sqrt{b\coth^3(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, Coth[c + d*x]^2])/(d*Sqrt
[b*Coth[c + d*x]^3])
```

fricas [B] time = 1.21, size = 907, normalized size = 8.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh
(d*x + c)^2 + b*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x +
c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
)/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x +
 c)^2 + b*d)]
```

giac [B] time = 0.50, size = 279, normalized size = 2.66

$$\frac{\frac{2\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\mathrm{sgn}\left(e^{(4dx+4c)}-1\right)}-\frac{\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\mathrm{sgn}\left(e^{(4dx+4c)}-1\right)}-\frac{\cdots}{\left(\sqrt{b}e^{(2dx+\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt
(b))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
 1)*sgn(e^(4*d*x + 4*c) - 1)) - log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e
^(4*d*x + 4*c) - b)))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*
e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)) - 8/((sqrt(b)*e^(2*d*x + 2*c
) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x
 + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

maple [A] time = 0.16, size = 92, normalized size = 0.88

$$-\frac{\coth\left(dx+c\right)\left(2b^{\frac{5}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth\left(dx+c\right)}+\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^{2}\sqrt{b\coth\left(dx+c\right)}\right)}{d\sqrt{b\left(\coth^{3}\left(dx+c\right)\right)}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(1/2),x)
```

```
[Out] -1/d*coth(d*x+c)*(2*b^(5/2)-arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*c
oth(d*x+c))^(1/2)+arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*coth(d*x+c))
^(1/2))/(b*coth(d*x+c)^3)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \coth \left(dx + c\right)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(1/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \coth^{3}{\left (c + dx \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**3), x)
```

3.32 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{2}{3bd\sqrt{b\coth^3(c+dx)}}-\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}}{bd\sqrt{b\coth^3}}$$

```
[Out] -2/3/b/d/(b*coth(d*x+c)^3)^(1/2)+arctan(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/2
)/b/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/2)/
b/d/(b*coth(d*x+c)^3)^(1/2)-2/7*tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3658, 3474, 3476, 329, 212, 206, 203}

$$-\frac{2}{3bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}-\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}+\frac{\coth^{\frac{3}{2}}(c+dx)\tanh^{-1}}{bd\sqrt{b\coth^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-3/2),x]
```

```
[Out] -2/(3*b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTan[Sqrt[Coth[c + d*x]]]*Coth[c +
d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c + d*x]]]*C
oth[c + d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) - (2*Tanh[c + d*x]^2)/(7*
b*d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{9}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{5}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\coth(c+dx)}}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right.}\right.}{bd\sqrt{b\coth^3(c+}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\right.}{bd\sqrt{b\coth^3(c}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,\right.}{bd\sqrt{b\coth^3(c+}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} + \frac{\tan^{-1}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt{\coth(\right.}{bd\sqrt{b\,c}}$$

Mathematica [C] time = 0.07, size = 43, normalized size = 0.30

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};\coth^2(c+dx)\right)}{7d\left(b\coth^3(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[-7/4, 1, -3/4, Coth[c + d*x]^2])/(7*d*(
b*Coth[c + d*x]^3)^(3/2))
```

fricas [B] time = 1.58, size = 3022, normalized size = 21.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x +
c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7
*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4
 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(
d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*c
osh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^
2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x
 + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d
*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
 + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x
 + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*c
osh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*
x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^
2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*
x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*l
og(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 +
4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*cosh(d*x + c)^8 + 40
*cosh(d*x + c)*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2
- 3)*sinh(d*x + c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)
*sinh(d*x + c)^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x
+ c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d
*x + c)^4 + 6*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*
(10*cosh(d*x + c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x +
c))*sinh(d*x + c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x
 + c)^8 + 8*b^2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4
*b^2*d*cosh(d*x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)
^2 + b^2*d)*sinh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c
)^4 + 30*b^2*d*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh
(d*x + c)^5 + 10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x +
c)^3 + b^2*d + 4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^
2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3
*b^2*d*cosh(d*x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sin
h(d*x + c)), 1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 +
sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x +
c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh
(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 +
 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 +
 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
```

```
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d
*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(
d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x +
c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 +
9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x +
c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c
)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*c
osh(d*x + c)/sinh(d*x + c)) - b) - 16*(5*cosh(d*x + c)^8 + 40*cosh(d*x + c)
*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2 - 3)*sinh(d*x
+ c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)
^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x + c)^3 + cosh(
d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d*x + c)^4 + 6*
cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*(10*cosh(d*x +
 c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x +
 c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^8 + 8*b^
2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4*b^2*d*cosh(d*
x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)^2 + b^2*d)*si
nh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*
x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c)^4 + 30*b^2*d
*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh(d*x + c)^5 +
10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*d +
 4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^2*d*cosh(d*x +
 c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3*b^2*d*cosh(d*
x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 2.33, size = 521, normalized size = 3.70

$$-\frac{\frac{42\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\mathrm{sgn}\left(e^{(4dx+4c)}-1\right)}+\frac{21\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right)}{\sqrt{b}\,\mathrm{sgn}\left(e^{(6dx+6c)}-3e^{(4dx+4c)}+3e^{(2dx+2c)}-1\right)\mathrm{sgn}\left(e^{(4dx+4c)}-1\right)}+\frac{16\left(21\left(\sqrt{b}e\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/42*(42*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/s
qrt(b))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c
) - 1)*sgn(e^(4*d*x + 4*c) - 1)) + 21*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sq
rt(b*e^(4*d*x + 4*c) - b)))/(sqrt(b)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c
) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)) + 16*(21*(sqrt(b)*e^(2
*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^6 + 42*(sqrt(b)*e^(2*d*x + 2*c)
- sqrt(b*e^(4*d*x + 4*c) - b))^5*sqrt(b) + 119*(sqrt(b)*e^(2*d*x + 2*c) - s
qrt(b*e^(4*d*x + 4*c) - b))^4*b + 56*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4
*d*x + 4*c) - b))^3*b^(3/2) + 63*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x
 + 4*c) - b))^2*b^2 + 14*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
- b))*b^(5/2) + 5*b^3)/((sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b) + sqrt(b))^7*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c
) - 1)*sgn(e^(4*d*x + 4*c) - 1)))/(b*d)
```

maple [A] time = 0.14, size = 106, normalized size = 0.75

$$\frac{\coth(dx+c)\left(-14b^{\frac{15}{2}}\left(\coth^2(dx+c)\right)-6b^{\frac{15}{2}}+21\,\mathrm{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^4\left(b\coth(dx+c)\right)^{\frac{7}{2}}+21\arctan\left(\cdots\right.}{21d\,b^{\frac{15}{2}}\left(b\left(\coth^3(dx+c)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(3/2),x)
```

```
[Out] 1/21/d*coth(d*x+c)/b^(15/2)*(-14*b^(15/2)*coth(d*x+c)^2-6*b^(15/2)+21*arcta
nh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2)+21*arctan((b*co
th(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2))/(b*coth(d*x+c)^3)^(3/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth(dx+c)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\coth(c+d\,x)^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(3/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-3/2), x)
```

3.33 $\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=74

$$-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+bx\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}$$

```
[Out] -b*(b*coth(d*x+c)^3)^(1/3)/d-1/3*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/3)/d+
b*x*(b*coth(d*x+c)^3)^(1/3)*tanh(d*x+c)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}+bx\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(4/3),x]
```

```
[Out] -((b*(b*Coth[c + d*x]^3)^(1/3))/d) - (b*Coth[c + d*x]^2*(b*Coth[c + d*x]^3)
^(1/3))/(3*d) + b*x*(b*Coth[c + d*x]^3)^(1/3)*Tanh[c + d*x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx=\left(b\sqrt[3]{b\coth^3(c+dx)}\,\tanh(c+dx)\right)\int \coth^4(c+dx)\,dx$$

$$=-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+\left(b\sqrt[3]{b\coth^3(c+dx)}\,\tanh(c+dx)\right)\int \coth^2(c$$

$$=-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+\left(b\sqrt[3]{b\coth^3(c+dx)}\,\tan\right.$$

$$=-\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+bx\sqrt[3]{b\coth^3(c+dx)}\,\tan$$

Mathematica [C] time = 0.07, size = 43, normalized size = 0.58

$$-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{4/3}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(4/3),x]
```

```
[Out] -1/3*((b*Coth[c + d*x]^3)^(4/3)*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d
*x]^2]*Tanh[c + d*x])/d
```

fricas [B] time = 1.45, size = 1046, normalized size = 14.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*d*x*cosh(d*x + c)^6 - 3*(b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x
+ c)^6 - 18*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sin
h(d*x + c)^5 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 + 3*(15*b*d*x*cosh(d*x + c
)^2 - 3*b*d*x - (15*b*d*x*cosh(d*x + c)^2 - 3*b*d*x - 4*b)*e^(2*d*x + 2*c)
- 4*b)*sinh(d*x + c)^4 + 12*(5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh
(d*x + c) - (5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c)^3 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 3*(15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2
- (15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 4*b)*e^(2*d*x + 2*c) + 4*b)*sinh(d*x + c)^2 - (3*b*d*x*cosh(d*x + c)^6 - 3
*(3*b*d*x + 4*b)*cosh(d*x + c)^4 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c
)^2 - 8*b)*e^(2*d*x + 2*c) + 6*(3*b*d*x*cosh(d*x + c)^5 - 2*(3*b*d*x + 4*b)
*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c) - (3*b*d*x*cosh(d*x + c)^5
 - 2*(3*b*d*x + 4*b)*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) - 8*b)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c)
+ 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*
x + 2*c) - 1))^(1/3)/(d*cosh(d*x + c)^6 + (d*e^(2*d*x + 2*c) + d)*sinh(d*x
+ c)^6 + 6*(d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c
)^5 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + (5*d*cosh(d*x + c)^2 -
 d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c) + (5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
+ c)^2 + (5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) +
d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*d*cosh(d*
x + c)^2 - d)*e^(2*d*x + 2*c) + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c) + (d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)^3\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(4/3), x)
```

maple [B] time = 0.40, size = 145, normalized size = 1.96

$$\frac{b\left(\mathrm{e}^{2dx+2c}-1\right)\left(\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^{3}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}} x}{1+\mathrm{e}^{2dx+2c}}-\frac{4b\left(\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^{3}}{\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(3\,\mathrm{e}^{4dx+4c}-3\,\mathrm{e}^{2dx+2c}+2\right)}{3\left(1+\mathrm{e}^{2dx+2c}\right)\left(\mathrm{e}^{2dx+2c}-1\right)^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(4/3),x)
```

```
[Out] b/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+
2*c)-1)^3)^(1/3)*x-4/3*b/(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)^2*(b*(1+exp(
2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(3*exp(4*d*x+4*c)-3*exp(2*d*x+2*c
)+2)/d
```

maxima [A] time = 0.42, size = 87, normalized size = 1.18

$$\frac{(dx+c)b^{\frac{4}{3}}}{d}-\frac{4\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)}-3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)}-2\,b^{\frac{4}{3}}\right)}{3\,d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(4/3)/d - 4/3*(3*b^(4/3)*e^(-2*d*x - 2*c) - 3*b^(4/3)*e^(-4*d*x
 - 4*c) - 2*b^(4/3))/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\coth\left(c+d\,x\right)^3\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(4/3),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(4/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(4/3),x)
```

```
[Out] Timed out
```

3.34 $\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=50

$$x\tanh^2(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}}{d}$$

```
[Out] -(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+c
)^2
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$x\tanh^2(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(2/3),x]
```

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x])/d) + x*(b*Coth[c + d*x]^3)^(2/3
)*Tanh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx &= \left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int \coth^2(c+dx)\,dx\\ &= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+\left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int 1\,dx\\ &= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+x\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.82

$$-\frac{\tanh(c+dx)\left(b\coth^3(c+dx)\right)^{2/3}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(2/3),x]
```

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^
2]*Tanh[c + d*x])/d)
```

fricas [B] time = 0.66, size = 392, normalized size = 7.84

$$\frac{\left(dx\cosh\left(dx+c\right)^{2}+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^{2}-dx+\left(dx\cosh\left(dx+c\right)^{2}-dx-2\right)e^{(4\,dx+4\,c}}{d\cosh\left(dx+c\right)^{2}+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx+c\right)^{2}+\left(d\cosh\left(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x +
2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2
/3)/(d*cosh(d*x + c)^2 + (d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh
(d*x + c)^2 + (d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^
2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x
+ c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)^{3}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(2/3), x)
```

maple [B] time = 0.40, size = 119, normalized size = 2.38

$$\frac{\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^{2}x}{\left(1+e^{2dx+2c}\right)^{2}}-\frac{2\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)}{\left(1+e^{2dx+2c}\right)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(2/3),x)
```

```
[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(1+exp(2*d*x+2*c))^2*(e
xp(2*d*x+2*c)-1)^2*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/
(1+exp(2*d*x+2*c))^2*(exp(2*d*x+2*c)-1)/d
```

maxima [A] time = 0.42, size = 34, normalized size = 0.68

$$\frac{(dx+c)b^{\frac{2}{3}}}{d}+\frac{2\,b^{\frac{2}{3}}}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
[Out] (d*x + c)*b^(2/3)/d + 2*b^(2/3)/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\coth\left(c+d\,x\right)^3\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(2/3),x)
[Out] int((b*coth(c + d*x)^3)^(2/3), x)
```

sympy [A] time = 23.76, size = 90, normalized size = 1.80

$$\begin{cases} \tilde{\infty} b^{\frac{2}{3}} x & \text{for}\: c = \log\left(-e^{-dx}\right) \vee c = \log\left(e^{-dx}\right) \\ x \left(b \coth^{3}\left(c\right)\right)^{\frac{2}{3}} & \text{for}\: d = 0 \\ b^{\frac{2}{3}} x \left(\frac{1}{\tanh^{3}\left(c + dx\right)}\right)^{\frac{2}{3}} \tanh^{2}\left(c + dx\right) - \frac{b^{\frac{2}{3}} \left(\frac{1}{\tanh^{3}\left(c + dx\right)}\right)^{\frac{2}{3}} \tanh\left(c + dx\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(2/3),x)
[Out] Piecewise((zoo*b**(2/3)*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))),
 (x*(b*coth(c)**3)**(2/3), Eq(d, 0)), (b**(2/3)*x*(tanh(c + d*x)**(-3))**(2
/3)*tanh(c + d*x)**2 - b**(2/3)*(tanh(c + d*x)**(-3))**(2/3)*tanh(c + d*x)/
d, True))
```

3.35 $\int \sqrt[3]{b \coth^3(c+dx)}\, dx$

Optimal. Leaf size=31

$$\frac{\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\,\log(\sinh(c+dx))}{d}$$

```
[Out] (b*coth(d*x+c)^3)^(1/3)*ln(sinh(d*x+c))*tanh(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3658, 3475}

$$\frac{\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\,\log(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\coth^3(c+dx)}\,dx &= \left(\sqrt[3]{b\coth^3(c+dx)}\,\tanh(c+dx)\right)\int \coth(c+dx)\,dx\\ &= \frac{\sqrt[3]{b\coth^3(c+dx)}\,\log(\sinh(c+dx))\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.26

$$\frac{\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}\,(\log(\tanh(c+dx))+\log(\cosh(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^3)^(1/3)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c
 + d*x])/d
```

fricas [B] time = 0.57, size = 148, normalized size = 4.77

$$-\frac{\left(dxe^{(2\,dx+2\,c)}-dx-\left(e^{(2\,dx+2\,c)}-1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c)}+3\,be^{(4\,dx+4\,c)}+3\,be^{(2\,dx+2\,c)}+b}{e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1}\right)^{\frac{1}{3}}}{de^{(2\,dx+2\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) +
3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x
+ 2*c) - 1))^(1/3)/(d*e^(2*d*x + 2*c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)^{3}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(1/3), x)
```

maple [B] time = 0.38, size = 192, normalized size = 6.19

$$\frac{\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)x}{1+e^{2dx+2c}}-\frac{2\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)(dx+c)}{\left(1+e^{2dx+2c}\right)d}+\frac{\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)\ln\left(e^{2dx+2c}\right.}{\left(1+e^{2dx+2c}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^3)^(1/3),x)
```

```
[Out] (b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp
(2*d*x+2*c)-1)*x-2*(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(1+e
xp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*(d*x+c)+(b*(1+exp(2*d*x+2*c))^3/(exp(2*
d*x+2*c)-1)^3)^(1/3)/(1+exp(2*d*x+2*c))*(exp(2*d*x+2*c)-1)/d*ln(exp(2*d*x+2
*c)-1)
```

maxima [A] time = 0.51, size = 51, normalized size = 1.65

$$\frac{(dx+c)b^{\frac{1}{3}}}{d}+\frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(1/3)/d + b^(1/3)*log(e^(-d*x - c) + 1)/d + b^(1/3)*log(e^(-d*x
 - c) - 1)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^3\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^3)^(1/3),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(1/3), x)
```

3.36 $\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^3)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3658, 3475}

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-1/3),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx &= \frac{\coth(c+dx)\int \tanh(c+dx)\,dx}{\sqrt[3]{b\coth^3(c+dx)}}\\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.00

$$\frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-1/3),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

fricas [B] time = 1.53, size = 187, normalized size = 6.03

$$-\frac{\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx-\left(e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c)}+3\,be^{(4\,dx+4\,c)}+3\,b}{e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e}\right.}{bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x - (e^(4*d*x + 4*c) - 2*
e^(2*d*x + 2*c) + 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))*
((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*
d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b*d*e^(4*d*
x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{3}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-1/3), x)
```

maple [B] time = 0.41, size = 192, normalized size = 6.19

$$\frac{\left(1+e^{2dx+2c}\right)x}{\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)}-\frac{2\left(1+e^{2dx+2c}\right)(dx+c)}{\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)d}+\frac{\left(1+e^{2dx+2c}\right)\ln\left(1+e^{2dx+2c}\right)}{\left(\frac{b\left(1+e^{2dx+2c}\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(1/3),x)
```

```
[Out] 1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1
+exp(2*d*x+2*c))*x-2/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(e
xp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*(d*x+c)+1/(b*(1+exp(2*d*x+2*c))^3/(ex
p(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(1+exp(2*d*x+2*c))/d*ln(1+exp(2
*d*x+2*c))
```

maxima [A] time = 0.83, size = 32, normalized size = 1.03

$$\frac{dx+c}{b^{\frac{1}{3}}d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b^(1/3)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^(1/3)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^{3}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(1/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b \coth^3(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-1/3), x)
```

3.37 $\int \frac{1}{\left(b \coth^3(c+dx)\right)^{2/3}} dx$

Optimal. Leaf size=50

$$\frac{x \coth^2(c+dx)}{\left(b \coth^3(c+dx)\right)^{2/3}}-\frac{\coth(c+dx)}{d \left(b \coth^3(c+dx)\right)^{2/3}}$$

```
[Out] -coth(d*x+c)/d/(b*coth(d*x+c)^3)^(2/3)+x*coth(d*x+c)^2/(b*coth(d*x+c)^3)^(2
/3)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x \coth^2(c+dx)}{\left(b \coth^3(c+dx)\right)^{2/3}}-\frac{\coth(c+dx)}{d \left(b \coth^3(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-2/3),x]
```

```
[Out] -(Coth[c + d*x]/(d*(b*Coth[c + d*x]^3)^(2/3))) + (x*Coth[c + d*x]^2)/(b*Cot
h[c + d*x]^3)^(2/3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(b \coth^3(c+dx)\right)^{2/3}} dx &= \frac{\coth^2(c+dx) \int \tanh^2(c+dx) \, dx}{\left(b \coth^3(c+dx)\right)^{2/3}}\\
&= -\frac{\coth(c+dx)}{d \left(b \coth^3(c+dx)\right)^{2/3}}+\frac{\coth^2(c+dx) \int 1 \, dx}{\left(b \coth^3(c+dx)\right)^{2/3}}\\
&= -\frac{\coth(c+dx)}{d \left(b \coth^3(c+dx)\right)^{2/3}}+\frac{x \coth^2(c+dx)}{\left(b \coth^3(c+dx)\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.80

$$\frac{\coth(c+dx)\left(\tanh^{-1}(\tanh(c+dx))\coth(c+dx)-1\right)}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-2/3),x]
```

```
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*(b*Coth[c +
d*x]^3)^(2/3))
```

fricas [B] time = 0.60, size = 287, normalized size = 5.74

$$-\frac{\left(dx\cosh\left(dx+c\right)^2-\left(dxe^{(2\,dx+2\,c)}-dx\right)\sinh\left(dx+c\right)^2+dx-\left(dx\cosh\left(dx+c\right)^2+dx+2\right)e^{(2\,dx+2\,c)}-2\left(dx\,\mathrm{c}\right.}{bd\cosh\left(dx+c\right)^2+\left(bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^2+bd+\left(bd\cosh\left(dx+c\right)^2+bd\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -(d*x*cosh(d*x + c)^2 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^2 + d*x -
 (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) - 2*(d*x*cosh(d*x + c)*e^(
2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c) + 2)*((b*e^(6*d*x + 6*c) +
3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*
x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(1/3)/(b*d*cosh(d*x + c)^2 + (b*d*e^(2*d
*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(
d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-2/3), x)
```

maple [B] time = 0.40, size = 119, normalized size = 2.38

$$\frac{\left(1+\mathrm{e}^{2dx+2c}\right)^2 x}{\left(\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^3}{\left(\mathrm{e}^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{2+2\,\mathrm{e}^{2dx+2c}}{\left(\frac{b\left(1+\mathrm{e}^{2dx+2c}\right)^3}{\left(\mathrm{e}^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(\mathrm{e}^{2dx+2c}-1\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(2/3),x)
```

```
[Out] 1/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(exp(2*d*x+2*c)-1)^2*
(1+exp(2*d*x+2*c))^2*x+2/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(2/3
)/(exp(2*d*x+2*c)-1)^2*(1+exp(2*d*x+2*c))/d
```

maxima [A] time = 1.06, size = 37, normalized size = 0.74

$$\frac{dx+c}{b^{\frac{2}{3}}d}-\frac{2}{\left(b^{\frac{2}{3}}e^{(-2\,dx-2\,c)}+b^{\frac{2}{3}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b^(2/3)*d) - 2/((b^(2/3)*e^(-2*d*x - 2*c) + b^(2/3))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(2/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-2/3), x)
```

3.38 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=80

$$\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] -1/b/d/(b*coth(d*x+c)^3)^(1/3)+x*coth(d*x+c)/b/(b*coth(d*x+c)^3)^(1/3)-1/3*
tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/3)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^3)^(-4/3),x]
```

```
[Out] -(1/(b*d*(b*Coth[c + d*x]^3)^(1/3))) + (x*Coth[c + d*x])/(b*(b*Coth[c + d*x
]^3)^(1/3)) - Tanh[c + d*x]^2/(3*b*d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \frac{\coth(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{\coth(c+dx)\int \tanh^2(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{\coth(c+dx)\int 1\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}$$
$$= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.64

$$\frac{-\tanh^2(c+dx) + 3\tanh^{-1}(\tanh(c+dx))\coth(c+dx) - 3}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-4/3),x]
```

```
[Out] (-3 + 3*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x] - Tanh[c + d*x]^2)/(3*b*d*(b*C
oth[c + d*x]^3)^(1/3))
```

fricas [B] time = 0.56, size = 1579, normalized size = 19.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cosh(d*x + c)^6 + 3*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c)
 + d*x)*sinh(d*x + c)^6 + 18*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d*x*cos
h(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(3*d*x
+ 4)*cosh(d*x + c)^4 + 3*(15*d*x*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x
 + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^2 + 3*d*x +
4)*e^(2*d*x + 2*c) + 4)*sinh(d*x + c)^4 + 12*(5*d*x*cosh(d*x + c)^3 + (3*d*
x + 4)*cosh(d*x + c) + (5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*
e^(4*d*x + 4*c) - 2*(5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(
2*d*x + 2*c))*sinh(d*x + c)^3 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*(15*d*x*c
osh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x +
 c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*
d*x*cosh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(2*d*x +
 2*c) + 4)*sinh(d*x + c)^2 + 3*d*x + (3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)
*cosh(d*x + c)^4 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(4*d*x + 4*
c) - 2*(3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)*cosh(d*x + c)^4 + 3*(3*d*x +
4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(2*d*x + 2*c) + 6*(3*d*x*cosh(d*x + c)^5
+ 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c) + (3*d*x*cosh(d
*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(4
*d*x + 4*c) - 2*(3*d*x*cosh(d*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3
*d*x + 4)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + 8)*((b*e^(6*d*x +
 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3
*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b^2*d*cosh(d*x + c)^6 + 3
*b^2*d*cosh(d*x + c)^4 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c) +
```

```
 b^2*d)*sinh(d*x + c)^6 + 6*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^2*d*
cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*b^
2*d*cosh(d*x + c)^2 + 3*(5*b^2*d*cosh(d*x + c)^2 + b^2*d + (5*b^2*d*cosh(d*
x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(
2*d*x + 2*c))*sinh(d*x + c)^4 + 4*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d
*x + c) + (5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c)
 + 2*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + b^2*d + 3*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2
 + b^2*d + (5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4
*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)
*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^6 + 3*b^2*d*cosh(d
*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cos
h(d*x + c)^6 + 3*b^2*d*cosh(d*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c) + 6*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d*c
osh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x
 + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-4/3), x)
```

maple [B] time = 0.39, size = 149, normalized size = 1.86

$$\frac{\left(1+e^{2dx+2c}\right)x}{b\left(e^{2dx+2c}-1\right)\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}}+\frac{4\,e^{4dx+4c}+4\,e^{2dx+2c}+\frac{8}{3}}{b\left(1+e^{2dx+2c}\right)^2\left(e^{2dx+2c}-1\right)\left(\frac{b\left(1+e^{2dx+2c}\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^3)^(4/3),x)
```

```
[Out] 1/b*(1+exp(2*d*x+2*c))/(exp(2*d*x+2*c)-1)/(b*(1+exp(2*d*x+2*c))^3/(exp(2*d*
x+2*c)-1)^3)^(1/3)*x+4/3/b/(1+exp(2*d*x+2*c))^2/(exp(2*d*x+2*c)-1)/(b*(1+ex
p(2*d*x+2*c))^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(3*exp(4*d*x+4*c)+3*exp(2*d*x+2
*c)+2)/d
```

maxima [A] time = 0.42, size = 89, normalized size = 1.11

$$-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{3\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)}+3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)}+b^{\frac{4}{3}}e^{(-6\,dx-6\,c)}+b^{\frac{4}{3}}\right)d}+\frac{dx+c}{b^{\frac{4}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/((3*b^(4/3)*e^(-2*d*x -
2*c) + 3*b^(4/3)*e^(-4*d*x - 4*c) + b^(4/3)*e^(-6*d*x - 6*c) + b^(4/3))*d)
+ (d*x + c)/(b^(4/3)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^3\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^3)^(4/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^{3}\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**3)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-4/3), x)
```

3.39 $\int \left(b\coth^4(c+dx)\right)^n\,dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);\coth^2(c+dx)\right)}{d(4n+1)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^4)^n*hypergeom([1, 1/2+2*n],[3/2+2*n],coth(d*x+c
)^2)/d/(1+4*n)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3658, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);\coth^2(c+dx)\right)}{d(4n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3 +
 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b\coth^4(c+dx)\right)^n\,dx &= \left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\int \coth^{4n}(c+dx)\,dx\\ &= -\frac{\left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\operatorname{Subst}\left(\int \frac{x^{4n}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\ &= \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(1+4n);\frac{1}{2}(3+4n);\coth^2(c+dx)\right)}{d(1+4n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.89

$$\frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n\,{}_2F_1\left(1,2n+\frac{1}{2};2n+\frac{3}{2};\coth^2(c+dx)\right)}{4dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, 1/2 + 2*n, 3/2 + 
2*n, Coth[c + d*x]^2])/(d + 4*d*n)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\coth\left(dx+c\right)^4\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^4)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)^4\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int\left(b\left(\coth^4\left(dx+c\right)\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^n,x)
```

```
[Out] int((b*coth(d*x+c)^4)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\coth\left(dx+c\right)^4\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(b\coth\left(c+d\,x\right)^4\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^n,x)
```

```
[Out] int((b*coth(c + d*x)^4)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4(c+dx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**n, x)
```

3.40 $\int \left(b \coth^4(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=110

$$-\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d}+bx\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}-\frac{b\tanh$$

```
[Out] -1/3*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/2)/d-1/5*b*coth(d*x+c)^3*(b*coth(d*
x+c)^4)^(1/2)/d-b*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+b*x*(b*coth(d*x+c)^
4)^(1/2)*tanh(d*x+c)^2
```

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d}-\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}+bx\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)}-\frac{b\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(3/2),x]
```

```
[Out] -(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^4])/(3*d) - (b*Coth[c + d*x]^3*Sqrt[
b*Coth[c + d*x]^4])/(5*d) - (b*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d + b
*x*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx = \left(b\sqrt{b\coth^4(c+dx)}\,\tanh^2(c+dx)\right)\int \coth^6(c+dx)\,dx$$
$$= -\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} + \left(b\sqrt{b\coth^4(c+dx)}\,\tanh^2(c+dx)\right)\int \operatorname{coth}$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} + \left(b\sqrt{b\operatorname{co}}\right.$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b\operatorname{co}}}{}$$
$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b\operatorname{co}}}{}$$

Mathematica [C] time = 0.07, size = 43, normalized size = 0.39

$$-\frac{\tanh(c+dx)\left(b\coth^4(c+dx)\right)^{3/2}\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\tanh^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(3/2),x]
```

```
[Out] -1/5*((b*Coth[c + d*x]^4)^(3/2)*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d
*x]^2]*Tanh[c + d*x])/d
```

fricas [B] time = 0.91, size = 3421, normalized size = 31.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*b*d*x*cosh(d*x + c)^10 + 15*(b*d*x*e^(4*d*x + 4*c) - 2*b*d*x*e^(2*
d*x + 2*c) + b*d*x)*sinh(d*x + c)^10 + 150*(b*d*x*cosh(d*x + c)*e^(4*d*x +
4*c) - 2*b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*x*cosh(d*x + c))*sinh(d*
x + c)^9 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 15*(45*b*d*x*cosh(d*x + c)^
2 - 5*b*d*x + (45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(4*d*x + 4*c) -
2*(45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(2*d*x + 2*c) - 6*b)*sinh(d*
x + c)^8 + 120*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c) +
(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c)
- 2*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(2*d*x + 2
*c))*sinh(d*x + c)^7 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 30*(105*b*d*x*c
osh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (105*b*d*x*
cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + 6*b)*e^(4*
d*x + 4*c) - 2*(105*b*d*x*cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*co
sh(d*x + c)^2 + 6*b)*e^(2*d*x + 2*c) + 6*b)*sinh(d*x + c)^6 + 60*(63*b*d*x*
cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*co
sh(d*x + c) + (63*b*d*x*cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^
3 + 3*(5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(63*b*d*x*cosh(d*x
 + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 - 10*(15*b*d*x + 28*b)*cosh(d*x + c)^
4 + 10*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 1
5*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (315*b*d*x*cosh(d*x + c)^6 -
 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d
```

```
*x + c)^2 - 28*b)*e^(4*d*x + 4*c) - 2*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b
*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2
 - 28*b)*e^(2*d*x + 2*c) - 28*b)*sinh(d*x + c)^4 + 40*(45*b*d*x*cosh(d*x +
c)^7 - 21*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c
)^3 - (15*b*d*x + 28*b)*cosh(d*x + c) + (45*b*d*x*cosh(d*x + c)^7 - 21*(5*b
*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15*b*d*
x + 28*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(45*b*d*x*cosh(d*x + c)^7 - 21
*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15
*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^3 - 15*b*d*x +
 5*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 5*(135*b*d*x*cosh(d*x + c)^8 - 84*(5
*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*b*d
*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + (135*b*d*x*cosh(d*x + c)^8 - 84
*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*
b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 28*b)*e^(4*d*x + 4*c) - 2*(1
35*b*d*x*cosh(d*x + c)^8 - 84*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x
 + 6*b)*cosh(d*x + c)^4 + 15*b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 +
 28*b)*e^(2*d*x + 2*c) + 28*b)*sinh(d*x + c)^2 + (15*b*d*x*cosh(d*x + c)^10
 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6
- 10*(15*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cos
h(d*x + c)^2 - 46*b)*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^10 - 15*(5
*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 - 10*(15
*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cosh(d*x +
c)^2 - 46*b)*e^(2*d*x + 2*c) + 10*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x +
 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x +
28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c) + (15*b*d*x*cosh(d*
x + c)^9 - 12*(5*b*d*x + 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x
 + c)^5 - 4*(15*b*d*x + 28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x
+ c))*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x + 6*b)*co
sh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x + 28*b)*co
sh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x
+ c) - 46*b)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*x +
 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) + 6*e
^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^10 + (d*e^(4*d*x
+ 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^10 + 10*(d*cosh(d*x + c)*e^
(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d
*x + c)^9 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + (9*d*cosh(d*x +
c)^2 - d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d
)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d
*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c)^3 - d*c
osh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*
(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^4 - 14*d
*cosh(d*x + c)^2 + d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^4 - 14*d*cosh
(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x +
c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c) + (63*d*cosh(d*x + c)^5 -
70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(63*d*cosh(d
*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^5 - 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 - 35*d*cosh(
d*x + c)^4 + 15*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^6 - 35*d*cosh(d*x +
 c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^6
 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*si
nh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*
x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c
)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*d*x
 + 2*c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 - 2
8*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + (9*d*co
sh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x
 + c)^2 + d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^
6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*s
```

```
inh(d*x + c)^2 + (d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x
+ c)^6 - 10*d*cosh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) +
2*(d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x + c)^6 - 10*d*c
osh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 10*(d*cosh(d*x
+ c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 +
d*cosh(d*x + c) + (d*cosh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x +
 c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(d*cosh(
d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

giac [A] time = 0.18, size = 77, normalized size = 0.70

$$\frac{\left(15\,dx + 15\,c - \frac{2\left(45\,e^{(8\,dx+8\,c)} - 90\,e^{(6\,dx+6\,c)} + 140\,e^{(4\,dx+4\,c)} - 70\,e^{(2\,dx+2\,c)} + 23\right)}{\left(e^{(2\,dx+2\,c)} - 1\right)^5}\right)b^{\frac{3}{2}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*d*x + 15*c - 2*(45*e^(8*d*x + 8*c) - 90*e^(6*d*x + 6*c) + 140*e^(4
*d*x + 4*c) - 70*e^(2*d*x + 2*c) + 23)/(e^(2*d*x + 2*c) - 1)^5)*b^(3/2)/d
```

maple [A] time = 0.16, size = 77, normalized size = 0.70

$$-\frac{\left(b\left(\coth^4(dx+c)\right)\right)^{\frac{3}{2}}\left(6\left(\coth^5(dx+c)\right) + 10\left(\coth^3(dx+c)\right) + 15\ln(\coth(dx+c)-1) - 15\ln(\coth(dx+c)+1) + 30\coth(dx+c)\right)}{30d\coth(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(3/2),x)
```

```
[Out] -1/30/d*(b*coth(d*x+c)^4)^(3/2)*(6*coth(d*x+c)^5+10*coth(d*x+c)^3+15*ln(cot
h(d*x+c)-1)-15*ln(coth(d*x+c)+1)+30*coth(d*x+c))/coth(d*x+c)^6
```

maxima [A] time = 0.42, size = 137, normalized size = 1.25

$$\frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{2\left(70\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)} - 140\,b^{\frac{3}{2}}e^{(-4\,dx-4\,c)} + 90\,b^{\frac{3}{2}}e^{(-6\,dx-6\,c)} - 45\,b^{\frac{3}{2}}e^{(-8\,dx-8\,c)} - 23\,b^{\frac{3}{2}}\right)}{15\,d\left(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(3/2)/d - 2/15*(70*b^(3/2)*e^(-2*d*x - 2*c) - 140*b^(3/2)*e^(-4
*d*x - 4*c) + 90*b^(3/2)*e^(-6*d*x - 6*c) - 45*b^(3/2)*e^(-8*d*x - 8*c) - 2
3*b^(3/2))/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\coth(c+d\,x)^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(3/2),x)
```

```
[Out] int((b*coth(c + d*x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4(c+dx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(3/2), x)
```

3.41 $\int \sqrt{b \coth^4(c+dx)}\, dx$

Optimal. Leaf size=50

$$x \tanh^2(c+dx)\sqrt{b \coth^4(c+dx)} - \frac{\tanh(c+dx)\sqrt{b \coth^4(c+dx)}}{d}$$

```
[Out] -(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c
)^2
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3658, 3473, 8}

$$x \tanh^2(c+dx)\sqrt{b \coth^4(c+dx)} - \frac{\tanh(c+dx)\sqrt{b \coth^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d) + x*Sqrt[b*Coth[c + d*x]^4]*Ta
nh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b \coth^4(c+dx)}\, dx &= \left(\sqrt{b \coth^4(c+dx)} \tanh^2(c+dx)\right)\int \coth^2(c+dx)\, dx\\
&= -\frac{\sqrt{b \coth^4(c+dx)} \tanh(c+dx)}{d} + \left(\sqrt{b \coth^4(c+dx)} \tanh^2(c+dx)\right)\int 1\, dx\\
&= -\frac{\sqrt{b \coth^4(c+dx)} \tanh(c+dx)}{d} + x\sqrt{b \coth^4(c+dx)} \tanh^2(c+dx)
\end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.82

$$-\frac{\tanh(c+dx)\sqrt{b \coth^4(c+dx)}\, {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2]
*Tanh[c + d*x])/d)
```

fricas [B] time = 1.10, size = 415, normalized size = 8.30

$$\frac{\left(dx\cosh\left(dx+c\right)^{2}+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^{2}-dx+\left(dx\cosh\left(dx+c\right)^{2}-dx-2\right)e^{(4\,dx+4\,c}}{d\cosh\left(dx+c\right)^{2}+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx+c\right)^{2}+\left(d\cosh\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^2 + (d*e^(4*d*
x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^2 -
d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(
d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x +
 c))*sinh(d*x + c) - d)
```

giac [A] time = 0.16, size = 27, normalized size = 0.54

$$\frac{\left(dx+c-\frac{2}{e^{(2\,dx+2\,c)}-1}\right)\sqrt{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] (d*x + c - 2/(e^(2*d*x + 2*c) - 1))*sqrt(b)/d
```

maple [A] time = 0.16, size = 55, normalized size = 1.10

$$-\frac{\sqrt{b\left(\coth^{4}\left(dx+c\right)\right)}\;\left(2\coth\left(dx+c\right)+\ln\left(\coth\left(dx+c\right)-1\right)-\ln\left(\coth\left(dx+c\right)+1\right)\right)}{2d\coth\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/2/d*(b*coth(d*x+c)^4)^(1/2)*(2*coth(d*x+c)+ln(coth(d*x+c)-1)-ln(coth(d*x
+c)+1))/coth(d*x+c)^2
```

maxima [A] time = 0.42, size = 34, normalized size = 0.68

$$\frac{(dx+c)\sqrt{b}}{d}+\frac{2\sqrt{b}}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)*sqrt(b)/d + 2*sqrt(b)/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \coth (c + d x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(1/2),x)
[Out] int((b*coth(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \coth^4 (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/2),x)
[Out] Integral(sqrt(b*coth(c + d*x)**4), x)
```

3.42 $\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx$

Optimal. Leaf size=50

$$\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}$$

```
[Out] -coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/(b*coth(d*x+c)^4)^(1
/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -(Coth[c + d*x]/(d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/Sqrt[b*C
oth[c + d*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx &= \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{\sqrt{b\coth^4(c+dx)}}\\ &= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}+\frac{\coth^2(c+dx)\int 1\,dx}{\sqrt{b\coth^4(c+dx)}}\\ &= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 0.80

$$\frac{\coth(c+dx)\left(\tanh^{-1}(\tanh(c+dx))\coth(c+dx)-1\right)}{d\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*Sqrt[b*Coth[
c + d*x]^4])
```

fricas [B] time = 1.30, size = 422, normalized size = 8.44

$$\frac{\left(dx\cosh\left(dx+c\right)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^2+dx+\left(dx\cosh\left(dx+c\right)^2+dx+2\right)e^{(4\,d}}{bd\cosh\left(dx+c\right)^2+\left(bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^2+bd+\left(bd\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 + d*x + (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) + 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b*d*cosh(d*x + c)^2 + (b*d*e^(
4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*co
sh(d*x + c)^2 + b*d)*e^(4*d*x + 4*c) + 2*(b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b*d*cosh(d*x + c)*e^(2
*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [A] time = 0.15, size = 32, normalized size = 0.64

$$\frac{\frac{dx+c}{\sqrt{b}}+\frac{2}{\sqrt{b}\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x + c)/sqrt(b) + 2/(sqrt(b)*(e^(2*d*x + 2*c) + 1)))/d
```

maple [A] time = 0.18, size = 59, normalized size = 1.18

$$-\frac{\coth\left(dx+c\right)\left(\ln\left(\coth\left(dx+c\right)-1\right)\coth\left(dx+c\right)-\ln\left(\coth\left(dx+c\right)+1\right)\coth\left(dx+c\right)+2\right)}{2d\sqrt{b\left(\coth^4\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)-1)*coth(d*x+c)-ln(coth(d*x+c)+1)*coth(d*
x+c)+2)/(b*coth(d*x+c)^4)^(1/2)
```

maxima [A] time = 0.42, size = 36, normalized size = 0.72

$$\frac{dx+c}{\sqrt{b}\,d}-\frac{2\,\sqrt{b}}{\left(be^{(-2\,dx-2\,c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(sqrt(b)*d) - 2*sqrt(b)/((b*e^(-2*d*x - 2*c) + b)*d)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(1/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^4)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)**4), x)
```

3.43 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}$$

```
[Out] -coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/b/(b*coth(d*x+c)^4
)^(1/2)-1/3*tanh(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)-1/5*tanh(d*x+c)^3/b/d/(
b*coth(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3658, 3473, 8}

$$\frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}-\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-3/2),x]
```

```
[Out] -(Coth[c + d*x]/(b*d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/(b*Sqr
t[b*Coth[c + d*x]^4]) - Tanh[c + d*x]/(3*b*d*Sqrt[b*Coth[c + d*x]^4]) - Tan
h[c + d*x]^3/(5*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx &= \frac{\coth^2(c+dx)\int \tanh^6(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}} \\
&= -\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}} \\
&= -\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}} \\
&= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} + \frac{\coth^2(c+}{b\sqrt{b\coth}} \\
&= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c}{5bd\sqrt{b\coth}}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 68, normalized size = 0.58

$$
\frac{-3\tanh^3(c+dx) - 5\tanh(c+dx) - 15\coth(c+dx) + 15\tanh^{-1}(\tanh(c+dx))\coth^2(c+dx)}{15bd\sqrt{b\coth^4(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-3/2),x]
```

```
[Out] (-15*Coth[c + d*x] + 15*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]^2 - 5*Tanh[c +
 d*x] - 3*Tanh[c + d*x]^3)/(15*b*d*Sqrt[b*Coth[c + d*x]^4])
```

fricas [B] time = 0.56, size = 3473, normalized size = 29.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cosh(d*x + c)^10 + 15*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x +
2*c) + d*x)*sinh(d*x + c)^10 + 150*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d
*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^9 + 15*
(5*d*x + 6)*cosh(d*x + c)^8 + 15*(45*d*x*cosh(d*x + c)^2 + 5*d*x + (45*d*x*
cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d*x + 4*c) - 2*(45*d*x*cosh(d*x + c)^2 +
5*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^8 + 120*(15*d*x*cosh(d*x + c)
^3 + (5*d*x + 6)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh
(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 30*(5*d*x + 6)*cosh(d*x + c)^6
+ 30*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + (1
05*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d
*x + 4*c) - 2*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5
*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^6 + 60*(63*d*x*cosh(d*x + c)^5
 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c) + (63*d*x*c
osh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x +
c))*e^(4*d*x + 4*c) - 2*(63*d*x*cosh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x +
 c)^3 + 3*(5*d*x + 6)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 10*
(15*d*x + 28)*cosh(d*x + c)^4 + 10*(315*d*x*cosh(d*x + c)^6 + 105*(5*d*x +
6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + (315*d*x*cos
h(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x +
```

```
c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(315*d*x*cosh(d*x + c)^6 + 105*(5*d
*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(
2*d*x + 2*c) + 28)*sinh(d*x + c)^4 + 40*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x
 + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh
(d*x + c) + (45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(
5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) -
 2*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6
)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x
+ c)^3 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 5*(135*d*x*cosh(d*x + c)^8 + 84*
(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x +
 28)*cosh(d*x + c)^2 + 15*d*x + (135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*c
osh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x
 + c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(135*d*x*cosh(d*x + c)^8 + 84*(5
*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 2
8)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(2*d*x + 2*c) + 28)*sinh(d*x + c)^2 + 1
5*d*x + (15*d*x*cosh(d*x + c)^10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d
*x + 6)*cosh(d*x + c)^6 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28
)*cosh(d*x + c)^2 + 15*d*x + 46)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^
10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d*x + 6)*cosh(d*x + c)^6 + 10*(
15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 4
6)*e^(2*d*x + 2*c) + 10*(15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x +
 c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 +
(15*d*x + 28)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh
(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c
)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c
)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(
15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c)
)*sinh(d*x + c) + 46)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e
^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6
*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b^2*d*cosh(d*x + c)^10 +
 5*b^2*d*cosh(d*x + c)^8 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c)
 + b^2*d)*sinh(d*x + c)^10 + 10*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^
2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^9 +
10*b^2*d*cosh(d*x + c)^6 + 5*(9*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2*d*co
sh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c)^2 + b^2*d
)*e^(2*d*x + 2*c))*sinh(d*x + c)^8 + 40*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c) + (3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(4*d*x + 4*
c) + 2*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^7 + 10*b^2*d*cosh(d*x + c)^4 + 10*(21*b^2*d*cosh(d*x + c)^4 + 14
*b^2*d*cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^4 + 14*b^2*d*cosh(
d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(d*x + c)^4 + 14*b^2*
d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^6 + 4*(63*b^2*d*c
osh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c) + (63*b^
2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c))*e^
(4*d*x + 4*c) + 2*(63*b^2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15
*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 5*b^2*d*cosh(d*x +
 c)^2 + 10*(21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*
cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c
)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(
d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c))*sinh(d*x + c)^4 + 40*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cos
h(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (3*b^2*d*cos
h(d*x + c)^7 + 7*b^2*d*cosh(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cosh(d*
x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*
sinh(d*x + c)^3 + b^2*d + 5*(9*b^2*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x +
c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2
*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 +
12*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c
```

```
)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d
*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^
10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d*cosh(d*x
 + c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(
d*x + c)^10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d
*cosh(d*x + c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 10*(b
^2*d*cosh(d*x + c)^9 + 4*b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 +
4*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^9 + 4*
b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 +
 b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^9 + 4*b^2*d*
cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

giac [A] time = 3.38, size = 99, normalized size = 0.84

$$\frac{\frac{15\,(dx+c)}{\sqrt{b}} + \frac{2\left(45\,\sqrt{b}\,e^{(8\,dx+8\,c)}+90\,\sqrt{b}\,e^{(6\,dx+6\,c)}+140\,\sqrt{b}\,e^{(4\,dx+4\,c)}+70\,\sqrt{b}\,e^{(2\,dx+2\,c)}+23\,\sqrt{b}\right)}{b\left(e^{(2\,dx+2\,c)}+1\right)^{5}}}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)/sqrt(b) + 2*(45*sqrt(b)*e^(8*d*x + 8*c) + 90*sqrt(b)*e^(
6*d*x + 6*c) + 140*sqrt(b)*e^(4*d*x + 4*c) + 70*sqrt(b)*e^(2*d*x + 2*c) + 2
3*sqrt(b))/(b*(e^(2*d*x + 2*c) + 1)^5))/(b*d)
```

maple [A] time = 0.13, size = 84, normalized size = 0.71

$$-\frac{\coth\left(dx+c\right)\left(15\ln\left(\coth\left(dx+c\right)-1\right)\left(\coth^{5}\left(dx+c\right)\right)-15\ln\left(\coth\left(dx+c\right)+1\right)\left(\coth^{5}\left(dx+c\right)\right)+30\left(\coth^{4}\right.\right.}{30d\left(b\left(\coth^{4}\left(dx+c\right)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(3/2),x)
```

```
[Out] -1/30/d*coth(d*x+c)*(15*ln(coth(d*x+c)-1)*coth(d*x+c)^5-15*ln(coth(d*x+c)+1
)*coth(d*x+c)^5+30*coth(d*x+c)^4+10*coth(d*x+c)^2+6)/(b*coth(d*x+c)^4)^(3/2
)
```

maxima [A] time = 0.45, size = 155, normalized size = 1.31

$$-\frac{2\left(70\,\sqrt{b}\,e^{(-2\,dx-2\,c)}+140\,\sqrt{b}\,e^{(-4\,dx-4\,c)}+90\,\sqrt{b}\,e^{(-6\,dx-6\,c)}+45\,\sqrt{b}\,e^{(-8\,dx-8\,c)}+23\,\sqrt{b}\right)}{15\left(5\,b^{2}e^{(-2\,dx-2\,c)}+10\,b^{2}e^{(-4\,dx-4\,c)}+10\,b^{2}e^{(-6\,dx-6\,c)}+5\,b^{2}e^{(-8\,dx-8\,c)}+b^{2}e^{(-10\,dx-10\,c)}+b^{2}\right)d}+\frac{dx+c}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(70*sqrt(b)*e^(-2*d*x - 2*c) + 140*sqrt(b)*e^(-4*d*x - 4*c) + 90*sqrt
(b)*e^(-6*d*x - 6*c) + 45*sqrt(b)*e^(-8*d*x - 8*c) + 23*sqrt(b))/((5*b^2*e^
(-2*d*x - 2*c) + 10*b^2*e^(-4*d*x - 4*c) + 10*b^2*e^(-6*d*x - 6*c) + 5*b^2*
e^(-8*d*x - 8*c) + b^2*e^(-10*d*x - 10*c) + b^2)*d) + (d*x + c)/(b^(3/2)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^{4}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(3/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-3/2), x)
```

3.44 $\int \left(b\coth^4(c+dx)\right)^{4/3} dx$

Optimal. Leaf size=353

$$-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}-\frac{3b\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{d}-\frac{b\sqrt[3]{\cdots}}{\cdots}$$

```
[Out] b*arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)-3/
7*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/3)/d-3/13*b*coth(d*x+c)^3*(b*coth(d*x+
c)^4)^(1/3)/d-1/4*b*(b*coth(d*x+c)^4)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x
+c)^(2/3))/d/coth(d*x+c)^(4/3)+1/4*b*(b*coth(d*x+c)^4)^(1/3)*ln(1+coth(d*x+
c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(4/3)-1/2*b*arctan(1/3*(1-2*coth(
d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+
1/2*b*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3
^(1/2)/d/coth(d*x+c)^(4/3)-3*b*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}-\frac{b\sqrt[3]{b\coth^4(c+dx)}\,\log\left(\coth^{\frac{2}{3}}(c+dx)\cdots\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] -(Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)
^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^
(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (b*A
rcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/
3)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^4)^(1/3))/(7*d) - (3*b*Coth[c + d
*x]^3*(b*Coth[c + d*x]^4)^(1/3))/(13*d) - (b*(b*Coth[c + d*x]^4)^(1/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) +
 (b*(b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*b*(b*Coth[c + d*x]^4)^(1/3)*Tanh[c +
d*x])/d
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
```

```
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx = \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{16}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{10}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} + \frac{\left(b\sqrt[3]{b\text{co}}\right.}{}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\text{co}}}{}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\text{co}}}{}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\text{co}}}{}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\text{co}}}{}$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} -$$

$$= \frac{b\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} -$$

$$= -\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\text{co}}}{2d\coth^{\frac{4}{3}}(c+dx)}$$

Mathematica [C] time = 0.14, size = 68, normalized size = 0.19

$$-\frac{3b\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}\left(-91\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)+7\coth^4(c+dx)+13\coth^2(c+dx)+91\right)}{91d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] (-3*b*(b*Coth[c + d*x]^4)^(1/3)*(91 + 13*Coth[c + d*x]^2 + 7*Coth[c + d*x]^
4 - 91*Hypergeometric2F1[1/6, 1, 7/6, Coth[c + d*x]^2])*Tanh[c + d*x])/(91*
d)
```

fricas [B] time = 1.33, size = 2864, normalized size = 8.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/364*(182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*sinh(d*x
 + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4*(7*sq
rt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b*cosh(
d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b*cosh(
d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x + c)^2
 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10*sqrt(
3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*sqrt(
3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*cosh(d
*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2 + 8*(s
qrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*cosh(d
*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*(-b)^(1/3)*
arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b) - 182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4
*(7*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b
*cosh(d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b
*cosh(d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x
+ c)^2 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10
*sqrt(3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4
*sqrt(3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*
cosh(d*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2
+ 8*(sqrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*
cosh(d*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*b^(1/
3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b) + 91*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 +
b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(
d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 +
6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)
*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 -
 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cos
h(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))
*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)
/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 91*(b*cosh
(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*c
osh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(
d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(
35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b
*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9
*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d
*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/
3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(2/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(1/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
```

```
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)
/sinh(d*x + c))^(1/3)) + 12*(111*b*cosh(d*x + c)^8 + 888*b*cosh(d*x + c)*si
nh(d*x + c)^7 + 111*b*sinh(d*x + c)^8 - 336*b*cosh(d*x + c)^6 + 84*(37*b*co
sh(d*x + c)^2 - 4*b)*sinh(d*x + c)^6 + 168*(37*b*cosh(d*x + c)^3 - 12*b*cos
h(d*x + c))*sinh(d*x + c)^5 + 562*b*cosh(d*x + c)^4 + 2*(3885*b*cosh(d*x +
c)^4 - 2520*b*cosh(d*x + c)^2 + 281*b)*sinh(d*x + c)^4 + 8*(777*b*cosh(d*x
+ c)^5 - 840*b*cosh(d*x + c)^3 + 281*b*cosh(d*x + c))*sinh(d*x + c)^3 - 336
*b*cosh(d*x + c)^2 + 12*(259*b*cosh(d*x + c)^6 - 420*b*cosh(d*x + c)^4 + 28
1*b*cosh(d*x + c)^2 - 28*b)*sinh(d*x + c)^2 + 8*(111*b*cosh(d*x + c)^7 - 25
2*b*cosh(d*x + c)^5 + 281*b*cosh(d*x + c)^3 - 84*b*cosh(d*x + c))*sinh(d*x
+ c) + 111*b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(d*cosh(d*x + c)^8 + 8
*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6
+ 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x +
 c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^
5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*
x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c
)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d
*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth \left(dx+c\right)^{4}\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(b \left(\coth^{4}\left(dx+c\right)\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth \left(dx+c\right)^{4}\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(4/3),x)
```

```
[Out] int((b*coth(c + d*x)^4)^(4/3), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4(c+dx)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(4/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(4/3), x)
```

3.45 $\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=291

$$-\frac{3\tanh(c+dx)\left(b\coth^4(c+dx)\right)^{2/3}}{5d}-\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)}+\frac{\left(b\coth^4(c\right.}{}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(2/3)/d/coth(d*x+c)^(8/3)-1/4*
(b*coth(d*x+c)^4)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(8/3)+1/4*(b*coth(d*x+c)^4)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(8/3)+1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^4)^(2/3)*3^(1/2)/d/coth(d*x+c)^(8/3)-1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(2/3)*3^(1/2)/d/coth(d*x+c)^(8/3)
-3/5*(b*coth(d*x+c)^4)^(2/3)*tanh(d*x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)}+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth\right.}{4d\coth^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2
/3))/(2*d*Coth[c + d*x]^(8/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(2/3))/(d*Coth[c + d*x]^(8/3)) - (
(b*Coth[c + d*x]^4)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(8/3)) + ((b*Coth[c + d*x]^4)^(2/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) - (3*(b*Coth[c
 + d*x]^4)^(2/3)*Tanh[c + d*x])/(5*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\coth^4(c+dx)\right)^{2/3} dx = \frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \text{co}\right.}{d\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x\right.}{d\coth^{\frac{8}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\text{co}}\right.}{d\coth^{\frac{8}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(}\right.}{4d\coth^{\frac{8}{3}}(c+}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4\right.}{2d\coth^{\frac{8}{3}}(c+dx)}$$

Mathematica [A] time = 0.42, size = 166, normalized size = 0.57

$$\frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(-12\coth^{\frac{5}{3}}(c+dx)+20\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)+5\left(-\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx}\right.\right.\right.}{20d\coth}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x]^4)^(2/3)*(20*ArcTanh[Coth[c + d*x]^(1/3)] - 12*Coth[c + d
*x]^(5/3) + 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*Sq
rt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x]^(
1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])))/(20*d*Coth[c + d*x]^(8/3))
```

fricas [B] time = 1.79, size = 618, normalized size = 2.12

$$-\frac{10\left(\sqrt{3}\cosh(dx+c)^2+2\sqrt{3}\cosh(dx+c)\sinh(dx+c)+\sqrt{3}\sinh(dx+c)^2-\sqrt{3}\right)\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
+ sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b -
2*sqrt(3)*(-b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 10*(sqrt
(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(
d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^
(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 5*(-b^2)^(1/3)*(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh
(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3)) + 5*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c
))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
)) - 10*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3
)) - 10*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3))
 + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x
 + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth \left(dx + c\right)^{4}\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(b \left(\coth^{4}\left(dx + c\right)\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth \left(dx + c\right)^{4}\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(2/3),x)
```

```
[Out] int((b*coth(c + d*x)^4)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^4(c+dx)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(2/3), x)
```

3.46 $\int \sqrt[3]{b\coth^4(c+dx)}\,dx$

Optimal. Leaf size=289

$$-\frac{3\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{d}-\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^4(c}{}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)-1/4*
(b*coth(d*x+c)^4)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(4/3)+1/4*(b*coth(d*x+c)^4)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(4/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)
-3*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3473, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth}\right.}{4d\coth^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(
1/3))/(2*d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3
))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTanh
[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/3)) -
((b*Coth[c + d*x]^4)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3
)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Coth[
c + d*x]^4)^(1/3)*Tanh[c + d*x])/d
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \frac{\sqrt[3]{b\coth^4(c+dx)}\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\,\tanh(c+dx)}{d} + \frac{\sqrt[3]{b\coth^4(c+dx)}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\,\tanh(c+dx)}{d} - \frac{\sqrt[3]{b\coth^4(c+dx)}\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \text{co}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\,\tanh(c+dx)}{d} - \frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{c}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\,\tanh(c+dx)}{d} + \frac{\sqrt[3]{b\coth^4(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(}\right.}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\sqrt[3]{b\coth^4(c+dx)}\,\tanh(c+dx)}{d} - \ldots$$

$$= \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^4(c+dx)}\,\log\left(1-\sqrt[3]{\coth(c}\right.}{4d\coth^{\frac{4}{3}}(c+d}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\,\text{cot}}}{2d\coth^{\frac{4}{3}}(c+dx)}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.15

$$\frac{3\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};\coth^2(c+dx)\right)-1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] (3*(b*Coth[c + d*x]^4)^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, Coth[c +
d*x]^2])*Tanh[c + d*x])/d
```

fricas [A] time = 3.08, size = 288, normalized size = 1.00

$$-\frac{2\sqrt{3}\,(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,b+2\sqrt{3}\,(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)-2\sqrt{3}\,b^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,b-2\sqrt{3}\,b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-b)^{\frac{1}{3}}\log\left((-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*
cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(1/3)*arctan(-1/3*(sqr
```

```
t(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-b)
^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(1/3)*log(b^(2/3) - b^(1/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3))
- 2*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*
b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(b*cosh(d
*x + c)/sinh(d*x + c))^(1/3))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth\left(dx + c\right)^{4}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(b \left(\coth^{4}\left(dx + c\right)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^4)^(1/3),x)
[Out] int((b*coth(d*x+c)^4)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth\left(dx + c\right)^{4}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^4)^(1/3),x)
[Out] int((b*coth(c + d*x)^4)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \coth^{4}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**4)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(1/3), x)
```

3.47 $\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx$

Optimal. Leaf size=289

$$-\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c-\right.}{4d\sqrt[3]{b\coth}}$$

```
[Out] -3*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*
x+c)^(4/3)/d/(b*coth(d*x+c)^4)^(1/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*x+c)
^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(1/3)+1/4*coth(d*x+c)^(4/3)*l
n(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(1/3)+1/2*arct
an(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/d/(b*coth
(d*x+c)^4)^(1/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c
)^(4/3)*3^(1/2)/d/(b*coth(d*x+c)^4)^(1/3)
```

Rubi [A] time = 0.22, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c-\right.}{4d\sqrt[3]{b\coth}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^4)^
(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^(1
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^4)^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
```

```
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\text{coth}}{}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{S}}{}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\text{lo}}{}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2}{}\right.}{2d\sqrt[3]{}}$$

Mathematica [C] time = 0.05, size = 41, normalized size = 0.14

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};\coth^2(c+dx)\right)}{d\sqrt[3]{b\coth^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-1/6, 1, 5/6, Coth[c + d*x]^2])/(d*(b*C
oth[c + d*x]^4)^(1/3))
```

fricas [B] time = 0.83, size = 3316, normalized size = 11.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
```

```
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh
(d*x + c) + b*d*sinh(d*x + c)^2 + b*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2
+ 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)
/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-1/b
^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*sqrt(-1/
b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*sinh(d*x
 + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1/b^(2/3)
) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3)
 - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh
(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(
d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*
d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2
 + b*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
+ b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*c
osh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
```

```
 c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-
b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/
3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(
d*x + c))^(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c
))^(1/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2
*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b^(1/3))/b^(1/3) - 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*d*cos
h(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2 + b*
d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) +
b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*s
qrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt
(-(-b)^(1/3)/b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/
3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(
d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c
))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 2
*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b^(1/3))/b^(1/3) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*d*cos
h(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2 + b*
d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{4}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^{4}\left(dx+c\right)\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{4}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b \coth \left(c+d x\right)^4\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(1/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^4)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b \coth^{4}{\left(c+dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-1/3), x)
```

3.48 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=291

$$-\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}\right.}{4d\left(b\coth\right.}$$

```
[Out] -3/5*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(
d*x+c)^(8/3)/d/(b*coth(d*x+c)^4)^(2/3)-1/4*coth(d*x+c)^(8/3)*ln(1-coth(d*x+
c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(2/3)+1/4*coth(d*x+c)^(8/3)
*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(2/3)-1/2*ar
ctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(8/3)*3^(1/2)/d/(b*co
th(d*x+c)^4)^(2/3)+1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x
+c)^(8/3)*3^(1/2)/d/(b*coth(d*x+c)^4)^(2/3)
```

Rubi [A] time = 0.18, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}\right.}{4d\left(b\coth\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(5*d*(b*Coth[c + d*x]^4)^(2/3)) - (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(8/3))/(2*d*(b*Coth[c + d*x]^4
)^(2/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*
x]^(8/3))/(2*d*(b*Coth[c + d*x]^4)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*C
oth[c + d*x]^(8/3))/(d*(b*Coth[c + d*x]^4)^(2/3)) - (Coth[c + d*x]^(8/3)*Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^
(2/3)) + (Coth[c + d*x]^(8/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*(b*Coth[c + d*x]^4)^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right.}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx}\right.}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}} +$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+}{}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+}{}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\sqrt{3}\tan^{-1}}{2}$$

Mathematica [C] time = 0.05, size = 43, normalized size = 0.15

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{5}{6},1;\frac{1}{6};\coth^2(c+dx)\right)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-5/6, 1, 1/6, Coth[c + d*x]^2])/(5*d*(b
*Coth[c + d*x]^4)^(2/3))
```

fricas [B] time = 0.49, size = 1159, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/20*(10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt
(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sqrt(-(-b^2)^(1/3)) - 2
```

```
*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b^2)^(1
/3)))/b^2) + 10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(b^2)^(1/6)*arctan(-1/3*sqrt(3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b^2) + 5*(cosh(d*x + c)^4 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*si
nh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sin
h(d*x + c) + 1)*(-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
(-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 5*(c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/si
nh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 10*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b^2)^(2/3)*log(
b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 10*(cosh(d*x + c)
^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)
^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x
 + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))
^(1/3) + (b^2)^(2/3)) - 12*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
 + b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(b^2*d*cosh(d*x + c)^4 + 4*b^2
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x
 + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(
b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth (dx+c)^4\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \left(\coth^4 (dx+c)\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth (dx+c)^4\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(2/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^4)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-2/3), x)
```

3.49 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=369

$$-\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\text{co}}\right.}{4bd\sqrt[3]{b\coth^4(c+dx)}}$$

```
[Out] -3*coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(
d*x+c)^(4/3)/b/d/(b*coth(d*x+c)^4)^(1/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*
x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^4)^(1/3)+1/4*coth(d*x+c)^(
4/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^4)^(1/3)+
1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/b
/d/(b*coth(d*x+c)^4)^(1/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*
coth(d*x+c)^(4/3)*3^(1/2)/b/d/(b*coth(d*x+c)^4)^(1/3)-3/7*tanh(d*x+c)/b/d/(
b*coth(d*x+c)^4)^(1/3)-3/13*tanh(d*x+c)^3/b/d/(b*coth(d*x+c)^4)^(1/3)
```

Rubi [A] time = 0.24, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3658, 3474, 3476, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+\right.}{4bd\sqrt[3]{b\coth^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^4)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(b*d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]
^4)^(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c +
d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3
)]*Coth[c + d*x]^(4/3))/(b*d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4
/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c +
d*x]^4)^(1/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c
+ d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x])/(7*b*d
*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x]^3)/(13*b*d*(b*Coth[c + d*x]^
4)^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
```

```
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{16}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{10}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c \cdots}{b \cdots}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c \cdots}{\cdots}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}} \cdots\right.}{\cdots}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c \cdots}{\cdots}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+d \cdots}{7bd\sqrt[3]{b\coth^4(c \cdots}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\tanh^{-1}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\,\mathrm{lo}\cdots}{\cdots}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+2\cdots}{\cdots}\right.}{2bd\sqrt[3]{\cdots}}$$

Mathematica [C] time = 0.04, size = 43, normalized size = 0.12

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(-\frac{13}{6},1;-\frac{7}{6};\coth^2(c+dx)\right)}{13d\left(b\coth^4(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[-13/6, 1, -7/6, Coth[c + d*x]^2])/(13*d
*(b*Coth[c + d*x]^4)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{4}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \left(\coth^{4}\left(dx+c\right)\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^4)^(4/3),x)
[Out] int(1/(b*coth(d*x+c)^4)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{4}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^4)^(4/3),x)
[Out] int(1/(b*coth(c + d*x)^4)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth^{4}{\left(c+dx\right)}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**4)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(-4/3), x)
```

3.50 $\int \left(b\coth^m(c+dx)\right)^n\,dx$

Optimal. Leaf size=57

$$\frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\coth^2(c+dx)\right)}{d(mn+1)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^m)^n*hypergeom([1, 1/2*m*n+1/2],[1/2*m*n+3/2],co
th(d*x+c)^2)/d/(m*n+1)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3659, 3476, 364}

$$\frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\coth^2(c+dx)\right)}{d(mn+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\coth^m(c+dx)\right)^n\,dx &= \left(\coth^{-mn}(c+dx)\left(b\coth^m(c+dx)\right)^n\right)\int \coth^{mn}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-mn}(c+dx)\left(b\coth^m(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{mn}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}\\
&= \frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\coth^2(c+dx)\right)}{d(1+mn)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.96

$$\frac{\coth(c+dx)\left(b\coth^m(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\coth^2(c+dx)\right)}{dmn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d + d*m*n)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\coth\left(dx+c\right)^m\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="fricas")
```

```
[Out] integral((b*coth(d*x + c)^m)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

maple [F] time = 5.40, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^m\left(dx+c\right)\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^n,x)
```

```
[Out] int((b*coth(d*x+c)^m)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\coth\left(c+d\,x\right)^m\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^n,x)
```

```
[Out] int((b*coth(c + d*x)^m)^n, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^m(c+dx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**n,x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**n, x)
```

3.51 $\int \left(b \coth^m(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=63

$$\frac{2b\coth^{m+1}(c+dx)\sqrt{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\coth^2(c+dx)\right)}{d(3m+2)}$$

```
[Out] 2*b*coth(d*x+c)^(1+m)*hypergeom([1, 1/2+3/4*m],[3/2+3/4*m],coth(d*x+c)^2)*(
b*coth(d*x+c)^m)^(1/2)/d/(2+3*m)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{2b\coth^{m+1}(c+dx)\sqrt{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\coth^2(c+dx)\right)}{d(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(3/2),x]
```

```
[Out] (2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2
+ 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\coth^m(c+dx)\right)^{3/2} dx &= \left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\int \coth^{\frac{3m}{2}}(c+dx)\,dx \\
&= -\frac{\left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{3m/2}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d} \\
&= \frac{2b\coth^{1+m}(c+dx)\sqrt{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\coth^2(c+dx)\right)}{d(2+3m)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 58, normalized size = 0.92

$$\frac{2\coth(c+dx)\left(b\coth^m(c+dx)\right)^{3/2}\,{}_2F_1\left(1,\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\coth^2(c+dx)\right)}{d(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(3/2),x]
```

```
[Out] (2*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(3/2)*Hypergeometric2F1[1, (2 + 3*m)/4
, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

maple [F] time = 1.57, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(3/2),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(3/2),x)
```

```
[Out] int((b*coth(c + d*x)^m)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth^{m}(c+dx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**m)**(3/2), x)
```

3.52 $\int \sqrt{b\coth^m(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+2}{4};\frac{m+6}{4};\coth^2(c+dx)\right)}{d(m+2)}$$

```
[Out] 2*coth(d*x+c)*hypergeom([1, 1/2+1/4*m],[3/2+1/4*m],coth(d*x+c)^2)*(b*coth(d
*x+c)^m)^(1/2)/d/(2+m)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+2}{4};\frac{m+6}{4};\coth^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b\coth^m(c+dx)}\,dx &= \left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\int \coth^{\frac{m}{2}}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{m/2}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\
&= \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{2+m}{4};\frac{6+m}{4};\coth^2(c+dx)\right)}{d(2+m)}
\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 54, normalized size = 1.00

$$\frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{m+2}{4};\frac{m+6}{4};\coth^2(c+dx)\right)}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

maple [**F**] time = 0.55, size = 0, normalized size = 0.00

$$\int \sqrt{b\left(\coth^m(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/2),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\coth(c+d\,x)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^m)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \coth^{m}(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**m), x)
```

3.53 $\int \frac{1}{\sqrt{b \coth^m(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

```
[Out] 2*coth(d*x+c)*hypergeom([1, 1/2-1/4*m],[3/2-1/4*m],coth(d*x+c)^2)/d/(2-m)/(
b*coth(d*x+c)^m)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^2
])/(d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{m}{2}}(c+dx)\,dx}{\sqrt{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{2}}(c+dx)\,\text{Subst}\left(\int \frac{x^{-m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt{b\coth^m(c+dx)}}$$
$$= \frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.97

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(1,\frac{2-m}{4};\frac{6-m}{4};\coth^2(c+dx)\right)}{d(m-2)\sqrt{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^
2])/(d*(-2 + m)*Sqrt[b*Coth[c + d*x]^m])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth\left(dx+c\right)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(\coth^m\left(dx+c\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth\left(dx+c\right)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(1/2),x)
[Out] int(1/(b*coth(c + d*x)^m)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth^{m}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/2),x)
[Out] Integral(1/sqrt(b*coth(c + d*x)**m), x)
```

3.54 $\int \frac{1}{\left(b\coth^m(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

```
[Out] 2*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-3/4*m],[3/2-3/4*m],coth(d*x+c)^2)/b/d
/(2-3*m)/(b*coth(d*x+c)^m)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{2\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-3/2),x]
```

```
[Out] (2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4, C
oth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{3/2}}\,dx = \frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{3m}{2}}(c+dx)\,dx}{b\sqrt{b\coth^m(c+dx)}}$$

$$= -\frac{\coth^{\frac{m}{2}}(c+dx)\,\text{Subst}\left(\int \frac{x^{-3m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt{b\coth^m(c+dx)}}$$

$$= \frac{2\coth^{1-m}(c+dx)\,{}_2F_1\left(1, \frac{1}{4}(2-3m); \frac{3(2-m)}{4}; \coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.84

$$-\frac{2\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{4}(2-3m); -\frac{3}{4}(m-2); \coth^2(c+dx)\right)}{d(3m-2)\left(b\coth^m(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - 3*m)/4, (-3*(-2 + m))/4, Coth[c
 + d*x]^2])/(d*(-2 + 3*m)*(b*Coth[c + d*x]^m)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth(dx+c)^m\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^m(dx+c)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{m}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^{m}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(3/2),x)
[Out] int(1/(b*coth(c + d*x)^m)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth^{m}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**m)**(-3/2), x)
```

3.55 $\int \left(b\coth^m(c+dx)\right)^{4/3}\,dx$

Optimal. Leaf size=65

$$\frac{3b\coth^{m+1}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{6}(4m+3);\frac{1}{6}(4m+9);\coth^2(c+dx)\right)}{d(4m+3)}$$

```
[Out] 3*b*coth(d*x+c)^(1+m)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+2/3*m],[3/2
+2/3*m],coth(d*x+c)^2)/d/(3+4*m)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3659, 3476, 364}

$$\frac{3b\coth^{m+1}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{6}(4m+3);\frac{1}{6}(4m+9);\coth^2(c+dx)\right)}{d(4m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(4/3),x]
```

```
[Out] (3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (
3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(b\coth^m(c+dx)\right)^{4/3}\,dx &= \left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\int \coth^{\frac{4m}{3}}(c+dx)\,dx\\ &= -\frac{\left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{4m/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\ &= \frac{3b\coth^{1+m}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\ {}_2F_1\left(1,\frac{1}{6}(3+4m);\frac{1}{6}(9+4m);\coth^2(c+dx\right.}{d(3+4m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.92

$$\frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{4/3}{}_2F_1\left(1,\frac{1}{6}(4m+3);\frac{1}{6}(4m+9);\coth^2(c+dx)\right)}{d(4m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(4/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(4/3)*Hypergeometric2F1[1, (3 + 4*m)/6
, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(4/3),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(4/3),x)
```

```
[Out] int((b*coth(c + d*x)^m)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth^{m}\left(c + dx\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(4/3), x)
```

3.56 $\int \left(b\coth^m(c+dx)\right)^{2/3}\,dx$

Optimal. Leaf size=60

$$\frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\,{}_2F_1\left(1,\frac{1}{6}(2m+3);\frac{1}{6}(2m+9);\coth^2(c+dx)\right)}{d(2m+3)}$$

```
[Out] 3*coth(d*x+c)*(b*coth(d*x+c)^m)^(2/3)*hypergeom([1, 1/2+1/3*m],[3/2+1/3*m],
coth(d*x+c)^2)/d/(3+2*m)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\,{}_2F_1\left(1,\frac{1}{6}(2m+3);\frac{1}{6}(2m+9);\coth^2(c+dx)\right)}{d(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(2/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(b\coth^m(c+dx)\right)^{2/3}\,dx &= \left(\coth^{-\frac{2m}{3}}(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\right)\int \coth^{\frac{2m}{3}}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-\frac{2m}{3}}(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^{2m/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\
&= \frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\,{}_2F_1\left(1,\frac{1}{6}(3+2m);\frac{1}{6}(9+2m);\coth^2(c+dx)\right)}{d(3+2m)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 1.00

$$\frac{3\coth(c+dx)\left(b\coth^m(c+dx)\right)^{2/3}\,{}_2F_1\left(1,\frac{1}{6}(2m+3);\frac{1}{6}(2m+9);\coth^2(c+dx)\right)}{d(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(2/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^{m}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^{m}\left(dx+c\right)\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(2/3),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^{m}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(2/3),x)
```

```
[Out] int((b*coth(c + d*x)^m)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \coth^m(c+dx)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(2/3), x)
```

3.57 $\int \sqrt[3]{b \coth^m(c+dx)}\,dx$

Optimal. Leaf size=54

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+3}{6};\frac{m+9}{6};\coth^2(c+dx)\right)}{d(m+3)}$$

```
[Out] 3*coth(d*x+c)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+1/6*m],[3/2+1/6*m],
coth(d*x+c)^2)/d/(3+m)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+3}{6};\frac{m+9}{6};\coth^2(c+dx)\right)}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(1/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{b\coth^m(c+dx)}\,dx &= \left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\int \coth^{\frac{m}{3}}(c+dx)\,dx\\
&= -\frac{\left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{m/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\
&= \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{3+m}{6};\frac{9+m}{6};\coth^2(c+dx)\right)}{d(3+m)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 1.00

$$\frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,{}_2F_1\left(1,\frac{m+3}{6};\frac{m+9}{6};\coth^2(c+dx)\right)}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(1/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(d*x+c)^m)^(1/3),x)
```

```
[Out] int((b*coth(d*x+c)^m)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\coth\left(dx+c\right)^m\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*coth(c + d*x)^m)^(1/3),x)
```

```
[Out] int((b*coth(c + d*x)^m)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*coth(d*x+c)**m)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(1/3), x)
```

3.58 $\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}} \, dx$

Optimal. Leaf size=60

$$\frac{3 \coth(c+dx) \, {}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(3-m) \sqrt[3]{b \coth^m(c+dx)}}$$

```
[Out] 3*coth(d*x+c)*hypergeom([1, 1/2-1/6*m],[3/2-1/6*m],coth(d*x+c)^2)/d/(3-m)/(
b*coth(d*x+c)^m)^(1/3)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3 \coth(c+dx) \, {}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(3-m) \sqrt[3]{b \coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-1/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2
])/(d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = \frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{m}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{3}}(c+dx)\,\mathrm{Subst}\left(\int \frac{x^{-m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^m(c+dx)}}$$
$$= \frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.97

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{3-m}{6}; \frac{9-m}{6}; \coth^2(c+dx)\right)}{d(m-3)\sqrt[3]{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^
2])/(d*(-3 + m)*(b*Coth[c + d*x]^m)^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^{m}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\coth\left(c+d\,x\right)^{m}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(1/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^m)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\coth^{m}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-1/3), x)
```

3.59 $\int \frac{1}{\left(b\coth^m(c+dx)\right)^{2/3}}\,dx$

Optimal. Leaf size=60

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

```
[Out] 3*coth(d*x+c)*hypergeom([1, 1/2-1/3*m],[3/2-1/3*m],coth(d*x+c)^2)/d/(3-2*m)
/(b*coth(d*x+c)^m)^(2/3)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-2m);\frac{1}{6}(9-2m);\coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-2/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*
x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{2/3}}\,dx = \frac{\coth^{\frac{2m}{3}}(c+dx)\int \coth^{-\frac{2m}{3}}(c+dx)\,dx}{\left(b\coth^m(c+dx)\right)^{2/3}}$$
$$= -\frac{\coth^{\frac{2m}{3}}(c+dx)\,\text{Subst}\left(\int \frac{x^{-2m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\left(b\coth^m(c+dx)\right)^{2/3}}$$
$$= \frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{6}(3-2m); \frac{1}{6}(9-2m); \coth^2(c+dx)\right)}{d(3-2m)\left(b\coth^m(c+dx)\right)^{2/3}}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 1.00

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1, \frac{1}{6}(3-2m); \frac{1}{6}(9-2m); \coth^2(c+dx)\right)}{d(2m-3)\left(b\coth^m(c+dx)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d
*x]^2])/(d*(-3 + 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth\left(dx + c\right)^{m}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^m\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(2/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^m)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth^{m}{\left(c + dx \right)}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-2/3), x)
```

3.60 $\int \frac{1}{\left(b\coth^m(c+dx)\right)^{4/3}}\,dx$

Optimal. Leaf size=69

$$\frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[Out] 3*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-2/3*m],[3/2-2/3*m],coth(d*x+c)^2)/b/d
/(3-4*m)/(b*coth(d*x+c)^m)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$\frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Coth[c + d*x]^m)^(-4/3),x]
```

```
[Out] (3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Cot
h[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\coth^m(c+dx)\right)^{4/3}}\,dx = \frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{4m}{3}}(c+dx)\,dx}{b\sqrt[3]{b\coth^m(c+dx)}}$$
$$= -\frac{\coth^{\frac{m}{3}}(c+dx)\,\mathrm{Subst}\left(\int \frac{x^{-4m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt[3]{b\coth^m(c+dx)}}$$
$$= \frac{3\coth^{1-m}(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.87

$$-\frac{3\coth(c+dx)\,{}_2F_1\left(1,\frac{1}{6}(3-4m);\frac{1}{6}(9-4m);\coth^2(c+dx)\right)}{d(4m-3)\left(b\coth^m(c+dx)\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d
*x]^2])/(d*(-3 + 4*m)*(b*Coth[c + d*x]^m)^(4/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\coth^m\left(dx+c\right)\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{m}\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b \coth \left(c+d\,x\right)^{m}\right)^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*coth(c + d*x)^m)^(4/3),x)
[Out] int(1/(b*coth(c + d*x)^m)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \coth^{m}\left(c+dx\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*coth(d*x+c)**m)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(-4/3), x)
```

3.61 $\int (1+\coth(x))^5\,dx$

Optimal. Leaf size=41

$$16x-\frac{1}{4}(\coth(x)+1)^4-\frac{2}{3}(\coth(x)+1)^3-2(\coth(x)+1)^2-8\coth(x)+16\log(\sinh(x))$$

```
[Out] 16*x-8*coth(x)-2*(1+coth(x))^2-2/3*(1+coth(x))^3-1/4*(1+coth(x))^4+16*ln(si
nh(x))
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3478, 3477, 3475}

$$16x-\frac{1}{4}(\coth(x)+1)^4-\frac{2}{3}(\coth(x)+1)^3-2(\coth(x)+1)^2-8\coth(x)+16\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^5,x]
```

```
[Out] 16*x - 8*Coth[x] - 2*(1 + Coth[x])^2 - (2*(1 + Coth[x])^3)/3 - (1 + Coth[x]
)^4/4 + 16*Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^5\,dx &= -\frac{1}{4}(1+\coth(x))^4+2\int (1+\coth(x))^4\,dx\\
&= -\frac{2}{3}(1+\coth(x))^3-\frac{1}{4}(1+\coth(x))^4+4\int (1+\coth(x))^3\,dx\\
&= -2(1+\coth(x))^2-\frac{2}{3}(1+\coth(x))^3-\frac{1}{4}(1+\coth(x))^4+8\int (1+\coth(x))^2\,dx\\
&= 16x-8\coth(x)-2(1+\coth(x))^2-\frac{2}{3}(1+\coth(x))^3-\frac{1}{4}(1+\coth(x))^4+16\int \coth(x)\\
&= 16x-8\coth(x)-2(1+\coth(x))^2-\frac{2}{3}(1+\coth(x))^3-\frac{1}{4}(1+\coth(x))^4+16\log(\sinh(x
\end{aligned}$$

Mathematica [C] time = 0.25, size = 94, normalized size = 2.29

$$\frac{\sinh(x)(\coth(x)+1)^5\left(-20\sinh(x)\cosh^3(x)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2(x)\right)-120\sinh^3(x)\cosh(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\text{ta}\right.\right.}{12(\sinh(x)+\text{cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^5,x]
```

```
[Out] ((1 + Coth[x])^5*Sinh[x]*(-3*Cosh[x]^4 - 20*Cosh[x]^3*Hypergeometric2F1[-3/
2, 1, -1/2, Tanh[x]^2]*Sinh[x] - 66*Cosh[x]^2*Sinh[x]^2 - 120*Cosh[x]*Hyper
geometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[x]^3 + 12*(x + 16*Log[Cosh[x]] +
 16*Log[Tanh[x]])*Sinh[x]^4))/(12*(Cosh[x] + Sinh[x])^5)
```

fricas [B] time = 0.39, size = 448, normalized size = 10.93

$$-\frac{4\left(48\,\cosh(x)^{6}+288\,\cosh(x)\sinh(x)^{5}+48\,\sinh(x)^{6}+36\left(20\,\cosh(x)^{2}-3\right)\sinh(x)^{4}-108\,\cosh(x)^{4}+48\,(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="fricas")
```

```
[Out] -4/3*(48*cosh(x)^6 + 288*cosh(x)*sinh(x)^5 + 48*sinh(x)^6 + 36*(20*cosh(x)^
2 - 3)*sinh(x)^4 - 108*cosh(x)^4 + 48*(20*cosh(x)^3 - 9*cosh(x))*sinh(x)^3
+ 8*(90*cosh(x)^4 - 81*cosh(x)^2 + 11)*sinh(x)^2 + 88*cosh(x)^2 - 12*(cosh(
x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*
cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*co
sh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*co
sh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)
^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh
(x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 16*(18*cosh(x)^5 - 27*cosh(x)
^3 + 11*cosh(x))*sinh(x) - 25)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x)
)*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15
*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*co
sh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.11, size = 41, normalized size = 1.00

$$-\frac{4\left(48\,e^{(6\,x)}-108\,e^{(4\,x)}+88\,e^{(2\,x)}-25\right)}{3\left(e^{(2\,x)}-1\right)^{4}}+16\,\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="giac")
```

```
[Out] -4/3*(48*e^(6*x) - 108*e^(4*x) + 88*e^(2*x) - 25)/(e^(2*x) - 1)^4 + 16*log(
abs(e^(2*x) - 1))
```

maple [A] time = 0.01, size = 31, normalized size = 0.76

$$-\frac{\left(\coth^{4}(x)\right)}{4}-\frac{5\left(\coth^{3}(x)\right)}{3}-\frac{11\left(\coth^{2}(x)\right)}{2}-15\coth(x)-16\ln\left(\coth(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^5,x)
```

```
[Out] -1/4*coth(x)^4-5/3*coth(x)^3-11/2*coth(x)^2-15*coth(x)-16*ln(coth(x)-1)
```

maxima [B] time = 0.33, size = 140, normalized size = 3.41

$$27\,x-\frac{20\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}-2\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}+\frac{4\left(e^{(-2\,x)}-e^{(-4\,x)}+e^{(-6\,x)}\right)}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}+\frac{20\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{20}{e^{(-2\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 27*x - 20/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*
x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e
^(-6*x) - e^(-8*x) - 1) + 20*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 20/(e^(
-2*x) - 1) + 11*log(e^(-x) + 1) + 11*log(e^(-x) - 1) + 5*log(sinh(x))
```

mupad [B] time = 1.13, size = 88, normalized size = 2.15

$$16\ln\left(e^{2x}-1\right)-\frac{64}{3\left(3e^{2x}-3e^{4x}+e^{6x}-1\right)}-\frac{48}{e^{4x}-2e^{2x}+1}-\frac{4}{6e^{4x}-4e^{2x}-4e^{6x}+e^{8x}+1}-\frac{64}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^5,x)
```

```
[Out] 16*log(exp(2*x) - 1) - 64/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 48
/(exp(4*x) - 2*exp(2*x) + 1) - 4/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + ex
p(8*x) + 1) - 64/(exp(2*x) - 1)
```

sympy [A] time = 1.22, size = 48, normalized size = 1.17

$$32x-16\log\left(\tanh(x)+1\right)+16\log\left(\tanh(x)\right)-\frac{15}{\tanh(x)}-\frac{11}{2\tanh^2(x)}-\frac{5}{3\tanh^3(x)}-\frac{1}{4\tanh^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**5,x)
```

```
[Out] 32*x - 16*log(tanh(x) + 1) + 16*log(tanh(x)) - 15/tanh(x) - 11/(2*tanh(x)**
2) - 5/(3*tanh(x)**3) - 1/(4*tanh(x)**4)
```

3.62 $\int (1+\coth(x))^4 \, dx$

Optimal. Leaf size=31

$$8x-\frac{1}{3}(\coth(x)+1)^3-(\coth(x)+1)^2-4\coth(x)+8\log(\sinh(x))$$

```
[Out] 8*x-4*coth(x)-(1+coth(x))^2-1/3*(1+coth(x))^3+8*ln(sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3478, 3477, 3475}

$$8x-\frac{1}{3}(\coth(x)+1)^3-(\coth(x)+1)^2-4\coth(x)+8\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^4,x]
```

```
[Out] 8*x - 4*Coth[x] - (1 + Coth[x])^2 - (1 + Coth[x])^3/3 + 8*Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^4\,dx &= -\frac{1}{3}(1+\coth(x))^3+2\int (1+\coth(x))^3\,dx\\ &= -(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+4\int (1+\coth(x))^2\,dx\\ &= 8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\int \coth(x)\,dx\\ &= 8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\log(\sinh(x))\end{aligned}$$

Mathematica [C] time = 0.19, size = 84, normalized size = 2.71

$$\frac{\sinh(x)(\coth(x)+1)^4\left(3\sinh(x)\left(-6\sinh(x)\cosh(x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)-2\cosh^2(x)+\sinh^2(x)(x+8\log(\right.\right.}{3(\sinh(x)+\cosh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^4,x]
```

```
[Out] ((1 + Coth[x])^4*Sinh[x]*(-(Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh
[x]^2]) + 3*Sinh[x]*(-2*Cosh[x]^2 - 6*Cosh[x]*Hypergeometric2F1[-1/2, 1, 1/
2, Tanh[x]^2]*Sinh[x] + (x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]])*Sinh[x]^2)))/
(3*(Cosh[x] + Sinh[x])^4)
```

fricas [B] time = 0.40, size = 273, normalized size = 8.81

$$-\frac{4\left(18\,\cosh(x)^4+72\,\cosh(x)\sinh(x)^3+18\,\sinh(x)^4+27\left(4\,\cosh(x)^2-1\right)\sinh(x)^2-27\,\cosh(x)^2-6\left(\cosh\right.\right.}{3\left(\cosh(x)^6+6\,\cosh(x)\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="fricas")
```

```
[Out] -4/3*(18*cosh(x)^4 + 72*cosh(x)*sinh(x)^3 + 18*sinh(x)^4 + 27*(4*cosh(x)^2
- 1)*sinh(x)^2 - 27*cosh(x)^2 - 6*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh
(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2
+ 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)*log(2*sinh(x)/(cosh(x)
 - sinh(x))) + 18*(4*cosh(x)^3 - 3*cosh(x))*sinh(x) + 11)/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*
sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1
)
```

giac [A] time = 0.11, size = 35, normalized size = 1.13

$$-\frac{4\left(18\,e^{(4\,x)}-27\,e^{(2\,x)}+11\right)}{3\left(e^{(2\,x)}-1\right)^3}+8\,\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="giac")
```

```
[Out] -4/3*(18*e^(4*x) - 27*e^(2*x) + 11)/(e^(2*x) - 1)^3 + 8*log(abs(e^(2*x) - 1
))
```

maple [A] time = 0.01, size = 25, normalized size = 0.81

$$-\frac{\left(\coth^3(x)\right)}{3}-2\left(\coth^2(x)\right)-7\coth(x)-8\ln\left(\coth(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^4,x)
```

```
[Out] -1/3*coth(x)^3-2*coth(x)^2-7*coth(x)-8*ln(coth(x)-1)
```

maxima [B] time = 0.32, size = 95, normalized size = 3.06

$$12\,x-\frac{4\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}-2\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}+\frac{8\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+\frac{12}{e^{(-2\,x)}-1}+4\,\log\left(e^{(-x)}+1\right)+4\,\log\left(e^{(-x)}-1\right)+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^4,x, algorithm="maxima")
```

```
[Out] 12*x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x
) - 1) + 8*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 12/(e^(-2*x) - 1) + 4*log
(e^(-x) + 1) + 4*log(e^(-x) - 1) + 4*log(sinh(x))
```

mupad [B] time = 1.14, size = 60, normalized size = 1.94

$$8\ \ln\left(\mathrm{e}^{2\,x}-1\right)-\frac{8}{3\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)}-\frac{12}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}-\frac{24}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^4,x)
```

```
[Out] 8*log(exp(2*x) - 1) - 8/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 12/(
exp(4*x) - 2*exp(2*x) + 1) - 24/(exp(2*x) - 1)
```

sympy [A] time = 0.92, size = 37, normalized size = 1.19

$$16x-8\log\left(\tanh\left(x\right)+1\right)+8\log\left(\tanh\left(x\right)\right)-\frac{7}{\tanh\left(x\right)}-\frac{2}{\tanh^{2}\left(x\right)}-\frac{1}{3\tanh^{3}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**4,x)
```

```
[Out] 16*x - 8*log(tanh(x) + 1) + 8*log(tanh(x)) - 7/tanh(x) - 2/tanh(x)**2 - 1/(
3*tanh(x)**3)
```

3.63 $\int (1+\coth(x))^3 \, dx$

Optimal. Leaf size=23

$$4x-\frac{1}{2}(\coth(x)+1)^2-2\coth(x)+4\log(\sinh(x))$$

```
[Out] 4*x-2*coth(x)-1/2*(1+coth(x))^2+4*ln(sinh(x))
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3478, 3477, 3475}

$$4x-\frac{1}{2}(\coth(x)+1)^2-2\coth(x)+4\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^3,x]
```

```
[Out] 4*x - 2*Coth[x] - (1 + Coth[x])^2/2 + 4*Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^3\,dx &= -\frac{1}{2}(1+\coth(x))^2+2\int (1+\coth(x))^2\,dx\\ &= 4x-2\coth(x)-\frac{1}{2}(1+\coth(x))^2+4\int \coth(x)\,dx\\ &= 4x-2\coth(x)-\frac{1}{2}(1+\coth(x))^2+4\log(\sinh(x))\end{aligned}$$

Mathematica [C] time = 0.16, size = 61, normalized size = 2.65

$$\frac{1}{4}\operatorname{csch}^2(x)\left(-6\sinh(2x)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(x)\right)-2x-8\log(\tanh(x))-8\log(\cosh(x))+\cosh(2x)(2x+8\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^3,x]
```

```
[Out] (Csch[x]^2*(-1 - 2*x - 8*Log[Cosh[x]] - 8*Log[Tanh[x]] + Cosh[2*x]*(-1 + 2*
x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]]) - 6*Hypergeometric2F1[-1/2, 1, 1/2, Ta
nh[x]^2]*Sinh[2*x]))/4
```

fricas [B] time = 0.39, size = 142, normalized size = 6.17

$$-\frac{2\left(4\cosh(x)^2 - 2\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2 - 2\cosh(x)^2 + 4\left(c\right.\right.\right.}{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 1\right)\sinh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*cosh(x)^2 - 2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*
log(2*sinh(x)/(cosh(x) - sinh(x))) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2
 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.11, size = 29, normalized size = 1.26

$$-\frac{2\left(4e^{(2x)} - 3\right)}{\left(e^{(2x)} - 1\right)^2} + 4\log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="giac")
```

```
[Out] -2*(4*e^(2*x) - 3)/(e^(2*x) - 1)^2 + 4*log(abs(e^(2*x) - 1))
```

maple [A] time = 0.01, size = 19, normalized size = 0.83

$$-\frac{\left(\coth^2(x)\right)}{2} - 3\coth(x) - 4\ln\left(\coth(x) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^3,x)
```

```
[Out] -1/2*coth(x)^2-3*coth(x)-4*ln(coth(x)-1)
```

maxima [B] time = 0.32, size = 55, normalized size = 2.39

$$5x + \frac{2e^{(-2x)}}{2e^{(-2x)} - e^{(-4x)} - 1} + \frac{6}{e^{(-2x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right) + 3\log\left(\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^3,x, algorithm="maxima")
```

```
[Out] 5*x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 6/(e^(-2*x) - 1) + log(e^(-x
) + 1) + log(e^(-x) - 1) + 3*log(sinh(x))
```

mupad [B] time = 0.04, size = 36, normalized size = 1.57

$$4\ln\left(e^{2x} - 1\right) - \frac{2}{e^{4x} - 2e^{2x} + 1} - \frac{8}{e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^3,x)
```

```
[Out] 4*log(exp(2*x) - 1) - 2/(exp(4*x) - 2*exp(2*x) + 1) - 8/(exp(2*x) - 1)
```

sympy [A] time = 0.68, size = 31, normalized size = 1.35

$$8x - 4\log(\tanh(x) + 1) + 4\log(\tanh(x)) - \frac{3}{\tanh(x)} - \frac{1}{2\tanh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**3,x)
[Out] 8*x - 4*log(tanh(x) + 1) + 4*log(tanh(x)) - 3/tanh(x) - 1/(2*tanh(x)**2)
```

3.64 $\int (1+\coth(x))^2\,dx$

Optimal. Leaf size=13

$$2x-\coth(x)+2\log(\sinh(x))$$

```
[Out] 2*x-coth(x)+2*ln(sinh(x))
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3477, 3475}

$$2x-\coth(x)+2\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^2,x]
```

```
[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^2\,dx &= 2x-\coth(x)+2\int \coth(x)\,dx\\ &= 2x-\coth(x)+2\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$2x-\coth(x)+2\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^2,x]
```

```
[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]
```

fricas [B] time = 0.41, size = 53, normalized size = 4.08

$$\frac{2\left(\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)-1\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
 sinh(x))) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [A] time = 0.13, size = 21, normalized size = 1.62

$$-\frac{2}{e^{(2x)}-1}+2\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="giac")
[Out] -2/(e^(2*x) - 1) + 2*log(abs(e^(2*x) - 1))
```

maple [A] time = 0.01, size = 13, normalized size = 1.00

$$-\coth(x)-2\ln\left(\coth(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^2,x)
[Out] -coth(x)-2*ln(coth(x)-1)
```

maxima [A] time = 0.31, size = 19, normalized size = 1.46

$$2x+\frac{2}{e^{(-2x)}-1}+2\log\left(\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^2,x, algorithm="maxima")
[Out] 2*x + 2/(e^(-2*x) - 1) + 2*log(sinh(x))
```

mupad [B] time = 1.14, size = 20, normalized size = 1.54

$$2\ln\left(e^{2x}-1\right)-\frac{2}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^2,x)
[Out] 2*log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

sympy [A] time = 0.37, size = 22, normalized size = 1.69

$$4x-2\log\left(\tanh\left(x\right)+1\right)+2\log\left(\tanh\left(x\right)\right)-\frac{1}{\tanh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**2,x)
[Out] 4*x - 2*log(tanh(x) + 1) + 2*log(tanh(x)) - 1/tanh(x)
```

3.65 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

```
[Out] 1/2*x-1/2/(1+coth(x))
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x-\sinh(2x)+\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-1),x]
```

```
[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4
```

fricas [B] time = 0.43, size = 26, normalized size = 1.62

$$\frac{(2\,x+1)\cosh(x)+(2\,x-1)\sinh(x)}{4\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

giac [**A**] time = 0.13, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
[Out] 1/2*x + 1/4*e^(-2*x)
```

maple [**A**] time = 0.05, size = 24, normalized size = 1.50

$$-\frac{\ln\left(\coth(x)-1\right)}{4} - \frac{1}{2\left(1+\coth(x)\right)} + \frac{\ln\left(1+\coth(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x)),x)
[Out] -1/4*ln(coth(x)-1)-1/2/(1+coth(x))+1/4*ln(1+coth(x))
```

maxima [**A**] time = 0.31, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/4*e^(-2*x)
```

mupad [**B**] time = 0.05, size = 14, normalized size = 0.88

$$\frac{x}{2} - \frac{1}{2\,\left(\coth(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1),x)
[Out] x/2 - 1/(2*(coth(x) + 1))
```

sympy [**B**] time = 0.44, size = 27, normalized size = 1.69

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2} + \frac{x}{2\tanh\left(x\right)+2} + \frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

3.66 $\int \frac{1}{(1+\coth(x))^2} dx$

Optimal. Leaf size=26

$$\frac{x}{4} - \frac{1}{4(\coth(x)+1)} - \frac{1}{4(\coth(x)+1)^2}$$

```
[Out] 1/4*x-1/4/(1+coth(x))^2-1/4/(1+coth(x))
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{4} - \frac{1}{4(\coth(x)+1)} - \frac{1}{4(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-2),x]
```

```
[Out] x/4 - 1/(4*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^2} dx &= -\frac{1}{4(1+\coth(x))^2} + \frac{1}{2}\int \frac{1}{1+\coth(x)} dx \\
&= -\frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))} + \frac{\int 1 \, dx}{4} \\
&= \frac{x}{4} - \frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 1.15

$$\frac{1}{16}(4x - 4\sinh(2x) + \sinh(4x) + 4\cosh(2x) - \cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-2),x]
```

```
[Out] (4*x + 4*Cosh[2*x] - Cosh[4*x] - 4*Sinh[2*x] + Sinh[4*x])/16
```

fricas [B] time = 0.38, size = 52, normalized size = 2.00

$$\frac{(4x-1)\cosh(x)^2 + 2(4x+1)\cosh(x)\sinh(x) + (4x-1)\sinh(x)^2 + 4}{16\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*x - 1)*cosh(x)^2 + 2*(4*x + 1)*cosh(x)*sinh(x) + (4*x - 1)*sinh(x)
^2 + 4)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.11, size = 18, normalized size = 0.69

$$\frac{1}{16}\left(4\,e^{(2\,x)}-1\right)e^{(-4\,x)}+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(4*e^(2*x) - 1)*e^(-4*x) + 1/4*x
```

maple [A] time = 0.05, size = 32, normalized size = 1.23

$$-\frac{\ln\left(\coth(x)-1\right)}{8}-\frac{1}{4\left(1+\coth(x)\right)^{2}}-\frac{1}{4\left(1+\coth(x)\right)}+\frac{\ln\left(1+\coth(x)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^2,x)
```

```
[Out] -1/8*ln(coth(x)-1)-1/4/(1+coth(x))^2-1/4/(1+coth(x))+1/8*ln(1+coth(x))
```

maxima [A] time = 0.31, size = 16, normalized size = 0.62

$$\frac{1}{4}\,x+\frac{1}{4}\,e^{(-2\,x)}-\frac{1}{16}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/4*e^(-2*x) - 1/16*e^(-4*x)
```

mupad [B] time = 0.05, size = 16, normalized size = 0.62

$$\frac{x}{4}+\frac{\mathrm{e}^{-2\,x}}{4}-\frac{\mathrm{e}^{-4\,x}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^2,x)
```

```
[Out] x/4 + exp(-2*x)/4 - exp(-4*x)/16
```

sympy [B] time = 0.81, size = 88, normalized size = 3.38

$$\frac{x\tanh^{2}\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{2x\tanh\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{x}{4\tanh^{2}\left(x\right)+8\tanh\left(x\right)+4}+\frac{3\tanh\left(x\right)}{4\tanh^{2}\left(x\right)+8\tanh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**2,x)
```

```
[Out] x*tanh(x)**2/(4*tanh(x)**2 + 8*tanh(x) + 4) + 2*x*tanh(x)/(4*tanh(x)**2 + 8
*tanh(x) + 4) + x/(4*tanh(x)**2 + 8*tanh(x) + 4) + 3*tanh(x)/(4*tanh(x)**2
+ 8*tanh(x) + 4) + 2/(4*tanh(x)**2 + 8*tanh(x) + 4)
```

3.67 $\int \frac{1}{(1+\coth(x))^3} \, dx$

Optimal. Leaf size=36

$$\frac{x}{8} - \frac{1}{8(\coth(x)+1)} - \frac{1}{8(\coth(x)+1)^2} - \frac{1}{6(\coth(x)+1)^3}$$

```
[Out] 1/8*x-1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/8/(1+coth(x))
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{8} - \frac{1}{8(\coth(x)+1)} - \frac{1}{8(\coth(x)+1)^2} - \frac{1}{6(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-3),x]
```

```
[Out] x/8 - 1/(6*(1 + Coth[x])^3) - 1/(8*(1 + Coth[x])^2) - 1/(8*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^3} \, dx &= -\frac{1}{6(1+\coth(x))^3} + \frac{1}{2}\int \frac{1}{(1+\coth(x))^2} \, dx \\
&= -\frac{1}{6(1+\coth(x))^3} - \frac{1}{8(1+\coth(x))^2} + \frac{1}{4}\int \frac{1}{1+\coth(x)} \, dx \\
&= -\frac{1}{6(1+\coth(x))^3} - \frac{1}{8(1+\coth(x))^2} - \frac{1}{8(1+\coth(x))} + \frac{\int 1 \, dx}{8} \\
&= \frac{x}{8} - \frac{1}{6(1+\coth(x))^3} - \frac{1}{8(1+\coth(x))^2} - \frac{1}{8(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 44, normalized size = 1.22

$$\frac{1}{96}(12x - 18\sinh(2x) + 9\sinh(4x) - 2\sinh(6x) + 18\cosh(2x) - 9\cosh(4x) + 2\cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-3),x]
```

```
[Out] (12*x + 18*Cosh[2*x] - 9*Cosh[4*x] + 2*Cosh[6*x] - 18*Sinh[2*x] + 9*Sinh[4*
x] - 2*Sinh[6*x])/96
```

fricas [B] time = 0.38, size = 86, normalized size = 2.39

$$\frac{2\,(6\,x+1)\cosh(x)^3 + 6\,(6\,x+1)\cosh(x)\sinh(x)^2 + 2\,(6\,x-1)\sinh(x)^3 + 3\left(2\,(6\,x-1)\cosh(x)^2 + 9\right)\sinh(x) + 9}{96\left(\cosh(x)^3 + 3\,\cosh(x)^2\sinh(x) + 3\,\cosh(x)\sinh(x)^2 + \sinh(x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(2*(6*x + 1)*cosh(x)^3 + 6*(6*x + 1)*cosh(x)*sinh(x)^2 + 2*(6*x - 1)*s
inh(x)^3 + 3*(2*(6*x - 1)*cosh(x)^2 + 9)*sinh(x) + 9*cosh(x))/(cosh(x)^3 +
3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

giac [A] time = 0.13, size = 24, normalized size = 0.67

$$\frac{1}{96}\left(18\,e^{(4\,x)} - 9\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="giac")
```

```
[Out] 1/96*(18*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/8*x
```

maple [A] time = 0.05, size = 40, normalized size = 1.11

$$-\frac{\ln\left(\coth(x)-1\right)}{16} - \frac{1}{6\left(1+\coth(x)\right)^{3}} - \frac{1}{8\left(1+\coth(x)\right)^{2}} - \frac{1}{8\left(1+\coth(x)\right)} + \frac{\ln\left(1+\coth(x)\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^3,x)
```

```
[Out] -1/16*ln(coth(x)-1)-1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/8/(1+coth(x))+1/1
6*ln(1+coth(x))
```

maxima [A] time = 0.31, size = 22, normalized size = 0.61

$$\frac{1}{8}\,x + \frac{3}{16}\,e^{(-2\,x)} - \frac{3}{32}\,e^{(-4\,x)} + \frac{1}{48}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="maxima")
```

```
[Out] 1/8*x + 3/16*e^(-2*x) - 3/32*e^(-4*x) + 1/48*e^(-6*x)
```

mupad [B] time = 0.06, size = 22, normalized size = 0.61

$$\frac{x}{8} + \frac{3\,\mathrm{e}^{-2\,x}}{16} - \frac{3\,\mathrm{e}^{-4\,x}}{32} + \frac{\mathrm{e}^{-6\,x}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^3,x)
```

```
[Out] x/8 + (3*exp(-2*x))/16 - (3*exp(-4*x))/32 + exp(-6*x)/48
```

sympy [B] time = 1.05, size = 182, normalized size = 5.06

$$\frac{3x\tanh^{3}(x)}{24\tanh^{3}(x) + 72\tanh^{2}(x) + 72\tanh(x) + 24} + \frac{9x\tanh^{2}(x)}{24\tanh^{3}(x) + 72\tanh^{2}(x) + 72\tanh(x) + 24} + \frac{}{24\tanh^{3}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**3,x)
```

```
[Out] 3*x*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh
(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh(x)/(24*
tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 3*x/(24*tanh(x)**3 + 72*tan
h(x)**2 + 72*tanh(x) + 24) + 21*tanh(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 +
 72*tanh(x) + 24) + 27*tanh(x)/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x)
+ 24) + 10/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24)
```

3.68 $\int \frac{1}{(1+\coth(x))^4}\,dx$

Optimal. Leaf size=46

$$\frac{x}{16}-\frac{1}{16(\coth(x)+1)}-\frac{1}{16(\coth(x)+1)^2}-\frac{1}{12(\coth(x)+1)^3}-\frac{1}{8(\coth(x)+1)^4}$$

```
[Out] 1/16*x-1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2-1/16/(1+coth
(x))
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{16}-\frac{1}{16(\coth(x)+1)}-\frac{1}{16(\coth(x)+1)^2}-\frac{1}{12(\coth(x)+1)^3}-\frac{1}{8(\coth(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-4),x]
```

```
[Out] x/16 - 1/(8*(1 + Coth[x])^4) - 1/(12*(1 + Coth[x])^3) - 1/(16*(1 + Coth[x])
^2) - 1/(16*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^4}\,dx &= -\frac{1}{8(1+\coth(x))^4}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^3}\,dx\\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}+\frac{1}{4}\int \frac{1}{(1+\coth(x))^2}\,dx\\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}+\frac{1}{8}\int \frac{1}{1+\coth(x)}\,dx\\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 53, normalized size = 1.15

$$\frac{1}{384}(\cosh(4x)-\sinh(4x))(32\sinh(2x)+24x\sinh(4x)+3\sinh(4x)+64\cosh(2x)+3(8x-1)\cosh(4x)-36)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-4),x]
```

```
[Out] ((Cosh[4*x] - Sinh[4*x])*(-36 + 64*Cosh[2*x] + 3*(-1 + 8*x)*Cosh[4*x] + 32*
Sinh[2*x] + 3*Sinh[4*x] + 24*x*Sinh[4*x]))/384
```

fricas [B] time = 0.39, size = 121, normalized size = 2.63

$$\frac{3\,(8\,x-1)\cosh(x)^4+12\,(8\,x+1)\cosh(x)\sinh(x)^3+3\,(8\,x-1)\sinh(x)^4+2\left(9\,(8\,x-1)\cosh(x)^2+32\right)\sinh(x)^2 \cdots}{384\left(\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)^2+4\,\cosh \cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(8*x - 1)*cosh(x)^4 + 12*(8*x + 1)*cosh(x)*sinh(x)^3 + 3*(8*x - 1)
*sinh(x)^4 + 2*(9*(8*x - 1)*cosh(x)^2 + 32)*sinh(x)^2 + 64*cosh(x)^2 + 4*(3
*(8*x + 1)*cosh(x)^3 + 16*cosh(x))*sinh(x) - 36)/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

giac [A] time = 0.13, size = 30, normalized size = 0.65

$$\frac{1}{384}\left(48\,e^{(6\,x)}-36\,e^{(4\,x)}+16\,e^{(2\,x)}-3\right)e^{(-8\,x)}+\frac{1}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="giac")
```

```
[Out] 1/384*(48*e^(6*x) - 36*e^(4*x) + 16*e^(2*x) - 3)*e^(-8*x) + 1/16*x
```

maple [A] time = 0.05, size = 48, normalized size = 1.04

$$-\frac{\ln\left(\coth(x)-1\right)}{32}-\frac{1}{8\left(1+\coth(x)\right)^4}-\frac{1}{12\left(1+\coth(x)\right)^3}-\frac{1}{16\left(1+\coth(x)\right)^2}-\frac{1}{16\left(1+\coth(x)\right)}+\frac{\ln\left(1+\coth(x)\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^4,x)
```

```
[Out] -1/32*ln(coth(x)-1)-1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2
-1/16/(1+coth(x))+1/32*ln(1+coth(x))
```

maxima [A] time = 0.32, size = 28, normalized size = 0.61

$$\frac{1}{16}\,x+\frac{1}{8}\,e^{(-2\,x)}-\frac{3}{32}\,e^{(-4\,x)}+\frac{1}{24}\,e^{(-6\,x)}-\frac{1}{128}\,e^{(-8\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="maxima")
```

```
[Out] 1/16*x + 1/8*e^(-2*x) - 3/32*e^(-4*x) + 1/24*e^(-6*x) - 1/128*e^(-8*x)
```

mupad [B] time = 1.14, size = 28, normalized size = 0.61

$$\frac{x}{16}+\frac{\mathrm{e}^{-2\,x}}{8}-\frac{3\,\mathrm{e}^{-4\,x}}{32}+\frac{\mathrm{e}^{-6\,x}}{24}-\frac{\mathrm{e}^{-8\,x}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^4,x)
```

```
[Out] x/16 + exp(-2*x)/8 - (3*exp(-4*x))/32 + exp(-6*x)/24 - exp(-8*x)/128
```

sympy [B] time = 1.42, size = 299, normalized size = 6.50

$$\frac{3x\tanh^4(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}+\frac{12x\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**4,x)
```

```
[Out] 3*x*tanh(x)**4/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(
x) + 48) + 12*x*tanh(x)**3/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2
 + 192*tanh(x) + 48) + 18*x*tanh(x)**2/(48*tanh(x)**4 + 192*tanh(x)**3 + 28
8*tanh(x)**2 + 192*tanh(x) + 48) + 12*x*tanh(x)/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 3*x/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 45*tanh(x)**3/(48*tanh(x)**4 +
192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 84*tanh(x)**2/(48*tan
h(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 61*tanh(x)/
(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 16/(
48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48)
```

3.69 $\int \frac{1}{(1+\coth(x))^5} \, dx$

Optimal. Leaf size=56

$$\frac{x}{32}-\frac{1}{32(\coth(x)+1)}-\frac{1}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}-\frac{1}{16(\coth(x)+1)^4}-\frac{1}{10(\coth(x)+1)^5}$$

```
[Out] 1/32*x-1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^3-1/32/(1+cot
h(x))^2-1/32/(1+coth(x))
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{32}-\frac{1}{32(\coth(x)+1)}-\frac{1}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}-\frac{1}{16(\coth(x)+1)^4}-\frac{1}{10(\coth(x)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-5),x]
```

```
[Out] x/32 - 1/(10*(1 + Coth[x])^5) - 1/(16*(1 + Coth[x])^4) - 1/(24*(1 + Coth[x]
)^3) - 1/(32*(1 + Coth[x])^2) - 1/(32*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^5} \, dx &= -\frac{1}{10(1+\coth(x))^5}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^4} \, dx \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}+\frac{1}{4}\int \frac{1}{(1+\coth(x))^3} \, dx \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}+\frac{1}{8}\int \frac{1}{(1+\coth(x))^2} \, dx \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}-\frac{1}{32(1+\coth(x))^2}+\frac{1}{16}\int \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}-\frac{1}{32(1+\coth(x))^2}-\frac{}{32(1} \\
&= \frac{x}{32}-\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}-\frac{1}{32(1+\coth(x))^2}-\frac{}{3}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 62, normalized size = 1.11

$$\frac{(\cosh(5x)-\sinh(5x))(-500\sinh(x)+375\sinh(3x)+120x\sinh(5x)-12\sinh(5x)-100\cosh(x)+225\cosh}{3840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-5),x]
```

```
[Out] ((Cosh[5*x] - Sinh[5*x])*(-100*Cosh[x] + 225*Cosh[3*x] + 12*Cosh[5*x] + 120
*x*Cosh[5*x] - 500*Sinh[x] + 375*Sinh[3*x] - 12*Sinh[5*x] + 120*x*Sinh[5*x]
))/3840
```

fricas [B] time = 0.39, size = 159, normalized size = 2.84

$$\frac{12\,(10\,x+1)\cosh(x)^5+60\,(10\,x+1)\cosh(x)\sinh(x)^4+12\,(10\,x-1)\sinh(x)^5+15\left(8\,(10\,x-1)\cosh(x)^2+25\right)\mathrm{s}}{3840\left(\cosh(x)^5+5\,\cosh(x)^4\sinh(x)+1\mathrm{0}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(12*(10*x + 1)*cosh(x)^5 + 60*(10*x + 1)*cosh(x)*sinh(x)^4 + 12*(10*
x - 1)*sinh(x)^5 + 15*(8*(10*x - 1)*cosh(x)^2 + 25)*sinh(x)^3 + 225*cosh(x)
^3 + 15*(8*(10*x + 1)*cosh(x)^3 + 45*cosh(x))*sinh(x)^2 + 5*(12*(10*x - 1)*
cosh(x)^4 + 225*cosh(x)^2 - 100)*sinh(x) - 100*cosh(x))/(cosh(x)^5 + 5*cosh
(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)
*sinh(x)^4 + sinh(x)^5)
```

giac [A] time = 0.12, size = 36, normalized size = 0.64

$$\frac{1}{3840}\left(300\,e^{(8\,x)}-300\,e^{(6\,x)}+200\,e^{(4\,x)}-75\,e^{(2\,x)}+12\right)e^{(-10\,x)}+\frac{1}{32}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="giac")
```

```
[Out] 1/3840*(300*e^(8*x) - 300*e^(6*x) + 200*e^(4*x) - 75*e^(2*x) + 12)*e^(-10*x
) + 1/32*x
```

maple [A] time = 0.05, size = 56, normalized size = 1.00

$$-\frac{\ln(\coth(x)-1)}{64}-\frac{1}{10\,(1+\coth(x))^5}-\frac{1}{16\,(1+\coth(x))^4}-\frac{1}{24\,(1+\coth(x))^3}-\frac{1}{32\,(1+\coth(x))^2}-\frac{1}{32\,(1+\coth(x))}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^5,x)
```

```
[Out] -1/64*ln(coth(x)-1)-1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^
3-1/32/(1+coth(x))^2-1/32/(1+coth(x))+1/64*ln(1+coth(x))
```

maxima [A] time = 0.32, size = 34, normalized size = 0.61

$$\frac{1}{32}\,x+\frac{5}{64}\,e^{(-2\,x)}-\frac{5}{64}\,e^{(-4\,x)}+\frac{5}{96}\,e^{(-6\,x)}-\frac{5}{256}\,e^{(-8\,x)}+\frac{1}{320}\,e^{(-10\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 1/32*x + 5/64*e^(-2*x) - 5/64*e^(-4*x) + 5/96*e^(-6*x) - 5/256*e^(-8*x) + 1
/320*e^(-10*x)
```

mupad [B] time = 1.15, size = 34, normalized size = 0.61

$$\frac{x}{32}+\frac{5\,\mathrm{e}^{-2\,x}}{64}-\frac{5\,\mathrm{e}^{-4\,x}}{64}+\frac{5\,\mathrm{e}^{-6\,x}}{96}-\frac{5\,\mathrm{e}^{-8\,x}}{256}+\frac{\mathrm{e}^{-10\,x}}{320}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^5,x)
```

```
[Out] x/32 + (5*exp(-2*x))/64 - (5*exp(-4*x))/64 + (5*exp(-6*x))/96 - (5*exp(-8*x
))/256 + exp(-10*x)/320
```

sympy [B] time = 1.80, size = 444, normalized size = 7.93

$$\frac{15x\tanh^5(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}+\frac{}{480\tanh^5(x)+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**5,x)
```

```
[Out] 15*x*tanh(x)**5/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*
tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tanh(x)**4/(480*tanh(x)**5 + 2400*t
anh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 150*x
*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh
(x)**2 + 2400*tanh(x) + 480) + 150*x*tanh(x)**2/(480*tanh(x)**5 + 2400*tanh
(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tan
h(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2
+ 2400*tanh(x) + 480) + 15*x/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(
x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 465*tanh(x)**4/(480*tanh(x)
**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) +
480) + 1125*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3
+ 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 1205*tanh(x)**2/(480*tanh(x)**5 +
 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
+ 625*tanh(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*ta
nh(x)**2 + 2400*tanh(x) + 480) + 128/(480*tanh(x)**5 + 2400*tanh(x)**4 + 48
00*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
```

3.70 $\int (1+\coth(x))^{7/2}\,dx$

Optimal. Leaf size=57

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-\frac{4}{3}(\coth(x)+1)^{3/2}-8\sqrt{\coth(x)+1}+8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] -4/3*(1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)+8*arctanh(1/2*(1+coth(x))^(1/2
)*2^(1/2))*2^(1/2)-8*(1+coth(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-\frac{4}{3}(\coth(x)+1)^{3/2}-8\sqrt{\coth(x)+1}+8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(7/2),x]
```

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 8*Sqrt[1 + Coth[x]] - (4*(1
+ Coth[x])^(3/2))/3 - (2*(1 + Coth[x])^(5/2))/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^{7/2}\,dx &= -\frac{2}{5}(1+\coth(x))^{5/2}+2\int (1+\coth(x))^{5/2}\,dx\\
&= -\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}+4\int (1+\coth(x))^{3/2}\,dx\\
&= -8\sqrt{1+\coth(x)}-\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}+8\int \sqrt{1+\coth(x)}\,dx\\
&= -8\sqrt{1+\coth(x)}-\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}+16\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1}\right.\\
&= 8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-8\sqrt{1+\coth(x)}-\frac{4}{3}(1+\coth(x))^{3/2}-\frac{2}{5}(1+\coth(x))^{5/2}
\end{aligned}$$

Mathematica [C] time = 0.27, size = 101, normalized size = 1.77

$$-\frac{2(\coth(x)+1)^{7/2}\left((8\sinh(2x)+3)\sinh(x)\sqrt{i(\coth(x)+1)}+4\sinh^3(x)\left(19\sqrt{i(\coth(x)+1)}-(15-15i)\tan\right.\right.}{15\sqrt{i(\coth(x)+1)}(\sinh(x)+\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(7/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(7/2)*(4*((-15 + 15*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Cot
h[x])]] + 19*Sqrt[I*(1 + Coth[x])])*Sinh[x]^3 + Sqrt[I*(1 + Coth[x])]*Sinh[
x]*(3 + 8*Sinh[2*x])))/(15*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x])^3)
```

fricas [B] time = 0.40, size = 438, normalized size = 7.68

$$-\frac{4\left(2\sqrt{2}\left(23\sqrt{2}\cosh(x)^5+115\sqrt{2}\cosh(x)\sinh(x)^4+23\sqrt{2}\sinh(x)^5+5\left(46\sqrt{2}\cosh(x)^2-7\sqrt{2}\right)\sinh(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^5 + 115*sqrt(2)*cosh(x)*sinh(x)^4 + 23
*sqrt(2)*sinh(x)^5 + 5*(46*sqrt(2)*cosh(x)^2 - 7*sqrt(2))*sinh(x)^3 - 35*sq
rt(2)*cosh(x)^3 + 5*(46*sqrt(2)*cosh(x)^3 - 21*sqrt(2)*cosh(x))*sinh(x)^2 +
 5*(23*sqrt(2)*cosh(x)^4 - 21*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x) + 15*s
qrt(2)*cosh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^6 +
 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 -
 sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt
(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqr
t(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*co
sh(x)^3 + sqrt(2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(c
osh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) +
2*sinh(x)^2 - 1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(
x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 +
 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 -
 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)
```

giac [B] time = 0.15, size = 160, normalized size = 2.81

$$-\frac{4}{15}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4+135\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3+170\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+100\sqrt{e}\right.}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="giac")
```

```
[Out] -4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 135*(sqrt(e^(4
*x) - e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 1
00*sqrt(e^(4*x) - e^(2*x)) - 100*e^(2*x) + 23)/(sqrt(e^(4*x) - e^(2*x)) - e
^(2*x) + 1)^5 + 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn
(e^(2*x) - 1)
```

maple [A] time = 0.08, size = 43, normalized size = 0.75

$$-\frac{4(1+\coth(x))^{\frac{3}{2}}}{3}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+8\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-8\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(7/2),x)
```

```
[Out] -4/3*(1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)+8*arctanh(1/2*(1+coth(x))^(1/2
)*2^(1/2))*2^(1/2)-8*(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x)+1)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(7/2), x)
```

mupad [B] time = 1.27, size = 44, normalized size = 0.77

$$-8\sqrt{\coth(x)+1}-\frac{4\,(\coth(x)+1)^{3/2}}{3}-\frac{2\,(\coth(x)+1)^{5/2}}{5}-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}\,1\mathrm{i}}{2}\right)8\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(7/2),x)
```

```
[Out] - 2^(1/2)*atan((2^(1/2)*(coth(x) + 1)^(1/2)*1i)/2)*8i - 8*(coth(x) + 1)^(1/
2) - (4*(coth(x) + 1)^(3/2))/3 - (2*(coth(x) + 1)^(5/2))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth{(x)}+1)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(7/2),x)
```

```
[Out] Integral((coth(x) + 1)**(7/2), x)
```

3.71 $\int (1+\coth(x))^{5/2}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-4\sqrt{\coth(x)+1}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(
1+coth(x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-4\sqrt{\coth(x)+1}+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(5/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 4*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (1+\coth(x))^{5/2}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2}+2\int (1+\coth(x))^{3/2}\,dx\\
&= -4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+4\int \sqrt{1+\coth(x)}\,dx\\
&= -4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+8\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\
&= 4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}
\end{aligned}$$

Mathematica [C] time = 0.16, size = 92, normalized size = 2.04

$$-\frac{2\sinh(x)(\coth(x)+1)^{5/2}\left(\cosh(x)\sqrt{i(\coth(x)+1)}+\sinh(x)\left(7\sqrt{i(\coth(x)+1)}-(6-6i)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i}\right.\right.\right.}{3\sqrt{i(\coth(x)+1)}\,(\sinh(x)+\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(5/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(5/2)*Sinh[x]*(Cosh[x]*Sqrt[I*(1 + Coth[x])] + ((-6 + 6*I
)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]] + 7*Sqrt[I*(1 + Coth[x])])*Sinh
[x]))/(3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x])^2)
```

fricas [B] time = 0.41, size = 259, normalized size = 5.76

$$-\frac{2\left(2\sqrt{2}\left(4\sqrt{2}\cosh(x)^3+12\sqrt{2}\cosh(x)\sinh(x)^2+4\sqrt{2}\sinh(x)^3+3\left(4\sqrt{2}\cosh(x)^2-\sqrt{2}\right)\sinh(x)-3\sqrt{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^3 + 12*sqrt(2)*cosh(x)*sinh(x)^2 + 4*sqr
t(2)*sinh(x)^3 + 3*(4*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.15, size = 112, normalized size = 2.49

$$-\frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+9\sqrt{e^{(4x)}-e^{(2x)}}-9e^{(2x)}+4\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 9*sqrt(e^(4*x) -
 e^(2*x)) - 9*e^(2*x) + 4)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3 + 3*lo
g(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

maple [A] time = 0.06, size = 35, normalized size = 0.78

$$-\frac{2\left(1+\coth(x)\right)^{\frac{3}{2}}}{3}+4\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-4\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(5/2),x)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(
1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(\coth(x)+1)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(5/2), x)
```

mupad [B] time = 1.19, size = 54, normalized size = 1.20

$$\sqrt{8}\ \ln\left(-2\sqrt{8}\ \sqrt{\coth(x)+1}\ -8\right)-\frac{2\left(\coth(x)+1\right)^{3/2}}{3}-2\sqrt{2}\ \ln\left(4\sqrt{2}\ \sqrt{\coth(x)+1}\ -8\right)-4\sqrt{\coth(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(5/2),x)
```

```
[Out] 8^(1/2)*log(- 2*8^(1/2)*(coth(x) + 1)^(1/2) - 8) - (2*(coth(x) + 1)^(3/2))/
3 - 2*2^(1/2)*log(4*2^(1/2)*(coth(x) + 1)^(1/2) - 8) - 4*(coth(x) + 1)^(1/2
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\coth\left(x\right)+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(5/2),x)
```

```
[Out] Integral((coth(x) + 1)**(5/2), x)
```

3.72 $\int (1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=33

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

```
[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3478, 3480, 206}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a + b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (1+\coth(x))^{3/2}\,dx &= -2\sqrt{1+\coth(x)}+2\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}+4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [C] time = 0.11, size = 69, normalized size = 2.09

$$-\frac{2\sinh(x)(\coth(x)+1)^{3/2}\left(\sqrt{i(\coth(x)+1)}-(1-i)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)\right)}{\sqrt{i(\coth(x)+1)}\,(\sinh(x)+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(3/2)*((-1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]
 + Sqrt[I*(1 + Coth[x])])*Sinh[x])/(Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x
]))
```

fricas [B] time = 0.40, size = 131, normalized size = 3.97

$$-\frac{2\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2-\right.}{\cosh(x)^2+2\cosh(x)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sin
h(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [B] time = 0.15, size = 63, normalized size = 1.91

$$-\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}+\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1) + log(abs(2*sqrt(e^(4*x
) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

maple [A] time = 0.06, size = 27, normalized size = 0.82

$$2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(3/2),x)
```

```
[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(\coth(x)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2), x)
```

mupad [B] time = 1.19, size = 26, normalized size = 0.79

$$2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)-2\sqrt{\coth(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(3/2),x)
```

```
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\coth(x) + 1)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2), x)
```

3.73 $\int \sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=21

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]],x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\coth(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.08, size = 45, normalized size = 2.14

$$\frac{(1+i)(\coth(x)+1)^{3/2}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{(i(\coth(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]],x]
[Out] ((1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2))/(I*
(1 + Coth[x]))^(3/2)
```

fricas [B] time = 0.39, size = 50, normalized size = 2.38

$$\frac{1}{2}\sqrt{2}\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}\left(\cosh(x)+\sinh(x)\right)+2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1)
```

giac [B] time = 0.14, size = 37, normalized size = 1.76

$$-\frac{1}{2}\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x
) - 1)
```

maple [A] time = 0.08, size = 17, normalized size = 0.81

$$\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+coth(x))^(1/2),x)
```

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1), x)
```

mupad [B] time = 1.20, size = 16, normalized size = 0.76

$$\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\coth{\left(x \right)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1), x)
```

3.74 $\int \frac{1}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx &= -\frac{1}{\sqrt{1+\coth(x)}}+\frac{1}{2}\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{1}{\sqrt{1+\coth(x)}}+\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}\end{aligned}$$

Mathematica [C] time = 0.31, size = 51, normalized size = 1.59

$$\frac{-2+(-1-i)\sqrt{i(\coth(x)+1)}\,\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{2\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Coth[x]],x]
```

```
[Out] (-2 - (1 + I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sqrt[I*(1 + Coth[x]
)])/(2*Sqrt[1 + Coth[x]])
```

fricas [B] time = 0.41, size = 85, normalized size = 2.66

$$\frac{\left(\sqrt{2}\,\cosh(x)+\sqrt{2}\,\sinh(x)\right)\log\left(2\,\sqrt{2}\,\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}\,(\cosh(x)+\sinh(x))+2\,\cosh(x)^2+4\,\cosh(x)\sinh(x)+2\cdots\right.}{4\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) - 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

giac [B] time = 0.15, size = 66, normalized size = 2.06

$$\frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}}-\log\left(\left|2\,\sqrt{e^{(4x)}-e^{(2x)}}-2\,e^{(2x)}+1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) - log(abs(2*sqrt(e^(4*x)
 - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

maple [A] time = 0.08, size = 27, normalized size = 0.84

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(coth(x) + 1), x)
```

mupad [B] time = 1.24, size = 26, normalized size = 0.81

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2}-\frac{1}{\sqrt{\coth(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 1/(coth(x) + 1)^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\coth(x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(1/2),x)
[Out] Integral(1/sqrt(coth(x) + 1), x)
```

3.75 $\int \frac{1}{(1+\coth(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1
/2/(1+coth(x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$-\frac{1}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^{3/2}}\,dx &= -\frac{1}{3(1+\coth(x))^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\
&= -\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{4}\int \sqrt{1+\coth(x)}\,dx\\
&= -\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] time = 0.32, size = 86, normalized size = 1.76

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\coth(x)+1}\left(\left(\frac{1}{6}-\frac{i}{6}\right)(-5\sinh(2x)+\sinh(4x)+5\cosh(2x)-\cosh(4x)-4)-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(}\right.}{\sqrt{i(\coth(x)}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-4 + 5*Cosh[2*x] - Cosh[4*x] - 5*
Sinh[2*x] + Sinh[4*x]))
```

fricas [B] time = 0.40, size = 168, normalized size = 3.43

$$-\frac{2\sqrt{2}\left(4\sqrt{2}\cosh(x)^2+8\sqrt{2}\cosh(x)\sinh(x)+4\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\right.}{24\left(\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^2 + 8*sqrt(2)*cosh(x)*sinh(x) + 4*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

giac [B] time = 0.16, size = 131, normalized size = 2.67

$$-\frac{1}{24}\sqrt{2}\left(\frac{3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{\operatorname{sgn}\left(e^{(2x)}-1\right)}-\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right.}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3\operatorname{sgn}\left(e^{(2x)}-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(e^
(2*x) - 1) - 2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x) -
e^(2*x)) - 3*e^(2*x) + 1)/((sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x
) - 1)) - 8*sgn(e^(2*x) - 1))
```

maple [A] time = 0.07, size = 35, normalized size = 0.71

$$-\frac{1}{3\left(1+\coth(x)\right)^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(3/2),x)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1
/2/(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(-3/2), x)
```

mupad [B] time = 1.20, size = 32, normalized size = 0.65

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4} - \frac{\frac{\coth(x)}{2}+\frac{5}{6}}{(\coth(x)+1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 5/6)/(cot
h(x) + 1)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(-3/2), x)
```

3.76 $\int \frac{1}{(1+\coth(x))^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{1}{4\sqrt{\coth(x)+1}}-\frac{1}{6(\coth(x)+1)^{3/2}}-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/5/(1+coth(x))^(5/2)-1/6/(1+coth(x))^(3/2)+1/8*arctanh(1/2*(1+coth(x))^(1
/2)*2^(1/2))*2^(1/2)-1/4/(1+coth(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3479, 3480, 206}

$$-\frac{1}{4\sqrt{\coth(x)+1}}-\frac{1}{6(\coth(x)+1)^{3/2}}-\frac{1}{5(\coth(x)+1)^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Coth[x])^(5/2))
- 1/(6*(1 + Coth[x])^(3/2)) - 1/(4*Sqrt[1 + Coth[x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+\coth(x))^{5/2}}\,dx &= -\frac{1}{5(1+\coth(x))^{5/2}}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^{3/2}}\,dx\\
&= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}+\frac{1}{4}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\
&= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}+\frac{1}{8}\int \sqrt{1+\coth(x)}\,dx\\
&= -\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}+\frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x,\right.\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{5(1+\coth(x))^{5/2}}-\frac{1}{6(1+\coth(x))^{3/2}}-\frac{1}{4\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] time = 0.80, size = 94, normalized size = 1.54

$$-\frac{1}{60}\sqrt{\coth(x)+1}\,(\cosh(3x)-\sinh(3x))(-24\sinh(x)+13\sinh(3x)-10\cosh(x)+10\cosh(3x))+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(\coth(x)+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-5/2),x]
```

```
[Out] ((1/8 + I/8)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*(1 + Coth[x])^(3/2))
/(I*(1 + Coth[x]))^(3/2) - (Sqrt[1 + Coth[x]]*(Cosh[3*x] - Sinh[3*x])*(-10*
Cosh[x] + 10*Cosh[3*x] - 24*Sinh[x] + 13*Sinh[3*x]))/60
```

fricas [B] time = 0.40, size = 266, normalized size = 4.36

$$-\frac{2\sqrt{2}\left(23\sqrt{2}\cosh(x)^4+92\sqrt{2}\cosh(x)\sinh(x)^3+23\sqrt{2}\sinh(x)^4+\left(138\sqrt{2}\cosh(x)^2-11\sqrt{2}\right)\sinh(x)^2-11\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^4 + 92*sqrt(2)*cosh(x)*sinh(x)^3 + 23
*sqrt(2)*sinh(x)^4 + (138*sqrt(2)*cosh(x)^2 - 11*sqrt(2))*sinh(x)^2 - 11*sq
rt(2)*cosh(x)^2 + 2*(46*sqrt(2)*cosh(x)^3 - 11*sqrt(2)*cosh(x))*sinh(x) + 3
*sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^5 + 5*sqr
t(2)*cosh(x)^4*sinh(x) + 10*sqrt(2)*cosh(x)^3*sinh(x)^2 + 10*sqrt(2)*cosh(x
)^2*sinh(x)^3 + 5*sqrt(2)*cosh(x)*sinh(x)^4 + sqrt(2)*sinh(x)^5)*log(2*sqrt
(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4
*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*
cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x
)^5)
```

giac [B] time = 0.15, size = 179, normalized size = 2.93

$$-\frac{1}{240}\sqrt{2}\left(\frac{15\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{\operatorname{sgn}\left(e^{(2x)}-1\right)}-\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4+45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3\ldots}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(2)*(15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(
e^(2*x) - 1) - 2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 45*(sqrt(e^(4*
x) - e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 15*
sqrt(e^(4*x) - e^(2*x)) - 15*e^(2*x) + 3)/((sqrt(e^(4*x) - e^(2*x)) - e^(2*
x))^5*sgn(e^(2*x) - 1)) - 46*sgn(e^(2*x) - 1))
```

maple [A] time = 0.07, size = 43, normalized size = 0.70

$$-\frac{1}{5\,(1+\coth(x))^{\frac{5}{2}}}-\frac{1}{6\,(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{4\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x))^(5/2),x)
```

```
[Out] -1/5/(1+coth(x))^(5/2)-1/6/(1+coth(x))^(3/2)+1/8*arctanh(1/2*(1+coth(x))^(1
/2)*2^(1/2))*2^(1/2)-1/4/(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\coth(x)+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(-5/2), x)
```

mupad [B] time = 1.20, size = 40, normalized size = 0.66

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{8}-\frac{\frac{\coth(x)}{6}+\frac{(\coth(x)+1)^2}{4}+\frac{11}{30}}{(\coth(x)+1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1)^(5/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/8 - (coth(x)/6 + (coth(x)
+ 1)^2/4 + 11/30)/(coth(x) + 1)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\coth{(x)}+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x))**(5/2),x)
```

```
[Out] Integral((coth(x) + 1)**(-5/2), x)
```

3.77 $\int (a + b\coth(c + dx))^5 \, dx$

Optimal. Leaf size=142

$$-\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d}+\frac{b\left(5a^4+10a^2b^2+b^4\right)\log(\sinh(c+dx))}{d}+ax\,(a$$

```
[Out] a*(a^4+10*a^2*b^2+5*b^4)*x-4*a*b^2*(a^2+b^2)*coth(d*x+c)/d-1/2*b*(3*a^2+b^2
)*(a+b*coth(d*x+c))^2/d-2/3*a*b*(a+b*coth(d*x+c))^3/d-1/4*b*(a+b*coth(d*x+c
))^4/d+b*(5*a^4+10*a^2*b^2+b^4)*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3482, 3528, 3525, 3475}

$$-\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d}-\frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d}+\frac{b\left(10a^2b^2+5a^4+b^4\right)\log(\sinh(c+dx))}{d}+ax\,(1$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^5,x]
```

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (b
*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*x]
)^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2 + b
^4)*Log[Sinh[c + d*x]])/d
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+b\coth(c+dx))^5\,dx = -\frac{b(a+b\coth(c+dx))^4}{4d} + \int (a+b\coth(c+dx))^3\left(a^2+b^2+2ab\coth(c+dx)\right)$$
$$= -\frac{2ab(a+b\coth(c+dx))^3}{3d} - \frac{b(a+b\coth(c+dx))^4}{4d} + \int (a+b\coth(c+dx))^2\left(a\right.$$
$$= -\frac{b\left(3a^2+b^2\right)(a+b\coth(c+dx))^2}{2d} - \frac{2ab(a+b\coth(c+dx))^3}{3d} - \frac{b(a+b\coth(c}{4d}$$
$$= a\left(a^4+10a^2b^2+5b^4\right)x - \frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d} - \frac{b\left(3a^2+b^2\right)(a+b\coth(}{2d}$$
$$= a\left(a^4+10a^2b^2+5b^4\right)x - \frac{4ab^2\left(a^2+b^2\right)\coth(c+dx)}{d} - \frac{b\left(3a^2+b^2\right)(a+b\coth(}{2d}$$

Mathematica [A] time = 0.82, size = 141, normalized size = 0.99

$$-\frac{60ab^2\left(2a^2+b^2\right)\coth(c+dx)+6b^3\left(10a^2+b^2\right)\coth^2(c+dx)-12b\left(5a^4+10a^2b^2+b^4\right)\log(\tanh(c+dx))-}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^5,x]
```

```
[Out] -1/12*(60*a*b^2*(2*a^2 + b^2)*Coth[c + d*x] + 6*b^3*(10*a^2 + b^2)*Coth[c +
 d*x]^2 + 20*a*b^4*Coth[c + d*x]^3 + 3*b^5*Coth[c + d*x]^4 + 6*(a + b)^5*Lo
g[1 - Tanh[c + d*x]] - 12*b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Tanh[c + d*x]] -
 6*(a - b)^5*Log[1 + Tanh[c + d*x]])/d
```

fricas [B] time = 0.46, size = 2748, normalized size = 19.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d
*x + c)^8 + 24*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d*x*sinh(d*x + c)^8 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^
4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*co
sh(d*x + c)^6 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 - 7*(a^5 - 5*a^4*
b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^2 + (a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*sinh(d*x + c)^6 + 24
*(7*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x
+ c)^3 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 60
*a^3*b^2 + 40*a*b^4 + 6*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^4
 + 6*(35*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh
(d*x + c)^4 + 30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b
 + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x - 30*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - 5*a^4*b + 10*a^3*b^2
 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^5 - 10*(5*a^3*b^2 + 5*a^2*
b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*d*x)*cosh(d*x + c)^3 + (30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
```

```
- b^5)*d*x - 4*(45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 3*(a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2 + 4*(21*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)
^6 - 45*a^3*b^2 - 15*a^2*b^3 - 25*a*b^4 - 3*b^5 - 45*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^4 - 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*d*x + 9*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*
a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 3*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^8 + 8*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a^4*b + 10*a^2*b^3 +
 b^5)*sinh(d*x + c)^8 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 4*
(5*a^4*b + 10*a^2*b^3 + b^5 - 7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^3 - 3*
(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^4*b + 10*
a^2*b^3 + b^5 + 6*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 2*(15*a^4*
b + 30*a^2*b^3 + 3*b^5 + 35*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 -
30*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*
a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 - 10*(5*a^4*b + 10*a^2*b^3 + b^5)
*cosh(d*x + c)^3 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 4*(7*(5*a^4*b + 10
*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 5*a^4*b - 10*a^2*b^3 - b^5 - 15*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 9*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
7 - 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 3*(5*a^4*b + 10*a^2*b^
3 + b^5)*cosh(d*x + c)^3 - (5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh
(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^5
 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^7 -
 9*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 1
0*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^5 + 3*(30*a^3*b^2 + 20*a^2*b^
3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d*x)*cosh(d*x + c)^3 - (45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5
+ 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^
7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*s
inh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)
^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d
*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

giac [A] time = 0.17, size = 226, normalized size = 1.59

$$\frac{3\left(a^{5}-5\,a^{4}b+10\,a^{3}b^{2}-10\,a^{2}b^{3}+5\,ab^{4}-b^{5}\right)(dx+c)+3\left(5\,a^{4}b+10\,a^{2}b^{3}+b^{5}\right)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{4\left(15\,a^{3}b^{2}+1\cdots\right.}{\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)
+ 3*(5*a^4*b + 10*a^2*b^3 + b^5)*log(abs(e^(2*d*x + 2*c) - 1)) + 4*(15*a^3*
b^2 + 10*a*b^4 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*c)
+ 3*(15*a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) - (45*a^3*b^
2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4
)/d
```

maple [B] time = 0.02, size = 322, normalized size = 2.27

$$\frac{\ln(\coth(dx+c)+1)\,a^5}{2d}-\frac{5\ln(\coth(dx+c)+1)\,a^4b}{2d}+\frac{5\ln(\coth(dx+c)+1)\,a^3b^2}{d}-\frac{5\ln(\coth(dx+c)+1)\,a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^5,x)
```

```
[Out] 1/2/d*ln(coth(d*x+c)+1)*a^5-5/2/d*ln(coth(d*x+c)+1)*a^4*b+5/d*ln(coth(d*x+c
)+1)*a^3*b^2-5/d*ln(coth(d*x+c)+1)*a^2*b^3+5/2/d*ln(coth(d*x+c)+1)*a*b^4-1/
2/d*ln(coth(d*x+c)+1)*b^5-1/2/d*coth(d*x+c)^2*b^5-5/3/d*coth(d*x+c)^3*a*b^4
-5/d*coth(d*x+c)^2*a^2*b^3-5/d*a*b^4*coth(d*x+c)-10/d*a^3*b^2*coth(d*x+c)-1
/2/d*ln(coth(d*x+c)-1)*a^5-5/2/d*ln(coth(d*x+c)-1)*a^4*b-5/d*ln(coth(d*x+c)
-1)*a^3*b^2-5/d*ln(coth(d*x+c)-1)*a^2*b^3-5/2/d*ln(coth(d*x+c)-1)*a*b^4-1/2
/d*ln(coth(d*x+c)-1)*b^5-1/4/d*b^5*coth(d*x+c)^4
```

maxima [B] time = 0.33, size = 348, normalized size = 2.45

$$\frac{5}{3}\,ab^4\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+b^5\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d
*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^5*(
x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 10*a^2*b^3*
(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 10*a^3*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^5*x + 5*a^4*b*log(sinh(d*x + c))/d
```

mupad [B] time = 1.28, size = 244, normalized size = 1.72

$$x\,(a-b)^5-\frac{4\,\left(5\,a^3\,b^2+5\,a^2\,b^3+5\,a\,b^4+b^5\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,\left(5\,a^4\,b+10\,a^2\,b^3+b^5\right)}{d}-\frac{4\,\left(5\,a^2\,b^3+5\,a\,b^4+\right.}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^5,x)
```

```
[Out] x*(a - b)^5 - (4*(5*a*b^4 + b^5 + 5*a^2*b^3 + 5*a^3*b^2))/(d*(exp(2*c + 2*d
*x) - 1)) + (log(exp(2*c)*exp(2*d*x) - 1)*(5*a^4*b + b^5 + 10*a^2*b^3))/d -
 (4*(5*a*b^4 + 2*b^5 + 5*a^2*b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x
) + 1)) - (4*b^5)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c +
 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*(5*a*b^4 + 3*b^5))/(3*d*(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

sympy [A] time = 13.42, size = 325, normalized size = 2.29

$$\begin{cases} a^5x+\tilde{\infty}a^4bx+\tilde{\infty}a^3b^2x+\tilde{\infty}a^2b^3x+\tilde{\infty}ab^4x+\tilde{\infty}b^5x \\ x\,(a+b\coth(c))^5 \\ a^5x+5a^4bx-\frac{5a^4b\log(\tanh(c+dx)+1)}{d}+\frac{5a^4b\log(\tanh(c+dx))}{d}+10a^3b^2x-\frac{10a^3b^2}{d\tanh(c+dx)}+10a^2b^3x-\frac{10a^2b^3\log(\tanh(c+d}{d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + zoo*a**4*b*x + zoo*a**3*b**2*x + zoo*a**2*b**3*x + zoo*
a*b**4*x + zoo*b**5*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))), (x*
(a + b*coth(c))**5, Eq(d, 0)), (a**5*x + 5*a**4*b*x - 5*a**4*b*log(tanh(c +
 d*x) + 1)/d + 5*a**4*b*log(tanh(c + d*x))/d + 10*a**3*b**2*x - 10*a**3*b**
2/(d*tanh(c + d*x)) + 10*a**2*b**3*x - 10*a**2*b**3*log(tanh(c + d*x) + 1)/
d + 10*a**2*b**3*log(tanh(c + d*x))/d - 5*a**2*b**3/(d*tanh(c + d*x)**2) +
5*a*b**4*x - 5*a*b**4/(d*tanh(c + d*x)) - 5*a*b**4/(3*d*tanh(c + d*x)**3) +
 b**5*x - b**5*log(tanh(c + d*x) + 1)/d + b**5*log(tanh(c + d*x))/d - b**5/
(2*d*tanh(c + d*x)**2) - b**5/(4*d*tanh(c + d*x)**4), True))
```

3.78 $\int (a + b\coth(c + dx))^4 \, dx$

Optimal. Leaf size=101

$$-\frac{b^2 (3a^2 + b^2) \coth(c + dx)}{d} + \frac{4ab (a^2 + b^2) \log(\sinh(c + dx))}{d} + x (a^4 + 6a^2b^2 + b^4) - \frac{b(a + b\coth(c + dx))^3}{3d} - ab\ldots$$

[Out] (a^4+6*a^2*b^2+b^4)*x-b^2*(3*a^2+b^2)*coth(d*x+c)/d-a*b*(a+b*coth(d*x+c))^2
/d-1/3*b*(a+b*coth(d*x+c))^3/d+4*a*b*(a^2+b^2)*ln(sinh(d*x+c))/d

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3482, 3528, 3525, 3475}

$$-\frac{b^2 (3a^2 + b^2) \coth(c + dx)}{d} + \frac{4ab (a^2 + b^2) \log(\sinh(c + dx))}{d} + x (6a^2b^2 + a^4 + b^4) - \frac{b(a + b\coth(c + dx))^3}{3d} - ab\ldots$$

Antiderivative was successfully verified.

[In] Int[(a + b*Coth[c + d*x])^4,x]

[Out] (a^4 + 6*a^2*b^2 + b^4)*x - (b^2*(3*a^2 + b^2)*Coth[c + d*x])/d - (a*b*(a +
 b*Coth[c + d*x])^2)/d - (b*(a + b*Coth[c + d*x])^3)/(3*d) + (4*a*b*(a^2 +
b^2)*Log[Sinh[c + d*x]])/d

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int (a+b\coth(c+dx))^4\,dx = -\frac{b(a+b\coth(c+dx))^3}{3d} + \int (a+b\coth(c+dx))^2\left(a^2+b^2+2ab\coth(c+dx)\right)dx$$
$$= -\frac{ab(a+b\coth(c+dx))^2}{d} - \frac{b(a+b\coth(c+dx))^3}{3d} + \int (a+b\coth(c+dx))\left(a\left(a^2+\right.\right.$$
$$= \left(a^4+6a^2b^2+b^4\right)x - \frac{b^2\left(3a^2+b^2\right)\coth(c+dx)}{d} - \frac{ab(a+b\coth(c+dx))^2}{d} - \frac{b(a+}{}$$
$$= \left(a^4+6a^2b^2+b^4\right)x - \frac{b^2\left(3a^2+b^2\right)\coth(c+dx)}{d} - \frac{ab(a+b\coth(c+dx))^2}{d} - \frac{b(a+}{}$$

Mathematica [**A**] time = 0.92, size = 109, normalized size = 1.08

$$-\frac{6b^2\left(6a^2+b^2\right)\coth(c+dx) - 24ab\left(a^2+b^2\right)\log(\tanh(c+dx)) + 12ab^3\coth^2(c+dx) - 3(a-b)^4\log(\tanh(c+}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^4,x]
```

```
[Out] -1/6*(6*b^2*(6*a^2 + b^2)*Coth[c + d*x] + 12*a*b^3*Coth[c + d*x]^2 + 2*b^4*
Coth[c + d*x]^3 + 3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 24*a*b*(a^2 + b^2)*L
og[Tanh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x]])/d
```

fricas [**B**] time = 0.42, size = 1396, normalized size = 13.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 + 18
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c
)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*sinh(d*x + c)^6 - 3
*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4
)*d*x*cosh(d*x + c)^2 - 12*a^2*b^2 - 8*a*b^3 - 4*b^4 - 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 - 36*a^2*b^2 - 8*b^4 + 12*(5
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 - (12*a^2*
b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x - 6*(12*a^2*b^2 + 8*a*b^3 + 4*b^4
+ 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b + a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6 - 3*(a^3*b + a*b^
3)*cosh(d*x + c)^4 - 3*(a^3*b + a*b^3 - 5*(a^3*b + a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 - a^3*b - a*b^3 + 4*(5*(a^3*b + a*b^3)*cosh(d*x + c)^3 - 3*
(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3*b + a*b^3)*cosh(d*x
 + c)^2 + 3*(5*(a^3*b + a*b^3)*cosh(d*x + c)^4 + a^3*b + a*b^3 - 6*(a^3*b +
 a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((a^3*b + a*b^3)*cosh(d*x + c)
^5 - 2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*cosh(d*x + c))*sin
h(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 - 2*(12*a^2*b^
2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*co
sh(d*x + c)^3 + (24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^
```

```
2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4
 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x +
 c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

giac [A] time = 0.14, size = 153, normalized size = 1.51

$$\frac{3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)(dx+c)+12\left(a^3b+ab^3\right)\log\left(\left|e^{(2dx+2c)}-1\right|\right)-\frac{4\left(9a^2b^2+2b^4+3\left(3a^2b^2+2ab^3+b^4\right)\cdots\right.}{\left(e^{(2d\cdots}\right.}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c) + 12*(a^3*b +
a*b^3)*log(abs(e^(2*d*x + 2*c) - 1)) - 4*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^2
+ 2*a*b^3 + b^4)*e^(4*d*x + 4*c) - 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*x +
 2*c))/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [B] time = 0.02, size = 246, normalized size = 2.44

$$-\frac{b^4\left(\coth^3(dx+c)\right)}{3d}-\frac{2\left(\coth^2(dx+c)\right)a\,b^3}{d}-\frac{6\coth(dx+c)\,a^2b^2}{d}-\frac{\coth(dx+c)\,b^4}{d}-\frac{\ln(\coth(dx+c)-1)\,a^4}{2d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^4,x)
```

```
[Out] -1/3/d*b^4*coth(d*x+c)^3-2/d*coth(d*x+c)^2*a*b^3-6/d*coth(d*x+c)*a^2*b^2-1/
d*coth(d*x+c)*b^4-1/2/d*ln(coth(d*x+c)-1)*a^4-2/d*ln(coth(d*x+c)-1)*a^3*b-3
/d*ln(coth(d*x+c)-1)*a^2*b^2-2/d*ln(coth(d*x+c)-1)*a*b^3-1/2/d*ln(coth(d*x+
c)-1)*b^4+1/2/d*ln(coth(d*x+c)+1)*a^4-2/d*ln(coth(d*x+c)+1)*a^3*b+3/d*ln(co
th(d*x+c)+1)*a^2*b^2-2/d*ln(coth(d*x+c)+1)*a*b^3+1/2/d*ln(coth(d*x+c)+1)*b^
4
```

maxima [B] time = 0.33, size = 219, normalized size = 2.17

$$\frac{1}{3}b^4\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+4ab^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-\cdots}\right.}{d}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b^3
*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
 - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 6*a^2*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^4*x + 4*a^3*b*log(sinh(d*x + c))/d
```

mupad [B] time = 1.24, size = 158, normalized size = 1.56

$$x\,(a-b)^4-\frac{4\left(3a^2b^2+2ab^3+b^4\right)}{d\left(e^{2c+2dx}-1\right)}-\frac{4\left(b^4+2ab^3\right)}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}+\frac{\ln\left(e^{2c}e^{2dx}-1\right)\left(4a^3b+4ab^3\right)}{d}-\frac{\cdots}{3d\left(3e^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^4,x)
```

```
[Out] x*(a - b)^4 - (4*(2*a*b^3 + b^4 + 3*a^2*b^2))/(d*(exp(2*c + 2*d*x) - 1)) -
(4*(2*a*b^3 + b^4))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(
exp(2*c)*exp(2*d*x) - 1)*(4*a*b^3 + 4*a^3*b))/d - (8*b^4)/(3*d*(3*exp(2*c +
 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

sympy [A] time = 6.23, size = 233, normalized size = 2.31

$$\begin{cases} a^4x + \tilde{\infty}a^3bx + \tilde{\infty}a^2b^2x + \tilde{\infty}ab^3x + \tilde{\infty}b^4x \\ x\,(a + b\coth{(c)})^4 \\ a^4x + 4a^3bx - \frac{4a^3b\log{(\tanh{(c+dx)}+1)}}{d} + \frac{4a^3b\log{(\tanh{(c+dx)})}}{d} + 6a^2b^2x - \frac{6a^2b^2}{d\tanh{(c+dx)}} + 4ab^3x - \frac{4ab^3\log{(\tanh{(c+dx)}+1)}}{d} + \frac{4}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + zoo*a**3*b*x + zoo*a**2*b**2*x + zoo*a*b**3*x + zoo*b**
4*x, Eq(c, log(exp(-d*x))) | Eq(c, log(-exp(-d*x)))), (x*(a + b*coth(c))**4
, Eq(d, 0)), (a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 4*a
**3*b*log(tanh(c + d*x))/d + 6*a**2*b**2*x - 6*a**2*b**2/(d*tanh(c + d*x))
+ 4*a*b**3*x - 4*a*b**3*log(tanh(c + d*x) + 1)/d + 4*a*b**3*log(tanh(c + d*
x))/d - 2*a*b**3/(d*tanh(c + d*x)**2) + b**4*x - b**4/(d*tanh(c + d*x)) - b
**4/(3*d*tanh(c + d*x)**3), True))
```

3.79 $\int (a+b\coth(c+dx))^3\,dx$

Optimal. Leaf size=69

$$\frac{b\left(3a^2+b^2\right)\log(\sinh(c+dx))}{d}+ax\left(a^2+3b^2\right)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

```
[Out] a*(a^2+3*b^2)*x-2*a*b^2*coth(d*x+c)/d-1/2*b*(a+b*coth(d*x+c))^2/d+b*(3*a^2+
b^2)*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3482, 3525, 3475}

$$\frac{b\left(3a^2+b^2\right)\log(\sinh(c+dx))}{d}+ax\left(a^2+3b^2\right)-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2)
/(2*d) + (b*(3*a^2 + b^2)*Log[Sinh[c + d*x]])/d
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(c+dx))^3\,dx &= -\frac{b(a+b\coth(c+dx))^2}{2d}+\int (a+b\coth(c+dx))\left(a^2+b^2+2ab\coth(c+dx)\right)\,d\\ &= a\left(a^2+3b^2\right)x-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}+\left(b\left(3a^2+b^2\right)\right)\int c\\ &= a\left(a^2+3b^2\right)x-\frac{2ab^2\coth(c+dx)}{d}-\frac{b(a+b\coth(c+dx))^2}{2d}+\frac{b\left(3a^2+b^2\right)\log(s}{d}\end{aligned}$$

Mathematica [A] time = 0.42, size = 86, normalized size = 1.25

$$-\frac{-2b\left(3a^2+b^2\right)\log(\tanh(c+dx))+6ab^2\coth(c+dx)+(a-b)^3(-\log(\tanh(c+dx)+1))+(a+b)^3\log(1-}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] -1/2*(6*a*b^2*Coth[c + d*x] + b^3*Coth[c + d*x]^2 + (a + b)^3*Log[1 - Tanh[
c + d*x]] - 2*b*(3*a^2 + b^2)*Log[Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh[c
 + d*x]])/d
```

fricas [B] time = 0.43, size = 654, normalized size = 9.48

$$\frac{\left(a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)dx\cosh\left(dx+c\right)^4 + 4\left(a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3 + \left(a^3 - 3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x
- 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 +
 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 - 3*a*b^2 - b^3 -
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b^3)*co
sh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b
+ b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 - 2*(3*a^2*b + b^3)*cosh(d*x + c)^2
- 2*(3*a^2*b + b^3 - 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((3*a^2*b + b^3)*cosh(d*x + c)^3 - (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 - (3*a*b^2 + b^3 + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4
*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2
+ 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*co
sh(d*x + c))*sinh(d*x + c) + d)
```

giac [A] time = 0.13, size = 99, normalized size = 1.43

$$\frac{\left(a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)(dx+c) + \left(3\,a^2b + b^3\right)\log\left(\left|e^{(2\,dx+2\,c)} - 1\right|\right) + \frac{2\left(3\,ab^2 - \left(3\,ab^2+b^3\right)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c) + (3*a^2*b + b^3)*log(abs(e^(2*d
*x + 2*c) - 1)) + 2*(3*a*b^2 - (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x +
 2*c) - 1)^2)/d
```

maple [B] time = 0.02, size = 173, normalized size = 2.51

$$-\frac{\left(\coth^2\left(dx+c\right)\right)b^3}{2d} - \frac{3a\,b^2\coth\left(dx+c\right)}{d} - \frac{\ln\left(\coth\left(dx+c\right)-1\right)a^3}{2d} - \frac{3\ln\left(\coth\left(dx+c\right)-1\right)a^2b}{2d} - \frac{3\ln\left(\coth\left(dx+\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^3,x)
```

```
[Out] -1/2/d*coth(d*x+c)^2*b^3-3*a*b^2*coth(d*x+c)/d-1/2/d*ln(coth(d*x+c)-1)*a^3-
3/2/d*ln(coth(d*x+c)-1)*a^2*b-3/2/d*ln(coth(d*x+c)-1)*a*b^2-1/2/d*ln(coth(d
*x+c)-1)*b^3+1/2/d*ln(coth(d*x+c)+1)*a^3-3/2/d*ln(coth(d*x+c)+1)*a^2*b+3/2/
d*ln(coth(d*x+c)+1)*a*b^2-1/2/d*ln(coth(d*x+c)+1)*b^3
```

maxima [B] time = 0.32, size = 136, normalized size = 1.97

$$b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)+3\,ab^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 3*a*b^2*(x +
c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^3*x + 3*a^2*b*log(sinh(d*x + c))/d
```

mupad [B] time = 0.11, size = 97, normalized size = 1.41

$$x\,(a-b)^3-\frac{2\,\left(b^3+3\,a\,b^2\right)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,b^3}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)\,\left(3\,a^2\,b+b^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^3,x)
```

```
[Out] x*(a - b)^3 - (2*(3*a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - (2*b^3)/(d*(
exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) - 1)
*(3*a^2*b + b^3))/d
```

sympy [A] time = 2.85, size = 175, normalized size = 2.54

$$\begin{cases} a^3x+\tilde{\infty}a^2bx+\tilde{\infty}ab^2x+\tilde{\infty}b^3x \\ x\,(a+b\coth(c))^3 \\ a^3x+3a^2bx-\frac{3a^2b\log(\tanh(c+dx)+1)}{d}+\frac{3a^2b\log(\tanh(c+dx))}{d}+3ab^2x-\frac{3ab^2}{d\tanh(c+dx)}+b^3x-\frac{b^3\log(\tanh(c+dx)+1)}{d}+\frac{b^3\,\mathrm{lo}}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + zoo*a**2*b*x + zoo*a*b**2*x + zoo*b**3*x, Eq(c, log(exp
(-d*x))) | Eq(c, log(-exp(-d*x)))), (x*(a + b*coth(c))**3, Eq(d, 0)), (a**3
*x + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d + 3*a**2*b*log(tanh(c +
 d*x))/d + 3*a*b**2*x - 3*a*b**2/(d*tanh(c + d*x)) + b**3*x - b**3*log(tanh
(c + d*x) + 1)/d + b**3*log(tanh(c + d*x))/d - b**3/(2*d*tanh(c + d*x)**2),
 True))
```

3.80 $\int (a+b\coth(c+dx))^2\,dx$

Optimal. Leaf size=38

$$x\left(a^2+b^2\right)+\frac{2ab\log(\sinh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

```
[Out] (a^2+b^2)*x-b^2*coth(d*x+c)/d+2*a*b*ln(sinh(d*x+c))/d
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3477, 3475}

$$x\left(a^2+b^2\right)+\frac{2ab\log(\sinh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^2,x]
```

```
[Out] (a^2 + b^2)*x - (b^2*Coth[c + d*x])/d + (2*a*b*Log[Sinh[c + d*x]])/d
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\coth(c+dx))^2\,dx &= \left(a^2+b^2\right)x-\frac{b^2\coth(c+dx)}{d}+(2ab)\int \coth(c+dx)\,dx\\ &= \left(a^2+b^2\right)x-\frac{b^2\coth(c+dx)}{d}+\frac{2ab\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.13, size = 65, normalized size = 1.71

$$\frac{(a-b)^2\log(\tanh(c+dx)+1)-(a+b)^2\log(1-\tanh(c+dx))+4ab\log(\tanh(c+dx))-2b^2\coth(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^2,x]
```

```
[Out] (-2*b^2*Coth[c + d*x] - (a + b)^2*Log[1 - Tanh[c + d*x]] + 4*a*b*Log[Tanh[c
 + d*x]] + (a - b)^2*Log[1 + Tanh[c + d*x]])/(2*d)
```

fricas [B] time = 0.41, size = 205, normalized size = 5.39

$$\frac{\left(a^2-2\,ab+b^2\right)dx\cosh\left(dx+c\right)^2+2\left(a^2-2\,ab+b^2\right)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(a^2-2\,ab+b^2\right)dx\sinh\left(dx\right.}{d\cosh\left(dx+c\right)^2+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 - (a^2 - 2*
a*b + b^2)*d*x - 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(
d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(2*sinh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*s
inh(d*x + c)^2 - d)
```

giac [A] time = 0.12, size = 57, normalized size = 1.50

$$\frac{2\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\left(a^2-2\,ab+b^2\right)(dx+c)-\frac{2\,b^2}{e^{(2\,dx+2\,c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(abs(e^(2*d*x + 2*c) - 1)) + (a^2 - 2*a*b + b^2)*(d*x + c) - 2*b^
2/(e^(2*d*x + 2*c) - 1))/d
```

maple [B] time = 0.02, size = 116, normalized size = 3.05

$$-\frac{b^2\coth(dx+c)}{d}-\frac{\ln(\coth(dx+c)-1)\,a^2}{2d}-\frac{\ln(\coth(dx+c)-1)\,ab}{d}-\frac{\ln(\coth(dx+c)-1)\,b^2}{2d}+\frac{\ln(\coth(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^2,x)
```

```
[Out] -b^2*coth(d*x+c)/d-1/2/d*ln(coth(d*x+c)-1)*a^2-1/d*ln(coth(d*x+c)-1)*a*b-1/
2/d*ln(coth(d*x+c)-1)*b^2+1/2/d*ln(coth(d*x+c)+1)*a^2-1/d*ln(coth(d*x+c)+1)
*a*b+1/2/d*ln(coth(d*x+c)+1)*b^2
```

maxima [A] time = 0.31, size = 49, normalized size = 1.29

$$b^2\left(x+\frac{c}{d}+\frac{2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}\right)+a^2x+\frac{2\,ab\log(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^2*x + 2*a*b*log(sinh(d*x +
 c))/d
```

mupad [B] time = 0.10, size = 51, normalized size = 1.34

$$x\,(a-b)^2-\frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,a\,b\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^2,x)
```

```
[Out] x*(a - b)^2 - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) + (2*a*b*log(exp(2*c)*exp(
2*d*x) - 1))/d
```

sympy [A] time = 1.36, size = 104, normalized size = 2.74

$$\begin{cases} a^2x+\tilde{\infty}abx+\tilde{\infty}b^2x & \text{for } c=\log\left(-e^{-dx}\right)\vee c=\log\left(e^{-dx}\right. \\ x\,(a+b\coth(c))^2 & \text{for } d=0 \\ a^2x+2abx-\frac{2ab\log(\tanh(c+dx)+1)}{d}+\frac{2ab\log(\tanh(c+dx))}{d}+b^2x-\frac{b^2}{d\tanh(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + zoo*a*b*x + zoo*b**2*x, Eq(c, log(exp(-d*x))) | Eq(c, l
og(-exp(-d*x)))), (x*(a + b*coth(c))**2, Eq(d, 0)), (a**2*x + 2*a*b*x - 2*a
*b*log(tanh(c + d*x) + 1)/d + 2*a*b*log(tanh(c + d*x))/d + b**2*x - b**2/(d
*tanh(c + d*x)), True))
```

3.81 $\int \frac{1}{a+b\coth(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)}$$

```
[Out] a*x/(a^2-b^2)-b*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)/d
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\coth(c+dx)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(c+dx)+a\sinh(c+dx))}{\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 1.28

$$\frac{(b-a)\log(1-\coth(c+dx))+(a+b)\log(\coth(c+dx)+1)-2b\log(a+b\coth(c+dx))}{2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-1),x]
```

```
[Out] ((-a + b)*Log[1 - Coth[c + d*x]] + (a + b)*Log[1 + Coth[c + d*x]] - 2*b*Log
[a + b*Coth[c + d*x]])/(2*(a - b)*(a + b)*d)
```

fricas [A] time = 0.45, size = 62, normalized size = 1.24

$$\frac{(a+b)dx - b\log\left(\frac{2\,(b\cosh(dx+c)+a\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a + b)*d*x - b*log(2*(b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 - b^2)*d)
```

giac [A] time = 0.15, size = 62, normalized size = 1.24

$$-\frac{\frac{b\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^2-b^2}-\frac{dx+c}{a-b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^2 - b^2) - (
d*x + c)/(a - b))/d
```

maple [A] time = 0.11, size = 76, normalized size = 1.52

$$-\frac{\ln\left(\coth\left(dx+c\right)-1\right)}{d\left(2b+2a\right)}+\frac{\ln\left(\coth\left(dx+c\right)+1\right)}{d\left(2a-2b\right)}-\frac{b\ln\left(a+b\coth\left(dx+c\right)\right)}{d\left(a-b\right)\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c)),x)
```

```
[Out] -1/d/(2*b+2*a)*ln(coth(d*x+c)-1)+1/d/(2*a-2*b)*ln(coth(d*x+c)+1)-1/d*b/(a-b
)/(a+b)*ln(a+b*coth(d*x+c))
```

maxima [A] time = 0.35, size = 52, normalized size = 1.04

$$-\frac{b\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{\left(a^2-b^2\right)d}+\frac{dx+c}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2 - b^2)*d) + (d*x + c)/((a +
 b)*d)
```

mupad [B] time = 1.22, size = 55, normalized size = 1.10

$$\frac{x}{a-b}-\frac{b\,\ln\left(b-a+a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{a^2\,d-b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x)),x)
```

```
[Out] x/(a - b) - (b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x)))/
(a^2*d - b^2*d)
```

sympy [A] time = 2.56, size = 236, normalized size = 4.72

$$\begin{cases} \frac{\tilde{\infty}x}{\coth(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ -\frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)-2bd} + \frac{dx}{2bd\tanh(c+dx)-2bd} - \frac{1}{2bd\tanh(c+dx)-2bd} & \text{for } a = -b \\ \frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)+2bd} + \frac{dx}{2bd\tanh(c+dx)+2bd} + \frac{1}{2bd\tanh(c+dx)+2bd} & \text{for } a = b \\ \frac{x}{a+b\coth(c)} & \text{for } d = 0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{b} & \text{for } a = 0 \\ \frac{adx}{a^2d-b^2d} - \frac{bdx}{a^2d-b^2d} + \frac{b\log(\tanh(c+dx)+1)}{a^2d-b^2d} - \frac{b\log\left(\tanh(c+dx)+\frac{b}{a}\right)}{a^2d-b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/coth(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x*tanh(c + d
*x)/(2*b*d*tanh(c + d*x) - 2*b*d) + d*x/(2*b*d*tanh(c + d*x) - 2*b*d) - 1/(
2*b*d*tanh(c + d*x) - 2*b*d), Eq(a, -b)), (d*x*tanh(c + d*x)/(2*b*d*tanh(c
+ d*x) + 2*b*d) + d*x/(2*b*d*tanh(c + d*x) + 2*b*d) + 1/(2*b*d*tanh(c + d*x
) + 2*b*d), Eq(a, b)), (x/(a + b*coth(c)), Eq(d, 0)), ((x - log(tanh(c + d*
x) + 1)/d)/b, Eq(a, 0)), (a*d*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b**2*d)
 + b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d) - b*log(tanh(c + d*x) + b/a)/
(a**2*d - b**2*d), True))
```

3.82 $\int \frac{1}{(a+b\coth(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}-\frac{2ab\log(a\sinh(c+dx)+b\cosh(c+dx))}{d(a^2-b^2)^2}+\frac{x(a^2+b^2)}{(a^2-b^2)^2}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2+b/(a^2-b^2)/d/(a+b*coth(d*x+c))-2*a*b*ln(b*cosh(d*x
+c)+a*sinh(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3483, 3531, 3530}

$$\frac{b}{d(a^2-b^2)(a+b\coth(c+dx))}-\frac{2ab\log(a\sinh(c+dx)+b\cosh(c+dx))}{d(a^2-b^2)^2}+\frac{x(a^2+b^2)}{(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-2),x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 + b/((a^2 - b^2)*d*(a + b*Coth[c + d*x])) - (
2*a*b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+b\coth(c+dx))^2}\,dx &= \frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))}+\frac{\int \frac{a-b\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))}-\frac{(2iab)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{(a^2-b^2)^2}\\ &= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))}-\frac{2ab\log(b\cosh(c+dx)+a\sinh(c}{(a^2-b^2)^2\,d}\end{aligned}$$

Mathematica [A] time = 1.56, size = 100, normalized size = 1.18

$$\frac{\frac{2b\left(\frac{b^3-a^2b}{a\tanh(c+dx)+b}-2a^2\log(a\tanh(c+dx)+b)\right)}{a\left(a^2-b^2\right)^2}-\frac{\log(1-\tanh(c+dx))}{(a+b)^2}+\frac{\log(\tanh(c+dx)+1)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-2),x]
```

```
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 + (
2*b*(-2*a^2*Log[b + a*Tanh[c + d*x]] + (-(a^2*b) + b^3)/(b + a*Tanh[c + d*x
])))/(a*(a^2 - b^2)^2))/(2*d)
```

fricas [B] time = 0.42, size = 426, normalized size = 5.01

$$\frac{\left(a^3+3a^2b+3ab^2+b^3\right)dx\cosh(dx+c)^2+2\left(a^3+3a^2b+3ab^2+b^3\right)dx\cosh(dx+c)\sinh(dx+c)+\left(a^3+\right.}{\left(a^5+a^4b-2a^3b^2-2a^2b^3+ab^4+b^5\right)d\cosh(dx+c)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x*sinh(d*x + c)^2 - 2*a*b^2 + 2*b^3 - (a^3 + a^2*b - a*b^2 - b^3)*d*
x + 2*(a^2*b - a*b^2 - (a^2*b + a*b^2)*cosh(d*x + c)^2 - 2*(a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c) - (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(b*cos
h(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a
^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^2 - (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)
```

giac [A] time = 0.13, size = 130, normalized size = 1.53

$$-\frac{\frac{2\,ab\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^4-2\,a^2b^2+b^4}-\frac{dx+c}{a^2-2\,ab+b^2}+\frac{2\left(ab^2-b^3\right)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right)(a+b)^2(a-b)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^4 - 2*a^
2*b^2 + b^4) - (d*x + c)/(a^2 - 2*a*b + b^2) + 2*(a*b^2 - b^3)/((a*e^(2*d*x
 + 2*c) + b*e^(2*d*x + 2*c) - a + b)*(a + b)^2*(a - b)^2))/d
```

maple [A] time = 0.12, size = 101, normalized size = 1.19

$$-\frac{\ln(\coth(dx+c)-1)}{2d(a+b)^2}+\frac{\ln(\coth(dx+c)+1)}{2d(a-b)^2}+\frac{b}{d(a-b)(a+b)(a+b\coth(dx+c))}-\frac{2ab\ln(a+b\coth(dx+}{d(a+b)^2(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(coth(d*x+c)-1)+1/2/d/(a-b)^2*ln(coth(d*x+c)+1)+1/d*b/(a-b
)/(a+b)/(a+b*coth(d*x+c))-2/d*a*b/(a+b)^2/(a-b)^2*ln(a+b*coth(d*x+c))
```

maxima [A] time = 0.33, size = 124, normalized size = 1.46

$$-\frac{2\,ab\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{\left(a^4-2\,a^2b^2+b^4\right)d}-\frac{2\,b^2}{\left(a^4-2\,a^2b^2+b^4-\left(a^4-2\,a^3b+2\,ab^3-b^4\right)e^{(-2\,dx-2\,c)}\right)d}+\frac{dx+c}{\left(a^2+2\,ab+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^4 - 2*a^2*b^2 + b^4)*d) -
 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*x
- 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

mupad [B] time = 1.30, size = 104, normalized size = 1.22

$$\frac{x}{(a-b)^2}-\frac{2\,a\,b\,\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)}{d\,a^4-2\,d\,a^2\,b^2+d\,b^4}-\frac{2\,b^2}{d\,(a+b)^2\,(a-b)\,\left(b-a+e^{2\,c+2\,d\,x}\,(a+b)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^2,x)
```

```
[Out] x/(a - b)^2 - (2*a*b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d
*x)))/(a^4*d + b^4*d - 2*a^2*b^2*d) - (2*b^2)/(d*(a + b)^2*(a - b)*(b - a +
 exp(2*c + 2*d*x)*(a + b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x/coth(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
+ d*x)/d)/b**2, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(4*b**2*d*tanh(c + d*x)**2
 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2*d*x*tanh(c + d*x)/(4*b**2*d*tanh(
c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + d*x/(4*b**2*d*tanh(c + d
*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + 3*tanh(c + d*x)/(4*b**2*d*tan
h(c + d*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2/(4*b**2*d*tanh(c + d
*x)**2 - 8*b**2*d*tanh(c + d*x) + 4*b**2*d), Eq(a, -b)), (d*x*tanh(c + d*x)
**2/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + 2*d*x
*tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2
*d) + d*x/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) +
 3*tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b*
*2*d) + 2/(4*b**2*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d),
Eq(a, b)), ((Integral(-2*exp(2*c)*exp(2*d*x)/(exp(4*c)*exp(4*d*x)*coth(c +
d*x)**2 - 2*exp(4*c)*exp(4*d*x)*coth(c + d*x) + exp(4*c)*exp(4*d*x) - 2*exp
(2*c)*exp(2*d*x)*coth(c + d*x)**2 + 2*exp(2*c)*exp(2*d*x) + coth(c + d*x)**
2 + 2*coth(c + d*x) + 1), x) + Integral(exp(4*c)*exp(4*d*x)/(exp(4*c)*exp(4
*d*x)*coth(c + d*x)**2 - 2*exp(4*c)*exp(4*d*x)*coth(c + d*x) + exp(4*c)*exp
(4*d*x) - 2*exp(2*c)*exp(2*d*x)*coth(c + d*x)**2 + 2*exp(2*c)*exp(2*d*x) +
coth(c + d*x)**2 + 2*coth(c + d*x) + 1), x) + Integral(1/(exp(4*c)*exp(4*d*
x)*coth(c + d*x)**2 - 2*exp(4*c)*exp(4*d*x)*coth(c + d*x) + exp(4*c)*exp(4*
d*x) - 2*exp(2*c)*exp(2*d*x)*coth(c + d*x)**2 + 2*exp(2*c)*exp(2*d*x) + cot
h(c + d*x)**2 + 2*coth(c + d*x) + 1), x))/b**2, Eq(a, (-b*exp(2*c)*exp(2*d*
x) - b)/(exp(2*c)*exp(2*d*x) - 1))), (x/(a + b*coth(c))**2, Eq(d, 0)), (a**
4*d*x*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c
 + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) - 2*a**3*b*
d*x*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c +
```

```
 d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + a**3*b*d*x/
(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c + d*x) - 2*a**3*b**
3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + 2*a**3*b*log(tanh(c + d*x) +
1)*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c +
d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) - 2*a**3*b*log
(tanh(c + d*x) + b/a)*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a*
*4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**
5*d) + a**2*b**2*d*x*tanh(c + d*x)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**
4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5
*d) - 2*a**2*b**2*d*x/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh
(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + 2*a**2*
b**2*log(tanh(c + d*x) + 1)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*
d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) - 2
*a**2*b**2*log(tanh(c + d*x) + b/a)/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a*
*4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**
5*d) - a**2*b**2/(a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c +
d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + a*b**3*d*x/(
a**6*d*tanh(c + d*x) + a**5*b*d - 2*a**4*b**2*d*tanh(c + d*x) - 2*a**3*b**3
*d + a**2*b**4*d*tanh(c + d*x) + a*b**5*d) + b**4/(a**6*d*tanh(c + d*x) + a
**5*b*d - 2*a**4*b**2*d*tanh(c + d*x) - 2*a**3*b**3*d + a**2*b**4*d*tanh(c
+ d*x) + a*b**5*d), True))
```

3.83 $\int \frac{1}{(a+b\coth(c+dx))^3} dx$

Optimal. Leaf size=129

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\coth(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a\sinh(c+dx)+b\cosh(c+$$

```
[Out] a*(a^2+3*b^2)*x/(a^2-b^2)^3+1/2*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^2+2*a*b/(a^
2-b^2)^2/d/(a+b*coth(d*x+c))-b*(3*a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/
(a^2-b^2)^3/d
```

Rubi [A] time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3483, 3529, 3531, 3530}

$$\frac{2ab}{d\left(a^2-b^2\right)^2(a+b\coth(c+dx))}+\frac{b}{2d\left(a^2-b^2\right)(a+b\coth(c+dx))^2}-\frac{b\left(3a^2+b^2\right)\log(a\sinh(c+dx)+b\cosh(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x]
)^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])) - (b*(3*a^2 + b^2)*L
og[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^2}\,dx}{a^2-b^2}$$

$$= \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))} + \frac{\int \frac{a^2+b^2-}{a+b}}{(a}$$

$$= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c-}$$

$$= \frac{a\left(a^2+3b^2\right)x}{\left(a^2-b^2\right)^3} + \frac{b}{2\left(a^2-b^2\right)d(a+b\coth(c+dx))^2} + \frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c-}$$

Mathematica [A] time = 3.62, size = 134, normalized size = 1.04

$$-\frac{\frac{b\left(2\left(3a^2+b^2\right)\log(a\tanh(c+dx)+b)+\frac{b\left(b^2-a^2\right)\left(\left(2ab^2-6a^3\right)\tanh(c+dx)-5a^2b+b^3\right)}{a^2(a\tanh(c+dx)+b)^2}\right)}{\left(a^2-b^2\right)^3}+\frac{\log(1-\tanh(c+dx))}{(a+b)^3}-\frac{\log(\tanh(c+dx)+1)}{(a-b)^3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] -1/2*(Log[1 - Tanh[c + d*x]]/(a + b)^3 - Log[1 + Tanh[c + d*x]]/(a - b)^3 +
 (b*(2*(3*a^2 + b^2)*Log[b + a*Tanh[c + d*x]] + (b*(-a^2 + b^2)*(-5*a^2*b +
 b^3 + (-6*a^3 + 2*a*b^2)*Tanh[c + d*x]))/(a^2*(b + a*Tanh[c + d*x])^2)))/(
a^2 - b^2)^3)/d
```

fricas [B] time = 0.44, size = 1431, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c
)^4 + 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*x*sinh(d*x + c)^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + (a^5 + a
^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*x - 2*(3*a^3*b^2 - a^2*b^3 -
3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x
)*cosh(d*x + c)^2 - 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 - 3*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^2 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*sinh(d*x + c)^2 - (3*
a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + 2*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^4*b + 6*a^3*b^2 + 4*
a^2*b^3 + 2*a*b^4 + b^5)*sinh(d*x + c)^4 - 2*(3*a^4*b - 2*a^2*b^3 - b^5)*co
sh(d*x + c)^2 - 2*(3*a^4*b - 2*a^2*b^3 - b^5 - 3*(3*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^3 - (3*a^4*b - 2*a^2*b^3
 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*cosh(d*x + c) + a*sinh(d*x +
 c))/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^3 - (3*a^3*b^2 - a^2*b^3 - 3*a*
b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*co
```

```
sh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3
*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^4 + 4*(a^8 + 2*a^7*b - 2*
a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*
a^2*b^6 - 2*a*b^7 - b^8)*d*sinh(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*d*cosh(d*x + c)^2 + 2*(3*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6
*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^2 - (a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sinh(d*x + c)^2 + (a^8 - 2*a^
7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)*d + 4*
((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 -
 b^8)*d*cosh(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d
*cosh(d*x + c))*sinh(d*x + c))
```

giac [A] time = 0.18, size = 203, normalized size = 1.57

$$-\frac{\frac{\left(3\,a^2b+b^3\right)\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{dx+c}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{2\left(\left(3\,a^2b^2-4\,ab^3+b^4\right)e^{(2\,dx+2\,c)}-\frac{3\left(a^3b^2-2\,a^2b^3+ab^4\right)}{a+b}\right)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right)^2(a+b)^2(a-b)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((3*a^2*b + b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + 2*((3*a^2*b^2 - 4*a*b^3 + b^4)*e^(2*d*x + 2*c) - 3*(a^3*b^2 - 2*a^2*b
^3 + a*b^4)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b)^2*(a
+ b)^2*(a - b)^3))/d
```

maple [A] time = 0.12, size = 166, normalized size = 1.29

$$-\frac{\ln\left(\coth\left(dx+c\right)-1\right)}{2d\left(a+b\right)^3}+\frac{\ln\left(\coth\left(dx+c\right)+1\right)}{2d\left(a-b\right)^3}+\frac{b}{2d\left(a-b\right)\left(a+b\right)\left(a+b\coth\left(dx+c\right)\right)^2}+\frac{2ab}{d\left(a+b\right)^2\left(a-b\right)^2\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^3,x)
```

```
[Out] -1/2/d/(a+b)^3*ln(coth(d*x+c)-1)+1/2/d/(a-b)^3*ln(coth(d*x+c)+1)+1/2/d*b/(a
-b)/(a+b)/(a+b*coth(d*x+c))^2+2/d*a*b/(a+b)^2/(a-b)^2/(a+b*coth(d*x+c))-3/d
*b/(a+b)^3/(a-b)^3*ln(a+b*coth(d*x+c))*a^2-1/d*b^3/(a+b)^3/(a-b)^3*ln(a+b*c
oth(d*x+c))
```

maxima [B] time = 0.34, size = 322, normalized size = 2.50

$$-\frac{\left(3\,a^2b+b^3\right)\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)d}-\frac{}{\left(a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7-2\left(a^7-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 - (3*a^2*b^2 - 2*a*b^3 - b^4)
*e^(-2*d*x - 2*c))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7 - 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*e^(-2*d*x - 2*c) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^
4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^(-4*d*x - 4*c))*d) + (d*x +
c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

mupad [B] time = 1.40, size = 195, normalized size = 1.51

$$\frac{x}{(a-b)^3}-\frac{\ln\left(b-a+a\,e^{2c}\,e^{2dx}+b\,e^{2c}\,e^{2dx}\right)\,\left(3\,a^2\,b+b^3\right)}{d\,a^6-3\,d\,a^4\,b^2+3\,d\,a^2\,b^4-d\,b^6}+\frac{2\,b^3}{d\,(a+b)^3\,(a-b)\,\left(e^{4c+4dx}\,(a+b)^2+(a-b)^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^3,x)
```

```
[Out] x/(a - b)^3 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*(
3*a^2*b + b^3))/(a^6*d - b^6*d + 3*a^2*b^4*d - 3*a^4*b^2*d) + (2*b^3)/(d*(a
 + b)^3*(a - b)*(exp(4*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*d*x
)*(a + b)*(a - b))) - (2*(3*a*b^2 - b^3))/(d*(a + b)^3*(a - b)^2*(b - a + e
xp(2*c + 2*d*x)*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.84 $\int \frac{1}{(a+b\coth(c+dx))^4}\,dx$

Optimal. Leaf size=169

$$\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3\left(a+b\coth(c+dx)\right)}+\frac{ab}{d\left(a^2-b^2\right)^2\left(a+b\coth(c+dx)\right)^2}+\frac{b}{3d\left(a^2-b^2\right)\left(a+b\coth(c+dx)\right)^3}-\frac{4ab\left(a^2 \cdots\right.}{}$$

```
[Out] (a^4+6*a^2*b^2+b^4)*x/(a^2-b^2)^4+1/3*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^3+a*b
/(a^2-b^2)^2/d/(a+b*coth(d*x+c))^2+b*(3*a^2+b^2)/(a^2-b^2)^3/d/(a+b*coth(d*
x+c))-4*a*b*(a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.26, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3483, 3529, 3531, 3530}

$$\frac{b\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3\left(a+b\coth(c+dx)\right)}+\frac{ab}{d\left(a^2-b^2\right)^2\left(a+b\coth(c+dx)\right)^2}+\frac{b}{3d\left(a^2-b^2\right)\left(a+b\coth(c+dx)\right)^3}-\frac{4ab\left(a^2 \cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 + b/(3*(a^2 - b^2)*d*(a + b*Coth[
c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])^2) + (b*(3*a^2
+ b^2))/((a^2 - b^2)^3*d*(a + b*Coth[c + d*x])) - (4*a*b*(a^2 + b^2)*Log[b*
Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^3}\,dx}{a^2-b^2}$$

$$= \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))^2} + \frac{\int \frac{a^2+b^2}{(a+}}{(}$$

$$= \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))^2} + \frac{}{\left(a^2-b\right.}$$

$$= \frac{\left(a^4+6a^2b^2+b^4\right)x}{\left(a^2-b^2\right)^4} + \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\,\mathrm{co}}$$

$$= \frac{\left(a^4+6a^2b^2+b^4\right)x}{\left(a^2-b^2\right)^4} + \frac{b}{3\left(a^2-b^2\right)d(a+b\coth(c+dx))^3} + \frac{ab}{\left(a^2-b^2\right)^2 d(a+b\,\mathrm{co}}$$

Mathematica [A] time = 6.22, size = 214, normalized size = 1.27

$$-\frac{4ab\left(a^2+b^2\right)\log(a\tanh(c+dx)+b)}{d\left(a^2-b^2\right)^4} - \frac{b^4}{3a^3d\left(a^2-b^2\right)(a\tanh(c+dx)+b)^3} + \frac{b^3\left(2a^2-b^2\right)}{a^3d\left(a^2-b^2\right)^2(a\tanh(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] -1/2*Log[1 - Tanh[c + d*x]]/((a + b)^4*d) + Log[1 + Tanh[c + d*x]]/(2*(a -
b)^4*d) - (4*a*b*(a^2 + b^2)*Log[b + a*Tanh[c + d*x]])/((a^2 - b^2)^4*d) -
b^4/(3*a^3*(a^2 - b^2)*d*(b + a*Tanh[c + d*x])^3) + (b^3*(2*a^2 - b^2))/(a^
3*(a^2 - b^2)^2*d*(b + a*Tanh[c + d*x])^2) - (b^2*(6*a^4 - 3*a^2*b^2 + b^4)
)/(a^3*(a^2 - b^2)^3*d*(b + a*Tanh[c + d*x]))
```

fricas [B] time = 0.50, size = 3698, normalized size = 21.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 +
 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^6 + 18*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a
^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)*sinh(d*
x + c)^5 + 3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
*b^5 + 7*a*b^6 + b^7)*d*x*sinh(d*x + c)^6 - 36*a^5*b^2 + 108*a^4*b^3 - 116*
a^3*b^4 + 60*a^2*b^5 - 24*a*b^6 + 8*b^7 - 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^
3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*
b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^4 - 3*(12*a^5*b^2 + 4*a
^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 - 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^2 + 3
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 -
 b^7)*d*x)*sinh(d*x + c)^4 + 12*(5*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3
 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^3 - (12*a^5*b
^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^
2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^3 - 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
```

```
2*b^5 - a*b^6 - b^7)*d*x + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2
*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^
4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c)^2 + 3*(24*a^5*b^2 - 32*a^4*
b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a
^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x
 + c)^4 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*
a*b^6 + b^7)*d*x - 6*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7
 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b
^6 - b^7)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*(a^6*b - 3*a^5*b^2 + 4
*a^4*b^3 - 4*a^3*b^4 + 3*a^2*b^5 - a*b^6 - (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 +
 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^6 - 6*(a^6*b + 3*a^5*b^2 + 4*
a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)*sinh(d*x + c)^5 - (a
^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*sinh(d*x + c)
^6 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^4 + 3*(a^6*b + a^5
*b^2 - a^2*b^5 - a*b^6 - 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a
^2*b^5 + a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(a^6*b + 3*a^5*b^2
+ 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^3 - 3*(a^6*b + a
^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^6*b - a^5*b
^2 - a^2*b^5 + a*b^6)*cosh(d*x + c)^2 - 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^
6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(
d*x + c)^4 - 6*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 - 6*((a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^
6)*cosh(d*x + c)^5 - 2*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^3
+ (a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(
b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*
(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^
6 + b^7)*d*x*cosh(d*x + c)^5 - 2*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a
*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2
*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^3 + (24*a^5*b^2 - 32*a^4*b^3 - 12*
a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/((a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14
*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x +
 c)^6 + 6*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5
+ 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b
^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10
+ b^11)*d*sinh(d*x + c)^6 - 3*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a
^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b
^10 - b^11)*d*cosh(d*x + c)^4 + 3*(5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^
3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9
+ 3*a*b^10 + b^11)*d*cosh(d*x + c)^2 - (a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b
^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*
b^9 - a*b^10 - b^11)*d)*sinh(d*x + c)^4 + 3*(a^11 - a^10*b - 5*a^9*b^2 + 5*
a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5
*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^11 + 3*a^10*b - a^9*b
^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*
b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^3 - 3*(a^11 + a^10*b - 5*a
^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*
a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5
*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^
4 - 6*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10
*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x +
 c)^2 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 -
10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d)*sinh(d*
x + c)^2 - (a^11 - 3*a^10*b - a^9*b^2 + 11*a^8*b^3 - 6*a^7*b^4 - 14*a^6*b^5
 + 14*a^5*b^6 + 6*a^4*b^7 - 11*a^3*b^8 + a^2*b^9 + 3*a*b^10 - b^11)*d + 6*(
(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b
```

```
^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^5
- 2*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a
^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c
)^3 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10
*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x +
 c))*sinh(d*x + c))
```

giac [A] time = 0.18, size = 303, normalized size = 1.79

$$-\frac{\frac{12\left(a^3b+ab^3\right)\log\left(\left|ae^{(2dx+2c)}+be^{(2dx+2c)}-a+b\right|\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{3(dx+c)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{4\left(3\left(3a^4b^2-2a^3b^3-2a^2b^4+2ab^5-b^6\right)e^{(4dx+4c)}-3\left(6a^4b^2-14\right.\right.}{\left(ae^{(2dx+2c)}+b\right.}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3*b + a*b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a
+ b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 3*(d*x + c)/(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 4*(3*(3*a^4*b^2 - 2*a^3*b^3 - 2*a^2*
b^4 + 2*a*b^5 - b^6)*e^(4*d*x + 4*c) - 3*(6*a^4*b^2 - 14*a^3*b^3 + 11*a^2*b
^4 - 4*a*b^5 + b^6)*e^(2*d*x + 2*c) + (9*a^5*b^2 - 27*a^4*b^3 + 29*a^3*b^4
- 15*a^2*b^5 + 6*a*b^6 - 2*b^7)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x +
 2*c) - a + b)^3*(a + b)^3*(a - b)^4))/d
```

maple [A] time = 0.13, size = 230, normalized size = 1.36

$$-\frac{\ln(\coth(dx+c)-1)}{2d(a+b)^4}+\frac{\ln(\coth(dx+c)+1)}{2d(a-b)^4}+\frac{b}{3d(a-b)(a+b)(a+b\coth(dx+c))^3}+\frac{ab}{d(a+b)^2(a-b)^2(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^4,x)
```

```
[Out] -1/2/d/(a+b)^4*ln(coth(d*x+c)-1)+1/2/d/(a-b)^4*ln(coth(d*x+c)+1)+1/3/d*b/(a
-b)/(a+b)/(a+b*coth(d*x+c))^3+1/d*a*b/(a+b)^2/(a-b)^2/(a+b*coth(d*x+c))^2+3
/d*b/(a+b)^3/(a-b)^3/(a+b*coth(d*x+c))*a^2+1/d*b^3/(a+b)^3/(a-b)^3/(a+b*cot
h(d*x+c))-4/d*b*a^3/(a+b)^4/(a-b)^4*ln(a+b*coth(d*x+c))-4/d*b^3*a/(a+b)^4/(
a-b)^4*ln(a+b*coth(d*x+c))
```

maxima [B] time = 0.37, size = 522, normalized size = 3.09

$$-\frac{4\left(a^3b+ab^3\right)\log\left(-(a-b)e^{(-2dx-2c)}+a+b\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)d}-\frac{}{3\left(a^{10}+2a^9b-3a^8b^2-8a^7b^3+2a^6b^4+12a^5b^5+2a^4b^6-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -4*(a^3*b + a*b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d) - 4/3*(9*a^4*b^2 + 18*a^3*b^3 + 11*a^2*b
^4 + 4*a*b^5 + 2*b^6 - 3*(6*a^4*b^2 + 2*a^3*b^3 - 5*a^2*b^4 - 2*a*b^5 - b^6
)*e^(-2*d*x - 2*c) + 3*(3*a^4*b^2 - 4*a^3*b^3 + b^6)*e^(-4*d*x - 4*c))/((a^
10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 -
 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 - 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4
- 10*a^4*b^6 + 5*a^2*b^8 - b^10)*e^(-2*d*x - 2*c) + 3*(a^10 - 2*a^9*b - 3*a
^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2
*b^8 - 2*a*b^9 + b^10)*e^(-4*d*x - 4*c) - (a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10)*
e^(-6*d*x - 6*c))*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d)
```

mupad [B] time = 1.39, size = 310, normalized size = 1.83

$$\frac{x}{(a-b)^4}-\frac{\ln\left(b-a+a\,e^{2c}\,e^{2dx}+b\,e^{2c}\,e^{2dx}\right)\left(4\,a^3\,b+4\,a\,b^3\right)}{d\,a^8-4\,d\,a^6\,b^2+6\,d\,a^4\,b^4-4\,d\,a^2\,b^6+d\,b^8}-\frac{4\left(3\,a^2\,b^2-2\,a\,b^3+b^4\right)}{d\,(a+b)^4\,(a-b)^3\,\left(b-a+e^{2c+2dx}\,(a+b)\right)}-\frac{}{3\,d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^4,x)
```

```
[Out] x/(a - b)^4 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*(
4*a*b^3 + 4*a^3*b))/(a^8*d + b^8*d - 4*a^2*b^6*d + 6*a^4*b^4*d - 4*a^6*b^2*
d) - (4*(b^4 - 2*a*b^3 + 3*a^2*b^2))/(d*(a + b)^4*(a - b)^3*(b - a + exp(2*
c + 2*d*x)*(a + b))) - (8*b^4)/(3*d*(a + b)^4*(a - b)*(exp(6*c + 6*d*x)*(a
+ b)^3 - (a - b)^3 + 3*exp(2*c + 2*d*x)*(a + b)*(a - b)^2 - 3*exp(4*c + 4*d
*x)*(a + b)^2*(a - b))) + (4*(2*a*b^3 - b^4))/(d*(a + b)^4*(a - b)^2*(exp(4
*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*d*x)*(a + b)*(a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.85 $\int \frac{1}{4+6\coth(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{3\log(2\sinh(c+dx)+3\cosh(c+dx))}{10d}-\frac{x}{5}$$

```
[Out] -1/5*x+3/10*ln(3*cosh(d*x+c)+2*sinh(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$\frac{3\log(2\sinh(c+dx)+3\cosh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -x/5 + (3*Log[3*Cosh[c + d*x] + 2*Sinh[c + d*x]])/(10*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+6\coth(c+dx)}\,dx &= -\frac{x}{5}+\frac{3}{10}i\int \frac{-6i-4i\coth(c+dx)}{4+6\coth(c+dx)}\,dx\\ &= -\frac{x}{5}+\frac{3\log(3\cosh(c+dx)+2\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.71

$$-\frac{\log(1-\tanh(c+dx))}{20d}-\frac{\log(\tanh(c+dx)+1)}{4d}+\frac{3\log(2\tanh(c+dx)+3)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -1/20*Log[1 - Tanh[c + d*x]]/d - Log[1 + Tanh[c + d*x]]/(4*d) + (3*Log[3 +
2*Tanh[c + d*x]])/(10*d)
```

fricas [A] time = 0.41, size = 49, normalized size = 1.58

$$-\frac{5\,dx-3\,\log\left(\frac{2\,(3\,\cosh(dx+c)+2\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/10*(5*d*x - 3*log(2*(3*cosh(d*x + c) + 2*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/d
```

giac [A] time = 0.14, size = 29, normalized size = 0.94

$$-\frac{5\,dx + 5\,c - 3\,\log\left(5\,e^{(2\,dx+2\,c)} + 1\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10*(5*d*x + 5*c - 3*log(5*e^(2*d*x + 2*c) + 1))/d
```

maple [A] time = 0.11, size = 46, normalized size = 1.48

$$-\frac{\ln\left(\coth\left(dx+c\right)-1\right)}{20d} - \frac{\ln\left(\coth\left(dx+c\right)+1\right)}{4d} + \frac{3\ln\left(2+3\coth\left(dx+c\right)\right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+6*coth(d*x+c)),x)
```

```
[Out] -1/20/d*ln(coth(d*x+c)-1)-1/4/d*ln(coth(d*x+c)+1)+3/10/d*ln(2+3*coth(d*x+c)
)
```

maxima [A] time = 0.31, size = 28, normalized size = 0.90

$$\frac{dx + c}{10\,d} + \frac{3\,\log\left(e^{(-2\,dx-2\,c)} + 5\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) + 5)/d
```

mupad [B] time = 0.04, size = 22, normalized size = 0.71

$$\frac{3\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+\frac{1}{5}\right)}{10\,d}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*coth(c + d*x) + 4),x)
```

```
[Out] (3*log(exp(2*c)*exp(2*d*x) + 1/5))/(10*d) - x/2
```

sympy [A] time = 0.81, size = 42, normalized size = 1.35

$$\begin{cases} \frac{x}{10} - \frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d} + \frac{3\log\left(\tanh\left(c+dx\right)+\frac{3}{2}\right)}{10d} & \text{for}\: d \neq 0 \\ \frac{x}{6\coth\left(c\right)+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+6*coth(d*x+c)),x)
```

```
[Out] Piecewise((x/10 - 3*log(tanh(c + d*x) + 1)/(10*d) + 3*log(tanh(c + d*x) + 3
/2)/(10*d), Ne(d, 0)), (x/(6*coth(c) + 4), True))
```

3.86 $\int \frac{1}{4-6\coth(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}-\frac{x}{5}$$

```
[Out] -1/5*x-3/10*ln(3*cosh(d*x+c)-2*sinh(d*x+c))/d
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3484, 3530}

$$-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}-\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -x/5 - (3*Log[3*Cosh[c + d*x] - 2*Sinh[c + d*x]])/(10*d)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-6\coth(c+dx)}\,dx &= -\frac{x}{5}-\frac{3}{10}i\int \frac{6i-4i\coth(c+dx)}{4-6\coth(c+dx)}\,dx\\ &= -\frac{x}{5}-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.71

$$-\frac{3\log(3-2\tanh(c+dx))}{10d}+\frac{\log(1-\tanh(c+dx))}{4d}+\frac{\log(\tanh(c+dx)+1)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*Coth[c + d*x])^(-1),x]
```

```
[Out] (-3*Log[3 - 2*Tanh[c + d*x]])/(10*d) + Log[1 - Tanh[c + d*x]]/(4*d) + Log[1
 + Tanh[c + d*x]]/(20*d)
```

fricas [A] time = 0.40, size = 48, normalized size = 1.55

$$\frac{dx-3\log\left(\frac{2\,(3\cosh(dx+c)-2\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(d*x - 3*log(2*(3*cosh(d*x + c) - 2*sinh(d*x + c))/(cosh(d*x + c) - si
nh(d*x + c))))/d
```

giac [A] time = 0.12, size = 24, normalized size = 0.77

$$\frac{dx + c - 3 \log\left(e^{(2\,dx+2\,c)} + 5\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10*(d*x + c - 3*log(e^(2*d*x + 2*c) + 5))/d
```

maple [A] time = 0.11, size = 46, normalized size = 1.48

$$\frac{\ln(\coth(dx+c)-1)}{4d} + \frac{\ln(\coth(dx+c)+1)}{20d} - \frac{3\ln(-2+3\coth(dx+c))}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*coth(d*x+c)),x)
```

```
[Out] 1/4/d*ln(coth(d*x+c)-1)+1/20/d*ln(coth(d*x+c)+1)-3/10/d*ln(-2+3*coth(d*x+c)
)
```

maxima [A] time = 0.30, size = 29, normalized size = 0.94

$$-\frac{1}{2}\,x - \frac{c}{2\,d} - \frac{3\,\log\left(5\,e^{(-2\,dx-2\,c)} + 1\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) + 1)/d
```

mupad [B] time = 0.04, size = 22, normalized size = 0.71

$$\frac{x}{10} - \frac{3\,\ln\left(e^{2\,c}\,e^{2\,d\,x} + 5\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(6*coth(c + d*x) - 4),x)
```

```
[Out] x/10 - (3*log(exp(2*c)*exp(2*d*x) + 5))/(10*d)
```

sympy [A] time = 0.81, size = 42, normalized size = 1.35

$$\begin{cases} -\frac{x}{2} - \frac{3\log\left(\tanh(c+dx)-\frac{3}{2}\right)}{10d} + \frac{3\log(\tanh(c+dx)+1)}{10d} & \text{for } d \neq 0 \\ \frac{x}{4-6\coth(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*coth(d*x+c)),x)
```

```
[Out] Piecewise((-x/2 - 3*log(tanh(c + d*x) - 3/2)/(10*d) + 3*log(tanh(c + d*x) +
 1)/(10*d), Ne(d, 0)), (x/(4 - 6*coth(c)), True))
```

3.87 $\int \sqrt{a+b\coth(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

```
[Out] -arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+arctanh((a+b*co
th(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 700, 1130, 207}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 700

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3485

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\coth(c+dx)}\,dx = -\frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt{a+x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$
$$= -\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$
$$= \frac{(a-b)\,\mathrm{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d} - \frac{(a+b)\,\mathrm{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx,\right.}{d}$$
$$= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Mathematica [C] time = 3.28, size = 128, normalized size = 1.73

$$\frac{\sqrt{a+b\coth(c+dx)}\left(\sqrt{i(a+b)}\,\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a+b)}}\right)-\sqrt{i(a-b)}\,\tanh^{-1}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a-b)}}\right)\right)}{d\sqrt{i(a+b\coth(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] ((-(Sqrt[I*(a - b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a - b)]])
 + Sqrt[I*(a + b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a + b)]])*
Sqrt[a + b*Coth[c + d*x]])/(d*Sqrt[I*(a + b*Coth[c + d*x])])
```

fricas [B] time = 0.51, size = 2231, normalized size = 30.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c
)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4
 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - (2*a
 + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)
^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)
+ a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*
b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*si
nh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 -
2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x +
 c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)) -
```

```
sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*
x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c
)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh
(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x
 + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a +
b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cos
h(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh
(d*x + c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a
^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^
2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x +
 c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2
 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/d, -1/2*(sqrt(-a + b)*arctan(-(a*cos
h(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)
*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2
 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2
- b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)))/d
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(exp
(2*(d*x+c))-1)]Error: Bad Argument Type
```

maple [A] time = 0.19, size = 63, normalized size = 0.85

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)\sqrt{a+b}}{d}-\frac{\sqrt{-a+b}\ \arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*coth(d*x+c))^(1/2),x)
```

```
[Out] arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d-1/d*(-a+b)^(1/2)
*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\coth\left(dx+c\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c) + a), x)
```

mupad [B] time = 1.41, size = 151, normalized size = 2.04

$$\frac{\operatorname{atan}\left(\frac{b^2\,\sqrt{a-b}\,\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}+a\,b\,\sqrt{a-b}\,\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\,\sqrt{a-b}\,1\mathrm{i}}{d}+\frac{\operatorname{atan}\left(\frac{b^2\,\sqrt{a+b}\,\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}-a\,b\,\sqrt{a+b}\,\sqrt{a+b\coth}}{a^2\,b-b^3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*coth(c + d*x))^(1/2),x)
```

```
[Out] (atan((b^2*(a - b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i + a*b*(a - b)^(1/2)
*(a + b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a - b)^(1/2)*1i)/d + (atan
((b^2*(a + b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i - a*b*(a + b)^(1/2)*(a +
 b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a + b)^(1/2)*1i)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \coth{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*coth(c + d*x)), x)
```

3.88 $\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[Out] -arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+arctanh((a+b*co
th(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 708, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 708

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3485

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\left(-b^2+x^2\right)}\,dx, x, b\coth(c+dx)\right)}{d}$$
$$= -\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.15, size = 73, normalized size = 0.99

$$-\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -((ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b] - ArcTanh[Sqr
t[a + b*Coth[c + d*x]]/Sqrt[a + b]]/Sqrt[a + b])/d)
```

fricas [B] time = 0.50, size = 2307, normalized size = 31.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh
(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(
d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d
*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
+ c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c)) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 +
4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)
^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2
*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(
d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(
d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sq
rt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*co
sh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a^2 - b^2)*d), -1/4*(
2*(a - b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
 + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cos
h(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x
```

```
+ c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b +
 b^2)*sinh(d*x + c)^2 - a^2 + b^2)) - (a + b)*sqrt(a - b)*log(((2*a^2 - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2
- b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)
*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 -
 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x +
 c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a
 - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a
^2 - b^2)*d), -1/4*(2*(a + b)*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt
((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x +
 c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x +
c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(
a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b
^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x
+ c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b
)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*s
inh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/((a^2 - b^2)*d), -1/2*((a + b)*sqrt
(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c
))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c
)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + (a -
b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 - a^2 + b^2)))/((a^2 - b^2)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(exp
(2*(d*x+c))-1)]Error: Bad Argument Type
```

maple [A] time = 0.11, size = 62, normalized size = 0.84

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}+\frac{\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(d*x+c))^(1/2),x)
```

```
[Out] arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/d/(-a+b)^(1/2)
*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\coth\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c) + a), x)
```

mupad [B] time = 1.47, size = 242, normalized size = 3.27

$$\frac{\operatorname{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\coth(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3-b\,d^3}-\frac{16\,a\,b^3\,d^3}{a\,d^3-b\,d^3}\right)\sqrt{a-b}}+\frac{\left(a\,d^3-b\,d^3\right)\sqrt{a+b\coth(c+d\,x)}}{b\,d^3\,\sqrt{a-b}}\right)}{d\,\sqrt{a-b}}-\frac{\operatorname{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\coth(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3+b\,d^3}+\frac{16\,a\,b^3\,d^3}{a\,d^3+b\,d^3}\right)\sqrt{a+b}}-\frac{\left(a\,d^3+b\,d^3\right)\sqrt{a+b\coth(c+d\,x)}}{b\,d^3\,\sqrt{a+b}}\right)}{d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(c + d*x))^(1/2),x)
```

```
[Out] atanh((16*a*b^2*(a + b*coth(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 - b*d^3)
 - (16*a*b^3*d^3)/(a*d^3 - b*d^3))*(a - b)^(1/2)) + ((a*d^3 - b*d^3)*(a + b
*coth(c + d*x))^(1/2))/(b*d^3*(a - b)^(1/2)))/(d*(a - b)^(1/2)) - atanh((16
*a*b^2*(a + b*coth(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 + b*d^3) + (16*a*
b^3*d^3)/(a*d^3 + b*d^3))*(a + b)^(1/2)) - ((a*d^3 + b*d^3)*(a + b*coth(c +
 d*x))^(1/2))/(b*d^3*(a + b)^(1/2)))/(d*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\coth\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*coth(c + d*x)), x)
```

3.89 $\int \frac{\sinh^4(x)}{1+\coth(x)} \, dx$

Optimal. Leaf size=60

$$\frac{5x}{16}+\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}-\frac{3}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

```
[Out] 5/16*x+1/32/(1-coth(x))^2+1/8/(1-coth(x))-1/24/(1+coth(x))^3-3/32/(1+coth(x
))^2-3/16/(1+coth(x))
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3487, 44, 207}

$$\frac{5x}{16}+\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}-\frac{3}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x
])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(x)}{1+\coth(x)} \, dx &= \text{Subst}\left(\int \frac{1}{(1-x)^3(1+x)^4} \, dx, x, \coth(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}+\frac{1}{8(1+x)^4}+\frac{3}{16(1+x)^3}+\frac{3}{16(1+x)^2}-\frac{5}{16\left(-1+\right.}\right.\right.\\
&= \frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth}\\
&= \frac{5x}{16}+\frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{}{16(1+}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 42, normalized size = 0.70

$$\frac{1}{192}(60x - 45\sinh(2x) + 9\sinh(4x) - \sinh(6x) + 15\cosh(2x) - 6\cosh(4x) + \cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (60*x + 15*Cosh[2*x] - 6*Cosh[4*x] + Cosh[6*x] - 45*Sinh[2*x] + 9*Sinh[4*x]
 - Sinh[6*x])/192
```

fricas [B] time = 0.39, size = 93, normalized size = 1.55

$$\frac{5\cosh(x)^5 + 25\cosh(x)\sinh(x)^4 + \sinh(x)^5 + 5\left(2\cosh(x)^2 - 3\right)\sinh(x)^3 - 45\cosh(x)^3 + 5\left(10\cosh(x)^3 - 27\right.}{384(\cosh(x) + \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + 5*(2*cosh(x)^2 - 3)
*sinh(x)^3 - 45*cosh(x)^3 + 5*(10*cosh(x)^3 - 27*cosh(x))*sinh(x)^2 + 60*(2
*x + 1)*cosh(x) + 5*(cosh(x)^4 - 9*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x)
 + sinh(x))
```

giac [A] time = 0.13, size = 42, normalized size = 0.70

$$-\frac{1}{384}\left(110\,e^{(6\,x)} - 60\,e^{(4\,x)} + 15\,e^{(2\,x)} - 2\right)e^{(-6\,x)} + \frac{5}{16}\,x + \frac{1}{128}\,e^{(4\,x)} - \frac{5}{64}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/384*(110*e^(6*x) - 60*e^(4*x) + 15*e^(2*x) - 2)*e^(-6*x) + 5/16*x + 1/12
8*e^(4*x) - 5/64*e^(2*x)
```

maple [B] time = 0.09, size = 110, normalized size = 1.83

$$\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(1+coth(x)),x)
```

```
[Out] 1/8/(tanh(1/2*x)-1)^4+1/4/(tanh(1/2*x)-1)^3-1/8/(tanh(1/2*x)-1)^2-1/4/(tanh
(1/2*x)-1)-5/16*ln(tanh(1/2*x)-1)+1/3/(tanh(1/2*x)+1)^6-1/(tanh(1/2*x)+1)^5
+5/8/(tanh(1/2*x)+1)^4+5/12/(tanh(1/2*x)+1)^3-3/8/(tanh(1/2*x)+1)+5/16*ln(t
anh(1/2*x)+1)
```

maxima [A] time = 0.32, size = 36, normalized size = 0.60

$$-\frac{1}{128}\left(10\,e^{(-2\,x)} - 1\right)e^{(4\,x)} + \frac{5}{16}\,x + \frac{5}{32}\,e^{(-2\,x)} - \frac{5}{128}\,e^{(-4\,x)} + \frac{1}{192}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -1/128*(10*e^(-2*x) - 1)*e^(4*x) + 5/16*x + 5/32*e^(-2*x) - 5/128*e^(-4*x)
+ 1/192*e^(-6*x)
```

mupad [B] time = 1.38, size = 34, normalized size = 0.57

$$\frac{5x}{16}+\frac{5\,\mathrm{e}^{-2x}}{32}-\frac{5\,\mathrm{e}^{2x}}{64}-\frac{5\,\mathrm{e}^{-4x}}{128}+\frac{\mathrm{e}^{4x}}{128}+\frac{\mathrm{e}^{-6x}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(coth(x) + 1),x)
```

```
[Out] (5*x)/16 + (5*exp(-2*x))/32 - (5*exp(2*x))/64 - (5*exp(-4*x))/128 + exp(4*x
)/128 + exp(-6*x)/192
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(coth(x) + 1), x)
```

3.90 $\int \frac{\sinh^3(x)}{1+\coth(x)} dx$

Optimal. Leaf size=29

$$\frac{4\cosh^3(x)}{15} - \frac{4\cosh(x)}{5} - \frac{\sinh^3(x)}{5(\coth(x)+1)}$$

```
[Out] -4/5*cosh(x)+4/15*cosh(x)^3-1/5*sinh(x)^3/(1+coth(x))
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3502, 2633}

$$\frac{4\cosh^3(x)}{15} - \frac{4\cosh(x)}{5} - \frac{\sinh^3(x)}{5(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (-4*Cosh[x])/5 + (4*Cosh[x]^3)/15 - Sinh[x]^3/(5*(1 + Coth[x]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{1+\coth(x)} dx &= -\frac{\sinh^3(x)}{5(1+\coth(x))} + \frac{4}{5}\int \sinh^3(x)\, dx \\
&= -\frac{\sinh^3(x)}{5(1+\coth(x))} - \frac{4}{5}\operatorname{Subst}\left(\int \left(1-x^2\right) dx, x, \cosh(x)\right) \\
&= -\frac{4\cosh(x)}{5} + \frac{4\cosh^3(x)}{15} - \frac{\sinh^3(x)}{5(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 1.24

$$\frac{\operatorname{csch}(x)(-40\sinh(2x) + 4\sinh(4x) - 20\cosh(2x) + \cosh(4x) - 45)}{120(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (Csch[x]*(-45 - 20*Cosh[2*x] + Cosh[4*x] - 40*Sinh[2*x] + 4*Sinh[4*x]))/(12
0*(1 + Coth[x]))
```

fricas [B] time = 0.40, size = 60, normalized size = 2.07

$$\frac{\cosh(x)^4 + 16 \cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3 \cosh(x)^2 - 10\right)\sinh(x)^2 - 20 \cosh(x)^2 + 16\left(\cosh(x)^3 - 5\right.}{120\left(\cosh(x) + \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 10)*
sinh(x)^2 - 20*cosh(x)^2 + 16*(cosh(x)^3 - 5*cosh(x))*sinh(x) - 45)/(cosh(x
) + sinh(x))
```

giac [A] time = 0.11, size = 31, normalized size = 1.07

$$-\frac{1}{240}\left(90\, e^{(4\,x)} - 20\, e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{48}\, e^{(3\,x)} - \frac{1}{4}\, e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/240*(90*e^(4*x) - 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) - 1/4*e^x
```

maple [B] time = 0.09, size = 80, normalized size = 2.76

$$-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+coth(x)),x)
```

```
[Out] -1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2+3/8/(tanh(1/2*x)-1)-2/5/(tanh(
1/2*x)+1)^5+1/(tanh(1/2*x)+1)^4-1/3/(tanh(1/2*x)+1)^3-1/2/(tanh(1/2*x)+1)^2
-3/8/(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 33, normalized size = 1.14

$$-\frac{1}{48}\left(12\, e^{(-2\,x)} - 1\right)e^{(3\,x)} - \frac{3}{8}\, e^{(-x)} + \frac{1}{12}\, e^{(-3\,x)} - \frac{1}{80}\, e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -1/48*(12*e^(-2*x) - 1)*e^(3*x) - 3/8*e^(-x) + 1/12*e^(-3*x) - 1/80*e^(-5*x
)
```

mupad [B] time = 1.27, size = 29, normalized size = 1.00

$$\frac{e^{-3\,x}}{12} - \frac{3\,e^{-x}}{8} + \frac{e^{3\,x}}{48} - \frac{e^{-5\,x}}{80} - \frac{e^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(coth(x) + 1),x)
```

```
[Out] exp(-3*x)/12 - (3*exp(-x))/8 + exp(3*x)/48 - exp(-5*x)/80 - exp(x)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{\coth(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**3/(coth(x) + 1), x)
```

3.91 $\int \frac{\sinh^2(x)}{1+\coth(x)} dx$

Optimal. Leaf size=38

$$-\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

```
[Out] -3/8*x-1/8/(1-coth(x))+1/8/(1+coth(x))^2+1/4/(1+coth(x))
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3487, 44, 207}

$$-\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/(4*(1 + Coth[x])
)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{1+\coth(x)} dx &= -\text{Subst}\left(\int \frac{1}{(1-x)^2(1+x)^3} dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\left(-1+x^2\right)}\right) dx, x, \coth(x)\right)\\
&= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \coth(x\right.\\
&= -\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 0.79

$$\frac{1}{32}(-12x+8\sinh(2x)-\sinh(4x)-4\cosh(2x)+\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-12*x - 4*Cosh[2*x] + Cosh[4*x] + 8*Sinh[2*x] - Sinh[4*x])/32
```

fricas [A] time = 0.40, size = 50, normalized size = 1.32

$$\frac{3\cosh(x)^3 + 9\cosh(x)\sinh(x)^2 + \sinh(x)^3 - 6(2x+1)\cosh(x) + 3\left(\cosh(x)^2 - 4x + 2\right)\sinh(x)}{32(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 - 6*(2*x + 1)*cosh(x) +
 3*(cosh(x)^2 - 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))
```

giac [A] time = 0.14, size = 30, normalized size = 0.79

$$\frac{1}{32}\left(9e^{(4x)} - 6e^{(2x)} + 1\right)e^{(-4x)} - \frac{3}{8}x + \frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 1/32*(9*e^(4*x) - 6*e^(2*x) + 1)*e^(-4*x) - 3/8*x + 1/16*e^(2*x)
```

maple [B] time = 0.09, size = 70, normalized size = 1.84

$$\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{1}{4\tanh\left(\frac{x}{2}\right)-4} + \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8} + \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} - \frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} + \frac{1}{2\tanh\left(\frac{x}{2}\right)+2} - \frac{3\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+coth(x)),x)
```

```
[Out] 1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)+3/8*ln(tanh(1/2*x)-1)+1/2/(tanh(1
/2*x)+1)^4-1/(tanh(1/2*x)+1)^3+1/2/(tanh(1/2*x)+1)-3/8*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 22, normalized size = 0.58

$$-\frac{3}{8}x + \frac{1}{16}e^{(2x)} - \frac{3}{16}e^{(-2x)} + \frac{1}{32}e^{(-4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) + 1/32*e^(-4*x)
```

mupad [B] time = 1.24, size = 22, normalized size = 0.58

$$\frac{e^{2x}}{16} - \frac{3\,e^{-2x}}{16} - \frac{3x}{8} + \frac{e^{-4x}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(coth(x) + 1),x)
```

```
[Out] exp(2*x)/16 - (3*exp(-2*x))/16 - (3*x)/8 + exp(-4*x)/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**2/(coth(x) + 1), x)
```

3.92 $\int \frac{\sinh(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=19

$$\frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(\coth(x)+1)}$$

[Out] 2/3*cosh(x)-1/3*sinh(x)/(1+coth(x))

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3502, 2638}

$$\frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(\coth(x)+1)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(1 + Coth[x]),x]

[Out] (2*Cosh[x])/3 - Sinh[x]/(3*(1 + Coth[x]))

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{1+\coth(x)}\,dx &= -\frac{\sinh(x)}{3(1+\coth(x))}+\frac{2}{3}\int \sinh(x)\,dx\\ &= \frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.05, size = 21, normalized size = 1.11

$$\frac{1}{12}\left(4\sinh^3(x)+9\cosh(x)-\cosh(3x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x]/(1 + Coth[x]),x]

[Out] (9*Cosh[x] - Cosh[3*x] + 4*Sinh[x]^3)/12

fricas [A] time = 0.39, size = 25, normalized size = 1.32

$$\frac{\cosh(x)^2+4\cosh(x)\sinh(x)+\sinh(x)^2+3}{6\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="fricas")
[Out] 1/6*(cosh(x)^2 + 4*cosh(x)*sinh(x) + sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 19, normalized size = 1.00

$$\frac{1}{12}\left(6e^{(2x)} - 1\right)e^{(-3x)} + \frac{1}{4}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="giac")
[Out] 1/12*(6*e^(2*x) - 1)*e^(-3*x) + 1/4*e^x
```

maple [B] time = 0.09, size = 40, normalized size = 2.11

$$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}} + \frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}} + \frac{1}{2\tanh\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+coth(x)),x)
[Out] -1/2/(tanh(1/2*x)-1)-2/3/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/
2*x)+1)
```

maxima [A] time = 0.31, size = 17, normalized size = 0.89

$$\frac{1}{2}e^{(-x)} - \frac{1}{12}e^{(-3x)} + \frac{1}{4}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*e^(-x) - 1/12*e^(-3*x) + 1/4*e^x
```

mupad [B] time = 1.24, size = 17, normalized size = 0.89

$$\frac{\mathrm{e}^{-x}}{2} - \frac{\mathrm{e}^{-3x}}{12} + \frac{\mathrm{e}^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(coth(x) + 1),x)
[Out] exp(-x)/2 - exp(-3*x)/12 + exp(x)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+coth(x)),x)
[Out] Integral(sinh(x)/(coth(x) + 1), x)
```

3.93 $\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\operatorname{csch}(x)}{\coth(x)+1}$$

```
[Out] -csch(x)/(1+coth(x))
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3488}

$$-\frac{\operatorname{csch}(x)}{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(1 + Coth[x]),x]
[Out] -(Csch[x]/(1 + Coth[x]))
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -\frac{\operatorname{csch}(x)}{1+\coth(x)}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 0.70

$$\sinh(x)-\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(1 + Coth[x]),x]
[Out] -Cosh[x] + Sinh[x]
```

fricas [A] time = 0.43, size = 9, normalized size = 0.90

$$-\frac{1}{\cosh(x)+\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="fricas")
[Out] -1/(cosh(x) + sinh(x))
```

giac [A] time = 0.11, size = 6, normalized size = 0.60

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="giac")
[Out] -e^(-x)
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$-\frac{\text{csch}(x)}{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(1+coth(x)),x)
[Out] -csch(x)/(1+coth(x))
```

maxima [A] time = 0.30, size = 6, normalized size = 0.60

$$-e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="maxima")
[Out] -e^(-x)
```

mupad [B] time = 1.20, size = 6, normalized size = 0.60

$$-\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(coth(x) + 1)),x)
[Out] -exp(-x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(1+coth(x)),x)
[Out] Integral(csch(x)/(coth(x) + 1), x)
```

3.94 $\int \frac{\mathrm{csch}^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=7

$$-\log(\coth(x)+1)$$

```
[Out] -ln(1+coth(x))
```

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3487, 31}

$$-\log(\coth(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(1 + Coth[x]),x]
```

```
[Out] -Log[1 + Coth[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{csch}^2(x)}{1+\coth(x)}\,dx &= -\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \coth(x)\right)\\ &= -\log(1+\coth(x))\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\log(\sinh(x))-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(1 + Coth[x]),x]
```

```
[Out] -x + Log[Sinh[x]]
```

fricas [B] time = 0.41, size = 18, normalized size = 2.57

$$-2\,x+\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -2*x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

giac [A] time = 0.13, size = 12, normalized size = 1.71

$$-2\,x + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] -2*x + log(abs(e^(2*x) - 1))
```

maple [A] time = 0.07, size = 8, normalized size = 1.14

$$-\ln(1 + \coth(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(1+coth(x)),x)
[Out] -ln(1+coth(x))
```

maxima [A] time = 0.30, size = 7, normalized size = 1.00

$$-\log(\coth(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] -log(coth(x) + 1)
```

mupad [B] time = 1.18, size = 11, normalized size = 1.57

$$\ln\left(e^{2\,x} - 1\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(coth(x) + 1)),x)
[Out] log(exp(2*x) - 1) - 2*x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{\coth(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(1+coth(x)),x)
[Out] Integral(csch(x)**2/(coth(x) + 1), x)
```

3.95 $\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=8

$$\tanh^{-1}(\cosh(x)) - \operatorname{csch}(x)$$

```
[Out] arctanh(cosh(x))-csch(x)
```

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3501, 3770}

$$\tanh^{-1}(\cosh(x)) - \operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(1 + Coth[x]),x]
```

```
[Out] ArcTanh[Cosh[x]] - Csch[x]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx &= -\operatorname{csch}(x) - \int \operatorname{csch}(x)\,dx \\ &= \tanh^{-1}(\cosh(x)) - \operatorname{csch}(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 14, normalized size = 1.75

$$-\operatorname{csch}(x) - \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(1 + Coth[x]),x]
```

```
[Out] -Csch[x] - Log[Tanh[x/2]]
```

fricas [B] time = 0.39, size = 77, normalized size = 9.62

$$\frac{\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\log\left(\cosh(x) + \sinh(x) + 1\right) - \left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \text{si}\right.}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] ((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) + 1)
 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) -
1) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [B] time = 0.13, size = 26, normalized size = 3.25

$$-\frac{2\, e^{x}}{e^{(2\, x)} - 1} + \log\left(e^{x} + 1\right) - \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*e^x/(e^(2*x) - 1) + log(e^x + 1) - log(abs(e^x - 1))
```

maple [B] time = 0.09, size = 23, normalized size = 2.88

$$\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)} - \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(1+coth(x)),x)
```

```
[Out] 1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

maxima [B] time = 0.30, size = 31, normalized size = 3.88

$$\frac{2\, e^{(-x)}}{e^{(-2\, x)} - 1} + \log\left(e^{(-x)} + 1\right) - \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(e^(-2*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

mupad [B] time = 0.08, size = 29, normalized size = 3.62

$$\ln\left(2\,\mathrm{e}^{x} + 2\right) - \ln\left(2\,\mathrm{e}^{x} - 2\right) - \frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2\,x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(coth(x) + 1)),x)
```

```
[Out] log(2*exp(x) + 2) - log(2*exp(x) - 2) - (2*exp(x))/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}(x)}{\coth(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(coth(x) + 1), x)
```

3.96 $\int \frac{\text{csch}^4(x)}{1+\text{coth}(x)}\,dx$

Optimal. Leaf size=11

$$\coth(x)-\frac{\coth^2(x)}{2}$$

```
[Out] coth(x)-1/2*coth(x)^2
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3487}

$$\coth(x)-\frac{\coth^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(1 + Coth[x]),x]
```

```
[Out] Coth[x] - Coth[x]^2/2
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}^4(x)}{1+\coth(x)}\,dx &= \text{Subst}\left(\int (1-x)\,dx, x, \coth(x)\right)\\ &= \coth(x)-\frac{\coth^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$\coth(x)-\frac{\text{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(1 + Coth[x]),x]
```

```
[Out] Coth[x] - Csch[x]^2/2
```

fricas [B] time = 0.40, size = 55, normalized size = 5.00

$$-\frac{2}{\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-1\right)\sinh(x)^2-2\cosh(x)^2+4\left(\cosh(x)^3-\cosh(x)\right)\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(
x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.13, size = 10, normalized size = 0.91

$$-\frac{2}{\left(e^{(2x)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) - 1)^2
```

maple [B] time = 0.10, size = 32, normalized size = 2.91

$$-\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)}{2}+\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(1+coth(x)),x)
```

```
[Out] -1/8*tanh(1/2*x)^2+1/2*tanh(1/2*x)+1/2/tanh(1/2*x)-1/8/tanh(1/2*x)^2
```

maxima [B] time = 0.30, size = 41, normalized size = 3.73

$$\frac{4\,e^{(-2x)}}{2\,e^{(-2x)}-e^{(-4x)}-1}-\frac{2}{2\,e^{(-2x)}-e^{(-4x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2/(2*e^(-2*x) - e^(-4*x) - 1)
```

mupad [B] time = 1.18, size = 16, normalized size = 1.45

$$-\frac{2}{e^{4x}-2\,e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(coth(x) + 1)),x)
```

```
[Out] -2/(exp(4*x) - 2*exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{4}(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(csch(x)**4/(coth(x) + 1), x)
```

3.97 $\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=155

$$-\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3}+\frac{(3a^2-9ab+8b^2)\log(\coth(x)+1)}{16(a-b)^3}-\frac{\sinh^4(x)(b-a\coth(x))}{4(a^2-b^2)}-\frac{b^5\log(a+b}{(a^2-b}$$

```
[Out] -1/16*(3*a^2+9*a*b+8*b^2)*ln(1-coth(x))/(a+b)^3+1/16*(3*a^2-9*a*b+8*b^2)*ln
(1+coth(x))/(a-b)^3-b^5*ln(a+b*coth(x))/(a^2-b^2)^3-1/8*(4*b^3-a*(7-3*a^2/b
^2)*b^2*coth(x))*sinh(x)^2/(a^2-b^2)^2-1/4*(b-a*coth(x))*sinh(x)^4/(a^2-b^2
)
```

Rubi [A] time = 0.24, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3506, 741, 823, 801}

$$-\frac{b^5\log(a+b\coth(x))}{(a^2-b^2)^3}-\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3}+\frac{(3a^2-9ab+8b^2)\log(\coth(x)+1)}{16(a-b)^3}-\frac{\sinh^4(x)(b-a}{4(a^2-}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Coth[x]])/(16*(a + b)^3) + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Coth[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Coth[x]])/(a^2
 - b^2)^3 - ((4*b^3 - a*(7 - (3*a^2)/b^2)*b^2*Coth[x])*Sinh[x]^2)/(8*(a^2 -
 b^2)^2) - ((b - a*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^3}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} + \frac{b\,\text{Subst}\left(\int \frac{-4+\frac{3a^2}{b^2}+\frac{3ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{4(a^2-b^2)}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} - \frac{b^5\,\text{Subst}\left(\int \frac{-\frac{3a^4-}{}}{}\right.}{}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} - \frac{b^5\,\text{Subst}\left(\int\left(-\frac{(a\cdots}{}\right.\right.}{}$$

$$= -\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(1+\coth(x))}{16(a-b)^3} - \frac{b^5\log(a\cdots}{(a^2\cdots}$$

Mathematica [A] time = 0.29, size = 156, normalized size = 1.01

$$\frac{12a^5x-8a^5\sinh(2x)+a^5\sinh(4x)-40a^3b^2x+24a^3b^2\sinh(2x)-2a^3b^2\sinh(4x)-b\left(a^2-b^2\right)^2\cosh(4x)+\cdots}{32(a-b)^3\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (12*a^5*x - 40*a^3*b^2*x + 60*a*b^4*x + 4*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Cosh[
2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*b^5*Log[b*Cosh[x] + a*Sinh[x]] - 8*a^
5*Sinh[2*x] + 24*a^3*b^2*Sinh[2*x] - 16*a*b^4*Sinh[2*x] + a^5*Sinh[4*x] - 2
*a^3*b^2*Sinh[4*x] + a*b^4*Sinh[4*x])/(32*(a - b)^3*(a + b)^3)
```

fricas [B] time = 0.43, size = 1279, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(2*a^5 - a^4*b
- 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^6 - 4*(2*a^5 - a^4*b - 6
*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
```

```
 + 8*b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^
5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 +
 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 30*(
2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^2 + 4*(3*a
^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(2*a^5 - a^4*b - 6*a^3*b^2
+ 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^3 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x))*sinh(x)^3 + 4*(2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 +
 4*a*b^4 + 3*b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^6 + 2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3
*b^5 - 15*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^4 + 12*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^2)*sinh(x)^2 - 64
*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b
^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x
) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^7 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^5 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^3 + (2*a^5 + a^4*b
 - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^
2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

giac [A] time = 0.13, size = 229, normalized size = 1.48

$$-\frac{b^5\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{\left(3a^2-9ab+8b^2\right)x}{8\left(a^3-3a^2b+3ab^2-b^3\right)}-\frac{\left(18a^2e^{(4x)}-54abe^{(4x)}+48b^2e^{(4x)}-8a^2e^{(2x)}+\right.}{64\left(a^3-3a^2b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6) + 1/8*(3*a^2 - 9*a*b + 8*b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/
64*(18*a^2*e^(4*x) - 54*a*b*e^(4*x) + 48*b^2*e^(4*x) - 8*a^2*e^(2*x) + 20*a
*b*e^(2*x) - 12*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b +
3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x) - 8*a*e^(2*x) - 12*b*e^(2*x))/
(a^2 + 2*a*b + b^2)
```

maple [B] time = 0.13, size = 354, normalized size = 2.28

$$-\frac{b^5\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a-b)^3(a+b)^3}+\frac{16}{(64a+64b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{64}{(128a+128b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*coth(x)),x)
```

```
[Out] -b^5/(a-b)^3/(a+b)^3*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)+16/(64*a+64*b)/(
tanh(1/2*x)-1)^4+64/(128*a+128*b)/(tanh(1/2*x)-1)^3-1/8/(a+b)^2/(tanh(1/2*x
)-1)^2*a-3/8/(a+b)^2/(tanh(1/2*x)-1)^2*b-3/8/(a+b)^2/(tanh(1/2*x)-1)*a-5/8/
(a+b)^2/(tanh(1/2*x)-1)*b-3/8/(a+b)^3*ln(tanh(1/2*x)-1)*a^2-9/8/(a+b)^3*ln(
tanh(1/2*x)-1)*a*b-1/(a+b)^3*ln(tanh(1/2*x)-1)*b^2-16/(64*a-64*b)/(tanh(1/2
*x)+1)^4+64/(128*a-128*b)/(tanh(1/2*x)+1)^3+1/8/(a-b)^2/(tanh(1/2*x)+1)^2*a
-3/8/(a-b)^2/(tanh(1/2*x)+1)^2*b-3/8/(a-b)^2/(tanh(1/2*x)+1)*a+5/8/(a-b)^2/
(tanh(1/2*x)+1)*b+3/8/(a-b)^3*ln(tanh(1/2*x)+1)*a^2-9/8/(a-b)^3*ln(tanh(1/2
*x)+1)*a*b+1/(a-b)^3*ln(tanh(1/2*x)+1)*b^2
```

maxima [A] time = 0.32, size = 166, normalized size = 1.07

$$-\frac{b^5 \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{\left(3a^2 + 9ab + 8b^2\right)x}{8\left(a^3 + 3a^2b + 3ab^2 + b^3\right)} - \frac{\left(4(2a + 3b)e^{(-2x)} - a - b\right)e^{(4x)}}{64\left(a^2 + 2ab + b^2\right)} + \frac{4(2a - 3b)e^{(-}}{64\left(a^2 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1
/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2*a
 + 3*b)*e^(-2*x) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - 3*b)
*e^(-2*x) - (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

mupad [B] time = 1.63, size = 143, normalized size = 0.92

$$\frac{e^{4x}}{64a + 64b} - \frac{e^{-4x}}{64a - 64b} + \frac{e^{-2x}\,(2a - 3b)}{16\,(a-b)^2} - \frac{b^5 \ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{x\left(3a^2 - 9ab + 8b^2\right)}{8\,(a-b)^3} - \frac{e^{2x}\,(2a + 3}{16\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*coth(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (exp(-2*x)*(2*a - 3*b))/
(16*(a - b)^2) - (b^5*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) + (x*(3*a^2 - 9*a*b + 8*b^2))/(8*(a - b)^3) - (exp(2*x
)*(2*a + 3*b))/(16*(a + b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(a + b*coth(x)), x)
```

3.98 $\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=134

$$-\frac{b\sinh^3(x)}{3(a^2-b^2)}+\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a\cosh(x)}{a^2-b^2}-\frac{b^4\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{b^3\sinh(x)}{(a^2-b^2)^2}$$

```
[Out] -b^4*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a*b^2*c
osh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+1/3*a*cosh(x)^3/(a^2-b^2)-b^3*sinh(x
)/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3511, 3486, 2633, 2638, 3509, 206}

$$-\frac{b\sinh^3(x)}{3(a^2-b^2)}-\frac{b^3\sinh(x)}{(a^2-b^2)^2}+\frac{a\cosh^3(x)}{3(a^2-b^2)}+\frac{ab^2\cosh(x)}{(a^2-b^2)^2}-\frac{a\cosh(x)}{a^2-b^2}-\frac{b^4\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b^4*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
)) + (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3
)/(3*(a^2 - b^2)) - (b^3*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 - b
^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \frac{\int (a-b\coth(x))\sinh^3(x)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\sinh(x)}{a+b\coth(x)}\,dx}{a^2-b^2}$$

$$= -\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)} + \frac{b^2\int (a-b\coth(x))\sinh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{b^4\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{a\int \sinh^3(x)\,dx}{a^2-b^2}$$

$$= -\frac{b^3\sinh(x)}{\left(a^2-b^2\right)^2} - \frac{b\sinh^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(ab^2\right)\int \sinh(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\,c\right.}{\left(a^2-b^2\right)^2}$$

$$= -\frac{b^4\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3\left(a^2-b^2\right)} - \frac{b^3\sinh(x)}{\left(a^2-b^2\right)^2} -$$

Mathematica [A] time = 0.83, size = 171, normalized size = 1.28

$$\frac{3b\sqrt{b-a}\left(a^3+a^2b-5ab^2-5b^3\right)\sinh(x) - 3a\sqrt{b-a}\left(3a^3+3a^2b-7ab^2-7b^3\right)\cosh(x) + 24b^4\sqrt{a+b}\,\tan^{-1}}{12(b-a)^{5/2}(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (24*b^4*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] -
3*a*Sqrt[-a + b]*(3*a^3 + 3*a^2*b - 7*a*b^2 - 7*b^3)*Cosh[x] - a*(-a + b)^(
3/2)*(a + b)^2*Cosh[3*x] + 3*b*Sqrt[-a + b]*(a^3 + a^2*b - 5*a*b^2 - 5*b^3)
*Sinh[x] + b*(-a + b)^(3/2)*(a + b)^2*Sinh[3*x])/(12*(-a + b)^(5/2)*(a + b)
^3)
```

fricas [B] time = 0.46, size = 1859, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^
5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4
 + 5*b^5)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 +
```

```
 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 +
6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sin
h(x)^2 + 24*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x
)^2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cos
h(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) +
a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
 a + b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5
 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^3 -
 (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*sinh(x
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b
 - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4
 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)
*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(
x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)
)*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*
cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 - 5
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 -
 a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sinh(x)^2 + 4
8*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*
sinh(x)^3)*sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a +
 b)*sinh(x))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x
)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

giac [A] time = 0.13, size = 163, normalized size = 1.22

$$-\frac{2\,b^4\arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{\left(9\,ae^{(2\,x)}-15\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-24\,abe}{24\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b^4*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) - a + b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^
x - 24*a*b*e^x - 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

maple [A] time = 0.13, size = 197, normalized size = 1.47

$$\frac{2b^4\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a-b)^2\,(a+b)^2\sqrt{-a^2+b^2}}-\frac{32}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3(32a+32b)}-\frac{16}{(32a+32b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{a}{2\,(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*coth(x)),x)
```

```
[Out] 2*b^4/(a-b)^2/(a+b)^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a
^2+b^2)^(1/2))-32/3/(tanh(1/2*x)-1)^3/(32*a+32*b)-16/(32*a+32*b)/(tanh(1/2*
```

```
x)-1)^2+1/2/(a+b)^2/(tanh(1/2*x)-1)*a+1/(a+b)^2/(tanh(1/2*x)-1)*b-16/(32*a-
32*b)/(tanh(1/2*x)+1)^2+32/3/(tanh(1/2*x)+1)^3/(32*a-32*b)-1/2/(a-b)^2/(tan
h(1/2*x)+1)*a+1/(a-b)^2/(tanh(1/2*x)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.86, size = 172, normalized size = 1.28

$$\frac{e^{-3x}}{24a-24b}+\frac{e^{3x}}{24a+24b}-\frac{e^{-x}\,(3a-5b)}{8\,(a-b)^2}-\frac{e^{x}\,(3a+5b)}{8\,(a+b)^2}-\frac{b^4\,\ln\left(2\,a^3\,b-2\,a\,b^3+a^4-b^4+e^{x}\,(a+b)^{7/2}\,\sqrt{a-b}\right.}{(a+b)^{5/2}\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*coth(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(-x)*(3*a - 5*b))/(8
*(a - b)^2) - (exp(x)*(3*a + 5*b))/(8*(a + b)^2) - (b^4*log(2*a^3*b - 2*a*b
^3 + a^4 - b^4 + exp(x)*(a + b)^(7/2)*(a - b)^(1/2)))/((a + b)^(5/2)*(a - b
)^(5/2)) + (b^4*log(2*a*b^3 - 2*a^3*b - a^4 + b^4 + exp(x)*(a + b)^(7/2)*(a
 - b)^(1/2)))/((a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*coth(x)), x)
```

3.99 $\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{\sinh^2(x)(b-a\coth(x))}{2\left(a^2-b^2\right)}-\frac{b^3\log(a+b\coth(x))}{\left(a^2-b^2\right)^2}+\frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2}-\frac{(a-2b)\log(\coth(x)+1)}{4(a-b)^2}$$

```
[Out] 1/4*(a+2*b)*ln(1-coth(x))/(a+b)^2-1/4*(a-2*b)*ln(1+coth(x))/(a-b)^2-b^3*ln(
a+b*coth(x))/(a^2-b^2)^2-1/2*(b-a*coth(x))*sinh(x)^2/(a^2-b^2)
```

Rubi [A] time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3506, 741, 801}

$$-\frac{b^3\log(a+b\coth(x))}{\left(a^2-b^2\right)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2\left(a^2-b^2\right)}+\frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2}-\frac{(a-2b)\log(\coth(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((a + 2*b)*Log[1 - Coth[x]])/(4*(a + b)^2) - ((a - 2*b)*Log[1 + Coth[x]])/(
4*(a - b)^2) - (b^3*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Si
nh[x]^2)/(2*(a^2 - b^2))
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{b\,\text{Subst}\left(\int \frac{-2+\frac{a^2}{b^2}+\frac{ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\coth(x)\right)}{2\left(a^2-b^2\right)}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{b\,\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right)dx, x,\right.}{2\left(a^2-b^2\right)}$$

$$= \frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2} - \frac{(a-2b)\log(1+\coth(x))}{4(a-b)^2} - \frac{b^3\log(a+b\coth(x))}{\left(a^2-b^2\right)^2} - \frac{(b-a\,c}{2}$$

Mathematica [A] time = 0.17, size = 75, normalized size = 0.82

$$\frac{-2a^3x+\left(b^3-a^2b\right)\cosh(2x)+a\left(a^2-b^2\right)\sinh(2x)-4b^3\log(a\sinh(x)+b\cosh(x))+6ab^2x}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (-2*a^3*x + 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] - 4*b^3*Log[b*Cosh[x] +
a*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.41, size = 331, normalized size = 3.60

$$\frac{\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^4+4\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)^3+\left(a^3-a^2b-ab^2+b^3\right)\sinh(x)^4-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 - 3*a*b^
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 - 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(b*cosh(x) + a*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

giac [A] time = 0.12, size = 114, normalized size = 1.24

$$-\frac{b^3\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}-\frac{(a-2b)x}{2\left(a^2-2ab+b^2\right)}+\frac{\left(2ae^{(2x)}-4be^{(2x)}-a+b\right)e^{(-2x)}}{8\left(a^2-2ab+b^2\right)}+\frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*(a - 2*b)*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) - a + b)*
e^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

maple [B] time = 0.13, size = 175, normalized size = 1.90

$$-\frac{b^{3}\ln\left(\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{\left(a-b\right)^{2}\left(a+b\right)^{2}}+\frac{8}{\left(16a+16b\right)\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{16}{\left(32a+32b\right)\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right.\right.}{2\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*coth(x)),x)
```

```
[Out] -b^3/(a-b)^2/(a+b)^2*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)+8/(16*a+16*b)/(t
anh(1/2*x)-1)^2+16/(32*a+32*b)/(tanh(1/2*x)-1)+1/2/(a+b)^2*ln(tanh(1/2*x)-1
)*a+1/(a+b)^2*ln(tanh(1/2*x)-1)*b-8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/(32*a-
32*b)/(tanh(1/2*x)+1)-1/2/(a-b)^2*ln(tanh(1/2*x)+1)*a+1/(a-b)^2*ln(tanh(1/2
*x)+1)*b
```

maxima [A] time = 0.33, size = 83, normalized size = 0.90

$$-\frac{b^{3}\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^{4}-2a^{2}b^{2}+b^{4}}-\frac{(a+2b)x}{2\left(a^{2}+2ab+b^{2}\right)}+\frac{e^{(2x)}}{8(a+b)}-\frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(a + 2*b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

mupad [B] time = 1.47, size = 85, normalized size = 0.92

$$\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}-\frac{b^{3}\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^{4}-2a^{2}b^{2}+b^{4}}-\frac{x\,(a-2b)}{2(a-b)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*coth(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (b^3*log(b - a + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) - (x*(a - 2*b))/(2*(a - b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^{2}(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*coth(x)), x)
```

3.100 $\int \frac{\sinh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=73

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b^2\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] -b^2*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh(
x)/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3511, 3486, 2638, 3509, 206}

$$-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b^2\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2
)) + (a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx = \frac{\int (a-b\coth(x))\sinh(x)\,dx}{a^2-b^2} + \frac{b^2\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{a^2-b^2}$$
$$= -\frac{b\sinh(x)}{a^2-b^2} + \frac{a\int \sinh(x)\,dx}{a^2-b^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\coth(x))\sinh(x)\right)}{a^2-b^2}$$
$$= -\frac{b^2\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a\cosh(x)}{a^2-b^2} - \frac{b\sinh(x)}{a^2-b^2}$$

Mathematica [A] time = 0.46, size = 80, normalized size = 1.10

$$\frac{a\cosh(x)}{a^2-b^2} + b\left(\frac{\sinh(x)}{b^2-a^2} - \frac{2b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\,\sqrt{a+b}}\right)}{(b-a)^{3/2}(a+b)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*Cosh[x])/(a^2 - b^2) + b*((-2*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*S
qrt[a + b])])/((-a + b)^(3/2)*(a + b)^(3/2)) + Sinh[x]/(-a^2 + b^2))
```

fricas [B] time = 0.44, size = 431, normalized size = 5.90

$$\left[\frac{a^3+a^2b-ab^2-b^3+\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^2+2\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)+\left(a^3-a^2b-ab^2\cdots\right.}{2\left(\left(a^4-2a^2b^2+b^4\right)\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cosh
(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(
cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*
a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3
 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2
 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

giac [A] time = 0.14, size = 72, normalized size = 0.99

$$\frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}} + \frac{e^{(-x)}}{2\,(a-b)} + \frac{e^x}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2
)) + 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

maple [A] time = 0.12, size = 93, normalized size = 1.27

$$\frac{2b^2 \arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a+b)(a-b)\sqrt{-a^2+b^2}} + \frac{8}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{8}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*coth(x)),x)
```

```
[Out] 2*b^2/(a+b)/(a-b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b
^2)^(1/2))+8/(8*a-8*b)/(tanh(1/2*x)+1)-8/(8*a+8*b)/(tanh(1/2*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.50, size = 156, normalized size = 2.14

$$\frac{e^x}{2\,a+2\,b}+\frac{e^{-x}}{2\,a-2\,b}-\frac{b^2\,\ln\left(\frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}-\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a-b}}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{b^2\,\ln\left(\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a-b}}+\frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*coth(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) - (b^2*log((2*b^2*exp(x))/(a*b^2 -
 a^2*b - a^3 + b^3) - (2*b^2)/((a + b)^(5/2)*(a - b)^(1/2))))/((a + b)^(3/2
)*(a - b)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(a - b)^(1/2)) + (2*b^2*
exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{a + b\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*coth(x)), x)
```

3.101 $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3509, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Coth[x]),x]
```

```
[Out] -(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\coth(x))\sinh(x)\right)\\ &= -\frac{i\tan^{-1}\left(\frac{(-ib-ia\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.21

$$\frac{2\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqrt
[a + b])
```

fricas [A] time = 0.40, size = 147, normalized size = 3.87

$$\left[\frac{\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{a^2-b^2}\,(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{\sqrt{a^2-b^2}},\ \frac{2\,\sqrt{-a^2+b^2}\,\arctan\left(\frac{\sqrt{-a^2}}{(a+b)\cosh(x)+}\right.}{a^2-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2
*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b))/sqrt(a^2 - b^2), 2*sqrt(-a^
2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/(a^2
- b^2)]
```

giac [A] time = 0.11, size = 35, normalized size = 0.92

$$\frac{2\,\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

maple [A] time = 0.07, size = 39, normalized size = 1.03

$$\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*coth(x)),x)
```

```
[Out] 2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.17, size = 35, normalized size = 0.92

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{b^2-a^2}}{a-b}\right)}{\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*coth(x))),x)
```

```
[Out] -(2*atan((exp(x)*(b^2 - a^2)^(1/2))/(a - b)))/(b^2 - a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*coth(x)), x)
```

3.102 $\int \frac{\text{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\coth(x))}{b}$$

[Out] -ln(a+b*coth(x))/b

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 31}

$$-\frac{\log(a+b\coth(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^2/(a + b*Coth[x]),x]

[Out] -(Log[a + b*Coth[x]]/b)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{csch}^2(x)}{a+b\coth(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{\log(a+b\coth(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 20, normalized size = 1.67

$$\frac{\log(\sinh(x)) - \log(a\sinh(x)+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]

[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b

fricas [B] time = 0.41, size = 43, normalized size = 3.58

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right) - \log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(
x) - sinh(x))))/b
```

giac [B] time = 0.11, size = 46, normalized size = 3.83

$$-\frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab+b^2}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2
*x) - 1))/b
```

maple [A] time = 0.09, size = 13, normalized size = 1.08

$$-\frac{\ln(a+b\coth(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*coth(x)),x)
```

```
[Out] -ln(a+b*coth(x))/b
```

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$-\frac{\log(b\coth(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -log(b*coth(x) + a)/b
```

mupad [B] time = 0.16, size = 51, normalized size = 4.25

$$-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}-a\,\sqrt{-b^2}+b\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*coth(x))),x)
```

```
[Out] -(2*atan((a*exp(2*x)*(-b^2)^(1/2) - a*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

3.103 $\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

```
[Out] a*arctanh(cosh(x))/b^2-csch(x)/b-arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1
/2))*(a^2-b^2)^(1/2)/b^2
```

Rubi [A] time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3510, 3486, 3770, 3509, 206}

$$-\frac{\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\tanh^{-1}(\cosh(x))}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x
])/Sqrt[a^2 - b^2]])/b^2 - Csch[x]/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3510

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = -\frac{\int (a-b\coth(x))\operatorname{csch}(x)\,dx}{b^2} + \frac{(a^2-b^2)\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{b^2}$$

$$= -\frac{\operatorname{csch}(x)}{b} - \frac{a\int \operatorname{csch}(x)\,dx}{b^2} - \frac{(a^2-b^2)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\coth(x))\sinh(x)\right)}{b^2}$$

$$= \frac{a\tanh^{-1}(\cosh(x))}{b^2} - \frac{\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^2} - \frac{\operatorname{csch}(x)}{b}$$

Mathematica [A] time = 0.11, size = 65, normalized size = 1.14

$$-\frac{2\sqrt{b-a}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\,\sqrt{a+b}}\right) + a\log\left(\tanh\left(\frac{x}{2}\right)\right) + b\operatorname{csch}(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((2*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a
 + b])] + b*Csch[x] + a*Log[Tanh[x/2]])/b^2)
```

fricas [B] time = 0.45, size = 384, normalized size = 6.74

$$\left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\log\left(\frac{(a+b)\cosh(x)^2 + 2(a+b)\cosh(x)\sinh(x) + (a+b)\sinh(x)^2 - 2\sqrt{a^2-b^2}}{(a+b)\cosh(x)^2 + 2(a+b)\cosh(x)\sinh(x) + (a+b)\sinh}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2
- b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*
sinh(x) + (a + b)*sinh(x)^2 - a + b)) - 2*b*cosh(x) + (a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (a*cosh(x)^2
+ 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) - 1) - 2*b*s
inh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2), (2*s
qrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(sqrt
(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 2*b*cosh(x) + (a*cosh(x
)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x)
- 1) - 2*b*sinh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)]
```

giac [A] time = 0.12, size = 85, normalized size = 1.49

$$\frac{a\log\left(e^x+1\right)}{b^2} - \frac{a\log\left(|e^x-1|\right)}{b^2} + \frac{2\left(a^2-b^2\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^2} - \frac{2\,e^x}{b\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1))/b^2 + 2*(a^2 - b^2)*arctan((a*e^x
+ b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - 2*e^x/(b*(e^(2*x) - 1))
```

maple [B] time = 0.11, size = 115, normalized size = 2.02

$$\frac{\tanh\left(\frac{x}{2}\right)}{2b}+\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)a^2}{b^2\sqrt{-a^2+b^2}}-\frac{2\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*coth(x)),x)
```

```
[Out] 1/2/b*tanh(1/2*x)+2/b^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(
-a^2+b^2)^(1/2))*a^2-2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-
a^2+b^2)^(1/2))-1/2/b/tanh(1/2*x)-1/b^2*a*ln(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.46, size = 230, normalized size = 4.04

$$\frac{2\,\mathrm{e}^{x}}{b-b\,\mathrm{e}^{2\,x}}-\frac{a\,\ln\left(32\,a\,b^2-64\,a^2\,b+32\,a^3-32\,a^3\,\mathrm{e}^{x}-32\,a\,b^2\,\mathrm{e}^{x}+64\,a^2\,b\,\mathrm{e}^{x}\right)}{b^2}+\frac{a\,\ln\left(32\,a\,b^2-64\,a^2\,b+32\,a^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*coth(x))),x)
```

```
[Out] (2*exp(x))/(b - b*exp(2*x)) - (a*log(32*a*b^2 - 64*a^2*b + 32*a^3 - 32*a^3*
exp(x) - 32*a*b^2*exp(x) + 64*a^2*b*exp(x)))/b^2 + (a*log(32*a*b^2 - 64*a^2
*b + 32*a^3 + 32*a^3*exp(x) + 32*a*b^2*exp(x) - 64*a^2*b*exp(x)))/b^2 + (lo
g(32*a*(a^2 - b^2)^(1/2) - 32*b*(a^2 - b^2)^(1/2) - 32*a^2*exp(x) + 32*b^2*
exp(x))*(a^2 - b^2)^(1/2))/b^2 - (log(32*a*(a^2 - b^2)^(1/2) - 32*b*(a^2 -
b^2)^(1/2) + 32*a^2*exp(x) - 32*b^2*exp(x))*(a^2 - b^2)^(1/2))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*coth(x)), x)
```

3.104 $\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

```
[Out] a*coth(x)/b^2-1/2*coth(x)^2/b-(a^2-b^2)*ln(a+b*coth(x))/b^3
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3506, 697}

$$-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - ((a^2 - b^2)*Log[a + b*Coth[x]])/b^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{b^2}}{a+x}\,dx,x,b\coth(x)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{b^2}-\frac{x}{b^2}+\frac{-a^2+b^2}{b^2(a+x)}\right)dx,x,b\coth(x)\right)}{b}\\ &= \frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}\end{aligned}$$

Mathematica [A] time = 0.17, size = 50, normalized size = 1.25

$$\frac{2\left(a^2-b^2\right)(\log(\sinh(x))-\log(a\sinh(x)+b\cosh(x)))+2ab\coth(x)-b^2\operatorname{csch}^2(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b*Coth[x] - b^2*Csch[x]^2 + 2*(a^2 - b^2)*(Log[Sinh[x]] - Log[b*Cosh[x
] + a*Sinh[x]]))/(2*b^3)
```

fricas [B] time = 0.42, size = 434, normalized size = 10.85

$$\frac{2\left(ab-b^2\right)\cosh(x)^2+4\left(ab-b^2\right)\cosh(x)\sinh(x)+2\left(ab-b^2\right)\sinh(x)^2-2\,ab-\left(\left(a^2-b^2\right)\cosh(x)^4+4\left(a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*si
nh(x)^2 - 2*a*b - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3
+ (a^2 - b^2)*sinh(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x
)^2 - a^2 + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 -
b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) +
 ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*sin
h(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 - a^2 + b^2)*
sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 - b^2)*cosh(x))*sin
h(x))*log(2*sinh(x)/(cosh(x) - sinh(x))))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)*si
nh(x)^3 + b^3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)
*sinh(x)^2 + 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))
```

giac [B] time = 0.12, size = 106, normalized size = 2.65

$$-\frac{\left(a^3+a^2b-ab^2-b^3\right)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab^3+b^4}+\frac{\left(a^2-b^2\right)\log\left(\left|e^{(2x)}-1\right|\right)}{b^3}-\frac{2\left(ab-\left(ab-b^2\right)e^{(2x)}\right)}{b^3\left(e^{(2x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + a^2*b - a*b^2 - b^3)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b^3
 + b^4) + (a^2 - b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2
*x))/(b^3*(e^(2*x) - 1)^2)
```

maple [B] time = 0.12, size = 116, normalized size = 2.90

$$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8b}+\frac{a\tanh\left(\frac{x}{2}\right)}{2b^2}-\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)a^2}{b^3}+\frac{\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{b}-\frac{}{8b\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*coth(x)),x)
```

```
[Out] -1/8/b*tanh(1/2*x)^2+1/2/b^2*a*tanh(1/2*x)-1/b^3*ln(tanh(1/2*x)^2*b+2*a*tan
h(1/2*x)+b)*a^2+1/b*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)-1/8/b/tanh(1/2*x)
^2+1/b^3*ln(tanh(1/2*x))*a^2-1/b*ln(tanh(1/2*x))+1/2/b^2*a/tanh(1/2*x)
```

maxima [B] time = 0.33, size = 110, normalized size = 2.75

$$\frac{2\left((a+b)e^{(-2x)}-a\right)}{2\,b^2e^{(-2x)}-b^2e^{(-4x)}-b^2}-\frac{\left(a^2-b^2\right)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{b^3}+\frac{\left(a^2-b^2\right)\log\left(e^{(-x)}+1\right)}{b^3}+\frac{\left(a^2-b^2\right)\log\left(e^{(-x)}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2*((a + b)*e^(-2*x) - a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) - (a^2 - b^2
)*log(-(a - b)*e^(-2*x) + a + b)/b^3 + (a^2 - b^2)*log(e^(-x) + 1)/b^3 + (a
^2 - b^2)*log(e^(-x) - 1)/b^3
```

mupad [B] time = 1.44, size = 88, normalized size = 2.20

$$\frac{2\,\left(a-b\right)}{b^2\,\left({\mathrm{e}}^{2\,x}-1\right)}-\frac{2}{b\,\left({\mathrm{e}}^{4\,x}-2\,{\mathrm{e}}^{2\,x}+1\right)}-\frac{\ln\left(b-a+a\,{\mathrm{e}}^{2\,x}+b\,{\mathrm{e}}^{2\,x}\right)\,\left(a+b\right)\,\left(a-b\right)}{b^3}+\frac{\ln\left({\mathrm{e}}^{2\,x}-1\right)\,\left(a+b\right)\,\left(a-b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*coth(x))),x)
```

```
[Out] (2*(a - b))/(b^2*(exp(2*x) - 1)) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) - (log
(b - a + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/b^3 + (log(exp(2*x) - 1)
*(a + b)*(a - b))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{4}{\left(x \right)}}{a + b \coth{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*coth(x)), x)
```

3.105 $\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{x}{16}-\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}+\frac{5}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

```
[Out] 1/16*x+1/32/(1-coth(x))^2-1/8/(1-coth(x))-1/24/(1+coth(x))^3+5/32/(1+coth(x
))^2-3/16/(1+coth(x))
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3516, 848, 88, 207}

$$\frac{x}{16}-\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)}+\frac{1}{32(1-\coth(x))^2}+\frac{5}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(1 + Coth[x]),x]
```

```
[Out] x/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3
) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = -\text{Subst}\left(\int \frac{x^4}{(1+x)\left(-1+x^2\right)^3}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \frac{x^4}{(-1+x)^3(1+x)^4}\,dx, x, \coth(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}-\frac{1}{8(1+x)^4}+\frac{5}{16(1+x)^3}-\frac{3}{16(1+x)^2}+\frac{1}{16\left(-1+x^2\right.}\right.\right.$$
$$= \frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x)}$$
$$= \frac{x}{16}+\frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\text{co}}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.70

$$\frac{1}{192}(12x+3\sinh(2x)-3\sinh(4x)-\sinh(6x)+15\cosh(2x)+6\cosh(4x)+\cosh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (12*x + 15*Cosh[2*x] + 6*Cosh[4*x] + Cosh[6*x] + 3*Sinh[2*x] - 3*Sinh[4*x]
- Sinh[6*x])/192
```

fricas [B] time = 0.41, size = 92, normalized size = 1.53

$$\frac{5\cosh(x)^5+25\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\cosh(x)^2+9\right)\sinh(x)^3+27\cosh(x)^3+\left(50\cosh(x)^3+81\right.}{384\,(\cosh(x)+\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + (10*cosh(x)^2 + 9)*
sinh(x)^3 + 27*cosh(x)^3 + (50*cosh(x)^3 + 81*cosh(x))*sinh(x)^2 + 12*(2*x
+ 1)*cosh(x) + (5*cosh(x)^4 + 27*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x) +
 sinh(x))
```

giac [A] time = 0.12, size = 42, normalized size = 0.70

$$-\frac{1}{384}\left(22\,e^{(6\,x)}-12\,e^{(4\,x)}-9\,e^{(2\,x)}-2\right)e^{(-6\,x)}+\frac{1}{16}\,x+\frac{1}{128}\,e^{(4\,x)}+\frac{3}{64}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/384*(22*e^(6*x) - 12*e^(4*x) - 9*e^(2*x) - 2)*e^(-6*x) + 1/16*x + 1/128*
e^(4*x) + 3/64*e^(2*x)
```

maple [B] time = 0.10, size = 118, normalized size = 1.97

$$\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1+coth(x)),x)
```

```
[Out] 1/8/(tanh(1/2*x)-1)^4+1/4/(tanh(1/2*x)-1)^3+3/8/(tanh(1/2*x)-1)^2+1/4/(tanh
(1/2*x)-1)-1/16*ln(tanh(1/2*x)-1)+1/3/(tanh(1/2*x)+1)^6-1/(tanh(1/2*x)+1)^5
+13/8/(tanh(1/2*x)+1)^4-19/12/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-3/8/(ta
nh(1/2*x)+1)+1/16*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.34, size = 36, normalized size = 0.60

$$\frac{1}{128}\left(6\,e^{(-2\,x)}+1\right)e^{(4\,x)}+\frac{1}{16}\,x+\frac{1}{32}\,e^{(-2\,x)}+\frac{3}{128}\,e^{(-4\,x)}+\frac{1}{192}\,e^{(-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/128*(6*e^(-2*x) + 1)*e^(4*x) + 1/16*x + 1/32*e^(-2*x) + 3/128*e^(-4*x) +
1/192*e^(-6*x)
```

mupad [B] time = 1.42, size = 34, normalized size = 0.57

$$\frac{x}{16}+\frac{e^{-2\,x}}{32}+\frac{3\,e^{2\,x}}{64}+\frac{3\,e^{-4\,x}}{128}+\frac{e^{4\,x}}{128}+\frac{e^{-6\,x}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(coth(x) + 1),x)
```

```
[Out] x/16 + exp(-2*x)/32 + (3*exp(2*x))/64 + (3*exp(-4*x))/128 + exp(4*x)/128 +
exp(-6*x)/192
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{4}(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)**4/(coth(x) + 1), x)
```

3.106 $\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{\sinh^5(x)}{5}-\frac{\sinh^3(x)}{3}+\frac{\cosh^5(x)}{5}$$

```
[Out] 1/5*cosh(x)^5-1/3*sinh(x)^3-1/5*sinh(x)^5
```

Rubi [A] time = 0.18, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2565, 30, 2564, 14}

$$-\frac{\sinh^5(x)}{5}-\frac{\sinh^3(x)}{3}+\frac{\cosh^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 + Coth[x]),x]
```

```
[Out] Cosh[x]^5/5 - Sinh[x]^3/3 - Sinh[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
```

```
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \cosh^3(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\
&= i\int \left(-i\cosh^4(x)\sinh(x)+i\cosh^3(x)\sinh^2(x)\right)\,dx\\
&= \int \cosh^4(x)\sinh(x)\,dx-\int \cosh^3(x)\sinh^2(x)\,dx\\
&= -\left(i\,\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,i\sinh(x)\right)\right)+\text{Subst}\left(\int x^4\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^5(x)}{5}-i\,\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,i\sinh(x)\right)\\
&= \frac{\cosh^5(x)}{5}-\frac{\sinh^3(x)}{3}-\frac{\sinh^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.36

$$\frac{1}{120}(\cosh(x)-\sinh(x))(10\sinh(2x)+\sinh(4x)+20\cosh(2x)+4\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 + Coth[x]),x]
```

```
[Out] ((Cosh[x] - Sinh[x])*(20*Cosh[2*x] + 4*Cosh[4*x] + 10*Sinh[2*x] + Sinh[4*x]
))/120
```

fricas [B] time = 0.37, size = 56, normalized size = 2.24

$$\frac{\cosh(x)^4+\cosh(x)\sinh(x)^3+\sinh(x)^4+\left(6\cosh(x)^2+5\right)\sinh(x)^2+5\cosh(x)^2+\left(\cosh(x)^3+5\cosh(x)\right)}{30\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 5)*sinh(x)
^2 + 5*cosh(x)^2 + (cosh(x)^3 + 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))
```

giac [A] time = 0.11, size = 25, normalized size = 1.00

$$\frac{1}{240}\left(10\,e^{(2\,x)}+3\right)e^{(-5\,x)}+\frac{1}{48}\,e^{(3\,x)}+\frac{1}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="giac")
```

[Out] 1/240*(10*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/8*e^x

maple [B] time = 0.09, size = 82, normalized size = 3.28

$$-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^3/(1+coth(x)),x)

[Out] -1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-3/8/(tanh(1/2*x)-1)-1/(tanh(1/2*x)+1)^4+2/5/(tanh(1/2*x)+1)^5+4/3/(tanh(1/2*x)+1)^3-1/(tanh(1/2*x)+1)^2+3/8/(tanh(1/2*x)+1)

maxima [A] time = 0.31, size = 27, normalized size = 1.08

$$\frac{1}{48}\left(6\,e^{(-2\,x)}+1\right)e^{(3\,x)}+\frac{1}{24}\,e^{(-3\,x)}+\frac{1}{80}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="maxima")

[Out] 1/48*(6*e^(-2*x) + 1)*e^(3*x) + 1/24*e^(-3*x) + 1/80*e^(-5*x)

mupad [B] time = 1.32, size = 23, normalized size = 0.92

$$\frac{\mathrm{e}^{-3\,x}}{24}+\frac{\mathrm{e}^{3\,x}}{48}+\frac{\mathrm{e}^{-5\,x}}{80}+\frac{\mathrm{e}^{x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^3/(coth(x) + 1),x)

[Out] exp(-3*x)/24 + exp(3*x)/48 + exp(-5*x)/80 + exp(x)/8

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^{3}(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**3/(1+coth(x)),x)

[Out] Integral(cosh(x)**3/(coth(x) + 1), x)

3.107 $\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{8}-\frac{1}{8(1-\coth(x))}-\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

```
[Out] 1/8*x-1/8/(1-coth(x))+1/8/(1+coth(x))^2-1/4/(1+coth(x))
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3516, 848, 88, 207}

$$\frac{x}{8}-\frac{1}{8(1-\coth(x))}-\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + Coth[x]),x]
```

```
[Out] x/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{x^2}{(1+x)\left(-1+x^2\right)^2}\,dx, x, \coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \frac{x^2}{(-1+x)^2(1+x)^3}\,dx, x, \coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{8\left(-1+x^2\right)}\right)dx, x, \coth(x)\right)\\
&= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \coth(x)\right)\\
&= \frac{x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.63

$$\frac{1}{32}(4x-\sinh(4x)+4\cosh(2x)+\cosh(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (4*x + 4*Cosh[2*x] + Cosh[4*x] - Sinh[4*x])/32
```

fricas [A] time = 0.40, size = 51, normalized size = 1.34

$$\frac{3\cosh(x)^3+9\cosh(x)\sinh(x)^2+\sinh(x)^3+2\,(2\,x+1)\cosh(x)+\left(3\cosh(x)^2+4\,x-2\right)\sinh(x)}{32\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 + 2*(2*x + 1)*cosh(x) +
 (3*cosh(x)^2 + 4*x - 2)*sinh(x))/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 30, normalized size = 0.79

$$-\frac{1}{32}\left(3\,e^{(4\,x)}-2\,e^{(2\,x)}-1\right)e^{(-4\,x)}+\frac{1}{8}\,x+\frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -1/32*(3*e^(4*x) - 2*e^(2*x) - 1)*e^(-4*x) + 1/8*x + 1/16*e^(2*x)
```

maple [B] time = 0.09, size = 78, normalized size = 2.05

$$\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{}{2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+coth(x)),x)
```

```
[Out] 1/4/(tanh(1/2*x)-1)^2+1/4/(tanh(1/2*x)-1)-1/8*ln(tanh(1/2*x)-1)+1/2/(tanh(1
/2*x)+1)^4-1/(tanh(1/2*x)+1)^3+1/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*x)+1)+1/8*
ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.30, size = 22, normalized size = 0.58

$$\frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)} + \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{32}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] 1/8*x + 1/16*e^(2*x) + 1/16*e^(-2*x) + 1/32*e^(-4*x)
```

mupad [B] time = 0.11, size = 22, normalized size = 0.58

$$\frac{x}{8} + \frac{\mathrm{e}^{-2\,x}}{16} + \frac{\mathrm{e}^{2\,x}}{16} + \frac{\mathrm{e}^{-4\,x}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(coth(x) + 1),x)
[Out] x/8 + exp(-2*x)/16 + exp(2*x)/16 + exp(-4*x)/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{\coth(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+coth(x)),x)
[Out] Integral(cosh(x)**2/(coth(x) + 1), x)
```

3.108 $\int \frac{\cosh(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}$$

```
[Out] 1/3*cosh(x)^3-1/3*sinh(x)^3
```

Rubi [A] time = 0.11, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3518, 3108, 3107, 2565, 30, 2564}

$$\frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 + Coth[x]),x]
```

```
[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
```

```
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\ &= -\int \cosh(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\ &= i\int \left(-i\cosh^2(x)\sinh(x)+i\cosh(x)\sinh^2(x)\right)\,dx\\ &= \int \cosh^2(x)\sinh(x)\,dx-\int \cosh(x)\sinh^2(x)\,dx\\ &= -\left(i\,\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\right)+\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)\\ &= \frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.12

$$\frac{1}{12}\left(-4\sinh^3(x)+3\cosh(x)+\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 + Coth[x]),x]
[Out] (3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12
```

fricas [A] time = 0.41, size = 23, normalized size = 1.35

$$\frac{\cosh(x)^2+\cosh(x)\sinh(x)+\sinh(x)^2}{3\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="fricas")
[Out] 1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 11, normalized size = 0.65

$$\frac{1}{12}\,e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="giac")
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

maple [B] time = 0.09, size = 42, normalized size = 2.47

$$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1+coth(x)),x)
```

[Out] -1/2/(tanh(1/2*x)-1)+2/3/(tanh(1/2*x)+1)^3-1/(tanh(1/2*x)+1)^2+1/2/(tanh(1/2*x)+1)

maxima [A] time = 0.30, size = 11, normalized size = 0.65

$$\frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

mupad [B] time = 1.19, size = 11, normalized size = 0.65

$$\frac{\mathrm{e}^{-3\,x}}{12} + \frac{\mathrm{e}^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(coth(x) + 1),x)
```

```
[Out] exp(-3*x)/12 + exp(x)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\coth(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+coth(x)),x)
```

```
[Out] Integral(cosh(x)/(coth(x) + 1), x)
```

3.109 $\int \frac{\text{sech}(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=10

$$-\sinh(x)+\cosh(x)+\tan^{-1}(\sinh(x))$$

```
[Out] arctan(sinh(x))+cosh(x)-sinh(x)
```

Rubi [A] time = 0.11, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3518, 3108, 3107, 2638, 2592, 321, 203}

$$-\sinh(x)+\cosh(x)+\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(1 + Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]] + Cosh[x] - Sinh[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
```

```
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\ &= -\int(-\cosh(x)+\sinh(x))\tanh(x)\,dx\\ &= i\int(-i\sinh(x)+i\sinh(x)\tanh(x))\,dx\\ &= \int \sinh(x)\,dx-\int \sinh(x)\tanh(x)\,dx\\ &= \cosh(x)-\operatorname{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\sinh(x)\right)\\ &= \cosh(x)-\sinh(x)+\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(x)\right)\\ &= \tan^{-1}(\sinh(x))+\cosh(x)-\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.60

$$-\sinh(x)+\cosh(x)+2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(1 + Coth[x]),x]
[Out] 2*ArcTan[Tanh[x/2]] + Cosh[x] - Sinh[x]
```

fricas [B] time = 0.39, size = 23, normalized size = 2.30

$$\frac{2(\cosh(x)+\sinh(x))\arctan(\cosh(x)+\sinh(x))+1}{\cosh(x)+\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="fricas")
[Out] (2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))
```

giac [A] time = 0.11, size = 10, normalized size = 1.00

$$2\arctan(e^x)+e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="giac")
[Out] 2*arctan(e^x) + e^(-x)
```

maple [A]   time = 0.10, size = 19, normalized size = 1.90

$$\frac{2}{\tanh\left(\frac{x}{2}\right)+1}+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(1+coth(x)),x)
[Out] 2/(tanh(1/2*x)+1)+2*arctan(tanh(1/2*x))
```

maxima [A]   time = 0.50, size = 12, normalized size = 1.20

$$-2\arctan\left(e^{(-x)}\right)+e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x)) + e^(-x)
```

mupad [B]   time = 1.28, size = 10, normalized size = 1.00

$$e^{-x}+2\,\mathrm{atan}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(coth(x) + 1)),x)
[Out] exp(-x) + 2*atan(exp(x))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(1+coth(x)),x)
[Out] Integral(sech(x)/(coth(x) + 1), x)
```

3.110 $\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=15

$$\tanh(x) - \log(\tanh(x)) - \log(\coth(x) + 1)$$

```
[Out] -ln(1+coth(x))-ln(tanh(x))+tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3516, 44}

$$\tanh(x) - \log(\tanh(x)) - \log(\coth(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(1 + Coth[x]),x]
```

```
[Out] -Log[1 + Coth[x]] - Log[Tanh[x]] + Tanh[x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x}\right) dx, x, \coth(x)\right)\\
&= -\log(1+\coth(x)) - \log(\tanh(x)) + \tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 9, normalized size = 0.60

$$-x + \tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(1 + Coth[x]),x]
```

```
[Out] -x + Log[Cosh[x]] + Tanh[x]
```

fricas [B] time = 0.41, size = 78, normalized size = 5.20

$$-\frac{2\,x\cosh(x)^2 + 4\,x\cosh(x)\sinh(x) + 2\,x\sinh(x)^2 - \left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log\left(\frac{2\cosh}{\cosh(x)-}\right.}{\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*x + 2)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [A] time = 0.11, size = 27, normalized size = 1.80

$$-2\,x - \frac{e^{(2\,x)} + 3}{e^{(2\,x)} + 1} + \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2*x - (e^(2*x) + 3)/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

maple [B] time = 0.11, size = 36, normalized size = 2.40

$$-2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)+1}+\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(1+coth(x)),x)
```

```
[Out] -2*ln(tanh(1/2*x)+1)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)+ln(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.40, size = 18, normalized size = 1.20

$$\frac{2}{e^{(-2\,x)} + 1} + \log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) + 1) + log(e^(-2*x) + 1)
```

mupad [B] time = 1.19, size = 21, normalized size = 1.40

$$\ln\left(\mathrm{e}^{2\,x}+1\right)-2\,x-\frac{2}{\mathrm{e}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(coth(x) + 1)),x)
```

```
[Out] log(exp(2*x) + 1) - 2*x - 2/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{\coth(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(coth(x) + 1), x)
```

3.111 $\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=20

$$-\operatorname{sech}(x)-\frac{1}{2}\tan^{-1}(\sinh(x))+\frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

```
[Out] -1/2*arctan(sinh(x))-sech(x)+1/2*sech(x)*tanh(x)
```

Rubi [A] time = 0.17, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3518, 3108, 3107, 2606, 8, 2611, 3770}

$$-\operatorname{sech}(x)-\frac{1}{2}\tan^{-1}(\sinh(x))+\frac{1}{2}\tanh(x)\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(1 + Coth[x]),x]
```

```
[Out] -ArcTan[Sinh[x]]/2 - Sech[x] + (Sech[x]*Tanh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3107

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3108

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
```

```
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx &= -\left(i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \operatorname{sech}^2(x)(-\cosh(x)+\sinh(x))\tanh(x)\,dx\\
&= i\int \left(-i\operatorname{sech}(x)\tanh(x)+i\operatorname{sech}(x)\tanh^2(x)\right)\,dx\\
&= \int \operatorname{sech}(x)\tanh(x)\,dx-\int \operatorname{sech}(x)\tanh^2(x)\,dx\\
&= \frac{1}{2}\operatorname{sech}(x)\tanh(x)-\frac{1}{2}\int \operatorname{sech}(x)\,dx-\text{Subst}\left(\int 1\,dx,x,\operatorname{sech}(x)\right)\\
&= -\frac{1}{2}\tan^{-1}(\sinh(x))-\operatorname{sech}(x)+\frac{1}{2}\operatorname{sech}(x)\tanh(x)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.00

$$\frac{1}{2}(\tanh(x)-2)\operatorname{sech}(x)-\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(1 + Coth[x]),x]
```

```
[Out] -ArcTan[Tanh[x/2]] + (Sech[x]*(-2 + Tanh[x]))/2
```

fricas [B] time = 0.38, size = 140, normalized size = 7.00

$$-\frac{\cosh(x)^3+3\cosh(x)\sinh(x)^2+\sinh(x)^3+\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+1\right.\right.}{\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\text{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x
)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(cosh(x)^2 + 1)*s
inh(x) + 3*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [A] time = 0.12, size = 25, normalized size = 1.25

$$-\frac{e^{(3x)}+3e^{x}}{\left(e^{(2x)}+1\right)^2}-\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -(e^(3*x) + 3*e^x)/(e^(2*x) + 1)^2 - arctan(e^x)
```

maple [B] time = 0.11, size = 45, normalized size = 2.25

$$\frac{-\left(\tanh^3\left(\frac{x}{2}\right)\right)-2\left(\tanh^2\left(\frac{x}{2}\right)\right)+\tanh\left(\frac{x}{2}\right)-2}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(1+coth(x)),x)
[Out] 4*(-1/4*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2+1/4*tanh(1/2*x)-1/2)/(tanh(1/2*x)^2
+1)^2-arctan(tanh(1/2*x))
```

maxima [B] time = 0.42, size = 33, normalized size = 1.65

$$-\frac{e^{(-x)}+3\,e^{(-3\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}+\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="maxima")
[Out] -(e^(-x) + 3*e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + arctan(e^(-x))
```

mupad [B] time = 1.25, size = 22, normalized size = 1.10

$$-\mathrm{atan}\left(\mathrm{e}^x\right)-\frac{1}{2\cosh(x)}-\frac{\mathrm{e}^{-x}}{2\cosh(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(coth(x) + 1)),x)
[Out] - atan(exp(x)) - 1/(2*cosh(x)) - exp(-x)/(2*cosh(x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^3(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(1+coth(x)),x)
[Out] Integral(sech(x)**3/(coth(x) + 1), x)
```

3.112 $\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}$$

```
[Out] 1/2*tanh(x)^2-1/3*tanh(x)^3
```

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3516, 848, 43}

$$\frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(1 + Coth[x]),x]
```

```
[Out] Tanh[x]^2/2 - Tanh[x]^3/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{-1+x^2}{x^4(1+x)}\,dx,x,\coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \frac{-1+x}{x^4}\,dx,x,\coth(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x^3}\right)\,dx,x,\coth(x)\right)\\
&= \frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 17, normalized size = 1.00

$$\frac{1}{6}\left(-2\tanh^3(x)-3\operatorname{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-3*Sech[x]^2 - 2*Tanh[x]^3)/6
```

fricas [B] time = 0.37, size = 84, normalized size = 4.94

$$-\frac{4\left(\cosh(x)+2\sinh(x)\right)}{3\left(\cosh(x)^5+5\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\cosh(x)^2+3\right)\sinh(x)^3+3\cosh(x)^3+\left(10\cosh(x)^3+9\,\mathrm{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(cosh(x) + 2*sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

giac [A] time = 0.11, size = 18, normalized size = 1.06

$$-\frac{2\left(3\,e^{(2\,x)}-1\right)}{3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) - 1)/(e^(2*x) + 1)^3
```

maple [B] time = 0.10, size = 38, normalized size = 2.24

$$-\frac{4\left(-\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{2}+\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(1+coth(x)),x)
```

```
[Out] -4*(-1/2*tanh(1/2*x)^4+2/3*tanh(1/2*x)^3-1/2*tanh(1/2*x)^2)/(tanh(1/2*x)^2+
1)^3
```

maxima [B] time = 0.30, size = 75, normalized size = 4.41

$$-\frac{2\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}-\frac{4\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}-\frac{2}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 4*e^(-4*x)/(3*e^(-2*
x) + 3*e^(-4*x) + e^(-6*x) + 1) - 2/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
 1)
```

mupad [B] time = 0.07, size = 18, normalized size = 1.06

$$-\frac{2\left(3\,\mathrm{e}^{2\,x}-1\right)}{3\left(\mathrm{e}^{2\,x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(coth(x) + 1)),x)
```

```
[Out] -(2*(3*exp(2*x) - 1))/(3*(exp(2*x) + 1)^3)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**4/(coth(x) + 1), x)
```

3.113 $\int \sqrt{1+\coth(x)}\, \operatorname{sech}^2(x)\, dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\sqrt{\coth(x)+1}\right)+\tanh(x)\sqrt{\coth(x)+1}$$

```
[Out] arctanh((1+coth(x))^(1/2))+(1+coth(x))^(1/2)*tanh(x)
```

Rubi [A] time = 0.05, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3516, 47, 63, 207}

$$\tanh^{-1}\left(\sqrt{\coth(x)+1}\right)+\tanh(x)\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+\coth(x)}\, \operatorname{sech}^2(x)\, dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\, dx, x, \coth(x)\right)\\
&= \sqrt{1+\coth(x)}\, \tanh(x)-\frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\, dx, x, \coth(x)\right)\\
&= \sqrt{1+\coth(x)}\, \tanh(x)-\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\, dx, x, \sqrt{1+\coth(x)}\right)\\
&= \tanh^{-1}\left(\sqrt{1+\coth(x)}\right)+\sqrt{1+\coth(x)}\, \tanh(x)
\end{aligned}$$

Mathematica [C] time = 5.27, size = 160, normalized size = 7.62

$$\frac{1}{2}\sqrt{\coth(x)+1}\left(2\tanh(x)+\frac{(1-i)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}}+\frac{\sinh\left(\frac{x}{2}\right)\left(4\tanh^{-1}\left(\sqrt{\tanh\left(\frac{x}{2}\right)}\right)+\sqrt{2}\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

```
[Out] (Sqrt[1 + Coth[x]]*(((1 - I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]])/Sqr
t[I*(1 + Coth[x])] + ((4*ArcTanh[Sqrt[Tanh[x/2]]] + Sqrt[2]*(Log[1 - Sqrt[2
]*Sqrt[Tanh[x/2]] + Tanh[x/2]] - Log[1 + Sqrt[2]*Sqrt[Tanh[x/2]] + Tanh[x/2
]]))*Sinh[x/2]*(-Cosh[x/2] + Sinh[x/2]))/Sqrt[Tanh[x/2]] + 2*Tanh[x]))/2
```

fricas [B] time = 0.43, size = 231, normalized size = 11.00

$$\frac{4\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}+\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{2\sqrt{2}(\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((2*sqrt(2)*
(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*c
osh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 1)/(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(-(2
*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x
))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 1)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [B] time = 0.13, size = 149, normalized size = 7.10

$$-\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2\sqrt{2}-\log\left(-\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)+\sqrt{2}\log\left(\frac{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^2-2\sqrt{2}+3}{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^2+2\sqrt{2}+3}\right)-\frac{8\left(3\left(\sqrt{e^{(2x)}-1}-\cdots\right.\right.}{\left(\sqrt{e^{(2x)}-1}-e^{x}\right)^4+6\left(\sqrt{e\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*(2*sqrt(2) - log(-(sqrt(2) - 1)/(sqrt(2) + 1))) + sqr
t(2)*log(((sqrt(e^(2*x) - 1) - e^x)^2 - 2*sqrt(2) + 3)/((sqrt(e^(2*x) - 1)
- e^x)^2 + 2*sqrt(2) + 3)) - 8*(3*(sqrt(e^(2*x) - 1) - e^x)^2 + 1)/((sqrt(e
^(2*x) - 1) - e^x)^4 + 6*(sqrt(e^(2*x) - 1) - e^x)^2 + 1))*sgn(e^(2*x) - 1)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(x)^2\sqrt{1+\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2*(1+coth(x))^(1/2),x)
```

```
[Out] int(sech(x)^2*(1+coth(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth(x)+1}\,\mathrm{sech}(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(coth(x) + 1)*sech(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\coth(x)+1}}{\cosh(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((coth(x) + 1)^(1/2)/cosh(x)^2,x)
[Out] int((coth(x) + 1)^(1/2)/cosh(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth(x)+1}\,\mathrm{sech}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2*(1+coth(x))**(1/2),x)
[Out] Integral(sqrt(coth(x) + 1)*sech(x)**2, x)
```

3.114 $\int \frac{\cosh^4(x)}{a+b\coth(x)} dx$

Optimal. Leaf size=147

$$-\frac{\sinh^4(x)(b-a\coth(x))}{4\left(a^2-b^2\right)}-\frac{\sinh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)}{8\left(a^2-b^2\right)^2}-\frac{a^4b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3}-\frac{a(3a+b)\text{l}}{16}$$

```
[Out] -1/16*a*(3*a+b)*ln(1-coth(x))/(a+b)^3+1/16*a*(3*a-b)*ln(1+coth(x))/(a-b)^3-
a^4*b*ln(a+b*coth(x))/(a^2-b^2)^3-1/8*(4*b*(2*a^2-b^2)-a*(5*a^2-b^2)*coth(x
))*sinh(x)^2/(a^2-b^2)^2-1/4*(b-a*coth(x))*sinh(x)^4/(a^2-b^2)
```

Rubi [A] time = 0.34, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3516, 1647, 801}

$$-\frac{a^4b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3}-\frac{\sinh^4(x)(b-a\coth(x))}{4\left(a^2-b^2\right)}-\frac{\sinh^2(x)\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)}{8\left(a^2-b^2\right)^2}-\frac{a(3a+b)\text{l}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -(a*(3*a + b)*Log[1 - Coth[x]])/(16*(a + b)^3) + (a*(3*a - b)*Log[1 + Coth[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Coth[x]])/(a^2 - b^2)^3 - ((4*b*(2*a
^2 - b^2) - a*(5*a^2 - b^2)*Coth[x])*Sinh[x]^2)/(8*(a^2 - b^2)^2) - ((b - a
*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^4}{(a+x)\left(-b^2+x^2\right)^3}\,dx, x, b\coth(x)\right)\right)$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2-b^2}-\frac{3ab^4x}{a^2-b^2}+4b^2x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\coth(x)\right)}{4b}$$

$$= -\frac{\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{a^2b \ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{\left(4b\left(2a^2-b^2\right)-a\left(5a^2-b^2\right)\coth(x)\right)\sinh^2(x)}{8\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \left(- \ldots\right.\right.}{\ldots}$$

$$= -\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\coth(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3} - \frac{(4b\,(\ldots}{\ldots}$$

Mathematica [A] time = 0.57, size = 144, normalized size = 0.98

$$\frac{12a^5x + a^5\sinh(4x) - 32a^4b\log(a\sinh(x)+b\cosh(x)) + 24a^3b^2x - 2a^3b^2\sinh(4x) - b\left(a^2-b^2\right)^2\cosh(4x) - 4b \ldots}{32(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (12*a^5*x + 24*a^3*b^2*x - 4*a*b^4*x - 4*b*(3*a^4 - 4*a^2*b^2 + b^4)*Cosh[2
*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*a^4*b*Log[b*Cosh[x] + a*Sinh[x]] + 8*a
^3*(a^2 - b^2)*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh
[4*x])/(32*(a - b)^3*(a + b)^3)
```

fricas [B] time = 0.43, size = 1229, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(2*a^5 - 3*a^4*
b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^6 + 4*(2*a^5 - 3*a^4*b - 2*a^3*b^2
 + 4*a^2*b^3 - b^5 + 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^4
+ 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(2
*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 30*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*
a^2*b^3 - b^5)*cosh(x)^2 + 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x)*sinh(
x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 +
 10*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^3 + 4*(3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x))*sinh(x)^3 - 4*(2*a^5 + 3*a^4*b - 2*
```

```
a^3*b^2 - 4*a^2*b^3 + b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^6 - 2*a^5 - 3*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 -
b^5 + 15*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^4 + 12*(3*
a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^4*b*cosh(
x)^4 + 4*a^4*b*cosh(x)^3*sinh(x) + 6*a^4*b*cosh(x)^2*sinh(x)^2 + 4*a^4*b*co
sh(x)*sinh(x)^3 + a^4*b*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) -
 sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^7 + 3*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^5 + 4*(3*a^5
 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^3 - (2*a^5 + 3*a^4*b - 2*a^3*b^2
- 4*a^2*b^3 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*si
nh(x)^4)
```

giac [A] time = 0.12, size = 216, normalized size = 1.47

$$-\frac{a^4 b \log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{\left(3a^2 - ab\right)x}{8\left(a^3 - 3a^2b + 3ab^2 - b^3\right)} - \frac{\left(18a^2e^{(4x)} - 6abe^{(4x)} + 8a^2e^{(2x)} - 12abe^{(2x}\right.}{64\left(a^3 - 3a^2b + 3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) + 1/8*(3*a^2 - a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/64*(18*a
^2*e^(4*x) - 6*a*b*e^(4*x) + 8*a^2*e^(2*x) - 12*a*b*e^(2*x) + 4*b^2*e^(2*x)
 + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^
(4*x) + b*e^(4*x) + 8*a*e^(2*x) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

maple [B] time = 0.13, size = 319, normalized size = 2.17

$$-\frac{a^4 b \ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right) b + 2a\tanh\left(\frac{x}{2}\right) + b\right)}{(a-b)^3 (a+b)^3} + \frac{1}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right) - 1\right)^4} + \frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} + \frac{}{8(a+b)^2 (}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*coth(x)),x)
```

```
[Out] -a^4*b/(a-b)^3/(a+b)^3*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)+1/(4*a+4*b)/(t
anh(1/2*x)-1)^4+4/(8*a+8*b)/(tanh(1/2*x)-1)^3+7/8/(a+b)^2/(tanh(1/2*x)-1)^2
*a+5/8/(a+b)^2/(tanh(1/2*x)-1)^2*b+5/8/(a+b)^2/(tanh(1/2*x)-1)*a+3/8/(a+b)^
2/(tanh(1/2*x)-1)*b-3/8/(a+b)^3*ln(tanh(1/2*x)-1)*a^2-1/8/(a+b)^3*ln(tanh(1
/2*x)-1)*a*b-1/(4*a-4*b)/(tanh(1/2*x)+1)^4+4/(8*a-8*b)/(tanh(1/2*x)+1)^3+5/
8/(a-b)^2/(tanh(1/2*x)+1)*a-3/8/(a-b)^2/(tanh(1/2*x)+1)*b-7/8/(a-b)^2/(tanh
(1/2*x)+1)^2*a+5/8/(a-b)^2/(tanh(1/2*x)+1)^2*b+3/8/(a-b)^3*ln(tanh(1/2*x)+1
)*a^2-1/8/(a-b)^3*ln(tanh(1/2*x)+1)*a*b
```

maxima [A] time = 0.34, size = 154, normalized size = 1.05

$$-\frac{a^4 b \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} + \frac{\left(3a^2 + ab\right)x}{8\left(a^3 + 3a^2b + 3ab^2 + b^3\right)} + \frac{\left(4(2a+b)e^{(-2x)} + a + b\right)e^{(4x)}}{64\left(a^2 + 2ab + b^2\right)} - \frac{4(2a-b)e^{(-2}}{64\left(a^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^4*b*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 1/8*(3*a^2 + a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*(2*a + b)*e^
```

```
(-2*x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - b)*e^(-2*x) +
(a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

mupad [B] time = 1.74, size = 135, normalized size = 0.92

$$\frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}-\frac{e^{-2x}\,(2a-b)}{16\,(a-b)^2}+\frac{e^{2x}\,(2a+b)}{16\,(a+b)^2}-\frac{a^4\,b\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{a\,x\,(3a-b)}{8\,(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*coth(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) - (exp(-2*x)*(2*a - b))/(1
6*(a - b)^2) + (exp(2*x)*(2*a + b))/(16*(a + b)^2) - (a^4*b*log(b - a + a*e
xp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*x*(3*a - b)
)/(8*(a - b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**4/(a + b*coth(x)), x)
```

3.115 $\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=135

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a\sinh(x)}{a^2-b^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[Out] a^3*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a^2*b*
cosh(x)/(a^2-b^2)^2-1/3*b*cosh(x)^3/(a^2-b^2)+a*b^2*sinh(x)/(a^2-b^2)^2+a*s
inh(x)/(a^2-b^2)+1/3*a*sinh(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3518, 3109, 2633, 2565, 30, 3100, 2637, 3074, 206}

$$\frac{a\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a\sinh(x)}{a^2-b^2}+\frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2}+\frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a^3*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
- (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*b^2*Si
nh[x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^
2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= \frac{a\int \cosh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(iab)\int \frac{\cosh^2(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}\\
&= -\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} + \frac{\left(ia^3b\right)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(ab^2\right)\int \cosh(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{(ia)\,\mathrm{Subst}\left(\int \left(1-x\right.\right.}{a^2 -}\\
&= -\frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(a^3b\right)\mathrm{Subst}\left(\int \frac{}{a^2-b}\right.}{}\\
&= \frac{a^3b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{a^2b\cosh(x)}{\left(a^2-b^2\right)^2} - \frac{b\cosh^3(x)}{3\left(a^2-b^2\right)} + \frac{ab^2\sinh(x)}{\left(a^2-b^2\right)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\,s}{3(}
\end{aligned}
$$

Mathematica [A] time = 1.48, size = 167, normalized size = 1.24

$$
\frac{1}{12}\left(\frac{a^3\sinh(3x)}{(a-b)^2(a+b)^2} - \frac{24a^3b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{(b-a)^{5/2}(a+b)^{5/2}} + \frac{3a\left(3a^2+b^2\right)\sinh(x)}{(a-b)^2(a+b)^2} + \frac{3b\left(b^2-5a^2\right)\cosh(x)}{(a-b)^2(a+b)^2} - \frac{ab^2\sinh(3x}{(a-b)^2(a+b}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ((-24*a^3*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)
^(5/2)*(a + b)^(5/2)) + (3*b*(-5*a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2)
+ (b*Cosh[3*x])/((-a + b)*(a + b)) + (3*a*(3*a^2 + b^2)*Sinh[x])/((a - b)^2
*(a + b)^2) + (a^3*Sinh[3*x])/((a - b)^2*(a + b)^2) - (a*b^2*Sinh[3*x])/((a
 - b)^2*(a + b)^2))/12
```

fricas [B] time = 0.44, size = 1873, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a
 + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) +
 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^
5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
+ (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)
^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 + 5*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 +
5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a^4
*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^
3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cosh(
x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(-a^2 + b^2
)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

giac [A] time = 0.14, size = 164, normalized size = 1.21

$$\frac{2\,a^3b\arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\left(a^2-2\,ab+b^2\right)}+\frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+9\,a^2e^x+12\,abe^x+}{24\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*a^3*b*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) + a - b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 9*a^2*e^
x + 12*a*b*e^x + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

maple [A] time = 0.13, size = 200, normalized size = 1.48

$$-\frac{2a^3b\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{-a^2+b^2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3(4a+4b)}-\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{a}{(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*coth(x)),x)
```

```
[Out] -2*a^3*b/(a-b)^2/(a+b)^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/
(-a^2+b^2)^(1/2))-4/3/(tanh(1/2*x)-1)^3/(4*a+4*b)-2/(4*a+4*b)/(tanh(1/2*x)-
1)^2-1/(a+b)^2/(tanh(1/2*x)-1)*a-1/2/(a+b)^2/(tanh(1/2*x)-1)*b-4/3/(tanh(1/
2*x)+1)^3/(4*a-4*b)+2/(4*a-4*b)/(tanh(1/2*x)+1)^2-1/(a-b)^2/(tanh(1/2*x)+1)
*a+1/2/(a-b)^2/(tanh(1/2*x)+1)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.04, size = 262, normalized size = 1.94

$$\frac{e^{3\,x}}{24\,a+24\,b}-\frac{e^{-3\,x}}{24\,a-24\,b}+\frac{e^x\,\left(3\,a+b\right)}{8\,\left(a+b\right)^2}-\frac{e^{-x}\,\left(3\,a-b\right)}{8\,\left(a-b\right)^2}+\frac{2\,\mathrm{atan}\left(\frac{a^3\,b\,e^x\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}{a^5\,\sqrt{a^6\,b^2}-b^5\,\sqrt{a^6\,b^2}+2\,a^2\,b^3\,\sqrt{a^6\,b^2}-2\,a^3\,b^2\,\sqrt{a^6\,b^2}+a\,b^4\,\sqrt{a^6\,b^2}-}\right.}{\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*coth(x)),x)
```

```
[Out] exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) + (exp(x)*(3*a + b))/(8*(a
 + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) + (2*atan((a^3*b*exp(x)*(b^10
- a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^5*(a^6*
b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^2*(a
^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b^2)^(
1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*coth(x)), x)
```

3.116 $\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{a^2b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2\left(a^2-b^2\right)}-\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(\coth(x)+1)}{4(a-b)^2}$$

```
[Out] -1/4*a*ln(1-coth(x))/(a+b)^2+1/4*a*ln(1+coth(x))/(a-b)^2-a^2*b*ln(a+b*coth(
x))/(a^2-b^2)^2-1/2*(b-a*coth(x))*sinh(x)^2/(a^2-b^2)
```

Rubi [A] time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3516, 1647, 801}

$$-\frac{a^2b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2\left(a^2-b^2\right)}-\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(\coth(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -(a*Log[1 - Coth[x]])/(4*(a + b)^2) + (a*Log[1 + Coth[x]])/(4*(a - b)^2) -
(a^2*b*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Sinh[x]^2)/(2*(
a^2 - b^2))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = -\left(b\,\text{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\coth(x)\right)\right)$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}}{(a+x)\left(-b^2+x^2\right)}\,dx, x, b\coth(x)\right)}{2b}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int\left(-\frac{ab}{2(a+b)^2(b-x)} + \frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{ab}{2(a-b)^2(b+x)}\right)dx, \ldots}{2b}$$

$$= -\frac{a\log(1-\coth(x))}{4(a+b)^2} + \frac{a\log(1+\coth(x))}{4(a-b)^2} - \frac{a^2b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sin\ldots}{2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.26, size = 73, normalized size = 0.86

$$\frac{\left(b^3-a^2b\right)\cosh(2x) + a\left(2x\left(a^2+b^2\right) + \left(a^2-b^2\right)\sinh(2x) - 4ab\log(a\sinh(x)+b\cosh(x))\right)}{4(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(2*(a^2 + b^2)*x - 4*a*b*Log[b*Cosh[x] + a*
Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.41, size = 334, normalized size = 3.93

$$\frac{\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^4 + 4\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)^3 + \left(a^3-a^2b-ab^2+b^3\right)\sinh(x)^4 + \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 + 2*a^2*
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 - 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(b*cosh(x) + a*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

giac [A] time = 0.12, size = 104, normalized size = 1.22

$$-\frac{a^2b\log\left(\left|-ae^{(2x)} - be^{(2x)} + a - b\right|\right)}{a^4 - 2a^2b^2 + b^4} + \frac{ax}{2\left(a^2-2ab+b^2\right)} - \frac{\left(2ae^{(2x)} + a - b\right)e^{(-2x)}}{8\left(a^2-2ab+b^2\right)} + \frac{e^{(2x)}}{8(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1
/2*a*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) + a - b)*e^(-2*x)/(a^2 - 2*a*
b + b^2) + 1/8*e^(2*x)/(a + b)
```

maple [A] time = 0.12, size = 146, normalized size = 1.72

$$-\frac{a^2 b \ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right) b + 2a \tanh\left(\frac{x}{2}\right) + b\right)}{(a-b)^2 (a+b)^2} + \frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - \right.}{2(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*coth(x)),x)
```

```
[Out] -a^2*b/(a-b)^2/(a+b)^2*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)+2/(4*a+4*b)/(t
anh(1/2*x)-1)^2+4/(8*a+8*b)/(tanh(1/2*x)-1)-1/2/(a+b)^2*ln(tanh(1/2*x)-1)*a
-2/(4*a-4*b)/(tanh(1/2*x)+1)^2+4/(8*a-8*b)/(tanh(1/2*x)+1)+1/2/(a-b)^2*ln(t
anh(1/2*x)+1)*a
```

maxima [A] time = 0.33, size = 80, normalized size = 0.94

$$-\frac{a^2 b \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^4 - 2a^2b^2 + b^4} + \frac{ax}{2\left(a^2 + 2ab + b^2\right)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

mupad [B] time = 1.40, size = 82, normalized size = 0.96

$$\frac{e^{2x}}{8a+8b} - \frac{e^{-2x}}{8a-8b} + \frac{a\,x}{2(a-b)^2} - \frac{a^2\,b\,\ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^4 - 2a^2b^2 + b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*coth(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (a*x)/(2*(a - b)^2) - (a^2*b
*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*coth(x)), x)
```

3.117 $\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=72

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}+\frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

```
[Out] a*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {3518, 3109, 2637, 2638, 3074, 206}

$$\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}+\frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
(b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\ &= \frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{b\int \sinh(x)\,dx}{a^2-b^2}+\frac{(iab)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}\\ &= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{(ab)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-a\cosh(x)-b\sinh(x)\right)}{a^2-b^2}\\ &= \frac{ab\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.31, size = 79, normalized size = 1.10

$$\frac{a\sinh(x)}{a^2-b^2}+\frac{b\cosh(x)}{b^2-a^2}+\frac{2ab\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{(b-a)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^(3/2
)*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

fricas [B] time = 0.42, size = 431, normalized size = 5.99

$$\left[-\frac{a^3+a^2b-ab^2-b^3-\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)^2-2\left(a^3-a^2b-ab^2+b^3\right)\cosh(x)\sinh(x)-\left(a^3-a^2b-al\right.}{2\left(\left(a^4-2a^2b^2+b^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*
(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a
^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

giac [A] time = 0.12, size = 71, normalized size = 0.99

$$-\frac{2ab\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{e^{(-x)}}{2(a-b)}+\frac{e^x}{2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*a*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^
2)) - 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

maple [A] time = 0.12, size = 92, normalized size = 1.28

$$-\frac{2ab\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{(a+b)(a-b)\sqrt{-a^2+b^2}}-\frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*coth(x)),x)
```

```
[Out] -2*a*b/(a+b)/(a-b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+
b^2)^(1/2))-4/(4*a-4*b)/(tanh(1/2*x)+1)-4/(4*a+4*b)/(tanh(1/2*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.53, size = 158, normalized size = 2.19

$$\frac{e^x}{2\,a+2\,b}-\frac{e^{-x}}{2\,a-2\,b}+\frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{a^3\,\sqrt{a^2\,b^2}+b^3\,\sqrt{a^2\,b^2}-a\,b^2\,\sqrt{a^2\,b^2}-a^2\,b\,\sqrt{a^2\,b^2}}\right)\sqrt{a^2\,b^2}}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*coth(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2) -
 a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*coth(x)), x)
```

3.118 $\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

[Out] arctan(sinh(x))/a+b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(1/2)

Rubi [A] time = 0.14, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {3518, 3110, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]/(a + b*Coth[x]),x]

[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= -\int \left(-\frac{\operatorname{sech}(x)}{a}+\frac{ib}{a(ib\cosh(x)+ia\sinh(x))}\right)dx\\
&= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{(ib)\int \frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\
&= \frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{a}\\
&= \frac{\tan^{-1}(\sinh(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 60, normalized size = 1.20

$$
\frac{2\left(\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{\sqrt{b-a}\sqrt{a+b}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

fricas [A] time = 0.46, size = 200, normalized size = 4.00

$$
\left[\frac{\sqrt{a^2-b^2}\,b\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{a^2-b^2}\,(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)+2\left(a^2-b^2\right)\arctan(c}{a^3-ab^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

giac [A] time = 0.12, size = 48, normalized size = 0.96

$$
-\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a}+\frac{2\,\arctan\left(e^x\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arctan(e^x)/a
```

maple [A] time = 0.12, size = 54, normalized size = 1.08

$$-\frac{2b\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*coth(x)),x)
```

```
[Out] -2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+
2/a*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.22, size = 164, normalized size = 3.28

$$\frac{b\,\ln\left(32\,a\,b^2\,e^x+32\,a^2\,b\,e^x+32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}-\frac{b\,\ln\left(32\,a\,b^2\,e^x+32\,a^2\,b\,e^x-32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}}+\frac{\ln\left(32\,a\,b\,e^x-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*coth(x))),x)
```

```
[Out] (log(a*b*32i - a^2*32i - 32*a^2*exp(x) + 32*a*b*exp(x))*1i)/a - (log(a*b*32
i - a^2*32i + 32*a^2*exp(x) - 32*a*b*exp(x))*1i)/a - (b*log(32*a*b^2*exp(x)
 + 32*a^2*b*exp(x) - 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) + (b*
log(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2
- b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*coth(x)), x)
```

3.119 $\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=29

$$-\frac{b\log(\tanh(x))}{a^2}-\frac{b\log(a+b\coth(x))}{a^2}+\frac{\tanh(x)}{a}$$

```
[Out] -b*ln(a+b*coth(x))/a^2-b*ln(tanh(x))/a^2+tanh(x)/a
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 44}

$$-\frac{b\log(\tanh(x))}{a^2}-\frac{b\log(a+b\coth(x))}{a^2}+\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((b*Log[a + b*Coth[x]])/a^2) - (b*Log[Tanh[x]])/a^2 + Tanh[x]/a
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx,x,b\coth(x)\right)\right)\\
&= -\left(b\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx,x,b\coth(x)\right)\right)\\
&= -\frac{b\log(a+b\coth(x))}{a^2}-\frac{b\log(\tanh(x))}{a^2}+\frac{\tanh(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 27, normalized size = 0.93

$$\frac{-b\log(a\sinh(x)+b\cosh(x))+a\tanh(x)+b\log(\cosh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (b*Log[Cosh[x]] - b*Log[b*Cosh[x] + a*Sinh[x]] + a*Tanh[x])/a^2
```

fricas [B] time = 0.44, size = 117, normalized size = 4.03

$$-\frac{\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2+b\right)\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(b\cosh(x)^2+2\,b\cosh(x)\sinh\right.}{a^2\cosh(x)^2+2\,a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*log(2*(b*cosh(x) +
a*sinh(x))/(cosh(x) - sinh(x))) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + b)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*a)/(a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

giac [B] time = 0.13, size = 76, normalized size = 2.62

$$-\frac{(ab+b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^3+a^2b}+\frac{b\log\left(e^{(2x)}+1\right)}{a^2}-\frac{be^{(2x)}+2a+b}{a^2\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 + a^2*b) + b*log(
e^(2*x) + 1)/a^2 - (b*e^(2*x) + 2*a + b)/(a^2*(e^(2*x) + 1))
```

maple [A] time = 0.14, size = 59, normalized size = 2.03

$$-\frac{b\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^2}+\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}+\frac{b\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*coth(x)),x)
```

```
[Out] -1/a^2*b*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)+2/a*tanh(1/2*x)/(tanh(1/2*x)
^2+1)+1/a^2*b*ln(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.42, size = 46, normalized size = 1.59

$$-\frac{b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2}+\frac{b\log\left(e^{(-2x)}+1\right)}{a^2}+\frac{2}{ae^{(-2x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/a^2 + b*log(e^(-2*x) + 1)/a^2 + 2/(a*e^(-
2*x) + a)
```

mupad [B] time = 1.58, size = 323, normalized size = 11.14

$$\frac{2\,\mathrm{atan}\left(\frac{b\left(a^4\,{\left(b^2\right)}^{3/2}-a^6\,\sqrt{b^2}\right)\left(b^6\,\sqrt{-a^4}-a\,b^5\,\sqrt{-a^4}-a^2\,b^4\,\sqrt{-a^4}+a^3\,b^3\,\sqrt{-a^4}+b^6\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}-2\,a^2\,b^4\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}+a^4\,b^2\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}\right)+b^2\left(a^3\,{\left(b^2\right)}^{3/2}-a^5 \cdots}{-a^{12}\,b^4+3\,a^{10}\,b^6-3\,a^8\,b^8+a^6\,b^{10} \cdots}\right.}{\sqrt{-a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*coth(x))),x)
```

```
[Out] (2*atan((b*(a^4*(b^2)^(3/2) - a^6*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(-
a^4)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a
^4)^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)
) + b^2*(a^3*(b^2)^(3/2) - a^5*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(-a^4
)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a^4)
^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)))/
```

```
(a^6*b^10 - 3*a^8*b^8 + 3*a^10*b^6 - a^12*b^4))*(b^2)^(1/2))/(-a^4)^(1/2) -
 2/(a*(exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*coth(x)), x)
```

3.120 $\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=83

$$-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}+\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

```
[Out] 1/2*arctan(sinh(x))/a-b^2*arctan(sinh(x))/a^3-b*sech(x)/a^2+b*arctanh((a*co
sh(x)+b*sinh(x))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^3+1/2*sech(x)*tanh(x)/a
```

Rubi [A] time = 0.24, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3518, 3110, 3768, 3770, 3104, 3074, 206}

$$-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) - (b^2*ArcTan[Sinh[x]])/a^3 + (b*Sqrt[a^2 - b^2]*ArcT
anh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (b*Sech[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\mathrm{sech}^3(x)}{a+b\coth(x)}\,dx &= -\left(i\int \frac{\mathrm{sech}^2(x)\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\ &= -\int\left(-\frac{\mathrm{sech}^3(x)}{a}+\frac{ib\,\mathrm{sech}^2(x)}{a(ib\cosh(x)+ia\sinh(x))}\right)dx\\ &= \frac{\int \mathrm{sech}^3(x)\,dx}{a}-\frac{(ib)\int \frac{\mathrm{sech}^2(x)}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\ &= -\frac{b\,\mathrm{sech}(x)}{a^2}+\frac{\mathrm{sech}(x)\tanh(x)}{2a}+\frac{\int \mathrm{sech}(x)\,dx}{2a}-\frac{b^2\int \mathrm{sech}(x)\,dx}{a^3}-\frac{\left(ib\left(a^2-b^2\right)\right)\int \frac{}{ib\cos\ldots}}{a^3}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}-\frac{b\,\mathrm{sech}(x)}{a^2}+\frac{\mathrm{sech}(x)\tanh(x)}{2a}+\frac{\left(b\left(a^2-b^2\right)\right)\mathrm{Sub}\ldots}{}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{b^2\tan^{-1}(\sinh(x))}{a^3}+\frac{b\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\,\mathrm{sech}(x)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.20, size = 85, normalized size = 1.02

$$\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+4b\sqrt{b-a}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{b-a}\,\sqrt{a+b}}\right)+a\,\mathrm{sech}(x)(a\tanh(x)-2b)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*ArcTan[Tanh[x/2]] + 4*b*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a
 + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] + a*Sech[x]*(-2*b + a*Tanh[x]))
/(2*a^3)
```

fricas [B] time = 0.47, size = 856, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*
b)*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cos
h(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh
(x) + b)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a +
```

```
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + (
(a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*b^2)
*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 + a^2
 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 - 2*b
^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x) + (
3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*co
sh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^
2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x)), ((a^2 - 2*a*
b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*b)*sinh(x)^3
- 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*
(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sq
rt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))
 + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*
b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 +
 a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 -
 2*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x)
 + (3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^
3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh
(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))]
```

giac [A] time = 0.12, size = 102, normalized size = 1.23

$$\frac{\left(a^2-2\,b^2\right)\arctan\left(e^x\right)}{a^3}-\frac{2\left(a^2b-b^3\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{ae^{(3\,x)}-2\,be^{(3\,x)}-ae^x-2\,be^x}{a^2\left(e^{(2\,x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] (a^2 - 2*b^2)*arctan(e^x)/a^3 - 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqrt
(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a*e^(3*x) - 2*b*e^(3*x) - a*e^x - 2
*b*e^x)/(a^2*(e^(2*x) + 1)^2)
```

maple [B] time = 0.18, size = 187, normalized size = 2.25

$$-\frac{2b\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{2b^3\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{a^3\sqrt{-a^2+b^2}}-\frac{\tanh^3\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{2\left(\tanh^2\left(\frac{x}{2}\right)\right)b}{a^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*coth(x)),x)
```

```
[Out] -2/a*b/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+
2*b^3/a^3/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tanh(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2
))-1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-2/a^2/(tanh(1/2*x)^2+1)^2*tanh(1/2
*x)^2*b+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)-2/a^2/(tanh(1/2*x)^2+1)^2*b+1/a
*arctan(tanh(1/2*x))-2/a^3*arctan(tanh(1/2*x))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.94, size = 166, normalized size = 2.00

$$\frac{e^x\,(a-2\,b)}{a^2\,\left(e^{2\,x}+1\right)}+\frac{\ln\left(e^x+1i\right)\,\left(a^2\,1i-b^2\,2i\right)}{2\,a^3}-\frac{2\,e^x}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}-\frac{\ln\left(e^x-i\right)\,\left(a^2\,1i-b^2\,2i\right)}{2\,a^3}+\frac{b\,\ln\left(a\,e^x+b\,e^x+\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*coth(x))),x)
```

```
[Out] (log(exp(x) + 1i)*(a^2*1i - b^2*2i))/(2*a^3) - (log(exp(x) - 1i)*(a^2*1i -
b^2*2i))/(2*a^3) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1)) + (exp(x)*(a
- 2*b))/(a^2*(exp(2*x) + 1)) + (b*log(a*exp(x) + b*exp(x) + (a^2 - b^2)^(1/
2))*((a + b)*(a - b))^(1/2))/a^3 - (b*log(a*exp(x) + b*exp(x) - (a^2 - b^2)
^(1/2))*((a + b)*(a - b))^(1/2))/a^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*coth(x)), x)
```

3.121 $\int \frac{\text{sech}^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=79

$$\frac{b\tanh^2(x)}{2a^2}-\frac{b\left(a^2-b^2\right)\log(\tanh(x))}{a^4}-\frac{b\left(a^2-b^2\right)\log(a+b\coth(x))}{a^4}+\frac{\left(a^2-b^2\right)\tanh(x)}{a^3}-\frac{\tanh^3(x)}{3a}$$

```
[Out] -b*(a^2-b^2)*ln(a+b*coth(x))/a^4-b*(a^2-b^2)*ln(tanh(x))/a^4+(a^2-b^2)*tanh
(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3516, 894}

$$\frac{\left(a^2-b^2\right)\tanh(x)}{a^3}-\frac{b\left(a^2-b^2\right)\log(\tanh(x))}{a^4}-\frac{b\left(a^2-b^2\right)\log(a+b\coth(x))}{a^4}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -((b*(a^2 - b^2)*Log[a + b*Coth[x]])/a^4) - (b*(a^2 - b^2)*Log[Tanh[x]])/a^
4 + ((a^2 - b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3516

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[(x^m*(a + x)^n)/(b^2 + x^2)^(m/2 + 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{sech}^4(x)}{a+b\coth(x)}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{-b^2+x^2}{x^4(a+x)}\,dx,x,b\coth(x)\right)\right)\\ &= -\left(b\,\text{Subst}\left(\int \left(-\frac{b^2}{ax^4}+\frac{b^2}{a^2x^3}+\frac{a^2-b^2}{a^3x^2}+\frac{-a^2+b^2}{a^4x}+\frac{a^2-b^2}{a^4(a+x)}\right)dx,x,b\coth(x)\right)\right)\\ &= -\frac{b\left(a^2-b^2\right)\log(a+b\coth(x))}{a^4}-\frac{b\left(a^2-b^2\right)\log(\tanh(x))}{a^4}+\frac{\left(a^2-b^2\right)\tanh(x)}{a^3}+\frac{b\tanh^2(}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.31, size = 68, normalized size = 0.86

$$\frac{\left(4a^3-6ab^2\right)\tanh(x)-6b\left(b^2-a^2\right)(\log(\cosh(x))-\log(a\sinh(x)+b\cosh(x)))+a^2\text{sech}^2(x)(2a\tanh(x)-3b)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (-6*b*(-a^2 + b^2)*(Log[Cosh[x]] - Log[b*Cosh[x] + a*Sinh[x]]) + (4*a^3 - 6
*a*b^2)*Tanh[x] + a^2*Sech[x]^2*(-3*b + 2*a*Tanh[x]))/(6*a^4)
```

fricas [B] time = 0.42, size = 909, normalized size = 11.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 +
6*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 + 6*(2*a^3 + a^2*b - 2*a*b^2)
*cosh(x)^2 + 6*(2*a^3 + a^2*b - 2*a*b^2 + 6*(a^2*b - a*b^2)*cosh(x)^2)*sinh
(x)^2 + 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a
^2*b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2
*b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b -
b^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a
^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2
+ 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*co
sh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^
2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*si
nh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2*b - b^3)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*
sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*co
sh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b -
b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x)
)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^2)*cosh(x)^3 + (2
*a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*si
nh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh
(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)^
3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^
5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

giac [B] time = 0.13, size = 201, normalized size = 2.54

$$-\frac{\left(a^3b + a^2b^2 - ab^3 - b^4\right)\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^5 + a^4b} + \frac{\left(a^2b - b^3\right)\log\left(e^{(2x)} + 1\right)}{a^4} - \frac{11\,a^2be^{(6x)} - 11\,b^3e^{(6x)} + 45}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3*b + a^2*b^2 - a*b^3 - b^4)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a
^5 + a^4*b) + (a^2*b - b^3)*log(e^(2*x) + 1)/a^4 - 1/6*(11*a^2*b*e^(6*x) -
11*b^3*e^(6*x) + 45*a^2*b*e^(4*x) - 12*a*b^2*e^(4*x) - 33*b^3*e^(4*x) + 24*
a^3*e^(2*x) + 45*a^2*b*e^(2*x) - 24*a*b^2*e^(2*x) - 33*b^3*e^(2*x) + 8*a^3
+ 11*a^2*b - 12*a*b^2 - 11*b^3)/(a^4*(e^(2*x) + 1)^3)
```

maple [B] time = 0.16, size = 257, normalized size = 3.25

$$-\frac{b\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b + 2a\tanh\left(\frac{x}{2}\right) + b\right)}{a^2} + \frac{b^3\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b + 2a\tanh\left(\frac{x}{2}\right) + b\right)}{a^4} + \frac{2\left(\tanh^5\left(\frac{x}{2}\right)\right)}{a\left(\tanh^2\left(\frac{x}{2}\right) + 1\right)^3} - \frac{2\,(\tan}{a^3\,(\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*coth(x)),x)
```

```
[Out] -1/a^2*b*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)+b^3/a^4*ln(tanh(1/2*x)^2*b+2
*a*tanh(1/2*x)+b)+2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5-2/a^3/(tanh(1/2*x)^
```

```
2+1)^3*tanh(1/2*x)^5*b^2+2/a^2/(tanh(1/2*x)^2+1)^3*b*tanh(1/2*x)^4+4/3/a/(t
anh(1/2*x)^2+1)^3*tanh(1/2*x)^3-4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2
+2/a^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^2*b+2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2
*x)-2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*b^2+1/a^2*b*ln(tanh(1/2*x)^2+1)-1
/a^4*ln(tanh(1/2*x)^2+1)*b^3
```

maxima [A] time = 0.41, size = 133, normalized size = 1.68

$$\frac{2\left(2\,a^2-3\,b^2+3\left(2\,a^2-ab-2\,b^2\right)e^{(-2\,x)}-3\left(ab+b^2\right)e^{(-4\,x)}\right)}{3\left(3\,a^3e^{(-2\,x)}+3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}+a^3\right)}-\frac{\left(a^2b-b^3\right)\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^4}+\frac{\left(a^2b-b^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2/3*(2*a^2 - 3*b^2 + 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-4
*x))/(3*a^3*e^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) - (a^2*b - b^3)
*log(-(a - b)*e^(-2*x) + a + b)/a^4 + (a^2*b - b^3)*log(e^(-2*x) + 1)/a^4
```

mupad [B] time = 1.45, size = 123, normalized size = 1.56

$$\frac{8}{3\,a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)}-\frac{2\,\left(2\,a-b\right)}{a^2\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}-\frac{2\,b\,\left(a-b\right)}{a^3\,\left(e^{2\,x}+1\right)}-\frac{b\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(a+b\right)\,\left(a-b\right)}{a^4}+\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*coth(x))),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (2*(2*a - b))/(a^2*(2*ex
p(2*x) + exp(4*x) + 1)) - (2*b*(a - b))/(a^3*(exp(2*x) + 1)) - (b*log(b - a
 + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/a^4 + (b*log(exp(2*x) + 1)*(a
+ b)*(a - b))/a^4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*coth(x)), x)
```

3.122 $\int \frac{\text{sech}(x)}{i+2\coth(x)}\,dx$

Optimal. Leaf size=31

$$-i\tan^{-1}(\sinh(x))-\frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -I*arctan(sinh(x))-2/5*arctanh(1/5*(cosh(x)-2*I*sinh(x))*5^(1/2))*5^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3518, 3110, 3770, 3074, 206}

$$-i\tan^{-1}(\sinh(x))-\frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + 2*Coth[x]),x]
```

```
[Out] (-I)*ArcTan[Sinh[x]] - (2*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])/Sqrt[
5]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3518

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[(Sin[e + f*x]^m*(a*Cos[e + f*x] + b*Sin[e + f*x])^n)/
Cos[e + f*x]^n, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = -\left(i\int \frac{\tanh(x)}{-2i\cosh(x)+\sinh(x)}\,dx\right)$$
$$= -\int \left(i\operatorname{sech}(x) - \frac{2i}{2\cosh(x)+i\sinh(x)}\right)dx$$
$$= -(i\int \operatorname{sech}(x)\,dx) + 2i\int \frac{1}{2\cosh(x)+i\sinh(x)}\,dx$$
$$= -i\tan^{-1}(\sinh(x)) - 2\,\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, \cosh(x)-2i\sinh(x)\right)$$
$$= -i\tan^{-1}(\sinh(x)) - \frac{2\tanh^{-1}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 1.23

$$-\frac{4\tanh^{-1}\left(\frac{1-2i\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}} - 2i\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + 2*Coth[x]),x]
```

```
[Out] (-2*I)*ArcTan[Tanh[x/2]] - (4*ArcTanh[(1 - (2*I)*Tanh[x/2])/Sqrt[5]])/Sqrt[
5]
```

fricas [A] time = 0.41, size = 41, normalized size = 1.32

$$-\frac{2}{5}\sqrt{5}\log\left(\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^x\right)+\frac{2}{5}\sqrt{5}\log\left(-\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^x\right)+\log\left(e^x+i\right)-\log\left(e^x-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(5)*log((2/5*I + 1/5)*sqrt(5) + e^x) + 2/5*sqrt(5)*log(-(2/5*I + 1
/5)*sqrt(5) + e^x) + log(e^x + I) - log(e^x - I)
```

giac [A] time = 0.13, size = 26, normalized size = 0.84

$$\frac{4}{5}i\sqrt{5}\arctan\left(\left(\frac{1}{5}i+\frac{2}{5}\right)\sqrt{5}e^x\right)+\log\left(e^x+i\right)-\log\left(e^x-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="giac")
```

```
[Out] 4/5*I*sqrt(5)*arctan((1/5*I + 2/5)*sqrt(5)*e^x) + log(e^x + I) - log(e^x -
I)
```

maple [A] time = 0.17, size = 41, normalized size = 1.32

$$\frac{4i\sqrt{5}\arctan\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+i\right)\sqrt{5}}{5}\right)}{5}+\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+2*coth(x)),x)
```

```
[Out] 4/5*I*5^(1/2)*arctan(1/5*(2*tanh(1/2*x)+I)*5^(1/2))+ln(tanh(1/2*x)+I)-ln(ta
nh(1/2*x)-I)
```

maxima [A] time = 0.43, size = 42, normalized size = 1.35

$$\frac{2}{5}\sqrt{5}\log\left(-\frac{2\sqrt{5}-(4i+2)e^{(-x)}}{2\sqrt{5}+(4i+2)e^{(-x)}}\right)+2i\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*log(-(2*sqrt(5) - (4*I + 2)*e^(-x))/(2*sqrt(5) + (4*I + 2)*e^(-
x))) + 2*I*arctan(e^(-x))
```

mupad [B] time = 0.51, size = 65, normalized size = 2.10

$$\ln\left(e^x\,(32+64i)-64+32i\right)-\ln\left(e^x\,(32+64i)+64-32i\right)-\frac{2\sqrt{5}\,\ln\left(e^x\left(-\frac{256}{5}+\frac{192}{5}i\right)+\sqrt{5}\left(-\frac{128}{5}-\frac{64}{5}i\right)\right)}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(2*coth(x) + 1i)),x)
```

```
[Out] log(exp(x)*(32 + 64i) - (64 - 32i)) - log(exp(x)*(32 + 64i) + (64 - 32i)) -
 (2*5^(1/2)*log(- exp(x)*(256/5 - 192i/5) - 5^(1/2)*(128/5 + 64i/5)))/5 + (
2*5^(1/2)*log(5^(1/2)*(128/5 + 64i/5) - exp(x)*(256/5 - 192i/5)))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{2\coth(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+2*coth(x)),x)
```

```
[Out] Integral(sech(x)/(2*coth(x) + I), x)
```

3.123 $\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=43

$$\frac{5x}{2}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

```
[Out] 5/2*x-2*ln(cosh(x))-5/2*tanh(x)+tanh(x)^2-5/6*tanh(x)^3+1/2*tanh(x)^3/(1+co
th(x))
```

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$\frac{5x}{2}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^3/(2*(1 + Coth[x]))
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx &= \frac{\tanh^3(x)}{2(1+\coth(x))}-\frac{1}{2}\int(-5+4\coth(x))\tanh^4(x)\,dx\\
&= -\frac{5}{6}\tanh^3(x)+\frac{\tanh^3(x)}{2(1+\coth(x))}-\frac{1}{2}i\int(-4i+5i\coth(x))\tanh^3(x)\,dx\\
&= \tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}+\frac{1}{2}\int(5-4\coth(x))\tanh^2(x)\,dx\\
&= -\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}+\frac{1}{2}i\int(4i-5i\coth(x))\tanh(x)\,d\\
&= \frac{5x}{2}-\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}-2\int\tanh(x)\,dx\\
&= \frac{5x}{2}-2\log(\cosh(x))-\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 40, normalized size = 0.93

$$\frac{1}{12}\left(30x-3\sinh(2x)+3\cosh(2x)-28\tanh(x)-24\log(\cosh(x))+(4\tanh(x)-6)\operatorname{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (30*x + 3*Cosh[2*x] - 24*Log[Cosh[x]] - 3*Sinh[2*x] - 28*Tanh[x] + Sech[x]^
2*(-6 + 4*Tanh[x]))/12
```

fricas [B] time = 0.42, size = 571, normalized size = 13.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x +
 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*cosh
(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*
cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^3 +
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh(x)^
7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^
3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh
(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))
```

giac [A] time = 0.13, size = 47, normalized size = 1.09

$$\frac{9}{2}\,x+\frac{\left(51\,e^{(6\,x)}+81\,e^{(4\,x)}+65\,e^{(2\,x)}+3\right)e^{(-2\,x)}}{12\,\left(e^{(2\,x)}+1\right)^3}-2\,\log\left(e^{(2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x) +
 1)^3 - 2*log(e^(2*x) + 1)
```

maple [B] time = 0.11, size = 96, normalized size = 2.23

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{9\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{4\left(\tanh^{5}\left(\frac{x}{2}\right)-\frac{\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{2}+\frac{8\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-\cdots\right.}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(1+coth(x)),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)-1)+1/(tanh(1/2*x)+1)^2-1/(tanh(1/2*x)+1)+9/2*ln(tanh(1/
2*x)+1)-4*(tanh(1/2*x)^5-1/2*tanh(1/2*x)^4+8/3*tanh(1/2*x)^3-1/2*tanh(1/2*x
)^2+tanh(1/2*x))/(tanh(1/2*x)^2+1)^3-2*ln(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.41, size = 55, normalized size = 1.28

$$\frac{1}{2}\,x-\frac{2\left(15\,e^{(-2\,x)}+12\,e^{(-4\,x)}+7\right)}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}+\frac{1}{4}\,e^{(-2\,x)}-2\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-
6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)
```

mupad [B] time = 1.30, size = 69, normalized size = 1.60

$$\frac{9\,x}{2}-2\,\ln\left(\mathrm{e}^{2\,x}+1\right)+\frac{\mathrm{e}^{-2\,x}}{4}+\frac{8}{3\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}-\frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{4}{\mathrm{e}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(coth(x) + 1),x)
```

```
[Out] (9*x)/2 - 2*log(exp(2*x) + 1) + exp(-2*x)/4 + 8/(3*(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1)) - 2/(2*exp(2*x) + exp(4*x) + 1) + 4/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**4/(coth(x) + 1), x)
```

3.124 $\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{3x}{2}-\tanh^2(x)+\frac{3\tanh(x)}{2}+2\log(\cosh(x))+\frac{\tanh^2(x)}{2(\coth(x)+1)}$$

```
[Out] -3/2*x+2*ln(cosh(x))+3/2*tanh(x)-tanh(x)^2+1/2*tanh(x)^2/(1+coth(x))
```

Rubi [A] time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$-\frac{3x}{2}-\tanh^2(x)+\frac{3\tanh(x)}{2}+2\log(\cosh(x))+\frac{\tanh^2(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^2/(2*(1 + C
oth[x]))
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \frac{\tanh^2(x)}{2(1+\coth(x))} - \frac{1}{2}\int (-4+3\coth(x))\tanh^3(x)\,dx$$
$$= -\tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} - \frac{1}{2}i\int (-3i+4i\coth(x))\tanh^2(x)\,dx$$
$$= \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} + \frac{1}{2}\int (4-3\coth(x))\tanh(x)\,dx$$
$$= -\frac{3x}{2} + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} + 2\int \tanh(x)\,dx$$
$$= -\frac{3x}{2} + 2\log(\cosh(x)) + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x+\sinh(2x)-\cosh(2x)+4\tanh(x)+2\operatorname{sech}^2(x)+8\log(\cosh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(1 + Coth[x]),x]
[Out] (-6*x - Cosh[2*x] + 8*Log[Cosh[x]] + 2*Sech[x]^2 + Sinh[2*x] + 4*Tanh[x])/4
```

fricas [B] time = 0.41, size = 354, normalized size = 9.57

$$-\frac{14\,x\cosh(x)^6 + 84\,x\cosh(x)\sinh(x)^5 + 14\,x\sinh(x)^6 + (28\,x+1)\cosh(x)^4 + \left(210\,x\cosh(x)^2 + 28\,x + 1\right)\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="fricas")
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 + (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 +
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh
(x)^3 + 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 + (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh(x)^3 + 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))
```

giac [A] time = 0.13, size = 39, normalized size = 1.05

$$-\frac{7}{2}\,x - \frac{\left(e^{(4\,x)} + 10\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} + 1\right)^2} + 2\,\log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] -7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e^
(2*x) + 1)
```

maple [B] time = 0.11, size = 80, normalized size = 2.16

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\frac{7\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2\left(\tanh^{3}\left(\frac{x}{2}\right)\right)-2\left(\tanh^{2}\left(\frac{x}{2}\right)\right)+2\,\mathrm{ta}}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(1+coth(x)),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-7/2*ln(tanh(1/
2*x)+1)+2*(tanh(1/2*x)^3-tanh(1/2*x)^2+tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+2*l
n(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.41, size = 43, normalized size = 1.16

$$\frac{1}{2}\,x+\frac{2\left(2\,e^{(-2\,x)}+1\right)}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}-\frac{1}{4}\,e^{(-2\,x)}+2\,\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*l
og(e^(-2*x) + 1)
```

mupad [B] time = 1.21, size = 35, normalized size = 0.95

$$2\,\ln\left(\mathrm{e}^{2\,x}+1\right)-\frac{7\,x}{2}-\frac{\mathrm{e}^{-2\,x}}{4}-\frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(coth(x) + 1),x)
```

```
[Out] 2*log(exp(2*x) + 1) - (7*x)/2 - exp(-2*x)/4 - 2/(2*exp(2*x) + exp(4*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^{3}(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**3/(coth(x) + 1), x)
```

3.125 $\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=29

$$\frac{3x}{2}-\frac{3\tanh(x)}{2}-\log(\cosh(x))+\frac{\tanh(x)}{2(\coth(x)+1)}$$

```
[Out] 3/2*x-ln(cosh(x))-3/2*tanh(x)+1/2*tanh(x)/(1+coth(x))
```

Rubi [A] time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3552, 3529, 3531, 3475}

$$\frac{3x}{2}-\frac{3\tanh(x)}{2}-\log(\cosh(x))+\frac{\tanh(x)}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth[x]))
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3552

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(a*(c + d*Tan[e + f*x])^(n + 1))/(2*f*(b*c
 - a*d)*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*Ta
n[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{\tanh(x)}{2(1+\coth(x))} - \frac{1}{2}\int (-3+2\coth(x))\tanh^2(x)\,dx$$
$$= -\frac{3\tanh(x)}{2} + \frac{\tanh(x)}{2(1+\coth(x))} - \frac{1}{2}i\int (-2i+3i\coth(x))\tanh(x)\,dx$$
$$= \frac{3x}{2} - \frac{3\tanh(x)}{2} + \frac{\tanh(x)}{2(1+\coth(x))} - \int \tanh(x)\,dx$$
$$= \frac{3x}{2} - \log(\cosh(x)) - \frac{3\tanh(x)}{2} + \frac{\tanh(x)}{2(1+\coth(x))}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.93

$$\frac{1}{4}(6x - \sinh(2x) + \cosh(2x) - 4\tanh(x) - 4\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(1 + Coth[x]),x]
[Out] (6*x + Cosh[2*x] - 4*Log[Cosh[x]] - Sinh[2*x] - 4*Tanh[x])/4
```

fricas [B] time = 0.44, size = 186, normalized size = 6.41

$$\frac{10\,x\cosh(x)^4 + 40\,x\cosh(x)\sinh(x)^3 + 10\,x\sinh(x)^4 + (10\,x+9)\cosh(x)^2 + \left(60\,x\cosh(x)^2 + 10\,x + 9\right)\sinh}{4\left(\cosh(x)^4 + 4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*co
sh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + c
osh(x))*sinh(x))
```

giac [A] time = 0.12, size = 35, normalized size = 1.21

$$\frac{5}{2}\,x + \frac{\left(9\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} + 1\right)} - \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] 5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)
```

maple [B] time = 0.10, size = 65, normalized size = 2.24

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2} + \frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{1}{\tanh\left(\frac{x}{2}\right)+1} + \frac{5\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2} - \frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1} - \ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(1+coth(x)),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)-1)+1/(tanh(1/2*x)+1)^2-1/(tanh(1/2*x)+1)+5/2*ln(tanh(1/
2*x)+1)-2*tanh(1/2*x)/(tanh(1/2*x)^2+1)-ln(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.42, size = 29, normalized size = 1.00

$$\frac{1}{2}\,x - \frac{2}{e^{(-2\,x)} + 1} + \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/(e^(-2*x) + 1) + 1/4*e^(-2*x) - log(e^(-2*x) + 1)
```

mupad [B] time = 1.22, size = 29, normalized size = 1.00

$$\frac{5\,x}{2} - \ln\left(\mathrm{e}^{2\,x} + 1\right) + \frac{\mathrm{e}^{-2\,x}}{4} + \frac{2}{\mathrm{e}^{2\,x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(coth(x) + 1),x)
```

```
[Out] (5*x)/2 - log(exp(2*x) + 1) + exp(-2*x)/4 + 2/(exp(2*x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{2}{\left(x \right)}}{\coth{\left(x \right)} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)**2/(coth(x) + 1), x)
```

3.126 $\int \frac{\tanh(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{x}{2}+\frac{1}{2(\coth(x)+1)}+\log(\cosh(x))$$

```
[Out] -1/2*x+1/2/(1+coth(x))+ln(cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3551, 3479, 8, 3475}

$$-\frac{x}{2}+\frac{1}{2(\coth(x)+1)}+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(1 + Coth[x]),x]
```

```
[Out] -x/2 + 1/(2*(1 + Coth[x])) + Log[Cosh[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3551

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{1+\coth(x)}\,dx &= -\int \frac{1}{1+\coth(x)}\,dx+\int \tanh(x)\,dx\\ &= \frac{1}{2(1+\coth(x))}+\log(\cosh(x))-\frac{\int 1\,dx}{2}\\ &= -\frac{x}{2}+\frac{1}{2(1+\coth(x))}+\log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x+\sinh(2x)-\cosh(2x)+4\log(\cosh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(1 + Coth[x]),x]
```

```
[Out] (-2*x - Cosh[2*x] + 4*Log[Cosh[x]] + Sinh[2*x])/4
```

fricas [B] time = 0.41, size = 73, normalized size = 3.84

$$-\frac{6\,x\cosh(x)^2 + 12\,x\cosh(x)\sinh(x) + 6\,x\sinh(x)^2 - 4\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) + 1}{4\left(\cosh(x)^2 + 2\,\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.13, size = 17, normalized size = 0.89

$$-\frac{3}{2}\,x - \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)
```

maple [B] time = 0.10, size = 47, normalized size = 2.47

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2} - \frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{1}{\tanh\left(\frac{x}{2}\right)+1} - \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2} + \ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(1+coth(x)),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)+1)^2+1/(tanh(1/2*x)+1)-3/2*ln(tanh(1/
2*x)+1)+ln(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.42, size = 17, normalized size = 0.89

$$\frac{1}{2}\,x - \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(-2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)
```

mupad [B] time = 1.17, size = 17, normalized size = 0.89

$$\ln\left(\mathrm{e}^{2\,x} + 1\right) - \frac{3\,x}{2} - \frac{\mathrm{e}^{-2\,x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(coth(x) + 1),x)
```

```
[Out] log(exp(2*x) + 1) - (3*x)/2 - exp(-2*x)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\coth(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)/(coth(x) + 1), x)
```

3.127 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

```
[Out] 1/2*x-1/2/(1+coth(x))
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3479, 8}

$$\frac{x}{2}-\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x-\sinh(2x)+\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Coth[x])^(-1),x]
```

```
[Out] (2*x + Cosh[2*x] - Sinh[2*x])/4
```

fricas [B] time = 0.41, size = 26, normalized size = 1.62

$$\frac{(2\,x+1)\cosh(x)+(2\,x-1)\sinh(x)}{4\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x+\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
[Out] 1/2*x + 1/4*e^(-2*x)
```

maple [A] time = 0.05, size = 24, normalized size = 1.50

$$-\frac{\ln\left(\coth(x)-1\right)}{4}-\frac{1}{2\left(1+\coth(x)\right)}+\frac{\ln\left(1+\coth(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+coth(x)),x)
[Out] -1/4*ln(coth(x)-1)-1/2/(1+coth(x))+1/4*ln(1+coth(x))
```

maxima [A] time = 0.31, size = 10, normalized size = 0.62

$$\frac{1}{2}\,x+\frac{1}{4}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/4*e^(-2*x)
```

mupad [B] time = 0.00, size = 14, normalized size = 0.88

$$\frac{x}{2}-\frac{1}{2\,\left(\coth(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(coth(x) + 1),x)
[Out] x/2 - 1/(2*(coth(x) + 1))
```

sympy [B] time = 0.45, size = 27, normalized size = 1.69

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}+\frac{1}{2\tanh\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+coth(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

3.128 $\int \frac{\coth(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}+\frac{1}{2(\coth(x)+1)}$$

```
[Out] 1/2*x+1/2/(1+coth(x))
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3526, 8}

$$\frac{x}{2}+\frac{1}{2(\coth(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 + Coth[x]),x]
[Out] x/2 + 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{1+\coth(x)}\,dx &= \frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.12

$$\frac{1}{4}(2x+\sinh(2x)-\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 + Coth[x]),x]
[Out] (2*x - Cosh[2*x] + Sinh[2*x])/4
```

fricas [B] time = 0.40, size = 26, normalized size = 1.62

$$\frac{(2\,x-1)\cosh(x)+(2\,x+1)\sinh(x)}{4\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*((2*x - 1)*cosh(x) + (2*x + 1)*sinh(x))/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 10, normalized size = 0.62

$$\frac{1}{2}x - \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="giac")
[Out] 1/2*x - 1/4*e^(-2*x)
```

maple [A] time = 0.05, size = 24, normalized size = 1.50

$$-\frac{\ln(\coth(x)-1)}{4} + \frac{1}{2+2\coth(x)} + \frac{\ln(1+\coth(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x)),x)
[Out] -1/4*ln(coth(x)-1)+1/2/(1+coth(x))+1/4*ln(1+coth(x))
```

maxima [A] time = 0.31, size = 10, normalized size = 0.62

$$\frac{1}{2}x - \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x - 1/4*e^(-2*x)
```

mupad [B] time = 0.05, size = 12, normalized size = 0.75

$$\frac{x}{2} + \frac{1}{2\,(\coth(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(coth(x) + 1),x)
[Out] x/2 + 1/(2*(coth(x) + 1))
```

sympy [B] time = 0.48, size = 27, normalized size = 1.69

$$\frac{x\tanh(x)}{2\tanh(x)+2} + \frac{x}{2\tanh(x)+2} - \frac{1}{2\tanh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

3.129 $\int \frac{\coth^2(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{x}{2}-\frac{1}{2(\coth(x)+1)}+\log(\sinh(x))$$

```
[Out] -1/2*x-1/2/(1+coth(x))+ln(sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3540, 3475}

$$-\frac{x}{2}-\frac{1}{2(\coth(x)+1)}+\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(1 + Coth[x]),x]
```

```
[Out] -x/2 - 1/(2*(1 + Coth[x])) + Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3540

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(a*c + b*d)^2*(a + b*Tan[e + f*x])^m)
/(2*a^3*f*m), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*
c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{1+\coth(x)}\,dx &= -\frac{1}{2(1+\coth(x))}-\frac{1}{2}\int (1-2\coth(x))\,dx\\
&= -\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\int \coth(x)\,dx\\
&= -\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\log(\sinh(x))
\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.21

$$\frac{1}{4}(-2x-\sinh(2x)+\cosh(2x)+4\log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-2*x + Cosh[2*x] + 4*Log[Sinh[x]] - Sinh[2*x])/4
```

fricas [B] time = 0.42, size = 73, normalized size = 3.84

$$-\frac{6\,x\cosh(x)^2+12\,x\cosh(x)\sinh(x)+6\,x\sinh(x)^2-4\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)\log\left(\frac{2\sinh(}{\cosh(x)-\mathrm{si}}\right.}{4\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [A] time = 0.12, size = 18, normalized size = 0.95

$$-\frac{3}{2}\,x + \frac{1}{4}\,e^{(-2\,x)} + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -3/2*x + 1/4*e^(-2*x) + log(abs(e^(2*x) - 1))
```

maple [A] time = 0.05, size = 24, normalized size = 1.26

$$-\frac{\ln\left(\coth(x)-1\right)}{4}-\frac{1}{2\left(1+\coth(x)\right)}-\frac{3\ln\left(1+\coth(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x)),x)
```

```
[Out] -1/4*ln(coth(x)-1)-1/2/(1+coth(x))-3/4*ln(1+coth(x))
```

maxima [A] time = 0.30, size = 24, normalized size = 1.26

$$\frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x) + log(e^(-x) + 1) + log(e^(-x) - 1)
```

mupad [B] time = 0.06, size = 21, normalized size = 1.11

$$\frac{x}{2}-\ln\left(\mathrm{coth}(x)+1\right)-\frac{1}{2\,\left(\mathrm{coth}(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(coth(x) + 1),x)
```

```
[Out] x/2 - log(coth(x) + 1) - 1/(2*(coth(x) + 1))
```

sympy [B] time = 0.62, size = 92, normalized size = 4.84

$$\frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2\tanh\left(x\right)+2}-\frac{2\log\left(\tanh\left(x\right)+1\right)}{2\tanh\left(x\right)+2}+\frac{2\log\left(\tanh\left(x\right)\right)\tanh\left(x\right)}{2\tanh\left(x\right)+2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) - 2*log(tanh(x) + 1)/(2*tanh(x) + 2) + 2*log(tanh(x))*tanh(
x)/(2*tanh(x) + 2) + 2*log(tanh(x))/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

3.130 $\int \frac{\coth^3(x)}{1+\coth(x)}\,dx$

Optimal. Leaf size=31

$$\frac{3x}{2}+\frac{\coth^2(x)}{2(\coth(x)+1)}-\frac{3\coth(x)}{2}-\log(\sinh(x))$$

```
[Out] 3/2*x-3/2*coth(x)+1/2*coth(x)^2/(1+coth(x))-ln(sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3550, 3525, 3475}

$$\frac{3x}{2}+\frac{\coth^2(x)}{2(\coth(x)+1)}-\frac{3\coth(x)}{2}-\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(1 + Coth[x]),x]
```

```
[Out] (3*x)/2 - (3*Coth[x])/2 + Coth[x]^2/(2*(1 + Coth[x])) - Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/
(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{1+\coth(x)}\,dx &= \frac{\coth^2(x)}{2(1+\coth(x))}-\frac{1}{2}\int (2-3\coth(x))\coth(x)\,dx\\ &= \frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{\coth^2(x)}{2(1+\coth(x))}-\int \coth(x)\,dx\\ &= \frac{3x}{2}-\frac{3\coth(x)}{2}+\frac{\coth^2(x)}{2(1+\coth(x))}-\log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 0.87

$$\frac{1}{4}(6x+\sinh(2x)-\cosh(2x)-4\coth(x)-4\log(\sinh(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(1 + Coth[x]),x]
[Out] (6*x - Cosh[2*x] - 4*Coth[x] - 4*Log[Sinh[x]] + Sinh[2*x])/4
```

fricas [B] time = 0.42, size = 196, normalized size = 6.32

$$\frac{10\,x\cosh(x)^4 + 40\,x\cosh(x)\sinh(x)^3 + 10\,x\sinh(x)^4 - (10\,x+9)\cosh(x)^2 + \left(60\,x\cosh(x)^2 - 10\,x - 9\right)\sinh(}{4\left(\cosh(x)^4 + 4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 - (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 - 10*x - 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*co
sh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 - (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - c
osh(x))*sinh(x))
```

giac [A] time = 0.12, size = 36, normalized size = 1.16

$$\frac{5}{2}\,x - \frac{\left(9\,e^{(2\,x)} - 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} - 1\right)} - \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] 5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

maple [A] time = 0.06, size = 28, normalized size = 0.90

$$-\coth(x) - \frac{\ln(\coth(x) - 1)}{4} + \frac{1}{2 + 2\coth(x)} + \frac{5\ln(1 + \coth(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(1+coth(x)),x)
[Out] -coth(x)-1/4*ln(coth(x)-1)+1/2/(1+coth(x))+5/4*ln(1+coth(x))
```

maxima [A] time = 0.35, size = 38, normalized size = 1.23

$$\frac{1}{2}\,x + \frac{2}{e^{(-2\,x)} - 1} - \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-x)} + 1\right) - \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 2/(e^(-2*x) - 1) - 1/4*e^(-2*x) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

mupad [B] time = 1.16, size = 21, normalized size = 0.68

$$\frac{x}{2} + \ln(\coth(x) + 1) - \coth(x) + \frac{1}{2\,(\coth(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(coth(x) + 1),x)
```

```
[Out] x/2 + log(coth(x) + 1) - coth(x) + 1/(2*(coth(x) + 1))
```

sympy [B] time = 0.93, size = 160, normalized size = 5.16

$$\frac{x\tanh^{2}(x)}{2\tanh^{2}(x)+2\tanh(x)}+\frac{x\tanh(x)}{2\tanh^{2}(x)+2\tanh(x)}+\frac{2\log(\tanh(x)+1)\tanh^{2}(x)}{2\tanh^{2}(x)+2\tanh(x)}+\frac{2\log(\tanh(x)+1)\tanh(x)}{2\tanh^{2}(x)+2\tanh(x)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + x*tanh(x)/(2*tanh(x)**2 + 2*tanh(
x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + 2*log(tanh
(x) + 1)*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*
tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*tanh(x)**2 + 2*tanh(x))
 - 3*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2/(2*tanh(x)**2 + 2*tanh(x))
```

3.131 $\int \frac{\coth^4(x)}{1+\coth(x)} dx$

Optimal. Leaf size=37

$$-\frac{3x}{2} + \frac{\coth^3(x)}{2(\coth(x)+1)} - \coth^2(x) + \frac{3\coth(x)}{2} + 2\log(\sinh(x))$$

```
[Out] -3/2*x+3/2*coth(x)-coth(x)^2+1/2*coth(x)^3/(1+coth(x))+2*ln(sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3550, 3528, 3525, 3475}

$$-\frac{3x}{2} + \frac{\coth^3(x)}{2(\coth(x)+1)} - \coth^2(x) + \frac{3\coth(x)}{2} + 2\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + 2*Log[
Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3550

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(c + d*Tan[e + f*x])^(n - 1))/
(2*a*f*(a + b*Tan[e + f*x])), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{\coth^3(x)}{2(1+\coth(x))} - \frac{1}{2}\int (3-4\coth(x))\coth^2(x)\,dx$$
$$= -\coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + \frac{1}{2}i\int (-4i+3i\coth(x))\coth(x)\,dx$$
$$= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + 2\int \coth(x)\,dx$$
$$= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + 2\log(\sinh(x))$$

Mathematica [**A**] time = 0.06, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(-6x - \sinh(2x) + \cosh(2x) + 4\coth(x) - 2\operatorname{csch}^2(x) + 8\log(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-6*x + Cosh[2*x] + 4*Coth[x] - 2*Csch[x]^2 + 8*Log[Sinh[x]] - Sinh[2*x])/4
```

fricas [**B**] time = 0.41, size = 357, normalized size = 9.65

$$-\frac{14\,x\cosh(x)^6 + 84\,x\cosh(x)\sinh(x)^5 + 14\,x\sinh(x)^6 - (28\,x+1)\cosh(x)^4 + \left(210\,x\cosh(x)^2 - 28\,x - 1\right)\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 - (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 - 28*x - 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 - (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 -
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh
(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*sinh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 - (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) - 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

giac [**A**] time = 0.13, size = 40, normalized size = 1.08

$$-\frac{7}{2}\,x + \frac{\left(e^{(4\,x)} - 10\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} - 1\right)^2} + 2\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] -7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(ab
s(e^(2*x) - 1))
```

maple [**A**] time = 0.07, size = 32, normalized size = 0.86

$$-\frac{\left(\coth^2(x)\right)}{2} + \coth(x) - \frac{\ln\left(\coth(x) - 1\right)}{4} - \frac{1}{2\left(1 + \coth(x)\right)} - \frac{7\ln\left(1 + \coth(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(1+coth(x)),x)
```

```
[Out] -1/2*coth(x)^2+coth(x)-1/4*ln(coth(x)-1)-1/2/(1+coth(x))-7/4*ln(1+coth(x))
```

maxima [A] time = 0.31, size = 54, normalized size = 1.46

$$\frac{1}{2}\,x + \frac{2\left(2\,e^{(-2\,x)} - 1\right)}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-x)} + 1\right) + 2\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) - 1)/(2*e^(-2*x) - e^(-4*x) - 1) + 1/4*e^(-2*x) + 2*l
og(e^(-x) + 1) + 2*log(e^(-x) - 1)
```

mupad [B] time = 0.07, size = 29, normalized size = 0.78

$$\frac{x}{2} - 2\,\ln\left(\coth(x) + 1\right) + \coth(x) - \frac{\coth(x)^2}{2} - \frac{1}{2\,\left(\coth(x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(coth(x) + 1),x)
```

```
[Out] x/2 - 2*log(coth(x) + 1) + coth(x) - coth(x)^2/2 - 1/(2*(coth(x) + 1))
```

sympy [B] time = 1.18, size = 197, normalized size = 5.32

$$\frac{x\tanh^3(x)}{2\tanh^3(x) + 2\tanh^2(x)} + \frac{x\tanh^2(x)}{2\tanh^3(x) + 2\tanh^2(x)} - \frac{4\log(\tanh(x) + 1)\tanh^3(x)}{2\tanh^3(x) + 2\tanh^2(x)} - \frac{4\log(\tanh(x) + 1)\tanh}{2\tanh^3(x) + 2\tanh^2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + x*tanh(x)**2/(2*tanh(x)**3 + 2
*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2)
- 4*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(
x))*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**2/(2
*tanh(x)**3 + 2*tanh(x)**2) + 3*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) +
tanh(x)/(2*tanh(x)**3 + 2*tanh(x)**2) - 1/(2*tanh(x)**3 + 2*tanh(x)**2)
```

3.132 $\int \coth(x)(1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(1+coth(x))^(3/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3527, 3478, 3480, 206}

$$-\frac{2}{3}(\coth(x)+1)^{3/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3527

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \coth(x)(1+\coth(x))^{3/2}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2} + \int (1+\coth(x))^{3/2}\,dx\\ &= -2\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2} + 2\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2} + 4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right.\\ &= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [C] time = 0.14, size = 90, normalized size = 2.00

$$-\frac{2(\coth(x)+1)^{3/2}\left(\cosh(x)\sqrt{i(\coth(x)+1)} + 4\sinh(x)\sqrt{i(\coth(x)+1)} - (3-3i)\sinh(x)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\right.\right.}{3\sqrt{i(\coth(x)+1)}\,(\sinh(x)+\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(1 + Coth[x])^(3/2)*(Cosh[x]*Sqrt[I*(1 + Coth[x])] - (3 - 3*I)*ArcTan[(
1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*Sinh[x] + 4*Sqrt[I*(1 + Coth[x])]*Sinh[x]
))/(3*Sqrt[I*(1 + Coth[x])]*(Cosh[x] + Sinh[x]))
```

fricas [B] time = 0.39, size = 259, normalized size = 5.76

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh(x)^3 + 15\sqrt{2}\cosh(x)\sinh(x)^2 + 5\sqrt{2}\sinh(x)^3 + 3\left(5\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x) - 3\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x)*sinh(x)^2 + 5*sqr
t(2)*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

giac [B] time = 0.15, size = 135, normalized size = 3.00

$$-\frac{1}{3}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}} - 2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right) + \frac{2\left(9\left(\sqrt{e^{(4x)}-e^{(2x)}} - e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)}-1\right)+1\right.}{\left(\sqrt{e^{(4x)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 2*(9*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 1
```

```
2*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + 5*sgn(e^(2*x) - 1)
)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

maple [A] time = 0.05, size = 35, normalized size = 0.78

$$-\frac{2\left(1+\coth(x)\right)^{\frac{3}{2}}}{3}+2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(1+coth(x))^(3/2),x)
[Out] -2/3*(1+coth(x))^(3/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\coth(x)+1\right)^{\frac{3}{2}}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="maxima")
[Out] integrate((coth(x) + 1)^(3/2)*coth(x), x)
```

mupad [B] time = 1.24, size = 34, normalized size = 0.76

$$2\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\mathrm{coth}(x)+1}}{2}\right)-2\,\sqrt{\mathrm{coth}(x)+1}-\frac{2\,{\left(\mathrm{coth}(x)+1\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(coth(x) + 1)^(3/2),x)
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
(2*(coth(x) + 1)^(3/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\coth{\left(x \right)}+1\right)^{\frac{3}{2}}\coth{\left(x \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))**(3/2),x)
[Out] Integral((coth(x) + 1)**(3/2)*coth(x), x)
```

3.133 $\int \coth(x)\sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=32

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3527, 3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-2\sqrt{\coth(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3527

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{1+\coth(x)}\,dx &= -2\sqrt{1+\coth(x)}+\int \sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}+2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [C] time = 0.12, size = 53, normalized size = 1.66

$$(1+i)\sqrt{\coth(x)+1}\left(-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}}-(1-i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[1 + Coth[x]],x]
```

```
[Out] (1 + I)*Sqrt[1 + Coth[x]]*((-1 + I) - (I*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Cot
h[x])]])/Sqrt[I*(1 + Coth[x])])
```

fricas [B] time = 0.40, size = 131, normalized size = 4.09

$$-\frac{4\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2-\sqrt{2}\right.}{2\left(\cosh(x)^2+2\cosh(x)\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
 sinh(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

giac [B] time = 0.16, size = 71, normalized size = 2.22

$$-\frac{1}{2}\sqrt{2}\left(\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{4\operatorname{sgn}\left(e^{(2x)}-1\right)}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*
x) - 1) + 4*sgn(e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1))
```

maple [A] time = 0.06, size = 26, normalized size = 0.81

$$\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(1+coth(x))^(1/2),x)
```

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\coth(x)+1}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x), x)
```

mupad [B] time = 1.20, size = 25, normalized size = 0.78

$$\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)-2\sqrt{\coth(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(coth(x) + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth(x) + 1} \coth(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*coth(x), x)
```

3.134 $\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=30

$$\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3526, 3480, 206}

$$\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/Sqrt[1 + Coth[x]],x]

[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx &= \frac{1}{\sqrt{1+\coth(x)}}+\frac{1}{2}\int \sqrt{1+\coth(x)}\,dx\\
&= \frac{1}{\sqrt{1+\coth(x)}}+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] time = 0.20, size = 97, normalized size = 3.23

$$\frac{\text{csch}(x)\left(\frac{1}{2}\sinh(2x)-\frac{1}{2}\cosh(2x)+\frac{1}{2}\right)(\sinh(x)+\cosh(x))}{\sqrt{\coth(x)+1}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\text{csch}(x)(\sinh(x)+\cosh(x))\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\right.}{\sqrt{i\coth(x)+i}\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(1/2 + I/2)*Sqrt[I + I*Coth[x]]]*Csch[x]*(Cosh[x] + Sin
h[x]))/(Sqrt[I + I*Coth[x]]*Sqrt[1 + Coth[x]]) + (Csch[x]*(Cosh[x] + Sinh[x
])*(1/2 - Cosh[2*x]/2 + Sinh[2*x]/2))/Sqrt[1 + Coth[x]]
```

fricas [B] time = 0.40, size = 85, normalized size = 2.83

$$\frac{\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x))+2\cosh(x)^2+4\cosh(x)\sinh(x)\right.}{4(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) + 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

giac [B] time = 0.17, size = 88, normalized size = 2.93

$$-\frac{1}{2}\sqrt{2}\,\mathrm{sgn}\left(e^{(2x)}-1\right)-\frac{\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}-\frac{\sqrt{2}}{2\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sgn(e^(2*x) - 1) - 1/4*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x
)) - 2*e^(2*x) + 1))/sgn(e^(2*x) - 1) - 1/2*sqrt(2)/((sqrt(e^(4*x) - e^(2*x
)) - e^(2*x))*sgn(e^(2*x) - 1))
```

maple [A] time = 0.08, size = 25, normalized size = 0.83

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1}{\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth(x)}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(coth(x) + 1), x)
```

mupad [B] time = 1.23, size = 24, normalized size = 0.80

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2}+\frac{1}{\sqrt{\coth(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(coth(x) + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 + 1/(coth(x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(coth(x) + 1), x)
```

3.135 $\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2\sqrt{\coth(x)+1}}+\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/
2/(1+coth(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3526, 3479, 3480, 206}

$$-\frac{1}{2\sqrt{\coth(x)+1}}+\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx &= \frac{1}{3(1+\coth(x))^{3/2}}+\frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\ &= \frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{4}\int \sqrt{1+\coth(x)}\,dx\\ &= \frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{1}{3(1+\coth(x))^{3/2}}-\frac{1}{2\sqrt{1+\coth(x)}}\end{aligned}$$

Mathematica [C] time = 0.35, size = 84, normalized size = 1.71

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\coth(x)+1}\left(\left(\frac{1}{6}-\frac{i}{6}\right)(-\sinh(2x)-\sinh(4x)+\cosh(2x)+\cosh(4x)-2)-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(}{\sqrt{i(\coth(x)+1)}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 + Coth[x])^(3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] + (1/6 - I/6)*(-2 + Cosh[2*x] + Cosh[4*x] - Sinh
[2*x] - Sinh[4*x]))
```

fricas [B] time = 0.40, size = 166, normalized size = 3.39

$$-\frac{2\sqrt{2}\left(2\sqrt{2}\cosh(x)^2+4\sqrt{2}\cosh(x)\sinh(x)+2\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\right.}{24\left(\cosh(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(2*sqrt(2)*cosh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x) + 2*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

giac [B] time = 0.22, size = 107, normalized size = 2.18

$$-\frac{1}{24}\sqrt{2}\left(\frac{3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{\operatorname{sgn}\left(e^{(2x)}-1\right)}+\frac{2\left(3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3\operatorname{sgn}\left(e^{(2x)}-1\right)}-4\operatorname{sgn}\left(e^{(2x)}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))/sgn(e^
(2*x) - 1) + 2*(3*sqrt(e^(4*x) - e^(2*x)) - 3*e^(2*x) + 1)/((sqrt(e^(4*x) -
 e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1)) - 4*sgn(e^(2*x) - 1))
```

maple [A] time = 0.08, size = 35, normalized size = 0.71

$$\frac{1}{3\left(1+\coth(x)\right)^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1+coth(x))^(3/2),x)
```

```
[Out] 1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/
2/(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\left(\coth(x)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(coth(x) + 1)^(3/2), x)
```

mupad [B] time = 1.22, size = 32, normalized size = 0.65

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4}-\frac{\frac{\coth(x)}{2}+\frac{1}{6}}{\left(\coth(x)+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 1/6)/(cot
h(x) + 1)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth{\left(x \right)}}{\left(\coth{\left(x \right)}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1+coth(x))**(3/2),x)
```

```
[Out] Integral(coth(x)/(coth(x) + 1)**(3/2), x)
```

3.136 $\int \coth^2(x)(1+\coth(x))^{3/2}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[Out] -2/5*(1+coth(x))^(5/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3543, 3478, 3480, 206}

$$-\frac{2}{5}(\coth(x)+1)^{5/2}-2\sqrt{\coth(x)+1}+2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(5/2))/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = -\frac{2}{5}(1+\coth(x))^{5/2} + \int (1+\coth(x))^{3/2}\,dx$$
$$= -2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2} + 2\int \sqrt{1+\coth(x)}\,dx$$
$$= -2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2} + 4\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x}\right.$$
$$= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2}$$

Mathematica [C] time = 0.26, size = 70, normalized size = 1.56

$$-\frac{2\left(2\coth^2(x) + \text{csch}^2(x) + (5+5i)\sqrt{i(\coth(x)+1)}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right) + \coth(x)\left(\text{csch}^2(x) + 9\right.\right.}{5\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] (-2*(7 + 2*Coth[x]^2 + (5 + 5*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]]*
Sqrt[I*(1 + Coth[x])] + Csch[x]^2 + Coth[x]*(9 + Csch[x]^2)))/(5*Sqrt[1 + C
oth[x]])
```

fricas [B] time = 0.40, size = 436, normalized size = 9.69

$$-\frac{2\sqrt{2}\left(9\sqrt{2}\cosh(x)^5 + 45\sqrt{2}\cosh(x)\sinh(x)^4 + 9\sqrt{2}\sinh(x)^5 + 10\left(9\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x)^3 - 10\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(2)*(9*sqrt(2)*cosh(x)^5 + 45*sqrt(2)*cosh(x)*sinh(x)^4 + 9*sqr
t(2)*sinh(x)^5 + 10*(9*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^3 - 10*sqrt(2)*
cosh(x)^3 + 30*(3*sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x)^2 + 5*(9*sqr
t(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 5*sqrt(2)*cosh(x)
)*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 5*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh
(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(
x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*si
nh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2
 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(
2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x
)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1
))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(
x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4
 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 +
cosh(x))*sinh(x) - 1)
```

giac [B] time = 0.17, size = 197, normalized size = 4.38

$$-\frac{1}{5}\sqrt{2}\left(5\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\text{sgn}\left(e^{(2x)}-1\right) + \frac{2\left(25\left(\sqrt{e^{(4x)}-e^{(2x)}} - e^{(2x)}\right)^4\text{sgn}\left(e^{(2x)}-1\right) + \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(2)*(5*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 2*(25*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4*sgn(e^(2*x) - 1) +
60*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1) + 70*(sqrt(e^(4*x
) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 40*(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x))*sgn(e^(2*x) - 1) + 9*sgn(e^(2*x) - 1))/(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x) + 1)^5)
```

maple [A] time = 0.07, size = 35, normalized size = 0.78

$$-\frac{2\left(1+\coth(x)\right)^{\frac{5}{2}}}{5}+2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(1+coth(x))^(3/2),x)
```

```
[Out] -2/5*(1+coth(x))^(5/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\coth(x)+1\right)^{\frac{3}{2}}\coth(x)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2)*coth(x)^2, x)
```

mupad [B] time = 1.25, size = 34, normalized size = 0.76

$$2\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\mathrm{coth}(x)+1}}{2}\right)-2\,\sqrt{\mathrm{coth}(x)+1}-\frac{2\,{\left(\mathrm{coth}(x)+1\right)}^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(coth(x) + 1)^(3/2),x)
```

```
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
(2*(coth(x) + 1)^(5/2))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\coth{\left(x \right)}+1\right)^{\frac{3}{2}}\coth^{2}{\left(x \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2)*coth(x)**2, x)
```

3.137 $\int \coth^2(x)\sqrt{1+\coth(x)}\,dx$

Optimal. Leaf size=34

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

```
[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3543, 3480, 206}

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\coth(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \coth^2(x)\sqrt{1+\coth(x)}\,dx &= -\frac{2}{3}(1+\coth(x))^{3/2}+\int \sqrt{1+\coth(x)}\,dx\\ &= -\frac{2}{3}(1+\coth(x))^{3/2}+2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [C] time = 0.18, size = 61, normalized size = 1.79

$$\frac{-2\coth^2(x)-4\coth(x)-(3+3i)\sqrt{i(\coth(x)+1)}\,\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)-2}{3\sqrt{\coth(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

```
[Out] (-2 - 4*Coth[x] - 2*Coth[x]^2 - (3 + 3*I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Co
th[x])]]*Sqrt[I*(1 + Coth[x])])/(3*Sqrt[1 + Coth[x]])
```

fricas [B] time = 0.40, size = 242, normalized size = 7.12

$$-\frac{8\sqrt{2}\left(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)^2\sinh(x) + 3\sqrt{2}\cosh(x)\sinh(x)^2 + \sqrt{2}\sinh(x)^3\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - 3\left(\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(8*sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2
)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*sqrt(sinh(x)/(cosh(x) - sinh(x)))
- 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt
(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh
(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh
(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) +
 1)
```

giac [B] time = 0.16, size = 133, normalized size = 3.91

$$-\frac{1}{6}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)} - e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\operatorname{sgn}\left(e^{(2x)} - 1\right) + \frac{8\left(3\left(\sqrt{e^{(4x)} - e^{(2x)}} - e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)} - 1\right) + 3\left(\right.\right.}{\left(\sqrt{e^{(4x)} - e^{(}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 8*(3*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 3
*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + sgn(e^(2*x) - 1))/(
sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

maple [A] time = 0.08, size = 26, normalized size = 0.76

$$-\frac{2\left(1+\coth(x)\right)^{\frac{3}{2}}}{3} + \operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(1+coth(x))^(1/2),x)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\coth(x)+1}\,\coth(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x)^2, x)
```

mupad [B] time = 1.19, size = 25, normalized size = 0.74

$$\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)-\frac{2\,(\coth(x)+1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2*(coth(x) + 1)^(1/2),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - (2*(coth(x) + 1)^(3/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\coth\left(x\right)+1}\,\coth^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2*(1+coth(x))**(1/2),x)
[Out] Integral(sqrt(coth(x) + 1)*coth(x)**2, x)
```

3.138 $\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal. Leaf size=42

$$-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1
+coth(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3543, 3479, 3480, 206}

$$-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[1
 + Coth[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3543

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(d^2*(a + b*Tan[e + f*x])^(m + 1))/(b*f*
(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx &= -2\sqrt{1+\coth(x)}\, + \int \frac{1}{\sqrt{1+\coth(x)}}\,dx\\
&= -\frac{1}{\sqrt{1+\coth(x)}} - 2\sqrt{1+\coth(x)}\, + \frac{1}{2}\int \sqrt{1+\coth(x)}\,dx\\
&= -\frac{1}{\sqrt{1+\coth(x)}} - 2\sqrt{1+\coth(x)}\, + \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1+\coth(x)}} - 2\sqrt{1+\coth(x)}
\end{aligned}
$$

Mathematica [C] time = 0.37, size = 81, normalized size = 1.93

$$
\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\operatorname{csch}(x)(\sinh(x)+\cosh(x))\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-\sinh(2x)+\cosh(2x)-5)-\frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(\coth(x)+1)}\right)}{\sqrt{i(\coth(x)+1)}}\right)}{\sqrt{\coth(x)+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

```
[Out] ((1/2 + I/2)*Csch[x]*(Cosh[x] + Sinh[x])*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(
1 + Coth[x])]])/Sqrt[I*(1 + Coth[x])] + (1/2 - I/2)*(-5 + Cosh[2*x] - Sinh[
2*x])))/Sqrt[1 + Coth[x]]
```

fricas [B] time = 0.40, size = 189, normalized size = 4.50

$$
-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh(x)^2+10\sqrt{2}\cosh(x)\sinh(x)+5\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^3+3\,\cdots\right.}{4\left(\mathrm{c}\cdots\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^2 + 10*sqrt(2)*cosh(x)*sinh(x) + 5*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x
)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^
2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x)
 - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh
(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1
)*sinh(x) - cosh(x))
```

giac [B] time = 0.18, size = 88, normalized size = 2.10

$$
-\frac{\frac{5\sqrt{2}e^{(2x)}}{\operatorname{sgn}\left(e^{(2x)}-1\right)}-\frac{\sqrt{2}}{\operatorname{sgn}\left(e^{(2x)}-1\right)}}{2\sqrt{e^{(4x)}-e^{(2x)}}}-\frac{\sqrt{2}\log\left(\left|4\sqrt{e^{(4x)}-e^{(2x)}}-4e^{(2x)}+2\right|\right)}{4\operatorname{sgn}\left(e^{(2x)}-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(2)*e^(2*x)/sgn(e^(2*x) - 1) - sqrt(2)/sgn(e^(2*x) - 1))/sqrt(e
^(4*x) - e^(2*x)) - 1/4*sqrt(2)*log(abs(4*sqrt(e^(4*x) - e^(2*x)) - 4*e^(2*
x) + 2))/sgn(e^(2*x) - 1)
```

maple [A] time = 0.09, size = 35, normalized size = 0.83

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x))^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1
+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/sqrt(coth(x) + 1), x)
```

mupad [B] time = 1.26, size = 36, normalized size = 0.86

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2}-\frac{3}{\sqrt{\coth(x)+1}}-\frac{2\coth(x)}{\sqrt{\coth(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(coth(x) + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 3/(coth(x) + 1)^(1/2)
- (2*coth(x))/(coth(x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}(x)}{\sqrt{\coth(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(coth(x)**2/sqrt(coth(x) + 1), x)
```

3.139 $\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx$

Optimal. Leaf size=49

$$\frac{3}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3
/2/(1+coth(x))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3540, 3526, 3480, 206}

$$\frac{3}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
+ 3/(2*Sqrt[1 + Coth[x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3526

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^m)/(2*a*
f*m), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0
] && LtQ[m, 0]
```

Rule 3540

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(b*(a*c + b*d)^2*(a + b*Tan[e + f*x])^m)
/(2*a^3*f*m), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*
c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = -\frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2}\int \frac{1-2\coth(x)}{\sqrt{1+\coth(x)}}\,dx$$
$$= -\frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}} + \frac{1}{4}\int \sqrt{1+\coth(x)}\,dx$$
$$= -\frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}}$$

Mathematica [C] time = 0.37, size = 86, normalized size = 1.76

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{\coth(x)+1}\left(-\left(\frac{1}{6}-\frac{i}{6}\right)(-7\sinh(2x)-\sinh(4x)+7\cosh(2x)+\cosh(4x)-8) - \frac{i\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(c}\right.}{\sqrt{i(\coth(x)}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

```
[Out] (1/4 + I/4)*Sqrt[1 + Coth[x]]*(((-I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x]
)]])/Sqrt[I*(1 + Coth[x])] - (1/6 - I/6)*(-8 + 7*Cosh[2*x] + Cosh[4*x] - 7*
Sinh[2*x] - Sinh[4*x]))
```

fricas [B] time = 0.41, size = 166, normalized size = 3.39

$$\frac{2\sqrt{2}\left(8\sqrt{2}\cosh(x)^2+16\sqrt{2}\cosh(x)\sinh(x)+8\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}+3\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\right.}{24\left(\cosh(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(2)*(8*sqrt(2)*cosh(x)^2 + 16*sqrt(2)*cosh(x)*sinh(x) + 8*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

giac [B] time = 0.19, size = 135, normalized size = 2.76

$$-\frac{2}{3}\sqrt{2}\operatorname{sgn}\left(e^{(2x)}-1\right) - \frac{\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)}{8\operatorname{sgn}\left(e^{(2x)}-1\right)} - \frac{\sqrt{2}\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2-3\sqrt{e^{(4x)}-e^{(2x)}}\right.}{12\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3\operatorname{sgn}\left(e^{(2x)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(2)*sgn(e^(2*x) - 1) - 1/8*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x
)) - 2*e^(2*x) + 1))/sgn(e^(2*x) - 1) - 1/12*sqrt(2)*(6*(sqrt(e^(4*x) - e^(
2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x) - e^(2*x)) + 3*e^(2*x) - 1)/((sqrt(e^(4
*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1))
```

maple [A] time = 0.08, size = 35, normalized size = 0.71

$$-\frac{1}{3\left(1+\coth(x)\right)^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{3}{2\sqrt{1+\coth(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(1+coth(x))^(3/2),x)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3
/2/(1+coth(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{\left(\coth(x)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(coth(x) + 1)^(3/2), x)
```

mupad [B] time = 1.23, size = 31, normalized size = 0.63

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4}+\frac{\frac{3\coth(x)}{2}+\frac{7}{6}}{\left(\coth(x)+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 + ((3*coth(x))/2 + 7/6)/
(coth(x) + 1)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}(x)}{\left(\coth(x)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(1+coth(x))**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(coth(x) + 1)**(3/2), x)
```

3.140 $\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=97

$$\frac{ax}{a^2-b^2}+\frac{b\tanh^2(x)}{2a^2}-\frac{b\left(a^2+b^2\right)\log(\cosh(x))}{a^4}-\frac{b^5\log(a\sinh(x)+b\cosh(x))}{a^4\left(a^2-b^2\right)}-\frac{\left(a^2+b^2\right)\tanh(x)}{a^3}-\frac{\tanh^3(x)}{3a}$$

```
[Out] a*x/(a^2-b^2)-b*(a^2+b^2)*ln(cosh(x))/a^4-b^5*ln(b*cosh(x)+a*sinh(x))/a^4/(
a^2-b^2)-(a^2+b^2)*tanh(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Rubi [A] time = 0.53, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3569, 3649, 3650, 3651, 3530, 3475}

$$\frac{ax}{a^2-b^2}-\frac{\left(a^2+b^2\right)\tanh(x)}{a^3}-\frac{b\left(a^2+b^2\right)\log(\cosh(x))}{a^4}-\frac{b^5\log(a\sinh(x)+b\cosh(x))}{a^4\left(a^2-b^2\right)}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*(a^2 + b^2)*Log[Cosh[x]])/a^4 - (b^5*Log[b*Cosh[x] +
 a*Sinh[x]])/(a^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/
(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
```

```
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n
+ 1))/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx &= -\frac{\tanh^3(x)}{3a} - \frac{i\int \frac{\left(-3ib+3ia\coth(x)+3ib\coth^2(x)\right)\tanh^3(x)}{a+b\coth(x)}\,dx}{3a} \\
&= \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{\int \frac{\left(-6\left(a^2+b^2\right)+6b^2\coth^2(x)\right)\tanh^2(x)}{a+b\coth(x)}\,dx}{6a^2} \\
&= -\frac{\left(a^2+b^2\right)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} + \frac{i\int \frac{\left(6ib\left(a^2+b^2\right)-6ia^3\coth(x)-6ib\left(a^2+b^2\right)\coth^2(x)\right)}{a+b\coth(x)}}{6a^3} \\
&= \frac{ax}{a^2-b^2} - \frac{\left(a^2+b^2\right)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{\left(ib^5\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^4\left(a^2-b^2\right)} - \frac{\left(b\left(a^2\right.\right.}{} \\
&= \frac{ax}{a^2-b^2} - \frac{b\left(a^2+b^2\right)\log(\cosh(x))}{a^4} - \frac{b^5\log(b\cosh(x)+a\sinh(x))}{a^4\left(a^2-b^2\right)} - \frac{\left(a^2+b^2\right)\tanh(x)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 105, normalized size = 1.08

$$\frac{6a^5x+6\left(b^5-a^4b\right)\log(\cosh(x))+a^2\left(a^2-b^2\right)\operatorname{sech}^2(x)(2a\tanh(x)-3b)+\left(-8a^5+2a^3b^2+6ab^4\right)\tanh(x)-}{6a^4(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (6*a^5*x + 6*(-(a^4*b) + b^5)*Log[Cosh[x]] - 6*b^5*Log[b*Cosh[x] + a*Sinh[x
]] + (-8*a^5 + 2*a^3*b^2 + 6*a*b^4)*Tanh[x] + a^2*(a^2 - b^2)*Sech[x]^2*(-3
*b + 2*a*Tanh[x]))/(6*a^4*(a - b)*(a + b))
```

fricas [B] time = 0.45, size = 1294, normalized size = 13.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 + a^4*b)*x*cosh(x)^6 + 18*(a^5 + a^4*b)*x*cosh(x)*sinh(x)^5 + 3
*(a^5 + a^4*b)*x*sinh(x)^6 + 8*a^5 - 2*a^3*b^2 - 6*a*b^4 + 3*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^4 + 3*(4*
a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 15*(a^5 + a^4*b)*x*cosh(x
)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^4 + 12*(5*(a^5 + a^4*b)*x*cosh(x)^3 + (4*a
^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)
*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*a^2*b^3
- 4*a*b^4 + 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 +
 a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 + 3*(a^5 + a^4*b)*x - 3
*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4
 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cosh(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*c
osh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2
+ b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4*b - b^5)*co
sh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)*sinh(x)^6 + a^4
*b - b^5 + 3*(a^4*b - b^5)*cosh(x)^4 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 + 3*(a^4*b - b^5)*cosh(x))
*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*c
osh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b - b^5)*cosh(x)^
5 + 2*(a^4*b - b^5)*cosh(x)^3 + (a^4*b - b^5)*cosh(x))*sinh(x))*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 6*(3*(a^5 + a^4*b)*x*cosh(x)^5 + 2*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^3 + (4*a^
5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x))*sinh(x))/((
a^6 - a^4*b^2)*cosh(x)^6 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4
*b^2)*sinh(x)^6 + a^6 - a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^
4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(
x)^3 + 3*(a^6 - a^4*b^2)*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 +
 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)
^2)*sinh(x)^2 + 6*((a^6 - a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3
+ (a^6 - a^4*b^2)*cosh(x))*sinh(x))
```

giac [A] time = 0.14, size = 141, normalized size = 1.45

$$-\frac{b^5\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^6-a^4b^2}+\frac{x}{a-b}-\frac{\left(a^2b+b^3\right)\log\left(e^{(2x)}+1\right)}{a^4}+\frac{2\left(4a^3+3ab^2+3\left(2a^3-a^2b+ab^2\right)e^{(4x)}+\right.}{3a^4\left(e^{(2x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^6 - a^4*b^2) + x/(a - b) -
(a^2*b + b^3)*log(e^(2*x) + 1)/a^4 + 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 - a^2*
b + a*b^2)*e^(4*x) + 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x) + 1
)^3)
```

maple [B] time = 0.15, size = 283, normalized size = 2.92

$$-\frac{b^5\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a+b)(a-b)a^4}-\frac{64\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{64a+64b}+\frac{64\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{64a-64b}-\frac{2\left(\tanh^5\left(\frac{x}{2}\right)\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}-\frac{2\left(\mathrm{ta}\right.}{a^3\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*coth(x)),x)
```

```
[Out] -b^5/(a+b)/(a-b)/a^4*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)-64/(64*a+64*b)*l
n(tanh(1/2*x)-1)+64/(64*a-64*b)*ln(tanh(1/2*x)+1)-2/a/(tanh(1/2*x)^2+1)^3*t
anh(1/2*x)^5-2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^2+2/a^2/(tanh(1/2*x)
^2+1)^3*b*tanh(1/2*x)^4-20/3/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3-4/a^3/(tan
h(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2+2/a^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^2*b
-2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)-2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*
b^2-1/a^2*b*ln(tanh(1/2*x)^2+1)-1/a^4*ln(tanh(1/2*x)^2+1)*b^3
```

maxima [A] time = 0.41, size = 146, normalized size = 1.51

$$-\frac{b^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-a^4b^2}-\frac{2\left(4a^2+3b^2+3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(2a^2+ab+b^2\right)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}+3a^3e^{(-4x)}+a^3e^{(-6x)}+a^3\right)}+\frac{x}{a+b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - a^4*b^2) - 2/3*(4*a^2 + 3*b^2 +
3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(2*a^2 + a*b + b^2)*e^(-4*x))/(3*a^3*e
^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) + x/(a + b) - (a^2*b + b^3)*
log(e^(-2*x) + 1)/a^4
```

mupad [B] time = 1.61, size = 163, normalized size = 1.68

$$\frac{8}{3a\left(3e^{2x}+3e^{4x}+e^{6x}+1\right)}+\frac{x}{a-b}-\frac{b^5\ln\left(b-a+ae^{2x}+be^{2x}\right)}{a^6-a^4b^2}-\frac{\ln\left(e^{2x}+1\right)\left(a^2b+b^3\right)}{a^4}+\frac{2\left(2a^3+a^2b+\right.}{a^3(a+b)\left(e^{2x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*coth(x)),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) + x/(a - b) - (b^5*log(b -
 a + a*exp(2*x) + b*exp(2*x)))/(a^6 - a^4*b^2) - (log(exp(2*x) + 1)*(a^2*b
+ b^3))/a^4 + (2*(a^2*b + 2*a^3 + b^3))/(a^3*(a + b)*(exp(2*x) + 1)) - (2*(
a*b + 2*a^2 - b^2))/(a^2*(a + b)*(2*exp(2*x) + exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*coth(x)), x)
```

3.141 $\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=76

$$-\frac{bx}{a^2-b^2}+\frac{b\tanh(x)}{a^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)}-\frac{\tanh^2(x)}{2a}$$

```
[Out] -b*x/(a^2-b^2)+(a^2+b^2)*ln(cosh(x))/a^3+b^4*ln(b*cosh(x)+a*sinh(x))/a^3/(a
^2-b^2)+b*tanh(x)/a^2-1/2*tanh(x)^2/a
```

Rubi [A] time = 0.33, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3569, 3649, 3652, 3530, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3}+\frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + ((a^2 + b^2)*Log[Cosh[x]])/a^3 + (b^4*Log[b*Cosh[x]
+ a*Sinh[x]])/(a^3*(a^2 - b^2)) + (b*Tanh[x])/a^2 - Tanh[x]^2/(2*a)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3649

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2 - a*(b*B - a*C))*(a + b*Tan[e
+ f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 +
 b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3652

```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[((a*
(A*c - c*C) - b*(A*d - C*d))*x)/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
 + f*x]), x], x] - Dist[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx &= -\frac{\tanh^2(x)}{2a}-\frac{i\int \frac{\left(-2ib+2ia\coth(x)+2ib\coth^2(x)\right)\tanh^2(x)}{a+b\coth(x)}\,dx}{2a}\\
&= \frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}-\frac{\int \frac{\left(-2\left(a^2+b^2\right)+2b^2\coth^2(x)\right)\tanh(x)}{a+b\coth(x)}\,dx}{2a^2}\\
&= -\frac{bx}{a^2-b^2}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}+\frac{\left(ib^4\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^3\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\int \tanh(x)\,dx}{a^3}\\
&= -\frac{bx}{a^2-b^2}+\frac{\left(a^2+b^2\right)\log(\cosh(x))}{a^3}+\frac{b^4\log(b\cosh(x)+a\sinh(x))}{a^3\left(a^2-b^2\right)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh}{2}\end{aligned}$$

Mathematica [A] time = 0.33, size = 88, normalized size = 1.16

$$\frac{a^2\left(a^2-b^2\right)\operatorname{sech}^2(x)+2\left(\left(a^4-b^4\right)\log(\cosh(x))-a^3bx+ab\left(a^2-b^2\right)\tanh(x)+b^4\log(a\sinh(x)+b\cosh(x)\right.}{2a^3(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a^2*(a^2 - b^2)*Sech[x]^2 + 2*(-(a^3*b*x) + (a^4 - b^4)*Log[Cosh[x]] + b^4
*Log[b*Cosh[x] + a*Sinh[x]] + a*b*(a^2 - b^2)*Tanh[x]))/(2*a^3*(a - b)*(a +
 b))
```

fricas [B] time = 0.43, size = 637, normalized size = 8.38

$$-\frac{\left(a^4+a^3b\right)x\cosh(x)^4+4\left(a^4+a^3b\right)x\cosh(x)\sinh(x)^3+\left(a^4+a^3b\right)x\sinh(x)^4+2a^3b-2ab^3-2\left(a^4-a^3b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + a^3*b)*x*cosh(x)^4 + 4*(a^4 + a^3*b)*x*cosh(x)*sinh(x)^3 + (a^4 +
a^3*b)*x*sinh(x)^4 + 2*a^3*b - 2*a*b^3 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 -
 (a^4 + a^3*b)*x)*cosh(x)^2 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 - 3*(a^4 + a
^3*b)*x*cosh(x)^2 - (a^4 + a^3*b)*x)*sinh(x)^2 + (a^4 + a^3*b)*x - (b^4*cos
h(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 +
```

```
2*(3*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - ((a^4 - b^4)*cosh(
x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4
+ 2*(a^4 - b^4)*cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 - b^4)*cosh(x)^3 + (a^4 - b^4)*cosh(x))*sinh(x))*log(2*cosh(x)
/(cosh(x) - sinh(x))) + 4*((a^4 + a^3*b)*x*cosh(x)^3 - (a^4 - a^3*b - a^2*b
^2 + a*b^3 - (a^4 + a^3*b)*x)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3
*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sin
h(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh
(x))*sinh(x))
```

giac [A] time = 0.12, size = 97, normalized size = 1.28

$$\frac{b^4\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^5-a^3b^2}-\frac{x}{a-b}+\frac{\left(a^2+b^2\right)\log\left(e^{(2x)}+1\right)}{a^3}-\frac{2\left(ab-\left(a^2-ab\right)e^{(2x)}\right)}{a^3\left(e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^5 - a^3*b^2) - x/(a - b) + (
a^2 + b^2)*log(e^(2*x) + 1)/a^3 - 2*(a*b - (a^2 - a*b)*e^(2*x))/(a^3*(e^(2*
x) + 1)^2)
```

maple [B] time = 0.14, size = 167, normalized size = 2.20

$$\frac{b^4\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a+b)(a-b)a^3}-\frac{32\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{32a+32b}-\frac{32\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{32a-32b}+\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{a^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{2(\mathrm{t}}{a(\mathrm{tan}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*coth(x)),x)
```

```
[Out] b^4/(a+b)/(a-b)/a^3*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)-32/(32*a+32*b)*ln
(tanh(1/2*x)-1)-32/(32*a-32*b)*ln(tanh(1/2*x)+1)+2/a^2/(tanh(1/2*x)^2+1)^2*
tanh(1/2*x)^3*b-2/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^2+2/a^2/(tanh(1/2*x)^2+
1)^2*b*tanh(1/2*x)+1/a*ln(tanh(1/2*x)^2+1)+1/a^3*ln(tanh(1/2*x)^2+1)*b^2
```

maxima [A] time = 0.41, size = 94, normalized size = 1.24

$$\frac{b^4\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^5-a^3b^2}+\frac{2\left((a+b)e^{(-2x)}+b\right)}{2a^2e^{(-2x)}+a^2e^{(-4x)}+a^2}+\frac{x}{a+b}+\frac{\left(a^2+b^2\right)\log\left(e^{(-2x)}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-(a - b)*e^(-2*x) + a + b)/(a^5 - a^3*b^2) + 2*((a + b)*e^(-2*x) +
b)/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^2) + x/(a + b) + (a^2 + b^2)*log(e^(-
2*x) + 1)/a^3
```

mupad [B] time = 1.51, size = 111, normalized size = 1.46

$$\frac{\ln\left(\mathrm{e}^{2x}+1\right)\left(a^2+b^2\right)}{a^3}-\frac{x}{a-b}-\frac{2}{a\left(2\,\mathrm{e}^{2x}+\mathrm{e}^{4x}+1\right)}+\frac{b^4\,\ln\left(b-a+a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}\right)}{a^5-a^3\,b^2}+\frac{2\,\left(a^2-b^2\right)}{a^2\,(a+b)\,\left(\mathrm{e}^{2x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*coth(x)),x)
```

```
[Out] (log(exp(2*x) + 1)*(a^2 + b^2))/a^3 - x/(a - b) - 2/(a*(2*exp(2*x) + exp(4*
x) + 1)) + (b^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^5 - a^3*b^2) + (2*
(a^2 - b^2))/(a^2*(a + b)*(exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*coth(x)), x)
```

3.142 $\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=60

$$\frac{ax}{a^2-b^2}-\frac{b^3\log(a\sinh(x)+b\cosh(x))}{a^2(a^2-b^2)}-\frac{b\log(\cosh(x))}{a^2}-\frac{\tanh(x)}{a}$$

```
[Out] a*x/(a^2-b^2)-b*ln(cosh(x))/a^2-b^3*ln(b*cosh(x)+a*sinh(x))/a^2/(a^2-b^2)-t
anh(x)/a
```

Rubi [A] time = 0.19, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3569, 3651, 3530, 3475}

$$\frac{ax}{a^2-b^2}-\frac{b^3\log(a\sinh(x)+b\cosh(x))}{a^2(a^2-b^2)}-\frac{b\log(\cosh(x))}{a^2}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/a^2 - (b^3*Log[b*Cosh[x] + a*Sinh[x]])
/(a^2*(a^2 - b^2)) - Tanh[x]/a
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3569

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m + 1)*(c
+ d*Tan[e + f*x])^(n + 1))/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d)), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3651

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[((a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*x)
/((a^2 + b^2)*(c^2 + d^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx &= -\frac{\tanh(x)}{a} - \frac{i\int \frac{\left(-ib+ia\coth(x)+ib\coth^2(x)\right)\tanh(x)}{a+b\coth(x)}\,dx}{a} \\
&= \frac{ax}{a^2-b^2} - \frac{\tanh(x)}{a} - \frac{b\int \tanh(x)\,dx}{a^2} - \frac{\left(ib^3\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2\left(a^2-b^2\right)} \\
&= \frac{ax}{a^2-b^2} - \frac{b\log(\cosh(x))}{a^2} - \frac{b^3\log(b\cosh(x)+a\sinh(x))}{a^2\left(a^2-b^2\right)} - \frac{\tanh(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 64, normalized size = 1.07

$$
\frac{\left(ab^2-a^3\right)\tanh(x)+a^3x+\left(b^3-a^2b\right)\log(\cosh(x))-b^3\log(a\sinh(x)+b\cosh(x))}{a^4-a^2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a^3*x + (-(a^2*b) + b^3)*Log[Cosh[x]] - b^3*Log[b*Cosh[x] + a*Sinh[x]] + (
-a^3 + a*b^2)*Tanh[x])/(a^4 - a^2*b^2)
```

fricas [B] time = 0.44, size = 264, normalized size = 4.40

$$
\frac{\left(a^3+a^2b\right)x\cosh(x)^2+2\left(a^3+a^2b\right)x\cosh(x)\sinh(x)+\left(a^3+a^2b\right)x\sinh(x)^2+2a^3-2ab^2+\left(a^3+a^2b\right)x-(b}{a^4-a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^2
*b)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 + (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*b^
3*cosh(x)*sinh(x) + b^3*sinh(x)^2 + b^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b - b^3)
*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x
))))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)
*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)
```

giac [A] time = 0.12, size = 74, normalized size = 1.23

$$
-\frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^4-a^2b^2}+\frac{x}{a-b}-\frac{b\log\left(e^{(2x)}+1\right)}{a^2}+\frac{2}{a\left(e^{(2x)}+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^4 - a^2*b^2) + x/(a - b) -
b*log(e^(2*x) + 1)/a^2 + 2/(a*(e^(2*x) + 1))
```

maple [A] time = 0.14, size = 110, normalized size = 1.83

$$
-\frac{b^3\ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a+b)(a-b)a^2}-\frac{16\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16a+16b}+\frac{16\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{16a-16b}-\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*coth(x)),x)
```

```
[Out] -b^3/(a+b)/(a-b)/a^2*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)-16/(16*a+16*b)*l
n(tanh(1/2*x)-1)+16/(16*a-16*b)*ln(tanh(1/2*x)+1)-2/a*tanh(1/2*x)/(tanh(1/2
*x)^2+1)-1/a^2*b*ln(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.46, size = 67, normalized size = 1.12

$$-\frac{b^3 \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^4 - a^2b^2} + \frac{x}{a+b} - \frac{b \log\left(e^{(-2x)} + 1\right)}{a^2} - \frac{2}{ae^{(-2x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^(
-2*x) + 1)/a^2 - 2/(a*e^(-2*x) + a)
```

mupad [B] time = 1.45, size = 73, normalized size = 1.22

$$\frac{2}{a\left(e^{2x}+1\right)} + \frac{x}{a-b} - \frac{b^3 \ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^4 - a^2\,b^2} - \frac{b\,\ln\left(e^{2x}+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*coth(x)),x)
```

```
[Out] 2/(a*(exp(2*x) + 1)) + x/(a - b) - (b^3*log(b - a + a*exp(2*x) + b*exp(2*x)
))/(a^4 - a^2*b^2) - (b*log(exp(2*x) + 1))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{a + b\coth(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*coth(x)), x)
```

3.143 $\int \frac{\tanh(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{bx}{a^2-b^2}+\frac{b^2\log(a\sinh(x)+b\cosh(x))}{a\left(a^2-b^2\right)}+\frac{\log(\cosh(x))}{a}$$

```
[Out] -b*x/(a^2-b^2)+ln(cosh(x))/a+b^2*ln(b*cosh(x)+a*sinh(x))/a/(a^2-b^2)
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3571, 3530, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{b^2\log(a\sinh(x)+b\cosh(x))}{a\left(a^2-b^2\right)}+\frac{\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Cosh[x]]/a + (b^2*Log[b*Cosh[x] + a*Sinh[x]])/(a
*(a^2 - b^2))
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3571

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[((a*c - b*d)*x)/((a^2 + b^2)*(c^2 + d^2)), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{a+b\coth(x)}\,dx&=-\frac{bx}{a^2-b^2}+\frac{\int \tanh(x)\,dx}{a}+\frac{\left(ib^2\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a\left(a^2-b^2\right)}\\&=-\frac{bx}{a^2-b^2}+\frac{\log(\cosh(x))}{a}+\frac{b^2\log(b\cosh(x)+a\sinh(x))}{a\left(a^2-b^2\right)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.90

$$\frac{\left(a^2-b^2\right)\log(\cosh(x))+b(b\log(a\sinh(x)+b\cosh(x))-ax)}{a^3-ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Coth[x]),x]
```

```
[Out] ((a^2 - b^2)*Log[Cosh[x]] + b*(-(a*x) + b*Log[b*Cosh[x] + a*Sinh[x]]))/(a^3
 - a*b^2)
```

fricas [A] time = 0.45, size = 73, normalized size = 1.43

$$\frac{b^2 \log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right) - \left(a^2+ab\right)x + \left(a^2-b^2\right)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (b^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x + (
a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
```

giac [A] time = 0.13, size = 57, normalized size = 1.12

$$\frac{b^2 \log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^3-ab^2} - \frac{x}{a-b} + \frac{\log\left(e^{(2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 - a*b^2) - x/(a - b) + log
(e^(2*x) + 1)/a
```

maple [A] time = 0.12, size = 88, normalized size = 1.73

$$\frac{b^2 \ln\left(\left(\tanh^2\left(\frac{x}{2}\right)\right)b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a+b)\,(a-b)\,a} - \frac{8\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a+8b} - \frac{8\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8a-8b} + \frac{\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*coth(x)),x)
```

```
[Out] b^2/(a+b)/(a-b)/a*ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)-8/(8*a+8*b)*ln(tanh
(1/2*x)-1)-8/(8*a-8*b)*ln(tanh(1/2*x)+1)+1/a*ln(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.42, size = 50, normalized size = 0.98

$$\frac{b^2 \log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^3-ab^2} + \frac{x}{a+b} + \frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-(a - b)*e^(-2*x) + a + b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-2*x)
 + 1)/a
```

mupad [B] time = 0.32, size = 58, normalized size = 1.14

$$\frac{\ln\left(e^{2\,x}+1\right)}{a} - \frac{x}{a-b} - \frac{b^2\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a\,b^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*coth(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - x/(a - b) - (b^2*log(b - a + a*exp(2*x) + b*exp(2*x))
)/(a*b^2 - a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{a + b\coth(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*coth(x)), x)
```

3.144 $\int \frac{1}{a+b\coth(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Coth[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\coth(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Coth[x])^(-1),x]
```

```
[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

fricas [A] time = 0.40, size = 42, normalized size = 1.08

$$\frac{(a+b)x-b\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="fricas")
[Out] ((a + b)*x - b*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

giac [A] time = 0.11, size = 43, normalized size = 1.10

$$-\frac{b\log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="giac")
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) + x/(a - b)
```

maple [A] time = 0.05, size = 55, normalized size = 1.41

$$-\frac{\ln(\coth(x) - 1)}{2b + 2a} + \frac{\ln(1 + \coth(x))}{2a - 2b} - \frac{b\ln(a + b\coth(x))}{(a - b)(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*coth(x)),x)
[Out] -1/(2*b+2*a)*ln(coth(x)-1)+1/(2*a-2*b)*ln(1+coth(x))-b/(a-b)/(a+b)*ln(a+b*c
oth(x))
```

maxima [A] time = 0.30, size = 37, normalized size = 0.95

$$-\frac{b\log\left(-(a - b)e^{(-2x)} + a + b\right)}{a^2 - b^2} + \frac{x}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="maxima")
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

mupad [B] time = 0.08, size = 42, normalized size = 1.08

$$\frac{x}{a - b} - \frac{b\ \ln\left(b - a + a\,\mathrm{e}^{2\,x} + b\,\mathrm{e}^{2\,x}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*coth(x)),x)
[Out] x/(a - b) - (b*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2)
```

sympy [A] time = 0.91, size = 148, normalized size = 3.79

$$\begin{cases} \tilde{\infty}\left(x - \log(\tanh(x) + 1)\right) & \text{for } a = 0 \wedge b = 0 \\ -\frac{x\tanh(x)}{2b\tanh(x) - 2b} + \frac{x}{2b\tanh(x) - 2b} - \frac{1}{2b\tanh(x) - 2b} & \text{for } a = -b \\ \frac{x\tanh(x)}{2b\tanh(x) + 2b} + \frac{x}{2b\tanh(x) + 2b} + \frac{1}{2b\tanh(x) + 2b} & \text{for } a = b \\ \frac{x - \log(\tanh(x) + 1)}{b} & \text{for } a = 0 \\ \frac{ax}{a^2 - b^2} - \frac{bx}{a^2 - b^2} + \frac{b\log(\tanh(x) + 1)}{a^2 - b^2} - \frac{b\log\left(\tanh(x) + \frac{b}{a}\right)}{a^2 - b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), (-x*tanh(x)/(2
*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b
)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x)
 + 2*b), Eq(a, b)), ((x - log(tanh(x) + 1))/b, Eq(a, 0)), (a*x/(a**2 - b**2
) - b*x/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2) - b*log(tanh(x) +
b/a)/(a**2 - b**2), True))
```

3.145 $\int \frac{\coth(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=39

$$\frac{a\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3531, 3530}

$$\frac{a\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+b\coth(x)}\,dx &= -\frac{bx}{a^2-b^2}+\frac{(ia)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= -\frac{bx}{a^2-b^2}+\frac{a\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 29, normalized size = 0.74

$$\frac{a\log(a\sinh(x)+b\cosh(x))-bx}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Coth[x]),x]
```

```
[Out] (-(b*x) + a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

fricas [A] time = 0.44, size = 43, normalized size = 1.10

$$-\frac{(a+b)x-a\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

giac [A] time = 0.11, size = 43, normalized size = 1.10

$$\frac{a \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2 - b^2} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) - x/(a - b)
```

maple [A] time = 0.05, size = 55, normalized size = 1.41

$$-\frac{\ln(\coth(x) - 1)}{2b + 2a} - \frac{\ln(1 + \coth(x))}{2a - 2b} + \frac{a \ln(a + b \coth(x))}{(a + b)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*coth(x)),x)
```

```
[Out] -1/(2*b+2*a)*ln(coth(x)-1)-1/(2*a-2*b)*ln(1+coth(x))+a/(a+b)/(a-b)*ln(a+b*c
oth(x))
```

maxima [A] time = 0.63, size = 36, normalized size = 0.92

$$\frac{a \log\left(-(a - b)e^{(-2x)} + a + b\right)}{a^2 - b^2} + \frac{x}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

mupad [B] time = 0.06, size = 42, normalized size = 1.08

$$\frac{a \ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^2 - b^2} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*coth(x)),x)
```

```
[Out] (a*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2) - x/(a - b)
```

sympy [A] time = 0.93, size = 134, normalized size = 3.44

$$\begin{cases} \tilde{\infty} x & \text{for}\ a = 0 \wedge b = 0 \\ \frac{x \tanh(x)}{2b \tanh(x) - 2b} - \frac{x}{2b \tanh(x) - 2b} - \frac{1}{2b \tanh(x) - 2b} & \text{for}\ a = -b \\ \frac{x \tanh(x)}{2b \tanh(x) + 2b} + \frac{x}{2b \tanh(x) + 2b} - \frac{1}{2b \tanh(x) + 2b} & \text{for}\ a = b \\ \frac{x}{b} & \text{for}\ a = 0 \\ \frac{ax}{a^2 - b^2} - \frac{a \log(\tanh(x) + 1)}{a^2 - b^2} + \frac{a \log\left(\tanh(x) + \frac{b}{a}\right)}{a^2 - b^2} - \frac{bx}{a^2 - b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*tanh(x)/(2*b*tanh(x) - 2*b) - x/
(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*ta
nh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (x
/b, Eq(a, 0)), (a*x/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) + a*lo
g(tanh(x) + b/a)/(a**2 - b**2) - b*x/(a**2 - b**2), True))
```

3.146 $\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=63

$$-\frac{a^2\log(a\sinh(x)+b\cosh(x))}{b\left(a^2-b^2\right)}+\frac{a^3x}{b^2\left(a^2-b^2\right)}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

```
[Out] -a*x/b^2+a^3*x/b^2/(a^2-b^2)+ln(sinh(x))/b-a^2*ln(b*cosh(x)+a*sinh(x))/b/(a
^2-b^2)
```

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3541, 3475, 3484, 3530}

$$\frac{a^3x}{b^2\left(a^2-b^2\right)}-\frac{a^2\log(a\sinh(x)+b\cosh(x))}{b\left(a^2-b^2\right)}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Sinh[x]]/b - (a^2*Log[b*Cosh
[x] + a*Sinh[x]])/(b*(a^2 - b^2))
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3541

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(d*(2*b*c - a*d)*x)/b^2, x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{ax}{b^2}+\frac{a^2\int \frac{1}{a+b\coth(x)}\,dx}{b^2}+\frac{\int \coth(x)\,dx}{b}$$
$$= -\frac{ax}{b^2}+\frac{a^3x}{b^2\left(a^2-b^2\right)}+\frac{\log(\sinh(x))}{b}-\frac{\left(ia^2\right)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{b\left(a^2-b^2\right)}$$
$$= -\frac{ax}{b^2}+\frac{a^3x}{b^2\left(a^2-b^2\right)}+\frac{\log(\sinh(x))}{b}-\frac{a^2\log(b\cosh(x)+a\sinh(x))}{b\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.09, size = 49, normalized size = 0.78

$$\frac{-a^2\log(a\sinh(x)+b\cosh(x))+a^2\log(\sinh(x))+abx-b^2\log(\sinh(x))}{a^2b-b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a*b*x + a^2*Log[Sinh[x]] - b^2*Log[Sinh[x]] - a^2*Log[b*Cosh[x] + a*Sinh[x
]])/(a^2*b - b^3)
```

fricas [A] time = 0.44, size = 76, normalized size = 1.21

$$-\frac{a^2\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(ab+b^2\right)x-\left(a^2-b^2\right)\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2b-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(a^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a*b + b^2)*x -
(a^2 - b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
```

giac [A] time = 0.13, size = 59, normalized size = 0.94

$$-\frac{a^2\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^2b-b^3}+\frac{x}{a-b}+\frac{\log\left(\left|e^{(2\,x)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b - b^3) + x/(a - b) + lo
g(abs(e^(2*x) - 1))/b
```

maple [A] time = 0.05, size = 60, normalized size = 0.95

$$-\frac{\ln(\coth(x)-1)}{2b+2a}+\frac{\ln(1+\coth(x))}{2a-2b}-\frac{a^2\ln(a+b\coth(x))}{(a+b)(a-b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*coth(x)),x)
```

```
[Out] -1/(2*b+2*a)*ln(coth(x)-1)+1/(2*a-2*b)*ln(1+coth(x))-a^2/(a+b)/(a-b)/b*ln(a
+b*coth(x))
```

maxima [A] time = 0.65, size = 63, normalized size = 1.00

$$-\frac{a^2 \log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^2 b - b^3} + \frac{x}{a+b} + \frac{\log\left(e^{(-x)} + 1\right)}{b} + \frac{\log\left(e^{(-x)} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b - b^3) + x/(a + b) + log(e^(-x)
+ 1)/b + log(e^(-x) - 1)/b
```

mupad [B] time = 1.48, size = 57, normalized size = 0.90

$$\frac{\ln\left(e^{2x} - 1\right)}{b} + \frac{x}{a-b} - \frac{a^2 \ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{a^2\,b - b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*coth(x)),x)
```

```
[Out] log(exp(2*x) - 1)/b + x/(a - b) - (a^2*log(b - a + a*exp(2*x) + b*exp(2*x))
)/(a^2*b - b^3)
```

sympy [A] time = 1.63, size = 372, normalized size = 5.90

$$\begin{cases} \tilde{\infty}\left(x - \log(\tanh(x) + 1) + \log(\tanh(x))\right) \\ \frac{x - \log(\tanh(x)+1) + \log(\tanh(x))}{b} \\ \frac{3x\tanh(x)}{2b\tanh(x)-2b} - \frac{3x}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x)+1)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x))}{2b\tanh(x)-2b} - \frac{1}{2b\tanh(x)} \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x))}{2b\tanh(x)+2b} + \frac{1}{2b\tanh(x} \\ \frac{x - \frac{1}{\tanh(x)}}{a} \\ -\frac{a^2\log\left(\tanh(x) + \frac{b}{a}\right)}{a^2b-b^3} + \frac{a^2\log(\tanh(x))}{a^2b-b^3} + \frac{abx}{a^2b-b^3} - \frac{b^2x}{a^2b-b^3} + \frac{b^2\log(\tanh(x)+1)}{a^2b-b^3} - \frac{b^2\log(\tanh(x))}{a^2b-b^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 ((x - log(tanh(x) + 1) + log(tanh(x)))/b, Eq(a, 0)), (3*x*tanh(x)/(2*b*tan
h(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tan
h(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x))*tanh(
x)/(2*b*tanh(x) - 2*b) - 2*log(tanh(x))/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x
) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b
) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)/(2*
b*tanh(x) + 2*b) + 2*log(tanh(x))*tanh(x)/(2*b*tanh(x) + 2*b) + 2*log(tanh(
x))/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x - 1/tanh(x)
)/a, Eq(b, 0)), (-a**2*log(tanh(x) + b/a)/(a**2*b - b**3) + a**2*log(tanh(x
))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) - b**2*x/(a**2*b - b**3) + b**2*
log(tanh(x) + 1)/(a**2*b - b**3) - b**2*log(tanh(x))/(a**2*b - b**3), True)
)
```

3.147 $\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=64

$$-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\log(a+b\coth(x))}{b^2\left(a^2-b^2\right)}-\frac{\coth(x)}{b}$$

```
[Out] -b*x/(a^2-b^2)-coth(x)/b+a^3*ln(a+b*coth(x))/b^2/(a^2-b^2)+a*ln(sinh(x))/(a
^2-b^2)
```

Rubi [A] time = 0.13, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3566, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\log(a+b\coth(x))}{b^2\left(a^2-b^2\right)}-\frac{\coth(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - Coth[x]/b + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2
)) + (a*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
```

```
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx &= -\frac{\coth(x)}{b}-\frac{\int \frac{-a-b\coth(x)+a\coth^2(x)}{a+b\coth(x)}\,dx}{b}\\ &= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a\int \coth(x)\,dx}{a^2-b^2}+\frac{a^3\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b\left(a^2-b^2\right)}\\ &= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\coth(x)\right)}{b^2\left(a^2-b^2\right)}\\ &= -\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a^3\log(a+b\coth(x))}{b^2\left(a^2-b^2\right)}+\frac{a\log(\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.14, size = 64, normalized size = 1.00

$$\frac{a^3(-\log(a\sinh(x)+b\cosh(x)))+b\left(a^2-b^2\right)\coth(x)+a\left(a^2-b^2\right)\log(\sinh(x))+b^3x}{b^2(b-a)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (b^3*x + b*(a^2 - b^2)*Coth[x] + a*(a^2 - b^2)*Log[Sinh[x]] - a^3*Log[b*Cos
h[x] + a*Sinh[x]])/(b^2*(-a + b)*(a + b))
```

fricas [B] time = 0.45, size = 271, normalized size = 4.23

$$\frac{\left(ab^2+b^3\right)x\cosh(x)^2+2\left(ab^2+b^3\right)x\cosh(x)\sinh(x)+\left(ab^2+b^3\right)x\sinh(x)^2+2\,a^2b-2\,b^3-\left(ab^2+b^3\right)x-\left(a^3\,\mathrm{c}\right.}{a^2b^2-b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 + b
^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 - (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*a^
3*cosh(x)*sinh(x) + a^3*sinh(x)^2 - a^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^3 - a*b^2 - (a^3 - a*b^2)*cosh(x)^2 - 2*(a^3 - a*b^2)
*cosh(x)*sinh(x) - (a^3 - a*b^2)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x
))))/(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cosh(x)^2 - 2*(a^2*b^2 - b^4)*cosh(x)
*sinh(x) - (a^2*b^2 - b^4)*sinh(x)^2)
```

giac [A] time = 0.12, size = 76, normalized size = 1.19

$$\frac{a^3\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^2b^2-b^4}-\frac{x}{a-b}-\frac{a\log\left(\left|e^{(2\,x)}-1\right|\right)}{b^2}-\frac{2}{b\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^2 - b^4) - x/(a - b) - a
*log(abs(e^(2*x) - 1))/b^2 - 2/(b*(e^(2*x) - 1))
```

maple [A] time = 0.07, size = 67, normalized size = 1.05

$$-\frac{\coth(x)}{b}-\frac{\ln(\coth(x)-1)}{2b+2a}-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*coth(x)),x)
```

```
[Out] -coth(x)/b-1/(2*b+2*a)*ln(coth(x)-1)-1/(2*a-2*b)*ln(1+coth(x))+1/b^2*a^3/(a
+b)/(a-b)*ln(a+b*coth(x))
```

maxima [A] time = 0.31, size = 82, normalized size = 1.28

$$\frac{a^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^2-b^4}+\frac{x}{a+b}-\frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{2}{be^{(-2x)}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a^3*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(-
x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(b*e^(-2*x) - b)
```

mupad [B] time = 1.51, size = 74, normalized size = 1.16

$$-\frac{2}{b\left(e^{2x}-1\right)}-\frac{x}{a-b}-\frac{a^3\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b^4-a^2\,b^2}-\frac{a\,\ln\left(e^{2x}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*coth(x)),x)
```

```
[Out] - 2/(b*(exp(2*x) - 1)) - x/(a - b) - (a^3*log(b - a + a*exp(2*x) + b*exp(2*
x)))/(b^4 - a^2*b^2) - (a*log(exp(2*x) - 1))/b^2
```

sympy [A] time = 2.40, size = 636, normalized size = 9.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x)), Eq(a, 0) & Eq(b, 0)), ((x - 1/tanh(x))/b, E
q(a, 0)), (5*x*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 5*x*tanh(x)/(2*b
*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2
- 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x))
+ 2*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x))
*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 3*tanh(x)/(2*b*tanh(x)**2 - 2*b*t
anh(x)) + 2/(2*b*tanh(x)**2 - 2*b*tanh(x)), Eq(a, -b)), (x*tanh(x)**2/(2*b*
tanh(x)**2 + 2*b*tanh(x)) + x*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*lo
g(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) +
1)*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*b*
tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*b*tanh(x)**2 + 2*b*ta
nh(x)) - 3*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2/(2*b*tanh(x)**2 + 2*b
*tanh(x)), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)*
*2))/a, Eq(b, 0)), (a**3*log(tanh(x) + b/a)*tanh(x)/(a**2*b**2*tanh(x) - b*
*4*tanh(x)) - a**3*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x))
```

```
- a**2*b/(a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*x*tanh(x)/(a**2*b**2*t
anh(x) - b**4*tanh(x)) - a*b**2*log(tanh(x) + 1)*tanh(x)/(a**2*b**2*tanh(x)
 - b**4*tanh(x)) + a*b**2*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*ta
nh(x)) - b**3*x*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) + b**3/(a**2*b**
2*tanh(x) - b**4*tanh(x)), True))
```

3.148 $\int \frac{\coth^4(x)}{a+b\coth(x)} \, dx$

Optimal. Leaf size=76

$$\frac{ax}{a^2-b^2}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\log(a+b\coth(x))}{b^3\left(a^2-b^2\right)}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

```
[Out] a*x/(a^2-b^2)+a*coth(x)/b^2-1/2*coth(x)^2/b-a^4*ln(a+b*coth(x))/b^3/(a^2-b^
2)-b*ln(sinh(x))/(a^2-b^2)
```

Rubi [A] time = 0.22, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3566, 3647, 3627, 3617, 31, 3475}

$$\frac{ax}{a^2-b^2}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\log(a+b\coth(x))}{b^3\left(a^2-b^2\right)}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) + (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - (a^4*Log[a + b*Coth
[x]])/(b^3*(a^2 - b^2)) - (b*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3627

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*(A - C)*x)/(a^2 + b^2), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[(b*(A - C))/(a^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
```

```
f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx &= -\frac{\coth^2(x)}{2b} - \frac{\int \frac{\coth(x)\left(-2a-2b\coth(x)+2a\coth^2(x)\right)}{a+b\coth(x)}\,dx}{2b} \\
&= \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{\int \frac{2a^2-2\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}\,dx}{2b^2} \\
&= \frac{ax}{a^2-b^2} + \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{a^4\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b^2\left(a^2-b^2\right)} - \frac{b\int \coth(x)\,dx}{a^2-b^2} \\
&= \frac{ax}{a^2-b^2} + \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{b\log(\sinh(x))}{a^2-b^2} - \frac{a^4\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b^3\left(a^2-b^2\right)} \\
&= \frac{ax}{a^2-b^2} + \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{a^4\log(a+b\coth(x))}{b^3\left(a^2-b^2\right)} - \frac{b\log(\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 88, normalized size = 1.16

$$
\frac{2\left(a^4-b^4\right)\log(\sinh(x)) - 2a^4\log(a\sinh(x)+b\cosh(x)) + 2ab\left(a^2-b^2\right)\coth(x) + \left(b^4-a^2b^2\right)\operatorname{csch}^2(x) + 2ab^3x}{2b^3(a-b)(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b^3*x + 2*a*b*(a^2 - b^2)*Coth[x] + (-(a^2*b^2) + b^4)*Csch[x]^2 + 2*(
a^4 - b^4)*Log[Sinh[x]] - 2*a^4*Log[b*Cosh[x] + a*Sinh[x]])/(2*(a - b)*b^3*
(a + b))
```

fricas [B] time = 0.45, size = 648, normalized size = 8.53

$$
\frac{\left(ab^3+b^4\right)x\cosh(x)^4 + 4\left(ab^3+b^4\right)x\cosh(x)\sinh(x)^3 + \left(ab^3+b^4\right)x\sinh(x)^4 - 2a^3b + 2ab^3 + 2\left(a^3b - a^2b^2 - a\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^3 + b^4)*x*cosh(x)^4 + 4*(a*b^3 + b^4)*x*cosh(x)*sinh(x)^3 + (a*b^3 +
 b^4)*x*sinh(x)^4 - 2*a^3*b + 2*a*b^3 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 -
```

```
(a*b^3 + b^4)*x)*cosh(x)^2 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 + 3*(a*b^3 +
b^4)*x*cosh(x)^2 - (a*b^3 + b^4)*x)*sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*cosh
(x)^4 + 4*a^4*cosh(x)*sinh(x)^3 + a^4*sinh(x)^4 - 2*a^4*cosh(x)^2 + a^4 + 2
*(3*a^4*cosh(x)^2 - a^4)*sinh(x)^2 + 4*(a^4*cosh(x)^3 - a^4*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + ((a^4 - b^4)*cosh(x
)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4 -
 2*(a^4 - b^4)*cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)^
2 + 4*((a^4 - b^4)*cosh(x)^3 - (a^4 - b^4)*cosh(x))*sinh(x))*log(2*sinh(x)/
(cosh(x) - sinh(x))) + 4*((a*b^3 + b^4)*x*cosh(x)^3 + (a^3*b - a^2*b^2 - a*
b^3 + b^4 - (a*b^3 + b^4)*x)*cosh(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 -
b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh
(x)^4 - 2*(a^2*b^3 - b^5)*cosh(x)^2 - 2*(a^2*b^3 - b^5 - 3*(a^2*b^3 - b^5)*
cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3 - (a^2*b^3 - b^5)*cosh(
x))*sinh(x))
```

giac [A] time = 0.14, size = 100, normalized size = 1.32

$$-\frac{a^{4} \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^{2} b^{3} - b^{5}} + \frac{x}{a - b} + \frac{\left(a^{2} + b^{2}\right) \log\left(\left|e^{(2x)} - 1\right|\right)}{b^{3}} - \frac{2\left(ab - \left(ab - b^{2}\right)e^{(2x)}\right)}{b^{3}\left(e^{(2x)} - 1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^3 - b^5) + x/(a - b) +
(a^2 + b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2*x))/(b^3*
(e^(2*x) - 1)^2)
```

maple [A] time = 0.06, size = 76, normalized size = 1.00

$$-\frac{\coth^{2}(x)}{2b} + \frac{a\coth(x)}{b^{2}} - \frac{\ln(\coth(x) - 1)}{2b + 2a} + \frac{\ln(1 + \coth(x))}{2a - 2b} - \frac{a^{4}\ln(a + b\coth(x))}{b^{3}(a + b)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*coth(x)),x)
```

```
[Out] -1/2*coth(x)^2/b+a*coth(x)/b^2-1/(2*b+2*a)*ln(coth(x)-1)+1/(2*a-2*b)*ln(1+c
oth(x))-1/b^3*a^4/(a+b)/(a-b)*ln(a+b*coth(x))
```

maxima [A] time = 0.98, size = 119, normalized size = 1.57

$$-\frac{a^{4} \log\left(-(a - b)e^{(-2x)} + a + b\right)}{a^{2} b^{3} - b^{5}} + \frac{2\left((a + b)e^{(-2x)} - a\right)}{2b^{2}e^{(-2x)} - b^{2}e^{(-4x)} - b^{2}} + \frac{x}{a + b} + \frac{\left(a^{2} + b^{2}\right)\log\left(e^{(-x)} + 1\right)}{b^{3}} + \frac{\left(a^{2} + b^{2}\right)\log\left(e^{(}\right.}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^4*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^3 - b^5) + 2*((a + b)*e^(-2*x) -
 a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) + x/(a + b) + (a^2 + b^2)*log(e^(
-x) + 1)/b^3 + (a^2 + b^2)*log(e^(-x) - 1)/b^3
```

mupad [B] time = 1.63, size = 110, normalized size = 1.45

$$\frac{x}{a - b} - \frac{2}{b\left(e^{4x} - 2e^{2x} + 1\right)} + \frac{\ln\left(e^{2x} - 1\right)\left(a^{2} + b^{2}\right)}{b^{3}} + \frac{a^{4}\ln\left(b - a + a\,e^{2x} + b\,e^{2x}\right)}{b^{5} - a^{2}b^{3}} + \frac{2\left(a^{2} - b^{2}\right)}{b^{2}(a + b)\left(e^{2x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*coth(x)),x)
```

```
[Out] x/(a - b) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) + (log(exp(2*x) - 1)*(a^2 + b
^2))/b^3 + (a^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(b^5 - a^2*b^3) + (2*
(a^2 - b^2))/(b^2*(a + b)*(exp(2*x) - 1))
```

sympy [A] time = 3.40, size = 882, normalized size = 11.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2)), Eq
(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2
))/b, Eq(a, 0)), (7*x*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 7*x*ta
nh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/
(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh
(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*
tanh(x)**2) - 4*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) -
 3*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 -
 2*b*tanh(x)**2) + 1/(2*b*tanh(x)**3 - 2*b*tanh(x)**2), Eq(a, -b)), (x*tanh
(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + x*tanh(x)**2/(2*b*tanh(x)**3 + 2
*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x
)**2) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4
*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4*log(tanh(x))
*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 3*tanh(x)**2/(2*b*tanh(x)**
3 + 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) - 1/(2*b*ta
nh(x)**3 + 2*b*tanh(x)**2), Eq(a, b)), ((x - 1/tanh(x) - 1/(3*tanh(x)**3))/
a, Eq(b, 0)), (-2*a**4*log(tanh(x) + b/a)*tanh(x)**2/(2*a**2*b**3*tanh(x)**
2 - 2*b**5*tanh(x)**2) + 2*a**4*log(tanh(x))*tanh(x)**2/(2*a**2*b**3*tanh(x
)**2 - 2*b**5*tanh(x)**2) + 2*a**3*b*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b*
*5*tanh(x)**2) - a**2*b**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2
*a*b**3*x*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*a*b**
3*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*x*tanh(x)**
2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*b**4*log(tanh(x) + 1)*ta
nh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*log(tanh(x))
*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + b**4/(2*a**2*b**
3*tanh(x)**2 - 2*b**5*tanh(x)**2), True))
```

3.149 $\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}-\frac{\left(a^2+b^2\right)\coth(x)}{b^3}+\frac{a^5\log(a+b\coth(x))}{b^4\left(a^2-b^2\right)}+\frac{a\coth^2(x)}{2b^2}-\frac{\coth^3(x)}{3b}$$

```
[Out] -b*x/(a^2-b^2)-(a^2+b^2)*coth(x)/b^3+1/2*a*coth(x)^2/b^2-1/3*coth(x)^3/b+a^
5*ln(a+b*coth(x))/b^4/(a^2-b^2)+a*ln(sinh(x))/(a^2-b^2)
```

Rubi [A] time = 0.39, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3566, 3647, 3648, 3626, 3617, 31, 3475}

$$-\frac{bx}{a^2-b^2}-\frac{\left(a^2+b^2\right)\coth(x)}{b^3}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^5\log(a+b\coth(x))}{b^4\left(a^2-b^2\right)}+\frac{a\coth^2(x)}{2b^2}-\frac{\coth^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - ((a^2 + b^2)*Coth[x])/b^3 + (a*Coth[x]^2)/(2*b^2) -
Coth[x]^3/(3*b) + (a^5*Log[a + b*Coth[x]])/(b^4*(a^2 - b^2)) + (a*Log[Sinh[
x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3566

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*(a + b*Tan[e + f*x])^(m - 2)*(c
+ d*Tan[e + f*x])^(n + 1))/(d*f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3617

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3626

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*A + b*B -
a*C)*x)/(a^2 + b^2), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
```

```
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[
e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3648

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(C*(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n + 1))/(d*f*(m + n +
 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx &= -\frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth^2(x)\left(-3a-3b\coth(x)+3a\coth^2(x)\right)}{a+b\coth(x)}\,dx}{3b}\\
&= \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth(x)\left(6a^2-6\left(a^2+b^2\right)\coth^2(x)\right)}{a+b\coth(x)}\,dx}{6b^2}\\
&= -\frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{-6a\left(a^2+b^2\right)-6b^3\coth(x)+6a\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}\,dx}{6b^3}\\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a\int \coth(x)\,dx}{a^2-b^2} + \frac{a^5\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,\cdots}{b^3\left(a^2-b^2\right)\cdots}\\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a\log(\sinh(x))}{a^2-b^2} + \frac{a^5\,\mathrm{Subst}\left(\int \frac{1}{a+\cdots}\cdots\right)}{b^4\left(a\cdots\right.}\\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a^5\log(a+b\coth(x))}{b^4\left(a^2-b^2\right)} + \frac{a\log(\sin\cdots}{a^2-\cdots}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 108, normalized size = 1.15

$$\frac{-6a^5\log(a\sinh(x)+b\cosh(x))+6a\left(a^4-b^4\right)\log(\sinh(x))-3ab^2\left(a^2-b^2\right)\operatorname{csch}^2(x)+2b\left(a^2-b^2\right)\coth(x)\left(3a^2\cdots\right.}{6b^4(b-a)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] (6*b^5*x - 3*a*b^2*(a^2 - b^2)*Csch[x]^2 + 2*b*(a^2 - b^2)*Coth[x]*(3*a^2 +
 4*b^2 + b^2*Csch[x]^2) + 6*a*(a^4 - b^4)*Log[Sinh[x]] - 6*a^5*Log[b*Cosh[x
] + a*Sinh[x]])/(6*b^4*(-a + b)*(a + b))
```

fricas [B] time = 0.46, size = 1299, normalized size = 13.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 +
3*(a*b^4 + b^5)*x*sinh(x)^6 + 6*a^4*b + 2*a^2*b^3 - 8*b^5 + 3*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 + 3*(2
*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 + 15*(a*b^4 + b^5)*x*cosh(
x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3 + (2*
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x
))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x
)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2 - 2*a*b
^4 + 4*b^5 + 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^
4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 - 3*(a*b^4 + b^5)*x -
3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 - 3*a^5*cosh(x)^
4 + 3*a^5*cosh(x)^2 - a^5 + 3*(5*a^5*cosh(x)^2 - a^5)*sinh(x)^4 + 4*(5*a^5*
cosh(x)^3 - 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 - 6*a^5*cosh(x)^2
 + a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 - 2*a^5*cosh(x)^3 + a^5*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^5 - a*b^4)*c
osh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4)*sinh(x)^6 - a^
5 + a*b^4 - 3*(a^5 - a*b^4)*cosh(x)^4 - 3*(a^5 - a*b^4 - 5*(a^5 - a*b^4)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 - 3*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*
cosh(x)^4 - 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^5 - a*b^4)*cosh(x)
^5 - 2*(a^5 - a*b^4)*cosh(x)^3 + (a^5 - a*b^4)*cosh(x))*sinh(x))*log(2*sinh
(x)/(cosh(x) - sinh(x))) + 6*(3*(a*b^4 + b^5)*x*cosh(x)^5 + 2*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^3 - (4*a
^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x))/(
(a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^2*b^4
- b^6)*sinh(x)^6 - a^2*b^4 + b^6 - 3*(a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^2*b^4
 - b^6 - 5*(a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b^4 - b^6)*cosh
(x)^3 - 3*(a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2
+ 3*(a^2*b^4 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^2*b^4 - b^6)*cosh(x
)^2)*sinh(x)^2 + 6*((a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^2*b^4 - b^6)*cosh(x)^3
 + (a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

giac [A] time = 0.12, size = 143, normalized size = 1.52

$$\frac{a^5 \log\left(\left|ae^{(2x)} + be^{(2x)} - a + b\right|\right)}{a^2b^4 - b^6} - \frac{x}{a-b} - \frac{\left(a^3 + ab^2\right)\log\left(\left|e^{(2x)} - 1\right|\right)}{b^4} - \frac{2\left(3\,a^2b + 4\,b^3 + 3\left(a^2b - ab^2 + 2\,b^3\right)e^{(4x)}}{3\,b^4\left(e^{(2x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^4 - b^6) - x/(a - b) - (
a^3 + a*b^2)*log(abs(e^(2*x) - 1))/b^4 - 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b -
a*b^2 + 2*b^3)*e^(4*x) - 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2*x)
 - 1)^3)
```

maple [A] time = 0.08, size = 96, normalized size = 1.02

$$-\frac{\coth^3(x)}{3b}+\frac{a\left(\coth^2(x)\right)}{2b^2}-\frac{a^2\coth(x)}{b^3}-\frac{\coth(x)}{b}-\frac{\ln(\coth(x)-1)}{2b+2a}-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{a^5\ln(a+b\coth(x))}{b^4(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*coth(x)),x)
```

```
[Out] -1/3*coth(x)^3/b+1/2*a*coth(x)^2/b^2-1/b^3*a^2*coth(x)-coth(x)/b-1/(2*b+2*a
)*ln(coth(x)-1)-1/(2*a-2*b)*ln(1+coth(x))+1/b^4*a^5/(a+b)/(a-b)*ln(a+b*coth
(x))
```

maxima [A] time = 0.52, size = 169, normalized size = 1.80

$$\frac{a^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^4-b^6}+\frac{2\left(3a^2+4b^2-3\left(2a^2+ab+2b^2\right)e^{(-2x)}+3\left(a^2+ab+2b^2\right)e^{(-4x)}\right)}{3\left(3b^3e^{(-2x)}-3b^3e^{(-4x)}+b^3e^{(-6x)}-b^3\right)}+\frac{x}{a+b}-\frac{\left(a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a^5*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^4 - b^6) + 2/3*(3*a^2 + 4*b^2 - 3
*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*e^
(-2*x) - 3*b^3*e^(-4*x) + b^3*e^(-6*x) - b^3) + x/(a + b) - (a^3 + a*b^2)*l
og(e^(-x) + 1)/b^4 - (a^3 + a*b^2)*log(e^(-x) - 1)/b^4
```

mupad [B] time = 1.63, size = 164, normalized size = 1.74

$$-\frac{8}{3b\left(3e^{2x}-3e^{4x}+e^{6x}-1\right)}-\frac{x}{a-b}-\frac{a^5\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b^6-a^2b^4}-\frac{\ln\left(e^{2x}-1\right)\left(a^3+a\,b^2\right)}{b^4}-\frac{2\left(a^3+a\,b^2+2\,b^3\right.}{b^3\,(a+b)\,\left(e^{2x}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b*coth(x)),x)
```

```
[Out] - 8/(3*b*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - x/(a - b) - (a^5*log(b
 - a + a*exp(2*x) + b*exp(2*x)))/(b^6 - a^2*b^4) - (log(exp(2*x) - 1)*(a*b^
2 + a^3))/b^4 - (2*(a*b^2 + a^3 + 2*b^3))/(b^3*(a + b)*(exp(2*x) - 1)) - (2
*(a*b - a^2 + 2*b^2))/(b^2*(a + b)*(exp(4*x) - 2*exp(2*x) + 1))
```

sympy [A] time = 5.02, size = 1013, normalized size = 10.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x) - 1/(3*tanh(x)**3)), Eq(a, 0) & Eq(b, 0)), ((
x - 1/tanh(x) - 1/(3*tanh(x)**3))/b, Eq(a, 0)), (27*x*tanh(x)**4/(6*b*tanh(
x)**4 - 6*b*tanh(x)**3) - 27*x*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3)
 - 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*lo
g(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*log(tanh(x
))*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**
3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 15*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*t
anh(x)**3) + 9*tanh(x)**2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + tanh(x)/(6*b*
tanh(x)**4 - 6*b*tanh(x)**3) + 2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3), Eq(a, -
b)), (3*x*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 3*x*tanh(x)**3/(6*
b*tanh(x)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x
)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 + 6
```

```
*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**
3) - 12*log(tanh(x))*tanh(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 15*tanh
(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 9*tanh(x)**2/(6*b*tanh(x)**4 + 6
*b*tanh(x)**3) + tanh(x)/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 2/(6*b*tanh(x)
**4 + 6*b*tanh(x)**3), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1
/(2*tanh(x)**2) - 1/(4*tanh(x)**4))/a, Eq(b, 0)), (6*a**5*log(tanh(x) + b/a
)*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**5*log(tanh
(x))*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**4*b*tan
h(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) + 3*a**3*b**2*tanh(x)/
(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 2*a**2*b**3/(6*a**2*b**4*tan
h(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**
3 - 6*b**6*tanh(x)**3) - 6*a*b**4*log(tanh(x) + 1)*tanh(x)**3/(6*a**2*b**4*
tanh(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*log(tanh(x))*tanh(x)**3/(6*a**2*
b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 3*a*b**4*tanh(x)/(6*a**2*b**4*tanh(x
)**3 - 6*b**6*tanh(x)**3) - 6*b**5*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6
*b**6*tanh(x)**3) + 6*b**5*tanh(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh
(x)**3) + 2*b**5/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3), True))
```

3.150 $\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{ax}{b\left(a^2-b^2\right)}+\frac{\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}+\frac{x}{b(a+b\coth(x))}$$

```
[Out] -a*x/b/(a^2-b^2)+x/b/(a+b*coth(x))+ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A] time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5467, 3484, 3530}

$$-\frac{ax}{b\left(a^2-b^2\right)}+\frac{\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}+\frac{x}{b(a+b\coth(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] -((a*x)/(b*(a^2 - b^2))) + x/(b*(a + b*Coth[x])) + Log[b*Cosh[x] + a*Sinh[x
]]/(a^2 - b^2)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 5467

```
Int[Csch[(c_.) + (d_.)*(x_)]^2*(Coth[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.
)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Coth[c
 + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*
x)^(m - 1)*(a + b*Coth[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f
, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx&=\frac{x}{b(a+b\coth(x))}-\frac{\int \frac{1}{a+b\coth(x)}\,dx}{b}\\&=-\frac{ax}{b\left(a^2-b^2\right)}+\frac{x}{b(a+b\coth(x))}+\frac{i\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\&=-\frac{ax}{b\left(a^2-b^2\right)}+\frac{x}{b(a+b\coth(x))}+\frac{\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.18, size = 49, normalized size = 0.91

$$\frac{ax - b\log(a\sinh(x) + b\cosh(x))}{b^3 - a^2b} + \frac{x\sinh(x)}{ab\sinh(x) + b^2\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(-(a^2*b) + b^3) + (x*Sinh[x])/(b^2*Co
sh[x] + a*b*Sinh[x])
```

fricas [B] time = 0.40, size = 184, normalized size = 3.41

$$\frac{2\,(a+b)x\cosh(x)^2 + 4\,(a+b)x\cosh(x)\sinh(x) + 2\,(a+b)x\sinh(x)^2 - \left((a+b)\cosh(x)^2 + 2\,(a+b)\cosh(x)\,\mathrm{s}}{a^3 - a^2b - ab^2 + b^3 - \left(a^3 + a^2b - ab^2 - b^3\right)\cosh(x)^2 - 2\left(a^3 + a^2b - ab^2 - b^3\right)\cosh(x)\,\mathrm{sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(a + b)*x*cosh(x)^2 + 4*(a + b)*x*cosh(x)*sinh(x) + 2*(a + b)*x*sinh(x)^
2 - ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a
+ b)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^3 - a^2*b - a*b
^2 + b^3 - (a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 - 2*(a^3 + a^2*b - a*b^2 -
 b^3)*cosh(x)*sinh(x) - (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

giac [B] time = 0.15, size = 169, normalized size = 3.13

$$-\frac{2\,axe^{(2\,x)} + 2\,bxe^{(2\,x)} - ae^{(2\,x)}\log\left(ae^{(2\,x)} + be^{(2\,x)} - a + b\right) - be^{(2\,x)}\log\left(ae^{(2\,x)} + be^{(2\,x)} - a + b\right) + a\log\left(ae^{(2\,x)}\right.}{a^3e^{(2\,x)} + a^2be^{(2\,x)} - ab^2e^{(2\,x)} - b^3e^{(2\,x)} - a^3 + a^2b + ab^2 - b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a*x*e^(2*x) + 2*b*x*e^(2*x) - a*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a +
 b) - b*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a + b) + a*log(a*e^(2*x) + b*e^
(2*x) - a + b) - b*log(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3*e^(2*x) + a^2*b
*e^(2*x) - a*b^2*e^(2*x) - b^3*e^(2*x) - a^3 + a^2*b + a*b^2 - b^3)
```

maple [A] time = 0.30, size = 73, normalized size = 1.35

$$-\frac{2x}{a^2 - b^2} - \frac{2x}{\left(a\,\mathrm{e}^{2x} + b\,\mathrm{e}^{2x} - a + b\right)(a+b)} + \frac{\ln\left(\mathrm{e}^{2x} - \frac{a-b}{a+b}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csch(x)^2/(a+b*coth(x))^2,x)
```

```
[Out] -2/(a^2-b^2)*x-2*x/(a*exp(2*x)+b*exp(2*x)-a+b)/(a+b)+1/(a^2-b^2)*ln(exp(2*x
)-(a-b)/(a+b))
```

maxima [A] time = 0.92, size = 68, normalized size = 1.26

$$\frac{2\,xe^{(2\,x)}}{a^2 - 2\,ab + b^2 - \left(a^2 - b^2\right)e^{(2\,x)}} + \frac{\log\left(\frac{(a+b)e^{(2\,x)} - a + b}{a+b}\right)}{a^2 - b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(2*x)/(a^2 - 2*a*b + b^2 - (a^2 - b^2)*e^(2*x)) + log(((a + b)*e^(2*x
) - a + b)/(a + b))/(a^2 - b^2)
```

mupad [B] time = 1.28, size = 68, normalized size = 1.26

$$\frac{\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}-\frac{2\,x}{a^2-b^2}-\frac{2\,x}{(a+b)\,\left(b-a+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(sinh(x)^2*(a + b*coth(x))^2),x)
```

```
[Out] log(b - a + a*exp(2*x) + b*exp(2*x))/(a^2 - b^2) - (2*x)/(a^2 - b^2) - (2*x
)/((a + b)*(b - a + exp(2*x)*(a + b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\operatorname{csch}^2(x)}{(a+b\coth(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csch(x)**2/(a+b*coth(x))**2,x)
```

```
[Out] Integral(x*csch(x)**2/(a + b*coth(x))**2, x)
```

3.151 $\int x^3 \coth(a+2\log(x))\,dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^{-2a}\log\left(1-e^{2a}x^4\right)+\frac{x^4}{4}$$

```
[Out] 1/4*x^4+1/2*ln(1-exp(2*a)*x^4)/exp(2*a)
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\coth(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][x^3*Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int x^3\coth(a+2\log(x))\,dx=\int x^3\coth(a+2\log(x))\,dx$$

Mathematica [B] time = 0.03, size = 64, normalized size = 2.13

$$\frac{1}{2}\cosh(2a)\log\left(x^4\sinh(a)+x^4\cosh(a)+\sinh(a)-\cosh(a)\right)-\frac{1}{2}\sinh(2a)\log\left(x^4\sinh(a)+x^4\cosh(a)+\text{sinh}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + 2*Log[x]],x]
```

```
[Out] x^4/4 + (Cosh[2*a]*Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]])/2 -
 (Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]]*Sinh[2*a])/2
```

fricas [A] time = 0.39, size = 28, normalized size = 0.93

$$\frac{1}{4}\left(x^4e^{(2\,a)}+2\,\log\left(x^4e^{(2\,a)}-1\right)\right)e^{(-2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4*e^(2*a) + 2*log(x^4*e^(2*a) - 1))*e^(-2*a)
```

giac [A] time = 0.13, size = 24, normalized size = 0.80

$$\frac{1}{4}\,x^4+\frac{1}{2}\,e^{(-2\,a)}\log\left(\left|x^4e^{(2\,a)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

maple [A] time = 0.10, size = 24, normalized size = 0.80

$$\frac{x^4}{4}+\frac{\mathrm{e}^{-2a}\ln\left(-1+\mathrm{e}^{2a}x^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a+2*ln(x)),x)
[Out] 1/4*x^4+1/2*exp(-2*a)*ln(-1+exp(2*a)*x^4)
```

maxima [A] time = 0.38, size = 36, normalized size = 1.20

$$\frac{1}{4}x^4 + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="maxima")
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(x^2*e^a + 1) + 1/2*e^(-2*a)*log(x^2*e^a - 1)
```

mupad [B] time = 1.25, size = 23, normalized size = 0.77

$$\frac{\ln\left(x^4 - e^{-2a}\right)e^{-2a}}{2} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a + 2*log(x)),x)
[Out] (log(x^4 - exp(-2*a))*exp(-2*a))/2 + x^4/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \coth\left(a + 2\log(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(a+2*ln(x)),x)
[Out] Integral(x**3*coth(a + 2*log(x)), x)
```

3.152 $\int x^2 \coth(a+2\log(x))\,dx$

Optimal. Leaf size=45

$$e^{-3a/2}\tan^{-1}\left(e^{a/2}x\right)-e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right)+\frac{x^3}{3}$$

```
[Out] 1/3*x^3+arctan(exp(1/2*a)*x)/exp(3/2*a)-arctanh(exp(1/2*a)*x)/exp(3/2*a)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\coth(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][x^2*Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int x^2\coth(a+2\log(x))\,dx=\int x^2\coth(a+2\log(x))\,dx$$

Mathematica [C] time = 0.24, size = 64, normalized size = 1.42

$$\frac{1}{6}\left(3(\sinh(2a)-\cosh(2a))\text{RootSum}\left[\#1^4\sinh(a)+\#1^4\cosh(a)+\sinh(a)-\cosh(a)\&,\frac{\log(x)-\log(x-\#1)}{\#1}\&\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[a + 2*Log[x]],x]
```

```
[Out] (2*x^3 + 3*RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Lo
g[x] - Log[x - #1])/#1 & ]*(-Cosh[2*a] + Sinh[2*a]))/6
```

fricas [A] time = 0.40, size = 62, normalized size = 1.38

$$\frac{1}{6}\left(2\,x^3e^{(2\,a)}+6\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+3\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)\right)e^{(-2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3*e^(2*a) + 6*arctan(x*e^(1/2*a))*e^(1/2*a) + 3*e^(1/2*a)*log((x^2
*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-2*a)
```

giac [A] time = 0.14, size = 54, normalized size = 1.20

$$\frac{1}{3}\,x^3+\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)}+\frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\left|\frac{2\,xe^a-2\,e^{\left(\frac{1}{2}\,a\right)}}{2\,xe^a+2\,e^{\left(\frac{1}{2}\,a\right)}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log(abs(2*x*e^a -
 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

maple [B] time = 0.15, size = 83, normalized size = 1.84

$$\frac{x^3}{3}+\frac{\ln\left(-e^{2a}x+(-e^{a})^{\frac{3}{2}}\right)}{2\,(-e^{a})^{\frac{3}{2}}}-\frac{\ln\left(e^{2a}x+(-e^{a})^{\frac{3}{2}}\right)}{2\,(-e^{a})^{\frac{3}{2}}}+\frac{\ln\left(-\sqrt{e^{a}}\,x+1\right)}{2\,(e^{a})^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{e^{a}}\,x+1\right)}{2\,(e^{a})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a+2*ln(x)),x)
```

```
[Out] 1/3*x^3+1/2/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/2/(-exp(a))^(
3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))+1/2/exp(a)^(3/2)*ln(-exp(a)^(1/2)*x+1)-
1/2/exp(a)^(3/2)*ln(exp(a)^(1/2)*x+1)
```

maxima [A] time = 1.07, size = 48, normalized size = 1.07

$$\frac{1}{3}\,x^3+\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)}+\frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}\,a\right)}}{xe^{a}+e^{\left(\frac{1}{2}\,a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log((x*e^a - e^(1
/2*a))/(x*e^a + e^(1/2*a)))
```

mupad [B] time = 1.21, size = 39, normalized size = 0.87

$$\frac{\mathrm{atan}\left(x\,{\left(e^{2\,a}\right)}^{1/4}\right)}{{\left(e^{2\,a}\right)}^{3/4}}-\frac{\mathrm{atanh}\left(x\,{\left(e^{2\,a}\right)}^{1/4}\right)}{{\left(e^{2\,a}\right)}^{3/4}}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a + 2*log(x)),x)
```

```
[Out] atan(x*exp(2*a)^(1/4))/exp(2*a)^(3/4) - atanh(x*exp(2*a)^(1/4))/exp(2*a)^(3
/4) + x^3/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\coth\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x**2*coth(a + 2*log(x)), x)
```

3.153 $\int x \coth(a + 2\log(x))\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2} - e^{-a}\tanh^{-1}\left(e^a x^2\right)$$

[Out] 1/2*x^2-arctanh(exp(a)*x^2)/exp(a)

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x \coth(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

[In] Int[x*Coth[a + 2*Log[x]],x]

[Out] Defer[Int][x*Coth[a + 2*Log[x]], x]

Rubi steps

$$\int x \coth(a + 2\log(x))\,dx = \int x \coth(a + 2\log(x))\,dx$$

Mathematica [A] time = 0.19, size = 26, normalized size = 1.13

$$(\sinh(a) - \cosh(a))\tanh^{-1}\left(x^2(\sinh(a) + \cosh(a))\right) + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Coth[a + 2*Log[x]],x]

[Out] x^2/2 + ArcTanh[x^2*(Cosh[a] + Sinh[a])]*(-Cosh[a] + Sinh[a])

fricas [A] time = 0.43, size = 33, normalized size = 1.43

$$\frac{1}{2}\left(x^2 e^a - \log\left(x^2 e^a + 1\right) + \log\left(x^2 e^a - 1\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(a+2*log(x)),x, algorithm="fricas")

[Out] 1/2*(x^2*e^a - log(x^2*e^a + 1) + log(x^2*e^a - 1))*e^(-a)

giac [A] time = 0.14, size = 37, normalized size = 1.61

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2 e^a + 1\right) + \frac{1}{2}e^{(-a)}\log\left(\left|x^2 e^a - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(a+2*log(x)),x, algorithm="giac")

[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1))

maple [A] time = 0.11, size = 37, normalized size = 1.61

$$\frac{x^2}{2} - \frac{e^{-a}\ln\left(e^a x^2 + 1\right)}{2} + \frac{e^{-a}\ln\left(e^a x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a+2*ln(x)),x)
```

```
[Out] 1/2*x^2-1/2*exp(-a)*ln(exp(a)*x^2+1)+1/2*exp(-a)*ln(exp(a)*x^2-1)
```

maxima [A] time = 0.31, size = 36, normalized size = 1.57

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}e^{(-a)}\log\left(x^2e^a - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1)
```

mupad [B] time = 1.23, size = 25, normalized size = 1.09

$$\frac{x^2}{2} - \frac{\operatorname{atanh}\left(x^2\sqrt{e^{2a}}\right)}{\sqrt{e^{2a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a + 2*log(x)),x)
```

```
[Out] x^2/2 - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth\left(a + 2\log(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*ln(x)),x)
```

```
[Out] Integral(x*coth(a + 2*log(x)), x)
```

3.154 $\int \coth(a+2\log(x))\,dx$

Optimal. Leaf size=40

$$-e^{-a/2}\tan^{-1}\left(e^{a/2}x\right)-e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right)+x$$

```
[Out] x-arctan(exp(1/2*a)*x)/exp(1/2*a)-arctanh(exp(1/2*a)*x)/exp(1/2*a)
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]],x]
[Out] Defer[Int][Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int \coth(a+2\log(x))\,dx=\int \coth(a+2\log(x))\,dx$$

Mathematica [C] time = 0.19, size = 58, normalized size = 1.45

$$\frac{1}{2}(\sinh(2a)-\cosh(2a))\text{RootSum}\left[\#1^4\sinh(a)+\#1^4\cosh(a)+\sinh(a)-\cosh(a)\&,\frac{\log(x)-\log(x-\#1)}{\#1^3}\&\right]+$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]],x]
[Out] x + (RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log[x] -
 Log[x - #1])/#1^3 & ]*(-Cosh[2*a] + Sinh[2*a]))/2
```

fricas [B] time = 0.41, size = 58, normalized size = 1.45

$$-\frac{1}{2}\left(2\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-2xe^{a}-e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a-2xe^{\left(\frac{1}{2}a\right)}+1}{x^2e^a-1}\right)\right)e^{(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="fricas")
[Out] -1/2*(2*arctan(x*e^(1/2*a))*e^(1/2*a) - 2*x*e^a - e^(1/2*a)*log((x^2*e^a -
2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-a)
```

giac [A] time = 0.14, size = 51, normalized size = 1.28

$$-\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2xe^a-2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a+2e^{\left(\frac{1}{2}a\right)}\right|}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^(1/2
*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x
```

maple [**B**] time = 0.14, size = 71, normalized size = 1.78

$$x+\frac{\ln\left(\sqrt{\mathrm{e}^{a}}\,x-1\right)}{2\sqrt{\mathrm{e}^{a}}}-\frac{\ln\left(\sqrt{\mathrm{e}^{a}}\,x+1\right)}{2\sqrt{\mathrm{e}^{a}}}-\frac{\ln\left(x\sqrt{-\mathrm{e}^{a}}\,+1\right)}{2\sqrt{-\mathrm{e}^{a}}}+\frac{\ln\left(x\sqrt{-\mathrm{e}^{a}}\,-1\right)}{2\sqrt{-\mathrm{e}^{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x)),x)
```

```
[Out] x+1/2/exp(a)^(1/2)*ln(exp(a)^(1/2)*x-1)-1/2/exp(a)^(1/2)*ln(exp(a)^(1/2)*x+
1)-1/2/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/2/(-exp(a))^(1/2)*ln(x*(-e
xp(a))^(1/2)-1)
```

maxima [**A**] time = 0.60, size = 45, normalized size = 1.12

$$-\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}+\frac{1}{2}\,e^{\left(-\frac{1}{2}\,a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}\,a\right)}}{xe^{a}+e^{\left(\frac{1}{2}\,a\right)}}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log((x*e^a - e^(1/2*a))/(x
*e^a + e^(1/2*a))) + x
```

mupad [**B**] time = 1.19, size = 36, normalized size = 0.90

$$x-\frac{\mathrm{atan}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\right)}{{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}}-\frac{\mathrm{atanh}\left(x\,{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}\right)}{{\left({\mathrm{e}}^{2\,a}\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x)),x)
```

```
[Out] x - atan(x*exp(2*a)^(1/4))/exp(2*a)^(1/4) - atanh(x*exp(2*a)^(1/4))/exp(2*a
)^(1/4)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+2\log\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x)),x)
```

```
[Out] Integral(coth(a + 2*log(x)), x)
```

3.155 $\int \frac{\coth(a+2\log(x))}{x}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log(\sinh(a+2\log(x)))$$

```
[Out] 1/2*ln(sinh(a+2*ln(x)))
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3475}

$$\frac{1}{2}\log(\sinh(a+2\log(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]/x,x]
[Out] Log[Sinh[a + 2*Log[x]]]/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(a+2\log(x))}{x}\,dx &= \text{Subst}\left(\int \coth(a+2x)\,dx, x, \log(x)\right)\\ &= \frac{1}{2}\log(\sinh(a+2\log(x)))\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 1.75

$$\frac{1}{2}(\log(\tanh(a+2\log(x)))+\log(\cosh(a+2\log(x))))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x,x]
[Out] (Log[Cosh[a + 2*Log[x]]] + Log[Tanh[a + 2*Log[x]]])/2
```

fricas [A] time = 0.42, size = 18, normalized size = 1.50

$$\frac{1}{2}\log\left(x^4e^{(2a)}-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="fricas")
[Out] 1/2*log(x^4*e^(2*a) - 1) - log(x)
```

giac [B] time = 0.13, size = 21, normalized size = 1.75

$$-\frac{1}{4}\log\left(x^4\right)+\frac{1}{2}\log\left(\left|x^4e^{(2a)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="giac")
[Out] -1/4*log(x^4) + 1/2*log(abs(x^4*e^(2*a) - 1))
```

maple [B] time = 0.01, size = 26, normalized size = 2.17

$$-\frac{\ln\left(\coth\left(a+2\ln(x)\right)-1\right)}{4}-\frac{\ln\left(\coth\left(a+2\ln(x)\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x,x)
[Out] -1/4*ln(coth(a+2*ln(x))-1)-1/4*ln(coth(a+2*ln(x))+1)
```

maxima [A] time = 0.30, size = 10, normalized size = 0.83

$$\frac{1}{2}\,\log\left(\sinh\left(a+2\,\log(x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="maxima")
[Out] 1/2*log(sinh(a + 2*log(x)))
```

mupad [B] time = 1.21, size = 18, normalized size = 1.50

$$\frac{\ln\left(x^4-\mathrm{e}^{-2\,a}\right)}{2}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))/x,x)
[Out] log(x^4 - exp(-2*a))/2 - log(x)
```

sympy [B] time = 0.98, size = 27, normalized size = 2.25

$$\log(x)-\frac{\log\left(\tanh\left(a+2\log(x)\right)+1\right)}{2}+\frac{\log\left(\tanh\left(a+2\log(x)\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x,x)
[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2 + log(tanh(a + 2*log(x)))/2
```

3.156 $\int \frac{\coth(a+2\log(x))}{x^2} dx$

Optimal. Leaf size=41

$$e^{a/2}\tan^{-1}\left(e^{a/2}x\right) - e^{a/2}\tanh^{-1}\left(e^{a/2}x\right) + \frac{1}{x}$$

```
[Out] 1/x+exp(1/2*a)*arctan(exp(1/2*a)*x)-exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+2\log(x))}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]/x^2,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]/x^2, x]
```

Rubi steps

$$\int \frac{\coth(a+2\log(x))}{x^2} dx = \int \frac{\coth(a+2\log(x))}{x^2} dx$$

Mathematica [C] time = 0.17, size = 62, normalized size = 1.51

$$\frac{x(\sinh(a)+\cosh(a))^2 \text{RootSum}\left[-\#1^4\sinh(a)+\#1^4\cosh(a)-\sinh(a)-\cosh(a)\&, \frac{\log\left(\frac{1}{x}-\#1\right)+\log(x)}{\#1^3}\&\right]+2}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x^2,x]
```

```
[Out] (2 + x*RootSum[-Cosh[a] - Sinh[a] + Cosh[a]*#1^4 - Sinh[a]*#1^4 & , (Log[x]
 + Log[x^(-1) - #1])/#1^3 & ]*(Cosh[a] + Sinh[a])^2)/(2*x)
```

fricas [A] time = 0.42, size = 54, normalized size = 1.32

$$\frac{2\,x\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} + xe^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a - 2\,xe^{\left(\frac{1}{2}\,a\right)} + 1}{x^2e^a - 1}\right) + 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*arctan(x*e^(1/2*a))*e^(1/2*a) + x*e^(1/2*a)*log((x^2*e^a - 2*x*e^(
1/2*a) + 1)/(x^2*e^a - 1)) + 2)/x
```

giac [A] time = 0.14, size = 52, normalized size = 1.27

$$\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} + \frac{1}{2}\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{\left|2\,xe^a - 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^a + 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="giac")
```

```
[Out] arctan(x*e^(1/2*a))*e^(1/2*a) + 1/2*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a)
)/abs(2*x*e^a + 2*e^(1/2*a))) + 1/x
```

maple [B] time = 0.13, size = 93, normalized size = 2.27

$$\frac{1}{x}+\frac{\sqrt{-\mathrm{e}^{a}}\,\ln\left(-\mathrm{e}^{2a}x+\left(-\mathrm{e}^{a}\right)^{\frac{3}{2}}\right)}{2}-\frac{\sqrt{-\mathrm{e}^{a}}\,\ln\left(-\mathrm{e}^{2a}x-\left(-\mathrm{e}^{a}\right)^{\frac{3}{2}}\right)}{2}+\frac{\sqrt{\mathrm{e}^{a}}\,\ln\left(-\mathrm{e}^{2a}x+\left(\mathrm{e}^{a}\right)^{\frac{3}{2}}\right)}{2}-\frac{\sqrt{\mathrm{e}^{a}}\,\ln\left(-\mathrm{e}^{2a}x-\left(\mathrm{e}^{a}\right)^{\frac{3}{2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x^2,x)
```

```
[Out] 1/x+1/2*(-exp(a))^(1/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))-1/2*(-exp(a))^(1/2)
*ln(-exp(2*a)*x-(-exp(a))^(3/2))+1/2*exp(a)^(1/2)*ln(-exp(2*a)*x+exp(a)^(3/
2))-1/2*exp(a)^(1/2)*ln(-exp(2*a)*x-exp(a)^(3/2))
```

maxima [A] time = 1.22, size = 47, normalized size = 1.15

$$-\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)}+\frac{1}{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x}-e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x}+e^{\left(\frac{1}{2}a\right)}}\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-1/2*a)/x)*e^(1/2*a) + 1/2*e^(1/2*a)*log((1/x - e^(1/2*a))/(1/x
+ e^(1/2*a))) + 1/x
```

mupad [B] time = 1.20, size = 37, normalized size = 0.90

$$\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)-\left(\mathrm{e}^{2\,a}\right)^{1/4}\,\mathrm{atanh}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))/x^2,x)
```

```
[Out] exp(2*a)^(1/4)*atan(x*exp(2*a)^(1/4)) - exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1/
4)) + 1/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(a+2\log\left(x\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))/x**2, x)
```

3.157 $\int \frac{\coth(a+2\log(x))}{x^3}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2x^2} - e^a \tanh^{-1}\left(e^a x^2\right)$$

```
[Out] 1/2/x^2-exp(a)*arctanh(exp(a)*x^2)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]/x^3,x]
[Out] Defer[Int][Coth[a + 2*Log[x]]/x^3, x]
```

Rubi steps

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \int \frac{\coth(a+2\log(x))}{x^3}\,dx$$

Mathematica [A] time = 0.16, size = 27, normalized size = 1.29

$$\frac{1}{2x^2} - (\sinh(a)+\cosh(a))\tanh^{-1}\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]/x^3,x]
[Out] 1/(2*x^2) - ArcTanh[(Cosh[a] - Sinh[a])/x^2]*(Cosh[a] + Sinh[a])
```

fricas [B] time = 0.41, size = 38, normalized size = 1.81

$$-\frac{x^2e^a\log\left(x^2e^a+1\right) - x^2e^a\log\left(x^2e^a-1\right) - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="fricas")
[Out] -1/2*(x^2*e^a*log(x^2*e^a + 1) - x^2*e^a*log(x^2*e^a - 1) - 1)/x^2
```

giac [A] time = 0.13, size = 33, normalized size = 1.57

$$-\frac{1}{2}e^a\log\left(x^2e^a+1\right) + \frac{1}{2}e^a\log\left(\left|x^2e^a-1\right|\right) + \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="giac")
[Out] -1/2*e^a*log(x^2*e^a + 1) + 1/2*e^a*log(abs(x^2*e^a - 1)) + 1/2/x^2
```

maple [A] time = 0.10, size = 35, normalized size = 1.67

$$\frac{1}{2x^2} - \frac{\mathrm{e}^a\ln\left(-\mathrm{e}^a x^2-1\right)}{2} + \frac{\mathrm{e}^a\ln\left(-\mathrm{e}^a x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))/x^3,x)
```

```
[Out] 1/2/x^2-1/2*exp(a)*ln(-exp(a)*x^2-1)+1/2*exp(a)*ln(-exp(a)*x^2+1)
```

maxima [A] time = 0.50, size = 30, normalized size = 1.43

$$-\frac{1}{2}\,e^{a}\log\left(\frac{1}{x^{2}}+e^{a}\right)+\frac{1}{2}\,e^{a}\log\left(\frac{1}{x^{2}}-e^{a}\right)+\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*e^a*log(1/x^2 + e^a) + 1/2*e^a*log(1/x^2 - e^a) + 1/2/x^2
```

mupad [B] time = 1.21, size = 25, normalized size = 1.19

$$\frac{1}{2\,x^{2}}-\mathrm{atanh}\left(x^{2}\,\sqrt{\mathrm{e}^{2\,a}}\right)\,\sqrt{\mathrm{e}^{2\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))/x^3,x)
```

```
[Out] 1/(2*x^2) - atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(a+2\log\left(x\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))/x**3,x)
```

```
[Out] Integral(coth(a + 2*log(x))/x**3, x)
```

3.158 $\int x^3 \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=47

$$\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)+\frac{x^4}{4}$$

```
[Out] 1/4*x^4+1/exp(2*a)/(1-exp(2*a)*x^4)+ln(1-exp(2*a)*x^4)/exp(2*a)
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\coth^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][x^3*Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int x^3\coth^2(a+2\log(x))\,dx=\int x^3\coth^2(a+2\log(x))\,dx$$

Mathematica [A] time = 0.11, size = 86, normalized size = 1.83

$$\frac{\sinh(3a)-\cosh(3a)}{\left(x^4+1\right)\sinh(a)+\left(x^4-1\right)\cosh(a)}+\cosh(2a)\log\left(\left(x^4+1\right)\sinh(a)+\left(x^4-1\right)\cosh(a)\right)-\sinh(2a)\log\left(\left(x^4+1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 + Cosh[2*a]*Log[(-1 + x^4)*Cosh[a] + (1 + x^4)*Sinh[a]] - Log[(-1 + x
^4)*Cosh[a] + (1 + x^4)*Sinh[a]]*Sinh[2*a] + (-Cosh[3*a] + Sinh[3*a])/((-1
+ x^4)*Cosh[a] + (1 + x^4)*Sinh[a])
```

fricas [A] time = 0.39, size = 61, normalized size = 1.30

$$\frac{x^8e^{(4a)}-x^4e^{(2a)}+4\left(x^4e^{(2a)}-1\right)\log\left(x^4e^{(2a)}-1\right)-4}{4\left(x^4e^{(4a)}-e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^8*e^(4*a) - x^4*e^(2*a) + 4*(x^4*e^(2*a) - 1)*log(x^4*e^(2*a) - 1) -
 4)/(x^4*e^(4*a) - e^(2*a))
```

giac [A] time = 0.11, size = 40, normalized size = 0.85

$$\frac{1}{4}x^4-\frac{x^4}{x^4e^{(2a)}-1}+e^{(-2a)}\log\left(\left|x^4e^{(2a)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - x^4/(x^4*e^(2*a) - 1) + e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

maple [A] time = 0.10, size = 41, normalized size = 0.87

$$\frac{x^4}{4} - \frac{e^{-2a}}{-1 + e^{2a}x^4} + e^{-2a}\ln\left(-1 + e^{2a}x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a+2*ln(x))^2,x)
[Out] 1/4*x^4-exp(-2*a)/(-1+exp(2*a)*x^4)+exp(-2*a)*ln(-1+exp(2*a)*x^4)
```

maxima [A] time = 0.33, size = 53, normalized size = 1.13

$$\frac{1}{4}x^4 + e^{(-2a)}\log\left(x^2e^a + 1\right) + e^{(-2a)}\log\left(x^2e^a - 1\right) - \frac{1}{x^4e^{(4a)} - e^{(2a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="maxima")
[Out] 1/4*x^4 + e^(-2*a)*log(x^2*e^a + 1) + e^(-2*a)*log(x^2*e^a - 1) - 1/(x^4*e^
(4*a) - e^(2*a))
```

mupad [B] time = 1.25, size = 40, normalized size = 0.85

$$\ln\left(x^4 - e^{-2a}\right)e^{-2a} - \frac{e^{-2a}}{x^4\,e^{2a} - 1} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(a + 2*log(x))^2,x)
[Out] log(x^4 - exp(-2*a))*exp(-2*a) - exp(-2*a)/(x^4*exp(2*a) - 1) + x^4/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\coth^2\left(a + 2\log(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(a+2*ln(x))**2,x)
[Out] Integral(x**3*coth(a + 2*log(x))**2, x)
```

3.159 $\int x^2 \coth^2(a + 2\log(x))\,dx$

Optimal. Leaf size=68

$$\frac{x^3}{1-e^{2a}x^4} + \frac{3}{2}e^{-3a/2}\tan^{-1}\left(e^{a/2}x\right) - \frac{3}{2}e^{-3a/2}\tanh^{-1}\left(e^{a/2}x\right) + \frac{x^3}{3}$$

```
[Out] 1/3*x^3+x^3/(1-exp(2*a)*x^4)+3/2*arctan(exp(1/2*a)*x)/exp(3/2*a)-3/2*arctan
h(exp(1/2*a)*x)/exp(3/2*a)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \coth^2(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][x^2*Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int x^2 \coth^2(a + 2\log(x))\,dx = \int x^2 \coth^2(a + 2\log(x))\,dx$$

Mathematica [C] time = 2.95, size = 154, normalized size = 2.26

$$\frac{16e^{2a}x^7\left(e^{2a}x^4+1\right)^2 {}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{2a}x^4\right)}{1155} + \frac{e^{-4a}\left(7\left(27e^{8a}x^{16} - 632e^{6a}x^{12} - 398e^{4a}x^8 + 1976e^{2a}x^4 + 133\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-9317 - 17825*E^(2*a)*x^4 - 4787*E^(4*a)*x^8 + 1481*E^(6*a)*x^12 + 7*(1331
 + 1976*E^(2*a)*x^4 - 398*E^(4*a)*x^8 - 632*E^(6*a)*x^12 + 27*E^(8*a)*x^16)
*Hypergeometric2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(2688*E^(4*a)*x^5) + (16*E^(2
*a)*x^7*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4},
 E^(2*a)*x^4])/1155
```

fricas [B] time = 0.41, size = 104, normalized size = 1.53

$$\frac{4\,x^7e^{(4\,a)} - 16\,x^3e^{(2\,a)} + 18\left(x^4e^{(2\,a)} - 1\right)\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} + 9\left(x^4e^{(2\,a)} - 1\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a - 2\,xe^{\left(\frac{1}{2}\,a\right)} + 1}{x^2e^a - 1}\right)}{12\left(x^4e^{(4\,a)} - e^{(2\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^7*e^(4*a) - 16*x^3*e^(2*a) + 18*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2
*a))*e^(1/2*a) + 9*(x^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a)
 + 1)/(x^2*e^a - 1)))/(x^4*e^(4*a) - e^(2*a))
```

giac [A] time = 0.11, size = 72, normalized size = 1.06

$$\frac{1}{3}x^3 - \frac{x^3}{x^4e^{(2a)} - 1} + \frac{3}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{3}{4}e^{\left(-\frac{3}{2}a\right)}\log\left(\frac{\left|2xe^a - 2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^a + 2e^{\left(\frac{1}{2}a\right)}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

maple [A] time = 0.12, size = 100, normalized size = 1.47

$$\frac{x^3}{3} - \frac{x^3}{-1 + e^{2a}x^4} + \frac{3\ln\left(-e^{2a}x + (-e^a)^{\frac{3}{2}}\right)}{4(-e^a)^{\frac{3}{2}}} - \frac{3\ln\left(e^{2a}x + (-e^a)^{\frac{3}{2}}\right)}{4(-e^a)^{\frac{3}{2}}} + \frac{3\ln\left(-\sqrt{e^a}\,x + 1\right)}{4(e^a)^{\frac{3}{2}}} - \frac{3\ln\left(\sqrt{e^a}\,x + 1\right)}{4(e^a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a+2*ln(x))^2,x)
```

```
[Out] 1/3*x^3-x^3/(-1+exp(2*a)*x^4)+3/4/(-exp(a))^(3/2)*ln(-exp(2*a)*x+(-exp(a))^
(3/2))-3/4/(-exp(a))^(3/2)*ln(exp(2*a)*x+(-exp(a))^(3/2))+3/4/exp(a)^(3/2)*
ln(-exp(a)^(1/2)*x+1)-3/4/exp(a)^(3/2)*ln(exp(a)^(1/2)*x+1)
```

maxima [A] time = 0.42, size = 66, normalized size = 0.97

$$\frac{1}{3}x^3 - \frac{x^3}{x^4e^{(2a)} - 1} + \frac{3}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} + \frac{3}{4}e^{\left(-\frac{3}{2}a\right)}\log\left(\frac{xe^a - e^{\left(\frac{1}{2}a\right)}}{xe^a + e^{\left(\frac{1}{2}a\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a)))
```

mupad [B] time = 1.24, size = 60, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(x\left(e^{2a}\right)^{1/4}\right)}{2\left(e^{2a}\right)^{3/4}} - \frac{x^3}{x^4\,e^{2a} - 1} + \frac{x^3}{3} + \frac{\mathrm{atan}\left(x\left(e^{2a}\right)^{1/4}\,1i\right)\,3i}{2\left(e^{2a}\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(a + 2*log(x))^2,x)
```

```
[Out] (3*atan(x*exp(2*a)^(1/4)))/(2*exp(2*a)^(3/4)) - x^3/(x^4*exp(2*a) - 1) + (a
tan(x*exp(2*a)^(1/4)*1i)*3i)/(2*exp(2*a)^(3/4)) + x^3/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\coth^2\left(a + 2\log(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**2*coth(a + 2*log(x))**2, x)
```

3.160 $\int x \coth^2(a + 2\log(x))\,dx$

Optimal. Leaf size=41

$$-e^{-a}\tanh^{-1}\left(e^a x^2\right) + \frac{x^2}{1-e^{2a}x^4} + \frac{x^2}{2}$$

[Out] 1/2*x^2+x^2/(1-exp(2*a)*x^4)-arctanh(exp(a)*x^2)/exp(a)

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x \coth^2(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

[In] Int[x*Coth[a + 2*Log[x]]^2,x]

[Out] Defer[Int][x*Coth[a + 2*Log[x]]^2, x]

Rubi steps

$$\int x \coth^2(a + 2\log(x))\,dx = \int x \coth^2(a + 2\log(x))\,dx$$

Mathematica [C] time = 3.07, size = 163, normalized size = 3.98

$$\frac{2}{105}e^{2a}x^6\left(e^{2a}x^4+1\right)^2 {}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};e^{2a}x^4\right)+\frac{e^{-4a}\left(61e^{6a}x^{12}-181e^{4a}x^8-713e^{2a}x^4+\frac{3\left(e^{8a}x^{16}-52e^{6a}x^{12}-14e^{4}\right.}{}\right.}{96x^6}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*Coth[a + 2*Log[x]]^2,x]

[Out] (-375 - 713*E^(2*a)*x^4 - 181*E^(4*a)*x^8 + 61*E^(6*a)*x^12 + (3*(125 + 196 *E^(2*a)*x^4 - 14*E^(4*a)*x^8 - 52*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqr t[E^(2*a)*x^4]])/Sqrt[E^(2*a)*x^4])/(96*E^(4*a)*x^6) + (2*E^(2*a)*x^6*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*a)*x^4]) /105

fricas [B] time = 0.40, size = 74, normalized size = 1.80

$$\frac{x^6 e^{(3a)} - 3x^2 e^a - \left(x^4 e^{(2a)} - 1\right)\log\left(x^2 e^a + 1\right) + \left(x^4 e^{(2a)} - 1\right)\log\left(x^2 e^a - 1\right)}{2\left(x^4 e^{(3a)} - e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="fricas")

[Out] 1/2*(x^6*e^(3*a) - 3*x^2*e^a - (x^4*e^(2*a) - 1)*log(x^2*e^a + 1) + (x^4*e^ (2*a) - 1)*log(x^2*e^a - 1))/(x^4*e^(3*a) - e^a)

giac [A] time = 0.13, size = 54, normalized size = 1.32

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2 e^a + 1\right) + \frac{1}{2}e^{(-a)}\log\left(\left|x^2 e^a - 1\right|\right) - \frac{x^2}{x^4 e^{(2a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1)) -
x^2/(x^4*e^(2*a) - 1)
```

maple [A] time = 0.09, size = 54, normalized size = 1.32

$$\frac{x^2}{2} - \frac{x^2}{-1 + e^{2a}x^4} + \frac{e^{-a}\ln\left(e^a x^2 - 1\right)}{2} - \frac{e^{-a}\ln\left(e^a x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a+2*ln(x))^2,x)
```

```
[Out] 1/2*x^2-x^2/(-1+exp(2*a)*x^4)+1/2*exp(-a)*ln(exp(a)*x^2-1)-1/2*exp(-a)*ln(e
xp(a)*x^2+1)
```

maxima [A] time = 0.32, size = 53, normalized size = 1.29

$$\frac{1}{2}x^2 - \frac{1}{2}e^{(-a)}\log\left(x^2e^a + 1\right) + \frac{1}{2}e^{(-a)}\log\left(x^2e^a - 1\right) - \frac{x^2}{x^4e^{(2a)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1) - x^2/(
x^4*e^(2*a) - 1)
```

mupad [B] time = 1.23, size = 42, normalized size = 1.02

$$\frac{x^2}{2} - \frac{x^2}{x^4\,e^{2\,a} - 1} - \frac{\operatorname{atanh}\left(x^2\,\sqrt{e^{2\,a}}\right)}{\sqrt{e^{2\,a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(a + 2*log(x))^2,x)
```

```
[Out] x^2/2 - x^2/(x^4*exp(2*a) - 1) - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth^2\left(a + 2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(x*coth(a + 2*log(x))**2, x)
```

3.161 $\int \coth^2(a+2\log(x))\,dx$

Optimal. Leaf size=60

$$\frac{x}{1-e^{2a}x^4}-\frac{1}{2}e^{-a/2}\tan^{-1}\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\tanh^{-1}\left(e^{a/2}x\right)+x$$

```
[Out] x+x/(1-exp(2*a)*x^4)-1/2*arctan(exp(1/2*a)*x)/exp(1/2*a)-1/2*arctanh(exp(1/
2*a)*x)/exp(1/2*a)
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^2(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int \coth^2(a+2\log(x))\,dx=\int \coth^2(a+2\log(x))\,dx$$

Mathematica [C] time = 2.24, size = 153, normalized size = 2.55

$$\frac{16}{585}e^{2a}x^5\left(e^{2a}x^4+1\right)^2{}_4F_3\left(\frac{5}{4},2,2,2;1,1,\frac{17}{4};e^{2a}x^4\right)+\frac{e^{-4a}\left(5\left(e^{8a}x^{16}-248e^{6a}x^{12}+102e^{4a}x^8+1208e^{2a}x^4+72\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-3645 - 6769*E^(2*a)*x^4 - 1483*E^(4*a)*x^8 + 681*E^(6*a)*x^12 + 5*(729 +
1208*E^(2*a)*x^4 + 102*E^(4*a)*x^8 - 248*E^(6*a)*x^12 + E^(8*a)*x^16)*Hyper
geometric2F1[1/4, 1, 5/4, E^(2*a)*x^4])/(640*E^(4*a)*x^7) + (16*E^(2*a)*x^5
*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, E^(2*a
)*x^4])/585
```

fricas [B] time = 0.41, size = 97, normalized size = 1.62

$$\frac{4\,x^5e^{(3\,a)}-2\left(x^4e^{(2\,a)}-1\right)\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+\left(x^4e^{(2\,a)}-1\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)-8\,xe^a}{4\left(x^4e^{(3\,a)}-e^a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^5*e^(3*a) - 2*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2*a))*e^(1/2*a) + (x
^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
- 8*x*e^a)/(x^4*e^(3*a) - e^a)
```

giac [A] time = 0.14, size = 66, normalized size = 1.10

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}a\right)}\right|}\right)+x-\frac{x}{x^{4}e^{(2\,a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^
(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

maple [A] time = 0.13, size = 86, normalized size = 1.43

$$x-\frac{x}{-1+e^{2a}x^{4}}+\frac{\ln\left(\sqrt{e^{a}}\,x-1\right)}{4\sqrt{e^{a}}}-\frac{\ln\left(\sqrt{e^{a}}\,x+1\right)}{4\sqrt{e^{a}}}-\frac{\ln\left(x\sqrt{-e^{a}}\,+1\right)}{4\sqrt{-e^{a}}}+\frac{\ln\left(x\sqrt{-e^{a}}\,-1\right)}{4\sqrt{-e^{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2,x)
```

```
[Out] x-x/(-1+exp(2*a)*x^4)+1/4/exp(a)^(1/2)*ln(exp(a)^(1/2)*x-1)-1/4/exp(a)^(1/2
)*ln(exp(a)^(1/2)*x+1)-1/4/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/4/(-ex
p(a))^(1/2)*ln(x*(-exp(a))^(1/2)-1)
```

maxima [A] time = 0.42, size = 60, normalized size = 1.00

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}+\frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}a\right)}}{xe^{a}+e^{\left(\frac{1}{2}a\right)}}\right)+x-\frac{x}{x^{4}e^{(2\,a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log((x*e^a - e^(1/2*a)
)/(x*e^a + e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

mupad [B] time = 1.21, size = 54, normalized size = 0.90

$$x-\frac{\mathrm{atan}\left(x\left(e^{2\,a}\right)^{1/4}\right)}{2\left(e^{2\,a}\right)^{1/4}}-\frac{x}{x^{4}\,e^{2\,a}-1}+\frac{\mathrm{atan}\left(x\left(e^{2\,a}\right)^{1/4}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\left(e^{2\,a}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2,x)
```

```
[Out] x - atan(x*exp(2*a)^(1/4))/(2*exp(2*a)^(1/4)) + (atan(x*exp(2*a)^(1/4)*1i)*
1i)/(2*exp(2*a)^(1/4)) - x/(x^4*exp(2*a) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\coth^{2}\left(a+2\log\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2, x)
```

3.162 $\int \frac{\coth^2(a+2\log(x))}{x}\,dx$

Optimal. Leaf size=14

$$\log(x) - \frac{1}{2}\coth(a+2\log(x))$$

```
[Out] -1/2*coth(a+2*ln(x))+ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3473, 8}

$$\log(x) - \frac{1}{2}\coth(a+2\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + 2*Log[x]]^2/x,x]
```

```
[Out] -Coth[a + 2*Log[x]]/2 + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(a+2\log(x))}{x}\,dx &= \text{Subst}\left(\int \coth^2(a+2x)\,dx, x, \log(x)\right)\\
&= -\frac{1}{2}\coth(a+2\log(x)) + \text{Subst}(\int 1\,dx, x, \log(x))\\
&= -\frac{1}{2}\coth(a+2\log(x)) + \log(x)
\end{aligned}$$

Mathematica [C] time = 0.05, size = 28, normalized size = 2.00

$$-\frac{1}{2}\coth(a+2\log(x))\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2(a+2\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x,x]
```

```
[Out] -1/2*(Coth[a + 2*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + 2*Log[x]]
^2])
```

fricas [B] time = 0.43, size = 28, normalized size = 2.00

$$\frac{\left(x^4e^{(2a)}-1\right)\log(x)-1}{x^4e^{(2a)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="fricas")
[Out] ((x^4*e^(2*a) - 1)*log(x) - 1)/(x^4*e^(2*a) - 1)
```

giac [A] time = 0.13, size = 21, normalized size = 1.50

$$-\frac{1}{x^4 e^{(2a)} - 1} + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="giac")
[Out] -1/(x^4*e^(2*a) - 1) + 1/4*log(x^4)
```

maple [B] time = 0.01, size = 35, normalized size = 2.50

$$-\frac{\coth(a+2\ln(x))}{2} - \frac{\ln(\coth(a+2\ln(x))-1)}{4} + \frac{\ln(\coth(a+2\ln(x))+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x,x)
[Out] -1/2*coth(a+2*ln(x))-1/4*ln(coth(a+2*ln(x))-1)+1/4*ln(coth(a+2*ln(x))+1)
```

maxima [A] time = 0.31, size = 19, normalized size = 1.36

$$\frac{1}{2}a + \frac{1}{e^{(-2a-4\log(x))} - 1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="maxima")
[Out] 1/2*a + 1/(e^(-2*a - 4*log(x)) - 1) + log(x)
```

mupad [B] time = 1.19, size = 28, normalized size = 2.00

$$\ln(x) - \frac{e^{2a}x^4 + 1}{2\left(x^4 e^{2a} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2/x,x)
[Out] log(x) - (x^4*exp(2*a) + 1)/(2*(x^4*exp(2*a) - 1))
```

sympy [A] time = 7.36, size = 32, normalized size = 2.29

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = \log\left(-\frac{1}{x^2}\right) \vee a = \log\left(\frac{1}{x^2}\right) \\ \log(x) - \frac{1}{2\tanh\left(a+2\log(x)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x,x)
[Out] Piecewise((zoo*log(x), Eq(a, log(x**(-2))) | Eq(a, log(-1/x**2))), (log(x)
- 1/(2*tanh(a + 2*log(x))), True))
```

3.163 $\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{x\left(1-e^{2a}x^4\right)}+\frac{2e^{2a}x^3}{1-e^{2a}x^4}-\frac{1}{2}e^{a/2}\tan^{-1}\left(e^{a/2}x\right)+\frac{1}{2}e^{a/2}\tanh^{-1}\left(e^{a/2}x\right)$$

```
[Out] -1/x/(1-exp(2*a)*x^4)+2*exp(2*a)*x^3/(1-exp(2*a)*x^4)-1/2*exp(1/2*a)*arctan
(exp(1/2*a)*x)+1/2*exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]^2/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx=\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$$

Mathematica [C] time = 2.85, size = 153, normalized size = 1.78

$$\frac{16}{231}e^{2a}x^3\left(e^{2a}x^4+1\right)^2{}_4F_3\left(\frac{3}{4},2,2,2;1,1,\frac{15}{4};e^{2a}x^4\right)+\frac{e^{-2a}\left(\left(-e^{8a}x^{16}-56e^{6a}x^{12}+362e^{4a}x^8+632e^{2a}x^4+343\right)\right.}{3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] (-343 - 1163*E^(2*a)*x^4 - 241*E^(4*a)*x^8 + 3*E^(6*a)*x^12 + (343 + 632*E^
(2*a)*x^4 + 362*E^(4*a)*x^8 - 56*E^(6*a)*x^12 - E^(8*a)*x^16)*Hypergeometri
c2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(384*E^(2*a)*x^5) + (16*E^(2*a)*x^3*(1 + E^
(2*a)*x^4)^2*HypergeometricPFQ[{3/4, 2, 2, 2}, {1, 1, 15/4}, E^(2*a)*x^4])/
231
```

fricas [A] time = 0.41, size = 97, normalized size = 1.13

$$-\frac{8x^4e^{(2a)}+2\left(x^5e^{(2a)}-x\right)\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\left(x^5e^{(2a)}-x\right)e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^2e^a+2xe^{\left(\frac{1}{2}a\right)}+1}{x^2e^a-1}\right)-4}{4\left(x^5e^{(2a)}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^4*e^(2*a) + 2*(x^5*e^(2*a) - x)*arctan(x*e^(1/2*a))*e^(1/2*a) - (
x^5*e^(2*a) - x)*e^(1/2*a)*log((x^2*e^a + 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
 - 4)/(x^5*e^(2*a) - x)
```

giac [A] time = 0.12, size = 77, normalized size = 0.90

$$-\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2xe^{a}-2e^{\left(\frac{1}{2}a\right)}\right|}{\left|2xe^{a}+2e^{\left(\frac{1}{2}a\right)}\right|}\right)-\frac{2x^{4}e^{(2a)}-1}{x^{5}e^{(2a)}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(1/2*a) - 1/4*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1
/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) - (2*x^4*e^(2*a) - 1)/(x^5*e^(2*a) - x)
```

maple [C] time = 0.13, size = 114, normalized size = 1.33

$$\frac{-2\,e^{2a}x^{4}+1}{x\left(-1+e^{2a}x^{4}\right)}+\frac{\sqrt{-e^{a}}\,\ln\left(-e^{2a}x-(-e^{a})^{\frac{3}{2}}\right)}{4}-\frac{\sqrt{-e^{a}}\,\ln\left(-e^{2a}x+(-e^{a})^{\frac{3}{2}}\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{2}-e^{a}\right)}\_R\ln\left(\left(-5\_R^{4}+4\,e^{2a}\right)x-\_R^{3}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x^2,x)
```

```
[Out] (-2*exp(2*a)*x^4+1)/x/(-1+exp(2*a)*x^4)+1/4*(-exp(a))^(1/2)*ln(-exp(2*a)*x-
(-exp(a))^(3/2))-1/4*(-exp(a))^(1/2)*ln(-exp(2*a)*x+(-exp(a))^(3/2))+1/4*su
m(_R*ln((-5*_R^4+4*exp(2*a))*x-_R^3),_R=RootOf(_Z^2-exp(a)))
```

maxima [A] time = 0.42, size = 69, normalized size = 0.80

$$\frac{1}{2}\arctan\left(\frac{e^{\left(-\frac{1}{2}a\right)}}{x}\right)e^{\left(\frac{1}{2}a\right)}-\frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\frac{1}{x}-e^{\left(\frac{1}{2}a\right)}}{\frac{1}{x}+e^{\left(\frac{1}{2}a\right)}}\right)-\frac{1}{x}+\frac{e^{(2a)}}{x\left(\frac{1}{x^{4}}-e^{(2a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*arctan(e^(-1/2*a)/x)*e^(1/2*a) - 1/4*e^(1/2*a)*log((1/x - e^(1/2*a))/(1
/x + e^(1/2*a))) - 1/x + e^(2*a)/(x*(1/x^4 - e^(2*a)))
```

mupad [B] time = 1.21, size = 60, normalized size = 0.70

$$\frac{\left(e^{2a}\right)^{1/4}\,\mathrm{atanh}\left(x\left(e^{2a}\right)^{1/4}\right)}{2}-\frac{\left(e^{2a}\right)^{1/4}\,\mathrm{atan}\left(x\left(e^{2a}\right)^{1/4}\right)}{2}+\frac{2\,x^{4}\,e^{2a}-1}{x-x^{5}\,e^{2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2/x^2,x)
```

```
[Out] (exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1/4)))/2 - (exp(2*a)^(1/4)*atan(x*exp(2*a
)^(1/4)))/2 + (2*x^4*exp(2*a) - 1)/(x - x^5*exp(2*a))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^{2}\left(a+2\log\left(x\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2/x**2, x)
```

3.164 $\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$

Optimal. Leaf size=60

$$e^a \tanh^{-1}\left(e^a x^2\right) + \frac{3e^{2a}x^2}{2\left(1-e^{2a}x^4\right)} - \frac{1}{2x^2\left(1-e^{2a}x^4\right)}$$

```
[Out] -1/2/x^2/(1-exp(2*a)*x^4)+3/2*exp(2*a)*x^2/(1-exp(2*a)*x^4)+exp(a)*arctanh(
exp(a)*x^2)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]^2/x^3, x]
```

Rubi steps

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$

Mathematica [C] time = 3.25, size = 155, normalized size = 2.58

$$\frac{64\left(e^{3a}x^6+e^a x^2\right)^2 {}_4F_3\left(\frac{1}{2},2,2,2;1,1,\frac{7}{2};e^{2a}x^4\right)+15\left(e^{4a}x^8-17e^{2a}x^4-\frac{27e^{-2a}}{x^4}-77\right)-\frac{15\left(e^{8a}x^{16}+4e^{6a}x^{12}-54e^{4a}x^8-52e\right.}{\left(e^{2a}x^4\right)^3}}{480x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] (15*(-77 - 27/(E^(2*a)*x^4) - 17*E^(2*a)*x^4 + E^(4*a)*x^8) - (15*(-27 - 52
*E^(2*a)*x^4 - 54*E^(4*a)*x^8 + 4*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqrt
[E^(2*a)*x^4]])/(E^(2*a)*x^4)^(3/2) + 64*(E^a*x^2 + E^(3*a)*x^6)^2*Hypergeo
metricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, E^(2*a)*x^4])/(480*x^2)
```

fricas [A] time = 0.40, size = 82, normalized size = 1.37

$$-\frac{3\,x^4e^{(2\,a)} - \left(x^6e^{(3\,a)} - x^2e^a\right)\log\left(x^2e^a+1\right) + \left(x^6e^{(3\,a)} - x^2e^a\right)\log\left(x^2e^a-1\right) - 1}{2\left(x^6e^{(2\,a)} - x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4*e^(2*a) - (x^6*e^(3*a) - x^2*e^a)*log(x^2*e^a + 1) + (x^6*e^(3*
a) - x^2*e^a)*log(x^2*e^a - 1) - 1)/(x^6*e^(2*a) - x^2)
```

giac [A] time = 0.14, size = 57, normalized size = 0.95

$$\frac{1}{2}\,e^a\log\left(x^2e^a+1\right) - \frac{1}{2}\,e^a\log\left(\left|x^2e^a-1\right|\right) - \frac{3\,x^4e^{(2\,a)}-1}{2\left(x^6e^{(2\,a)}-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*e^a*log(x^2*e^a + 1) - 1/2*e^a*log(abs(x^2*e^a - 1)) - 1/2*(3*x^4*e^(2*
a) - 1)/(x^6*e^(2*a) - x^2)
```

maple [A] time = 0.09, size = 55, normalized size = 0.92

$$\frac{-\frac{3\,e^{2a}x^4}{2}+\frac{1}{2}}{x^2\left(-1+e^{2a}x^4\right)}-\frac{e^a\ln\left(e^a x^2-1\right)}{2}+\frac{e^a\ln\left(e^a x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^2/x^3,x)
```

```
[Out] (-3/2*exp(2*a)*x^4+1/2)/x^2/(-1+exp(2*a)*x^4)-1/2*exp(a)*ln(exp(a)*x^2-1)+1
/2*exp(a)*ln(exp(a)*x^2+1)
```

maxima [A] time = 0.33, size = 50, normalized size = 0.83

$$\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}+e^a\right)-\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}-e^a\right)-\frac{1}{2\,x^2}+\frac{e^{(2\,a)}}{x^2\left(\frac{1}{x^4}-e^{(2\,a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*e^a*log(1/x^2 + e^a) - 1/2*e^a*log(1/x^2 - e^a) - 1/2/x^2 + e^(2*a)/(x^
2*(1/x^4 - e^(2*a)))
```

mupad [B] time = 1.23, size = 48, normalized size = 0.80

$$\mathrm{atanh}\left(x^2\,\sqrt{e^{2\,a}}\right)\,\sqrt{e^{2\,a}}-\frac{\frac{3\,x^4\,e^{2\,a}}{2}-\frac{1}{2}}{x^6\,e^{2\,a}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2/x^3,x)
```

```
[Out] atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2) - ((3*x^4*exp(2*a))/2 - 1/2)/(x^6*
exp(2*a) - x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^2\left(a+2\log\left(x\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**2/x**3,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2/x**3, x)
```

3.165 $\int (ex)^m \coth(a+2\log(x))\,dx$

Optimal. Leaf size=59

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{e(m+1)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],exp(
2*a)*x^4)/e/(1+m)
```

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]],x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]], x]
```

Rubi steps

$$\int (ex)^m \coth(a+2\log(x))\,dx=\int (ex)^m \coth(a+2\log(x))\,dx$$

Mathematica [A] time = 0.10, size = 46, normalized size = 0.78

$$-\frac{x(ex)^m\left(2\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};x^4(\cosh(2a)+\sinh(2a))\right)-1\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]],x]
```

```
[Out] -((x*(e*x)^m*(-1 + 2*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])]))/(1 + m))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \coth\left(a+2\,\log(x)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a+2\,\log(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (ex)^m \coth(a + 2\ln(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x)),x)
[Out] int((e*x)^m*coth(a+2*ln(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a + 2\log(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="maxima")
[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(a + 2\ln(x))\,(e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))*(e*x)^m,x)
[Out] int(coth(a + 2*log(x))*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a + 2\log(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x)),x)
[Out] Integral((e*x)**m*coth(a + 2*log(x)), x)
```

3.166 $\int (ex)^m \coth^2(a + 2\log(x))\,dx$

Optimal. Leaf size=79

$$-\frac{(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{e}+\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}+\frac{(ex)^{m+1}}{e(m+1)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)+(e*x)^(1+m)/e/(1-exp(2*a)*x^4)-(e*x)^(1+m)*hypergeom([1
, 1/4+1/4*m],[5/4+1/4*m],exp(2*a)*x^4)/e
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth^2(a + 2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]]^2, x]
```

Rubi steps

$$\int (ex)^m \coth^2(a + 2\log(x))\,dx = \int (ex)^m \coth^2(a + 2\log(x))\,dx$$

Mathematica [A] time = 0.18, size = 77, normalized size = 0.97

$$-\frac{x(ex)^m\left(4\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};x^4(\cosh(2a)+\sinh(2a))\right)-4\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4(\cosh(2a)+\sinh(2a))\right)-1\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] -((x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])] - 4*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Cosh[
2*a] + Sinh[2*a])]))/(1 + m))
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((ex)^m \coth\left(a + 2\,\log(x)\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x))^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a + 2\,\log(x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^2 (a + 2\ln(x))\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x))^2,x)
```

```
[Out] int((e*x)^m*coth(a+2*ln(x))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a + 2\log(x)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth (a + 2\ln(x))^2 (e\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^2*(e*x)^m,x)
```

```
[Out] int(coth(a + 2*log(x))^2*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^2 \left(a + 2\log (x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**2,x)
```

```
[Out] Integral((e*x)**m*coth(a + 2*log(x))**2, x)
```

3.167 $\int (ex)^m \coth^3(a+2\log(x))\,dx$

Optimal. Leaf size=177

$$-\frac{(m^2+2m+9)(ex)^{m+1}{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};e^{2a}x^4\right)}{4e(m+1)}-\frac{(e^{2a}x^4+1)^2(ex)^{m+1}}{4e(1-e^{2a}x^4)^2}-\frac{e^{-2a}\left(e^{2a}(3-m)-e^{4a}(m+5)x^4\right)(ex)^{m}}{8e(1-e^{2a}x^4)}$$

```
[Out] 1/8*(3+m)*(5+m)*(e*x)^(1+m)/e/(1+m)-1/4*(e*x)^(1+m)*(1+exp(2*a)*x^4)^2/e/(1
-exp(2*a)*x^4)^2-1/8*(e*x)^(1+m)*(exp(2*a)*(3-m)-exp(4*a)*(5+m)*x^4)/e/exp(
2*a)/(1-exp(2*a)*x^4)-1/4*(m^2+2*m+9)*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],
[5/4+1/4*m],exp(2*a)*x^4)/e/(1+m)
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth^3(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[a + 2*Log[x]]^3, x]
```

Rubi steps

$$\int (ex)^m \coth^3(a+2\log(x))\,dx=\int (ex)^m \coth^3(a+2\log(x))\,dx$$

Mathematica [A] time = 0.23, size = 108, normalized size = 0.61

$$-\frac{x(ex)^m\left(6\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};x^4(\cosh(2a)+\sinh(2a))\right)-12\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4(\cosh(2a)+\sinh(2a))\right)+8\,{}_2F_1\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

```
[Out] -((x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])] - 12*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Cosh
[2*a] + Sinh[2*a])] + 8*Hypergeometric2F1[3, (1 + m)/4, (5 + m)/4, x^4*(Cos
h[2*a] + Sinh[2*a])]))/(1 + m))
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \coth\left(a+2\log(x)\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(a + 2*log(x))^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a+2\log(x)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^3 (a + 2\ln(x))\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+2*ln(x))^3,x)
```

```
[Out] int((e*x)^m*coth(a+2*ln(x))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(a + 2\log(x)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth (a + 2\ \ln(x))^3 (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^3*(e*x)^m,x)
```

```
[Out] int(coth(a + 2*log(x))^3*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^3\left(a + 2\log(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**3,x)
```

```
[Out] Integral((e*x)**m*coth(a + 2*log(x))**3, x)
```

3.168 $\int \coth^p(a+b\log(x))\,dx$

Optimal. Leaf size=79

$$x\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}F_1\left(\frac{1}{2b};p,-p;\frac{1}{2}\left(2+\frac{1}{b}\right);e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[Out] x*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2/b,p,-p,1+1/2/b,exp(2*a)*x^(2*b),-exp
(2*a)*x^(2*b))/((1+exp(2*a)*x^(2*b))^p)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p(a+b\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + b*Log[x]]^p,x]
[Out] Defer[Int][Coth[a + b*Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a+b\log(x))\,dx=\int \coth^p(a+b\log(x))\,dx$$

Mathematica [B] time = 2.06, size = 259, normalized size = 3.28

$$\frac{(2b+1)x\left(\frac{e^{2a}x^{2b}+1}{e^{2a}x^{2b}-1}\right)^pF_1\left(\frac{1}{2b};p,-p;1+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{2e^{2a}bpx^{2b}F_1\left(1+\frac{1}{2b};p,1-p;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)+2e^{2a}bpx^{2b}F_1\left(1+\frac{1}{2b};p+1,-p;2+\frac{1}{2b};e^{2a}x^{2b},-e^{2a}x^{2b}\right)+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + b*Log[x]]^p,x]
[Out] ((1 + 2*b)*x*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(2*b*E^(2*a)
*p*x^(2*b)*AppellF1[1 + 1/(2*b), p, 1 - p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -(
E^(2*a)*x^(2*b))] + 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 + p, -p,
2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(b\log(x)+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(b*log(x) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(b\log(x)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(b*log(x) + a)^p, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \coth^{p}(a + b\ln(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(x))^p,x)
[Out] int(coth(a+b*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(b\log(x) + a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(b*log(x) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(a + b\,\ln(x))^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(x))^p,x)
[Out] int(coth(a + b*log(x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a + b\log{\left(x \right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(x))**p,x)
[Out] Integral(coth(a + b*log(x))**p, x)
```

3.169 $\int (ex)^m \coth^p(a+b\log(x))\,dx$

Optimal. Leaf size=99

$$\frac{(ex)^{m+1}\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}F_1\left(\frac{m+1}{2b};p,-p;\frac{m+1}{2b}+1;e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(m+1)}$$

```
[Out] (e*x)^(1+m)*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2*(1+m)/b,p,-p,1+1/2*(1+m)/b
,exp(2*a)*x^(2*b),-exp(2*a)*x^(2*b))/e/(1+m)/((1+exp(2*a)*x^(2*b))^p)
```

Rubi [F] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth^p(a+b\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[a + b*Log[x]]^p,x]
[Out] Defer[Int][(e*x)^m*Coth[a + b*Log[x]]^p, x]
```

Rubi steps

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int (ex)^m \coth^p(a+b\log(x))\,dx$$

Mathematica [A] time = 3.23, size = 126, normalized size = 1.27

$$\frac{x(ex)^m\left(1-e^{2a}x^{2b}\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}\left(\frac{e^{2a}x^{2b}+1}{e^{2a}x^{2b}-1}\right)^p F_1\left(\frac{m+1}{2b};p,-p;\frac{m+1}{2b}+1;e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[a + b*Log[x]]^p,x]
[Out] (x*(e*x)^m*(1 - E^(2*a)*x^(2*b))^p*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(
2*b)))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 + (1 + m)/(2*b), E^(2*a)*x^(2*b),
 -(E^(2*a)*x^(2*b))])/((1 + m)*(1 + E^(2*a)*x^(2*b))^p)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \coth\left(b\log(x)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*coth(b*log(x) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(b\log(x)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^p (a + b\ln(x))\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(a+b*ln(x))^p,x)
```

```
[Out] int((e*x)^m*coth(a+b*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(b\log(x) + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(a + b\ \ln(x)\right)^p\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(x))^p*(e*x)^m,x)
```

```
[Out] int(coth(a + b*log(x))^p*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^p\left(a + b\log\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(a+b*ln(x))**p,x)
```

```
[Out] Integral((e*x)**m*coth(a + b*log(x))**p, x)
```

3.170 $\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx$

Optimal. Leaf size=52

$$-\frac{e^{-2a}2^{-p}\left(-e^{2a}x-1\right)^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}x+1\right)\right)}{p+1}$$

```
[Out] -(-1-exp(2*a)*x)^(1+p)*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x)/(2^p)/e
xp(2*a)/(1+p)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/2]^p,x]
```

```
[Out] Defer[Int][Coth[(2*a + Log[x])/2]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth^p\left(\frac{1}{2}(2a+\log(x))\right)\,dx$$

Mathematica [A] time = 0.43, size = 83, normalized size = 1.60

$$-\frac{e^{-2a}2^p\left(e^{2a}x+1\right)^{1-p}\left(\frac{e^{2a}x+1}{e^{2a}x-1}\right)^{p-1}\,{}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}x\right)}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/2]^p,x]
```

```
[Out] -((2^p*(1 + E^(2*a)*x)^(1 - p)*((1 + E^(2*a)*x)/(-1 + E^(2*a)*x))^(-1 + p)*
Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x)/2])/(E^(2*a)*(-1 + p)
))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(a+\frac{1}{2}\log(x)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/2*log(x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+\frac{1}{2}\log(x)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a+\frac{\ln(x)}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/2*ln(x))^p,x)
```

```
[Out] int(coth(a+1/2*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+\frac{1}{2}\,\log(x)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth\left(a+\frac{\ln(x)}{2}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/2)^p,x)
```

```
[Out] int(coth(a + log(x)/2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a+\frac{\log{(x)}}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/2*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/2)**p, x)
```

3.171 $\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$

Optimal. Leaf size=108

$$e^{-4a}\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt{x}\right)^{1-p}-\frac{e^{-4a}2^{1-p}p\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\,{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt{x}+1\right)\right)}{p+1}$$

```
[Out] -2^(1-p)*p*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x^(1/2))*(-1-exp(2*a)*
x^(1/2))^(1+p)/exp(4*a)/(1+p)+(-1-exp(2*a)*x^(1/2))^(1+p)*(1-exp(2*a)*x^(1/
2))^(1-p)/exp(4*a)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/4]^p,x]
```

```
[Out] Defer[Int][Coth[(4*a + Log[x])/4]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth^p\left(\frac{1}{4}(4a+\log(x))\right)\,dx$$

Mathematica [A] time = 0.56, size = 125, normalized size = 1.16

$$\frac{e^{-4a}\left(e^{2a}\sqrt{x}+1\right)^{1-p}\left(\frac{e^{2a}\sqrt{x}+1}{e^{2a}\sqrt{x}-1}\right)^{p-1}\left((p-1)\left(e^{2a}\sqrt{x}+1\right)^{p+1}-2^{p+1}p\,{}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt{x}\right)\right)}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/4]^p,x]
```

```
[Out] ((1 + E^(2*a)*Sqrt[x])^(1 - p)*((1 + E^(2*a)*Sqrt[x])/(-1 + E^(2*a)*Sqrt[x]
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*Sqrt[x])^(1 + p) - 2^(1 + p)*p*Hypergeom
etric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*Sqrt[x])/2]))/(E^(4*a)*(-1 + p))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(a+\frac{1}{4}\,\log(x)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/4*log(x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+\frac{1}{4}\,\log(x)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \coth^p\left(a + \frac{\ln(x)}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/4*ln(x))^p,x)
[Out] int(coth(a+1/4*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a + \frac{1}{4}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(a + \frac{\ln(x)}{4}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/4)^p,x)
[Out] int(coth(a + log(x)/4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^p\left(a + \frac{\log(x)}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/4*ln(x))**p,x)
[Out] Integral(coth(a + log(x)/4)**p, x)
```

3.172 $\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx$

Optimal. Leaf size=162

$$-\frac{e^{-6a}2^{-p}\left(2p^2+1\right)\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt[3]{x}+1\right)\right)}{p+1}+e^{-6a}p\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1}$$

```
[Out] p*(-1-exp(2*a)*x^(1/3))^(1+p)*(1-exp(2*a)*x^(1/3))^(1-p)/exp(6*a)+(-1-exp(2
*a)*x^(1/3))^(1+p)*(1-exp(2*a)*x^(1/3))^(1-p)*x^(1/3)/exp(4*a)-(2*p^2+1)*(-
1-exp(2*a)*x^(1/3))^(1+p)*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x^(1/3)
)/(2^p)/exp(6*a)/(1+p)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/6]^p,x]
```

```
[Out] Defer[Int][Coth[(6*a + Log[x])/6]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)dx=\int \coth^p\left(\frac{1}{6}(6a+\log(x))\right)dx$$

Mathematica [A] time = 0.60, size = 142, normalized size = 0.88

$$\frac{e^{-6a}\left(e^{2a}\sqrt[3]{x}+1\right)^{1-p}\left(\frac{e^{2a}\sqrt[3]{x}+1}{e^{2a}\sqrt[3]{x}-1}\right)^{p-1}\left((p-1)\left(e^{2a}\sqrt[3]{x}+1\right)^{p+1}\left(e^{2a}\sqrt[3]{x}+p\right)-2^p\left(2p^2+1\right){}_2F_1\left(1-p,-p;2-p;\frac{1}{2}-\right.\right.}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/6]^p,x]
```

```
[Out] ((1 + E^(2*a)*x^(1/3))^(1 - p)*((1 + E^(2*a)*x^(1/3))/(-1 + E^(2*a)*x^(1/3)
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*x^(1/3))^(1 + p)*(p + E^(2*a)*x^(1/3)) -
 2^p*(1 + 2*p^2)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x^(1/3)
)/2]))/(E^(6*a)*(-1 + p))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(a+\frac{1}{6}\log(x)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/6*log(x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+\frac{1}{6}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="giac")
```

```
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \coth^p \left(a + \frac{\ln(x)}{6}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/6*ln(x))^p,x)
```

```
[Out] int(coth(a+1/6*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a + \frac{1}{6}\log(x)\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="maxima")
```

```
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(a + \frac{\ln(x)}{6}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/6)^p,x)
```

```
[Out] int(coth(a + log(x)/6)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^p \left(a + \frac{\log(x)}{6}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/6*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/6)**p, x)
```

3.173 $\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$

Optimal. Leaf size=194

$$-\frac{e^{-8a}2^{2-p}p\left(p^2+2\right)\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}{}_2F_1\left(p,p+1;p+2;\frac{1}{2}\left(e^{2a}\sqrt[4]{x}+1\right)\right)}{3(p+1)}+\frac{1}{3}e^{-12a}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{4a}\left(2p^2+\right.\right.$$

```
[Out] 1/3*(-1-exp(2*a)*x^(1/4))^(1+p)*(1-exp(2*a)*x^(1/4))^(1-p)*(exp(4*a)*(2*p^2
+3)+2*exp(6*a)*p*x^(1/4))/exp(12*a)-1/3*2^(2-p)*p*(p^2+2)*(-1-exp(2*a)*x^(1
/4))^(1+p)*hypergeom([p, 1+p],[2+p],1/2+1/2*exp(2*a)*x^(1/4))/exp(8*a)/(1+p
)+(-1-exp(2*a)*x^(1/4))^(1+p)*(1-exp(2*a)*x^(1/4))^(1-p)*x^(1/2)/exp(4*a)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]/8]^p,x]
```

```
[Out] Defer[Int][Coth[(8*a + Log[x])/8]^p, x]
```

Rubi steps

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth^p\left(\frac{1}{8}(8a+\log(x))\right)\,dx$$

Mathematica [A] time = 0.95, size = 223, normalized size = 1.15

$$\frac{e^{-8a}\left(e^{2a}\sqrt[4]{x}+1\right)^{1-p}\left(\frac{e^{2a}\sqrt[4]{x}+1}{e^{2a}\sqrt[4]{x}-1}\right)^{p-1}\left(-2^{p+3}p\,{}_2F_1\left(-p-2,1-p;2-p;\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt[4]{x}\right)+2^{p+2}(2p-1)\,{}_2F_1\left(-p-1,1\right.\right.}{p-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]/8]^p,x]
```

```
[Out] ((1 + E^(2*a)*x^(1/4))^(1 - p)*((1 + E^(2*a)*x^(1/4))/(-1 + E^(2*a)*x^(1/4)
))^(-1 + p)*(-(2^(3 + p)*p*Hypergeometric2F1[-2 - p, 1 - p, 2 - p, 1/2 - (E
^(2*a)*x^(1/4))/2]) + 2^(2 + p)*(-1 + 2*p)*Hypergeometric2F1[-1 - p, 1 - p,
 2 - p, 1/2 - (E^(2*a)*x^(1/4))/2] + (-1 + p)*(E^(4*a)*(1 + E^(2*a)*x^(1/4)
)^(1 + p)*Sqrt[x] - 2^(1 + p)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^
(2*a)*x^(1/4))/2])))/(E^(8*a)*(-1 + p))
```

fricas [F] time = 1.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(a+\frac{1}{8}\log(x)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/8*log(x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a + \frac{1}{8}\log(x)\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \coth^p\left(a + \frac{\ln(x)}{8}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+1/8*ln(x))^p,x)
[Out] int(coth(a+1/8*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a + \frac{1}{8}\log(x)\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(a + \frac{\ln(x)}{8}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x)/8)^p,x)
[Out] int(coth(a + log(x)/8)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^p\left(a + \frac{\log(x)}{8}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+1/8*ln(x))**p,x)
[Out] Integral(coth(a + log(x)/8)**p, x)
```

3.174 $\int \coth^p(a+\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-e^{2a}x^2-1\right)^p\left(e^{2a}x^2+1\right)^{-p}F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)$$

```
[Out] x*(-1-exp(2*a)*x^2)^p*AppellF1(1/2,p,-p,3/2,exp(2*a)*x^2,-exp(2*a)*x^2)/((1
+exp(2*a)*x^2)^p)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p(a+\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + Log[x]]^p,x]
```

```
[Out] Defer[Int][Coth[a + Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a+\log(x))\,dx=\int \coth^p(a+\log(x))\,dx$$

Mathematica [B] time = 1.76, size = 171, normalized size = 2.80

$$\frac{3x\left(\frac{e^{2a}x^2+1}{e^{2a}x^2-1}\right)^pF_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)}{2e^{2a}px^2\left(F_1\left(\frac{3}{2};p,1-p;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)+F_1\left(\frac{3}{2};p+1,-p;\frac{5}{2};e^{2a}x^2,-e^{2a}x^2\right)\right)+3F_1\left(\frac{1}{2};p,-p;\frac{3}{2};e^{2a}x^2,-e^{2a}x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + Log[x]]^p,x]
```

```
[Out] (3*x*((1 + E^(2*a)*x^2)/(-1 + E^(2*a)*x^2))^p*AppellF1[1/2, p, -p, 3/2, E^(
2*a)*x^2, -(E^(2*a)*x^2)])/(3*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2
*a)*x^2)] + 2*E^(2*a)*p*x^2*(AppellF1[3/2, p, 1 - p, 5/2, E^(2*a)*x^2, -(E^
(2*a)*x^2)] + AppellF1[3/2, 1 + p, -p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(a+\log(x)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + log(x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+\log(x)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + log(x))^p, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \coth^p(a + \ln(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+ln(x))^p,x)
[Out] int(coth(a+ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a + \log(x)\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + log(x))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(a + \ln(x))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + log(x))^p,x)
[Out] int(coth(a + log(x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^p\left(a + \log(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+ln(x))**p,x)
[Out] Integral(coth(a + log(x))**p, x)
```

3.175 $\int \coth^p(a+2\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-e^{2a}x^4-1\right)^p\left(e^{2a}x^4+1\right)^{-p}F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)$$

```
[Out] x*(-1-exp(2*a)*x^4)^p*AppellF1(1/4,p,-p,5/4,exp(2*a)*x^4,-exp(2*a)*x^4)/((1
+exp(2*a)*x^4)^p)
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p(a+2\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] Defer[Int][Coth[a + 2*Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a+2\log(x))\,dx=\int \coth^p(a+2\log(x))\,dx$$

Mathematica [B] time = 1.95, size = 171, normalized size = 2.80

$$\frac{5x\left(\frac{e^{2a}x^4+1}{e^{2a}x^4-1}\right)^pF_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)}{4e^{2a}px^4\left(F_1\left(\frac{5}{4};p,1-p;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)+F_1\left(\frac{5}{4};p+1,-p;\frac{9}{4};e^{2a}x^4,-e^{2a}x^4\right)\right)+5F_1\left(\frac{1}{4};p,-p;\frac{5}{4};e^{2a}x^4,-e^{2a}x^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] (5*x*((1 + E^(2*a)*x^4)/(-1 + E^(2*a)*x^4))^p*AppellF1[1/4, p, -p, 5/4, E^(
2*a)*x^4, -(E^(2*a)*x^4)])/(5*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2
*a)*x^4)] + 4*E^(2*a)*p*x^4*(AppellF1[5/4, p, 1 - p, 9/4, E^(2*a)*x^4, -(E^
(2*a)*x^4)] + AppellF1[5/4, 1 + p, -p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(a+2\log(x)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 2*log(x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+2\log(x)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 2*log(x))^p, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a+2\ln(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+2*ln(x))^p,x)
[Out] int(coth(a+2*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+2\,\log(x)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 2*log(x))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth\left(a+2\,\ln(x)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 2*log(x))^p,x)
[Out] int(coth(a + 2*log(x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}{\left(a+2\log{\left(x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+2*ln(x))**p,x)
[Out] Integral(coth(a + 2*log(x))**p, x)
```

3.176 $\int \coth^p(a+3\log(x))\,dx$

Optimal. Leaf size=61

$$x\left(-e^{2a}x^6-1\right)^p\left(e^{2a}x^6+1\right)^{-p}F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)$$

```
[Out] x*(-1-exp(2*a)*x^6)^p*AppellF1(1/6,p,-p,7/6,exp(2*a)*x^6,-exp(2*a)*x^6)/((1
+exp(2*a)*x^6)^p)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p(a+3\log(x))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] Defer[Int][Coth[a + 3*Log[x]]^p, x]
```

Rubi steps

$$\int \coth^p(a+3\log(x))\,dx=\int \coth^p(a+3\log(x))\,dx$$

Mathematica [B] time = 2.01, size = 171, normalized size = 2.80

$$\frac{7x\left(\frac{e^{2a}x^6+1}{e^{2a}x^6-1}\right)^pF_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)}{6e^{2a}px^6\left(F_1\left(\frac{7}{6};p,1-p;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)+F_1\left(\frac{7}{6};p+1,-p;\frac{13}{6};e^{2a}x^6,-e^{2a}x^6\right)\right)+7F_1\left(\frac{1}{6};p,-p;\frac{7}{6};e^{2a}x^6,-e^{2a}x^6\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] (7*x*((1 + E^(2*a)*x^6)/(-1 + E^(2*a)*x^6))^p*AppellF1[1/6, p, -p, 7/6, E^(
2*a)*x^6, -(E^(2*a)*x^6)])/(7*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2
*a)*x^6)] + 6*E^(2*a)*p*x^6*(AppellF1[7/6, p, 1 - p, 13/6, E^(2*a)*x^6, -(E
^(2*a)*x^6)] + AppellF1[7/6, 1 + p, -p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)])
)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(a+3\,\log(x)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 3*log(x))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+3\,\log(x)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 3*log(x))^p, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \coth^{p}\left(a+3\ln\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+3*ln(x))^p,x)
[Out] int(coth(a+3*ln(x))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(a+3\,\log\left(x\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 3*log(x))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\mathrm{coth}\left(a+3\,\ln\left(x\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + 3*log(x))^p,x)
[Out] int(coth(a + 3*log(x))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^{p}{\left(a+3\log{\left(x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+3*ln(x))**p,x)
[Out] Integral(coth(a + 3*log(x))**p, x)
```

3.177 $\int x^3 \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=58

$$\frac{x^4}{4}-\frac{1}{2}x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] 1/4*x^4-1/2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*
d))
```

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3 \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][x^3*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x^3 \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int x^3 \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [B] time = 7.16, size = 198, normalized size = 3.41

$$-\frac{x^4\left(2e^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(1,1+\frac{2}{bdn};2+\frac{2}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)+(bdn+2)\left({}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^4*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2
+ 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(Coth[d*(a + b*Log[c
*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 2/
(b*d*n), 1 + 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x
^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(8 + 4*
b*d*n))
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\coth\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 1.25, size = 0, normalized size = 0.00

$$\int x^3 \coth\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,x^4-\int\frac{x^3}{c^{bd}e^{\left(bd\log(x^n)+ad\right)}+1}\,dx+\int\frac{x^3}{c^{bd}e^{\left(bd\log(x^n)+ad\right)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - integrate(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^3*coth(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \coth{\left(ad + bd \log{\left(cx^{n} \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**3*coth(a*d + b*d*log(c*x**n)), x)
```

3.178 $\int x^2 \coth \left(d \left(a+b \log \left(c x^n\right)\right)\right) \, dx$

Optimal. Leaf size=62

$$\frac{x^3}{3}-\frac{2}{3} x^3 \, _2F_1\left(1,\frac{3}{2 b d n};1+\frac{3}{2 b d n};e^{2 a d} \left(c x^n\right)^{2 b d}\right)$$

```
[Out] 1/3*x^3-2/3*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*(c*x^n)^(
2*b*d))
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \coth \left(d \left(a+b \log \left(c x^n\right)\right)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][x^2*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x^2 \coth \left(d \left(a+b \log \left(c x^n\right)\right)\right) \, dx=\int x^2 \coth \left(d \left(a+b \log \left(c x^n\right)\right)\right) \, dx$$

Mathematica [B] time = 5.00, size = 207, normalized size = 3.34

$$-\frac{x^3 \left(3 e^{2 d \left(a+b \log \left(c x^n\right)\right)} \, _2F_1\left(1,1+\frac{3}{2 b d n};2+\frac{3}{2 b d n};e^{2 d \left(a+b \log \left(c x^n\right)\right)}\right)+(2 b d n+3) \left(\, _2F_1\left(1,\frac{3}{2 b d n};1+\frac{3}{2 b d n};e^{2 d \left(a+b \log \left(c x^n\right)\right.}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n),
2 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(Coth[d*(a + b
*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1
[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
 b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
))/(9 + 6*b*d*n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(x^{2} \coth \left(b d \log \left(c x^{n}\right) + a d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \coth \left({\left(b \log \left(c x^{n}\right) + a\right)} d\right)\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 1.15, size = 0, normalized size = 0.00

$$\int x^2 \coth\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3 - \int \frac{x^2}{c^{bd}e^{\left(bd\log(x^n)+ad\right)}+1}\,dx + \int \frac{x^2}{c^{bd}e^{\left(bd\log(x^n)+ad\right)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - integrate(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x^2*coth(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \coth{\left(ad + bd \log{\left(cx^{n} \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n)), x)
```

3.179 $\int x \coth \left(d \left(a + b \log \left(cx^n\right)\right)\right) dx$

Optimal. Leaf size=54

$$\frac{x^2}{2} - x^2\,{}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] 1/2*x^2-x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x \coth \left(d \left(a + b \log \left(cx^n\right)\right)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])],x]
[Out] Defer[Int][x*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int x \coth \left(d \left(a + b \log \left(cx^n\right)\right)\right) dx = \int x \coth \left(d \left(a + b \log \left(cx^n\right)\right)\right) dx$$

Mathematica [B] time = 6.99, size = 193, normalized size = 3.57

$$-\frac{x^2 \left(e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1, 1 + \frac{1}{bdn}; 2 + \frac{1}{bdn}; e^{2d(a+b\log(cx^n))}\right) + (bdn+1)\left({}_2F_1\left(1, \frac{1}{bdn}; 1 + \frac{1}{bdn}; e^{2d(a+b\log(cx^n))}\right) + \cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])],x]
[Out] -((x^2*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(Coth[d*(a + b*Log[c*x
^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 1/(b
*d*n), 1 + 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n
])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(2 + 2*b*
d*n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(x \coth \left(bd \log \left(cx^n\right) + ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x*coth(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \coth \left(\left(b \log \left(cx^n\right) + a\right)d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(x*coth((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int x \coth \left(d \left(a+b \ln \left(c\,x^{n}\right)\right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x*coth(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^{2} - \int \frac{x}{c^{bd}e^{\left(bd\log\left(x^{n}\right)+ad\right)}+1}\,dx + \int \frac{x}{c^{bd}e^{\left(bd\log\left(x^{n}\right)+ad\right)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(
x/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a + b*log(c*x^n))),x)
```

```
[Out] int(x*coth(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \coth{\left(ad + bd \log{\left(cx^{n} \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x*coth(a*d + b*d*log(c*x**n)), x)
```

3.180 $\int \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=52

$$x-2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int \coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [B] time = 8.36, size = 198, normalized size = 3.81

$$-\frac{xe^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(1,1+\frac{1}{2bdn};2+\frac{1}{2bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)}{2bdn+1}-x\left({}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)+\coth\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1
/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n)) - x*(Coth[d*(a + b*
Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[
1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(bd\log\left(cx^n\right)+ad\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \coth \left(d \left(a +b \ln \left(c \,x^{n}\right)\right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n))),x)
[Out] int(coth(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x - \int \frac{1}{c^{bd}e^{\left(bd\log\left(x^{n}\right)+ad\right)} + 1}\,dx + \int \frac{1}{c^{bd}e^{\left(bd\log\left(x^{n}\right)+ad\right)} - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] x - integrate(1/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(1/(c^(
b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n))),x)
[Out] int(coth(d*(a + b*log(c*x^n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth{\left(d \left(a + b \log{\left(c x^{n} \right)}\right) \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n))),x)
[Out] Integral(coth(d*(a + b*log(c*x**n))), x)
```

3.181 $\int \frac{\coth\left(d\left(a+b\log(cx^n)\right)\right)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{\log\left(\sinh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

```
[Out] ln(sinh(a*d+b*d*ln(c*x^n)))/b/d/n
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3475}

$$\frac{\log\left(\sinh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \coth(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\log\left(\sinh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}\end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 1.60

$$\frac{\log\left(\tanh\left(ad+bd\log\left(cx^n\right)\right)\right)+\log\left(\cosh\left(d\left(a+b\log\left(cx^n\right)\right)\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] (Log[Cosh[d*(a + b*Log[c*x^n])]] + Log[Tanh[a*d + b*d*Log[c*x^n]]])/(b*d*n)
```

fricas [B] time = 0.41, size = 76, normalized size = 3.04

$$-\frac{bdn\log(x)-\log\left(\frac{2\sinh\left(bdn\log(x)+bd\log(c)+ad\right)}{\cosh\left(bdn\log(x)+bd\log(c)+ad\right)-\sinh\left(bdn\log(x)+bd\log(c)+ad\right)}\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

```
[Out] -(b*d*n*log(x) - log(2*sinh(b*d*n*log(x) + b*d*log(c) + a*d)/(cosh(b*d*n*lo
g(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(x) + b*d*log(c) + a*d))))/(b*d*n)
```

giac [B] time = 0.31, size = 74, normalized size = 2.96

$$\frac{\log\left(\sqrt{-2\,x^{2\,bdn}|c|^{2\,bd}\cos\left(\pi bd\mathrm{sgn}(c)-\pi bd\right)e^{(2\,ad)}+x^{4\,bdn}|c|^{4\,bd}e^{(4\,ad)}+1}\right)}{bdn}-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*d*n)*abs(c)^(2*b*d)*cos(pi*b*d*sgn(c) - pi*b*d)*e^(2*a*d
) + x^(4*b*d*n)*abs(c)^(4*b*d)*e^(4*a*d) + 1))/(b*d*n) - log(x)
```

maple [B] time = 0.02, size = 56, normalized size = 2.24

$$-\frac{\ln\left(\coth\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)-1\right)}{2bdn}-\frac{\ln\left(\coth\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+1\right)}{2bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x,x)
```

```
[Out] -1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))-1)-1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)
))+1)
```

maxima [A] time = 0.31, size = 24, normalized size = 0.96

$$\frac{\log\left(\sinh\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)\right)}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

```
[Out] log(sinh((b*log(c*x^n) + a)*d))/(b*d*n)
```

mupad [B] time = 1.19, size = 34, normalized size = 1.36

$$\frac{\ln\left(\mathrm{e}^{2\,a\,d}\,\left(c\,x^{n}\right)^{2\,b\,d}-1\right)}{b\,d\,n}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))/x,x)
```

```
[Out] log(exp(2*a*d)*(c*x^n)^(2*b*d) - 1)/(b*d*n) - log(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth\left(ad+bd\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x, x)
```

3.182 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^2} dx$

Optimal. Leaf size=58

$$\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{x}-\frac{1}{x}$$

```
[Out] -1/x+2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/
x
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]/x^2, x]
```

Rubi steps

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx=\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Mathematica [B] time = 3.79, size = 197, normalized size = 3.40

$$-\frac{e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)}{2bdn-1}+{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)+\coth\left(d\left(a+b\log(cx^n\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^
(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*b*d
*n), E^(2*d*(a + b*Log[c*x^n]))])/(-1 + 2*b*d*n) + Hypergeometric2F1[1, -1/
2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*L
og[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/x
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right)+ad\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^2, x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int \frac{1}{c^{bd}x^{2}e^{\left(bd\log\left(x^{n}\right)+ad\right)}+x^{2}}\,dx+\int \frac{1}{c^{bd}x^{2}e^{\left(bd\log\left(x^{n}\right)+ad\right)}-x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - integrate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) + x^2), x) + integra
te(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x**2, x)
```

3.183 $\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$

Optimal. Leaf size=55

$$\frac{{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{x^2}-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2+hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/x^
2
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]/x^3, x]
```

Rubi steps

$$\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx=\int \frac{\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Mathematica [B] time = 3.76, size = 191, normalized size = 3.47

$$-\frac{e^{2d\left(a+b\log\left(cx^n\right)\right)}{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)}{bdn-1}+{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)+\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^
(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/(b*d*
n)), 1 - 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n])
]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(2*x^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right)+ad\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^3, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\,x^{2}}-\int \frac{1}{c^{bd}x^{3}e^{\left(bd\log\left(x^{n}\right)+ad\right)}+x^{3}}\,dx+\int \frac{1}{c^{bd}x^{3}e^{\left(bd\log\left(x^{n}\right)+ad\right)}-x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) + x^3), x) + int
egrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) - x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x**3, x)
```

3.184 $\int x^3 \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=132

$$-\frac{2x^4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdn}+\frac{x^4\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{1}{4}x^4\left(\frac{4}{bdn}+1\right)$$

```
[Out] 1/4*(1+4/b/d/n)*x^4+x^4*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x^n
)^(2*b*d))/b/d/n
```

Rubi [F] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x^3*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x^3\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int x^3\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [A] time = 6.72, size = 155, normalized size = 1.17

$$\frac{x^4\left((bdn+2)\left(-4\,{}_2F_1\left(1,\frac{2}{bdn};1+\frac{2}{bdn};e^{2d(a+b\log(cx^n))}\right)-4\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)+bdn\right)-8e^{2d(a+b\log(cx^n))}\,{}_2F\cdots\right)}{4bdn(bdn+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^4*(-8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Coth[d*(a
+ b*Log[c*x^n])] - 4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(4*b*d*n*(2 + b*d*n))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d)^2, x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int x^3 \left(\coth^2 \left(d \left(a + b \ln \left(c x^n\right)\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bc^{2\,bd}dnx^4e^{\left(2\,bd\log(x^n)+2\,ad\right)} - (bdn+8)x^4}{4\left(bc^{2\,bd}dne^{\left(2\,bd\log(x^n)+2\,ad\right)} - bdn\right)} - 4\int \frac{x^3}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)} + bdn}\,dx + 4\int \frac{x^3}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)} - bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 8)*x^4)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 4*integrate(x^3/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 4*integrate(x^3/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x^3*coth(d*(a + b*log(c*x^n)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \coth^{2}{\left(ad + bd \log{\left(cx^{n} \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**3*coth(a*d + b*d*log(c*x**n))**2, x)
```

3.185 $\int x^2 \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=136

$$-\frac{2x^3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdn}+\frac{x^3\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{1}{3}x^3\left(\frac{3}{bdn}+1\right)$$

```
[Out] 1/3*(1+3/b/d/n)*x^3+x^3*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*(c
*x^n)^(2*b*d))/b/d/n
```

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][x^2*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x^2\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int x^2\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [A] time = 4.62, size = 165, normalized size = 1.21

$$\frac{x^3\left((2bdn+3)\left(-3\,{}_2F_1\left(1,\frac{3}{2bdn};1+\frac{3}{2bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)-3\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)+bdn\right)-9e^{2d\left(a+b\log\left(cx^n\right)\right)}\right.}{3bdn(2bdn+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^3*(-9*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(b*d*n - 3*Coth
[d*(a + b*Log[c*x^n])] - 3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n
), E^(2*d*(a + b*Log[c*x^n]))])))/(3*b*d*n*(3 + 2*b*d*n))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d)^2, x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int x^2 \left(\coth^2\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bc^{2\,bd}dnx^3e^{\left(2\,bd\log(x^n)+2\,ad\right)}-(bdn+6)x^3}{3\left(bc^{2\,bd}dne^{\left(2\,bd\log(x^n)+2\,ad\right)}-bdn\right)}-3\int\frac{x^2}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}+bdn}\,dx+3\int\frac{x^2}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}-bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 6)*x^3)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 3*integrate(x^2/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 3*integrate(x^2/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*coth(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x^2*coth(d*(a + b*log(c*x^n)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\coth^{2}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n))**2, x)
```

3.186 $\int x \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=130

$$-\frac{2x^2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdn}+\frac{x^2\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{1}{2}x^2\left(\frac{2}{bdn}+1\right)$$

```
[Out] 1/2*(1+2/b/d/n)*x^2+x^2*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x^n
)^(2*b*d))/b/d/n
```

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
[Out] Defer[Int][x*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int x\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int x\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [A] time = 6.63, size = 151, normalized size = 1.16

$$\frac{x^2\left((bdn+1)\left(-2\,{}_2F_1\left(1,\frac{1}{bdn};1+\frac{1}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)-2\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)+bdn\right)-2e^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F\right.}{2bdn(bdn+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
[Out] (x^2*(-2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(b*d*n - 2*Coth[d*(a
+ b*Log[c*x^n])] - 2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(2*b*d*n*(1 + b*d*n))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(x*coth(b*d*log(c*x^n) + a*d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x*coth((b*log(c*x^n) + a)*d)^2, x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int x\left(\coth^2\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bc^{2\,bd}dnx^2e^{\left(2\,bd\log(x^n)+2\,ad\right)}-(bdn+4)x^2}{2\left(bc^{2\,bd}dne^{\left(2\,bd\log(x^n)+2\,ad\right)}-bdn\right)}-2\int\frac{x}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}+bdn}\,dx+2\int\frac{x}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}-bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 4)*x^2)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 2*integrate(x/(b*c^(b*d)
*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 2*integrate(x/(b*c^(b*d)*d*n*e^(
b*d*log(x^n) + a*d) - b*d*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\coth\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(x*coth(d*(a + b*log(c*x^n)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\coth^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x*coth(a*d + b*d*log(c*x**n))**2, x)
```

3.187 $\int \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=126

$$-\frac{2x\,{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdn}+\frac{x\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+x\left(\frac{1}{bdn}+1\right)$$

```
[Out] (1+1/b/d/n)*x+x*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*(c*x^n)^
(2*b*d))-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b
*d))/b/d/n
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [A] time = 7.88, size = 160, normalized size = 1.27

$$\frac{x\left((2bdn+1)\left(-{}_2F_1\left(1,\frac{1}{2bdn};1+\frac{1}{2bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)-\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)+bdn\right)-e^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(\right.\right.}{bdn(2bdn+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(-(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 +
1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))]) + (1 + 2*b*d*n)*(b*d*n - Coth[d*(
a + b*Log[c*x^n])] - Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(
2*d*(a + b*Log[c*x^n]))])))/(b*d*n*(1 + 2*b*d*n))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(bd\log\left(cx^n\right)+ad\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2, x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \coth^2(d(a+b\ln(cx^n)))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bc^{2\,bd}dnxe^{\left(2\,bd\log(x^n)+2\,ad\right)}-(bdn+2)x}{bc^{2\,bd}dne^{\left(2\,bd\log(x^n)+2\,ad\right)}-bdn}-\int \frac{1}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}+bdn}\,dx+\int \frac{1}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}-bdn}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] (b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 2)*x)/(b*c^(2*b*d)
*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - integrate(1/(b*c^(b*d)*d*n*e^(b*
d*log(x^n) + a*d) + b*d*n), x) + integrate(1/(b*c^(b*d)*d*n*e^(b*d*log(x^n)
 + a*d) - b*d*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^2\left(d\left(a+b\log(cx^n)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(coth(d*(a + b*log(c*x**n)))**2, x)
```

3.188 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal. Leaf size=28

$$\log(x)-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}$$

```
[Out] -coth(a*d+b*d*ln(c*x^n))/b/d/n+ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3473, 8}

$$\log(x)-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -(Coth[a*d + b*d*Log[c*x^n]]/(b*d*n)) + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \coth^2(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}+\frac{\operatorname{Subst}\left(\int 1\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn}+\log(x)
\end{aligned}$$

Mathematica [C] time = 0.11, size = 49, normalized size = 1.75

$$-\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2\left(ad+b\log\left(cx^n\right)d\right)\right)}{bdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -((Coth[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a*d + b*
d*Log[c*x^n]]^2])/(b*d*n))
```

fricas [B] time = 0.43, size = 72, normalized size = 2.57

$$\frac{\left(bdn\log(x)+1\right)\sinh\left(bdn\log(x)+bd\log(c)+ad\right)-\cosh\left(bdn\log(x)+bd\log(c)+ad\right)}{bdn\sinh\left(bdn\log(x)+bd\log(c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

```
[Out] ((b*d*n*log(x) + 1)*sinh(b*d*n*log(x) + b*d*log(c) + a*d) - cosh(b*d*n*log(
x) + b*d*log(c) + a*d))/(b*d*n*sinh(b*d*n*log(x) + b*d*log(c) + a*d))
```

giac [A] time = 0.30, size = 37, normalized size = 1.32

$$-\frac{2}{\left(c^{2\,bd}x^{2\,bdn}e^{(2\,ad)}-1\right)bdn}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] -2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) - 1)*b*d*n) + log(x)
```

maple [B] time = 0.02, size = 80, normalized size = 2.86

$$-\frac{\coth\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{bdn}-\frac{\ln\left(\coth\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)-1\right)}{2bdn}+\frac{\ln\left(\coth\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)+1\right)}{2bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x,x)
```

```
[Out] -1/b/d/n*coth(d*(a+b*ln(c*x^n)))-1/2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))-1)+1/
2/b/d/n*ln(coth(d*(a+b*ln(c*x^n)))+1)
```

maxima [A] time = 0.83, size = 37, normalized size = 1.32

$$-\frac{2}{bc^{2\,bd}dne^{\left(2\,bd\log(x^{n})+2\,ad\right)}-bdn}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) + log(x)
```

mupad [B] time = 1.19, size = 34, normalized size = 1.21

$$\ln(x)-\frac{2}{b\,d\,n\,\left(\mathrm{e}^{2\,a\,d}\,{\left(c\,x^{n}\right)}^{2\,b\,d}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x,x)
```

```
[Out] log(x) - 2/(b*d*n*(exp(2*a*d)*(c*x^n)^(2*b*d) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^{2}\left(ad+bd\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x, x)
```

3.189 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal. Leaf size=134

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdnx}+\frac{e^{2ad}\left(cx^n\right)^{2bd}+1}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{1-\frac{1}{bdn}}{x}$$

```
[Out] (-1+1/b/d/n)/x+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x/(1-exp(2*a*d)*(c*x^n)
^(2*b*d))-2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b
*d))/b/d/n/x
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2/x^2, x]
```

Rubi steps

$$\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx=\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^2}\,dx$$

Mathematica [A] time = 3.63, size = 158, normalized size = 1.18

$$\frac{e^{2d(a+b\log(cx^n))}\,{}_2F_1\left(1,1-\frac{1}{2bdn};2-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)-(2bdn-1)\left({}_2F_1\left(1,-\frac{1}{2bdn};1-\frac{1}{2bdn};e^{2d(a+b\log(cx^n))}\right)+\right.}{bdnx(2bdn-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + 2*b*d*n)*(b*d*n + Coth[d*(a + b
*Log[c*x^n])] + Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*
d*(a + b*Log[c*x^n]))]))/(b*d*n*(-1 + 2*b*d*n)*x)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right)+ad\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^2, x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{\coth^2 \left(d \left(a + b \ln \left(c\, x^n\right)\right)\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bc^{2\,bd} dne^{\left(2\,bd \log(x^n)+2\,ad\right)} - bdn + 2}{bc^{2\,bd} dnxe^{\left(2\,bd \log(x^n)+2\,ad\right)} - bdnx} + \int \frac{1}{bc^{bd} dnx^2 e^{\left(bd \log(x^n)+ad\right)} + bdnx^2}\, dx - \int \frac{1}{bc^{bd} dnx^2 e^{\left(bd \log(x^n)+ad\right)} - bdnx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 2)/(b*c^(2*b*d)*d*n*
x*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x) + integrate(1/(b*c^(b*d)*d*n*x^2*e^
(b*d*log(x^n) + a*d) + b*d*n*x^2), x) - integrate(1/(b*c^(b*d)*d*n*x^2*e^(b
*d*log(x^n) + a*d) - b*d*n*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x^2,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}{\left(ad + bd \log{\left(cx^{n} \right)} \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x**2, x)
```

3.190 $\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{bdnx^2}+\frac{e^{2ad}\left(cx^n\right)^{2bd}+1}{bdnx^2\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{2-bdn}{2bdnx^2}$$

```
[Out] 1/2*(-b*d*n+2)/b/d/n/x^2+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x^2/(1-exp(2*
a*d)*(c*x^n)^(2*b*d))-2*hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c*x
^n)^(2*b*d))/b/d/n/x^2
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^2/x^3, x]
```

Rubi steps

$$\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx=\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x^3}\,dx$$

Mathematica [A] time = 3.59, size = 156, normalized size = 1.16

$$\frac{2e^{2d\left(a+b\log\left(cx^n\right)\right)}\,{}_2F_1\left(1,1-\frac{1}{bdn};2-\frac{1}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)-(bdn-1)\left(2\,{}_2F_1\left(1,-\frac{1}{bdn};1-\frac{1}{bdn};e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)+2\,c\right.}{2bdnx^2(bdn-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*
d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + b*d*n)*(b*d*n + 2*Coth[d*(a + b*L
og[c*x^n])] + 2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*d*(a
 + b*Log[c*x^n]))]))/(2*b*d*n*(-1 + b*d*n)*x^2)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\coth\left(bd\log\left(cx^n\right)+ad\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^3, x)
```

maple [F] time = 1.15, size = 0, normalized size = 0.00

$$\int \frac{\coth^2\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bc^{2\,bd}dne^{\left(2\,bd\log\left(x^n\right)+2\,ad\right)}-bdn+4}{2\left(bc^{2\,bd}dnx^2e^{\left(2\,bd\log\left(x^n\right)+2\,ad\right)}-bdnx^2\right)}+2\int\frac{1}{bc^{bd}dnx^3e^{\left(bd\log\left(x^n\right)+ad\right)}+bdnx^3}\,dx-2\int\frac{1}{bc^{bd}dnx^3e^{\left(bd\log\left(x^n\right)+ad\right)}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 4)/(b*c^(2*b*d)*
d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x^2) + 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) + b*d*n*x^3), x) - 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) - b*d*n*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x^3,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^2/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

```
[Out] Timed out
```

3.191 $\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n+ln(sinh(a+b*ln(c*x^n)))/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 3475}

$$\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -Coth[a + b*Log[c*x^n]]^2/(2*b*n) + Log[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\text{Subst}\left(\int \coth(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 52, normalized size = 1.21

$$-\frac{-2\log\left(\tanh\left(a+b\log\left(cx^n\right)\right)\right)-2\log\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)+\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/2*(Coth[a + b*Log[c*x^n]]^2 - 2*Log[Cosh[a + b*Log[c*x^n]]] - 2*Log[Tanh
[a + b*Log[c*x^n]]])/(b*n)
```

fricas [B] time = 0.63, size = 572, normalized size = 13.30

$$-\frac{bn\cosh\left(bn\log(x)+b\log(c)+a\right)^4\log(x)+4\,bn\cosh\left(bn\log(x)+b\log(c)+a\right)\log(x)\sinh\left(bn\log(x)+b\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 4*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^4 - 2*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^2 + b*n*log(x) + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n
*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c
) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) +
 a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(b*n*log(x) + b*log(c
) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)))
 + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) - (b*n*log(x) - 1)*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cos
h(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 - b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) +
b*log(c) + a)^3 - b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*
log(c) + a))
```

giac [B] time = 0.30, size = 127, normalized size = 2.95

$$\frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\mathrm{sgn}(c)-\pi b\right)e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1}\right)}{bn}-\frac{3\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{2\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^2bn}-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b
*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/2*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 2
*c^(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*n) - l
og(x)
```

maple [A] time = 0.02, size = 67, normalized size = 1.56

$$-\frac{\coth^2\left(a+b\ln\left(c\,x^n\right)\right)}{2bn}-\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^n\right)\right)-1\right)}{2nb}-\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^n\right)\right)+1\right)}{2nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(coth(a+b*ln(c*x^n))-1)-1/2/n/b*ln
(coth(a+b*ln(c*x^n))+1)
```

maxima [B] time = 0.43, size = 330, normalized size = 7.67

$$-\frac{4\,c^{2\,b}e^{\left(2\,b\log(x^n)+2\,a\right)}-3}{4\left(bc^{4\,b}ne^{\left(4\,b\log(x^n)+4\,a\right)}-2\,bc^{2\,b}ne^{\left(2\,b\log(x^n)+2\,a\right)}+bn\right)}-\frac{3\left(2\,c^{2\,b}e^{\left(2\,b\log(x^n)+2\,a\right)}-1\right)}{4\left(bc^{4\,b}ne^{\left(4\,b\log(x^n)+4\,a\right)}-2\,bc^{2\,b}ne^{\left(2\,b\log(x^n)+2\,a\right)}+bn\right)}+\frac{}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) +
4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 3/4*(2*c^(2*b)*e^(2*b*
log(x^n) + 2*a) - 1)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^
(2*b*log(x^n) + 2*a) + b*n) + 1/4*(2*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b
*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) +
b*n) - 3/4/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x
^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + lo
g((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

mupad [**B**] time = 1.23, size = 95, normalized size = 2.21

$$\frac{2}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}-\ln(x)-\frac{2}{b\,n-2\,b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}+\frac{\ln\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^3/x,x)
```

```
[Out] 2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) - log(x) - 2/(b*n - 2*b*n*exp(2*a)*(c*
x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*b) - 1)/
(b*n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Timed out
```

3.192 $\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=45

$$-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}+\log(x)$$

```
[Out] -coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n+ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{3473, 8\}$

$$-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n)) - Coth[a + b*Log[c*x^n]]^3/(3*b*n) + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\text{Subst}\left(\int \coth^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\text{Subst}\left(\int 1\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\log(x)
\end{aligned}$$

Mathematica [C] time = 0.12, size = 44, normalized size = 0.98

$$-\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\tanh^2\left(a+b\log\left(cx^n\right)\right)\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] -1/3*(Coth[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b*
Log[c*x^n]]^2])/(b*n)
```

fricas [B] time = 0.51, size = 171, normalized size = 3.80

$$\frac{\left(3\,bn\log(x)+4\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^{3}-4\,\cosh\left(bn\log(x)+b\log(c)+a\right)^{3}-12\,\cosh\left(bn\log(x)+\cdots\right.}{3\left(bn\sinh\left(bn\log(x)+b\log(c)+a\right)^{3}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*n*log(x) + 4)*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(
x) + b*log(c) + a)^3 - 12*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + 3*((3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*log(c) + a)^2
- 3*b*n*log(x) - 4)*sinh(b*n*log(x) + b*log(c) + a))/(b*n*sinh(b*n*log(x) +
 b*log(c) + a)^3 + 3*(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n
*log(x) + b*log(c) + a))
```

giac [A] time = 0.31, size = 67, normalized size = 1.49

$$-\frac{4\left(3\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+2\right)}{3\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^{3}bn}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 3*c^(2*b)*x^(2*b*n)*e^(2*a) + 2)/((c^(2
*b)*x^(2*b*n)*e^(2*a) - 1)^3*b*n) + log(x)
```

maple [A] time = 0.02, size = 86, normalized size = 1.91

$$-\frac{\coth^{3}\left(a+b\ln\left(c\,x^{n}\right)\right)}{3bn}-\frac{\coth\left(a+b\ln\left(c\,x^{n}\right)\right)}{bn}-\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2nb}+\frac{\ln\left(\coth\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] -1/3*coth(a+b*ln(c*x^n))^3/b/n-coth(a+b*ln(c*x^n))/b/n-1/2/n/b*ln(coth(a+b*
ln(c*x^n))-1)+1/2/n/b*ln(coth(a+b*ln(c*x^n))+1)
```

maxima [B] time = 0.45, size = 499, normalized size = 11.09

$$-\frac{18\,c^{4\,b}e^{\left(4\,b\log(x^{n})+4\,a\right)}-27\,c^{2\,b}e^{\left(2\,b\log(x^{n})+2\,a\right)}+11}{12\left(bc^{6\,b}ne^{\left(6\,b\log(x^{n})+6\,a\right)}-3\,bc^{4\,b}ne^{\left(4\,b\log(x^{n})+4\,a\right)}+3\,bc^{2\,b}ne^{\left(2\,b\log(x^{n})+2\,a\right)}-bn\right)}-\frac{6\,c^{4\,b}e^{\left(4\,b\log(x^{n}\cdots\right.}}{12\left(bc^{6\,b}ne^{\left(6\,b\log(x^{n})+6\,a\right)}-3\,b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -1/12*(18*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 27*c^(2*b)*e^(2*b*log(x^n) + 2*a
) + 11)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n)
 + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 1/12*(6*c^(4*b)*e^(
4*b*log(x^n) + 4*a) - 15*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 11)/(b*c^(6*b)*n*
e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)
*n*e^(2*b*log(x^n) + 2*a) - b*n) - 2/3*(3*c^(4*b)*e^(4*b*log(x^n) + 4*a) -
3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) -
 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a
) - b*n) - 1/2*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*l
```

```
og(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*
b*log(x^n) + 2*a) - b*n) - 2/3/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^
(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n
) + log(x)
```

mupad [B] time = 1.21, size = 163, normalized size = 3.62

$$\ln(x)-\frac{\frac{4}{3\,b\,n}+\frac{4\,e^{4\,a}\,(c\,x^n)^{4\,b}}{3\,b\,n}}{3\,e^{2\,a}\,(c\,x^n)^{2\,b}-3\,e^{4\,a}\,(c\,x^n)^{4\,b}+e^{6\,a}\,(c\,x^n)^{6\,b}-1}-\frac{4}{3\,b\,n\,\left(e^{2\,a}\,(c\,x^n)^{2\,b}-1\right)}-\frac{4\,e^{2\,a}\,(c\,x^n)^{2\,b}}{3\,b\,n\,\left(e^{4\,a}\,(c\,x^n)^{4\,b}-2\,e^{2\,a}\,(c\,x^n)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^4/x,x)
```

```
[Out] log(x) - (4/(3*b*n) + (4*exp(4*a)*(c*x^n)^(4*b))/(3*b*n))/(3*exp(2*a)*(c*x^
n)^(2*b) - 3*exp(4*a)*(c*x^n)^(4*b) + exp(6*a)*(c*x^n)^(6*b) - 1) - 4/(3*b*
n*(exp(2*a)*(c*x^n)^(2*b) - 1)) - (4*exp(2*a)*(c*x^n)^(2*b))/(3*b*n*(exp(4*
a)*(c*x^n)^(4*b) - 2*exp(2*a)*(c*x^n)^(2*b) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Timed out
```

3.193 $\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=66

$$\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\coth^4\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n-1/4*coth(a+b*ln(c*x^n))^4/b/n+ln(sinh(a+b*ln
(c*x^n)))/b/n
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3473, 3475}

$$\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}-\frac{\coth^4\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -Coth[a + b*Log[c*x^n]]^2/(2*b*n) - Coth[a + b*Log[c*x^n]]^4/(4*b*n) + Log[
Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \coth^5(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{\coth^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\text{Subst}\left(\int \coth^3(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\coth^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\text{Subst}\left(\int \coth(a+bx)\,d\right.}{n}\\ &= -\frac{\coth^2\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\coth^4\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\log\left(\sinh\left(a+b\log\left(cx^n\right.\right.\right.}{bn}\end{aligned}$$

Mathematica [A] time = 0.29, size = 67, normalized size = 1.02

$$-\frac{-4\log\left(\tanh\left(a+b\log\left(cx^n\right)\right)\right)-4\log\left(\cosh\left(a+b\log\left(cx^n\right)\right)\right)+\coth^4\left(a+b\log\left(cx^n\right)\right)+2\coth^2\left(a+b\log\right.}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -1/4*(2*Coth[a + b*Log[c*x^n]]^2 + Coth[a + b*Log[c*x^n]]^4 - 4*Log[Cosh[a
+ b*Log[c*x^n]]] - 4*Log[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

fricas [B] time = 0.42, size = 1576, normalized size = 23.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^8*log(x) + 8*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^8 - 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n*log(x) + 1)*
sinh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a
)^3*log(x) - 3*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*l
og(x) + b*log(c) + a)^5 + 2*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) +
 a)^4 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) - 30*(b*n*log(x)
 - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*log(x) - 2)*sinh(b*n*log(x)
 + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5*log(x) - 10
*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^3 + (3*b*n*log(x) - 2)*co
sh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*(b*n*l
og(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n*log(x) + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6*log(x) - 15*(b*n*log(x) - 1)*cosh(b*n*log(x) +
b*log(c) + a)^4 + 3*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a)^2 -
b*n*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*lo
g(c) + a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c)
+ a)^7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c
) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log
(c) + a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b
*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b
*log(c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x)
+ b*log(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log
(x) + b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*l
og(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*lo
g(x) + b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*
log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(
b*n*log(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b
*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(
b*n*log(x) + b*log(c) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(
x) + b*log(c) + a))) + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7*log(x) - 3*
(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^5 + (3*b*n*log(x) - 2)*cos
h(b*n*log(x) + b*log(c) + a)^3 - (b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c
) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) +
a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^7 + b*n*sinh(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log
(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log
(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*
cosh(b*n*log(x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) +
a)^4 - 30*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) +
b*log(c) + a)^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*
(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c
) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3
- b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

giac [**B**] time = 0.34, size = 161, normalized size = 2.44

$$\frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\mathrm{sgn}(c)-\pi b\right)e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1}\right)}{bn}-\frac{25\,c^{8\,b}x^{8\,bn}e^{(8\,a)}-52\,c^{6\,b}x^{6\,bn}e^{(6\,a)}+102\,c^{4}}{12\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b
*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/12*(25*c^(8*b)*x^(8*b*n)*e^(8*a) -
 52*c^(6*b)*x^(6*b*n)*e^(6*a) + 102*c^(4*b)*x^(4*b*n)*e^(4*a) - 52*c^(2*b)*
x^(2*b*n)*e^(2*a) + 25)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*n) - log(x)
```

maple [**A**] time = 0.02, size = 88, normalized size = 1.33

$$-\frac{\coth^4(a+b\ln(c\,x^n))}{4bn}-\frac{\coth^2(a+b\ln(c\,x^n))}{2bn}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2nb}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] -1/4*coth(a+b*ln(c*x^n))^4/b/n-1/2*coth(a+b*ln(c*x^n))^2/b/n-1/2/n/b*ln(cot
h(a+b*ln(c*x^n))-1)-1/2/n/b*ln(coth(a+b*ln(c*x^n))+1)
```

maxima [**B**] time = 0.50, size = 855, normalized size = 12.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] -1/24*(48*c^(6*b)*e^(6*b*log(x^n) + 6*a) - 108*c^(4*b)*e^(4*b*log(x^n) + 4*
a) + 88*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) +
 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n
) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/24*(12*c^(6*b)*e
^(6*b*log(x^n) + 6*a) - 42*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 52*c^(2*b)*e^(2
*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*
n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*
b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 5/8*(4*c^(6*b)*e^(6*b*log(x^n) + 6*a)
- 6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n) - 5/12*(6*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 4*c^(2*b)*e^(2*b*log(x
^n) + 2*a) + 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*
log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2
*b*log(x^n) + 2*a) + b*n) - 5/12*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*
c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6
*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a)
+ b*n) - 5/8/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log
(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*
log(x^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n)
 + log((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

mupad [**B**] time = 1.20, size = 229, normalized size = 3.47

$$\frac{8}{b\,n-3\,b\,n\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}+3\,b\,n\,\mathrm{e}^{4\,a}\,(c\,x^n)^{4\,b}-b\,n\,\mathrm{e}^{6\,a}\,(c\,x^n)^{6\,b}}-\ln(x)+\frac{4}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,(c\,x^n)^{2\,b}}-\frac{}{b\,n-4\,b\,n\,\mathrm{e}^{2\,a}\,(c\,x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^5/x,x)
```

```
[Out] 8/(b*n - 3*b*n*exp(2*a)*(c*x^n)^(2*b) + 3*b*n*exp(4*a)*(c*x^n)^(4*b) - b*n*
exp(6*a)*(c*x^n)^(6*b)) - log(x) + 4/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) - 4
/(b*n - 4*b*n*exp(2*a)*(c*x^n)^(2*b) + 6*b*n*exp(4*a)*(c*x^n)^(4*b) - 4*b*n
*exp(6*a)*(c*x^n)^(6*b) + b*n*exp(8*a)*(c*x^n)^(8*b)) - 8/(b*n - 2*b*n*exp(
2*a)*(c*x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*
b) - 1)/(b*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Timed out
```

3.194 $\int (ex)^m \coth \left(d \left(a+b \log \left(c x^n\right)\right)\right) \, dx$

Optimal. Leaf size=87

$$\frac{(ex)^{m+1}}{e(m+1)}-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd}\right)}{e(m+1)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+
m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/e/(1+m)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth \left(d \left(a+b \log \left(cx^n\right)\right)\right) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])], x]
```

Rubi steps

$$\int (ex)^m \coth \left(d \left(a+b \log \left(cx^n\right)\right)\right) \, dx=\int (ex)^m \coth \left(d \left(a+b \log \left(cx^n\right)\right)\right) \, dx$$

Mathematica [A] time = 13.48, size = 158, normalized size = 1.82

$$\frac{x(ex)^m\left(-\frac{(m+1)e^{2ad}\left(cx^n\right)^{2bd}\,{}_2F_1\left(1,\frac{m+2bdn+1}{2bdn};\frac{m+4bdn+1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd}\right)}{2bdn+m+1}-{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2d\left(a+b\log\left(cx^n\right)\right)}\right)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
[Out] (x*(e*x)^m*(-Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))] - (E^(2*a*d)*(1 + m)*(c*x^n)^(2*b*d)*Hypergeom
etric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*
a*d)*(c*x^n)^(2*b*d)])/(1 + m + 2*b*d*n)))/(1 + m)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex\right)^{m} \coth \left(bd \log \left(cx^n\right)+ad\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{m} \coth \left(\left(b \log \left(cx^n\right)+a\right)d\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d), x)
```

maple [F] time = 1.58, size = 0, normalized size = 0.00

$$\int (ex)^{m} \coth \left(d \left(a+b \ln \left(c \,x^{n}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{m} x x^{m}}{m+1} - e^{m} \int \frac{x^{m}}{c^{bd} e^{\left(bd \log\left(x^{n}\right) + ad\right)} + 1}\, dx + e^{m} \int \frac{x^{m}}{c^{bd} e^{\left(bd \log\left(x^{n}\right) + ad\right)} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] e^m*x*x^m/(m + 1) - e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1),
 x) + e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,{\left(e\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{m} \coth{\left(ad + bd \log{\left(cx^{n} \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n)), x)
```

3.195 $\int (ex)^m \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right) dx$

Optimal. Leaf size=168

$$-\frac{2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd}\right)}{bden}+\frac{(ex)^{m+1}\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}{bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{(ex)^{m+1}(bdn+m+1)}{bde(m+1)n}$$

```
[Out] (b*d*n+m+1)*(e*x)^(1+m)/b/d/e/(1+m)/n+(e*x)^(1+m)*(1+exp(2*a*d)*(c*x^n)^(2*
b*d))/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-2*(e*x)^(1+m)*hypergeom([1, 1/
2*(1+m)/b/d/n],[1+1/2*(1+m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/e/n
```

Rubi [F] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int (ex)^m \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right) dx=\int (ex)^m \coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right) dx$$

Mathematica [A] time = 14.88, size = 312, normalized size = 1.86

$$(ex)^m\left(\frac{x}{m+1}-\frac{x^{-2m}\exp\left(-\frac{(2m+1)\left(a+b\log(cx^n)-bn\log(x)\right)}{bn}\right)\left((m+1)x^{2bdn+2m+1}\exp\left(\frac{(2bdn+2m+1)\left(a+b\log(cx^n)-bn\log(x)\right)}{bn}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (e*x)^m*(x/(1 + m) - (E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1 + m + 2*b
*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))
*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b
*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^(((1 + 2*m + 2*b*d*n)*(a - b*n*Log[x
] + b*Log[c*x^n]))/(b*n))*(1 + m)*x^(1 + 2*m + 2*b*d*n)*Hypergeometric2F1[1
, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a + b*L
og[c*x^n]))])/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*n*
(1 + m + 2*b*d*n)*x^(2*m)))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex\right)^m \coth\left(bd\log\left(cx^n\right)+ad\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^2, x)
```

maple [F] time = 1.56, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^2\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-e^m(m+1)\int \frac{x^m}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}+bdn}\,dx+e^m(m+1)\int \frac{x^m}{bc^{bd}dne^{\left(bd\log(x^n)+ad\right)}-bdn}\,dx+\frac{bc^{2\,bd}de^m nxe^{\left(2\,bd\log(x^n)\right.}}{(mn+n)bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] -e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n),
x) + e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) - b*d*
n), x) + (b*c^(2*b*d)*d*e^m*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x)) - (b*
d*e^m*n + 2*e^m*(m + 1))*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log(x^n)
+ 2*a*d) - (m*n + n)*b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{coth}\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}^2\,{\left(e\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n))**2, x)
```

3.196 $\int (ex)^m \coth^3\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=306

$$-\frac{(ex)^{m+1}\left(2b^2d^2n^2+m^2+2m+1\right){}_2F_1\left(1,\frac{m+1}{2bdn};\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd}\right)}{b^2d^2e(m+1)n^2}+\frac{e^{-2ad}(ex)^{m+1}\left(\frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}+\cdots\right.}{2b^2d^2en\left(1-e^{2ad}\left(cx^n\right)^{2bd}\cdots\right.}$$

```
[Out] 1/2*(b*d*n+m+1)*(2*b*d*n+m+1)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2-1/2*(e*x)^(1+
m)*(1+exp(2*a*d)*(c*x^n)^(2*b*d))^2/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))^
2+1/2*(e*x)^(1+m)*(exp(2*a*d)*(-2*b*d*n+m+1)/n+exp(4*a*d)*(2*b*d*n+m+1)*(c*
x^n)^(2*b*d)/n)/b^2/d^2/e/exp(2*a*d)/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-(2*b^
2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+m
)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b^2/d^2/e/(1+m)/n^2
```

Rubi [F] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth^3\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3, x]
```

Rubi steps

$$\int (ex)^m \coth^3\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int (ex)^m \coth^3\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [A] time = 16.83, size = 600, normalized size = 1.96

$$-\frac{x^{-m}(ex)^m\left(2b^2d^2n^2+m^2+2m+1\right)\operatorname{csch}\left(d\left(a+b\left(\log\left(cx^n\right)-n\log(x)\right)\right)\right)\left(\frac{\sinh\left(d\left(a+b\left(\log\left(cx^n\right)-n\log(x)\right)\right)\right)\exp\left(-\cdots\right)}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*Coth[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m) - (x*(e*x)^m
*Csch[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) + (
(1 + m)*x*(e*x)^m*Csch[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csch[b*d*n*Log
[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(2*b^2*d^2*
n^2) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csch[d*(a + b*(-(n*Log[x])
+ Log[c*x^n]))]*((x^(1 + m)*Csch[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/
(1 + m) + ((E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) +
 Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^((a +
 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n)
)*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1
 + m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n
)*(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*
n)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])*Sinh[d*(a + b*(-(n*Log[x]) + Log
```

```
[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n]))))/(b*n))*(1 +
m)*(1 + m + 2*b*d*n))))/(2*b^2*d^2*n^2*x^m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ex\right)^{m}\coth\left(bd\log\left(cx^{n}\right)+ad\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex\right)^{m}\coth\left(\left(b\log\left(cx^{n}\right)+a\right)d\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^3, x)
```

maple [F] time = 1.69, size = 0, normalized size = 0.00

$$\int\left(ex\right)^{m}\left(\coth^{3}\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(2\,b^{2}d^{2}e^{m}n^{2}+\left(m^{2}+2\,m+1\right)e^{m}\right)\int\frac{x^{m}}{2\left(b^{2}c^{bd}d^{2}n^{2}e^{\left(bd\log(x^{n})+ad\right)}+b^{2}d^{2}n^{2}\right)}\,dx+\left(2\,b^{2}d^{2}e^{m}n^{2}+\left(m^{2}+2\,m+1\right)e^{m}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] -(2*b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d
^2*n^2*e^(b*d*log(x^n) + a*d) + b^2*d^2*n^2), x) + (2*b^2*d^2*e^m*n^2 + (m^
2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d^2*n^2*e^(b*d*log(x^n) +
a*d) - b^2*d^2*n^2), x) + (b^2*c^(4*b*d)*d^2*e^m*n^2*x*e^(4*b*d*log(x^n) +
4*a*d + m*log(x)) + (b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*x*x^m - (2*b^2*
c^(2*b*d)*d^2*e^m*n^2*e^(2*a*d) + 2*(m*n + n)*b*c^(2*b*d)*d*e^m*e^(2*a*d) +
 (m^2 + 2*m + 1)*c^(2*b*d)*e^m*e^(2*a*d))*x*e^(2*b*d*log(x^n) + m*log(x)))/
((m*n^2 + n^2)*b^2*c^(4*b*d)*d^2*e^(4*b*d*log(x^n) + 4*a*d) - 2*(m*n^2 + n^
2)*b^2*c^(2*b*d)*d^2*e^(2*b*d*log(x^n) + 2*a*d) + (m*n^2 + n^2)*b^2*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\coth\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)^{3}\,\left(e\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth^3 \left(ad + bd \log (cx^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n))**3, x)
```

3.197 $\int \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=115

$$x\left(-e^{2ad}\left(cx^n\right)^{2bd}-1\right)^p\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)^{-p}F_1\left(\frac{1}{2bdn};p,-p;1+\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd},-e^{2ad}\left(cx^n\right)^{2bd}\right)$$

```
[Out] x*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2/b/d/n,p,-p,1+1/2/b/d/n,exp
(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/((1+exp(2*a*d)*(c*x^n)
^(2*b*d))^p)
```

Rubi [F] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] Defer[Int][Coth[d*(a + b*Log[c*x^n])]^p, x]
```

Rubi steps

$$\int \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx=\int \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx$$

Mathematica [B] time = 3.97, size = 387, normalized size = 3.37

$$\frac{x(2bdn+1)\left(\frac{e^{2ad}(cx^n)^{2bd}+1}{e^{2ad}(cx^n)^{2bd}-1}\right)^p F_1\left(\frac{1}{2bdn};p,-\right.}{2bdnpe^{2ad}\left(cx^n\right)^{2bd}F_1\left(1+\frac{1}{2bdn};p,1-p;2+\frac{1}{2bdn};e^{2ad}\left(cx^n\right)^{2bd},-e^{2ad}\left(cx^n\right)^{2bd}\right)+2bdnpe^{2ad}\left(cx^n\right)^{2bd}F_1\left(1+\frac{1}{2bdn};p\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((1 + 2*b*d*n)*x*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(-1 + E^(2*a*d)*(c*x^n)^(
2*b*d)))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^
(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))]])/(2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d
)*AppellF1[1 + 1/(2*b*d*n), p, 1 - p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2
*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d)*
AppellF1[1 + 1/(2*b*d*n), 1 + p, -p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*
b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*d*n), p
, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*
d))])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\coth\left(bd\log\left(cx^n\right)+ad\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \coth^p\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a+b*ln(c*x^n)))^p,x)
[Out] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^p,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**p,x)
[Out] Integral(coth(d*(a + b*log(c*x**n)))**p, x)
```

3.198 $\int (ex)^m \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$

Optimal. Leaf size=135

$$\frac{(ex)^{m+1}\left(-e^{2ad}\left(cx^n\right)^{2bd}-1\right)^p\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)^{-p}F_1\left(\frac{m+1}{2bdn};p,-p;\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd},-e^{2ad}\left(cx^n\right)^{2bd}\right)}{e(m+1)}$$

```
[Out] (e*x)^(1+m)*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2*(1+m)/b/d/n,p,-p
,1+1/2*(1+m)/b/d/n,exp(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/
e/(1+m)/((1+exp(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rubi [F] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] Defer[Int][(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p, x]
```

Rubi steps

$$\int (ex)^m \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx=\int (ex)^m \coth^p\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$

Mathematica [A] time = 5.42, size = 174, normalized size = 1.29

$$\frac{x(ex)^m\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^p\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)^{-p}\left(\frac{e^{2ad}\left(cx^n\right)^{2bd}+1}{e^{2ad}\left(cx^n\right)^{2bd}-1}\right)^p F_1\left(\frac{m+1}{2bdn};p,-p;\frac{m+1}{2bdn}+1;e^{2ad}\left(cx^n\right)^{2bd},-e^{2ad}\left(cx^n\right)^{2bd}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*((1 + E^(2*a*d)*(c*x^n)^(2*b*d
))/(-1 + E^(2*a*d)*(c*x^n)^(2*b*d)))^p*AppellF1[(1 + m)/(2*b*d*n), p, -p, 1
 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d
))])/((1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex\right)^m\coth\left(bd\log\left(cx^n\right)+ad\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^m\coth\left(\left(b\log\left(cx^n\right)+a\right)d\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\coth^p \left(d \left(a + b\ln \left(c\,x^n\right)\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \coth\left(\left(b\log\left(cx^n\right) + a\right)d\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(d\,\left(a + b\,\ln\left(c\,x^n\right)\right)\right)^p\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**p,x)
[Out] Timed out
```

3.199 $\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}-\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2/3*coth(a+b*ln(c*x^n))^(3/2)/b/n
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3473, 3476, 329, 298, 203, 206}

$$-\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}-\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \coth^{\frac{5}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{2\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\operatorname{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{2\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\operatorname{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\
&= -\frac{2\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{2\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} \\
&= -\frac{2\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \\
&= -\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{2\,\mathrm{c}}{}
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 64, normalized size = 0.88

$$
-\frac{2\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)+3\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)-3\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{3bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -1/3*(3*ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - 3*ArcTanh[Sqrt[Coth[a + b*Lo
g[c*x^n]]]] + 2*Coth[a + b*Log[c*x^n]]^(3/2))/(b*n)
```

fricas [B] time = 0.45, size = 626, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)^2 - 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x)
+ b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c
```

```
) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(
c) + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a) - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) +
a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)
/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
+ b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.22, size = 93, normalized size = 1.27

$$-\frac{2\left(\coth^{\frac{3}{2}}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{3bn}-\frac{\ln\left(\sqrt{\coth}\,\left(a+b\ln\left(c\,x^{n}\right)\right)-1\right)}{2bn}+\frac{\ln\left(\sqrt{\coth}\,\left(a+b\ln\left(c\,x^{n}\right)\right)+1\right)}{2bn}-\frac{\arctan\left(\sqrt{\coth}\,\left(a\right.\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] -2/3*coth(a+b*ln(c*x^n))^(3/2)/b/n-1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+
1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b
/n
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(coth(b*log(c*x^n) + a)^(5/2)/x, x)
```

mupad [**B**] time = 2.24, size = 65, normalized size = 0.89

$$\frac{\mathrm{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\right)}{b\,n}-\frac{\mathrm{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\right)}{b\,n}-\frac{2\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{3/2}}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^(5/2)/x,x)
```

```
[Out] atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - (2*coth(a + b*log(c*x^n))^(3/2))/(3*b*n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

3.200 $\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=70

$$-\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n-2*coth(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3473, 3476, 329, 212, 206, 203}

$$-\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}+\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\text{Subst}\left(\int \coth^{\frac{3}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, \coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$
$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$
$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{S}}{}$$
$$= \frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{2\sqrt{\coth}}{}$$

Mathematica [A] time = 0.15, size = 57, normalized size = 0.81

$$\frac{-2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}+\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)+\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]
]] - 2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

fricas [B] time = 0.47, size = 334, normalized size = 4.77

$$-\frac{4\sqrt{\frac{\cosh\left(bn\log(x)+b\log(c)+a\right)}{\sinh\left(bn\log(x)+b\log(c)+a\right)}}+2\arctan\left(-\cosh\left(bn\log(x)+b\log(c)+a\right)^2-2\cosh\left(bn\log(x)+b\log(c)+a\right)\text{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a
)) + 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
```

```
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.13, size = 92, normalized size = 1.31

$$-\frac{2\left(\sqrt{\coth}\ (a+b\ln(c\,x^n))\right)}{bn}-\frac{\ln\left(\sqrt{\coth}\ (a+b\ln(c\,x^n))-1\right)}{2bn}+\frac{\ln\left(\sqrt{\coth}\ (a+b\ln(c\,x^n))+1\right)}{2bn}+\frac{\arctan\left(\sqrt{\coth}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(3/2)/x,x)
[Out] -2*coth(a+b*ln(c*x^n))^(1/2)/b/n-1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/
2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)+1)+arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(coth(b*log(c*x^n) + a)^(3/2)/x, x)
```

mupad [B] time = 1.86, size = 51, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)+\operatorname{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)-2\,\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^(3/2)/x,x)
[Out] (atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2)) -
 2*coth(a + b*log(c*x^n))^(1/2))/(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(3/2)/x,x)
[Out] Integral(coth(a + b*log(c*x**n))**(3/2)/x, x)
```

3.201 $\int \frac{\sqrt{\coth\left(a+b\log(cx^n)\right)}}{x}\, dx$

Optimal. Leaf size=48

$$\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3476, 329, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}-\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth}\right.}{bn}$$
$$= -\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

fricas [B] time = 0.45, size = 305, normalized size = 6.35

$$2\arctan\left(-\cosh\left(bn\log(x)+b\log(c)+a\right)^2 - 2\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.13, size = 72, normalized size = 1.50

$$-\frac{\ln\left(\sqrt{\coth}\ (a+b\ln(c\,x^{n}))-1\right)}{2bn}+\frac{\ln\left(\sqrt{\coth}\ (a+b\ln(c\,x^{n}))+1\right)}{2bn}-\frac{\arctan\left(\sqrt{\coth}\ (a+b\ln(c\,x^{n}))\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] -1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/
2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\coth\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(b*log(c*x^n) + a))/x, x)
```

mupad [B] time = 1.50, size = 39, normalized size = 0.81

$$-\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\ \ln\left(c\,x^{n}\right)\right)}\right)-\operatorname{atanh}\left(\sqrt{\coth\left(a+b\ \ln\left(c\,x^{n}\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] -(atan(coth(a + b*log(c*x^n))^(1/2)) - atanh(coth(a + b*log(c*x^n))^(1/2)))
/(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\coth\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(coth(a + b*log(c*x**n)))/x, x)
```

3.202 $\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=47

$$\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3476, 329, 212, 206, 203}

$$\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, \coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth\left(\right.}\right.}{bn}$$
$$= \frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Mathematica [A] time = 0.12, size = 47, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

fricas [B] time = 0.49, size = 303, normalized size = 6.45

$$-\frac{2\arctan\left(-\cosh\left(bn\log(x)+b\log(c)+a\right)^2 - 2\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a
)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*
sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + lo
g(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b
*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n
*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\coth\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)
```

maple [A] time = 0.14, size = 44, normalized size = 0.94

$$\frac{\arctan\left(\sqrt{\coth}\ (a+b\ln(c\,x^n))\right)}{bn}+\frac{\operatorname{arctanh}\left(\sqrt{\coth}\ (a+b\ln(c\,x^n))\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\coth\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)
```

mupad [B] time = 1.64, size = 36, normalized size = 0.77

$$\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\ \ln\left(c\,x^n\right)\right)}\right)+\operatorname{atanh}\left(\sqrt{\coth\left(a+b\ \ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(1/2)),x)
```

```
[Out] (atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2)))/
(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(coth(a + b*log(c*x**n)))), x)
```

3.203 $\int \frac{1}{x \coth^{\frac{3}{2}}(a+b\log(cx^n))} \, dx$

Optimal. Leaf size=71

$$-\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}-\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2/b/n/coth(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3474, 3476, 329, 298, 203, 206}

$$-\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}-\frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x \coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\ldots}{b\ldots}$$

Mathematica [C] time = 0.15, size = 44, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\coth^2(a+b\log(cx^n))\right)}{bn\sqrt{\coth(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, Coth[a + b*Log[c*x^n]]^2])/(b*n*Sqrt[Co
th[a + b*Log[c*x^n]]])
```

fricas [B] time = 0.46, size = 625, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
```

```
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x)
+ b*log(c) + a)^2 + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) +
b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
+ a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c)
 + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + si
nh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/s
inh(b*n*log(x) + b*log(c) + a)) - 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \coth\left(b \log\left(cx^n\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

maple [A] time = 0.14, size = 93, normalized size = 1.31

$$-\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))-1\right)}{2bn}+\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))+1\right)}{2bn}-\frac{2}{bn\sqrt{\coth(a+b\ln(c\,x^n))}}-\frac{\arctan\left(\sqrt{\coth}\,(a\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] -1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/
2)+1)-2/b/n/coth(a+b*ln(c*x^n))^(1/2)-arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \coth\left(b \log\left(cx^n\right) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

mupad [B] time = 1.74, size = 65, normalized size = 0.92

$$\frac{\mathrm{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{\mathrm{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}-\frac{2}{b\,n\,\sqrt{\coth\left(a+b\,\ln\left(c\,x^n\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(b*n*coth(a + b*log(c*x^n))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\coth^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*coth(a + b*log(c*x**n))**(3/2)), x)
```

3.204 $\int \frac{1}{x \coth^{\frac{5}{2}}(a+b\log(cx^n))} \, dx$

Optimal. Leaf size=72

$$-\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n-2/3/b/n/coth(a+b*ln(c*x^n))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3474, 3476, 329, 212, 206, 203}

$$-\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{\tan^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}+\frac{\tanh^{-1}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{x\coth^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{5}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+x^2\right)}\,dx, x, \coth\left(a+b\log\left(cx\right.\right.\right.}{bn}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx\right.\right.}\right.}{bn}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right.}{bn}$$

$$= \frac{\tan^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\tanh^{-1}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{}{3bn}$$

Mathematica [C] time = 0.21, size = 46, normalized size = 0.64

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\coth^2\left(a+b\log\left(cx^n\right)\right)\right)}{3bn\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, Coth[a + b*Log[c*x^n]]^2])/(3*b*n*Coth[
a + b*Log[c*x^n]]^(3/2))
```

fricas [B] time = 0.53, size = 1104, normalized size = 15.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*n*log(x) + b*log(c) + a)^4 + 16*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*sinh(b*n*log(x) + b*log(c) + a)^4
 + 8*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 6*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c)
```

```
 + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c)
 + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)
+ 1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*s
inh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqr
t(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + 8*cos
h(b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sin
h(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(c
osh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*lo
g(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*
log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b
*log(c) + a))) + 16*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) +
b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 4*(cosh(b*n*log(x) + b*log
(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
 + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*sqrt(cosh(b*n*log(x) + b*log(
c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) +
b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*
n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3
 + b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2/sign(c)/2)>(-2/sign
(c)/2)Unable to check sign: (2/sign(c)/2)>(-2/sign(c)/2)Unable to check sig
n: (2/sign(c)/2)>(-2/sign(c)/2)Unable to check sign: (2/sign(c)/2)>(-2/sign
(c)/2)Unable to check sign: (2/sign(c)/2)>(-2/sign(c)/2)Unable to check sig
n: (2/sign(c)/2)>(-2/sign(c)/2)Error index.cc index_gcd Error: Bad Argument
 ValueEvaluation time: 2.68Error index.cc index_gcd Error: Bad Argument Val
ue
```

maple [A] time = 0.14, size = 92, normalized size = 1.28

$$-\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))-1\right)}{2bn}+\frac{\ln\left(\sqrt{\coth}\,(a+b\ln(c\,x^n))+1\right)}{2bn}-\frac{2}{3bn\coth(a+b\ln(c\,x^n))^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{\coth}\,(\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] -1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2/b/n*ln(coth(a+b*ln(c*x^n))^(1/
2)+1)-2/3/b/n/coth(a+b*ln(c*x^n))^(3/2)+arctan(coth(a+b*ln(c*x^n))^(1/2))/b
/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\coth\left(b\log\left(cx^{n}\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)
```

mupad [B] time = 2.39, size = 64, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\right)}{b\,n}+\frac{\operatorname{atanh}\left(\sqrt{\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)}\right)}{b\,n}-\frac{2}{3\,b\,n\coth\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] atan(coth(a + b*log(c*x^n))^(1/2))/(b*n) + atanh(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(3*b*n*coth(a + b*log(c*x^n))^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

3.205 $\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=135

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}}-\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] 1/4*(b-2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)^
4)^(1/2))/c^(3/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(
a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*coth(x
)^4)^(1/2)/c
```

Rubi [A] time = 0.35, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3701, 1251, 1653, 843, 621, 206, 724}

$$\frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}}-\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*
Coth[x]^4])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Coth[x]^2 + c*Coth[x]^4]/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\text{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{\text{Subst}\left(\int \frac{\frac{b}{2}+\frac{1}{2}(b-2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\text{coth}\right.}{2c}$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,d\right.$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{(b-2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-}{\sqrt{a+b\,\text{cot}}}\right.}{2c}$$

$$= \frac{(b-2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}} + \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\,\text{c}}{2\sqrt{a+b+c}\sqrt{a+b\,\text{coth}^2}}\right.}{2\sqrt{a+b+c}}$$

Mathematica [A] time = 9.08, size = 266, normalized size = 1.97

$$\frac{\frac{2\operatorname{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\left(2c^{3/2}\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}}\right)+(b-2c)\sqrt{a+b+c}\tanh^{-1}\left(\frac{\quad}{2\sqrt{c}\sqrt{\sinh}}\right.\right.}{\sqrt{a+b+c}}}{8c^{3/2}\sqrt{\operatorname{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((2*(2*c^(3/2)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*
Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])] + (b - 2*c)*Sqrt[a
+ b + c]*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*
Sinh[x]^2 + (a + b + c)*Sinh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2
*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/Sqrt[a + b + c] - Sqrt[2]*Sqrt[c]*(
3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4)/(8*
c^(3/2)*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*
Csch[x]^4])
```

fricas [B] time = 2.03, size = 8951, normalized size = 66.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a +
 b)*c - 2*c^2)*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x
)^4 - 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*
a + b)*c - 2*c^2)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 +
a*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh
(x)^3 - (a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(c)*log(((b
^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cos
h(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c -
 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*
a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(
b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c
^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(
7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*cosh(x
)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^2 + 4*a*
c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b
^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x
)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(
b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 +
2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b
 - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4 + (
a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2
*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2*b)*
c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8
*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b^2 + 4*a
*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
+ 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))
*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*
cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cos
```

```
h(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) - 2*(c^2*cosh(x)^4 + 4*c^2*co
sh(x)*sinh(x)^3 + c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 - c^
2)*sinh(x)^2 + c^2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(a + b +
c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^
6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b -
 b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2
 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x
)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*
b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*s
inh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*
a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(
x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a
 + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
+ 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c
 - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2
+ a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*((a + b)
*c + c^2)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*c
osh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*
c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sin
h(x)^3 + sinh(x)^4)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3
)*cosh(x)*sinh(x)^3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2
*((a + b)*c^2 + c^3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^
3)*cosh(x)^2)*sinh(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 +
 c^3)*cosh(x))*sinh(x)), -1/8*(4*(c^2*cosh(x)^4 + 4*c^2*cosh(x)*sinh(x)^3 +
 c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 - c^2)*sinh(x)^2 + c^
2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*arctan(sqrt(2
)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*si
nh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)
^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(
-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 +
 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c
 + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3
 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2
*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2
 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
 + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b -
 b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2
)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(
a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 -
3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
```

```
+ 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) + ((a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a + b)*c - 2*c^2)*c
osh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^4 - 2*(a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a + b)*c - 2*c^2
)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 + a*b + b^2 - (2*a
 + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^3 - (a*b + b
^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(c)*log(((b^2 + 4*(a + 2*b)
*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 +
 (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^
6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sin
h(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*
c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 +
2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4*a*c - 8*c^2)*co
sh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a +
2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4
*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(
x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b^2 + 4*a*c - 8*c
^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^2 - b^2 - 4*a*
c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)
*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*c
osh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))
*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(
x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x
)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((
b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x)^5
+ (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b^2 + 4*a*c - 8*c^2)*cosh
(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2
 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(
35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 -
 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*co
sh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(
x)^3 - cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*((a + b)*c + c^2)*sqrt(((a + b +
c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(((
a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3)*cosh(x)*sinh(x)^3 + ((a
 + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2*((a + b)*c^2 + c^3)*cosh
(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^3)*cosh(x)^2)*sinh(x)^2 +
 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 + c^3)*cosh(x))*sinh(x)),
-1/4*(((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a +
b)*c - 2*c^2)*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)
^4 - 2*(a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a
 + b)*c - 2*c^2)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 + a
*b + b^2 - (2*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(
x)^3 - (a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(-c)*arctan(
1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c
)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*s
inh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*s
qrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cos
h(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)
/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(
x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*cosh(
x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*cosh(x)^6 + 4*(7
*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*((a + b)*c + c^2
)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2)*cosh(x)^2 + (3
*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 - 10*(a*c -
 c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a*c - c^2)*
```

```
cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^2)*cosh(x)^4 + 3
*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (a + b)*c + c^2 +
 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*c +
3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))) - (c^2*cosh(x)^4 + 4*c^2*
cosh(x)*sinh(x)^3 + c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 -
c^2)*sinh(x)^2 + c^2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(a + b
+ c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 2*sqrt(2)*((a +
b)*c + c^2)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c
^3)*cosh(x)*sinh(x)^3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 -
 2*((a + b)*c^2 + c^3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 +
c^3)*cosh(x)^2)*sinh(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2
 + c^3)*cosh(x))*sinh(x)), -1/4*(2*(c^2*cosh(x)^4 + 4*c^2*cosh(x)*sinh(x)^3
 + c^2*sinh(x)^4 - 2*c^2*cosh(x)^2 + 2*(3*c^2*cosh(x)^2 - c^2)*sinh(x)^2 +
c^2 + 4*(c^2*cosh(x)^3 - c^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*arctan(sqrt
(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*
sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(
x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqr
t(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a -
c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b
+ 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)
*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8
 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a
*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)
^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b
*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 +
 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b
^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)
*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b
 - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c
^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15
*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
```

```
 + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7
- 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) + ((a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^4 + 4*(a*b + b^2 - (2*a + b)*c - 2*c^2)
*cosh(x)*sinh(x)^3 + (a*b + b^2 - (2*a + b)*c - 2*c^2)*sinh(x)^4 - 2*(a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^2 + 2*(3*(a*b + b^2 - (2*a + b)*c - 2*c
^2)*cosh(x)^2 - a*b - b^2 + (2*a + b)*c + 2*c^2)*sinh(x)^2 + a*b + b^2 - (2
*a + b)*c - 2*c^2 + 4*((a*b + b^2 - (2*a + b)*c - 2*c^2)*cosh(x)^3 - (a*b +
 b^2 - (2*a + b)*c - 2*c^2)*cosh(x))*sinh(x))*sqrt(-c)*arctan(1/2*sqrt(2)*(
(b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 -
 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*
((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(
((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3
*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 -
 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x
)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7
+ ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c +
 c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 -
 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(
35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3
*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 - 10*(a*c - c^2)*cosh(x)
^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a*c - c^2)*cosh(x)^2 + 4
*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c
 + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*
c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x
)^3 - (a*c - c^2)*cosh(x))*sinh(x))) + 2*sqrt(2)*((a + b)*c + c^2)*sqrt(((a
 + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a
 + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*
cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4
)))/(((a + b)*c^2 + c^3)*cosh(x)^4 + 4*((a + b)*c^2 + c^3)*cosh(x)*sinh(x)^
3 + ((a + b)*c^2 + c^3)*sinh(x)^4 + (a + b)*c^2 + c^3 - 2*((a + b)*c^2 + c^
3)*cosh(x)^2 - 2*((a + b)*c^2 + c^3 - 3*((a + b)*c^2 + c^3)*cosh(x)^2)*sinh
(x)^2 + 4*(((a + b)*c^2 + c^3)*cosh(x)^3 - ((a + b)*c^2 + c^3)*cosh(x))*sin
h(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

maple [A] time = 0.22, size = 149, normalized size = 1.10

$$-\frac{\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\left(\coth^2(x)\right)}{\sqrt{c}}+\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}\right)}{4c^{\frac{3}{2}}}-\frac{\ln\left(\frac{\frac{b}{2}+c\left(\coth^2(x)\right)}{\sqrt{c}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] -1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)/c+1/4*b/c^(3/2)*ln((1/2*b+c*coth(x)^
2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))-1/2*ln((1/2*b+c*coth(x)^2)/c^
(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1/2)+1/2/(a+b+c)^(1/2)*arctanh(
```

```
1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x
)^4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)**5/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

3.206 $\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\,\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2
))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*cot
h(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3701, 1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\,\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[c]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{}{(1+x)\sqrt{a}}\right.$$
$$= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) + \text{Subst}\left(\int \frac{}{4a}\right.$$
$$= \frac{\tanh^{-1}\left(\frac{-b-2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth}}\right.}{2\sqrt{a+b+c}}$$

Mathematica [A] time = 26.60, size = 199, normalized size = 1.90

$$\frac{\text{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\left(\frac{\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}}\right)}{\sqrt{a+b+c}} - \frac{\text{tan}}{}\right.}{2\sqrt{\text{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b +
 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]/Sqrt[a + b + c] - ArcTanh[(2*c +
 (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)
*Sinh[x]^4])]/Sqrt[c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b +
c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a
+ b + c)*Cosh[4*x])*Csch[x]^4])
```

fricas [B] time = 1.42, size = 6695, normalized size = 63.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)

[Out] [1/4*((a + b + c)*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*
(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*
c^2)*sinh(x)^8 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b
)*c + 8*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a
+ 2*b)*c + 8*c^2)*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 +
2*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c
 + 8*c^2)*cosh(x)^4 - 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a -
 2*b)*c + 24*c^2)*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5
- 10*(b^2 + 4*a*c - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*c
osh(x))*sinh(x)^3 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a +
2*b)*c + 8*c^2)*cosh(x)^6 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 +
 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*s
qrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sin
h(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x
)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c
)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2
 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh
(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4)) + b^2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^
2)*cosh(x)^7 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c
 + 24*c^2)*cosh(x)^3 - (b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 +
 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x
)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^
2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))
*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4
*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) +
1)) + sqrt(a + b + c)*c*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh
(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b
+ b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(
x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*
(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a +
 b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*
c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c
^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a
*b + b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 +
(a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7
 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c
^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*co
sh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))
)/((a + b)*c + c^2), -1/4*(2*sqrt(-a - b - c)*c*arctan(sqrt(2)*((a + b + c)
*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a
 - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b
 + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
```

```
h(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)
*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x
)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a
+ b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 +
4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*
c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x
)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b -
 b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(
x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(c)*log
(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*c^2)
*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 + 4*a
*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 - b^2
- 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^3 -
 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*c +
24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*(b^2
 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)^4 +
 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2)*co
sh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^2 +
4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6 - 1
5*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*co
sh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)^4 +
 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^
2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3
- (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(x)^4
 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2
 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) +
 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a + 2
*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4*a*c
 - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b^2 +
 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x
)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh
(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^
4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 -
 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3
*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)))/((a + b)*c + c^2), 1/4*(
2*(a + b + c)*sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c
)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b
 + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)
*cosh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b
+ c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2
*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)
^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)
^8 + 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 -
4*(a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sin
h(x)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^
5 + 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 -
 30*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*
c + c^2)*cosh(x)^5 - 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(
x))*sinh(x)^3 - 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6
- 15*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)
*sinh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^
```

```
2)*cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(
x))) + sqrt(a + b + c)*c*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(
x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cos
h(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b
 + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh
(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2
*(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a
+ b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*co
sh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)
*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c -
c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 -
a*b + b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)
*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a
+ b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*
cosh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*
c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
))/((a + b)*c + c^2), -1/2*(sqrt(-a - b - c)*c*arctan(sqrt(2)*((a + b + c)*
cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a
- c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b
+ c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqr
t(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*
(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4
 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh
(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)
^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
- b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4
- 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)
^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a +
 b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c
 - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)
^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c
^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b -
b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x
)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(-c)*arc
tan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b +
 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*
c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*
c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b +
3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*s
inh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b)*c + c^2)*c
osh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*cosh(x)^6 +
4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*((a + b)*c +
 c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a - b)*c + 3*c^2
```

```
)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2)*cosh(x)^2
+ (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(x)^5 - 10*(a
*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a*c - c
^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^2)*cosh(x)^4
 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (a + b)*c + c
^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 + ((3*a - b)*
c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))))/((a + b)*c + c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

maple [A] time = 0.20, size = 90, normalized size = 0.86

$$-\frac{\ln\left(\frac{\frac{b}{2}+c\left(\coth^2(x)\right)}{\sqrt{c}}+\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b\left(\coth^2(x)\right)+2c\left(\coth^2(x)\right)+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}\right)}{2\sqrt{a+b+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] -1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1
/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)
^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{coth}\left(x\right)}^3}{\sqrt{c\,{\mathrm{coth}\left(x\right)}^4+b\,{\mathrm{coth}\left(x\right)}^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}{\left(x \right)}}{\sqrt{a + b \coth^{2}{\left(x \right)} + c \coth^{4}{\left(x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

3.207 $\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] 1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*co
th(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3701, 1247, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)$$
$$= \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Mathematica [B] time = 18.60, size = 141, normalized size = 2.43

$$\frac{\text{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\,\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)+(b+2c)\sinh^2(x)+c}}\right)}{2\sqrt{a+b+c}\sqrt{\text{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[c + (b +
2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*Sqrt[3*a - b + 3*c - 4*(a - c)*Co
sh[2*x] + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/(2*Sqrt[a + b + c]*Sqrt[(3*a -
b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4])
```

fricas [B] time = 1.10, size = 1752, normalized size = 30.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
```

```
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sqrt(a + b + c), -
1/2*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^
2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(
7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
 c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*si
nh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c
^2)*cosh(x))*sinh(x)))/(a + b + c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.16, size = 52, normalized size = 0.90

$$\frac{\operatorname{arctanh}\left(\frac{b\left(\coth^2(x)\right)+2c\left(\coth^2(x)\right)+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}\right)}{2\sqrt{a+b+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] 1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/
2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

3.208 $\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=106

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2
))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*cot
h(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3701, 1251, 960, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[a]) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a-bx+cx^2}} + \frac{1}{x\sqrt{a-bx+cx^2}}\right) dx, x, -\coth^2(x\right.$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{}{x\sqrt{a}}\right.$$

$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) - \text{Subst}\left(\int \frac{}{4a}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} - \frac{\tanh^{-1}\left(\frac{-2a-b+(-b-2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(}}\right.}{2\sqrt{a+b+c}}$$

Mathematica [A] time = 7.74, size = 203, normalized size = 1.92

$$-\frac{\operatorname{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\left(-\frac{\tanh^{-1}\left(\frac{2a-(2a+b)\cosh^2(x)}{2\sqrt{a}\sqrt{\cosh^4(x)(a+b+c)-(2a+b)\cosh^2(x)+a}}\right)}{\sqrt{a}} - \frac{\tanh^{-}}{}\right.}{2\sqrt{\operatorname{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*((-(ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*Sqrt[a - (2*a + b)*
Cosh[x]^2 + (a + b + c)*Cosh[x]^4])]/Sqrt[a]) - ArcTanh[(-a + c + (a + b +
c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)
*Cosh[x]^4])]/Sqrt[a + b + c])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (
a + b + c)*Cosh[4*x]]*Csch[x]^2)/Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x]
+ (a + b + c)*Cosh[4*x])*Csch[x]^4]
```

fricas [B] time = 1.39, size = 6705, normalized size = 63.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b + c)*sqrt(a)*log(((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*
```

```
a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a
*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 +
2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2
+ 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b +
3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 -
10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh
(x))*sinh(x)^3 - 4*(8*a^2 - b^2 - 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b +
b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8
*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 - 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 - 4*sqrt(
2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)
^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2
+ 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sq
rt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2
*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^
4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sin
h(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*c
osh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12
*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*co
sh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 +
 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3
)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh
(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh
(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) +
 sqrt(a + b + c)*a*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 +
 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6
 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c
 + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3
- 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a +
b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^
5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*c
osh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b +
b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(
x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x
))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a +
b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*
(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*c
osh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(a^
2 + a*b + a*c), -1/4*(2*a*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh
(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)
*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)
*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((
a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(
a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 +
```

```
8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^
2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30
*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c +
3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 -
 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*
c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c
^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 -
 a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 +
 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c
- c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3
- (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))) - (a + b + c)*sqrt(a)*log(((8*
a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh
(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 -
4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b
^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8
*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c
)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b
^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(
8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3
 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 -
4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2
 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 -
 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 - 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a +
 b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*
(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a -
b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b
 + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4
*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)
*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4
*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7
*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(
x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*c
osh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x
)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)))/(a^2 + a*b + a*c), 1/4*(2*sqrt(-a)
*(a + b + c)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*
sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*co
sh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*
sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(
x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x
)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 + 8*(a^
2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 - 4*(a^2 - a
*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^6 + 8
*(7*(a^2 + a*b + a*c)*cosh(x)^3 - 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2*(3*a
^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 - 30*(a^2 -
 a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*c
osh(x)^5 - 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*sinh(x
)^3 - 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 - 15*(a^2
- a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^2
 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^7 - 3*(a^2 - a*c)*cosh(x)
^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 - (a^2 - a*c)*cosh(x))*sinh(x))) + sqr
t(a + b + c)*a*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4
```

```
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c
^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*
(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)
^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3
*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 -
10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2
)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*
a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(
x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c
+ c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*s
inh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*c
osh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*s
inh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c
)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2
*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2
 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(
x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(a^2 +
a*b + a*c), 1/2*(sqrt(-a)*(a + b + c)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)
^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh
(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x
)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh
(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh
(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh
(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b
 + a*c)*cosh(x)^8 + 8*(a^2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*
c)*sinh(x)^8 - 4*(a^2 - a*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 -
 a^2 + a*c)*sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^3 - 3*(a^2 - a*c)*co
sh(x))*sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a
*c)*cosh(x)^4 - 30*(a^2 - a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 +
 8*(7*(a^2 + a*b + a*c)*cosh(x)^5 - 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b
 + 3*a*c)*cosh(x))*sinh(x)^3 - 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b +
a*c)*cosh(x)^6 - 15*(a^2 - a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)
^2 - a^2 + a*c)*sinh(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^
7 - 3*(a^2 - a*c)*cosh(x)^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 - (a^2 - a*c)
*cosh(x))*sinh(x))) - a*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x
)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*c
osh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*c
osh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a
+ b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a
+ b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*c
osh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
)/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c
- c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)
*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(
a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*
c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 1
0*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c
+ 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2
)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a
^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8
```

```
*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c -
c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 -
(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))))/(a^2 + a*b + a*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

3.209 $\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal. Leaf size=183

$$\frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}+\frac{\tanh^{-1}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] 1/4*b*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/
2))/a^(3/2)-1/2*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth
(x)^4)^(1/2))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/
2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*co
th(x)^4)^(1/2)*tanh(x)^2/a
```

Rubi [A] time = 0.33, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3701, 1251, 960, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}}-\frac{\tanh^2(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2a}-\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]
)]/(2*Sqrt[a]) + (b*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b +
c]) - (Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2)/(2*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x^3\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{x^2\sqrt{a-bx+cx^2}} - \frac{1}{x\sqrt{a-bx+cx^2}} + \frac{1}{(1+x)\sqrt{a-bx+}\right.\right.\right.$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{}{x\sqrt{a-}}\right.$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,\tanh^2(x)}{2a} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\right.}{4a}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{\tanh^{-1}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(}}\right.}{2\sqrt{a+b+c}}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{b\tanh^{-1}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right.}{4a^{3/2}}$$

Mathematica [A] time = 11.08, size = 278, normalized size = 1.52

$$\frac{\operatorname{csch}^2(x)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\left(2a^{3/2}\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c)-a+c}{2\sqrt{a+b+c}\sqrt{\cosh^4(x)(a+b+c)-(2a+b)\cosh^2}}\right.\right.}{4a^{3/2}\sqrt{a+b+c}\sqrt{\operatorname{csch}^4(x)(\cosh(4x)(a+b+c)-4(a-c)\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (((2*a - b)*Sqrt[a + b + c]*ArcTanh[(2*a - (2*a + b)*Cosh[x]^2)/(2*Sqrt[a]*
Sqrt[a - (2*a + b)*Cosh[x]^2 + (a + b + c)*Cosh[x]^4])] + 2*a^(3/2)*ArcTanh
```

```
[(-a + c + (a + b + c)*Cosh[2*x])/(2*Sqrt[a + b + c]*Sqrt[a - (2*a + b)*Cos
h[x]^2 + (a + b + c)*Cosh[x]^4])])*Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x]
 + (a + b + c)*Cosh[4*x]]*Csch[x]^2)/(4*a^(3/2)*Sqrt[a + b + c]*Sqrt[(3*a -
 b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4]) - (Sqrt
[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^4]*T
anh[x]^2)/(4*Sqrt[2]*a)
```

fricas [B] time = 1.92, size = 9148, normalized size = 49.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2
+ (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x
)^4 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b
^2 + (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 +
2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh
(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8
*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cos
h(x)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 +
b^2 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(
8*a^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*
c)*cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 -
b^2 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*
(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^
3 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 -
 4*a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^
2 - b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2
- 8*a^2 + b^2 + 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a
+ b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3
*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a -
 b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a +
b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a +
 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 +
4*a*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c
)*cosh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 -
4*a*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(
7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh
(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*
cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(
x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^
5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) - 2*(a^2*cosh(x)^4 + 4*a^2*cosh(x)
*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)*si
nh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(a + b + c)*lo
g(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8
*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c
- c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)
^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*
b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 +
 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b
*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x
)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a
```

```
+ b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 +
 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2
+ sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b
 + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)
*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b +
c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*
(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a
 - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*(
(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^
2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 4*sqrt(2)*(a^2 + a*b +
a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x
)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(c
osh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^
3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)
*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sinh(x)^4 + a^3 + a^2*b + a^2*c
+ 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 + a^2*b + a^2*c + 3*(a^3 + a^2
*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)*cosh(x)^3 + (a^
3 + a^2*b + a^2*c)*cosh(x))*sinh(x)), -1/8*(4*(a^2*cosh(x)^4 + 4*a^2*cosh(x
)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)*s
inh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(-a - b - c)*
arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a
 + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^
4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
- 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 +
 (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c
^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^
2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*
a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2
+ 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*
b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 + 2*
(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 + 2
*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))
) + ((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (
2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^4
 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2
+ (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 + 2*a
^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)
^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(a)*log(((8*a^
2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^8 + 8*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x
)*sinh(x)^7 + (8*a^2 + 8*a*b + b^2 + 4*a*c)*sinh(x)^8 - 4*(8*a^2 - b^2 - 4*
a*c)*cosh(x)^6 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^2 - 8*a^2 + b^2
 + 4*a*c)*sinh(x)^6 + 8*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^3 - 3*(8*a
^2 - b^2 - 4*a*c)*cosh(x))*sinh(x)^5 + 2*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*
cosh(x)^4 + 2*(35*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^4 - 30*(8*a^2 - b^2
 - 4*a*c)*cosh(x)^2 + 24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*sinh(x)^4 + 8*(7*(8*
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a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^5 - 10*(8*a^2 - b^2 - 4*a*c)*cosh(x)^3 +
 (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x))*sinh(x)^3 - 4*(8*a^2 - b^2 - 4*
a*c)*cosh(x)^2 + 4*(7*(8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^6 - 15*(8*a^2 -
 b^2 - 4*a*c)*cosh(x)^4 + 3*(24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^2 - 8
*a^2 + b^2 + 4*a*c)*sinh(x)^2 + 4*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b
)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2
*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)
*cosh(x))*sinh(x) + 2*a + b)*sqrt(a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
 c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*
c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + 8*a^2 + 8*a*b + b^2 + 4*a
*c + 8*((8*a^2 + 8*a*b + b^2 + 4*a*c)*cosh(x)^7 - 3*(8*a^2 - b^2 - 4*a*c)*c
osh(x)^5 + (24*a^2 - 8*a*b + 3*b^2 + 12*a*c)*cosh(x)^3 - (8*a^2 - b^2 - 4*a
*c)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*c
osh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)
^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cos
h(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^
4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 +
 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((
a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(
a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4
*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^
4)))/((a^3 + a^2*b + a^2*c)*cosh(x)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sin
h(x)^3 + (a^3 + a^2*b + a^2*c)*sinh(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a
^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 + a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*
cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b + a^2*c)*cosh(x)^3 + (a^3 + a^2*b +
a^2*c)*cosh(x))*sinh(x)), 1/4*(((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4
 + 4*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b
^2 + (2*a - b)*c)*sinh(x)^4 + 2*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2
 + 2*(3*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (
2*a - b)*c)*sinh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b -
 b^2 + (2*a - b)*c)*cosh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*
sinh(x))*sqrt(-a)*arctan(1/2*sqrt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cos
h(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a +
b)*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh
(x))*sinh(x) + 2*a + b)*sqrt(-a)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*
sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*s
inh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*si
nh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 +
8*(a^2 + a*b + a*c)*cosh(x)*sinh(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 - 4*(a^
2 - a*c)*cosh(x)^6 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^
6 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^3 - 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2
*(3*a^2 - a*b + 3*a*c)*cosh(x)^4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 - 30*(
a^2 - a*c)*cosh(x)^2 + 3*a^2 - a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a
*c)*cosh(x)^5 - 10*(a^2 - a*c)*cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*s
inh(x)^3 - 4*(a^2 - a*c)*cosh(x)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 - 15*
(a^2 - a*c)*cosh(x)^4 + 3*(3*a^2 - a*b + 3*a*c)*cosh(x)^2 - a^2 + a*c)*sinh
(x)^2 + a^2 + a*b + a*c + 8*((a^2 + a*b + a*c)*cosh(x)^7 - 3*(a^2 - a*c)*co
sh(x)^5 + (3*a^2 - a*b + 3*a*c)*cosh(x)^3 - (a^2 - a*c)*cosh(x))*sinh(x)))
+ (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^
2 + 2*(3*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh
(x))*sinh(x))*sqrt(a + b + c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*
cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2
)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2
- a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*
b + 2*(a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
 + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b +
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2*(a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a
 + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 +
 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b
*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a
^2 - a*b + b*c + c^2)*sinh(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b
 + c)*cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(
3*(a + b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a
- c)*cosh(x))*sinh(x) + a + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x
)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x
)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x
) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cos
h(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c
 + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4
+ 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4)) - 2*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b +
 c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*
c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cos
h(x)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*
sinh(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^
3 + a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3
+ a^2*b + a^2*c)*cosh(x)^3 + (a^3 + a^2*b + a^2*c)*cosh(x))*sinh(x)), 1/4*(
((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2 + (2*a
- b)*c)*cosh(x)*sinh(x)^3 + (2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^4 + 2
*(2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2 + (2
*a - b)*c)*cosh(x)^2 + 2*a^2 + a*b - b^2 + (2*a - b)*c)*sinh(x)^2 + 2*a^2 +
 a*b - b^2 + (2*a - b)*c + 4*((2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x)^3 +
 (2*a^2 + a*b - b^2 + (2*a - b)*c)*cosh(x))*sinh(x))*sqrt(-a)*arctan(1/2*sq
rt(2)*((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh
(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 - 2*a + b)*sinh(x)
^2 + 4*((2*a + b)*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 2*a + b)*sqrt(-a
)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2
 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh
(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
 sinh(x)^4))/((a^2 + a*b + a*c)*cosh(x)^8 + 8*(a^2 + a*b + a*c)*cosh(x)*sin
h(x)^7 + (a^2 + a*b + a*c)*sinh(x)^8 - 4*(a^2 - a*c)*cosh(x)^6 + 4*(7*(a^2
+ a*b + a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^6 + 8*(7*(a^2 + a*b + a*c)*cosh
(x)^3 - 3*(a^2 - a*c)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - a*b + 3*a*c)*cosh(x)^
4 + 2*(35*(a^2 + a*b + a*c)*cosh(x)^4 - 30*(a^2 - a*c)*cosh(x)^2 + 3*a^2 -
a*b + 3*a*c)*sinh(x)^4 + 8*(7*(a^2 + a*b + a*c)*cosh(x)^5 - 10*(a^2 - a*c)*
cosh(x)^3 + (3*a^2 - a*b + 3*a*c)*cosh(x))*sinh(x)^3 - 4*(a^2 - a*c)*cosh(x
)^2 + 4*(7*(a^2 + a*b + a*c)*cosh(x)^6 - 15*(a^2 - a*c)*cosh(x)^4 + 3*(3*a^
2 - a*b + 3*a*c)*cosh(x)^2 - a^2 + a*c)*sinh(x)^2 + a^2 + a*b + a*c + 8*((a
^2 + a*b + a*c)*cosh(x)^7 - 3*(a^2 - a*c)*cosh(x)^5 + (3*a^2 - a*b + 3*a*c)
*cosh(x)^3 - (a^2 - a*c)*cosh(x))*sinh(x))) - 2*(a^2*cosh(x)^4 + 4*a^2*cosh
(x)*sinh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)
*sinh(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(-a - b - c
)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x
)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)
^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^
2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c
^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7
 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c -
 c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 -
```

```
a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 +
2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 +
 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^
2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(x)^3 + (
3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4*(a^2 +
a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*c
osh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b - b^2 +
2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b - b^2 +
 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x
))) - 2*sqrt(2)*(a^2 + a*b + a*c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)
*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*
sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^3 + a^2*b + a^2*c)*cosh(x
)^4 + 4*(a^3 + a^2*b + a^2*c)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b + a^2*c)*sin
h(x)^4 + a^3 + a^2*b + a^2*c + 2*(a^3 + a^2*b + a^2*c)*cosh(x)^2 + 2*(a^3 +
 a^2*b + a^2*c + 3*(a^3 + a^2*b + a^2*c)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a
^2*b + a^2*c)*cosh(x)^3 + (a^3 + a^2*b + a^2*c)*cosh(x))*sinh(x))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{3}(x)}{\sqrt{a+b\left(\coth^{2}(x)\right)+c\left(\coth^{4}(x)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^{3}}{\sqrt{c\coth(x)^{4}+b\coth(x)^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(tanh(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^{3}}{\sqrt{c\coth(x)^{4}+b\coth(x)^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(tanh(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

3.210 $\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{2}{2\sqrt{a+b}}\right.$$

```
[Out] -1/4*(b+2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)
^4)^(1/2))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/
(a+b*coth(x)^2+c*coth(x)^4)^(1/2))*(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*coth(
x)^4)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3701, 1247, 734, 843, 621, 206, 724}

$$-\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}-\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}+\frac{1}{2}\sqrt{a+b+c}\tanh^{-1}\left(\frac{2}{2\sqrt{a+b}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -((b + 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c
*Coth[x]^4])])/(4*Sqrt[c]) + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])])/2 - Sq
rt[a + b*Coth[x]^2 + c*Coth[x]^4]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
```

```
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3701

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> -Dist[f/e, Subst[Int[((x/f)^m*(a + b*x^n + c*x^(2*n))^p)/(f^2 + x^2), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = -\text{Subst}\left(\int \frac{x\sqrt{a-bx^2+cx^4}}{1+x^2}dx, x, -i\coth(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a-bx+cx^2}}{1+x}dx, x, -\coth^2(x)\right)\right)$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{-2a-b+(b+2c}{(1+x)\sqrt{a-bx+c}}\right.$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)} + \frac{1}{2}(-a-b-c)\,\text{Subst}\left(\int \frac{}{(1+x)}\right.$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)} + (a+b+c)\,\text{Subst}\left(\int \frac{1}{4a+4b+}\right.$$

$$= -\frac{(b+2c)\tanh^{-1}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}} + \frac{1}{2}\sqrt{a+b+c}\,\tanh^{-}$$

Mathematica [B] time = 8.42, size = 304, normalized size = 2.30

$$\frac{\text{csch}^2(x)\left(4\sqrt{c}\,(a+b+c)\sqrt{\cosh(4x)(a+b+c)-4(a-c)\cosh(2x)+3a-b+3c}\,\tanh^{-1}\left(\frac{\cosh(2x)(a+b+c}{2\sqrt{a+b+c}\sqrt{\sinh^4(x)(a+b+c)}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (Csch[x]^2*(4*Sqrt[c]*(a + b + c)*ArcTanh[(-a + c + (a + b + c)*Cosh[2*x])/
(2*Sqrt[a + b + c]*Sqrt[c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*
Sqrt[3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] + Sqrt[a
+ b + c]*(-2*(b + 2*c)*ArcTanh[(2*c + (b + 2*c)*Sinh[x]^2)/(2*Sqrt[c]*Sqrt[
c + (b + 2*c)*Sinh[x]^2 + (a + b + c)*Sinh[x]^4])]*Sqrt[3*a - b + 3*c - 4*(
a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x]] - Sqrt[2]*Sqrt[c]*(3*a - b + 3*c
- 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*x])*Csch[x]^2)))/(8*Sqrt[c]*Sqrt
[a + b + c]*Sqrt[(3*a - b + 3*c - 4*(a - c)*Cosh[2*x] + (a + b + c)*Cosh[4*
x])*Csch[x]^4])
```

fricas [B] time = 2.31, size = 7964, normalized size = 60.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh
(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b - 2*c)*sinh(x)
^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)
*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b)*c + 8*
c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 - 4*(b^2 +
 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^2 -
b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)
^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a - 2*b)*
c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^4 - 30*
(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*sinh(x)
^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c - 8*c^2
)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3 - 4*(b^
2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^6
 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c + 24*c^2
)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)*cosh(x)
^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh
(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x
)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b + c)*cosh(
x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(
x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(
x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^2 + 4*(a
 + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*(b^2 + 4
*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x)^3 - (b
^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + si
nh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*
cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh
(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)
^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7
 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*(c*cosh(x)^4 + 4*
c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cosh(x)^2 - c)*s
inh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(a + b + c)*log(((a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2
)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^4 +
2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b
*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 -
 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*
c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*
```

```
b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2 + sqr
t(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (a + b + c)
*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a + c)*sinh
(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a + b + c)*sq
rt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a -
c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b
+ 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)
*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2
+ 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*co
sh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*
c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*s
inh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 4*sqrt(2)*c*sqrt(((a + b + c
)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c
)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(c*c
osh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cos
h(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c), -1/8*(4*(
c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*
cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(-a
 - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sin
h(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cos
h(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sin
h(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c
)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)
*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*
sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*s
inh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b
 - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh
(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a +
 b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(
3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b
 + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2)*cosh(x)^
2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7*(a^2 + 2*
a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c - c^2)*cosh(
x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sinh(x)^3 - 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b
- b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^2
 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b
)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b
 - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^2)*cosh(x)
)*sinh(x))) - ((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2
*c)*sinh(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b - 2*c)
*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)
*sqrt(c)*log(((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^8 + 8*(b^2 + 4*(a + 2*b
)*c + 8*c^2)*cosh(x)*sinh(x)^7 + (b^2 + 4*(a + 2*b)*c + 8*c^2)*sinh(x)^8 -
4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^6 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh
(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^6 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)
*cosh(x)^3 - 3*(b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x)^5 + 2*(3*b^2 + 4*(3*a
 - 2*b)*c + 24*c^2)*cosh(x)^4 + 2*(35*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)
^4 - 30*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)
*sinh(x)^4 + 8*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^5 - 10*(b^2 + 4*a*c
 - 8*c^2)*cosh(x)^3 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x))*sinh(x)^3
 - 4*(b^2 + 4*a*c - 8*c^2)*cosh(x)^2 + 4*(7*(b^2 + 4*(a + 2*b)*c + 8*c^2)*c
osh(x)^6 - 15*(b^2 + 4*a*c - 8*c^2)*cosh(x)^4 + 3*(3*b^2 + 4*(3*a - 2*b)*c
+ 24*c^2)*cosh(x)^2 - b^2 - 4*a*c + 8*c^2)*sinh(x)^2 - 4*sqrt(2)*((b + 2*c)
*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2
*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c
```

```
)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh(x) + b + 2*c)*sqrt(c)*sqrt(((a + b +
c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b +
c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)
^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + b^
2 + 4*(a + 2*b)*c + 8*c^2 + 8*((b^2 + 4*(a + 2*b)*c + 8*c^2)*cosh(x)^7 - 3*
(b^2 + 4*a*c - 8*c^2)*cosh(x)^5 + (3*b^2 + 4*(3*a - 2*b)*c + 24*c^2)*cosh(x
)^3 - (b^2 + 4*a*c - 8*c^2)*cosh(x))*sinh(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x
)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x
)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4
+ 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7
*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(c
osh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 4*sqrt(2)*c
*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2
+ 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(
x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 +
sinh(x)^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(
x)^2 + 2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x
) + c), 1/4*(((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^3 + (b + 2*
c)*sinh(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2 - b - 2*c)*
sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x) + b + 2*c)*
sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh
(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x
)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*cosh(x))*sinh
(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8 + 8*((a + b
)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(a*c - c^2)*
cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^6 + 8*(7*
((a + b)*c + c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 + 2*((3*a -
b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30*(a*c - c^2
)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c + c^2)*cosh(
x)^5 - 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))*sinh(x)^3
- 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 15*(a*c - c^
2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*sinh(x)^2 + (
a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*cosh(x)^5 +
 ((3*a - b)*c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))) + (c*cosh
(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 + 2*(3*c*cosh(x
)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c)*sqrt(a + b +
c)*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b
+ b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a +
b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2
*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^
6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b -
 b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*co
sh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2
 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh(x
)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*
b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))*s
inh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 +
2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*
a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(
x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 + (
a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - a
 + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a +
 b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4
 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2
+ 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 -
 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
```

```
+ 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c
 - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^2
+ a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*c
osh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) - 2*sqrt(2)*c*sqrt((
(a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*
(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 -
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)
^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2 +
2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) + c),
 -1/4*(2*(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x)^4 - 2*c*cosh(x)^2
 + 2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*cosh(x))*sinh(x) +
c)*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*c
osh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*co
sh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
(a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*
cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)
*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4*
(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c +
 c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh(
x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(a
^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^2
)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(7
*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*sin
h(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2*
(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a^
2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)
*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c^
2)*cosh(x))*sinh(x))) - ((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*cosh(x)*sinh(x)^
3 + (b + 2*c)*sinh(x)^4 - 2*(b + 2*c)*cosh(x)^2 + 2*(3*(b + 2*c)*cosh(x)^2
- b - 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b + 2*c)*cosh(x))*sinh(x)
+ b + 2*c)*sqrt(-c)*arctan(1/2*sqrt(2)*((b + 2*c)*cosh(x)^4 + 4*(b + 2*c)*c
osh(x)*sinh(x)^3 + (b + 2*c)*sinh(x)^4 - 2*(b - 2*c)*cosh(x)^2 + 2*(3*(b +
2*c)*cosh(x)^2 - b + 2*c)*sinh(x)^2 + 4*((b + 2*c)*cosh(x)^3 - (b - 2*c)*co
sh(x))*sinh(x) + b + 2*c)*sqrt(-c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c
)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)
*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*
sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/(((a + b)*c + c^2)*cosh(x)^8
+ 8*((a + b)*c + c^2)*cosh(x)*sinh(x)^7 + ((a + b)*c + c^2)*sinh(x)^8 - 4*(
a*c - c^2)*cosh(x)^6 + 4*(7*((a + b)*c + c^2)*cosh(x)^2 - a*c + c^2)*sinh(x
)^6 + 8*(7*((a + b)*c + c^2)*cosh(x)^3 - 3*(a*c - c^2)*cosh(x))*sinh(x)^5 +
 2*((3*a - b)*c + 3*c^2)*cosh(x)^4 + 2*(35*((a + b)*c + c^2)*cosh(x)^4 - 30
*(a*c - c^2)*cosh(x)^2 + (3*a - b)*c + 3*c^2)*sinh(x)^4 + 8*(7*((a + b)*c +
 c^2)*cosh(x)^5 - 10*(a*c - c^2)*cosh(x)^3 + ((3*a - b)*c + 3*c^2)*cosh(x))
*sinh(x)^3 - 4*(a*c - c^2)*cosh(x)^2 + 4*(7*((a + b)*c + c^2)*cosh(x)^6 - 1
5*(a*c - c^2)*cosh(x)^4 + 3*((3*a - b)*c + 3*c^2)*cosh(x)^2 - a*c + c^2)*si
nh(x)^2 + (a + b)*c + c^2 + 8*(((a + b)*c + c^2)*cosh(x)^7 - 3*(a*c - c^2)*
cosh(x)^5 + ((3*a - b)*c + 3*c^2)*cosh(x)^3 - (a*c - c^2)*cosh(x))*sinh(x))
) + 2*sqrt(2)*c*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)^4 - 4*(a
- c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^2 + 3*a -
b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(
x)*sinh(x)^3 + sinh(x)^4)))/(c*cosh(x)^4 + 4*c*cosh(x)*sinh(x)^3 + c*sinh(x
)^4 - 2*c*cosh(x)^2 + 2*(3*c*cosh(x)^2 - c)*sinh(x)^2 + 4*(c*cosh(x)^3 - c*
```

```
cosh(x))*sinh(x) + c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\coth(x)^4 + b\coth(x)^2 + a}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

maple [C] time = 0.15, size = 559, normalized size = 4.23

$$-\frac{\sqrt{a+b\left(\coth^2(x)\right)+c\left(\coth^4(x)\right)}}{2}-\frac{(b+c)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)\left(\coth^2(x)\right)}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\left(\coth^2(x)\right)}{a}}\ \mathrm{Ell}}{8\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{a+b\left(\coth^2(x)\right)+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
[Out] -1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)-1/8*(b+c)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2)*Ellip
ticF(1/2*coth(x)*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*ln((b+2*c*coth(x)^2)/c^(1/2)+2*(a+b*coth
(x)^2+c*coth(x)^4)^(1/2))*c^(1/2)-1/4*ln((b+2*c*coth(x)^2)/c^(1/2)+2*(a+b*c
oth(x)^2+c*coth(x)^4)^(1/2))/c^(1/2)*b+1/2*a/(a+b+c)^(1/2)*arctanh(1/2*(b*c
oth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/
2))+1/2*b/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+
c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))+1/2*c/(a+b+c)^(1/2)*arctanh(1/2
*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x)^4
)^(1/2))-1/8*(-b-c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*coth(x)^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*coth(x)^2)
^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2)*EllipticF(1/2*coth(x)*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\coth(x)^4 + b\coth(x)^2 + a}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)\sqrt{c\,{\coth(x)}^4 + b\,{\coth(x)}^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\coth^2(x) + c\coth^4(x)}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*coth(x)**2 + c*coth(x)**4)*coth(x), x)
```

3.211 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=319

$$-\frac{15\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{25e^{c(a+bx)}}{}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-4*exp(c*(b*x+a
))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4+26/
3*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(
b*x+a)))^3-55/6*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/
c/(1-exp(2*c*(b*x+a)))^2+25/4*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh
(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))-15/4*arctanh(exp(c*(b*x+a)))*(coth(b*c
*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Rubi [A] time = 0.91, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6720, 2282, 390, 1814, 1157, 385, 207}

$$-\frac{15\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{25e^{c(a+bx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (4*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c +
b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Sqrt[Coth[a
*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(
2*c*(a + b*x)))) - (15*ArcTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*T
anh[a*c + b*c*x])/(4*b*c)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\text{Subst}\left(\int\frac{(1+x^2)^5}{(-1+x^2)^5}\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\text{Subst}\left(\int\left(1+\frac{2(1+10x^4+5x^8)}{(-1+x^2)^5}\right)dx,x,e^{c(a+\cdots}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+\cdots}{\cdots}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh\cdots}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh\cdots}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh\cdots}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh\cdots}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh\cdots}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

Mathematica [A] time = 10.28, size = 164, normalized size = 0.51

$$\frac{\left(66e^{c(a+bx)} - 314e^{3c(a+bx)} + 374e^{5c(a+bx)} - 246e^{7c(a+bx)} + 24e^{9c(a+bx)} + 45\left(e^{2c(a+bx)}-1\right)^4\log\left(1-e^{c(a+bx)}\right) - 45\left(\cdots\right.}{24bc\left(e^{2c(a+bx)}-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[Coth[c*(a + b*x)]^2]*(66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 37
4*E^(5*c*(a + b*x)) - 246*E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(-1
 + E^(2*c*(a + b*x)))^4*Log[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a + b*x
)))^4*Log[1 + E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(24*b*c*(-1 + E^(2*c*(a
+ b*x)))^4)
```

fricas [B] time = 0.44, size = 1617, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*c)
^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*c*x
+ a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(1512*cos
h(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)^4 + 18
70*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x + a*c)^3
 + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x
+ a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*
c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7
*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(
7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*co
sh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cos
h(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b
*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c
*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x
 + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c)
 + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*si
nh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(
b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh
(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x
 + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x
+ a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c
))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) +
2*(108*cosh(b*c*x + a*c)^8 - 861*cosh(b*c*x + a*c)^6 + 935*cosh(b*c*x + a*c
)^4 - 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 66*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 +
 b*c*sinh(b*c*x + a*c)^8 - 4*b*c*cosh(b*c*x + a*c)^6 + 4*(7*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)^4 + 8*(7*b*c*
cosh(b*c*x + a*c)^3 - 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*
b*c*cosh(b*c*x + a*c)^4 - 30*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x +
a*c)^4 - 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x + a*c)^5 - 10*b*c*
cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*b
*c*cosh(b*c*x + a*c)^6 - 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*c*cosh(b*c*x + a*
c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x + a*c)^7 - 3*b*c*
cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c))
```

giac [A] time = 1.07, size = 181, normalized size = 0.57

$$\frac{\frac{24\,e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{45\,\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}+\frac{45\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{2\left(75\,e^{(7\,bcx+7\,ac)}-115\,e^{(5\,bcx+5\,ac)}+109\,e^{(3\,bcx+3\,ac)}-21\,e^{(bcx+ac)}\right)}{\left(e^{(2\,bcx+2\,ac)}-1\right)^{4}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{24\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 45*log(e^(b*c*x + a
*c) + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 45*log(abs(e^(b*c*x + a*c) - 1))/sg
n(e^(2*b*c*x + 2*a*c) - 1) - 2*(75*e^(7*b*c*x + 7*a*c) - 115*e^(5*b*c*x + 5
*a*c) + 109*e^(3*b*c*x + 3*a*c) - 21*e^(b*c*x + a*c))/((e^(2*b*c*x + 2*a*c)
 - 1)^4*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

maple [A] time = 0.98, size = 320, normalized size = 1.00

$$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^{2}}{\left(e^{2c(bx+a)}-1\right)^{2}}}\, e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}-\frac{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^{2}}{\left(e^{2c(bx+a)}-1\right)^{2}}}\, e^{c(bx+a)}\left(75\,e^{6c(bx+a)}-115\,e^{4c(bx+a)}+109\,e^{2c(bx+a)}-21\right)}{12\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)^{3}cb}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/c/b-1/12/(1+exp(2*c*(b*x+a)))/(exp(2*
c*(b*x+a))-1)^3*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))-115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))-21
)/c/b+15/8/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^
2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-1)-15/8/(1+exp(2*c*(b
*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2
)^(1/2)/c/b*ln(1+exp(c*(b*x+a)))
```

maxima [A] time = 0.43, size = 167, normalized size = 0.52

$$-\frac{15\,\log\left(e^{(bcx+ac)}+1\right)}{8\,bc}+\frac{15\,\log\left(e^{(bcx+ac)}-1\right)}{8\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}-123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}-157\,e^{(3\,bcx+3\,ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
+ 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x +
 5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x +
8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a
*c) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int e^{c\,(a+b\,x)}\,\left(\mathrm{coth}\left(a\,c+b\,c\,x\right)^{2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.212 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=197

$$-\frac{3\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{3e^{c(a+bx)}\tanh(}{b}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*exp(c*(b*x+a
))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2+3*e
xp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x
+a)))-3*arctanh(exp(c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b
/c
```

Rubi [A] time = 0.28, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6720, 2282, 390, 1158, 12, 288, 207}

$$-\frac{3\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}+\frac{3e^{c(a+bx)}\tanh(}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b
*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))) - (3*ArcTanh[E^(c*(a + b*x))]*Sqrt[Cot
h[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^3(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(-1+x^2\right)^3}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\text{Subst}\left(\int \left(1+\frac{2\left(1+3x^4\right)}{\left(-1+x^2\right)^3}\right)dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+\right.}{}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

Mathematica [C] time = 3.82, size = 334, normalized size = 1.70

$$-\frac{e^{-5c(a+bx)}\tanh^3(c(a+bx))\coth^2(c(a+bx))^{3/2}\left(256e^{8c(a+bx)}\left(e^{2c(a+bx)}+1\right)^3{}_6F_5\left(\frac{3}{2},2,2,2,2,2;1,1,1,1,\frac{11}{2};e^{2c(a+b}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -1/60480*((Coth[c*(a + b*x)]^2)^(3/2)*(-21*(252105 + 507305*E^(2*c*(a + b*x
)) + 173916*E^(4*c*(a + b*x)) - 154296*E^(6*c*(a + b*x)) - 73885*E^(8*c*(a
+ b*x)) + 4887*E^(10*c*(a + b*x))) - (315*(-16807 - 28218*E^(2*c*(a + b*x))
 + 1173*E^(4*c*(a + b*x)) + 17748*E^(6*c*(a + b*x)) + 4299*E^(8*c*(a + b*x)
) - 1434*E^(10*c*(a + b*x)) + 7*E^(12*c*(a + b*x)))*ArcTanh[Sqrt[E^(2*c*(a
+ b*x))]])/Sqrt[E^(2*c*(a + b*x))] + 384*E^(8*c*(a + b*x))*(1 + E^(2*c*(a +
 b*x)))^2*(7 + 5*E^(2*c*(a + b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1
, 1, 1, 11/2}, E^(2*c*(a + b*x))] + 256*E^(8*c*(a + b*x))*(1 + E^(2*c*(a +
b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*c
*(a + b*x))])*Tanh[c*(a + b*x)]^3)/(b*c*E^(5*c*(a + b*x)))
```

fricas [B] time = 0.41, size = 613, normalized size = 3.11

$$\frac{2\cosh(bcx+ac)^5+10\cosh(bcx+ac)\sinh(bcx+ac)^4+2\sinh(bcx+ac)^5+10\left(2\cosh(bcx+ac)^2-1\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + 2*s
inh(b*c*x + a*c)^5 + 10*(2*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^3 - 1
0*cosh(b*c*x + a*c)^3 + 10*(2*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x
 + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x
+ a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*
c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) +
 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*
c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 2*cosh(b
*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*
c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + 2*(5*cosh(b*c*x +
a*c)^4 - 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 4*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
 b*c*sinh(b*c*x + a*c)^4 - 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3 - b*
c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

giac [A] time = 0.99, size = 155, normalized size = 0.79

$$\frac{\frac{2e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)}-\frac{3\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)}+\frac{3\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)}-\frac{2\left(3e^{(3bcx+3ac)}-e^{(bcx+ac)}\right)}{\left(e^{(2bcx+2ac)}-1\right)^2\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 3*log(e^(b*c*x + a*c)
 + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 3*log(abs(e^(b*c*x + a*c) - 1))/sgn(e^
(2*b*c*x + 2*a*c) - 1) - 2*(3*e^(3*b*c*x + 3*a*c) - e^(b*c*x + a*c))/((e^(2
*b*c*x + 2*a*c) - 1)^2*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

maple [A] time = 0.85, size = 298, normalized size = 1.51

$$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\; e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}-\frac{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\; e^{c(bx+a)}\left(3\,e^{2c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)cb}-\frac{3\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\;\ln}{2\left(1+e^{2c(bx+a)}\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/c/b-1/(1+exp(2*c*(b*x+a)))/(exp(2*c*(
b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))-1)/c/b-3/2/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-
1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(1+exp(c*(b*
x+a)))+3/2/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^
2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-1)
```

maxima [A] time = 0.42, size = 112, normalized size = 0.57

$$-\frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{2\,bc}+\frac{3\,\log\left(e^{(bcx+ac)}-1\right)}{2\,bc}+\frac{e^{(5\,bcx+5\,ac)}-5\,e^{(3\,bcx+3\,ac)}+2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(b*c*x + a*c) + 1)/(b*c) + 3/2*log(e^(b*c*x + a*c) - 1)/(b*c) +
(e^(5*b*c*x + 5*a*c) - 5*e^(3*b*c*x + 3*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(
4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{c\,(a+b\,x)}\,\left(\mathrm{coth}\left(a\,c+b\,c\,x\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.213 $\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*arctanh(exp(
c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Rubi [A] time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 388, 206}

$$\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}-\frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*Ar
cTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{\left(2\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+\right.}{}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,\tanh(ac+bcx)}{bc} - \frac{2\tanh^{-1}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+}}{bc}$$

Mathematica [A] time = 0.06, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)}-2\tanh^{-1}\left(e^{c(a+bx)}\right)\right)\tanh(c(a+bx))\sqrt{\coth^2(c(a+bx))}}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Sqrt[Coth[c*(a + b*x)]^2]*T
anh[c*(a + b*x)])/(b*c)
```

fricas [A] time = 0.41, size = 70, normalized size = 0.84

$$\frac{\cosh(bcx+ac)-\log(\cosh(bcx+ac)+\sinh(bcx+ac)+1)+\log(\cosh(bcx+ac)+\sinh(bcx+ac)-1)+\text{s}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c) - log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + log(c
osh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + sinh(b*c*x + a*c))/(b*c)
```

giac [A] time = 0.37, size = 94, normalized size = 1.13

$$\frac{\frac{e^{(bcx+ac)}}{\text{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}-\frac{\log\left(e^{(bcx+ac)}+1\right)}{\text{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}+\frac{\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\text{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - log(e^(b*c*x + a*c) + 1)/sg
n(e^(2*b*c*x + 2*a*c) - 1) + log(abs(e^(b*c*x + a*c) - 1))/sgn(e^(2*b*c*x +
 2*a*c) - 1))/(b*c)
```

maple [B] time = 0.98, size = 213, normalized size = 2.57

$$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)cb}+\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,\ln\left(e^{c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)cb}-\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+}\right.}{\left(e^{2c(bx+a)}-\right.}}}{\left(1+e^{2c(bx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/c/b+1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(
b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp
(c*(b*x+a))-1)-1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x
+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(1+exp(c*(b*x+a)))
```

maxima [A] time = 0.42, size = 56, normalized size = 0.67

$$\frac{e^{(bcx+ac)}}{bc} - \frac{\log\left(e^{(bcx+ac)} + 1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)} - 1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(b*c*x + a*c)/(b*c) - log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c
) - 1)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{c\,(a+b\,x)}\,\sqrt{\coth\left(a\,c+b\,c\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\sqrt{\coth^2\left(ac+bcx\right)}\,e^{bcx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(sqrt(coth(a*c + b*c*x)**2)*exp(b*c*x), x)
```

3.214 $\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx$

Optimal. Leaf size=83

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*arctan(exp(c
*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 388, 203}

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$
$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$
$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$
$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2\tan^{-1}\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

Mathematica [A] time = 0.13, size = 51, normalized size = 0.61

$$\frac{\left(e^{c(a+bx)} - 2\tan^{-1}\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*Sqrt
[Coth[c*(a + b*x)]^2])
```

fricas [A] time = 0.39, size = 53, normalized size = 0.64

$$-\frac{2\arctan\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right) - \cosh\left(bcx+ac\right) - \sinh\left(bcx+ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))/(b*c)
```

giac [A] time = 1.33, size = 60, normalized size = 0.72

$$-\frac{2\arctan\left(e^{(bcx+ac)}\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - e^{(bcx+ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)
```

maple [C] time = 0.93, size = 218, normalized size = 2.63

$$\frac{\left(1+\mathrm{e}^{2c(bx+a)}\right)\mathrm{e}^{c(bx+a)}}{\sqrt{\frac{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}\left(\mathrm{e}^{2c(bx+a)}-1\right)cb} + \frac{i\left(1+\mathrm{e}^{2c(bx+a)}\right)\ln\left(\mathrm{e}^{c(bx+a)}-i\right)}{\sqrt{\frac{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}\left(\mathrm{e}^{2c(bx+a)}-1\right)cb} - \frac{i\left(1+\mathrm{e}^{2c(bx+a)}\right)\ln\left(\mathrm{e}^{c(bx+a)}+i\right)}{\sqrt{\frac{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2}}\left(\mathrm{e}^{2c(bx+a)}-1\right)cb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))*exp(c*(b*x+a))/c/b+I/((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp
(c*(b*x+a))-I)-I/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp
(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp(c*(b*x+a))+I)
```

maxima [A] time = 0.43, size = 35, normalized size = 0.42

$$-\frac{2\arctan\left(e^{(bcx+ac)}\right)}{bc}+\frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(b*c*x + a*c))/(b*c) + e^(b*c*x + a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\mathrm{coth}\left(a\,c+b\,c\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\sqrt{\coth^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(coth(a*c + b*c*x)**2), x)
```

3.215 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=193

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}-\frac{3\tan^{-1}\left(e^{c(a+\cdots}\right.}{bc\sqrt{\cot\cdots}}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*exp(c*(b*x+a
))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(coth(b*c*x+a*c)^2)^(1/2)+3*e
xp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))/(coth(b*c*x+a*c)^2)^
(1/2)-3*arctan(exp(c*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2
)
```

Rubi [A] time = 0.86, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6720, 2282, 390, 1158, 12, 288, 203}

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}-\frac{3\tan^{-1}\left(e^{c(a+\cdots}\right.}{bc\sqrt{\cot\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a
 + b*x)))*Sqrt[Coth[a*c + b*c*x]^2]) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1158

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(
q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*E
xpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^3(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \left(1-\frac{2\left(1+3x^4\right)}{\left(1+x^2\right)^3}\right)dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\,\text{Subst}\left(\int \frac{1+3x^4}{\left(1+x^2\right)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)\,\text{Su}\ldots}{2bc\ldots}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} - \frac{(6\coth(ac+bcx)\ldots}{bc\ldots}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\text{cot}\ldots}{bc\left(1+e^{2c(a+bx)}\right)\ldots}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\text{cot}\ldots}{bc\left(1+e^{2c(a+bx)}\right)\ldots}$$

Mathematica [A] time = 0.31, size = 104, normalized size = 0.54

$$\frac{\left(e^{c(a+bx)}\left(5e^{2c(a+bx)}+e^{4c(a+bx)}+2\right)-3\left(e^{2c(a+bx)}+1\right)^2\tan^{-1}\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(2 + 5*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))) - 3*(1 + E^
(2*c*(a + b*x)))^2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*(1 + E^
(2*c*(a + b*x)))^2*Sqrt[Coth[c*(a + b*x)]^2])
```

fricas [B] time = 0.43, size = 458, normalized size = 2.37

$$\frac{\cosh\left(bcx+ac\right)^5+5\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^4+\sinh\left(bcx+ac\right)^5+5\left(2\cosh\left(bcx+ac\right)^2+1\right)\sinh\left(bcx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*x
 + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*c*
x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a
*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 2
*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c
*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*cosh(b*c*
x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh(b*c*x +
 a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*
c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3
+ b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

giac [A] time = 0.33, size = 130, normalized size = 0.67

$$-\frac{\left(3\arctan\left(e^{(bcx+ac)}\right)e^{(-ac)}\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)-e^{(bcx)}\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)-\frac{3e^{(3bcx+2ac)}\mathrm{sgn}\left(e^{(2bcx+2ac)}-1\right)+e^{(bcx)}\mathrm{sgn}\left(e^{(2b\right.}}{\left(e^{(2bcx+2ac)}+1\right)^2}\right.}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*
x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (3*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2
*a*c) - 1) + e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) +
 1)^2)*e^(a*c)/(b*c)
```

maple [C] time = 0.88, size = 301, normalized size = 1.56

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\left(e^{2c(bx+a)}-1\right)cb}+\frac{e^{c(bx+a)}\left(3e^{2c(bx+a)}+1\right)}{\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,cb}+\frac{3i\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(bx+a)}-i\right.}{2\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\left(e^{2c(bx+a)}-1\right)cl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))*exp(c*(b*x+a))/c/b+1/(1+exp(2*c*(b*x+a)))/(exp(2*c*(
b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x
+a))*(3*exp(2*c*(b*x+a))+1)/c/b+3/2*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a)
)-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*
x+a))-I)-3/2*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a
)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

maxima [A] time = 0.42, size = 90, normalized size = 0.47

$$-\frac{3\arctan\left(e^{(bcx+ac)}\right)}{bc}+\frac{e^{(5\,bcx+5\,ac)}+5\,e^{(3\,bcx+3\,ac)}+2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*c*x + a*c))/(b*c) + (e^(5*b*c*x + 5*a*c) + 5*e^(3*b*c*x + 3*
a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c)
 + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\left(\mathrm{coth}\left(a\,c+b\,c\,x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\left(\coth^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(coth(a*c + b*c*x)**2)**(3/2), x)
```

3.216 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=311

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}+\frac{26e^{c}}{3bc\left(e^{2c(a+l}\right.}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-4*exp(c*(b*x+a
))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(coth(b*c*x+a*c)^2)^(1/2)+26/
3*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(coth(b*c*x+a*c
)^2)^(1/2)-55/6*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(
coth(b*c*x+a*c)^2)^(1/2)+25/4*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c
*(b*x+a)))/(coth(b*c*x+a*c)^2)^(1/2)-15/4*arctan(exp(c*(b*x+a)))*coth(b*c*x
+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 1.74, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6720, 2282, 390, 1814, 1157, 385, 203}

$$\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}}-\frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}+\frac{26e^{c}}{3bc\left(e^{2c(a+l}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3*Sqrt[Coth[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x])/(4*b*c*(1 + E^(2*c*(a + b*x)))*Sqrt[Coth[
a*c + b*c*x]^2]) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(4*b*c*Sq
rt[Coth[a*c + b*c*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^5(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \frac{\left(-1+x^2\right)^5}{\left(1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\,\text{Subst}\left(\int \left(1-\frac{2\left(1+10x^4+5x^8\right)}{\left(1+x^2\right)^5}\right)dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\,\text{Subst}\left(\int \frac{1+10x^4+5x^8}{\left(1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)\,\text{Subs}}{4bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{}}$$

Mathematica [A] time = 0.48, size = 133, normalized size = 0.43

$$\frac{\left(e^{c(a+bx)}\left(157e^{2c(a+bx)}+187e^{4c(a+bx)}+123e^{6c(a+bx)}+12e^{8c(a+bx)}+33\right)-45\left(e^{2c(a+bx)}+1\right)^4\tan^{-1}\left(e^{c(a+bx)}\right)\right)\coth(c(}{12bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\coth^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 123
*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4*A
rcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(12*b*c*(1 + E^(2*c*(a + b*x)))^
4*Sqrt[Coth[c*(a + b*x)]^2])
```

fricas [B] time = 0.49, size = 1226, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*c)
^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*x +
a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*cosh(b*
c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x
 + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1870*cos
h(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)^3 + (43
2*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3
 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8
*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*
c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b
*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x
 + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x
+ a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x
 + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 15*cosh(b*c*x + a
*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4*cosh(b*c*x + a*c
)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^
3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + si
nh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*cosh(b*c*x + a*c)^6 + 935
*cosh(b*c*x + a*c)^4 + 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 33
*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh
(b*c*x + a*c)^7 + b*c*sinh(b*c*x + a*c)^8 + 4*b*c*cosh(b*c*x + a*c)^6 + 4*(
7*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a
*c)^4 + 8*(7*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^5 + 2*(35*b*c*cosh(b*c*x + a*c)^4 + 30*b*c*cosh(b*c*x + a*c)^2 + 3*b
*c)*sinh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x +
 a*c)^5 + 10*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*b*c*cosh(b*c*x + a*c)^6 + 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*
c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x
+ a*c)^7 + 3*b*c*cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh
(b*c*x + a*c))*sinh(b*c*x + a*c))
```

giac [A] time = 0.69, size = 185, normalized size = 0.59

$$-\frac{\left(45 \arctan\left(e^{(bcx+ac)}\right) e^{(-ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - 12\,e^{(bcx)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - \frac{75\,e^{(7\,bcx+6\,ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+115 \cdots}{\cdots}\right.}{12\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(45*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 1
2*e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (75*e^(7*b*c*x + 6*a*c)*sgn(e^(2
*b*c*x + 2*a*c) - 1) + 115*e^(5*b*c*x + 4*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1)
 + 109*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 21*e^(b*c*x)*sgn(
e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) + 1)^4)*e^(a*c)/(b*c)
```

maple [C] time = 0.87, size = 324, normalized size = 1.04

$$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\left(e^{2c(bx+a)}-1\right)cb} + \frac{e^{c(bx+a)}\left(75\,e^{6c(bx+a)}+115\,e^{4c(bx+a)}+109\,e^{2c(bx+a)}+21\right)}{12\left(1+e^{2c(bx+a)}\right)^3\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,cb} + \frac{15i\left(1+e^{2c(bx+a)}\right)\ln\cdots}{8\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\left(e^{2\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/(exp(2*c*(b*x+a))-1
)*(1+exp(2*c*(b*x+a)))*exp(c*(b*x+a))/c/b+1/12/(1+exp(2*c*(b*x+a)))^3/(exp(
2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)*exp(c
*(b*x+a))*(75*exp(6*c*(b*x+a))+115*exp(4*c*(b*x+a))+109*exp(2*c*(b*x+a))+21
)/c/b+15/8*I*(1+exp(2*c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a))
)^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))-I)-15/8*I*(1+exp(2*
c*(b*x+a)))/(exp(2*c*(b*x+a))-1)/((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-
1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+I)
```

maxima [A] time = 0.42, size = 145, normalized size = 0.47

$$-\frac{15\arctan\left(e^{(bcx+ac)}\right)}{4\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}+123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}+157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bcx+ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e^
(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 33*
e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4
*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{{\left({\mathrm{coth}\left(a\,c+b\,c\,x\right)}^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.217 $\int \sin^3(\coth(a+bx))\,dx$

Optimal. Leaf size=157

$$\frac{\sin(3)\text{Ci}(3-3\coth(a+bx))}{8b}+\frac{\sin(3)\text{Ci}(3\coth(a+bx)+3)}{8b}-\frac{3\sin(1)\text{Ci}(1-\coth(a+bx))}{8b}-\frac{3\sin(1)\text{Ci}(\coth(a}{8b}$$

```
[Out] 1/8*cos(3)*Si(-3+3*coth(b*x+a))/b-3/8*cos(1)*Si(-1+coth(b*x+a))/b+3/8*cos(1
)*Si(1+coth(b*x+a))/b-1/8*cos(3)*Si(3+3*coth(b*x+a))/b-3/8*Ci(1-coth(b*x+a)
)*sin(1)/b-3/8*Ci(1+coth(b*x+a))*sin(1)/b+1/8*Ci(3-3*coth(b*x+a))*sin(3)/b+
1/8*Ci(3+3*coth(b*x+a))*sin(3)/b
```

Rubi [A] time = 0.37, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\sin(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}+\frac{\sin(3)\text{CosIntegral}(3\coth(a+bx)+3)}{8b}-\frac{3\sin(1)\text{CosIntegral}(1-\text{co}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Coth[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sin^3(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sin^3(x)}{2(-1+x)}+\frac{\sin^3(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(-1+x)}-\frac{\sin(3x)}{4(-1+x)}\right)dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(1+x)}-\frac{\sin(3x)}{4(1+x)}\right)dx, x\right.}{2b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{\text{Subst}\left(\int \frac{\sin(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{3\,\text{Subst}}{}$$
$$= \frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \text{co}\right.}{8b}$$
$$= -\frac{3\text{Ci}(1-\coth(a+bx))\sin(1)}{8b}-\frac{3\text{Ci}(1+\coth(a+bx))\sin(1)}{8b}+\frac{\text{Ci}(3-3\coth(a+b}{8b}$$

Mathematica [A] time = 0.26, size = 124, normalized size = 0.79

$$\frac{2\sin(3)\text{Ci}(3-3\coth(a+bx))+2\sin(3)\text{Ci}(3\coth(a+bx)+3)-6\sin(1)\text{Ci}(1-\coth(a+bx))-6\sin(1)\text{Ci}(\text{coth}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-6*CosIntegral[1 - Coth[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Coth[a + b*x]
]*Sin[1] + 2*CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3] + 2*CosIntegral[3 + 3*
Coth[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Coth[a + b*x]] + 6*Cos[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Coth[a + b*x]]
- 2*Cos[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

fricas [B] time = 0.44, size = 296, normalized size = 1.89

$$\frac{\text{Ci}\left(\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(3)+\text{Ci}\left(-\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(3)+\text{Ci}\left(\frac{6}{e^{(2bx+2a)}-1}\right)\sin(3)+\text{Ci}\left(-\frac{6}{e^{(2bx+2a)}-1}\right)\sin(3)-3\,\text{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(3) + cos_in
tegral(-6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(3) + cos_integral(6/(e
^(2*b*x + 2*a) - 1))*sin(3) + cos_integral(-6/(e^(2*b*x + 2*a) - 1))*sin(3)
 - 3*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_i
ntegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_integral(2
/(e^(2*b*x + 2*a) - 1))*sin(1) - 3*cos_integral(-2/(e^(2*b*x + 2*a) - 1))*s
in(1) - 2*cos(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 6*
cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*cos(3)*sin
_integral(6/(e^(2*b*x + 2*a) - 1)) - 6*cos(1)*sin_integral(2/(e^(2*b*x + 2*
a) - 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\coth\left(bx+a\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^3, x)
```

maple [A] time = 0.13, size = 118, normalized size = 0.75

$$\frac{-\frac{\mathrm{Si}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{Si}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(-3+3\coth(bx+a))\sin(3)}{8}+\frac{3\,\mathrm{Si}(1+\coth(bx+a)}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a))^3,x)
```

```
[Out] 1/b*(-1/8*Si(3+3*coth(b*x+a))*cos(3)+1/8*Ci(3+3*coth(b*x+a))*sin(3)+1/8*Si(
-3+3*coth(b*x+a))*cos(3)+1/8*Ci(-3+3*coth(b*x+a))*sin(3)+3/8*Si(1+coth(b*x+
a))*cos(1)-3/8*Ci(1+coth(b*x+a))*sin(1)-3/8*Si(-1+coth(b*x+a))*cos(1)-3/8*C
i(-1+coth(b*x+a))*sin(1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\coth\left(bx+a\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(coth(b*x + a))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(\mathrm{coth}\left(a+b\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(a + b*x))^3,x)
```

```
[Out] int(sin(coth(a + b*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{3}{\left(\coth{\left(a + b x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))**3,x)
```

```
[Out] Integral(sin(coth(a + b*x))**3, x)
```

3.218 $\int \sin^2(\coth(a+bx))\,dx$

Optimal. Leaf size=115

$$\frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\text{Ci}(2\coth(a+bx)+2)}{4b}+\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}-\frac{\sin(2)\text{Si}(2\coth(a+\ldots}{4b}$$

```
[Out] 1/4*Ci(2-2*coth(b*x+a))*cos(2)/b-1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*ln(1-
coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b-1/4*Si(-2+2*coth(b*x+a))*sin(2)/b-1/
4*Si(2+2*coth(b*x+a))*sin(2)/b
```

Rubi [A] time = 0.26, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b}+\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2
*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b*
x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Sin[2]*SinIn
tegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \sin^2(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sin^2(x)}{2(-1+x)}+\frac{\sin^2(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}-\frac{\cos(2x)}{2(-1+x)}\right)dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}-\frac{\cos(2x)}{2(1+x)}\right)dx\right.}{2b}$$
$$= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx, x, \coth(a+\right.}{4b}$$
$$= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}+\frac{\cos(2)\,\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx, x,\right.}{4b}$$
$$= \frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\text{Ci}(2+2\coth(a+bx))}{4b}-\frac{\log(1-\coth(a+}{4b}$$

Mathematica [A] time = 0.17, size = 88, normalized size = 0.77

$$\frac{\cos(2)\text{Ci}(2-2\coth(a+bx))-\cos(2)\text{Ci}(2(\coth(a+bx)+1))+\sin(2)\text{Si}(2-2\coth(a+bx))-\sin(2)\text{Si}(2(\cot}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Coth[a
 + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] + Sin[2]*SinInt
egral[2 - 2*Coth[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/(4*
b)
```

fricas [A] time = 0.48, size = 155, normalized size = 1.35

$$\frac{4\,bx-\cos(2)\,\text{Ci}\left(\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)-\cos(2)\,\text{Ci}\left(-\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right)+\cos(2)\,\text{Ci}\left(\frac{4}{e^{(2\,bx+2\,a)}-1}\right)+\cos(2)\,\text{Ci}\left(-\frac{4}{e^{(2\,bx+2\,a)}-1}\right)-2\,\text{si}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x - cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) + cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) - 1)) - 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) -
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\coth\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a))^2, x)
```

maple [A] time = 0.14, size = 102, normalized size = 0.89

$$-\frac{\ln\left(-1+\coth\left(bx+a\right)\right)}{4b}+\frac{\ln\left(1+\coth\left(bx+a\right)\right)}{4b}-\frac{\mathrm{Si}\left(2+2\coth\left(bx+a\right)\right)\sin(2)}{4b}-\frac{\mathrm{Ci}\left(2+2\coth\left(bx+a\right)\right)\cos(2)}{4b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a))^2,x)
```

```
[Out] -1/4/b*ln(-1+coth(b*x+a))+1/4*ln(1+coth(b*x+a))/b-1/4*Si(2+2*coth(b*x+a))*s
in(2)/b-1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*Si(-2+2*coth(b*x+a))*sin(2)/b+
1/4/b*Ci(-2+2*coth(b*x+a))*cos(2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x-\frac{1}{2}\int\cos\left(\frac{2\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,bx+2\,a)}-1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(\coth\left(a+b\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(a + b*x))^2,x)
```

```
[Out] int(sin(coth(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sin^{2}\left(\coth\left(a+bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a))**2,x)
```

```
[Out] Integral(sin(coth(a + b*x))**2, x)
```

3.219 $\int \sin(\coth(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\sin(1)\text{Ci}(1-\coth(a+bx))}{2b}-\frac{\sin(1)\text{Ci}(\coth(a+bx)+1)}{2b}+\frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{Si}(\coth(a+bx)}{2b}$$

```
[Out] -1/2*cos(1)*Si(-1+coth(b*x+a))/b+1/2*cos(1)*Si(1+coth(b*x+a))/b-1/2*Ci(1-co
th(b*x+a))*sin(1)/b-1/2*Ci(1+coth(b*x+a))*sin(1)/b
```

Rubi [A] time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3333, 3303, 3299, 3302}

$$-\frac{\sin(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}-\frac{\sin(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}+\frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b}+\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Coth[a + b*x]],x]
```

```
[Out] -(CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(2*b) - (CosIntegral[1 + Coth[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \sin(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{2(1-x)} + \frac{\sin(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\cos(1)\,\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \coth(a -\right.}{2b}$$
$$= -\frac{\text{Ci}(1-\coth(a+bx))\sin(1)}{2b} - \frac{\text{Ci}(1+\coth(a+bx))\sin(1)}{2b} + \frac{\cos(1)\text{Si}(1-\coth(a+b}{2b}$$

Mathematica [A] time = 0.14, size = 59, normalized size = 0.77

$$-\frac{\sin(1)\text{Ci}(1-\coth(a+bx)) + \sin(1)\text{Ci}(\coth(a+bx)+1) - \cos(1)(\text{Si}(1-\coth(a+bx)) + \text{Si}(\coth(a+bx)+1))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Coth[a + b*x]],x]
```

```
[Out] -1/2*(CosIntegral[1 - Coth[a + b*x]]*Sin[1] + CosIntegral[1 + Coth[a + b*x]
]*Sin[1] - Cos[1]*(SinIntegral[1 - Coth[a + b*x]] + SinIntegral[1 + Coth[a
+ b*x]]))/b
```

fricas [B] time = 0.44, size = 149, normalized size = 1.94

$$-\frac{\text{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1) + \text{Ci}\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)\sin(1) + \text{Ci}\left(\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1) + \text{Ci}\left(-\frac{2}{e^{(2bx+2a)}-1}\right)\sin(1) - 2\cos(1)\,\text{Si}\left(\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_in
tegral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(2/(e
^(2*b*x + 2*a) - 1))*sin(1) + cos_integral(-2/(e^(2*b*x + 2*a) - 1))*sin(1)
 - 2*cos(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*cos(1
)*sin_integral(2/(e^(2*b*x + 2*a) - 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(sin(coth(b*x + a)), x)
```

maple [A] time = 0.13, size = 58, normalized size = 0.75

$$\frac{\frac{\text{Si}(1+\coth(bx+a))\cos(1)}{2} - \frac{\text{Ci}(1+\coth(bx+a))\sin(1)}{2} - \frac{\text{Si}(-1+\coth(bx+a))\cos(1)}{2} - \frac{\text{Ci}(-1+\coth(bx+a))\sin(1)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(b*x+a)),x)
```

```
[Out] 1/b*(1/2*Si(1+coth(b*x+a))*cos(1)-1/2*Ci(1+coth(b*x+a))*sin(1)-1/2*Si(-1+co
th(b*x+a))*cos(1)-1/2*Ci(-1+coth(b*x+a))*sin(1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(\coth(bx + a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(sin(coth(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(\coth(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(coth(a + b*x)),x)
```

```
[Out] int(sin(coth(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(\coth(a + bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(coth(b*x+a)),x)
```

```
[Out] Integral(sin(coth(a + b*x)), x)
```

3.220 $\int \csc(\coth(a+bx))\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\csc(\coth(a+bx))}{\coth(a+bx)-1},x\right)-\frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\csc(\coth(a+bx))}{\coth(a+bx)+1},x\right)$$

```
[Out] 1/2*Unintegrable(csc(coth(b*x+a))*csch(b*x+a)^2/(-1+coth(b*x+a)),x)-1/2*Uni
ntegrable(csc(coth(b*x+a))*csch(b*x+a)^2/(1+coth(b*x+a)),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[Coth[a + b*x]],x]
```

```
[Out] -Defer[Subst][Defer[Int][Csc[x]/(-1 + x), x], x, Coth[a + b*x]]/(2*b) + Def
er[Subst][Defer[Int][Csc[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}\int \csc(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\csc(x)}{1-x^2}\,dx,x,\coth(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{\csc(x)}{2(-1+x)}+\frac{\csc(x)}{2(1+x)}\right)dx,x,\coth(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{\csc(x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\csc(x)}{1+x}\,dx,x,\coth(a+bx)\right)}{2b}\end{aligned}$$

Mathematica [A] time = 3.55, size = 0, normalized size = 0.00

$$\int \csc(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[Coth[a + b*x]],x]
```

```
[Out] Integrate[Csc[Coth[a + b*x]], x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\csc\left(\coth\left(bx+a\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(csc(coth(b*x + a)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(csc(coth(b*x + a)), x)
```

maple [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(coth(b*x+a)),x)
[Out] int(csc(coth(b*x+a)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(csc(coth(b*x + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(\coth\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(coth(a + b*x)),x)
[Out] int(1/sin(coth(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(coth(b*x+a)),x)
[Out] Integral(csc(coth(a + b*x)), x)
```

3.221 $\int \cos^3(\coth(a+bx))\,dx$

Optimal. Leaf size=157

$$-\frac{\cos(3)\text{Ci}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{Ci}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{Ci}(\coth(a+bx)+1)}{8b}+\frac{\cos(3)\text{Ci}(3\coth(a}{8b}$$

```
[Out] -3/8*Ci(1-coth(b*x+a))*cos(1)/b+3/8*Ci(1+coth(b*x+a))*cos(1)/b-1/8*Ci(3-3*c
oth(b*x+a))*cos(3)/b+1/8*Ci(3+3*coth(b*x+a))*cos(3)/b+3/8*Si(-1+coth(b*x+a)
)*sin(1)/b+3/8*Si(1+coth(b*x+a))*sin(1)/b+1/8*Si(-3+3*coth(b*x+a))*sin(3)/b
+1/8*Si(3+3*coth(b*x+a))*sin(3)/b
```

Rubi [A] time = 0.37, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6725, 3312, 3303, 3299, 3302}

$$-\frac{\cos(3)\text{CosIntegral}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{CosIntegral}(\coth(a}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] -(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]])/(8*b) - (3*Cos[1]*CosIntegral[1
- Coth[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Coth[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \cos^3(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\cos^3(x)}{2(-1+x)}+\frac{\cos^3(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(-1+x)}+\frac{\cos(3x)}{4(-1+x)}\right)dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(1+x)}+\frac{\cos(3x)}{4(1+x)}\right)\ldots\right.}{2b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{\text{Subst}\left(\int \frac{\cos(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{3\ldots}{\ldots}$$
$$= -\frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{(3\cos(1))\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx\ldots\right.}{8b}$$
$$= -\frac{\cos(3)\text{Ci}(3-3\coth(a+bx))}{8b}-\frac{3\cos(1)\text{Ci}(1-\coth(a+bx))}{8b}+\frac{3\cos(1)\text{Ci}(1+\text{c}\ldots}{8b}$$

Mathematica [A] time = 0.26, size = 124, normalized size = 0.79

$$-2\cos(3)\text{Ci}(3-3\coth(a+bx))-6\cos(1)\text{Ci}(1-\coth(a+bx))+6\cos(1)\text{Ci}(\coth(a+bx)+1)+2\cos(3)\text{Ci}(\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] (-2*Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]] - 6*Cos[1]*CosIntegral[1 - Coth
[a + b*x]] + 6*Cos[1]*CosIntegral[1 + Coth[a + b*x]] + 2*Cos[3]*CosIntegral
[3 + 3*Coth[a + b*x]] - 2*Sin[3]*SinIntegral[3 - 3*Coth[a + b*x]] - 6*Sin[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Sin[1]*SinIntegral[1 + Coth[a + b*x]]
+ 2*Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

fricas [B] time = 0.48, size = 298, normalized size = 1.90

$$\cos(3)\,\text{Ci}\left(\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)+3\cos(1)\,\text{Ci}\left(\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)+3\cos(1)\,\text{Ci}\left(-\frac{2e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)+\cos(3)\,\text{Ci}\left(-\frac{6e^{(2bx+2a)}}{e^{(2bx+2a)}-1}\right)-\cos(3)\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(cos(3)*cos_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(
1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 3*cos(1)*cos_int
egral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(3)*cos_integral(-6*e^
(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(6/(e^(2*b*x + 2*
a) - 1)) - 3*cos(1)*cos_integral(2/(e^(2*b*x + 2*a) - 1)) - 3*cos(1)*cos_in
tegral(-2/(e^(2*b*x + 2*a) - 1)) - cos(3)*cos_integral(-6/(e^(2*b*x + 2*a)
- 1)) + 2*sin(3)*sin_integral(6*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 6*
sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*sin(3)*sin
_integral(6/(e^(2*b*x + 2*a) - 1)) + 6*sin(1)*sin_integral(2/(e^(2*b*x + 2*
a) - 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\coth\left(bx+a\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^3, x)
```

maple [A] time = 0.14, size = 118, normalized size = 0.75

$$\frac{\frac{\mathrm{Si}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Si}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Ci}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{3\,\mathrm{Si}(1+\coth(bx+a))\sin(1}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a))^3,x)
```

```
[Out] 1/b*(1/8*Si(3+3*coth(b*x+a))*sin(3)+1/8*Ci(3+3*coth(b*x+a))*cos(3)+1/8*Si(-
3+3*coth(b*x+a))*sin(3)-1/8*Ci(-3+3*coth(b*x+a))*cos(3)+3/8*Si(1+coth(b*x+a
))*sin(1)+3/8*Ci(1+coth(b*x+a))*cos(1)+3/8*Si(-1+coth(b*x+a))*sin(1)-3/8*Ci
(-1+coth(b*x+a))*cos(1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\coth\left(bx+a\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(coth(b*x + a))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(\mathrm{coth}\left(a+b\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(a + b*x))^3,x)
```

```
[Out] int(cos(coth(a + b*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{3}{\left(\coth{\left(a + bx \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))**3,x)
```

```
[Out] Integral(cos(coth(a + b*x))**3, x)
```

3.222 $\int \cos^2(\coth(a+bx))\,dx$

Optimal. Leaf size=115

$$-\frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{Ci}(2\coth(a+bx)+2)}{4b}-\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}+\frac{\sin(2)\text{Si}(2\coth}{4b}$$

```
[Out] -1/4*Ci(2-2*coth(b*x+a))*cos(2)/b+1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*ln(1
-coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b+1/4*Si(-2+2*coth(b*x+a))*sin(2)/b+1
/4*Si(2+2*coth(b*x+a))*sin(2)/b
```

Rubi [A] time = 0.25, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6725, 3312, 3303, 3299, 3302}

$$-\frac{\cos(2)\text{CosIntegral}(2-2\coth(a+bx))}{4b}+\frac{\cos(2)\text{CosIntegral}(2\coth(a+bx)+2)}{4b}-\frac{\sin(2)\text{Si}(2-2\coth(a+b}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] -(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \cos^2(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\cos^2(x)}{2(-1+x)} + \frac{\cos^2(x)}{2(1+x)}\right) dx, x, \coth(a+bx)\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)} + \frac{\cos(2x)}{2(-1+x)}\right) dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)} + \frac{\cos(2x)}{2(1+x)}\right) dx, x\right.}{2b}$$

$$= -\frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx, x, \coth(a+b\right.}{4b}$$

$$= -\frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} - \frac{\cos(2)\,\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx, x, \text{co}\right.}{4b}$$

$$= -\frac{\cos(2)\text{Ci}(2-2\coth(a+bx))}{4b} + \frac{\cos(2)\text{Ci}(2+2\coth(a+bx))}{4b} - \frac{\log(1-\coth(a+b}{4b}$$

Mathematica [A] time = 0.18, size = 88, normalized size = 0.77

$$\frac{-\cos(2)\text{Ci}(2-2\coth(a+bx)) + \cos(2)\text{Ci}(2(\coth(a+bx)+1)) - \sin(2)\text{Si}(2-2\coth(a+bx)) + \sin(2)\text{Si}(2(\text{coth}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] (-(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]]) + Cos[2]*CosIntegral[2*(1 + Cot
h[a + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] - Sin[2]*Sin
Integral[2 - 2*Coth[a + b*x]] + Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/
(4*b)
```

fricas [A] time = 0.46, size = 155, normalized size = 1.35

$$\frac{4\,bx + \cos(2)\,\text{Ci}\left(\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right) + \cos(2)\,\text{Ci}\left(-\frac{4\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right) - \cos(2)\,\text{Ci}\left(\frac{4}{e^{(2\,bx+2\,a)}-1}\right) - \cos(2)\,\text{Ci}\left(-\frac{4}{e^{(2\,bx+2\,a)}-1}\right) + 2\,\sin(}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x + cos(2)*cos_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
 cos(2)*cos_integral(-4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos
_integral(4/(e^(2*b*x + 2*a) - 1)) - cos(2)*cos_integral(-4/(e^(2*b*x + 2*a
) - 1)) + 2*sin(2)*sin_integral(4*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) +
2*sin(2)*sin_integral(4/(e^(2*b*x + 2*a) - 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\coth\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a))^2, x)
```

maple [A] time = 0.14, size = 102, normalized size = 0.89

$$\frac{\mathrm{Si}\left(2+2\coth\left(bx+a\right)\right)\sin(2)}{4b}+\frac{\mathrm{Ci}\left(2+2\coth\left(bx+a\right)\right)\cos(2)}{4b}+\frac{\mathrm{Si}\left(-2+2\coth\left(bx+a\right)\right)\sin(2)}{4b}-\frac{\mathrm{Ci}\left(-2+2\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a))^2,x)
```

```
[Out] 1/4*Si(2+2*coth(b*x+a))*sin(2)/b+1/4*Ci(2+2*coth(b*x+a))*cos(2)/b+1/4*Si(-2
+2*coth(b*x+a))*sin(2)/b-1/4/b*Ci(-2+2*coth(b*x+a))*cos(2)-1/4/b*ln(-1+coth
(b*x+a))+1/4*ln(1+coth(b*x+a))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x+\frac{1}{2}\int\cos\left(\frac{2\left(e^{(2\,bx+2\,a)}+1\right)}{e^{(2\,bx+2\,a)}-1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(\coth\left(a+b\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(a + b*x))^2,x)
```

```
[Out] int(cos(coth(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cos^2\left(\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a))**2,x)
```

```
[Out] Integral(cos(coth(a + b*x))**2, x)
```

3.223 $\int \cos(\coth(a+bx))\,dx$

Optimal. Leaf size=77

$$-\frac{\cos(1)\text{Ci}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{Ci}(\coth(a+bx)+1)}{2b}-\frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\sin(1)\text{Si}(\coth(a+bx)+1)}{2b}$$

```
[Out] -1/2*Ci(1-coth(b*x+a))*cos(1)/b+1/2*Ci(1+coth(b*x+a))*cos(1)/b+1/2*Si(-1+co
th(b*x+a))*sin(1)/b+1/2*Si(1+coth(b*x+a))*sin(1)/b
```

Rubi [A] time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3334, 3303, 3299, 3302}

$$-\frac{\cos(1)\text{CosIntegral}(1-\coth(a+bx))}{2b}+\frac{\cos(1)\text{CosIntegral}(\coth(a+bx)+1)}{2b}-\frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b}+\frac{\sin(1)\text{Si}(\coth(a+bx)+1)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Coth[a + b*x]],x]
```

```
[Out] -(Cos[1]*CosIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[1]*CosIntegral[1 + Co
th[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \cos(\coth(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{2(1-x)} + \frac{\cos(x)}{2(1+x)}\right)dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$
$$= \frac{\cos(1)\,\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\cos(1)\,\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth(\right.}{2b}$$
$$= -\frac{\cos(1)\text{Ci}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\text{Ci}(1+\coth(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\coth(a+}{2b}$$

Mathematica [A] time = 0.11, size = 62, normalized size = 0.81

$$-\frac{\cos(1)\text{Ci}(1-\coth(a+bx)) - \cos(1)\text{Ci}(\coth(a+bx)+1) + \sin(1)\text{Si}(1-\coth(a+bx)) - \sin(1)\text{Si}(\coth(a+b}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Coth[a + b*x]],x]
```

```
[Out] -1/2*(Cos[1]*CosIntegral[1 - Coth[a + b*x]] - Cos[1]*CosIntegral[1 + Coth[a
 + b*x]] + Sin[1]*SinIntegral[1 - Coth[a + b*x]] - Sin[1]*SinIntegral[1 + C
oth[a + b*x]])/b
```

fricas [B] time = 0.47, size = 151, normalized size = 1.96

$$\frac{\cos(1)\,\text{Ci}\left(\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right) + \cos(1)\,\text{Ci}\left(-\frac{2\,e^{(2\,bx+2\,a)}}{e^{(2\,bx+2\,a)}-1}\right) - \cos(1)\,\text{Ci}\left(\frac{2}{e^{(2\,bx+2\,a)}-1}\right) - \cos(1)\,\text{Ci}\left(-\frac{2}{e^{(2\,bx+2\,a)}-1}\right) + 2\,\sin(1)\,\text{Si}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(1)*cos_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + cos(1)*
cos_integral(-2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) - cos(1)*cos_integra
l(2/(e^(2*b*x + 2*a) - 1)) - cos(1)*cos_integral(-2/(e^(2*b*x + 2*a) - 1))
+ 2*sin(1)*sin_integral(2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)) + 2*sin(1)
*sin_integral(2/(e^(2*b*x + 2*a) - 1)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\coth\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(cos(coth(b*x + a)), x)
```

maple [A] time = 0.14, size = 58, normalized size = 0.75

$$\frac{\frac{\text{Si}(1+\coth(bx+a))\sin(1)}{2} + \frac{\text{Ci}(1+\coth(bx+a))\cos(1)}{2} + \frac{\text{Si}(-1+\coth(bx+a))\sin(1)}{2} - \frac{\text{Ci}(-1+\coth(bx+a))\cos(1)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(b*x+a)),x)
```

```
[Out] 1/b*(1/2*Si(1+coth(b*x+a))*sin(1)+1/2*Ci(1+coth(b*x+a))*cos(1)+1/2*Si(-1+co
th(b*x+a))*sin(1)-1/2*Ci(-1+coth(b*x+a))*cos(1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(\coth(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(cos(coth(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(\coth(a+b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(coth(a + b*x)),x)
```

```
[Out] int(cos(coth(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(coth(b*x+a)),x)
```

```
[Out] Integral(cos(coth(a + b*x)), x)
```

3.224 $\int \sec(\coth(a + bx))\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{\coth(a+bx)-1},x\right)-\frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{\coth(a+bx)+1},x\right)$$

```
[Out] 1/2*Unintegrable(csch(b*x+a)^2*sec(coth(b*x+a))/(-1+coth(b*x+a)),x)-1/2*Uni
ntegrable(csch(b*x+a)^2*sec(coth(b*x+a))/(1+coth(b*x+a)),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[Coth[a + b*x]],x]
```

```
[Out] -Defer[Subst][Defer[Int][Sec[x]/(-1 + x), x], x, Coth[a + b*x]]/(2*b) + Def
er[Subst][Defer[Int][Sec[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}\int \sec(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{\sec(x)}{1-x^2}\,dx,x,\coth(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{\sec(x)}{2(-1+x)}+\frac{\sec(x)}{2(1+x)}\right)dx,x,\coth(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{\sec(x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sec(x)}{1+x}\,dx,x,\coth(a+bx)\right)}{2b}\end{aligned}$$

Mathematica [A] time = 6.63, size = 0, normalized size = 0.00

$$\int \sec(\coth(a+bx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[Coth[a + b*x]],x]
```

```
[Out] Integrate[Sec[Coth[a + b*x]], x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sec\left(\coth\left(bx+a\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(sec(coth(b*x + a)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(\coth\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(sec(coth(b*x + a)), x)
```

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \sec(\coth(bx + a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(coth(b*x+a)),x)
[Out] int(sec(coth(b*x+a)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(\coth(bx + a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(sec(coth(b*x + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(\coth(a + b\,x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(coth(a + b*x)),x)
[Out] int(1/cos(coth(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(\coth(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(coth(b*x+a)),x)
[Out] Integral(sec(coth(a + b*x)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```