Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.5-Hyperbolic-cosine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [336]. This is test number [169].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (336)	% 0.00 (0)
Mathematica	% 99.70 (335)	% 0.30 (1)
Maple	% 87.20 (293)	% 12.80 (43)
Maxima	% 61.90 (208)	% 38.10 (128)
Fricas	% 84.23 (283)	% 15.77 (53)
Sympy	% 30.65 (103)	% 69.35 (233)
Giac	% 77.08 (259)	% 22.92 (77)
Mupad	% 56.55 (190)	% 43.45 (146)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

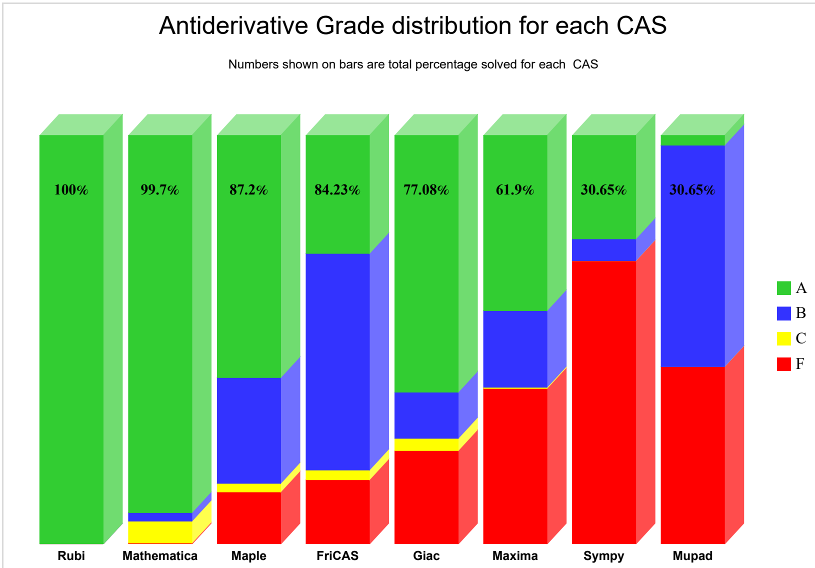
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

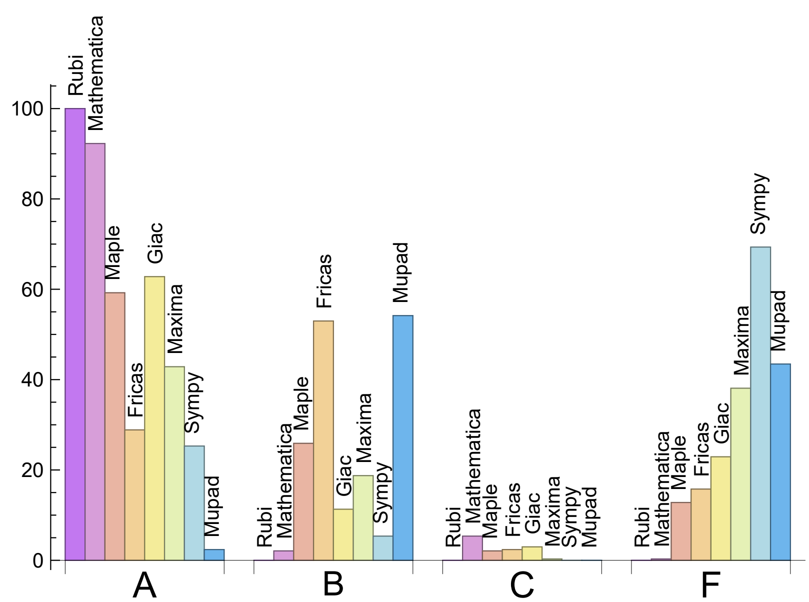
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	92.26	2.08	5.36	0.30
Maple	59.23	25.89	2.08	12.80
Maxima	42.86	18.75	0.30	38.10
Fricas	28.87	52.98	2.38	15.77
Sympy	25.30	5.36	0.00	69.35
Giac	62.80	11.31	2.98	22.92
Mupad	2.38	54.17	0.00	43.45

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	0.00 %	100.00 %	0.00 %
Maple	43	76.74 %	0.00 %	23.26 %
Maxima	128	63.28 %	0.00 %	36.72 %
Fricas	53	94.34 %	0.00 %	5.66 %
Sympy	233	69.10 %	30.90 %	0.00 %
Giac	77	92.21 %	5.19 %	2.60 %
Mupad	146	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

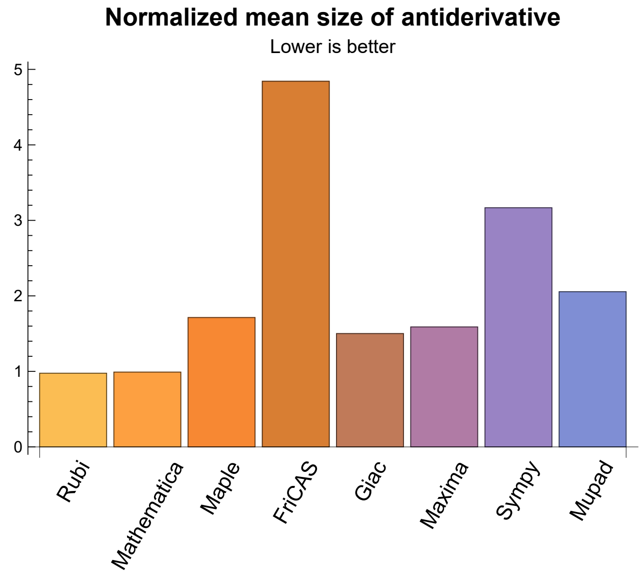
1.3 Performance

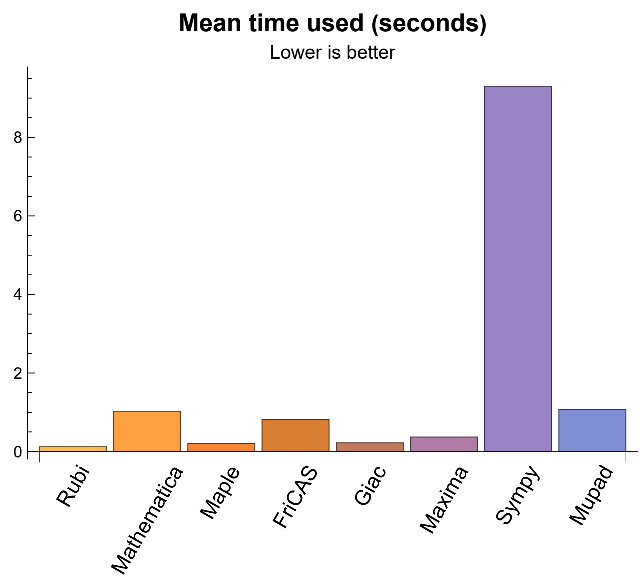
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	89.23	0.98	63.50	1.00
Mathematica	1.03	99.53	0.99	55.00	0.92
Maple	0.20	140.89	1.71	81.00	1.16
Maxima	0.37	104.63	1.59	80.00	1.23
Fricas	0.81	507.89	4.84	190.00	2.53
Sympy	9.30	165.68	3.17	51.00	1.52
Giac	0.22	152.87	1.50	65.00	1.14
Mupad	1.07	137.66	2.06	67.50	1.29

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{216, 217, 221, 226, 227, 232, 233, 238}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {258, 259, 319, 326, 327, 329, 331, 335, 336}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

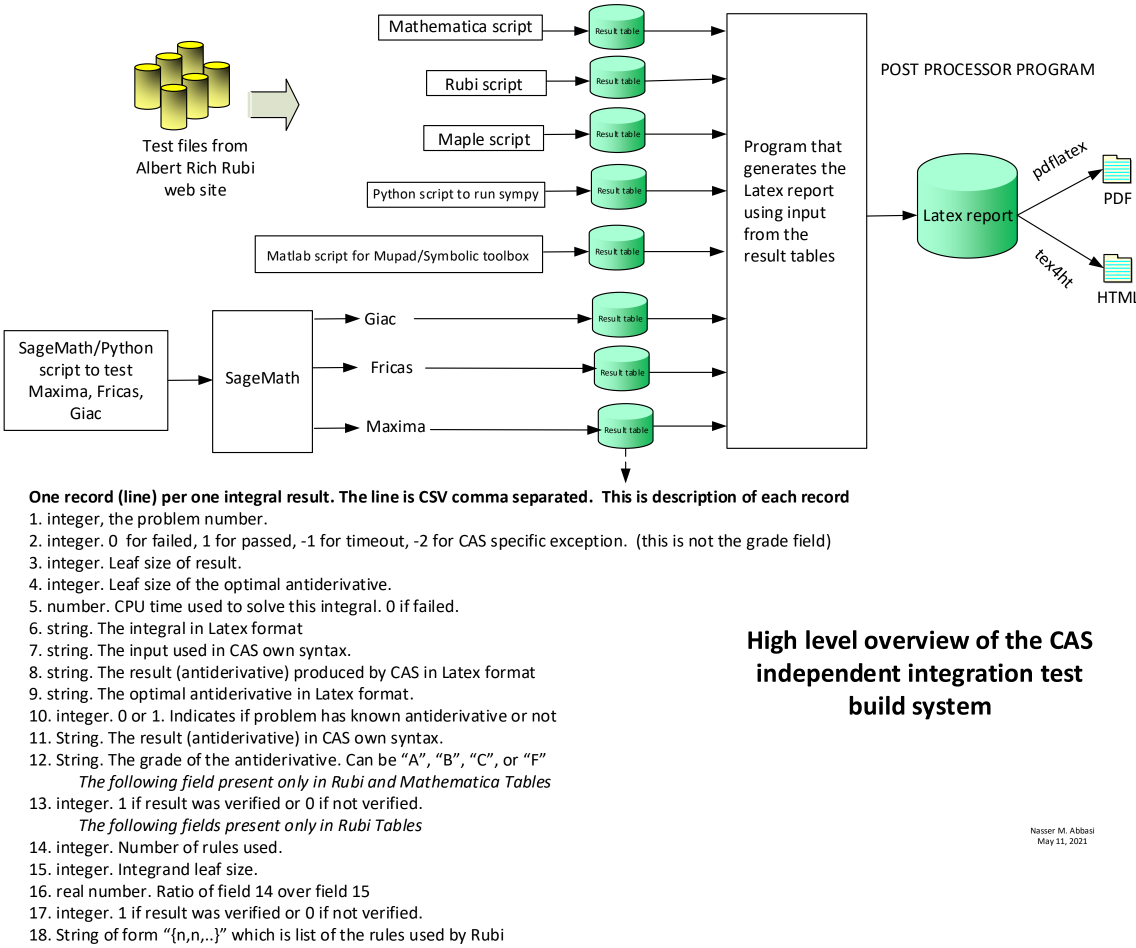
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 254, 255, 256, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 281, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 330, 331, 333, 334 }

B grade: { 1, 75, 247, 262, 325, 328, 329 }

C grade: { 9, 13, 17, 21, 130, 143, 210, 253, 257, 258, 275, 279, 280, 283, 284, 332, 335, 336 }

F grade: { 238 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 12, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 111, 112, 113, 115, 117, 118, 121, 122, 123, 124, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 159, 160, 161, 162, 163, 164, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 203, 204, 205, 207, 208, 209, 216, 217, 220, 221, 225, 226, 227, 232, 233, 238, 247, 248, 249, 250, 251, 256, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 281, 282, 285, 286, 287, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334, 335, 336 }

B grade: { 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 40, 45, 46, 47, 54, 55, 61, 79, 80, 81, 82, 83, 84, 85, 101, 102, 103, 107, 108, 109, 110, 114, 116, 119, 120, 125, 126, 127, 135, 136, 137, 152, 153, 154, 155, 156, 157, 158, 165, 166, 167, 168, 169, 170, 179, 180, 181, 187, 189, 199, 202, 206, 210, 211, 212, 218, 219, 224, 230, 231, 236, 237, 252, 253, 254, 255, 257, 262, 263 }

C grade: { 275, 276, 279, 280, 283, 284, 302 }

F grade: { 23, 128, 129, 130, 131, 132, 133, 213, 214, 215, 222, 223, 228, 229, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 258, 259, 260, 261, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 329, 330, 331, 332 }

2.1.4 Maxima

A grade: { 1, 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 32, 36, 42, 43, 44, 62, 63, 64, 65, 66, 71, 72, 73, 74, 75, 76, 77, 78, 93, 97, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 140, 141, 142, 143, 145, 146, 147, 151, 153, 155, 158, 159, 171, 172, 174, 180, 182, 183, 185, 191, 192, 193, 200, 201, 216, 217, 220, 221, 225, 226, 227, 232, 233, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 292, 293, 294, 295, 296, 297, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334 }

B grade: { 3, 5, 31, 33, 34, 35, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 87, 88, 89, 90, 91, 92, 94, 95, 96, 98, 99, 100, 101, 102, 103, 138, 139, 144, 148, 149, 150, 152, 154, 156, 157, 160, 161, 162, 163, 164, 165, 167, 169, 176, 187, 188, 189, 190, 194, 195, 196, 237, 249, 251, 270, 272, 298 }

C grade: { 302 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 128, 129, 130, 131, 132, 133, 166, 168, 170, 173, 175, 177, 178, 179, 181, 184, 186, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 231, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 283, 284, 288, 289, 290, 291, 299, 329, 330, 331, 332, 335, 336 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 25, 26, 27, 28, 32, 36, 40, 44, 45, 50, 51, 57, 58, 62, 63, 64, 65, 66, 67, 71, 75, 93, 94, 97, 98, 101, 110, 114, 115, 117, 142, 143, 154, 155, 156, 157, 158, 159, 172, 182, 183, 192, 203, 204, 205, 206, 216, 217, 220, 221, 226, 227, 232, 233, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 264, 266, 268, 274, 275, 279, 280, 292, 293, 294, 295, 299, 300, 301, 302, 303, 304, 312, 333, 334 }

B grade: { 24, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 48, 49, 52, 53, 54, 55, 56, 59, 60, 61, 68, 69, 70, 72, 73, 74, 76, 77, 78, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 102, 103, 104, 105, 106, 111, 112, 113, 116, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208, 209, 210, 218, 219, 224, 225, 230, 231, 236, 237, 245, 246, 263, 265, 267, 269, 270, 271, 272, 273, 276, 277, 278, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 335, 336 }

C grade: { 211, 212, 222, 223, 228, 229, 234, 235 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 213, 214, 215, 252, 253, 254, 255, 256, 257, 288, 289, 290, 291, 329, 331, 332 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 27, 32, 33, 34, 35, 36, 37, 38, 39, 56, 57, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 110, 114, 115, 117, 124, 140, 141, 142, 143, 146, 147, 148, 149, 159, 170, 171, 199, 200, 201, 206, 216, 217, 221, 225, 226, 227, 231, 232, 233, 238, 247, 249, 251, 264, 265, 266, 267, 274, 278, 282, 286, 287, 294, 299, 333, 334 }

B grade: { 24, 25, 26, 116, 144, 145, 150, 151, 152, 153, 154, 155, 156, 157, 158, 273, 277, 281 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 68, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 268, 269, 270, 271, 272, 275, 276, 279, 280, 283, 284, 285, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336 }

2.1.7 Giac

A grade: { 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 110, 111, 114, 115, 117, 121, 122, 123, 124, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 216, 217, 220, 221, 225, 226, 227, 231, 232, 233, 237, 238, 239, 240, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 333, 334 }

B grade: { 1, 3, 5, 48, 49, 50, 66, 88, 89, 90, 91, 92, 105, 106, 112, 113, 116, 144, 160, 162, 174, 176, 177, 180, 195, 208, 209, 241, 242, 243, 244, 245, 246, 247, 248, 249, 262, 263 }

C grade: { 40, 41, 101, 104, 285, 286, 287, 302, 306, 309 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 125, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 288, 289, 290, 291, 329, 330, 331, 332, 335, 336 }

2.1.8 Mupad

A grade: { 216, 217, 221, 226, 227, 232, 233, 238 }

B grade: { 1, 2, 3, 4, 5, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 110, 111, 114, 115, 116, 117, 124, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 218, 225, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 294, 296, 297, 298, 299, 329, 330, 331, 333, 334 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 197, 198, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 288, 289, 290, 291, 292, 293, 295, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 332, 335, 336 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10
normalized size	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00
time (sec)	N/A	0.005	0.011	0.041	0.317	0.591	0.133	0.135	0.045

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	22	46	32	18
normalized size	1	1.00	0.92	1.08	1.28	0.88	1.84	1.28	0.72
time (sec)	N/A	0.010	0.022	0.085	0.315	0.564	0.207	0.134	0.882

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	54	32	36	54	22
normalized size	1	1.00	1.00	0.88	2.08	1.23	1.38	2.08	0.85
time (sec)	N/A	0.012	0.008	0.205	0.311	0.734	0.407	0.122	0.885

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	39	60	49	95	60	31
normalized size	1	1.00	0.72	0.85	1.30	1.07	2.07	1.30	0.67
time (sec)	N/A	0.020	0.045	0.213	0.309	0.812	0.847	0.121	0.083

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	82	66	58	82	31
normalized size	1	1.00	1.00	0.80	2.00	1.61	1.41	2.00	0.76
time (sec)	N/A	0.013	0.015	0.215	0.315	0.469	1.568	0.141	0.918

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	49	86	90	139	88	42
normalized size	1	1.00	0.64	0.73	1.28	1.34	2.07	1.31	0.63
time (sec)	N/A	0.033	0.041	0.223	0.312	0.613	2.988	0.136	0.966

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	55	201	0	0	0	0	-1
normalized size	1	1.00	0.80	2.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.116	0.353	0.000	0.464	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	188	0	0	0	0	-1
normalized size	1	1.00	0.96	4.09	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.056	0.298	0.000	0.540	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	81	174	0	0	0	0	-1
normalized size	1	1.00	1.76	3.78	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.112	0.293	0.000	0.499	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	0	0	0	-1
normalized size	1	1.00	1.00	6.75	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.030	0.261	0.000	0.487	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	0	0	0	-1
normalized size	1	1.00	1.00	6.75	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.029	0.289	0.000	0.490	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	103	0	0	0	0	-1
normalized size	1	1.00	1.00	2.45	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.063	0.355	0.000	0.518	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	217	0	0	0	0	-1
normalized size	1	1.00	1.83	4.72	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.072	0.334	0.000	0.484	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	363	0	0	0	0	-1
normalized size	1	1.00	0.91	5.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.142	0.628	0.000	0.454	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	145	0	0	0	0	-1
normalized size	1	1.00	0.82	2.23	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.053	0.349	0.000	0.475	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	184	0	0	0	0	-1
normalized size	1	1.00	0.85	3.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.046	0.368	0.000	0.462	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	130	0	0	0	0	-1
normalized size	1	1.00	1.19	2.71	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.064	0.376	0.000	0.549	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	118	0	0	0	0	-1
normalized size	1	1.00	1.00	4.37	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.010	0.373	0.000	0.580	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	100	0	0	0	0	-1
normalized size	1	1.00	1.00	3.70	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.013	0.253	0.000	0.494	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	159	0	0	0	0	-1
normalized size	1	1.00	0.74	3.46	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.026	0.368	0.000	0.662	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	177	0	0	0	0	-1
normalized size	1	1.00	1.12	3.54	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.043	0.344	0.000	0.517	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	254	0	0	0	0	-1
normalized size	1	1.00	0.64	3.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.051	0.602	0.000	0.688	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.067	0.269	0.000	0.997	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	111	66	100	337	70	70
normalized size	1	1.00	0.98	2.06	1.22	1.85	6.24	1.30	1.30
time (sec)	N/A	0.076	0.086	0.066	0.372	0.515	1.893	0.122	0.961

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	87	56	70	189	51	52
normalized size	1	1.00	1.05	2.02	1.30	1.63	4.40	1.19	1.21
time (sec)	N/A	0.051	0.056	0.068	0.350	3.488	1.113	0.116	0.917

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	59	41	47	63	35	34
normalized size	1	1.00	1.28	2.36	1.64	1.88	2.52	1.40	1.36
time (sec)	N/A	0.069	0.059	0.068	0.370	1.463	0.600	0.120	0.901

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	14	34	18	24	8	17	17
normalized size	1	1.00	0.78	1.89	1.00	1.33	0.44	0.94	0.94
time (sec)	N/A	0.032	0.029	0.054	0.309	0.379	0.335	0.122	0.875

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	21	23	29	0	20	31
normalized size	1	1.00	1.10	1.05	1.15	1.45	0.00	1.00	1.55
time (sec)	N/A	0.042	0.026	0.065	0.397	0.607	0.000	0.122	0.878

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	43	39	45	127	0	36	58
normalized size	1	1.00	1.54	1.39	1.61	4.54	0.00	1.29	2.07
time (sec)	N/A	0.068	0.088	0.088	0.802	0.459	0.000	0.147	0.895

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	61	73	325	0	48	73
normalized size	1	1.00	1.14	1.42	1.70	7.56	0.00	1.12	1.70
time (sec)	N/A	0.074	0.092	0.089	0.455	0.487	0.000	0.146	0.909

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	60	81	101	600	0	57	107
normalized size	1	1.00	1.07	1.45	1.80	10.71	0.00	1.02	1.91
time (sec)	N/A	0.077	0.195	0.092	0.409	0.522	0.000	0.140	0.897

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	14	14	18	22	17	15	15
normalized size	1	1.00	0.70	0.70	0.90	1.10	0.85	0.75	0.75
time (sec)	N/A	0.010	0.017	0.053	0.308	0.427	0.518	0.141	0.895

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	30	90	113	36	25	25
normalized size	1	1.00	0.72	0.64	1.91	2.40	0.77	0.53	0.53
time (sec)	N/A	0.022	0.031	0.062	0.308	0.449	1.005	0.129	0.059

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	43	205	174	51	36	36
normalized size	1	1.00	0.63	0.61	2.93	2.49	0.73	0.51	0.51
time (sec)	N/A	0.036	0.061	0.059	0.354	0.501	2.166	0.142	0.934

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	54	56	364	347	68	47	283
normalized size	1	1.00	0.58	0.60	3.91	3.73	0.73	0.51	3.04
time (sec)	N/A	0.054	0.087	0.054	0.487	0.522	5.187	0.138	0.919

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	16	18	24	32	15	15
normalized size	1	1.00	0.61	0.70	0.78	1.04	1.39	0.65	0.65
time (sec)	N/A	0.011	0.026	0.063	0.388	0.584	0.626	0.136	0.893

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	31	32	90	117	53	25	25
normalized size	1	1.00	0.61	0.63	1.76	2.29	1.04	0.49	0.49
time (sec)	N/A	0.025	0.030	0.073	0.879	0.768	1.226	0.119	0.059

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	41	45	205	174	70	36	36
normalized size	1	1.00	0.54	0.59	2.70	2.29	0.92	0.47	0.47
time (sec)	N/A	0.040	0.056	0.076	0.408	0.485	2.525	0.125	0.911

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	51	58	364	347	87	47	283
normalized size	1	1.00	0.50	0.57	3.60	3.44	0.86	0.47	2.80
time (sec)	N/A	0.057	0.080	0.076	0.332	0.544	5.607	0.133	0.085

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	92	114	62	0	46	-1
normalized size	1	1.00	0.67	1.80	2.24	1.22	0.00	0.90	-0.02
time (sec)	N/A	0.045	0.023	0.268	1.111	0.520	0.000	0.173	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	40	0	92	0	90	-1
normalized size	1	1.00	0.66	0.75	0.00	1.74	0.00	1.70	-0.02
time (sec)	N/A	0.049	0.031	0.338	0.000	0.553	0.000	0.159	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	121	327	0	105	-1
normalized size	1	1.00	0.80	0.82	1.36	3.67	0.00	1.18	-0.01
time (sec)	N/A	0.047	0.135	0.283	1.058	0.612	0.000	0.153	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	81	140	0	75	-1
normalized size	1	1.00	0.93	0.98	1.37	2.37	0.00	1.27	-0.02
time (sec)	N/A	0.029	0.067	0.169	0.606	0.460	0.000	0.128	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	40	41	0	35	26
normalized size	1	1.00	1.12	1.65	1.54	1.58	0.00	1.35	1.00
time (sec)	N/A	0.013	0.033	0.137	0.408	0.496	0.000	0.115	0.115

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	103	86	149	0	21	-1
normalized size	1	1.00	0.87	2.24	1.87	3.24	0.00	0.46	-0.02
time (sec)	N/A	0.023	0.017	0.189	0.472	0.777	0.000	0.142	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	144	170	219	0	67	-1
normalized size	1	1.00	0.82	1.87	2.21	2.84	0.00	0.87	-0.01
time (sec)	N/A	0.041	0.090	0.273	0.502	0.529	0.000	0.199	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	91	178	250	522	0	97	-1
normalized size	1	1.00	0.85	1.66	2.34	4.88	0.00	0.91	-0.01
time (sec)	N/A	0.062	0.283	0.286	0.512	0.419	0.000	0.270	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	72	71	190	328	0	189	-1
normalized size	1	1.00	0.78	0.77	2.07	3.57	0.00	2.05	-0.01
time (sec)	N/A	0.052	0.145	0.257	0.421	0.511	0.000	0.161	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	56	124	139	0	119	-1
normalized size	1	1.00	0.92	0.92	2.03	2.28	0.00	1.95	-0.02
time (sec)	N/A	0.031	0.094	0.250	0.434	0.515	0.000	0.162	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	41	58	42	0	61	27
normalized size	1	1.00	1.11	1.52	2.15	1.56	0.00	2.26	1.00
time (sec)	N/A	0.014	0.035	0.215	0.425	0.399	0.000	0.150	0.941

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	41	0	154	0	40	-1
normalized size	1	1.00	0.85	0.85	0.00	3.21	0.00	0.83	-0.02
time (sec)	N/A	0.025	0.032	0.219	0.000	0.526	0.000	0.140	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	87	0	274	0	107	-1
normalized size	1	1.00	1.08	1.10	0.00	3.47	0.00	1.35	-0.01
time (sec)	N/A	0.041	0.175	0.369	0.000	0.610	0.000	0.245	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	115	137	0	580	0	164	-1
normalized size	1	1.00	1.05	1.25	0.00	5.27	0.00	1.49	-0.01
time (sec)	N/A	0.063	0.199	0.336	0.000	1.254	0.000	0.301	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	264	0	1625	0	133	209
normalized size	1	1.00	0.88	2.36	0.00	14.51	0.00	1.19	1.87
time (sec)	N/A	0.309	0.200	0.067	0.000	0.801	0.000	0.125	1.263

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	174	0	903	0	92	167
normalized size	1	1.00	0.92	2.05	0.00	10.62	0.00	1.08	1.96
time (sec)	N/A	0.169	0.132	0.063	0.000	0.892	0.000	0.120	1.117

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	94	0	449	1275	62	139
normalized size	1	1.00	0.92	1.52	0.00	7.24	20.56	1.00	2.24
time (sec)	N/A	0.106	0.113	0.060	0.000	0.776	93.904	0.150	1.041

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	0	218	241	42	109
normalized size	1	1.00	0.92	1.23	0.00	4.19	4.63	0.81	2.10
time (sec)	N/A	0.053	0.048	0.057	0.000	0.485	25.019	0.122	0.219

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	227	0	45	286
normalized size	1	1.00	1.00	0.94	0.00	4.20	0.00	0.83	5.30
time (sec)	N/A	0.067	0.053	0.076	0.000	0.634	0.000	0.147	3.456

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	73	0	515	0	61	294
normalized size	1	1.00	0.98	1.14	0.00	8.05	0.00	0.95	4.59
time (sec)	N/A	0.117	0.110	0.087	0.000	0.652	0.000	0.145	3.098

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	146	0	1370	0	89	476
normalized size	1	1.00	0.94	1.68	0.00	15.75	0.00	1.02	5.47
time (sec)	N/A	0.300	0.210	0.091	0.000	0.850	0.000	0.146	4.126

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	101	239	0	2483	0	123	547
normalized size	1	1.00	0.89	2.10	0.00	21.78	0.00	1.08	4.80
time (sec)	N/A	0.472	0.427	0.112	0.000	1.112	0.000	0.120	4.533

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	133	155	273	190	314	263	160
normalized size	1	1.00	0.73	0.85	1.49	1.04	1.72	1.44	0.87
time (sec)	N/A	0.260	0.367	0.303	0.364	0.510	2.253	0.155	1.144

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	104	119	183	123	240	196	114
normalized size	1	1.00	0.76	0.87	1.34	0.90	1.75	1.43	0.83
time (sec)	N/A	0.151	0.223	0.254	0.300	0.871	1.102	0.132	0.191

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	80	77	116	78	128	131	73
normalized size	1	1.00	0.89	0.86	1.29	0.87	1.42	1.46	0.81
time (sec)	N/A	0.068	0.132	0.204	0.301	0.628	0.537	0.141	0.955

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	55	40	78	75	41
normalized size	1	1.00	0.92	1.02	1.10	0.80	1.56	1.50	0.82
time (sec)	N/A	0.017	0.079	0.074	0.504	0.519	0.265	0.137	0.923

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	32	15
normalized size	1	1.00	1.73	1.07	1.00	1.13	1.13	2.13	1.00
time (sec)	N/A	0.008	0.009	0.028	0.294	0.386	0.132	0.110	0.057

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	237	163	39	53
normalized size	1	1.00	0.98	0.90	0.00	4.84	3.33	0.80	1.08
time (sec)	N/A	0.035	0.053	0.065	0.000	0.584	4.620	0.135	1.225

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	118	0	743	0	99	215
normalized size	1	1.00	0.98	1.37	0.00	8.64	0.00	1.15	2.50
time (sec)	N/A	0.084	0.227	0.076	0.000	0.727	0.000	0.140	1.301

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	186	0	2591	0	195	-1
normalized size	1	1.00	0.85	1.40	0.00	19.48	0.00	1.47	-0.01
time (sec)	N/A	0.147	0.410	0.086	0.000	0.547	0.000	0.149	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	160	284	0	5705	0	329	-1
normalized size	1	1.00	0.87	1.54	0.00	31.01	0.00	1.79	-0.01
time (sec)	N/A	0.253	1.101	0.094	0.000	0.679	0.000	0.137	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	18	19	24	24	16	34
normalized size	1	1.00	1.05	0.82	0.86	1.09	1.09	0.73	1.55
time (sec)	N/A	0.015	0.039	0.063	0.408	0.526	0.777	0.135	0.125

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	48	64	147	316	54	74
normalized size	1	1.00	0.94	1.00	1.33	3.06	6.58	1.12	1.54
time (sec)	N/A	0.032	0.105	0.069	0.399	0.598	2.519	0.120	0.939

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	79	108	408	530	76	137
normalized size	1	1.00	0.75	1.08	1.48	5.59	7.26	1.04	1.88
time (sec)	N/A	0.065	0.165	0.072	0.447	0.457	6.592	0.120	0.959

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	65	110	152	793	809	98	223
normalized size	1	1.00	0.66	1.12	1.55	8.09	8.26	1.00	2.28
time (sec)	N/A	0.097	0.273	0.072	0.406	0.673	14.132	0.145	0.949

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	77	36	37	42	41	28	40
normalized size	1	1.00	2.48	1.16	1.19	1.35	1.32	0.90	1.29
time (sec)	N/A	0.013	0.032	0.085	0.662	1.604	0.633	0.119	0.943

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	45	72	81	212	199	65	77
normalized size	1	1.00	0.80	1.29	1.45	3.79	3.55	1.16	1.38
time (sec)	N/A	0.036	0.121	0.073	0.311	1.026	1.652	0.139	0.938

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	58	108	125	563	445	87	141
normalized size	1	1.00	0.72	1.33	1.54	6.95	5.49	1.07	1.74
time (sec)	N/A	0.063	0.199	0.072	0.335	0.495	3.669	0.125	0.954

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	68	144	169	1078	784	109	226
normalized size	1	1.00	0.64	1.36	1.59	10.17	7.40	1.03	2.13
time (sec)	N/A	0.097	0.265	0.084	0.336	0.660	7.954	0.122	0.114

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	150	685	0	0	0	0	-1
normalized size	1	1.00	0.98	4.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.547	0.494	0.000	0.687	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	458	0	0	0	0	-1
normalized size	1	1.00	0.90	3.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.244	0.527	0.000	1.232	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	276	0	0	0	0	-1
normalized size	1	1.00	1.00	4.52	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.093	0.431	0.000	0.809	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	146	0	0	0	0	-1
normalized size	1	1.00	1.00	3.17	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.039	0.358	0.000	1.323	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	296	0	0	0	0	-1
normalized size	1	1.00	0.81	3.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.128	0.498	0.000	0.881	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	135	459	0	0	0	0	-1
normalized size	1	1.00	0.76	2.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.574	0.842	0.000	0.885	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	165	566	0	0	0	0	-1
normalized size	1	1.00	0.73	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.719	1.115	0.000	1.014	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	181	0	0	0	0	-1
normalized size	1	1.00	0.73	1.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.383	0.422	0.000	0.751	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	60	71	237	563	0	153	-1
normalized size	1	1.00	0.64	0.76	2.52	5.99	0.00	1.63	-0.01
time (sec)	N/A	0.092	0.135	0.224	0.494	0.700	0.000	0.145	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	57	163	279	0	113	-1
normalized size	1	1.00	0.68	0.84	2.40	4.10	0.00	1.66	-0.01
time (sec)	N/A	0.074	0.093	0.205	0.463	0.510	0.000	0.128	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	90	100	0	71	-1
normalized size	1	1.00	0.78	0.98	2.25	2.50	0.00	1.78	-0.02
time (sec)	N/A	0.052	0.040	0.234	0.457	0.495	0.000	0.145	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	61	69	288	564	0	295	-1
normalized size	1	1.00	0.62	0.70	2.94	5.76	0.00	3.01	-0.01
time (sec)	N/A	0.104	0.148	0.261	0.468	0.517	0.000	0.191	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	47	55	199	279	0	212	-1
normalized size	1	1.00	0.66	0.77	2.80	3.93	0.00	2.99	-0.01
time (sec)	N/A	0.080	0.106	0.316	0.466	0.524	0.000	0.153	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	32	39	109	107	0	131	-1
normalized size	1	1.00	0.73	0.89	2.48	2.43	0.00	2.98	-0.02
time (sec)	N/A	0.058	0.055	0.270	0.465	0.494	0.000	0.151	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	23	34	26	29	15	17	19
normalized size	1	1.00	1.28	1.89	1.44	1.61	0.83	0.94	1.06
time (sec)	N/A	0.036	0.059	0.043	0.311	0.837	0.336	0.118	0.046

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	34	129	50	36	30	30
normalized size	1	1.00	0.71	0.97	3.69	1.43	1.03	0.86	0.86
time (sec)	N/A	0.038	0.059	0.044	0.322	0.845	0.612	0.118	0.076

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	42	38	263	127	46	46	141
normalized size	1	1.00	0.75	0.68	4.70	2.27	0.82	0.82	2.52
time (sec)	N/A	0.048	0.095	0.040	0.336	1.104	1.198	0.116	0.920

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	55	449	175	78	60	231
normalized size	1	1.00	0.76	0.73	5.99	2.33	1.04	0.80	3.08
time (sec)	N/A	0.059	0.111	0.040	0.340	0.731	2.362	0.120	0.917

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	35	37	27	31	15	16	19
normalized size	1	1.00	1.75	1.85	1.35	1.55	0.75	0.80	0.95
time (sec)	N/A	0.040	0.058	0.059	0.312	0.722	0.488	0.130	0.053

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	26	131	48	36	32	32
normalized size	1	1.00	0.68	0.70	3.54	1.30	0.97	0.86	0.86
time (sec)	N/A	0.041	0.059	0.059	0.322	1.204	0.842	0.114	0.933

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	39	267	127	46	46	143
normalized size	1	1.00	0.70	0.65	4.45	2.12	0.77	0.77	2.38
time (sec)	N/A	0.055	0.085	0.061	0.331	1.487	1.461	0.113	0.081

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	57	56	451	175	78	60	233
normalized size	1	1.00	0.70	0.69	5.57	2.16	0.96	0.74	2.88
time (sec)	N/A	0.066	0.104	0.062	0.341	0.837	2.725	0.116	0.917

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	41	128	174	72	0	61	-1
normalized size	1	1.00	0.73	2.29	3.11	1.29	0.00	1.09	-0.02
time (sec)	N/A	0.066	0.037	0.321	0.622	3.967	0.000	0.153	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	159	300	189	0	78	-1
normalized size	1	1.00	0.68	2.45	4.62	2.91	0.00	1.20	-0.02
time (sec)	N/A	0.068	0.084	0.331	0.577	1.036	0.000	0.176	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	57	209	427	509	0	118	-1
normalized size	1	1.00	0.61	2.25	4.59	5.47	0.00	1.27	-0.01
time (sec)	N/A	0.089	0.170	0.406	0.929	0.854	0.000	0.225	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	63	0	99	0	103	-1
normalized size	1	1.00	0.70	1.11	0.00	1.74	0.00	1.81	-0.02
time (sec)	N/A	0.065	0.060	0.370	0.000	0.863	0.000	0.157	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	71	83	0	217	0	111	-1
normalized size	1	1.00	1.09	1.28	0.00	3.34	0.00	1.71	-0.02
time (sec)	N/A	0.073	0.162	0.342	0.000	0.900	0.000	0.188	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	118	0	548	0	189	-1
normalized size	1	1.00	1.15	1.26	0.00	5.83	0.00	2.01	-0.01
time (sec)	N/A	0.096	0.387	0.351	0.000	1.124	0.000	0.240	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	203	1365	0	0	0	0	-1
normalized size	1	1.00	0.87	5.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.626	0.553	0.000	0.778	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	124	973	0	0	0	0	-1
normalized size	1	1.00	0.69	5.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	0.705	0.578	0.000	0.606	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	123	605	0	0	0	0	-1
normalized size	1	1.00	0.89	4.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.354	0.525	0.000	1.430	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	103	0	240	403	50	242
normalized size	1	1.00	0.98	1.72	0.00	4.00	6.72	0.83	4.03
time (sec)	N/A	0.068	0.103	0.062	0.000	0.771	27.029	0.121	1.123

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	81	108	0	828	0	107	246
normalized size	1	1.00	0.99	1.32	0.00	10.10	0.00	1.30	3.00
time (sec)	N/A	0.078	0.201	0.065	0.000	1.380	0.000	0.119	1.420

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	134	207	0	3166	0	249	-1
normalized size	1	1.00	0.99	1.53	0.00	23.45	0.00	1.84	-0.01
time (sec)	N/A	0.169	0.428	0.069	0.000	0.819	0.000	0.135	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	196	342	0	7603	0	453	-1
normalized size	1	1.00	0.99	1.74	0.00	38.59	0.00	2.30	-0.01
time (sec)	N/A	0.348	0.830	0.076	0.000	1.989	0.000	0.154	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	107	0	190	170	57	205
normalized size	1	1.00	1.00	1.91	0.00	3.39	3.04	1.02	3.66
time (sec)	N/A	0.078	0.075	0.067	0.000	1.796	28.168	0.139	0.494

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	6	3	6	6
normalized size	1	1.00	1.00	1.17	0.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.001	0.000	0.006	0.000	0.697	0.296	0.124	0.020

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	29	0	54	26	26	51
normalized size	1	1.00	1.00	2.64	0.00	4.91	2.36	2.36	4.64
time (sec)	N/A	0.029	0.054	0.056	0.000	1.783	146.491	0.129	1.020

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	32	34	45	44	37	48
normalized size	1	1.00	0.67	0.89	0.94	1.25	1.22	1.03	1.33
time (sec)	N/A	0.050	0.077	0.063	0.408	2.705	0.793	0.138	0.111

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	80	218	0	0	0	0	-1
normalized size	1	1.00	0.74	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.484	0.458	0.000	0.719	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	133	483	0	0	0	0	-1
normalized size	1	1.00	0.88	3.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.381	1.046	0.000	0.767	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	172	797	0	0	0	0	-1
normalized size	1	1.00	0.74	3.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.350	0.922	1.551	0.000	0.965	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	42	38	71	817	0	79	-1
normalized size	1	1.00	0.58	0.53	0.99	11.35	0.00	1.10	-0.01
time (sec)	N/A	0.055	0.027	0.164	0.430	2.764	0.000	0.120	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	53	501	0	61	-1
normalized size	1	1.00	0.68	0.60	1.00	9.45	0.00	1.15	-0.02
time (sec)	N/A	0.036	0.019	0.177	0.441	0.862	0.000	0.119	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	35	222	0	29	-1
normalized size	1	1.00	0.76	0.71	1.03	6.53	0.00	0.85	-0.03
time (sec)	N/A	0.023	0.009	0.178	0.419	0.808	0.000	0.121	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	69	19	14	17
normalized size	1	1.00	1.00	1.15	1.31	5.31	1.46	1.08	1.31
time (sec)	N/A	0.012	0.004	0.146	0.423	1.055	0.450	0.127	0.053

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	55	8	186	0	0	-1
normalized size	1	1.00	1.31	3.44	0.50	11.62	0.00	0.00	-0.06
time (sec)	N/A	0.015	0.007	0.218	0.460	1.321	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	82	41	299	0	56	-1
normalized size	1	1.00	0.74	1.95	0.98	7.12	0.00	1.33	-0.02
time (sec)	N/A	0.025	0.016	0.300	0.460	1.263	0.000	0.141	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	40	102	75	837	0	67	-1
normalized size	1	1.00	0.66	1.67	1.23	13.72	0.00	1.10	-0.02
time (sec)	N/A	0.039	0.037	0.304	0.468	1.000	0.000	0.166	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	65	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.121	0.192	0.000	1.704	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.075	0.217	0.000	1.980	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	0	0	0	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.046	0.229	0.000	0.846	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.023	0.181	0.000	0.999	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	48	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.064	0.172	0.000	1.769	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	61	0	0	0	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.109	0.176	0.000	1.652	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	53	177	100	1597	0	114	-1
normalized size	1	1.00	0.40	1.34	0.76	12.10	0.00	0.86	-0.01
time (sec)	N/A	0.053	0.125	0.593	0.414	0.867	0.000	0.126	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	38	131	62	659	0	52	-1
normalized size	1	1.00	0.49	1.68	0.79	8.45	0.00	0.67	-0.01
time (sec)	N/A	0.035	0.070	0.500	0.415	0.830	0.000	0.144	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	89	27	180	0	28	-1
normalized size	1	1.00	0.69	2.47	0.75	5.00	0.00	0.78	-0.03
time (sec)	N/A	0.016	0.015	0.446	0.431	0.853	0.000	0.150	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	56	16	116	0	13	39
normalized size	1	1.00	1.00	3.73	1.07	7.73	0.00	0.87	2.60
time (sec)	N/A	0.017	0.006	0.372	0.422	2.328	0.000	0.133	0.065

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	30	80	165	1137	0	27	48
normalized size	1	1.00	0.45	1.19	2.46	16.97	0.00	0.40	0.72
time (sec)	N/A	0.025	0.028	0.386	0.427	1.329	0.000	0.161	0.970

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	47	96	457	3065	0	39	256
normalized size	1	1.00	0.40	0.82	3.91	26.20	0.00	0.33	2.19
time (sec)	N/A	0.035	0.050	0.361	0.426	0.642	0.000	0.209	0.986

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	8
normalized size	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.00
time (sec)	N/A	0.021	0.011	0.031	0.301	1.535	0.335	0.142	0.074

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	8
normalized size	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.00
time (sec)	N/A	0.022	0.010	0.033	0.305	0.401	0.543	0.124	0.908

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	24	12	20	7	10	10
normalized size	1	1.00	1.50	2.00	1.00	1.67	0.58	0.83	0.83
time (sec)	N/A	0.031	0.007	0.049	0.309	2.085	0.545	0.130	0.897

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	26	12	22	7	10	10
normalized size	1	1.00	1.71	1.86	0.86	1.57	0.50	0.71	0.71
time (sec)	N/A	0.031	0.011	0.071	0.309	2.154	1.045	0.158	0.045

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	23	48	58	21	10
normalized size	1	1.00	1.30	1.10	2.30	4.80	5.80	2.10	1.00
time (sec)	N/A	0.038	0.020	0.041	0.303	2.698	0.519	0.122	0.940

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	23	54	58	22	10
normalized size	1	1.00	1.08	0.92	1.92	4.50	4.83	1.83	0.83
time (sec)	N/A	0.037	0.018	0.062	0.314	1.087	0.528	0.131	0.944

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	55	15	12	8
normalized size	1	1.00	1.20	0.90	0.80	5.50	1.50	1.20	0.80
time (sec)	N/A	0.021	0.010	0.030	0.295	0.816	0.635	0.120	0.920

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	8	55	14	12	8
normalized size	1	1.00	1.00	0.92	0.67	4.58	1.17	1.00	0.67
time (sec)	N/A	0.021	0.012	0.029	0.296	1.710	0.583	0.132	0.076

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	49	33	7	16	16
normalized size	1	1.00	0.86	0.64	3.50	2.36	0.50	1.14	1.14
time (sec)	N/A	0.032	0.028	0.050	0.309	0.496	0.960	0.128	0.925

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	49	33	8	16	16
normalized size	1	1.00	0.75	0.56	3.06	2.06	0.50	1.00	1.00
time (sec)	N/A	0.033	0.029	0.056	0.311	3.354	1.492	0.140	0.922

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	20	15	31	89	126	21	14
normalized size	1	1.00	1.43	1.07	2.21	6.36	9.00	1.50	1.00
time (sec)	N/A	0.040	0.010	0.051	0.302	0.553	0.591	0.123	0.984

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	27	17	35	90	126	20	16
normalized size	1	1.00	1.35	0.85	1.75	4.50	6.30	1.00	0.80
time (sec)	N/A	0.040	0.014	0.075	0.304	0.697	0.598	0.144	0.994

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	208	102	101	1253	90	131
normalized size	1	1.00	0.89	3.65	1.79	1.77	21.98	1.58	2.30
time (sec)	N/A	0.059	0.072	0.096	0.310	1.115	8.767	0.122	1.262

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	107	84	94	284	75	107
normalized size	1	1.00	0.59	2.33	1.83	2.04	6.17	1.63	2.33
time (sec)	N/A	0.065	0.033	0.089	0.316	1.383	5.623	0.118	1.133

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	156	78	57	692	66	95
normalized size	1	1.00	0.89	3.55	1.77	1.30	15.73	1.50	2.16
time (sec)	N/A	0.054	0.055	0.085	0.306	1.039	3.712	0.133	1.045

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	87	60	52	150	51	71
normalized size	1	1.00	0.64	2.64	1.82	1.58	4.55	1.55	2.15
time (sec)	N/A	0.056	0.022	0.082	0.315	0.629	2.220	0.126	0.979

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	103	54	27	294	40	59
normalized size	1	1.00	0.81	3.32	1.74	0.87	9.48	1.29	1.90
time (sec)	N/A	0.046	0.037	0.080	0.321	2.017	1.303	0.138	0.942

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	13	47	36	18	49	27	35
normalized size	1	1.00	0.68	2.47	1.89	0.95	2.58	1.42	1.84
time (sec)	N/A	0.043	0.013	0.072	0.313	0.729	0.734	0.133	0.936

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	51	23	11	46	17	23
normalized size	1	1.00	1.31	3.92	1.77	0.85	3.54	1.31	1.77
time (sec)	N/A	0.039	0.009	0.070	0.311	1.689	0.410	0.117	0.913

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	12	11	16	7	17	9
normalized size	1	1.00	1.33	1.33	1.22	1.78	0.78	1.89	1.00
time (sec)	N/A	0.024	0.006	0.036	0.304	0.784	0.132	0.160	0.888

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	42	23	47	103	0	52	51
normalized size	1	1.00	1.83	1.00	2.04	4.48	0.00	2.26	2.22
time (sec)	N/A	0.052	0.032	0.072	0.300	1.748	0.000	0.153	0.928

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	29	59	94	0	35	89
normalized size	1	1.00	1.25	1.21	2.46	3.92	0.00	1.46	3.71
time (sec)	N/A	0.048	0.050	0.081	0.304	1.617	0.000	0.153	0.919

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	45	103	631	0	94	114
normalized size	1	1.00	1.22	0.92	2.10	12.88	0.00	1.92	2.33
time (sec)	N/A	0.081	0.170	0.092	0.312	0.867	0.000	0.178	0.933

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	45	233	250	0	59	263
normalized size	1	1.00	1.03	1.22	6.30	6.76	0.00	1.59	7.11
time (sec)	N/A	0.051	0.057	0.092	0.314	1.091	0.000	0.147	0.953

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	89	67	155	1551	0	116	244
normalized size	1	1.00	1.14	0.86	1.99	19.88	0.00	1.49	3.13
time (sec)	N/A	0.106	0.287	0.096	0.326	0.827	0.000	0.136	1.048

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	144	1039	310	2134	0	229	289
normalized size	1	1.00	1.03	7.42	2.21	15.24	0.00	1.64	2.06
time (sec)	N/A	0.167	0.187	0.078	0.321	0.839	0.000	0.169	1.715

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	679	0	2913	0	266	348
normalized size	1	1.00	1.00	4.41	0.00	18.92	0.00	1.73	2.26
time (sec)	N/A	0.425	0.244	0.085	0.000	1.508	0.000	0.164	1.701

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	84	599	178	866	0	124	169
normalized size	1	1.00	1.01	7.22	2.14	10.43	0.00	1.49	2.04
time (sec)	N/A	0.107	0.115	0.069	0.335	1.029	0.000	0.129	1.310

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	95	338	0	1099	0	146	222
normalized size	1	1.00	0.91	3.25	0.00	10.57	0.00	1.40	2.13
time (sec)	N/A	0.241	0.190	0.085	0.000	2.778	0.000	0.125	1.310

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	283	84	234	0	56	79
normalized size	1	1.00	1.00	7.08	2.10	5.85	0.00	1.40	1.98
time (sec)	N/A	0.069	0.060	0.060	0.312	1.416	0.000	0.127	1.044

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	129	0	279	892	68	139
normalized size	1	1.00	0.92	2.19	0.00	4.73	15.12	1.15	2.36
time (sec)	N/A	0.112	0.086	0.062	0.000	1.288	93.140	0.133	1.045

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	19	11
normalized size	1	1.00	1.00	1.09	1.00	2.45	1.27	1.73	1.00
time (sec)	N/A	0.027	0.016	0.029	0.301	0.843	0.312	0.142	0.058

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	37	52	59	58	0	67	160
normalized size	1	1.00	0.70	0.98	1.11	1.09	0.00	1.26	3.02
time (sec)	N/A	0.077	0.071	0.072	0.378	0.620	0.000	0.149	1.286

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	78	0	470	0	76	327
normalized size	1	1.00	1.15	1.16	0.00	7.01	0.00	1.13	4.88
time (sec)	N/A	0.090	0.212	0.083	0.000	0.602	0.000	0.139	1.479

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	100	97	154	818	0	179	291
normalized size	1	1.00	1.10	1.07	1.69	8.99	0.00	1.97	3.20
time (sec)	N/A	0.159	0.280	0.086	0.351	0.555	0.000	0.146	1.477

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	141	127	0	2339	0	156	642
normalized size	1	1.00	1.28	1.15	0.00	21.26	0.00	1.42	5.84
time (sec)	N/A	0.246	0.571	0.092	0.000	0.496	0.000	0.132	1.975

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	148	191	348	3450	0	338	559
normalized size	1	1.00	0.98	1.26	2.30	22.85	0.00	2.24	3.70
time (sec)	N/A	0.254	0.910	0.089	0.402	0.591	0.000	0.161	1.831

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	201	213	0	6381	0	303	1031
normalized size	1	1.00	1.26	1.34	0.00	40.13	0.00	1.91	6.48
time (sec)	N/A	0.477	1.813	0.094	0.000	0.649	0.000	0.187	2.605

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	99	0	700	0	68	139
normalized size	1	1.00	0.91	1.48	0.00	10.45	0.00	1.01	2.07
time (sec)	N/A	0.104	0.109	0.068	0.000	0.585	0.000	0.152	1.123

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	315	0	2003	0	144	722
normalized size	1	1.00	0.88	2.79	0.00	17.73	0.00	1.27	6.39
time (sec)	N/A	0.406	0.444	0.098	0.000	1.575	0.000	0.155	5.941

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	140	96	450	0	115	1221
normalized size	1	1.00	0.81	2.46	1.68	7.89	0.00	2.02	21.42
time (sec)	N/A	0.099	0.104	0.095	0.443	0.710	0.000	0.128	1.610

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	108	0	326	0	67	285
normalized size	1	1.00	1.00	1.77	0.00	5.34	0.00	1.10	4.67
time (sec)	N/A	0.232	0.121	0.086	0.000	0.632	0.000	0.130	3.533

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	33	40	0	33	201
normalized size	1	1.00	1.00	1.05	1.65	2.00	0.00	1.65	10.05
time (sec)	N/A	0.042	0.009	0.073	0.453	1.435	0.000	0.122	0.425

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	53	59	60	0	67	148
normalized size	1	1.00	0.70	0.98	1.09	1.11	0.00	1.24	2.74
time (sec)	N/A	0.070	0.074	0.086	0.303	0.556	0.000	0.123	0.427

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	78	0	470	0	76	337
normalized size	1	1.00	1.00	1.01	0.00	6.10	0.00	0.99	4.38
time (sec)	N/A	0.094	0.207	0.093	0.000	0.415	0.000	0.145	1.338

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	101	97	156	839	0	178	291
normalized size	1	1.00	1.07	1.03	1.66	8.93	0.00	1.89	3.10
time (sec)	N/A	0.199	0.225	0.096	0.322	0.437	0.000	0.128	1.519

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	127	0	2417	0	172	666
normalized size	1	1.00	0.96	0.93	0.00	17.64	0.00	1.26	4.86
time (sec)	N/A	0.196	0.542	0.109	0.000	0.635	0.000	0.147	1.806

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	58	115	89	750	0	58	183
normalized size	1	1.00	1.26	2.50	1.93	16.30	0.00	1.26	3.98
time (sec)	N/A	0.093	0.091	0.128	0.498	0.525	0.000	0.149	1.062

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	30	223	174	0	48	117
normalized size	1	1.00	0.83	1.00	7.43	5.80	0.00	1.60	3.90
time (sec)	N/A	0.086	0.030	0.112	0.369	0.464	0.000	0.148	1.013

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	71	57	315	0	39	95
normalized size	1	1.00	1.39	2.15	1.73	9.55	0.00	1.18	2.88
time (sec)	N/A	0.080	0.065	0.099	0.424	0.482	0.000	0.131	0.962

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	18	70	66	0	22	25
normalized size	1	1.00	0.89	0.95	3.68	3.47	0.00	1.16	1.32
time (sec)	N/A	0.066	0.023	0.101	0.342	0.637	0.000	0.129	0.919

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	31	23	50	0	22	33
normalized size	1	1.00	1.20	2.07	1.53	3.33	0.00	1.47	2.20
time (sec)	N/A	0.049	0.050	0.085	0.434	0.539	0.000	0.141	0.921

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	12	19	24	28	0	22	26
normalized size	1	1.00	0.67	1.06	1.33	1.56	0.00	1.22	1.44
time (sec)	N/A	0.040	0.019	0.084	0.330	0.520	0.000	0.140	0.076

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	42	23	48	103	0	52	51
normalized size	1	1.00	1.27	0.70	1.45	3.12	0.00	1.58	1.55
time (sec)	N/A	0.065	0.043	0.099	0.328	0.530	0.000	0.128	0.922

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	29	121	91	0	35	92
normalized size	1	1.00	0.83	0.97	4.03	3.03	0.00	1.17	3.07
time (sec)	N/A	0.079	0.052	0.090	0.333	0.447	0.000	0.148	0.927

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	60	45	103	631	0	94	132
normalized size	1	1.00	1.30	0.98	2.24	13.72	0.00	2.04	2.87
time (sec)	N/A	0.107	0.134	0.101	0.335	0.526	0.000	0.148	0.962

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	45	469	224	0	59	263
normalized size	1	1.00	1.00	1.10	11.44	5.46	0.00	1.44	6.41
time (sec)	N/A	0.081	0.078	0.099	0.345	0.436	0.000	0.159	1.029

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	0	376	0	0	-1
normalized size	1	1.00	1.00	0.81	0.00	10.16	0.00	0.00	-0.03
time (sec)	N/A	0.063	0.024	0.063	0.000	0.722	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	356	0	0	-1
normalized size	1	1.00	1.00	0.79	0.00	14.83	0.00	0.00	-0.04
time (sec)	N/A	0.057	0.012	0.074	0.000	0.652	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	137	0	291	741	60	197
normalized size	1	1.00	0.98	2.45	0.00	5.20	13.23	1.07	3.52
time (sec)	N/A	0.128	0.091	0.067	0.000	0.549	27.721	0.134	2.982

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	28	19	46	20	22	22
normalized size	1	1.00	1.06	1.56	1.06	2.56	1.11	1.22	1.22
time (sec)	N/A	0.077	0.035	0.047	0.373	0.486	0.342	0.121	0.056

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	36	20	48	31	22	21
normalized size	1	1.00	0.79	1.50	0.83	2.00	1.29	0.92	0.88
time (sec)	N/A	0.086	0.050	0.071	0.374	0.483	0.507	0.123	0.909

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	125	0	315	0	66	160
normalized size	1	1.00	0.94	1.92	0.00	4.85	0.00	1.02	2.46
time (sec)	N/A	0.148	0.158	0.102	0.000	0.581	0.000	0.124	12.141

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	81	139	0	303	0	90	974
normalized size	1	1.00	0.81	1.39	0.00	3.03	0.00	0.90	9.74
time (sec)	N/A	0.166	0.262	0.106	0.000	2.754	0.000	0.129	3.679

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	63	89	0	249	0	53	636
normalized size	1	1.00	1.02	1.44	0.00	4.02	0.00	0.85	10.26
time (sec)	N/A	0.133	0.125	0.095	0.000	0.984	0.000	0.126	6.413

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	138	0	298	0	90	983
normalized size	1	1.00	0.82	1.39	0.00	3.01	0.00	0.91	9.93
time (sec)	N/A	0.305	0.200	0.115	0.000	3.246	0.000	0.154	3.328

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	276	0	405	695	97	653
normalized size	1	1.00	0.94	3.21	0.00	4.71	8.08	1.13	7.59
time (sec)	N/A	0.154	0.256	0.173	0.000	0.474	31.240	0.158	2.188

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	115	144	0	1044	0	161	301
normalized size	1	1.00	0.95	1.19	0.00	8.63	0.00	1.33	2.49
time (sec)	N/A	0.176	0.446	0.172	0.000	0.754	0.000	0.185	1.588

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	175	273	0	3636	0	387	-1
normalized size	1	1.00	0.94	1.46	0.00	19.44	0.00	2.07	-0.01
time (sec)	N/A	0.264	0.849	0.166	0.000	0.658	0.000	0.212	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	245	459	0	8531	0	688	-1
normalized size	1	1.00	0.94	1.77	0.00	32.81	0.00	2.65	-0.00
time (sec)	N/A	0.448	2.574	0.176	0.000	0.690	0.000	0.246	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	536	487	0	780	0	0	-1
normalized size	1	1.00	2.81	2.55	0.00	4.08	0.00	0.00	-0.01
time (sec)	N/A	0.377	0.634	0.129	0.000	0.558	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	221	686	0	1162	0	0	-1
normalized size	1	1.00	0.76	2.36	0.00	3.99	0.00	0.00	-0.00
time (sec)	N/A	0.567	0.774	0.124	0.000	1.380	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	295	889	0	1542	0	0	-1
normalized size	1	1.00	0.75	2.27	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	0.599	0.693	0.126	0.000	0.569	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.119	0.084	0.602	0.000	0.720	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.038	0.346	0.000	0.924	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.035	0.197	0.000	0.715	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	6.337	0.201	0.000	0.565	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	26.141	0.198	0.000	0.444	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	138	0	480	0	0	110
normalized size	1	1.00	0.98	2.30	0.00	8.00	0.00	0.00	1.83
time (sec)	N/A	0.057	0.136	0.176	0.000	0.504	0.000	0.000	1.091

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	231	0	1692	0	0	-1
normalized size	1	1.00	1.00	2.66	0.00	19.45	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.264	0.229	0.000	0.522	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	-1
normalized size	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	-0.02
time (sec)	N/A	0.462	0.113	0.292	0.433	0.500	0.000	0.142	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	5.528	0.140	0.000	2.070	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	326	0	0	624	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.036	0.562	0.000	0.570	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	244	0	0	497	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.023	0.533	0.000	0.654	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	160	368	0	354	0	0	-1
normalized size	1	1.00	0.99	2.29	0.00	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.013	0.210	0.000	1.633	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	44	41	31	18
normalized size	1	1.00	1.00	1.06	1.00	2.44	2.28	1.72	1.00
time (sec)	N/A	0.032	0.038	0.027	0.319	1.003	0.970	0.144	0.072

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	14.214	0.385	0.000	0.613	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	25.213	0.304	0.000	0.526	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	386	0	0	1174	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	2.37	0.00	0.00	-0.00
time (sec)	N/A	0.839	1.513	0.422	0.000	0.687	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	293	0	0	937	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	2.53	0.00	0.00	-0.00
time (sec)	N/A	0.704	1.305	0.426	0.000	1.701	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	187	862	0	669	0	0	-1
normalized size	1	1.00	0.77	3.53	0.00	2.74	0.00	0.00	-0.00
time (sec)	N/A	0.418	1.030	0.287	0.000	0.733	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	177	0	415	1122	89	176
normalized size	1	1.00	0.95	2.42	0.00	5.68	15.37	1.22	2.41
time (sec)	N/A	0.123	0.180	0.079	0.000	0.502	122.503	0.140	1.115

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	116.728	0.290	0.000	0.583	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	32.313	0.209	0.000	0.444	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	1082	0	0	2025	0	0	-1
normalized size	1	1.00	1.85	0.00	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	0.686	11.572	0.743	0.000	0.529	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	831	0	0	1622	0	0	-1
normalized size	1	1.00	1.92	0.00	0.00	3.75	0.00	0.00	-0.00
time (sec)	N/A	0.563	8.423	0.655	0.000	0.698	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	414	860	0	1196	0	0	-1
normalized size	1	1.00	1.44	2.99	0.00	4.15	0.00	0.00	-0.00
time (sec)	N/A	0.336	2.961	0.300	0.000	0.526	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	415	130	340	0	88	122
normalized size	1	1.00	0.90	6.80	2.13	5.57	0.00	1.44	2.00
time (sec)	N/A	0.073	0.110	0.081	0.328	0.645	0.000	0.150	1.065

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	180.000	0.505	0.000	1.352	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	0	51	44	0	47	44
normalized size	1	1.00	0.76	0.00	0.94	0.81	0.00	0.87	0.81
time (sec)	N/A	0.012	0.072	0.100	0.337	0.970	0.000	0.124	0.995

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	0	67	90	0	169	53
normalized size	1	1.00	0.64	0.00	0.76	1.02	0.00	1.92	0.60
time (sec)	N/A	0.020	0.106	0.498	0.343	0.535	0.000	0.174	1.006

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	117	0	115	199	0	665	94
normalized size	1	1.00	0.79	0.00	0.77	1.34	0.00	4.46	0.63
time (sec)	N/A	0.039	0.562	0.502	0.382	0.495	0.000	0.218	1.047

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	129	293	0	777	102
normalized size	1	1.00	0.87	0.00	0.68	1.53	0.00	4.07	0.53
time (sec)	N/A	0.051	0.448	0.545	0.397	0.564	0.000	0.238	1.038

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	0	64	99	0	235	55
normalized size	1	1.00	0.74	0.00	0.88	1.36	0.00	3.22	0.75
time (sec)	N/A	0.023	0.141	0.083	0.362	0.469	0.000	0.163	1.045

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	87	0	87	250	0	759	73
normalized size	1	1.00	0.72	0.00	0.72	2.08	0.00	6.32	0.61
time (sec)	N/A	0.048	0.303	0.428	0.353	0.618	0.000	0.218	1.088

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	197	292	0	138	584	0	3225	117
normalized size	1	0.97	1.44	0.00	0.68	2.88	0.00	15.89	0.58
time (sec)	N/A	0.083	1.480	0.439	0.923	0.520	0.000	0.285	1.182

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	260	311	0	161	1123	0	6880	134
normalized size	1	0.98	1.17	0.00	0.61	4.22	0.00	25.86	0.50
time (sec)	N/A	0.128	3.445	0.467	0.484	0.452	0.000	0.413	1.195

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	41	42	18
normalized size	1	1.00	2.06	1.06	1.00	1.06	2.28	2.33	1.00
time (sec)	N/A	0.016	0.014	0.049	0.473	0.482	0.980	0.122	1.027

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	49	39	0	80	32
normalized size	1	1.00	0.92	1.33	1.26	1.00	0.00	2.05	0.82
time (sec)	N/A	0.031	0.025	0.084	0.313	0.426	0.000	0.130	1.046

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	86	53	87	81	35
normalized size	1	1.00	1.00	0.86	2.05	1.26	2.07	1.93	0.83
time (sec)	N/A	0.032	0.010	0.213	0.328	0.525	10.569	0.156	1.049

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	84	93	84	0	114	50
normalized size	1	1.00	0.70	1.15	1.27	1.15	0.00	1.56	0.68
time (sec)	N/A	0.047	0.047	0.228	0.325	0.530	0.000	0.148	1.107

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	51	130	105	128	116	49
normalized size	1	1.00	1.00	0.78	2.00	1.62	1.97	1.78	0.75
time (sec)	N/A	0.037	0.020	0.215	0.323	0.555	97.056	0.170	1.165

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	62	256	0	0	0	0	-1
normalized size	1	1.00	0.93	3.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.056	0.434	0.000	0.564	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	114	237	0	0	0	0	-1
normalized size	1	1.00	1.70	3.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.131	0.418	0.000	0.611	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	0	0	0	-1
normalized size	1	1.00	1.00	6.54	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.023	0.333	0.000	0.572	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	0	0	0	-1
normalized size	1	1.00	1.00	6.54	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.021	0.299	0.000	0.768	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	141	0	0	0	0	-1
normalized size	1	1.00	0.92	2.24	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.059	0.431	0.000	0.505	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	122	295	0	0	0	0	-1
normalized size	1	1.00	1.82	4.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.087	0.395	0.000	0.451	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	85	0	0	187	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.485	0.656	0.000	0.491	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	74	0	0	141	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.331	0.482	0.000	0.432	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	61	0	0	68	0	0	-1
normalized size	1	1.00	1.45	0.00	0.00	1.62	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.156	0.474	0.000	0.492	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	121	0	0	128	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.267	0.472	0.000	0.585	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	373	347	0	171	0	764	-1
normalized size	1	1.00	3.69	3.44	0.00	1.69	0.00	7.56	-0.01
time (sec)	N/A	0.177	0.369	0.125	0.000	0.521	0.000	5.288	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	358	0	366	0	749	-1
normalized size	1	1.00	1.04	3.35	0.00	3.42	0.00	7.00	-0.01
time (sec)	N/A	0.194	0.309	0.419	0.000	0.641	0.000	15.289	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	45	68	113	139	60	58
normalized size	1	1.00	0.75	0.54	0.82	1.36	1.67	0.72	0.70
time (sec)	N/A	0.039	0.047	0.217	0.320	0.519	54.778	0.122	0.502

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	49	53	95	207	57	42
normalized size	1	1.00	0.82	0.86	0.93	1.67	3.63	1.00	0.74
time (sec)	N/A	0.037	0.041	0.227	0.316	0.478	15.947	0.140	0.262

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	35	40	54	78	34	34
normalized size	1	1.00	0.80	0.71	0.82	1.10	1.59	0.69	0.69
time (sec)	N/A	0.029	0.023	0.213	0.306	0.935	4.342	0.137	0.965

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	24	50	63	22	18
normalized size	1	1.00	1.00	1.61	1.04	2.17	2.74	0.96	0.78
time (sec)	N/A	0.015	0.013	0.043	0.313	0.630	1.020	0.115	0.921

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	16	30	0	16	16
normalized size	1	1.00	1.00	1.12	0.94	1.76	0.00	0.94	0.94
time (sec)	N/A	0.017	0.015	0.029	0.419	0.427	0.000	0.115	0.922

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	27	37	105	0	35	48
normalized size	1	1.00	0.90	0.68	0.92	2.62	0.00	0.88	1.20
time (sec)	N/A	0.029	0.067	0.075	0.411	0.519	0.000	0.140	0.079

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	68	86	0	31	31
normalized size	1	1.00	1.00	0.76	2.34	2.97	0.00	1.07	1.07
time (sec)	N/A	0.027	0.019	0.213	0.312	0.685	0.000	0.118	0.927

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	64	43	83	513	0	60	130
normalized size	1	1.00	0.67	0.45	0.87	5.40	0.00	0.63	1.37
time (sec)	N/A	0.047	0.091	0.218	0.406	0.647	0.000	0.117	0.956

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	35	172	233	0	42	42
normalized size	1	1.00	0.73	0.58	2.87	3.88	0.00	0.70	0.70
time (sec)	N/A	0.048	0.037	0.224	0.333	0.478	0.000	0.125	0.955

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	47	42	17	17
normalized size	1	1.00	1.00	1.31	0.65	1.81	1.62	0.65	0.65
time (sec)	N/A	0.020	0.015	0.096	0.306	0.435	0.651	0.138	0.076

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	22	13	26	20	13	12
normalized size	1	1.00	0.84	1.16	0.68	1.37	1.05	0.68	0.63
time (sec)	N/A	0.011	0.010	0.070	0.307	0.564	0.255	0.117	0.052

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	24	25	76	113	0	76	77
normalized size	1	1.00	0.26	0.27	0.83	1.23	0.00	0.83	0.84
time (sec)	N/A	0.064	0.011	0.141	0.416	0.585	0.000	0.115	1.114

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	36	88	162	0	88	85
normalized size	1	1.00	0.95	0.32	0.79	1.46	0.00	0.79	0.77
time (sec)	N/A	0.075	0.093	0.151	0.424	0.540	0.000	0.123	1.088

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	67	42	17	17
normalized size	1	1.00	1.00	1.31	0.65	2.58	1.62	0.65	0.65
time (sec)	N/A	0.019	0.017	0.098	0.311	0.470	0.638	0.133	0.076

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	26	13	38	20	13	12
normalized size	1	1.00	0.84	1.37	0.68	2.00	1.05	0.68	0.63
time (sec)	N/A	0.012	0.011	0.096	0.314	0.628	0.248	0.113	0.926

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	24	79	71	83	0	44	65
normalized size	1	1.00	0.44	1.44	1.29	1.51	0.00	0.80	1.18
time (sec)	N/A	0.058	0.012	0.137	0.426	0.528	0.000	0.113	1.039

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	34	59	79	154	0	79	84
normalized size	1	1.00	0.31	0.54	0.72	1.40	0.00	0.72	0.76
time (sec)	N/A	0.205	0.022	0.240	0.416	0.767	0.000	0.124	0.309

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	87	42	17	17
normalized size	1	1.00	1.00	1.31	0.65	3.35	1.62	0.65	0.65
time (sec)	N/A	0.019	0.017	0.095	0.330	0.506	0.627	0.130	0.961

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	26	13	46	20	13	14
normalized size	1	1.00	1.00	1.37	0.68	2.42	1.05	0.68	0.74
time (sec)	N/A	0.011	0.011	0.092	0.319	1.414	0.249	0.136	0.049

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	24	25	0	1087	0	249	479
normalized size	1	1.00	0.06	0.07	0.00	2.93	0.00	0.67	1.29
time (sec)	N/A	0.321	0.011	0.142	0.000	0.611	0.000	0.208	4.560

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	34	36	0	1367	0	261	473
normalized size	1	1.00	0.09	0.09	0.00	3.61	0.00	0.69	1.25
time (sec)	N/A	0.319	0.022	0.164	0.000	0.536	0.000	0.145	3.467

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	159	326	134	2218	0	1239	154
normalized size	1	1.00	0.79	1.61	0.66	10.98	0.00	6.13	0.76
time (sec)	N/A	0.078	0.702	0.293	0.354	0.653	0.000	0.272	1.811

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	85	143	94	699	604	903	100
normalized size	1	1.00	0.64	1.08	0.71	5.30	4.58	6.84	0.76
time (sec)	N/A	0.053	0.236	0.167	0.351	1.135	30.568	0.237	1.249

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	74	63	246	316	611	74
normalized size	1	1.00	0.67	0.99	0.84	3.28	4.21	8.15	0.99
time (sec)	N/A	0.018	0.117	0.070	0.344	0.549	6.474	0.206	1.007

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.020	0.080	0.000	0.469	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.017	0.102	0.000	0.500	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	96	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.255	0.112	0.000	0.598	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.203	0.148	0.000	0.608	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	0	112	218	0	101	-1
normalized size	1	1.00	0.42	0.00	0.45	0.87	0.00	0.40	-0.00
time (sec)	N/A	0.229	0.106	180.000	0.336	0.649	0.000	0.133	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	78	0	74	126	0	73	-1
normalized size	1	1.00	0.48	0.00	0.46	0.78	0.00	0.45	-0.01
time (sec)	N/A	0.124	0.116	180.000	0.345	0.524	0.000	0.144	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	0	29	66	204	23	76
normalized size	1	1.00	0.65	0.00	0.39	0.89	2.76	0.31	1.03
time (sec)	N/A	0.099	0.042	180.000	0.340	0.518	16.611	0.112	0.121

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	21	42	0	20	-1
normalized size	1	1.00	0.95	0.00	0.48	0.95	0.00	0.45	-0.02
time (sec)	N/A	0.113	0.056	180.000	0.434	0.560	0.000	0.122	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	0	84	120	0	38	76
normalized size	1	1.00	0.82	0.00	1.50	2.14	0.00	0.68	1.36
time (sec)	N/A	0.129	0.076	180.000	0.338	0.485	0.000	0.119	0.944

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	72	0	209	315	0	51	89
normalized size	1	1.00	0.51	0.00	1.48	2.23	0.00	0.36	0.63
time (sec)	N/A	0.197	0.073	180.000	0.334	0.596	0.000	0.137	0.104

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	84	0	386	589	0	64	345
normalized size	1	1.00	0.44	0.00	2.02	3.08	0.00	0.34	1.81
time (sec)	N/A	0.259	0.086	180.000	0.332	0.592	0.000	0.119	0.111

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	62	0	45	99	32	45
normalized size	1	1.00	0.68	1.51	0.00	1.10	2.41	0.78	1.10
time (sec)	N/A	0.013	0.049	0.091	0.000	0.605	0.649	0.134	0.087

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	72	65	104	0	73	-1
normalized size	1	1.00	0.93	0.85	0.76	1.22	0.00	0.86	-0.01
time (sec)	N/A	0.081	0.082	0.230	0.311	0.585	0.000	0.141	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	97	81	130	0	91	-1
normalized size	1	1.00	0.90	0.96	0.80	1.29	0.00	0.90	-0.01
time (sec)	N/A	0.125	0.154	0.453	0.322	0.529	0.000	0.142	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	44	0	45	-1
normalized size	1	1.00	0.78	0.80	0.69	0.68	0.00	0.69	-0.02
time (sec)	N/A	0.064	0.070	0.256	0.323	0.534	0.000	0.125	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	71	48	47	76	0	49	-1
normalized size	1	1.00	1.09	0.74	0.72	1.17	0.00	0.75	-0.02
time (sec)	N/A	0.079	0.102	0.740	0.316	0.631	0.000	0.139	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	122	105	89	165	0	101	-1
normalized size	1	1.00	1.06	0.91	0.77	1.43	0.00	0.88	-0.01
time (sec)	N/A	0.159	0.370	0.753	0.328	0.554	0.000	0.126	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	100	90	211	0	106	-1
normalized size	1	1.00	0.93	0.91	0.82	1.92	0.00	0.96	-0.01
time (sec)	N/A	0.148	0.128	0.158	0.324	0.469	0.000	0.148	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	355	-1
normalized size	1	1.00	1.01	0.85	0.86	1.88	0.00	2.40	-0.01
time (sec)	N/A	0.191	0.700	0.286	0.416	0.621	0.000	0.185	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	286	207	200	443	0	223	-1
normalized size	1	1.00	1.20	0.87	0.84	1.85	0.00	0.93	-0.00
time (sec)	N/A	0.286	0.429	0.415	0.431	0.453	0.000	0.166	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	126	102	251	0	134	-1
normalized size	1	1.00	1.07	1.10	0.89	2.18	0.00	1.17	-0.01
time (sec)	N/A	0.220	0.276	0.156	0.333	0.640	0.000	0.152	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	143	334	0	389	-1
normalized size	1	1.00	1.37	0.98	0.89	2.07	0.00	2.42	-0.01
time (sec)	N/A	0.273	0.649	0.283	0.435	0.643	0.000	0.201	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	353	265	228	539	0	285	-1
normalized size	1	1.00	1.37	1.03	0.89	2.10	0.00	1.11	-0.00
time (sec)	N/A	0.466	0.798	0.434	0.444	0.525	0.000	0.164	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	216	0	132	-1
normalized size	1	1.00	0.78	0.88	0.79	1.62	0.00	0.99	-0.01
time (sec)	N/A	0.194	0.153	0.161	0.324	0.621	0.000	0.149	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	242	0	150	-1
normalized size	1	1.00	0.81	0.86	0.81	1.50	0.00	0.93	-0.01
time (sec)	N/A	0.220	0.230	0.252	0.333	0.604	0.000	0.156	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	426	0	264	-1
normalized size	1	1.00	0.79	0.86	0.78	1.57	0.00	0.97	-0.00
time (sec)	N/A	0.338	0.468	0.446	0.337	0.495	0.000	0.168	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	70	69	145	0	75	-1
normalized size	1	1.00	0.93	0.86	0.85	1.79	0.00	0.93	-0.01
time (sec)	N/A	0.158	0.333	0.138	0.326	0.434	0.000	0.130	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	-1
normalized size	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	-0.01
time (sec)	N/A	0.203	0.564	0.267	0.335	0.496	0.000	0.141	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	270	144	143	491	0	155	-1
normalized size	1	1.00	1.58	0.84	0.84	2.87	0.00	0.91	-0.01
time (sec)	N/A	0.297	1.235	0.370	0.333	0.689	0.000	0.157	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	165	147	127	321	0	172	-1
normalized size	1	1.00	1.18	1.05	0.91	2.29	0.00	1.23	-0.01
time (sec)	N/A	0.311	0.673	0.175	0.342	0.431	0.000	0.146	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	420	0	198	-1
normalized size	1	1.00	1.41	0.97	0.88	2.30	0.00	1.08	-0.01
time (sec)	N/A	0.332	1.489	0.291	0.336	0.643	0.000	0.159	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	478	302	263	847	0	352	-1
normalized size	1	1.00	1.59	1.01	0.88	2.82	0.00	1.17	-0.00
time (sec)	N/A	0.581	5.998	0.420	0.344	0.529	0.000	0.193	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	156	129	262	0	169	-1
normalized size	1	1.00	0.88	1.02	0.84	1.71	0.00	1.10	-0.01
time (sec)	N/A	0.291	0.321	0.167	0.345	0.590	0.000	0.154	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	211	185	341	0	225	-1
normalized size	1	1.00	0.84	0.96	0.84	1.56	0.00	1.03	-0.00
time (sec)	N/A	0.369	0.564	0.283	0.340	0.513	0.000	0.147	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	262	316	263	526	0	343	-1
normalized size	1	1.00	0.83	1.00	0.83	1.67	0.00	1.09	-0.00
time (sec)	N/A	0.457	1.023	0.387	0.340	0.635	0.000	0.163	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	160	139	324	0	181	-1
normalized size	1	1.00	1.20	1.04	0.90	2.10	0.00	1.18	-0.01
time (sec)	N/A	0.321	0.679	0.183	0.334	0.542	0.000	0.158	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	-1
normalized size	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	-0.00
time (sec)	N/A	0.350	2.279	0.365	0.341	0.480	0.000	0.156	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	2511	326	287	851	0	369	-1
normalized size	1	1.00	7.77	1.01	0.89	2.63	0.00	1.14	-0.00
time (sec)	N/A	0.512	6.509	0.425	0.353	0.634	0.000	0.185	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	251	186	151	362	0	209	-1
normalized size	1	1.00	1.56	1.16	0.94	2.25	0.00	1.30	-0.01
time (sec)	N/A	0.426	1.514	0.187	0.335	0.536	0.000	0.168	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	215	516	0	273	-1
normalized size	1	1.00	1.42	1.04	0.90	2.16	0.00	1.14	-0.00
time (sec)	N/A	0.506	6.167	0.327	0.331	0.473	0.000	0.185	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	315	939	0	431	-1
normalized size	1	1.00	8.69	1.12	0.92	2.73	0.00	1.25	-0.00
time (sec)	N/A	0.734	6.649	0.440	0.347	0.478	0.000	0.211	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0	39
normalized size	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	1.95
time (sec)	N/A	0.050	0.362	180.000	0.000	0.000	0.000	0.000	0.148

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	109	0	0	42
normalized size	1	1.00	0.67	0.00	0.00	4.54	0.00	0.00	1.75
time (sec)	N/A	0.051	0.082	0.005	0.000	0.499	0.000	0.000	0.967

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0	110
normalized size	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	2.34
time (sec)	N/A	0.071	0.610	180.000	0.000	0.000	0.000	0.000	1.106

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0	-1
normalized size	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.094	0.189	180.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	36	23	41	36	24
normalized size	1	1.00	0.87	0.83	1.20	0.77	1.37	1.20	0.80
time (sec)	N/A	0.038	0.066	0.031	0.306	0.665	0.189	0.131	0.049

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	52	81	54	85	75	48
normalized size	1	1.00	0.91	0.93	1.45	0.96	1.52	1.34	0.86
time (sec)	N/A	0.075	0.093	0.220	0.318	0.568	0.315	0.117	0.069

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	316	0	0	-1
normalized size	1	1.00	0.85	1.00	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.299	0.118	0.000	0.517	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	248	376	0	671	0	0	-1
normalized size	1	1.00	0.92	1.39	0.00	2.48	0.00	0.00	-0.00
time (sec)	N/A	0.751	0.499	0.105	0.000	0.533	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [279] had the largest ratio of [1.125]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	3	2	1.00	10	0.200
8	A	2	2	1.00	10	0.200
9	A	2	2	1.00	10	0.200
10	A	1	1	1.00	10	0.100
11	A	1	1	1.00	10	0.100
12	A	2	2	1.00	10	0.200
13	A	2	2	1.00	10	0.200
14	A	3	2	1.00	10	0.200
15	A	4	3	1.00	8	0.375
16	A	3	3	1.00	8	0.375
17	A	3	3	1.00	8	0.375
18	A	2	2	1.00	8	0.250
19	A	2	2	1.00	8	0.250
20	A	3	3	1.00	8	0.375
21	A	3	3	1.00	8	0.375
22	A	4	3	1.00	8	0.375
23	A	1	1	1.00	10	0.100
24	A	6	5	1.00	13	0.385
25	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	4	1.00	13	0.308
27	A	2	2	1.00	11	0.182
28	A	3	3	1.00	11	0.273
29	A	5	5	1.00	13	0.385
30	A	6	6	1.00	13	0.462
31	A	6	5	1.00	13	0.385
32	A	1	1	1.00	10	0.100
33	A	2	2	1.00	10	0.200
34	A	3	2	1.00	10	0.200
35	A	4	2	1.00	10	0.200
36	A	1	1	1.00	12	0.083
37	A	2	2	1.00	12	0.167
38	A	3	2	1.00	12	0.167
39	A	4	2	1.00	12	0.167
40	A	3	3	1.00	13	0.231
41	A	3	3	1.00	14	0.214
42	A	3	2	1.00	14	0.143
43	A	2	2	1.00	14	0.143
44	A	1	1	1.00	14	0.071
45	A	2	2	1.00	14	0.143
46	A	3	3	1.00	14	0.214
47	A	4	3	1.00	14	0.214
48	A	3	2	1.00	15	0.133
49	A	2	2	1.00	15	0.133
50	A	1	1	1.00	15	0.067
51	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	3	3	1.00	15	0.200
53	A	4	3	1.00	15	0.200
54	A	6	6	1.00	13	0.462
55	A	5	5	1.00	13	0.385
56	A	5	5	1.00	13	0.385
57	A	3	3	1.00	11	0.273
58	A	4	4	1.00	11	0.364
59	A	6	6	1.00	13	0.462
60	A	6	6	1.00	13	0.462
61	A	7	6	1.00	13	0.462
62	A	4	3	1.00	12	0.250
63	A	3	3	1.00	12	0.250
64	A	2	2	1.00	12	0.167
65	A	1	1	1.00	12	0.083
66	A	2	1	1.00	10	0.100
67	A	2	2	1.00	12	0.167
68	A	4	4	1.00	12	0.333
69	A	5	5	1.00	12	0.417
70	A	6	5	1.00	12	0.417
71	A	2	2	1.00	12	0.167
72	A	4	4	1.00	12	0.333
73	A	5	5	1.00	12	0.417
74	A	6	5	1.00	12	0.417
75	A	1	1	1.00	12	0.083
76	A	3	3	1.00	12	0.250
77	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	5	4	1.00	12	0.333
79	A	7	7	1.00	10	0.700
80	A	6	6	1.00	10	0.600
81	A	2	2	1.00	14	0.143
82	A	2	2	1.00	10	0.200
83	A	4	4	1.00	10	0.400
84	A	7	7	1.00	10	0.700
85	A	8	7	1.00	10	0.700
86	A	5	5	1.00	13	0.385
87	A	4	3	1.00	17	0.176
88	A	3	3	1.00	17	0.176
89	A	2	2	1.00	17	0.118
90	A	4	3	1.00	18	0.167
91	A	3	3	1.00	18	0.167
92	A	2	2	1.00	18	0.111
93	A	2	2	1.00	13	0.154
94	A	2	2	1.00	13	0.154
95	A	3	3	1.00	13	0.231
96	A	4	3	1.00	13	0.231
97	A	2	2	1.00	15	0.133
98	A	2	2	1.00	15	0.133
99	A	3	3	1.00	15	0.200
100	A	4	3	1.00	15	0.200
101	A	3	3	1.00	17	0.176
102	A	3	3	1.00	17	0.176
103	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
104	A	3	3	1.00	18	0.167
105	A	3	3	1.00	18	0.167
106	A	4	4	1.00	18	0.222
107	A	8	6	1.00	17	0.353
108	A	7	6	1.00	17	0.353
109	A	6	6	1.00	17	0.353
110	A	3	3	1.00	15	0.200
111	A	4	4	1.00	15	0.267
112	A	5	4	1.00	15	0.267
113	A	6	4	1.00	15	0.267
114	A	3	3	1.00	20	0.150
115	A	2	2	1.00	20	0.100
116	A	2	2	1.00	15	0.133
117	A	2	2	1.00	13	0.154
118	A	5	5	1.00	17	0.294
119	A	6	6	1.00	17	0.353
120	A	7	6	1.00	17	0.353
121	A	5	3	1.00	10	0.300
122	A	4	3	1.00	10	0.300
123	A	3	3	1.00	10	0.300
124	A	2	2	1.00	10	0.200
125	A	2	2	1.00	10	0.200
126	A	3	3	1.00	10	0.300
127	A	4	3	1.00	10	0.300
128	A	6	3	1.00	10	0.300
129	A	4	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	3	1.00	10	0.300
131	A	3	3	1.00	10	0.300
132	A	4	3	1.00	10	0.300
133	A	6	3	1.00	10	0.300
134	A	7	3	1.00	10	0.300
135	A	5	3	1.00	10	0.300
136	A	3	3	1.00	10	0.300
137	A	3	3	1.00	10	0.300
138	A	3	2	1.00	10	0.200
139	A	3	2	1.00	10	0.200
140	A	2	2	1.00	9	0.222
141	A	2	2	1.00	11	0.182
142	A	2	2	1.00	11	0.182
143	A	2	2	1.00	13	0.154
144	A	3	2	1.00	11	0.182
145	A	3	2	1.00	13	0.154
146	A	2	2	1.00	9	0.222
147	A	2	2	1.00	11	0.182
148	A	1	1	1.00	11	0.091
149	A	1	1	1.00	13	0.077
150	A	3	2	1.00	11	0.182
151	A	3	2	1.00	13	0.154
152	A	5	3	1.00	13	0.231
153	A	3	2	1.00	13	0.154
154	A	4	3	1.00	13	0.231
155	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	3	3	1.00	13	0.231
157	A	2	1	1.00	13	0.077
158	A	2	2	1.00	13	0.154
159	A	2	2	1.00	11	0.182
160	A	4	3	1.00	11	0.273
161	A	3	3	1.00	13	0.231
162	A	4	3	1.00	13	0.231
163	A	3	2	1.00	13	0.154
164	A	4	3	1.00	13	0.231
165	A	3	2	1.00	13	0.154
166	A	6	5	1.00	13	0.385
167	A	3	2	1.00	13	0.154
168	A	5	5	1.00	13	0.385
169	A	3	2	1.00	13	0.154
170	A	4	4	1.00	13	0.308
171	A	2	2	1.00	11	0.182
172	A	6	4	1.00	11	0.364
173	A	4	4	1.00	13	0.308
174	A	4	3	1.00	13	0.231
175	A	5	5	1.00	13	0.385
176	A	5	4	1.00	13	0.308
177	A	6	5	1.00	13	0.385
178	A	4	4	1.00	13	0.308
179	A	6	6	1.00	13	0.462
180	A	3	2	1.00	13	0.154
181	A	6	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	4	1.00	11	0.364
183	A	3	2	1.00	11	0.182
184	A	7	6	1.00	13	0.462
185	A	4	3	1.00	13	0.231
186	A	12	8	1.00	13	0.615
187	A	6	5	1.00	13	0.385
188	A	5	4	1.00	13	0.308
189	A	5	5	1.00	13	0.385
190	A	5	4	1.00	13	0.308
191	A	4	4	1.00	13	0.308
192	A	4	4	1.00	11	0.364
193	A	5	5	1.00	11	0.454
194	A	5	4	1.00	13	0.308
195	A	6	5	1.00	13	0.385
196	A	6	5	1.00	13	0.385
197	A	4	4	1.00	13	0.308
198	A	3	3	1.00	13	0.231
199	A	6	5	1.00	15	0.333
200	A	5	4	1.00	13	0.308
201	A	5	4	1.00	15	0.267
202	A	8	7	1.00	15	0.467
203	A	7	5	1.00	15	0.333
204	A	5	5	1.00	15	0.333
205	A	11	8	1.00	15	0.533
206	A	6	6	1.00	31	0.194
207	A	7	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	8	7	1.00	31	0.226
209	A	9	7	1.00	31	0.226
210	A	9	6	1.00	12	0.500
211	A	11	7	1.00	14	0.500
212	A	13	8	1.00	14	0.571
213	A	5	3	1.00	36	0.083
214	A	4	3	1.00	36	0.083
215	A	2	2	1.00	34	0.059
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	3	3	1.00	12	0.250
219	A	5	5	1.00	12	0.417
220	A	13	5	1.00	20	0.250
221	A	0	0	0.00	0	0.000
222	A	11	6	1.00	22	0.273
223	A	9	5	1.00	22	0.227
224	A	7	4	1.00	20	0.200
225	A	2	2	1.00	19	0.105
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	18	11	1.00	24	0.458
229	A	15	10	1.00	24	0.417
230	A	12	9	1.00	22	0.409
231	A	4	4	1.00	21	0.190
232	A	0	0	0.00	0	0.000
233	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
234	A	21	14	1.00	24	0.583
235	A	16	11	1.00	24	0.458
236	A	13	10	1.00	22	0.454
237	A	3	2	1.00	21	0.095
238	A	0	0	0.00	0	0.000
239	A	1	1	1.00	11	0.091
240	A	2	2	1.00	13	0.154
241	A	2	2	1.00	13	0.154
242	A	3	2	1.00	13	0.154
243	A	1	1	1.00	15	0.067
244	A	2	2	1.00	17	0.118
245	A	2	2	0.97	17	0.118
246	A	3	2	0.98	17	0.118
247	A	2	1	1.00	15	0.067
248	A	3	2	1.00	17	0.118
249	A	3	1	1.00	17	0.059
250	A	4	2	1.00	17	0.118
251	A	3	1	1.00	17	0.059
252	A	3	2	1.00	19	0.105
253	A	3	2	1.00	19	0.105
254	A	2	1	1.00	19	0.053
255	A	2	1	1.00	19	0.053
256	A	3	2	1.00	19	0.105
257	A	3	2	1.00	19	0.105
258	A	8	8	1.00	18	0.444
259	A	6	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	3	3	1.00	18	0.167
261	A	4	4	1.00	18	0.222
262	A	5	5	1.00	14	0.357
263	A	6	6	1.00	16	0.375
264	A	4	3	1.00	16	0.188
265	A	5	4	1.00	16	0.250
266	A	4	3	1.00	16	0.188
267	A	4	3	1.00	14	0.214
268	A	3	3	1.00	14	0.214
269	A	4	4	1.00	16	0.250
270	A	3	3	1.00	16	0.188
271	A	6	5	1.00	16	0.312
272	A	5	4	1.00	16	0.250
273	A	4	3	1.00	10	0.300
274	A	4	3	1.00	8	0.375
275	A	11	8	1.00	8	1.000
276	A	12	9	1.00	10	0.900
277	A	4	3	1.00	10	0.300
278	A	4	3	1.00	8	0.375
279	A	9	9	1.00	8	1.125
280	A	13	9	1.00	10	0.900
281	A	4	3	1.00	10	0.300
282	A	4	3	1.00	8	0.375
283	A	21	9	1.00	8	1.125
284	A	22	9	1.00	10	0.900
285	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
286	A	2	2	1.00	18	0.111
287	A	1	1	1.00	16	0.062
288	A	1	1	1.00	16	0.062
289	A	1	1	1.00	18	0.056
290	A	2	2	1.00	18	0.111
291	A	2	2	1.00	18	0.111
292	A	6	5	1.00	25	0.200
293	A	6	5	1.00	25	0.200
294	A	5	4	1.00	25	0.160
295	A	4	4	1.00	25	0.160
296	A	4	4	1.00	25	0.160
297	A	6	5	1.00	25	0.200
298	A	6	5	1.00	25	0.200
299	A	1	1	1.00	10	0.100
300	A	6	4	1.00	12	0.333
301	A	6	4	1.00	15	0.267
302	A	6	3	1.00	12	0.250
303	A	4	2	1.00	14	0.143
304	A	6	3	1.00	17	0.176
305	A	8	5	1.00	16	0.312
306	A	9	6	1.00	18	0.333
307	A	14	5	1.00	18	0.278
308	A	8	5	1.00	19	0.263
309	A	9	6	1.00	21	0.286
310	A	14	5	1.00	21	0.238
311	A	8	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	9	4	1.00	18	0.222
313	A	14	4	1.00	18	0.222
314	A	6	4	1.00	18	0.222
315	A	7	4	1.00	20	0.200
316	A	10	4	1.00	20	0.200
317	A	8	5	1.00	21	0.238
318	A	9	5	1.00	23	0.217
319	A	14	5	1.00	23	0.217
320	A	8	4	1.00	19	0.210
321	A	10	4	1.00	21	0.190
322	A	14	4	1.00	21	0.190
323	A	8	5	1.00	21	0.238
324	A	10	5	1.00	23	0.217
325	A	14	5	1.00	23	0.217
326	A	8	5	1.00	24	0.208
327	A	10	5	1.00	26	0.192
328	A	14	5	1.00	26	0.192
329	A	2	1	1.00	17	0.059
330	A	2	1	1.00	20	0.050
331	A	3	1	1.00	20	0.050
332	A	3	2	1.00	21	0.095
333	A	6	5	1.00	6	0.833
334	A	9	6	1.00	6	1.000
335	A	8	4	1.00	16	0.250
336	A	8	4	1.00	19	0.210

Chapter 3

Listing of integrals

3.1 $\int \cosh(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sinh(a+bx)}{b}$$

```
[Out] sinh(b*x+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2637}

$$\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x],x]
```

```
[Out] Sinh[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx)\,dx = \frac{\sinh(a+bx)}{b}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.10

$$\frac{\sinh(a)\cosh(bx)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x],x]
[Out] (Cosh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

fricas [A] time = 0.59, size = 10, normalized size = 1.00

$$\frac{\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="fricas")
[Out] sinh(b*x + a)/b
```

giac [B] time = 0.13, size = 26, normalized size = 2.60

$$\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="giac")
[Out] 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

maple [A] time = 0.04, size = 11, normalized size = 1.10

$$\frac{\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a),x)
[Out] sinh(b*x+a)/b
```

maxima [A] time = 0.32, size = 10, normalized size = 1.00

$$\frac{\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] sinh(b*x + a)/b
```

mupad [B] time = 0.05, size = 10, normalized size = 1.00

$$\frac{\sinh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x),x)
```

```
[Out] sinh(a + b*x)/b
```

sympy [A] time = 0.13, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\sinh(a+bx)}{b} & \text{for}\ b \neq 0 \\ x\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a),x)
```

```
[Out] Piecewise((sinh(a + b*x)/b, Ne(b, 0)), (x*cosh(a), True))
```

3.2 $\int \cosh^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}$$

```
[Out] 1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2,x]
```

```
[Out] x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sinh(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

fricas [A] time = 0.56, size = 22, normalized size = 0.88

$$\frac{bx + \cosh\left(bx + a\right)\sinh\left(bx + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cosh(b*x + a)*sinh(b*x + a))/b
```

giac [A] time = 0.13, size = 32, normalized size = 1.28

$$\frac{1}{2}\,x + \frac{e^{\left(2\,bx+2\,a\right)}}{8\,b} - \frac{e^{\left(-2\,bx-2\,a\right)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

maple [A] time = 0.08, size = 27, normalized size = 1.08

$$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)
```

maxima [A] time = 0.32, size = 32, normalized size = 1.28

$$\frac{1}{2}\,x + \frac{e^{\left(2\,bx+2\,a\right)}}{8\,b} - \frac{e^{\left(-2\,bx-2\,a\right)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

mupad [B] time = 0.88, size = 18, normalized size = 0.72

$$\frac{x}{2}+\frac{\sinh\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2,x)
```

```
[Out] x/2 + sinh(2*a + 2*b*x)/(4*b)
```

sympy [A] time = 0.21, size = 46, normalized size = 1.84

$$\begin{cases} -\frac{x\sinh^{2}\left(a+bx\right)}{2}+\frac{x\cosh^{2}\left(a+bx\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b} & \text{for } b\neq 0 \\ x\cosh^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cos
h(a + b*x)/(2*b), Ne(b, 0)), (x*cosh(a)**2, True))
```

3.3 $\int \cosh^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh(a+bx)}{b}+\frac{\sinh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3,x]
```

```
[Out] Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

fricas [A] time = 0.73, size = 32, normalized size = 1.23

$$\frac{\sinh(bx+a)^3 + 3\left(\cosh(bx+a)^2 + 3\right)\sinh(bx+a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

giac [B] time = 0.12, size = 54, normalized size = 2.08

$$\frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

maple [A] time = 0.20, size = 23, normalized size = 0.88

$$\frac{\left(\frac{2}{3} + \frac{\left(\cosh^2(bx+a)\right)}{3}\right)\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)
```

maxima [B] time = 0.31, size = 54, normalized size = 2.08

$$\frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

mupad [B] time = 0.89, size = 22, normalized size = 0.85

$$\frac{\sinh(a+b\,x)^3+3\sinh(a+b\,x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3,x)
```

```
[Out] (3*sinh(a + b*x) + sinh(a + b*x)^3)/(3*b)
```

sympy [A] time = 0.41, size = 36, normalized size = 1.38

$$\begin{cases} -\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)\cosh^2(a+bx)}{b} & \text{for } b\neq 0 \\ x\cosh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*sinh(a + b*x)**3/(3*b) + sinh(a + b*x)*cosh(a + b*x)**2/b, Ne
(b, 0)), (x*cosh(a)**3, True))
```

3.4 $\int \cosh^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] 3/8*x+3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a +
 b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \cosh^4(a+bx)\,dx &= \frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3}{4}\int \cosh^2(a+bx)\,dx\\
&= \frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\
&= \frac{3x}{8}+\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 33, normalized size = 0.72

$$\frac{12(a+bx)+8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) + 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

fricas [A] time = 0.81, size = 49, normalized size = 1.07

$$\frac{\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+3\,bx+\left(\cosh\left(bx+a\right)^{3}+4\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 + 4*cosh(b*x
+ a))*sinh(b*x + a))/b
```

giac [A] time = 0.12, size = 60, normalized size = 1.30

$$\frac{3}{8}\,x+\frac{e^{(4\,bx+4\,a)}}{64\,b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

maple [A] time = 0.21, size = 39, normalized size = 0.85

$$\frac{\left(\frac{\left(\cosh^{3}(bx+a)\right)}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh\left(bx+a\right)+\frac{3bx}{8}+\frac{3a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^4,x)
```

```
[Out] 1/b*((1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/8*b*x+3/8*a)
```

maxima [A] time = 0.31, size = 60, normalized size = 1.30

$$\frac{3}{8}\,x+\frac{e^{(4\,bx+4\,a)}}{64\,b}+\frac{e^{(2\,bx+2\,a)}}{8\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

mupad [B] time = 0.08, size = 31, normalized size = 0.67

$$\frac{3\,x}{8}+\frac{\frac{\sinh(2\,a+2\,b\,x)}{4}+\frac{\sinh(4\,a+4\,b\,x)}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4,x)
```

```
[Out] (3*x)/8 + (sinh(2*a + 2*b*x)/4 + sinh(4*a + 4*b*x)/32)/b
```

sympy [A] time = 0.85, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sinh^4(a+bx)}{8}-\frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4}+\frac{3x\cosh^4(a+bx)}{8}-\frac{3\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b\neq 0 \\ x\cosh^4(a) & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/4
 + 3*x*cosh(a + b*x)**4/8 - 3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + 5*sinh
(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*cosh(a)**4, True))
```

3.5 $\int \cosh^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\sinh^5(a+bx)}{5b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

```
[Out] sinh(b*x+a)/b+2/3*sinh(b*x+a)^3/b+1/5*sinh(b*x+a)^5/b
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sinh^5(a+bx)}{5b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^5,x]
```

```
[Out] Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^5(a+bx)\,dx &= \frac{i\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -i\sinh(a+bx)\right)}{b}\\ &= \frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$\frac{\sinh^5(a+bx)}{5b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^5,x]
```

```
[Out] Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

fricas [A] time = 0.47, size = 66, normalized size = 1.61

$$\frac{3 \sinh(bx+a)^5 + 5\left(6 \cosh(bx+a)^2 + 5\right)\sinh(bx+a)^3 + 15\left(\cosh(bx+a)^4 + 5 \cosh(bx+a)^2 + 10\right)\sinh(bx}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

giac [B] time = 0.14, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

maple [A] time = 0.22, size = 33, normalized size = 0.80

$$\frac{\left(\frac{8}{15} + \frac{\left(\cosh^4(bx+a)\right)}{5} + \frac{4\left(\cosh^2(bx+a)\right)}{15}\right)\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^5,x)
```

```
[Out] 1/b*(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*x+a)
```

maxima [B] time = 0.32, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

mupad [B] time = 0.92, size = 31, normalized size = 0.76

$$\frac{\frac{\sinh(a+b\,x)^5}{5}+\frac{2\sinh(a+b\,x)^3}{3}+\sinh\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^5,x)
```

```
[Out] (sinh(a + b*x) + (2*sinh(a + b*x)^3)/3 + sinh(a + b*x)^5/5)/b
```

sympy [A] time = 1.57, size = 58, normalized size = 1.41

$$\begin{cases} \frac{8\sinh^5\left(a+bx\right)}{15b}-\frac{4\sinh^3\left(a+bx\right)\cosh^2\left(a+bx\right)}{3b}+\frac{\sinh\left(a+bx\right)\cosh^4\left(a+bx\right)}{b} & \text{for } b\neq 0 \\ x\cosh^5\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**5,x)
```

```
[Out] Piecewise((8*sinh(a + b*x)**5/(15*b) - 4*sinh(a + b*x)**3*cosh(a + b*x)**2/
(3*b) + sinh(a + b*x)*cosh(a + b*x)**4/b, Ne(b, 0)), (x*cosh(a)**5, True))
```

3.6 $\int \cosh^6(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{5x}{16}$$

```
[Out] 5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b+5/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6*
cosh(b*x+a)^5*sinh(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5\sinh(a+bx)\cosh^3(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}+\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) + (5*Cosh[a + b*x]^3*Sinh
[a + b*x])/(24*b) + (Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cosh^6(a+bx)\,dx = \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} + \frac{5}{6}\int \cosh^4(a+bx)\,dx$$
$$= \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b} + \frac{5}{8}\int \cosh^2(a+bx)\,dx$$
$$= \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+b}{6b}$$
$$= \frac{5x}{16} + \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh}{6b}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.64

$$\frac{45\sinh(2(a+bx)) + 9\sinh(4(a+bx)) + \sinh(6(a+bx)) + 60a + 60bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^6,x]
```

```
[Out] (60*a + 60*b*x + 45*Sinh[2*(a + b*x)] + 9*Sinh[4*(a + b*x)] + Sinh[6*(a + b
*x)])/(192*b)
```

fricas [A] time = 0.61, size = 90, normalized size = 1.34

$$\frac{3\cosh(bx+a)\sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^3 + 9\cosh(bx+a)\right)\sinh(bx+a)^3 + 30\,bx + 3\left(\cosh(bx+\right.}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 9*cosh(b*x +
 a))*sinh(b*x + a)^3 + 30*b*x + 3*(cosh(b*x + a)^5 + 6*cosh(b*x + a)^3 + 15
*cosh(b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.14, size = 88, normalized size = 1.31

$$\frac{5}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{3\,e^{(4\,bx+4\,a)}}{128\,b} + \frac{15\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{15\,e^{(-2\,bx-2\,a)}}{128\,b} - \frac{3\,e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/384*e^(6*b*x + 6*a)/b + 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*b*
x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b - 3/128*e^(-4*b*x - 4*a)/b - 1/384*e
^(-6*b*x - 6*a)/b
```

maple [A] time = 0.22, size = 49, normalized size = 0.73

$$\frac{\left(\frac{\left(\cosh^{5}(bx+a)\right)}{6}+\frac{5\left(\cosh^{3}(bx+a)\right)}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh\left(bx+a\right)+\frac{5bx}{16}+\frac{5a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^6,x)
```

```
[Out] 1/b*((1/6*cosh(b*x+a)^5+5/24*cosh(b*x+a)^3+5/16*cosh(b*x+a))*sinh(b*x+a)+5/
16*b*x+5/16*a)
```

maxima [A] time = 0.31, size = 86, normalized size = 1.28

$$\frac{\left(9\,e^{(-2\,bx-2\,a)}+45\,e^{(-4\,bx-4\,a)}+1\right)e^{(6\,bx+6\,a)}}{384\,b}+\frac{5\,(bx+a)}{16\,b}-\frac{45\,e^{(-2\,bx-2\,a)}+9\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/384*(9*e^(-2*b*x - 2*a) + 45*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b + 5/
16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) + 9*e^(-4*b*x - 4*a) + e^(-6*b*
x - 6*a))/b
```

mupad [B] time = 0.97, size = 42, normalized size = 0.63

$$\frac{5\,x}{16}+\frac{\frac{15\sinh(2\,a+2\,b\,x)}{64}+\frac{3\sinh(4\,a+4\,b\,x)}{64}+\frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^6,x)
```

```
[Out] (5*x)/16 + ((15*sinh(2*a + 2*b*x))/64 + (3*sinh(4*a + 4*b*x))/64 + sinh(6*a
 + 6*b*x)/192)/b
```

sympy [A] time = 2.99, size = 139, normalized size = 2.07

$$\begin{cases} -\frac{5x\sinh^{6}(a+bx)}{16}+\frac{15x\sinh^{4}(a+bx)\cosh^{2}(a+bx)}{16}-\frac{15x\sinh^{2}(a+bx)\cosh^{4}(a+bx)}{16}+\frac{5x\cosh^{6}(a+bx)}{16}+\frac{5\sinh^{5}(a+bx)\cosh(a+bx)}{16b}-\frac{5\sinh}{} \\ x\cosh^{6}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**6,x)
```

```
[Out] Piecewise((-5*x*sinh(a + b*x)**6/16 + 15*x*sinh(a + b*x)**4*cosh(a + b*x)**
2/16 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + 5*x*cosh(a + b*x)**6/16
+ 5*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a + b*x
)**3/(6*b) + 11*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*cosh(a
)**6, True))
```

3.7 $\int \cosh^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b}+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sinh(a+bx)\sqrt{\cosh(a+bx)}}{21b}$$

```
[Out] -10/21*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh
(1/2*a+1/2*b*x),2^(1/2))/b+2/7*cosh(b*x+a)^(5/2)*sinh(b*x+a)/b+10/21*sinh(b
*x+a)*cosh(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2635, 2641}

$$-\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b}+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sinh(a+bx)\sqrt{\cosh(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I/2)*(a + b*x), 2])/b + (10*Sqrt[Cosh[a + b*x]]*Si
nh[a + b*x])/(21*b) + (2*Cosh[a + b*x]^(5/2)*Sinh[a + b*x])/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b} + \frac{5}{7}\int \cosh^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{10\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{21b} + \frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\cosh(a+bx)}}$$
$$= -\frac{10iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{21b} + \frac{10\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{21b} + \frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.80

$$\frac{(23\sinh(a+bx)+3\sinh(3(a+bx)))\sqrt{\cosh(a+bx)} - 20iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(7/2),x]
```

```
[Out] ((-20*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(23*Sinh[a + b
*x] + 3*Sinh[3*(a + b*x)]))/(42*b)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(7/2), x)
```

<u>**maple**</u> **[B]** time = 0.35, size = 201, normalized size = 2.91

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(48\left(\cosh^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cosh^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+128\left(\cosh^5\left(\frac{bx}{2}+\right.\right.\right.}{21\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(7/2),x)
```

```
[Out] 2/21*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(48*cosh(1/2
*b*x+1/2*a)^9-120*cosh(1/2*b*x+1/2*a)^7+128*cosh(1/2*b*x+1/2*a)^5-72*cosh(1
/2*b*x+1/2*a)^3+5*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+
1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+16*cosh(1/2*b*x+1/2*a))/(2*
sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*c
osh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

<u>**maxima**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(7/2), x)
```

<u>**mupad**</u> **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(a+b\,x\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(7/2),x)
```

```
[Out] int(cosh(a + b*x)^(7/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.8 $\int \cosh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

```
[Out] -6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1
/2*a+1/2*b*x),2^(1/2))/b+2/5*cosh(b*x+a)^(3/2)*sinh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2635, 2639}

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Cosh[a + b*x]^(3/2)*Sinh[
a + b*x])/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^{\frac{5}{2}}(a+bx)\,dx&=\frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\cosh(a+bx)}\,dx\\&=-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.96

$$\frac{\sinh(2(a+bx))\sqrt{\cosh(a+bx)}-6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(5/2),x]
```

```
[Out] ((-6*I)*EllipticE[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*Sinh[2*(a + b*x
)])/(5*b)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\cosh\left(bx+a\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(5/2), x)
```

maple [B] time = 0.30, size = 188, normalized size = 4.09

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\ \left(8\left(\cosh^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-16\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\ \sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(5/2),x)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*b
*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/2*
```

```
b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh(bx+a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(cosh(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cosh(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(5/2),x)
[Out] int(cosh(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(5/2),x)
[Out] Timed out
```

3.9 $\int \cosh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1
/2*a+1/2*b*x),2^(1/2))/b+2/3*sinh(b*x+a)*cosh(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2635, 2641}

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(3/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh[
a + b*x])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^{\frac{3}{2}}(a+bx)\,dx&=\frac{2\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\&=-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cosh(a+bx)}\,\sinh(a+bx)}{3b}\end{aligned}$$

Mathematica [C] time = 0.11, size = 81, normalized size = 1.76

$$\frac{2\sqrt{\sinh(2(a+bx))+\cosh(2(a+bx))+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+bx))-\sinh(2(a+bx))\right)+\sinh(2(a+bx))}{3b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(3/2),x]
```

```
[Out] (Sinh[2*(a + b*x)] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a + b*x)]
- Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]])/(3*b*
Sqrt[Cosh[a + b*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\cosh\left(bx+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(3/2), x)
```

maple [B] time = 0.29, size = 174, normalized size = 3.78

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(4\left(\cosh^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-6\left(\cosh^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}}{3\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(3/2),x)
```

```
[Out] 2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*b
*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh
(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1/
2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/
2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\mathrm{cosh}\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(3/2),x)
```

```
[Out] int(cosh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh^{\frac{3}{2}}{\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**(3/2), x)
```

3.10 $\int \sqrt{\cosh(a+bx)}\,dx$

Optimal. Leaf size=20

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2639}

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cosh(a+bx)}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{\cosh\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cosh(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cosh\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(b*x + a)), x)
```

maple [B] time = 0.26, size = 135, normalized size = 6.75

$$-\frac{2\sqrt{\left(2\left(\cosh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-\left(\sinh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-2\left(\cosh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,\mathrm{EllipticE}\left(\mathrm{cosh}\cdots\right.}{\sqrt{2\left(\sinh^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^(1/2),x)
```

```
[Out] -2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*x+
1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(
1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cosh\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{\cosh\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^(1/2),x)
[Out] int(cosh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**(1/2),x)
[Out] Integral(sqrt(cosh(a + b*x)), x)
```

3.11 $\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2
*a+1/2*b*x),2^(1/2))/b
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2641}

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cosh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\cosh\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(cosh(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cosh(b*x + a)), x)
```

maple [B] time = 0.29, size = 135, normalized size = 6.75

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\;\text{EllipticF}\left(\cosh\right.}{\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\;\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(1/2),x)
```

```
[Out] 2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4
+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sinh(1
/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cosh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(1/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cosh\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cosh(a + b*x)), x)
```

3.12 $\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[Out] 2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*
a+1/2*b*x),2^(1/2))/b+2*sinh(b*x+a)/b/cosh(b*x+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2636, 2639}

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
+ b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}} - \int \sqrt{\cosh(a+bx)}\,dx$$
$$= \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b} + \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 1.00

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}} + \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
+ b*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\cosh\left(bx+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(-3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-3/2), x)
```

maple [A] time = 0.36, size = 103, normalized size = 2.45

$$\frac{2\sqrt{-2\left(\sinh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \operatorname{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{-\left(\sinh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}+4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sinh^{2}\left(\frac{bx}{2}\right.\right.}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cosh^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(3/2),x)
```

```
[Out] 2*((-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2)
)*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^
2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cosh\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(3/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**(-3/2), x)
```

3.13 $\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1
/2*a+1/2*b*x),2^(1/2))/b+2/3*sinh(b*x+a)/b/cosh(b*x+a)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2636, 2641}

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-5/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(3*b*Cosh[
a + b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)} + \frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$$
$$= -\frac{2iF\left(\frac{1}{2}i(a+bx)\middle|2\right)}{3b} + \frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Mathematica [C] time = 0.07, size = 84, normalized size = 1.83

$$\frac{2\left(\cosh(a+bx)\sqrt{\sinh(2(a+bx))+\cosh(2(a+bx))+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2(a+bx))-\sinh(2(a+bx))\right)+\sin\right.}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-5/2),x]
```

```
[Out] (2*(Sinh[a + b*x] + Cosh[a + b*x]*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*
(a + b*x)] - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*
x)]]))/(3*b*Cosh[a + b*x]^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\cosh\left(bx+a\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(-5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-5/2), x)
```

maple [B] time = 0.33, size = 217, normalized size = 4.72

$$\frac{2\left(2\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{-\left(\sin\right.}}{3\sqrt{2\left(\sinh^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(5/2),x)
```

```
[Out] 2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1
/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/
2*a),2^(1/2))+2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2)*((2*cosh(1/2*b*x
+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2
*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(3/2)/sinh(1/2*b*x+1/2*a)/
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cosh\left(a+b\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(5/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(5/2),x)
```

```
[Out] Integral(cosh(a + b*x)**(-5/2), x)
```

3.14 $\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=69

$$\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

```
[Out] 6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/
2*a+1/2*b*x),2^(1/2))/b+2/5*sinh(b*x+a)/b/cosh(b*x+a)^(5/2)+6/5*sinh(b*x+a)
/b/cosh(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2636, 2639}

$$\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^(-7/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(5*b*Cosh[a
 + b*x]^(5/2)) + (6*Sinh[a + b*x])/(5*b*Sqrt[Cosh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx &= \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx \\ &= \frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}-\frac{3}{5}\int \sqrt{\cosh(a+bx)}\,dx \\ &= \frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 63, normalized size = 0.91

$$\frac{3\sinh(2(a+bx))+2\tanh(a+bx)+6i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b\cosh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^(-7/2),x]
```

```
[Out] ((6*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + 3*Sinh[2*(a + b*
x)] + 2*Tanh[a + b*x])/(5*b*Cosh[a + b*x]^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\cosh\left(bx+a\right)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(b*x + a)^(-7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^(-7/2), x)
```

maple [B] time = 0.63, size = 363, normalized size = 5.26

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(12\sqrt{-\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{-2\left(\sinh^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(b*x+a)^(7/2),x)
```

```
[Out] 2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(8*sinh(1/2*b
*x+1/2*a)^6+12*sinh(1/2*b*x+1/2*a)^4+6*sinh(1/2*b*x+1/2*a)^2+1)/sinh(1/2*b*
x+1/2*a)^3*(12*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^
(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^4+24*sinh(
1/2*b*x+1/2*a)^6*cosh(1/2*b*x+1/2*a)+12*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*
sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(
1/2*b*x+1/2*a)^2+24*sinh(1/2*b*x+1/2*a)^4*cosh(1/2*b*x+1/2*a)+3*(-2*sinh(1/
2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*(-sinh(1/2*b
*x+1/2*a)^2)^(1/2)+8*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2)*(2*sinh(1/2
*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2
)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a+b\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + b*x)^(7/2),x)
```

```
[Out] int(1/cosh(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.15 $\int (a\cosh(x))^{7/2}\,dx$

Optimal. Leaf size=65

$$-\frac{10ia^4\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}}+\frac{10}{21}a^3\sinh(x)\sqrt{a\cosh(x)}+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

```
[Out] 2/7*a*(a*cosh(x))^(5/2)*sinh(x)-10/21*I*a^4*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*
x)*EllipticF(I*sinh(1/2*x),2^(1/2))*cosh(x)^(1/2)/(a*cosh(x))^(1/2)+10/21*a
^3*sinh(x)*(a*cosh(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2635, 2642, 2641}

$$\frac{10}{21}a^3\sinh(x)\sqrt{a\cosh(x)}-\frac{10ia^4\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}}+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*a^4*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (1
0*a^3*Sqrt[a*Cosh[x]]*Sinh[x])/21 + (2*a*(a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a\cosh(x))^{7/2}\,dx &= \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)+\frac{1}{7}\left(5a^2\right)\int (a\cosh(x))^{3/2}\,dx\\
&= \frac{10}{21}a^3\sqrt{a\cosh(x)}\,\sinh(x)+\frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)+\frac{1}{21}\left(5a^4\right)\int \frac{1}{\sqrt{a\cosh(x)}}\,dx\\
&= \frac{10}{21}a^3\sqrt{a\cosh(x)}\,\sinh(x)+\frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)+\frac{\left(5a^4\sqrt{\cosh(x)}\right)\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{21\sqrt{a\cosh(x)}}\\
&= -\frac{10ia^4\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{21\sqrt{a\cosh(x)}}+\frac{10}{21}a^3\sqrt{a\cosh(x)}\,\sinh(x)+\frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.82

$$
\frac{a^3\sqrt{a\cosh(x)}\left((23\sinh(x)+3\sinh(3x))\sqrt{\cosh(x)}-20iF\left(\frac{ix}{2}\middle|2\right)\right)}{42\sqrt{\cosh(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(7/2),x]
```

```
[Out] (a^3*Sqrt[a*Cosh[x]]*((-20*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(23*Sin
h[x] + 3*Sinh[3*x])))/(42*Sqrt[Cosh[x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\sqrt{a\cosh(x)}\,a^3\cosh(x)^3, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))*a^3*cosh(x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int (a\cosh(x))^{\frac{7}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(7/2), x)
```

maple [B] time = 0.35, size = 145, normalized size = 2.23

$$\frac{\sqrt{a\left(2\left(\cosh^{2}\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\; a^{4}\left(96\left(\cosh^{9}\left(\frac{x}{2}\right)\right)-240\left(\cosh^{7}\left(\frac{x}{2}\right)\right)+256\left(\cosh^{5}\left(\frac{x}{2}\right)\right)+5\sqrt{2}\;\sqrt{-2\left(\cosh^{2}\left(\right.\right.}\right.}{21\sqrt{a\left(2\left(\sinh^{4}\left(\frac{x}{2}\right)\right)+\sinh^{2}\left(\frac{x}{2}\right)\right)}\;\sinh\left(\frac{x}{2}\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(7/2),x)
```

```
[Out] 1/21*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^4*(96*cosh(1/2*x)^9-240*
cosh(1/2*x)^7+256*cosh(1/2*x)^5+5*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1/2)*(-sinh
(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-144*cosh(1/2*x)
^3+32*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a
*(2*cosh(1/2*x)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x))^{\frac{7}{2}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a\cosh(x))^{7/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(7/2),x)
```

```
[Out] int((a*cosh(x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(7/2),x)
```

```
[Out] Timed out
```

3.16 $\int (a\cosh(x))^{5/2}\,dx$

Optimal. Leaf size=48

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}-\frac{6ia^2E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5\sqrt{\cosh(x)}}$$

```
[Out] 2/5*a*(a*cosh(x))^(3/2)*sinh(x)-6/5*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)
*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))^(1/2)/cosh(x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2635, 2640, 2639}

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}-\frac{6ia^2E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(5/2),x]
```

```
[Out] (((-6*I)/5)*a^2*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]] + (2*a
*(a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int (a\cosh(x))^{5/2}\,dx = \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x) + \frac{1}{5}\left(3a^2\right)\int \sqrt{a\cosh(x)}\,dx$$
$$= \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x) + \frac{\left(3a^2\sqrt{a\cosh(x)}\right)\int \sqrt{\cosh(x)}\,dx}{5\sqrt{\cosh(x)}}$$
$$= -\frac{6ia^2\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{5\sqrt{\cosh(x)}} + \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)$$

Mathematica [A] time = 0.05, size = 41, normalized size = 0.85

$$\frac{2(a\cosh(x))^{5/2}\left(\sinh(x)\cosh^{\frac{3}{2}}(x) - 3iE\left(\frac{ix}{2}\middle|2\right)\right)}{5\cosh^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(5/2),x]
```

```
[Out] (2*(a*Cosh[x])^(5/2)*((-3*I)*EllipticE[(I/2)*x, 2] + Cosh[x]^(3/2)*Sinh[x])
)/(5*Cosh[x]^(5/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\cosh(x)}\,a^2\cosh(x)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))*a^2*cosh(x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(5/2), x)
```

maple [B] time = 0.37, size = 184, normalized size = 3.83

$$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\, a^3\left(16\left(\sinh^6\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+16\left(\sinh^4\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+3\sqrt{2}\,\sqrt{-2\left(\sinh^2\left(\right.\right.}}{5\sqrt{a\left(2\left(\mathrm{si}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(5/2),x)
```

```
[Out] 1/5*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^3*(16*sinh(1/2*x)^6*cosh(
1/2*x)+16*sinh(1/2*x)^4*cosh(1/2*x)+3*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-
sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-6*2^(1/2)*(
-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(1/2)*cosh(1/2
*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*
x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a\cosh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(5/2),x)
```

```
[Out] int((a*cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

3.17 $\int (a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=48

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\sqrt{a\cosh(x)}}$$

```
[Out] -2/3*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2
))*cosh(x)^(1/2)/(a*cosh(x))^(1/2)+2/3*a*sinh(x)*(a*cosh(x))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2635, 2642, 2641}

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*a^2*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*a
*Sqrt[a*Cosh[x]]*Sinh[x])/3
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (a\cosh(x))^{3/2}\,dx = \frac{2}{3}a\sqrt{a\cosh(x)}\,\sinh(x) + \frac{1}{3}a^2\int\frac{1}{\sqrt{a\cosh(x)}}\,dx$$
$$= \frac{2}{3}a\sqrt{a\cosh(x)}\,\sinh(x) + \frac{\left(a^2\sqrt{\cosh(x)}\right)\int\frac{1}{\sqrt{\cosh(x)}}\,dx}{3\sqrt{a\cosh(x)}}$$
$$= -\frac{2ia^2\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\sqrt{a\cosh(x)}} + \frac{2}{3}a\sqrt{a\cosh(x)}\,\sinh(x)$$

Mathematica [C] time = 0.06, size = 57, normalized size = 1.19

$$\frac{2}{3}(a\cosh(x))^{3/2}\left(\operatorname{sech}^2(x)\sqrt{\sinh(2x)+\cosh(2x)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(3/2),x]
```

```
[Out] (2*(a*Cosh[x])^(3/2)*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*
x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\cosh(x)}\,a\cosh(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))*a*cosh(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(3/2), x)
```

maple [B] time = 0.38, size = 130, normalized size = 2.71

$$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\, a^2\left(8\left(\sinh^4\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+\sqrt{2}\,\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)-1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\ \mathrm{Ellipt}}{3\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\ \sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(3/2),x)
```

```
[Out] 1/3*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a^2*(8*sinh(1/2*x)^4*cosh(1
/2*x)+2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2
^(1/2)*cosh(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2
*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a\cosh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(3/2),x)
```

```
[Out] int((a*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh{(x)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x))**(3/2), x)
```

3.18 $\int \sqrt{a\cosh(x)}\,dx$

Optimal. Leaf size=27

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{\sqrt{\cosh(x)}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*
cosh(x))^(1/2)/cosh(x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2640, 2639}

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\cosh(x)}\,dx &= \frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}\,dx}{\sqrt{\cosh(x)}}\\ &= -\frac{2i\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\cosh(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x)), x)
```

maple [B] time = 0.37, size = 118, normalized size = 4.37

$$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,a\sqrt{2}\,\sqrt{-2\left(\cosh^2\left(\frac{x}{2}\right)\right)+1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\left(\text{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)-\right.}{\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(1/2),x)
```

```
[Out] (a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*a*2^(1/2)*(-2*cosh(1/2*x)^2+1)^
(1/2)*(-sinh(1/2*x)^2)^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-2*
EllipticE(2^(1/2)*cosh(1/2*x),1/2*2^(1/2)))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)
^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*cosh(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a \cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x))^(1/2),x)
[Out] int((a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cosh{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x))**(1/2),x)
[Out] Integral(sqrt(a*cosh(x)), x)
```

3.19 $\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$

Optimal. Leaf size=27

$$-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*cos
h(x)^(1/2)/(a*cosh(x))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cosh(x)}}\,dx &= \frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{\sqrt{a\cosh(x)}}\\ &= -\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2i\sqrt{\cosh(x)}F\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]],x]
[Out] ((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)}}{a\cosh(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cosh(x))/(a*cosh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*cosh(x)), x)
```

maple [B] time = 0.25, size = 100, normalized size = 3.70

$$\frac{\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sqrt{2}\,\sqrt{-2\left(\cosh^2\left(\frac{x}{2}\right)\right)+1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\text{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right), \frac{\sqrt{2}}{2}\right)}{\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(1/2),x)
[Out] (a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1
/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)*Ellipt
```

```
icF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*cosh(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(1/2),x)
[Out] int(1/(a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(1/2),x)
[Out] Integral(1/sqrt(a*cosh(x)), x)
```

3.20 $\int \frac{1}{(a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}$$

```
[Out] 2*sinh(x)/a/(a*cosh(x))^(1/2)+2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*Ellipti
cE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))^(1/2)/a^2/cosh(x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2636, 2640, 2639}

$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-3/2),x]
```

```
[Out] ((2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sinh
[x])/(a*Sqrt[a*Cosh[x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\cosh(x))^{3/2}}\,dx &= \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}} - \frac{\int \sqrt{a\cosh(x)}\,dx}{a^2} \\
&= \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}} - \frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}\,dx}{a^2\sqrt{\cosh(x)}} \\
&= \frac{2i\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{a^2\sqrt{\cosh(x)}} + \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.74

$$
\frac{2\cosh(x)\left(\sinh(x) + i\sqrt{\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)\right)}{(a\cosh(x))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-3/2),x]
[Out] (2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/(a*Cosh[x])^(
3/2)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{\sqrt{a\cosh(x)}}{a^2\cosh(x)^2}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cosh(x))/(a^2*cosh(x)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="giac")
[Out] integrate((a*cosh(x))^(-3/2), x)
```

maple [B] time = 0.37, size = 159, normalized size = 3.46

$$-\frac{\sqrt{2a\left(\sinh^4\left(\frac{x}{2}\right)\right)+a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\left(\sqrt{2}\,\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)-1}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\mathrm{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)-2}{a\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(3/2),x)
```

```
[Out] -1/a*(2*a*sinh(1/2*x)^4+a*sinh(1/2*x)^2)^(1/2)*(2^(1/2)*(-2*sinh(1/2*x)^2-1
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-2
*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(1/2
)*cosh(1/2*x),1/2*2^(1/2))-4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4
+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(3/2),x)
```

```
[Out] int(1/(a*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(3/2),x)
```

```
[Out] Integral((a*cosh(x))**(-3/2), x)
```

3.21 $\int \frac{1}{(a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}}$$

```
[Out] 2/3*sinh(x)/a/(a*cosh(x))^(3/2)-2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*Ell
ipticF(I*sinh(1/2*x),2^(1/2))*cosh(x)^(1/2)/a^2/(a*cosh(x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2636, 2642, 2641}

$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}-\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]]) + (2
*Sinh[x])/(3*a*(a*Cosh[x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}} + \frac{\int \frac{1}{\sqrt{a\cosh(x)}}\,dx}{3a^2}$$
$$= \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}} + \frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3a^2\sqrt{a\cosh(x)}}$$
$$= -\frac{2i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3a^2\sqrt{a\cosh(x)}} + \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$

Mathematica [C] time = 0.04, size = 56, normalized size = 1.12

$$\frac{2\left(\sqrt{\sinh(2x)+\cosh(2x)+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)+\tanh(x)\right)}{3a^2\sqrt{a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-5/2),x]
```

```
[Out] (2*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*x]]*Sqrt[1 + Cosh[
2*x] + Sinh[2*x]] + Tanh[x]))/(3*a^2*Sqrt[a*Cosh[x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)}}{a^3\cosh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))/(a^3*cosh(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-5/2), x)
```

maple [B] time = 0.34, size = 177, normalized size = 3.54

$$\frac{\left(2\sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)-1}\ \sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\ \mathrm{EllipticF}\left(\sqrt{2}\ \cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)\sqrt{2}\ \left(\sinh^2\left(\frac{x}{2}\right)\right)+\sqrt{2}\ \sqrt{-2\left(\sinh^2\left(\frac{x}{2}\right)\right)-}\right.}{3a^2\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}\ \left(2\left(\cosh^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(5/2),x)
```

```
[Out] 1/3*(2*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*
cosh(1/2*x),1/2*2^(1/2))*2^(1/2)*sinh(1/2*x)^2+2^(1/2)*(-2*sinh(1/2*x)^2-1)
^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))+4*
sinh(1/2*x)^2*cosh(1/2*x))/a^2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)/
(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(2*cosh(1/2*x)^2-1)/sinh(1/2*x)/(
a*(2*cosh(1/2*x)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(5/2),x)
```

```
[Out] int(1/(a*cosh(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh{(x)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(5/2),x)
```

```
[Out] Integral((a*cosh(x))**(-5/2), x)
```

3.22 $\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$

Optimal. Leaf size=67

$$\frac{6iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5a^4\sqrt{\cosh(x)}}+\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

```
[Out] 2/5*sinh(x)/a/(a*cosh(x))^(5/2)+6/5*sinh(x)/a^3/(a*cosh(x))^(1/2)+6/5*I*(co
sh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))*(a*cosh(x))
^(1/2)/a^4/cosh(x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2636, 2640, 2639}

$$\frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}+\frac{6iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5a^4\sqrt{\cosh(x)}}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x])^(-7/2),x]
```

```
[Out] (((6*I)/5)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^4*Sqrt[Cosh[x]]) + (2*
Sinh[x])/(5*a*(a*Cosh[x])^(5/2)) + (6*Sinh[x])/(5*a^3*Sqrt[a*Cosh[x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{3\int \frac{1}{(a\cosh(x))^{3/2}}\,dx}{5a^2}$$
$$= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}} - \frac{3\int \sqrt{a\cosh(x)}\,dx}{5a^4}$$
$$= \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}} - \frac{\left(3\sqrt{a\cosh(x)}\right)\int \sqrt{\cosh(x)}\,dx}{5a^4\sqrt{\cosh(x)}}$$
$$= \frac{6i\sqrt{a\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{5a^4\sqrt{\cosh(x)}} + \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.64

$$\frac{2\left(\tanh(x) + 3i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 3\sinh(x)\cosh(x)\right)}{5a^2(a\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x])^(-7/2),x]
```

```
[Out] (2*((3*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 3*Cosh[x]*Sinh[x] + Tanh[x]
))/(5*a^2*(a*Cosh[x])^(3/2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)}}{a^4\cosh(x)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x))/(a^4*cosh(x)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x))^(-7/2), x)
```

maple [B] time = 0.60, size = 254, normalized size = 3.79

$$\frac{2\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(\frac{\cosh\left(\frac{x}{2}\right)\sqrt{a\left(2\left(\sinh^4\left(\frac{x}{2}\right)\right)+\sinh^2\left(\frac{x}{2}\right)\right)}}{20a\left(\cosh^2\left(\frac{x}{2}\right)-\frac{1}{2}\right)^3}+\frac{6\left(\sinh^2\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{5\sqrt{a\left(2\left(\cosh^2\left(\frac{x}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}}+\frac{3\sqrt{2}\ \sqrt{-2\left(\cosh^2\right.}}{}}{a^3\sinh\left(\frac{x}{2}\right)\surd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(7/2),x)
```

```
[Out] 2*(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)/a^3*(1/20*cosh(1/2*x)/a*(a*(2
*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(cosh(1/2*x)^2-1/2)^3+6/5*sinh(1/2*x)^
2*cosh(1/2*x)/(a*(2*cosh(1/2*x)^2-1)*sinh(1/2*x)^2)^(1/2)+3/10*2^(1/2)*(-2*
cosh(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*
x)^2))^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-3/5*2^(1/2)*(-2*cos
h(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^
2))^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-EllipticE(2^(1/2)*cos
h(1/2*x),1/2*2^(1/2))))/sinh(1/2*x)/(a*(2*cosh(1/2*x)^2-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x))^(7/2),x)
```

```
[Out] int(1/(a*cosh(x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x))**(7/2),x)
```

```
[Out] Timed out
```

3.23 $\int (b\cosh(c+dx))^n\,dx$

Optimal. Leaf size=71

$$-\frac{\sinh(c+dx)(b\cosh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(c+dx)\right)}{bd(n+1)\sqrt{-\sinh^2(c+dx)}}$$

```
[Out] -(b*cosh(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cosh(d*x+c)^2
)*sinh(d*x+c)/b/d/(1+n)/(-sinh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2643}

$$-\frac{\sinh(c+dx)(b\cosh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(c+dx)\right)}{bd(n+1)\sqrt{-\sinh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cosh[c + d*x])^n,x]
```

```
[Out] -(((b*Cosh[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, C
osh[c + d*x]^2]*Sinh[c + d*x])/(b*d*(1 + n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cosh(c+dx))^n\,dx = -\frac{(b\cosh(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cosh^2(c+dx)\right)\sinh(c+dx)}{bd(1+n)\sqrt{-\sinh^2(c+dx)}}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.92

$$\frac{\sqrt{-\sinh^2(c+dx)}\,\coth(c+dx)(b\cosh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cosh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cosh[c + d*x])^n,x]
```

```
[Out] ((b*Cosh[c + d*x])^n*Coth[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cosh[c + d*x]^2]*Sqrt[-Sinh[c + d*x]^2])/(d*(1 + n))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cosh\left(dx+c\right))^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cosh(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh\left(dx+c\right))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(d*x + c))^n, x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int (b\cosh\left(dx+c\right))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(d*x+c))^n,x)
```

```
[Out] int((b*cosh(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh\left(dx+c\right))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(d*x + c))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b\cosh (c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(c + d*x))^n,x)
```

```
[Out] int((b*cosh(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh (c+dx))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cosh(d*x+c))**n,x)
```

```
[Out] Integral((b*cosh(c + d*x))**n, x)
```

3.24 $\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3\sinh(x)\cosh(x)}{2a}$$

```
[Out] -3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^3*sinh(x)/(a+a*cosh(x))+
4/3*sinh(x)^3/a
```

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2767, 2748, 2635, 8, 2633}

$$-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^3*Sinh[
x])/(a + a*Cosh[x]) + (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx &= -\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}-\frac{\int \cosh^2(x)(3a-4a\cosh(x))\,dx}{a^2}\\ &= -\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}-\frac{3\int \cosh^2(x)\,dx}{a}+\frac{4\int \cosh^3(x)\,dx}{a}\\ &= -\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{(4i)\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}-\frac{3\int}{2}\\ &= -\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{4\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.09, size = 53, normalized size = 0.98

$$\frac{\text{sech}\left(\frac{x}{2}\right)\left(45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)-36x\cosh\left(\frac{x}{2}\right)\right)}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*x
)/2] + Sinh[(7*x)/2]))/(24*a)
```

fricas [B] time = 0.51, size = 100, normalized size = 1.85

$$\frac{\cosh(x)^4+(4\cosh(x)-1)\sinh(x)^3+\sinh(x)^4-3\cosh(x)^3+\left(6\cosh(x)^2-9\cosh(x)+20\right)\sinh(x)^2-3\,(12}{24\,(a\cosh(x)+a\,s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6*
cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)^2
 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x - 69
)/(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.12, size = 70, normalized size = 1.30

$$-\frac{3\,x}{2\,a}-\frac{\left(69\,e^{(3\,x)}+18\,e^{(2\,x)}-2\,e^{x}+1\right)e^{(-3\,x)}}{24\,a(e^{x}+1)}+\frac{a^{2}e^{(3\,x)}-3\,a^{2}e^{(2\,x)}+21\,a^{2}e^{x}}{24\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1)
) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

maple [B] time = 0.07, size = 111, normalized size = 2.06

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+-\frac{\cdots}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/3/a/(tanh(1/2*x)-1)^3-1/a/(tanh(1/2*x)-1)^2-5/2/a/(tanh(1
/2*x)-1)+3/2/a*ln(tanh(1/2*x)-1)-1/3/a/(tanh(1/2*x)+1)^3+1/a/(tanh(1/2*x)+1
)^2-5/2/a/(tanh(1/2*x)+1)-3/2/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.37, size = 66, normalized size = 1.22

$$-\frac{3\,x}{2\,a}-\frac{21\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}}{24\,a}-\frac{2\,e^{(-x)}-18\,e^{(-2\,x)}-69\,e^{(-3\,x)}-1}{24\left(ae^{(-3\,x)}+ae^{(-4\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 18
*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

mupad [B] time = 0.96, size = 70, normalized size = 1.30

$$\frac{e^{-2x}}{8a} - \frac{7e^{-x}}{8a} - \frac{e^{2x}}{8a} - \frac{e^{-3x}}{24a} + \frac{e^{3x}}{24a} - \frac{3x}{2a} - \frac{2}{a\,(e^{x}+1)} + \frac{7e^{x}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + a*cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + e
xp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

sympy [B] time = 1.89, size = 337, normalized size = 6.24

$$-\frac{9x\tanh^{6}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}+\frac{27x\tanh^{4}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}-\frac{}{6a\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] -9*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
 - 6*a) + 27*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*ta
nh(x/2)**2 - 6*a) - 27*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) + 9*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 1
8*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)**7/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)
**4 + 18*a*tanh(x/2)**2 - 6*a) - 48*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*t
anh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 50*tanh(x/2)**3/(6*a*tanh(x/2)**6
- 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 24*tanh(x/2)/(6*a*tanh(x/2
)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

3.25 $\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=43

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}+\frac{3\sinh(x)\cosh(x)}{2a}$$

```
[Out] 3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*cosh(x))
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2767, 2734}

$$\frac{3x}{2a}-\frac{2\sinh(x)}{a}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}+\frac{3\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[x
])/(a + a*Cosh[x])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = -\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)} - \frac{\int \cosh(x)(2a-3a\cosh(x))\,dx}{a^2}$$
$$= \frac{3x}{2a} - \frac{2\sinh(x)}{a} + \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 1.05

$$\frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)+12x\cosh\left(\frac{x}{2}\right)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2]
))/(8*a)
```

fricas [A] time = 3.49, size = 70, normalized size = 1.63

$$\frac{\cosh(x)^3 + (3\cosh(x)-4)\sinh(x)^2 + \sinh(x)^3 + (12x-1)\cosh(x) - 4\cosh(x)^2 + \left(3\cosh(x)^2 + 12x - 4\cos\right.}{8\,(a\cosh(x)+a\sinh(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x)
 - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20)/
(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.12, size = 51, normalized size = 1.19

$$\frac{3\,x}{2\,a} + \frac{\left(20\,e^{(2\,x)} + 3\,e^x - 1\right)e^{(-2\,x)}}{8\,a(e^x+1)} + \frac{ae^{(2\,x)} - 4\,ae^x}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(2
*x) - 4*a*e^x)/a^2
```

maple [B] time = 0.07, size = 87, normalized size = 2.02

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*cosh(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)+1/2/a/(tanh(1/2*x)-1)^2+3/2/a/(tanh(1/2*x)-1)-3/2/a*ln(tan
h(1/2*x)-1)-1/2/a/(tanh(1/2*x)+1)^2+3/2/a/(tanh(1/2*x)+1)+3/2/a*ln(tanh(1/2
*x)+1)
```

maxima [A] time = 0.35, size = 56, normalized size = 1.30

$$\frac{3\,x}{2\,a}+\frac{4\,e^{(-x)}-e^{(-2\,x)}}{8\,a}-\frac{3\,e^{(-x)}+20\,e^{(-2\,x)}-1}{8\left(ae^{(-2\,x)}+ae^{(-3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(a
*e^(-2*x) + a*e^(-3*x))
```

mupad [B] time = 0.92, size = 52, normalized size = 1.21

$$\frac{e^{-x}}{2\,a}-\frac{e^{-2\,x}}{8\,a}+\frac{e^{2\,x}}{8\,a}+\frac{3\,x}{2\,a}+\frac{2}{a\,\left(e^{x}+1\right)}-\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + a*cosh(x)),x)
```

```
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp(
x) + 1)) - exp(x)/(2*a)
```

sympy [B] time = 1.11, size = 189, normalized size = 4.40

$$\frac{3x\tanh^{4}\left(\frac{x}{2}\right)}{2a\tanh^{4}\left(\frac{x}{2}\right)-4a\tanh^{2}\left(\frac{x}{2}\right)+2a}-\frac{6x\tanh^{2}\left(\frac{x}{2}\right)}{2a\tanh^{4}\left(\frac{x}{2}\right)-4a\tanh^{2}\left(\frac{x}{2}\right)+2a}+\frac{3x}{2a\tanh^{4}\left(\frac{x}{2}\right)-4a\tanh^{2}\left(\frac{x}{2}\right)+2a}-\frac{}{2a\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] 3*x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 6*x*tanh(x/2
)**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + 3*x/(2*a*tanh(x/2)**4 -
4*a*tanh(x/2)**2 + 2*a) - 2*tanh(x/2)**5/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)*
*2 + 2*a) + 10*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 4
*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

3.26 $\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(\cosh(x)+1)}$$

```
[Out] -x/a+sinh(x)/a+sinh(x)/a/(1+cosh(x))
```

Rubi [A] time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2746, 12, 2735, 2648}

$$-\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(x/a) + Sinh[x]/a + Sinh[x]/(a*(1 + Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh(x)}{a}-\frac{\int \frac{a\cosh(x)}{a+a\cosh(x)}\,dx}{a}\\ &= \frac{\sinh(x)}{a}-\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx\\ &= -\frac{x}{a}+\frac{\sinh(x)}{a}+\int \frac{1}{a+a\cosh(x)}\,dx\\ &= -\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 1.28

$$\frac{-2x+3\tanh\left(\frac{x}{2}\right)+\sinh\left(\frac{3x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + a*Cosh[x]),x]
[Out] (-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

fricas [A] time = 1.46, size = 47, normalized size = 1.88

$$-\frac{2\,x\cosh(x)-\cosh(x)^2+2\,(x-\cosh(x)-1)\sinh(x)-\sinh(x)^2+2\,x+5}{2\,(a\cosh(x)+a\sinh(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
[Out] -1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 + 2
*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.12, size = 35, normalized size = 1.40

$$-\frac{x}{a}-\frac{(5\,e^x+1)e^{(-x)}}{2\,a(e^x+1)}+\frac{e^x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

maple [B] time = 0.07, size = 59, normalized size = 2.36

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*cosh(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)-1/a/(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)-1)-1/a/(tanh(1/2*x)+
1)-1/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.37, size = 41, normalized size = 1.64

$$-\frac{x}{a}+\frac{5\,e^{(-x)}+1}{2\left(ae^{(-x)}+ae^{(-2\,x)}\right)}-\frac{e^{(-x)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

mupad [B] time = 0.90, size = 34, normalized size = 1.36

$$\frac{\mathrm{e}^x}{2\,a}-\frac{x}{a}-\frac{2}{a\,\left(\mathrm{e}^x+1\right)}-\frac{\mathrm{e}^{-x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + a*cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

sympy [B] time = 0.60, size = 63, normalized size = 2.52

$$-\frac{x\tanh^2\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}+\frac{x}{a\tanh^2\left(\frac{x}{2}\right)-a}+\frac{\tanh^3\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}-\frac{3\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] -x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) + tanh(x/2)**
3/(a*tanh(x/2)**2 - a) - 3*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

3.27 $\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{x}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

[Out] x/a-sinh(x)/(a+a*cosh(x))

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$\frac{x}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + a*Cosh[x]),x]

[Out] x/a - Sinh[x]/(a + a*Cosh[x])

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\cosh(x)}\,dx\\ &= \frac{x}{a}-\frac{\sinh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 0.78

$$\frac{x-\tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Cosh[x]),x]
```

```
[Out] (x - Tanh[x/2])/a
```

fricas [A] time = 0.38, size = 24, normalized size = 1.33

$$\frac{x\cosh(x) + x\sinh(x) + x + 2}{a\cosh(x) + a\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] (x*cosh(x) + x*sinh(x) + x + 2)/(a*cosh(x) + a*sinh(x) + a)
```

giac [A] time = 0.12, size = 17, normalized size = 0.94

$$\frac{x}{a} + \frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] x/a + 2/(a*(e^x + 1))
```

maple [A] time = 0.05, size = 34, normalized size = 1.89

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*cosh(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 18, normalized size = 1.00

$$\frac{x}{a} - \frac{2}{ae^{(-x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] x/a - 2/(a*e^(-x) + a)
```

mupad [**B**] time = 0.87, size = 17, normalized size = 0.94

$$\frac{x}{a} + \frac{2}{a\,(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a*cosh(x)),x)
```

```
[Out] x/a + 2/(a*(exp(x) + 1))
```

sympy [**A**] time = 0.33, size = 8, normalized size = 0.44

$$\frac{x}{a} - \frac{\tanh\left(\frac{x}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x)),x)
```

```
[Out] x/a - tanh(x/2)/a
```

3.28 $\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}(\sinh(x))}{a} - \frac{\sinh(x)}{a\cosh(x)+a}$$

```
[Out] arctan(sinh(x))/a-sinh(x)/(a+a*cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2747, 3770, 2648}

$$\frac{\tan^{-1}(\sinh(x))}{a} - \frac{\sinh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Sinh[x]/(a + a*Cosh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{a + a\cosh(x)}\,dx = \frac{\int \operatorname{sech}(x)\,dx}{a} - \int \frac{1}{a + a\cosh(x)}\,dx$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{\sinh(x)}{a + a\cosh(x)}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.10

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - \tanh\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Cosh[x]),x]
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

fricas [A] time = 0.61, size = 29, normalized size = 1.45

$$\frac{2\left((\cosh(x) + \sinh(x) + 1)\arctan\left(\cosh(x) + \sinh(x)\right) + 1\right)}{a\cosh(x) + a\sinh(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="fricas")
[Out] 2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*si
nh(x) + a)
```

giac [A] time = 0.12, size = 20, normalized size = 1.00

$$\frac{2\arctan\left(e^x\right)}{a} + \frac{2}{a(e^x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="giac")
[Out] 2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

maple [A] time = 0.06, size = 21, normalized size = 1.05

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a} + \frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*cosh(x)),x)
[Out] -1/a*tanh(1/2*x)+2/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.40, size = 23, normalized size = 1.15

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}-\frac{2}{ae^{(-x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

mupad [B] time = 0.88, size = 31, normalized size = 1.55

$$\frac{2}{a\left(e^{x}+1\right)}+\frac{2\operatorname{atan}\left(\frac{e^{x}\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + a*cosh(x))),x)
[Out] 2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\operatorname{sech}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*cosh(x)),x)
[Out] Integral(sech(x)/(cosh(x) + 1), x)/a
```

3.29 $\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\cosh(x)+a}$$

[Out] -arctan(sinh(x))/a+2*tanh(x)/a-tanh(x)/(a+a*cosh(x))

Rubi [A] time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 8, 3770}

$$\frac{2\tanh(x)}{a}-\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2/(a + a*Cosh[x]),x]

[Out] -(ArcTan[Sinh[x]]/a) + (2*Tanh[x])/a - Tanh[x]/(a + a*Cosh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx &= -\frac{\tanh(x)}{a+a\cosh(x)}-\frac{\int(-2a+a\cosh(x))\operatorname{sech}^2(x)\,dx}{a^2}\\ &= -\frac{\tanh(x)}{a+a\cosh(x)}-\frac{\int\operatorname{sech}(x)\,dx}{a}+\frac{2\int\operatorname{sech}^2(x)\,dx}{a}\\ &= -\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a+a\cosh(x)}+\frac{(2i)\operatorname{Subst}(\int 1\,dx,x,-i\tanh(x))}{a}\\ &= -\frac{\tan^{-1}(\sinh(x))}{a}+\frac{2\tanh(x)}{a}-\frac{\tanh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 43, normalized size = 1.54

$$\frac{2\cosh\left(\frac{x}{2}\right)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(\tanh(x)-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)\right)}{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (2*Cosh[x/2]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x])))/(a*(
1 + Cosh[x]))
```

fricas [B] time = 0.46, size = 127, normalized size = 4.54

$$-\frac{2\left(\left(\cosh(x)^3+(3\cosh(x)+1)\sinh(x)^2+\sinh(x)^3+\cosh(x)^2+\left(3\cosh(x)^2+2\cosh(x)+1\right)\sinh(x)+\cosh\right.}{a\cosh(x)^3+a\sinh(x)^3+a\cosh(x)^2+(3a\cosh(x)+a)\sinh(x)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*cos
h(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) +
```

```
cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x)^3
 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (3
*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.15, size = 36, normalized size = 1.29

$$-\frac{2\,\arctan\left(e^{x}\right)}{a}-\frac{2\,\left(e^{(2\,x)}+e^{x}+2\right)}{a\left(e^{(3\,x)}+e^{(2\,x)}+e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="giac")
[Out] -2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

maple [A] time = 0.09, size = 39, normalized size = 1.39

$$\frac{\tanh\left(\frac{x}{2}\right)}{a}+\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+a*cosh(x)),x)
[Out] 1/a*tanh(1/2*x)+2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.80, size = 45, normalized size = 1.61

$$\frac{2\,\left(e^{(-x)}+e^{(-2\,x)}+2\right)}{ae^{(-x)}+ae^{(-2\,x)}+ae^{(-3\,x)}+a}+\frac{2\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
[Out] 2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arct
an(e^(-x))/a
```

mupad [B] time = 0.89, size = 58, normalized size = 2.07

$$-\frac{\frac{2\,\mathrm{e}^{2\,x}}{a}+\frac{4}{a}+\frac{2\,\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+\mathrm{e}^{x}+1}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + a*cosh(x))),x)
```

```
[Out] - ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
- (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^2(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**2/(cosh(x) + 1), x)/a
```

3.30 $\int \frac{\text{sech}^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=43

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}+\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

```
[Out] 3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)*tanh(x)/(a+
a*cosh(x))
```

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$-\frac{2\tanh(x)}{a}+\frac{3\tan^{-1}(\sinh(x))}{2a}+\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]*Tanh[x])/(a + a*Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{\int(-3a+2a\cosh(x))\operatorname{sech}^3(x)\,dx}{a^2}\\ &= -\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{2\int\operatorname{sech}^2(x)\,dx}{a}+\frac{3\int\operatorname{sech}^3(x)\,dx}{a}\\ &= \frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}-\frac{(2i)\operatorname{Subst}(\int 1\,dx,x,-i\tanh(x))}{a}+\frac{3\int\operatorname{sech}(x)\,dx}{2a}\\ &= \frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 49, normalized size = 1.14

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\cosh\left(\frac{x}{2}\right)\left(6\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)(\operatorname{sech}(x)-2)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + Sech[x])*
Tanh[x])))/(a*(1 + Cosh[x]))
```

fricas [B] time = 0.49, size = 325, normalized size = 7.56

$$\frac{3\,\cosh(x)^4+3\,(4\,\cosh(x)+1)\,\sinh(x)^3+3\,\sinh(x)^4+3\,\cosh(x)^3+\left(18\,\cosh(x)^2+9\,\cosh(x)+5\right)\sinh(x)^2 -}{a\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] (3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (1
8*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*sin
h(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^3
+ 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 2
*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)*sinh
(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh(x)^3
+ 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5 + a*sin
h(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3 + 2*(5*a
*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*cosh(x)^3
+ 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*cosh(x)^4 +
 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.15, size = 48, normalized size = 1.12

$$\frac{3\,\arctan\left(e^x\right)}{a}+\frac{e^{(3\,x)}+2\,e^{(2\,x)}-e^x+2}{a\left(e^{(2\,x)}+1\right)^2}+\frac{2}{a(e^x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/(
a*(e^x + 1))
```

maple [A] time = 0.09, size = 61, normalized size = 1.42

$$-\frac{\tanh\left(\frac{x}{2}\right)}{a}-\frac{3\left(\tanh^3\left(\frac{x}{2}\right)\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*cosh(x)),x)
```

```
[Out] -1/a*tanh(1/2*x)-3/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-1/a/(tanh(1/2*x)^2+1
)^2*tanh(1/2*x)+3/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.46, size = 73, normalized size = 1.70

$$-\frac{e^{(-x)} + 5\,e^{(-2\,x)} + 3\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 4}{ae^{(-x)} + 2\,ae^{(-2\,x)} + 2\,ae^{(-3\,x)} + ae^{(-4\,x)} + ae^{(-5\,x)} + a} - \frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2*
x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

mupad [B] time = 0.91, size = 73, normalized size = 1.70

$$\frac{2}{a\,\left(e^x+1\right)} + \frac{\frac{2}{a}+\frac{e^x}{a}}{e^{2\,x}+1} + \frac{3\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,e^x}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + a*cosh(x))),x)
```

```
[Out] 2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2
)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^{3}{\left(x \right)}}{\cosh{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**3/(cosh(x) + 1), x)/a
```

3.31 $\int \frac{\text{sech}^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{4\tanh^3(x)}{3a}+\frac{4\tanh(x)}{a}-\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

```
[Out] -3/2*arctan(sinh(x))/a+4*tanh(x)/a-3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)/
(a+a*cosh(x))-4/3*tanh(x)^3/a
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 3768, 3770}

$$-\frac{4\tanh^3(x)}{3a}+\frac{4\tanh(x)}{a}-\frac{3\tan^{-1}(\sinh(x))}{2a}-\frac{3\tanh(x)\text{sech}(x)}{2a}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (-3*ArcTan[Sinh[x]])/(2*a) + (4*Tanh[x])/a - (3*Sech[x]*Tanh[x])/(2*a) - (S
ech[x]^2*Tanh[x])/(a + a*Cosh[x]) - (4*Tanh[x]^3)/(3*a)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{\int(-4a+3a\cosh(x))\operatorname{sech}^4(x)\,dx}{a^2}\\ &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{3\int\operatorname{sech}^3(x)\,dx}{a}+\frac{4\int\operatorname{sech}^4(x)\,dx}{a}\\ &= -\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}+\frac{(4i)\operatorname{Subst}\left(\int\left(1+x^2\right)dx,x,-i\tanh(x)\right)}{a}-\frac{3\int \operatorname{s}}{}\\ &= -\frac{3\tan^{-1}(\sinh(x))}{2a}+\frac{4\tanh(x)}{a}-\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{4\tanh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.20, size = 60, normalized size = 1.07

$$\frac{\cosh\left(\frac{x}{2}\right)\left(6\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(\tanh(x)\left(2\operatorname{sech}^2(x)-3\operatorname{sech}(x)+10\right)-18\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)\right)}{3a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]*(6*Sinh[x/2] + Cosh[x/2]*(-18*ArcTan[Tanh[x/2]] + (10 - 3*Sech[x
] + 2*Sech[x]^2)*Tanh[x])))/(3*a*(1 + Cosh[x]))
```

fricas [B] time = 0.52, size = 600, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(9*cosh(x)^6 + 9*(6*cosh(x) + 1)*sinh(x)^5 + 9*sinh(x)^6 + 9*cosh(x)^5
 + 3*(45*cosh(x)^2 + 15*cosh(x) + 8)*sinh(x)^4 + 24*cosh(x)^4 + 6*(30*cosh(
x)^3 + 15*cosh(x)^2 + 16*cosh(x) + 4)*sinh(x)^3 + 24*cosh(x)^3 + 3*(45*cosh
(x)^4 + 30*cosh(x)^3 + 48*cosh(x)^2 + 24*cosh(x) + 13)*sinh(x)^2 + 9*(cosh(
x)^7 + (7*cosh(x) + 1)*sinh(x)^6 + sinh(x)^7 + cosh(x)^6 + 3*(7*cosh(x)^2 +
 2*cosh(x) + 1)*sinh(x)^5 + 3*cosh(x)^5 + (35*cosh(x)^3 + 15*cosh(x)^2 + 15
*cosh(x) + 3)*sinh(x)^4 + 3*cosh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^3 + 30*c
osh(x)^2 + 12*cosh(x) + 3)*sinh(x)^3 + 3*cosh(x)^3 + 3*(7*cosh(x)^5 + 5*cos
h(x)^4 + 10*cosh(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 3*cosh(x)^
2 + (7*cosh(x)^6 + 6*cosh(x)^5 + 15*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2
+ 6*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 39*cosh
(x)^2 + (54*cosh(x)^5 + 45*cosh(x)^4 + 96*cosh(x)^3 + 72*cosh(x)^2 + 78*cos
h(x) + 7)*sinh(x) + 7*cosh(x) + 16)/(a*cosh(x)^7 + a*sinh(x)^7 + a*cosh(x)^
6 + (7*a*cosh(x) + a)*sinh(x)^6 + 3*a*cosh(x)^5 + 3*(7*a*cosh(x)^2 + 2*a*co
sh(x) + a)*sinh(x)^5 + 3*a*cosh(x)^4 + (35*a*cosh(x)^3 + 15*a*cosh(x)^2 + 1
5*a*cosh(x) + 3*a)*sinh(x)^4 + 3*a*cosh(x)^3 + (35*a*cosh(x)^4 + 20*a*cosh(
x)^3 + 30*a*cosh(x)^2 + 12*a*cosh(x) + 3*a)*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(
7*a*cosh(x)^5 + 5*a*cosh(x)^4 + 10*a*cosh(x)^3 + 6*a*cosh(x)^2 + 3*a*cosh(x
) + a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 6*a*cosh(x)^5 + 15*a*cosh(x
)^4 + 12*a*cosh(x)^3 + 9*a*cosh(x)^2 + 6*a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.14, size = 57, normalized size = 1.02

$$-\frac{3 \arctan\left(e^{x}\right)}{a} - \frac{2}{a(e^{x} + 1)} - \frac{3 e^{(5x)} + 6 e^{(4x)} + 24 e^{(2x)} - 3 e^{x} + 10}{3 a\left(e^{(2x)} + 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3*arctan(e^x)/a - 2/(a*(e^x + 1)) - 1/3*(3*e^(5*x) + 6*e^(4*x) + 24*e^(2*x
) - 3*e^x + 10)/(a*(e^(2*x) + 1)^3)
```

maple [A] time = 0.09, size = 81, normalized size = 1.45

$$\frac{\tanh\left(\frac{x}{2}\right)}{a} + \frac{5\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}} + \frac{16\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}} + \frac{3\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}} - \frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/a*tanh(1/2*x)+5/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5+16/3/a/(tanh(1/2*x)^2
+1)^3*tanh(1/2*x)^3+3/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)-3/a*arctan(tanh(1/2
*x))
```

maxima [B] time = 0.41, size = 101, normalized size = 1.80

$$\frac{7 e^{(-x)} + 39 e^{(-2x)} + 24 e^{(-3x)} + 24 e^{(-4x)} + 9 e^{(-5x)} + 9 e^{(-6x)} + 16}{3 \left(ae^{(-x)} + 3 ae^{(-2x)} + 3 ae^{(-3x)} + 3 ae^{(-4x)} + 3 ae^{(-5x)} + ae^{(-6x)} + ae^{(-7x)} + a\right)} + \frac{3 \arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(7*e^(-x) + 39*e^(-2*x) + 24*e^(-3*x) + 24*e^(-4*x) + 9*e^(-5*x) + 9*e^
(-6*x) + 16)/(a*e^(-x) + 3*a*e^(-2*x) + 3*a*e^(-3*x) + 3*a*e^(-4*x) + 3*a*e
^(-5*x) + a*e^(-6*x) + a*e^(-7*x) + a) + 3*arctan(e^(-x))/a
```

mupad [B] time = 0.90, size = 107, normalized size = 1.91

$$\frac{8}{3\,a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)}-\frac{\frac{4}{a}-\frac{2\,e^x}{a}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{2}{a\,\left(e^x+1\right)}-\frac{\frac{2}{a}+\frac{e^x}{a}}{e^{2\,x}+1}-\frac{3\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + a*cosh(x))),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2*
exp(2*x) + exp(4*x) + 1) - 2/(a*(exp(x) + 1)) - (2/a + exp(x)/a)/(exp(2*x)
+ 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^{4}{\left(x \right)}}{\cosh{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(sech(x)**4/(cosh(x) + 1), x)/a
```

3.32 $\int \frac{1}{1+\cosh(c+dx)}\,dx$

Optimal. Leaf size=20

$$\frac{\sinh(c+dx)}{d(\cosh(c+dx)+1)}$$

```
[Out] sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2648}

$$\frac{\sinh(c+dx)}{d(\cosh(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-1),x]
```

```
[Out] Sinh[c + d*x]/(d*(1 + Cosh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cosh(c+dx)}\,dx=\frac{\sinh(c+dx)}{d(1+\cosh(c+dx))}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.70

$$\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-1),x]
```

```
[Out] Tanh[(c + d*x)/2]/d
```

fricas [A] time = 0.43, size = 22, normalized size = 1.10

$$-\frac{2}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="fricas")
[Out] -2/(d*cosh(d*x + c) + d*sinh(d*x + c) + d)
```

giac [A] time = 0.14, size = 15, normalized size = 0.75

$$-\frac{2}{d\left(e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="giac")
[Out] -2/(d*(e^(d*x + c) + 1))
```

maple [A] time = 0.05, size = 14, normalized size = 0.70

$$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c)),x)
[Out] 1/d*tanh(1/2*d*x+1/2*c)
```

maxima [A] time = 0.31, size = 18, normalized size = 0.90

$$\frac{2}{d\left(e^{(-dx-c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x, algorithm="maxima")
[Out] 2/(d*(e^(-d*x - c) + 1))
```

mupad [B] time = 0.89, size = 15, normalized size = 0.75

$$-\frac{2}{d\,\left(\mathrm{e}^{c+d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1),x)
```

```
[Out] -2/(d*(exp(c + d*x) + 1))
```

sympy [A] time = 0.52, size = 17, normalized size = 0.85

$$\begin{cases} \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} & \text{for } d \neq 0 \\ \frac{x}{\cosh(c)+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c)),x)
```

```
[Out] Piecewise((tanh(c/2 + d*x/2)/d, Ne(d, 0)), (x/(cosh(c) + 1), True))
```

3.33 $\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

```
[Out] 1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2650, 2648}

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-2),x]
```

```
[Out] Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c
+ d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\cosh(c+dx))^2}\,dx &= \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1+\cosh(c+dx)}\,dx\\ &= \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2}+\frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.72

$$\frac{4\sinh(c+dx)+\sinh(2(c+dx))}{6d(\cosh(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-2),x]
```

```
[Out] (4*Sinh[c + d*x] + Sinh[2*(c + d*x)])/(6*d*(1 + Cosh[c + d*x])^2)
```

fricas [B] time = 0.45, size = 113, normalized size = 2.40

$$-\frac{2\,(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1}{3\,\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}+3\,(d\cosh\left(dx+c\right)+d)\sinh\left(dx+c\right)^{2}+3\,d\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1)/(d*cosh(d*x + c)^3 + d*sinh(d*
x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) + d)*sinh(d*x + c)^2 +
3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c) + d)*sinh(d*x
+ c) + d)
```

giac [A] time = 0.13, size = 25, normalized size = 0.53

$$-\frac{2\left(3\,e^{(dx+c)}+1\right)}{3\,d\left(e^{(dx+c)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^3)
```

maple [A] time = 0.06, size = 30, normalized size = 0.64

$$\frac{-\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/6*tanh(1/2*d*x+1/2*c)^3+1/2*tanh(1/2*d*x+1/2*c))
```

maxima [B] time = 0.31, size = 90, normalized size = 1.91

$$\frac{2\,e^{(-dx-c)}}{d\left(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1\right)}+\frac{2}{3\,d\left(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) +
 1)) + 2/3/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) + 1))
```

mupad [B] time = 0.06, size = 25, normalized size = 0.53

$$-\frac{2\left(3\,e^{c+d\,x}+1\right)}{3\,d\left(e^{c+d\,x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1)^2,x)
```

```
[Out] -(2*(3*exp(c + d*x) + 1))/(3*d*(exp(c + d*x) + 1)^3)
```

sympy [A] time = 1.01, size = 36, normalized size = 0.77

$$\begin{cases} -\frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} & \text{for } d\neq 0 \\ \frac{x}{(\cosh(c)+1)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((-tanh(c/2 + d*x/2)**3/(6*d) + tanh(c/2 + d*x/2)/(2*d), Ne(d, 0))
, (x/(cosh(c) + 1)**2, True))
```

3.34 $\int \frac{1}{(1+\cosh(c+dx))^3} \, dx$

Optimal. Leaf size=70

$$\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)} + \frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)^2} + \frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

```
[Out] 1/5*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2/15*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2/
15*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2650, 2648}

$$\frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)} + \frac{2\sinh(c+dx)}{15d(\cosh(c+dx)+1)^2} + \frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-3),x]
```

```
[Out] Sinh[c + d*x]/(5*d*(1 + Cosh[c + d*x])^3) + (2*Sinh[c + d*x])/(15*d*(1 + Co
sh[c + d*x])^2) + (2*Sinh[c + d*x])/(15*d*(1 + Cosh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3} + \frac{2}{5}\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$$
$$= \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2} + \frac{2}{15}\int \frac{1}{1+\cosh(c+dx)}\,dx$$
$$= \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2} + \frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))}$$

Mathematica [**A**] time = 0.06, size = 44, normalized size = 0.63

$$\frac{15\sinh(c+dx)+6\sinh(2(c+dx))+\sinh(3(c+dx))}{30d(\cosh(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-3),x]
```

```
[Out] (15*Sinh[c + d*x] + 6*Sinh[2*(c + d*x)] + Sinh[3*(c + d*x)])/(30*d*(1 + Cos
h[c + d*x])^3)
```

fricas [**B**] time = 0.50, size = 174, normalized size = 2.49

$$-\frac{}{15\left(d\cosh(dx+c)^4+d\sinh(dx+c)^4+5\,d\cosh(dx+c)^3+(4\,d\cosh(dx+c)+5\,d)\sinh(dx+c)^3+10\,d\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) + 5)/(d*cosh(d*x + c)^4 + d*sinh(
d*x + c)^4 + 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^
3 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 + 15*d*cosh(d*x + c) + 10*d
)*sinh(d*x + c)^2 + 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 + 15*d*cosh(d
*x + c)^2 + 20*d*cosh(d*x + c) + 9*d)*sinh(d*x + c) + 5*d)
```

giac [**A**] time = 0.14, size = 36, normalized size = 0.51

$$-\frac{4\left(10\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}+1\right)}{15\,d\left(e^{(dx+c)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -4/15*(10*e^(2*d*x + 2*c) + 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^5)
```

maple [A] time = 0.06, size = 43, normalized size = 0.61

$$\frac{\frac{\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20}-\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/20*tanh(1/2*d*x+1/2*c)^5-1/6*tanh(1/2*d*x+1/2*c)^3+1/4*tanh(1/2*d*x+
1/2*c))
```

maxima [B] time = 0.35, size = 205, normalized size = 2.93

$$\frac{4\,e^{(-dx-c)}}{3\,d\left(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1\right)}+\frac{8\,e^{(-}}{3\,d\left(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1)) + 8/3*e^(-2*d*x - 2*c)/(
d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x
 - 4*c) + e^(-5*d*x - 5*c) + 1)) + 4/15/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x -
 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1))
```

mupad [B] time = 0.93, size = 36, normalized size = 0.51

$$-\frac{4\,\left(5\,\mathrm{e}^{c+d\,x}+10\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}{15\,d\,{\left(\mathrm{e}^{c+d\,x}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1)^3,x)
```

```
[Out] -(4*(5*exp(c + d*x) + 10*exp(2*c + 2*d*x) + 1))/(15*d*(exp(c + d*x) + 1)^5)
```

sympy [A] time = 2.17, size = 51, normalized size = 0.73

$$\begin{cases} \frac{\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20d}-\frac{\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} & \text{for } d\neq 0 \\ \frac{x}{(\cosh(c)+1)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((tanh(c/2 + d*x/2)**5/(20*d) - tanh(c/2 + d*x/2)**3/(6*d) + tanh(
c/2 + d*x/2)/(4*d), Ne(d, 0)), (x/(cosh(c) + 1)**3, True))
```

3.35 $\int \frac{1}{(1+\cosh(c+dx))^4} \, dx$

Optimal. Leaf size=93

$$\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)^2}+\frac{3\sinh(c+dx)}{35d(\cosh(c+dx)+1)^3}+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

```
[Out] 1/7*sinh(d*x+c)/d/(1+cosh(d*x+c))^4+3/35*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2/
35*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2/35*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2650, 2648}

$$\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)}+\frac{2\sinh(c+dx)}{35d(\cosh(c+dx)+1)^2}+\frac{3\sinh(c+dx)}{35d(\cosh(c+dx)+1)^3}+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[c + d*x])^(-4),x]
```

```
[Out] Sinh[c + d*x]/(7*d*(1 + Cosh[c + d*x])^4) + (3*Sinh[c + d*x])/(35*d*(1 + Co
sh[c + d*x])^3) + (2*Sinh[c + d*x])/(35*d*(1 + Cosh[c + d*x])^2) + (2*Sinh[
c + d*x])/(35*d*(1 + Cosh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1+\cosh(c+dx))^3}\,dx$$
$$= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1+\cosh(c+dx))^2}\,dx$$
$$= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \frac{2}{35}\int$$
$$= \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \frac{2}{35d(1}$$

Mathematica [A] time = 0.09, size = 54, normalized size = 0.58

$$\frac{56\sinh(c+dx)+28\sinh(2(c+dx))+8\sinh(3(c+dx))+\sinh(4(c+dx))}{140d(\cosh(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[c + d*x])^(-4),x]
```

```
[Out] (56*Sinh[c + d*x] + 28*Sinh[2*(c + d*x)] + 8*Sinh[3*(c + d*x)] + Sinh[4*(c
+ d*x)])/(140*d*(1 + Cosh[c + d*x])^4)
```

fricas [B] time = 0.52, size = 347, normalized size = 3.73

$$-\frac{}{35\left(d\cosh(dx+c)^6 + d\sinh(dx+c)^6 + 7\,d\cosh(dx+c)^5 + (6\,d\cosh(dx+c)+7\,d)\sinh(dx+c)^5 + 21\,d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) + 2)*sinh(d*x + c) + 35*sin
h(d*x + c)^2 + 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)^6
 + 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) + 7*d)*sinh(d*x + c)^5 + 21*d*c
osh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 35*d*cosh(d*x + c) + 21*d)*sinh(d*
x + c)^4 + 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 + 70*d*cosh(d*x + c
)^2 + 84*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^2 + (
15*d*cosh(d*x + c)^4 + 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2 + 105*d
*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 + 22*d*cosh(d*x + c) + (6*d*cosh(d*x
 + c)^5 + 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 105*d*cosh(d*x + c)
^2 + 70*d*cosh(d*x + c) + 20*d)*sinh(d*x + c) + 7*d)
```

giac [A] time = 0.14, size = 47, normalized size = 0.51

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}+21\,e^{(2\,dx+2\,c)}+7\,e^{(dx+c)}+1\right)}{35\,d\left(e^{(dx+c)}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) + 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) + 1)/(d*(e^(
d*x + c) + 1)^7)
```

maple [A] time = 0.05, size = 56, normalized size = 0.60

$$\frac{-\frac{\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56}+\frac{3\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40}-\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(-1/56*tanh(1/2*d*x+1/2*c)^7+3/40*tanh(1/2*d*x+1/2*c)^5-1/8*tanh(1/2*d*
x+1/2*c)^3+1/8*tanh(1/2*d*x+1/2*c))
```

maxima [B] time = 0.49, size = 364, normalized size = 3.91

$$\frac{4\,e^{(-dx-c)}}{5\,d\left(7\,e^{(-dx-c)}+21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}+35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}+1\right)}+\frac{}{5\,d\left(7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(
-7*d*x - 7*c) + 1)) + 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d
*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c)
 + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4*e^(-3*d*x - 3*c)/(d*(7*e
^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*
c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4/
35/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-
4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c)
+ 1))
```

mupad [B] time = 0.92, size = 283, normalized size = 3.04

$$-\frac{4}{35\,d\,\left(4\,{\mathrm{e}}^{c+d\,x}+6\,{\mathrm{e}}^{2\,c+2\,d\,x}+4\,{\mathrm{e}}^{3\,c+3\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{16\,{\mathrm{e}}^{c+d\,x}}{35\,d\,\left(5\,{\mathrm{e}}^{c+d\,x}+10\,{\mathrm{e}}^{2\,c+2\,d\,x}+10\,{\mathrm{e}}^{3\,c+3\,d\,x}+5\,{\mathrm{e}}^{4\,c+4\,d\,x}+\mathrm{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) + 1)^4,x)
```

```
[Out] - 4/(35*d*(4*exp(c + d*x) + 6*exp(2*c + 2*d*x) + 4*exp(3*c + 3*d*x) + exp(4
*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) + 10*exp(2*c +
2*d*x) + 10*exp(3*c + 3*d*x) + 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) + 1))
- (8*exp(2*c + 2*d*x))/(7*d*(6*exp(c + d*x) + 15*exp(2*c + 2*d*x) + 20*exp(
3*c + 3*d*x) + 15*exp(4*c + 4*d*x) + 6*exp(5*c + 5*d*x) + exp(6*c + 6*d*x)
+ 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) + 21*exp(2*c + 2*d*x) +
35*exp(3*c + 3*d*x) + 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) + 7*exp(6*c
 + 6*d*x) + exp(7*c + 7*d*x) + 1))
```

sympy [A] time = 5.19, size = 68, normalized size = 0.73

$$\begin{cases} -\frac{\tanh^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56d}+\frac{3\tanh^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40d}-\frac{\tanh^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} & \text{for}\ d\neq 0\\ \frac{x}{(\cosh(c)+1)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((-tanh(c/2 + d*x/2)**7/(56*d) + 3*tanh(c/2 + d*x/2)**5/(40*d) - t
anh(c/2 + d*x/2)**3/(8*d) + tanh(c/2 + d*x/2)/(8*d), Ne(d, 0)), (x/(cosh(c)
 + 1)**4, True))
```

3.36 $\int \frac{1}{1-\cosh(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

```
[Out] -sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-1),x]
```

```
[Out] -(Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cosh(c+dx)}\,dx=-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 0.61

$$\frac{\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-1),x]
```

```
[Out] Coth[(c + d*x)/2]/d
```

fricas [A] time = 0.58, size = 24, normalized size = 1.04

$$\frac{2}{d \cosh\left(dx + c\right) + d \sinh\left(dx + c\right) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="fricas")
[Out] 2/(d*cosh(d*x + c) + d*sinh(d*x + c) - d)
```

giac [A] time = 0.14, size = 15, normalized size = 0.65

$$\frac{2}{d\left(e^{(dx+c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="giac")
[Out] 2/(d*(e^(d*x + c) - 1))
```

maple [A] time = 0.06, size = 16, normalized size = 0.70

$$\frac{1}{d \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c)),x)
[Out] 1/d/tanh(1/2*d*x+1/2*c)
```

maxima [A] time = 0.39, size = 18, normalized size = 0.78

$$-\frac{2}{d\left(e^{(-dx-c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x, algorithm="maxima")
[Out] -2/(d*(e^(-d*x - c) - 1))
```

mupad [B] time = 0.89, size = 15, normalized size = 0.65

$$\frac{2}{d\,\left(\mathrm{e}^{c+d\,x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(c + d*x) - 1),x)
```

```
[Out] 2/(d*(exp(c + d*x) - 1))
```

sympy [A] time = 0.63, size = 32, normalized size = 1.39

$$\begin{cases} \tilde{\infty}x & \text{for } (c = 0 \vee c = -dx) \wedge (c = -dx \vee d = 0) \\ \frac{x}{1-\cosh(c)} & \text{for } d = 0 \\ \frac{1}{d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x/
(1 - cosh(c)), Eq(d, 0)), (1/(d*tanh(c/2 + d*x/2)), True))
```

3.37 $\int \frac{1}{(1-\cosh(c+dx))^2} \, dx$

Optimal. Leaf size=51

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

```
[Out] -1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-2),x]
```

```
[Out] -Sinh[c + d*x]/(3*d*(1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cosh[c
 + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-\cosh(c+dx))^2} \, dx &= -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)} \, dx\\ &= -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.61

$$\frac{\sinh(c+dx)(\cosh(c+dx)-2)}{3d(\cosh(c+dx)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-2),x]
```

```
[Out] ((-2 + Cosh[c + d*x])*Sinh[c + d*x])/(3*d*(-1 + Cosh[c + d*x])^2)
```

fricas [B] time = 0.77, size = 117, normalized size = 2.29

$$-\frac{2\,(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)-1}{3\,\left(d\cosh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{3}-3\,d\cosh\left(dx+c\right)^{2}+3\,\left(d\cosh\left(dx+c\right)-d\right)\sinh\left(dx+c\right)^{2}+3\,d\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) - 1)/(d*cosh(d*x + c)^3 + d*sinh(d*
x + c)^3 - 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) - d)*sinh(d*x + c)^2 +
3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 - 2*d*cosh(d*x + c) + d)*sinh(d*x
+ c) - d)
```

giac [A] time = 0.12, size = 25, normalized size = 0.49

$$-\frac{2\left(3\,e^{(dx+c)}-1\right)}{3\,d\left(e^{(dx+c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(d*x + c) - 1)/(d*(e^(d*x + c) - 1)^3)
```

maple [A] time = 0.07, size = 32, normalized size = 0.63

$$\frac{-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/6/tanh(1/2*d*x+1/2*c)^3+1/2/tanh(1/2*d*x+1/2*c))
```

maxima [B] time = 0.88, size = 90, normalized size = 1.76

$$\frac{2\,e^{(-dx-c)}}{d\left(3\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}-1\right)}-\frac{2}{3\,d\left(3\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) -
 1)) - 2/3/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) - 1))
```

mupad [B] time = 0.06, size = 25, normalized size = 0.49

$$-\frac{2\,\left(3\,\mathrm{e}^{c+d\,x}-1\right)}{3\,d\,\left(\mathrm{e}^{c+d\,x}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) - 1)^2,x)
```

```
[Out] -(2*(3*exp(c + d*x) - 1))/(3*d*(exp(c + d*x) - 1)^3)
```

sympy [A] time = 1.23, size = 53, normalized size = 1.04

$$\begin{cases} \tilde{\infty}x & \text{for}\ (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ \frac{x}{(1-\cosh(c))^2} & \text{for}\ d=0 \\ \frac{1}{2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{1}{6d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x/
(1 - cosh(c))**2, Eq(d, 0)), (1/(2*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 +
 d*x/2)**3), True))
```

3.38 $\int \frac{1}{(1-\cosh(c+dx))^3} \, dx$

Optimal. Leaf size=76

$$-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

```
[Out] -1/5*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-2/15*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-2
/15*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2}-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-3),x]
```

```
[Out] -Sinh[c + d*x]/(5*d*(1 - Cosh[c + d*x])^3) - (2*Sinh[c + d*x])/(15*d*(1 - C
osh[c + d*x])^2) - (2*Sinh[c + d*x])/(15*d*(1 - Cosh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} + \frac{2}{5}\int \frac{1}{(1-\cosh(c+dx))^2}\,dx$$
$$= -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2} + \frac{2}{15}\int \frac{1}{1-\cosh(c+dx)}\,dx$$
$$= -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 0.54

$$\frac{\sinh(c+dx)(-6\cosh(c+dx)+\cosh(2(c+dx))+8)}{15d(\cosh(c+dx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-3),x]
```

```
[Out] ((8 - 6*Cosh[c + d*x] + Cosh[2*(c + d*x)])*Sinh[c + d*x])/(15*d*(-1 + Cosh[
c + d*x])^3)
```

fricas [B] time = 0.48, size = 174, normalized size = 2.29

$$\frac{}{15\left(d\cosh\left(dx+c\right)^{4} + d\sinh\left(dx+c\right)^{4} - 5\,d\cosh\left(dx+c\right)^{3} + \left(4\,d\cosh\left(dx+c\right) - 5\,d\right)\sinh\left(dx+c\right)^{3} + 10\,d\cosh\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) - 5)/(d*cosh(d*x + c)^4 + d*sinh(d
*x + c)^4 - 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) - 5*d)*sinh(d*x + c)^3
 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 - 15*d*cosh(d*x + c) + 10*d)
*sinh(d*x + c)^2 - 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 - 15*d*cosh(d*
x + c)^2 + 20*d*cosh(d*x + c) - 9*d)*sinh(d*x + c) + 5*d)
```

giac [A] time = 0.13, size = 36, normalized size = 0.47

$$\frac{4\left(10\,e^{(2\,dx+2\,c)} - 5\,e^{(dx+c)} + 1\right)}{15\,d\left(e^{(dx+c)} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/15*(10*e^(2*d*x + 2*c) - 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) - 1)^5)
```

maple [A] time = 0.08, size = 45, normalized size = 0.59

$$\frac{-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{1}{20\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/6/tanh(1/2*d*x+1/2*c)^3+1/4/tanh(1/2*d*x+1/2*c)+1/20/tanh(1/2*d*x+1
/2*c)^5)
```

maxima [B] time = 0.41, size = 205, normalized size = 2.70

$$\frac{4\,e^{(-dx-c)}}{3\,d\left(5\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}-1\right)}-\frac{8\,e^{(-2}}{3\,d\left(5\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10\,e^{(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1)) - 8/3*e^(-2*d*x - 2*c)/(
d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x
 - 4*c) + e^(-5*d*x - 5*c) - 1)) - 4/15/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x -
 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1))
```

mupad [B] time = 0.91, size = 36, normalized size = 0.47

$$\frac{4\,\left(10\,\mathrm{e}^{2\,c+2\,d\,x}-5\,\mathrm{e}^{c+d\,x}+1\right)}{15\,d\,\left(\mathrm{e}^{c+d\,x}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh(c + d*x) - 1)^3,x)
```

```
[Out] (4*(10*exp(2*c + 2*d*x) - 5*exp(c + d*x) + 1))/(15*d*(exp(c + d*x) - 1)^5)
```

sympy [A] time = 2.52, size = 70, normalized size = 0.92

$$\begin{cases} \tilde{\infty}x & \text{for } (c = 0 \lor c = -dx) \land (c = -dx \lor d = 0) \\ \frac{x}{(1-\cosh(c))^3} & \text{for } d = 0 \\ \frac{1}{4d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{6d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{1}{20d\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((zoo*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x/
(1 - cosh(c))**3, Eq(d, 0)), (1/(4*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 +
 d*x/2)**3) + 1/(20*d*tanh(c/2 + d*x/2)**5), True))
```

3.39 $\int \frac{1}{(1-\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=101

$$-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

```
[Out] -1/7*sinh(d*x+c)/d/(1-cosh(d*x+c))^4-3/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-2
/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-2/35*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cosh[c + d*x])^(-4),x]
```

```
[Out] -Sinh[c + d*x]/(7*d*(1 - Cosh[c + d*x])^4) - (3*Sinh[c + d*x])/(35*d*(1 - C
osh[c + d*x])^3) - (2*Sinh[c + d*x])/(35*d*(1 - Cosh[c + d*x])^2) - (2*Sinh
[c + d*x])/(35*d*(1 - Cosh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}+\frac{3}{7}\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$$
$$= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}+\frac{6}{35}\int \frac{1}{(1-\cosh(c+dx))^2}\,dx$$
$$= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}+\frac{2}{35}\int$$
$$= -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}-\frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3}-\frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2}-\frac{2}{35d(1}$$

Mathematica [A] time = 0.08, size = 51, normalized size = 0.50

$$\frac{\sinh(c+dx)(29\cosh(c+dx)-8\cosh(2(c+dx))+\cosh(3(c+dx))-32)}{70d(\cosh(c+dx)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cosh[c + d*x])^(-4),x]
```

```
[Out] ((-32 + 29*Cosh[c + d*x] - 8*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)])*Sinh[c
+ d*x])/(70*d*(-1 + Cosh[c + d*x])^4)
```

fricas [B] time = 0.54, size = 347, normalized size = 3.44

$$-\frac{}{35\left(d\cosh(dx+c)^6+d\sinh(dx+c)^6-7\,d\cosh(dx+c)^5+(6\,d\cosh(dx+c)-7\,d)\sinh(dx+c)^5+21\,d\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) - 2)*sinh(d*x + c) + 35*sin
h(d*x + c)^2 - 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)^6
 - 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) - 7*d)*sinh(d*x + c)^5 + 21*d*c
osh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - 35*d*cosh(d*x + c) + 21*d)*sinh(d*
x + c)^4 - 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 - 70*d*cosh(d*x + c
)^2 + 84*d*cosh(d*x + c) - 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^2 + (
15*d*cosh(d*x + c)^4 - 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2 - 105*d
*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 - 22*d*cosh(d*x + c) + (6*d*cosh(d*x
 + c)^5 - 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 - 105*d*cosh(d*x + c)
^2 + 70*d*cosh(d*x + c) - 20*d)*sinh(d*x + c) + 7*d)
```

giac [A] time = 0.13, size = 47, normalized size = 0.47

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}-21\,e^{(2\,dx+2\,c)}+7\,e^{(dx+c)}-1\right)}{35\,d\left(e^{(dx+c)}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) - 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) - 1)/(d*(e^(
d*x + c) - 1)^7)
```

maple [A] time = 0.08, size = 58, normalized size = 0.57

$$\frac{-\frac{1}{56\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{3}{40\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(-1/56/tanh(1/2*d*x+1/2*c)^7-1/8/tanh(1/2*d*x+1/2*c)^3+1/8/tanh(1/2*d*x
+1/2*c)+3/40/tanh(1/2*d*x+1/2*c)^5)
```

maxima [B] time = 0.33, size = 364, normalized size = 3.60

$$\frac{4\,e^{(-dx-c)}}{5\,d\left(7\,e^{(-dx-c)}-21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}-35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}-7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}-1\right)}-\frac{}{5\,d\left(7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(
-7*d*x - 7*c) - 1)) - 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d
*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c)
 - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) + 4*e^(-3*d*x - 3*c)/(d*(7*e
^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*
c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) - 4/
35/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-
4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c)
- 1))
```

mupad [B] time = 0.08, size = 283, normalized size = 2.80

$$-\frac{4}{35\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{c+d\,x}-4\,\mathrm{e}^{3\,c+3\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{16\,\mathrm{e}^{c+d\,x}}{35\,d\,\left(5\,\mathrm{e}^{c+d\,x}-10\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{3\,c+3\,d\,x}-5\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x) - 1)^4,x)
```

```
[Out] - 4/(35*d*(6*exp(2*c + 2*d*x) - 4*exp(c + d*x) - 4*exp(3*c + 3*d*x) + exp(4
*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) - 10*exp(2*c +
2*d*x) + 10*exp(3*c + 3*d*x) - 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) - 1))
- (8*exp(2*c + 2*d*x))/(7*d*(15*exp(2*c + 2*d*x) - 6*exp(c + d*x) - 20*exp(
3*c + 3*d*x) + 15*exp(4*c + 4*d*x) - 6*exp(5*c + 5*d*x) + exp(6*c + 6*d*x)
+ 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) - 21*exp(2*c + 2*d*x) +
35*exp(3*c + 3*d*x) - 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) - 7*exp(6*c
 + 6*d*x) + exp(7*c + 7*d*x) - 1))
```

sympy [A] time = 5.61, size = 87, normalized size = 0.86

$$\begin{cases} \tilde{\infty}x & \text{for } (c=0 \vee c=-dx) \wedge (c=-dx \vee d=0) \\ \frac{x}{(1-\cosh(c))^4} & \text{for } d=0 \\ \frac{1}{8d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{1}{8d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{3}{40d\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{1}{56d\tanh^7\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*x, (Eq(c, 0) | Eq(c, -d*x)) & (Eq(d, 0) | Eq(c, -d*x))), (x/
(1 - cosh(c))**4, Eq(d, 0)), (1/(8*d*tanh(c/2 + d*x/2)) - 1/(8*d*tanh(c/2 +
 d*x/2)**3) + 3/(40*d*tanh(c/2 + d*x/2)**5) - 1/(56*d*tanh(c/2 + d*x/2)**7)
, True))
```

3.40 $\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal. Leaf size=51

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] -arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(1/2)/a^(1/2)+2*
sinh(x)/(a+a*cosh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2751, 2649, 206}

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx &= \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}-\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx\\ &= \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}-2i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)\\ &= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.67

$$-\frac{2\cosh\left(\frac{x}{2}\right)\left(\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(\cosh(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (-2*Cosh[x/2]*(ArcTan[Sinh[x/2]] - 2*Sinh[x/2]))/Sqrt[a*(1 + Cosh[x])]
```

fricas [A] time = 0.52, size = 62, normalized size = 1.22

$$\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\,(\cosh(x)+\sinh(x)-1)-\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\,(\cosh(x)+\sinh(x))}{\sqrt{a}}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) - 1) - sqrt(2)*
sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sin
h(x))/sqrt(a)))/a
```

giac [C] time = 0.17, size = 46, normalized size = 0.90

$$-\sqrt{2}\left(\frac{2\,(-i\arctan(-i)+1)}{\sqrt{-a}}+\frac{2\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}\,x\right)}}{\sqrt{a}}+\frac{e^{\left(-\frac{1}{2}\,x\right)}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(2*(-I*arctan(-I) + 1)/sqrt(-a) + 2*arctan(e^(1/2*x))/sqrt(a) - e^
(1/2*x)/sqrt(a) + e^(-1/2*x)/sqrt(a))
```

maple [B] time = 0.27, size = 92, normalized size = 1.80

$$\frac{\cosh\left(\frac{x}{2}\right)\sqrt{a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(2\sqrt{a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\sqrt{-a}+\ln\left(\frac{2\sqrt{a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)a\right)\sqrt{2}}{a\sqrt{-a}\sinh\left(\frac{x}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*cosh(x))^(1/2),x)
```

```
[Out] cosh(1/2*x)*(a*sinh(1/2*x)^2)^(1/2)*(2*(a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2)+l
n(2/cosh(1/2*x)*((a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2)-a))*a)/a/(-a)^(1/2)/sin
h(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

maxima [B] time = 1.11, size = 114, normalized size = 2.24

$$-\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)+\frac{1}{3}\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}e^{(-x)}+\sqrt{a}}\right)+\frac{3\sqrt{2}\sqrt{a}e^{\left(\frac{3}{2}x\right)}-}{3\left(ae^{x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) +
1/3*sqrt(2)*(3*arctan(e^(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*
x)/(sqrt(a)*e^(-x) + sqrt(a))) + 1/3*(3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)
*sqrt(a)*e^(-1/2*x))/(a*e^x + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a*cosh(x))^(1/2),x)
```

```
[Out] int(cosh(x)/(a + a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{a(\cosh(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(a*(cosh(x) + 1)), x)
```

3.41 $\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal. Leaf size=53

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

```
[Out] -arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^(1/2)/a^(1/2)+2*
sinh(x)/(a-a*cosh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2751, 2649, 206}

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/Sqrt[a]
) + (2*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx &= \frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx\\ &= \frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+2i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)\\ &= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.66

$$\frac{2\sinh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)+\log\left(\tanh\left(\frac{x}{4}\right)\right)\right)}{\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] (2*(2*Cosh[x/2] + Log[Tanh[x/4]])*Sinh[x/2])/Sqrt[a - a*Cosh[x]]
```

fricas [B] time = 0.55, size = 92, normalized size = 1.74

$$\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right)-2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))
*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) + sinh(x)
 - 1)) - 2*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) + 1))/
a
```

giac [C] time = 0.16, size = 90, normalized size = 1.70

$$-\sqrt{2}\left(\frac{2\,(i\arctan(-i)-1)\text{sgn}\left(-e^x+1\right)}{\sqrt{-a}}+\frac{2\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{\sqrt{a}\,\text{sgn}\left(-e^x+1\right)}+\frac{1}{\sqrt{-ae^x}\,\text{sgn}\left(-e^x+1\right)}-\frac{\sqrt{-ae^x}}{a\text{sgn}\left(-e^x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(2*(I*arctan(-I) - 1)*sgn(-e^x + 1)/sqrt(-a) + 2*arctan(sqrt(-a*e^
x)/sqrt(a))/(sqrt(a)*sgn(-e^x + 1)) + 1/(sqrt(-a*e^x)*sgn(-e^x + 1)) - sqrt
(-a*e^x)/(a*sgn(-e^x + 1)))
```

maple [A] time = 0.34, size = 40, normalized size = 0.75

$$\frac{\sinh\left(\frac{x}{2}\right)\left(4\cosh\left(\frac{x}{2}\right)+\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)}{\sqrt{-2a\left(\sinh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a-a*cosh(x))^(1/2),x)
```

```
[Out] sinh(1/2*x)*(4*cosh(1/2*x)+ln(-1+cosh(1/2*x))-ln(cosh(1/2*x)+1))/(-2*a*sinh
(1/2*x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{-a\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(-a*cosh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a - a*cosh(x))^(1/2),x)
```

```
[Out] int(cosh(x)/(a - a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(x\right)}{\sqrt{-a\left(\cosh\left(x\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a-a*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(-a*(cosh(x) - 1)), x)
```

3.42 $\int (a+a\cosh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\sinh(c+dx)}{15d\sqrt{a\cosh(c+dx)+a}}+\frac{16a^2\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{15d}+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

```
[Out] 2/5*a*(a+a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d+64/15*a^3*sinh(d*x+c)/d/(a+a*co
sh(d*x+c))^(1/2)+16/15*a^2*sinh(d*x+c)*(a+a*cosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{64a^3\sinh(c+dx)}{15d\sqrt{a\cosh(c+dx)+a}}+\frac{16a^2\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{15d}+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a + a*Cosh[c + d*x]]) + (16*a^2*Sqrt[a +
a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) + (2*a*(a + a*Cosh[c + d*x])^(3/2)*S
inh[c + d*x])/(5*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\int (a + a\cosh(c + dx))^{5/2}\,dx = \frac{2a(a + a\cosh(c + dx))^{3/2}\sinh(c + dx)}{5d} + \frac{1}{5}(8a)\int (a + a\cosh(c + dx))^{3/2}\,dx$$
$$= \frac{16a^2\sqrt{a + a\cosh(c + dx)}\,\sinh(c + dx)}{15d} + \frac{2a(a + a\cosh(c + dx))^{3/2}\sinh(c + dx)}{5d} +$$
$$= \frac{64a^3\sinh(c + dx)}{15d\sqrt{a + a\cosh(c + dx)}} + \frac{16a^2\sqrt{a + a\cosh(c + dx)}\,\sinh(c + dx)}{15d} + \frac{2a(a + a\cos}{}$$

Mathematica [A] time = 0.13, size = 71, normalized size = 0.80

$$\frac{a^2\left(150\sinh\left(\frac{1}{2}(c + dx)\right) + 25\sinh\left(\frac{3}{2}(c + dx)\right) + 3\sinh\left(\frac{5}{2}(c + dx)\right)\right)\operatorname{sech}\left(\frac{1}{2}(c + dx)\right)\sqrt{a(\cosh(c + dx) + 1)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(150*Sinh[(c + d*x)/2] +
 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(30*d)
```

fricas [B] time = 0.61, size = 327, normalized size = 3.67

$$\frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx + c\right)^5 + 3\,a^2\sinh\left(dx + c\right)^5 + 25\,a^2\cosh\left(dx + c\right)^4 + 150\,a^2\cosh\left(dx + c\right)^3 + 5\left(3\,a^2\cosh\left(dx + \right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 + 25*a^2*cosh
(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) + 5*a^2)*sin
h(d*x + c)^4 - 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 + 10*a^2
*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a^2*c
osh(d*x + c)^3 + 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) - 5*a^2)*sinh
(d*x + c)^2 - 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x + c)^3 + 9
0*a^2*cosh(d*x + c)^2 - 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))*sqrt(a
/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*si
nh(d*x + c) + d*sinh(d*x + c)^2)
```

giac [A] time = 0.15, size = 105, normalized size = 1.18

$$-\frac{\sqrt{2}\left(\left(\left(150\,a^{\frac{5}{2}}e^{\left(2\,dx+\frac{5}{2}\,c\right)} + 25\,a^{\frac{5}{2}}e^{\left(dx+\frac{3}{2}\,c\right)} + 3\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{5}{2}\,dx-3\,c\right)} - \left(3\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,dx+\frac{35}{2}\,c\right)} + 25\,a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,dx+\frac{33}{2}\,c\right)} + 150\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,d\right.}\right.\right.\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*((150*a^(5/2)*e^(2*d*x + 5/2*c) + 25*a^(5/2)*e^(d*x + 3/2*c)
+ 3*a^(5/2)*e^(1/2*c))*e^(-5/2*d*x - 3*c) - (3*a^(5/2)*e^(5/2*d*x + 35/2*c)
 + 25*a^(5/2)*e^(3/2*d*x + 33/2*c) + 150*a^(5/2)*e^(1/2*d*x + 31/2*c))*e^(-
15*c))/d
```

maple [A] time = 0.28, size = 73, normalized size = 0.82

$$\frac{8a^3 \cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\left(\cosh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)\sqrt{2}}{15\sqrt{a\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(5/2),x)
```

```
[Out] 8/15*a^3*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(3*cosh(1/2*d*x+1/2*c)^4+4
*cosh(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.06, size = 121, normalized size = 1.36

$$\frac{\sqrt{2}\, a^{\frac{5}{2}} e^{\left(\frac{5}{2}\, dx+\frac{5}{2}\, c\right)}}{20\, d}+\frac{5\, \sqrt{2}\, a^{\frac{5}{2}} e^{\left(\frac{3}{2}\, dx+\frac{3}{2}\, c\right)}}{12\, d}+\frac{5\, \sqrt{2}\, a^{\frac{5}{2}} e^{\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)}}{2\, d}-\frac{5\, \sqrt{2}\, a^{\frac{5}{2}} e^{\left(-\frac{1}{2}\, dx-\frac{1}{2}\, c\right)}}{2\, d}-\frac{5\, \sqrt{2}\, a^{\frac{5}{2}} e^{\left(-\frac{3}{2}\, dx-\frac{3}{2}\, c\right)}}{12\, d}-\frac{\sqrt{2}\, a^{\frac{5}{2}} e^{\left(-\frac{5}{2}\, dx-\frac{5}{2}\, c\right)}}{20\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*a^(5/2)*e^(5/2*d*x + 5/2*c)/d + 5/12*sqrt(2)*a^(5/2)*e^(3/2*d*
x + 3/2*c)/d + 5/2*sqrt(2)*a^(5/2)*e^(1/2*d*x + 1/2*c)/d - 5/2*sqrt(2)*a^(5
/2)*e^(-1/2*d*x - 1/2*c)/d - 5/12*sqrt(2)*a^(5/2)*e^(-3/2*d*x - 3/2*c)/d -
1/20*sqrt(2)*a^(5/2)*e^(-5/2*d*x - 5/2*c)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\cosh(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*cosh(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.43 $\int (a+a\cosh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{8a^2\sinh(c+dx)}{3d\sqrt{a\cosh(c+dx)+a}}+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}$$

```
[Out] 8/3*a^2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)+2/3*a*sinh(d*x+c)*(a+a*cosh(d
*x+c))^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{8a^2\sinh(c+dx)}{3d\sqrt{a\cosh(c+dx)+a}}+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Cos
h[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cosh(c+dx))^{3/2}\,dx&=\frac{2a\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{3d}+\frac{1}{3}(4a)\int\sqrt{a+a\cosh(c+dx)}\,dx\\&=\frac{8a^2\sinh(c+dx)}{3d\sqrt{a+a\cosh(c+dx)}}+\frac{2a\sqrt{a+a\cosh(c+dx)}\,\sinh(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.93

$$\frac{a\left(9\sinh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{3}{2}(c+dx)\right)\right)\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cosh(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(9*Sinh[(c + d*x)/2] + Sin
h[(3*(c + d*x))/2]))/(3*d)
```

fricas [B] time = 0.46, size = 140, normalized size = 2.37

$$\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{3}+9\,a\cosh\left(dx+c\right)^{2}+3\left(a\cosh\left(dx+c\right)+3\,a\right)\sinh\left(dx+c\right)^{2}-9\,a\cosh\left(\right.\right.}{3\left(d\cosh\left(dx+c\right)+d\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 + 9*a*cosh(d*x + c)^2
+ 3*(a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*cosh
(d*x + c)^2 + 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x
+ c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

giac [A] time = 0.13, size = 75, normalized size = 1.27

$$-\frac{\sqrt{2}\left(\left(9\,a^{\frac{3}{2}}e^{\left(dx+\frac{3}{2}\,c\right)}+a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,c\right)}\right)e^{\left(-\frac{3}{2}\,dx-2\,c\right)}-\left(a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{15}{2}\,c\right)}+9\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{13}{2}\,c\right)}\right)e^{\left(-6\,c\right)}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((9*a^(3/2)*e^(d*x + 3/2*c) + a^(3/2)*e^(1/2*c))*e^(-3/2*d*x -
 2*c) - (a^(3/2)*e^(3/2*d*x + 15/2*c) + 9*a^(3/2)*e^(1/2*d*x + 13/2*c))*e^(
-6*c))/d
```

maple [A] time = 0.17, size = 58, normalized size = 0.98

$$\frac{4a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\sqrt{2}}{3\sqrt{a\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(3/2),x)
```

```
[Out] 4/3*a^2*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2+2)*2
^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 0.61, size = 81, normalized size = 1.37

$$\frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}}{6d}+\frac{3\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}-\frac{3\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{2d}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}dx-\frac{3}{2}c\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*a^(3/2)*e^(3/2*d*x + 3/2*c)/d + 3/2*sqrt(2)*a^(3/2)*e^(1/2*d*x
+ 1/2*c)/d - 3/2*sqrt(2)*a^(3/2)*e^(-1/2*d*x - 1/2*c)/d - 1/6*sqrt(2)*a^(3/
2)*e^(-3/2*d*x - 3/2*c)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a + a\cosh(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cosh(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(3/2), x)
```

3.44 $\int \sqrt{a+a\cosh(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sinh(c+dx)}{d\sqrt{a\cosh(c+dx)+a}}$$

```
[Out] 2*a*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$\frac{2a\sinh(c+dx)}{d\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*a*Sinh[c + d*x])/(d*Sqrt[a + a*Cosh[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \frac{2a\sinh(c+dx)}{d\sqrt{a+a\cosh(c+dx)}}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.12

$$\frac{2\tanh\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cosh(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*Tanh[(c + d*x)/2])/d
```

fricas [A] time = 0.50, size = 41, normalized size = 1.58

$$\frac{2\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d
*x + c) - 1)/d
```

giac [A] time = 0.12, size = 35, normalized size = 1.35

$$\frac{\sqrt{2}\left(\sqrt{a}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\sqrt{a}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*e^(1/2*d*x + 1/2*c) - sqrt(a)*e^(-1/2*d*x - 1/2*c))/d
```

maple [A] time = 0.14, size = 43, normalized size = 1.65

$$\frac{2a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] 2*a*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

maxima [A] time = 0.41, size = 40, normalized size = 1.54

$$\frac{\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{d}-\frac{\sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*sqrt(a)*e^(1/2*d*x + 1/2*c)/d - sqrt(2)*sqrt(a)*e^(-1/2*d*x - 1/2*c
)/d
```

mupad [B] time = 0.11, size = 26, normalized size = 1.00

$$\frac{2 \tanh\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,\sqrt{a + a\cosh\left(c + d\,x\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] (2*tanh(c/2 + (d*x)/2)*(a + a*cosh(c + d*x))^(1/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cosh\left(c + dx\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cosh(c + d*x) + a), x)
```

3.45 $\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

[Out] arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + a*Cosh[c + d*x]],x]

[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])])/(Sqrt[a]*d)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(c+dx)}{\sqrt{a+a\cosh(c+dx)}}\right)}{d}$$

$$= \frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a+a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.87

$$\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cosh(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2])/(d*Sqrt[a*(1 + Cosh[c + d*x
])])
```

fricas [A] time = 0.78, size = 149, normalized size = 3.24

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}\,(\cosh(dx+c)+\sinh(dx+c))+\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{d}, \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{c}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(
d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) + cosh(d*x + c) + sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/d, 2*sqrt(2)*arctan(sq
rt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + si
nh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

giac [A] time = 0.14, size = 21, normalized size = 0.46

$$\frac{2\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d)
```

maple [B] time = 0.19, size = 103, normalized size = 2.24

$$-\frac{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\ln\left(\frac{2\sqrt{a\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{-a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}}{\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] -cosh(1/2*d*x+1/2*c)*(a*sinh(1/2*d*x+1/2*c)^2)^(1/2)/(-a)^(1/2)*ln(2/cosh(1
/2*d*x+1/2*c)*((a*sinh(1/2*d*x+1/2*c)^2)^(1/2)*(-a)^(1/2)-a))/sinh(1/2*d*x+
1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 0.47, size = 86, normalized size = 1.87

$$2\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{\sqrt{a}\,d}+\frac{e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(\sqrt{a}\,e^{(dx+c)}+\sqrt{a}\right)d}\right)-\frac{2\sqrt{2}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\sqrt{a}\,de^{(dx+c)}+\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*(arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d) + e^(1/2*d*x + 1/2*c)/((
sqrt(a)*e^(d*x + c) + sqrt(a))*d)) - 2*sqrt(2)*e^(1/2*d*x + 1/2*c)/(sqrt(a)
*d*e^(d*x + c) + sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + a*cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh{(c+dx)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(c + d*x) + a), x)
```

3.46 $\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(3/2)+1/4*arctan(1/2*sinh(d*x+c)*a^(1/2
)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqrt
[2]*a^(3/2)*d) + Sinh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx}{4a}$$
$$= \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}} + \frac{i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(c+dx)}{\sqrt{a+a\cosh(c+dx)}}\right)}{2ad}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a+a\cosh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}$$

Mathematica [A] time = 0.09, size = 63, normalized size = 0.82

$$\frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\left(\tanh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d(a(\cosh(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^2*(ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2] + Tanh[(c
 + d*x)/2]))/(d*(a*(1 + Cosh[c + d*x]))^(3/2))
```

fricas [B] time = 0.53, size = 219, normalized size = 2.84

$$-\frac{\sqrt{2}\left(\cosh(dx+c)^2+2(\cosh(dx+c)+1)\sinh(dx+c)+\sinh(dx+c)^2+2\cosh(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}}{\ }\right.}{2\left(a^2d\cosh(dx+c)^2+a^2d\sinh(dx+c)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/
(cosh(d*x + c) + sinh(d*x + c)))/sqrt(a)) - 2*sqrt(1/2)*(cosh(d*x + c)^2 +
(2*cosh(d*x + c) - 1)*sinh(d*x + c) + sinh(d*x + c)^2 - cosh(d*x + c))*sqrt
(a/(cosh(d*x + c) + sinh(d*x + c))))/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*
x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a^2*d*cosh(d*x + c) + a^2*d)*
sinh(d*x + c))
```

giac [A] time = 0.20, size = 67, normalized size = 0.87

$$\frac{\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{\sqrt{a}}+\frac{a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(ae^{(dx+c)}+a\right)^{2}}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(arctan(e^(1/2*d*x + 1/2*c))/sqrt(a) + (a^(3/2)*e^(3/2*d*x + 3/
2*c) - a^(3/2)*e^(1/2*d*x + 1/2*c))/(a*e^(d*x + c) + a)^2)/(a*d)
```

maple [B] time = 0.27, size = 144, normalized size = 1.87

$$-\frac{\sqrt{a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\ln\left(\frac{2\sqrt{a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{-a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{-a}\right)\sqrt{2}}{4a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(3/2),x)
```

```
[Out] -1/4*(a*sinh(1/2*d*x+1/2*c)^2)^(1/2)*(ln(2/cosh(1/2*d*x+1/2*c)*((a*sinh(1/2
*d*x+1/2*c)^2)^(1/2)*(-a)^(1/2)-a))*a*cosh(1/2*d*x+1/2*c)^2-(a*sinh(1/2*d*x
+1/2*c)^2)^(1/2)*(-a)^(1/2))/a^2/cosh(1/2*d*x+1/2*c)/(-a)^(1/2)/sinh(1/2*d*
x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 0.50, size = 170, normalized size = 2.21

$$\frac{1}{6}\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}+8\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-3\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(a^{\frac{3}{2}}e^{(3\,dx+3\,c)}+3\,a^{\frac{3}{2}}e^{(2\,dx+2\,c)}+3\,a^{\frac{3}{2}}e^{(dx+c)}+a^{\frac{3}{2}}\right)d}+\frac{3\,\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{a^{\frac{3}{2}}d}\right)-\frac{4\sqrt{2}\,e^{\left(\frac{3}{2}\,\ldots\right.}}{3\left(a^{\frac{3}{2}}de^{(3\,dx+3\,c)}+3\,a^{\frac{3}{2}}de^{(2\,dx+\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*((3*e^(5/2*d*x + 5/2*c) + 8*e^(3/2*d*x + 3/2*c) - 3*e^(1/2*d*x
+ 1/2*c))/((a^(3/2)*e^(3*d*x + 3*c) + 3*a^(3/2)*e^(2*d*x + 2*c) + 3*a^(3/2)
*e^(d*x + c) + a^(3/2))*d) + 3*arctan(e^(1/2*d*x + 1/2*c))/(a^(3/2)*d)) - 4
```

```
/3*sqrt(2)*e^(3/2*d*x + 3/2*c)/(a^(3/2)*d*e^(3*d*x + 3*c) + 3*a^(3/2)*d*e^(
2*d*x + 2*c) + 3*a^(3/2)*d*e^(d*x + c) + a^(3/2)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + a\cosh(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a*cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cosh(c + dx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(-3/2), x)
```

3.47 $\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{3\sinh(c+dx)}{16ad(a\cosh(c+dx)+a)^{3/2}}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

```
[Out] 1/4*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(5/2)+3/16*sinh(d*x+c)/a/d/(a+a*cosh(d*
x+c))^(3/2)+3/32*arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cosh(d*x+c))^(
1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{3\sinh(c+dx)}{16ad(a\cosh(c+dx)+a)^{3/2}}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Sinh[c + d*x]/(4*d*(a + a*Cosh[c + d*x])^(5/2)) + (3*
Sinh[c + d*x])/(16*a*d*(a + a*Cosh[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx\right.}{16a}$$
$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+d}$$

Mathematica [A] time = 0.28, size = 91, normalized size = 0.85

$$\frac{\cosh^5\left(\frac{1}{2}(c+dx)\right)\left(32\sinh^5\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^4(c+dx)+3\left(\tan^{-1}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+\tanh\left(\frac{1}{2}(c+dx)\right)\operatorname{sech}\left(\frac{1}{2}(\right.\right.\right.}{4d(a(\cosh(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^5*(32*Csch[c + d*x]^4*Sinh[(c + d*x)/2]^5 + 3*(ArcTan[Si
nh[(c + d*x)/2]] + Sech[(c + d*x)/2]*Tanh[(c + d*x)/2])))/(4*d*(a*(1 + Cosh
[c + d*x]))^(5/2))
```

fricas [B] time = 0.42, size = 522, normalized size = 4.88

$$-\frac{3\sqrt{2}\left(\cosh(dx+c)^4+4(\cosh(dx+c)+1)\sinh(dx+c)^3+\sinh(dx+c)^4+4\cosh(dx+c)^3+6\left(\cosh(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
```

```
 + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d*x
 + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)*sqrt(a)
*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/sqrt(a))
- 2*sqrt(1/2)*(3*cosh(d*x + c)^4 + (12*cosh(d*x + c) + 11)*sinh(d*x + c)^3
+ 3*sinh(d*x + c)^4 + 11*cosh(d*x + c)^3 + (18*cosh(d*x + c)^2 + 33*cosh(d*
x + c) - 11)*sinh(d*x + c)^2 - 11*cosh(d*x + c)^2 + (12*cosh(d*x + c)^3 + 3
3*cosh(d*x + c)^2 - 22*cosh(d*x + c) - 3)*sinh(d*x + c) - 3*cosh(d*x + c))*
sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*sin
h(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)^3 + 6*a^3*d*cosh(d*x + c)^2 + 4*a^3*d*
cosh(d*x + c) + a^3*d + 4*(a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^3 + 6
*(a^3*d*cosh(d*x + c)^2 + 2*a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^2 +
4*(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c)
+ a^3*d)*sinh(d*x + c))
```

giac [A] time = 0.27, size = 97, normalized size = 0.91

$$\frac{\sqrt{2}\left(\frac{3\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{a^{\frac{5}{2}}}+\frac{3\,a^{\frac{7}{2}}e^{\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}+11\,a^{\frac{7}{2}}e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}-11\,a^{\frac{7}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-3\,a^{\frac{7}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(ae^{(dx+c)}+a\right)^{4}a^{2}}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*arctan(e^(1/2*d*x + 1/2*c))/a^(5/2) + (3*a^(7/2)*e^(7/2*d*x
 + 7/2*c) + 11*a^(7/2)*e^(5/2*d*x + 5/2*c) - 11*a^(7/2)*e^(3/2*d*x + 3/2*c)
 - 3*a^(7/2)*e^(1/2*d*x + 1/2*c))/((a*e^(d*x + c) + a)^4*a^2))/d
```

maple [B] time = 0.29, size = 178, normalized size = 1.66

$$-\frac{\sqrt{a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(3\ln\left(\frac{2\sqrt{a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{-a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cosh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3\sqrt{a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cosh^{2}\left(\frac{dx}{2}\right.\right.\right.}{32a^{3}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cosh(d*x+c))^(5/2),x)
```

```
[Out] -1/32*(a*sinh(1/2*d*x+1/2*c)^2)^(1/2)*(3*ln(2/cosh(1/2*d*x+1/2*c)*((a*sinh(
1/2*d*x+1/2*c)^2)^(1/2)*(-a)^(1/2)-a))*a*cosh(1/2*d*x+1/2*c)^4-3*(a*sinh(1/
2*d*x+1/2*c)^2)^(1/2)*cosh(1/2*d*x+1/2*c)^2*(-a)^(1/2)-2*(a*sinh(1/2*d*x+1/
```

```
2*c)^2)^(1/2)*(-a)^(1/2))/a^3/cosh(1/2*d*x+1/2*c)^3/(-a)^(1/2)/sinh(1/2*d*x
+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 0.51, size = 250, normalized size = 2.34

$$\frac{1}{80}\sqrt{2}\left(\frac{15\,e^{\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}+70\,e^{\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}+128\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}-70\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-15\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(a^{\frac{5}{2}}e^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}e^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}e^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}e^{(2\,dx+2\,c)}+5\,a^{\frac{5}{2}}e^{(dx+c)}+a^{\frac{5}{2}}\right)d}+\frac{15\,\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}\right.}{a^{\frac{5}{2}}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/80*sqrt(2)*((15*e^(9/2*d*x + 9/2*c) + 70*e^(7/2*d*x + 7/2*c) + 128*e^(5/2
*d*x + 5/2*c) - 70*e^(3/2*d*x + 3/2*c) - 15*e^(1/2*d*x + 1/2*c))/((a^(5/2)*
e^(5*d*x + 5*c) + 5*a^(5/2)*e^(4*d*x + 4*c) + 10*a^(5/2)*e^(3*d*x + 3*c) +
10*a^(5/2)*e^(2*d*x + 2*c) + 5*a^(5/2)*e^(d*x + c) + a^(5/2))*d) + 15*arcta
n(e^(1/2*d*x + 1/2*c))/(a^(5/2)*d)) - 8/5*sqrt(2)*e^(5/2*d*x + 5/2*c)/(a^(5
/2)*d*e^(5*d*x + 5*c) + 5*a^(5/2)*d*e^(4*d*x + 4*c) + 10*a^(5/2)*d*e^(3*d*x
 + 3*c) + 10*a^(5/2)*d*e^(2*d*x + 2*c) + 5*a^(5/2)*d*e^(d*x + c) + a^(5/2)*
d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\mathrm{cosh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cosh(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + a*cosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cosh\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*cosh(c + d*x) + a)**(-5/2), x)
```

3.48 $\int (a - a\cosh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=92

$$-\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{15d}-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

```
[Out] -2/5*a*(a-a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d-64/15*a^3*sinh(d*x+c)/d/(a-a*c
osh(d*x+c))^(1/2)-16/15*a^2*sinh(d*x+c)*(a-a*cosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.133, Rules used = {2647, 2646}

$$-\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}}-\frac{16a^2\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{15d}-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (-64*a^3*Sinh[c + d*x])/(15*d*Sqrt[a - a*Cosh[c + d*x]]) - (16*a^2*Sqrt[a -
 a*Cosh[c + d*x]]*Sinh[c + d*x])/(15*d) - (2*a*(a - a*Cosh[c + d*x])^(3/2)*
Sinh[c + d*x])/(5*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\int (a-a\cosh(c+dx))^{5/2}\,dx = -\frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d} + \frac{1}{5}(8a)\int (a-a\cosh(c+dx))^{3/2}\,dx$$
$$= -\frac{16a^2\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{15d} - \frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}$$
$$= -\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}} - \frac{16a^2\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{15d} - \frac{2a(a-a\cos$$

Mathematica [A] time = 0.14, size = 72, normalized size = 0.78

$$\frac{a^2\left(150\cosh\left(\frac{1}{2}(c+dx)\right)-25\cosh\left(\frac{3}{2}(c+dx)\right)+3\cosh\left(\frac{5}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cosh(c+dx)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a - a*Cosh[c + d*x]]*(150*Cosh[(c + d*x)/2] - 25*Cosh[(3*(c + d*x
))/2] + 3*Cosh[(5*(c + d*x))/2])*Csch[(c + d*x)/2])/(30*d)
```

fricas [B] time = 0.51, size = 328, normalized size = 3.57

$$\frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5+3\,a^2\sinh\left(dx+c\right)^5-25\,a^2\cosh\left(dx+c\right)^4+150\,a^2\cosh\left(dx+c\right)^3+5\left(3\,a^2\cosh\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 - 25*a^2*cosh
(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) - 5*a^2)*sin
h(d*x + c)^4 + 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 - 10*a^2
*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a^2*c
osh(d*x + c)^3 - 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) + 5*a^2)*sinh
(d*x + c)^2 + 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)^3 + 9
0*a^2*cosh(d*x + c)^2 + 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))*sqrt(-
a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*s
inh(d*x + c) + d*sinh(d*x + c)^2)
```

giac [B] time = 0.16, size = 189, normalized size = 2.05

$$-\frac{\sqrt{2}\left(3\sqrt{-a}\,a^2e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}\operatorname{sgn}\left(-e^{(dx+c)}+1\right)-25\sqrt{-a}\,a^2e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}\operatorname{sgn}\left(-e^{(dx+c)}+1\right)+150\sqrt{-a}\,a^2e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\operatorname{sgn}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*(3*sqrt(-a)*a^2*e^(5/2*d*x + 5/2*c)*sgn(-e^(d*x + c) + 1) - 2
5*sqrt(-a)*a^2*e^(3/2*d*x + 3/2*c)*sgn(-e^(d*x + c) + 1) + 150*sqrt(-a)*a^2
*e^(1/2*d*x + 1/2*c)*sgn(-e^(d*x + c) + 1) + 150*sqrt(-a)*a^2*e^(-1/2*d*x -
 1/2*c)*sgn(-e^(d*x + c) + 1) - 25*sqrt(-a)*a^2*e^(-3/2*d*x - 3/2*c)*sgn(-e
^(d*x + c) + 1) + 3*sqrt(-a)*a^2*e^(-5/2*d*x - 5/2*c)*sgn(-e^(d*x + c) + 1)
)/d
```

maple [A] time = 0.26, size = 71, normalized size = 0.77

$$-\frac{16\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\left(\sinh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\right)}{15\sqrt{-2a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(5/2),x)
```

```
[Out] -16/15*sinh(1/2*d*x+1/2*c)*a^3*cosh(1/2*d*x+1/2*c)*(3*sinh(1/2*d*x+1/2*c)^4
-4*sinh(1/2*d*x+1/2*c)^2+8)/(-2*a*sinh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 0.42, size = 190, normalized size = 2.07

$$\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-dx-c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-3\,dx-3\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}+\frac{5\sqrt{2}\,a^{\frac{5}{2}}e^{(-4\,dx-4\,c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{\sqrt{2}\,a^{\frac{5}{2}}e^{(-5\,dx-5\,c)}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}-\frac{\sqrt{2}\,a^{\frac{5}{2}}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*sqrt(2)*a^(5/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a
^(5/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a^(5/2)*e^(
-3*d*x - 3*c)/(d*(-e^(-d*x - c))^(5/2)) + 5/12*sqrt(2)*a^(5/2)*e^(-4*d*x -
4*c)/(d*(-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)*e^(-5*d*x - 5*c)/(d*(
-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)/(d*(-e^(-d*x - c))^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a-a\cosh\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cosh(c + d*x))^(5/2),x)
```

```
[Out] int((a - a*cosh(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.49 $\int (a - a\cosh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=61

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

```
[Out] -8/3*a^2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)-2/3*a*sinh(d*x+c)*(a-a*cosh(
d*x+c))^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2647, 2646}

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] (-8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*Co
sh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a-a\cosh(c+dx))^{3/2}\,dx &= -\frac{2a\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{3d}+\frac{1}{3}(4a)\int\sqrt{a-a\cosh(c+dx)}\,dx\\ &= -\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sqrt{a-a\cosh(c+dx)}\,\sinh(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 56, normalized size = 0.92

$$-\frac{a\left(\cosh\left(\frac{3}{2}(c+dx)\right)-9\cosh\left(\frac{1}{2}(c+dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cosh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(a*Sqrt[a - a*Cosh[c + d*x]]*(-9*Cosh[(c + d*x)/2] + Cosh[(3*(c + d*x)
)/2])*Csch[(c + d*x)/2])/d
```

fricas [B] time = 0.51, size = 139, normalized size = 2.28

$$-\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^{3}+a\sinh\left(dx+c\right)^{3}-9\,a\cosh\left(dx+c\right)^{2}+3\left(a\cosh\left(dx+c\right)-3\,a\right)\sinh\left(dx+c\right)^{2}-9\,a\cosh}{3\left(d\cosh\left(dx+c\right)+d\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 - 9*a*cosh(d*x + c)^2
 + 3*(a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*cos
h(d*x + c)^2 - 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) + a)*sqrt(-a/(cosh(d*
x + c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

giac [B] time = 0.16, size = 119, normalized size = 1.95

$$\frac{\sqrt{2}\left(\sqrt{-a}\,ae^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}\operatorname{sgn}\left(-e^{(dx+c)}+1\right)-9\,\sqrt{-a}\,ae^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\operatorname{sgn}\left(-e^{(dx+c)}+1\right)-9\,\sqrt{-a}\,ae^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\operatorname{sgn}\left(-e^{(dx+c)}+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(sqrt(-a)*a*e^(3/2*d*x + 3/2*c)*sgn(-e^(d*x + c) + 1) - 9*sqrt(
-a)*a*e^(1/2*d*x + 1/2*c)*sgn(-e^(d*x + c) + 1) - 9*sqrt(-a)*a*e^(-1/2*d*x
- 1/2*c)*sgn(-e^(d*x + c) + 1) + sqrt(-a)*a*e^(-3/2*d*x - 3/2*c)*sgn(-e^(d*
x + c) + 1))/d
```

maple [A] time = 0.25, size = 56, normalized size = 0.92

$$\frac{8\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cosh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-3\right)}{3\sqrt{-2a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(3/2),x)
```

```
[Out] 8/3*sinh(1/2*d*x+1/2*c)*a^2*cosh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2-3)/(
-2*a*sinh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 0.43, size = 124, normalized size = 2.03

$$\frac{3\sqrt{2}\,a^{\frac{3}{2}}e^{(-dx-c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}+\frac{3\sqrt{2}\,a^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\,a^{\frac{3}{2}}e^{(-3\,dx-3\,c)}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\,a^{\frac{3}{2}}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(2)*a^(3/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(3/2)) + 3/2*sqrt(2)*a^
(3/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)*e^(-
3*d*x - 3*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)/(d*(-e^(-d*x -
 c))^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a-a\cosh(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cosh(c + d*x))^(3/2),x)
```

```
[Out] int((a - a*cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-a\cosh(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((-a*cosh(c + d*x) + a)**(3/2), x)
```

3.50 $\int \sqrt{a-a\cosh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

[Out] -2*a*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2646}

$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - a*Cosh[c + d*x]],x]

[Out] (-2*a*Sinh[c + d*x])/(d*Sqrt[a - a*Cosh[c + d*x]])

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a-a\cosh(c+dx)}\,dx=-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 1.11

$$\frac{2\coth\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cosh(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a - a*Cosh[c + d*x]],x]

[Out] (2*Sqrt[a - a*Cosh[c + d*x]]*Coth[(c + d*x)/2])/d

fricas [A] time = 0.40, size = 42, normalized size = 1.56

$$\frac{2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(
d*x + c) + 1)/d
```

giac [B] time = 0.15, size = 61, normalized size = 2.26

$$-\frac{\sqrt{2}\left(\sqrt{-a}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\operatorname{sgn}\left(-e^{(dx+c)}+1\right)+\sqrt{-a}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\operatorname{sgn}\left(-e^{(dx+c)}+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(-a)*e^(1/2*d*x + 1/2*c)*sgn(-e^(d*x + c) + 1) + sqrt(-a)*e^(
-1/2*d*x - 1/2*c)*sgn(-e^(d*x + c) + 1))/d
```

maple [A] time = 0.22, size = 41, normalized size = 1.52

$$-\frac{4\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{-2a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(d*x+c))^(1/2),x)
```

```
[Out] -4*sinh(1/2*d*x+1/2*c)*a*cosh(1/2*d*x+1/2*c)/(-2*a*sinh(1/2*d*x+1/2*c)^2)^(
1/2)/d
```

maxima [B] time = 0.43, size = 58, normalized size = 2.15

$$-\frac{\sqrt{2}\sqrt{a}\,e^{(-dx-c)}}{d\sqrt{-e^{(-dx-c)}}}-\frac{\sqrt{2}\sqrt{a}}{d\sqrt{-e^{(-dx-c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*sqrt(a)*e^(-d*x - c)/(d*sqrt(-e^(-d*x - c))) - sqrt(2)*sqrt(a)/(d*
sqrt(-e^(-d*x - c)))
```

mupad [B] time = 0.94, size = 27, normalized size = 1.00

$$\frac{2\coth\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-a\cosh\left(c+d\,x\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*cosh(c + d*x))^(1/2),x)
```

```
[Out] (2*coth(c/2 + (d*x)/2)*(a - a*cosh(c + d*x))^(1/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\cosh\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*cosh(c + d*x) + a), x)
```

3.51 $\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))*2^(1/2)/d/
a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2649, 206}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]
]])])/(Sqrt[a]*d))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.85

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)\log\left(\tanh\left(\frac{1}{4}(c+dx)\right)\right)}{d\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

```
[Out] (2*Log[Tanh[(c + d*x)/4]]*Sinh[(c + d*x)/2])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

fricas [A] time = 0.53, size = 154, normalized size = 3.21

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\,\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}\,(\cosh(dx+c)+\sinh(dx+c))-\cosh(dx+c)-\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)-1}\right)}{d}, \frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\cdots}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(
d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) - cosh(d*x + c) - sin
h(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) - 1))/d, 2*sqrt(2)*arctan(sq
rt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + s
inh(d*x + c))/sqrt(a))/(sqrt(a)*d)]
```

giac [A] time = 0.14, size = 40, normalized size = 0.83

$$-\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d\,\text{sgn}\left(-e^{(dx+c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(sqrt(a)*d*sgn(-e^(d*x + c)
 + 1))
```

maple [A] time = 0.22, size = 41, normalized size = 0.85

$$-\frac{2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{arctanh}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{-2a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(1/2),x)
```

```
[Out] -2*sinh(1/2*d*x+1/2*c)*arctanh(cosh(1/2*d*x+1/2*c))/(-2*a*sinh(1/2*d*x+1/2*
c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\cosh\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-a*cosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a-a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*cosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - a*cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\cosh\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*cosh(c + d*x) + a), x)
```

3.52 $\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

```
[Out] -1/2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(3/2)-1/4*arctan(1/2*sinh(d*x+c)*a^(1/
2)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2650, 2649, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] -ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(2*Sqr
t[2]*a^(3/2)*d) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = -\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx}{4a}$$
$$= -\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}} + \frac{i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}\right)}{2ad}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(c+dx)}{\sqrt{2}\,\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

Mathematica [A] time = 0.17, size = 85, normalized size = 1.08

$$\frac{\sinh^3\left(\frac{1}{2}(c+dx)\right)\left(\operatorname{csch}^2\left(\frac{1}{4}(c+dx)\right)+\operatorname{sech}^2\left(\frac{1}{4}(c+dx)\right)+4\log\left(\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)}{4ad(\cosh(c+dx)-1)\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(-3/2),x]
```

```
[Out] ((Csch[(c + d*x)/4]^2 + 4*Log[Tanh[(c + d*x)/4]] + Sech[(c + d*x)/4]^2)*Sin
h[(c + d*x)/2]^3)/(4*a*d*(-1 + Cosh[c + d*x])*Sqrt[a - a*Cosh[c + d*x]])
```

fricas [B] time = 0.61, size = 274, normalized size = 3.47

$$-\frac{\sqrt{2}\left(\cosh(dx+c)^2+2(\cosh(dx+c)-1)\sinh(dx+c)+\sinh(dx+c)^2-2\cosh(dx+c)+1\right)\sqrt{-a}\,\log\left(-\frac{2\sqrt{2}}{}\right.}{4\left(a^2d\cosh(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x + c) + sinh
(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(
-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c)
) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c) -
 1)) + 4*sqrt(1/2)*(cosh(d*x + c)^2 + (2*cosh(d*x + c) + 1)*sinh(d*x + c) +
 sinh(d*x + c)^2 + cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c))))
/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 - 2*a^2*d*cosh(d*x + c) + a
^2*d + 2*(a^2*d*cosh(d*x + c) - a^2*d)*sinh(d*x + c))
```

giac [A] time = 0.25, size = 107, normalized size = 1.35

$$-\frac{\sqrt{2}\left(\frac{\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}\,\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}-\frac{\sqrt{-ae^{(dx+c)}}\,ae^{(dx+c)}+\sqrt{-ae^{(dx+c)}}\,a}{\left(ae^{(dx+c)}-a\right)^{2}\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(sqrt(a)*sgn(-e^(d*x + c
) + 1)) - (sqrt(-a*e^(d*x + c))*a*e^(d*x + c) + sqrt(-a*e^(d*x + c))*a)/((a
*e^(d*x + c) - a)^2*sgn(-e^(d*x + c) + 1)))/(a*d)
```

maple [A] time = 0.37, size = 87, normalized size = 1.10

$$\frac{2\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\ln\left(-1+\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2a\left(\sinh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(3/2),x)
```

```
[Out] 1/4/a*(2*cosh(1/2*d*x+1/2*c)+(ln(-1+cosh(1/2*d*x+1/2*c))-ln(cosh(1/2*d*x+1/
2*c)+1))*sinh(1/2*d*x+1/2*c)^2)/sinh(1/2*d*x+1/2*c)/(-2*a*sinh(1/2*d*x+1/2*
c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-a\cosh\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*cosh(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(a-a\cosh\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*cosh(c + d*x))^(3/2),x)
[Out] int(1/(a - a*cosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a\cosh(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(3/2),x)
[Out] Integral((-a*cosh(c + d*x) + a)**(-3/2), x)
```

3.53 $\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

```
[Out] -1/4*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(5/2)-3/16*sinh(d*x+c)/a/d/(a-a*cosh(d
*x+c))^(3/2)-3/32*arctan(1/2*sinh(d*x+c)*a^(1/2)*2^(1/2)/(a-a*cosh(d*x+c))^
(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2650, 2649, 206}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - Sinh[c + d*x]/(4*d*(a - a*Cosh[c + d*x])^(5/2)) - (3
*Sinh[c + d*x])/(16*a*d*(a - a*Cosh[c + d*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,d\right.}{16a}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+a}$$

Mathematica [A] time = 0.20, size = 115, normalized size = 1.05

$$\frac{\sinh^5\left(\frac{1}{2}(c+dx)\right)\left(-\mathrm{csch}^4\left(\frac{1}{4}(c+dx)\right)+6\mathrm{csch}^2\left(\frac{1}{4}(c+dx)\right)+\mathrm{sech}^4\left(\frac{1}{4}(c+dx)\right)+6\mathrm{sech}^2\left(\frac{1}{4}(c+dx)\right)+24\log\left(\right.\right.}{32a^2d(\cosh(c+dx)-1)^2\sqrt{a-a\cosh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[c + d*x])^(-5/2),x]
```

```
[Out] ((6*Csch[(c + d*x)/4]^2 - Csch[(c + d*x)/4]^4 + 24*Log[Tanh[(c + d*x)/4]] +
 6*Sech[(c + d*x)/4]^2 + Sech[(c + d*x)/4]^4)*Sinh[(c + d*x)/2]^5)/(32*a^2*
d*(-1 + Cosh[c + d*x])^2*Sqrt[a - a*Cosh[c + d*x]])
```

fricas [B] time = 1.25, size = 580, normalized size = 5.27

$$-\frac{3\sqrt{2}\left(\cosh(dx+c)^4+4(\cosh(dx+c)-1)\sinh(dx+c)^3+\sinh(dx+c)^4-4\cosh(dx+c)^3+6\left(\cosh(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x + c)
```

```
 + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(d*x
 + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)*sqrt(-a
)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)
))*(cosh(d*x + c) + sinh(d*x + c)) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)
/(cosh(d*x + c) + sinh(d*x + c) - 1)) + 4*sqrt(1/2)*(3*cosh(d*x + c)^4 + (1
2*cosh(d*x + c) - 11)*sinh(d*x + c)^3 + 3*sinh(d*x + c)^4 - 11*cosh(d*x + c
)^3 + (18*cosh(d*x + c)^2 - 33*cosh(d*x + c) - 11)*sinh(d*x + c)^2 - 11*cos
h(d*x + c)^2 + (12*cosh(d*x + c)^3 - 33*cosh(d*x + c)^2 - 22*cosh(d*x + c)
+ 3)*sinh(d*x + c) + 3*cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c
))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*sinh(d*x + c)^4 - 4*a^3*d*cosh(d*x + c)
^3 + 6*a^3*d*cosh(d*x + c)^2 - 4*a^3*d*cosh(d*x + c) + a^3*d + 4*(a^3*d*cos
h(d*x + c) - a^3*d)*sinh(d*x + c)^3 + 6*(a^3*d*cosh(d*x + c)^2 - 2*a^3*d*co
sh(d*x + c) + a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - 3*a^3*d*c
osh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c) - a^3*d)*sinh(d*x + c))
```

giac [A] time = 0.30, size = 164, normalized size = 1.49

$$-\frac{\sqrt{2}\left(\frac{3\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(-e^{(dx+c)}+1\right)}-\frac{3\sqrt{-ae^{(dx+c)}}\,a^3e^{(3\,dx+3\,c)}-11\sqrt{-ae^{(dx+c)}}\,a^3e^{(2\,dx+2\,c)}-11\sqrt{-ae^{(dx+c)}}\,a^3e^{(dx+c)}+3\sqrt{-ae^{(dx+c)}}\,a^3}{\left(ae^{(dx+c)}-a\right)^4a^2\operatorname{sgn}\left(-e^{(dx+c)}+1\right)}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(5/2)*sgn(-e^(d*x
+ c) + 1)) - (3*sqrt(-a*e^(d*x + c))*a^3*e^(3*d*x + 3*c) - 11*sqrt(-a*e^(d*
x + c))*a^3*e^(2*d*x + 2*c) - 11*sqrt(-a*e^(d*x + c))*a^3*e^(d*x + c) + 3*s
qrt(-a*e^(d*x + c))*a^3)/((a*e^(d*x + c) - a)^4*a^2*sgn(-e^(d*x + c) + 1)))
/d
```

maple [A] time = 0.34, size = 137, normalized size = 1.25

$$\frac{6\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-4\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3\ln\left(-1+\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)\left(s\right.}{32a^2\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(-1+\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2a\left(\sinh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*cosh(d*x+c))^(5/2),x)
```

```
[Out] 1/32/a^2*(6*sinh(1/2*d*x+1/2*c)^2*cosh(1/2*d*x+1/2*c)-4*cosh(1/2*d*x+1/2*c)
+(3*ln(-1+cosh(1/2*d*x+1/2*c))-3*ln(cosh(1/2*d*x+1/2*c)+1))*sinh(1/2*d*x+1/
```

```
2*c)^4)/(cosh(1/2*d*x+1/2*c)+1)/(-1+cosh(1/2*d*x+1/2*c))/sinh(1/2*d*x+1/2*c
)/(-2*a*sinh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a\cosh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((-a*cosh(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\cosh(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*cosh(c + d*x))^(5/2),x)
[Out] int(1/(a - a*cosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a\cosh(c+dx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*cosh(d*x+c))**(5/2),x)
[Out] Integral((-a*cosh(c + d*x) + a)**(-5/2), x)
```

3.54 $\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=112

$$\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4\sqrt{a-b}\,\sqrt{a+b}}-\frac{ax\left(2a^2+b^2\right)}{2b^4}+\frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+2*b^2)*sinh(x)/b^3-1/2*a*cosh(x)*sinh(x
)/b^2+1/3*cosh(x)^2*sinh(x)/b+2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(
1/2))/b^4/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2793, 3049, 3023, 2735, 2659, 208}

$$-\frac{ax\left(2a^2+b^2\right)}{2b^4}+\frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4\sqrt{a-b}\,\sqrt{a+b}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[
a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]) + ((3*a^2 + 2*b^2)*Sinh[x])/(3*b^3)
- (a*Cosh[x]*Sinh[x])/(2*b^2) + (Cosh[x]^2*Sinh[x])/(3*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{\cosh(x)\left(2a+2b\cosh(x)-3a\cosh^2(x)\right)}{a+b\cosh(x)}\,dx}{3b}$$

$$= -\frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{-3a^2+ab\cosh(x)+2\left(3a^2+2b^2\right)\cosh^2(x)}{a+b\cosh(x)}\,dx}{6b^2}$$

$$= \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\int \frac{-3a^2b-3a\left(2a^2+b^2\right)\cosh(x)}{a+b\cosh(x)}\,d}{6b^3}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{a^4\int \ldots}{}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh^2(x)\sinh(x)}{3b} + \frac{\left(2a^4\right)\ldots}{}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}} + \frac{\left(3a^2+2b^2\right)\sinh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2}$$

<u>Mathematica</u> [A] time = 0.20, size = 99, normalized size = 0.88

$$\frac{-6ax\left(2a^2+b^2\right)+3b\left(4a^2+3b^2\right)\sinh(x)-\frac{24a^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-3ab^2\sinh(2x)+b^3\sinh(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Cosh[x]),x]

[Out] (-6*a*(2*a^2 + b^2)*x - (24*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]
])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sinh[x] - 3*a*b^2*Sinh[2*x] + b^3
*Sinh[3*x])/(12*b^4)
```

<u>fricas</u> [B] time = 0.80, size = 1625, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")

[Out] [1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)^6 - 3*(a^3*b^2 -
 a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)*cosh(x))*sinh(x)
```

```
^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*b
- a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^5 + 5*(a^2*b^3 -
b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^3 -
b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5 - a^3*b^2 - a*b^
4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - 3*(4*a^4*b - a^2*
b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*(a^2*b^3 - b^5)*c
osh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^3*b^2 - a*b^4)*x*
cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^4*cosh
(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)*
sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) + b)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 - b^5)*cosh(x)^5
+ a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a^5 - a^3*b^2 - a
*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^3 - 2*(4*a^4*b -
a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^2*b^4
- b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^2*b^4 -
 b^6)*sinh(x)^3), 1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)
^6 - 3*(a^3*b^2 - a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)
*cosh(x))*sinh(x)^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x
)^3 + 3*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^
5 + 5*(a^2*b^3 - b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 +
2*(10*(a^2*b^3 - b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5
 - a^3*b^2 - a*b^4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 -
3*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*
(a^2*b^3 - b^5)*cosh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^
3*b^2 - a*b^4)*x*cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)
^2 - 48*(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2
+ a^4*sinh(x)^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a)/(a^2 - b^2)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 - b^
5)*cosh(x)^5 + a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a^5
- a^3*b^2 - a*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^3 -
2*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(x)^3
+ 3*(a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
 + (a^2*b^4 - b^6)*sinh(x)^3)]
```

giac [A] time = 0.12, size = 133, normalized size = 1.19

$$\frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^4}+\frac{b^2e^{(3\,x)}-3\,abe^{(2\,x)}+12\,a^2e^x+9\,b^2e^x}{24\,b^3}-\frac{\left(2\,a^3+ab^2\right)x}{2\,b^4}+\frac{\left(3\,ab^2e^x-b^3-3\left(4\,a^2b+3\,b^3\right)e^{(2\,x)}\right.}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^4) + 1/24*(b
^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 9*b^2*e^x)/b^3 - 1/2*(2*a^3 + a*b
^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b + 3*b^3)*e^(2*x))*e^(-3*x)
/b^4
```

maple [B] time = 0.07, size = 264, normalized size = 2.36

$$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{a}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{a^{2}}{b^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{a}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{b\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*cosh(x)),x)
```

```
[Out] -1/3/b/(tanh(1/2*x)-1)^3-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)^
2-1/b^3/(tanh(1/2*x)-1)*a^2-1/2/b^2/(tanh(1/2*x)-1)*a-1/b/(tanh(1/2*x)-1)+a
^3/b^4*ln(tanh(1/2*x)-1)+1/2*a/b^2*ln(tanh(1/2*x)-1)-1/3/b/(tanh(1/2*x)+1)^
3+1/2/b^2/(tanh(1/2*x)+1)^2*a+1/2/b/(tanh(1/2*x)+1)^2-1/b^3/(tanh(1/2*x)+1)
*a^2-1/2/b^2/(tanh(1/2*x)+1)*a-1/b/(tanh(1/2*x)+1)-a^3/b^4*ln(tanh(1/2*x)+1
)-1/2*a/b^2*ln(tanh(1/2*x)+1)+2*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
anh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.26, size = 209, normalized size = 1.87

$$\frac{e^{3\,x}}{24\,b}-\frac{e^{-3\,x}}{24\,b}-\frac{x\,\left(2\,a^3+a\,b^2\right)}{2\,b^4}+\frac{e^{x}\,\left(4\,a^2+3\,b^2\right)}{8\,b^3}+\frac{a\,e^{-2\,x}}{8\,b^2}-\frac{a\,e^{2\,x}}{8\,b^2}-\frac{e^{-x}\,\left(4\,a^2+3\,b^2\right)}{8\,b^3}+\frac{a^4\,\ln\left(-\frac{2\,a^4\,e^{x}}{b^5}-\frac{2\,a^4\,\left(b+a\,e^{x}\right)}{b^5\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^4\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*cosh(x)),x)
```

```
[Out] exp(3*x)/(24*b) - exp(-3*x)/(24*b) - (x*(a*b^2 + 2*a^3))/(2*b^4) + (exp(x)*
(4*a^2 + 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) - (
exp(-x)*(4*a^2 + 3*b^2))/(8*b^3) + (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^4*(
b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)^(1/2))))/(b^4*(a + b)^(1/2)*(a -
b)^(1/2)) - (a^4*log((2*a^4*(b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)^(1/2
)) - (2*a^4*exp(x))/b^5))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.55 $\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\,\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

```
[Out] 1/2*(2*a^2+b^2)*x/b^3-a*sinh(x)/b^2+1/2*cosh(x)*sinh(x)/b-2*a^3*arctanh((a-
b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^3/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2793, 3023, 2735, 2659, 208}

$$\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\,\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Sinh[x])/b^2 + (Cosh[x]*Sinh[x])/(
2*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2b}+\frac{\int \frac{a+b\cosh(x)-2a\cosh^2(x)}{a+b\cosh(x)}\,dx}{2b}\\
&= -\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}+\frac{\int \frac{ab+\left(2a^2+b^2\right)\cosh(x)}{a+b\cosh(x)}\,dx}{2b^2}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}-\frac{a^3\int \frac{1}{a+b\cosh(x)}\,dx}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}-\frac{\left(2a^3\right)\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^3}\\
&= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}}-\frac{a\sinh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 78, normalized size = 0.92

$$\frac{4a^2x+\frac{8a^3\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-4ab\sinh(x)+2b^2x+b^2\sinh(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (4*a^2*x + 2*b^2*x + (8*a^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] - 4*a*b*Sinh[x] + b^2*Sinh[2*x])/(4*b^3)
```

fricas [B] time = 0.89, size = 903, normalized size = 10.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh(x)^4 - a^2*b^2 + b^4
 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*
(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 - b^4)*
cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x
)^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^2 -
b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(
b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))
/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))
 + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3
 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)*sinh(
x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x)*sinh(x) + (a^2*b
^3 - b^5)*sinh(x)^2), 1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh
(x)^4 - a^2*b^2 + b^4 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b -
a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 +
2*(3*(a^2*b^2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a
*b^3)*cosh(x))*sinh(x)^2 + 16*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*
sinh(x)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a)/(a^2 - b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^
2 - b^4)*cosh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3
)*cosh(x)^2)*sinh(x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x
)*sinh(x) + (a^2*b^3 - b^5)*sinh(x)^2)]
```

giac [A] time = 0.12, size = 92, normalized size = 1.08

$$-\frac{2\,a^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^3}+\frac{be^{(2\,x)}-4\,ae^x}{8\,b^2}+\frac{\left(2\,a^2+b^2\right)x}{2\,b^3}+\frac{\left(4\,abe^x-b^2\right)e^{(-2\,x)}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^3) + 1/8*(b
*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2*a^2 + b^2)*x/b^3 + 1/8*(4*a*b*e^x - b^2)*e
^(-2*x)/b^3
```

maple [B] time = 0.06, size = 174, normalized size = 2.05

$$\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{a}{b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)a^{2}}{b^{3}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*cosh(x)),x)
```

```
[Out] 1/2/b/(tanh(1/2*x)-1)^2+1/b^2/(tanh(1/2*x)-1)*a+1/2/b/(tanh(1/2*x)-1)-1/b^3
*ln(tanh(1/2*x)-1)*a^2-1/2/b*ln(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^2+1/b^
2/(tanh(1/2*x)+1)*a+1/2/b/(tanh(1/2*x)+1)+1/b^3*ln(tanh(1/2*x)+1)*a^2+1/2/b
*ln(tanh(1/2*x)+1)-2*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/
((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.12, size = 167, normalized size = 1.96

$$\frac{e^{2x}}{8b}-\frac{e^{-2x}}{8b}-\frac{a\,e^{x}}{2b^{2}}+\frac{a\,e^{-x}}{2b^{2}}+\frac{x\left(2a^{2}+b^{2}\right)}{2b^{3}}+\frac{a^{3}\ln\left(\frac{2a^{3}e^{x}}{b^{4}}-\frac{2a^{3}(b+a\,e^{x})}{b^{4}\sqrt{a+b}\sqrt{a-b}}\right)}{b^{3}\sqrt{a+b}\sqrt{a-b}}-\frac{a^{3}\ln\left(\frac{2a^{3}e^{x}}{b^{4}}+\frac{2a^{3}(b+a\,e^{x})}{b^{4}\sqrt{a+b}\sqrt{a-b}}\right)}{b^{3}\sqrt{a+b}\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*cosh(x)),x)
```

```
[Out] exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (a*exp(x))/(2*b^2) + (a*exp(-x))/(2*b^2)
 + (x*(2*a^2 + b^2))/(2*b^3) + (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b + a*
exp(x)))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^(1/
2)) - (a^3*log((2*a^3*exp(x))/b^4 + (2*a^3*(b + a*exp(x)))/(b^4*(a + b)^(1/
2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.56 $\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sinh(x)}{b}$$

```
[Out] -a*x/b^2+sinh(x)/b+2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^2/(
a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2746, 12, 2735, 2659, 208}

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sinh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a
 - b]*b^2*Sqrt[a + b]) + Sinh[x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx &= \frac{\sinh(x)}{b}-\frac{\int \frac{a\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\
&= \frac{\sinh(x)}{b}-\frac{a\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}+\frac{a^2\int \frac{1}{a+b\cosh(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}+\frac{\sinh(x)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 57, normalized size = 0.92

$$
\frac{a\left(-\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-x\right)+b\sinh(x)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (a*(-x - (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2
]) + b*Sinh[x])/b^2
```

fricas [B] time = 0.78, size = 449, normalized size = 7.24

$$\left[-\frac{a^2b - b^3 + 2\left(a^3 - ab^2\right)x\cosh(x) - \left(a^2b - b^3\right)\cosh(x)^2 - \left(a^2b - b^3\right)\sinh(x)^2 - 2\left(a^2\cosh(x) + a^2\sinh(x)\right)\sqrt{a^2}}{2\left(\left(a^2b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (a^2*b - b^3)*cosh(x)^2 -
(a^2*b - b^3)*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 - b^2)*log
((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh
(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*((a
^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*b^2 - b^4)*cosh(x) +
(a^2*b^2 - b^4)*sinh(x)), -1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (
a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 + 4*(a^2*cosh(x) + a^2*sin
h(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)
/(a^2 - b^2)) + 2*((a^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*
b^2 - b^4)*cosh(x) + (a^2*b^2 - b^4)*sinh(x))]
```

giac [A] time = 0.15, size = 62, normalized size = 1.00

$$\frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^2} - \frac{ax}{b^2} - \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - a*x/b^2
 - 1/2*e^(-x)/b + 1/2*e^x/b
```

maple [A] time = 0.06, size = 94, normalized size = 1.52

$$-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*cosh(x)),x)
```

```
[Out] -1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)-1/b/(tanh(1/2*x)+1)-a/b^2*ln(t
anh(1/2*x)+1)+2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b
)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.04, size = 139, normalized size = 2.24

$$\frac{e^x}{2\,b}-\frac{e^{-x}}{2\,b}-\frac{a\,x}{b^2}+\frac{a^2\,\ln\left(-\frac{2\,a^2\,e^x}{b^3}-\frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a^2\,\ln\left(\frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{2\,a^2\,e^x}{b^3}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*cosh(x)),x)
```

```
[Out] exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (a^2*log(- (2*a^2*exp(x))/b^3 -
(2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(1/
2)*(a - b)^(1/2)) - (a^2*log((2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a -
 b)^(1/2)) - (2*a^2*exp(x))/b^3))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))
```

sympy [A] time = 93.90, size = 1275, normalized size = 20.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*sinh(x), Eq(a, 0) & Eq(b, 0)), (x*tanh(x/2)**3/(b*tanh(x/2)*
*3 - b*tanh(x/2)) - x*tanh(x/2)/(b*tanh(x/2)**3 - b*tanh(x/2)) - 3*tanh(x/2
)**2/(b*tanh(x/2)**3 - b*tanh(x/2)) + 1/(b*tanh(x/2)**3 - b*tanh(x/2)), Eq(
a, -b)), ((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0
)), (-x*tanh(x/2)**2/(b*tanh(x/2)**2 - b) + x/(b*tanh(x/2)**2 - b) + tanh(x
/2)**3/(b*tanh(x/2)**2 - b) - 3*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b)),
(-a**2*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) +
```

```
b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/
(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a**
2*x*sqrt(a/(a - b) + b/(a - b))/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/
2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b
))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a**2*log(-sqrt(a/(a -
 b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) + b/(a -
b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a**2*log(-
sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b**2*sqrt(a/(a - b) + b/(a - b)
)*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) +
 b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a**2*log(sqr
t(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) +
 b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a
/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a*
*2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b**2*sqrt(a/(a - b) + b/
(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a
 - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) + a*b*x
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(a*b**2*sqrt(a/(a - b) + b/(a - b
))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b)
+ b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a*b*x*sqrt(
a/(a - b) + b/(a - b))/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a
*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x
/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - 2*a*b*sqrt(a/(a - b) + b/(a -
b))*tanh(x/2)/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqr
t(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 +
b**3*sqrt(a/(a - b) + b/(a - b))) + 2*b**2*sqrt(a/(a - b) + b/(a - b))*tanh
(x/2)/(a*b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a -
 b) + b/(a - b)) - b**3*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqr
t(a/(a - b) + b/(a - b))), True))
```

3.57 $\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] x/b-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2)/(a+b)^(1
/2)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2735, 2659, 208}

$$\frac{x}{b}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Cosh[x]),x]
```

```
[Out] x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqr
t[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{b}$$
$$= \frac{x}{b} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{x}{b} - \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.92

$$\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Cosh[x]),x]
```

```
[Out] (x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])/b
```

fricas [A] time = 0.48, size = 218, normalized size = 4.19

$$\left[\frac{\sqrt{a^2-b^2}\,a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+\left(a^2-b^2\right)}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + (a^2 - b^2)*x)/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*a*arcta
n(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 - b^2)*
x)/(a^2*b - b^3)]
```

giac [A] time = 0.12, size = 42, normalized size = 0.81

$$-\frac{2\,a\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) + x/b
```

maple [A] time = 0.06, size = 64, normalized size = 1.23

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*cosh(x)),x)
```

```
[Out] -1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)-2*a/b/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.22, size = 109, normalized size = 2.10

$$\frac{x}{b}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^2}-\frac{2\,a\,\left(b+a\,\mathrm{e}^x\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^2}+\frac{2\,a\,\left(b+a\,\mathrm{e}^x\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*cosh(x)),x)
```

```
[Out] x/b + (a*log((2*a*exp(x))/b^2 - (2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a
- b)^(1/2))))/(b*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^2 + (
2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))))/(b*(a + b)^(1/2)*(a
 - b)^(1/2))
```

sympy [A] time = 25.02, size = 241, normalized size = 4.63

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \\ \frac{x}{b} - \frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for } a = - \\ \frac{\sinh(x)}{a} & \text{for } b = 0 \\ \frac{x}{b} - \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for } a = b \\ \frac{ax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{bx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b - 1/(b*tanh(x/2)), Eq(a, -b)),
 (sinh(x)/a, Eq(b, 0)), (x/b - tanh(x/2)/b, Eq(a, b)), (a*x*sqrt(a/(a - b)
+ b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a
- b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b
) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - a*log(sqrt(a/(a - b) +
 b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
- b) + b/(a - b))) - b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))), True))
```

3.58 $\int \frac{\text{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] arctan(sinh(x))/a-2*b*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)^
(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2747, 3770, 2659, 208}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*S
qrt[a - b]*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\cosh(x)}\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 1.00

$$\frac{2\left(\frac{b\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Cosh[x]),x]
```

```
[Out] (2*(ArcTan[Tanh[x/2]] + (b*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2]))/a
```

fricas [A] time = 0.63, size = 227, normalized size = 4.20

$$\left[\frac{\sqrt{a^2-b^2}\,b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+2\left(a^2-b^2\right.}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), 2
*(sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(
a^2 - b^2)) + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

giac [A] time = 0.15, size = 45, normalized size = 0.83

$$-\frac{2\,b\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,a}+\frac{2\,\arctan\left(e^{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arctan(e
^x)/a
```

maple [A] time = 0.08, size = 51, normalized size = 0.94

$$-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)\,(a-b)}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*cosh(x)),x)
```

```
[Out] -2/a*b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2
/a*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.46, size = 286, normalized size = 5.30

$$\frac{b\ \ln\Big(64\,a^4\,b-64\,a^2\,b^3+128\,a^5\,e^x+32\,a\,b^3\,\sqrt{a^2-b^2}\,-64\,a^3\,b\,\sqrt{a^2-b^2}\,+32\,a\,b^4\,e^x-128\,a^4\,e^x\,\sqrt{a^2-b^2}\,-160}{a\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*cosh(x))),x)
```

```
[Out] (b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) + 32*a*b^3*(a^2 - b^2)^(1/2)
- 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) - 128*a^4*exp(x)*(a^2 - b^2)
^(1/2) - 160*a^3*b^2*exp(x) + 96*a^2*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(a*(a^2
 - b^2)^(1/2)) - (b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) - 32*a*b^3*(
a^2 - b^2)^(1/2) + 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) + 128*a^4*e
xp(x)*(a^2 - b^2)^(1/2) - 160*a^3*b^2*exp(x) - 96*a^2*b^2*exp(x)*(a^2 - b^2
)^(1/2)))/(a*(a^2 - b^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1
i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*cosh(x)), x)
```

3.59 $\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=64

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{\tanh(x)}{a}$$

```
[Out] -b*arctan(sinh(x))/a^2+2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a
^2/(a-b)^(1/2)/(a+b)^(1/2)+tanh(x)/a
```

Rubi [A] time = 0.12, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2802, 12, 2747, 3770, 2659, 208}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
+ b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]) + Tanh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx &= \frac{\tanh(x)}{a}-\frac{\int \frac{b\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}\\
&= \frac{\tanh(x)}{a}-\frac{b\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}\\
&= \frac{\tanh(x)}{a}-\frac{b\int \operatorname{sech}(x)\,dx}{a^2}+\frac{b^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2}\\
&= -\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{\tanh(x)}{a}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\tanh(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 63, normalized size = 0.98

$$\frac{-\frac{2b^2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+a\tanh(x)-2b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (-2*b*ArcTan[Tanh[x/2]] - (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + a*Tanh[x])/a^2
```

fricas [B] time = 0.65, size = 515, normalized size = 8.05

$$\left[-\frac{2a^3-2ab^2-\left(b^2\cosh(x)^2+2b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\,\mathrm{co}}{b\cosh(x}\right.}{a^4-a^2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(2*a^3 - 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
 + b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
+ 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x
) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
 + a)*sinh(x) + b)) + 2*(a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b -
 b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*arctan(cosh(x) + sinh(x)))
/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)*sin
h(x) + (a^4 - a^2*b^2)*sinh(x)^2), -2*(a^3 - a*b^2 + (b^2*cosh(x)^2 + 2*b^2
*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3 + (a^2*b - b
^3)*cosh(x)^2 + 2*(a^2*b - b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*
arctan(cosh(x) + sinh(x)))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(
a^4 - a^2*b^2)*cosh(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)]
```

giac [A] time = 0.15, size = 61, normalized size = 0.95

$$\frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^2}-\frac{2\,b\arctan\left(e^x\right)}{a^2}-\frac{2}{a\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^2) - 2*b*arc
tan(e^x)/a^2 - 2/(a*(e^(2*x) + 1))
```

maple [A] time = 0.09, size = 73, normalized size = 1.14

$$\frac{2b^2 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)\,(a-b)}} + \frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)} - \frac{2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*cosh(x)),x)
```

```
[Out] 2*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)+2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2/a^2*b*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.10, size = 294, normalized size = 4.59

$$\frac{b^2\,\ln\Big(64\,a\,b^3 - 64\,a^3\,b + 32\,b^3\,\sqrt{a^2-b^2} - 128\,a^4\,e^x - 32\,b^4\,e^x - 64\,a^2\,b\,\sqrt{a^2-b^2} - 128\,a^3\,e^x\,\sqrt{a^2-b^2} + 160\,a^2\,b}{a^2\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*cosh(x))),x)
```

```
[Out] (b*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/a^2 - 2/(a + a*exp(
2*x)) - (b^2*log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^2)^(1/2) + 128*a^4*e
xp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^3*exp(x)*(a^2 -
b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(a^2*
(a^2 - b^2)^(1/2)) + (b^2*log(64*a*b^3 - 64*a^3*b + 32*b^3*(a^2 - b^2)^(1/2
) - 128*a^4*exp(x) - 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^3*e
```

```
xp(x)*(a^2 - b^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^2)^
(1/2)))/(a^2*(a^2 - b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*cosh(x)), x)
```

3.60 $\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tanh(x)}{a^2}+\frac{\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^3}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

```
[Out] 1/2*(a^2+2*b^2)*arctan(sinh(x))/a^3-2*b^3*arctanh((a-b)^(1/2)*tanh(1/2*x)/(
a+b)^(1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)-b*tanh(x)/a^2+1/2*sech(x)*tanh(x)/a
```

Rubi [A] time = 0.30, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2802, 3055, 3001, 3770, 2659, 208}

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}}+\frac{\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^3}-\frac{b\tanh(x)}{a^2}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[Sinh[x]])/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tanh[
x/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Tanh[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
```

```
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = \frac{\operatorname{sech}(x)\tanh(x)}{2a} + \frac{\int \frac{(-2b+a\cosh(x)+b\cosh^2(x))\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx}{2a}$$

$$= -\frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} + \frac{\int \frac{(a^2+2b^2+ab\cosh(x))\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{2a^2}$$

$$= -\frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} - \frac{b^3\int \frac{1}{a+b\cosh(x)}\,dx}{a^3} + \frac{(a^2+2b^2)\int \operatorname{sech}(x)\,dx}{2a^3}$$

$$= \frac{(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^3} - \frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a} - \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,\right.}{a^3}$$

$$= \frac{(a^2+2b^2)\tan^{-1}(\sinh(x))}{2a^3} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\tanh(x)}{a^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2a}$$

Mathematica [A] time = 0.21, size = 82, normalized size = 0.94

$$\frac{2(a^2+2b^2)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{4b^3\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + a\tanh(x)(a\operatorname{sech}(x)-2b)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] + (4*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt
[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(-2*b + a*Sech[x])*Tanh[x])/(2*a^3)
```

fricas [B] time = 0.85, size = 1370, normalized size = 15.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3
 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cos
h(x))*sinh(x)^2 + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4
+ 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh
(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh
```

```
(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqr
t(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*
cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + ((a^4 + a^2*b^2 - 2*b^4)*cosh(x
)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*sinh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)
*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2 +
2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 +
 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)*cosh(x))*si
nh(x))*arctan(cosh(x) + sinh(x)) - (a^4 - a^2*b^2)*cosh(x) - (a^4 - a^2*b^2
 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*sinh(x))/(a^5 -
 a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3
+ (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^
2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 +
 (a^5 - a^3*b^2)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*co
sh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*
b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^2 + 2*(b^3*cosh(x)^4 + 4*b
^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cos
h(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
+ ((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*si
nh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^
4 + a^2*b^2 - 2*b^4)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^
2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a
^4 + a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^4 -
a^2*b^2)*cosh(x) - (a^4 - a^2*b^2 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b
- a*b^3)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(
a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3
*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2
 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh(x))*sinh(x))]
```

giac [A] time = 0.15, size = 89, normalized size = 1.02

$$-\frac{2\,b^{3}\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{3}}+\frac{\left(a^{2}+2\,b^{2}\right)\arctan\left(e^{x}\right)}{a^{3}}+\frac{ae^{(3\,x)}+2\,be^{(2\,x)}-ae^{x}+2\,b}{a^{2}\left(e^{(2\,x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a^2 +
 2*b^2)*arctan(e^x)/a^3 + (a*e^(3*x) + 2*b*e^(2*x) - a*e^x + 2*b)/(a^2*(e^(
2*x) + 1)^2)
```

maple [A] time = 0.09, size = 146, normalized size = 1.68

$$-\frac{2b^3 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}}-\frac{\tanh^3\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{2\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{a^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+\frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-\frac{2\tanh\left(\frac{x}{2}\right)b}{a^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3-2/a^2/(tanh(1/2*x)^2+1)^2*tanh(1/2
*x)^3*b+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)-2/a^2/(tanh(1/2*x)^2+1)^2*tanh(
1/2*x)*b+1/a*arctan(tanh(1/2*x))+2/a^3*arctan(tanh(1/2*x))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.13, size = 476, normalized size = 5.47

$$\frac{e^x}{a+a\,e^{2\,x}}-\frac{2\,e^x}{a+2\,a\,e^{2\,x}+a\,e^{4\,x}}+\frac{2\,b}{a^2\,e^{2\,x}+a^2}-\frac{\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i}{2\,a}-\frac{b^2\,\left(\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*cosh(x))),x)
```

```
[Out] exp(x)/(a + a*exp(2*x)) - (2*exp(x))/(a + 2*a*exp(2*x) + a*exp(4*x)) + (2*b
)/(a^2*exp(2*x) + a^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2*a
) - (b^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/a^3 - (b^3*log(16*a
^5*b - 48*a*b^5 - 24*b^5*(a^2 - b^2)^(1/2) + 32*a^3*b^3 + 32*a^6*exp(x) + 2
4*b^6*exp(x) + 16*a^4*b*(a^2 - b^2)^(1/2) + 40*a^2*b^3*(a^2 - b^2)^(1/2) +
32*a^5*exp(x)*(a^2 - b^2)^(1/2) - 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) +
72*a^3*b^2*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 - b^2)^(1/2)))/(
a^3*(a^2 - b^2)^(1/2)) + (b^3*log(16*a^5*b - 48*a*b^5 + 24*b^5*(a^2 - b^2)^
```

```
(1/2) + 32*a^3*b^3 + 32*a^6*exp(x) + 24*b^6*exp(x) - 16*a^4*b*(a^2 - b^2)^(
1/2) - 40*a^2*b^3*(a^2 - b^2)^(1/2) - 32*a^5*exp(x)*(a^2 - b^2)^(1/2) - 112
*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 72*a^3*b^2*exp(x)*(a^2 - b^2)^(1/2) +
 72*a*b^4*exp(x)*(a^2 - b^2)^(1/2)))/(a^3*(a^2 - b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*cosh(x)), x)
```

3.61 $\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=114

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}-\frac{b\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^4}+\frac{\left(2a^2+3b^2\right)\tanh(x)}{3a^3}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

```
[Out] -1/2*b*(a^2+2*b^2)*arctan(sinh(x))/a^4+2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*x
)/(a+b)^(1/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(2*a^2+3*b^2)*tanh(x)/a^3-1/
2*b*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

Rubi [A] time = 0.47, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2802, 3055, 3001, 3770, 2659, 208}

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}}+\frac{\left(2a^2+3b^2\right)\tanh(x)}{3a^3}-\frac{b\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^4}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -(b*(a^2 + 2*b^2)*ArcTan[Sinh[x]])/(2*a^4) + (2*b^4*ArcTanh[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Tan
h[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{\left(-3b+2a\cosh(x)+2b\cosh^2(x)\right)\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx}{3a}$$

$$= -\frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{\left(2\left(2a^2+3b^2\right)+ab\cosh(x)-3b^2\cosh^2(x)\right)\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx}{6a^2}$$

$$= \frac{\left(2a^2+3b^2\right)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{\int \frac{\left(-3b\left(a^2+2b^2\right)-3ab^2\cosh(x)\right)\operatorname{se}}{a+b\cosh(x)}}{6a^3}$$

$$= \frac{\left(2a^2+3b^2\right)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a} + \frac{b^4\int \frac{1}{a+b\cosh(x)}\,dx}{a^4} - \frac{\left(b\left(a^2\right.\right.}{}$$

$$= -\frac{b\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^4} + \frac{\left(2a^2+3b^2\right)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh}{3a}$$

$$= -\frac{b\left(a^2+2b^2\right)\tan^{-1}(\sinh(x))}{2a^4} + \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}} + \frac{\left(2a^2+3b^2\right)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}}{}$$

Mathematica [A] time = 0.43, size = 101, normalized size = 0.89

$$\frac{-6b\left(a^2+2b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + a\tanh(x)\left(2a^2\operatorname{sech}^2(x)+4a^2-3ab\operatorname{sech}(x)+6b^2\right) - \frac{12b^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] - (12*b^4*ArcTan[((a - b)*Tanh[x/2])/
Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(4*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*
a^2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

fricas [B] time = 1.11, size = 2483, normalized size = 21.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 + 4*a^
5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*b^2 - 2*
```

```
a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*b^3)*cos
h(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*cosh(x)^
2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 - a*b^4
)*cosh(x)^2)*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*s
inh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*si
nh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*co
sh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(
x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(
x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt
(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*c
osh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*((a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4*b + a^2*b^3 -
 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2*b^3 - 2*b^5)*c
osh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + 3*(a^4*b + a^2*b
^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*
(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^4 + 6*(a^4*b
 + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + (a^4*b + a^2*b^3 - 2*b^5)*
cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - 3*(a^4*b - a^2*b^3)*cosh(x) -
 3*(a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 8*(a^3*b^2 - a*b^4)*c
osh(x)^3 - 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^6 + 6
*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4*b^2)*sinh(x)^6 + a^6 - a^4*
b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a^4*b^2)*cos
h(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 + 3*(a^6 - a^4*b^2 + 5*(a^6 -
 a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - a^
4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3 + (a^6 - a^4*b^2)*cosh(x))*s
inh(x)), -1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^
5 + 4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*
b^2 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*
b^3)*cosh(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*
cosh(x)^2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2
 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5
 + b^4*sinh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 +
 b^4)*sinh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(
5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b
^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*((a^4*b + a^2*b^3 - 2*b^5)
*cosh(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4*b + a^2*b
^3 - 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2*b^3 - 2*b^
5)*cosh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + 3*(a^4*b + a
^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2
+ 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^4 + 6*(a
```

```
^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b + a^2*b^3 - 2*b^5)
*cosh(x)^5 + 2*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 + (a^4*b + a^2*b^3 - 2*b
^5)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - 3*(a^4*b - a^2*b^3)*cosh(
x) - 3*(a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 8*(a^3*b^2 - a*b^
4)*cosh(x)^3 - 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^6
 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4*b^2)*sinh(x)^6 + a^6 -
a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2
)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a^4*b^2)
*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 + 3*(a^6 - a^4*b^2 + 5*(a
^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^6
- a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3 + (a^6 - a^4*b^2)*cosh(x
))*sinh(x))]
```

giac [A] time = 0.12, size = 123, normalized size = 1.08

$$\frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^4}-\frac{\left(a^2b+2\,b^3\right)\arctan\left(e^x\right)}{a^4}-\frac{3\,abe^{(5\,x)}+6\,b^2e^{(4\,x)}+12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2+6\,b^2}{3\,a^3\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - (a^2*b
+ 2*b^3)*arctan(e^x)/a^4 - 1/3*(3*a*b*e^(5*x) + 6*b^2*e^(4*x) + 12*a^2*e^(2
*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

maple [B] time = 0.11, size = 239, normalized size = 2.10

$$\frac{2b^4\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4\sqrt{(a+b)\,(a-b)}}+\frac{2\left(\tanh^5\left(\frac{x}{2}\right)\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)b}{a^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{2\left(\tanh^5\left(\frac{x}{2}\right)\right)b^2}{a^3\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{4\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*cosh(x)),x)
```

```
[Out] 2*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)
)+2/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5+1/a^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*
x)^5*b+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^2+4/3/a/(tanh(1/2*x)^2+1)^
3*tanh(1/2*x)^3+4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2+2/a/(tanh(1/2*x
)^2+1)^3*tanh(1/2*x)+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*b^2-1/a^2/(tanh(
1/2*x)^2+1)^3*tanh(1/2*x)*b-1/a^2*b*arctan(tanh(1/2*x))-2/a^4*arctan(tanh(1
/2*x))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.53, size = 547, normalized size = 4.80

$$\frac{8}{3\left(a+3\,a\,e^{2\,x}+3\,a\,e^{4\,x}+a\,e^{6\,x}\right)}-\frac{4}{a+2\,a\,e^{2\,x}+a\,e^{4\,x}}-\frac{2\,b^2}{a^3\,e^{2\,x}+a^3}+\frac{b^3\,\left(\ln\left(e^x-i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{a^4}-\frac{b\,e^x}{a^2\,e^{2\,x}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*cosh(x))),x)
```

```
[Out] 8/(3*(a + 3*a*exp(2*x) + 3*a*exp(4*x) + a*exp(6*x))) - 4/(a + 2*a*exp(2*x)
+ a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) + a^3) + (b^3*(log(exp(x) - 1i)*1i -
log(exp(x) + 1i)*1i))/a^4 - (b*exp(x))/(a^2*exp(2*x) + a^2) + (2*b*exp(x))/
(2*a^2*exp(2*x) + a^2*exp(4*x) + a^2) + (b*(log(exp(x) - 1i)*1i - log(exp(x
) + 1i)*1i))/(2*a^2) + (b^4*log(32*a^3*b^4 - 24*b^6*(a^2 - b^2)^(1/2) - 48*
a*b^6 + 16*a^5*b^2 + 24*b^7*exp(x) + 32*a^6*b*exp(x) + 40*a^2*b^4*(a^2 - b^
2)^(1/2) + 16*a^4*b^2*(a^2 - b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^4*b^3*e
xp(x) + 72*a^3*b^3*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^5*exp(x)*(a^2 - b^2)^(
1/2) + 32*a^5*b*exp(x)*(a^2 - b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2)) - (b^4*l
og(24*b^6*(a^2 - b^2)^(1/2) - 48*a*b^6 + 32*a^3*b^4 + 16*a^5*b^2 + 24*b^7*e
xp(x) + 32*a^6*b*exp(x) - 40*a^2*b^4*(a^2 - b^2)^(1/2) - 16*a^4*b^2*(a^2 -
b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a^3*b^3*exp(x)*(a^
2 - b^2)^(1/2) + 72*a*b^5*exp(x)*(a^2 - b^2)^(1/2) - 32*a^5*b*exp(x)*(a^2 -
 b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*cosh(x)), x)
```

3.62 $\int (a + b\cosh(c + dx))^5 \, dx$

Optimal. Leaf size=183

$$\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{60d}+\frac{7ab^2\left(22a^2+23b^2\right)\sinh(c+dx)\cosh(c+dx)}{120d}+\frac{b\left(107a^4+19\right.}{}$$

```
[Out] 1/8*a*(8*a^4+40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4+192*a^2*b^2+16*b^4)*sinh(
d*x+c)/d+7/120*a*b^2*(22*a^2+23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47*a
^2+16*b^2)*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d+9/20*a*b*(a+b*cosh(d*x+c))^3*s
inh(d*x+c)/d+1/5*b*(a+b*cosh(d*x+c))^4*sinh(d*x+c)/d
```

Rubi [A] time = 0.26, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2656, 2753, 2734}

$$\frac{b\left(192a^2b^2+107a^4+16b^4\right)\sinh(c+dx)}{30d}+\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{60d}+\frac{7ab^2\left(22a^2+23b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^5,x]
```

```
[Out] (a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 + 192*a^2*b^2 + 16*b^4)
*Sinh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 + 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 + 16*b^2)*(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])
/(60*d) + (9*a*b*(a + b*Cosh[c + d*x])^3*Sinh[c + d*x])/(20*d) + (b*(a + b*
Cosh[c + d*x])^4*Sinh[c + d*x])/(5*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^5\,dx = \frac{b(a+b\cosh(c+dx))^4\sinh(c+dx)}{5d} + \frac{1}{5}\int (a+b\cosh(c+dx))^3\left(5a^2+4b^2+9ab\right.$$

$$= \frac{9ab(a+b\cosh(c+dx))^3\sinh(c+dx)}{20d} + \frac{b(a+b\cosh(c+dx))^4\sinh(c+dx)}{5d} + \frac{1}{20}$$

$$= \frac{b\left(47a^2+16b^2\right)(a+b\cosh(c+dx))^2\sinh(c+dx)}{60d} + \frac{9ab(a+b\cosh(c+dx))^3\sin}{20d}$$

$$= \frac{1}{8}a\left(8a^4+40a^2b^2+15b^4\right)x + \frac{b\left(107a^4+192a^2b^2+16b^4\right)\sinh(c+dx)}{30d} + \frac{7ab^2\left(2\right.}{}$$

Mathematica [A] time = 0.37, size = 133, normalized size = 0.73

$$\frac{600ab^2\left(2a^2+b^2\right)\sinh(2(c+dx)) + 50b^3\left(8a^2+b^2\right)\sinh(3(c+dx)) + 60a\left(8a^4+40a^2b^2+15b^4\right)(c+dx) + 30}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^5,x]
```

```
[Out] (60*a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*(c + d*x) + 300*b*(8*a^4 + 12*a^2*b^2 +
 b^4)*Sinh[c + d*x] + 600*a*b^2*(2*a^2 + b^2)*Sinh[2*(c + d*x)] + 50*b^3*(8
*a^2 + b^2)*Sinh[3*(c + d*x)] + 75*a*b^4*Sinh[4*(c + d*x)] + 6*b^5*Sinh[5*(
c + d*x)])/(480*d)
```

fricas [A] time = 0.51, size = 190, normalized size = 1.04

$$\frac{3\,b^5\sinh\left(dx+c\right)^5 + 5\left(6\,b^5\cosh\left(dx+c\right)^2 + 30\,ab^4\cosh\left(dx+c\right) + 40\,a^2b^3 + 5\,b^5\right)\sinh\left(dx+c\right)^3 + 30\left(8\,a^5+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^5*sinh(d*x + c)^5 + 5*(6*b^5*cosh(d*x + c)^2 + 30*a*b^4*cosh(d*x
 + c) + 40*a^2*b^3 + 5*b^5)*sinh(d*x + c)^3 + 30*(8*a^5 + 40*a^3*b^2 + 15*a
*b^4)*d*x + 15*(b^5*cosh(d*x + c)^4 + 10*a*b^4*cosh(d*x + c)^3 + 80*a^4*b +
 120*a^2*b^3 + 10*b^5 + 5*(8*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 40*(2*a^3*b^2
 + a*b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.16, size = 263, normalized size = 1.44

$$\frac{b^5e^{(5\,dx+5\,c)}}{160\,d}+\frac{5\,ab^4e^{(4\,dx+4\,c)}}{64\,d}-\frac{5\,ab^4e^{(-4\,dx-4\,c)}}{64\,d}-\frac{b^5e^{(-5\,dx-5\,c)}}{160\,d}+\frac{1}{8}\left(8\,a^5+40\,a^3b^2+15\,ab^4\right)x+\frac{5\left(8\,a^2b^3+b^5\right)e^{(3\,dx+3\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^(
-4*d*x - 4*c)/d - 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 + 40*a^3*b^2 +
15*a*b^4)*x + 5/96*(8*a^2*b^3 + b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2 + a
*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(d*x + c)/d -
 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 + a*b^4)
*e^(-2*d*x - 2*c)/d - 5/96*(8*a^2*b^3 + b^5)*e^(-3*d*x - 3*c)/d
```

maple [A] time = 0.30, size = 155, normalized size = 0.85

$$\frac{b^5\left(\frac{8}{15}+\frac{\left(\cosh^4(dx+c)\right)}{5}+\frac{4\left(\cosh^2(dx+c)\right)}{15}\right)\sinh\left(dx+c\right)+5a\,b^4\left(\left(\frac{\left(\cosh^3(dx+c)\right)}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(8/15+1/5*cosh(d*x+c)^4+4/15*cosh(d*x+c)^2)*sinh(d*x+c)+5*a*b^4*((
1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+10*a^2*b^3*(2
/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+10*a^3*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1
/2*d*x+1/2*c)+5*a^4*b*sinh(d*x+c)+a^5*(d*x+c))
```

maxima [A] time = 0.36, size = 273, normalized size = 1.49

$$\frac{5}{64}\,ab^4\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{5}{4}\,a^3b^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^5x+\frac{1}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + 5/4*a^3*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2
```

```
*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d + 25*e^(3*d*x + 3*c
)/d + 150*e^(d*x + c)/d - 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d - 3*e^
(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 5*a^4*b*sinh(d*x + c)/d
```

mupad [B] time = 1.14, size = 160, normalized size = 0.87

$$75\,b^5\sinh\left(c+d\,x\right)+\frac{25\,b^5\sinh(3\,c+3\,d\,x)}{2}+\frac{3\,b^5\sinh(5\,c+5\,d\,x)}{2}+150\,a\,b^4\sinh\left(2\,c+2\,d\,x\right)+\frac{75\,a\,b^4\sinh(4\,c+4\,d\,x)}{4}+900\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^5,x)
```

```
[Out] (75*b^5*sinh(c + d*x) + (25*b^5*sinh(3*c + 3*d*x))/2 + (3*b^5*sinh(5*c + 5*
d*x))/2 + 150*a*b^4*sinh(2*c + 2*d*x) + (75*a*b^4*sinh(4*c + 4*d*x))/4 + 90
0*a^2*b^3*sinh(c + d*x) + 300*a^3*b^2*sinh(2*c + 2*d*x) + 100*a^2*b^3*sinh(
3*c + 3*d*x) + 600*a^4*b*sinh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x + 600*
a^3*b^2*d*x)/(120*d)
```

sympy [A] time = 2.25, size = 314, normalized size = 1.72

$$\begin{cases} a^5x+\frac{5a^4b\sinh(c+dx)}{d}-5a^3b^2x\sinh^2(c+dx)+5a^3b^2x\cosh^2(c+dx)+\frac{5a^3b^2\sinh(c+dx)\cosh(c+dx)}{d}-\frac{20a^2b^3\sinh^3(c+d}{3d} \\ x\left(a+b\cosh(c)\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*sinh(c + d*x)/d - 5*a**3*b**2*x*sinh(c + d*x)*
*2 + 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c + d*
x)/d - 20*a**2*b**3*sinh(c + d*x)**3/(3*d) + 10*a**2*b**3*sinh(c + d*x)*cos
h(c + d*x)**2/d + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 - 15*a*b**4*sinh(c
 + d*x)**3*cosh(c + d*x)/(8*d) + 25*a*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(
8*d) + 8*b**5*sinh(c + d*x)**5/(15*d) - 4*b**5*sinh(c + d*x)**3*cosh(c + d*
x)**2/(3*d) + b**5*sinh(c + d*x)*cosh(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*c
osh(c))**5, True))
```

3.63 $\int (a+b\cosh(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{ab\left(19a^2+16b^2\right)\sinh(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(8a^4+24a^2b^2+3b^4\right)+\frac{b\sinh(c+$$

```
[Out] 1/8*(8*a^4+24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2+16*b^2)*sinh(d*x+c)/d+1/24*b
^2*(26*a^2+9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*(a+b*cosh(d*x+c))^2*si
nh(d*x+c)/d+1/4*b*(a+b*cosh(d*x+c))^3*sinh(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2656, 2753, 2734}

$$\frac{ab\left(19a^2+16b^2\right)\sinh(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)+\frac{b\sinh(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sinh[c + d*x])/
(6*d) + (b^2*(26*a^2 + 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])/(12*d) + (b*(a + b*Cosh[c + d*x])^3*
Sinh[c + d*x])/(4*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^4\,dx = \frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d} + \frac{1}{4}\int (a+b\cosh(c+dx))^2\left(4a^2+3b^2+7ab\right.$$

$$= \frac{7ab(a+b\cosh(c+dx))^2\sinh(c+dx)}{12d} + \frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d} + \frac{1}{12}$$

$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x + \frac{ab\left(19a^2+16b^2\right)\sinh(c+dx)}{6d} + \frac{b^2\left(26a^2+9b^2\right)\cosh}{24}$$

Mathematica [A] time = 0.22, size = 104, normalized size = 0.76

$$\frac{24b^2\left(6a^2+b^2\right)\sinh(2(c+dx)) + 96ab\left(4a^2+3b^2\right)\sinh(c+dx) + 12\left(8a^4+24a^2b^2+3b^4\right)(c+dx) + 32ab^3\sin}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sinh[c
+ d*x] + 24*b^2*(6*a^2 + b^2)*Sinh[2*(c + d*x)] + 32*a*b^3*Sinh[3*(c + d*x)
] + 3*b^4*Sinh[4*(c + d*x)])/(96*d)
```

fricas [A] time = 0.87, size = 123, normalized size = 0.90

$$\frac{\left(3\,b^4\cosh(dx+c)+8\,ab^3\right)\sinh(dx+c)^3 + 3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)dx + 3\left(b^4\cosh(dx+c)^3 + 8\,ab^3\cosh(dx\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*((3*b^4*cosh(d*x + c) + 8*a*b^3)*sinh(d*x + c)^3 + 3*(8*a^4 + 24*a^2*b
^2 + 3*b^4)*d*x + 3*(b^4*cosh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^2 + 32*a^3
*b + 24*a*b^3 + 4*(6*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.13, size = 196, normalized size = 1.43

$$\frac{b^4e^{(4\,dx+4\,c)}}{64\,d}+\frac{ab^3e^{(3\,dx+3\,c)}}{6\,d}-\frac{ab^3e^{(-3\,dx-3\,c)}}{6\,d}-\frac{b^4e^{(-4\,dx-4\,c)}}{64\,d}+\frac{1}{8}\left(8\,a^4+24\,a^2b^2+3\,b^4\right)x+\frac{\left(6\,a^2b^2+b^4\right)e^{(2\,dx+2\,c)}}{8\,d}+\frac{(4\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d - 1/6*a*b^3*e^(-3*
d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 + 24*a^2*b^2 + 3*b^
4)*x + 1/8*(6*a^2*b^2 + b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b + 3*a*b^3)*e^
(d*x + c)/d - 1/2*(4*a^3*b + 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 + b^4
)*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.25, size = 119, normalized size = 0.87

$$\frac{b^4\left(\left(\frac{\left(\cosh^3(dx+c)\right)}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+4a\,b^3\left(\frac{2}{3}+\frac{\left(\cosh^2(dx+c)\right)}{3}\right)\sinh\left(dx+c\right)+6a^2b^2\left(\frac{\cosh(dx+}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+4*
a*b^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+6*a^2*b^2*(1/2*cosh(d*x+c)*sinh(d
*x+c)+1/2*d*x+1/2*c)+4*a^3*b*sinh(d*x+c)+a^4*(d*x+c))
```

maxima [A] time = 0.30, size = 183, normalized size = 1.34

$$\frac{1}{64}\,b^4\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{3}{4}\,a^2b^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^4x+\frac{1}{6}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/64*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/4*a^2*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d
*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 4*a^3*b*sinh(d*x + c)/d
```

mupad [B] time = 0.19, size = 114, normalized size = 0.83

$$\frac{6\,b^4\sinh\left(2\,c+2\,d\,x\right)+\frac{3\,b^4\sinh(4\,c+4\,d\,x)}{4}+8\,a\,b^3\sinh\left(3\,c+3\,d\,x\right)+36\,a^2\,b^2\sinh\left(2\,c+2\,d\,x\right)+72\,a\,b^3\sinh\left(c+d\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^4,x)
```

```
[Out] (6*b^4*sinh(2*c + 2*d*x) + (3*b^4*sinh(4*c + 4*d*x))/4 + 8*a*b^3*sinh(3*c +
 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) + 72*a*b^3*sinh(c + d*x) + 96*a^3*b*
sinh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x + 72*a^2*b^2*d*x)/(24*d)
```

sympy [A] time = 1.10, size = 240, normalized size = 1.75

$$\begin{cases} a^4x + \frac{4a^3b\sinh(c+dx)}{d} - 3a^2b^2x\sinh^2(c+dx) + 3a^2b^2x\cosh^2(c+dx) + \frac{3a^2b^2\sinh(c+dx)\cosh(c+dx)}{d} - \frac{8ab^3\sinh^3(c+dx)}{3d} \\ x(a+b\cosh(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sinh(c + d*x)/d - 3*a**2*b**2*x*sinh(c + d*x)*
*2 + 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c + d*
x)/d - 8*a*b**3*sinh(c + d*x)**3/(3*d) + 4*a*b**3*sinh(c + d*x)*cosh(c + d*
x)**2/d + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 - 3*b**4*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) + 5*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a
+ b*cosh(c))**4, True))
```

3.64 $\int (a+b\cosh(c+dx))^3\,dx$

Optimal. Leaf size=90

$$\frac{2b\left(4a^2+b^2\right)\sinh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\sinh(c+dx)(a+b\cosh(c+dx)}{3d}$$

```
[Out] 1/2*a*(2*a^2+3*b^2)*x+2/3*b*(4*a^2+b^2)*sinh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c)
*sinh(d*x+c)/d+1/3*b*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$\frac{2b\left(4a^2+b^2\right)\sinh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\sinh(c+dx)(a+b\cosh(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 + (2*b*(4*a^2 + b^2)*Sinh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*(a + b*Cosh[c + d*x])^2*Sinh[c +
d*x])/(3*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{b(a+b\cosh(c+dx))^2\sinh(c+dx)}{3d} + \frac{1}{3}\int (a+b\cosh(c+dx))\left(3a^2+2b^2+5ab\right.$$
$$= \frac{1}{2}a\left(2a^2+3b^2\right)x + \frac{2b\left(4a^2+b^2\right)\sinh(c+dx)}{3d} + \frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}$$

Mathematica [A] time = 0.13, size = 80, normalized size = 0.89

$$\frac{12a^3c+12a^3dx+9b\left(4a^2+b^2\right)\sinh(c+dx)+9ab^2\sinh(2(c+dx))+18ab^2c+18ab^2dx+b^3\sinh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^3,x]
```

```
[Out] (12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sinh
[c + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + b^3*Sinh[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.63, size = 78, normalized size = 0.87

$$\frac{b^3\sinh(dx+c)^3+6\left(2a^3+3ab^2\right)dx+3\left(b^3\cosh(dx+c)^2+6ab^2\cosh(dx+c)+12a^2b+3b^3\right)\sinh(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b^3*sinh(d*x + c)^3 + 6*(2*a^3 + 3*a*b^2)*d*x + 3*(b^3*cosh(d*x + c)^
2 + 6*a*b^2*cosh(d*x + c) + 12*a^2*b + 3*b^3)*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 131, normalized size = 1.46

$$\frac{b^3e^{(3dx+3c)}}{24d}+\frac{3ab^2e^{(2dx+2c)}}{8d}-\frac{3ab^2e^{(-2dx-2c)}}{8d}-\frac{b^3e^{(-3dx-3c)}}{24d}+\frac{1}{2}\left(2a^3+3ab^2\right)x+\frac{3\left(4a^2b+b^3\right)e^{(dx+c)}}{8d}-\frac{3\left(4a^2b+b^3\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2*
d*x - 2*c)/d - 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 + 3*a*b^2)*x + 3/8*
(4*a^2*b + b^3)*e^(d*x + c)/d - 3/8*(4*a^2*b + b^3)*e^(-d*x - c)/d
```

maple [A] time = 0.20, size = 77, normalized size = 0.86

$$\frac{b^3\left(\frac{2}{3}+\frac{\left(\cosh^2(dx+c)\right)}{3}\right)\sinh(dx+c)+3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\sinh(dx+c)+a^3(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh(
d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*sinh(d*x+c)+a^3*(d*x+c))
```

maxima [A] time = 0.30, size = 116, normalized size = 1.29

$$\frac{3}{8}ab^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+a^3x+\frac{1}{24}b^3\left(\frac{e^{(3dx+3c)}}{d}+\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}-\frac{e^{(-3dx-3c)}}{d}\right)+\frac{3a^2b\sinh(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^3
*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)
/d) + 3*a^2*b*sinh(d*x + c)/d
```

mupad [B] time = 0.95, size = 73, normalized size = 0.81

$$\frac{\frac{9b^3\sinh(c+dx)}{2}+\frac{b^3\sinh(3c+3dx)}{2}+\frac{9ab^2\sinh(2c+2dx)}{2}+18a^2b\sinh(c+dx)+6a^3dx+9ab^2dx}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^3,x)
```

```
[Out] ((9*b^3*sinh(c + d*x))/2 + (b^3*sinh(3*c + 3*d*x))/2 + (9*a*b^2*sinh(2*c +
2*d*x))/2 + 18*a^2*b*sinh(c + d*x) + 6*a^3*d*x + 9*a*b^2*d*x)/(6*d)
```

sympy [A] time = 0.54, size = 128, normalized size = 1.42

$$\begin{cases} a^3x+\frac{3a^2b\sinh(c+dx)}{d}-\frac{3ab^2x\sinh^2(c+dx)}{2}+\frac{3ab^2x\cosh^2(c+dx)}{2}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{2b^3\sinh^3(c+dx)}{3d}+\frac{b^3\sinh(c+dx)\cos}{d} \\ x(a+b\cosh(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)/d - 3*a*b**2*x*sinh(c + d*x)**2/
2 + 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/(2
*d) - 2*b**3*sinh(c + d*x)**3/(3*d) + b**3*sinh(c + d*x)*cosh(c + d*x)**2/d
, Ne(d, 0)), (x*(a + b*cosh(c))**3, True))
```

3.65 $\int (a+b\cosh(c+dx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2+b^2)*x+2*a*b*sinh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + (2*a*b*Sinh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\cosh(c+dx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 0.92

$$\frac{2\left(2a^2+b^2\right)(c+dx)+8ab\sinh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sinh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(
4*d)
```

fricas [A] time = 0.52, size = 40, normalized size = 0.80

$$\frac{\left(2\,a^2+b^2\right)dx+\left(b^2\cosh\left(dx+c\right)+4\,ab\right)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a^2 + b^2)*d*x + (b^2*cosh(d*x + c) + 4*a*b)*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 75, normalized size = 1.50

$$\frac{1}{2}\left(2\,a^2+b^2\right)x+\frac{b^2e^{(2\,dx+2\,c)}}{8\,d}+\frac{abe^{(dx+c)}}{d}-\frac{abe^{(-dx-c)}}{d}-\frac{b^2e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d - a*b*e
^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.07, size = 51, normalized size = 1.02

$$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sinh\left(dx+c\right)+a^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*sinh(d*x+c)+a^2*
(d*x+c))
```

maxima [A] time = 0.50, size = 55, normalized size = 1.10

$$\frac{1}{8}\,b^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^2x+\frac{2\,ab\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

[Out] 1/8*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*sinh
(d*x + c)/d

mupad [B] time = 0.92, size = 41, normalized size = 0.82

$$\frac{\frac{\sinh(2\,c+2\,d\,x)\,b^2}{4}+2\,a\,\sinh\left(c+d\,x\right)\,b}{d}+a^2\,x+\frac{b^2\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cosh(c + d*x))^2,x)

[Out] ((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*sinh(c + d*x))/d + a^2*x + (b^2*x)/2

sympy [A] time = 0.26, size = 78, normalized size = 1.56

$$\begin{cases} a^2x+\frac{2ab\sinh\left(c+dx\right)}{d}-\frac{b^2x\sinh^2\left(c+dx\right)}{2}+\frac{b^2x\cosh^2\left(c+dx\right)}{2}+\frac{b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x\left(a+b\cosh\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cosh(d*x+c))**2,x)

[Out] Piecewise((a**2*x + 2*a*b*sinh(c + d*x)/d - b**2*x*sinh(c + d*x)**2/2 + b**
2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a + b*cosh(c))**2, True))

3.66 $\int (a+b\cosh(c+dx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\sinh(c+dx)}{d}$$

```
[Out] a*x+b*sinh(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2637}

$$ax+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cosh[c + d*x],x]
```

```
[Out] a*x + (b*Sinh[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cosh(c+dx))\,dx &= ax+b\int \cosh(c+dx)\,dx\\ &= ax+\frac{b\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.73

$$ax+\frac{b\sinh(c)\cosh(dx)}{d}+\frac{b\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cosh[c + d*x],x]
```

```
[Out] a*x + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d
```

fricas [A] time = 0.39, size = 17, normalized size = 1.13

$$\frac{adx+b\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + b*sinh(d*x + c))/d
```

giac [B] time = 0.11, size = 32, normalized size = 2.13

$$ax + \frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

maple [A] time = 0.03, size = 16, normalized size = 1.07

$$ax + \frac{b\sinh\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cosh(d*x+c),x)
```

```
[Out] a*x+b*sinh(d*x+c)/d
```

maxima [A] time = 0.29, size = 15, normalized size = 1.00

$$ax + \frac{b\sinh\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*sinh(d*x + c)/d
```

mupad [B] time = 0.06, size = 15, normalized size = 1.00

$$a\,x + \frac{b\sinh\left(c + d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cosh(c + d*x),x)
```

```
[Out] a*x + (b*sinh(c + d*x))/d
```

sympy [A] time = 0.13, size = 17, normalized size = 1.13

$$ax + b\left(\begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d \neq 0 \\ x\cosh(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cosh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True))
```

3.67 $\int \frac{1}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] 2*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2659, 205}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d} = \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.98

$$-\frac{2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2]
*d)
```

fricas [A] time = 0.58, size = 237, normalized size = 4.84

$$\left[\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2-b^2+2\,\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)-2\,\sqrt{a^2-b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)+b}\right)}{\sqrt{a^2-b^2}\,d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] [log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a
^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b
*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b))/(sqrt(
a^2 - b^2)*d), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c
) + b*sinh(d*x + c) + a)/(a^2 - b^2))/((a^2 - b^2)*d)]
```

giac [A] time = 0.14, size = 39, normalized size = 0.80

$$\frac{2\,\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*d)
```

maple [A] time = 0.06, size = 44, normalized size = 0.90

$$\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c)),x)
```

```
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.22, size = 53, normalized size = 1.08

$$\frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{\sqrt{b^2\,d^2-a^2\,d^2}}\right)}{\sqrt{b^2\,d^2-a^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x)),x)
```

```
[Out] (2*atan((a*d + b*d*exp(d*x)*exp(c))/(b^2*d^2 - a^2*d^2)^(1/2)))/(b^2*d^2 -
a^2*d^2)^(1/2)
```

sympy [A] time = 4.62, size = 163, normalized size = 3.33

$$\begin{cases} \frac{\tilde{\infty}x}{\cosh(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for } a = b \\ -\frac{1}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } a = -b \\ \frac{x}{a+b\cosh(c)} & \text{for } d = 0 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (tanh(c/2 + d*x/
2)/(b*d), Eq(a, b)), (-1/(b*d*tanh(c/2 + d*x/2)), Eq(a, -b)), (x/(a + b*cos
h(c)), Eq(d, 0)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(
a*d*sqrt(a/(a - b) + b/(a - b)) - b*d*sqrt(a/(a - b) + b/(a - b))) + log(sq
rt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(a*d*sqrt(a/(a - b) + b/(a -
 b)) - b*d*sqrt(a/(a - b) + b/(a - b))), True))
```

3.68 $\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

```
[Out] 2*a*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)/d-b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 12, 2659, 205}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-2),x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) - (b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx &= -\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}-\frac{\int \frac{a}{a+b\cosh(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}+\frac{a\int \frac{1}{a+b\cosh(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}-\frac{(2ia)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+id\right.\right.}{\left(a^2-b^2\right)d}\\
&= \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\sinh(c+dx)}{\left(a^2-b^2\right)d(a+b\cosh(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 84, normalized size = 0.98

$$
\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{b\sinh(c+dx)}{(a-b)(a+b)(a+b\cosh(c+dx))}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-2),x]
```

```
[Out] ((2*a*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3
/2) - (b*Sinh[c + d*x])/((a - b)*(a + b)*(a + b*Cosh[c + d*x])))/d
```

fricas [B] time = 0.73, size = 743, normalized size = 8.64

$$
\left[\frac{2\,a^2b-2\,b^3-\left(ab\cosh\left(dx+c\right)^2+ab\sinh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)+ab+2\left(ab\cosh\left(dx+c\right)+a^2\right)\sinh}{\left(a^4b-2\,a^2b^3+b^5\right)d\cosh\left(dx+c\right)^2+\left(a^4b-2\,a^2b^3+b^5\right)d\sinh\left(dx+\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b - 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cosh
(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 - b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 - b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(
a^3 - a*b^2)*cosh(d*x + c) + 2*(a^3 - a*b^2)*sinh(d*x + c))/((a^4*b - 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2
 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*
d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4
)*d)*sinh(d*x + c)), 2*(a^2*b - b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x +
 c)^2 + 2*a^2*cosh(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x +
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x
 + c) + a)/(a^2 - b^2)) + (a^3 - a*b^2)*cosh(d*x + c) + (a^3 - a*b^2)*sinh(
d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3
 + b^5)*d*sinh(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (
a^4*b - 2*a^2*b^3 + b^5)*d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) +
 (a^5 - 2*a^3*b^2 + a*b^4)*d)*sinh(d*x + c))]
```

giac [A] time = 0.14, size = 99, normalized size = 1.15

$$\frac{2\left(\frac{a\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{ae^{(dx+c)}+b}{(a^2-b^2)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 +
b^2)) + (a*e^(d*x + c) + b)/((a^2 - b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x +
c) + b)))/d
```

maple [A] time = 0.08, size = 118, normalized size = 1.37

$$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}+\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^2,x)
```

```
[Out] 1/d*(2*b/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*
x+1/2*c)^2*b-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.30, size = 215, normalized size = 2.50

$$\frac{\frac{2\,b^2}{d\,\left(a^2\,b-b^3\right)}+\frac{2\,a\,b\,{\mathrm{e}}^{c+d\,x}}{d\,\left(a^2\,b-b^3\right)}}{b+2\,a\,{\mathrm{e}}^{c+d\,x}+b\,{\mathrm{e}}^{2\,c+2\,d\,x}}+\frac{a\,\ln\left(-\frac{2\,a\,{\mathrm{e}}^{c+d\,x}}{b\,\left(a^2-b^2\right)}-\frac{2\,a\,\left(b+a\,{\mathrm{e}}^{c+d\,x}\right)}{b\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{a\,\ln\left(\frac{2\,a\,\left(b+a\,{\mathrm{e}}^{c+d\,x}\right)}{b\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,a\,{\mathrm{e}}^{c+d\,x}}{b\,\left(a^2-b^2\right)}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x))^2,x)
```

```
[Out] ((2*b^2)/(d*(a^2*b - b^3)) + (2*a*b*exp(c + d*x))/(d*(a^2*b - b^3)))/(b + 2
*a*exp(c + d*x) + b*exp(2*c + 2*d*x)) + (a*log(- (2*a*exp(c + d*x))/(b*(a^2
 - b^2)) - (2*a*(b + a*exp(c + d*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2))))/(d*
(a + b)^(3/2)*(a - b)^(3/2)) - (a*log((2*a*(b + a*exp(c + d*x)))/(b*(a + b)
^(3/2)*(a - b)^(3/2)) - (2*a*exp(c + d*x))/(b*(a^2 - b^2))))/(d*(a + b)^(3/
2)*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \cosh{\left(c + d x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*cosh(c + d*x))**(-2), x)
```

3.69 $\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sinh(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cosh(c+dx))}-\frac{b\sinh(c+dx)}{2d\left(a^2-b^2\right)(a+b\cosh(c+dx))^2}$$

```
[Out] (2*a^2+b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2
)/(a+b)^(5/2)/d-1/2*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^2-3/2*a*b*s
inh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sinh(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cosh(c+dx))}-\frac{b\sinh(c+dx)}{2d\left(a^2-b^2\right)(a+b\cosh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((a -
b)^(5/2)*(a + b)^(5/2)*d) - (b*Sinh[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cosh[
c + d*x])^2) - (3*a*b*Sinh[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cosh[c + d*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{\int \frac{-2a+b\cosh(c+dx)}{(a+b\cosh(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} + \frac{\int \frac{\dots}{a+b\dots}}{2(\dots}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} + \frac{\left(2a^2 \dots\right.}{\dots}$$

$$= -\frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))} - \frac{\left(i\left(2a^2 \dots\right.\right.}{\dots}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{2\left(a^2-b^2\right)d(a+b\cosh(c+dx))^2} - \frac{\dots}{2(a\dots}$$

Mathematica [A] time = 0.41, size = 113, normalized size = 0.85

$$\frac{\frac{b\sinh(c+dx)\left(-4a^2-3ab\cosh(c+dx)+b^2\right)}{(a-b)^2(a+b)^2(a+b\cosh(c+dx))^2}-\frac{2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cosh[c + d*x])*Sinh[c + d*x])/(
(a - b)^2*(a + b)^2*(a + b*Cosh[c + d*x])^2))/(2*d)
```

fricas [B] time = 0.55, size = 2591, normalized size = 19.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + 2*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2
*(2*a^4*b - a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 - a^3*b^2 - a*b^4)*co
sh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh
(d*x + c))*sinh(d*x + c)^2 + ((2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^2*b^
2 + b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(d*x +
 c)^3 + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)
^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*b^2 + b
^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^2 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c) + 4*(2*a^3*b + a*b^3
 + (2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)^2
+ (4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*l
og((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*c
osh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(10*
a^4*b - 11*a^2*b^3 + b^5)*cosh(d*x + c) + 2*(10*a^4*b - 11*a^2*b^3 + b^5 +
3*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4)*c
osh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos
h(d*x + c)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*sinh(d*x + c)^4 +
4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 - 5*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 - 3*a^
4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d)*sinh(d*x + c)^3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*co
```

```
sh(d*x + c) + 2*(3*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^
2 + 6*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + (2*a^8 - 5*
a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^4*
b^4 + 3*a^2*b^6 - b^8)*d + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos
h(d*x + c)^3 + 3*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^2
+ (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)), (3*a^3*b^2 - 3*a*b^4 +
(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 + (2*a^4*b - a^2*b^3 - b^5)*sinh(
d*x + c)^3 + 3*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c)^2 + 3*(2*a^5 - a^3*b
^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 - ((2
*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + 2*a^2
*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 4*(2*a^3*b + a*b^3 + (2*
a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)
*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*b^2 + b^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*
x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(2*a^3*b
+ a*b^3)*cosh(d*x + c) + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x +
c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)^2 + (4*a^4 + 4*a^2*b^2 + b^4)*cosh
(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh
(d*x + c) + b*sinh(d*x + c) + a)/(a^2 - b^2)) + (10*a^4*b - 11*a^2*b^3 + b^
5)*cosh(d*x + c) + (10*a^4*b - 11*a^2*b^3 + b^5 + 3*(2*a^4*b - a^2*b^3 - b^
5)*cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x +
c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 -
 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*sinh(d*x + c)^4 + 4*(a^7*b - 3*a^5*b^3 + 3*
a^3*b^5 - a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d)*sinh(d*x + c)^
3 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + 2*(3*(a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 6*(a^7*b - 3*a^5*b^3
+ 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c) + (2*a^8 - 5*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
+ 4*((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cosh(d*x + c)^3 + 3*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x + c)^2 + (2*a^8 - 5*a^6*b^2 + 3*
a^4*b^4 + a^2*b^6 - b^8)*d*cosh(d*x + c) + (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d)*sinh(d*x + c))]
```

giac [A] time = 0.15, size = 195, normalized size = 1.47

$$\frac{\frac{\left(2\,a^{2}+b^{2}\right)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sqrt{-a^{2}+b^{2}}}+\frac{2\,a^{2}be^{(3\,dx+3\,c)}+b^{3}e^{(3\,dx+3\,c)}+6\,a^{3}e^{(2\,dx+2\,c)}+3\,ab^{2}e^{(2\,dx+2\,c)}+10\,a^{2}be^{(dx+c)}-b^{3}e^{(dx+c)}+3\,ab^{2}}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^2 + b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b
^2 + b^4)*sqrt(-a^2 + b^2)) + (2*a^2*b*e^(3*d*x + 3*c) + b^3*e^(3*d*x + 3*c
) + 6*a^3*e^(2*d*x + 2*c) + 3*a*b^2*e^(2*d*x + 2*c) + 10*a^2*b*e^(d*x + c)
- b^3*e^(d*x + c) + 3*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c) +
2*a*e^(d*x + c) + b)^2))/d
```

maple [A] time = 0.09, size = 186, normalized size = 1.40

$$\frac{-\frac{2\left(-\frac{(4a+b)b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{(4a-b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/2*(4*
a-b)*b/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2*a-
tanh(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*cosh(c + d*x))^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.70 $\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cosh(c+dx))}-\frac{5ab\sinh(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cosh(c+dx))^2}\cdot$$

```
[Out] a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/(a+b)^(7/2)/d-1/3*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^3-5/6*a
*b*sinh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*sinh(
d*x+c)/(a^2-b^2)^3/d/(a+b*cosh(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cosh(c+dx))}-\frac{5ab\sinh(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cosh(c+dx))^2}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(7/2)*(a + b)^(7/2)*d) - (b*Sinh[c + d*x])/(3*(a^2 - b^2)*d*(a + b*C
osh[c + d*x])^3) - (5*a*b*Sinh[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cosh[c +
 d*x])^2) - (b*(11*a^2 + 4*b^2)*Sinh[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Co
sh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{\int \frac{-3a+2b\cosh(c+dx)}{(a+b\cosh(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} + \frac{\int \frac{2(3a^2}{}}{}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} - \frac{b(1}{6(a^2-}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} - \frac{b(1}{6(a^2-}$$

$$= -\frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cosh(c+dx))^2} - \frac{b(1}{6(a^2-}$$

$$= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\sinh(c+dx)}{3\left(a^2-b^2\right)d(a+b\cosh(c+dx))^3} - \frac{}{6(}$$

Mathematica [A] time = 1.10, size = 160, normalized size = 0.87

$$\frac{\frac{6a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \frac{b\sinh(c+dx)\left(36a^4+6ab\left(9a^2+b^2\right)\cosh(c+dx)+a^2b^2+\left(11a^2b^2+4b^4\right)\cosh(2(c+dx))+8b^4\right)}{2(a-b)^3(a+b)^3(a+b\cosh(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(7/2) - (b*(36*a^4 + a^2*b^2 + 8*b^4 + 6*a*b*(9*a^2 + b^2)*Co
sh[c + d*x] + (11*a^2*b^2 + 4*b^4)*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(2*(a
- b)^3*(a + b)^3*(a + b*Cosh[c + d*x])^3))/(6*d)
```

fricas [B] time = 0.68, size = 5705, normalized size = 31.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/6*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 + 6*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*c
osh(d*x + c)^5 + 6*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 + 30*(2*
a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 + 30*(2*a^6*b + a^4*b^3 - 3*a^
2*b^5 + (2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*
(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c)^3 + 4*(22*a^7 +
 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6 + 15*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*co
sh(d*x + c)^2 + 30*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 12*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(d*x + c)^2 + 1
2*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a
*b^6)*cosh(d*x + c)^3 + 15*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2
+ (22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c
)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 + 3*a*b^5)*sinh
(d*x + c)^6 + 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)
^5 + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d
*x + c)^5 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b
 + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a
^4*b^2 + 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 + 12*a^4*b^2
+ 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3
*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^
2 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 3*(8*a^5*b + 14*a^3*b^3 +
 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 20*(2*a^4*b^2 + 3*a^2*
b^4)*cosh(d*x + c)^3 + 6*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 +
 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a
^4*b^2 + 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 +
 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^4 + 2*(
8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 2*(4*a^6 + 12*a^4*b^2 + 9
*a^2*b^4)*cosh(d*x + c)^2 + (8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(a^2 - b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c
)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) + 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c)
+ a)*sinh(d*x + c) + b)) + 30*(4*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cosh(d*x + c)
 + 6*(20*a^5*b^2 - 15*a^3*b^4 - 5*a*b^6 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)
*cosh(d*x + c)^4 + 20*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(
22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c)^2 + 4*(17*a^6*b
- 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(d*x + c))*sinh(d*x + c))/((a^8*b^3 -
 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^6 + (a^8*b^3 - 4
*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 - 4
*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b
- 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^4 + 6*((a^8*
```

b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c) + (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^2 + 10*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^4 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^3 + 15*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^2 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c) + (2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d)*sinh(d*x + c)^3 + 6*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c) + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^2 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d + 6*((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^5 + 5*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^4 + 2*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c)^2 + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c) + (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)), 1/3*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 3*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^5 + 3*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 + 15*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 + 15*(2*a^6*b + a^4*b^3 - 3*a^2*b^5 + (2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c)^3 + 2*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6 + 15*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^2 + 30*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(d*x + c)^2 + 6*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^3 + 15*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2 + (22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 + 3*a*b^5)*sinh(d*x + c)^6 + 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^5 + 6*(2*a^4*b^2 + 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^2 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 +

```
3*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4
 + 20*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c)^3 + 6*(8*a^5*b + 14*a^3*b^3 + 3
*a*b^5)*cosh(d*x + c)^2 + 4*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))
*sinh(d*x + c)^2 + 6*(2*a^4*b^2 + 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 +
 3*a^2*b^4 + (2*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 + 3*a^2*b
^4)*cosh(d*x + c)^4 + 2*(8*a^5*b + 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 +
2*(4*a^6 + 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c)^2 + (8*a^5*b + 14*a^3*b^3
+ 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a)/(a^2 - b^2)) + 15*(4*a^5*b^
2 - 3*a^3*b^4 - a*b^6)*cosh(d*x + c) + 3*(20*a^5*b^2 - 15*a^3*b^4 - 5*a*b^6
 + 5*(2*a^5*b^2 + a^3*b^4 - 3*a*b^6)*cosh(d*x + c)^4 + 20*(2*a^6*b + a^4*b^
3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(22*a^7 + 19*a^5*b^2 - 29*a^3*b^4 - 12*a
*b^6)*cosh(d*x + c)^2 + 4*(17*a^6*b - 11*a^4*b^3 - 4*a^2*b^5 - 2*b^7)*cosh(
d*x + c))*sinh(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cosh(d*x + c)^6 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11
)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^
10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7
+ b^11)*d*cosh(d*x + c)^4 + 6*((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9
 + b^11)*d*cosh(d*x + c) + (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a
*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^11 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8
 + 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*
a^2*b^9 + b^11)*d*cosh(d*x + c)^2 + 10*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4
*a^3*b^8 + a*b^10)*d*cosh(d*x + c) + (4*a^10*b - 15*a^8*b^3 + 20*a^6*b^5 -
10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^4 + 3*(4*a^10*b - 15*a^8*b^3 + 20*a^6*b
^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^3 + 15*(a^9*b^2 - 4*a^7*b^4 + 6*
a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^2 + 3*(4*a^10*b - 15*a^8*b^3
+ 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c) + (2*a^11 - 5*a^9*b^2 + 1
0*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d)*sinh(d*x + c)^3 + 6*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 - 4*a
^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 - 4*
a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b -
 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11
 - 5*a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c) + (4*a^1
0*b - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d)*sinh(d*x + c)^2 + (a^
8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d + 6*((a^8*b^3 - 4*a^6*b
^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^5 + 5*(a^9*b^2 - 4*a^7*b
^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^4 + 2*(4*a^10*b - 15*a
^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 - 5*
a^9*b^2 + 10*a^5*b^6 - 10*a^3*b^8 + 3*a*b^10)*d*cosh(d*x + c)^2 + (4*a^10*b
 - 15*a^8*b^3 + 20*a^6*b^5 - 10*a^4*b^7 + b^11)*d*cosh(d*x + c) + (a^9*b^2
- 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c))]
```

giac [A] time = 0.14, size = 329, normalized size = 1.79

$$\frac{\frac{3\left(2\,a^{3}+3\,ab^{2}\right)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{-a^{2}+b^{2}}}+\frac{6\,a^{3}b^{2}e^{(5\,dx+5\,c)}+9\,ab^{4}e^{(5\,dx+5\,c)}+30\,a^{4}be^{(4\,dx+4\,c)}+45\,a^{2}b^{3}e^{(4\,dx+4\,c)}+44\,a^{5}e^{(3\,dx+3\,c)}+82\,a^{3}b^{2}e^{(3\,dx+3\,c)}+24\,ab^{4}e^{(3\,d}}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(be^{(2\,dx}\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^3 + 3*a*b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^3*b^2*e^(5*d*x + 5
*c) + 9*a*b^4*e^(5*d*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) + 45*a^2*b^3*e^(4*
d*x + 4*c) + 44*a^5*e^(3*d*x + 3*c) + 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a*b^4
*e^(3*d*x + 3*c) + 102*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2*c) +
 12*b^5*e^(2*d*x + 2*c) + 60*a^3*b^2*e^(d*x + c) + 15*a*b^4*e^(d*x + c) + 1
1*a^2*b^3 + 4*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*e^(2*d*x + 2*c)
+ 2*a*e^(d*x + c) + b)^3))/d
```

maple [A] time = 0.09, size = 284, normalized size = 1.54

$$\frac{-\frac{2\left(-\frac{\left(6a^{2}+3ab+2b^{2}\right)b\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)}+\frac{2\left(9a^{2}+b^{2}\right)b\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^{2}+2ab+b^{2}\right)\left(a^{2}-2ab+b^{2}\right)}-\frac{\left(6a^{2}-3ab+2b^{2}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)}\right)}{\left(\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^{3}}+\frac{a\left(2a^{2}+3b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)\sqrt{(a+b)(a-b)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/
2*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(
1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b-a-b)^3+a*(
2*a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*cosh(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.71 $\int \frac{1}{3+5\cosh(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

```
[Out] 1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2659, 206}

$$\frac{\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-1),x]
```

```
[Out] ArcTan[Tanh[(c + d*x)/2]/2]/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+5\cosh(c+dx)}\,dx &= -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.05

$$-\frac{\tan^{-1}\left(2\coth\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-1),x]
[Out] -1/2*ArcTan[2*Coth[c/2 + (d*x)/2]]/d
```

fricas [A] time = 0.53, size = 24, normalized size = 1.09

$$\frac{\arctan\left(\frac{5}{4}\,\cosh\left(dx+c\right)+\frac{5}{4}\,\sinh\left(dx+c\right)+\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="fricas")
[Out] 1/2*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)/d
```

giac [A] time = 0.14, size = 16, normalized size = 0.73

$$\frac{\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="giac")
[Out] 1/2*arctan(5/4*e^(d*x + c) + 3/4)/d
```

maple [A] time = 0.06, size = 18, normalized size = 0.82

$$\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c)),x)
[Out] 1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

maxima [A] time = 0.41, size = 19, normalized size = 0.86

$$-\frac{\arctan\left(\frac{5}{4}e^{(-dx-c)} + \frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(5/4*e^(-d*x - c) + 3/4)/d
```

mupad [B] time = 0.13, size = 34, normalized size = 1.55

$$\frac{\operatorname{atan}\left(\frac{3\,\sqrt{d^2}+5\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{4\,d}\right)}{2\,\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3),x)
```

```
[Out] atan((3*(d^2)^(1/2) + 5*exp(d*x)*exp(c)*(d^2)^(1/2))/(4*d))/(2*(d^2)^(1/2))
```

sympy [A] time = 0.78, size = 24, normalized size = 1.09

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{2d} & \text{for } d \neq 0 \\ \frac{x}{5\cosh(c)+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c)),x)
```

```
[Out] Piecewise((atan(tanh(c/2 + d*x/2)/2)/(2*d), Ne(d, 0)), (x/(5*cosh(c) + 3),
True))
```

3.72 $\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

```
[Out] -3/32*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/16*sinh(d*x+c)/d/(3+5*cosh(d*x+c)
)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 12, 2659, 206}

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-2),x]
```

```
[Out] (-3*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d) + (5*Sinh[c + d*x])/(16*d*(3 + 5*Co
sh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}+\frac{1}{16}\int -\frac{3}{3+5\cosh(c+dx)}\,dx\\ &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}-\frac{3}{16}\int \frac{1}{3+5\cosh(c+dx)}\,dx\\ &= \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}+\frac{(3i)\,\text{Subst}\left(\int \frac{1}{8-2x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{8d}\\ &= -\frac{3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}+\frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.11, size = 45, normalized size = 0.94

$$\frac{\frac{10\sinh(c+dx)}{5\cosh(c+dx)+3}-3\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-2),x]
```

```
[Out] (-3*ArcTan[Tanh[(c + d*x)/2]/2] + (10*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x]))
/(32*d)
```

fricas [B] time = 0.60, size = 147, normalized size = 3.06

$$-\frac{3\left(5\cosh(dx+c)^2+2\,(5\cosh(dx+c)+3)\sinh(dx+c)+5\sinh(dx+c)^2+6\cosh(dx+c)+5\right)\arctan\left(\frac{5}{4}\right.}{32\left(5\,d\cosh(dx+c)^2+5\,d\sinh(dx+c)^2+6\,d\cosh(dx+c)+2\,(5\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/32*(3*(5*cosh(d*x + c)^2 + 2*(5*cosh(d*x + c) + 3)*sinh(d*x + c) + 5*sin
h(d*x + c)^2 + 6*cosh(d*x + c) + 5)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x
```

```
 + c) + 3/4) + 12*cosh(d*x + c) + 12*sinh(d*x + c) + 20)/(5*d*cosh(d*x + c)
^2 + 5*d*sinh(d*x + c)^2 + 6*d*cosh(d*x + c) + 2*(5*d*cosh(d*x + c) + 3*d)*
sinh(d*x + c) + 5*d)
```

giac [A] time = 0.12, size = 54, normalized size = 1.12

$$-\frac{\frac{4\left(3 e^{(dx+c)}+5\right)}{5 e^{(2 dx+2 c)}+6 e^{(dx+c)}+5}+3 \arctan\left(\frac{5}{4} e^{(dx+c)}+\frac{3}{4}\right)}{32 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*(3*e^(d*x + c) + 5)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5) + 3*ar
ctan(5/4*e^(d*x + c) + 3/4))/d
```

maple [A] time = 0.07, size = 48, normalized size = 1.00

$$\frac{5 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}-\frac{3 \arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^2,x)
```

```
[Out] 5/16/d*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+4)-3/32*arctan(1/2*tanh(1
/2*d*x+1/2*c))/d
```

maxima [A] time = 0.40, size = 64, normalized size = 1.33

$$\frac{3 \arctan\left(\frac{5}{4} e^{(-dx-c)}+\frac{3}{4}\right)}{32 d}+\frac{3 e^{(-dx-c)}+5}{8 d\left(6 e^{(-dx-c)}+5 e^{(-2 dx-2 c)}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/32*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/8*(3*e^(-d*x - c) + 5)/(d*(6*e^(-
d*x - c) + 5*e^(-2*d*x - 2*c) + 5))
```

mupad [B] time = 0.94, size = 74, normalized size = 1.54

$$-\frac{\frac{3 e^{c+dx}}{8 d}+\frac{5}{8 d}}{6 e^{c+dx}+5 e^{2 c+2 dx}+5}-\frac{3 \operatorname{atan}\left(\left(\frac{3}{4 d}+\frac{5 e^{dx} e^{c}}{4 d}\right) \sqrt{d^2}\right)}{32 \sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3)^2,x)
```

```
[Out] - ((3*exp(c + d*x))/(8*d) + 5/(8*d))/(6*exp(c + d*x) + 5*exp(2*c + 2*d*x) +
 5) - (3*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(32*(d^2)
^(1/2))
```

sympy [A] time = 2.52, size = 316, normalized size = 6.58

$$\begin{cases} \frac{x}{\left(5\cosh\left(\log\left(-\frac{3}{5}-\frac{4i}{5}\right)\right)+3\right)^2} \\ -\frac{\log\left(-3e^{-dx}-4ie^{-dx}\right)}{25d\cosh^2\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+30d\cosh\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+9d}+\frac{\log(}{25d\cosh^2\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+30} \\ \frac{x}{(5\cosh(c)+3)^2} \\ -\frac{3\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}+\frac{10\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}-\frac{12\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((x/(5*cosh(log(-3/5 - 4*I/5)) + 3)**2, Eq(d, 0) & Eq(c, log(-3/5
- 4*I/5))), (-log(-3*exp(-d*x) - 4*I*exp(-d*x))/(25*d*cosh(d*x + log(-3*exp
(-d*x) - 4*I*exp(-d*x)) - log(5))**2 + 30*d*cosh(d*x + log(-3*exp(-d*x) - 4
*I*exp(-d*x)) - log(5)) + 9*d) + log(5)/(25*d*cosh(d*x + log(-3*exp(-d*x) -
 4*I*exp(-d*x)) - log(5))**2 + 30*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-
d*x)) - log(5)) + 9*d), Eq(c, log((-3 - 4*I)*exp(-d*x)/5))), (x/(5*cosh(c)
+ 3)**2, Eq(d, 0)), (-3*tanh(c/2 + d*x/2)**2*atan(tanh(c/2 + d*x/2)/2)/(32*
d*tanh(c/2 + d*x/2)**2 + 128*d) + 10*tanh(c/2 + d*x/2)/(32*d*tanh(c/2 + d*x
/2)**2 + 128*d) - 12*atan(tanh(c/2 + d*x/2)/2)/(32*d*tanh(c/2 + d*x/2)**2 +
 128*d), True))
```

3.73 $\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$\frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}-\frac{45\sinh(c+dx)}{512d(5\cosh(c+dx)+3)}+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

```
[Out] 43/1024*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/32*sinh(d*x+c)/d/(3+5*cosh(d*x+
c))^2-45/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 206}

$$\frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}-\frac{45\sinh(c+dx)}{512d(5\cosh(c+dx)+3)}+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-3),x]
```

```
[Out] (43*ArcTan[Tanh[(c + d*x)/2]/2])/(1024*d) + (5*Sinh[c + d*x])/(32*d*(3 + 5*
Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}+\frac{1}{32}\int \frac{-6+5\cosh(c+dx)}{(3+5\cosh(c+dx))^2}\,dx\\ &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}+\frac{1}{512}\int \frac{43}{3+5\cosh(c+dx}\\ &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}+\frac{43}{512}\int \frac{1}{3+5\cosh(c+dx}\\ &= \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}-\frac{(43i)\,\mathrm{Subst}\left(\int \frac{1}{8-2x^2}\,dx,x\right.}{256d}\\ &= \frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d}+\frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2}-\frac{45\sinh(c+dx)}{512d(3+5\cosh(c+}\end{aligned}$$

Mathematica [A] time = 0.16, size = 55, normalized size = 0.75

$$\frac{43\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)-\frac{10\sinh(c+dx)(45\cosh(c+dx)+11)}{(5\cosh(c+dx)+3)^2}}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-3),x]
```

```
[Out] (43*ArcTan[Tanh[(c + d*x)/2]/2] - (10*(11 + 45*Cosh[c + d*x])*Sinh[c + d*x]
)/(3 + 5*Cosh[c + d*x])^2)/(1024*d)
```

fricas [B] time = 0.46, size = 408, normalized size = 5.59

$$\frac{860 \cosh(dx+c)^3 + 516(5 \cosh(dx+c)+3)\sinh(dx+c)^2 + 860 \sinh(dx+c)^3 + 43\left(25 \cosh(dx+c)^4 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(860*cosh(d*x + c)^3 + 516*(5*cosh(d*x + c) + 3)*sinh(d*x + c)^2 + 8
60*sinh(d*x + c)^3 + 43*(25*cosh(d*x + c)^4 + 20*(5*cosh(d*x + c) + 3)*sinh
(d*x + c)^3 + 25*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(75*cosh(d*x + c)
^2 + 90*cosh(d*x + c) + 43)*sinh(d*x + c)^2 + 86*cosh(d*x + c)^2 + 4*(25*co
sh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 43*cosh(d*x + c) + 15)*sinh(d*x + c) +
 60*cosh(d*x + c) + 25)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)
 + 1548*cosh(d*x + c)^2 + 4*(645*cosh(d*x + c)^2 + 774*cosh(d*x + c) + 325)
*sinh(d*x + c) + 1300*cosh(d*x + c) + 900)/(25*d*cosh(d*x + c)^4 + 25*d*sin
h(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 20*(5*d*cosh(d*x + c) + 3*d)*sinh(d*x
 + c)^3 + 86*d*cosh(d*x + c)^2 + 2*(75*d*cosh(d*x + c)^2 + 90*d*cosh(d*x +
c) + 43*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x + c) + 4*(25*d*cosh(d*x + c)^3 +
 45*d*cosh(d*x + c)^2 + 43*d*cosh(d*x + c) + 15*d)*sinh(d*x + c) + 25*d)
```

giac [A] time = 0.12, size = 76, normalized size = 1.04

$$\frac{\frac{4\left(215\, e^{(3\,dx+3\,c)}+387\, e^{(2\,dx+2\,c)}+325\, e^{(dx+c)}+225\right)}{\left(5\, e^{(2\,dx+2\,c)}+6\, e^{(dx+c)}+5\right)^2} + 43\,\arctan\left(\frac{5}{4}\, e^{(dx+c)} + \frac{3}{4}\right)}{1024\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1024*(4*(215*e^(3*d*x + 3*c) + 387*e^(2*d*x + 2*c) + 325*e^(d*x + c) + 22
5)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5)^2 + 43*arctan(5/4*e^(d*x + c) +
3/4))/d
```

maple [A] time = 0.07, size = 79, normalized size = 1.08

$$-\frac{85\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512d\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^2} - \frac{35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{128d\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^2} + \frac{43\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^3,x)
```

```
[Out] -85/512/d/(tanh(1/2*d*x+1/2*c)^2+4)^2*tanh(1/2*d*x+1/2*c)^3-35/128/d/(tanh(
1/2*d*x+1/2*c)^2+4)^2*tanh(1/2*d*x+1/2*c)+43/1024*arctan(1/2*tanh(1/2*d*x+1
/2*c))/d
```

maxima [A] time = 0.45, size = 108, normalized size = 1.48

$$-\frac{43 \arctan\left(\frac{5}{4} e^{(-dx-c)} + \frac{3}{4}\right)}{1024 d} - \frac{325 e^{(-dx-c)} + 387 e^{(-2 dx-2 c)} + 215 e^{(-3 dx-3 c)} + 225}{256 d\left(60 e^{(-dx-c)} + 86 e^{(-2 dx-2 c)} + 60 e^{(-3 dx-3 c)} + 25 e^{(-4 dx-4 c)} + 25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -43/1024*arctan(5/4*e^(-d*x - c) + 3/4)/d - 1/256*(325*e^(-d*x - c) + 387*e
^(-2*d*x - 2*c) + 215*e^(-3*d*x - 3*c) + 225)/(d*(60*e^(-d*x - c) + 86*e^(-
2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 25*e^(-4*d*x - 4*c) + 25))
```

mupad [B] time = 0.96, size = 137, normalized size = 1.88

$$\frac{\frac{43 e^{c+dx}}{256 d} + \frac{129}{1280 d}}{6 e^{c+dx} + 5 e^{2c+2dx} + 5} - \frac{\frac{7 e^{c+dx}}{40 d} - \frac{3}{8 d}}{60 e^{c+dx} + 86 e^{2c+2dx} + 60 e^{3c+3dx} + 25 e^{4c+4dx} + 25} + \frac{43 \operatorname{atan}\left(\left(\frac{3}{4 d} + \frac{5 e^{dx} e^{c}}{4 d}\right)\sqrt{d^2}\right)}{1024 \sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3)^3,x)
```

```
[Out] ((43*exp(c + d*x))/(256*d) + 129/(1280*d))/(6*exp(c + d*x) + 5*exp(2*c + 2*
d*x) + 5) - ((7*exp(c + d*x))/(40*d) - 3/(8*d))/(60*exp(c + d*x) + 86*exp(2
*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*c + 4*d*x) + 25) + (43*atan((3
/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(1024*(d^2)^(1/2))
```

sympy [A] time = 6.59, size = 530, normalized size = 7.26

$$\begin{cases} -\frac{\log\left(-3e^{-dx}+4ie^{-dx}\right)}{125d\cosh^3\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+225d\cosh^2\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+135d\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+27d} \\ -\frac{\log\left(-3e^{-dx}-4ie^{-dx}\right)}{125d\cosh^3\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+225d\cosh^2\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+135d\cosh\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+27d} \\ \frac{x}{(5\cosh(c)+3)^3} \\ \frac{43\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} - \frac{170\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+16384d} + \frac{344\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh}{}\right.}{1024d\tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)+8192d\tanh^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((-log(-3*exp(-d*x) + 4*I*exp(-d*x))/(125*d*cosh(d*x + log(-3*exp(
-d*x) + 4*I*exp(-d*x)) - log(5))**3 + 225*d*cosh(d*x + log(-3*exp(-d*x) + 4
*I*exp(-d*x)) - log(5))**2 + 135*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d
*x)) - log(5)) + 27*d), Eq(c, log((-3 + 4*I)*exp(-d*x)) - log(5))), (-log(-
3*exp(-d*x) - 4*I*exp(-d*x))/(125*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-
d*x)) - log(5))**3 + 225*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - l
og(5))**2 + 135*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5)) +
27*d), Eq(c, log(-(3 + 4*I)*exp(-d*x)) - log(5))), (x/(5*cosh(c) + 3)**3, E
q(d, 0)), (43*tanh(c/2 + d*x/2)**4*atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c
/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) - 170*tanh(c/2 + d*
x/2)**3/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*
d) + 344*tanh(c/2 + d*x/2)**2*atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c/2 +
d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) - 280*tanh(c/2 + d*x/2)/
(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) + 688
*atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 +
 d*x/2)**2 + 16384*d), True))
```

3.74 $\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{279\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{995\sinh(c+dx)}{24576d(5\cosh(c+dx)+3)}-\frac{25\sinh(c+dx)}{512d(5\cosh(c+dx)+3)^2}+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)}$$

```
[Out] -279/16384*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/48*sinh(d*x+c)/d/(3+5*cosh(d
*x+c))^3-25/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))^2+995/24576*sinh(d*x+c)/d/(
3+5*cosh(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 206}

$$-\frac{279\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d}+\frac{995\sinh(c+dx)}{24576d(5\cosh(c+dx)+3)}-\frac{25\sinh(c+dx)}{512d(5\cosh(c+dx)+3)^2}+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Cosh[c + d*x])^(-4),x]
```

```
[Out] (-279*ArcTan[Tanh[(c + d*x)/2]/2])/(16384*d) + (5*Sinh[c + d*x])/(48*d*(3 +
 5*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*(3 + 5*Cosh[c + d*x])^2) +
 (995*Sinh[c + d*x])/(24576*d*(3 + 5*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} + \frac{1}{48}\int \frac{-9+10\cosh(c+dx)}{(3+5\cosh(c+dx))^3}\,dx$$

$$= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2} + \frac{\int \frac{154-75\cosh(c+dx)}{(3+5\cosh(c+dx))^2}\,dx}{1536}$$

$$= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2} + \frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+$$

$$= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2} + \frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+$$

$$= \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2} + \frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+$$

$$= -\frac{279\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d} + \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} - \frac{25\sinh(c+$$

Mathematica [A] time = 0.27, size = 65, normalized size = 0.66

$$\frac{\frac{5\sinh(c+dx)(9540\cosh(c+dx)+4975\cosh(2(c+dx))+8141)}{(5\cosh(c+dx)+3)^3} - 837\tan^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{49152d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Cosh[c + d*x])^(-4),x]
```

```
[Out] (-837*ArcTan[Tanh[(c + d*x)/2]/2] + (5*(8141 + 9540*Cosh[c + d*x] + 4975*Co
sh[2*(c + d*x)])*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x])^3)/(49152*d)
```

fricas [B] time = 0.67, size = 793, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/49152*(83700*cosh(d*x + c)^5 + 83700*(5*cosh(d*x + c) + 3)*sinh(d*x + c)
^4 + 83700*sinh(d*x + c)^5 + 251100*cosh(d*x + c)^4 + 2232*(375*cosh(d*x +
c)^2 + 450*cosh(d*x + c) + 199)*sinh(d*x + c)^3 + 444168*cosh(d*x + c)^3 +
24*(34875*cosh(d*x + c)^3 + 62775*cosh(d*x + c)^2 + 55521*cosh(d*x + c) + 1
9885)*sinh(d*x + c)^2 + 837*(125*cosh(d*x + c)^6 + 150*(5*cosh(d*x + c) + 3
)*sinh(d*x + c)^5 + 125*sinh(d*x + c)^6 + 450*cosh(d*x + c)^5 + 15*(125*cos
h(d*x + c)^2 + 150*cosh(d*x + c) + 61)*sinh(d*x + c)^4 + 915*cosh(d*x + c)^
4 + 4*(625*cosh(d*x + c)^3 + 1125*cosh(d*x + c)^2 + 915*cosh(d*x + c) + 279
)*sinh(d*x + c)^3 + 1116*cosh(d*x + c)^3 + 3*(625*cosh(d*x + c)^4 + 1500*co
sh(d*x + c)^3 + 1830*cosh(d*x + c)^2 + 1116*cosh(d*x + c) + 305)*sinh(d*x +
 c)^2 + 915*cosh(d*x + c)^2 + 6*(125*cosh(d*x + c)^5 + 375*cosh(d*x + c)^4
+ 610*cosh(d*x + c)^3 + 558*cosh(d*x + c)^2 + 305*cosh(d*x + c) + 75)*sinh(
d*x + c) + 450*cosh(d*x + c) + 125)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x
 + c) + 3/4) + 477240*cosh(d*x + c)^2 + 12*(34875*cosh(d*x + c)^4 + 83700*c
osh(d*x + c)^3 + 111042*cosh(d*x + c)^2 + 79540*cosh(d*x + c) + 22875)*sinh
(d*x + c) + 274500*cosh(d*x + c) + 99500)/(125*d*cosh(d*x + c)^6 + 125*d*si
nh(d*x + c)^6 + 450*d*cosh(d*x + c)^5 + 150*(5*d*cosh(d*x + c) + 3*d)*sinh(
d*x + c)^5 + 915*d*cosh(d*x + c)^4 + 15*(125*d*cosh(d*x + c)^2 + 150*d*cosh
(d*x + c) + 61*d)*sinh(d*x + c)^4 + 1116*d*cosh(d*x + c)^3 + 4*(625*d*cosh(
d*x + c)^3 + 1125*d*cosh(d*x + c)^2 + 915*d*cosh(d*x + c) + 279*d)*sinh(d*x
 + c)^3 + 915*d*cosh(d*x + c)^2 + 3*(625*d*cosh(d*x + c)^4 + 1500*d*cosh(d*
x + c)^3 + 1830*d*cosh(d*x + c)^2 + 1116*d*cosh(d*x + c) + 305*d)*sinh(d*x
+ c)^2 + 450*d*cosh(d*x + c) + 6*(125*d*cosh(d*x + c)^5 + 375*d*cosh(d*x +
c)^4 + 610*d*cosh(d*x + c)^3 + 558*d*cosh(d*x + c)^2 + 305*d*cosh(d*x + c)
+ 75*d)*sinh(d*x + c) + 125*d)
```

giac [A] time = 0.15, size = 98, normalized size = 1.00

$$-\frac{\frac{4\left(20925\,e^{(5\,dx+5\,c)}+62775\,e^{(4\,dx+4\,c)}+111042\,e^{(3\,dx+3\,c)}+119310\,e^{(2\,dx+2\,c)}+68625\,e^{(dx+c)}+24875\right)}{\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)^{3}}+837\,\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{49152\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/49152*(4*(20925*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) + 111042*e^(3*d*
x + 3*c) + 119310*e^(2*d*x + 2*c) + 68625*e^(d*x + c) + 24875)/(5*e^(2*d*x
+ 2*c) + 6*e^(d*x + c) + 5)^3 + 837*arctan(5/4*e^(d*x + c) + 3/4))/d
```

maple [A] time = 0.07, size = 110, normalized size = 1.12

$$\frac{745\left(\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8192d\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}+\frac{265\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}+\frac{295\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{512d\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)^{3}}-\frac{279\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*cosh(d*x+c))^4,x)
```

```
[Out] 745/8192/d/(tanh(1/2*d*x+1/2*c)^2+4)^3*tanh(1/2*d*x+1/2*c)^5+265/768/d/(tan
h(1/2*d*x+1/2*c)^2+4)^3*tanh(1/2*d*x+1/2*c)^3+295/512/d/(tanh(1/2*d*x+1/2*c
)^2+4)^3*tanh(1/2*d*x+1/2*c)-279/16384*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

maxima [A] time = 0.41, size = 152, normalized size = 1.55

$$\frac{279\ \arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{16384\,d}+\frac{68625\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}+111042\,e^{(-3\,dx-3\,c)}+62775\,e^{(-4\,dx-4\,c)}+20\cdots}{12288\,d\left(450\,e^{(-dx-c)}+915\,e^{(-2\,dx-2\,c)}+1116\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}+450\,e^{(-5\,d\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 279/16384*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/12288*(68625*e^(-d*x - c) +
119310*e^(-2*d*x - 2*c) + 111042*e^(-3*d*x - 3*c) + 62775*e^(-4*d*x - 4*c)
+ 20925*e^(-5*d*x - 5*c) + 24875)/(d*(450*e^(-d*x - c) + 915*e^(-2*d*x - 2*
c) + 1116*e^(-3*d*x - 3*c) + 915*e^(-4*d*x - 4*c) + 450*e^(-5*d*x - 5*c) +
125*e^(-6*d*x - 6*c) + 125))
```

mupad [B] time = 0.95, size = 223, normalized size = 2.28

$$\frac{\frac{39\,\mathrm{e}^{c+d\,x}}{50\,d}+\frac{7}{30\,d}}{450\,\mathrm{e}^{c+d\,x}+915\,\mathrm{e}^{2\,c+2\,d\,x}+1116\,\mathrm{e}^{3\,c+3\,d\,x}+915\,\mathrm{e}^{4\,c+4\,d\,x}+450\,\mathrm{e}^{5\,c+5\,d\,x}+125\,\mathrm{e}^{6\,c+6\,d\,x}+125}-\frac{\cdots}{60\,\mathrm{e}^{c+d\,x}+86\,\mathrm{e}^{2\,c+2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*cosh(c + d*x) + 3)^4,x)
```

```
[Out] ((39*exp(c + d*x))/(50*d) + 7/(30*d))/(450*exp(c + d*x) + 915*exp(2*c + 2*d
*x) + 1116*exp(3*c + 3*d*x) + 915*exp(4*c + 4*d*x) + 450*exp(5*c + 5*d*x) +
 125*exp(6*c + 6*d*x) + 125) - ((93*exp(c + d*x))/(640*d) + 791/(3200*d))/(
60*exp(c + d*x) + 86*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*c +
4*d*x) + 25) - (279*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2))
)/(16384*(d^2)^(1/2)) - ((279*exp(c + d*x))/(4096*d) + 837/(20480*d))/(6*ex
p(c + d*x) + 5*exp(2*c + 2*d*x) + 5)
```

sympy [A] time = 14.13, size = 809, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((-log(-3*exp(-d*x) + 4*I*exp(-d*x))/(625*d*cosh(d*x + log(-3*exp(
-d*x) + 4*I*exp(-d*x)) - log(5))**4 + 1500*d*cosh(d*x + log(-3*exp(-d*x) +
4*I*exp(-d*x)) - log(5))**3 + 1350*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(
-d*x)) - log(5))**2 + 540*d*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) -
log(5)) + 81*d), Eq(c, log((-3 + 4*I)*exp(-d*x)) - log(5))), (-log(-3*exp(-
d*x) - 4*I*exp(-d*x))/(625*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) -
 log(5))**4 + 1500*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))
**3 + 1350*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5))**2 + 54
0*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)) - log(5)) + 81*d), Eq(c, l
og(-(3 + 4*I)*exp(-d*x)) - log(5))), (x/(5*cosh(c) + 3)**4, Eq(d, 0)), (-83
7*tanh(c/2 + d*x/2)**6*atan(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)
**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145
728*d) + 4470*tanh(c/2 + d*x/2)**5/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d
*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 10044
*tanh(c/2 + d*x/2)**4*atan(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)*
*6 + 589824*d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 31457
28*d) + 16960*tanh(c/2 + d*x/2)**3/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d
*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 40176
*tanh(c/2 + d*x/2)**2*atan(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)*
*6 + 589824*d*tanh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 31457
28*d) + 28320*tanh(c/2 + d*x/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*ta
nh(c/2 + d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 53568*at
an(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 +
 d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d), True))
```

3.75 $\int \frac{1}{5+3\cosh(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}$$

```
[Out] 1/4*x-1/2*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2657}

$$\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-1),x]
```

```
[Out] x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx=\frac{x}{4}-\frac{\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{2d}$$

Mathematica [B] time = 0.03, size = 77, normalized size = 2.48

$$\frac{\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}-\frac{\log\left(2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)-\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-1),x]
```

```
[Out] -1/4*Log[2*Cosh[c/2 + (d*x)/2] - Sinh[c/2 + (d*x)/2]]/d + Log[2*Cosh[c/2 +
(d*x)/2] + Sinh[c/2 + (d*x)/2]]/(4*d)
```

fricas [A] time = 1.60, size = 42, normalized size = 1.35

$$\frac{\log\left(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1\right)-\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(
d*x + c) + 3))/d
```

giac [A] time = 0.12, size = 28, normalized size = 0.90

$$\frac{\log\left(3\,e^{(dx+c)}+1\right)-\log\left(e^{(dx+c)}+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(3*e^(d*x + c) + 1) - log(e^(d*x + c) + 3))/d
```

maple [A] time = 0.08, size = 36, normalized size = 1.16

$$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4d}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c)),x)
```

```
[Out] 1/4/d*ln(tanh(1/2*d*x+1/2*c)+2)-1/4/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

maxima [A] time = 0.66, size = 37, normalized size = 1.19

$$-\frac{\log\left(3\,e^{(-dx-c)}+1\right)}{4\,d}+\frac{\log\left(e^{(-dx-c)}+3\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*e^(-d*x - c) + 1)/d + 1/4*log(e^(-d*x - c) + 3)/d
```

mupad [B] time = 0.94, size = 40, normalized size = 1.29

$$-\frac{\operatorname{atan}\left(\frac{5\sqrt{-d^2}+3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{4\,d}\right)}{2\,\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cosh(c + d*x) + 5),x)
```

```
[Out] -atan((5*(-d^2)^(1/2) + 3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(4*d))/(2*(-d^2)^(1
/2))
```

sympy [A] time = 0.63, size = 41, normalized size = 1.32

$$\begin{cases} -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{4d}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{4d} & \text{for } d\neq 0 \\ \frac{x}{3\cosh(c)+5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c)),x)
```

```
[Out] Piecewise((-log(tanh(c/2 + d*x/2) - 2)/(4*d) + log(tanh(c/2 + d*x/2) + 2)/(
4*d), Ne(d, 0)), (x/(3*cosh(c) + 5), True))
```

3.76 $\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}-\frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{32d}+\frac{5x}{64}$$

```
[Out] 5/64*x-5/32*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/16*sinh(d*x+c)/d/(5+3*
cosh(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2664, 12, 2657}

$$-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}-\frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{32d}+\frac{5x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (5*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(32*d) - (3*Sinh[
c + d*x])/(16*d*(5 + 3*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx &= -\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))} - \frac{1}{16}\int -\frac{5}{5+3\cosh(c+dx)}\,dx\\ &= -\frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))} + \frac{5}{16}\int \frac{1}{5+3\cosh(c+dx)}\,dx\\ &= \frac{5x}{64} - \frac{5\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{32d} - \frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.12, size = 45, normalized size = 0.80

$$\frac{5\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{6\sinh(c+dx)}{3\cosh(c+dx)+5}}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-2),x]
```

```
[Out] (5*ArcTanh[Tanh[(c + d*x)/2]/2] - (6*Sinh[c + d*x])/(5 + 3*Cosh[c + d*x]))/
(32*d)
```

fricas [B] time = 1.03, size = 212, normalized size = 3.79

$$\frac{5\left(3\,\cosh\left(dx+c\right)^{2} + 2\left(3\,\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right) + 3\,\sinh\left(dx+c\right)^{2} + 10\,\cosh\left(dx+c\right) + 3\right)\log\left(3\,\mathrm{co}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sinh
(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(3*cosh(d*x + c) + 3*sinh(d*x + c) +
 1) - 5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sinh
(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(cosh(d*x + c) + sinh(d*x + c) + 3)
+ 40*cosh(d*x + c) + 40*sinh(d*x + c) + 24)/(3*d*cosh(d*x + c)^2 + 3*d*sinh
(d*x + c)^2 + 10*d*cosh(d*x + c) + 2*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c
) + 3*d)
```

giac [A] time = 0.14, size = 65, normalized size = 1.16

$$\frac{\frac{8\left(5\,e^{(dx+c)}+3\right)}{3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3} + 5\,\log\left(3\,e^{(dx+c)}+1\right) - 5\,\log\left(e^{(dx+c)}+3\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*(5*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3) + 5*lo
g(3*e^(d*x + c) + 1) - 5*log(e^(d*x + c) + 3))/d
```

maple [A] time = 0.07, size = 72, normalized size = 1.29

$$\frac{3}{32d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64d}+\frac{3}{32d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^2,x)
```

```
[Out] 3/32/d/(tanh(1/2*d*x+1/2*c)+2)+5/64/d*ln(tanh(1/2*d*x+1/2*c)+2)+3/32/d/(tan
h(1/2*d*x+1/2*c)-2)-5/64/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

maxima [A] time = 0.31, size = 81, normalized size = 1.45

$$-\frac{5\log\left(3e^{(-dx-c)}+1\right)}{64d}+\frac{5\log\left(e^{(-dx-c)}+3\right)}{64d}-\frac{5e^{(-dx-c)}+3}{8d\left(10e^{(-dx-c)}+3e^{(-2dx-2c)}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -5/64*log(3*e^(-d*x - c) + 1)/d + 5/64*log(e^(-d*x - c) + 3)/d - 1/8*(5*e^(
-d*x - c) + 3)/(d*(10*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + 3))
```

mupad [B] time = 0.94, size = 77, normalized size = 1.38

$$\frac{\frac{5\,e^{c+d\,x}}{8\,d}+\frac{3}{8\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3}-\frac{5\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{-d^2}\right)}{32\,\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cosh(c + d*x) + 5)^2,x)
```

```
[Out] ((5*exp(c + d*x))/(8*d) + 3/(8*d))/(10*exp(c + d*x) + 3*exp(2*c + 2*d*x) +
3) - (5*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))/(32*(-d^2
)^(1/2))
```

sympy [A] time = 1.65, size = 199, normalized size = 3.55

$$\begin{cases} -\frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \cdots \\ \frac{x}{(3\cosh(c)+5)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**2,x)
```

```
[Out] Piecewise((-5*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(64*d*tanh(c/
2 + d*x/2)**2 - 256*d) + 20*log(tanh(c/2 + d*x/2) - 2)/(64*d*tanh(c/2 + d*x
/2)**2 - 256*d) + 5*log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(64*d*t
anh(c/2 + d*x/2)**2 - 256*d) - 20*log(tanh(c/2 + d*x/2) + 2)/(64*d*tanh(c/2
 + d*x/2)**2 - 256*d) + 12*tanh(c/2 + d*x/2)/(64*d*tanh(c/2 + d*x/2)**2 - 2
56*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**2, True))
```

3.77 $\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$

Optimal. Leaf size=81

$$-\frac{45\sinh(c+dx)}{512d(3\cosh(c+dx)+5)}-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{1024d}+\frac{59x}{2048}$$

```
[Out] 59/2048*x-59/1024*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/32*sinh(d*x+c)/d
/(5+3*cosh(d*x+c))^2-45/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 2754, 12, 2657}

$$-\frac{45\sinh(c+dx)}{512d(3\cosh(c+dx)+5)}-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}-\frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{1024d}+\frac{59x}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (59*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(1024*d) - (3
*Sinh[c + d*x])/(32*d*(5 + 3*Cosh[c + d*x])^2) - (45*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2} - \frac{1}{32}\int \frac{-10+3\cosh(c+dx)}{(5+3\cosh(c+dx))^2}\,dx$$
$$= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2} - \frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))} + \frac{1}{512}\int \frac{59}{5+3\cosh(c+$$
$$= -\frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2} - \frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))} + \frac{59}{512}\int \frac{1}{5+3\cosh(c+$$
$$= \frac{59x}{2048} - \frac{59\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{1024d} - \frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2} - \frac{45\sinh(c+$$

Mathematica [A] time = 0.20, size = 58, normalized size = 0.72

$$\frac{59\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{3(182\sinh(c+dx)+45\sinh(2(c+dx)))}{(3\cosh(c+dx)+5)^2}}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-3),x]
```

```
[Out] (59*ArcTanh[Tanh[(c + d*x)/2]/2] - (3*(182*Sinh[c + d*x] + 45*Sinh[2*(c + d
*x)]))/(5 + 3*Cosh[c + d*x])^2)/(1024*d)
```

fricas [B] time = 0.50, size = 563, normalized size = 6.95

$$\frac{1416\cosh(dx+c)^3 + 1416(3\cosh(dx+c)+5)\sinh(dx+c)^2 + 1416\sinh(dx+c)^3 + 7080\cosh(dx+c)^2 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2048*(1416*cosh(d*x + c)^3 + 1416*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^2 +
 1416*sinh(d*x + c)^3 + 7080*cosh(d*x + c)^2 + 59*(9*cosh(d*x + c)^4 + 12*(
3*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)
^3 + 2*(27*cosh(d*x + c)^2 + 90*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*c
osh(d*x + c)^2 + 4*(9*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x +
c) + 15)*sinh(d*x + c) + 60*cosh(d*x + c) + 9)*log(3*cosh(d*x + c) + 3*sinh
(d*x + c) + 1) - 59*(9*cosh(d*x + c)^4 + 12*(3*cosh(d*x + c) + 5)*sinh(d*x
+ c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(27*cosh(d*x + c)^2 + 9
0*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*cosh(d*x + c)^2 + 4*(9*cosh(d*x
 + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x + c) + 15)*sinh(d*x + c) + 60*co
sh(d*x + c) + 9)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24*(177*cosh(d*x
+ c)^2 + 590*cosh(d*x + c) + 241)*sinh(d*x + c) + 5784*cosh(d*x + c) + 1080
)/(9*d*cosh(d*x + c)^4 + 9*d*sinh(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 12*(3
*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^3 + 118*d*cosh(d*x + c)^2 + 2*(27*d*c
osh(d*x + c)^2 + 90*d*cosh(d*x + c) + 59*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x
 + c) + 4*(9*d*cosh(d*x + c)^3 + 45*d*cosh(d*x + c)^2 + 59*d*cosh(d*x + c)
+ 15*d)*sinh(d*x + c) + 9*d)
```

giac [A] time = 0.13, size = 87, normalized size = 1.07

$$\frac{\frac{24\left(59\,e^{(3\,dx+3\,c)}+295\,e^{(2\,dx+2\,c)}+241\,e^{(dx+c)}+45\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^{2}}+59\,\log\left(3\,e^{(dx+c)}+1\right)-59\,\log\left(e^{(dx+c)}+3\right)}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2048*(24*(59*e^(3*d*x + 3*c) + 295*e^(2*d*x + 2*c) + 241*e^(d*x + c) + 45
)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3)^2 + 59*log(3*e^(d*x + c) + 1) -
59*log(e^(d*x + c) + 3))/d
```

maple [A] time = 0.07, size = 108, normalized size = 1.33

$$-\frac{9}{512d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}+\frac{69}{1024d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{59\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{2048d}+\frac{9}{512d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^3,x)
```

```
[Out] -9/512/d/(tanh(1/2*d*x+1/2*c)+2)^2+69/1024/d/(tanh(1/2*d*x+1/2*c)+2)+59/204
8/d*ln(tanh(1/2*d*x+1/2*c)+2)+9/512/d/(tanh(1/2*d*x+1/2*c)-2)^2+69/1024/d/(
tanh(1/2*d*x+1/2*c)-2)-59/2048/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

maxima [A] time = 0.34, size = 125, normalized size = 1.54

$$-\frac{59 \log\left(3 e^{(-dx-c)}+1\right)}{2048 d}+\frac{59 \log\left(e^{(-dx-c)}+3\right)}{2048 d}-\frac{3\left(241 e^{(-dx-c)}+295 e^{(-2 dx-2 c)}+59 e^{(-3 dx-3 c)}+45\right)}{256 d\left(60 e^{(-dx-c)}+118 e^{(-2 dx-2 c)}+60 e^{(-3 dx-3 c)}+9 e^{(-4 dx-4 c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -59/2048*log(3*e^(-d*x - c) + 1)/d + 59/2048*log(e^(-d*x - c) + 3)/d - 3/25
6*(241*e^(-d*x - c) + 295*e^(-2*d*x - 2*c) + 59*e^(-3*d*x - 3*c) + 45)/(d*(
60*e^(-d*x - c) + 118*e^(-2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 9*e^(-4*d*x
- 4*c) + 9))
```

mupad [B] time = 0.95, size = 141, normalized size = 1.74

$$\frac{\frac{59 e^{c+d x}}{256 d}+\frac{295}{768 d}}{10 e^{c+d x}+3 e^{2 c+2 d x}+3}-\frac{59 \operatorname{atan}\left(\left(\frac{5}{4 d}+\frac{3 e^{d x} e^{c}}{4 d}\right) \sqrt{-d^2}\right)}{1024 \sqrt{-d^2}}-\frac{\frac{41 e^{c+d x}}{24 d}+\frac{5}{8 d}}{60 e^{c+d x}+118 e^{2 c+2 d x}+60 e^{3 c+3 d x}+9 e^{4 c+4 d x}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cosh(c + d*x) + 5)^3,x)
```

```
[Out] ((59*exp(c + d*x))/(256*d) + 295/(768*d))/(10*exp(c + d*x) + 3*exp(2*c + 2*
d*x) + 3) - (59*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))/(
1024*(-d^2)^(1/2)) - ((41*exp(c + d*x))/(24*d) + 5/(8*d))/(60*exp(c + d*x)
+ 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*c + 4*d*x) + 9)
```

sympy [A] time = 3.67, size = 445, normalized size = 5.49

$$\begin{cases} -\frac{59 \log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right) \tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048 d \tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384 d \tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768 d}+\frac{472 \log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right) \tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2048 d \tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384 d \tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+32768 d}-\frac{944 \log\left(\tanh\left(\right.\right.}{2048 d \tanh^4\left(\frac{c}{2}+\frac{dx}{2}\right)-16384 d} \\ \frac{x}{(3 \cosh(c)+5)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**3,x)
```

```
[Out] Piecewise((-59*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh
(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 472*log(tanh(c
/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*
d*tanh(c/2 + d*x/2)**2 + 32768*d) - 944*log(tanh(c/2 + d*x/2) - 2)/(2048*d*
```

```
tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 59*log(tan
h(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh(c/2 + d*x/2)**4 - 163
84*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 472*log(tanh(c/2 + d*x/2) + 2)*tanh(
c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2
 + 32768*d) + 944*log(tanh(c/2 + d*x/2) + 2)/(2048*d*tanh(c/2 + d*x/2)**4 -
 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 276*tanh(c/2 + d*x/2)**3/(2048*d
*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 816*tanh(
c/2 + d*x/2)/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 +
32768*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**3, True))
```

3.78 $\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$-\frac{311\sinh(c+dx)}{8192d(3\cosh(c+dx)+5)}-\frac{25\sinh(c+dx)}{512d(3\cosh(c+dx)+5)^2}-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{16384d}+$$

```
[Out] 385/32768*x-385/16384*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-1/16*sinh(d*x+
c)/d/(5+3*cosh(d*x+c))^3-25/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))^2-311/8192*
sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 2754, 12, 2657}

$$-\frac{311\sinh(c+dx)}{8192d(3\cosh(c+dx)+5)}-\frac{25\sinh(c+dx)}{512d(3\cosh(c+dx)+5)^2}-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}-\frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{16384d}+$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Cosh[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (385*ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])])/(16384*d)
- Sinh[c + d*x]/(16*d*(5 + 3*Cosh[c + d*x])^3) - (25*Sinh[c + d*x])/(512*d*
(5 + 3*Cosh[c + d*x])^2) - (311*Sinh[c + d*x])/(8192*d*(5 + 3*Cosh[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
```

```
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3} - \frac{1}{48}\int \frac{-15+6\cosh(c+dx)}{(5+3\cosh(c+dx))^3}\,dx$$

$$= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2} + \frac{\int \frac{186-75\cosh(c+dx)}{(5+3\cosh(c+dx))^2}\,dx}{1536}$$

$$= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2} - \frac{311\sinh(c+dx)}{8192d(5+3\cosh(c+a$$

$$= -\frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2} - \frac{311\sinh(c+dx)}{8192d(5+3\cosh(c+a$$

$$= \frac{385x}{32768} - \frac{385\tanh^{-1}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{16384d} - \frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3} - \frac{25\sinh(c+}{512d(5+3\cosh$$

Mathematica [A] time = 0.27, size = 68, normalized size = 0.64

$$\frac{770\tanh^{-1}\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{9(4883\sinh(c+dx)+2340\sinh(2(c+dx))+311\sinh(3(c+dx)))}{(3\cosh(c+dx)+5)^3}}{32768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Cosh[c + d*x])^(-4),x]
```

```
[Out] (770*ArcTanh[Tanh[(c + d*x)/2]/2] - (9*(4883*Sinh[c + d*x] + 2340*Sinh[2*(c
 + d*x)] + 311*Sinh[3*(c + d*x)]))/(5 + 3*Cosh[c + d*x])^3)/(32768*d)
```

fricas [B] time = 0.66, size = 1078, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/98304*(83160*cosh(d*x + c)^5 + 138600*(3*cosh(d*x + c) + 5)*sinh(d*x + c)
^4 + 83160*sinh(d*x + c)^5 + 693000*cosh(d*x + c)^4 + 6160*(135*cosh(d*x +
c)^2 + 450*cosh(d*x + c) + 311)*sinh(d*x + c)^3 + 1915760*cosh(d*x + c)^3 +
 48*(17325*cosh(d*x + c)^3 + 86625*cosh(d*x + c)^2 + 119735*cosh(d*x + c) +
 36411)*sinh(d*x + c)^2 + 1747728*cosh(d*x + c)^2 + 1155*(27*cosh(d*x + c)^
6 + 54*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cos
h(d*x + c)^5 + 9*(45*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x +
c)^4 + 981*cosh(d*x + c)^4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 +
 981*cosh(d*x + c) + 385)*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*c
osh(d*x + c)^4 + 900*cosh(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x
 + c) + 327)*sinh(d*x + c)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5
+ 225*cosh(d*x + c)^4 + 654*cosh(d*x + c)^3 + 770*cosh(d*x + c)^2 + 327*cos
h(d*x + c) + 45)*sinh(d*x + c) + 270*cosh(d*x + c) + 27)*log(3*cosh(d*x + c
) + 3*sinh(d*x + c) + 1) - 1155*(27*cosh(d*x + c)^6 + 54*(3*cosh(d*x + c) +
 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cosh(d*x + c)^5 + 9*(45*cosh
(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x + c)^4 + 981*cosh(d*x + c)^
4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 + 981*cosh(d*x + c) + 385)
*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*cosh(d*x + c)^4 + 900*cosh
(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x + c) + 327)*sinh(d*x + c
)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5 + 225*cosh(d*x + c)^4 + 6
54*cosh(d*x + c)^3 + 770*cosh(d*x + c)^2 + 327*cosh(d*x + c) + 45)*sinh(d*x
 + c) + 270*cosh(d*x + c) + 27)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24
*(17325*cosh(d*x + c)^4 + 115500*cosh(d*x + c)^3 + 239470*cosh(d*x + c)^2 +
 145644*cosh(d*x + c) + 24525)*sinh(d*x + c) + 588600*cosh(d*x + c) + 67176
)/(27*d*cosh(d*x + c)^6 + 27*d*sinh(d*x + c)^6 + 270*d*cosh(d*x + c)^5 + 54
*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^5 + 981*d*cosh(d*x + c)^4 + 9*(45*
d*cosh(d*x + c)^2 + 150*d*cosh(d*x + c) + 109*d)*sinh(d*x + c)^4 + 1540*d*c
osh(d*x + c)^3 + 4*(135*d*cosh(d*x + c)^3 + 675*d*cosh(d*x + c)^2 + 981*d*c
osh(d*x + c) + 385*d)*sinh(d*x + c)^3 + 981*d*cosh(d*x + c)^2 + 3*(135*d*co
sh(d*x + c)^4 + 900*d*cosh(d*x + c)^3 + 1962*d*cosh(d*x + c)^2 + 1540*d*cos
h(d*x + c) + 327*d)*sinh(d*x + c)^2 + 270*d*cosh(d*x + c) + 6*(27*d*cosh(d*
x + c)^5 + 225*d*cosh(d*x + c)^4 + 654*d*cosh(d*x + c)^3 + 770*d*cosh(d*x +
 c)^2 + 327*d*cosh(d*x + c) + 45*d)*sinh(d*x + c) + 27*d)
```

giac [A] time = 0.12, size = 109, normalized size = 1.03

$$\frac{\frac{8\left(10395\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}+239470\,e^{(3\,dx+3\,c)}+218466\,e^{(2\,dx+2\,c)}+73575\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^{3}}+1155\,\log\left(3\,e^{(dx+c)}+1\right)-1155\,\log\left(e\right.}{98304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/98304*(8*(10395*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) + 239470*e^(3*d*x
 + 3*c) + 218466*e^(2*d*x + 2*c) + 73575*e^(d*x + c) + 8397)/(3*e^(2*d*x +
2*c) + 10*e^(d*x + c) + 3)^3 + 1155*log(3*e^(d*x + c) + 1) - 1155*log(e^(d*
x + c) + 3))/d
```

maple [A] time = 0.08, size = 144, normalized size = 1.36

$$\frac{9}{2048d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^3}-\frac{81}{4096d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}+\frac{639}{16384d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{385\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{32768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*cosh(d*x+c))^4,x)
```

```
[Out] 9/2048/d/(tanh(1/2*d*x+1/2*c)+2)^3-81/4096/d/(tanh(1/2*d*x+1/2*c)+2)^2+639/
16384/d/(tanh(1/2*d*x+1/2*c)+2)+385/32768/d*ln(tanh(1/2*d*x+1/2*c)+2)+9/204
8/d/(tanh(1/2*d*x+1/2*c)-2)^3+81/4096/d/(tanh(1/2*d*x+1/2*c)-2)^2+639/16384
/d/(tanh(1/2*d*x+1/2*c)-2)-385/32768/d*ln(tanh(1/2*d*x+1/2*c)-2)
```

maxima [A] time = 0.34, size = 169, normalized size = 1.59

$$-\frac{385\,\log\left(3\,e^{(-dx-c)}+1\right)}{32768\,d}+\frac{385\,\log\left(e^{(-dx-c)}+3\right)}{32768\,d}-\frac{73575\,e^{(-dx-c)}+218466\,e^{(-2\,dx-2\,c)}+239470\,e^{(-3\,dx-3\,c)}+8}{12288\,d\left(270\,e^{(-dx-c)}+981\,e^{(-2\,dx-2\,c)}+1540\,e^{(-3\,dx-3\,c)}+981\,e^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -385/32768*log(3*e^(-d*x - c) + 1)/d + 385/32768*log(e^(-d*x - c) + 3)/d -
1/12288*(73575*e^(-d*x - c) + 218466*e^(-2*d*x - 2*c) + 239470*e^(-3*d*x -
3*c) + 86625*e^(-4*d*x - 4*c) + 10395*e^(-5*d*x - 5*c) + 8397)/(d*(270*e^(-
d*x - c) + 981*e^(-2*d*x - 2*c) + 1540*e^(-3*d*x - 3*c) + 981*e^(-4*d*x - 4
*c) + 270*e^(-5*d*x - 5*c) + 27*e^(-6*d*x - 6*c) + 27))
```

mupad [B] time = 0.11, size = 226, normalized size = 2.13

$$\frac{\frac{385\,e^{c+d\,x}}{4096\,d}+\frac{1925}{12288\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3}-\frac{385\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{-d^2}\right)}{16384\,\sqrt{-d^2}}-\frac{\frac{385\,e^{c+d\,x}}{1152\,d}+\frac{3461}{3456\,d}}{60\,e^{c+d\,x}+118\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+9\,e^{4\,c+4\,d\,x}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cosh(c + d*x) + 5)^4,x)
```

```
[Out] ((385*exp(c + d*x))/(4096*d) + 1925/(12288*d))/(10*exp(c + d*x) + 3*exp(2*c
 + 2*d*x) + 3) - (385*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/
2)))/(16384*(-d^2)^(1/2)) - ((385*exp(c + d*x))/(1152*d) + 3461/(3456*d))/(
60*exp(c + d*x) + 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*c +
4*d*x) + 9) + ((365*exp(c + d*x))/(54*d) + 41/(18*d))/(270*exp(c + d*x) + 9
81*exp(2*c + 2*d*x) + 1540*exp(3*c + 3*d*x) + 981*exp(4*c + 4*d*x) + 270*ex
p(5*c + 5*d*x) + 27*exp(6*c + 6*d*x) + 27)
```

sympy [A] time = 7.95, size = 784, normalized size = 7.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*cosh(d*x+c))**4,x)
```

```
[Out] Piecewise((-385*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**6/(32768*d*ta
nh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d
*x/2)**2 - 2097152*d) + 4620*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**
4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d
*tanh(c/2 + d*x/2)**2 - 2097152*d) - 18480*log(tanh(c/2 + d*x/2) - 2)*tanh(
c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)*
*4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 24640*log(tanh(c/2 + d*x
/2) - 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 15
72864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 385*log(tanh(c/2 + d*x/2) + 2)*
tanh(c/2 + d*x/2)**6/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*
x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 4620*log(tanh(c/2 +
 d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*
tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 18480*
log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)*
*6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 20971
52*d) - 24640*log(tanh(c/2 + d*x/2) + 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 39
3216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) +
 2556*tanh(c/2 + d*x/2)**5/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/
2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 14976*tanh(c/
2 + d*x/2)**3/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4
 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 23616*tanh(c/2 + d*x/2)/(3
2768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tan
h(c/2 + d*x/2)**2 - 2097152*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**4, True))
```

3.79 $\int (a+b\cosh(x))^{5/2}\,dx$

Optimal. Leaf size=153

$$\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{16}{15}$$

```
[Out] 2/5*b*(a+b*cosh(x))^(3/2)*sinh(x)+16/15*a*b*sinh(x)*(a+b*cosh(x))^(1/2)-2/1
5*I*(23*a^2+9*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x
),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/((a+b*cosh(x))/(a+b))^(1/2)+
16/15*I*a*(a^2-b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*
x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{16}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/Sqrt[(a + b*Cosh[x])/(a + b)] + (((16*I)/15)*a*(a^2 - b^2)*Sqrt[(
a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[
x]] + (16*a*b*Sqrt[a + b*Cosh[x]]*Sinh[x])/15 + (2*b*(a + b*Cosh[x])^(3/2)*
Sinh[x])/5
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(x))^{5/2}\,dx = \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{5}\int \sqrt{a+b\cosh(x)}\left(\frac{1}{2}\left(5a^2+3b^2\right)+4ab\cosh(x)\right)dx$$
$$=\frac{16}{15}ab\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)+\frac{4}{15}\int\frac{\frac{1}{4}a\left(15a^2+17b^2\right)}{\sqrt{a}}$$
$$=\frac{16}{15}ab\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)-\frac{1}{15}\left(8a\left(a^2-b^2\right)\right)\int\frac{}{\sqrt{a}}$$
$$=\frac{16}{15}ab\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)+\frac{\left(\left(23a^2+9b^2\right)\sqrt{a+b\,\mathrm{co}}\right.}{15\sqrt{}}$$
$$=-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}}+\frac{1}{1}$$

Mathematica [A] time = 0.55, size = 150, normalized size = 0.98

$$\frac{b\sinh(x)\left(22a^2+28ab\cosh(x)+3b^2\cosh(2x)+3b^2\right)+16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-2i\left(23a^3+23a^2b\right.}{15\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(5/2),x]
```

```
[Out] ((-2*I)*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cosh[x])/(a + b)]
*EllipticE[(I/2)*x, (2*b)/(a + b)] + (16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cosh[
x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2 + 28*a*b
*Cosh[x] + 3*b^2*Cosh[2*x])*Sinh[x])/(15*Sqrt[a + b*Cosh[x]])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\cosh(x)^2+2\,ab\cosh(x)+a^2\right)\sqrt{b\cosh(x)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cosh(x)^2 + 2*a*b*cosh(x) + a^2)*sqrt(b*cosh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(5/2), x)
```

maple [B] time = 0.49, size = 685, normalized size = 4.48

$$\frac{2\left(24\sqrt{-\frac{2b}{a-b}}\, b^3\cosh\left(\frac{x}{2}\right)\left(\sinh^6\left(\frac{x}{2}\right)\right)+\left(56\sqrt{-\frac{2b}{a-b}}\, a\, b^2+24\sqrt{-\frac{2b}{a-b}}\, b^3\right)\left(\sinh^4\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+\left(22\sqrt{-\frac{2b}{a-b}}\, a^2 b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(5/2),x)
```

```
[Out] 2/15*(24*(-2*b/(a-b))^(1/2)*b^3*cosh(1/2*x)*sinh(1/2*x)^6+(56*(-2*b/(a-b))^
(1/2)*a*b^2+24*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(22*(-2*b/
(a-b))^(1/2)*a^2*b+28*(-2*b/(a-b))^(1/2)*a*b^2+6*(-2*b/(a-b))^(1/2)*b^3)*si
nh(1/2*x)^2*cosh(1/2*x)+15*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*
(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-
b)/b)^(1/2))+23*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/
2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/
2))+17*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+9*b^3
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipti
cF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-46*(2*b/(a-b)*sin
h(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2*b-18*(2*b/(a-b)*sinh(1/2*x)^
2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-
b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*b^3)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)
^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/
sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x)+a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\cosh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x))^(5/2),x)
```

```
[Out] int((a + b*cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

3.80 $\int (a+b\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] 2/3*b*sinh(x)*(a+b*cosh(x))^(1/2)-8/3*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)
*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/((a+b
*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*El
lipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/
(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2656, 2752, 2663, 2661, 2655, 2653}

$$\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}-\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(3/2),x]
```

```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(
a + b*Cosh[x])/(a + b)] + (((2*I)/3)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a +
b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]] + (2*b*Sqrt[a +
b*Cosh[x]]*Sinh[x])/3
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\cosh(x))^{3/2}\,dx = \frac{2}{3}b\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2+b^2\right)+2ab\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$
$$= \frac{2}{3}b\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{1}{3}(4a)\int \sqrt{a+b\cosh(x)}\,dx+\frac{1}{3}\left(-a^2+b^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}$$
$$= \frac{2}{3}b\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{\left(4a\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{\left(\left(-a^2+b^2\right.\right.}{}$$
$$= -\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}+\frac{2}{3}b\sqrt{a+b\cosh}$$

Mathematica [A] time = 0.24, size = 111, normalized size = 0.90

$$\frac{2i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)+2b\sinh(x)(a+b\cosh(x))-8ia(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((-8*I)*a*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a
 + b)] + (2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x,
 (2*b)/(a + b)] + 2*b*(a + b*Cosh[x])*Sinh[x])/(3*Sqrt[a + b*Cosh[x]])
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left((b\cosh(x)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(x) + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(3/2), x)
```

maple [B] time = 0.53, size = 458, normalized size = 3.69

$$2\left(4\sqrt{-\frac{2b}{a-b}}\, b^2 \cosh\left(\frac{x}{2}\right)\left(\sinh^4\left(\frac{x}{2}\right)\right) + \left(2\sqrt{-\frac{2b}{a-b}}\, ab + 2\sqrt{-\frac{2b}{a-b}}\, b^2\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right) + 3a^2\sqrt{\frac{2b\left(\sinh^2\left(\frac{x}{2}\right)\right)}{a-b} + \frac{a+b}{a-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(3/2),x)
```

```
[Out] 2/3*(4*(-2*b/(a-b))^(1/2)*b^2*cosh(1/2*x)*sinh(1/2*x)^4+(2*(-2*b/(a-b))^(1/
2)*a*b+2*(-2*b/(a-b))^(1/2)*b^2)*sinh(1/2*x)^2*cosh(1/2*x)+3*a^2*(2*b/(a-b)
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2
*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+4*a*b*(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b)
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2*(a-b)/b)^(1/2))-8*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1
/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2))*a*b)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/
(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^
2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(x) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\cosh(x))^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x))^(3/2),x)
```

```
[Out] int((a + b*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cosh(x))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x))**(3/2), x)
```

3.81 $\int \sqrt{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=61

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(d*x+c))^(1/2)/d/((a+b*cosh(d
*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2655, 2653}

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \frac{\sqrt{a+b\cosh(c+dx)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$
$$= -\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.09, size = 61, normalized size = 1.00

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}\,E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)]
)/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cosh\left(dx+c\right)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c) + a), x)
```

maple [B] time = 0.43, size = 276, normalized size = 4.52

$$\frac{2\left(a\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+b\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2b\,\mathrm{EllipticE}\left(\mathrm{co}\right.\right.}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2b\left(\sinh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a+b)\left(\mathrm{sinh}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(d*x+c))^(1/2),x)
```

```
[Out] 2*(a*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2
))+b*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2
))-2*b*EllipticE(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2)))*(-sinh(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)/(-2*b
/(a-b))^(1/2)/(2*b*sinh(1/2*d*x+1/2*c)^4+(a+b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)
/sinh(1/2*d*x+1/2*c)/(2*b*sinh(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh\left(dx+c\right)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+b\cosh\left(c+d\,x\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*cosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cosh\left(c+dx\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*cosh(c + d*x)), x)
```

3.82 $\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=46

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2663, 2661}

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sq
rt[a + b*Cosh[x]]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \frac{\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}\,dx}{\sqrt{a+b\cosh(x)}}$$
$$= -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cosh[x]],x]
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/Sq
rt[a + b*Cosh[x]]
```

fricas [F] time = 1.32, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\cosh(x)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*cosh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*cosh(x) + a), x)
```

maple [B] time = 0.36, size = 146, normalized size = 3.17

$$\frac{2\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sqrt{\frac{2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b}{a-b}}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2b\left(\sinh^4\left(\frac{x}{2}\right)\right)+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\left(\sinh^2\left(\frac{x}{2}\right)\right)+a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(1/2),x)
```

```
[Out] 2*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)*((2*b*co
sh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^4+(a+
b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a
-b)/b)^(1/2))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cosh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(1/2),x)
```

```
[Out] int(1/(a + b*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cosh(x)), x)
```

3.83 $\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] -2*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(1/2)-2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1
/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/
(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2664, 21, 2655, 2653}

$$-\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-3/2),x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
*Sqrt[(a + b*Cosh[x])/(a + b)]) - (2*b*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cos
h[x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx &= -\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx}{a^2-b^2}\\
&= -\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\int \sqrt{a+b\cosh(x)}\,dx}{a^2-b^2}\\
&= -\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}\\
&= -\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2b\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 68, normalized size = 0.81

$$-\frac{2\left(b\sinh(x)+i(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)\right)}{(a-b)(a+b)\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-3/2),x]
```

```
[Out] (-2*(I*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a +
b)] + b*Sinh[x]))/((a - b)*(a + b)*Sqrt[a + b*Cosh[x]])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\cosh(x)+a}}{b^2\cosh(x)^2+2\,ab\cosh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + a)/(b^2*cosh(x)^2 + 2*a*b*cosh(x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(-3/2), x)
```

maple [B] time = 0.50, size = 296, normalized size = 3.52

$$\frac{-4\sqrt{-\frac{2b}{a-b}}\,b\cosh\left(\frac{x}{2}\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)+2\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)\sqrt{\frac{2b\left(\sinh^2\left(\frac{x}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(3/2),x)
```

```
[Out] 2*(-2*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^2+EllipticF(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(
a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*a+EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/
2),1/2*(-2*(a-b)/b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-si
nh(1/2*x)^2)^(1/2)*b-2*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-
b)/b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*b)/(-2*b/(a-b))^(1/2)/(a-b)/(a+b)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\cosh(x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(3/2),x)
```

```
[Out] int(1/(a + b*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\cosh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(3/2),x)
```

```
[Out] Integral((a + b*cosh(x))**(-3/2), x)
```

3.84 $\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}\,E}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b}{}}}$$

```
[Out] -2/3*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(3/2)-8/3*a*b*sinh(x)/(a^2-b^2)^2/(a
+b*cosh(x))^(1/2)-8/3*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sin
h(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/(a^2-b^2)^2/((a+b*cos
h(x))/(a+b))^(1/2)+2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh
(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a^2-b^2)/(a+b
*cosh(x))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}\,E}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-5/2),x]
```

```
[Out] (((-8*I)/3)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2
- b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*Sqrt[(a + b*Cosh[x])/(
a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]]
) - (2*b*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (8*a*b*Sinh[x])/(
3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2+b^2\right)+ab\cosh(}{\sqrt{a+b\cosh(x)}}}{3\left(a^2-b^2\right)^2}$$

$$= -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{(4a)\int \sqrt{a+b\cosh(}}{3\left(a^2-b^2\right)^2}$$

$$= -\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{\left(4a\sqrt{a+b\cosh(x)}\right)}{3\left(a^2-b^2\right)^2}$$

$$= -\frac{8ia\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} - \frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cos}$$

Mathematica [A] time = 0.57, size = 135, normalized size = 0.76

$$\frac{2b\sinh(x)\left(-5a^2-4ab\cosh(x)+b^2\right)+2i(a-b)(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-8ia(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}E}{3(a-b)^2(a+b)^2(a+b\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-5/2),x]
```

```
[Out] ((-8*I)*a*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*EllipticE[(I/2)*x, (2*b
)/(a + b)] + (2*I)*(a - b)*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*Ellipt
icF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cosh[x])*Sinh[x])/(
3*(a - b)^2*(a + b)^2*(a + b*Cosh[x])^(3/2))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cosh(x)+a}}{b^3\cosh(x)^3+3\,ab^2\cosh(x)^2+3\,a^2b\cosh(x)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + a)/(b^3*cosh(x)^3 + 3*a*b^2*cosh(x)^2 + 3*a^2*b*c
osh(x) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(-5/2), x)
```

maple [B] time = 0.84, size = 459, normalized size = 2.59

$$\frac{\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right) + a - b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2b\left(\sinh^4\left(\frac{x}{2}\right)\right)+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}}{3b(a-b)(a+b)\left(\cosh^2\left(\frac{x}{2}\right)+\frac{a-b}{2b}\right)^2} - \frac{16b\left(\sinh^2\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)a}{3(a-b)^2(a+b)^2\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(5/2),x)
```

```
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/3/b/(a-b)/(a+b)*cosh(1/2*
x)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b
)^2-16/3*b*sinh(1/2*x)^2/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*b*cosh(1/2*x)^2+
a-b)*sinh(1/2*x)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*b/(a-
b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b
*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b)
)^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-32/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(-2*b/(a-
b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b
*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\cosh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(5/2),x)
```

```
[Out] int(1/(a + b*cosh(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cosh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(5/2),x)
```

```
[Out] Integral((a + b*cosh(x))**(-5/2), x)
```

3.85 $\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh(x)}}-\frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{16ia\sqrt{\frac{a+b\cosh(x}{a+b}}}{15\left(a^2-b^2\right)^2\sqrt{a}}$$

```
[Out] -2/5*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(5/2)-16/15*a*b*sinh(x)/(a^2-b^2)^2/
(a+b*cosh(x))^(3/2)-2/15*b*(23*a^2+9*b^2)*sinh(x)/(a^2-b^2)^3/(a+b*cosh(x))
^(1/2)-2/15*I*(23*a^2+9*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*
sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/(a^2-b^2)^3/((a+b*
cosh(x))/(a+b))^(1/2)+16/15*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF
(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/(a^2-b^
2)^2/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh(x)}}-\frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{16ia\sqrt{\frac{a+b\cosh(x}{a+b}}}{15\left(a^2-b^2\right)^2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-7/2),x]
```

```
[Out] (((-2*I)/15)*(23*a^2 + 9*b^2)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/((a^2 - b^2)^3*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((16*I)/15)*a*Sq
rt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2)
^2*Sqrt[a + b*Cosh[x]]) - (2*b*Sinh[x])/(5*(a^2 - b^2)*(a + b*Cosh[x])^(5/2
)) - (16*a*b*Sinh[x])/(15*(a^2 - b^2)^2*(a + b*Cosh[x])^(3/2)) - (2*b*(23*a
^2 + 9*b^2)*Sinh[x])/(15*(a^2 - b^2)^3*Sqrt[a + b*Cosh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx}{5\left(a^2-b^2\right)}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} + \frac{4\int \frac{\frac{3}{4}\left(5a^2+3b^2\right)-2ab\,\mathrm{co}}{(a+b\cosh(x))^{3}}}{15\left(a^2-b^2\right)}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} - \frac{2b\left(23a^2+9b^2\right)}{15\left(a^2-b^2\right)^3\sqrt{a+}}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} - \frac{2b\left(23a^2+9b^2\right)}{15\left(a^2-b^2\right)^3\sqrt{a+}}$$

$$= -\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} - \frac{2b\left(23a^2+9b^2\right)}{15\left(a^2-b^2\right)^3\sqrt{a+}}$$

$$= -\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^3\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} - \frac{}{5\left(a^2-\right.}$$

Mathematica [A] time = 0.72, size = 165, normalized size = 0.73

$$\frac{2\left(\frac{b\sinh(x)\left(34a^4+b^2\left(23a^2+9b^2\right)\cosh^2(x)+2ab\left(27a^2+5b^2\right)\cosh(x)-5a^2b^2+3b^4\right)}{\left(b^2-a^2\right)^3} - \frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{5/2}\left(\left(23a^2+9b^2\right)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+8a(b-a)F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{(a-b)^3}\right)}{15(a+b\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-7/2),x]
```

```
[Out] (2*(((-I)*((a + b*Cosh[x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(I/2)
*x, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(a -
b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cosh[x] + b^
2*(23*a^2 + 9*b^2)*Cosh[x]^2)*Sinh[x])/(-a^2 + b^2)^3))/(15*(a + b*Cosh[x])
^(5/2))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cosh(x)+a}}{b^4\cosh(x)^4+4\,ab^3\cosh(x)^3+6\,a^2b^2\cosh(x)^2+4\,a^3b\cosh(x)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cosh(x) + a)/(b^4*cosh(x)^4 + 4*a*b^3*cosh(x)^3 + 6*a^2*b^2
*cosh(x)^2 + 4*a^3*b*cosh(x) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(x) + a)^(-7/2), x)
```

maple [B] time = 1.12, size = 566, normalized size = 2.49

$$\frac{\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2b\left(\sinh^4\left(\frac{x}{2}\right)\right)+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}}{10b^2(a-b)(a+b)\left(\cosh^2\left(\frac{x}{2}\right)+\frac{a-b}{2b}\right)^3}-\frac{8a\cosh\left(\frac{x}{2}\right)\sqrt{2b\left(\sinh^4\left(\frac{x}{2}\right)\right)+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}}{15b(a+b)^2(a-b)^2\left(\cosh^2\left(\frac{x}{2}\right)+\frac{a-b}{2b}\right)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x))^(7/2),x)
```

```
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/10/b^2/(a-b)/(a+b)*cosh(1
/2*x)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b
)/b)^3-8/15*a/b/(a+b)^2/(a-b)^2*cosh(1/2*x)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1
/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2*(a-b)/b)^2-4/15*b*sinh(1/2*x)^2/(a-b)^3/(
a+b)^3*cosh(1/2*x)*(23*a^2+9*b^2)/((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(
1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15*a*b^4
+15*b^5)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/
2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(
1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-8/15*b*(23*a^2+9*b^2)/(
```

```
a+b)^3/(a-b)^3*(-a+b)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1
/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(2
*a-2*b)*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))
-EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(
1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cosh(x) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cosh(x))^(7/2),x)
```

```
[Out] int(1/(a + b*cosh(x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x))**(7/2),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=100

$$\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] -2*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(
a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)+2*I*a*(cosh(
1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*a*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I
/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx &= \frac{\int \sqrt{a+b\cosh(x)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b}\\
&= \frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\cosh(x)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}\,dx}{b\sqrt{a+b\cosh(x)}}\\
&= -\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.38, size = 73, normalized size = 0.73

$$
-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-aF\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*EllipticE[(I/2)*x, (2*b)/(a + b)] - a*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cosh[x]])

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(x)}{\sqrt{b\cosh(x)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")

[Out] integral(cosh(x)/sqrt(b*cosh(x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")

[Out] integrate(cosh(x)/sqrt(b*cosh(x) + a), x)

maple [A] time = 0.42, size = 181, normalized size = 1.81

$$\frac{2\left(\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2\,\text{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)\right)\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sqrt{\frac{2b\left(\cosh^2\left(\frac{x}{2}\right)\right.}{a-b}}}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2b\left(\sinh^4\left(\frac{x}{2}\right)\right)+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\left(\sinh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)/(a+b*cosh(x))^(1/2),x)

[Out] 2*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)^2)^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(x)}{\sqrt{a + b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*cosh(x))^(1/2),x)
```

```
[Out] int(cosh(x)/(a + b*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(x\right)}{\sqrt{a + b\cosh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(a + b*cosh(x)), x)
```

3.87 $\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=94

$$\frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a\cosh(x)+a}}+\frac{16}{105}a^2(7A+5B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{35}a(7A+5B)\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{2}{7}B\,\mathrm{si}$$

```
[Out] 2/35*a*(7*A+5*B)*(a+a*cosh(x))^(3/2)*sinh(x)+2/7*B*(a+a*cosh(x))^(5/2)*sinh
(x)+64/105*a^3*(7*A+5*B)*sinh(x)/(a+a*cosh(x))^(1/2)+16/105*a^2*(7*A+5*B)*s
inh(x)*(a+a*cosh(x))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2751, 2647, 2646}

$$\frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a\cosh(x)+a}}+\frac{16}{105}a^2(7A+5B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{35}a(7A+5B)\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{2}{7}B\,\mathrm{si}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Sinh[x])/(105*Sqrt[a + a*Cosh[x]]) + (16*a^2*(7*A + 5*B
)*Sqrt[a + a*Cosh[x]]*Sinh[x])/105 + (2*a*(7*A + 5*B)*(a + a*Cosh[x])^(3/2)
*Sinh[x])/35 + (2*B*(a + a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{2}{7}B(a+a\cosh(x))^{5/2}\sinh(x)+\frac{1}{7}(7A+5B)\int (a+a\cosh(x))^{5/2}\,dx$$
$$= \frac{2}{35}a(7A+5B)(a+a\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+a\cosh(x))^{5/2}\sinh(x$$
$$= \frac{16}{105}a^2(7A+5B)\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{2}{35}a(7A+5B)(a+a\cosh(x$$
$$= \frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a+a\cosh(x)}}+\frac{16}{105}a^2(7A+5B)\sqrt{a+a\cosh(x)}\,\sinh(x)+$$

Mathematica [A] time = 0.13, size = 60, normalized size = 0.64

$$\frac{1}{210}a^2\tanh\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}\,((392A+505B)\cosh(x)+6(7A+20B)\cosh(2x)+1246A+15B\cosh(3x)+1040B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cosh[x])]*(1246*A + 1040*B + (392*A + 505*B)*Cosh[x] + 6*(
7*A + 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Tanh[x/2])/210
```

fricas [B] time = 0.70, size = 563, normalized size = 5.99

$$\frac{\sqrt{\frac{1}{2}}\left(15\,Ba^2\cosh(x)^7+15\,Ba^2\sinh(x)^7+21\,(2\,A+5\,B)a^2\cosh(x)^6+35\,(10\,A+11\,B)a^2\cosh(x)^5+525\,(4\,A+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A + 5*B)*a
^2*cosh(x)^6 + 35*(10*A + 11*B)*a^2*cosh(x)^5 + 525*(4*A + 3*B)*a^2*cosh(x)
^4 + 21*(5*B*a^2*cosh(x) + (2*A + 5*B)*a^2)*sinh(x)^6 - 525*(4*A + 3*B)*a^2
*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A + 5*B)*a^2*cosh(x) + 5*(10*A +
 11*B)*a^2)*sinh(x)^5 - 35*(10*A + 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*cosh(
x)^3 + 9*(2*A + 5*B)*a^2*cosh(x)^2 + 5*(10*A + 11*B)*a^2*cosh(x) + 15*(4*A
+ 3*B)*a^2)*sinh(x)^4 - 21*(2*A + 5*B)*a^2*cosh(x) + 35*(15*B*a^2*cosh(x)^4
 + 12*(2*A + 5*B)*a^2*cosh(x)^3 + 10*(10*A + 11*B)*a^2*cosh(x)^2 + 60*(4*A
```

```
+ 3*B)*a^2*cosh(x) - 15*(4*A + 3*B)*a^2)*sinh(x)^3 - 15*B*a^2 + 35*(9*B*a^2
*cosh(x)^5 + 9*(2*A + 5*B)*a^2*cosh(x)^4 + 10*(10*A + 11*B)*a^2*cosh(x)^3 +
 90*(4*A + 3*B)*a^2*cosh(x)^2 - 45*(4*A + 3*B)*a^2*cosh(x) - (10*A + 11*B)*
a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A + 5*B)*a^2*cosh(x)^5 + 25*
(10*A + 11*B)*a^2*cosh(x)^4 + 300*(4*A + 3*B)*a^2*cosh(x)^3 - 225*(4*A + 3*
B)*a^2*cosh(x)^2 - 10*(10*A + 11*B)*a^2*cosh(x) - 3*(2*A + 5*B)*a^2)*sinh(x
))*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)
*sinh(x)^2 + sinh(x)^3)
```

giac [A] time = 0.14, size = 153, normalized size = 1.63

$$-\frac{1}{840}\sqrt{2}\left(\frac{\left(2100\,Aa^6e^{(3\,x)}+1575\,Ba^6e^{(3\,x)}+350\,Aa^6e^{(2\,x)}+385\,Ba^6e^{(2\,x)}+42\,Aa^6e^{x}+105\,Ba^6e^{x}+15\,Ba^6\right)e^{\left(-\frac{7}{2}\right.}}{a^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/840*sqrt(2)*((2100*A*a^6*e^(3*x) + 1575*B*a^6*e^(3*x) + 350*A*a^6*e^(2*x
) + 385*B*a^6*e^(2*x) + 42*A*a^6*e^x + 105*B*a^6*e^x + 15*B*a^6)*e^(-7/2*x)
/a^(7/2) - (15*B*a^(19/2)*e^(7/2*x) + 42*A*a^(19/2)*e^(5/2*x) + 105*B*a^(19
/2)*e^(5/2*x) + 350*A*a^(19/2)*e^(3/2*x) + 385*B*a^(19/2)*e^(3/2*x) + 2100*
A*a^(19/2)*e^(1/2*x) + 1575*B*a^(19/2)*e^(1/2*x))/a^7)
```

maple [A] time = 0.22, size = 71, normalized size = 0.76

$$\frac{8\cosh\left(\frac{x}{2}\right)a^3\sinh\left(\frac{x}{2}\right)\left(30B\left(\sinh^6\left(\frac{x}{2}\right)\right)+(21A+105B)\left(\sinh^4\left(\frac{x}{2}\right)\right)+(70A+140B)\left(\sinh^2\left(\frac{x}{2}\right)\right)+105A+105\right.}{105\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

```
[Out] 8/105*cosh(1/2*x)*a^3*sinh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A+105*B)*sinh(1/2
*x)^4+(70*A+140*B)*sinh(1/2*x)^2+105*A+105*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/
2)
```

maxima [B] time = 0.49, size = 237, normalized size = 2.52

$$\frac{1}{60}\left(3\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,x\right)}+25\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,x\right)}+150\sqrt{2}\,a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,x\right)}-150\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{1}{2}\,x\right)}-25\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{3}{2}\,x\right)}-3\sqrt{2}\,a^{\frac{5}{2}}e^{\left(-\frac{5}{2}\,x\right)}\right)A+\frac{}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*a^(5/2)*e^(5/2*x) + 25*sqrt(2)*a^(5/2)*e^(3/2*x) + 150*sqrt
(2)*a^(5/2)*e^(1/2*x) - 150*sqrt(2)*a^(5/2)*e^(-1/2*x) - 25*sqrt(2)*a^(5/2)
*e^(-3/2*x) - 3*sqrt(2)*a^(5/2)*e^(-5/2*x))*A + 1/168*((3*sqrt(2)*a^(5/2)*e
^(-x) + 21*sqrt(2)*a^(5/2)*e^(-2*x) + 70*sqrt(2)*a^(5/2)*e^(-3*x) + 210*sqr
t(2)*a^(5/2)*e^(-4*x) - 105*sqrt(2)*a^(5/2)*e^(-5*x) - 7*sqrt(2)*a^(5/2)*e^
(-6*x))*e^(9/2*x) + (7*sqrt(2)*a^(5/2)*e^(-x) + 105*sqrt(2)*a^(5/2)*e^(-2*x
) - 210*sqrt(2)*a^(5/2)*e^(-3*x) - 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*sqrt(2)
*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)*e^(-6*x))*e^(5/2*x))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cosh(x))\ (a + a\cosh(x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + a*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + a*cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

3.88 $\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=68

$$\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a\cosh(x)+a}}+\frac{2}{15}a(5A+3B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

```
[Out] 2/5*B*(a+a*cosh(x))^(3/2)*sinh(x)+8/15*a^2*(5*A+3*B)*sinh(x)/(a+a*cosh(x))^
(1/2)+2/15*a*(5*A+3*B)*sinh(x)*(a+a*cosh(x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2751, 2647, 2646}

$$\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a\cosh(x)+a}}+\frac{2}{15}a(5A+3B)\sinh(x)\sqrt{a\cosh(x)+a}+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Sinh[x])/(15*Sqrt[a + a*Cosh[x]]) + (2*a*(5*A + 3*B)*Sqr
t[a + a*Cosh[x]]*Sinh[x])/15 + (2*B*(a + a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a+a\cosh(x))^{3/2}\sinh(x)+\frac{1}{5}(5A+3B)\int (a+a\cosh(x))^{3/2}\,dx$$
$$=\frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a+a\cosh(x))^{3/2}\sinh(x)$$
$$=\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a+a\cosh(x)}}+\frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.68

$$\frac{1}{15}a\tanh\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}\,(2(5A+9B)\cosh(x)+50A+3B\cosh(2x)+39B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cosh[x] + 3*B*Cosh[2*
x])*Tanh[x/2])/15
```

fricas [B] time = 0.51, size = 279, normalized size = 4.10

$$\sqrt{\frac{1}{2}}\left(3\,Ba\cosh(x)^5+3\,Ba\sinh(x)^5+5\,(2\,A+3\,B)a\cosh(x)^4+30\,(3\,A+2\,B)a\cosh(x)^3+5\,(3\,Ba\cosh(x)+(2\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A + 3*B)*a*cosh(x)
^4 + 30*(3*A + 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A + 3*B)*a)*sinh(x)
^4 - 30*(3*A + 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A + 3*B)*a*cos
h(x) + 3*(3*A + 2*B)*a)*sinh(x)^3 - 5*(2*A + 3*B)*a*cosh(x) + 30*(B*a*cosh(
x)^3 + (2*A + 3*B)*a*cosh(x)^2 + 3*(3*A + 2*B)*a*cosh(x) - (3*A + 2*B)*a)*s
inh(x)^2 - 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A + 3*B)*a*cosh(x)^3 + 18*(3*A
 + 2*B)*a*cosh(x)^2 - 12*(3*A + 2*B)*a*cosh(x) - (2*A + 3*B)*a)*sinh(x))*sq
rt(a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [B] time = 0.13, size = 113, normalized size = 1.66

$$-\frac{1}{60}\sqrt{2}\left(\frac{\left(90\,Aa^4e^{(2\,x)}+60\,Ba^4e^{(2\,x)}+10\,Aa^4e^x+15\,Ba^4e^x+3\,Ba^4\right)e^{\left(-\frac{5}{2}\,x\right)}}{a^{\frac{5}{2}}}-\frac{3\,Ba^{\frac{13}{2}}e^{\left(\frac{5}{2}\,x\right)}+10\,Aa^{\frac{13}{2}}e^{\left(\frac{3}{2}\,x\right)}+15\,Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(2)*((90*A*a^4*e^(2*x) + 60*B*a^4*e^(2*x) + 10*A*a^4*e^x + 15*B*a
^4*e^x + 3*B*a^4)*e^(-5/2*x)/a^(5/2) - (3*B*a^(13/2)*e^(5/2*x) + 10*A*a^(13
/2)*e^(3/2*x) + 15*B*a^(13/2)*e^(3/2*x) + 90*A*a^(13/2)*e^(1/2*x) + 60*B*a^
(13/2)*e^(1/2*x))/a^5)
```

maple [A] time = 0.20, size = 57, normalized size = 0.84

$$\frac{4 \cosh\left(\frac{x}{2}\right) a^2 \sinh\left(\frac{x}{2}\right) \left(6B \left(\sinh^4\left(\frac{x}{2}\right)\right) + (5A + 15B)\left(\sinh^2\left(\frac{x}{2}\right)\right) + 15A + 15B\right)\sqrt{2}}{15\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

```
[Out] 4/15*cosh(1/2*x)*a^2*sinh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A+15*B)*sinh(1/2*x)^
2+15*A+15*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

maxima [B] time = 0.46, size = 163, normalized size = 2.40

$$\frac{1}{6}\left(\sqrt{2}\, a^{\frac{3}{2}} e^{\left(\frac{3}{2}x\right)} + 9\sqrt{2}\, a^{\frac{3}{2}} e^{\left(\frac{1}{2}x\right)} - 9\sqrt{2}\, a^{\frac{3}{2}} e^{\left(-\frac{1}{2}x\right)} - \sqrt{2}\, a^{\frac{3}{2}} e^{\left(-\frac{3}{2}x\right)}\right)A + \frac{1}{20}\left(\left(\sqrt{2}\, a^{\frac{3}{2}} e^{(-x)} + 5\sqrt{2}\, a^{\frac{3}{2}} e^{(-2x)} + 15\sqrt{2}\, a^{\frac{3}{2}} e^{(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*a^(3/2)*e^(3/2*x) + 9*sqrt(2)*a^(3/2)*e^(1/2*x) - 9*sqrt(2)*a^
(3/2)*e^(-1/2*x) - sqrt(2)*a^(3/2)*e^(-3/2*x))*A + 1/20*((sqrt(2)*a^(3/2)*e
^(-x) + 5*sqrt(2)*a^(3/2)*e^(-2*x) + 15*sqrt(2)*a^(3/2)*e^(-3*x) - 5*sqrt(2
)*a^(3/2)*e^(-4*x))*e^(7/2*x) + (5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^(3
/2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2)*e^(-4*x))*e^(3/
2*x))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cosh(x))\,(a + a\cosh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + a*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + a*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\cosh (x) + 1))^{\frac{3}{2}} (A + B \cosh (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral((a*(cosh(x) + 1))**(3/2)*(A + B*cosh(x)), x)
```

3.89 $\int \sqrt{a+a\cosh(x)}\,(A+B\cosh(x))\,dx$

Optimal. Leaf size=40

$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

```
[Out] 2/3*a*(3*A+B)*sinh(x)/(a+a*cosh(x))^(1/2)+2/3*B*sinh(x)*(a+a*cosh(x))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2751, 2646}

$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*a*(3*A + B)*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*B*Sqrt[a + a*Cosh[x]]*
Sinh[x])/3
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cosh(x)}\,(A+B\cosh(x))\,dx &= \frac{2}{3}B\sqrt{a+a\cosh(x)}\,\sinh(x)+\frac{1}{3}(3A+B)\int \sqrt{a+a\cosh(x)}\,dx\\ &= \frac{2a(3A+B)\sinh(x)}{3\sqrt{a+a\cosh(x)}}+\frac{2}{3}B\sqrt{a+a\cosh(x)}\,\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 0.78

$$\frac{2}{3}\tanh\left(\frac{x}{2}\right)\sqrt{a(\cosh(x)+1)}\,(3A+B\cosh(x)+2B)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[x])]*(3*A + 2*B + B*Cosh[x])*Tanh[x/2])/3
```

fricas [B] time = 0.49, size = 100, normalized size = 2.50

$$\frac{\sqrt{\frac{1}{2}}\left(B\cosh(x)^{3}+B\sinh(x)^{3}+3\,(2\,A+B)\cosh(x)^{2}+3\,(B\cosh(x)+2\,A+B)\sinh(x)^{2}-3\,(2\,A+B)\cosh(x)+\right.}{3\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A + B)*cosh(x)^2 + 3*(B*cos
h(x) + 2*A + B)*sinh(x)^2 - 3*(2*A + B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A +
 B)*cosh(x) - 2*A - B)*sinh(x) - B)*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x) +
sinh(x))
```

giac [B] time = 0.15, size = 71, normalized size = 1.78

$$-\frac{1}{6}\,\sqrt{2}\left(\frac{\left(6\,Aa^{2}e^{x}+3\,Ba^{2}e^{x}+Ba^{2}\right)e^{\left(-\frac{3}{2}\,x\right)}}{a^{\frac{3}{2}}}-\frac{Ba^{\frac{7}{2}}e^{\left(\frac{3}{2}\,x\right)}+6\,Aa^{\frac{7}{2}}e^{\left(\frac{1}{2}\,x\right)}+3\,Ba^{\frac{7}{2}}e^{\left(\frac{1}{2}\,x\right)}}{a^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*((6*A*a^2*e^x + 3*B*a^2*e^x + B*a^2)*e^(-3/2*x)/a^(3/2) - (B*a
^(7/2)*e^(3/2*x) + 6*A*a^(7/2)*e^(1/2*x) + 3*B*a^(7/2)*e^(1/2*x))/a^3)
```

maple [A] time = 0.23, size = 39, normalized size = 0.98

$$\frac{2\cosh\left(\frac{x}{2}\right)a\sinh\left(\frac{x}{2}\right)\left(2B\left(\cosh^{2}\left(\frac{x}{2}\right)\right)+3A+B\right)\sqrt{2}}{3\sqrt{a\left(\cosh^{2}\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/3*cosh(1/2*x)*a*sinh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A+B)*2^(1/2)/(a*cosh(1/2
*x)^2)^(1/2)
```

maxima [B] time = 0.46, size = 90, normalized size = 2.25

$$\left(\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}x\right)} - \sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}x\right)}\right)A + \frac{1}{6}\left(\left(\sqrt{2}\sqrt{a}e^{(-x)} + 3\sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{5}{2}x\right)} - \left(3\sqrt{2}\sqrt{a}e^{(-x)} + \sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{1}{2}x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] (sqrt(2)*sqrt(a)*e^(1/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))*A + 1/6*((sqrt(2)*
sqrt(a)*e^(-x) + 3*sqrt(2)*sqrt(a)*e^(-2*x))*e^(5/2*x) - (3*sqrt(2)*sqrt(a)
*e^(-x) + sqrt(2)*sqrt(a)*e^(-2*x))*e^(1/2*x))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (A + B\cosh(x))\sqrt{a + a\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + a*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cosh(x) + 1)}\,(A + B\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))*(A + B*cosh(x)), x)
```

3.90 $\int (a - a\cosh(x))^{5/2}(A + B\cosh(x))\,dx$

Optimal. Leaf size=98

$$-\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2}{35}a(7A-5B)\sinh(x)(a-a\cosh(x))^{3/2}+\frac{2}{7}B\sin$$

```
[Out] -2/35*a*(7*A-5*B)*(a-a*cosh(x))^(3/2)*sinh(x)+2/7*B*(a-a*cosh(x))^(5/2)*sin
h(x)-64/105*a^3*(7*A-5*B)*sinh(x)/(a-a*cosh(x))^(1/2)-16/105*a^2*(7*A-5*B)*
sinh(x)*(a-a*cosh(x))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2}{35}a(7A-5B)\sinh(x)(a-a\cosh(x))^{3/2}+\frac{2}{7}B\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (-64*a^3*(7*A - 5*B)*Sinh[x])/(105*Sqrt[a - a*Cosh[x]]) - (16*a^2*(7*A - 5*
B)*Sqrt[a - a*Cosh[x]]*Sinh[x])/105 - (2*a*(7*A - 5*B)*(a - a*Cosh[x])^(3/2
)*Sinh[x])/35 + (2*B*(a - a*Cosh[x])^(5/2)*Sinh[x])/7
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{2}{7}B(a-a\cosh(x))^{5/2}\sinh(x)-\frac{1}{7}(-7A+5B)\int (a-a\cosh(x))^{5/2}\,dx$$
$$= -\frac{2}{35}a(7A-5B)(a-a\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a-a\cosh(x))^{5/2}\sinh$$
$$= -\frac{16}{105}a^2(7A-5B)\sqrt{a-a\cosh(x)}\,\sinh(x)-\frac{2}{35}a(7A-5B)(a-a\cosh$$
$$= -\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}}-\frac{16}{105}a^2(7A-5B)\sqrt{a-a\cosh(x)}\,\sinh(x)-$$

Mathematica [A] time = 0.15, size = 61, normalized size = 0.62

$$\frac{1}{210}a^2\coth\left(\frac{x}{2}\right)\sqrt{a-a\cosh(x)}\,((505B-392A)\cosh(x)+6(7A-20B)\cosh(2x)+1246A+15B\cosh(3x)-1040B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (a^2*Sqrt[a - a*Cosh[x]]*(1246*A - 1040*B + (-392*A + 505*B)*Cosh[x] + 6*(7
*A - 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Coth[x/2])/210
```

fricas [B] time = 0.52, size = 564, normalized size = 5.76

$$\frac{\sqrt{\frac{1}{2}}\left(15\,Ba^2\cosh(x)^7+15\,Ba^2\sinh(x)^7+21\,(2\,A-5\,B)a^2\cosh(x)^6-35\,(10\,A-11\,B)a^2\cosh(x)^5+525\,(4\,A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A - 5*B)*a
^2*cosh(x)^6 - 35*(10*A - 11*B)*a^2*cosh(x)^5 + 525*(4*A - 3*B)*a^2*cosh(x)
^4 + 21*(5*B*a^2*cosh(x) + (2*A - 5*B)*a^2)*sinh(x)^6 + 525*(4*A - 3*B)*a^2
*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A - 5*B)*a^2*cosh(x) - 5*(10*A -
 11*B)*a^2)*sinh(x)^5 - 35*(10*A - 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*cosh(
x)^3 + 9*(2*A - 5*B)*a^2*cosh(x)^2 - 5*(10*A - 11*B)*a^2*cosh(x) + 15*(4*A
- 3*B)*a^2)*sinh(x)^4 + 21*(2*A - 5*B)*a^2*cosh(x) + 35*(15*B*a^2*cosh(x)^4
 + 12*(2*A - 5*B)*a^2*cosh(x)^3 - 10*(10*A - 11*B)*a^2*cosh(x)^2 + 60*(4*A
```

```
- 3*B)*a^2*cosh(x) + 15*(4*A - 3*B)*a^2)*sinh(x)^3 + 15*B*a^2 + 35*(9*B*a^2
*cosh(x)^5 + 9*(2*A - 5*B)*a^2*cosh(x)^4 - 10*(10*A - 11*B)*a^2*cosh(x)^3 +
 90*(4*A - 3*B)*a^2*cosh(x)^2 + 45*(4*A - 3*B)*a^2*cosh(x) - (10*A - 11*B)*
a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A - 5*B)*a^2*cosh(x)^5 - 25*
(10*A - 11*B)*a^2*cosh(x)^4 + 300*(4*A - 3*B)*a^2*cosh(x)^3 + 225*(4*A - 3*
B)*a^2*cosh(x)^2 - 10*(10*A - 11*B)*a^2*cosh(x) + 3*(2*A - 5*B)*a^2)*sinh(x
))*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x
)*sinh(x)^2 + sinh(x)^3)
```

giac [B] time = 0.19, size = 295, normalized size = 3.01

$$\frac{1}{840}\sqrt{2}\left(\frac{\left(2100\,Aa^{6}e^{(3\,x)}\mathrm{sgn}\left(-e^{x}+1\right)-1575\,Ba^{6}e^{(3\,x)}\mathrm{sgn}\left(-e^{x}+1\right)-350\,Aa^{6}e^{(2\,x)}\mathrm{sgn}\left(-e^{x}+1\right)+385\,Ba^{6}e^{(2\,x)}\mathrm{sg}\right.}{\sqrt{-ae^{x}}\,a^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/840*sqrt(2)*((2100*A*a^6*e^(3*x)*sgn(-e^x + 1) - 1575*B*a^6*e^(3*x)*sgn(-
e^x + 1) - 350*A*a^6*e^(2*x)*sgn(-e^x + 1) + 385*B*a^6*e^(2*x)*sgn(-e^x + 1
) + 42*A*a^6*e^x*sgn(-e^x + 1) - 105*B*a^6*e^x*sgn(-e^x + 1) + 15*B*a^6*sgn
(-e^x + 1))*e^(-3*x)/(sqrt(-a*e^x)*a^3) - (15*sqrt(-a*e^x)*B*a^9*e^(3*x)*sg
n(-e^x + 1) + 42*sqrt(-a*e^x)*A*a^9*e^(2*x)*sgn(-e^x + 1) - 105*sqrt(-a*e^x
)*B*a^9*e^(2*x)*sgn(-e^x + 1) - 350*sqrt(-a*e^x)*A*a^9*e^x*sgn(-e^x + 1) +
385*sqrt(-a*e^x)*B*a^9*e^x*sgn(-e^x + 1) + 2100*sqrt(-a*e^x)*A*a^9*sgn(-e^x
 + 1) - 1575*sqrt(-a*e^x)*B*a^9*sgn(-e^x + 1))/a^7)
```

maple [A] time = 0.26, size = 69, normalized size = 0.70

$$-\frac{16\sinh\left(\frac{x}{2}\right)a^{3}\cosh\left(\frac{x}{2}\right)\left(30B\left(\sinh^{6}\left(\frac{x}{2}\right)\right)+\left(21A-15B\right)\left(\sinh^{4}\left(\frac{x}{2}\right)\right)+\left(-28A+20B\right)\left(\sinh^{2}\left(\frac{x}{2}\right)\right)+56A-40B\right)}{105\sqrt{-2a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

```
[Out] -16/105*sinh(1/2*x)*a^3*cosh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A-15*B)*sinh(1/
2*x)^4+(-28*A+20*B)*sinh(1/2*x)^2+56*A-40*B)/(-2*a*sinh(1/2*x)^2)^(1/2)
```

maxima [B] time = 0.47, size = 288, normalized size = 2.94

$$\frac{1}{60}\left(\frac{25\sqrt{2}\,a^{\frac{5}{2}}e^{(-x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{150\sqrt{2}\,a^{\frac{5}{2}}e^{(-2\,x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{150\sqrt{2}\,a^{\frac{5}{2}}e^{(-3\,x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}+\frac{25\sqrt{2}\,a^{\frac{5}{2}}e^{(-4\,x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{3\sqrt{2}\,a^{\frac{5}{2}}e^{(-5\,x)}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}-\frac{3\sqrt{2}\,a^{\frac{5}{2}}}{\left(-e^{(-x)}\right)^{\frac{5}{2}}}\right)A+\frac{1}{168}B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/60*(25*sqrt(2)*a^(5/2)*e^(-x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-2
*x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-3*x)/(-e^(-x))^(5/2) + 25*sqr
t(2)*a^(5/2)*e^(-4*x)/(-e^(-x))^(5/2) - 3*sqrt(2)*a^(5/2)*e^(-5*x)/(-e^(-x)
)^(5/2) - 3*sqrt(2)*a^(5/2)/(-e^(-x))^(5/2))*A + 1/168*B*((21*sqrt(2)*a^(5/
2)*e^(-x) - 70*sqrt(2)*a^(5/2)*e^(-2*x) + 210*sqrt(2)*a^(5/2)*e^(-3*x) + 10
5*sqrt(2)*a^(5/2)*e^(-4*x) - 7*sqrt(2)*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)
)*e^x/(-e^(-x))^(5/2) - (7*sqrt(2)*a^(5/2)*e^(-x) - 105*sqrt(2)*a^(5/2)*e^(
-2*x) - 210*sqrt(2)*a^(5/2)*e^(-3*x) + 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*sqr
t(2)*a^(5/2)*e^(-5*x) + 3*sqrt(2)*a^(5/2)*e^(-6*x))/(-e^(-x))^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cosh(x))\ (a - a\cosh(x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a - a*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))*(a - a*cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

3.91 $\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=71

$$-\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sinh(x)\sqrt{a-a\cosh(x)}+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

```
[Out] 2/5*B*(a-a*cosh(x))^(3/2)*sinh(x)-8/15*a^2*(5*A-3*B)*sinh(x)/(a-a*cosh(x))^
(1/2)-2/15*a*(5*A-3*B)*sinh(x)*(a-a*cosh(x))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sinh(x)\sqrt{a-a\cosh(x)}+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (-8*a^2*(5*A - 3*B)*Sinh[x])/(15*Sqrt[a - a*Cosh[x]]) - (2*a*(5*A - 3*B)*Sq
rt[a - a*Cosh[x]]*Sinh[x])/15 + (2*B*(a - a*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x)-\frac{1}{5}(-5A+3B)\int (a-a\cosh(x))^{3/2}\,dx$$
$$= -\frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\,\sinh(x)+\frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x$$
$$= -\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\,\sinh(x)+\frac{2}{5}B$$

Mathematica [A] time = 0.11, size = 47, normalized size = 0.66

$$-\frac{1}{15}a\coth\left(\frac{x}{2}\right)\sqrt{a-a\cosh(x)}\,(2(5A-9B)\cosh(x)-50A+3B\cosh(2x)+39B)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] -1/15*(a*Sqrt[a - a*Cosh[x]]*(-50*A + 39*B + 2*(5*A - 9*B)*Cosh[x] + 3*B*Co
sh[2*x])*Coth[x/2])
```

fricas [B] time = 0.52, size = 279, normalized size = 3.93

$$-\frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh(x)^5+3\,Ba\sinh(x)^5+5\,(2\,A-3\,B)a\cosh(x)^4-30\,(3\,A-2\,B)a\cosh(x)^3+5\,(3\,Ba\cosh(x)+(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A - 3*B)*a*cosh(x
)^4 - 30*(3*A - 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A - 3*B)*a)*sinh(x
)^4 - 30*(3*A - 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A - 3*B)*a*co
sh(x) - 3*(3*A - 2*B)*a)*sinh(x)^3 + 5*(2*A - 3*B)*a*cosh(x) + 30*(B*a*cosh
(x)^3 + (2*A - 3*B)*a*cosh(x)^2 - 3*(3*A - 2*B)*a*cosh(x) - (3*A - 2*B)*a)*
sinh(x)^2 + 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A - 3*B)*a*cosh(x)^3 - 18*(3*
A - 2*B)*a*cosh(x)^2 - 12*(3*A - 2*B)*a*cosh(x) + (2*A - 3*B)*a)*sinh(x))*s
qrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [B] time = 0.15, size = 212, normalized size = 2.99

$$\frac{1}{60}\sqrt{2}\left(\frac{\left(90\,Aa^4e^{(2\,x)}\mathrm{sgn}\left(-e^x+1\right)-60\,Ba^4e^{(2\,x)}\mathrm{sgn}\left(-e^x+1\right)-10\,Aa^4e^x\mathrm{sgn}\left(-e^x+1\right)+15\,Ba^4e^x\mathrm{sgn}\left(-e^x+1\right)\right.}{\sqrt{-ae^x}\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(2)*((90*A*a^4*e^(2*x)*sgn(-e^x + 1) - 60*B*a^4*e^(2*x)*sgn(-e^x +
 1) - 10*A*a^4*e^x*sgn(-e^x + 1) + 15*B*a^4*e^x*sgn(-e^x + 1) - 3*B*a^4*sgn
(-e^x + 1))*e^(-2*x)/(sqrt(-a*e^x)*a^2) + (3*sqrt(-a*e^x)*B*a^6*e^(2*x)*sgn
(-e^x + 1) + 10*sqrt(-a*e^x)*A*a^6*e^x*sgn(-e^x + 1) - 15*sqrt(-a*e^x)*B*a^
6*e^x*sgn(-e^x + 1) - 90*sqrt(-a*e^x)*A*a^6*sgn(-e^x + 1) + 60*sqrt(-a*e^x)
*B*a^6*sgn(-e^x + 1))/a^5)
```

maple [A] time = 0.32, size = 55, normalized size = 0.77

$$\frac{8\sinh\left(\frac{x}{2}\right)a^{2}\cosh\left(\frac{x}{2}\right)\left(6B\left(\sinh^{4}\left(\frac{x}{2}\right)\right)+(5A-3B)\left(\sinh^{2}\left(\frac{x}{2}\right)\right)-10A+6B\right)}{15\sqrt{-2a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

```
[Out] 8/15*sinh(1/2*x)*a^2*cosh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A-3*B)*sinh(1/2*x)^2
-10*A+6*B)/(-2*a*sinh(1/2*x)^2)^(1/2)
```

maxima [B] time = 0.47, size = 199, normalized size = 2.80

$$\frac{1}{6}\left(\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}+\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}\right)A+\frac{1}{20}B\left(\frac{\left(5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}-15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}-5\sqrt{2}a^{\frac{3}{2}}e^{(-}\right.}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(9*sqrt(2)*a^(3/2)*e^(-x)/(-e^(-x))^(3/2) + 9*sqrt(2)*a^(3/2)*e^(-2*x)/
(-e^(-x))^(3/2) - sqrt(2)*a^(3/2)*e^(-3*x)/(-e^(-x))^(3/2) - sqrt(2)*a^(3/2
)/(-e^(-x))^(3/2))*A + 1/20*B*((5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^(3/
2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2))*e^x/(-e^(-x))^(
3/2) - (5*sqrt(2)*a^(3/2)*e^(-x) + 15*sqrt(2)*a^(3/2)*e^(-2*x) - 5*sqrt(2)*
a^(3/2)*e^(-3*x) + sqrt(2)*a^(3/2)*e^(-4*x))/(-e^(-x))^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cosh(x))\,(a-a\cosh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a - a*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))*(a - a*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-a(\cosh(x)-1))^{\frac{3}{2}}(A+B\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral((-a*(cosh(x) - 1))**(3/2)*(A + B*cosh(x)), x)
```

3.92 $\int \sqrt{a-a\cosh(x)}\,(A+B\cosh(x))\,dx$

Optimal. Leaf size=44

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}$$

```
[Out] -2/3*a*(3*A-B)*sinh(x)/(a-a*cosh(x))^(1/2)+2/3*B*sinh(x)*(a-a*cosh(x))^(1/2
)
```

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2751, 2646}

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (-2*a*(3*A - B)*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) + (2*B*Sqrt[a - a*Cosh[x]]
*Sinh[x])/3
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\cosh(x)}\,(A+B\cosh(x))\,dx &= \frac{2}{3}B\sqrt{a-a\cosh(x)}\,\sinh(x)-\frac{1}{3}(-3A+B)\int \sqrt{a-a\cosh(x)}\,dx\\ &= -\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}+\frac{2}{3}B\sqrt{a-a\cosh(x)}\,\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.73

$$\frac{2}{3}\coth\left(\frac{x}{2}\right)\sqrt{a-a\cosh(x)}\,(3A+B\cosh(x)-2B)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a - a*Cosh[x]]*(3*A - 2*B + B*Cosh[x])*Coth[x/2])/3
```

fricas [B] time = 0.49, size = 107, normalized size = 2.43

$$\frac{\sqrt{\frac{1}{2}}\left(B\cosh(x)^3+B\sinh(x)^3+3\,(2\,A-B)\cosh(x)^2+3\,(B\cosh(x)+2\,A-B)\sinh(x)^2+3\,(2\,A-B)\cosh(x)\right.}{3\,(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A - B)*cosh(x)^2 + 3*(B*cos
h(x) + 2*A - B)*sinh(x)^2 + 3*(2*A - B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A -
 B)*cosh(x) + 2*A - B)*sinh(x) + B)*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x) +
 sinh(x))
```

giac [B] time = 0.15, size = 131, normalized size = 2.98

$$\frac{1}{6}\sqrt{2}\left(\frac{\left(6\,Aa^2e^x\mathrm{sgn}\left(-e^x+1\right)-3\,Ba^2e^x\mathrm{sgn}\left(-e^x+1\right)+Ba^2\mathrm{sgn}\left(-e^x+1\right)\right)e^{(-x)}}{\sqrt{-ae^x}\,a}-\frac{\sqrt{-ae^x}\,Ba^3e^x\mathrm{sgn}\left(-e^x+1\right)+6}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*((6*A*a^2*e^x*sgn(-e^x + 1) - 3*B*a^2*e^x*sgn(-e^x + 1) + B*a^2
*sgn(-e^x + 1))*e^(-x)/(sqrt(-a*e^x)*a) - (sqrt(-a*e^x)*B*a^3*e^x*sgn(-e^x
+ 1) + 6*sqrt(-a*e^x)*A*a^3*sgn(-e^x + 1) - 3*sqrt(-a*e^x)*B*a^3*sgn(-e^x +
 1))/a^3)
```

maple [A] time = 0.27, size = 39, normalized size = 0.89

$$-\frac{4\sinh\left(\frac{x}{2}\right)a\cosh\left(\frac{x}{2}\right)\left(2B\left(\cosh^2\left(\frac{x}{2}\right)\right)+3A-3B\right)}{3\sqrt{-2a\left(\sinh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

```
[Out] -4/3*sinh(1/2*x)*a*cosh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A-3*B)/(-2*a*sinh(1/2*x
)^2)^(1/2)
```

maxima [B] time = 0.46, size = 109, normalized size = 2.48

$$-\left(\frac{\sqrt{2}\sqrt{a}e^{(-x)}}{\sqrt{-e^{(-x)}}}+\frac{\sqrt{2}\sqrt{a}}{\sqrt{-e^{(-x)}}}\right)A+\frac{1}{6}\left(\frac{\left(3\sqrt{2}\sqrt{a}e^{(-x)}-\sqrt{2}\sqrt{a}\right)e^{x}}{\sqrt{-e^{(-x)}}}+\frac{3\sqrt{2}\sqrt{a}e^{(-x)}-\sqrt{2}\sqrt{a}e^{(-2x)}}{\sqrt{-e^{(-x)}}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*e^(-x)/sqrt(-e^(-x)) + sqrt(2)*sqrt(a)/sqrt(-e^(-x)))*A +
 1/6*((3*sqrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a))*e^x/sqrt(-e^(-x)) + (3*s
qrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a)*e^(-2*x))/sqrt(-e^(-x)))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (A+B\cosh(x))\,\sqrt{a-a\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a - a*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))*(a - a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\,(\cosh(x)-1)}\,(A+B\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral(sqrt(-a*(cosh(x) - 1))*(A + B*cosh(x)), x)
```

3.93 $\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{(A-B)\sinh(x)}{\cosh(x)+1}+Bx$$

```
[Out] B*x+(A-B)*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2648}

$$\frac{(A-B)\sinh(x)}{\cosh(x)+1}+Bx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

```
[Out] B*x + ((A - B)*Sinh[x])/(1 + Cosh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx &= Bx-(-A+B)\int \frac{1}{1+\cosh(x)}\,dx\\ &= Bx+\frac{(A-B)\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 1.28

$$\frac{\sinh(x)\left(A+Bx\coth\left(\frac{x}{2}\right)-B\right)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x]),x]
[Out] ((A - B + B*x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

fricas [A] time = 0.84, size = 29, normalized size = 1.61

$$\frac{Bx\cosh(x) + Bx\sinh(x) + Bx - 2A + 2B}{\cosh(x) + \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="fricas")
[Out] (B*x*cosh(x) + B*x*sinh(x) + B*x - 2*A + 2*B)/(cosh(x) + sinh(x) + 1)
```

giac [A] time = 0.12, size = 17, normalized size = 0.94

$$Bx - \frac{2(A - B)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="giac")
[Out] B*x - 2*(A - B)/(e^x + 1)
```

maple [A] time = 0.04, size = 34, normalized size = 1.89

$$A\tanh\left(\frac{x}{2}\right) - B\tanh\left(\frac{x}{2}\right) - B\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + B\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x)),x)
[Out] A*tanh(1/2*x)-B*tanh(1/2*x)-B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 26, normalized size = 1.44

$$B\left(x - \frac{2}{e^{(-x)} + 1}\right) + \frac{2A}{e^{(-x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="maxima")
[Out] B*(x - 2/(e^(-x) + 1)) + 2*A/(e^(-x) + 1)
```

mupad [B] time = 0.05, size = 19, normalized size = 1.06

$$B\,x-\frac{2\,A-2\,B}{{\mathrm{e}}^x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1),x)
[Out] B*x - (2*A - 2*B)/(exp(x) + 1)
```

sympy [A] time = 0.34, size = 15, normalized size = 0.83

$$A\tanh\left(\frac{x}{2}\right)+Bx-B\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x)),x)
[Out] A*tanh(x/2) + B*x - B*tanh(x/2)
```

3.94 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=35

$$\frac{(A+2B)\sinh(x)}{3(\cosh(x)+1)}+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

```
[Out] 1/3*(A-B)*sinh(x)/(1+cosh(x))^2+1/3*(A+2*B)*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2750, 2648}

$$\frac{(A+2B)\sinh(x)}{3(\cosh(x)+1)}+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

```
[Out] ((A - B)*Sinh[x])/(3*(1 + Cosh[x])^2) + ((A + 2*B)*Sinh[x])/(3*(1 + Cosh[x]
))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx &= \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2}+\frac{1}{3}(A+2B)\int \frac{1}{1+\cosh(x)}\,dx\\ &= \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2}+\frac{(A+2B)\sinh(x)}{3(1+\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.06, size = 25, normalized size = 0.71

$$\frac{\sinh(x)((A+2B)\cosh(x)+2A+B)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

```
[Out] ((2*A + B + (A + 2*B)*Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

fricas [A] time = 0.85, size = 50, normalized size = 1.43

$$-\frac{2\,((A+5\,B)\cosh(x)-(A-B)\sinh(x)+3\,A+3\,B)}{3\,\left(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*((A + 5*B)*cosh(x) - (A - B)*sinh(x) + 3*A + 3*B)/(cosh(x)^2 + 2*(cosh
(x) + 1)*sinh(x) + sinh(x)^2 + 4*cosh(x) + 3)
```

giac [A] time = 0.12, size = 30, normalized size = 0.86

$$-\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}+3\,Be^{x}+A+2\,B\right)}{3\,(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x + 3*B*e^x + A + 2*B)/(e^x + 1)^3
```

maple [A] time = 0.04, size = 34, normalized size = 0.97

$$-\frac{A\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{B\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x))^2,x)
```

```
[Out] -1/6*A*tanh(1/2*x)^3+1/6*B*tanh(1/2*x)^3+1/2*A*tanh(1/2*x)+1/2*B*tanh(1/2*x
)
```

maxima [B] time = 0.32, size = 129, normalized size = 3.69

$$\frac{2}{3}\,B\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}\right)+\frac{2}{3}\,A\left(\frac{}{3\,e^{(-x)}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*B*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 3*e^(-2*x)/(3*e^(-
x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1))
 + 2/3*A*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 1/(3*e^(-x) + 3
*e^(-2*x) + e^(-3*x) + 1))
```

mupad [B] time = 0.08, size = 30, normalized size = 0.86

$$-\frac{2\,\left(A+2\,B+3\,A\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{2\,x}\right)}{3\,{\left(\mathrm{e}^{x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1)^2,x)
```

```
[Out] -(2*(A + 2*B + 3*A*exp(x) + 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) + 1)^3)
```

sympy [A] time = 0.61, size = 36, normalized size = 1.03

$$-\frac{A\tanh^{3}\left(\frac{x}{2}\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh^{3}\left(\frac{x}{2}\right)}{6}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**2,x)
```

```
[Out] -A*tanh(x/2)**3/6 + A*tanh(x/2)/2 + B*tanh(x/2)**3/6 + B*tanh(x/2)/2
```

3.95 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx$

Optimal. Leaf size=56

$$\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)}+\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)^2}+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

```
[Out] 1/5*(A-B)*sinh(x)/(1+cosh(x))^3+1/15*(2*A+3*B)*sinh(x)/(1+cosh(x))^2+1/15*(
2*A+3*B)*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2750, 2650, 2648}

$$\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)}+\frac{(2A+3B)\sinh(x)}{15(\cosh(x)+1)^2}+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

```
[Out] ((A - B)*Sinh[x])/(5*(1 + Cosh[x])^3) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh
[x])^2) + ((2*A + 3*B)*Sinh[x])/(15*(1 + Cosh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{1}{5}(2A+3B)\int \frac{1}{(1+\cosh(x))^2}\,dx$$
$$= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{1}{15}(2A+3B)\int \frac{1}{1+\cosh(x)}\,dx$$
$$= \frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))}$$

Mathematica [A] time = 0.10, size = 42, normalized size = 0.75

$$\frac{\sinh(x)(6(2A+3B)\cosh(x)+(2A+3B)\cosh(2x)+16A+9B)}{30(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

```
[Out] ((16*A + 9*B + 6*(2*A + 3*B)*Cosh[x] + (2*A + 3*B)*Cosh[2*x])*Sinh[x])/(30*
(1 + Cosh[x])^3)
```

fricas [B] time = 1.10, size = 127, normalized size = 2.27

$$-\frac{2\left(15\,B\cosh(x)^2+15\,B\sinh(x)^2+2\,(11\,A+9\,B)\cosh(x)+6\,(5\,B\,c\right.}{15\left(\cosh(x)^4+(4\,\cosh(x)+5)\sinh(x)^3+\sinh(x)^4+5\,\cosh(x)^3+\left(6\,\cosh(x)^2+15\,\cosh(x)+10\right)\sinh(x)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A + 9*B)*cosh(x) + 6*(5*B*co
sh(x) + 3*A + 2*B)*sinh(x) + 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) + 5)*sinh
(x)^3 + sinh(x)^4 + 5*cosh(x)^3 + (6*cosh(x)^2 + 15*cosh(x) + 10)*sinh(x)^2
 + 10*cosh(x)^2 + (4*cosh(x)^3 + 15*cosh(x)^2 + 20*cosh(x) + 9)*sinh(x) + 1
1*cosh(x) + 5)
```

giac [A] time = 0.12, size = 46, normalized size = 0.82

$$-\frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}+15\,Be^{(2\,x)}+10\,Ae^{x}+15\,Be^{x}+2\,A+3\,B\right)}{15\,(e^{x}+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*B*e^(2*x) + 10*A*e^x + 15*B*e^x + 2
*A + 3*B)/(e^x + 1)^5
```

maple [A] time = 0.04, size = 38, normalized size = 0.68

$$\frac{(A-B)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{20}-\frac{A\left(\tanh^3\left(\frac{x}{2}\right)\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{4}+\frac{B\tanh\left(\frac{x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x))^3,x)
```

```
[Out] 1/20*(A-B)*tanh(1/2*x)^5-1/6*A*tanh(1/2*x)^3+1/4*A*tanh(1/2*x)+1/4*B*tanh(1
/2*x)
```

maxima [B] time = 0.34, size = 263, normalized size = 4.70

$$\frac{4}{15}A\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)}+10\,e^{(-2\,x)}+10\,e^{(-3\,x)}+5\,e^{(-4\,x)}+e^{(-5\,x)}+1}+\frac{10\,e^{(-2\,x)}}{5\,e^{(-x)}+10\,e^{(-2\,x)}+10\,e^{(-3\,x)}+5\,e^{(-4\,x)}+e^{(-5\,x)}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 4/15*A*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(-5
*x) + 1) + 10*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) +
 e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(
-5*x) + 1)) + 2/5*B*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(
-4*x) + e^(-5*x) + 1) + 5*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) +
5*e^(-4*x) + e^(-5*x) + 1) + 5*e^(-3*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*
x) + 5*e^(-4*x) + e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) +
 5*e^(-4*x) + e^(-5*x) + 1))
```

mupad [B] time = 0.92, size = 141, normalized size = 2.52

$$-\frac{\frac{4\,B\,e^{x}}{5}+\frac{8\,A\,e^{2\,x}}{5}+\frac{4\,B\,e^{3\,x}}{5}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^{x}+1}-\frac{\frac{B}{5}+\frac{4\,A\,e^{x}}{5}+\frac{3\,B\,e^{2\,x}}{5}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}-\frac{\frac{4\,A}{15}+\frac{2\,B\,e^{x}}{5}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{B}{5\,\left(e^{2\,x}+2\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1)^3,x)
```

```
[Out] - ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(10*exp(2*x) + 10*
exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1) - (B/5 + (4*A*exp(x))/5 +
```

```
(3*B*exp(2*x))/5)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) - ((4
*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - B/(5*(exp
(2*x) + 2*exp(x) + 1))
```

sympy [A] time = 1.20, size = 46, normalized size = 0.82

$$\frac{A\tanh^{5}\left(\frac{x}{2}\right)}{20}-\frac{A\tanh^{3}\left(\frac{x}{2}\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{4}-\frac{B\tanh^{5}\left(\frac{x}{2}\right)}{20}+\frac{B\tanh\left(\frac{x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**3,x)
```

```
[Out] A*tanh(x/2)**5/20 - A*tanh(x/2)**3/6 + A*tanh(x/2)/4 - B*tanh(x/2)**5/20 +
B*tanh(x/2)/4
```

3.96 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx$

Optimal. Leaf size=75

$$\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)}+\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)^2}+\frac{(3A+4B)\sinh(x)}{35(\cosh(x)+1)^3}+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

```
[Out] 1/7*(A-B)*sinh(x)/(1+cosh(x))^4+1/35*(3*A+4*B)*sinh(x)/(1+cosh(x))^3+2/105*
(3*A+4*B)*sinh(x)/(1+cosh(x))^2+2/105*(3*A+4*B)*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2750, 2650, 2648}

$$\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)}+\frac{2(3A+4B)\sinh(x)}{105(\cosh(x)+1)^2}+\frac{(3A+4B)\sinh(x)}{35(\cosh(x)+1)^3}+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

```
[Out] ((A - B)*Sinh[x])/(7*(1 + Cosh[x])^4) + ((3*A + 4*B)*Sinh[x])/(35*(1 + Cosh
[x])^3) + (2*(3*A + 4*B)*Sinh[x])/(105*(1 + Cosh[x])^2) + (2*(3*A + 4*B)*Si
nh[x])/(105*(1 + Cosh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4} + \frac{1}{7}(3A+4B)\int \frac{1}{(1+\cosh(x))^3}\,dx$$
$$= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4} + \frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3} + \frac{1}{35}(2(3A+4B))\int \frac{1}{(1+\cosh(x))^2}\,dx$$
$$= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4} + \frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3} + \frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2} + \frac{1}{105}(2(3A+4B))\int \frac{}{1+c}$$
$$= \frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4} + \frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3} + \frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2} + \frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))}$$

Mathematica [A] time = 0.11, size = 57, normalized size = 0.76

$$\frac{\sinh(x)(29(3A+4B)\cosh(x)+8(3A+4B)\cosh(2x)+3A\cosh(3x)+96A+4B\cosh(3x)+58B)}{210(\cosh(x)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

```
[Out] ((96*A + 58*B + 29*(3*A + 4*B)*Cosh[x] + 8*(3*A + 4*B)*Cosh[2*x] + 3*A*Cosh
[3*x] + 4*B*Cosh[3*x])*Sinh[x])/(210*(1 + Cosh[x])^4)
```

fricas [B] time = 0.73, size = 175, normalized size = 2.33

$$-\frac{4\left((3A+74B)\cosh(x)^2+(3A\right.}{105\left(\cosh(x)^5+(5\cosh(x)+7)\sinh(x)^4+\sinh(x)^5+7\cosh(x)^4+\left(10\cosh(x)^2+28\cosh(x)+21\right)\sinh(x)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="fricas")
```

```
[Out] -4/105*((3*A + 74*B)*cosh(x)^2 + (3*A + 74*B)*sinh(x)^2 + 14*(9*A + 7*B)*co
sh(x) - 6*((A - 22*B)*cosh(x) - 14*A - 7*B)*sinh(x) + 63*A + 84*B)/(cosh(x)
^5 + (5*cosh(x) + 7)*sinh(x)^4 + sinh(x)^5 + 7*cosh(x)^4 + (10*cosh(x)^2 +
28*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 + 42*cosh(x)^2 +
63*cosh(x) + 36)*sinh(x)^2 + 36*cosh(x)^2 + (5*cosh(x)^4 + 28*cosh(x)^3 + 6
3*cosh(x)^2 + 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) + 21)
```

giac [A] time = 0.12, size = 60, normalized size = 0.80

$$-\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}+70\,Be^{(3\,x)}+63\,Ae^{(2\,x)}+84\,Be^{(2\,x)}+21\,Ae^{x}+28\,Be^{x}+3\,A+4\,B\right)}{105\,(e^{x}+1)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*B*e^(3*x) + 63*A*e^(2*x) + 84*B*e
^(2*x) + 21*A*e^x + 28*B*e^x + 3*A + 4*B)/(e^x + 1)^7
```

maple [A] time = 0.04, size = 55, normalized size = 0.73

$$-\frac{(A-B)\left(\tanh^7\left(\frac{x}{2}\right)\right)}{56}-\frac{(-3A+B)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{40}-\frac{(3A+B)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1+cosh(x))^4,x)
```

```
[Out] -1/56*(A-B)*tanh(1/2*x)^7-1/40*(-3*A+B)*tanh(1/2*x)^5-1/24*(3*A+B)*tanh(1/2
*x)^3+1/8*A*tanh(1/2*x)+1/8*B*tanh(1/2*x)
```

maxima [B] time = 0.34, size = 449, normalized size = 5.99

$$\frac{8}{105}B\left(\frac{14\,e^{(-x)}}{7\,e^{(-x)}+21\,e^{(-2\,x)}+35\,e^{(-3\,x)}+35\,e^{(-4\,x)}+21\,e^{(-5\,x)}+7\,e^{(-6\,x)}+e^{(-7\,x)}+1}+\frac{}{7\,e^{(-x)}+21\,e^{(-2\,x)}+35\,e^{(-3\,x)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="maxima")
```

```
[Out] 8/105*B*(14*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21
*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 42*e^(-2*x)/(7*e^(-x) + 21*e^(-2*x
) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) +
35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*
x) + 7*e^(-6*x) + e^(-7*x) + 1) + 35*e^(-4*x)/(7*e^(-x) + 21*e^(-2*x) + 35*
e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 2/(7*e^
(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) +
 e^(-7*x) + 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) +
35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 21*e^(-2*x)/(7*e^(
-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) +
e^(-7*x) + 1) + 35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-
4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 1/(7*e^(-x) + 21*e^(-2*x)
 + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1))
```

mupad [B] time = 0.92, size = 231, normalized size = 3.08

$$-\frac{\frac{4\,A}{35}+\frac{8\,B\,e^{x}}{35}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}-\frac{8\,B}{105\,\left(3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1\right)}-\frac{\frac{16\,A\,e^{3\,x}}{7}+\frac{8\,B\,e^{2\,x}}{7}+\frac{8\,B\,e^{4\,x}}{7}}{21\,e^{2\,x}+35\,e^{3\,x}+35\,e^{4\,x}+21\,e^{5\,x}+7\,e^{6\,x}+e^{7\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) + 1)^4,x)
```

```
[Out] - ((4*A)/35 + (8*B*exp(x))/35)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(
x) + 1) - (8*B)/(105*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1)) - ((16*A*exp(3
*x))/7 + (8*B*exp(2*x))/7 + (8*B*exp(4*x))/7)/(21*exp(2*x) + 35*exp(3*x) +
35*exp(4*x) + 21*exp(5*x) + 7*exp(6*x) + exp(7*x) + 7*exp(x) + 1) - ((8*B*e
xp(x))/21 + (8*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) + 20*exp(3*
x) + 15*exp(4*x) + 6*exp(5*x) + exp(6*x) + 6*exp(x) + 1) - ((8*B)/105 + (16
*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x)
 + exp(5*x) + 5*exp(x) + 1)
```

sympy [A] time = 2.36, size = 78, normalized size = 1.04

$$-\frac{A\tanh^{7}\left(\frac{x}{2}\right)}{56}+\frac{3A\tanh^{5}\left(\frac{x}{2}\right)}{40}-\frac{A\tanh^{3}\left(\frac{x}{2}\right)}{8}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh^{7}\left(\frac{x}{2}\right)}{56}-\frac{B\tanh^{5}\left(\frac{x}{2}\right)}{40}-\frac{B\tanh^{3}\left(\frac{x}{2}\right)}{24}+\frac{B\tanh\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1+cosh(x))**4,x)
```

```
[Out] -A*tanh(x/2)**7/56 + 3*A*tanh(x/2)**5/40 - A*tanh(x/2)**3/8 + A*tanh(x/2)/8
 + B*tanh(x/2)**7/56 - B*tanh(x/2)**5/40 - B*tanh(x/2)**3/24 + B*tanh(x/2)/
8
```

3.97 $\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx$

Optimal. Leaf size=20

$$-\frac{(A+B)\sinh(x)}{1-\cosh(x)}-Bx$$

```
[Out] -B*x-(A+B)*sinh(x)/(1-cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2735, 2648}

$$-\frac{(A+B)\sinh(x)}{1-\cosh(x)}-Bx$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

```
[Out] -(B*x) - ((A + B)*Sinh[x])/(1 - Cosh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx &= -Bx-(-A-B)\int \frac{1}{1-\cosh(x)}\,dx\\ &= -Bx-\frac{(A+B)\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 1.75

$$\frac{2\sinh\left(\frac{x}{2}\right)\left((A+B)\cosh\left(\frac{x}{2}\right)-Bx\sinh\left(\frac{x}{2}\right)\right)}{\cosh(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x]),x]
[Out] (2*Sinh[x/2]*((A + B)*Cosh[x/2] - B*x*Sinh[x/2]))/(-1 + Cosh[x])
```

fricas [A] time = 0.72, size = 31, normalized size = 1.55

$$-\frac{Bx\cosh(x) + Bx\sinh(x) - Bx - 2A - 2B}{\cosh(x) + \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="fricas")
[Out] -(B*x*cosh(x) + B*x*sinh(x) - B*x - 2*A - 2*B)/(cosh(x) + sinh(x) - 1)
```

giac [A] time = 0.13, size = 16, normalized size = 0.80

$$-Bx + \frac{2(A+B)}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="giac")
[Out] -B*x + 2*(A + B)/(e^x - 1)
```

maple [A] time = 0.06, size = 37, normalized size = 1.85

$$B\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - B\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + \frac{A}{\tanh\left(\frac{x}{2}\right)} + \frac{B}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x)),x)
[Out] B*ln(tanh(1/2*x)-1)-B*ln(tanh(1/2*x)+1)+1/tanh(1/2*x)*A+1/tanh(1/2*x)*B
```

maxima [A] time = 0.31, size = 27, normalized size = 1.35

$$-B\left(x + \frac{2}{e^{(-x)} - 1}\right) - \frac{2A}{e^{(-x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="maxima")
[Out] -B*(x + 2/(e^(-x) - 1)) - 2*A/(e^(-x) - 1)
```

mupad [B] time = 0.05, size = 19, normalized size = 0.95

$$\frac{2\,A+2\,B}{\mathrm{e}^x-1}-B\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*cosh(x))/(cosh(x) - 1),x)
[Out] (2*A + 2*B)/(exp(x) - 1) - B*x
```

sympy [A] time = 0.49, size = 15, normalized size = 0.75

$$\frac{A}{\tanh\left(\frac{x}{2}\right)}-Bx+\frac{B}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x)),x)
[Out] A/tanh(x/2) - B*x + B/tanh(x/2)
```

3.98 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx$

Optimal. Leaf size=37

$$-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

```
[Out] -1/3*(A+B)*sinh(x)/(1-cosh(x))^2-1/3*(A-2*B)*sinh(x)/(1-cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2750, 2648}

$$-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

```
[Out] -((A + B)*Sinh[x])/(3*(1 - Cosh[x])^2) - ((A - 2*B)*Sinh[x])/(3*(1 - Cosh[x
]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx &= -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}(A-2B)\int \frac{1}{1-\cosh(x)}\,dx\\ &= -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.06, size = 25, normalized size = 0.68

$$\frac{\sinh(x)((A-2B)\cosh(x)-2A+B)}{3(\cosh(x)-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

```
[Out] ((-2*A + B + (A - 2*B)*Cosh[x])*Sinh[x])/(3*(-1 + Cosh[x])^2)
```

fricas [A] time = 1.20, size = 48, normalized size = 1.30

$$\frac{2\,((A-5\,B)\cosh(x)-(A+B)\sinh(x)-3\,A+3\,B)}{3\,\left(\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] 2/3*((A - 5*B)*cosh(x) - (A + B)*sinh(x) - 3*A + 3*B)/(cosh(x)^2 + 2*(cosh(
x) - 1)*sinh(x) + sinh(x)^2 - 4*cosh(x) + 3)
```

giac [A] time = 0.11, size = 32, normalized size = 0.86

$$-\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}-3\,Be^{x}-A+2\,B\right)}{3\,(e^{x}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x - 3*B*e^x - A + 2*B)/(e^x - 1)^3
```

maple [A] time = 0.06, size = 26, normalized size = 0.70

$$-\frac{A+B}{6\tanh\left(\frac{x}{2}\right)^3}-\frac{-A+B}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^2,x)
```

```
[Out] -1/6*(A+B)/tanh(1/2*x)^3-1/2*(-A+B)/tanh(1/2*x)
```

maxima [B] time = 0.32, size = 131, normalized size = 3.54

$$-\frac{2}{3}\,B\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{2}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}\right)+\frac{2}{3}\,A\left(\frac{}{3\,e^{(-x)}\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*B*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 3*e^(-2*x)/(3*e^(
-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1)
) + 2/3*A*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 1/(3*e^(-x) -
3*e^(-2*x) + e^(-3*x) - 1))
```

mupad [B] time = 0.93, size = 32, normalized size = 0.86

$$-\frac{2\left(2B - A + 3Ae^{x} - 3Be^{x} + 3Be^{2x}\right)}{3\left(e^{x}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) - 1)^2,x)
```

```
[Out] -(2*(2*B - A + 3*A*exp(x) - 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) - 1)^3)
```

sympy [A] time = 0.84, size = 36, normalized size = 0.97

$$\frac{A}{2\tanh\left(\frac{x}{2}\right)} - \frac{A}{6\tanh^{3}\left(\frac{x}{2}\right)} - \frac{B}{2\tanh\left(\frac{x}{2}\right)} - \frac{B}{6\tanh^{3}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**2,x)
```

```
[Out] A/(2*tanh(x/2)) - A/(6*tanh(x/2)**3) - B/(2*tanh(x/2)) - B/(6*tanh(x/2)**3)
```

3.99 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

```
[Out] -1/5*(A+B)*sinh(x)/(1-cosh(x))^3-1/15*(2*A-3*B)*sinh(x)/(1-cosh(x))^2-1/15*
(2*A-3*B)*sinh(x)/(1-cosh(x))
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2750, 2650, 2648}

$$-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

```
[Out] -((A + B)*Sinh[x])/(5*(1 - Cosh[x])^3) - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cos
h[x])^2) - ((2*A - 3*B)*Sinh[x])/(15*(1 - Cosh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}+\frac{1}{5}(2A-3B)\int\frac{1}{(1-\cosh(x))^2}\,dx$$
$$= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}+\frac{1}{15}(2A-3B)\int\frac{1}{1-\cosh(x)}\,dx$$
$$= -\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.70

$$\frac{\sinh(x)(-6(2A-3B)\cosh(x)+(2A-3B)\cosh(2x)+16A-9B)}{30(\cosh(x)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

```
[Out] ((16*A - 9*B - 6*(2*A - 3*B)*Cosh[x] + (2*A - 3*B)*Cosh[2*x])*Sinh[x])/(30*
(-1 + Cosh[x])^3)
```

fricas [B] time = 1.49, size = 127, normalized size = 2.12

$$\frac{2\left(15\,B\cosh(x)^2+15\,B\sinh(x)^2+2\,(11\,A-9\,B)\cosh(x)+6\,(5\,B\,\mathrm{co}\right.}{15\left(\cosh(x)^4+(4\,\cosh(x)-5)\sinh(x)^3+\sinh(x)^4-5\,\cosh(x)^3+\left(6\,\cosh(x)^2-15\,\cosh(x)+10\right)\sinh(x)^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A - 9*B)*cosh(x) + 6*(5*B*cos
h(x) + 3*A - 2*B)*sinh(x) - 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) - 5)*sinh(
x)^3 + sinh(x)^4 - 5*cosh(x)^3 + (6*cosh(x)^2 - 15*cosh(x) + 10)*sinh(x)^2
+ 10*cosh(x)^2 + (4*cosh(x)^3 - 15*cosh(x)^2 + 20*cosh(x) - 9)*sinh(x) - 11
*cosh(x) + 5)
```

giac [A] time = 0.11, size = 46, normalized size = 0.77

$$\frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}-15\,Be^{(2\,x)}-10\,Ae^{x}+15\,Be^{x}+2\,A-3\,B\right)}{15\,(e^{x}-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*B*e^(2*x) - 10*A*e^x + 15*B*e^x + 2*
A - 3*B)/(e^x - 1)^5
```

maple [A] time = 0.06, size = 39, normalized size = 0.65

$$-\frac{A}{6\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-A+B}{4\tanh\left(\frac{x}{2}\right)}-\frac{-A-B}{20\tanh\left(\frac{x}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^3,x)
```

```
[Out] -1/6*A/tanh(1/2*x)^3-1/4*(-A+B)/tanh(1/2*x)-1/20*(-A-B)/tanh(1/2*x)^5
```

maxima [B] time = 0.33, size = 267, normalized size = 4.45

$$-\frac{2}{5}B\left(\frac{5e^{(-x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}-\frac{5e^{(-2x)}}{5e^{(-x)}-10e^{(-2x)}+10e^{(-3x)}-5e^{(-4x)}+e^{(-5x)}-1}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*B*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) + e^(-5
*x) - 1) - 5*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) +
e^(-5*x) - 1) + 5*e^(-3*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*
x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) +
 e^(-5*x) - 1)) + 4/15*A*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) -
5*e^(-4*x) + e^(-5*x) - 1) - 10*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3
*x) - 5*e^(-4*x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x)
- 5*e^(-4*x) + e^(-5*x) - 1))
```

mupad [B] time = 0.08, size = 143, normalized size = 2.38

$$\frac{\frac{B}{5}+\frac{4A\mathrm{e}^{x}}{5}+\frac{3B\mathrm{e}^{2x}}{5}}{6\mathrm{e}^{2x}-4\mathrm{e}^{3x}+\mathrm{e}^{4x}-4\mathrm{e}^{x}+1}-\frac{\frac{4A}{15}+\frac{2B\mathrm{e}^{x}}{5}}{3\mathrm{e}^{2x}-\mathrm{e}^{3x}-3\mathrm{e}^{x}+1}-\frac{\frac{4B\mathrm{e}^{x}}{5}+\frac{8A\mathrm{e}^{2x}}{5}+\frac{4B\mathrm{e}^{3x}}{5}}{10\mathrm{e}^{2x}-10\mathrm{e}^{3x}+5\mathrm{e}^{4x}-\mathrm{e}^{5x}-5\mathrm{e}^{x}+1}+\frac{B}{5\left(\mathrm{e}^{2x}-2\mathrm{e}^{x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*cosh(x))/(cosh(x) - 1)^3,x)
```

```
[Out] (B/5 + (4*A*exp(x))/5 + (3*B*exp(2*x))/5)/(6*exp(2*x) - 4*exp(3*x) + exp(4*
x) - 4*exp(x) + 1) - ((4*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) - exp(3*x) - 3
*exp(x) + 1) - ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(10*e
```

```
xp(2*x) - 10*exp(3*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) + B/(5*(exp(2
*x) - 2*exp(x) + 1))
```

sympy [A] time = 1.46, size = 46, normalized size = 0.77

$$\frac{A}{4\tanh\left(\frac{x}{2}\right)} - \frac{A}{6\tanh^{3}\left(\frac{x}{2}\right)} + \frac{A}{20\tanh^{5}\left(\frac{x}{2}\right)} - \frac{B}{4\tanh\left(\frac{x}{2}\right)} + \frac{B}{20\tanh^{5}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**3,x)
```

```
[Out] A/(4*tanh(x/2)) - A/(6*tanh(x/2)**3) + A/(20*tanh(x/2)**5) - B/(4*tanh(x/2)
) + B/(20*tanh(x/2)**5)
```

3.100 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$

Optimal. Leaf size=81

$$-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

```
[Out] -1/7*(A+B)*sinh(x)/(1-cosh(x))^4-1/35*(3*A-4*B)*sinh(x)/(1-cosh(x))^3-2/105
*(3*A-4*B)*sinh(x)/(1-cosh(x))^2-2/105*(3*A-4*B)*sinh(x)/(1-cosh(x))
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2750, 2650, 2648}

$$-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}-\frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2}-\frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3}-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

```
[Out] -((A + B)*Sinh[x])/(7*(1 - Cosh[x])^4) - ((3*A - 4*B)*Sinh[x])/(35*(1 - Cos
h[x])^3) - (2*(3*A - 4*B)*Sinh[x])/(105*(1 - Cosh[x])^2) - (2*(3*A - 4*B)*S
inh[x])/(105*(1 - Cosh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4} + \frac{1}{7}(3A-4B)\int \frac{1}{(1-\cosh(x))^3}\,dx$$
$$= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4} - \frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3} + \frac{1}{35}(2(3A-4B))\int \frac{1}{(1-\cosh(x))^2}\,dx$$
$$= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4} - \frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3} - \frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2} + \frac{1}{105}(2(3A-4B))\int \frac{}{1-}$$
$$= -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4} - \frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3} - \frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2} - \frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}$$

Mathematica [A] time = 0.10, size = 57, normalized size = 0.70

$$\frac{\sinh(x)(29(3A-4B)\cosh(x)-8(3A-4B)\cosh(2x)+3A\cosh(3x)-96A-4B\cosh(3x)+58B)}{210(\cosh(x)-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

```
[Out] ((-96*A + 58*B + 29*(3*A - 4*B)*Cosh[x] - 8*(3*A - 4*B)*Cosh[2*x] + 3*A*Cos
h[3*x] - 4*B*Cosh[3*x])*Sinh[x])/(210*(-1 + Cosh[x])^4)
```

fricas [B] time = 0.84, size = 175, normalized size = 2.16

$$\frac{4\left((3\,A-74\,B)\cosh(x)^2+(3\,A-\right.}{105\left(\cosh(x)^5+(5\,\cosh(x)-7)\sinh(x)^4+\sinh(x)^5-7\,\cosh(x)^4+\left(10\,\cosh(x)^2-28\,\cosh(x)+21\right)\sinh(x)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="fricas")
```

```
[Out] 4/105*((3*A - 74*B)*cosh(x)^2 + (3*A - 74*B)*sinh(x)^2 - 14*(9*A - 7*B)*cos
h(x) - 6*((A + 22*B)*cosh(x) + 14*A - 7*B)*sinh(x) + 63*A - 84*B)/(cosh(x)^
5 + (5*cosh(x) - 7)*sinh(x)^4 + sinh(x)^5 - 7*cosh(x)^4 + (10*cosh(x)^2 - 2
8*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 - 42*cosh(x)^2 + 6
3*cosh(x) - 36)*sinh(x)^2 - 36*cosh(x)^2 + (5*cosh(x)^4 - 28*cosh(x)^3 + 63
*cosh(x)^2 - 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) - 21)
```

giac [A] time = 0.12, size = 60, normalized size = 0.74

$$-\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}-70\,Be^{(3\,x)}-63\,Ae^{(2\,x)}+84\,Be^{(2\,x)}+21\,Ae^{x}-28\,Be^{x}-3\,A+4\,B\right)}{105\,(e^{x}-1)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*B*e^(3*x) - 63*A*e^(2*x) + 84*B*e
^(2*x) + 21*A*e^x - 28*B*e^x - 3*A + 4*B)/(e^x - 1)^7
```

maple [A] time = 0.06, size = 56, normalized size = 0.69

$$-\frac{A+B}{56\tanh\left(\frac{x}{2}\right)^{7}}-\frac{3A-B}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-A+B}{8\tanh\left(\frac{x}{2}\right)}-\frac{-3A-B}{40\tanh\left(\frac{x}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(1-cosh(x))^4,x)
```

```
[Out] -1/56*(A+B)/tanh(1/2*x)^7-1/24*(3*A-B)/tanh(1/2*x)^3-1/8*(-A+B)/tanh(1/2*x)
-1/40*(-3*A-B)/tanh(1/2*x)^5
```

maxima [B] time = 0.34, size = 451, normalized size = 5.57

$$-\frac{8}{105}B\left(\frac{14\,e^{(-x)}}{7\,e^{(-x)}-21\,e^{(-2\,x)}+35\,e^{(-3\,x)}-35\,e^{(-4\,x)}+21\,e^{(-5\,x)}-7\,e^{(-6\,x)}+e^{(-7\,x)}-1}-\frac{}{7\,e^{(-x)}-21\,e^{(-2\,x)}+35\,e^{(-3\,x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="maxima")
```

```
[Out] -8/105*B*(14*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 2
1*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 42*e^(-2*x)/(7*e^(-x) - 21*e^(-2*
x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) +
 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5
*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 35*e^(-4*x)/(7*e^(-x) - 21*e^(-2*x) + 35
*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 2/(7*e
^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x)
+ e^(-7*x) - 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) -
 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 21*e^(-2*x)/(7*e^
(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) +
 e^(-7*x) - 1) + 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(
-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 1/(7*e^(-x) - 21*e^(-2*x
) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1))
```

mupad [B] time = 0.92, size = 233, normalized size = 2.88

$$\frac{\frac{8\,B}{105}+\frac{16\,A\,e^{x}}{35}+\frac{16\,B\,e^{2\,x}}{35}}{10\,e^{2\,x}-10\,e^{3\,x}+5\,e^{4\,x}-e^{5\,x}-5\,e^{x}+1}-\frac{\frac{4\,A}{35}+\frac{8\,B\,e^{x}}{35}}{6\,e^{2\,x}-4\,e^{3\,x}+e^{4\,x}-4\,e^{x}+1}-\frac{\frac{8\,B\,e^{x}}{21}+\frac{8\,A\,e^{2\,x}}{7}+\frac{16\,B\,e^{3\,x}}{21}}{15\,e^{2\,x}-20\,e^{3\,x}+15\,e^{4\,x}-6\,e^{5\,x}+e^{6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(cosh(x) - 1)^4,x)
```

```
[Out] ((8*B)/105 + (16*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) - 10*exp(3
*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) - ((4*A)/35 + (8*B*exp(x))/35)/
(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1) - ((8*B*exp(x))/21 + (8
*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) - 20*exp(3*x) + 15*exp(4*
x) - 6*exp(5*x) + exp(6*x) - 6*exp(x) + 1) + (8*B)/(105*(3*exp(2*x) - exp(3
*x) - 3*exp(x) + 1)) + ((16*A*exp(3*x))/7 + (8*B*exp(2*x))/7 + (8*B*exp(4*x
))/7)/(21*exp(2*x) - 35*exp(3*x) + 35*exp(4*x) - 21*exp(5*x) + 7*exp(6*x) -
 exp(7*x) - 7*exp(x) + 1)
```

sympy [A] time = 2.72, size = 78, normalized size = 0.96

$$\frac{A}{8\tanh\left(\frac{x}{2}\right)}-\frac{A}{8\tanh^{3}\left(\frac{x}{2}\right)}+\frac{3A}{40\tanh^{5}\left(\frac{x}{2}\right)}-\frac{A}{56\tanh^{7}\left(\frac{x}{2}\right)}-\frac{B}{8\tanh\left(\frac{x}{2}\right)}+\frac{B}{24\tanh^{3}\left(\frac{x}{2}\right)}+\frac{B}{40\tanh^{5}\left(\frac{x}{2}\right)}-\frac{B}{56\tanh^{7}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(1-cosh(x))**4,x)
```

```
[Out] A/(8*tanh(x/2)) - A/(8*tanh(x/2)**3) + 3*A/(40*tanh(x/2)**5) - A/(56*tanh(x
/2)**7) - B/(8*tanh(x/2)) + B/(24*tanh(x/2)**3) + B/(40*tanh(x/2)**5) - B/(
56*tanh(x/2)**7)
```

3.101 $\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

```
[Out] (A-B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(1/2)/a^(1/
2)+2*B*sinh(x)/(a+a*cosh(x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/S
qrt[a] + (2*B*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}} + (A-B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx$$
$$= \frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}} + (2i(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)$$
$$= \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}} + \frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.73

$$\frac{2\cosh\left(\frac{x}{2}\right)\left((A-B)\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)+2B\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(\cosh(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

```
[Out] (2*Cosh[x/2]*((A - B)*ArcTan[Sinh[x/2]] + 2*B*Sinh[x/2]))/Sqrt[a*(1 + Cosh[
x])]
```

fricas [A] time = 3.97, size = 72, normalized size = 1.29

$$\frac{2\left(\sqrt{2}\,(A-B)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\,(\cosh(x)+\sinh(x))}{\sqrt{a}}\right)+\sqrt{\frac{1}{2}}\,(B\cosh(x)+B\sinh(x)-B)\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(
x)))*(cosh(x) + sinh(x))/sqrt(a)) + sqrt(1/2)*(B*cosh(x) + B*sinh(x) - B)*s
qrt(a/(cosh(x) + sinh(x))))/a
```

giac [C] time = 0.15, size = 61, normalized size = 1.09

$$\frac{1}{4}\sqrt{2}\left(\frac{8\,(A-B)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{4\,Be^{\left(\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{4\,Be^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}+\frac{-8i\,A\arctan(-i)+8i\,B\arctan(-i)-8\,B}{\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(8*(A - B)*arctan(e^(1/2*x))/sqrt(a) + 4*B*e^(1/2*x)/sqrt(a) -
4*B*e^(-1/2*x)/sqrt(a) + (-8*I*A*arctan(-I) + 8*I*B*arctan(-I) - 8*B)/sqrt(
-a))
```

maple [B] time = 0.32, size = 128, normalized size = 2.29

$$-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(\ln\left(\frac{2\sqrt{a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)aA-2B\sqrt{a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\sqrt{-a}-\ln\left(\frac{2\sqrt{a\left(\sinh^2\left(\frac{x}{2}\right)\right)}\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)a}{a\sqrt{-a}\sinh\left(\frac{x}{2}\right)\sqrt{a\left(\cosh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x)
```

```
[Out] -cosh(1/2*x)*(a*sinh(1/2*x)^2)^(1/2)*(ln(2/cosh(1/2*x)*((a*sinh(1/2*x)^2)^(
1/2)*(-a)^(1/2)-a))*a*A-2*B*(a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2)-ln(2/cosh(1/
2*x)*((a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2)-a))*a*B)/a/(-a)^(1/2)/sinh(1/2*x)*
2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

maxima [B] time = 0.62, size = 174, normalized size = 3.11

$$2\left(\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)A-\frac{1}{3}\left(3\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\sqrt{2}\left(\frac{3\,\mathrm{arc}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) + e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))
- sqrt(2)*e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))*A - 1/3*(3*sqrt(2)*(arctan(e^(
1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) - sqrt(2)*(3*arctan(e^
(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*x)/(sqrt(a)*e^(-x) + sqr
t(a))) - (3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))/(a*e^x
+ a))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + a*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cosh(x)}{\sqrt{a(\cosh(x) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(a*(cosh(x) + 1)), x)
```

3.102 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

```
[Out] 1/2*(A-B)*sinh(x)/(a+a*cosh(x))^(3/2)+1/4*(A+3*B)*arctan(1/2*sinh(x)*a^(1/2
)*2^(1/2)/(a+a*cosh(x))^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2750, 2649, 206}

$$\frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(2*Sqrt
[2]*a^(3/2)) + ((A - B)*Sinh[x])/(2*(a + a*Cosh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}} + \frac{(A+3B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx}{4a}$$
$$= \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}} + \frac{(i(A+3B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia\sinh(x)}{\sqrt{a+a\cosh(x)}}\right)}{2a}$$
$$= \frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}} + \frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}$$

Mathematica [**A**] time = 0.08, size = 44, normalized size = 0.68

$$\frac{\frac{1}{2}(A-B)\sinh(x)+(A+3B)\cosh^3\left(\frac{x}{2}\right)\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)}{(a(\cosh(x)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^3 + ((A - B)*Sinh[x])/2)/(a*(1 + Cos
h[x]))^(3/2)
```

fricas [**B**] time = 1.04, size = 189, normalized size = 2.91

$$-\frac{\sqrt{2}\left((A+3B)\cosh(x)^2+(A+3B)\sinh(x)^2+2(A+3B)\cosh(x)+2((A+3B)\cosh(x)+A+3B)\sinh(x)+\right.}{2\left(a^2\cosh(x)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A + 3*B)*cosh(x)^2 + (A + 3*B)*sinh(x)^2 + 2*(A + 3*B)*cosh
(x) + 2*((A + 3*B)*cosh(x) + A + 3*B)*sinh(x) + A + 3*B)*sqrt(a)*arctan(sqr
t(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))/sqrt(a)) - 2*sqrt(1/2)*((A - B)*
cosh(x)^2 + (A - B)*sinh(x)^2 - (A - B)*cosh(x) + (2*(A - B)*cosh(x) - A +
B)*sinh(x))*sqrt(a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2
*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) + a^2)*sinh(x))
```

giac [A] time = 0.18, size = 78, normalized size = 1.20

$$\frac{\left(\sqrt{2}\,A+3\,\sqrt{2}\,B\right)\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,a^{\frac{3}{2}}}+\frac{\sqrt{2}\left(Aa^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Ba^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Aa^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}+Ba^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,(ae^{x}+a)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*A + 3*sqrt(2)*B)*arctan(e^(1/2*x))/a^(3/2) + 1/2*sqrt(2)*(A*a^
(3/2)*e^(3/2*x) - B*a^(3/2)*e^(3/2*x) - A*a^(3/2)*e^(1/2*x) + B*a^(3/2)*e^(
1/2*x))/((a*e^x + a)^2*a)
```

maple [B] time = 0.33, size = 159, normalized size = 2.45

$$-\frac{\sqrt{a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\left(A\ln\left(\frac{2\sqrt{a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\left(\cosh^{2}\left(\frac{x}{2}\right)\right)a+3B\ln\left(\frac{2\sqrt{a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)a\left(\cosh^{2}\left(\frac{x}{2}\right)\right)-A\sqrt{a}}{4\cosh\left(\frac{x}{2}\right)a^{2}\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\left(\cosh^{2}\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x)
```

```
[Out] -1/4*(a*sinh(1/2*x)^2)^(1/2)*(A*ln(2/cosh(1/2*x)*((a*sinh(1/2*x)^2)^(1/2)*(
-a)^(1/2)-a))*cosh(1/2*x)^2*a+3*B*ln(2/cosh(1/2*x)*((a*sinh(1/2*x)^2)^(1/2)
*(-a)^(1/2)-a))*a*cosh(1/2*x)^2-A*(a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2)+B*(a*s
inh(1/2*x)^2)^(1/2)*(-a)^(1/2))/cosh(1/2*x)/a^2/(-a)^(1/2)/sinh(1/2*x)*2^(1
/2)/(a*cosh(1/2*x)^2)^(1/2)
```

maxima [B] time = 0.58, size = 300, normalized size = 4.62

$$\frac{1}{6}\left(\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,x\right)}+8\,e^{\left(\frac{3}{2}\,x\right)}-3\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{3}{2}}}\right)-\frac{8\,\sqrt{2}\,e^{\left(\frac{3}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}\right)A+\frac{1}{20}\left(\sqrt{2}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
 - 8*sqrt(2)*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x
```

```
 + a^(3/2)))*A + 1/20*(sqrt(2)*((15*e^(5/2*x) + 40*e^(3/2*x) + 33*e^(1/2*x)
)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 15*arct
an(e^(1/2*x))/a^(3/2)) + 5*sqrt(2)*((3*e^(5/2*x) - 8*e^(3/2*x) - 3*e^(1/2*x
))/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arct
an(e^(1/2*x))/a^(3/2)) - 8*(5*sqrt(2)*sqrt(a)*e^(5/2*x) + sqrt(2)*sqrt(a)*e
^(1/2*x))/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^2*e^x + a^2))*B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + a*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + a*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cosh(x)}{(a(\cosh(x) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral((A + B*cosh(x))/(a*(cosh(x) + 1))**(3/2), x)
```

3.103 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(3A+5B)\sinh(x)}{16a(a\cosh(x)+a)^{3/2}}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

```
[Out] 1/4*(A-B)*sinh(x)/(a+a*cosh(x))^(5/2)+1/16*(3*A+5*B)*sinh(x)/a/(a+a*cosh(x)
)^(3/2)+1/32*(3*A+5*B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a+a*cosh(x))^(1/
2))/a^(5/2)*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2750, 2650, 2649, 206}

$$\frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(3A+5B)\sinh(x)}{16a(a\cosh(x)+a)^{3/2}}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(16*S
qrt[2]*a^(5/2)) + ((A - B)*Sinh[x])/(4*(a + a*Cosh[x])^(5/2)) + ((3*A + 5*B
)*Sinh[x])/(16*a*(a + a*Cosh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx &= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\cosh(x))^{3/2}}\,dx}{8a} \\
&= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\cosh(x)}}\,dx}{32a^2} \\
&= \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}} + \frac{(i(3A+5B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{ia}{\sqrt{a+}}\right.}{16a^2} \\
&= \frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{16\sqrt{2}\,a^{5/2}} + \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 57, normalized size = 0.61

$$\frac{\sinh(x)((3A+5B)\cosh(x)+7A+B)+4(3A+5B)\cosh^5\left(\frac{x}{2}\right)\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)}{16(a(\cosh(x)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

```
[Out] (4*(3*A + 5*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^5 + (7*A + B + (3*A + 5*B)*Cosh[
x])*Sinh[x])/(16*(a*(1 + Cosh[x]))^(5/2))
```

fricas [B] time = 0.85, size = 509, normalized size = 5.47

$$-\frac{\sqrt{2}\left((3\,A+5\,B)\cosh(x)^4+(3\,A+5\,B)\sinh(x)^4+4\,(3\,A+5\,B)\cosh(x)^3+4\,((3\,A+5\,B)\cosh(x)+3\,A+5\,B)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + 4*(3*A + 5*
B)*cosh(x)^3 + 4*((3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x)^3 + 6*(3*A + 5*B
)*cosh(x)^2 + 6*((3*A + 5*B)*cosh(x)^2 + 2*(3*A + 5*B)*cosh(x) + 3*A + 5*B)
*sinh(x)^2 + 4*(3*A + 5*B)*cosh(x) + 4*((3*A + 5*B)*cosh(x)^3 + 3*(3*A + 5*
B)*cosh(x)^2 + 3*(3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x) + 3*A + 5*B)*sqrt
(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))/sqrt(a)) - 2*sqrt(
1/2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + (11*A - 3*B)*cosh(x)^
3 + (4*(3*A + 5*B)*cosh(x) + 11*A - 3*B)*sinh(x)^3 - (11*A - 3*B)*cosh(x)^2
 + (6*(3*A + 5*B)*cosh(x)^2 + 3*(11*A - 3*B)*cosh(x) - 11*A + 3*B)*sinh(x)^
2 - (3*A + 5*B)*cosh(x) + (4*(3*A + 5*B)*cosh(x)^3 + 3*(11*A - 3*B)*cosh(x)
^2 - 2*(11*A - 3*B)*cosh(x) - 3*A - 5*B)*sinh(x))*sqrt(a/(cosh(x) + sinh(x)
)))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 + 4*a^3*cosh(x)^3 + 6*a^3*cosh(x)^2 + 4*
a^3*cosh(x) + 4*(a^3*cosh(x) + a^3)*sinh(x)^3 + a^3 + 6*(a^3*cosh(x)^2 + 2*
a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + 3*a^3*cosh(x)^2 + 3*a^3*c
osh(x) + a^3)*sinh(x))
```

giac [A] time = 0.22, size = 118, normalized size = 1.27

$$\frac{\sqrt{2}\,(3\,A+5\,B)\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{16\,a^{\frac{5}{2}}}+\frac{\sqrt{2}\left(3\,Aa^{\frac{7}{2}}e^{\left(\frac{7}{2}\,x\right)}+5\,Ba^{\frac{7}{2}}e^{\left(\frac{7}{2}\,x\right)}+11\,Aa^{\frac{7}{2}}e^{\left(\frac{5}{2}\,x\right)}-3\,Ba^{\frac{7}{2}}e^{\left(\frac{5}{2}\,x\right)}-11\,Aa^{\frac{7}{2}}e^{\left(\frac{3}{2}\,x\right)}+3\,Ba}{16\,(ae^{x}+a)^{4}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*A + 5*B)*arctan(e^(1/2*x))/a^(5/2) + 1/16*sqrt(2)*(3*A*a^(7
/2)*e^(7/2*x) + 5*B*a^(7/2)*e^(7/2*x) + 11*A*a^(7/2)*e^(5/2*x) - 3*B*a^(7/2
)*e^(5/2*x) - 11*A*a^(7/2)*e^(3/2*x) + 3*B*a^(7/2)*e^(3/2*x) - 3*A*a^(7/2)*
e^(1/2*x) - 5*B*a^(7/2)*e^(1/2*x))/((a*e^x + a)^4*a^2)
```

maple [B] time = 0.41, size = 209, normalized size = 2.25

$$-\frac{\sqrt{a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\left(3A\ln\left(\frac{2\sqrt{a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\left(\cosh^{4}\left(\frac{x}{2}\right)\right)a+5B\ln\left(\frac{2\sqrt{a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\left(\cosh^{4}\left(\frac{x}{2}\right)\right)a-3A}{32\cosh\left(\frac{x}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x)
```

```
[Out] -1/32*(a*sinh(1/2*x)^2)^(1/2)*(3*A*ln(2/cosh(1/2*x)*((a*sinh(1/2*x)^2)^(1/2
)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a+5*B*ln(2/cosh(1/2*x)*((a*sinh(1/2*x)^2)^(1
/2)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a-3*A*(a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2)*c
osh(1/2*x)^2-5*B*(a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2)*cosh(1/2*x)^2-2*A*(a*si
nh(1/2*x)^2)^(1/2)*(-a)^(1/2)+2*B*(a*sinh(1/2*x)^2)^(1/2)*(-a)^(1/2))/cosh(
1/2*x)^3/a^3/(-a)^(1/2)/sinh(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

maxima [B] time = 0.93, size = 427, normalized size = 4.59

$$\frac{1}{80}\left(\sqrt{2}\left(\frac{15\,e^{\left(\frac{9}{2}\,x\right)} + 70\,e^{\left(\frac{7}{2}\,x\right)} + 128\,e^{\left(\frac{5}{2}\,x\right)} - 70\,e^{\left(\frac{3}{2}\,x\right)} - 15\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{5}{2}}e^{(5\,x)} + 5\,a^{\frac{5}{2}}e^{(4\,x)} + 10\,a^{\frac{5}{2}}e^{(3\,x)} + 10\,a^{\frac{5}{2}}e^{(2\,x)} + 5\,a^{\frac{5}{2}}e^{x} + a^{\frac{5}{2}}} + \frac{15\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{5}{2}}}\right) - \frac{}{a^{\frac{5}{2}}e^{(5\,x)} + 5\,a^{\frac{5}{2}}e^{(4\,x)} +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/80*(sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) + 128*e^(5/2*x) - 70*e^(3/2*x)
- 15*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) +
 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^(5/
2)) - 128*sqrt(2)*e^(5/2*x)/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/
2)*e^(3*x) + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)))*A + 1/672*(sqrt
(2)*((105*e^(9/2*x) + 490*e^(7/2*x) + 896*e^(5/2*x) + 790*e^(3/2*x) - 105*e
^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) + 10*a^
(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 105*arctan(e^(1/2*x))/a^(5/2)) +
 7*sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) - 128*e^(5/2*x) - 70*e^(3/2*x) - 1
5*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x) + 10
*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^(5/2))
 - 128*(7*sqrt(2)*sqrt(a)*e^(7/2*x) + 3*sqrt(2)*sqrt(a)*e^(3/2*x))/(a^3*e^(
5*x) + 5*a^3*e^(4*x) + 10*a^3*e^(3*x) + 10*a^3*e^(2*x) + 5*a^3*e^x + a^3))*
B
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + a*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + a*cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+a*cosh(x))**(5/2),x)
```

```
[Out] Timed out
```

3.104 $\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal. Leaf size=57

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\,(A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

```
[Out] -(A+B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^(1/2)/a^(1
/2)+2*B*sinh(x)/(a-a*cosh(x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2751, 2649, 206}

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\,(A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

```
[Out] -((Sqrt[2]*(A + B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])
/Sqrt[a]) + (2*B*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx &= \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+(A+B)\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx\\ &= \frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+(2i(A+B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)\\ &= -\frac{\sqrt{2}\,(A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.70

$$\frac{2\sinh\left(\frac{x}{2}\right)\left((A+B)\log\left(\tanh\left(\frac{x}{4}\right)\right)+2B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
[Out] (2*(2*B*Cosh[x/2] + (A + B)*Log[Tanh[x/4]])*Sinh[x/2])/Sqrt[a - a*Cosh[x]]
```

fricas [B] time = 0.86, size = 99, normalized size = 1.74

$$\frac{\sqrt{2}\,(A+B)a\sqrt{-\frac{1}{a}}\,\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}\,(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right)-2\sqrt{\frac{1}{2}}\,(B\cosh(x)+B\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
[Out] (sqrt(2)*(A + B)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + s
inh(x)))*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) +
 sinh(x) - 1)) - 2*sqrt(1/2)*(B*cosh(x) + B*sinh(x) + B)*sqrt(-a/(cosh(x) +
 sinh(x))))/a
```

giac [C] time = 0.16, size = 103, normalized size = 1.81

$$\frac{1}{4}\sqrt{2}\left(\frac{(-8i\,A\arctan(-i)-8i\,B\arctan(-i)+8\,B)\text{sgn}(-e^x+1)}{\sqrt{-a}}-\frac{8\,(A+B)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{\sqrt{a}\,\text{sgn}(-e^x+1)}-\frac{4\,B}{\sqrt{-ae^x}\,\text{sgn}(-e^x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((-8*I*A*arctan(-I) - 8*I*B*arctan(-I) + 8*B)*sgn(-e^x + 1)/sqr
t(-a) - 8*(A + B)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x + 1)) - 4*
B/(sqrt(-a*e^x)*sgn(-e^x + 1)) + 4*sqrt(-a*e^x)*B/(a*sgn(-e^x + 1)))
```

maple [A] time = 0.37, size = 63, normalized size = 1.11

$$\frac{\sinh\left(\frac{x}{2}\right)\left(\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A+B\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)-B\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)+4B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{-2a\left(\sinh^2\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x)
```

```
[Out] sinh(1/2*x)*(ln(-1+cosh(1/2*x))*A-ln(cosh(1/2*x)+1)*A+B*ln(-1+cosh(1/2*x))-
B*ln(cosh(1/2*x)+1)+4*B*cosh(1/2*x))/(-2*a*sinh(1/2*x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x)+A}{\sqrt{-a\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(-a*cosh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a - a*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))/(a - a*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cosh\left(x\right)}{\sqrt{-a\left(\cosh\left(x\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(1/2),x)
```

```
[Out] Integral((A + B*cosh(x))/sqrt(-a*(cosh(x) - 1)), x)
```

3.105 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(A-3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

```
[Out] -1/2*(A+B)*sinh(x)/(a-a*cosh(x))^(3/2)-1/4*(A-3*B)*arctan(1/2*sinh(x)*a^(1/
2)*2^(1/2)/(a-a*cosh(x))^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2750, 2649, 206}

$$-\frac{(A-3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

```
[Out] -((A - 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(2*Sqr
t[2]*a^(3/2)) - ((A + B)*Sinh[x])/(2*(a - a*Cosh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = -\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(A-3B)\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx}{4a}$$
$$= -\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(i(A-3B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}\right)}{2a}$$
$$= -\frac{(A-3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sinh(x)}{\sqrt{2}\,\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

Mathematica [A] time = 0.16, size = 71, normalized size = 1.09

$$\frac{\sinh^3\left(\frac{x}{2}\right)\left((A+B)\text{csch}^2\left(\frac{x}{4}\right)+(A+B)\text{sech}^2\left(\frac{x}{4}\right)+4(A-3B)\log\left(\tanh\left(\frac{x}{4}\right)\right)\right)}{4a(\cosh(x)-1)\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

```
[Out] (((A + B)*Csch[x/4]^2 + 4*(A - 3*B)*Log[Tanh[x/4]] + (A + B)*Sech[x/4]^2)*S
inh[x/2]^3)/(4*a*(-1 + Cosh[x])*Sqrt[a - a*Cosh[x]])
```

fricas [B] time = 0.90, size = 217, normalized size = 3.34

$$\frac{\sqrt{2}\left((A-3\,B)\cosh(x)^2+(A-3\,B)\sinh(x)^2-2\,(A-3\,B)\cosh(x)+2\,((A-3\,B)\cosh(x)-A+3\,B)\sinh(x)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A - 3*B)*cosh(x)^2 + (A - 3*B)*sinh(x)^2 - 2*(A - 3*B)*cosh(
x) + 2*((A - 3*B)*cosh(x) - A + 3*B)*sinh(x) + A - 3*B)*sqrt(-a)*log((2*sqr
t(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x)) -
a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(1/2)*((A + B)*
cosh(x)^2 + (A + B)*sinh(x)^2 + (A + B)*cosh(x) + (2*(A + B)*cosh(x) + A +
B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x)^2 -
2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) - a^2)*sinh(x))
```

giac [B] time = 0.19, size = 111, normalized size = 1.71

$$-\frac{\left(\sqrt{2}\,A-3\,\sqrt{2}\,B\right)\arctan\left(\frac{\sqrt{-ae^{x}}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}\left(-e^{x}+1\right)}+\frac{\sqrt{2}\left(\sqrt{-ae^{x}}\,Aae^{x}+\sqrt{-ae^{x}}\,Bae^{x}+\sqrt{-ae^{x}}\,Aa+\sqrt{-ae^{x}}\,Ba\right)}{2\left(ae^{x}-a\right)^{2}a\mathrm{sgn}\left(-e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*A - 3*sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(3/2)*sgn(-e
^x + 1)) + 1/2*sqrt(2)*(sqrt(-a*e^x)*A*a*e^x + sqrt(-a*e^x)*B*a*e^x + sqrt(
-a*e^x)*A*a + sqrt(-a*e^x)*B*a)/((a*e^x - a)^2*a*sgn(-e^x + 1))
```

maple [A] time = 0.34, size = 83, normalized size = 1.28

$$\frac{\cosh\left(\frac{x}{2}\right)(2A+2B)+\left(\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-3B\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)+3B\ln\left(\cosh\left(\frac{x}{2}\right)+1\cdots\right.\right.}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2a\left(\sinh^{2}\left(\frac{x}{2}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x)
```

```
[Out] 1/4/a*(cosh(1/2*x)*(2*A+2*B)+(ln(-1+cosh(1/2*x))*A-ln(cosh(1/2*x)+1)*A-3*B*
ln(-1+cosh(1/2*x))+3*B*ln(cosh(1/2*x)+1))*sinh(1/2*x)^2)/sinh(1/2*x)/(-2*a*
sinh(1/2*x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cosh(x)+A}{(-a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a - a*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))/(a - a*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cosh(x)}{(-a(\cosh(x) - 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(3/2),x)
```

```
[Out] Integral((A + B*cosh(x))/(-a*(cosh(x) - 1))**(3/2), x)
```

3.106 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{(3A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

```
[Out] -1/4*(A+B)*sinh(x)/(a-a*cosh(x))^(5/2)-1/16*(3*A-5*B)*sinh(x)/a/(a-a*cosh(x
))^(3/2)-1/32*(3*A-5*B)*arctan(1/2*sinh(x)*a^(1/2)*2^(1/2)/(a-a*cosh(x))^(1
/2))/a^(5/2)*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2750, 2650, 2649, 206}

$$-\frac{(3A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

```
[Out] -((3*A - 5*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(16*
Sqrt[2]*a^(5/2)) - ((A + B)*Sinh[x])/(4*(a - a*Cosh[x])^(5/2)) - ((3*A - 5*
B)*Sinh[x])/(16*a*(a - a*Cosh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\int \frac{1}{(a-a\cosh(x))^{3/2}}\,dx}{8a}$$

$$= -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}+\frac{(3A-5B)\int \frac{1}{\sqrt{a-a\cosh(x)}}\,dx}{32a^2}$$

$$= -\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}+\frac{(i(3A-5B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{i\ldots}{\sqrt{a\ldots}}\right.}{16a^2}$$

$$= -\frac{(3A-5B)\tan^{-1}\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}-\frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}$$

Mathematica [A] time = 0.39, size = 108, normalized size = 1.15

$$\frac{\sinh^5\left(\frac{x}{2}\right)\left(-\left((A+B)\operatorname{csch}^4\left(\frac{x}{4}\right)\right)+2(3A-5B)\operatorname{csch}^2\left(\frac{x}{4}\right)+(A+B)\operatorname{sech}^4\left(\frac{x}{4}\right)+2(3A-5B)\operatorname{sech}^2\left(\frac{x}{4}\right)+8(3A-\ldots\right.}{32a^2(\cosh(x)-1)^2\sqrt{a-a\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

```
[Out] ((2*(3*A - 5*B)*Csch[x/4]^2 - (A + B)*Csch[x/4]^4 + 8*(3*A - 5*B)*Log[Tanh[
x/4]] + 2*(3*A - 5*B)*Sech[x/4]^2 + (A + B)*Sech[x/4]^4)*Sinh[x/2]^5)/(32*a
^2*(-1 + Cosh[x])^2*Sqrt[a - a*Cosh[x]])
```

fricas [B] time = 1.12, size = 548, normalized size = 5.83

$$\frac{\sqrt{2}\left((3A-5B)\cosh(x)^4+(3A-5B)\sinh(x)^4-4(3A-5B)\cosh(x)^3+4((3A-5B)\cosh(x)-3A+5B)\,\mathrm{s}\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - 4*(3*A - 5*B
)*cosh(x)^3 + 4*((3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x)^3 + 6*(3*A - 5*B)
*cosh(x)^2 + 6*((3*A - 5*B)*cosh(x)^2 - 2*(3*A - 5*B)*cosh(x) + 3*A - 5*B)*
sinh(x)^2 - 4*(3*A - 5*B)*cosh(x) + 4*((3*A - 5*B)*cosh(x)^3 - 3*(3*A - 5*B
)*cosh(x)^2 + 3*(3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x) + 3*A - 5*B)*sqrt(
-a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x)
 + sinh(x)) - a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(
1/2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - (11*A + 3*B)*cosh(x)^
3 + (4*(3*A - 5*B)*cosh(x) - 11*A - 3*B)*sinh(x)^3 - (11*A + 3*B)*cosh(x)^2
 + (6*(3*A - 5*B)*cosh(x)^2 - 3*(11*A + 3*B)*cosh(x) - 11*A - 3*B)*sinh(x)^
2 + (3*A - 5*B)*cosh(x) + (4*(3*A - 5*B)*cosh(x)^3 - 3*(11*A + 3*B)*cosh(x)
^2 - 2*(11*A + 3*B)*cosh(x) + 3*A - 5*B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x
))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 - 4*a^3*cosh(x)^3 + 6*a^3*cosh(x)^2 - 4
*a^3*cosh(x) + 4*(a^3*cosh(x) - a^3)*sinh(x)^3 + a^3 + 6*(a^3*cosh(x)^2 - 2
*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 - 3*a^3*cosh(x)^2 + 3*a^3*
cosh(x) - a^3)*sinh(x))
```

giac [B] time = 0.24, size = 189, normalized size = 2.01

$$-\frac{\sqrt{2}\,(3\,A-5\,B)\arctan\left(\frac{\sqrt{-ae^{x}}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(-e^{x}+1\right)}+\frac{\sqrt{2}\left(3\,\sqrt{-ae^{x}}\,Aa^{3}e^{(3\,x)}-5\,\sqrt{-ae^{x}}\,Ba^{3}e^{(3\,x)}-11\,\sqrt{-ae^{x}}\,Aa^{3}e^{(2\,x)}-3\,\sqrt{-ae^{x}}\,Ba^{3}e^{(2}\right.}{16\,(ae^{x}-a)^{4}a^{2}\mathrm{sgn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(3*A - 5*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(5/2)*sgn(-e^x +
1)) + 1/16*sqrt(2)*(3*sqrt(-a*e^x)*A*a^3*e^(3*x) - 5*sqrt(-a*e^x)*B*a^3*e^(
3*x) - 11*sqrt(-a*e^x)*A*a^3*e^(2*x) - 3*sqrt(-a*e^x)*B*a^3*e^(2*x) - 11*sq
rt(-a*e^x)*A*a^3*e^x - 3*sqrt(-a*e^x)*B*a^3*e^x + 3*sqrt(-a*e^x)*A*a^3 - 5*
sqrt(-a*e^x)*B*a^3)/((a*e^x - a)^4*a^2*sgn(-e^x + 1))
```

maple [A] time = 0.35, size = 118, normalized size = 1.26

$$\frac{(6A-10B)\cosh\left(\frac{x}{2}\right)\left(\sinh^{2}\left(\frac{x}{2}\right)\right)+(-4A-4B)\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(-1+\cosh\left(\frac{x}{2}\right)\right)A-3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-5B\right.}{32a^{2}\left(\cosh\left(\frac{x}{2}\right)+1\right)\left(-1+\cosh\left(\frac{x}{2}\right)\right)\sinh\left(\frac{x}{2}\right)\sqrt{-2a\left(\sinh^{2}\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x)
```

```
[Out] 1/32/a^2*((6*A-10*B)*cosh(1/2*x)*sinh(1/2*x)^2+(-4*A-4*B)*cosh(1/2*x)+(3*ln
(-1+cosh(1/2*x))*A-3*ln(cosh(1/2*x)+1)*A-5*B*ln(-1+cosh(1/2*x))+5*B*ln(cosh
(1/2*x)+1))*sinh(1/2*x)^4)/(cosh(1/2*x)+1)/(-1+cosh(1/2*x))/sinh(1/2*x)/(-2
*a*sinh(1/2*x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x) + A}{(-a\cosh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cosh(x)}{(a - a\cosh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a - a*cosh(x))^(5/2),x)
[Out] int((A + B*cosh(x))/(a - a*cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a-a*cosh(x))**(5/2),x)
[Out] Timed out
```

3.107 $\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=233

$$\frac{2}{105}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{2i\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\right.}{105b\sqrt{a+b\cosh(x)}}$$

```
[Out] 2/35*(7*A*b+5*B*a)*(a+b*cosh(x))^(3/2)*sinh(x)+2/7*B*(a+b*cosh(x))^(5/2)*si
nh(x)+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*sinh(x)*(a+b*cosh(x))^(1/2)-2/105*
I*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/
2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b
/((a+b*cosh(x))/(a+b))^(1/2)+2/105*I*(a^2-b^2)*(56*A*a*b+15*B*a^2+25*B*b^2)
*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2}{105}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{2i\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\right.}{105b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)/105)*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*
Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)
]) + (((2*I)/105)*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*
Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]
) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cosh[x]]*Sinh[x])/105 +
(2*(7*A*b + 5*a*B)*(a + b*Cosh[x])^(3/2)*Sinh[x])/35 + (2*B*(a + b*Cosh[x])
^(5/2)*Sinh[x])/7
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{2}{7}B(a+b\cosh(x))^{5/2}\sinh(x)+\frac{2}{7}\int (a+b\cosh(x))^{3/2}\left(\frac{1}{2}(7aA+5bB)\right.$$

$$= \frac{2}{35}(7Ab+5aB)(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+b\cosh(x))^{5/2}\sinh($$

$$= \frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{35}(7Ab+5a$$

$$= \frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{35}(7Ab+5a$$

$$= \frac{2}{105}\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{35}(7Ab+5a$$

$$= -\frac{2i\left(161a^2Ab+63Ab^3+15a^3B+145ab^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{105b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Mathematica [A] time = 0.63, size = 203, normalized size = 0.87

$$\frac{\sinh(x)(a+b\cosh(x))\left(90a^2B+6b\cosh(x)(15aB+7Ab)+154aAb+15b^2B\cosh(2x)+65b^2B\right)-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left(b(\right.}{}}{105\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*(b*(105*a^3*A + 119*a*A*b^2 + 135*a^
2*b*B + 25*b^3*B)*EllipticF[(I/2)*x, (2*b)/(a + b)] + (161*a^2*A*b + 63*A*b
^3 + 15*a^3*B + 145*a*b^2*B)*((a + b)*EllipticE[(I/2)*x, (2*b)/(a + b)] - a
*EllipticF[(I/2)*x, (2*b)/(a + b)])))/b + (a + b*Cosh[x])*(154*a*A*b + 90*a
^2*B + 65*b^2*B + 6*b*(7*A*b + 15*a*B)*Cosh[x] + 15*b^2*B*Cosh[2*x])*Sinh[x
])/(105*Sqrt[a + b*Cosh[x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cosh(x)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cosh(x)^2+\left(Ba^2+2\,Aab\right)\cosh(x)\right)\sqrt{b\cosh(x)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cosh(x)^3 + A*a^2 + (2*B*a*b + A*b^2)*cosh(x)^2 + (B*a^2 +
2*A*a*b)*cosh(x))*sqrt(b*cosh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

maple [B] time = 0.55, size = 1365, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/105*(240*B*(-2*b/(a-b))^(1/2)*b^3*cosh(1/2*x)*sinh(1/2*x)^8+(168*A*(-2*b/
(a-b))^(1/2)*b^3+480*B*(-2*b/(a-b))^(1/2)*a*b^2+360*B*(-2*b/(a-b))^(1/2)*b^
3)*sinh(1/2*x)^6*cosh(1/2*x)+(392*A*(-2*b/(a-b))^(1/2)*a*b^2+168*A*(-2*b/(a
-b))^(1/2)*b^3+360*B*(-2*b/(a-b))^(1/2)*a^2*b+480*B*(-2*b/(a-b))^(1/2)*a*b^
2+280*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(154*A*(-2*b/(a-b
))^(1/2)*a^2*b+196*A*(-2*b/(a-b))^(1/2)*a*b^2+42*A*(-2*b/(a-b))^(1/2)*b^3+9
0*B*(-2*b/(a-b))^(1/2)*a^3+180*B*(-2*b/(a-b))^(1/2)*a^2*b+170*B*(-2*b/(a-b)
)^(1/2)*a*b^2+80*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^2*cosh(1/2*x)+105*a^
3*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Elli
pticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+161*A*a^2*b*(2
*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(
cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+119*A*a*b^2*(2*b/(a-
b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1
/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+63*A*b^3*(2*b/(a-b)*sinh(1
/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2
*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-322*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b
)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1
/2),1/2*(-2*(a-b)/b)^(1/2))*a^2*b-126*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b
))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/
2*(-2*(a-b)/b)^(1/2))*b^3+15*a^3*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1
/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2
*(a-b)/b)^(1/2))+135*B*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-
sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)
```

```
/b)^(1/2))+145*B*a*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1
/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1
/2))+25*B*b^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^
(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-30*B
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipti
cE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^3-290*B*(2*b/(a
-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(
1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a*b^2)*((2*b*cosh(1/2*x)^
2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sin
h(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\cosh(x))\,(a + b\cosh(x))^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + b*cosh(x))^(5/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + b*cosh(x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

```
[Out] Timed out
```

3.108 $\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal. Leaf size=181

$$\frac{2i\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}}-\frac{2i\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{15}\,\text{sinh}$$

```
[Out] 2/5*B*(a+b*cosh(x))^(3/2)*sinh(x)+2/15*(5*A*b+3*B*a)*sinh(x)*(a+b*cosh(x))^
(1/2)-2/15*I*(20*A*a*b+3*B*a^2+9*B*b^2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*E
llipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b
*cosh(x))/(a+b))^(1/2)+2/15*I*(a^2-b^2)*(5*A*b+3*B*a)*(cosh(1/2*x)^2)^(1/2)
/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x)
)/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2i\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}}-\frac{2i\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{15}\,\text{sinh}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)/15)*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(
I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/15)*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (
2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Cosh
[x]]*Sinh[x])/15 + (2*B*(a + b*Cosh[x])^(3/2)*Sinh[x])/5
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{5}\int \sqrt{a+b\cosh(x)}\left(\frac{1}{2}(5aA+3bB)\right.$$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh($$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh($$

$$= \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x)+\frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh($$

$$= -\frac{2i\left(20aAb+3a^2B+9b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\left(a^2-b^2\right)}{}$$

Mathematica [A] time = 0.70, size = 124, normalized size = 0.69

$$\frac{2}{15}\sqrt{a+b\cosh(x)}\left(\sinh(x)(6aB+5Ab+3bB\cosh(x))-\frac{i\left(\left(3a^2B+20aAb+9b^2B\right)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-(a-b)(3aB+\right.}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

```
[Out] (2*Sqrt[a + b*Cosh[x]]*(((-I)*((20*a*A*b + 3*a^2*B + 9*b^2*B)*EllipticE[(I/
2)*x, (2*b)/(a + b)] - (a - b)*(5*A*b + 3*a*B)*EllipticF[(I/2)*x, (2*b)/(a
+ b)]))/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (5*A*b + 6*a*B + 3*b*B*Cosh[x])
*Sinh[x]))/15
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb\cosh(x)^2+Aa+(Ba+Ab)\cosh(x)\right)\sqrt{b\cosh(x)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cosh(x)^2 + A*a + (B*a + A*b)*cosh(x))*sqrt(b*cosh(x) + a), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

maple [B] time = 0.58, size = 973, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/15*(24*B*(-2*b/(a-b))^(1/2)*b^2*cosh(1/2*x)*sinh(1/2*x)^6+(20*A*(-2*b/(a-
b))^(1/2)*b^2+36*B*(-2*b/(a-b))^(1/2)*a*b+24*B*(-2*b/(a-b))^(1/2)*b^2)*sinh
(1/2*x)^4*cosh(1/2*x)+(10*A*(-2*b/(a-b))^(1/2)*a*b+10*A*(-2*b/(a-b))^(1/2)*
b^2+12*B*(-2*b/(a-b))^(1/2)*a^2+18*B*(-2*b/(a-b))^(1/2)*a*b+6*B*(-2*b/(a-b)
)^(1/2)*b^2)*sinh(1/2*x)^2*cosh(1/2*x)+15*a^2*A*(2*b/(a-b)*sinh(1/2*x)^2+(a
+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^
(1/2),1/2*(-2*(a-b)/b)^(1/2))+20*A*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b)
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2*(a-b)/b)^(1/2))+5*A*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-
sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)
/b)^(1/2))-40*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2
)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a*
b+3*B*a^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2
)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+12*b*B*a
*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ellipti
cF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+9*B*b^2*(2*b/(a-b
)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/
2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-6*B*(2*b/(a-b)*sinh(1/2*x)^
2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-
b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a^2-18*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(
a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2)
,1/2*(-2*(a-b)/b)^(1/2))*b^2)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)
/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*
x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cosh(x) + A)(b\cosh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cosh(x))\ (a + b\cosh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + b*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + b*cosh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cosh(x))\,(a + b\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral((A + B*cosh(x))*(a + b*cosh(x))**(3/2), x)
```

3.109 $\int \sqrt{a+b\cosh(x)}\,(A+B\cosh(x))\,dx$

Optimal. Leaf size=138

$$\frac{2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

```
[Out] 2/3*B*sinh(x)*(a+b*cosh(x))^(1/2)-2/3*I*(3*A*b+B*a)*(cosh(1/2*x)^2)^(1/2)/c
osh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cosh(x))^(
1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*B*(cosh(1/2*x)^2)^(1/2)/
cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cosh(x))
/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] (((-2*I)/3)*(3*A*b + a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a +
 b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*I)/3)*(a^2 - b^2)*B*Sqrt[(a
+ b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh
[x]]) + (2*B*Sqrt[a + b*Cosh[x]]*Sinh[x])/3
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{a+b\cosh(x)}\,(A+B\cosh(x))\,dx = \frac{2}{3}B\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{1}{2}(3Ab+aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}$$

$$= \frac{2}{3}B\sqrt{a+b\cosh(x)}\,\sinh(x)-\frac{\left(\left(a^2-b^2\right)B\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{3b}+\frac{(3Ab+a}{}$$

$$= \frac{2}{3}B\sqrt{a+b\cosh(x)}\,\sinh(x)+\frac{\left((3Ab+aB)\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\,c}{}}}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$= -\frac{2i(3Ab+aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2i\left(a^2-b^2\right)B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\right.}{3b\sqrt{a+b\cosh(x)}}$$

Mathematica [A] time = 0.35, size = 123, normalized size = 0.89

$$\frac{2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-2i(a+b)(aB+3Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)+2bB\sinh(x)(a+b\cosh(x))}{3b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

```
[Out] ((-2*I)*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)
*x, (2*b)/(a + b)] + (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*B*(a + b*Cosh[x])*Sinh[x])/(3*b*Sqrt[a
+ b*Cosh[x]])
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cosh(x)+A)\sqrt{b\cosh(x)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cosh(x)+A)\sqrt{b\cosh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

maple [B] time = 0.52, size = 605, normalized size = 4.38

$$\frac{2\left(4B\sqrt{-\frac{2b}{a-b}}\, b\cosh\left(\frac{x}{2}\right)\left(\sinh^4\left(\frac{x}{2}\right)\right)+\left(2B\sqrt{-\frac{2b}{a-b}}\, a+2B\sqrt{-\frac{2b}{a-b}}\, b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)+3Aa\sqrt{\frac{2b\left(\sinh^2\left(\frac{x}{2}\right)\right)}{a-b}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

```
[Out] 2/3*(4*B*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^4+(2*B*(-2*b/(a-b))^(
1/2)*a+2*B*(-2*b/(a-b))^(1/2)*b)*sinh(1/2*x)^2*cosh(1/2*x)+3*A*a*(2*b/(a-b)
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2
*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+3*A*b*(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a
-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-6*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b)
)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2*(a-b)/b)^(1/2))*b+a*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-si
nh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b
)^(1/2))+b*B*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*B*(
2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE
(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))*a)*((2*b*cosh(1/2*x
)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*s
inh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cosh(x)+A)\sqrt{b\cosh(x)+a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cosh(x))\,\sqrt{a+b\cosh(x)}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))*(a + b*cosh(x))^(1/2),x)
```

```
[Out] int((A + B*cosh(x))*(a + b*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cosh(x))\sqrt{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

```
[Out] Integral((A + B*cosh(x))*sqrt(a + b*cosh(x)), x)
```

3.110 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}+\frac{Bx}{b}$$

```
[Out] B*x/b+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2
)/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2735, 2659, 208}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+b\cosh(x)}\,dx}{b}$$
$$= \frac{Bx}{b} - \frac{(2(-Ab+aB))\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{Bx}{b} + \frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}$$

Mathematica [A] time = 0.10, size = 59, normalized size = 0.98

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\sqrt{b^2-a^2}} + \frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(
b*Sqrt[-a^2 + b^2])
```

fricas [A] time = 0.77, size = 240, normalized size = 4.00

$$\left[-\frac{(Ba-Ab)\sqrt{a^2-b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right) -}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x)/(a^2*b - b^3), (2*(B*a - A*
b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a
^2 - b^2)) + (B*a^2 - B*b^2)*x)/(a^2*b - b^3)]
```

giac [A] time = 0.12, size = 50, normalized size = 0.83

$$\frac{Bx}{b} - \frac{2\,(Ba - Ab)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a - A*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2
)*b)
```

maple [B] time = 0.06, size = 103, normalized size = 1.72

$$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)A}{\sqrt{(a+b)\,(a-b)}}-\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)aB}{b\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] -B/b*ln(tanh(1/2*x)-1)+B/b*ln(tanh(1/2*x)+1)+2/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A-2/b/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*a*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.12, size = 242, normalized size = 4.03

$$\frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^x\,\sqrt{b^4-a^2\,b^2}\,\left(\frac{2\left(A\,b\,\sqrt{b^4-a^2\,b^2}-B\,a\,\sqrt{b^4-a^2\,b^2}\right)}{b^4\,\sqrt{b^4-a^2\,b^2}\,\sqrt{{(A\,b-B\,a)}^2}}+\frac{2\,a^2\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{b^2\,\left(b^4-a^2\,b^2\right)\,(A\,b-B\,a)}\right)}{2}+\frac{a\,b\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{b^4-a^2\,b^2}\,(A\,b-B\,a)}\right)\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b}}{\sqrt{b^4-a^2\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((b^2*exp(x)*(b^4 - a^2*b^2)^(1/2)*((2*(A*b*(b^4 - a^2*b^2)^(1/2) -
B*a*(b^4 - a^2*b^2)^(1/2)))/(b^4*(b^4 - a^2*b^2)^(1/2)*((A*b - B*a)^2)^(1/2
)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^2*(b^4 - a^2*b^2)*(A*
b - B*a))))/2 + (a*b*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((b^4 - a^2*b^2
)^(1/2)*(A*b - B*a)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4 - a^2*b^2
)^(1/2) + (B*x)/b
```

sympy [A] time = 27.03, size = 403, normalized size = 6.72

$$\begin{cases} \tilde{\infty}\left(2A\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)+Bx\right) \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)}+\frac{Bx}{b}-\frac{B}{b\tanh\left(\frac{x}{2}\right)} \\ \frac{Ax+B\sinh(x)}{a} \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}-\frac{B\tanh\left(\frac{x}{2}\right)}{b} \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ab\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Bax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ba\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{Ba}{a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), (-A/(b*ta
nh(x/2)) + B*x/b - B/(b*tanh(x/2)), Eq(a, -b)), ((A*x + B*sinh(x))/a, Eq(b,
 0)), (A*tanh(x/2)/b + B*x/b - B*tanh(x/2)/b, Eq(a, b)), (-A*b*log(-sqrt(a/
(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/
2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) +
B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*s
qrt(a/(a - b) + b/(a - b))) + B*a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x
/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) -
 B*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(
a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a -
 b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))),
True))
```

3.111 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=82

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sinh(x)(Ab-aB)}{\left(a^2-b^2\right)(a+b\cosh(x))}$$

```
[Out] 2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)-(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2754, 12, 2659, 208}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sinh(x)(Ab-aB)}{\left(a^2-b^2\right)(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)) - ((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx = -\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{\int \frac{-aA+bB}{a+b\cosh(x)}\,dx}{-a^2+b^2}$$

$$= -\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{(2(aA-bB))\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}$$

$$= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)(a+b\cosh(x))}$$

Mathematica [A] time = 0.20, size = 81, normalized size = 0.99

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{\sinh(x)(aB-Ab)}{(a-b)(a+b)(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*Cosh[x]))
```

fricas [B] time = 1.38, size = 828, normalized size = 10.10

$$\left[-\frac{2\,Ba^3b-2\,Aa^2b^2-2\,Bab^3+2\,Ab^4-\left(Aab^2-Bb^3+\left(Aab^2-Bb^3\right)\cosh(x)^2+\left(Aab^2-Bb^3\right)\sinh(x)^2+2\left(Aa^2\right.\right.}{a^4b^2-2\,a^2b^4+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a*
b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2
)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh(x))*sinh(x))*sqrt
(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^
2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x
) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x
) + b)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cosh(x) + 2*(B*a^4 - A*
a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 + (a^4*b^2
 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(x)^2 + 2*(
a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (a^4*b^
2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 +
 A*b^4 + (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)
*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^
2 - B*b^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b
^3)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2
*a^2*b^4 + b^6 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b
^4 + b^6)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*
a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.12, size = 107, normalized size = 1.30

$$\frac{2\,(Aa - Bb)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}}} - \frac{2\left(Ba^{2}e^{x} - Aabe^{x} + Bab - Ab^{2}\right)}{\left(a^{2}b-b^{3}\right)\left(be^{(2\,x)} + 2\,ae^{x} + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 +
 b^2)) - 2*(B*a^2*e^x - A*a*b*e^x + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2*x
) + 2*a*e^x + b))
```

maple [A] time = 0.06, size = 108, normalized size = 1.32

$$\frac{2\,(Ab - aB)\tanh\left(\frac{x}{2}\right)}{\left(a^{2}-b^{2}\right)\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right) - \left(\tanh^{2}\left(\frac{x}{2}\right)\right)b - a - b\right)} + \frac{2\,(Aa - Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^2,x)
```

```
[Out] 2*(A*b-B*a)/(a^2-b^2)*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+2*(
A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.42, size = 246, normalized size = 3.00

$$\frac{\frac{2\left(A\,b^3-B\,a\,b^2\right)}{b\left(a^2\,b-b^3\right)}-\frac{2\,e^x\left(B\,a^2\,b^2-A\,a\,b^3\right)}{b^2\left(a^2\,b-b^3\right)}}{b+2\,a\,e^x+b\,e^{2\,x}}+\frac{\ln\left(-\frac{2\,e^x\,(A\,a-B\,b)}{b\left(a^2-b^2\right)}-\frac{2\,(b+a\,e^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{\ln\left(\frac{2\,(b+a\,e^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{2\,e^x\,(A\,a-B\,b)}{b\left(a^2-b^2\right)}\right)(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^2,x)
```

```
[Out] ((2*(A*b^3 - B*a*b^2))/(b*(a^2*b - b^3)) - (2*exp(x)*(B*a^2*b^2 - A*a*b^3))
/(b^2*(a^2*b - b^3)))/(b + 2*a*exp(x) + b*exp(2*x)) + (log(- (2*exp(x)*(A*a
 - B*b))/(b*(a^2 - b^2)) - (2*(b + a*exp(x))*(A*a - B*b))/(b*(a + b)^(3/2)*
(a - b)^(3/2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(b + a
*exp(x))*(A*a - B*b))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(x)*(A*a - B*
b))/(b*(a^2 - b^2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

3.112 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx$

Optimal. Leaf size=135

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

```
[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^
(5/2)/(a+b)^(5/2)-1/2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^2-1/2*(3*A*
a*b-B*a^2-2*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))
```

Rubi [A] time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2754, 12, 2659, 208}

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/
((a - b)^(5/2)*(a + b)^(5/2)) - ((A*b - a*B)*Sinh[x])/(2*(a^2 - b^2)*(a + b
*Cosh[x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/(2*(a^2 - b^2)^2*(a +
b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh(x)}{(a+b\cosh(x))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sinh(x)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}+\frac{\int \frac{2a^2A+Ab^2-3abB}{a+b\cosh(x)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sinh(x)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}+\frac{\left(2a^2A+Ab^2-3abB\right)\ldots}{2\left(a^2-b^2\right)\ldots}$$

$$= -\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a^2B-2b^2B\right)\sinh(x)}{2\left(a^2-b^2\right)^2(a+b\cosh(x))}+\frac{\left(2a^2A+Ab^2-3abB\right)\ldots}{\ldots}$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{(Ab-aB)\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}-\frac{\left(3aAb-a\ldots\right.}{2\left(a^2-b\ldots\right.}$$

Mathematica [A] time = 0.43, size = 134, normalized size = 0.99

$$\frac{1}{2}\left(-\frac{2\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+\frac{\sinh(x)\left(a^2B-3aAb+2b^2B\right)}{(a-b)^2(a+b)^2(a+b\cosh(x))}+\frac{\sinh(x)(aB-Ab)}{(a-b)(a+b)(a+b\cosh(x)\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*C
osh[x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[x])/((a - b)^2*(a + b)^2*(a
 + b*Cosh[x])))/2
```

fricas [B] time = 0.82, size = 3166, normalized size = 23.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2*
A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2*B
*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*
B*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B
*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 +
 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 +
 (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A
*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^3 + 4*(2*
A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(
x))*sinh(x)^3 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^
5)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2
*b^3 + A*a*b^4)*cosh(x))*sinh(x)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4
)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b
^4 + A*b^5)*cosh(x)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^2 +
 (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(
x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*
a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) +
b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a
)*sinh(x) + b)) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 -
5*B*a*b^5 - A*b^6)*cosh(x) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A
*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3
*B*a*b^5 - A*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*A*a^
3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*cosh(x))*sinh(x))/(a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)^
4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*sinh(x)^4 + 4*(a^7*b^2 - 3*a^5*
b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^3 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a
*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x))*sinh(x)^3 + 2*(2*a^
8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x)^2 + 2*(2*a^8*b - 5*a^6
*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 + 3*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cosh(x)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x))*sinh(x)^
2 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x) + 4*(a^7*b^2 - 3*a^
```

```
5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)
^3 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^2 + (2*a^8*b - 5*a
^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x))*sinh(x)), -(B*a^4*b^2 - 3*A*a^
3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^
2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b
^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + (2*B*a^6 - 6*A*a^5*b + 3*B*a^4*b^2 + 3*
A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*cosh(x)^2 + (2*B*a^6 - 6*A*a
^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6 - 3*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^
2 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*co
sh(x)^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 - 3*
B*a^2*b^3 + A*a*b^4)*cosh(x)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (
2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x)^3 + 2*(4*A*a^4*b - 6*B*a^
3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^2 + 2*(4*A*a^4*b - 6*B*a^3
*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x))*sinh(x)^2 +
4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x) + 4*(2*A*a^3*b^2 - 3*B*a^2*
b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x)^3 + 3*(2*A*a^3*b^
2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(x)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b
^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (4*B*a^5*b - 10*A*a^4*b
^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(x) + (4*B*a^5*b - 1
0*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2
 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*
A*a^5*b + 3*B*a^4*b^2 + 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*co
sh(x))*sinh(x))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9 + (a^6*b^3 - 3*a^4*b
^5 + 3*a^2*b^7 - b^9)*cosh(x)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*s
inh(x)^4 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cosh(x)^3 + 4*(a^7*b
^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9
)*cosh(x))*sinh(x)^3 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*
cosh(x)^2 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9 + 3*(a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cosh(x)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8)*cosh(x))*sinh(x)^2 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8)*cosh(x) + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8 + (a^6*b^3 - 3*a^4*
b^5 + 3*a^2*b^7 - b^9)*cosh(x)^3 + 3*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b
^8)*cosh(x)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*cosh(x))*
sinh(x))]
```

giac [B] time = 0.14, size = 249, normalized size = 1.84

$$\frac{\left(2\,Aa^2 - 3\,Bab + Ab^2\right)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^4 - 2\,a^2b^2 + b^4\right)\sqrt{-a^2+b^2}} + \frac{2\,Aa^2b^2e^{(3\,x)} - 3\,Bab^3e^{(3\,x)} + Ab^4e^{(3\,x)} - 2\,Ba^4e^{(2\,x)} + 6\,Aa^3be^{(2\,x)} - 5\,B}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="giac")
```

```
[Out] (2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*
a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*x)
 + A*b^4*e^(3*x) - 2*B*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(2*x
) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*e^x
- 5*B*a*b^3*e^x - A*b^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b - 2*
a^2*b^3 + b^5)*(b*e^(2*x) + 2*a*e^x + b)^2)
```

maple [A] time = 0.07, size = 207, normalized size = 1.53

$$-\frac{2\left(-\frac{\left(4Aab+A\,b^{2}-2B\,a^{2}-bBa-2B\,b^{2}\right)\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(4Aab-A\,b^{2}-2B\,a^{2}+bBa-2B\,b^{2}\right)\tanh\left(\frac{x}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)}\right)}{\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-a-b\right)^{2}}+\frac{\left(2a^{2}A+A\,b^{2}-3bBa\right)\operatorname{arctanh}\left(\frac{(a-}{\sqrt{}}\right.}{\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\sqrt{(a+b)\,(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^3,x)
```

```
[Out] -2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*tanh(1
/2*x)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan
h(1/2*x))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b)/(
a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-
b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{\left(a+b\cosh(x)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^3,x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**3,x)
```

```
[Out] Timed out
```

3.113 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$

Optimal. Leaf size=197

$$-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\ \ldots}{\sqrt{a\ldots}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}$$

```
[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(
1/2))/(a-b)^(7/2)/(a+b)^(7/2)-1/3*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^2-1/6*(1
1*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*sinh(x)/(a^2-b^2)^3/(a+b*cosh(x))
```

Rubi [A] time = 0.35, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2754, 12, 2659, 208}

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sinh(x)\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)}{6\left(a^2-b^2\right)^3(a+b\cosh(x))}-\frac{\sinh(x)\left(-\ldots\right)}{6\left(a^2-b\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/
Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)) - ((A*b - a*B)*Sinh[x])/(3*(a^2
 - b^2)*(a + b*Cosh[x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[x])/(6*(a^
2 - b^2)^2*(a + b*Cosh[x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2
*B)*Sinh[x])/(6*(a^2 - b^2)^3*(a + b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3} - \frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cosh(x)}{(a+b\cosh(x))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2} + \frac{\int \frac{2\left(3a^2A+2Ab^2-5abB\right)-(5 \cdots}{(a+b\cos \cdots}}{6\left(a^2 \cdots\right.}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2} - \frac{\left(11a^2Ab+4Ab^3-2a \cdots\right.}{6\left(a^2-b^2\right)^3(a \cdots}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2} - \frac{\left(11a^2Ab+4Ab^3-2a \cdots\right.}{6\left(a^2-b^2\right)^3(a \cdots}$$

$$= -\frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sinh(x)}{6\left(a^2-b^2\right)^2(a+b\cosh(x))^2} - \frac{\left(11a^2Ab+4Ab^3-2a \cdots\right.}{6\left(a^2-b^2\right)^3(a \cdots}$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} - \frac{(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3} - \cdots$$

Mathematica [A] time = 0.83, size = 196, normalized size = 0.99

$$\frac{1}{6}\left(\frac{\sinh(x)\left(2a^2B-5aAb+3b^2B\right)}{(a-b)^2(a+b)^2(a+b\cosh(x))^2}+\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}+\frac{\sinh(x)\left(2a^3B-\right.}{(a-b)^3(a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[x/2])/Sq
rt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sinh[x])/((a - b)*(
a + b)*(a + b*Cosh[x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*B)*Sinh[x])/((a -
b)^2*(a + b)^2*(a + b*Cosh[x])^2) + ((-11*a^2*A*b - 4*A*b^3 + 2*a^3*B + 13*
a*b^2*B)*Sinh[x])/((a - b)^3*(a + b)^3*(a + b*Cosh[x])))/6
```

fricas [B] time = 1.99, size = 7603, normalized size = 38.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 + 22*B*a^3*b^5 + 14*A*a^2*b^6 - 26*B*a*b^
7 + 8*A*b^8 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*
a*b^7 + B*b^8)*cosh(x)^5 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a
^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(x)^5 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a
^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^4 - 30*(2*A*a^6*b^2 -
 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 + (2*A*a^5*b
^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x))*si
nh(x)^4 + 4*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b
^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x)^3 + 4*(4*B*a^8 - 22*
A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a
^2*b^6 + 12*A*a*b^7 - 15*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b
^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b
^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x))*sinh(x)^3 + 12*(4*B*a^7*
b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6
 - 4*B*a*b^7 + 2*A*b^8)*cosh(x)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5
*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*
(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*c
osh(x)^3 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^
2*b^6 + B*a*b^7)*cosh(x)^2 + (4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^
5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x))*si
nh(x)^2 - 3*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 -
```

4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 +
3*A*a*b^6 - B*b^7)*sinh(x)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 -
B*a*b^6)*cosh(x)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6
+ (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 + 3*(8
*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)
cosh(x)^4 + 3(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x
)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(
x)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b
^5 - 3*B*a*b^6)*cosh(x)^3 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*
B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*
b^6 - B*b^7)*cosh(x)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*
b^6)*cosh(x)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5
+ 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 +
14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2
- 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^
3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 20*(2*A*a^4*b^3 - 4*B*
a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3
+ 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 4*(4*A*a^6*b
- 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh
(x))*sinh(x)^2 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh
(x) + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 -
4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4
+ 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a
^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b - 8*B*a^
5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x)^2 +
(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^
7)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2
*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b
^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4
*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(x) + 6*(4*B*a^
6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b
^7 + B*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 + B*b^8)*cosh(x)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a
^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B*
a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a
*b^7)*cosh(x)^2 + 4*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4
- 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x))*sinh(x))/(a^8
*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12 + (a^8*b^4 - 4*a^6*b^6 + 6
*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8
- 4*a^2*b^10 + b^12)*sinh(x)^6 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3
*b^9 + a*b^11)*cosh(x)^5 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 +
a*b^11 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x))*si
nh(x)^5 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh

(x)^4 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^2 + 10*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x))*sinh(x)^4 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^3 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x))*sinh(x)^3 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^4 + 20*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^3 + 6*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^2 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x))*sinh(x)^2 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x) + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^5 + 5*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^4 + 2*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^3 + 2*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^2 + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x))*sinh(x)), -1/3*(2*B*a^5*b^3 - 11*A*a^4*b^4 + 11*B*a^3*b^5 + 7*A*a^2*b^6 - 13*B*a*b^7 + 4*A*b^8 - 3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^5 - 3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(x)^5 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^4 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 + (2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x))*sinh(x)^4 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7 - 15*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x))*sinh(x)^3 + 6*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x)^2 + 6*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x)^3 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(x)^2 + (4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x))*sinh(x)^2 + 3*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*sinh(x)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 + 3*(8*A*a^5*b^2 - 16*

B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 3*(
8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7
 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 10*(2*A*a^
4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(x)^4 + 4*(4*A*a^
6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*
cosh(x)^3 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*
a^2*b^5 - 3*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cos
h(x)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^2 +
 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B
*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8
*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 +
 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*
b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^4 + 20*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^
2*b^5 - B*a*b^6)*cosh(x)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 -
 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^2 + 4*(4*A*a^6*b - 8*B*a^5*b^2 +
12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x))*sinh(x)^2 +
 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(x) + 6*(2*A*a^4
*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 -
B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2
*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4
*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(x)^2 + (8*A*a^5*b^2 - 1
6*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(x))*sinh
(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/
(a^2 - b^2)) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5
- 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(x) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3
 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8 - 5*(2*A*
a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(x
)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6
 + B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b + 28*B*a^6*b^2 - 19*A*a^5*b
^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 - 39*B*a^2*b^6 + 12*A*a*b^7)*cosh(x)^2 + 4*
(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*
A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(x))*sinh(x))/(a^8*b^4 - 4*a^6*b^6 + 6
*a^4*b^8 - 4*a^2*b^10 + b^12 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^1
0 + b^12)*cosh(x)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)
sinh(x)^6 + 6(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(
x)^5 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11 + (a^8*b^4 -
 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x))*sinh(x)^5 + 3*(4*a^10*
b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^4 + 3*(4*a^10*b^
2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12 + 5*(a^8*b^4 - 4*a^6*b^6 +
6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^2 + 10*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*
b^7 - 4*a^3*b^9 + a*b^11)*cosh(x))*sinh(x)^4 + 4*(2*a^11*b - 5*a^9*b^3 + 10
*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^3 + 4*(2*a^11*b - 5*a^9*b^3 + 10*
a^5*b^7 - 10*a^3*b^9 + 3*a*b^11 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^
2*b^10 + b^12)*cosh(x)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9

```
+ a*b^11)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8
+ b^12)*cosh(x))*sinh(x)^3 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a
^4*b^8 + b^12)*cosh(x)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4
*b^8 + b^12 + 5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(
x)^4 + 20*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^3
+ 6*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^2 +
4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x))*sinh
(x)^2 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x) +
6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11 + (a^8*b^4 - 4*a^6*
b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cosh(x)^5 + 5*(a^9*b^3 - 4*a^7*b^5 + 6
*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cosh(x)^4 + 2*(4*a^10*b^2 - 15*a^8*b^4 + 20*
a^6*b^6 - 10*a^4*b^8 + b^12)*cosh(x)^3 + 2*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b
^7 - 10*a^3*b^9 + 3*a*b^11)*cosh(x)^2 + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b
^6 - 10*a^4*b^8 + b^12)*cosh(x))*sinh(x))]
```

giac [B] time = 0.15, size = 453, normalized size = 2.30

$$\frac{\left(2\,Aa^3-4\,Ba^2b+3\,Aab^2-Bb^3\right)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{6\,Aa^3b^3e^{(5\,x)}-12\,Ba^2b^4e^{(5\,x)}+9\,Aab^5e^{(5\,x)}-3\,Bb^6e^{(5\,x)}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="giac")
```

```
[Out] (2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^x + a)/sqrt(-a^2 + b^
2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + 1/3*(6*A*a^3*b
^3*e^(5*x) - 12*B*a^2*b^4*e^(5*x) + 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 3
0*A*a^4*b^2*e^(4*x) - 60*B*a^3*b^3*e^(4*x) + 45*A*a^2*b^4*e^(4*x) - 15*B*a*
b^5*e^(4*x) - 8*B*a^6*e^(3*x) + 44*A*a^5*b*e^(3*x) - 64*B*a^4*b^2*e^(3*x) +
 82*A*a^3*b^3*e^(3*x) - 78*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) - 24*B*a^
5*b*e^(2*x) + 102*A*a^4*b^2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*
e^(2*x) - 24*B*a*b^5*e^(2*x) + 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a
^3*b^3*e^x - 66*B*a^2*b^4*e^x + 15*A*a*b^5*e^x + 3*B*b^6*e^x - 2*B*a^3*b^3
+ 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*(b*e^(2*x) + 2*a*e^x + b)^3)
```

maple [A] time = 0.08, size = 342, normalized size = 1.74

$$-\frac{2\left(-\frac{\left(6A\,a^2b+3Aa\,b^2+2A\,b^3-2a^3B-2B\,a^2b-6Ba\,b^2-B\,b^3\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(9A\,a^2b+A\,b^3-3a^3B-7Ba\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6A\,a^2b-3Aa\,b^2+2A\,b^3-2\right.}{2(a+b)\left(a\right.}}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^4,x)
```

```
[Out] -2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^3)/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*x)^5+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*
B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*x)^3-1/2*(6*A*a^2*b-3*A*a
*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tanh(1/2*x))/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)^3+(2*A*a^3+3*A*a*b^
2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^4,x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**4,x)
```

```
[Out] Timed out
```

3.114 $\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=56

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

```
[Out] B*x/b-2*B*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1
/2)/a/b
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2735, 2659, 208}

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{Bx}{b} - \frac{\left(aB - \frac{b^2B}{a}\right)\int \frac{1}{a+b\cosh(x)}\,dx}{b}$$
$$= \frac{Bx}{b} - \frac{\left(2\left(aB - \frac{b^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{Bx}{b} - \frac{2\sqrt{a-b}\,\sqrt{a+b}\,B\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

Mathematica [A] time = 0.08, size = 56, normalized size = 1.00

$$\frac{B\left(\frac{2\sqrt{b^2-a^2}\,\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b} + \frac{ax}{b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*((a*x)/b + (2*Sqrt[-a^2 + b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[-a^2 + b
^2]])/b))/a
```

fricas [A] time = 1.80, size = 190, normalized size = 3.39

$$\left[\frac{Bax + \sqrt{a^2-b^2}\,B\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{ab}, \frac{Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(B*a*x + sqrt(a^2 - b^2)*B*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)))/(a*b), (B*a*x + 2*sqrt(-a^2 + b^2)*B*arctan(-sqrt(
-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a*b)]
```

giac [A] time = 0.14, size = 57, normalized size = 1.02

$$\frac{Bx}{b} - \frac{2\left(Ba^2 - Bb^2\right)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - 2*(B*a^2 - B*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 +
 b^2)*a*b)
```

maple [B] time = 0.07, size = 107, normalized size = 1.91

$$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)aB}{b\sqrt{(a+b)\,(a-b)}}+\frac{2Bb\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] -B/b*ln(tanh(1/2*x)-1)+B/b*ln(tanh(1/2*x)+1)-2/b/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*a*B+2*B/a*b/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.49, size = 205, normalized size = 3.66

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\sqrt{a^2\,b^2}\,\sqrt{B^2\,b^2-B^2\,a^2}}{B\left(b^4-a^2\,b^2\right)}+\frac{a\,b^2\,\mathrm{e}^x\left(\frac{2\,\sqrt{B^2\,b^2-B^2\,a^2}}{B\,b^2\left(b^4-a^2\,b^2\right)}-\frac{2\left(B\,a^2\,\sqrt{a^2\,b^2}-B\,b^2\,\sqrt{a^2\,b^2}\right)}{a^2\,b^4\,\sqrt{-B^2\left(a^2-b^2\right)}\,\sqrt{a^2\,b^2}}\right)\sqrt{a^2\,b^2}}{2}\right)\sqrt{B^2\,b^2-B^2\,a^2}}{\sqrt{a^2\,b^2}}+\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + (B*b)/a)/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((b*(a^2*b^2)^(1/2)*(B^2*b^2 - B^2*a^2)^(1/2))/(B*(b^4 - a^2*b^2)) +
 (a*b^2*exp(x)*((2*(B^2*b^2 - B^2*a^2)^(1/2))/(B*b^2*(b^4 - a^2*b^2)) - (2*
(B*a^2*(a^2*b^2)^(1/2) - B*b^2*(a^2*b^2)^(1/2)))/(a^2*b^4*(-B^2*(a^2 - b^2)
)^(1/2)*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/2)*(B^2*b^2 - B^2*a^2)^(1/2))/(a
^2*b^2)^(1/2) + (B*x)/b
```

sympy [A] time = 28.17, size = 170, normalized size = 3.04

$$\begin{cases} \text{NaN} & \text{fo} \\ \frac{Bx}{b} & \text{fo} \\ \frac{B\sinh(x)}{a} & \text{fo} \\ \frac{Bx}{b} & \text{fo} \\ \frac{Bx}{b} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} - \frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*x/b, Eq(a, -b)), (B*sinh(x)/a, Eq(
b, 0)), (B*x/b, Eq(a, b)), (B*x/b + B*log(-sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))/(b*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) + b/(a - b)
) + tanh(x/2))/(b*sqrt(a/(a - b) + b/(a - b))) + B*log(-sqrt(a/(a - b) + b/
(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b
) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))), True))
```

3.115 $\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] B*x/b
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*x)/b
```

fricas [A] time = 0.70, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

giac [A] time = 0.12, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

maple [A] time = 0.01, size = 7, normalized size = 1.17

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] B*x/b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 0.02, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cosh(x) + (B*a)/b)/(a + b*cosh(x)),x)
```

```
[Out] (B*x)/b
```

sympy [A] time = 0.30, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

```
[Out] B*x/b
```

3.116 $\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx$

Optimal. Leaf size=11

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

[Out] sinh(x)/(b+a*cosh(x))

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2754, 8}

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]

[Out] Sinh[x]/(b + a*Cosh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx &= \frac{\sinh(x)}{b+a\cosh(x)}+\frac{\int 0\,dx}{a^2-b^2}\\ &= \frac{\sinh(x)}{b+a\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 11, normalized size = 1.00

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

```
[Out] Sinh[x]/(b + a*Cosh[x])
```

fricas [B] time = 1.78, size = 54, normalized size = 4.91

$$-\frac{2\,(b\cosh(x)+b\sinh(x)+a)}{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)+a^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(
x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

giac [B] time = 0.13, size = 26, normalized size = 2.36

$$-\frac{2\,(be^x+a)}{\left(ae^{(2\,x)}+2\,be^x+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

maple [B] time = 0.06, size = 29, normalized size = 2.64

$$\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))/(b+a*cosh(x))^2,x)
```

```
[Out] 2*tanh(1/2*x)/(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b+a+b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.02, size = 51, normalized size = 4.64

$$-\frac{\frac{2\,{\mathrm{e}}^x\,\left(a\,b^3-a^3\,b\right)}{a\,\left(a\,b^2-a^3\right)}+2}{a+2\,b\,{\mathrm{e}}^x+a\,{\mathrm{e}}^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(x))/(b + a*cosh(x))^2,x)
```

```
[Out] -((2*exp(x)*(a*b^3 - a^3*b))/(a*(a*b^2 - a^3)) + 2)/(a + 2*b*exp(x) + a*exp
(2*x))
```

sympy [B] time = 146.49, size = 26, normalized size = 2.36

$$\frac{2\tanh{\left(\frac{x}{2}\right)}}{a\tanh^{2}{\left(\frac{x}{2}\right)}+a-b\tanh^{2}{\left(\frac{x}{2}\right)}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))/(b+a*cosh(x))**2,x)
```

```
[Out] 2*tanh(x/2)/(a*tanh(x/2)**2 + a - b*tanh(x/2)**2 + b)
```

3.117 $\int \frac{3+\cosh(x)}{2-\cosh(x)}\, dx$

Optimal. Leaf size=36

$$\frac{5x}{\sqrt{3}} - x + \frac{10\tanh^{-1}\left(\frac{\sinh(x)}{-\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] -x+5/3*x*3^(1/2)+10/3*arctanh(sinh(x)/(2-cosh(x)+3^(1/2)))*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2657}

$$\frac{5x}{\sqrt{3}} - x + \frac{10\tanh^{-1}\left(\frac{\sinh(x)}{-\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

```
[Out] -x + (5*x)/Sqrt[3] + (10*ArcTanh[Sinh[x]/(2 + Sqrt[3] - Cosh[x])])/Sqrt[3]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+\cosh(x)}{2-\cosh(x)}\, dx &= -x + 5\int \frac{1}{2-\cosh(x)}\, dx\\ &= -x + \frac{5x}{\sqrt{3}} + \frac{10\tanh^{-1}\left(\frac{\sinh(x)}{2+\sqrt{3}-\cosh(x)}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 24, normalized size = 0.67

$$\frac{10 \tanh^{-1}\left(\sqrt{3} \tanh\left(\frac{x}{2}\right)\right)}{\sqrt{3}} - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

```
[Out] -x + (10*ArcTanh[Sqrt[3]*Tanh[x/2]])/Sqrt[3]
```

fricas [A] time = 2.70, size = 45, normalized size = 1.25

$$\frac{5}{3} \sqrt{3} \log\left(-\frac{2\left(\sqrt{3} - 2\right)\cosh(x) - \left(2\sqrt{3} - 3\right)\sinh(x) - \sqrt{3} + 2}{\cosh(x) - 2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="fricas")
```

```
[Out] 5/3*sqrt(3)*log(-(2*(sqrt(3) - 2)*cosh(x) - (2*sqrt(3) - 3)*sinh(x) - sqrt(
3) + 2)/(cosh(x) - 2)) - x
```

giac [A] time = 0.14, size = 37, normalized size = 1.03

$$-\frac{5}{3} \sqrt{3} \log\left(\frac{\left|-2\sqrt{3} + 2 e^{x} - 4\right|}{\left|2\sqrt{3} + 2 e^{x} - 4\right|}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="giac")
```

```
[Out] -5/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*e^x - 4)/abs(2*sqrt(3) + 2*e^x - 4)) -
x
```

maple [A] time = 0.06, size = 32, normalized size = 0.89

$$\frac{10\sqrt{3}\ \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)}{3} + \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+cosh(x))/(2-cosh(x)),x)
```

```
[Out] 10/3*3^(1/2)*arctanh(tanh(1/2*x)*3^(1/2))+ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+
1)
```

maxima [A] time = 0.41, size = 34, normalized size = 0.94

$$\frac{5}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}+2}{\sqrt{3}+e^{(-x)}-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="maxima")
```

```
[Out] 5/3*sqrt(3)*log(-(sqrt(3) - e^(-x) + 2)/(sqrt(3) + e^(-x) - 2)) - x
```

mupad [B] time = 0.11, size = 48, normalized size = 1.33

$$\frac{5\sqrt{3}\ \ln\left(10\,e^x+\frac{5\sqrt{3}\,(4\,e^x-2)}{3}\right)}{3}-\frac{5\sqrt{3}\ \ln\left(10\,e^x-\frac{5\sqrt{3}\,(4\,e^x-2)}{3}\right)}{3}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cosh(x) + 3)/(cosh(x) - 2),x)
```

```
[Out] (5*3^(1/2)*log(10*exp(x) + (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - (5*3^(1/2)*lo
g(10*exp(x) - (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - x
```

sympy [A] time = 0.79, size = 44, normalized size = 1.22

$$-x-\frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{\sqrt{3}}{3}\right)}{3}+\frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cosh(x))/(2-cosh(x)),x)
```

```
[Out] -x - 5*sqrt(3)*log(tanh(x/2) - sqrt(3)/3)/3 + 5*sqrt(3)*log(tanh(x/2) + sqr
t(3)/3)/3
```

3.118 $\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=108

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

```
[Out] -2*I*B*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*cosh(x))^(1/2)/b/((a+b*cosh(x))/(a+b))^(1/2)-2*I*(A*b-B
*a)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a
+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2752, 2663, 2661, 2655, 2653}

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*B*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a
 + b*Cosh[x])/(a + b)]) - ((2*I)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*
EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx &= \frac{B\int \sqrt{a+b\cosh(x)}\,dx}{b}+\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b}\\
&= \frac{\left(B\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{\left((Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}\,dx}{b\sqrt{a+b\cosh(x)}}\\
&= -\frac{2iB\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.48, size = 80, normalized size = 0.74

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((Ab-aB)F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+B(a+b)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] ((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*B*EllipticE[(I/2)*x, (2*b)/(
a + b)] + (A*b - a*B)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cos
h[x]])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cosh(x)+A}{\sqrt{b\cosh(x)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x)+A}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

maple [A] time = 0.46, size = 218, normalized size = 2.02

$$\frac{2\left(A\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+B\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2B\,\text{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{\right.\right.\right.}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2b\left(\sinh^4\left(\frac{x}{2}\right)\right)+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\;\text{sinh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x)
```

```
[Out] 2*(A*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))+B*Ell
ipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2))-2*B*EllipticE
(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2*(a-b)/b)^(1/2)))*(-sinh(1/2*x)^2)^(
1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/
2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1
/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x)+A}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
[Out] integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cosh(x)}{\sqrt{a + b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^(1/2),x)
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cosh{(x)}}{\sqrt{a + b\cosh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(1/2),x)
[Out] Integral((A + B*cosh(x))/sqrt(a + b*cosh(x)), x)
```

3.119 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

```
[Out] -2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(1/2)-2*I*(A*b-B*a)*(cosh(1/2*
x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*(a
+b*cosh(x))^(1/2)/b/(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)-2*I*B*(cosh(1/2*x
)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*cosh(x))/(a+b))^(1/2)/b/(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((-2*I)*(A*b - a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/
(b*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]) - ((2*I)*B*Sqrt[(a + b*Cosh[x
])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]) - (2
*(A*b - a*B)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = -\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} - \frac{2\int \frac{\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} + \frac{B\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b\cosh(x)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}} + \frac{\left((Ab-aB)\sqrt{a+b\cosh(x)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}\,dx}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{(B\cdots}{\cdots}$$

$$= -\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}} - \frac{2(Ab-aB)\,\mathrm{s}\cdots}{\left(a^2-b^2\right)\sqrt{a+\cdots}}$$

Mathematica [A] time = 0.38, size = 133, normalized size = 0.88

$$\frac{-2iB\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)+2b\sinh(x)(aB-Ab)+2i(a+b)(aB-Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b(a-b)(a+b)\sqrt{a+b\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

```
[Out] ((2*I)*(a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)
*x, (2*b)/(a + b)] - (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-(A*b) + a*B)*Sinh[x])/((a - b)*b*(a +
 b)*Sqrt[a + b*Cosh[x]])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cosh(x)+A)\sqrt{b\cosh(x)+a}}{b^2\cosh(x)^2+2\,ab\cosh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)*sqrt(b*cosh(x) + a)/(b^2*cosh(x)^2 + 2*a*b*cosh(x)
 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="giac")
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

maple [B] time = 1.05, size = 483, normalized size = 3.18

$$\frac{\sqrt{\left(2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b\right)\left(\sinh^2\left(\frac{x}{2}\right)\right)}\left(\frac{2B\sqrt{\frac{2b\left(\cosh^2\left(\frac{x}{2}\right)\right)+a-b}{a-b}}\,\sqrt{-\left(\sinh^2\left(\frac{x}{2}\right)\right)}\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{\frac{-2a+2b}{b}}}{2}\right)}{b\sqrt{-\frac{2b}{a-b}}\,\sqrt{2b\left(\sinh^4\left(\frac{x}{2}\right)\right)+(a+b)\left(\sinh^2\left(\frac{x}{2}\right)\right)}}+\frac{2(Ab-aB)\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x)
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(2*B/b/(-2*b/(a-b))^(1/2)*((2
*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*b*sinh(1/2*x)^
4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(
(-2*a+2*b)/b)^(1/2))+2*(A*b-B*a)/b/sinh(1/2*x)^2/(2*b*sinh(1/2*x)^2+a+b)/(-
2*b/(a-b))^(1/2)/(a^2-b^2)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*(-
2*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^2+(-sinh(1/2*x)^2)^(1/2)*Ell
ipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*(2*b/(a-b)*
sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*a+(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/
2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)^2+
(a+b)/(a-b))^(1/2)*b-2*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(
a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))
^(1/2)*b))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^(3/2),x)
```

```
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(3/2),x)
```

```
[Out] Timed out
```

3.120 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal. Leaf size=231

$$-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(\cdot\right.}{}$$

```
[Out] -2/3*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(3/2)-2/3*(4*A*a*b-B*a^2-3*B
*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^(1/2)-2/3*I*(4*A*a*b-B*a^2-3*B*b^2)
*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*cosh(x))^(1/2)/b/(a^2-b^2)^2/((a+b*cosh(x))/(a+b))^(1/2)+2/3*
I*(A*b-B*a)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/
2)*(b/(a+b))^(1/2))*((a+b*cosh(x))/(a+b))^(1/2)/b/(a^2-b^2)/(a+b*cosh(x))^(
1/2)
```

Rubi [A] time = 0.35, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(\cdot\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)
*x, (2*b)/(a + b)])/(b*(a^2 - b^2)^2*Sqrt[(a + b*Cosh[x])/(a + b)]) + (((2*
I)/3)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a
 + b)])/(b*(a^2 - b^2)*Sqrt[a + b*Cosh[x]]) - (2*(A*b - a*B)*Sinh[x])/(3*(a
^2 - b^2)*(a + b*Cosh[x])^(3/2)) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sinh[x])/
(3*(a^2 - b^2)^2*Sqrt[a + b*Cosh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2A+Ab^2-4a\right.}{}}{}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} - \frac{(Ab-aB)\int \frac{}{\sqrt{a+b}}}{3b\left(a^2-b\right.}$$

$$= -\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} + \frac{\left(\left(4aAb-a^2B-\right.\right.}{}$$

$$= -\frac{2i\left(4aAb-a^2B-3b^2B\right)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,F\left(\frac{ix}{2}|\right.}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x}}$$

Mathematica [A] time = 0.92, size = 172, normalized size = 0.74

$$\frac{2\left(\frac{\sinh(x)\left(2a^3B+b\cosh(x)\left(a^2B-4aAb+3b^2B\right)-5a^2Ab+2ab^2B+Ab^3\right)}{\left(a^2-b^2\right)^2}+\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\left(\left(a^2B-4aAb+3b^2B\right)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-(a-b)(aB-Ab)F\left(\frac{ix}{2}|\frac{2b}{a+b}\right)\right)}{b(a-b)^2}\right)}{3(a+b\cosh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

```
[Out] (2*((I*((a + b*Cosh[x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*Ellipt
icE[(I/2)*x, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(I/2)*x, (2*
b)/(a + b)]))/((a - b)^2*b) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B +
b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cosh[x])*Sinh[x])/(a^2 - b^2)^2))/(3*(a + b*
Cosh[x])^(3/2))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cosh(x)+A)\sqrt{b\cosh(x)+a}}{b^3\cosh(x)^3+3\,ab^2\cosh(x)^2+3\,a^2b\cosh(x)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cosh(x) + A)*sqrt(b*cosh(x) + a)/(b^3*cosh(x)^3 + 3*a*b^2*cosh(
x)^2 + 3*a^2*b*cosh(x) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

maple [B] time = 1.55, size = 797, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x)
```

```
[Out] ((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(2*(A*b-B*a)/b*(-1/6/b/(a-b)/
(a+b)*cosh(1/2*x)*(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x
)^2+1/2*(a-b)/b)^2-8/3*b*sinh(1/2*x)^2/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*b*
cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^
3)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2
)^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)
*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-16/3*a*b/(a+b)^2/(a-b)^2*(-a+
b)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2
)^(1/2)/(2*b*sinh(1/2*x)^4+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(
cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2
*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))-2*B/b/(-2*b/(a-b))^(1/2)
/sinh(1/2*x)^2/(2*b*sinh(1/2*x)^2+a+b)/(a^2-b^2)*(2*b*sinh(1/2*x)^4+(a+b)*s
inh(1/2*x)^2)^(1/2)*(2*(-2*b/(a-b))^(1/2)*b*cosh(1/2*x)*sinh(1/2*x)^2-(-sin
h(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/
b)^(1/2))*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*a-(-sinh(1/2*x)^2)^(1
/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*(2*b
/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*b+2*(-sinh(1/2*x)^2)^(1/2)*Elliptic
E(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*(2*b/(a-b)*sinh(
1/2*x)^2+(a+b)/(a-b))^(1/2)*b))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cosh(x) + A}{(b\cosh(x) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*cosh(x))^(5/2),x)
[Out] int((A + B*cosh(x))/(a + b*cosh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*cosh(x))**(5/2),x)
[Out] Timed out
```

3.121 $\int \left(a\cosh^2(x)\right)^{7/2}\,dx$

Optimal. Leaf size=72

$$\frac{16}{35}a^3\tanh(x)\sqrt{a\cosh^2(x)}+\frac{8}{35}a^2\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{35}a\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

```
[Out] 8/35*a^2*(a*cosh(x)^2)^(3/2)*tanh(x)+6/35*a*(a*cosh(x)^2)^(5/2)*tanh(x)+1/7
*(a*cosh(x)^2)^(7/2)*tanh(x)+16/35*a^3*(a*cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3203, 3207, 2637}

$$\frac{8}{35}a^2\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{16}{35}a^3\tanh(x)\sqrt{a\cosh^2(x)}+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{35}a\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(7/2),x]
```

```
[Out] (16*a^3*Sqrt[a*Cosh[x]^2]*Tanh[x])/35 + (8*a^2*(a*Cosh[x]^2)^(3/2)*Tanh[x])
/35 + (6*a*(a*Cosh[x]^2)^(5/2)*Tanh[x])/35 + ((a*Cosh[x]^2)^(7/2)*Tanh[x])/
7
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\cosh^2(x)\right)^{7/2}\,dx &= \frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x)+\frac{1}{7}(6a)\int \left(a\cosh^2(x)\right)^{5/2}\,dx\\
&= \frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x)+\frac{1}{35}\left(24a^2\right)\int \left(a\cosh^2(x)\right)^{3/2}\\
&= \frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x\\
&= \frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x\\
&= \frac{16}{35}a^3\sqrt{a\cosh^2(x)}\,\tanh(x)+\frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.58

$$\frac{a^3(1225\sinh(x)+245\sinh(3x)+49\sinh(5x)+5\sinh(7x))\operatorname{sech}(x)\sqrt{a\cosh^2(x)}}{2240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(7/2),x]
```

```
[Out] (a^3*Sqrt[a*Cosh[x]^2]*Sech[x]*(1225*Sinh[x] + 245*Sinh[3*x] + 49*Sinh[5*x]
 + 5*Sinh[7*x]))/2240
```

fricas [B] time = 2.76, size = 817, normalized size = 11.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(70*a^3*cosh(x)*e^x*sinh(x)^13 + 5*a^3*e^x*sinh(x)^14 + 7*(65*a^3*co
sh(x)^2 + 7*a^3)*e^x*sinh(x)^12 + 28*(65*a^3*cosh(x)^3 + 21*a^3*cosh(x))*e^
x*sinh(x)^11 + 7*(715*a^3*cosh(x)^4 + 462*a^3*cosh(x)^2 + 35*a^3)*e^x*sinh(
x)^10 + 70*(143*a^3*cosh(x)^5 + 154*a^3*cosh(x)^3 + 35*a^3*cosh(x))*e^x*sin
h(x)^9 + 35*(429*a^3*cosh(x)^6 + 693*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 35
*a^3)*e^x*sinh(x)^8 + 8*(2145*a^3*cosh(x)^7 + 4851*a^3*cosh(x)^5 + 3675*a^3
*cosh(x)^3 + 1225*a^3*cosh(x))*e^x*sinh(x)^7 + 7*(2145*a^3*cosh(x)^8 + 6468
*a^3*cosh(x)^6 + 7350*a^3*cosh(x)^4 + 4900*a^3*cosh(x)^2 - 175*a^3)*e^x*sin
h(x)^6 + 14*(715*a^3*cosh(x)^9 + 2772*a^3*cosh(x)^7 + 4410*a^3*cosh(x)^5 +
4900*a^3*cosh(x)^3 - 525*a^3*cosh(x))*e^x*sinh(x)^5 + 35*(143*a^3*cosh(x)^1
```

```
0 + 693*a^3*cosh(x)^8 + 1470*a^3*cosh(x)^6 + 2450*a^3*cosh(x)^4 - 525*a^3*c
osh(x)^2 - 7*a^3)*e^x*sinh(x)^4 + 140*(13*a^3*cosh(x)^11 + 77*a^3*cosh(x)^9
 + 210*a^3*cosh(x)^7 + 490*a^3*cosh(x)^5 - 175*a^3*cosh(x)^3 - 7*a^3*cosh(x
))*e^x*sinh(x)^3 + 7*(65*a^3*cosh(x)^12 + 462*a^3*cosh(x)^10 + 1575*a^3*cos
h(x)^8 + 4900*a^3*cosh(x)^6 - 2625*a^3*cosh(x)^4 - 210*a^3*cosh(x)^2 - 7*a^
3)*e^x*sinh(x)^2 + 14*(5*a^3*cosh(x)^13 + 42*a^3*cosh(x)^11 + 175*a^3*cosh(
x)^9 + 700*a^3*cosh(x)^7 - 525*a^3*cosh(x)^5 - 70*a^3*cosh(x)^3 - 7*a^3*cos
h(x))*e^x*sinh(x) + (5*a^3*cosh(x)^14 + 49*a^3*cosh(x)^12 + 245*a^3*cosh(x)
^10 + 1225*a^3*cosh(x)^8 - 1225*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 - 49*a^3*
cosh(x)^2 - 5*a^3)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^7
*e^(2*x) + (e^(2*x) + 1)*sinh(x)^7 + cosh(x)^7 + 7*(cosh(x)*e^(2*x) + cosh(
x))*sinh(x)^6 + 21*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^5 + 35*(cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^4 + 35*(cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(
x)^3 + 21*(cosh(x)^5*e^(2*x) + cosh(x)^5)*sinh(x)^2 + 7*(cosh(x)^6*e^(2*x)
+ cosh(x)^6)*sinh(x))
```

giac [A] time = 0.12, size = 79, normalized size = 1.10

$$\frac{1}{4480}\left(5\,a^3e^{(7\,x)}+49\,a^3e^{(5\,x)}+245\,a^3e^{(3\,x)}+1225\,a^3e^{x}-\left(1225\,a^3e^{(6\,x)}+245\,a^3e^{(4\,x)}+49\,a^3e^{(2\,x)}+5\,a^3\right)e^{(-7\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*e^(7*x) + 49*a^3*e^(5*x) + 245*a^3*e^(3*x) + 1225*a^3*e^x - (
1225*a^3*e^(6*x) + 245*a^3*e^(4*x) + 49*a^3*e^(2*x) + 5*a^3)*e^(-7*x))*sqrt
(a)
```

maple [A] time = 0.16, size = 38, normalized size = 0.53

$$\frac{a^4\cosh(x)\sinh(x)\left(5\left(\cosh^6(x)\right)+6\left(\cosh^4(x)\right)+8\left(\cosh^2(x)\right)+16\right)}{35\sqrt{a\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(7/2),x)
```

```
[Out] 1/35*a^4*cosh(x)*sinh(x)*(5*cosh(x)^6+6*cosh(x)^4+8*cosh(x)^2+16)/(a*cosh(x
)^2)^(1/2)
```

maxima [A] time = 0.43, size = 71, normalized size = 0.99

$$\frac{1}{896}\,a^{\frac{7}{2}}e^{(7\,x)}+\frac{7}{640}\,a^{\frac{7}{2}}e^{(5\,x)}+\frac{7}{128}\,a^{\frac{7}{2}}e^{(3\,x)}-\frac{35}{128}\,a^{\frac{7}{2}}e^{(-x)}-\frac{7}{128}\,a^{\frac{7}{2}}e^{(-3\,x)}-\frac{7}{640}\,a^{\frac{7}{2}}e^{(-5\,x)}-\frac{1}{896}\,a^{\frac{7}{2}}e^{(-7\,x)}+\frac{35}{128}\,a^{\frac{7}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/896*a^(7/2)*e^(7*x) + 7/640*a^(7/2)*e^(5*x) + 7/128*a^(7/2)*e^(3*x) - 35/
128*a^(7/2)*e^(-x) - 7/128*a^(7/2)*e^(-3*x) - 7/640*a^(7/2)*e^(-5*x) - 1/89
6*a^(7/2)*e^(-7*x) + 35/128*a^(7/2)*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a \cosh(x)^2\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(7/2),x)
```

```
[Out] int((a*cosh(x)^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

3.122 $\int \left(a\cosh^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\tanh(x)\sqrt{a\cosh^2(x)}+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{15}a\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

```
[Out] 4/15*a*(a*cosh(x)^2)^(3/2)*tanh(x)+1/5*(a*cosh(x)^2)^(5/2)*tanh(x)+8/15*a^2
*(a*cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3203, 3207, 2637}

$$\frac{8}{15}a^2\tanh(x)\sqrt{a\cosh^2(x)}+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{15}a\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cosh[x]^2]*Tanh[x])/15 + (4*a*(a*Cosh[x]^2)^(3/2)*Tanh[x])/15
 + ((a*Cosh[x]^2)^(5/2)*Tanh[x])/5
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx = \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{5}(4a)\int \left(a\cosh^2(x)\right)^{3/2}\,dx$$
$$= \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{15}\left(8a^2\right)\int \sqrt{a\cosh^2(x)}\,dx$$
$$= \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x) + \frac{1}{15}\left(8a^2\sqrt{a\cosh^2(x)}\,\mathrm{sech}(x\right.$$
$$= \frac{8}{15}a^2\sqrt{a\cosh^2(x)}\tanh(x) + \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\sinh(x) + 25\sinh(3x) + 3\sinh(5x))\mathrm{sech}(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*Cosh[x]^2]*Sech[x]*(150*Sinh[x] + 25*Sinh[3*x] + 3*Sinh[5*x]))/
240
```

fricas [B] time = 0.86, size = 501, normalized size = 9.45

$$\left(30\,a^2\cosh(x)e^x\sinh(x)^9 + 3\,a^2e^x\sinh(x)^{10} + 5\left(27\,a^2\cosh(x)^2 + 5\,a^2\right)e^x\sinh(x)^8 + 40\left(9\,a^2\cosh(x)^3 + 5\,a^2\,\mathrm{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cosh
(x)^2 + 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 + 5*a^2*cosh(x))*e^x*sin
h(x)^7 + 10*(63*a^2*cosh(x)^4 + 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6 +
4*(189*a^2*cosh(x)^5 + 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)^5 +
 10*(63*a^2*cosh(x)^6 + 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 - 15*a^2)*e^x
*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 + 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^3 - 15
*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 + 140*a^2*cosh(x)^6 + 450
*a^2*cosh(x)^4 - 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a^2*cosh(
x)^9 + 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 - 60*a^2*cosh(x)^3 - 5*a^2*cosh(
x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 + 25*a^2*cosh(x)^8 + 150*a^2*cosh(x)^6
```

```
- 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 - 3*a^2)*e^x)*sqrt(a*e^(4*x) + 2*a*e
^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) + 1)*sinh(x)^5 + cosh(x)^5
 + 5*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(2*x) + cosh(x
)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^2 + 5*(cosh(x)^
4*e^(2*x) + cosh(x)^4)*sinh(x))
```

giac [A] time = 0.12, size = 61, normalized size = 1.15

$$\frac{1}{480}\left(3\,a^2e^{(5\,x)}+25\,a^2e^{(3\,x)}+150\,a^2e^x-\left(150\,a^2e^{(4\,x)}+25\,a^2e^{(2\,x)}+3\,a^2\right)e^{(-5\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*e^(5*x) + 25*a^2*e^(3*x) + 150*a^2*e^x - (150*a^2*e^(4*x) + 25
*a^2*e^(2*x) + 3*a^2)*e^(-5*x))*sqrt(a)
```

maple [A] time = 0.18, size = 32, normalized size = 0.60

$$\frac{a^3\cosh(x)\sinh(x)\left(3\left(\cosh^4(x)\right)+4\left(\cosh^2(x)\right)+8\right)}{15\sqrt{a\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(5/2),x)
```

```
[Out] 1/15*a^3*cosh(x)*sinh(x)*(3*cosh(x)^4+4*cosh(x)^2+8)/(a*cosh(x)^2)^(1/2)
```

maxima [A] time = 0.44, size = 53, normalized size = 1.00

$$\frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)}+\frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)}-\frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)}-\frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)}-\frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)}+\frac{5}{16}\,a^{\frac{5}{2}}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) - 5/96*a
^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) + 5/16*a^(5/2)*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(a\cosh(x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(5/2),x)
```

```
[Out] int((a*cosh(x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.123 $\int \left(a\cosh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}$$

```
[Out] 1/3*(a*cosh(x)^2)^(3/2)*tanh(x)+2/3*a*(a*cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3203, 3207, 2637}

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\cosh^2(x)\right)^{3/2}\,dx &= \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{1}{3}(2a)\int \sqrt{a\cosh^2(x)}\,dx\\ &= \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{1}{3}\left(2a\sqrt{a\cosh^2(x)}\,\mathrm{sech}(x)\right)\int \cosh(x)\,dx\\ &= \frac{2}{3}a\sqrt{a\cosh^2(x)}\,\tanh(x)+\frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.76

$$\frac{1}{12}a(9\sinh(x)+\sinh(3x))\mathrm{sech}(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12
```

fricas [B] time = 0.81, size = 222, normalized size = 6.53

$$\frac{\left(6\,a\cosh(x)e^{x}\sinh(x)^{5}+ae^{x}\sinh(x)^{6}+3\left(5\,a\cosh(x)^{2}+3\,a\right)e^{x}\sinh(x)^{4}+4\left(5\,a\cosh(x)^{3}+9\,a\cosh(x)\right)e^{x}\sin}{24\left(\cosh(x)^{3}e^{(2\,x)}+\left(e^{(2\,x)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 + 3*a)
*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*cos
h(x)^4 + 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 + 6*a*cosh(x)
^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 + 9*a*cosh(x)^4 - 9*a*cosh(x)^
2 - a)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x) + (
e^(2*x) + 1)*sinh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^
2 + 3*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x))
```

giac [A] time = 0.12, size = 29, normalized size = 0.85

$$-\frac{1}{24}\left(\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)*a^(3/2)
```

maple [A] time = 0.18, size = 24, normalized size = 0.71

$$\frac{a^2 \cosh(x)\sinh(x)\left(\cosh^2(x)+2\right)}{3\sqrt{a\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(3/2),x)
[Out] 1/3*a^2*cosh(x)*sinh(x)*(cosh(x)^2+2)/(a*cosh(x)^2)^(1/2)
```

maxima [A] time = 0.42, size = 35, normalized size = 1.03

$$\frac{1}{24}\,a^{\frac{3}{2}}e^{(3\,x)} - \frac{3}{8}\,a^{\frac{3}{2}}e^{(-x)} - \frac{1}{24}\,a^{\frac{3}{2}}e^{(-3\,x)} + \frac{3}{8}\,a^{\frac{3}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(3/2),x, algorithm="maxima")
[Out] 1/24*a^(3/2)*e^(3*x) - 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^(
3/2)*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(a\cosh(x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(3/2),x)
[Out] int((a*cosh(x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(3/2),x)
[Out] Timed out
```

3.124 $\int \sqrt{a\cosh^2(x)}\,dx$

Optimal. Leaf size=13

$$\tanh(x)\sqrt{a\cosh^2(x)}$$

[Out] (a*cosh(x)^2)^(1/2)*tanh(x)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 2637}

$$\tanh(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*Cosh[x]^2],x]

[Out] Sqrt[a*Cosh[x]^2]*Tanh[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\cosh^2(x)}\,dx &= \left(\sqrt{a\cosh^2(x)}\,\mathrm{sech}(x)\right)\int \cosh(x)\,dx\\ &= \sqrt{a\cosh^2(x)}\,\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tanh(x)\sqrt{a\cosh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^2],x]
```

```
[Out] Sqrt[a*Cosh[x]^2]*Tanh[x]
```

fricas [B] time = 1.06, size = 69, normalized size = 5.31

$$\frac{\left(2\cosh(x)e^x\sinh(x)+e^x\sinh(x)^2+\left(\cosh(x)^2-1\right)e^x\right)\sqrt{ae^{(4x)}+2ae^{(2x)}+a}\,e^{(-x)}}{2\left(\cosh(x)e^{(2x)}+\left(e^{(2x)}+1\right)\sinh(x)+\cosh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 - 1)*e^x)*sqrt(a*e^
(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) + 1)*sinh(x) +
cosh(x))
```

giac [A] time = 0.13, size = 14, normalized size = 1.08

$$-\frac{1}{2}\sqrt{a}\left(e^{(-x)}-e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(e^(-x) - e^x)
```

maple [A] time = 0.15, size = 15, normalized size = 1.15

$$\frac{a\cosh(x)\sinh(x)}{\sqrt{a\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(1/2),x)
```

```
[Out] 1/(a*cosh(x)^2)^(1/2)*a*cosh(x)*sinh(x)
```

maxima [**A**] time = 0.42, size = 17, normalized size = 1.31

$$-\frac{1}{2}\sqrt{a}e^{(-x)} + \frac{1}{2}\sqrt{a}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*e^(-x) + 1/2*sqrt(a)*e^x
```

mupad [**B**] time = 0.05, size = 17, normalized size = 1.31

$$\sqrt{a}\tanh(x)\left(\frac{e^{-x}}{2} + \frac{e^{x}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^2)^(1/2),x)
```

```
[Out] a^(1/2)*tanh(x)*(exp(-x)/2 + exp(x)/2)
```

sympy [**A**] time = 0.45, size = 19, normalized size = 1.46

$$\frac{\sqrt{a}\sqrt{\cosh^{2}(x)}\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**2)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(cosh(x)**2)*sinh(x)/cosh(x)
```

3.125 $\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{a\cosh^2(x)}}$$

```
[Out] arctan(sinh(x))*cosh(x)/(a*cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3770}

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{\sqrt{a\cosh^2(x)}}$$
$$= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{a\cosh^2(x)}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.31

$$\frac{2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^2],x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

fricas [B] time = 1.32, size = 186, normalized size = 11.62

$$\left[-\frac{\sqrt{-a}\,\log\left(\frac{a\cosh(x)^2-2\sqrt{ae^{(4x)}+2ae^{(2x)}+a}\,(\cosh(x)e^x+e^x\sinh(x))\sqrt{-a}\,e^{(-x)}+\left(ae^{(2x)}+a\right)\sinh(x)^2+\left(a\cosh(x)^2-a\right)e^{(2x)}+2\left(a\cosh(x)e^{(2x)}+a\,c\right.}{\left(e^{(2x)}+1\right)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)+1}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log((a*cosh(x)^2 - 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*(cosh(x)*
e^x + e^x*sinh(x))*sqrt(-a)*e^(-x) + (a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)
^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x) - a)/((e^(2*x)
+ 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*x) +
 cosh(x))*sinh(x) + 1))/a, 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*arctan(cosh(
x) + sinh(x))/(a*e^(2*x) + a)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.22, size = 55, normalized size = 3.44

$$-\frac{\cosh(x)\sqrt{a\left(\sinh^{2}(x)\right)}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\left(\sinh^{2}(x)\right)}-2a}{\cosh(x)}\right)}{\sqrt{-a}\,\sinh(x)\sqrt{a\left(\cosh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(1/2),x)
```

```
[Out] -cosh(x)*(a*sinh(x)^2)^(1/2)/(-a)^(1/2)*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2
)-a)/cosh(x))/sinh(x)/(a*cosh(x)^2)^(1/2)
```

maxima [A] time = 0.46, size = 8, normalized size = 0.50

$$\frac{2\arctan\left(e^{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^x)/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a\cosh(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(1/2),x)
```

```
[Out] int(1/(a*cosh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cosh^{2}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x)**2), x)
```

3.126 $\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\tan^{-1}(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}$$

```
[Out] 1/2*arctan(sinh(x))*cosh(x)/a/(a*cosh(x)^2)^(1/2)+1/2*tanh(x)/a/(a*cosh(x)^
2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3204, 3207, 3770}

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\tan^{-1}(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(-3/2),x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Cos
h[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}} + \frac{\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx}{2a}$$
$$= \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}} + \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{2a\sqrt{a\cosh^2(x)}}$$
$$= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{2a\sqrt{a\cosh^2(x)}} + \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.74

$$\frac{\tanh(x) + 2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(-3/2),x]
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x])/(2*a*Sqrt[a*Cosh[x]^2])
```

fricas [B] time = 1.26, size = 299, normalized size = 7.12

$$\frac{\left(3\ \cosh(x)e^{x}\sinh(x)^{2} + e^{x}\sinh(x)^{3} + \left(3\ \cosh(x)^{2} - 1\right)e^{x}\sinh(x) + \left(4\ \cosh(x)e^{x}\sinh(x)^{3} + e^{x}\sinh(x)^{4} + 2\left(3\right.\right.\right.}{a^{2}\cosh(x)^{4} + \left(a^{2}e^{(2x)} + a^{2}\right)\sinh(x)^{4} + 2\,a^{2}\cosh(x)^{2} + 4\left(a^{2}\cosh(x)e^{(2x)} + a^{2}\cosh(x)\right)\sinh(x)^{3} + 2\left(3\,a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="fricas")
[Out] (3*cosh(x)*e^x*sinh(x)^2 + e^x*sinh(x)^3 + (3*cosh(x)^2 - 1)*e^x*sinh(x) +
(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^
2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x
)*arctan(cosh(x) + sinh(x)) + (cosh(x)^3 - cosh(x))*e^x)*sqrt(a*e^(4*x) + 2
*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4 + (a^2*e^(2*x) + a^2)*sinh(x)^4 + 2*a
```

```
^2*cosh(x)^2 + 4*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*c
osh(x)^2 + a^2 + (3*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*co
sh(x)^4 + 2*a^2*cosh(x)^2 + a^2)*e^(2*x) + 4*(a^2*cosh(x)^3 + a^2*cosh(x) +
 (a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))
```

giac [A] time = 0.14, size = 56, normalized size = 1.33

$$\frac{\frac{\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)}{\sqrt{a}}-\frac{4\left(e^{(-x)}-e^{x}\right)}{\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)\sqrt{a}}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/sqrt(a) - 4*(e^(-x) - e^x)/(
((e^(-x) - e^x)^2 + 4)*sqrt(a)))/a
```

maple [B] time = 0.30, size = 82, normalized size = 1.95

$$\frac{\sqrt{a\left(\sinh^{2}(x)\right)}\left(-\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\left(\sinh^{2}(x)\right)}-2a}{\cosh(x)}\right)a\left(\cosh^{2}(x)\right)+\sqrt{-a}\,\sqrt{a\left(\sinh^{2}(x)\right)}\right)}{2a^{2}\cosh(x)\sqrt{-a}\,\sinh(x)\sqrt{a\left(\cosh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(3/2),x)
```

```
[Out] 1/2/a^2/cosh(x)*(a*sinh(x)^2)^(1/2)*(-ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2)-
a)/cosh(x))*a*cosh(x)^2+(-a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/
(a*cosh(x)^2)^(1/2)
```

maxima [A] time = 0.46, size = 41, normalized size = 0.98

$$\frac{e^{(3\,x)}-e^{x}}{a^{\frac{3}{2}}e^{(4\,x)}+2\,a^{\frac{3}{2}}e^{(2\,x)}+a^{\frac{3}{2}}}+\frac{\arctan\left(e^{x}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(3*x) - e^x)/(a^(3/2)*e^(4*x) + 2*a^(3/2)*e^(2*x) + a^(3/2)) + arctan(e^
x)/a^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\cosh(x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(3/2),x)
[Out] int(1/(a*cosh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(3/2),x)
[Out] Integral((a*cosh(x)**2)**(-3/2), x)
```

3.127 $\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{3\cosh(x)\tan^{-1}(\sinh(x))}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

```
[Out] 3/8*arctan(sinh(x))*cosh(x)/a^2/(a*cosh(x)^2)^(1/2)+1/4*tanh(x)/a/(a*cosh(x
)^2)^(3/2)+3/8*tanh(x)/a^2/(a*cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3204, 3207, 3770}

$$\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{3\cosh(x)\tan^{-1}(\sinh(x))}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTan[Sinh[x]]*Cosh[x])/(8*a^2*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(4*a*(a*Cos
h[x]^2)^(3/2)) + (3*Tanh[x])/(8*a^2*Sqrt[a*Cosh[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}} + \frac{3\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx}{4a}$$

$$= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}} + \frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx}{8a^2}$$

$$= \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}} + \frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}} + \frac{(3\cosh(x))\int \operatorname{sech}(x)\,dx}{8a^2\sqrt{a\cosh^2(x)}}$$

$$= \frac{3\tan^{-1}(\sinh(x))\cosh(x)}{8a^2\sqrt{a\cosh^2(x)}} + \frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}} + \frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.66

$$\frac{\tanh(x)\left(2\operatorname{sech}^2(x)+3\right)+6\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{8a^2\sqrt{a\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^2)^(-5/2),x]
```

```
[Out] (6*ArcTan[Tanh[x/2]]*Cosh[x] + (3 + 2*Sech[x]^2)*Tanh[x])/(8*a^2*Sqrt[a*Cos
h[x]^2])
```

fricas [B] time = 1.00, size = 837, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(21*cosh(x)*e^x*sinh(x)^6 + 3*e^x*sinh(x)^7 + (63*cosh(x)^2 + 11)*e^x*s
inh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*e^x*sinh(x)^4 + (105*cosh(x)^4 + 1
```

```
10*cosh(x)^2 - 11)*e^x*sinh(x)^3 + (63*cosh(x)^5 + 110*cosh(x)^3 - 33*cosh(
x))*e^x*sinh(x)^2 + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*e^x*si
nh(x) + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*e^
x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 +
 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x
))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^x*sin
h(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*e^x*sinh(x) +
(cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4*cosh(x)^2 + 1)*e^x)*arctan(cosh(
x) + sinh(x)) + (3*cosh(x)^7 + 11*cosh(x)^5 - 11*cosh(x)^3 - 3*cosh(x))*e^x
)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(a^3*cosh(x)^8 + 4*a^3*cosh(x)^6
 + (a^3*e^(2*x) + a^3)*sinh(x)^8 + 8*(a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*si
nh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3*cosh(x)^2 + a^3 + (7*a^3*cosh(x)^2 + a
^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (7*a^3*cosh(x
)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(
x)^4 + 30*a^3*cosh(x)^2 + 3*a^3 + (35*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + 3*
a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cos
h(x) + (7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x
)^3 + a^3 + 4*(7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3 +
 (7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh
(x)^2 + (a^3*cosh(x)^8 + 4*a^3*cosh(x)^6 + 6*a^3*cosh(x)^4 + 4*a^3*cosh(x)^
2 + a^3)*e^(2*x) + 8*(a^3*cosh(x)^7 + 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 + a
^3*cosh(x) + (a^3*cosh(x)^7 + 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 + a^3*cosh(
x))*e^(2*x))*sinh(x))
```

giac [A] time = 0.17, size = 67, normalized size = 1.10

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)}{16\,a^{\frac{5}{2}}}-\frac{3\left(e^{(-x)}-e^{x}\right)^{3}+20\,e^{(-x)}-20\,e^{x}}{4\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)^{2}a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a^(5/2) - 1/4*(3*(e^(-x) - e
^x)^3 + 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a^(5/2))
```

maple [B] time = 0.30, size = 102, normalized size = 1.67

$$\frac{\sqrt{a\left(\sinh^{2}(x)\right)}\,\left(-3\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\left(\sinh^{2}(x)\right)}-2a}{\cosh(x)}\right)a\left(\cosh^{4}(x)\right)+3\sqrt{a\left(\sinh^{2}(x)\right)}\,\left(\cosh^{2}(x)\right)\sqrt{-a}\,+2\sqrt{-a}\,\sqrt{a\left(\sinh^{2}\right.}}{8a^{3}\cosh(x)^{3}\sqrt{-a}\,\sinh(x)\sqrt{a\left(\cosh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(5/2),x)
```

```
[Out] 1/8/a^3/cosh(x)^3*(a*sinh(x)^2)^(1/2)*(-3*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1
/2)-a)/cosh(x))*a*cosh(x)^4+3*(a*sinh(x)^2)^(1/2)*cosh(x)^2*(-a)^(1/2)+2*(-
a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/(a*cosh(x)^2)^(1/2)
```

maxima [A] time = 0.47, size = 75, normalized size = 1.23

$$\frac{3\,e^{(7\,x)} + 11\,e^{(5\,x)} - 11\,e^{(3\,x)} - 3\,e^{x}}{4\left(a^{\frac{5}{2}}e^{(8\,x)} + 4\,a^{\frac{5}{2}}e^{(6\,x)} + 6\,a^{\frac{5}{2}}e^{(4\,x)} + 4\,a^{\frac{5}{2}}e^{(2\,x)} + a^{\frac{5}{2}}\right)} + \frac{3\,\arctan\left(e^{x}\right)}{4\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(7*x) + 11*e^(5*x) - 11*e^(3*x) - 3*e^x)/(a^(5/2)*e^(8*x) + 4*a^(5
/2)*e^(6*x) + 6*a^(5/2)*e^(4*x) + 4*a^(5/2)*e^(2*x) + a^(5/2)) + 3/4*arctan
(e^x)/a^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\cosh(x)^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*cosh(x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.128 $\int \left(a\cosh^3(x)\right)^{5/2} dx$

Optimal. Leaf size=121

$$\frac{26}{165}a^2\sinh(x)\cosh^3(x)\sqrt{a\cosh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\cosh^3(x)}+\frac{26}{77}a^2\tanh(x)\sqrt{a\cosh^3(x)}-\frac{26ia^2F\big(}{7}$$

```
[Out] -26/77*I*a^2*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1
/2))*(a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+78/385*a^2*cosh(x)*sinh(x)*(a*cosh(x
)^3)^(1/2)+26/165*a^2*cosh(x)^3*sinh(x)*(a*cosh(x)^3)^(1/2)+2/15*a^2*cosh(x
)^5*sinh(x)*(a*cosh(x)^3)^(1/2)+26/77*a^2*(a*cosh(x)^3)^(1/2)*tanh(x)
```

Rubi [A] time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 2641}

$$\frac{2}{15}a^2\sinh(x)\cosh^5(x)\sqrt{a\cosh^3(x)}+\frac{26}{165}a^2\sinh(x)\cosh^3(x)\sqrt{a\cosh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\cosh^3(x)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(5/2),x]
```

```
[Out] (((-26*I)/77)*a^2*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) +
(78*a^2*Cosh[x]*Sqrt[a*Cosh[x]^3]*Sinh[x])/385 + (26*a^2*Cosh[x]^3*Sqrt[a*C
osh[x]^3]*Sinh[x])/165 + (2*a^2*Cosh[x]^5*Sqrt[a*Cosh[x]^3]*Sinh[x])/15 + (
26*a^2*Sqrt[a*Cosh[x]^3]*Tanh[x])/77
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{15}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(13a^2\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{11}{2}}(x)\,dx}{15\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(39a^2\sqrt{a\right.}}{}$$

$$= \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{15}a^2\cos$$

$$= \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{15}a^2\cos$$

$$= -\frac{26ia^2\sqrt{a\cosh^3(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{77\cosh^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{}$$

Mathematica [A] time = 0.12, size = 65, normalized size = 0.54

$$\frac{a\left(a\cosh^3(x)\right)^{3/2}\left((15465\sinh(x) + 3657\sinh(3x) + 749\sinh(5x) + 77\sinh(7x))\sqrt{\cosh(x)} - 12480iF\left(\frac{ix}{2}\middle|2\right)\right)}{36960\cosh^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(5/2),x]
```

```
[Out] (a*(a*Cosh[x]^3)^(3/2)*((-12480*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(1
5465*Sinh[x] + 3657*Sinh[3*x] + 749*Sinh[5*x] + 77*Sinh[7*x])))/(36960*Cosh
[x]^(9/2))
```

fricas [F] time = 1.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\cosh(x)^3}\,a^2\cosh(x)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)*a^2*cosh(x)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cosh(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(5/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int\left(a\left(\cosh^3(x)\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(5/2),x)
```

```
[Out] int((a*cosh(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cosh(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\cosh(x)^3\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(5/2),x)
[Out] int((a*cosh(x)^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(5/2),x)
[Out] Timed out
```

3.129 $\int \left(a\cosh^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=71

$$\frac{14}{45}a\sinh(x)\sqrt{a\cosh^3(x)} - \frac{14iaE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{15\cosh^{\frac{3}{2}}(x)} + \frac{2}{9}a\sinh(x)\cosh^2(x)\sqrt{a\cosh^3(x)}$$

```
[Out] -14/15*I*a*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2
))*(a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+14/45*a*sinh(x)*(a*cosh(x)^3)^(1/2)+2/
9*a*cosh(x)^2*sinh(x)*(a*cosh(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 2639}

$$\frac{2}{9}a\sinh(x)\cosh^2(x)\sqrt{a\cosh^3(x)} + \frac{14}{45}a\sinh(x)\sqrt{a\cosh^3(x)} - \frac{14iaE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{15\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(3/2),x]
```

```
[Out] (((-14*I)/15)*a*Sqrt[a*Cosh[x]^3]*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + (1
4*a*Sqrt[a*Cosh[x]^3]*Sinh[x])/45 + (2*a*Cosh[x]^2*Sqrt[a*Cosh[x]^3]*Sinh[x
])/9
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{9}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(7a\sqrt{a\cosh^3(x)}\right)\int \cosh^{\frac{5}{2}}(x)\,dx}{9\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{14}{45}a\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{\left(7a\sqrt{a\cosh^3(x)}\right)\int \sqrt{c}}{15\cosh^{\frac{3}{2}}(x)}$$

$$= -\frac{14ia\sqrt{a\cosh^3(x)}\,E\left(\frac{ix}{2}\middle|2\right)}{15\cosh^{\frac{3}{2}}(x)} + \frac{14}{45}a\sqrt{a\cosh^3(x)}\,\sinh(x) + \frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\,\sin$$

Mathematica [A] time = 0.08, size = 54, normalized size = 0.76

$$\frac{\left(a\cosh^3(x)\right)^{3/2}\left((38\sinh(2x)+5\sinh(4x))\sqrt{\cosh(x)}-168iE\left(\frac{ix}{2}\middle|2\right)\right)}{180\cosh^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(3/2),x]
```

```
[Out] ((a*Cosh[x]^3)^(3/2)*((-168*I)*EllipticE[(I/2)*x, 2] + Sqrt[Cosh[x]]*(38*Si
nh[2*x] + 5*Sinh[4*x])))/(180*Cosh[x]^(9/2))
```

fricas [F] time = 1.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{a\cosh(x)^3}\,a\cosh(x)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)*a*cosh(x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cosh(x)^3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*cosh(x)^3)^(3/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \left(a\left(\cosh^3(x)\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(3/2),x)
[Out] int((a*cosh(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cosh(x)^3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*cosh(x)^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\mathrm{cosh}\left(x\right)}^3\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(3/2),x)
[Out] int((a*cosh(x)^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

3.130 $\int \sqrt{a\cosh^3(x)}\,dx$

Optimal. Leaf size=48

$$\frac{2}{3}\tanh(x)\sqrt{a\cosh^3(x)}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{3\cosh^{\frac{3}{2}}(x)}$$

```
[Out] -2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))*(
a*cosh(x)^3)^(1/2)/cosh(x)^(3/2)+2/3*(a*cosh(x)^3)^(1/2)*tanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 2641}

$$\frac{2}{3}\tanh(x)\sqrt{a\cosh^3(x)}-\frac{2iF\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh^3(x)}}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^3],x]
```

```
[Out] (((-2*I)/3)*Sqrt[a*Cosh[x]^3]*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + (2*Sqr
t[a*Cosh[x]^3]*Tanh[x])/3
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a\cosh^3(x)}\,dx = \frac{\sqrt{a\cosh^3(x)}\int \cosh^{\frac{3}{2}}(x)\,dx}{\cosh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{3}\sqrt{a\cosh^3(x)}\tanh(x) + \frac{\sqrt{a\cosh^3(x)}\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3\cosh^{\frac{3}{2}}(x)}$$

$$= -\frac{2i\sqrt{a\cosh^3(x)}\,F\left(\frac{ix}{2}\middle|2\right)}{3\cosh^{\frac{3}{2}}(x)} + \frac{2}{3}\sqrt{a\cosh^3(x)}\tanh(x)$$

Mathematica [C] time = 0.05, size = 59, normalized size = 1.23

$$\frac{2}{3}\sqrt{a\cosh^3(x)}\left(\operatorname{sech}^2(x)\sqrt{\sinh(2x)+\cosh(2x)+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh(2x)-\sinh(2x)\right)+\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^3],x]
```

```
[Out] (2*Sqrt[a*Cosh[x]^3]*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*
x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\cosh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\cosh(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x)^3), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cosh^{3}(x)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(1/2),x)
```

```
[Out] int((a*cosh(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cosh(x)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a \cosh(x)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^3)^(1/2),x)
```

```
[Out] int((a*cosh(x)^3)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**3)**(1/2),x)
```

```
[Out] Timed out
```

3.131 $\int \frac{1}{\sqrt{a \cosh^3(x)}} \, dx$

Optimal. Leaf size=46

$$\frac{2 \sinh(x) \cosh(x)}{\sqrt{a \cosh^3(x)}} + \frac{2i \cosh^{\frac{3}{2}}(x) E\left(\frac{ix}{2} \middle| 2\right)}{\sqrt{a \cosh^3(x)}}$$

```
[Out] 2*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x)
,2^(1/2))/(a*cosh(x)^3)^(1/2)+2*cosh(x)*sinh(x)/(a*cosh(x)^3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2636, 2639}

$$\frac{2 \sinh(x) \cosh(x)}{\sqrt{a \cosh^3(x)}} + \frac{2i \cosh^{\frac{3}{2}}(x) E\left(\frac{ix}{2} \middle| 2\right)}{\sqrt{a \cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^3],x]
```

```
[Out] ((2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/Sqrt[a*Cosh[x]^3] + (2*Cosh[x]*
Sinh[x])/Sqrt[a*Cosh[x]^3]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx &= \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\cosh^3(x)}}\\ &= \frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}} - \frac{\cosh^{\frac{3}{2}}(x)\int \sqrt{\cosh(x)}\,dx}{\sqrt{a\cosh^3(x)}}\\ &= \frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}} + \frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.78

$$\frac{2\cosh(x)\left(\sinh(x) + i\sqrt{\cosh(x)}\,E\left(\frac{ix}{2}\middle|2\right)\right)}{\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^3],x]
[Out] (2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/Sqrt[a*Cosh[x
]^3]
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{a\cosh(x)^3}}{a\cosh(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cosh(x)^3)/(a*cosh(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cosh(x)^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*cosh(x)^3), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \left(\cosh^3(x)\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(1/2),x)
[Out] int(1/(a*cosh(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cosh(x)^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*cosh(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a \cosh(x)^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(1/2),x)
[Out] int(1/(a*cosh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cosh^{3}{(x)}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cosh(x)**3), x)
```

3.132 $\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}-\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\text{sech}(x)}{7a\sqrt{a\cosh^3(x)}}$$

```
[Out] -10/21*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1
/2*x),2^(1/2))/a/(a*cosh(x)^3)^(1/2)+10/21*sinh(x)/a/(a*cosh(x)^3)^(1/2)+2/
7*sech(x)*tanh(x)/a/(a*cosh(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2636, 2641}

$$\frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}}-\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\text{sech}(x)}{7a\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(-3/2),x]
```

```
[Out] (((-10*I)/21)*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2])/(a*Sqrt[a*Cosh[x]^3]) +
(10*Sinh[x])/(21*a*Sqrt[a*Cosh[x]^3]) + (2*Sech[x]*Tanh[x])/(7*a*Sqrt[a*Cos
h[x]^3])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
```

```
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\cosh^3(x)}}$$

$$= \frac{2\operatorname{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}} + \frac{\left(5\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\cosh^3(x)}}$$

$$= \frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}} + \frac{2\operatorname{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}} + \frac{\left(5\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{21a\sqrt{a\cosh^3(x)}}$$

$$= -\frac{10i\cosh^{\frac{3}{2}}(x)F\left(\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\cosh^3(x)}} + \frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}} + \frac{2\operatorname{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.64

$$\frac{2\cosh^2(x)\left(3\tanh(x) - 5i\cosh^{\frac{5}{2}}(x)F\left(\frac{ix}{2}\middle|2\right) + 5\sinh(x)\cosh(x)\right)}{21\left(a\cosh^3(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(-3/2),x]
```

```
[Out] (2*Cosh[x]^2*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2] + 5*Cosh[x]*Sinh[x
] + 3*Tanh[x]))/(21*(a*Cosh[x]^3)^(3/2))
```

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{a\cosh(x)^3}}{a^2\cosh(x)^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)/(a^2*cosh(x)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cosh(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(-3/2), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\cosh^3(x)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cosh(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\cosh(x)^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

3.133 $\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\mathrm{sech}^4(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{22\tanh(x)\mathrm{sech}^2(x)}{117a^2\sqrt{a\cosh^3(x)}}$$

```
[Out] 154/195*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(
1/2*x),2^(1/2))/a^2/(a*cosh(x)^3)^(1/2)+154/195*cosh(x)*sinh(x)/a^2/(a*cosh
(x)^3)^(1/2)+154/585*tanh(x)/a^2/(a*cosh(x)^3)^(1/2)+22/117*sech(x)^2*tanh(
x)/a^2/(a*cosh(x)^3)^(1/2)+2/13*sech(x)^4*tanh(x)/a^2/(a*cosh(x)^3)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2636, 2639}

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{2\tanh(x)\mathrm{sech}^4(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{22\tanh(x)\mathrm{sech}^2(x)}{117a^2\sqrt{a\cosh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^3)^(-5/2),x]
```

```
[Out] (((154*I)/195)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]^3])
 + (154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Cosh[x]^3]) + (154*Tanh[x])/(585*a
^2*Sqrt[a*Cosh[x]^3]) + (22*Sech[x]^2*Tanh[x])/(117*a^2*Sqrt[a*Cosh[x]^3])
+ (2*Sech[x]^4*Tanh[x])/(13*a^2*Sqrt[a*Cosh[x]^3])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx &= \frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\cosh^3(x)}}\\
&= \frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{\left(11\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\cosh^3(x)}}\\
&= \frac{22\text{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}}+\frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{\left(77\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\cosh^3(x)}}\\
&= \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{22\text{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}}+\frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}}+\frac{\left(77\cosh^{\frac{3}{2}}(x)\right)\int \frac{1}{\cosh^{\frac{3}{2}}}}{195a^2\sqrt{a\cosh^3(x}}\\
&= \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{22\text{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}}+\frac{2\text{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}}\\
&= \frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}}+\frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}}+\frac{22\text{sech}^2(x)\tanh}{117a^2\sqrt{a\cosh^3(}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 61, normalized size = 0.50

$$
\frac{462i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)+462\sinh(x)\cosh(x)+2\tanh(x)\left(45\text{sech}^4(x)+55\text{sech}^2(x)+77\right)}{585a^2\sqrt{a\cosh^3(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^3)^(-5/2),x]
```

```
[Out] ((462*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 462*Cosh[x]*Sinh[x] + 2*(77
+ 55*Sech[x]^2 + 45*Sech[x]^4)*Tanh[x])/(585*a^2*Sqrt[a*Cosh[x]^3])
```

fricas [F] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\cosh(x)^3}}{a^3\cosh(x)^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x)^3)/(a^3*cosh(x)^9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cosh(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(x)^3)^(-5/2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\cosh^3(x)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*cosh(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\cosh(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x)^3)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\cosh(x)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^3)^(5/2),x)
[Out] int(1/(a*cosh(x)^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**3)**(5/2),x)
[Out] Timed out
```

3.134 $\int \left(a\cosh^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=132

$$\frac{21}{128}a^2\sinh(x)\cosh(x)\sqrt{a\cosh^4(x)}+\frac{63}{256}a^2\tanh(x)\sqrt{a\cosh^4(x)}+\frac{63}{256}a^2x\mathrm{sech}^2(x)\sqrt{a\cosh^4(x)}+\frac{1}{10}a^2\sinh(x)$$

```
[Out] 63/256*a^2*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+21/128*a^2*cosh(x)*sinh(x)*(a*co
sh(x)^4)^(1/2)+21/160*a^2*cosh(x)^3*sinh(x)*(a*cosh(x)^4)^(1/2)+9/80*a^2*co
sh(x)^5*sinh(x)*(a*cosh(x)^4)^(1/2)+1/10*a^2*cosh(x)^7*sinh(x)*(a*cosh(x)^4
)^(1/2)+63/256*a^2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

Rubi [A] time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$\frac{1}{10}a^2\sinh(x)\cosh^7(x)\sqrt{a\cosh^4(x)}+\frac{9}{80}a^2\sinh(x)\cosh^5(x)\sqrt{a\cosh^4(x)}+\frac{21}{160}a^2\sinh(x)\cosh^3(x)\sqrt{a\cosh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/256 + (21*a^2*Cosh[x]*Sqrt[a*Cosh[x]
^4]*Sinh[x])/128 + (21*a^2*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/160 + (9*a^
2*Cosh[x]^5*Sqrt[a*Cosh[x]^4]*Sinh[x])/80 + (a^2*Cosh[x]^7*Sqrt[a*Cosh[x]^4
]*Sinh[x])/10 + (63*a^2*Sqrt[a*Cosh[x]^4]*Tanh[x])/256
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx = \left(a^2\sqrt{a\cosh^4(x)}\,\text{sech}^2(x)\right)\int \cosh^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{10}\left(9a^2\sqrt{a\cosh^4(x)}\,\text{sech}^2(x)\right)\int \cosh^8(x)\,dx$$
$$= \frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{80}\left(63a^2\sqrt{a\right.$$
$$= \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{1}{10}a^2\cosh$$
$$= \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{9}{80}a^2\cosh$$
$$= \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{9}{80}a^2\cosh$$
$$= \frac{63}{256}a^2x\sqrt{a\cosh^4(x)}\,\text{sech}^2(x) + \frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\,\sinh(x) + \frac{21}{160}a^2\cosh^3(x)\sqrt{}$$

Mathematica [A] time = 0.12, size = 53, normalized size = 0.40

$$\frac{a(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) + 25\sinh(8x) + 2\sinh(10x))\text{sech}^6(x)\left(a\cosh^4(x)\right)^{3/2}}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(5/2),x]

[Out] (a*(a*Cosh[x]^4)^(3/2)*Sech[x]^6*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] +
 150*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

fricas [B] time = 0.87, size = 1597, normalized size = 12.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="fricas")

[Out] 1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*(
76*a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 + 15*a^
```

2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 + 255*a^2*cosh(x)^2 +
10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 + 425*a^2*cosh(x)^3 + 5
0*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 + 1275*a^2*cosh(
x)^4 + 300*a^2*cosh(x)^2 + 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2*cosh(
x)^7 + 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 + 70*a^2*cosh(x))*e^(2*x)*sin
h(x)^13 + 60*(4199*a^2*cosh(x)^8 + 7735*a^2*cosh(x)^6 + 4550*a^2*cosh(x)^4
+ 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*cosh(x)^9 +
9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 + 2730*a^2*cosh(x)^3 + 315*a^2*cos
h(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 + 546975*a^2*cosh(x)^8
+ 600600*a^2*cosh(x)^6 + 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(x)^2 + 2520*
a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 + 60775*a^2*cosh(x)^9
+ 85800*a^2*cosh(x)^7 + 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(x)^3 + 2520*a^
2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 + 36465*a^2*cosh(x
)^10 + 64350*a^2*cosh(x)^8 + 60060*a^2*cosh(x)^6 + 34650*a^2*cosh(x)^4 + 75
60*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*a^2*cosh(x)^13 +
3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 + 8580*a^2*cosh(x)^7 + 6930*a^2*co
sh(x)^5 + 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^7 + 60*(12
92*a^2*cosh(x)^14 + 7735*a^2*cosh(x)^12 + 20020*a^2*cosh(x)^10 + 30030*a^2*
cosh(x)^8 + 32340*a^2*cosh(x)^6 + 17640*a^2*x*cosh(x)^4 - 980*a^2*cosh(x)^2
- 10*a^2)*e^(2*x)*sinh(x)^6 + 24*(1292*a^2*cosh(x)^15 + 8925*a^2*cosh(x)^1
3 + 27300*a^2*cosh(x)^11 + 50050*a^2*cosh(x)^9 + 69300*a^2*cosh(x)^7 + 5292
0*a^2*x*cosh(x)^5 - 4900*a^2*cosh(x)^3 - 150*a^2*cosh(x))*e^(2*x)*sinh(x)^5
+ 30*(323*a^2*cosh(x)^16 + 2550*a^2*cosh(x)^14 + 9100*a^2*cosh(x)^12 + 200
20*a^2*cosh(x)^10 + 34650*a^2*cosh(x)^8 + 35280*a^2*x*cosh(x)^6 - 4900*a^2*
cosh(x)^4 - 300*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^4 + 120*(19*a^2*cosh
(x)^17 + 170*a^2*cosh(x)^15 + 700*a^2*cosh(x)^13 + 1820*a^2*cosh(x)^11 + 38
50*a^2*cosh(x)^9 + 5040*a^2*x*cosh(x)^7 - 980*a^2*cosh(x)^5 - 100*a^2*cosh(
x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x)^3 + 5*(76*a^2*cosh(x)^18 + 765*a^2*co
sh(x)^16 + 3600*a^2*cosh(x)^14 + 10920*a^2*cosh(x)^12 + 27720*a^2*cosh(x)^1
0 + 45360*a^2*x*cosh(x)^8 - 11760*a^2*cosh(x)^6 - 1800*a^2*cosh(x)^4 - 180*
a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^2 + 10*(4*a^2*cosh(x)^19 + 45*a^2*co
sh(x)^17 + 240*a^2*cosh(x)^15 + 840*a^2*cosh(x)^13 + 2520*a^2*cosh(x)^11 +
5040*a^2*x*cosh(x)^9 - 1680*a^2*cosh(x)^7 - 360*a^2*cosh(x)^5 - 60*a^2*cosh
(x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x) + (2*a^2*cosh(x)^20 + 25*a^2*cosh(x)
^18 + 150*a^2*cosh(x)^16 + 600*a^2*cosh(x)^14 + 2100*a^2*cosh(x)^12 + 5040*
a^2*x*cosh(x)^10 - 2100*a^2*cosh(x)^8 - 600*a^2*cosh(x)^6 - 150*a^2*cosh(x)
^4 - 25*a^2*cosh(x)^2 - 2*a^2)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*
e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^10*e^(4*x) + 2*cosh(x)^10*e^(2
*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^10 + cosh(x)^10 + 10*(cosh(x)*e^(4*
x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 + 45*(cosh(x)^2*e^(4*x) + 2*cos
h(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^8 + 120*(cosh(x)^3*e^(4*x) + 2*cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^7 + 210*(cosh(x)^4*e^(4*x) + 2*cosh(x)^4*e^(
2*x) + cosh(x)^4)*sinh(x)^6 + 252*(cosh(x)^5*e^(4*x) + 2*cosh(x)^5*e^(2*x)
+ cosh(x)^5)*sinh(x)^5 + 210*(cosh(x)^6*e^(4*x) + 2*cosh(x)^6*e^(2*x) + cos
h(x)^6)*sinh(x)^4 + 120*(cosh(x)^7*e^(4*x) + 2*cosh(x)^7*e^(2*x) + cosh(x)^

```
7)*sinh(x)^3 + 45*(cosh(x)^8*e^(4*x) + 2*cosh(x)^8*e^(2*x) + cosh(x)^8)*sin
h(x)^2 + 10*(cosh(x)^9*e^(4*x) + 2*cosh(x)^9*e^(2*x) + cosh(x)^9)*sinh(x))
```

giac [A] time = 0.13, size = 114, normalized size = 0.86

$$\frac{1}{20480}\left(5040\,a^2x + 2\,a^2e^{(10\,x)} + 25\,a^2e^{(8\,x)} + 150\,a^2e^{(6\,x)} + 600\,a^2e^{(4\,x)} + 2100\,a^2e^{(2\,x)} - \left(5754\,a^2e^{(10\,x)} + 2100\,a^2e^{(8\,x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20480*(5040*a^2*x + 2*a^2*e^(10*x) + 25*a^2*e^(8*x) + 150*a^2*e^(6*x) + 6
00*a^2*e^(4*x) + 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) + 2100*a^2*e^(8*x) +
 600*a^2*e^(6*x) + 150*a^2*e^(4*x) + 25*a^2*e^(2*x) + 2*a^2)*e^(-10*x))*sqr
t(a)
```

maple [A] time = 0.59, size = 177, normalized size = 1.34

$$\frac{\sqrt{8}\,\left(\cosh\left(2x\right)+1\right)\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(\cosh\left(2x\right)+1\right)}\,\sqrt{2}\,a^{\frac{3}{2}}\left(8\sqrt{a\left(\sinh^{2}\left(2x\right)\right)}\,\sqrt{a}\,\left(\sinh^{4}\left(2x\right)\right)+50\sqrt{a\left(\sin\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(5/2),x)
```

```
[Out] 1/10240*8^(1/2)*(cosh(2*x)+1)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2
)*a^(3/2)*(8*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4+50*(a*sinh(2*x)^2)^(
1/2)*a^(1/2)*cosh(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2
*x)^2+325*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)
*a^(1/2)+315*ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/((cos
h(2*x)+1)^2*a)^(1/2)
```

maxima [A] time = 0.41, size = 100, normalized size = 0.76

$$\frac{63}{256}\,a^{\frac{5}{2}}x+\frac{1}{20480}\left(25\,a^{\frac{5}{2}}e^{(-2\,x)} + 150\,a^{\frac{5}{2}}e^{(-4\,x)} + 600\,a^{\frac{5}{2}}e^{(-6\,x)} + 2100\,a^{\frac{5}{2}}e^{(-8\,x)} - 2100\,a^{\frac{5}{2}}e^{(-12\,x)} - 600\,a^{\frac{5}{2}}e^{(-14\,x)} - 15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x + 1/20480*(25*a^(5/2)*e^(-2*x) + 150*a^(5/2)*e^(-4*x) + 60
0*a^(5/2)*e^(-6*x) + 2100*a^(5/2)*e^(-8*x) - 2100*a^(5/2)*e^(-12*x) - 600*a
^(5/2)*e^(-14*x) - 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) - 2*a^(5/2)
*e^(-20*x) + 2*a^(5/2))*e^(10*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a \cosh(x)^4\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(5/2),x)
```

```
[Out] int((a*cosh(x)^4)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

3.135 $\int \left(a\cosh^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\cosh^4(x)}+\frac{5}{16}a\tanh(x)\sqrt{a\cosh^4(x)}+\frac{5}{16}ax\operatorname{sech}^2(x)\sqrt{a\cosh^4(x)}+\frac{1}{6}a\sinh(x)\cosh^3(x)\sqrt{}$$

```
[Out] 5/16*a*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+5/24*a*cosh(x)*sinh(x)*(a*cosh(x)^4)
^(1/2)+1/6*a*cosh(x)^3*sinh(x)*(a*cosh(x)^4)^(1/2)+5/16*a*(a*cosh(x)^4)^(1/
2)*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$\frac{1}{6}a\sinh(x)\cosh^3(x)\sqrt{a\cosh^4(x)}+\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\cosh^4(x)}+\frac{5}{16}a\tanh(x)\sqrt{a\cosh^4(x)}+\frac{5}{16}ax\operatorname{sech}^2(x)\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(3/2),x]
```

```
[Out] (5*a*x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/16 + (5*a*Cosh[x]*Sqrt[a*Cosh[x]^4]*Sin
h[x])/24 + (a*Cosh[x]^3*Sqrt[a*Cosh[x]^4]*Sinh[x])/6 + (5*a*Sqrt[a*Cosh[x]^
4]*Tanh[x])/16
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a \cosh^4(x)\right)^{3/2} dx = \left(a\sqrt{a \cosh^4(x)}\,\text{sech}^2(x)\right) \int \cosh^6(x)\, dx$$
$$= \frac{1}{6} a \cosh^3(x)\sqrt{a \cosh^4(x)}\, \sinh(x) + \frac{1}{6}\left(5a\sqrt{a \cosh^4(x)}\,\text{sech}^2(x)\right) \int \cosh^4(x)\, dx$$
$$= \frac{5}{24} a \cosh(x)\sqrt{a \cosh^4(x)}\, \sinh(x) + \frac{1}{6} a \cosh^3(x)\sqrt{a \cosh^4(x)}\, \sinh(x) + \frac{1}{8}\left(5a\sqrt{a \cosh$$
$$= \frac{5}{24} a \cosh(x)\sqrt{a \cosh^4(x)}\, \sinh(x) + \frac{1}{6} a \cosh^3(x)\sqrt{a \cosh^4(x)}\, \sinh(x) + \frac{5}{16} a\sqrt{a \cosh^4(}$$
$$= \frac{5}{16} ax\sqrt{a \cosh^4(x)}\,\text{sech}^2(x) + \frac{5}{24} a \cosh(x)\sqrt{a \cosh^4(x)}\, \sinh(x) + \frac{1}{6} a \cosh^3(x)\sqrt{a \cosh}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 0.49

$$\frac{1}{192}(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x))\text{sech}^6(x)\left(a \cosh^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(3/2),x]
```

```
[Out] ((a*Cosh[x]^4)^(3/2)*Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*
x]))/192
```

fricas [B] time = 0.83, size = 659, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*cos
h(x)^2 + 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 + 9*a*cosh(x))*e^(2*x
)*sinh(x)^9 + 45*(11*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8 + 7
2*(11*a*cosh(x)^5 + 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6*(15
4*a*cosh(x)^6 + 315*a*cosh(x)^4 + 210*a*cosh(x)^2 + 20*a*x)*e^(2*x)*sinh(x)
^6 + 36*(22*a*cosh(x)^7 + 63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 20*a*x*cosh(x))
*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 + 42*a*cosh(x)^6 + 70*a*cosh(x)^4 +
 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 + 54*a*cosh(x
)^7 + 126*a*cosh(x)^5 + 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*sinh(x)^3
```

```
+ 3*(22*a*cosh(x)^10 + 135*a*cosh(x)^8 + 420*a*cosh(x)^6 + 600*a*x*cosh(x)^
4 - 90*a*cosh(x)^2 - 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11 + 15*a*cosh
(x)^9 + 60*a*cosh(x)^7 + 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 - 3*a*cosh(x))*
e^(2*x)*sinh(x) + (a*cosh(x)^12 + 9*a*cosh(x)^10 + 45*a*cosh(x)^8 + 120*a*x
*cosh(x)^6 - 45*a*cosh(x)^4 - 9*a*cosh(x)^2 - a)*e^(2*x))*sqrt(a*e^(8*x) +
4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^6*e^(4*x) +
2*cosh(x)^6*e^(2*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(
cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 + 15*(cosh(x)^2*e^
(4*x) + 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(cosh(x)^3*e^(4*x)
+ 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(x)^4*e^(4*x) + 2*co
sh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^5*e^(4*x) + 2*cosh(x)^5
*e^(2*x) + cosh(x)^5)*sinh(x))
```

giac [A] time = 0.14, size = 52, normalized size = 0.67

$$-\frac{1}{384}\left(\left(110\,e^{(6\,x)}+45\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)e^{(-6\,x)}-120\,x-e^{(6\,x)}-9\,e^{(4\,x)}-45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6*
x) - 9*e^(4*x) - 45*e^(2*x))*a^(3/2)
```

maple [B] time = 0.50, size = 131, normalized size = 1.68

$$\frac{\sqrt{8}\,\left(\cosh\left(2x\right)+1\right)\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(\cosh\left(2x\right)+1\right)}\,\sqrt{2}\,\sqrt{a}\,\left(2\sqrt{a\left(\sinh^{2}\left(2x\right)\right)}\,\sqrt{a}\,\left(\sinh^{2}\left(2x\right)\right)+9\cosh\left(2\ldots\right.}{384\sinh\left(2x\right)\sqrt{\left(\cosh\left(2x\right)\ldots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(3/2),x)
```

```
[Out] 1/384*8^(1/2)*(cosh(2*x)+1)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2)*
a^(1/2)*(2*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2+9*cosh(2*x)*(a*sinh(2*
x)^2)^(1/2)*a^(1/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+15*ln(a^(1/2)*cosh(2*x
)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/((cosh(2*x)+1)^2*a)^(1/2)
```

maxima [A] time = 0.42, size = 62, normalized size = 0.79

$$\frac{5}{16}\,a^{\frac{3}{2}}x+\frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)}+45\,a^{\frac{3}{2}}e^{(-4\,x)}-45\,a^{\frac{3}{2}}e^{(-8\,x)}-9\,a^{\frac{3}{2}}e^{(-10\,x)}-a^{\frac{3}{2}}e^{(-12\,x)}+a^{\frac{3}{2}}\right)e^{(6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^(3/2)*x + 1/384*(9*a^(3/2)*e^(-2*x) + 45*a^(3/2)*e^(-4*x) - 45*a^(3/
2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) - a^(3/2)*e^(-12*x) + a^(3/2))*e^(6*x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\cosh(x)^4\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(3/2),x)
```

```
[Out] int((a*cosh(x)^4)^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(3/2),x)
```

```
[Out] Timed out
```

3.136 $\int \sqrt{a \cosh^4(x)}\, dx$

Optimal. Leaf size=36

$$\frac{1}{2}\tanh(x)\sqrt{a\cosh^4(x)}+\frac{1}{2}x\operatorname{sech}^2(x)\sqrt{a\cosh^4(x)}$$

```
[Out] 1/2*x*sech(x)^2*(a*cosh(x)^4)^(1/2)+1/2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$\frac{1}{2}\tanh(x)\sqrt{a\cosh^4(x)}+\frac{1}{2}x\operatorname{sech}^2(x)\sqrt{a\cosh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (x*Sqrt[a*Cosh[x]^4]*Sech[x]^2)/2 + (Sqrt[a*Cosh[x]^4]*Tanh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a\cosh^4(x)}\,dx = \left(\sqrt{a\cosh^4(x)}\,\mathrm{sech}^2(x)\right)\int \cosh^2(x)\,dx$$
$$= \frac{1}{2}\sqrt{a\cosh^4(x)}\,\tanh(x) + \frac{1}{2}\left(\sqrt{a\cosh^4(x)}\,\mathrm{sech}^2(x)\right)\int 1\,dx$$
$$= \frac{1}{2}x\sqrt{a\cosh^4(x)}\,\mathrm{sech}^2(x) + \frac{1}{2}\sqrt{a\cosh^4(x)}\,\tanh(x)$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.69

$$\frac{1}{2}\mathrm{sech}^2(x)\sqrt{a\cosh^4(x)}\,(x+\sinh(x)\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (Sqrt[a*Cosh[x]^4]*Sech[x]^2*(x + Cosh[x]*Sinh[x]))/2
```

fricas [B] time = 0.85, size = 180, normalized size = 5.00

$$\frac{(4\,\cosh(x)e^{(2\,x)}\sinh(x)^3 + e^{(2\,x)}\sinh(x)^4 + 2\,(3\,\cosh(x)^2 + 2\,x)e^{(2\,x)}\sinh(x)^2 + 4\,(\cosh(x)^3 + 2\,x\cosh(x))e^{(2\,x)}}{8\,(\cosh(x)^2e^{(4\,x)} + 2\,\cosh(x)^2e^{(2\,x)} + (e^{(4\,x)} + 2\,e^{(2\,x)} + 1)\sinh(x)^2 + \cosh(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 2*x
)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh(x
)^4 + 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x
) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) + 2*cosh(x)^2*e^(2*x) + (e
^(4*x) + 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) + 2*cosh
(x)*e^(2*x) + cosh(x))*sinh(x))
```

giac [A] time = 0.15, size = 28, normalized size = 0.78

$$-\frac{1}{8}\left(\left(2\,e^{(2\,x)}+1\right)e^{(-2\,x)} - 4\,x - e^{(2\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))*sqrt(a)
```

maple [B] time = 0.45, size = 89, normalized size = 2.47

$$\frac{\sqrt{8}\,\left(\cosh\left(2x\right)+1\right)\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(\cosh\left(2x\right)+1\right)}\,\sqrt{2}\,\left(\sqrt{a\left(\sinh^{2}\left(2x\right)\right)}\,\sqrt{a}+\ln\left(\sqrt{a}\,\cosh\left(2x\right)+\sqrt{a\left(\sin\right.}\right.\right.}{16\sqrt{a}\,\sinh\left(2x\right)\sqrt{\left(\cosh\left(2x\right)+1\right)^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(1/2),x)
```

```
[Out] 1/16*8^(1/2)*(cosh(2*x)+1)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*2^(1/2)*(
(a*sinh(2*x)^2)^(1/2)*a^(1/2)+ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a
)/a^(1/2)/sinh(2*x)/((cosh(2*x)+1)^2*a)^(1/2)
```

maxima [A] time = 0.43, size = 27, normalized size = 0.75

$$-\frac{1}{8}\left(\sqrt{a}\,e^{(-4\,x)}-\sqrt{a}\right)e^{(2\,x)}+\frac{1}{2}\,\sqrt{a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) + 1/2*sqrt(a)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a\cosh(x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cosh(x)^4)^(1/2),x)
```

```
[Out] int((a*cosh(x)^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cosh(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

3.137 $\int \frac{1}{\sqrt{a \cosh^4(x)}} \, dx$

Optimal. Leaf size=15

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

```
[Out] cosh(x)*sinh(x)/(a*cosh(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 3767, 8}

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{\cosh^2(x)\int \operatorname{sech}^2(x)\,dx}{\sqrt{a\cosh^4(x)}}$$
$$= \frac{\left(i\cosh^2(x)\right)\text{Subst}(\int 1\,dx, x, -i\tanh(x))}{\sqrt{a\cosh^4(x)}}$$
$$= \frac{\cosh(x)\sinh(x)}{\sqrt{a\cosh^4(x)}}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cosh[x]^4],x]
```

```
[Out] (Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

fricas [B] time = 2.33, size = 116, normalized size = 7.73

$$-\frac{2\sqrt{ae^{(8x)} + 4\,ae^{(6x)} + 6\,ae^{(4x)} + 4\,ae^{(2x)} + a}}{a\cosh(x)^2 + \left(ae^{(4x)} + 2\,ae^{(2x)} + a\right)\sinh(x)^2 + \left(a\cosh(x)^2 + a\right)e^{(4x)} + 2\left(a\cosh(x)^2 + a\right)e^{(2x)} + 2\left(a\cosh(x)e^{(4x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)/(a*cosh(x)
^2 + (a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 + a)*e^(4*x) +
2*(a*cosh(x)^2 + a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) +
a*cosh(x))*sinh(x) + a)
```

giac [A] time = 0.13, size = 13, normalized size = 0.87

$$-\frac{2}{\sqrt{a}\left(e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(a)*(e^(2*x) + 1))
```

maple [B] time = 0.37, size = 56, normalized size = 3.73

$$\frac{\sqrt{8}\,\sqrt{2}\,\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(\cosh\left(2x\right)+1\right)}\,\sqrt{a\left(\sinh^{2}\left(2x\right)\right)}}{4a\sinh\left(2x\right)\sqrt{\left(\cosh\left(2x\right)+1\right)^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(1/2),x)
```

```
[Out] 1/4*8^(1/2)*2^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/a*(a*sinh(2*x)^2
)^(1/2)/sinh(2*x)/((cosh(2*x)+1)^2*a)^(1/2)
```

maxima [A] time = 0.42, size = 16, normalized size = 1.07

$$\frac{2}{\sqrt{a}\,e^{(-2\,x)}+\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(a)*e^(-2*x) + sqrt(a))
```

mupad [B] time = 0.06, size = 39, normalized size = 2.60

$$-\frac{e^{-x}\sqrt{a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^{4}}}{a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(1/2),x)
```

```
[Out] -(exp(-x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 + exp(x)/2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

3.138 $\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(x)\cosh(x)}{a\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}-\frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}}$$

```
[Out] cosh(x)*sinh(x)/a/(a*cosh(x)^4)^(1/2)-2/3*sinh(x)^2*tanh(x)/a/(a*cosh(x)^4)
^(1/2)+1/5*sinh(x)^2*tanh(x)^3/a/(a*cosh(x)^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3767}

$$\frac{\sinh(x)\cosh(x)}{a\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}-\frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(-3/2),x]
```

```
[Out] (Cosh[x]*Sinh[x])/(a*Sqrt[a*Cosh[x]^4]) - (2*Sinh[x]^2*Tanh[x])/(3*a*Sqrt[a
*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^3)/(5*a*Sqrt[a*Cosh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \frac{\cosh^2(x)\int \operatorname{sech}^6(x)\,dx}{a\sqrt{a\cosh^4(x)}}$$
$$= \frac{\left(i\cosh^2(x)\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(x)\right)}{a\sqrt{a\cosh^4(x)}}$$
$$= \frac{\cosh(x)\sinh(x)}{a\sqrt{a\cosh^4(x)}} - \frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}} + \frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.45

$$\frac{\sinh(x)\cosh(x)(6\cosh(2x)+\cosh(4x)+8)}{15\left(a\cosh^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(-3/2),x]
[Out] (Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*Sinh[x])/(15*(a*Cosh[x]^4)^(3/2))
```

fricas [B] time = 1.33, size = 1137, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="fricas")
[Out] -16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x)
^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (1
0*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*
e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) + 2*a^2*
e^(2*x) + a^2)*sinh(x)^10 + 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x) + 2*a
^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 + a^2 + (9
*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(9*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^
8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^3
 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x
)^7 + 10*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^4 + 1
4*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a
^2)*e^(2*x))*sinh(x)^6 + 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5 + 70*a^2*co
sh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*a^2*co
```

```
sh(x))*e^(4*x) + 2*(63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e
^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x
)^2 + a^2 + (21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*
e^(4*x) + 2*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*
e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^
5 + 5*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^5 + 5*
a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^7 + 7*a^2*cosh(x)^5
 + 5*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(9*a^2*cosh(x)^8 +
 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh(x)^2 + a^2 + (9*a^2*cosh
(x)^8 + 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh(x)^2 + a^2)*e^(4*
x) + 2*(9*a^2*cosh(x)^8 + 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 + 12*a^2*cosh
(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^10 + 5*a^2*cosh(x)^8 +
 10*a^2*cosh(x)^6 + 10*a^2*cosh(x)^4 + 5*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(
a^2*cosh(x)^10 + 5*a^2*cosh(x)^8 + 10*a^2*cosh(x)^6 + 10*a^2*cosh(x)^4 + 5*
a^2*cosh(x)^2 + a^2)*e^(2*x) + 10*(a^2*cosh(x)^9 + 4*a^2*cosh(x)^7 + 6*a^2*
cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^9 + 4*a^2*cosh(x)^
7 + 6*a^2*cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(a^2*cosh(
x)^9 + 4*a^2*cosh(x)^7 + 6*a^2*cosh(x)^5 + 4*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(2*x))*sinh(x))
```

giac [A] time = 0.16, size = 27, normalized size = 0.40

$$-\frac{16\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) + 1)^5)
```

maple [A] time = 0.39, size = 80, normalized size = 1.19

$$\frac{\sqrt{8}\,\sqrt{2}\,\left(2\left(\cosh^2(2x)\right)+6\cosh(2x)+7\right)\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a\,(-1+\cosh(2x))\,(\cosh(2x)+1)}}{15a^2\,(\cosh(2x)+1)^2\sinh(2x)\sqrt{(\cosh(2x)+1)^2\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(3/2),x)
```

```
[Out] 1/15*8^(1/2)*2^(1/2)/a^2*(2*cosh(2*x)^2+6*cosh(2*x)+7)*(a*sinh(2*x)^2)^(1/2
)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/(cosh(2*x)+1)^2/sinh(2*x)/((cosh(2
*x)+1)^2*a)^(1/2)
```

maxima [B] time = 0.43, size = 165, normalized size = 2.46

$$\frac{16\,e^{(-2\,x)}}{3\,\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}+5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}\right)}+\frac{32\,e^{(-4\,x}}{3\,\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}+10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*
x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 32/3*e^(-4*x)/(5*a
^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(
-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) + 10*a^(3/
2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x)
+ a^(3/2))
```

mupad [B] time = 0.97, size = 48, normalized size = 0.72

$$-\frac{64\,{\mathrm{e}}^{2\,x}\,\sqrt{a\,{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^{x}}{2}\right)}^4}\,\left(5\,{\mathrm{e}}^{2\,x}+10\,{\mathrm{e}}^{4\,x}+1\right)}{15\,a^2\,{\left({\mathrm{e}}^{2\,x}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(3/2),x)
```

```
[Out] -(64*exp(2*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*x)
+ 1))/(15*a^2*(exp(2*x) + 1)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(3/2),x)
```

```
[Out] Timed out
```

3.139 $\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}}+\frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}}$$

```
[Out] cosh(x)*sinh(x)/a^2/(a*cosh(x)^4)^(1/2)-4/3*sinh(x)^2*tanh(x)/a^2/(a*cosh(x
)^4)^(1/2)+6/5*sinh(x)^2*tanh(x)^3/a^2/(a*cosh(x)^4)^(1/2)-4/7*sinh(x)^2*ta
nh(x)^5/a^2/(a*cosh(x)^4)^(1/2)+1/9*sinh(x)^2*tanh(x)^7/a^2/(a*cosh(x)^4)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3767}

$$\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\cosh^4(x)}}+\frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}}+\frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}}-\frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cosh[x]^4)^(-5/2),x]
```

```
[Out] (Cosh[x]*Sinh[x])/(a^2*Sqrt[a*Cosh[x]^4]) - (4*Sinh[x]^2*Tanh[x])/(3*a^2*Sq
rt[a*Cosh[x]^4]) + (6*Sinh[x]^2*Tanh[x]^3)/(5*a^2*Sqrt[a*Cosh[x]^4]) - (4*S
inh[x]^2*Tanh[x]^5)/(7*a^2*Sqrt[a*Cosh[x]^4]) + (Sinh[x]^2*Tanh[x]^7)/(9*a^
2*Sqrt[a*Cosh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \frac{\cosh^2(x)\int \text{sech}^{10}(x)\,dx}{a^2\sqrt{a\cosh^4(x)}}$$
$$= \frac{\left(i\cosh^2(x)\right)\text{Subst}\left(\int\left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\tanh(x)\right)}{a^2\sqrt{a\cosh^4(x)}}$$
$$= \frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}} + \frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}} +$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.40

$$\frac{(130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x)+128)\tanh(x)\text{sech}^6(x)}{315a^2\sqrt{a\cosh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cosh[x]^4)^(-5/2),x]
```

```
[Out] ((128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*Sech[x]^6*
Tanh[x])/(315*a^2*Sqrt[a*Cosh[x]^4])
```

fricas [B] time = 0.64, size = 3065, normalized size = 26.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*c
osh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh
(x)^5 + 36*(245*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(147*c
osh(x)^5 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)^6 +
 140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^7 + 2
8*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8 + 84*
cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e
^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 + 9*a^3*co
sh(x)^16 + (a^3*e^(4*x) + 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^3*cosh(x)
*e^(4*x) + 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a^3*cosh(x)
```

```
^14 + 9*(17*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(17*
a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3 + 3*a^3*cos
h(x) + (17*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^3 + 3
*a^3*cosh(x))*e^(2*x))*sinh(x)^15 + 84*a^3*cosh(x)^12 + 36*(85*a^3*cosh(x)^
4 + 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3)*e^
(4*x) + 2*(85*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^14 +
 504*(17*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^5
 + 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^5 + 10*a^3*c
osh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*cosh(x)^10 + 84*(221*
a^3*cosh(x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3 + (221*a^3*cosh(
x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(221*a^3*cos
h(x)^6 + 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^12 +
144*(221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 + 7*a^3*cosh(
x) + (221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 + 7*a^3*cosh
(x))*e^(4*x) + 2*(221*a^3*cosh(x)^7 + 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3
+ 7*a^3*cosh(x))*e^(2*x))*sinh(x)^11 + 126*a^3*cosh(x)^8 + 18*(2431*a^3*cos
h(x)^8 + 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 + 308*a^3*cosh(x)^2 + 7*a^
3 + (2431*a^3*cosh(x)^8 + 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 + 308*a^3
*cosh(x)^2 + 7*a^3)*e^(4*x) + 2*(2431*a^3*cosh(x)^8 + 4004*a^3*cosh(x)^6 +
2002*a^3*cosh(x)^4 + 308*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^10 + 4*(12
155*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 + 4620*a^3*co
sh(x)^3 + 315*a^3*cosh(x) + (12155*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18
018*a^3*cosh(x)^5 + 4620*a^3*cosh(x)^3 + 315*a^3*cosh(x))*e^(4*x) + 2*(1215
5*a^3*cosh(x)^9 + 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 + 4620*a^3*cosh
(x)^3 + 315*a^3*cosh(x))*e^(2*x))*sinh(x)^9 + 84*a^3*cosh(x)^6 + 18*(2431*a
^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 + 2310*a^3*cosh(x)^
4 + 315*a^3*cosh(x)^2 + 7*a^3 + (2431*a^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 +
 6006*a^3*cosh(x)^6 + 2310*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 7*a^3)*e^(4*
x) + 2*(2431*a^3*cosh(x)^10 + 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 + 231
0*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^8 + 144*(221*
a^3*cosh(x)^11 + 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^7 + 462*a^3*cosh(x)^5
+ 105*a^3*cosh(x)^3 + 7*a^3*cosh(x) + (221*a^3*cosh(x)^11 + 715*a^3*cosh(x)
^9 + 858*a^3*cosh(x)^7 + 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 + 7*a^3*cosh
(x))*e^(4*x) + 2*(221*a^3*cosh(x)^11 + 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^
7 + 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 + 7*a^3*cosh(x))*e^(2*x))*sinh(x)
^7 + 36*a^3*cosh(x)^4 + 84*(221*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*
a^3*cosh(x)^8 + 924*a^3*cosh(x)^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 +
a^3 + (221*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 + 924*a
^3*cosh(x)^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(221
*a^3*cosh(x)^12 + 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 + 924*a^3*cosh(x)
^6 + 315*a^3*cosh(x)^4 + 42*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 504*(
17*a^3*cosh(x)^13 + 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)
^7 + 63*a^3*cosh(x)^5 + 14*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^13
 + 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)^7 + 63*a^3*cosh(
x)^5 + 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x)^13 + 78*
```

```
a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 + 132*a^3*cosh(x)^7 + 63*a^3*cosh(x)^5 +
 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 9*a^3*cosh(x)^2 + 36*
(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 + 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 + a^3 +
(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 + 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 + a^3)*e
^(4*x) + 2*(85*a^3*cosh(x)^14 + 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 +
1155*a^3*cosh(x)^8 + 735*a^3*cosh(x)^6 + 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)
^2 + a^3)*e^(2*x))*sinh(x)^4 + 48*(17*a^3*cosh(x)^15 + 105*a^3*cosh(x)^13 +
 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 + 147*a^3*cosh(
x)^5 + 35*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (17*a^3*cosh(x)^15 + 105*a^3*cosh
(x)^13 + 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 + 147*a
^3*cosh(x)^5 + 35*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(17*a^3*cosh(x
)^15 + 105*a^3*cosh(x)^13 + 273*a^3*cosh(x)^11 + 385*a^3*cosh(x)^9 + 315*a^
3*cosh(x)^7 + 147*a^3*cosh(x)^5 + 35*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x)
)*sinh(x)^3 + a^3 + 9*(17*a^3*cosh(x)^16 + 120*a^3*cosh(x)^14 + 364*a^3*cos
h(x)^12 + 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 + 392*a^3*cosh(x)^6 + 140*
a^3*cosh(x)^4 + 24*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^16 + 120*a^3*cosh(
x)^14 + 364*a^3*cosh(x)^12 + 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 + 392*a
^3*cosh(x)^6 + 140*a^3*cosh(x)^4 + 24*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(17*
a^3*cosh(x)^16 + 120*a^3*cosh(x)^14 + 364*a^3*cosh(x)^12 + 616*a^3*cosh(x)^
10 + 630*a^3*cosh(x)^8 + 392*a^3*cosh(x)^6 + 140*a^3*cosh(x)^4 + 24*a^3*cos
h(x)^2 + a^3)*e^(2*x))*sinh(x)^2 + (a^3*cosh(x)^18 + 9*a^3*cosh(x)^16 + 36*
a^3*cosh(x)^14 + 84*a^3*cosh(x)^12 + 126*a^3*cosh(x)^10 + 126*a^3*cosh(x)^8
 + 84*a^3*cosh(x)^6 + 36*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2
*(a^3*cosh(x)^18 + 9*a^3*cosh(x)^16 + 36*a^3*cosh(x)^14 + 84*a^3*cosh(x)^12
 + 126*a^3*cosh(x)^10 + 126*a^3*cosh(x)^8 + 84*a^3*cosh(x)^6 + 36*a^3*cosh(
x)^4 + 9*a^3*cosh(x)^2 + a^3)*e^(2*x) + 18*(a^3*cosh(x)^17 + 8*a^3*cosh(x)^
15 + 28*a^3*cosh(x)^13 + 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)^9 + 56*a^3*cosh
(x)^7 + 28*a^3*cosh(x)^5 + 8*a^3*cosh(x)^3 + a^3*cosh(x) + (a^3*cosh(x)^17
+ 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 + 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)
^9 + 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 + 8*a^3*cosh(x)^3 + a^3*cosh(x))*e
^(4*x) + 2*(a^3*cosh(x)^17 + 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 + 56*a^3*
cosh(x)^11 + 70*a^3*cosh(x)^9 + 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 + 8*a^3
*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x))
```

giac [A] time = 0.21, size = 39, normalized size = 0.33

$$-\frac{256\left(126\,e^{(8\,x)}+84\,e^{(6\,x)}+36\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}{315\,a^{\frac{5}{2}}\left(e^{(2\,x)}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(a^(5/2)*(
e^(2*x) + 1)^9)
```

maple [A] time = 0.36, size = 96, normalized size = 0.82

$$\frac{4\sqrt{8}\,\sqrt{2}\,\left(8\left(\cosh^4(2x)\right)+40\left(\cosh^3(2x)\right)+84\left(\cosh^2(2x)\right)+100\cosh(2x)+83\right)\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a\,(-1+\mathrm{co}}}{315a^3\left(\cosh(2x)+1\right)^4\sinh(2x)\sqrt{\left(\cosh(2x)+1\right)^2 a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(5/2),x)
```

```
[Out] 4/315*8^(1/2)*2^(1/2)/a^3*(8*cosh(2*x)^4+40*cosh(2*x)^3+84*cosh(2*x)^2+100*
cosh(2*x)+83)*(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/
(cosh(2*x)+1)^4/sinh(2*x)/((cosh(2*x)+1)^2*a)^(1/2)
```

maxima [B] time = 0.43, size = 457, normalized size = 3.91

$$\frac{256\,e^{(-2\,x)}}{35\left(9\,a^{\frac{5}{2}}e^{(-2\,x)}+36\,a^{\frac{5}{2}}e^{(-4\,x)}+84\,a^{\frac{5}{2}}e^{(-6\,x)}+126\,a^{\frac{5}{2}}e^{(-8\,x)}+126\,a^{\frac{5}{2}}e^{(-10\,x)}+84\,a^{\frac{5}{2}}e^{(-12\,x)}+36\,a^{\frac{5}{2}}e^{(-14\,x)}+9\,a^{\frac{5}{2}}e^{(-16}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-
6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12*x)
+ 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5/2))
 + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*
e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12
*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5
/2)) + 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5
/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^
(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) +
a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^
(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*
e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
+ a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)
*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-1
2*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(
5/2))
```

mupad [B] time = 0.99, size = 256, normalized size = 2.19

$$\frac{4096\,\mathrm{e}^{4\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}}{3\,a^3\,{\left({\mathrm{e}}^{2\,x}+1\right)}^6\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{2048\,\mathrm{e}^{4\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}}{5\,a^3\,{\left({\mathrm{e}}^{2\,x}+1\right)}^5\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}-\frac{12288\,\mathrm{e}^{4\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}}{7\,a^3\,{\left({\mathrm{e}}^{2\,x}+1\right)}^7\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cosh(x)^4)^(5/2),x)
```

```
[Out] (4096*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) + 1)^6*
(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*exp(4*x)*(a*(exp(-x)/2 + exp(x)
/2)^4)^(1/2))/(5*a^3*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*x) + exp(6*x))) -
 (12288*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x) + 1)^
7*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*exp(4*x)*(a*(exp(-x)/2 + exp(
x)/2)^4)^(1/2))/(a^3*(exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6*x))) -
 (2048*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*x) + 1)^9
*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cosh(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

3.140 $\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx$

Optimal. Leaf size=8

$$-\frac{1}{\cosh(x)+1}$$

```
[Out] -1/(1+cosh(x))
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2667, 32}

$$-\frac{1}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 + Cosh[x])^2,x]
```

```
[Out] -(1 + Cosh[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \cosh(x)\right)\\ &= -\frac{1}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-\frac{1}{2}\text{sech}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x])^2,x]
```

```
[Out] -1/2*Sech[x/2]^2
```

fricas [B] time = 1.54, size = 31, normalized size = 3.88

$$-\frac{2\left(\cosh(x)+\sinh(x)\right)}{\cosh(x)^{2}+2\left(\cosh(x)+1\right)\sinh(x)+\sinh(x)^{2}+2\,\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2
*cosh(x) + 1)
```

giac [A] time = 0.14, size = 10, normalized size = 1.25

$$-\frac{2\,e^{x}}{\left(e^{x}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x + 1)^2
```

maple [A] time = 0.03, size = 9, normalized size = 1.12

$$-\frac{1}{1+\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+cosh(x))^2,x)
```

```
[Out] -1/(1+cosh(x))
```

maxima [A] time = 0.30, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(cosh(x) + 1)
```

mupad [B] time = 0.07, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) + 1)^2,x)
```

```
[Out] -1/(cosh(x) + 1)
```

sympy [A] time = 0.34, size = 7, normalized size = 0.88

$$-\frac{1}{\cosh{\left(x \right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))**2,x)
```

```
[Out] -1/(cosh(x) + 1)
```

3.141 $\int \frac{\sinh(x)}{(1-\cosh(x))^2} \, dx$

Optimal. Leaf size=8

$$\frac{1}{1-\cosh(x)}$$

```
[Out] 1/(1-cosh(x))
```

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 32}

$$\frac{1}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 - Cosh[x])^2,x]
```

```
[Out] (1 - Cosh[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(1-\cosh(x))^2} \, dx &= -\text{Subst}\left(\int \frac{1}{(1+x)^2} \, dx, x, -\cosh(x)\right)\\ &= \frac{1}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-\frac{1}{2}\text{csch}^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 - Cosh[x])^2,x]
```

```
[Out] -1/2*Csch[x/2]^2
```

fricas [B] time = 0.40, size = 31, normalized size = 3.88

$$-\frac{2\,(\cosh(x)+\sinh(x))}{\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-2\,\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 - 2
*cosh(x) + 1)
```

giac [A] time = 0.12, size = 10, normalized size = 1.25

$$-\frac{2\,e^x}{(e^x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x - 1)^2
```

maple [A] time = 0.03, size = 9, normalized size = 1.12

$$\frac{1}{1-\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1-cosh(x))^2,x)
```

```
[Out] 1/(1-cosh(x))
```

maxima [A] time = 0.30, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(cosh(x) - 1)
```

mupad [B] time = 0.91, size = 8, normalized size = 1.00

$$-\frac{1}{\cosh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) - 1)^2,x)
[Out] -1/(cosh(x) - 1)
```

sympy [A] time = 0.54, size = 7, normalized size = 0.88

$$-\frac{1}{\cosh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))**2,x)
[Out] -1/(cosh(x) - 1)
```

3.142 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx$

Optimal. Leaf size=12

$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

[Out] x-2*sinh(x)/(1+cosh(x))

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2680, 8}

$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]^2/(1 + Cosh[x])^2,x]

[Out] x - (2*Sinh[x])/(1 + Cosh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx &= -\frac{2\sinh(x)}{1+\cosh(x)}+\int 1 \, dx\\ &= x-\frac{2\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.50

$$2\tanh^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Cosh[x])^2,x]
[Out] 2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

fricas [A] time = 2.08, size = 20, normalized size = 1.67

$$\frac{x\cosh(x) + x\sinh(x) + x + 4}{\cosh(x) + \sinh(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

giac [A] time = 0.13, size = 10, normalized size = 0.83

$$x + \frac{4}{e^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="giac")
[Out] x + 4/(e^x + 1)
```

maple [A] time = 0.05, size = 24, normalized size = 2.00

$$-2\tanh\left(\frac{x}{2}\right) - \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+cosh(x))^2,x)
[Out] -2*tanh(1/2*x)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$x - \frac{4}{e^{(-x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="maxima")
[Out] x - 4/(e^(-x) + 1)
```

mupad [B] time = 0.90, size = 10, normalized size = 0.83

$$x + \frac{4}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x) + 1)^2,x)
```

```
[Out] x + 4/(exp(x) + 1)
```

sympy [A] time = 0.55, size = 7, normalized size = 0.58

$$x - 2\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+cosh(x))**2,x)
```

```
[Out] x - 2*tanh(x/2)
```

3.143 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx$

Optimal. Leaf size=14

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

```
[Out] x+2*sinh(x)/(1-cosh(x))
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2680, 8}

$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

```
[Out] x + (2*Sinh[x])/(1 - Cosh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx &= \frac{2\sinh(x)}{1-\cosh(x)}+\int 1\,dx\\ &= x+\frac{2\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 1.71

$$-2\coth\left(\frac{x}{2}\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\tanh^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 - Cosh[x])^2,x]
[Out] -2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

fricas [A] time = 2.15, size = 22, normalized size = 1.57

$$\frac{x\cosh(x) + x\sinh(x) - x - 4}{\cosh(x) + \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="fricas")
[Out] (x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

giac [A] time = 0.16, size = 10, normalized size = 0.71

$$x - \frac{4}{e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="giac")
[Out] x - 4/(e^x - 1)
```

maple [A] time = 0.07, size = 26, normalized size = 1.86

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1-cosh(x))^2,x)
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-2/tanh(1/2*x)
```

maxima [A] time = 0.31, size = 12, normalized size = 0.86

$$x + \frac{4}{e^{(-x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4/(e^(-x) - 1)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.71

$$x-\frac{4}{e^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x) - 1)^2,x)
```

```
[Out] x - 4/(exp(x) - 1)
```

sympy [A] time = 1.04, size = 7, normalized size = 0.50

$$x-\frac{2}{\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1-cosh(x))**2,x)
```

```
[Out] x - 2/tanh(x/2)
```

3.144 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} \, dx$

Optimal. Leaf size=10

$$\cosh(x) - 2\log(\cosh(x) + 1)$$

```
[Out] cosh(x)-2*ln(1+cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 43}

$$\cosh(x) - 2\log(\cosh(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

```
[Out] Cosh[x] - 2*Log[1 + Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} \, dx &= -\text{Subst}\left(\int \frac{1-x}{1+x} \, dx, x, \cosh(x)\right)\\
&= -\text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right) \, dx, x, \cosh(x)\right)\\
&= \cosh(x) - 2\log(1+\cosh(x))
\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.30

$$\cosh(x) - 4\log\left(\cosh\left(\frac{x}{2}\right)\right) - 1$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

```
[Out] -1 + Cosh[x] - 4*Log[Cosh[x/2]]
```

fricas [B] time = 2.70, size = 48, normalized size = 4.80

$$\frac{4\,x\cosh(x) + \cosh(x)^2 - 8\,(\cosh(x) + \sinh(x))\log\left(\cosh(x) + \sinh(x) + 1\right) + 2\,(2\,x + \cosh(x))\sinh(x) + \sinh(}{2\,(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*x*cosh(x) + cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 2*(2*x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

giac [B] time = 0.12, size = 21, normalized size = 2.10

$$2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4\,\log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] 2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^x + 1)
```

maple [A] time = 0.04, size = 11, normalized size = 1.10

$$\cosh(x) - 2\ln\left(1 + \cosh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+cosh(x))^2,x)
```

```
[Out] cosh(x)-2*ln(1+cosh(x))
```

maxima [B] time = 0.30, size = 23, normalized size = 2.30

$$-2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4\,\log\left(e^{(-x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^(-x) + 1)
```

mupad [B] time = 0.94, size = 10, normalized size = 1.00

$$\cosh(x) - 2\ \ln\left(\cosh(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x) + 1)^2,x)
```

```
[Out] cosh(x) - 2*log(cosh(x) + 1)
```

sympy [B] time = 0.52, size = 58, normalized size = 5.80

$$-\frac{2\log\left(\cosh\left(x\right)+1\right)\cosh\left(x\right)}{\cosh\left(x\right)+1}-\frac{2\log\left(\cosh\left(x\right)+1\right)}{\cosh\left(x\right)+1}-\frac{\sinh^{2}\left(x\right)}{\cosh\left(x\right)+1}+\frac{2\cosh^{2}\left(x\right)}{\cosh\left(x\right)+1}-\frac{2}{\cosh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+cosh(x))**2,x)
```

```
[Out] -2*log(cosh(x) + 1)*cosh(x)/(cosh(x) + 1) - 2*log(cosh(x) + 1)/(cosh(x) + 1
) - sinh(x)**2/(cosh(x) + 1) + 2*cosh(x)**2/(cosh(x) + 1) - 2/(cosh(x) + 1)
```

3.145 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx$

Optimal. Leaf size=12

$$\cosh(x)+2\log(1-\cosh(x))$$

```
[Out] cosh(x)+2*ln(1-cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\cosh(x)+2\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

```
[Out] Cosh[x] + 2*Log[1 - Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx &= \text{Subst}\left(\int \frac{1-x}{1+x}\,dx, x, -\cosh(x)\right)\\
&= \text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right)\,dx, x, -\cosh(x)\right)\\
&= \cosh(x)+2\log(1-\cosh(x))
\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.08

$$\cosh(x) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right) - 1$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

```
[Out] -1 + Cosh[x] + 4*Log[Sinh[x/2]]
```

fricas [B] time = 1.09, size = 54, normalized size = 4.50

$$-\frac{4\,x\cosh(x) - \cosh(x)^2 - 8\,(\cosh(x) + \sinh(x))\log\left(\cosh(x) + \sinh(x) - 1\right) + 2\,(2\,x - \cosh(x))\sinh(x) - \sinh(x}{2\,(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*x*cosh(x) - cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
 - 1) + 2*(2*x - cosh(x))*sinh(x) - sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

giac [A] time = 0.13, size = 22, normalized size = 1.83

$$-2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} + 4\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(abs(e^x - 1))
```

maple [A] time = 0.06, size = 11, normalized size = 0.92

$$\cosh(x) + 2\ln\left(-1 + \cosh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1-cosh(x))^2,x)
```

```
[Out] cosh(x)+2*ln(-1+cosh(x))
```

maxima [A] time = 0.31, size = 23, normalized size = 1.92

$$2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} + 4\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(e^(-x) - 1)
```

mupad [B] time = 0.94, size = 10, normalized size = 0.83

$$2\ \ln(\cosh(x)-1)+\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x) - 1)^2,x)
```

```
[Out] 2*log(cosh(x) - 1) + cosh(x)
```

sympy [B] time = 0.53, size = 58, normalized size = 4.83

$$\frac{2\log\left(\cosh\left(x\right)-1\right)\cosh\left(x\right)}{\cosh\left(x\right)-1}-\frac{2\log\left(\cosh\left(x\right)-1\right)}{\cosh\left(x\right)-1}-\frac{\sinh^{2}\left(x\right)}{\cosh\left(x\right)-1}+\frac{2\cosh^{2}\left(x\right)}{\cosh\left(x\right)-1}-\frac{2}{\cosh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1-cosh(x))**2,x)
```

```
[Out] 2*log(cosh(x) - 1)*cosh(x)/(cosh(x) - 1) - 2*log(cosh(x) - 1)/(cosh(x) - 1)
 - sinh(x)**2/(cosh(x) - 1) + 2*cosh(x)**2/(cosh(x) - 1) - 2/(cosh(x) - 1)
```

3.146 $\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2(\cosh(x)+1)^2}$$

[Out] -1/2/(1+cosh(x))^2

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2667, 32}

$$-\frac{1}{2(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(1 + Cosh[x])^3,x]

[Out] -1/(2*(1 + Cosh[x])^2)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, \cosh(x)\right)\\ &= -\frac{1}{2(1+\cosh(x))^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.20

$$-\frac{1}{8}\text{sech}^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 + Cosh[x])^3,x]
```

```
[Out] -1/8*Sech[x/2]^4
```

fricas [B] time = 0.82, size = 55, normalized size = 5.50

$$-\frac{2\,(\cosh(x)+\sinh(x))}{\cosh(x)^3+(3\,\cosh(x)+4)\sinh(x)^2+\sinh(x)^3+4\,\cosh(x)^2+\left(3\,\cosh(x)^2+8\,\cosh(x)+5\right)\sinh(x)+7\,\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) + 4)*sinh(x)^2 + sinh(x)^3 +
 4*cosh(x)^2 + (3*cosh(x)^2 + 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) + 4)
```

giac [A] time = 0.12, size = 12, normalized size = 1.20

$$-\frac{2\,e^{(2\,x)}}{(e^x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2*e^(2*x)/(e^x + 1)^4
```

maple [A] time = 0.03, size = 9, normalized size = 0.90

$$-\frac{1}{2\,(1+\cosh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1+cosh(x))^3,x)
```

```
[Out] -1/2/(1+cosh(x))^2
```

maxima [A] time = 0.29, size = 8, normalized size = 0.80

$$-\frac{1}{2\,(\cosh(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2/(cosh(x) + 1)^2
```

mupad [B] time = 0.92, size = 8, normalized size = 0.80

$$-\frac{1}{2\,{\left(\cosh(x)+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) + 1)^3,x)
```

```
[Out] -1/(2*(cosh(x) + 1)^2)
```

sympy [A] time = 0.63, size = 15, normalized size = 1.50

$$-\frac{1}{2\cosh^2(x)+4\cosh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1+cosh(x))**3,x)
```

```
[Out] -1/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

3.147 $\int \frac{\sinh(x)}{(1-\cosh(x))^3} dx$

Optimal. Leaf size=12

$$\frac{1}{2(1-\cosh(x))^2}$$

```
[Out] 1/2/(1-cosh(x))^2
```

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 32}

$$\frac{1}{2(1-\cosh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(1 - Cosh[x])^3,x]
```

```
[Out] 1/(2*(1 - Cosh[x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh(x)}{(1-\cosh(x))^3} dx &= -\text{Subst}\left(\int \frac{1}{(1+x)^3} dx, x, -\cosh(x)\right) \\ &= \frac{1}{2(1-\cosh(x))^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{1}{8}\text{csch}^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(1 - Cosh[x])^3,x]
```

```
[Out] Csch[x/2]^4/8
```

fricas [B] time = 1.71, size = 55, normalized size = 4.58

$$\frac{2\left(\cosh(x)+\sinh(x)\right)}{\cosh(x)^3+\left(3\cosh(x)-4\right)\sinh(x)^2+\sinh(x)^3-4\cosh(x)^2+\left(3\cosh(x)^2-8\cosh(x)+5\right)\sinh(x)+7\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 -
4*cosh(x)^2 + (3*cosh(x)^2 - 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) - 4)
```

giac [A] time = 0.13, size = 12, normalized size = 1.00

$$\frac{2e^{(2x)}}{\left(e^{x}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2*e^(2*x)/(e^x - 1)^4
```

maple [A] time = 0.03, size = 11, normalized size = 0.92

$$\frac{1}{2\left(1-\cosh(x)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(1-cosh(x))^3,x)
```

```
[Out] 1/2/(1-cosh(x))^2
```

maxima [A] time = 0.30, size = 8, normalized size = 0.67

$$\frac{1}{2\left(\cosh(x)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2/(cosh(x) - 1)^2
```

mupad **[B]** time = 0.08, size = 8, normalized size = 0.67

$$\frac{1}{2\,(\cosh(x)-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(x)/(cosh(x) - 1)^3,x)
[Out] 1/(2*(cosh(x) - 1)^2)
```

sympy **[A]** time = 0.58, size = 14, normalized size = 1.17

$$\frac{1}{2\cosh^2(x)-4\cosh(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(1-cosh(x))**3,x)
[Out] 1/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

3.148 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{\sinh^3(x)}{3(\cosh(x)+1)^3}$$

```
[Out] 1/3*sinh(x)^3/(1+cosh(x))^3
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2671}

$$\frac{\sinh^3(x)}{3(\cosh(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

```
[Out] Sinh[x]^3/(3*(1 + Cosh[x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} \, dx = \frac{\sinh^3(x)}{3(1+\cosh(x))^3}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 0.86

$$\frac{1}{3} \tanh^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

```
[Out] Tanh[x/2]^3/3
```

fricas [B] time = 0.50, size = 33, normalized size = 2.36

$$-\frac{4\,(2\,\cosh(x)+\sinh(x))}{3\left(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
 + 4*cosh(x) + 3)
```

giac [A] time = 0.13, size = 16, normalized size = 1.14

$$-\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\,(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 1)/(e^x + 1)^3
```

maple [A] time = 0.05, size = 9, normalized size = 0.64

$$\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1+cosh(x))^3,x)
```

```
[Out] 1/3*tanh(1/2*x)^3
```

maxima [B] time = 0.31, size = 49, normalized size = 3.50

$$\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2
*x) + e^(-3*x) + 1)
```

mupad [B] time = 0.92, size = 16, normalized size = 1.14

$$-\frac{2\,\left(3\,{\mathrm{e}}^{2\,x}+1\right)}{3\,{\left({\mathrm{e}}^x+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(cosh(x) + 1)^3,x)
[Out] -(2*(3*exp(2*x) + 1))/(3*(exp(x) + 1)^3)
```

sympy [A] time = 0.96, size = 7, normalized size = 0.50

$$\frac{\tanh^{3}{\left(\frac{x}{2}\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1+cosh(x))**3,x)
[Out] tanh(x/2)**3/3
```

3.149 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} \, dx$

Optimal. Leaf size=16

$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

```
[Out] -1/3*sinh(x)^3/(1-cosh(x))^3
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2671}

$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

```
[Out] -Sinh[x]^3/(3*(1 - Cosh[x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} \, dx = -\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 0.75

$$\frac{1}{3} \coth^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

```
[Out] Coth[x/2]^3/3
```

fricas [B] time = 3.35, size = 33, normalized size = 2.06

$$\frac{4\,(2\,\cosh(x)+\sinh(x))}{3\left(\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-4\,\cosh(x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
- 4*cosh(x) + 3)
```

giac [A] time = 0.14, size = 16, normalized size = 1.00

$$\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\,(e^x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*e^(2*x) + 1)/(e^x - 1)^3
```

maple [A] time = 0.06, size = 9, normalized size = 0.56

$$\frac{1}{3\tanh\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(1-cosh(x))^3,x)
```

```
[Out] 1/3/tanh(1/2*x)^3
```

maxima [B] time = 0.31, size = 49, normalized size = 3.06

$$-\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{2}{3\left(3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -2*e^(-2*x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/3/(3*e^(-x) - 3*e^(-
2*x) + e^(-3*x) - 1)
```

mupad [B] time = 0.92, size = 16, normalized size = 1.00

$$\frac{2\left(3\,e^{2\,x}+1\right)}{3\,{\left(e^x-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(x)^2/(cosh(x) - 1)^3,x)
```

```
[Out] (2*(3*exp(2*x) + 1))/(3*(exp(x) - 1)^3)
```

sympy [A] time = 1.49, size = 8, normalized size = 0.50

$$\frac{1}{3\tanh^{3}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(1-cosh(x))**3,x)
```

```
[Out] 1/(3*tanh(x/2)**3)
```

3.150 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

```
[Out] 2/(1+cosh(x))+ln(1+cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 43}

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

```
[Out] 2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = -\operatorname{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, \cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{-1-x} + \frac{2}{(1+x)^2}\right)dx, x, \cosh(x)\right)$$
$$= \frac{2}{1+\cosh(x)} + \log(1+\cosh(x))$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.43

$$2\log\left(\cosh\left(\frac{x}{2}\right)\right) - \tanh^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

```
[Out] 2*Log[Cosh[x/2]] - Tanh[x/2]^2
```

fricas [B] time = 0.55, size = 89, normalized size = 6.36

$$-\frac{x\cosh(x)^2 + x\sinh(x)^2 + 2\,(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2\,(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\,\cosh(x) + \right.}{\cosh(x)^2 + 2\,(\cosh(x)+1)\sinh(x) + \sinh(x)^2 + 2\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2*(
x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh
(x)^2 + 2*cosh(x) + 1)
```

giac [A] time = 0.12, size = 21, normalized size = 1.50

$$-x + \frac{4\,e^x}{(e^x+1)^2} + 2\,\log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="giac")
```

```
[Out] -x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

maple [A] time = 0.05, size = 15, normalized size = 1.07

$$\frac{2}{1+\cosh(x)}+\ln\left(1+\cosh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1+cosh(x))^3,x)
[Out] 2/(1+cosh(x))+ln(1+cosh(x))
```

maxima [B] time = 0.30, size = 31, normalized size = 2.21

$$x+\frac{4\,e^{(-x)}}{2\,e^{(-x)}+e^{(-2\,x)}+1}+2\,\log\left(e^{(-x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="maxima")
[Out] x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

mupad [B] time = 0.98, size = 14, normalized size = 1.00

$$\ln\left(\cosh(x)+1\right)+\frac{2}{\cosh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(cosh(x) + 1)^3,x)
[Out] log(cosh(x) + 1) + 2/(cosh(x) + 1)
```

sympy [B] time = 0.59, size = 126, normalized size = 9.00

$$\frac{2\log\left(\cosh\left(x\right)+1\right)\cosh^{2}\left(x\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}+\frac{4\log\left(\cosh\left(x\right)+1\right)\cosh\left(x\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}+\frac{2\log\left(\cosh\left(x\right)+1\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right)+2}-\frac{\sinh^{2}\left(x\right)}{2\cosh^{2}\left(x\right)+4\cosh\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1+cosh(x))**3,x)
[Out] 2*log(cosh(x) + 1)*cosh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 4*log(cosh(x
) + 1)*cosh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*log(cosh(x) + 1)/(2*cosh(
x)**2 + 4*cosh(x) + 2) - sinh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*cosh
(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

3.151 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))$$

```
[Out] -2/(1-cosh(x))-ln(1-cosh(x))
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$-\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

```
[Out] -2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = \text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -\cosh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, -\cosh(x)\right)$$
$$= -\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.35

$$\coth^2\left(\frac{x}{2}\right) - 2\log\left(\tanh\left(\frac{x}{2}\right)\right) - 2\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

```
[Out] Coth[x/2]^2 - 2*Log[Cosh[x/2]] - 2*Log[Tanh[x/2]]
```

fricas [B] time = 0.70, size = 90, normalized size = 4.50

$$\frac{x\cosh(x)^2 + x\sinh(x)^2 - 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(x) + 1\right)\,\mathrm{l}}{\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
- 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(x
*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(
x)^2 - 2*cosh(x) + 1)
```

giac [A] time = 0.14, size = 20, normalized size = 1.00

$$x + \frac{4e^x}{(e^x-1)^2} - 2\log\left(|e^x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="giac")
```

```
[Out] x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

maple [A] time = 0.08, size = 17, normalized size = 0.85

$$-\ln(-1+\cosh(x)) + \frac{2}{-1+\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(1-cosh(x))^3,x)
```

```
[Out] -ln(-1+cosh(x))+2/(-1+cosh(x))
```

maxima [A] time = 0.30, size = 35, normalized size = 1.75

$$-x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="maxima")
```

```
[Out] -x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

mupad [B] time = 0.99, size = 16, normalized size = 0.80

$$\frac{2}{\cosh(x) - 1} - \ln(\cosh(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh(x)^3/(cosh(x) - 1)^3,x)
```

```
[Out] 2/(cosh(x) - 1) - log(cosh(x) - 1)
```

sympy [B] time = 0.60, size = 126, normalized size = 6.30

$$-\frac{2\log(\cosh(x)-1)\cosh^2(x)}{2\cosh^2(x)-4\cosh(x)+2} + \frac{4\log(\cosh(x)-1)\cosh(x)}{2\cosh^2(x)-4\cosh(x)+2} - \frac{2\log(\cosh(x)-1)}{2\cosh^2(x)-4\cosh(x)+2} + \frac{\sinh^2(x)}{2\cosh^2(x)-4\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(1-cosh(x))**3,x)
```

```
[Out] -2*log(cosh(x) - 1)*cosh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 4*log(cosh(
x) - 1)*cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2*log(cosh(x) - 1)/(2*cosh
(x)**2 - 4*cosh(x) + 2) + sinh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 2*cos
h(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

3.152 $\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=57

$$\frac{5x}{16a}+\frac{\sinh^7(x)}{7a}-\frac{\sinh^5(x)\cosh(x)}{6a}+\frac{5\sinh^3(x)\cosh(x)}{24a}-\frac{5\sinh(x)\cosh(x)}{16a}$$

```
[Out] 5/16*x/a-5/16*cosh(x)*sinh(x)/a+5/24*cosh(x)*sinh(x)^3/a-1/6*cosh(x)*sinh(x
)^5/a+1/7*sinh(x)^7/a
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$\frac{5x}{16a}+\frac{\sinh^7(x)}{7a}-\frac{\sinh^5(x)\cosh(x)}{6a}+\frac{5\sinh^3(x)\cosh(x)}{24a}-\frac{5\sinh(x)\cosh(x)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^8/(a + a*Cosh[x]),x]
```

```
[Out] (5*x)/(16*a) - (5*Cosh[x]*Sinh[x])/(16*a) + (5*Cosh[x]*Sinh[x]^3)/(24*a) -
(Cosh[x]*Sinh[x]^5)/(6*a) + Sinh[x]^7/(7*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh^7(x)}{7a}-\frac{\int \sinh^6(x)\,dx}{a}\\
&= -\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}+\frac{5\int \sinh^4(x)\,dx}{6a}\\
&= \frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}-\frac{5\int \sinh^2(x)\,dx}{8a}\\
&= -\frac{5\cosh(x)\sinh(x)}{16a}+\frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}+\frac{5\int 1\,dx}{16a}\\
&= \frac{5x}{16a}-\frac{5\cosh(x)\sinh(x)}{16a}+\frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 51, normalized size = 0.89

$$
\frac{420x-105\sinh(x)-315\sinh(2x)+63\sinh(3x)+63\sinh(4x)-21\sinh(5x)-7\sinh(6x)+3\sinh(7x)}{1344a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^8/(a + a*Cosh[x]),x]
```

```
[Out] (420*x - 105*Sinh[x] - 315*Sinh[2*x] + 63*Sinh[3*x] + 63*Sinh[4*x] - 21*Sin
h[5*x] - 7*Sinh[6*x] + 3*Sinh[7*x])/(1344*a)
```

fricas [B] time = 1.12, size = 101, normalized size = 1.77

$$
\frac{3\sinh(x)^7+21\left(3\cosh(x)^2-2\cosh(x)-1\right)\sinh(x)^5+7\left(15\cosh(x)^4-20\cosh(x)^3-30\cosh(x)^2+36\cos\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/1344*(3*sinh(x)^7 + 21*(3*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^5 + 7*(15*co
sh(x)^4 - 20*cosh(x)^3 - 30*cosh(x)^2 + 36*cosh(x) + 9)*sinh(x)^3 + 21*(cos
h(x)^6 - 2*cosh(x)^5 - 5*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2 - 30*cosh(x
) - 5)*sinh(x) + 420*x)/a
```

giac [A] time = 0.12, size = 90, normalized size = 1.58

$$
\frac{\left(105\,e^{(6\,x)}+315\,e^{(5\,x)}-63\,e^{(4\,x)}-63\,e^{(3\,x)}+21\,e^{(2\,x)}+7\,e^{x}-3\right)e^{(-7\,x)}+840\,x+3\,e^{(7\,x)}-7\,e^{(6\,x)}-21\,e^{(5\,x)}+63\,e^{(4}}{2688\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/2688*((105*e^(6*x) + 315*e^(5*x) - 63*e^(4*x) - 63*e^(3*x) + 21*e^(2*x) +
 7*e^x - 3)*e^(-7*x) + 840*x + 3*e^(7*x) - 7*e^(6*x) - 21*e^(5*x) + 63*e^(4
*x) + 63*e^(3*x) - 315*e^(2*x) - 105*e^x)/a
```

maple [B] time = 0.10, size = 208, normalized size = 3.65

$$-\frac{1}{7a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{7}}-\frac{2}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{6}}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{11}{24a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{8a\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^8/(a+a*cosh(x)),x)
```

```
[Out] -1/7/a/(tanh(1/2*x)-1)^7-2/3/a/(tanh(1/2*x)-1)^6-1/a/(tanh(1/2*x)-1)^5-1/4/
a/(tanh(1/2*x)-1)^4+11/24/a/(tanh(1/2*x)-1)^3-1/8/a/(tanh(1/2*x)-1)^2-5/16/
a/(tanh(1/2*x)-1)-5/16/a*ln(tanh(1/2*x)-1)-1/7/a/(tanh(1/2*x)+1)^7+2/3/a/(t
anh(1/2*x)+1)^6-1/a/(tanh(1/2*x)+1)^5+1/4/a/(tanh(1/2*x)+1)^4+11/24/a/(tanh
(1/2*x)+1)^3+1/8/a/(tanh(1/2*x)+1)^2-5/16/a/(tanh(1/2*x)+1)+5/16/a*ln(tanh(
1/2*x)+1)
```

maxima [B] time = 0.31, size = 102, normalized size = 1.79

$$-\frac{\left(7\,e^{(-x)}+21\,e^{(-2\,x)}-63\,e^{(-3\,x)}-63\,e^{(-4\,x)}+315\,e^{(-5\,x)}+105\,e^{(-6\,x)}-3\right)e^{(7\,x)}}{2688\,a}+\frac{5\,x}{16\,a}+\frac{105\,e^{(-x)}+315\,e^{(-2\,x)}-63\,e^{(-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/2688*(7*e^(-x) + 21*e^(-2*x) - 63*e^(-3*x) - 63*e^(-4*x) + 315*e^(-5*x)
+ 105*e^(-6*x) - 3)*e^(7*x)/a + 5/16*x/a + 1/2688*(105*e^(-x) + 315*e^(-2*x
) - 63*e^(-3*x) - 63*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) - 3*e^(-7*x))/a
```

mupad [B] time = 1.26, size = 131, normalized size = 2.30

$$\frac{5\,\mathrm{e}^{-x}}{128\,a}+\frac{15\,\mathrm{e}^{-2\,x}}{128\,a}-\frac{15\,\mathrm{e}^{2\,x}}{128\,a}-\frac{3\,\mathrm{e}^{-3\,x}}{128\,a}+\frac{3\,\mathrm{e}^{3\,x}}{128\,a}-\frac{3\,\mathrm{e}^{-4\,x}}{128\,a}+\frac{3\,\mathrm{e}^{4\,x}}{128\,a}+\frac{\mathrm{e}^{-5\,x}}{128\,a}-\frac{\mathrm{e}^{5\,x}}{128\,a}+\frac{\mathrm{e}^{-6\,x}}{384\,a}-\frac{\mathrm{e}^{6\,x}}{384\,a}-\frac{\mathrm{e}^{-7\,x}}{896\,a}+\frac{\mathrm{e}^{7\,x}}{896\,a}+\frac{5\,x}{16\,a}-\frac{5\,\mathrm{e}^{x}}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^8/(a + a*cosh(x)),x)
```

```
[Out] (5*exp(-x))/(128*a) + (15*exp(-2*x))/(128*a) - (15*exp(2*x))/(128*a) - (3*e
xp(-3*x))/(128*a) + (3*exp(3*x))/(128*a) - (3*exp(-4*x))/(128*a) + (3*exp(4
*x))/(128*a) + exp(-5*x)/(128*a) - exp(5*x)/(128*a) + exp(-6*x)/(384*a) - e
xp(6*x)/(384*a) - exp(-7*x)/(896*a) + exp(7*x)/(896*a) + (5*x)/(16*a) - (5*
exp(x))/(128*a)
```

sympy [B] time = 8.77, size = 1253, normalized size = 21.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**8/(a+a*cosh(x)),x)
```

```
[Out] 105*x*tanh(x/2)**14/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*ta
nh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2
)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 735*x*tanh(x/2)**12/(336*a*tanh(x/2)*
*14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 +
11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) +
 2205*x*tanh(x/2)**10/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*
tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x
/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3675*x*tanh(x/2)**8/(336*a*tanh(x/2
)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8
+ 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a)
 + 3675*x*tanh(x/2)**6/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a
*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(
x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 2205*x*tanh(x/2)**4/(336*a*tanh(x/
2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8
 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a
) + 735*x*tanh(x/2)**2/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a
*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(
x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 105*x/(336*a*tanh(x/2)**14 - 2352*
a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tan
h(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 210*tanh(x
/2)**13/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10
- 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*
a*tanh(x/2)**2 - 336*a) + 1400*tanh(x/2)**11/(336*a*tanh(x/2)**14 - 2352*a*
tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(
x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3962*tanh(x/
2)**9/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 -
11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*
tanh(x/2)**2 - 336*a) - 6144*tanh(x/2)**7/(336*a*tanh(x/2)**14 - 2352*a*tan
h(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2
)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) + 3962*tanh(x/2)*
*5/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 117
60*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tan
```

```
h(x/2)**2 - 336*a) - 1400*tanh(x/2)**3/(336*a*tanh(x/2)**14 - 2352*a*tanh(x
/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**
6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) + 210*tanh(x/2)/(336
*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*ta
nh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)*
*2 - 336*a)
```

3.153 $\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{(a-a\cosh(x))^6}{6a^7}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^4}{a^5}$$

```
[Out] (a-a*cosh(x))^4/a^5-4/5*(a-a*cosh(x))^5/a^6+1/6*(a-a*cosh(x))^6/a^7
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{(a-a\cosh(x))^6}{6a^7}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^4}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^7/(a + a*Cosh[x]),x]
```

```
[Out] (a - a*Cosh[x])^4/a^5 - (4*(a - a*Cosh[x])^5)/(5*a^6) + (a - a*Cosh[x])^6/(
6*a^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = -\frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx, x, a\cosh(x)\right)}{a^7}$$
$$= -\frac{\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)\,dx, x, a\cosh(x)\right)}{a^7}$$
$$= \frac{(a-a\cosh(x))^4}{a^5} - \frac{4(a-a\cosh(x))^5}{5a^6} + \frac{(a-a\cosh(x))^6}{6a^7}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.59

$$\frac{4\sinh^8\left(\frac{x}{2}\right)(28\cosh(x)+5\cosh(2x)+27)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^7/(a + a*Cosh[x]),x]
```

```
[Out] (4*(27 + 28*Cosh[x] + 5*Cosh[2*x])*Sinh[x/2]^8)/(15*a)
```

fricas [B] time = 1.38, size = 94, normalized size = 2.04

$$\frac{5\cosh(x)^6+5\sinh(x)^6-12\cosh(x)^5+15\left(5\cosh(x)^2-4\cosh(x)-2\right)\sinh(x)^4-30\cosh(x)^4+100\cosh(x}{960\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(5*cosh(x)^6 + 5*sinh(x)^6 - 12*cosh(x)^5 + 15*(5*cosh(x)^2 - 4*cosh(
x) - 2)*sinh(x)^4 - 30*cosh(x)^4 + 100*cosh(x)^3 + 15*(5*cosh(x)^4 - 8*cosh
(x)^3 - 12*cosh(x)^2 + 20*cosh(x) + 5)*sinh(x)^2 + 75*cosh(x)^2 - 600*cosh(
x))/a
```

giac [A] time = 0.12, size = 75, normalized size = 1.63

$$-\frac{\left(600\,e^{(5\,x)}-75\,e^{(4\,x)}-100\,e^{(3\,x)}+30\,e^{(2\,x)}+12\,e^{x}-5\right)e^{(-6\,x)}-5\,e^{(6\,x)}+12\,e^{(5\,x)}+30\,e^{(4\,x)}-100\,e^{(3\,x)}-75\,e^{(2\,x)}+}{1920\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/1920*((600*e^(5*x) - 75*e^(4*x) - 100*e^(3*x) + 30*e^(2*x) + 12*e^x - 5)
*e^(-6*x) - 5*e^(6*x) + 12*e^(5*x) + 30*e^(4*x) - 100*e^(3*x) - 75*e^(2*x)
+ 600*e^x)/a
```

maple [B] time = 0.09, size = 107, normalized size = 2.33

$$\frac{\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{6}}+\frac{7}{10\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}+\frac{7}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{7}{16\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{7}{16\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^{6}}-\frac{7}{10\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\cdots}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a+a*cosh(x)),x)
```

```
[Out] 128/a*(1/768/(tanh(1/2*x)-1)^6+7/1280/(tanh(1/2*x)-1)^5+7/1024/(tanh(1/2*x)
-1)^4-7/2048/(tanh(1/2*x)-1)^2+7/2048/(tanh(1/2*x)-1)+1/768/(tanh(1/2*x)+1)
^6-7/1280/(tanh(1/2*x)+1)^5+7/1024/(tanh(1/2*x)+1)^4-7/2048/(tanh(1/2*x)+1)
^2-7/2048/(tanh(1/2*x)+1))
```

maxima [A] time = 0.32, size = 84, normalized size = 1.83

$$-\frac{\left(12\,e^{(-x)}+30\,e^{(-2\,x)}-100\,e^{(-3\,x)}-75\,e^{(-4\,x)}+600\,e^{(-5\,x)}-5\right)e^{(6\,x)}}{1920\,a}-\frac{600\,e^{(-x)}-75\,e^{(-2\,x)}-100\,e^{(-3\,x)}+30\,e^{(-4\,x)}\cdots}{1920\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*e^(-x) + 30*e^(-2*x) - 100*e^(-3*x) - 75*e^(-4*x) + 600*e^(-5*x
) - 5)*e^(6*x)/a - 1/1920*(600*e^(-x) - 75*e^(-2*x) - 100*e^(-3*x) + 30*e^(
-4*x) + 12*e^(-5*x) - 5*e^(-6*x))/a
```

mupad [B] time = 1.13, size = 107, normalized size = 2.33

$$\frac{5\,e^{-2\,x}}{128\,a}-\frac{5\,e^{-x}}{16\,a}+\frac{5\,e^{2\,x}}{128\,a}+\frac{5\,e^{-3\,x}}{96\,a}+\frac{5\,e^{3\,x}}{96\,a}-\frac{e^{-4\,x}}{64\,a}-\frac{e^{4\,x}}{64\,a}-\frac{e^{-5\,x}}{160\,a}-\frac{e^{5\,x}}{160\,a}+\frac{e^{-6\,x}}{384\,a}+\frac{e^{6\,x}}{384\,a}-\frac{5\,e^{x}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a + a*cosh(x)),x)
```

```
[Out] (5*exp(-2*x))/(128*a) - (5*exp(-x))/(16*a) + (5*exp(2*x))/(128*a) + (5*exp(
-3*x))/(96*a) + (5*exp(3*x))/(96*a) - exp(-4*x)/(64*a) - exp(4*x)/(64*a) -
exp(-5*x)/(160*a) - exp(5*x)/(160*a) + exp(-6*x)/(384*a) + exp(6*x)/(384*a)
 - (5*exp(x))/(16*a)
```

sympy [B] time = 5.62, size = 284, normalized size = 6.17

$$\frac{320\tanh^{6}\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right)-90a\tanh^{10}\left(\frac{x}{2}\right)+225a\tanh^{8}\left(\frac{x}{2}\right)-300a\tanh^{6}\left(\frac{x}{2}\right)+225a\tanh^{4}\left(\frac{x}{2}\right)-90a\tanh^{2}\left(\frac{x}{2}\right)+15a}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**7/(a+a*cosh(x)),x)
```

```
[Out] 320*tanh(x/2)**6/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2)
**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a) -
 240*tanh(x/2)**4/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2
)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
+ 96*tanh(x/2)**2/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2
)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
- 16/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2)**8 - 300*a*
tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
```

3.154 $\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=44

$$-\frac{3x}{8a}+\frac{\sinh^5(x)}{5a}-\frac{\sinh^3(x)\cosh(x)}{4a}+\frac{3\sinh(x)\cosh(x)}{8a}$$

```
[Out] -3/8*x/a+3/8*cosh(x)*sinh(x)/a-1/4*cosh(x)*sinh(x)^3/a+1/5*sinh(x)^5/a
```

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{3x}{8a}+\frac{\sinh^5(x)}{5a}-\frac{\sinh^3(x)\cosh(x)}{4a}+\frac{3\sinh(x)\cosh(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (-3*x)/(8*a) + (3*Cosh[x]*Sinh[x])/(8*a) - (Cosh[x]*Sinh[x]^3)/(4*a) + Sinh
[x]^5/(5*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{\sinh^5(x)}{5a} - \frac{\int \sinh^4(x)\,dx}{a}$$
$$= -\frac{\cosh(x)\sinh^3(x)}{4a} + \frac{\sinh^5(x)}{5a} + \frac{3\int \sinh^2(x)\,dx}{4a}$$
$$= \frac{3\cosh(x)\sinh(x)}{8a} - \frac{\cosh(x)\sinh^3(x)}{4a} + \frac{\sinh^5(x)}{5a} - \frac{3\int 1\,dx}{8a}$$
$$= -\frac{3x}{8a} + \frac{3\cosh(x)\sinh(x)}{8a} - \frac{\cosh(x)\sinh^3(x)}{4a} + \frac{\sinh^5(x)}{5a}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 0.89

$$\frac{-60x + 20\sinh(x) + 40\sinh(2x) - 10\sinh(3x) - 5\sinh(4x) + 2\sinh(5x)}{160a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (-60*x + 20*Sinh[x] + 40*Sinh[2*x] - 10*Sinh[3*x] - 5*Sinh[4*x] + 2*Sinh[5*
x])/(160*a)
```

fricas [A] time = 1.04, size = 57, normalized size = 1.30

$$\frac{\sinh(x)^5 + 5\left(2\cosh(x)^2 - 2\cosh(x) - 1\right)\sinh(x)^3 + 5\left(\cosh(x)^4 - 2\cosh(x)^3 - 3\cosh(x)^2 + 8\cosh(x) + 2\right)\text{si}}{80\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/80*(sinh(x)^5 + 5*(2*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^3 + 5*(cosh(x)^4
- 2*cosh(x)^3 - 3*cosh(x)^2 + 8*cosh(x) + 2)*sinh(x) - 30*x)/a
```

giac [A] time = 0.13, size = 66, normalized size = 1.50

$$-\frac{\left(20\,e^{(4\,x)} + 40\,e^{(3\,x)} - 10\,e^{(2\,x)} - 5\,e^{x} + 2\right)e^{(-5\,x)} + 120\,x - 2\,e^{(5\,x)} + 5\,e^{(4\,x)} + 10\,e^{(3\,x)} - 40\,e^{(2\,x)} - 20\,e^{x}}{320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/320*((20*e^(4*x) + 40*e^(3*x) - 10*e^(2*x) - 5*e^x + 2)*e^(-5*x) + 120*x
 - 2*e^(5*x) + 5*e^(4*x) + 10*e^(3*x) - 40*e^(2*x) - 20*e^x)/a
```

maple [B] time = 0.08, size = 156, normalized size = 3.55

$$-\frac{1}{5a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{3}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{3}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{8a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a+a*cosh(x)),x)
```

```
[Out] -1/5/a/(tanh(1/2*x)-1)^5-3/4/a/(tanh(1/2*x)-1)^4-3/4/a/(tanh(1/2*x)-1)^3+1/
4/a/(tanh(1/2*x)-1)^2+3/8/a/(tanh(1/2*x)-1)+3/8/a*ln(tanh(1/2*x)-1)-1/5/a/(
tanh(1/2*x)+1)^5+3/4/a/(tanh(1/2*x)+1)^4-3/4/a/(tanh(1/2*x)+1)^3-1/4/a/(tan
h(1/2*x)+1)^2+3/8/a/(tanh(1/2*x)+1)-3/8/a*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.31, size = 78, normalized size = 1.77

$$-\frac{\left(5\,e^{(-x)}+10\,e^{(-2\,x)}-40\,e^{(-3\,x)}-20\,e^{(-4\,x)}-2\right)e^{(5\,x)}}{320\,a}-\frac{3\,x}{8\,a}-\frac{20\,e^{(-x)}+40\,e^{(-2\,x)}-10\,e^{(-3\,x)}-5\,e^{(-4\,x)}+2\,e^{(-5\,x)}}{320\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/320*(5*e^(-x) + 10*e^(-2*x) - 40*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)/a -
 3/8*x/a - 1/320*(20*e^(-x) + 40*e^(-2*x) - 10*e^(-3*x) - 5*e^(-4*x) + 2*e^
(-5*x))/a
```

mupad [B] time = 1.05, size = 95, normalized size = 2.16

$$\frac{e^{2\,x}}{8\,a}-\frac{e^{-2\,x}}{8\,a}-\frac{e^{-x}}{16\,a}+\frac{e^{-3\,x}}{32\,a}-\frac{e^{3\,x}}{32\,a}+\frac{e^{-4\,x}}{64\,a}-\frac{e^{4\,x}}{64\,a}-\frac{e^{-5\,x}}{160\,a}+\frac{e^{5\,x}}{160\,a}-\frac{3\,x}{8\,a}+\frac{e^{x}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a + a*cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - exp(-x)/(16*a) + exp(-3*x)/(32*a) - exp(
3*x)/(32*a) + exp(-4*x)/(64*a) - exp(4*x)/(64*a) - exp(-5*x)/(160*a) + exp(
5*x)/(160*a) - (3*x)/(8*a) + exp(x)/(16*a)
```

sympy [B] time = 3.71, size = 692, normalized size = 15.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**6/(a+a*cosh(x)),x)
```

```
[Out] -15*x*tanh(x/2)**10/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x
/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 75*x*tanh(x/2)**
8/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tan
h(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 150*x*tanh(x/2)**6/(40*a*tanh(x/2)
**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a
*tanh(x/2)**2 - 40*a) + 150*x*tanh(x/2)**4/(40*a*tanh(x/2)**10 - 200*a*tanh
(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 4
0*a) - 75*x*tanh(x/2)**2/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*t
anh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 15*x/(40*a*
tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**
4 + 200*a*tanh(x/2)**2 - 40*a) + 30*tanh(x/2)**9/(40*a*tanh(x/2)**10 - 200*
a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)*
*2 - 40*a) - 140*tanh(x/2)**7/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 40
0*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 256*ta
nh(x/2)**5/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 -
400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 140*tanh(x/2)**3/(40*a*ta
nh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4
+ 200*a*tanh(x/2)**2 - 40*a) - 30*tanh(x/2)/(40*a*tanh(x/2)**10 - 200*a*tan
h(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 -
40*a)
```

3.155 $\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$\frac{(a-a\cosh(x))^4}{4a^5}-\frac{2(a-a\cosh(x))^3}{3a^4}$$

```
[Out] -2/3*(a-a*cosh(x))^3/a^4+1/4*(a-a*cosh(x))^4/a^5
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{(a-a\cosh(x))^4}{4a^5}-\frac{2(a-a\cosh(x))^3}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (-2*(a - a*Cosh[x])^3)/(3*a^4) + (a - a*Cosh[x])^4/(4*a^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)\,dx, x, a\cosh(x)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx, x, a\cosh(x)\right)}{a^5}$$
$$= -\frac{2(a-a\cosh(x))^3}{3a^4}+\frac{(a-a\cosh(x))^4}{4a^5}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 0.64

$$\frac{2\sinh^6\left(\frac{x}{2}\right)(3\cosh(x)+5)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (2*(5 + 3*Cosh[x])*Sinh[x/2]^6)/(3*a)
```

fricas [A] time = 0.63, size = 52, normalized size = 1.58

$$\frac{3\cosh(x)^4+3\sinh(x)^4-8\cosh(x)^3+6\left(3\cosh(x)^2-4\cosh(x)-2\right)\sinh(x)^2-12\cosh(x)^2+72\cosh(x)}{96\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(x)^4 + 3*sinh(x)^4 - 8*cosh(x)^3 + 6*(3*cosh(x)^2 - 4*cosh(x)
- 2)*sinh(x)^2 - 12*cosh(x)^2 + 72*cosh(x))/a
```

giac [A] time = 0.13, size = 51, normalized size = 1.55

$$\frac{\left(72\,e^{(3\,x)}-12\,e^{(2\,x)}-8\,e^{x}+3\right)e^{(-4\,x)}+3\,e^{(4\,x)}-8\,e^{(3\,x)}-12\,e^{(2\,x)}+72\,e^{x}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/192*((72*e^(3*x) - 12*e^(2*x) - 8*e^x + 3)*e^(-4*x) + 3*e^(4*x) - 8*e^(3*
x) - 12*e^(2*x) + 72*e^x)/a
```

maple [B] time = 0.08, size = 87, normalized size = 2.64

$$\frac{\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)\right.}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a+a*cosh(x)),x)
```

```
[Out] 32/a*(1/128/(tanh(1/2*x)-1)^4+5/192/(tanh(1/2*x)-1)^3+5/256/(tanh(1/2*x)-1)
^2-5/256/(tanh(1/2*x)-1)+1/128/(tanh(1/2*x)+1)^4-5/192/(tanh(1/2*x)+1)^3+5/
256/(tanh(1/2*x)+1)^2+5/256/(tanh(1/2*x)+1))
```

maxima [A] time = 0.31, size = 60, normalized size = 1.82

$$-\frac{\left(8\,e^{(-x)}+12\,e^{(-2\,x)}-72\,e^{(-3\,x)}-3\right)e^{(4\,x)}}{192\,a}+\frac{72\,e^{(-x)}-12\,e^{(-2\,x)}-8\,e^{(-3\,x)}+3\,e^{(-4\,x)}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*e^(-x) + 12*e^(-2*x) - 72*e^(-3*x) - 3)*e^(4*x)/a + 1/192*(72*e^(
-x) - 12*e^(-2*x) - 8*e^(-3*x) + 3*e^(-4*x))/a
```

mupad [B] time = 0.98, size = 71, normalized size = 2.15

$$\frac{3\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{16\,a}-\frac{\mathrm{e}^{2\,x}}{16\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}+\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}+\frac{3\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a + a*cosh(x)),x)
```

```
[Out] (3*exp(-x))/(8*a) - exp(-2*x)/(16*a) - exp(2*x)/(16*a) - exp(-3*x)/(24*a) -
 exp(3*x)/(24*a) + exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (3*exp(x))/(8*a)
```

sympy [B] time = 2.22, size = 150, normalized size = 4.55

$$\frac{24\tanh^{4}\left(\frac{x}{2}\right)}{3a\tanh^{8}\left(\frac{x}{2}\right)-12a\tanh^{6}\left(\frac{x}{2}\right)+18a\tanh^{4}\left(\frac{x}{2}\right)-12a\tanh^{2}\left(\frac{x}{2}\right)+3a}-\frac{16\tanh^{2}\left(\frac{x}{2}\right)}{3a\tanh^{8}\left(\frac{x}{2}\right)-12a\tanh^{6}\left(\frac{x}{2}\right)+18a\tanh^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5/(a+a*cosh(x)),x)
```

```
[Out] 24*tanh(x/2)**4/(3*a*tanh(x/2)**8 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4 -
 12*a*tanh(x/2)**2 + 3*a) - 16*tanh(x/2)**2/(3*a*tanh(x/2)**8 - 12*a*tanh(x
/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a) + 4/(3*a*tanh(x/2)**8
 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a)
```

3.156 $\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{2a}+\frac{\sinh^3(x)}{3a}-\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] 1/2*x/a-1/2*cosh(x)*sinh(x)/a+1/3*sinh(x)^3/a
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$\frac{x}{2a}+\frac{\sinh^3(x)}{3a}-\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] x/(2*a) - (Cosh[x]*Sinh[x])/(2*a) + Sinh[x]^3/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{\sinh^3(x)}{3a} - \frac{\int \sinh^2(x)\,dx}{a}$$
$$= -\frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a} + \frac{\int 1\,dx}{2a}$$
$$= \frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.81

$$\frac{6x - 3\sinh(x) - 3\sinh(2x) + \sinh(3x)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + a*Cosh[x]),x]
[Out] (6*x - 3*Sinh[x] - 3*Sinh[2*x] + Sinh[3*x])/(12*a)
```

fricas [A] time = 2.02, size = 27, normalized size = 0.87

$$\frac{\sinh(x)^3 + 3\left(\cosh(x)^2 - 2\cosh(x) - 1\right)\sinh(x) + 6x}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
[Out] 1/12*(sinh(x)^3 + 3*(cosh(x)^2 - 2*cosh(x) - 1)*sinh(x) + 6*x)/a
```

giac [A] time = 0.14, size = 40, normalized size = 1.29

$$\frac{\left(3e^{(2x)} + 3e^x - 1\right)e^{(-3x)} + 12x + e^{(3x)} - 3e^{(2x)} - 3e^x}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
[Out] 1/24*((3*e^(2*x) + 3*e^x - 1)*e^(-3*x) + 12*x + e^(3*x) - 3*e^(2*x) - 3*e^x
)/a
```

maple [B] time = 0.08, size = 103, normalized size = 3.32

$$-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+a*cosh(x)),x)
```

```
[Out] -1/3/a/(tanh(1/2*x)-1)^3-1/a/(tanh(1/2*x)-1)^2-1/2/a/(tanh(1/2*x)-1)-1/2/a*
ln(tanh(1/2*x)-1)-1/3/a/(tanh(1/2*x)+1)^3+1/a/(tanh(1/2*x)+1)^2-1/2/a/(tanh
(1/2*x)+1)+1/2/a*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.32, size = 54, normalized size = 1.74

$$-\frac{\left(3\,e^{(-x)}+3\,e^{(-2\,x)}-1\right)e^{(3\,x)}}{24\,a}+\frac{x}{2\,a}+\frac{3\,e^{(-x)}+3\,e^{(-2\,x)}-e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*e^(-x) + 3*e^(-2*x) - 1)*e^(3*x)/a + 1/2*x/a + 1/24*(3*e^(-x) + 3*
e^(-2*x) - e^(-3*x))/a
```

mupad [B] time = 0.94, size = 59, normalized size = 1.90

$$\frac{e^{-x}}{8\,a}+\frac{e^{-2\,x}}{8\,a}-\frac{e^{2\,x}}{8\,a}-\frac{e^{-3\,x}}{24\,a}+\frac{e^{3\,x}}{24\,a}+\frac{x}{2\,a}-\frac{e^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + a*cosh(x)),x)
```

```
[Out] exp(-x)/(8*a) + exp(-2*x)/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + exp(3
*x)/(24*a) + x/(2*a) - exp(x)/(8*a)
```

sympy [B] time = 1.30, size = 294, normalized size = 9.48

$$\frac{3x\tanh^{6}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}-\frac{9x\tanh^{4}\left(\frac{x}{2}\right)}{6a\tanh^{6}\left(\frac{x}{2}\right)-18a\tanh^{4}\left(\frac{x}{2}\right)+18a\tanh^{2}\left(\frac{x}{2}\right)-6a}+\frac{}{6a\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] 3*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
- 6*a) - 9*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh
(x/2)**2 - 6*a) + 9*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 +
18*a*tanh(x/2)**2 - 6*a) - 3*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a
*tanh(x/2)**2 - 6*a) - 6*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) - 16*tanh(x/2)**3/(6*a*tanh(x/2)**6 - 18*a*tanh
(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)/(6*a*tanh(x/2)**6 - 18*a*
tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

3.157 $\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\cosh^2(x)}{2a}-\frac{\cosh(x)}{a}$$

```
[Out] -cosh(x)/a+1/2*cosh(x)^2/a
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2667}

$$\frac{\cosh^2(x)}{2a}-\frac{\cosh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] -(Cosh[x]/a) + Cosh[x]^2/(2*a)
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx &= -\frac{\text{Subst}\left(\int (a-x)\,dx,x,a\cosh(x)\right)}{a^3}\\ &= -\frac{\cosh(x)}{a}+\frac{\cosh^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.68

$$\frac{2\sinh^4\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + a*Cosh[x]),x]
[Out] (2*Sinh[x/2]^4)/a
```

fricas [A] time = 0.73, size = 18, normalized size = 0.95

$$\frac{\cosh(x)^2 + \sinh(x)^2 - 4\cosh(x)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
[Out] 1/4*(cosh(x)^2 + sinh(x)^2 - 4*cosh(x))/a
```

giac [A] time = 0.13, size = 27, normalized size = 1.42

$$-\frac{(4e^x - 1)e^{(-2x)} - e^{(2x)} + 4e^x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
[Out] -1/8*((4*e^x - 1)*e^(-2*x) - e^(2*x) + 4*e^x)/a
```

maple [B] time = 0.07, size = 47, normalized size = 2.47

$$\frac{\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+a*cosh(x)),x)
[Out] 8/a*(1/16/(tanh(1/2*x)-1)^2+3/16/(tanh(1/2*x)-1)+1/16/(tanh(1/2*x)+1)^2-3/1
6/(tanh(1/2*x)+1))
```

maxima [B] time = 0.31, size = 36, normalized size = 1.89

$$-\frac{\left(4e^{(-x)} - 1\right)e^{(2x)}}{8a} - \frac{4e^{(-x)} - e^{(-2x)}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
[Out] -1/8*(4*e^(-x) - 1)*e^(2*x)/a - 1/8*(4*e^(-x) - e^(-2*x))/a
```

mupad [B] time = 0.94, size = 35, normalized size = 1.84

$$\frac{e^{-2x}}{8a} - \frac{e^{-x}}{2a} + \frac{e^{2x}}{8a} - \frac{e^{x}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + a*cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*a) - exp(-x)/(2*a) + exp(2*x)/(8*a) - exp(x)/(2*a)
```

sympy [B] time = 0.73, size = 49, normalized size = 2.58

$$\frac{4\tanh^{2}\left(\frac{x}{2}\right)}{a\tanh^{4}\left(\frac{x}{2}\right) - 2a\tanh^{2}\left(\frac{x}{2}\right) + a} - \frac{2}{a\tanh^{4}\left(\frac{x}{2}\right) - 2a\tanh^{2}\left(\frac{x}{2}\right) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] 4*tanh(x/2)**2/(a*tanh(x/2)**4 - 2*a*tanh(x/2)**2 + a) - 2/(a*tanh(x/2)**4
- 2*a*tanh(x/2)**2 + a)
```

3.158 $\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=13

$$\frac{\sinh(x)}{a}-\frac{x}{a}$$

```
[Out] -x/a+sinh(x)/a
```

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2682, 8}

$$\frac{\sinh(x)}{a}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -(x/a) + Sinh[x]/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx &= \frac{\sinh(x)}{a}-\frac{\int 1\,dx}{a}\\ &= -\frac{x}{a}+\frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.31

$$\frac{2\left(\frac{\sinh(x)}{2}-\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (2*(-1/2*x + Sinh[x]/2))/a
```

fricas [A] time = 1.69, size = 11, normalized size = 0.85

$$-\frac{x-\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(x - sinh(x))/a
```

giac [A] time = 0.12, size = 17, normalized size = 1.31

$$-\frac{2\,x+e^{(-x)}-e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/2*(2*x + e^(-x) - e^x)/a
```

maple [B] time = 0.07, size = 51, normalized size = 3.92

$$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+a*cosh(x)),x)
```

```
[Out] -1/a/(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)-1)-1/a/(tanh(1/2*x)+1)-1/a*ln(tanh(
1/2*x)+1)
```

maxima [A] time = 0.31, size = 23, normalized size = 1.77

$$-\frac{x}{a}-\frac{e^{(-x)}}{2\,a}+\frac{e^{x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -x/a - 1/2*e^(-x)/a + 1/2*e^x/a
```

mupad [B] time = 0.91, size = 23, normalized size = 1.77

$$\frac{e^x}{2\,a}-\frac{x}{a}-\frac{e^{-x}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + a*cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - x/a - exp(-x)/(2*a)
```

sympy [B] time = 0.41, size = 46, normalized size = 3.54

$$-\frac{x\tanh^2\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}+\frac{x}{a\tanh^2\left(\frac{x}{2}\right)-a}-\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right)-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+a*cosh(x)),x)
```

```
[Out] -x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) - 2*tanh(x/2)
/(a*tanh(x/2)**2 - a)
```

3.159 $\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\log(\cosh(x)+1)}{a}$$

[Out] ln(1+cosh(x))/a

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 31}

$$\frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(a + a*Cosh[x]),x]

[Out] Log[1 + Cosh[x]]/a

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cosh(x)\right)}{a}\\ &= \frac{\log(1+\cosh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.33

$$\frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + a*Cosh[x]),x]
[Out] (2*Log[Cosh[x/2]])/a
```

fricas [A] time = 0.78, size = 16, normalized size = 1.78

$$-\frac{x - 2\log\left(\cosh(x) + \sinh(x) + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="fricas")
[Out] -(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

giac [A] time = 0.16, size = 17, normalized size = 1.89

$$-\frac{x}{a} + \frac{2\log\left(e^{x} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="giac")
[Out] -x/a + 2*log(e^x + 1)/a
```

maple [A] time = 0.04, size = 12, normalized size = 1.33

$$\frac{\ln\left(a + a\cosh(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+a*cosh(x)),x)
[Out] ln(a+a*cosh(x))/a
```

maxima [A] time = 0.30, size = 11, normalized size = 1.22

$$\frac{\log\left(a\cosh(x) + a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(a*cosh(x) + a)/a
```

mupad [B] time = 0.89, size = 9, normalized size = 1.00

$$\frac{\ln\left(\cosh(x)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + a*cosh(x)),x)
```

```
[Out] log(cosh(x) + 1)/a
```

sympy [A] time = 0.13, size = 7, normalized size = 0.78

$$\frac{\log\left(\cosh\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+a*cosh(x)),x)
```

```
[Out] log(cosh(x) + 1)/a
```

3.160 $\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2(a\cosh(x)+a)}-\frac{\tanh^{-1}(\cosh(x))}{2a}$$

```
[Out] -1/2*arctanh(cosh(x))/a+1/2/(a+a*cosh(x))
```

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2667, 44, 206}

$$\frac{1}{2(a\cosh(x)+a)}-\frac{\tanh^{-1}(\cosh(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + a*Cosh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/(2*a) + 1/(2*(a + a*Cosh[x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx &= -\left(a\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx,x,a\cosh(x)\right)\right)\\
&= -\left(a\,\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\left(a^2-x^2\right)}\right)dx,x,a\cosh(x)\right)\right)\\
&= \frac{1}{2(a+a\cosh(x))}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,a\cosh(x)\right)\\
&= -\frac{\tanh^{-1}(\cosh(x))}{2a}+\frac{1}{2(a+a\cosh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.83

$$\frac{1-2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + a*Cosh[x]),x]
```

```
[Out] (1 - 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(2*a*(1 + Cosh[x]))
```

fricas [B] time = 1.75, size = 103, normalized size = 4.48

$$-\frac{\left(\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+2\,\cosh(x)+1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(\cosh(x)^2+2\,(\cdots\right.}{2\left(a\cosh(x)^2+a\sinh(x)^2+2\,a\cosh(x)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) - 2*cosh(x) - 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

giac [B] time = 0.15, size = 52, normalized size = 2.26

$$-\frac{\log\left(e^{(-x)}+e^x+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{4\,a}+\frac{e^{(-x)}+e^x+6}{4\,a\left(e^{(-x)}+e^x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x + 6)/(a*(e^(-x) + e^x + 2))
```

maple [A] time = 0.07, size = 23, normalized size = 1.00

$$-\frac{\tanh^2\left(\frac{x}{2}\right)}{4a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+a*cosh(x)),x)
```

```
[Out] -1/4/a*tanh(1/2*x)^2+1/2/a*ln(tanh(1/2*x))
```

maxima [B] time = 0.30, size = 47, normalized size = 2.04

$$\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(-
x) - 1)/a
```

mupad [B] time = 0.93, size = 51, normalized size = 2.22

$$\frac{1}{a\,\left(\mathrm{e}^x+1\right)}-\frac{1}{a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^x+1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + a*cosh(x))),x)
```

```
[Out] 1/(a*(exp(x) + 1)) - 1/(a*(exp(2*x) + 2*exp(x) + 1)) - atan((exp(x)*(-a^2)^
(1/2))/a)/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+a*cosh(x)),x)
[Out] Integral(csch(x)/(cosh(x) + 1), x)/a
```

3.161 $\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=24

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\coth(x)}{3a}$$

```
[Out] -2/3*coth(x)/a+1/3*csch(x)/(a+a*cosh(x))
```

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 3767, 8}

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\coth(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (-2*Coth[x])/(3*a) + Csch[x]/(3*(a + a*Cosh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))} + \frac{2\int \operatorname{csch}^2(x)\,dx}{3a}$$
$$= \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))} - \frac{(2i)\operatorname{Subst}(\int 1\,dx, x, -i\coth(x))}{3a}$$
$$= -\frac{2\coth(x)}{3a} + \frac{\operatorname{csch}(x)}{3(a+a\cosh(x))}$$

Mathematica [A] time = 0.05, size = 30, normalized size = 1.25

$$-\frac{(2\cosh(x)+\cosh(2x))\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] -1/12*((2*Cosh[x] + Cosh[2*x])*Csch[x/2]*Sech[x/2]^3)/a
```

fricas [B] time = 1.62, size = 94, normalized size = 3.92

$$-\frac{4\,(2\,\cosh(x)+2\,\sinh(x)+1)}{3\left(a\cosh(x)^4+a\sinh(x)^4+2\,a\cosh(x)^3+2\,(2\,a\cosh(x)+a)\sinh(x)^3+6\left(a\cosh(x)^2+a\cosh(x)\right)\sinh(x)^2 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*cosh(x) + 2*sinh(x) + 1)/(a*cosh(x)^4 + a*sinh(x)^4 + 2*a*cosh(x)^3
 + 2*(2*a*cosh(x) + a)*sinh(x)^3 + 6*(a*cosh(x)^2 + a*cosh(x))*sinh(x)^2 -
2*a*cosh(x) + 2*(2*a*cosh(x)^3 + 3*a*cosh(x)^2 - a)*sinh(x) - a)
```

giac [A] time = 0.15, size = 35, normalized size = 1.46

$$-\frac{1}{2\,a(e^x-1)} + \frac{3\,e^{(2\,x)}+12\,e^x+5}{6\,a(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 12*e^x + 5)/(a*(e^x + 1)^3)
```

maple [A] time = 0.08, size = 29, normalized size = 1.21

$$\frac{\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-2\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+a*cosh(x)),x)
```

```
[Out] 1/4/a*(1/3*tanh(1/2*x)^3-2*tanh(1/2*x)-1/tanh(1/2*x))
```

maxima [B] time = 0.30, size = 59, normalized size = 2.46

$$-\frac{8\,e^{(-x)}}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}-\frac{4}{3\left(2\,ae^{(-x)}-2\,ae^{(-3\,x)}-ae^{(-4\,x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 4/3/(2*a*e^(-x)
- 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

mupad [B] time = 0.92, size = 89, normalized size = 3.71

$$\frac{\frac{e^{2\,x}}{6\,a}+\frac{1}{6\,a}+\frac{e^{x}}{a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}+\frac{\frac{1}{2\,a}+\frac{e^{x}}{6\,a}}{e^{2\,x}+2\,e^{x}+1}-\frac{1}{2\,a\,\left(e^{x}-1\right)}+\frac{1}{6\,a\,\left(e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + a*cosh(x))),x)
```

```
[Out] (exp(2*x)/(6*a) + 1/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1
) + (1/(2*a) + exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(x) - 1
)) + 1/(6*a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{2}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**2/(cosh(x) + 1), x)/a
```

3.162 $\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=49

$$-\frac{a}{8(a\cosh(x)+a)^2}+\frac{1}{8(a-a\cosh(x))}-\frac{1}{4(a\cosh(x)+a)}+\frac{3\tanh^{-1}(\cosh(x))}{8a}$$

```
[Out] 3/8*arctanh(cosh(x))/a+1/8/(a-a*cosh(x))-1/8*a/(a+a*cosh(x))^2-1/4/(a+a*cos
h(x))
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 206}

$$-\frac{a}{8(a\cosh(x)+a)^2}+\frac{1}{8(a-a\cosh(x))}-\frac{1}{4(a\cosh(x)+a)}+\frac{3\tanh^{-1}(\cosh(x))}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/(8*a) + 1/(8*(a - a*Cosh[x])) - a/(8*(a + a*Cosh[x])^2
) - 1/(4*(a + a*Cosh[x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = a^3\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, a\cosh(x)\right)$$
$$= a^3\operatorname{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right)dx, x, a\cosh(x\right)$$
$$= \frac{1}{8(a-a\cosh(x))}-\frac{a}{8(a+a\cosh(x))^2}-\frac{1}{4(a+a\cosh(x))}+\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\,c\right.$$
$$= \frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{1}{8(a-a\cosh(x))}-\frac{a}{8(a+a\cosh(x))^2}-\frac{1}{4(a+a\cosh(x))}$$

Mathematica [A] time = 0.17, size = 60, normalized size = 1.22

$$-\frac{2\coth^2\left(\frac{x}{2}\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+4}{16a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] -1/16*(4 + 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]])
 + Sech[x/2]^2)/(a*(1 + Cosh[x]))
```

fricas [B] time = 0.87, size = 631, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(6*cosh(x)^5 + 6*(5*cosh(x) + 2)*sinh(x)^4 + 6*sinh(x)^5 + 12*cosh(x)^
4 + 4*(15*cosh(x)^2 + 12*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + 12*(5*cosh(
x)^3 + 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)^2 + 12*cosh(x)^2 - 3*(cosh(x)^6 +
 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10
*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x
) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2
 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2
*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x
) + sinh(x) + 1) + 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 +
 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5
```

```
*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(
x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 +
 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*si
nh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(5*cosh(x)^4 + 8*cosh
(x)^3 - 2*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 6*cosh(x))/(a*cosh(x)^6 + a*
sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4 + (
15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*cosh(
x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*cosh(
x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*a*c
osh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^2 -
 a*cosh(x) + a)*sinh(x) + a)
```

giac [B] time = 0.18, size = 94, normalized size = 1.92

$$\frac{3\log\left(e^{(-x)}+e^{x}+2\right)}{16a}-\frac{3\log\left(e^{(-x)}+e^{x}-2\right)}{16a}+\frac{3e^{(-x)}+3e^{x}-10}{16a\left(e^{(-x)}+e^{x}-2\right)}-\frac{9\left(e^{(-x)}+e^{x}\right)^{2}+52e^{(-x)}+52e^{x}+84}{32a\left(e^{(-x)}+e^{x}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/16*log(e^(-x) + e^x + 2)/a - 3/16*log(e^(-x) + e^x - 2)/a + 1/16*(3*e^(-x
) + 3*e^x - 10)/(a*(e^(-x) + e^x - 2)) - 1/32*(9*(e^(-x) + e^x)^2 + 52*e^(-
x) + 52*e^x + 84)/(a*(e^(-x) + e^x + 2)^2)
```

maple [A] time = 0.09, size = 45, normalized size = 0.92

$$-\frac{\tanh^{4}\left(\frac{x}{2}\right)}{32a}+\frac{3\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{16a}-\frac{1}{16a\tanh\left(\frac{x}{2}\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+a*cosh(x)),x)
```

```
[Out] -1/32/a*tanh(1/2*x)^4+3/16/a*tanh(1/2*x)^2-1/16/a/tanh(1/2*x)^2-3/8/a*ln(ta
nh(1/2*x))
```

maxima [B] time = 0.31, size = 103, normalized size = 2.10

$$-\frac{3e^{(-x)}+6e^{(-2x)}-2e^{(-3x)}+6e^{(-4x)}+3e^{(-5x)}}{4\left(2ae^{(-x)}-ae^{(-2x)}-4ae^{(-3x)}-ae^{(-4x)}+2ae^{(-5x)}+ae^{(-6x)}+a\right)}+\frac{3\log\left(e^{(-x)}+1\right)}{8a}-\frac{3\log\left(e^{(-x)}-1\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(3*e^(-x) + 6*e^(-2*x) - 2*e^(-3*x) + 6*e^(-4*x) + 3*e^(-5*x))/(2*a*e^
(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) +
 a) + 3/8*log(e^(-x) + 1)/a - 3/8*log(e^(-x) - 1)/a
```

mupad [B] time = 0.93, size = 114, normalized size = 2.33

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}-\frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{x}-1\right)}-\frac{1}{2\,a\,\left(\mathrm{e}^{x}+1\right)}-\frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)}+\frac{}{a\,\left(3\,\mathrm{e}^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + a*cosh(x))),x)
```

```
[Out] (3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) - 1/(2*a*(6*exp(2*x) + 4
*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) - 1)) - 1/(2*a*(exp(
x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(a*(3*exp(2*x) + exp(3*x)
+ 3*exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{3}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**3/(cosh(x) + 1), x)/a
```

3.163 $\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{4\coth^3(x)}{15a}+\frac{4\coth(x)}{5a}+\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}$$

[Out] 4/5*coth(x)/a-4/15*coth(x)^3/a+1/5*csch(x)^3/(a+a*cosh(x))

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2672, 3767}

$$-\frac{4\coth^3(x)}{15a}+\frac{4\coth(x)}{5a}+\frac{\operatorname{csch}^3(x)}{5(a\cosh(x)+a)}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^4/(a + a*Cosh[x]),x]

[Out] (4*Coth[x])/(5*a) - (4*Coth[x]^3)/(15*a) + Csch[x]^3/(5*(a + a*Cosh[x]))

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))} + \frac{4\int \operatorname{csch}^4(x)\,dx}{5a}$$
$$= \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))} + \frac{(4i)\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\coth(x)\right)}{5a}$$
$$= \frac{4\coth(x)}{5a} - \frac{4\coth^3(x)}{15a} + \frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}$$

Mathematica [A] time = 0.06, size = 38, normalized size = 1.03

$$\frac{(-6\cosh(x) - 2\cosh(2x) + 2\cosh(3x) + \cosh(4x))\operatorname{csch}^3(x)}{15a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] ((-6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(15*a*(1 +
 Cosh[x]))
```

fricas [B] time = 1.09, size = 250, normalized size = 6.76

$$-\frac{}{15\left(a\cosh(x)^7 + a\sinh(x)^7 + 2a\cosh(x)^6 + (7a\cosh(x) + 2a)\sinh(x)^6 - 2a\cosh(x)^5 + \left(21a\cosh(x)^2 + 12\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -16/15*(6*cosh(x)^2 + 3*(4*cosh(x) + 1)*sinh(x) + 6*sinh(x)^2 + cosh(x) - 2
)/(a*cosh(x)^7 + a*sinh(x)^7 + 2*a*cosh(x)^6 + (7*a*cosh(x) + 2*a)*sinh(x)^
6 - 2*a*cosh(x)^5 + (21*a*cosh(x)^2 + 12*a*cosh(x) - 2*a)*sinh(x)^5 - 6*a*c
osh(x)^4 + (35*a*cosh(x)^3 + 30*a*cosh(x)^2 - 10*a*cosh(x) - 6*a)*sinh(x)^4
 + (35*a*cosh(x)^4 + 40*a*cosh(x)^3 - 20*a*cosh(x)^2 - 24*a*cosh(x))*sinh(x
)^3 + 6*a*cosh(x)^2 + (21*a*cosh(x)^5 + 30*a*cosh(x)^4 - 20*a*cosh(x)^3 - 3
6*a*cosh(x)^2 + 6*a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 12*a*cosh(x)^
5 - 10*a*cosh(x)^4 - 24*a*cosh(x)^3 + 12*a*cosh(x) + 3*a)*sinh(x) - 2*a)
```

giac [A] time = 0.15, size = 59, normalized size = 1.59

$$\frac{9e^{(2x)} - 24e^x + 11}{24a(e^x-1)^3} - \frac{45e^{(4x)} + 240e^{(3x)} + 490e^{(2x)} + 320e^x + 73}{120a(e^x+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(9*e^(2*x) - 24*e^x + 11)/(a*(e^x - 1)^3) - 1/120*(45*e^(4*x) + 240*e^
(3*x) + 490*e^(2*x) + 320*e^x + 73)/(a*(e^x + 1)^5)
```

maple [A] time = 0.09, size = 45, normalized size = 1.22

$$\frac{\frac{\left(\tanh^5\left(\frac{x}{2}\right)\right)}{5}-\frac{4\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}+6\tanh\left(\frac{x}{2}\right)-\frac{1}{3\tanh\left(\frac{x}{2}\right)^3}+\frac{4}{\tanh\left(\frac{x}{2}\right)}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/16/a*(1/5*tanh(1/2*x)^5-4/3*tanh(1/2*x)^3+6*tanh(1/2*x)-1/3/tanh(1/2*x)^3
+4/tanh(1/2*x))
```

maxima [B] time = 0.31, size = 233, normalized size = 6.30

$$\frac{32\,e^{(-x)}}{15\left(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-3\,x)}+6\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-2\,ae^{(-7\,x)}-ae^{(-8\,x)}+a\right)}-\frac{}{15\left(2\,ae^{(-x)}-2\,ae^{(-2\,x)}-6\,ae^{(-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 32/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a
*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 32/15*e^(-2*x)/(2*a*e^(-x) - 2
*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a
*e^(-8*x) + a) - 32/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) +
6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 16/15/(2*a*e
^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(
-7*x) - a*e^(-8*x) + a)
```

mupad [B] time = 0.95, size = 263, normalized size = 7.11

$$\frac{1}{6\,a\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}-\frac{\frac{3\,e^{2\,x}}{8\,a}+\frac{3\,e^{3\,x}}{40\,a}+\frac{1}{8\,a}+\frac{5\,e^{x}}{8\,a}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}-\frac{\frac{3\,e^{2\,x}}{40\,a}+\frac{5}{24\,a}+\frac{e^{x}}{4\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{\frac{1}{8\,a}+\frac{3\,e^{x}}{40\,a}}{e^{2\,x}+2\,e^{x}+1}-\frac{\frac{5\,e^{2\,x}}{4\,a}+}{10\,e^{2\,x}+10\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + a*cosh(x))),x)
```

```
[Out] 1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (3*e
xp(3*x))/(40*a) + 1/(8*a) + (5*exp(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) + ex
p(4*x) + 4*exp(x) + 1) - ((3*exp(2*x))/(40*a) + 5/(24*a) + exp(x)/(4*a))/(3
*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (3*exp(x))/(40*a))/(exp(2
*x) + 2*exp(x) + 1) - ((5*exp(2*x))/(4*a) + exp(3*x)/(2*a) + (3*exp(4*x))/(
40*a) + 3/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) +
exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 3/(8*a*(exp(
x) - 1)) - 3/(40*a*(exp(x) + 1))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{4}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**4/(cosh(x) + 1), x)/a
```

3.164 $\int \frac{\mathrm{csch}^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=78

$$\frac{a^2}{24(a\cosh(x)+a)^3}-\frac{a}{32(a-a\cosh(x))^2}+\frac{3a}{32(a\cosh(x)+a)^2}-\frac{1}{8(a-a\cosh(x))}+\frac{3}{16(a\cosh(x)+a)}-\frac{5\tanh^{-1}(\cos}{16a}$$

```
[Out] -5/16*arctanh(cosh(x))/a-1/32*a/(a-a*cosh(x))^2-1/8/(a-a*cosh(x))+1/24*a^2/
(a+a*cosh(x))^3+3/32*a/(a+a*cosh(x))^2+3/16/(a+a*cosh(x))
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 206}

$$\frac{a^2}{24(a\cosh(x)+a)^3}-\frac{a}{32(a-a\cosh(x))^2}+\frac{3a}{32(a\cosh(x)+a)^2}-\frac{1}{8(a-a\cosh(x))}+\frac{3}{16(a\cosh(x)+a)}-\frac{5\tanh^{-1}(\cos}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (-5*ArcTanh[Cosh[x]])/(16*a) - a/(32*(a - a*Cosh[x])^2) - 1/(8*(a - a*Cosh[
x])) + a^2/(24*(a + a*Cosh[x])^3) + (3*a)/(32*(a + a*Cosh[x])^2) + 3/(16*(a
 + a*Cosh[x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = -\left(a^5\operatorname{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, a\cosh(x)\right)\right)$$
$$= -\left(a^5\operatorname{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3}+\frac{1}{8a^5(a-x)^2}+\frac{1}{8a^3(a+x)^4}+\frac{3}{16a^4(a+x)^3}+\frac{3}{16a^5(a+x\right.\right.\right.$$
$$= -\frac{a}{32(a-a\cosh(x))^2}-\frac{1}{8(a-a\cosh(x))}+\frac{a^2}{24(a+a\cosh(x))^3}+\frac{3a}{32(a+a\cosh(x))^2}+\frac{}{16}$$
$$= -\frac{5\tanh^{-1}(\cosh(x))}{16a}-\frac{a}{32(a-a\cosh(x))^2}-\frac{1}{8(a-a\cosh(x))}+\frac{a^2}{24(a+a\cosh(x))^3}+\frac{}{32}$$

Mathematica [A] time = 0.29, size = 89, normalized size = 1.14

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(-3\operatorname{csch}^4\left(\frac{x}{2}\right)+24\operatorname{csch}^2\left(\frac{x}{2}\right)+2\operatorname{sech}^6\left(\frac{x}{2}\right)+9\operatorname{sech}^4\left(\frac{x}{2}\right)+36\operatorname{sech}^2\left(\frac{x}{2}\right)+120\log\left(\sinh\left(\frac{x}{2}\right)\right)-120\log\right.}{192(a\cosh(x)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(24*Csch[x/2]^2 - 3*Csch[x/2]^4 - 120*Log[Cosh[x/2]] + 120*Log
[Sinh[x/2]] + 36*Sech[x/2]^2 + 9*Sech[x/2]^4 + 2*Sech[x/2]^6))/(192*(a + a*
Cosh[x]))
```

fricas [B] time = 0.83, size = 1551, normalized size = 19.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/48*(30*cosh(x)^9 + 30*(9*cosh(x) + 2)*sinh(x)^8 + 30*sinh(x)^9 + 60*cosh(
x)^8 + 40*(27*cosh(x)^2 + 12*cosh(x) - 2)*sinh(x)^7 - 80*cosh(x)^7 + 20*(12
6*cosh(x)^3 + 84*cosh(x)^2 - 28*cosh(x) - 11)*sinh(x)^6 - 220*cosh(x)^6 + 1
2*(315*cosh(x)^4 + 280*cosh(x)^3 - 140*cosh(x)^2 - 110*cosh(x) + 3)*sinh(x)
^5 + 36*cosh(x)^5 + 20*(189*cosh(x)^5 + 210*cosh(x)^4 - 140*cosh(x)^3 - 165
*cosh(x)^2 + 9*cosh(x) - 11)*sinh(x)^4 - 220*cosh(x)^4 + 40*(63*cosh(x)^6 +
 84*cosh(x)^5 - 70*cosh(x)^4 - 110*cosh(x)^3 + 9*cosh(x)^2 - 22*cosh(x) - 2
```

```
)*sinh(x)^3 - 80*cosh(x)^3 + 60*(18*cosh(x)^7 + 28*cosh(x)^6 - 28*cosh(x)^5
 - 55*cosh(x)^4 + 6*cosh(x)^3 - 22*cosh(x)^2 - 4*cosh(x) + 1)*sinh(x)^2 + 6
0*cosh(x)^2 - 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9 + sinh(x)^10 + 2
*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 - 3*cosh(x)^8 + 8*(
15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*cosh(x)^7 + 2*(10
5*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1)*sinh(x)^6 + 2*c
osh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3 - 14*cosh(x)^2 +
cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 + 126*cosh(x)^5 -
105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x) + 1)*sinh(x)^4 +
2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^5 - 35*cosh(x)^4
+ 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8*cosh(x)^3 + 3*(15
*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5 + 10*cosh(x)^4 + 40
*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 2*(5*co
sh(x)^9 + 9*cosh(x)^8 - 12*cosh(x)^7 - 28*cosh(x)^6 + 6*cosh(x)^5 + 30*cosh
(x)^4 + 4*cosh(x)^3 - 12*cosh(x)^2 - 3*cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1
)*log(cosh(x) + sinh(x) + 1) + 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9
 + sinh(x)^10 + 2*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 -
3*cosh(x)^8 + 8*(15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*
cosh(x)^7 + 2*(105*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1
)*sinh(x)^6 + 2*cosh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3
- 14*cosh(x)^2 + cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 +
 126*cosh(x)^5 - 105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x)
+ 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^
5 - 35*cosh(x)^4 + 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8*
cosh(x)^3 + 3*(15*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5 +
10*cosh(x)^4 + 40*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 - 3*co
sh(x)^2 + 2*(5*cosh(x)^9 + 9*cosh(x)^8 - 12*cosh(x)^7 - 28*cosh(x)^6 + 6*co
sh(x)^5 + 30*cosh(x)^4 + 4*cosh(x)^3 - 12*cosh(x)^2 - 3*cosh(x) + 1)*sinh(x
) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 10*(27*cosh(x)^8 + 48*cosh(
x)^7 - 56*cosh(x)^6 - 132*cosh(x)^5 + 18*cosh(x)^4 - 88*cosh(x)^3 - 24*cosh
(x)^2 + 12*cosh(x) + 3)*sinh(x) + 30*cosh(x))/(a*cosh(x)^10 + a*sinh(x)^10
+ 2*a*cosh(x)^9 + 2*(5*a*cosh(x) + a)*sinh(x)^9 - 3*a*cosh(x)^8 + 3*(15*a*c
osh(x)^2 + 6*a*cosh(x) - a)*sinh(x)^8 - 8*a*cosh(x)^7 + 8*(15*a*cosh(x)^3 +
 9*a*cosh(x)^2 - 3*a*cosh(x) - a)*sinh(x)^7 + 2*a*cosh(x)^6 + 2*(105*a*cosh
(x)^4 + 84*a*cosh(x)^3 - 42*a*cosh(x)^2 - 28*a*cosh(x) + a)*sinh(x)^6 + 12*
a*cosh(x)^5 + 12*(21*a*cosh(x)^5 + 21*a*cosh(x)^4 - 14*a*cosh(x)^3 - 14*a*c
osh(x)^2 + a*cosh(x) + a)*sinh(x)^5 + 2*a*cosh(x)^4 + 2*(105*a*cosh(x)^6 +
126*a*cosh(x)^5 - 105*a*cosh(x)^4 - 140*a*cosh(x)^3 + 15*a*cosh(x)^2 + 30*a
*cosh(x) + a)*sinh(x)^4 - 8*a*cosh(x)^3 + 8*(15*a*cosh(x)^7 + 21*a*cosh(x)^
6 - 21*a*cosh(x)^5 - 35*a*cosh(x)^4 + 5*a*cosh(x)^3 + 15*a*cosh(x)^2 + a*co
sh(x) - a)*sinh(x)^3 - 3*a*cosh(x)^2 + 3*(15*a*cosh(x)^8 + 24*a*cosh(x)^7 -
 28*a*cosh(x)^6 - 56*a*cosh(x)^5 + 10*a*cosh(x)^4 + 40*a*cosh(x)^3 + 4*a*co
sh(x)^2 - 8*a*cosh(x) - a)*sinh(x)^2 + 2*a*cosh(x) + 2*(5*a*cosh(x)^9 + 9*a
*cosh(x)^8 - 12*a*cosh(x)^7 - 28*a*cosh(x)^6 + 6*a*cosh(x)^5 + 30*a*cosh(x)
^4 + 4*a*cosh(x)^3 - 12*a*cosh(x)^2 - 3*a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.14, size = 116, normalized size = 1.49

$$-\frac{5\,\log\left(e^{(-x)}+e^{x}+2\right)}{32\,a}+\frac{5\,\log\left(e^{(-x)}+e^{x}-2\right)}{32\,a}-\frac{15\left(e^{(-x)}+e^{x}\right)^{2}-76\,e^{(-x)}-76\,e^{x}+100}{64\,a\left(e^{(-x)}+e^{x}-2\right)^{2}}+\frac{55\left(e^{(-x)}+e^{x}\right)^{3}+402\left(\right.}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -5/32*log(e^(-x) + e^x + 2)/a + 5/32*log(e^(-x) + e^x - 2)/a - 1/64*(15*(e^
(-x) + e^x)^2 - 76*e^(-x) - 76*e^x + 100)/(a*(e^(-x) + e^x - 2)^2) + 1/192*
(55*(e^(-x) + e^x)^3 + 402*(e^(-x) + e^x)^2 + 1020*e^(-x) + 1020*e^x + 936)
/(a*(e^(-x) + e^x + 2)^3)
```

maple [A] time = 0.10, size = 67, normalized size = 0.86

$$-\frac{\tanh^{6}\left(\frac{x}{2}\right)}{192a}+\frac{5\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{128a}-\frac{5\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{32a}-\frac{1}{128a\tanh\left(\frac{x}{2}\right)^{4}}+\frac{5}{64a\tanh\left(\frac{x}{2}\right)^{2}}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+a*cosh(x)),x)
```

```
[Out] -1/192/a*tanh(1/2*x)^6+5/128/a*tanh(1/2*x)^4-5/32/a*tanh(1/2*x)^2-1/128/a/t
anh(1/2*x)^4+5/64/a/tanh(1/2*x)^2+5/16/a*ln(tanh(1/2*x))
```

maxima [B] time = 0.33, size = 155, normalized size = 1.99

$$\frac{15\,e^{(-x)}+30\,e^{(-2\,x)}-40\,e^{(-3\,x)}-110\,e^{(-4\,x)}+18\,e^{(-5\,x)}-110\,e^{(-6\,x)}-40\,e^{(-7\,x)}+30\,e^{(-8\,x)}+15\,e^{(-9\,x)}}{24\left(2\,ae^{(-x)}-3\,ae^{(-2\,x)}-8\,ae^{(-3\,x)}+2\,ae^{(-4\,x)}+12\,ae^{(-5\,x)}+2\,ae^{(-6\,x)}-8\,ae^{(-7\,x)}-3\,ae^{(-8\,x)}+2\,ae^{(-9\,x)}+ae^{(-10\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/24*(15*e^(-x) + 30*e^(-2*x) - 40*e^(-3*x) - 110*e^(-4*x) + 18*e^(-5*x) -
110*e^(-6*x) - 40*e^(-7*x) + 30*e^(-8*x) + 15*e^(-9*x))/(2*a*e^(-x) - 3*a*e
^(-2*x) - 8*a*e^(-3*x) + 2*a*e^(-4*x) + 12*a*e^(-5*x) + 2*a*e^(-6*x) - 8*a*
e^(-7*x) - 3*a*e^(-8*x) + 2*a*e^(-9*x) + a*e^(-10*x) + a) - 5/16*log(e^(-x)
 + 1)/a + 5/16*log(e^(-x) - 1)/a
```

mupad [B] time = 1.05, size = 244, normalized size = 3.13

$$\frac{1}{a\left(10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^{x}+1\right)}+\frac{1}{4\,a\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}+\frac{1}{8\,a\left(e^{2\,x}-2\,e^{x}+1\right)}-\frac{}{8\,a\left(6\,e^{2\,x}-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^5*(a + a*cosh(x))),x)
```

```
[Out] 1/(a*(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1)) +
1/(4*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) + 1/(8*a*(exp(2*x) - 2*exp(x
) + 1)) - 1/(8*a*(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1)) - 5/(
8*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) + 1/(4*a*(exp(x) -
 1)) + 3/(8*a*(exp(x) + 1)) - (5*atan((exp(x)*(-a^2)^(1/2))/a))/(8*(-a^2)^(
1/2)) - 1/(3*a*(15*exp(2*x) + 20*exp(3*x) + 15*exp(4*x) + 6*exp(5*x) + exp(
6*x) + 6*exp(x) + 1)) - 5/(12*a*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{csch}^{5}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+a*cosh(x)),x)
```

```
[Out] Integral(csch(x)**5/(cosh(x) + 1), x)/a
```

3.165 $\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=140

$$\frac{\left(a^2-b^2\right)^3\log(a+b\cosh(x))}{b^7}-\frac{a\left(a^2-3b^2\right)\cosh^3(x)}{3b^4}+\frac{\left(a^2-3b^2\right)\cosh^4(x)}{4b^3}-\frac{a\left(a^4-3a^2b^2+3b^4\right)\cosh(x)}{b^6}+\frac{\left(a^4-\right.}{}$$

```
[Out] -a*(a^4-3*a^2*b^2+3*b^4)*cosh(x)/b^6+1/2*(a^4-3*a^2*b^2+3*b^4)*cosh(x)^2/b^
5-1/3*a*(a^2-3*b^2)*cosh(x)^3/b^4+1/4*(a^2-3*b^2)*cosh(x)^4/b^3-1/5*a*cosh(
x)^5/b^2+1/6*cosh(x)^6/b+(a^2-b^2)^3*ln(a+b*cosh(x))/b^7
```

Rubi [A] time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{\left(a^2-3b^2\right)\cosh^4(x)}{4b^3}-\frac{a\left(a^2-3b^2\right)\cosh^3(x)}{3b^4}+\frac{\left(-3a^2b^2+a^4+3b^4\right)\cosh^2(x)}{2b^5}-\frac{a\left(-3a^2b^2+a^4+3b^4\right)\cosh(x)}{b^6}+\frac{\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^7/(a + b*Cosh[x]),x]
```

```
[Out] -((a*(a^4 - 3*a^2*b^2 + 3*b^4)*Cosh[x])/b^6) + ((a^4 - 3*a^2*b^2 + 3*b^4)*C
osh[x]^2)/(2*b^5) - (a*(a^2 - 3*b^2)*Cosh[x]^3)/(3*b^4) + ((a^2 - 3*b^2)*Co
sh[x]^4)/(4*b^3) - (a*Cosh[x]^5)/(5*b^2) + Cosh[x]^6/(6*b) + ((a^2 - b^2)^3
*Log[a + b*Cosh[x]])/b^7
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^3}{a+x}\,dx, x, b\cosh(x)\right)}{b^7}$$

$$= -\frac{\text{Subst}\left(\int \left(a^5\left(1+\frac{3b^2\left(-a^2+b^2\right)}{a^4}\right)-\left(a^4-3a^2b^2+3b^4\right)x+a\left(a^2-3b^2\right)x^2-\left(a^2-3b^2\right)x^3+\right.\right.}{b^7}$$

$$= -\frac{a\left(a^4-3a^2b^2+3b^4\right)\cosh(x)}{b^6}+\frac{\left(a^4-3a^2b^2+3b^4\right)\cosh^2(x)}{2b^5}-\frac{a\left(a^2-3b^2\right)\cosh^3(x)}{3b^4}+$$

Mathematica [A] time = 0.19, size = 144, normalized size = 1.03

$$\frac{960\left(a^2-b^2\right)^3\log(a+b\cosh(x))-30b^4\left(2b^2-a^2\right)\cosh(4x)+15b^2\left(16a^4-40a^2b^2+29b^4\right)\cosh(2x)-120ab\left(8\right.}{960b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^7/(a + b*Cosh[x]),x]
```

```
[Out] (-120*a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cosh[x] + 15*b^2*(16*a^4 - 40*a^2*b
^2 + 29*b^4)*Cosh[2*x] - 20*a*(2*a - 3*b)*b^3*(2*a + 3*b)*Cosh[3*x] - 30*b^
4*(-a^2 + 2*b^2)*Cosh[4*x] - 12*a*b^5*Cosh[5*x] + 5*b^6*Cosh[6*x] + 960*(a^
2 - b^2)^3*Log[a + b*Cosh[x]])/(960*b^7)
```

fricas [B] time = 0.84, size = 2134, normalized size = 15.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*b
^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^10 + 6*(55*b^
6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 - 10*b^6)*sinh(x)^10 - 20*(4*a^3
*b^3 - 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 - 4*a
^3*b^3 + 9*a*b^5 + 15*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^4*b^2
 - 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5*cosh(
x)^3 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 - 2*b^6)*cosh(x)^2 -
12*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*x*cosh(x)^6 - 120*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^7 +
 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b + 22*a^3*b^3 - 19*a*b
```

```
^5 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^2 + (
16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 - 12*a*b^5*cosh(x) + 1
2*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 - 2*b^6)*cosh(x)^
4 - 140*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 - 40*a^2*b^4 + 29*
b^6)*cosh(x)^2 - 160*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 70*(8*a^5*b -
22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 - 120*(8*a^5*b - 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b^5*cosh(x)^6 - 40
*a^5*b + 110*a^3*b^3 - 95*a*b^5 + 315*(a^2*b^4 - 2*b^6)*cosh(x)^5 - 105*(4*
a^3*b^3 - 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x
)^3 - 480*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x) - 105*(8*a^5*b - 22
*a^3*b^3 + 19*a*b^5)*cosh(x)^2)*sinh(x)^5 + 15*(16*a^4*b^2 - 40*a^2*b^4 + 2
9*b^6)*cosh(x)^4 + 15*(165*b^6*cosh(x)^8 - 264*a*b^5*cosh(x)^7 + 420*(a^2*b
^4 - 2*b^6)*cosh(x)^6 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 - 168*(4*a^3*b^3 -
 9*a*b^5)*cosh(x)^5 + 70*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^4 - 192
0*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 - 280*(8*a^5*b - 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^3 - 40*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x))*si
nh(x)^4 - 20*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 20*(55*b^6*cosh(x)^9 - 99*a*
b^5*cosh(x)^8 + 180*(a^2*b^4 - 2*b^6)*cosh(x)^7 - 84*(4*a^3*b^3 - 9*a*b^5)*
cosh(x)^6 - 4*a^3*b^3 + 9*a*b^5 + 42*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cos
h(x)^5 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 210*(8*a^5*
b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 - 60*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)
*cosh(x)^2 + 3*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^3 + 30*(
a^2*b^4 - 2*b^6)*cosh(x)^2 + 30*(11*b^6*cosh(x)^10 - 22*a*b^5*cosh(x)^9 + 4
5*(a^2*b^4 - 2*b^6)*cosh(x)^8 - 24*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^7 + 14*(16
*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^6 + a^2*b^4 - 2*b^6 - 960*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^4 - 84*(8*a^5*b - 22*a^3*b^3 + 19*a*b^
5)*cosh(x)^5 - 40*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 + 3*(16*a^4*b
^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^2 - 2*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh
(x)^2 + 1920*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5*sinh(x) + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^4*sinh(x)^2 + 20*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(
x)^3*sinh(x)^3 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^4
 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*sinh(x)^6)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x)))
 + 12*(5*b^6*cosh(x)^11 - 11*a*b^5*cosh(x)^10 + 25*(a^2*b^4 - 2*b^6)*cosh(x
)^9 - 15*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^8 + 10*(16*a^4*b^2 - 40*a^2*b^4 + 29
*b^6)*cosh(x)^7 - 960*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 - 70*
(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^6 - a*b^5 - 50*(8*a^5*b - 22*a^3*
b^3 + 19*a*b^5)*cosh(x)^4 + 5*(16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^3
- 5*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^2 + 5*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x))
/(b^7*cosh(x)^6 + 6*b^7*cosh(x)^5*sinh(x) + 15*b^7*cosh(x)^4*sinh(x)^2 + 20
*b^7*cosh(x)^3*sinh(x)^3 + 15*b^7*cosh(x)^2*sinh(x)^4 + 6*b^7*cosh(x)*sinh(
x)^5 + b^7*sinh(x)^6)
```

giac [A] time = 0.17, size = 229, normalized size = 1.64

$$\frac{5\,b^5\left(e^{(-x)}+e^x\right)^6-12\,ab^4\left(e^{(-x)}+e^x\right)^5+30\,a^2b^3\left(e^{(-x)}+e^x\right)^4-90\,b^5\left(e^{(-x)}+e^x\right)^4-80\,a^3b^2\left(e^{(-x)}+e^x\right)^3+240\,ab^4\left(e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^5*(e^(-x) + e^x)^6 - 12*a*b^4*(e^(-x) + e^x)^5 + 30*a^2*b^3*(e^
(-x) + e^x)^4 - 90*b^5*(e^(-x) + e^x)^4 - 80*a^3*b^2*(e^(-x) + e^x)^3 + 240
*a*b^4*(e^(-x) + e^x)^3 + 240*a^4*b*(e^(-x) + e^x)^2 - 720*a^2*b^3*(e^(-x)
+ e^x)^2 + 720*b^5*(e^(-x) + e^x)^2 - 960*a^5*(e^(-x) + e^x) + 2880*a^3*b^2
*(e^(-x) + e^x) - 2880*a*b^4*(e^(-x) + e^x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*log(abs(b*(e^(-x) + e^x) + 2*a))/b^7
```

maple [B] time = 0.08, size = 1039, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a+b*cosh(x)),x)
```

```
[Out] 1/6/b/(tanh(1/2*x)-1)^6+1/6/b/(tanh(1/2*x)+1)^6+1/2/b/(tanh(1/2*x)-1)^5+1/8
/b/(tanh(1/2*x)-1)^4-1/2/b/(tanh(1/2*x)+1)^5+1/8/b/(tanh(1/2*x)+1)^4+1/b^7/
(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^7-1/b^6/(a-b)*ln(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-a-b)*a^6-3/b^5/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2
*b-a-b)*a^5+3/b^4/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^4+3/b^3/(
a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^3-3/b^2/(a-b)*ln(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-a-b)*a^2-1/b/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-
a-b)*a-7/12/b/(tanh(1/2*x)-1)^3+5/16/b/(tanh(1/2*x)-1)^2+11/16/b/(tanh(1/2*
x)-1)+7/12/b/(tanh(1/2*x)+1)^3+5/16/b/(tanh(1/2*x)+1)^2-11/16/b/(tanh(1/2*x
)+1)+1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)-3/b^3*ln(tanh(1/2*x)-1)*a^
2-3/b^3*ln(tanh(1/2*x)+1)*a^2-1/b^6/(tanh(1/2*x)+1)*a^5-1/2/b^5/(tanh(1/2*x
)+1)*a^4+5/2/b^4/(tanh(1/2*x)+1)*a^3-1/5/b^2/(tanh(1/2*x)+1)^5*a+1/4/b^3/(t
anh(1/2*x)+1)^4*a^2+1/2/b^2/(tanh(1/2*x)+1)^4*a-1/3/b^4/(tanh(1/2*x)+1)^3*a
^3-1/2/b^3/(tanh(1/2*x)+1)^3*a^2+1/4/b^2/(tanh(1/2*x)+1)^3*a-1/b^7*ln(tanh(
1/2*x)+1)*a^6+3/b^5*ln(tanh(1/2*x)+1)*a^4+1/2/b^5/(tanh(1/2*x)+1)^2*a^4+1/2
/b^4/(tanh(1/2*x)+1)^2*a^3-7/8/b^3/(tanh(1/2*x)+1)^2*a^2+1/3/b^4/(tanh(1/2*
x)-1)^3*a^3+1/2/b^3/(tanh(1/2*x)-1)^3*a^2-1/4/b^2/(tanh(1/2*x)-1)^3*a-1/b^7
*ln(tanh(1/2*x)-1)*a^6+3/b^5*ln(tanh(1/2*x)-1)*a^4+1/2/b^5/(tanh(1/2*x)-1)^
2*a^4+1/2/b^4/(tanh(1/2*x)-1)^2*a^3-7/8/b^3/(tanh(1/2*x)-1)^2*a^2+1/b^6/(ta
nh(1/2*x)-1)*a^5+1/2/b^5/(tanh(1/2*x)-1)*a^4-5/2/b^4/(tanh(1/2*x)-1)*a^3+1/
5/b^2/(tanh(1/2*x)-1)^5*a+1/4/b^3/(tanh(1/2*x)-1)^4*a^2+1/2/b^2/(tanh(1/2*x
)-1)^4*a+1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)-15/8/b^2/(tanh(1/2
```

```
*x)+1)*a-7/8/b^2/(tanh(1/2*x)-1)^2*a-9/8/b^3/(tanh(1/2*x)-1)*a^2+15/8/b^2/(
tanh(1/2*x)-1)*a-7/8/b^2/(tanh(1/2*x)+1)^2*a+9/8/b^3/(tanh(1/2*x)+1)*a^2
```

maxima [B] time = 0.32, size = 310, normalized size = 2.21

$$-\frac{\left(12\,ab^4e^{(-x)}-5\,b^5-30\left(a^2b^3-2\,b^5\right)e^{(-2\,x)}+20\left(4\,a^3b^2-9\,ab^4\right)e^{(-3\,x)}-15\left(16\,a^4b-40\,a^2b^3+29\,b^5\right)e^{(-4\,x)}+12}{1920\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 - 2*b^5)*e^(-2*x) + 20*(4*a^
3*b^2 - 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-4*x) +
120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 - 1/1920*(12*a*b^
4*e^(-5*x) - 5*b^5*e^(-6*x) + 120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-x) -
15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 - 9*a*b^4)*e^(
-3*x) - 30*(a^2*b^3 - 2*b^5)*e^(-4*x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*x/b^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(2*a*e^(-x) + b*e^(-2*
x) + b)/b^7
```

mupad [B] time = 1.71, size = 289, normalized size = 2.06

$$\frac{e^{-6\,x}}{384\,b}+\frac{e^{6\,x}}{384\,b}-\frac{x\left(a^2-b^2\right)^3}{b^7}-\frac{e^{-x}\left(8\,a^5-22\,a^3\,b^2+19\,a\,b^4\right)}{16\,b^6}+\frac{e^{-3\,x}\left(9\,a\,b^2-4\,a^3\right)}{96\,b^4}+\frac{e^{3\,x}\left(9\,a\,b^2-4\,a^3\right)}{96\,b^4}+\frac{e^{-4\,x}\left(a^2\right.}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^7/(a + b*cosh(x)),x)
```

```
[Out] exp(-6*x)/(384*b) + exp(6*x)/(384*b) - (x*(a^2 - b^2)^3)/b^7 - (exp(-x)*(19
*a*b^4 + 8*a^5 - 22*a^3*b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 - 4*a^3))/(96*
b^4) + (exp(3*x)*(9*a*b^2 - 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 - 2*b^2))/(6
4*b^3) + (exp(4*x)*(a^2 - 2*b^2))/(64*b^3) - (a*exp(-5*x))/(160*b^2) - (a*e
xp(5*x))/(160*b^2) + (exp(-2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b^5) +
 (exp(2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*b^4 +
8*a^5 - 22*a^3*b^2))/(16*b^6) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**7/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.166 $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=154

$$\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{8b^5}+\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^3}-\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{8b^6}$$

```
[Out] -1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*x/b^6+2*(a-b)^(5/2)*(a+b)^(5/2)*arctanh((a
-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^6+1/8*(8*(a^2-b^2)^2-a*b*(4*a^2-7*b^2)
*cosh(x))*sinh(x)/b^5+1/12*(4*a^2-4*b^2-3*a*b*cosh(x))*sinh(x)^3/b^3+1/5*si
nh(x)^5/b
```

Rubi [A] time = 0.43, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2695, 2865, 2735, 2659, 208}

$$-\frac{ax\left(-20a^2b^2+8a^4+15b^4\right)}{8b^6}+\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^3}+\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] -(a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) + (2*(a - b)^(5/2)*(a + b)^(5/
2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^6 + ((8*(a^2 - b^2)^2 -
a*b*(4*a^2 - 7*b^2)*Cosh[x])*Sinh[x])/(8*b^5) + ((4*(a^2 - b^2) - 3*a*b*Cos
h[x])*Sinh[x]^3)/(12*b^3) + Sinh[x]^5/(5*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = \frac{\sinh^5(x)}{5b} + \frac{\int \frac{(-b-a\cosh(x))\sinh^4(x)}{a+b\cosh(x)}\,dx}{b}$$

$$= \frac{\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)\sinh^3(x)}{12b^3} + \frac{\sinh^5(x)}{5b} - \frac{\int \frac{\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\cosh(x)\right)\sinh^2(x)}{a+b\cosh(x)}\,dx}{4b^3}$$

$$= \frac{\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)\sinh^3(x)}{12b^3} + \cdots$$

$$= -\frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{\left(4\left(a^2-b\right.\right.}{}$$

$$= -\frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{\left(4\left(a^2-b\right.\right.}{}$$

$$= -\frac{a\left(8a^4-20a^2b^2+15b^4\right)x}{8b^6} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^6} + \frac{\left(8\left(a^2-b^2\right)^2-\right.}{}$$

Mathematica [A] time = 0.24, size = 154, normalized size = 1.00

$$\frac{-120ab^2\left(a^2-2b^2\right)\sinh(2x)+960\left(b^2-a^2\right)^{5/2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)-10b^3\left(7b^2-4a^2\right)\sinh(3x)-60ax\left(8a^4-20\right.}{480b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] (-60*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x + 960*(-a^2 + b^2)^(5/2)*ArcTan[((a
- b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + 60*b*(8*a^4 - 18*a^2*b^2 + 11*b^4)*Sinh
[x] - 120*a*b^2*(a^2 - 2*b^2)*Sinh[2*x] - 10*b^3*(-4*a^2 + 7*b^2)*Sinh[3*x]
 - 15*a*b^4*Sinh[4*x] + 6*b^5*Sinh[5*x])/(480*b^6)
```

fricas [B] time = 1.51, size = 2913, normalized size = 18.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^
5*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b^5
```

```
*cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3*b^
2 - 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*
b^2 + 12*a*b^4 + 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 - 20
*a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(
x)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2*b^3 +
33*b^5 + 14*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*cosh(x))
*sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4
+ 280*(4*a^2*b^3 - 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*cosh(x)^2 -
60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*
cosh(x))*sinh(x)^5 - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^4 +
 10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*a^2*b^3 - 66*
b^5 + 70*(4*a^2*b^3 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b^4)*cosh(x)^3
- 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b - 18*a^2*b^3 +
 11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 + 20*(36*
b^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^5 + 6*a
^3*b^2 - 12*a*b^4 - 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 - 20*a^3*
b^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3
- 12*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 - 10*(4*a^2*b^3 - 7
*b^5)*cosh(x)^2 + 10*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3
 - 7*b^5)*cosh(x)^6 - 252*(a^3*b^2 - 2*a*b^4)*cosh(x)^5 - 4*a^2*b^3 + 7*b^5
 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b - 18*a^2*b
^3 + 11*b^5)*cosh(x)^4 - 36*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*
(a^3*b^2 - 2*a*b^4)*cosh(x))*sinh(x)^2 + 960*((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^5 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^3*sinh(x)^2 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 +
 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 - 2*a^2*b^2 + b^4)*sinh
(x)^5)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) + 5*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b
^3 - 7*b^5)*cosh(x)^7 - 168*(a^3*b^2 - 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 - 20
*a^3*b^2 + 15*a*b^4)*x*cosh(x)^4 + 72*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(
x)^5 + 3*a*b^4 - 48*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2
 - 2*a*b^4)*cosh(x)^2 - 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x
)^5 + 5*b^6*cosh(x)^4*sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)
^2*sinh(x)^3 + 5*b^6*cosh(x)*sinh(x)^4 + b^6*sinh(x)^5), 1/960*(6*b^5*cosh(
x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^5*cosh(x) - a*b^4)*
sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b^5*cosh(x)^2 - 27*a*b
^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3*b^2 - 2*a*b^4)*cosh(x
)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*b^2 + 12*a*b^4 + 4*
(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^6 + 20*(63*b^5*c
osh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2*b^3 + 33*b^5 + 14*(4*a^2*
b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*cosh(x))*sinh(x)^6 + 15*a*b
^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4 + 280*(4*a^2*b^3 -
```

```
7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 60*(8*a^5 - 20*a^3*
b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^5
- 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^4 + 10*(126*b^5*cosh(x
)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*a^2*b^3 - 66*b^5 + 70*(4*a^2*b^3
 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 60*(8*a^5 - 20*a^
3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2)
*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cosh(x)^3 + 20*(36*b^5*cosh(x)^7 - 63*
a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^5 + 6*a^3*b^2 - 12*a*b^4 -
 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*c
osh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 - 12*(8*a^4*b - 18*
a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 - 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 + 1
0*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3 - 7*b^5)*cosh(x)^6
 - 252*(a^3*b^2 - 2*a*b^4)*cosh(x)^5 - 4*a^2*b^3 + 7*b^5 - 120*(8*a^5 - 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x
)^4 - 36*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*(a^3*b^2 - 2*a*b^4)
*cosh(x))*sinh(x)^2 - 1920*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 5*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3*sin
h(x)^2 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 + 5*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^5)*sqrt(-a^2 +
 b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 5
*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b^3 - 7*b^5)*cosh(x)^7
- 168*(a^3*b^2 - 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x
*cosh(x)^4 + 72*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^5 + 3*a*b^4 - 48*(8
*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2 - 2*a*b^4)*cosh(x)^2
- 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x)^5 + 5*b^6*cosh(x)^4*
sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)^2*sinh(x)^3 + 5*b^6*c
osh(x)*sinh(x)^4 + b^6*sinh(x)^5)]
```

giac [A] time = 0.16, size = 266, normalized size = 1.73

$$\frac{6\,b^{4}e^{(5\,x)} - 15\,ab^{3}e^{(4\,x)} + 40\,a^{2}b^{2}e^{(3\,x)} - 70\,b^{4}e^{(3\,x)} - 120\,a^{3}be^{(2\,x)} + 240\,ab^{3}e^{(2\,x)} + 480\,a^{4}e^{x} - 1080\,a^{2}b^{2}e^{x} + 660\,b^{4}e}{960\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) - 70*b^4*e^(3*
x) - 120*a^3*b*e^(2*x) + 240*a*b^3*e^(2*x) + 480*a^4*e^x - 1080*a^2*b^2*e^x
 + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*(15
*a*b^4*e^x - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(a^3*
b^2 - 2*a*b^4)*e^(3*x) - 10*(4*a^2*b^3 - 7*b^5)*e^(2*x))*e^(-5*x)/b^6 + 2*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sq
rt(-a^2 + b^2)*b^6)
```

maple [B] time = 0.08, size = 679, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a+b*cosh(x)),x)
```

```
[Out] -1/5/b/(tanh(1/2*x)-1)^5-1/2/b/(tanh(1/2*x)-1)^4-1/5/b/(tanh(1/2*x)+1)^5+1/
2/b/(tanh(1/2*x)+1)^4+1/12/b/(tanh(1/2*x)-1)^3+5/8/b/(tanh(1/2*x)-1)^2-1/b/
(tanh(1/2*x)-1)+1/12/b/(tanh(1/2*x)+1)^3-5/8/b/(tanh(1/2*x)+1)^2-1/b/(tanh(
1/2*x)+1)+2/b^6/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))
^(1/2))*a^6-6/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b
))^(1/2))*a^4+6/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a
-b))^(1/2))*a^2-1/b^5/(tanh(1/2*x)+1)*a^4-1/2/b^4/(tanh(1/2*x)+1)*a^3+1/4/b
^2/(tanh(1/2*x)+1)^4*a-1/3/b^3/(tanh(1/2*x)+1)^3*a^2-1/2/b^2/(tanh(1/2*x)+1
)^3*a+1/2/b^4/(tanh(1/2*x)+1)^2*a^3+1/2/b^3/(tanh(1/2*x)+1)^2*a^2-1/3/b^3/(
tanh(1/2*x)-1)^3*a^2-1/2/b^2/(tanh(1/2*x)-1)^3*a-1/2/b^4/(tanh(1/2*x)-1)^2*
a^3-1/2/b^3/(tanh(1/2*x)-1)^2*a^2-1/b^5/(tanh(1/2*x)-1)*a^4-1/2/b^4/(tanh(1
/2*x)-1)*a^3-1/4/b^2/(tanh(1/2*x)-1)^4*a+7/8/b^2/(tanh(1/2*x)+1)*a+5/2*a^3/
b^4*ln(tanh(1/2*x)+1)-15/8*a/b^2*ln(tanh(1/2*x)+1)+5/8/b^2/(tanh(1/2*x)-1)^
2*a+2/b^3/(tanh(1/2*x)-1)*a^2+7/8/b^2/(tanh(1/2*x)-1)*a-5/2*a^3/b^4*ln(tanh
(1/2*x)-1)+15/8*a/b^2*ln(tanh(1/2*x)-1)-5/8/b^2/(tanh(1/2*x)+1)^2*a+2/b^3/(
tanh(1/2*x)+1)*a^2-2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))-a^5/b^6*ln(tanh(1/2*x)+1)+a^5/b^6*ln(tanh(1/2*x)-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.70, size = 348, normalized size = 2.26

$$\frac{e^{5x}}{160\,b}-\frac{e^{-5x}}{160\,b}-\frac{e^{-2x}\,\left(2\,a\,b^2-a^3\right)}{8\,b^4}+\frac{e^{2x}\,\left(2\,a\,b^2-a^3\right)}{8\,b^4}-\frac{x\,\left(8\,a^5-20\,a^3\,b^2+15\,a\,b^4\right)}{8\,b^6}+\frac{e^{x}\,\left(8\,a^4-18\,a^2\,b^2+11\,b^4\right)}{16\,b^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^6/(a + b*cosh(x)),x)
```

```
[Out] exp(5*x)/(160*b) - exp(-5*x)/(160*b) - (exp(-2*x)*(2*a*b^2 - a^3))/(8*b^4)
+ (exp(2*x)*(2*a*b^2 - a^3))/(8*b^4) - (x*(15*a*b^4 + 8*a^5 - 20*a^3*b^2))/
(8*b^6) + (exp(x)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x))/(
64*b^2) - (a*exp(4*x))/(64*b^2) - (exp(-x)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(
16*b^5) - (exp(-3*x)*(4*a^2 - 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 - 7*b^2))
/(96*b^3) + (log(- (2*exp(x)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7 - (2*
(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7)*(a + b)^(5/2)*(a - b)^(5/2
))/b^6 - (log((2*(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7 - (2*exp(x
)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7)*(a + b)^(5/2)*(a - b)^(5/2))/b^
6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**6/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.167 $\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=83

$$\frac{(a^2-b^2)^2\log(a+b\cosh(x))}{b^5}-\frac{a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^2-2b^2)\cosh^2(x)}{2b^3}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}$$

```
[Out] -a*(a^2-2*b^2)*cosh(x)/b^4+1/2*(a^2-2*b^2)*cosh(x)^2/b^3-1/3*a*cosh(x)^3/b^
2+1/4*cosh(x)^4/b+(a^2-b^2)^2*ln(a+b*cosh(x))/b^5
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2-2b^2)\cosh^2(x)}{2b^3}-\frac{a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^2-b^2)^2\log(a+b\cosh(x))}{b^5}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2)*Cosh[x])/b^4) + ((a^2 - 2*b^2)*Cosh[x]^2)/(2*b^3) - (a*C
osh[x]^3)/(3*b^2) + Cosh[x]^4/(4*b) + ((a^2 - b^2)^2*Log[a + b*Cosh[x]])/b^
5
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\cosh(x)\right)}{b^5}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3\left(1-\frac{2b^2}{a^2}\right)+\left(a^2-2b^2\right)x-ax^2+x^3+\frac{\left(a^2-b^2\right)^2}{a+x}\right)dx, x, b\cosh(x)\right)}{b^5}$$
$$= -\frac{a\left(a^2-2b^2\right)\cosh(x)}{b^4}+\frac{\left(a^2-2b^2\right)\cosh^2(x)}{2b^3}-\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}+\frac{\left(a^2-b^2\right)^2\log(a}{b^5}$$

Mathematica [A] time = 0.12, size = 84, normalized size = 1.01

$$\frac{-12b^2\left(3b^2-2a^2\right)\cosh(2x)-24ab\left(4a^2-7b^2\right)\cosh(x)+96\left(a^2-b^2\right)^2\log(a+b\cosh(x))-8ab^3\cosh(3x)+3b^4}{96b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] (-24*a*b*(4*a^2 - 7*b^2)*Cosh[x] - 12*b^2*(-2*a^2 + 3*b^2)*Cosh[2*x] - 8*a*
b^3*Cosh[3*x] + 3*b^4*Cosh[4*x] + 96*(a^2 - b^2)^2*Log[a + b*Cosh[x]])/(96*
b^5)
```

fricas [B] time = 1.03, size = 866, normalized size = 10.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*cos
h(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^6 + 4*(21*b^4*cosh
(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 - 9*b^4)*sinh(x)^6 - 192*(a^4 - 2*a^2*
b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*cosh(
x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b + 7*a*b^3 + 3*(2*a^2*b^2 - 3*b^4)*cosh(x
))*sinh(x)^5 - 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cosh(x)^3
 + 90*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 - 96*(a^4 - 2*a^2*b^2 + b^4)*x - 60*(4*
a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x
)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 - 12*a^3*b + 21*a*b^3 + 30*(
2*a^2*b^2 - 3*b^4)*cosh(x)^3 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x) - 30*(4
*a^3*b - 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 +
```

```
12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 - 3*b^4)*cosh(x)^
4 + 2*a^2*b^2 - 3*b^4 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 20*(4*a^3*
b - 7*a*b^3)*cosh(x)^3 - 6*(4*a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3*sinh(
x) + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4)*log(2*(b*cosh(x)
 + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3*cosh(x)^6 + 9*(2*
a^2*b^2 - 3*b^4)*cosh(x)^5 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^3 - 15*(4
*a^3*b - 7*a*b^3)*cosh(x)^4 - a*b^3 - 9*(4*a^3*b - 7*a*b^3)*cosh(x)^2 + 3*(
2*a^2*b^2 - 3*b^4)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(
x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)
```

giac [A] time = 0.13, size = 124, normalized size = 1.49

$$\frac{3\,b^3\left(e^{(-x)}+e^x\right)^4-8\,ab^2\left(e^{(-x)}+e^x\right)^3+24\,a^2b\left(e^{(-x)}+e^x\right)^2-48\,b^3\left(e^{(-x)}+e^x\right)^2-96\,a^3\left(e^{(-x)}+e^x\right)+192\,ab^2\left(e^{(-x)}\right.}{192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b^3*(e^(-x) + e^x)^4 - 8*a*b^2*(e^(-x) + e^x)^3 + 24*a^2*b*(e^(-x)
 + e^x)^2 - 48*b^3*(e^(-x) + e^x)^2 - 96*a^3*(e^(-x) + e^x) + 192*a*b^2*(e^
(-x) + e^x))/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(b*(e^(-x) + e^x) + 2*a))
/b^5
```

maple [B] time = 0.07, size = 599, normalized size = 7.22

$$\frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{3}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{5}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{2b\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a+b*cosh(x)),x)
```

```
[Out] 1/4/b/(tanh(1/2*x)-1)^4+1/4/b/(tanh(1/2*x)+1)^4+1/b^5/(a-b)*ln(a*tanh(1/2*x
)^2-tanh(1/2*x)^2*b-a-b)*a^5-1/b^4/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b
-a-b)*a^4-2/b^3/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^3+2/b^2/(a-
b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^2+1/b/(a-b)*ln(a*tanh(1/2*x)^2
-tanh(1/2*x)^2*b-a-b)*a+1/2/b/(tanh(1/2*x)-1)^3-3/8/b/(tanh(1/2*x)-1)^2-5/8
/b/(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^3-3/8/b/(tanh(1/2*x)+1)^2+5/8/b/(t
anh(1/2*x)+1)-1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)+2/b^3*ln(tanh(1/2
*x)-1)*a^2+2/b^3*ln(tanh(1/2*x)+1)*a^2-1/b^4/(tanh(1/2*x)+1)*a^3-1/3/b^2/(t
anh(1/2*x)+1)^3*a-1/b^5*ln(tanh(1/2*x)+1)*a^4+1/2/b^3/(tanh(1/2*x)+1)^2*a^2
+1/3/b^2/(tanh(1/2*x)-1)^3*a-1/b^5*ln(tanh(1/2*x)-1)*a^4+1/2/b^3/(tanh(1/2*
```

```
x)-1)^2*a^2+1/b^4/(tanh(1/2*x)-1)*a^3-1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x
)^2*b-a-b)+3/2/b^2/(tanh(1/2*x)+1)*a+1/2/b^2/(tanh(1/2*x)-1)^2*a+1/2/b^3/(t
anh(1/2*x)-1)*a^2-3/2/b^2/(tanh(1/2*x)-1)*a+1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2
/b^3/(tanh(1/2*x)+1)*a^2
```

maxima [B] time = 0.33, size = 178, normalized size = 2.14

$$-\frac{\left(8\,ab^2e^{(-x)}-3\,b^3-12\left(2\,a^2b-3\,b^3\right)e^{(-2\,x)}+24\left(4\,a^3-7\,ab^2\right)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4}-\frac{8\,ab^2e^{(-3\,x)}-3\,b^3e^{(-4\,x)}+24\left(4\,a^3-7\,a\right.}{192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b - 3*b^3)*e^(-2*x) + 24*(4*a^3
- 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 - 1/192*(8*a*b^2*e^(-3*x) - 3*b^3*e^(-4*x)
 + 24*(4*a^3 - 7*a*b^2)*e^(-x) - 12*(2*a^2*b - 3*b^3)*e^(-2*x))/b^4 + (a^4
- 2*a^2*b^2 + b^4)*x/b^5 + (a^4 - 2*a^2*b^2 + b^4)*log(2*a*e^(-x) + b*e^(-2
*x) + b)/b^5
```

mupad [B] time = 1.31, size = 169, normalized size = 2.04

$$\frac{e^{-4\,x}}{64\,b}+\frac{e^{4\,x}}{64\,b}-\frac{x\left(a^2-b^2\right)^2}{b^5}+\frac{e^{-x}\left(7\,a\,b^2-4\,a^3\right)}{8\,b^4}+\frac{\ln\left(b+2\,a\,e^{x}+b\,e^{2\,x}\right)\left(a^4-2\,a^2\,b^2+b^4\right)}{b^5}-\frac{a\,e^{-3\,x}}{24\,b^2}-\frac{a\,e^{3\,x}}{24\,b^2}+\frac{e^{-2\,x}\left(2\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5/(a + b*cosh(x)),x)
```

```
[Out] exp(-4*x)/(64*b) + exp(4*x)/(64*b) - (x*(a^2 - b^2)^2)/b^5 + (exp(-x)*(7*a*
b^2 - 4*a^3))/(8*b^4) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^4 + b^4 - 2*a^
2*b^2))/b^5 - (a*exp(-3*x))/(24*b^2) - (a*exp(3*x))/(24*b^2) + (exp(-2*x)*(
2*a^2 - 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 - 3*b^2))/(16*b^3) + (exp(x)*(7
*a*b^2 - 4*a^3))/(8*b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.168 $\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=104

$$-\frac{ax\left(2a^2-3b^2\right)}{2b^4}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^3}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{\sinh^3(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh((a-b)^(1/2)*ta
nh(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cosh(x))*sinh(x)/b^3+1/3*si
nh(x)^3/b
```

Rubi [A] time = 0.24, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2695, 2865, 2735, 2659, 208}

$$-\frac{ax\left(2a^2-3b^2\right)}{2b^4}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^3}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{\sinh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] -(a*(2*a^2 - 3*b^2)*x)/(2*b^4) + (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTanh[(Sq
rt[a - b]*Tanh[x/2])/Sqrt[a + b]])/b^4 + ((2*(a^2 - b^2) - a*b*Cosh[x])*Sin
h[x])/(2*b^3) + Sinh[x]^3/(3*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{\sinh^3(x)}{3b} + \frac{\int \frac{(-b-a\cosh(x))\sinh^2(x)}{a+b\cosh(x)}\,dx}{b}$$

$$= \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2b^3} + \frac{\sinh^3(x)}{3b} - \frac{\int \frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cosh(x)}{a+b\cosh(x)}\,dx}{2b^3}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2b^3} + \frac{\sinh^3(x)}{3b} + \frac{\left(a^2-b^2\right)^2\int \frac{1}{a+b\cosh(x}}{b^4}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)\sinh(x)}{2b^3} + \frac{\sinh^3(x)}{3b} + \frac{\left(2\left(a^2-b^2\right)^2\right)\text{Subst}\,(}{}$$

$$= -\frac{a\left(2a^2-3b^2\right)x}{2b^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4} + \frac{\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^3}$$

Mathematica [A] time = 0.19, size = 95, normalized size = 0.91

$$\frac{-12a^3x-24\left(b^2-a^2\right)^{3/2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)+12a^2b\sinh(x)+18ab^2x-3ab^2\sinh(2x)-15b^3\sinh(x)+b^3\sinh}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTan[((a - b)*Tanh[x/2])/
Sqrt[-a^2 + b^2]] + 12*a^2*b*Sinh[x] - 15*b^3*Sinh[x] - 3*a*b^2*Sinh[2*x] +
 b^3*Sinh[3*x])/(12*b^4)
```

fricas [B] time = 2.78, size = 1099, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
 - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*b^3
)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*sinh(
x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2*a^
3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a^2*b
- 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*b + 5*
b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2)*sinh(
x)^2 - 24*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x) + 3*(a^2
 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(a^2 - b^2)*log((b^2
*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) +
 a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*(2*b^3*c
osh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*
b - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x))/(b^4*c
osh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^
3), 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh
(x) - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*
b^3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*si
nh(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2
*a^3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a^2
*b - 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*b +
 5*b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2)*si
nh(x)^2 - 48*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x) + 3*(
a^2 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(-a^2 + b^2)*arct
```

```
an(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*(2*b^3*co
sh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*b
 - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x))/(b^4*co
sh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3
)]
```

giac [A] time = 0.12, size = 146, normalized size = 1.40

$$\frac{b^2e^{(3x)} - 3\,abe^{(2x)} + 12\,a^2e^x - 15\,b^2e^x}{24\,b^3} - \frac{\left(2\,a^3 - 3\,ab^2\right)x}{2\,b^4} + \frac{\left(3\,ab^2e^x - b^3 - 3\left(4\,a^2b - 5\,b^3\right)e^{(2x)}\right)e^{(-3x)}}{24\,b^4} + \frac{2\left(a^4 - 2\,a^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x - 15*b^2*e^x)/b^3 - 1/2*(2*a
^3 - 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b - 5*b^3)*e^(2*x)
)*e^(-3*x)/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x + a)/sqrt(-a^2 + b
^2))/(sqrt(-a^2 + b^2)*b^4)
```

maple [B] time = 0.08, size = 338, normalized size = 3.25

$$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{a^2}{b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{1}{b\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*cosh(x)),x)
```

```
[Out] -1/3/b/(tanh(1/2*x)-1)^3-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)^
2-1/b^3/(tanh(1/2*x)-1)*a^2-1/2/b^2/(tanh(1/2*x)-1)*a+1/b/(tanh(1/2*x)-1)+a
^3/b^4*ln(tanh(1/2*x)-1)-3/2*a/b^2*ln(tanh(1/2*x)-1)-1/3/b/(tanh(1/2*x)+1)^
3+1/2/b^2/(tanh(1/2*x)+1)^2*a+1/2/b/(tanh(1/2*x)+1)^2-1/b^3/(tanh(1/2*x)+1)
*a^2-1/2/b^2/(tanh(1/2*x)+1)*a+1/b/(tanh(1/2*x)+1)-a^3/b^4*ln(tanh(1/2*x)+1
)+3/2*a/b^2*ln(tanh(1/2*x)+1)+2/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(
1/2*x)/((a+b)*(a-b))^(1/2))*a^4-4/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
h(1/2*x)/((a+b)*(a-b))^(1/2))*a^2+2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(
1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 1.31, size = 222, normalized size = 2.13

$$\frac{e^{3x}}{24b}-\frac{e^{-3x}}{24b}+\frac{x\left(3ab^2-2a^3\right)}{2b^4}+\frac{e^{x}\left(4a^2-5b^2\right)}{8b^3}+\frac{a\,e^{-2x}}{8b^2}-\frac{a\,e^{2x}}{8b^2}-\frac{e^{-x}\left(4a^2-5b^2\right)}{8b^3}+\frac{\ln\left(-\frac{2e^{x}\left(a^4-2a^2b^2+b^4\right)}{b^5}-\frac{2(a+b)}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*cosh(x)),x)
```

```
[Out] exp(3*x)/(24*b) - exp(-3*x)/(24*b) + (x*(3*a*b^2 - 2*a^3))/(2*b^4) + (exp(x
)*(4*a^2 - 5*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) -
 (exp(-x)*(4*a^2 - 5*b^2))/(8*b^3) + (log(- (2*exp(x)*(a^4 + b^4 - 2*a^2*b^
2))/b^5 - (2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/2))/b^5)*(a + b)^(3/2)
*(a - b)^(3/2))/b^4 - (log((2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/2))/b
^5 - (2*exp(x)*(a^4 + b^4 - 2*a^2*b^2))/b^5)*(a + b)^(3/2)*(a - b)^(3/2))/b
^4
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.169 $\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=40

$$\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}-\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}$$

```
[Out] -a*cosh(x)/b^2+1/2*cosh(x)^2/b+(a^2-b^2)*ln(a+b*cosh(x))/b^3
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}-\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Cosh[x])/b^2) + Cosh[x]^2/(2*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^3
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx, x, b\cosh(x)\right)}{b^3}$$
$$= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)dx, x, b\cosh(x)\right)}{b^3}$$
$$= -\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}+\frac{\left(a^2-b^2\right)\log(a+b\cosh(x))}{b^3}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 1.00

$$\frac{\left(a^2-b^2\right)\log(a+b\cosh(x))}{b^3}-\frac{a\cosh(x)}{b^2}+\frac{\cosh(2x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Cosh[x])/b^2) + Cosh[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^3
```

fricas [B] time = 1.42, size = 234, normalized size = 5.85

$$b^2\cosh(x)^4+b^2\sinh(x)^4-4\,ab\cosh(x)^3-8\left(a^2-b^2\right)x\cosh(x)^2+4\left(b^2\cosh(x)-ab\right)\sinh(x)^3-4\,ab\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 - b^2)*x*cosh
(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*b^2*cosh(x)^
2 - 6*a*b*cosh(x) - 4*(a^2 - b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 - b^2)*cosh(
x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2)*log(2*(b*cosh
(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4*(a^2
 - b^2)*x*cosh(x) - a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) +
b^3*sinh(x)^2)
```

giac [A] time = 0.13, size = 56, normalized size = 1.40

$$\frac{b\left(e^{(-x)}+e^x\right)^2-4\,a\left(e^{(-x)}+e^x\right)}{8\,b^2}+\frac{\left(a^2-b^2\right)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*(e^(-x) + e^x)^2 - 4*a*(e^(-x) + e^x))/b^2 + (a^2 - b^2)*log(abs(b*(
e^(-x) + e^x) + 2*a))/b^3
```

maple [B] time = 0.06, size = 283, normalized size = 7.08

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)a^{2}}{b^{3}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{a}{b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{1}{2b\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -1/b^3*ln(tanh(1/2*x)-1)*a^2+1/b*ln(tanh(1/2*x)-1)+1/2/b/(tanh(1/2*x)-1)^2+
1/b^2/(tanh(1/2*x)-1)*a+1/2/b/(tanh(1/2*x)-1)+1/2/b/(tanh(1/2*x)+1)^2-1/b^2
/(tanh(1/2*x)+1)*a-1/2/b/(tanh(1/2*x)+1)-1/b^3*ln(tanh(1/2*x)+1)*a^2+1/b*ln
(tanh(1/2*x)+1)+1/b^3/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^3-1/b
^2/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*a^2-1/b/(a-b)*ln(a*tanh(1/
2*x)^2-tanh(1/2*x)^2*b-a-b)*a+1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-
b)
```

maxima [B] time = 0.31, size = 84, normalized size = 2.10

$$-\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^{2}}-\frac{4\,ae^{(-x)}-be^{(-2\,x)}}{8\,b^{2}}+\frac{\left(a^{2}-b^{2}\right)x}{b^{3}}+\frac{\left(a^{2}-b^{2}\right)\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x) - b*e^(-2*x))/b^2 + (a^
2 - b^2)*x/b^3 + (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/b^3
```

mupad [B] time = 1.04, size = 79, normalized size = 1.98

$$\frac{\mathrm{e}^{-2\,x}}{8\,b}+\frac{\mathrm{e}^{2\,x}}{8\,b}+\frac{\ln\left(b+2\,a\,\mathrm{e}^{x}+b\,\mathrm{e}^{2\,x}\right)\,\left(a^{2}-b^{2}\right)}{b^{3}}-\frac{a\,\mathrm{e}^{x}}{2\,b^{2}}-\frac{a\,\mathrm{e}^{-x}}{2\,b^{2}}-\frac{x\,\left(a^{2}-b^{2}\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^2 -
 b^2))/b^3 - (a*exp(x))/(2*b^2) - (a*exp(-x))/(2*b^2) - (x*(a^2 - b^2))/b^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.170 $\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}$$

```
[Out] -a*x/b^2+sinh(x)/b+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/
2)*(a+b)^(1/2)/b^2
```

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2695, 2735, 2659, 208}

$$-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/S
qrt[a + b]])/b^2 + Sinh[x]/b
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
```

```
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx &= \frac{\sinh(x)}{b}+\frac{\int \frac{-b-a\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}-\left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cosh(x)}\,dx\\
&= -\frac{ax}{b^2}+\frac{\sinh(x)}{b}-\left(2\left(1-\frac{a^2}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{ax}{b^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 54, normalized size = 0.92

$$
\frac{2\sqrt{b^2-a^2}\,\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)-ax+b\sinh(x)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (-(a*x) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] +
 b*Sinh[x])/b^2
```

fricas [B] time = 1.29, size = 279, normalized size = 4.73

$$
\left[-\frac{2\,ax\cosh(x)-b\cosh(x)^2-b\sinh(x)^2-2\,\sqrt{a^2-b^2}\,(\cosh(x)+\sinh(x))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a}{b\cosh(x)^2+b\sinh}\right.}{2\left(b^2\cosh(x)+b^2\sinh(x\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(
x) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 -
b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh
(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh
(x) + b)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x)), -
1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 + 4*sqrt(-a^2 + b^2)*(cosh(x
) + sinh(x))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^
2)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x))]
```

giac [A] time = 0.13, size = 68, normalized size = 1.15

$$-\frac{ax}{b^2} - \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{2\left(a^2 - b^2\right)\arctan\left(\frac{be^x + a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2 + b^2}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a*x/b^2 - 1/2*e^(-x)/b + 1/2*e^x/b + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2)
```

maple [B] time = 0.06, size = 129, normalized size = 2.19

$$-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)a^2}{b^2\sqrt{(a+b)(a-b)}}-\frac{2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(x)),x)
```

```
[Out] -1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)-1/b/(tanh(1/2*x)+1)-a/b^2*ln(t
anh(1/2*x)+1)+2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a
-b))^(1/2))*a^2-2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b
))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.04, size = 139, normalized size = 2.36

$$\frac{e^x}{2b}-\frac{e^{-x}}{2b}-\frac{a\,x}{b^2}+\frac{\ln\left(-\frac{2\,e^x\left(a^2-b^2\right)}{b^3}-\frac{2\,\sqrt{a+b}\,\left(b+a\,e^x\right)\,\sqrt{a-b}}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2}-\frac{\ln\left(\frac{2\,\sqrt{a+b}\,\left(b+a\,e^x\right)\,\sqrt{a-b}}{b^3}-\frac{2\,e^x\left(a^2-b^2\right)}{b^3}\right)\sqrt{a+}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*cosh(x)),x)
```

```
[Out] exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (log(- (2*exp(x)*(a^2 - b^2))/b^
3 - (2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3)*(a + b)^(1/2)*(a -
b)^(1/2))/b^2 - (log((2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3 - (
2*exp(x)*(a^2 - b^2))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/b^2
```

sympy [A] time = 93.14, size = 892, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(-2*tanh(x/2)**2*atan(tanh(x/2))/(tanh(x/2)**2 - 1) - 2*tanh
(x/2)/(tanh(x/2)**2 - 1) + 2*atan(tanh(x/2))/(tanh(x/2)**2 - 1)), Eq(a, 0)
& Eq(b, 0)), (x*tanh(x/2)**2/(b*tanh(x/2)**2 - b) - x/(b*tanh(x/2)**2 - b)
- 2*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, -b)), ((x*sinh(x)**2/2 - x*cosh(x
)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), (-x*tanh(x/2)**2/(b*tanh(x/2)**2
- b) + x/(b*tanh(x/2)**2 - b) - 2*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b))
, (-a*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(
a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) + a*x*sqrt(a/(a -
b) + b/(a - b))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(
a/(a - b) + b/(a - b))) - a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*t
anh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a
 - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b)
)) + a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(b**2*sqrt
(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) -
a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2*sqrt(a/(a - b) + b/(a
- b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*b*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b*
```

```
*2*sqrt(a/(a - b) + b/(a - b))) - b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh
(x/2))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*s
qrt(a/(a - b) + b/(a - b))) + b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2
))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b
/(a - b))) + b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(b
**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a -
 b))) - b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))), True))
```

3.171 $\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\cosh(x))}{b}$$

[Out] ln(a+b*cosh(x))/b

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(a + b*Cosh[x]),x]

[Out] Log[a + b*Cosh[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{b}\\ &= \frac{\log(a+b\cosh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$\frac{\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Cosh[x]),x]
[Out] Log[a + b*Cosh[x]]/b
```

fricas [B] time = 0.84, size = 27, normalized size = 2.45

$$-\frac{x-\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="fricas")
[Out] -(x - log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/b
```

giac [A] time = 0.14, size = 19, normalized size = 1.73

$$\frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="giac")
[Out] log(abs(b*(e^(-x) + e^x) + 2*a))/b
```

maple [A] time = 0.03, size = 12, normalized size = 1.09

$$\frac{\ln\left(a+b\cosh(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*cosh(x)),x)
[Out] ln(a+b*cosh(x))/b
```

maxima [A] time = 0.30, size = 11, normalized size = 1.00

$$\frac{\log\left(b\cosh(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(b*cosh(x) + a)/b
```

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\cosh(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*cosh(x)),x)
[Out] log(a + b*cosh(x))/b
```

sympy [A] time = 0.31, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log\left(\frac{a}{b}+\cosh\left(x\right)\right)}{b} & \text{for } b\neq 0 \\ \frac{\cosh\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*cosh(x)),x)
[Out] Piecewise((log(a/b + cosh(x))/b, Ne(b, 0)), (cosh(x)/a, True))
```

3.172 $\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=53

$$\frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] 1/2*ln(1-cosh(x))/(a+b)-1/2*ln(1+cosh(x))/(a-b)+b*ln(a+b*cosh(x))/(a^2-b^2)
```

Rubi **[A]** time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2668, 706, 31, 633}

$$\frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx,x,b\cosh(x)\right)\right)\\
&= \frac{b\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cosh(x)\right)}{a^2-b^2}+\frac{b\,\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx,x,b\cosh(x)\right)}{a^2-b^2}\\
&= \frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx,x,b\cosh(x)\right)}{2(a-b)}-\frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx,x,b\cosh(x)\right)}{2(a+b)}\\
&= \frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(a+b\cosh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 37, normalized size = 0.70

$$
\frac{b\log(a+b\cosh(x))+a\log\left(\tanh\left(\frac{x}{2}\right)\right)-b\log(\sinh(x))}{a^2-b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Cosh[x]),x]
```

```
[Out] (b*Log[a + b*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]])/(a^2 - b^2)
```

fricas [A] time = 0.62, size = 58, normalized size = 1.09

$$
\frac{b\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh(x)+\sinh(x)+1\right)+(a-b)\log\left(\cosh(x)+\sinh(x)-1\right)}{a^2-b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] (b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh(
x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

giac [A] time = 0.15, size = 67, normalized size = 1.26

$$
\frac{b^2\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^2b-b^3}-\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) - 1/2*log(e^(-x) + e^x +
 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

maple [A] time = 0.07, size = 52, normalized size = 0.98

$$\frac{b \ln \left(a \left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right) b-a-b\right)}{(a+b)(a-b)}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*cosh(x)),x)
```

```
[Out] b/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+1/(a+b)*ln(tanh(1/2*x
))
```

maxima [A] time = 0.38, size = 59, normalized size = 1.11

$$\frac{b \log \left(2 a e^{(-x)}+b e^{(-2 x)}+b\right)}{a^{2}-b^{2}}-\frac{\log \left(e^{(-x)}+1\right)}{a-b}+\frac{\log \left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
log(e^(-x) - 1)/(a + b)
```

mupad [B] time = 1.29, size = 160, normalized size = 3.02

$$\frac{\ln \left(128\, a\, b^{2}-128\, a^{2}\, b+32\, a^{3}-32\, a^{3}\, e^{x}-128\, a\, b^{2}\, e^{x}+128\, a^{2}\, b\, e^{x}\right)}{a+b}-\frac{\ln \left(128\, a\, b^{2}+128\, a^{2}\, b+32\, a^{3}+32\, a^{3}\, e^{x}+\right.}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*cosh(x))),x)
```

```
[Out] log(128*a*b^2 - 128*a^2*b + 32*a^3 - 32*a^3*exp(x) - 128*a*b^2*exp(x) + 128
*a^2*b*exp(x))/(a + b) - log(128*a*b^2 + 128*a^2*b + 32*a^3 + 32*a^3*exp(x)
 + 128*a*b^2*exp(x) + 128*a^2*b*exp(x))/(a - b) + (b*log(16*b^3*exp(2*x) -
4*a^2*b + 16*b^3 - 8*a^3*exp(x) + 32*a*b^2*exp(x) - 4*a^2*b*exp(2*x)))/(a^2
 - b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*cosh(x)), x)
```

3.173 $\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=67

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] 2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+
(b-a*cosh(x))*csch(x)/(a^2-b^2)
```

Rubi [A] time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2696, 12, 2659, 208}

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx &= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}-\frac{\int \frac{b^2}{a+b\cosh(x)}\,dx}{-a^2+b^2}\\ &= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}+\frac{b^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}\\ &= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\ &= \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.21, size = 77, normalized size = 1.15

$$\frac{2b^2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - C
oth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

fricas [B] time = 0.60, size = 470, normalized size = 7.01

$$\left[\frac{2\,a^3-2\,ab^2+\left(b^2\cosh(x)^2+2\,b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2-b^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh}{b\cosh(x)^2}\right.}{a^4-2\,a^2b^2+b^4-\left(a^4-2\,a^2b^2+b^4\right)\cosh(x)^2-2\left(a^4-2\,a^2b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(a^
4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a*b
^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x
) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

giac [A] time = 0.14, size = 76, normalized size = 1.13

$$\frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}+\frac{2\,(be^x-a)}{\left(a^2-b^2\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) +
 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

maple [A] time = 0.08, size = 78, normalized size = 1.16

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2\,(a-b)}-\frac{1}{2\,(a+b)\tanh\left(\frac{x}{2}\right)}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*cosh(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a+b)/(a-b)*b^2/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.48, size = 327, normalized size = 4.88

$$-\frac{\frac{2a}{a^2-b^2}-\frac{2b\,e^x}{a^2-b^2}}{e^{2x}-1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2}{\left(a^2-b^2\right)^2\sqrt{b^4}}+\frac{2a\left(a^3\sqrt{b^4}-a\,b^2\sqrt{b^4}\right)}{b^4\left(a^2-b^2\right)\sqrt{-\left(a^2-b^2\right)^3}\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}}\right)-\frac{2a\left(b^3\sqrt{b^4}-a^2b\sqrt{b\cdots}\right.}{b^4\left(a^2-b^2\right)\sqrt{-\left(a^2-b^2\right)^3}\sqrt{-a^6+3\cdots}}\right.\right.\right.}{\sqrt{-a^6+3a^4b^2-3a^2b^4+\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*cosh(x))),x)
```

```
[Out] - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan((
exp(x)*(2/((a^2 - b^2)^2*(b^4)^(1/2)) + (2*a*(a^3*(b^4)^(1/2) - a*b^2*(b^4)
^(1/2)))/(b^4*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) - a^2*b*(b^4)^(1/2)))/(b^4*(a^2 -
 b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)))*((
b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*cosh(x)), x)
```

3.174 $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=91

$$\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2\left(a^2-b^2\right)}+\frac{b^3\log(a+b\cosh(x))}{\left(a^2-b^2\right)^2}-\frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\cosh(x)+1)}{4(a-b)^2}$$

```
[Out] 1/2*(b-a*cosh(x))*csch(x)^2/(a^2-b^2)-1/4*(a+2*b)*ln(1-cosh(x))/(a+b)^2+1/4
*(a-2*b)*ln(1+cosh(x))/(a-b)^2+b^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2668, 741, 801}

$$\frac{b^3\log(a+b\cosh(x))}{\left(a^2-b^2\right)^2}+\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2\left(a^2-b^2\right)}-\frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - ((a + 2*b)*Log[1 - Cosh[x]])/
(4*(a + b)^2) + ((a - 2*b)*Log[1 + Cosh[x]])/(4*(a - b)^2) + (b^3*Log[a + b
*Cosh[x]])/(a^2 - b^2)^2
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = b^3\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right) dx, x, b\ c\right.}{2\left(a^2-b^2\right)}$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(a-2b)\log(1+\cosh(x))}{4(a-b)^2} + \frac{b^3\ \mathrm{lo}}{}$$

Mathematica [A] time = 0.28, size = 100, normalized size = 1.10

$$-\frac{4a^3\log\left(\tanh\left(\frac{x}{2}\right)\right) - 8b^3\log(a+b\cosh(x)) - 12ab^2\log\left(\tanh\left(\frac{x}{2}\right)\right) + (a-b)^2(a+b)\operatorname{csch}^2\left(\frac{x}{2}\right) + (a-b)(a+b)}{8(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -1/8*((a - b)^2*(a + b)*Csch[x/2]^2 - 8*b^3*Log[a + b*Cosh[x]] + 8*b^3*Log[
Sinh[x]] + 4*a^3*Log[Tanh[x/2]] - 12*a*b^2*Log[Tanh[x/2]] + (a - b)*(a + b)
^2*Sech[x/2]^2)/((a - b)^2*(a + b)^2)
```

fricas [B] time = 0.55, size = 818, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b^
3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 + 2*
```

```
(a^3 - a*b^2)*cosh(x) - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*si
nh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 + 4*(
b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh
(x))) - ((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 - 2*b^3)*cosh
(x)*sinh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 -
 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3 - 3*(a^3 -
3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^
3 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) +
((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 + 2*b^3)*cosh(x)*sinh
(x)^3 + (a^3 - 3*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 + 2*b^3 - 2*(a^3
- 3*a*b^2 + 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 + 2*b^3 - 3*(a^3 - 3*a*b^2
+ 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^3 - (a^3
 - 3*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^3 -
 a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh(x))*sinh(x))/((a^
4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^
3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(a^4 - 2*a^2*b^2 +
b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 - 2
*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

giac [B] time = 0.15, size = 179, normalized size = 1.97

$$\frac{b^4\log\left(\left|b\left(e^{(-x)}+e^x\right)+2a\right|\right)}{a^4b-2a^2b^3+b^5}+\frac{(a-2b)\log\left(e^{(-x)}+e^x+2\right)}{4\left(a^2-2ab+b^2\right)}-\frac{(a+2b)\log\left(e^{(-x)}+e^x-2\right)}{4\left(a^2+2ab+b^2\right)}+\frac{b^3\left(e^{(-x)}+e^x\right)^2-2a^3\left(e^{(\ldots}\right.}{2\left(a^4-2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(a - 2
*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*(a + 2*b)*log(e^(-x) +
e^x - 2)/(a^2 + 2*a*b + b^2) + 1/2*(b^3*(e^(-x) + e^x)^2 - 2*a^3*(e^(-x) +
e^x) + 2*a*b^2*(e^(-x) + e^x) + 4*a^2*b - 8*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(
(e^(-x) + e^x)^2 - 4))
```

maple [A] time = 0.09, size = 97, normalized size = 1.07

$$\frac{\tanh^2\left(\frac{x}{2}\right)}{8a-8b}+\frac{b^3\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)}{(a+b)^2(a-b)^2}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)a}{2(a+b)^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)b}{(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*cosh(x)),x)
```

```
[Out] 1/8*tanh(1/2*x)^2/(a-b)+b^3/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^
2*b-a-b)-1/8/(a+b)/tanh(1/2*x)^2-1/2/(a+b)^2*ln(tanh(1/2*x))*a-1/(a+b)^2*ln
(tanh(1/2*x))*b
```

maxima [A] time = 0.35, size = 154, normalized size = 1.69

$$\frac{b^3 \log\left(2\, ae^{(-x)} + be^{(-2\,x)} + b\right)}{a^4 - 2\,a^2b^2 + b^4} + \frac{(a-2\,b)\log\left(e^{(-x)}+1\right)}{2\left(a^2 - 2\,ab + b^2\right)} - \frac{(a+2\,b)\log\left(e^{(-x)}-1\right)}{2\left(a^2 + 2\,ab + b^2\right)} - \frac{ae^{(-x)} - 2\,be^{(-2\,x)} + ae^{(}}{a^2 - b^2 - 2\left(a^2-b^2\right)e^{(-2\,x)} + \left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a - 2*b
)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*(a + 2*b)*log(e^(-x) - 1)/(a^2
+ 2*a*b + b^2) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2
 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

mupad [B] time = 1.48, size = 291, normalized size = 3.20

$$\frac{\frac{2\left(a^2\,b-b^3\right)}{\left(a^2-b^2\right)^2} + \frac{e^x\left(a\,b^2-a^3\right)}{\left(a^2-b^2\right)^2}}{e^{2\,x}-1} + \frac{\frac{2\,b}{a^2-b^2} - \frac{2\,a\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1} + \frac{b^3\,\ln\left(16\,b^7\,e^{2\,x} - a^6\,b + 16\,b^7 - 9\,a^2\,b^5 + 6\,a^4\,b^3 - 2\,a^7\,e^x - 9\,a^2\,b^5\,e^{2\,x}\right.}{a^4 - 2\,a^2\,b^2 +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*cosh(x))),x)
```

```
[Out] ((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(e
xp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) - 2
*exp(2*x) + 1) + (b^3*log(16*b^7*exp(2*x) - a^6*b + 16*b^7 - 9*a^2*b^5 + 6*
a^4*b^3 - 2*a^7*exp(x) - 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a*b^6
*exp(x) - a^6*b*exp(2*x) - 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^4 + b
^4 - 2*a^2*b^2) - (log(exp(x) - 1)*(a + 2*b))/(4*a*b + 2*a^2 + 2*b^2) + (lo
g(exp(x) + 1)*(a - 2*b))/(2*a^2 - 4*a*b + 2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*cosh(x)), x)
```

3.175 $\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=110

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{3\left(a^2-b^2\right)^2}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] 2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)+
1/3*(3*b^3+a*(2*a^2-5*b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+1/3*(b-a*cosh(x))*c
sch(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.25, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2696, 2866, 12, 2659, 208}

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{3\left(a^2-b^2\right)^2}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) + ((3*b^3 + a*(2*a^2 - 5*b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2) +
 ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\int \frac{\left(-2a^2+3b^2-2ab\cosh(x)\right)\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\int \frac{3b^4}{a+b\cosh(x)}\,dx}{3\left(a^2-b^2\right)^2}$$

$$= \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{b^4\int \frac{1}{a+b\cosh(x)}\,dx}{\left(a^2-b^2\right)^2}$$

$$= \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} + \frac{\left(2b^4\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-}\right.}{\left(a^2-\right.}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{\left(3b^3+a\left(2a^2-5b^2\right)\cosh(x)\right)\operatorname{csch}(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.57, size = 141, normalized size = 1.28

$$\frac{1}{24}\left(-\frac{48b^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} - \frac{14b\tanh\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{2(4a+7b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} - \frac{\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right)}{2(a+b)} + \frac{8\sinh}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] ((-48*b^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
+ (2*(4*a + 7*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) -
 (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])/(a - b)^2 - (14*b*Tanh
[x/2])/(a - b)^2)/24
```

fricas [B] time = 0.50, size = 2339, normalized size = 21.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(a^2*b^3 - b^5)*cosh(x)^5 + 6*(a^2*b^3 - b^5)*sinh(x)^5 + 4*a^5 - 1
4*a^3*b^2 + 10*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^4 -
 5*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cos
h(x)^3 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2 - 6*
(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(
x)^2 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 + 3*(a^3
*b^2 - a*b^4)*cosh(x)^2 - (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2
+ 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^4*cosh(x
)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4 + 4*(5*b^
4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b^4*cosh(x)
^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*cosh(x))*sin
h(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) + b)) + 6*(a^2*b^3 - b^5)*cosh(x) + 6*(a^2*b^3 - b^5 + 5*(a^2*b
^3 - b^5)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 + 2*(2*a^4*b - 7*a^2*b^
3 + 5*b^5)*cosh(x)^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 -
 a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3
*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 +
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), 2/3*(3*(a^2*b^3 - b^
5)*cosh(x)^5 + 3*(a^2*b^3 - b^5)*sinh(x)^5 + 2*a^5 - 7*a^3*b^2 + 5*a*b^4 -
3*(a^3*b^2 - a*b^4)*cosh(x)^4 - 3*(a^3*b^2 - a*b^4 - 5*(a^2*b^3 - b^5)*cosh
(x))*sinh(x)^4 + 2*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^3 + 2*(2*a^4*b - 7
*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2 - 6*(a^3*b^2 - a*b^4)*cosh(
x))*sinh(x)^3 - 6*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 6*(a^5 - 3*a^3*b^
2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 + 3*(a^3*b^2 - a*b^4)*cosh(x)^2 -
 (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^
4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3
*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(
x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(
b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 3*(a^2*b^
3 - b^5)*cosh(x) + 3*(a^2*b^3 - b^5 + 5*(a^2*b^3 - b^5)*cosh(x)^4 - 4*(a^3*
b^2 - a*b^4)*cosh(x)^3 + 2*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^2 - 4*(a^5
 - 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4
```

```
+ b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh
(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
h(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x))*sinh(x))]
```

giac [A] time = 0.13, size = 156, normalized size = 1.42

$$\frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{2\left(3\,b^3e^{(5\,x)}-3\,ab^2e^{(4\,x)}+4\,a^2be^{(3\,x)}-10\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}+12\,ab^2e^{(2\,x)}+3\,b^3e^x+\right.}{3\left(a^4-2\,a^2b^2+b^4\right)\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a
^2 + b^2)) + 2/3*(3*b^3*e^(5*x) - 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) - 10*b^
3*e^(3*x) - 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*b^3*e^x + 2*a^3 - 5*a*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

maple [A] time = 0.09, size = 127, normalized size = 1.15

$$-\frac{\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{3}-3a\tanh\left(\frac{x}{2}\right)+5\tanh\left(\frac{x}{2}\right)b}{8\left(a-b\right)^2}+\frac{2b^4\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a-b\right)^2\left(a+b\right)^2\sqrt{\left(a+b\right)\left(a-b\right)}}-\frac{1}{24\left(a+b\right)\tanh\left(\frac{x}{2}\right)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*cosh(x)),x)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*tanh(1/2*x)^3*b-3*a*tanh(1/2*x)+5*tan
h(1/2*x)*b)+2/(a-b)^2/(a+b)^2*b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-5*b)/t
anh(1/2*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.97, size = 642, normalized size = 5.84

$$\frac{\frac{4\left(a\,b^2-a^3\right)}{\left(a^2-b^2\right)^2}+\frac{8\,e^x\left(a^2\,b-b^3\right)}{3\left(a^2-b^2\right)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{2\,a\,b^2}{\left(a^2-b^2\right)^2}-\frac{2\,b^3\,e^x}{\left(a^2-b^2\right)^2}}{e^{2\,x}-1}-\frac{\frac{8\,a}{3\left(a^2-b^2\right)}-\frac{8\,b\,e^x}{3\left(a^2-b^2\right)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}+\frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,b^2}{\left(a^2-b^2\right)^2\sqrt{b^8}\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\cdots}{b^6\,\sqrt{\cdots}}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*cosh(x))),x)
```

```
[Out] ((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^
2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*a*b^2)/(a^2 - b^2)^2 - (2*b^3*exp(x))
/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*(
a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + (2*atan((exp(x)*((2
*b^2)/((a^2 - b^2)^2*(b^8)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*(a^5*(b^8)
^(1/2) - 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 - b^2)^5)^
(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^
6*b^4 + 5*a^8*b^2)^(1/2))) + (2*a*(b^5*(b^8)^(1/2) - 2*a^2*b^3*(b^8)^(1/2)
+ a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(
b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)))*((b^
5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2
- a^2*b^3*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(
1/2) + (a^4*b*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^
2)^(1/2))/2))*(b^8)^(1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b
^4 + 5*a^8*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*cosh(x)), x)
```

3.176 $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=151

$$\frac{(3a^2+9ab+8b^2)\log(1-\cosh(x))}{16(a+b)^3}-\frac{(3a^2-9ab+8b^2)\log(\cosh(x)+1)}{16(a-b)^3}+\frac{\operatorname{csch}^4(x)(b-a\cosh(x))}{4(a^2-b^2)}+\frac{b^5\log(a+b}{(a^2-b}$$

```
[Out] 1/8*(4*b^3+a*(3*a^2-7*b^2)*cosh(x))*csch(x)^2/(a^2-b^2)^2+1/4*(b-a*cosh(x))
*csch(x)^4/(a^2-b^2)+1/16*(3*a^2+9*a*b+8*b^2)*ln(1-cosh(x))/(a+b)^3-1/16*(3
*a^2-9*a*b+8*b^2)*ln(1+cosh(x))/(a-b)^3+b^5*ln(a+b*cosh(x))/(a^2-b^2)^3
```

Rubi [A] time = 0.25, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2668, 741, 823, 801}

$$\frac{b^5\log(a+b\cosh(x))}{(a^2-b^2)^3}+\frac{(3a^2+9ab+8b^2)\log(1-\cosh(x))}{16(a+b)^3}-\frac{(3a^2-9ab+8b^2)\log(\cosh(x)+1)}{16(a-b)^3}+\frac{\operatorname{csch}^4(x)(b-a}{4(a^2-b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] ((4*b^3 + a*(3*a^2 - 7*b^2)*Cosh[x])*Csch[x]^2)/(8*(a^2 - b^2)^2) + ((b - a
*Cosh[x])*Csch[x]^4)/(4*(a^2 - b^2)) + ((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Cos
h[x]])/(16*(a + b)^3) - ((3*a^2 - 9*a*b + 8*b^2)*Log[1 + Cosh[x]])/(16*(a -
 b)^3) + (b^5*Log[a + b*Cosh[x]])/(a^2 - b^2)^3
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = -\left(b^5\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\cosh(x)\right)\right)$$

$$= \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} - \frac{b^3\operatorname{Subst}\left(\int \frac{3a^2-4b^2+3ax}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)}{4\left(a^2-b^2\right)}$$

$$= \frac{\left(4b^3+a\left(3a^2-7b^2\right)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \frac{-3a^4+7a^2b}{(a}\right.}{}$$

$$= \frac{\left(4b^3+a\left(3a^2-7b^2\right)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{b\operatorname{Subst}\left(\int \left(-\frac{(a-b)^2(}{2b(}\right.\right.}{}$$

$$= \frac{\left(4b^3+a\left(3a^2-7b^2\right)\cosh(x)\right)\operatorname{csch}^2(x)}{8\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^4(x)}{4\left(a^2-b^2\right)} + \frac{\left(3a^2+9ab+8b^2\right)\text{l}}{16(a+}$$

Mathematica [A] time = 0.91, size = 148, normalized size = 0.98

$$\frac{1}{64}\left(\frac{2\left(3a^2-8ab+5b^2\right)\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{8\left(a\left(3a^4-10a^2b^2+15b^4\right)\log\left(\tanh\left(\frac{x}{2}\right)\right)+8b^5\log(a+b\cosh(x))-8b^5\log(\sinh(x))\right)}{(a+b)^3}+(a-b)^2\operatorname{sech}}{(a-b)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^5/(a + b*Cosh[x]),x]
```

```
[Out] ((2*(3*a + 5*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) + ((8*(8*b^5*L
og[a + b*Cosh[x]] - 8*b^5*Log[Sinh[x]] + a*(3*a^4 - 10*a^2*b^2 + 15*b^4)*Lo
g[Tanh[x/2]]))/(a + b)^3 + 2*(3*a^2 - 8*a*b + 5*b^2)*Sech[x/2]^2 + (a - b)^
2*Sech[x/2]^4)/(a - b)^3)/64
```

fricas [B] time = 0.59, size = 3450, normalized size = 22.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + 2*(3*a^5 - 10*a^3*b^2 + 7
*a*b^4)*sinh(x)^7 + 16*(a^2*b^3 - b^5)*cosh(x)^6 + 2*(8*a^2*b^3 - 8*b^5 + 7
*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 - 2*(11*a^5 - 26*a^3*b^2
 + 15*a*b^4)*cosh(x)^5 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4 - 21*(3*a^5 - 10
*a^3*b^2 + 7*a*b^4)*cosh(x)^2 - 48*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 + 32*
(a^4*b - 3*a^2*b^3 + 2*b^5)*cosh(x)^4 + 2*(16*a^4*b - 48*a^2*b^3 + 32*b^5 +
 35*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 - b^5)*cosh(x)^
2 - 5*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - 2*(11*a^5 - 26*
a^3*b^2 + 15*a*b^4)*cosh(x)^3 - 2*(11*a^5 - 26*a^3*b^2 + 15*a*b^4 - 35*(3*a
^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 - b^5)*cosh(x)^3 + 10*(
11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b - 3*a^2*b^3 + 2*b^5)*
cosh(x))*sinh(x)^3 + 16*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(21*(3*a^5 - 10*a^3*b
^2 + 7*a*b^4)*cosh(x)^5 + 8*a^2*b^3 - 8*b^5 + 120*(a^2*b^3 - b^5)*cosh(x)^4
 - 10*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b - 3*a^2*b^3 +
2*b^5)*cosh(x)^2 - 3*(11*a^5 - 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 +
2*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x) + 8*(b^5*cosh(x)^8 + 8*b^5*cosh(x)
*sinh(x)^7 + b^5*sinh(x)^8 - 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 - 4*b^5*cosh
(x)^2 + 4*(7*b^5*cosh(x)^2 - b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 - 3*b^5*co
sh(x))*sinh(x)^5 + b^5 + 2*(35*b^5*cosh(x)^4 - 30*b^5*cosh(x)^2 + 3*b^5)*si
nh(x)^4 + 8*(7*b^5*cosh(x)^5 - 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3
+ 4*(7*b^5*cosh(x)^6 - 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 - b^5)*sinh(x)^2
+ 8*(b^5*cosh(x)^7 - 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 - b^5*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - ((3*a^5 - 10*a^3*b^2 + 15*
a*b^4 + 8*b^5)*cosh(x)^8 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x
)*sinh(x)^7 + (3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*sinh(x)^8 - 4*(3*a^5
- 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^6 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b
^4 + 8*b^5 - 7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^6
 + 8*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^3 - 3*(3*a^5 - 10*a
^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x))*sinh(x)^5 + 3*a^5 - 10*a^3*b^2 + 15*a*b
```

^4 + 8*b^5 + 6*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^4 + 2*(9*a^5
 - 30*a^3*b^2 + 45*a*b^4 + 24*b^5 + 35*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b
^5)*cosh(x)^4 - 30*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(
x)^4 + 8*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^5 - 10*(3*a^5 -
 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^3 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^
4 + 8*b^5)*cosh(x))*sinh(x)^3 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*c
osh(x)^2 + 4*(7*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^6 - 3*a^5 +
 10*a^3*b^2 - 15*a*b^4 - 8*b^5 - 15*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)
cosh(x)^4 + 9(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^2)*sinh(x)^2
 + 8*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^7 - 3*(3*a^5 - 10*a^3
*b^2 + 15*a*b^4 + 8*b^5)*cosh(x)^5 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b
^5)*cosh(x)^3 - (3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cosh(x))*sinh(x))*l
og(cosh(x) + sinh(x) + 1) + ((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x
)^8 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)*sinh(x)^7 + (3*a^5
- 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*sinh(x)^8 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b
^4 - 8*b^5)*cosh(x)^6 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5 - 7*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 - 10*a
^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^3 - 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8
*b^5)*cosh(x))*sinh(x)^5 + 3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5 + 6*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 + 2*(9*a^5 - 30*a^3*b^2 + 45*a*
b^4 - 24*b^5 + 35*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 - 30*(3
*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 -
10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^5 - 10*(3*a^5 - 10*a^3*b^2 + 15*a*b^
4 - 8*b^5)*cosh(x)^3 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x))*s
inh(x)^3 - 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2 + 4*(7*(3*a^
5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^6 - 3*a^5 + 10*a^3*b^2 - 15*a*b^
4 + 8*b^5 - 15*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^4 + 9*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5 - 10*a^3
*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^7 - 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b
^5)*cosh(x)^5 + 3*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x)^3 - (3*a^
5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
- 1) + 2*(7*(3*a^5 - 10*a^3*b^2 + 7*a*b^4)*cosh(x)^6 + 48*(a^2*b^3 - b^5)*c
osh(x)^5 + 3*a^5 - 10*a^3*b^2 + 7*a*b^4 - 5*(11*a^5 - 26*a^3*b^2 + 15*a*b^4
)*cosh(x)^4 + 64*(a^4*b - 3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(11*a^5 - 26*a^3
*b^2 + 15*a*b^4)*cosh(x)^2 + 16*(a^2*b^3 - b^5)*cosh(x))*sinh(x))/((a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 8*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^8 - 4*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 - 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6 - 7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^6 + a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 8*(7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5 + 6*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 2*(3*a^6 - 9*a^4*b^2 + 9*a^2*b^
4 - 3*b^6 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 30*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^5 - 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^

```
3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 - 4*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 4*(7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 15*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^4 + 9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2
)*sinh(x)^2 + 8*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 - 3*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^3 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

giac [B] time = 0.16, size = 338, normalized size = 2.24

$$\frac{b^6 \log\left(\left|b\left(e^{(-x)}+e^x\right)+2a\right|\right)}{a^6b-3a^4b^3+3a^2b^5-b^7}-\frac{\left(3a^2-9ab+8b^2\right)\log\left(e^{(-x)}+e^x+2\right)}{16\left(a^3-3a^2b+3ab^2-b^3\right)}+\frac{\left(3a^2+9ab+8b^2\right)\log\left(e^{(-x)}+e^x-2\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
- 1/16*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + 1/16*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) + e^x - 2)/(a^3 + 3*a^2
*b + 3*a*b^2 + b^3) + 1/4*(3*b^5*(e^(-x) + e^x)^4 + 3*a^5*(e^(-x) + e^x)^3
- 10*a^3*b^2*(e^(-x) + e^x)^3 + 7*a*b^4*(e^(-x) + e^x)^3 + 8*a^2*b^3*(e^(-x
) + e^x)^2 - 32*b^5*(e^(-x) + e^x)^2 - 20*a^5*(e^(-x) + e^x) + 56*a^3*b^2*(
e^(-x) + e^x) - 36*a*b^4*(e^(-x) + e^x) + 16*a^4*b - 64*a^2*b^3 + 96*b^5)/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

maple [A] time = 0.09, size = 191, normalized size = 1.26

$$\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)a}{64\left(a-b\right)^2}-\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)b}{64\left(a-b\right)^2}-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)a}{8\left(a-b\right)^2}+\frac{3\left(\tanh^2\left(\frac{x}{2}\right)\right)b}{16\left(a-b\right)^2}+\frac{b^5\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)}{\left(a-b\right)^3\left(a+b\right)^3}-\frac{}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^5/(a+b*cosh(x)),x)
```

```
[Out] 1/64/(a-b)^2*tanh(1/2*x)^4*a-1/64/(a-b)^2*tanh(1/2*x)^4*b-1/8/(a-b)^2*tanh(
1/2*x)^2*a+3/16/(a-b)^2*tanh(1/2*x)^2*b+1/(a-b)^3*b^5/(a+b)^3*ln(a*tanh(1/2
*x)^2-tanh(1/2*x)^2*b-a-b)-1/64/(a+b)/tanh(1/2*x)^4+1/8/(a+b)^2/tanh(1/2*x)
^2*a+3/16/(a+b)^2/tanh(1/2*x)^2*b+3/8/(a+b)^3*ln(tanh(1/2*x))*a^2+9/8/(a+b)
^3*ln(tanh(1/2*x))*a*b+1/(a+b)^3*ln(tanh(1/2*x))*b^2
```

maxima [B] time = 0.40, size = 348, normalized size = 2.30

$$\frac{b^5\log\left(2ae^{(-x)}+be^{(-2x)}+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{\left(3a^2-9ab+8b^2\right)\log\left(e^{(-x)}+1\right)}{8\left(a^3-3a^2b+3ab^2-b^3\right)}+\frac{\left(3a^2+9ab+8b^2\right)\log\left(e^{(-x)}-1\right)}{8\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{8b^3e^{(-2x)}}{4\left(a^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] b^5*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
1/8*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3)
 + 1/8*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) + 1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 - 7*a*b^2)*e^(-x) - (11
*a^3 - 15*a*b^2)*e^(-3*x) + 16*(a^2*b - 2*b^3)*e^(-4*x) - (11*a^3 - 15*a*b^
2)*e^(-5*x) + (3*a^3 - 7*a*b^2)*e^(-7*x))/(a^4 - 2*a^2*b^2 + b^4 - 4*(a^4 -
 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2 + b^4)*e^(-4*x) - 4*(a^4 -
2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b^4)*e^(-8*x))
```

mupad [B] time = 1.83, size = 559, normalized size = 3.70

$$\frac{\frac{4b}{a^2-b^2}-\frac{4ae^x}{a^2-b^2}}{6e^{4x}-4e^{2x}-4e^{6x}+e^{8x}+1}-\frac{\frac{2\left(b^5-a^2b^3\right)}{\left(a^2-b^2\right)^3}-\frac{e^x\left(3a^5-10a^3b^2+7ab^4\right)}{4\left(a^2-b^2\right)^3}}{e^{2x}-1}+\frac{\frac{8\left(a^2b-b^3\right)}{\left(a^2-b^2\right)^2}+\frac{6e^x\left(ab^2-a^3\right)}{\left(a^2-b^2\right)^2}}{3e^{2x}-3e^{4x}+e^{6x}-1}+\frac{\frac{2\left(2a^2b-b^3\right)}{\left(a^2-b^2\right)^2}-\frac{e^x\left(a^3+3a\cdots\right.}{2\left(a^2-b^2\cdots\right.}}{e^{4x}-2e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^5*(a + b*cosh(x))),x)
```

```
[Out] ((4*b)/(a^2 - b^2) - (4*a*exp(x))/(a^2 - b^2))/(6*exp(4*x) - 4*exp(2*x) - 4
*exp(6*x) + exp(8*x) + 1) - ((2*(b^5 - a^2*b^3))/(a^2 - b^2)^3 - (exp(x)*(7
*a*b^4 + 3*a^5 - 10*a^3*b^2))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) + ((8*(a^2*
b - b^3))/(a^2 - b^2)^2 + (6*exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(3*exp(2*
x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*(2*a^2*b - b^3))/(a^2 - b^2)^2 - (exp
(x)*(3*a*b^2 + a^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) + (b^5*
log(256*b^11*exp(2*x) - 9*a^10*b + 256*b^11 - 225*a^2*b^9 + 300*a^4*b^7 - 1
90*a^6*b^5 + 60*a^8*b^3 - 18*a^11*exp(x) - 225*a^2*b^9*exp(2*x) + 300*a^4*b
^7*exp(2*x) - 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b^10*exp(x
) - 9*a^10*b*exp(2*x) - 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) - 380*a^7*b
^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (log
(exp(x) - 1)*(9*a*b + 3*a^2 + 8*b^2))/(24*a*b^2 + 24*a^2*b + 8*a^3 + 8*b^3)
 - (log(exp(x) + 1)*(3*a^2 - 9*a*b + 8*b^2))/(24*a*b^2 - 24*a^2*b + 8*a^3 -
 8*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**5/(a+b*cosh(x)),x)
```

```
[Out] Integral(csch(x)**5/(a + b*cosh(x)), x)
```

3.177 $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=159

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}+\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{15(a^2-b^2)^2}+\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(\right.}{15(a^2-b^2)^3}$$

```
[Out] 2*b^6*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)+
1/15*(15*b^5-a*(8*a^4-26*a^2*b^2+33*b^4)*cosh(x))*csch(x)/(a^2-b^2)^3+1/15*
(5*b^3+a*(4*a^2-9*b^2)*cosh(x))*csch(x)^3/(a^2-b^2)^2+1/5*(b-a*cosh(x))*csc
h(x)^5/(a^2-b^2)
```

Rubi [A] time = 0.48, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2696, 2866, 12, 2659, 208}

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}+\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{15(a^2-b^2)^2}+\frac{\operatorname{csch}(x)\left(15b^5-a\left(-26a^2b^2+8a^4+33b^4\right)\cos\right.}{15(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] (2*b^6*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)
^(7/2)) + ((15*b^5 - a*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Cosh[x])*Csch[x])/(15*
(a^2 - b^2)^3) + ((5*b^3 + a*(4*a^2 - 9*b^2)*Cosh[x])*Csch[x]^3)/(15*(a^2 -
 b^2)^2) + ((b - a*Cosh[x])*Csch[x]^5)/(5*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{(b-a\cosh(x))\operatorname{csch}^5(x)}{5(a^2-b^2)} + \frac{\int \frac{(-4a^2+5b^2-4ab\cosh(x))\operatorname{csch}^4(x)}{a+b\cosh(x)}dx}{5(a^2-b^2)}$$

$$= \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\operatorname{csch}^3(x)}{15\left(a^2-b^2\right)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^5(x)}{5(a^2-b^2)} + \frac{\int \frac{(8a^4-18a^2b^2+15b^4+2ab(\ldots}{a+b\ldots}}{15(a\ldots}$$

$$= \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\operatorname{csch}^3(x\ldots}{15\left(a^2-b^2\right)^2}$$

$$= \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\operatorname{csch}^3(x\ldots}{15\left(a^2-b^2\right)^2}$$

$$= \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{\left(5b^3+a\left(4a^2-9b^2\right)\cosh(x)\right)\operatorname{csch}^3(x\ldots}{15\left(a^2-b^2\right)^2}$$

$$= \frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} + \frac{\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)\operatorname{csch}(x)}{15\left(a^2-b^2\right)^3} + \frac{(5b^3+a\ldots}{}$$

Mathematica [A] time = 1.81, size = 201, normalized size = 1.26

$$\frac{1}{480}\left(-\frac{2\left(64a^2-183ab+149b^2\right)\tanh\left(\frac{x}{2}\right)}{(a-b)^3} - \frac{2\left(64a^2+183ab+149b^2\right)\coth\left(\frac{x}{2}\right)}{(a+b)^3} + \frac{960b^6\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^6/(a + b*Cosh[x]),x]
```

```
[Out] ((960*b^6*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2)
- (2*(64*a^2 + 183*a*b + 149*b^2)*Coth[x/2])/(a + b)^3 - (8*(19*a - 29*b)*C
sch[x]^3*Sinh[x/2]^4)/(a - b)^2 - (96*Csch[x]^5*Sinh[x/2]^6)/(a - b) + ((19
*a + 29*b)*Csch[x/2]^4*Sinh[x])/(2*(a + b)^2) - (3*Csch[x/2]^6*Sinh[x])/(2*
(a + b)) - (2*(64*a^2 - 183*a*b + 149*b^2)*Tanh[x/2])/(a - b)^3)/480
```

fricas [B] time = 0.65, size = 6381, normalized size = 40.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="fricas")

[Out] [1/15*(30*(a^2*b^5 - b^7)*cosh(x)^9 + 30*(a^2*b^5 - b^7)*sinh(x)^9 - 30*(a^
3*b^4 - a*b^6)*cosh(x)^8 - 30*(a^3*b^4 - a*b^6 - 9*(a^2*b^5 - b^7)*cosh(x))
*sinh(x)^8 + 40*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^7 + 40*(a^4*b^3 - 5*a
^2*b^5 + 4*b^7 + 27*(a^2*b^5 - b^7)*cosh(x)^2 - 6*(a^3*b^4 - a*b^6)*cosh(x)
)*sinh(x)^7 - 16*a^7 + 68*a^5*b^2 - 118*a^3*b^4 + 66*a*b^6 - 60*(a^5*b^2 -
4*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 20*(3*a^5*b^2 - 12*a^3*b^4 + 9*a*b^6 - 126
*(a^2*b^5 - b^7)*cosh(x)^3 + 42*(a^3*b^4 - a*b^6)*cosh(x)^2 - 14*(a^4*b^3 -
 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 4*(24*a^6*b - 92*a^4*b^3 + 157*a^2
*b^5 - 89*b^7)*cosh(x)^5 + 4*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7
+ 945*(a^2*b^5 - b^7)*cosh(x)^4 - 420*(a^3*b^4 - a*b^6)*cosh(x)^3 + 210*(a^
4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*c
osh(x))*sinh(x)^5 - 20*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)
^4 - 20*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6 - 189*(a^2*b^5 - b^7)*c
osh(x)^5 + 105*(a^3*b^4 - a*b^6)*cosh(x)^4 - 70*(a^4*b^3 - 5*a^2*b^5 + 4*b^
7)*cosh(x)^3 + 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^2 - (24*a^6*b - 9
2*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x))*sinh(x)^4 + 40*(a^4*b^3 - 5*a^2*
b^5 + 4*b^7)*cosh(x)^3 + 40*(a^4*b^3 - 5*a^2*b^5 + 4*b^7 + 63*(a^2*b^5 - b^
7)*cosh(x)^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^5 + 35*(a^4*b^3 - 5*a^2*b^5 + 4
*b^7)*cosh(x)^4 - 30*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^3 + (24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^2 - 2*(8*a^7 - 31*a^5*b^2 + 47
*a^3*b^4 - 24*a*b^6)*cosh(x))*sinh(x)^3 + 20*(4*a^7 - 17*a^5*b^2 + 28*a^3*b
^4 - 15*a*b^6)*cosh(x)^2 + 20*(54*(a^2*b^5 - b^7)*cosh(x)^7 + 4*a^7 - 17*a^
5*b^2 + 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^6 + 42*(a^4*b^
3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(
x)^4 + 2*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^3 - 6*(8*a^
7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)^2 + 6*(a^4*b^3 - 5*a^2*b^5
+ 4*b^7)*cosh(x))*sinh(x)^2 - 15*(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9
 + b^6*sinh(x)^10 - 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 - 10*b^6*cosh(x)^4 +
 5*(9*b^6*cosh(x)^2 - b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^
3 - b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 - 14*b^6*cosh(x)^2 + b^6)
*sinh(x)^6 - b^6 + 4*(63*b^6*cosh(x)^5 - 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))
*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 - 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 - b
^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 - 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 - b
^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 - 28*b^6*cosh(x)^6 + 30*b^6*cosh
(x)^4 - 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 - 4*b^6*cosh(
x)^7 + 6*b^6*cosh(x)^5 - 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x))*sqrt(a^2 -
 b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*
(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a)
)/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)
) + 30*(a^2*b^5 - b^7)*cosh(x) + 10*(27*(a^2*b^5 - b^7)*cosh(x)^8 - 24*(a^3
*b^4 - a*b^6)*cosh(x)^7 + 3*a^2*b^5 - 3*b^7 + 28*(a^4*b^3 - 5*a^2*b^5 + 4*b
```

^7)*cosh(x)^6 - 36*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^5 + 2*(24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^4 - 8*(8*a^7 - 31*a^5*b^2 + 47
*a^3*b^4 - 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2
 + 4*(4*a^7 - 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6)*cosh(x))*sinh(x))/((a^8 -
 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*(a^8 - 4*a^6*b^2
+ 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*sinh(x)^10 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*cosh(x)^8 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 -
9*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^8 - a^
8 + 4*a^6*b^2 - 6*a^4*b^4 + 4*a^2*b^6 - b^8 + 40*(3*(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b
^6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
+ b^8)*cosh(x)^6 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 + 21*(
a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 14*(a^8 - 4*a^6*
b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 - 4*a^
6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 70*(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^
2*b^6 + b^8)*cosh(x))*sinh(x)^5 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b
^6 + b^8)*cosh(x)^4 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 - 2
1*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^6 + 35*(a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 15*(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^8 - 4*a^6*b^2 + 6
*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 - 7*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*
a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh
(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2 + 5*(9*
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^8 + a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 - 28*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*cosh(x)^6 + 30*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cos
h(x)^4 - 12*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh
(x)^2 + 10*((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^9 - 4*(
a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 + 6*(a^8 - 4*a^6*b
^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 4*(a^8 - 4*a^6*b^2 + 6*a^4*b^
4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*cosh(x))*sinh(x)), 2/15*(15*(a^2*b^5 - b^7)*cosh(x)^9 + 15*(a^2*b^5 -
 b^7)*sinh(x)^9 - 15*(a^3*b^4 - a*b^6)*cosh(x)^8 - 15*(a^3*b^4 - a*b^6 - 9*
(a^2*b^5 - b^7)*cosh(x))*sinh(x)^8 + 20*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(
x)^7 + 20*(a^4*b^3 - 5*a^2*b^5 + 4*b^7 + 27*(a^2*b^5 - b^7)*cosh(x)^2 - 6*(
a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^7 - 8*a^7 + 34*a^5*b^2 - 59*a^3*b^4 + 33*
a*b^6 - 30*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 10*(3*a^5*b^2 - 12*a
^3*b^4 + 9*a*b^6 - 126*(a^2*b^5 - b^7)*cosh(x)^3 + 42*(a^3*b^4 - a*b^6)*cos
h(x)^2 - 14*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 2*(24*a^6*b
- 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^5 + 2*(24*a^6*b - 92*a^4*b^3 +
 157*a^2*b^5 - 89*b^7 + 945*(a^2*b^5 - b^7)*cosh(x)^4 - 420*(a^3*b^4 - a*b^
6)*cosh(x)^3 + 210*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 -

```
4*a^3*b^4 + 3*a*b^6)*cosh(x))*sinh(x)^5 - 10*(8*a^7 - 31*a^5*b^2 + 47*a^3*b
^4 - 24*a*b^6)*cosh(x)^4 - 10*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6 -
 189*(a^2*b^5 - b^7)*cosh(x)^5 + 105*(a^3*b^4 - a*b^6)*cosh(x)^4 - 70*(a^4*
b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^3 + 45*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cos
h(x)^2 - (24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x))*sinh(x)^4
+ 20*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^3 + 20*(a^4*b^3 - 5*a^2*b^5 + 4*
b^7 + 63*(a^2*b^5 - b^7)*cosh(x)^6 - 42*(a^3*b^4 - a*b^6)*cosh(x)^5 + 35*(a
^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^4 - 30*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*
cosh(x)^3 + (24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^2 - 2*(8
*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x))*sinh(x)^3 + 10*(4*a^7 -
 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6)*cosh(x)^2 + 10*(54*(a^2*b^5 - b^7)*cos
h(x)^7 + 4*a^7 - 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 - a*b^6)*
cosh(x)^6 + 42*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 - 4*a^
3*b^4 + 3*a*b^6)*cosh(x)^4 + 2*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^
7)*cosh(x)^3 - 6*(8*a^7 - 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)^2 + 6
*(a^4*b^3 - 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^2 - 15*(b^6*cosh(x)^10 + 10
*b^6*cosh(x)*sinh(x)^9 + b^6*sinh(x)^10 - 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^
6 - 10*b^6*cosh(x)^4 + 5*(9*b^6*cosh(x)^2 - b^6)*sinh(x)^8 + 5*b^6*cosh(x)^
2 + 40*(3*b^6*cosh(x)^3 - b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 - 1
4*b^6*cosh(x)^2 + b^6)*sinh(x)^6 - b^6 + 4*(63*b^6*cosh(x)^5 - 70*b^6*cosh(
x)^3 + 15*b^6*cosh(x))*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 - 35*b^6*cosh(x)^4
+ 15*b^6*cosh(x)^2 - b^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 - 7*b^6*cosh(x)^5
 + 5*b^6*cosh(x)^3 - b^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 - 28*b^6*c
osh(x)^6 + 30*b^6*cosh(x)^4 - 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*c
osh(x)^9 - 4*b^6*cosh(x)^7 + 6*b^6*cosh(x)^5 - 4*b^6*cosh(x)^3 + b^6*cosh(x
))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x
) + a)/(a^2 - b^2)) + 15*(a^2*b^5 - b^7)*cosh(x) + 5*(27*(a^2*b^5 - b^7)*co
sh(x)^8 - 24*(a^3*b^4 - a*b^6)*cosh(x)^7 + 3*a^2*b^5 - 3*b^7 + 28*(a^4*b^3
- 5*a^2*b^5 + 4*b^7)*cosh(x)^6 - 36*(a^5*b^2 - 4*a^3*b^4 + 3*a*b^6)*cosh(x)
^5 + 2*(24*a^6*b - 92*a^4*b^3 + 157*a^2*b^5 - 89*b^7)*cosh(x)^4 - 8*(8*a^7
- 31*a^5*b^2 + 47*a^3*b^4 - 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 - 5*a^2*b^5 +
 4*b^7)*cosh(x)^2 + 4*(4*a^7 - 17*a^5*b^2 + 28*a^3*b^4 - 15*a*b^6)*cosh(x))
*sinh(x))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 -
4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sinh(x)^10 - 5*(a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^8 - 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4
*a^2*b^6 + b^8 - 9*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^
2)*sinh(x)^8 - a^8 + 4*a^6*b^2 - 6*a^4*b^4 + 4*a^2*b^6 - b^8 + 40*(3*(a^8 -
 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^6 + 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8 + 21*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 -
 14*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 +
4*(63*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 70*(a^8 -
 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 - 4*a^6*b^2 +
```

```
 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh(x)^5 - 10*(a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 10*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*
a^2*b^6 + b^8 - 21*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^
6 + 35*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^4 - 15*(a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^
8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 - 7*(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
- 4*a^2*b^6 + b^8)*cosh(x)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b
^8)*cosh(x))*sinh(x)^3 + 5*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*
cosh(x)^2 + 5*(9*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^8
+ a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8 - 28*(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^6 + 30*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a
^2*b^6 + b^8)*cosh(x)^4 - 12*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*cosh(x)^2)*sinh(x)^2 + 10*((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8
)*cosh(x)^9 - 4*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^7 +
 6*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^5 - 4*(a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*
b^4 - 4*a^2*b^6 + b^8)*cosh(x))*sinh(x))]
```

giac [B] time = 0.19, size = 303, normalized size = 1.91

$$\frac{2\,b^{6}\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{-a^{2}+b^{2}}}+\frac{2\left(15\,b^{5}e^{(9\,x)}-15\,ab^{4}e^{(8\,x)}+20\,a^{2}b^{3}e^{(7\,x)}-80\,b^{5}e^{(7\,x)}-30\,a^{3}b^{2}e^{(6\,x)}+90\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b^6*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*sqrt(-a^2 + b^2)) + 2/15*(15*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b
^3*e^(7*x) - 80*b^5*e^(7*x) - 30*a^3*b^2*e^(6*x) + 90*a*b^4*e^(6*x) + 48*a^
4*b*e^(5*x) - 136*a^2*b^3*e^(5*x) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) + 230*
a^3*b^2*e^(4*x) - 240*a*b^4*e^(4*x) + 20*a^2*b^3*e^(3*x) - 80*b^5*e^(3*x) +
 40*a^5*e^(2*x) - 130*a^3*b^2*e^(2*x) + 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*
a^5 + 26*a^3*b^2 - 33*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(e^(2*x)
- 1)^5)
```

maple [A] time = 0.09, size = 213, normalized size = 1.34

$$-\frac{\frac{a^{2}\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{5}-\frac{2\left(\tanh^{5}\left(\frac{x}{2}\right)\right)ab}{5}+\frac{b^{2}\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{5}-\frac{5a^{2}\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}+4a\left(\tanh^{3}\left(\frac{x}{2}\right)\right)b-\frac{7b^{2}\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}+10a^{2}\tanh\left(\frac{x}{2}\right)-28ab}{32\left(a-b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^6/(a+b*cosh(x)),x)
```

```
[Out] -1/32/(a-b)^3*(1/5*a^2*tanh(1/2*x)^5-2/5*tanh(1/2*x)^5*a*b+1/5*b^2*tanh(1/2
*x)^5-5/3*a^2*tanh(1/2*x)^3+4*a*tanh(1/2*x)^3*b-7/3*b^2*tanh(1/2*x)^3+10*a^
2*tanh(1/2*x)-28*a*b*tanh(1/2*x)+22*b^2*tanh(1/2*x))+2/(a-b)^3/(a+b)^3*b^6/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/160/(a
+b)/tanh(1/2*x)^5-1/96*(-5*a-7*b)/(a+b)^2/tanh(1/2*x)^3-1/32/(a+b)^3*(10*a^
2+28*a*b+22*b^2)/tanh(1/2*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.61, size = 1031, normalized size = 6.48

$$\frac{\frac{16\left(a\,b^2-a^3\right)}{{\left(a^2-b^2\right)}^2}+\frac{64\,{\mathrm{e}}^x\,\left(a^2\,b-b^3\right)}{5\,{\left(a^2-b^2\right)}^2}}{6\,{\mathrm{e}}^{4\,x}-4\,{\mathrm{e}}^{2\,x}-4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1}-\frac{\frac{2\,a\,b^4}{{\left(a^2-b^2\right)}^3}-\frac{2\,b^5\,{\mathrm{e}}^x}{{\left(a^2-b^2\right)}^3}}{{\mathrm{e}}^{2\,x}-1}-\frac{\frac{32\,a}{5\,\left(a^2-b^2\right)}-\frac{32\,b\,{\mathrm{e}}^x}{5\,\left(a^2-b^2\right)}}{5\,{\mathrm{e}}^{2\,x}-10\,{\mathrm{e}}^{4\,x}+10\,{\mathrm{e}}^{6\,x}-5\,{\mathrm{e}}^{8\,x}+{\mathrm{e}}^{10\,x}-1}+\frac{\frac{8\left(3\,a\,b^2-4\,a^3\right)}{3\,{\left(a^2-b^2\right)}^2}+\frac{8\,\mathrm{e}}{}}{3\,{\mathrm{e}}^{2\,x}-3\,{\mathrm{e}}^{4\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^6*(a + b*cosh(x))),x)
```

```
[Out] ((16*(a*b^2 - a^3))/(a^2 - b^2)^2 + (64*exp(x)*(a^2*b - b^3))/(5*(a^2 - b^2
)^2))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - ((2*a*b^4)/(a
^2 - b^2)^3 - (2*b^5*exp(x))/(a^2 - b^2)^3)/(exp(2*x) - 1) - ((32*a)/(5*(a^
2 - b^2)) - (32*b*exp(x))/(5*(a^2 - b^2)))/(5*exp(2*x) - 10*exp(4*x) + 10*e
xp(6*x) - 5*exp(8*x) + exp(10*x) - 1) + ((8*(3*a*b^2 - 4*a^3))/(3*(a^2 - b^
2)^2) + (8*exp(x)*(12*a^2*b - 7*b^3))/(15*(a^2 - b^2)^2))/(3*exp(2*x) - 3*e
xp(4*x) + exp(6*x) - 1) + ((4*(a*b^4 - a^3*b^2))/(a^2 - b^2)^3 - (8*exp(x)*
(b^5 - a^2*b^3))/(3*(a^2 - b^2)^3))/(exp(4*x) - 2*exp(2*x) + 1) - (2*atan((
exp(x)*((2*b^4)/((a^2 - b^2)^3*(b^12)^(1/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) - 3*a^5*b^2*(b^12)^
(1/2) - a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 - b^2)^7)^(1/2)*(a^6 - b^6 + 3*a^2
*b^4 - 3*a^4*b^2)*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35
*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^12)^(1/2) - 3*a
```

```
^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) - a^6*b*(b^12)^(1/2)))/(b^8*(-
(a^2 - b^2)^7)^(1/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)*(b^14 - a^14 - 7*a
^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)
^(1/2)))*((b^7*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^
8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))/2 - (a^6*b*(b^14 - a^14 - 7*a^2*b^
12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2
))/2 - (3*a^2*b^5*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35
*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))/2 + (3*a^4*b^3*(b^14 - a^14 - 7
*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^
2)^(1/2))/2))*(b^12)^(1/2))/(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^
6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**6/(a+b*cosh(x)),x)
```

```
[Out] Timed out
```

3.178 $\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2-sinh(x)/b/(a+b*cosh(x))-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/
2))/b^2/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2693, 2735, 2659, 208}

$$-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

```
[Out] x/b^2 - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2
*Sqrt[a + b]) - Sinh[x]/(b*(a + b*Cosh[x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
```

```
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx &= -\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx}{b}\\ &= \frac{x}{b^2}-\frac{\sinh(x)}{b(a+b\cosh(x))}-\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{b^2}\\ &= \frac{x}{b^2}-\frac{\sinh(x)}{b(a+b\cosh(x))}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\ &= \frac{x}{b^2}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}}-\frac{\sinh(x)}{b(a+b\cosh(x))}\end{aligned}$$

Mathematica [A] time = 0.11, size = 61, normalized size = 0.91

$$\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-\frac{b\sinh(x)}{a+b\cosh(x)}+x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

```
[Out] (x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] -
(b*Sinh[x])/(a + b*Cosh[x]))/b^2
```

fricas [B] time = 0.59, size = 700, normalized size = 10.45

$$\left[\frac{\left(a^2b-b^3\right)x\cosh(x)^2+\left(a^2b-b^3\right)x\sinh(x)^2+2\,a^2b-2\,b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,a^2\cosh(x)+ab+2\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*
x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 - a*b^2)*x)*si
nh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)*sinh(x)
^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*cos
h(x))*sinh(x)), ((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*
a^2*b - 2*b^3 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*
(a*b*cosh(x) + a^2)*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
osh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (
a^3 - a*b^2)*x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 -
 a*b^2)*x)*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 -
 b^5)*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b
^3 - b^5)*cosh(x))*sinh(x))]
```

giac [A] time = 0.15, size = 68, normalized size = 1.01

$$-\frac{2\,a\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}+\frac{x}{b^{2}}+\frac{2\,(ae^{x}+b)}{\left(be^{(2\,x)}+2\,ae^{x}+b\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) + x/b^2 +
2*(a*e^x + b)/((b*e^(2*x) + 2*a*e^x + b)*b^2)
```

maple [A] time = 0.07, size = 99, normalized size = 1.48

$$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}+\frac{2\tanh\left(\frac{x}{2}\right)}{b\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-a-b\right)}-\frac{2a\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{2}\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*cosh(x))^2,x)
```

```
[Out] -1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)+2/b*tanh(1/2*x)/(a*tanh(1/
2*x)^2-tanh(1/2*x)^2*b-a-b)-2/b^2*a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(
1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.12, size = 139, normalized size = 2.07

$$\frac{x}{b^2}+\frac{\frac{2}{b}+\frac{2\,a\,\mathrm{e}^x}{b^2}}{b+2\,a\,\mathrm{e}^x+b\,\mathrm{e}^{2\,x}}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^3}-\frac{2\,a\,\left(b+a\,\mathrm{e}^x\right)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^3}+\frac{2\,a\,\left(b+a\,\mathrm{e}^x\right)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*cosh(x))^2,x)
```

```
[Out] x/b^2 + (2/b + (2*a*exp(x))/b^2)/(b + 2*a*exp(x) + b*exp(2*x)) + (a*log((2*
a*exp(x))/b^3 - (2*a*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b
^2*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^3 + (2*a*(b + a*exp
(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

3.179 $\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=113

$$\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{b\left(3a^2-2b^2\right)\tan^{-1}(\sinh(x))}{2a^4}-\frac{\left(4a^2-3b^2\right)\tanh(}{3a^3}$$

```
[Out] 1/2*b*(3*a^2-2*b^2)*arctan(sinh(x))/a^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh((
a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^4-1/3*(4*a^2-3*b^2)*tanh(x)/a^3-1/2*b
*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

Rubi [A] time = 0.41, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2725, 3055, 3001, 3770, 2659, 208}

$$-\frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3}+\frac{b\left(3a^2-2b^2\right)\tan^{-1}(\sinh(x))}{2a^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{b\tanh(x)\text{sech}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (b*(3*a^2 - 2*b^2)*ArcTan[Sinh[x]])/(2*a^4) + (2*(a - b)^(3/2)*(a + b)^(3/2
)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^4 - ((4*a^2 - 3*b^2)*Tanh
[x])/(3*a^3) - (b*Sech[x]*Tanh[x])/(2*a^2) + (Sech[x]^2*Tanh[x])/(3*a)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
```

```
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = -\frac{b\text{sech}(x)\tanh(x)}{2a^2}+\frac{\text{sech}^2(x)\tanh(x)}{3a}-\frac{\int \frac{\left(2\left(4a^2-3b^2\right)-ab\cosh(x)-3\left(2a^2-b^2\right)\cosh^2(x)\right)\text{sech}^2(x)}{a+b\cosh(x)}\,dx}{6a^2}$$

$$= -\frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3}-\frac{b\text{sech}(x)\tanh(x)}{2a^2}+\frac{\text{sech}^2(x)\tanh(x)}{3a}-\frac{\int \frac{\left(-3b\left(3a^2-2b^2\right)-3a\left(2a^2-b^2\right)\right.}{a+b\cosh(x)}}{6a^3}$$

$$= -\frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3}-\frac{b\text{sech}(x)\tanh(x)}{2a^2}+\frac{\text{sech}^2(x)\tanh(x)}{3a}+\frac{\left(b\left(3a^2-2b^2\right)\right)\int \text{sech}(}{2a^4}$$

$$= \frac{b\left(3a^2-2b^2\right)\tan^{-1}(\sinh(x))}{2a^4}-\frac{\left(4a^2-3b^2\right)\tanh(x)}{3a^3}-\frac{b\text{sech}(x)\tanh(x)}{2a^2}+\frac{\text{sech}^2(x)\tanh}{3a}$$

$$= \frac{b\left(3a^2-2b^2\right)\tan^{-1}(\sinh(x))}{2a^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{\left(4a^2-3b^2\right)}{3a^3}$$

Mathematica [A] time = 0.44, size = 100, normalized size = 0.88

$$\frac{-12\left(b^2-a^2\right)^{3/2}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)+6b\left(3a^2-2b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+a\tanh(x)\left(2a^2\text{sech}^2(x)-8a^2-3ab\text{sec}\right.}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (6*b*(3*a^2 - 2*b^2)*ArcTan[Tanh[x/2]] - 12*(-a^2 + b^2)^(3/2)*ArcTan[((a -
 b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + a*(-8*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*a^
2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

fricas [B] time = 1.57, size = 2003, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4
+ 3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(5*
a^2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2 -
12*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6*(2*
a^3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 3*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2
```

```
)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(
5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3
 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 -
 b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^
2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x
))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*(3*a^2*b - 2*b
^3)*cosh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a^2*b - 2*b^3)*c
osh(x)^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^4 +
4*(5*(3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cosh(x))*sinh(x)^3 +
 3*a^2*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b - 2*b^3)*c
osh(x)^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 6*(
(3*a^2*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*cosh(x)^3 + (3*a^2*b - 2*
b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(5*a^2*b*cosh(x)^4 - 8
*(2*a^3 - a*b^2)*cosh(x)^3 - a^2*b - 8*(a^3 - a*b^2)*cosh(x))*sinh(x))/(a^4
*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*
a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x
)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4
)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x)), -
1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4 +
3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(5*a^
2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2 - 12
*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6*(2*a^
3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2)*
cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(5*
(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3 +
 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 - b
^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2
+ 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))
*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x)
+ a)/(a^2 - b^2)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*(3*a^2*b - 2*b^3)*co
sh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a^2*b - 2*b^3)*cosh(x)
^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(
3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cosh(x))*sinh(x)^3 + 3*a^2
*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b - 2*b^3)*cosh(x)
^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^2
*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*cosh(x)^3 + (3*a^2*b - 2*b^3)*c
osh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(5*a^2*b*cosh(x)^4 - 8*(2*a^
3 - a*b^2)*cosh(x)^3 - a^2*b - 8*(a^3 - a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(
x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*co
sh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 +
3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh
(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))]
```

giac [A] time = 0.15, size = 144, normalized size = 1.27

$$\frac{\left(3\,a^{2}b-2\,b^{3}\right)\arctan\left(e^{x}\right)}{a^{4}}+\frac{2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,a^{4}}-\frac{3\,abe^{(5\,x)}-12\,a^{2}e^{(4\,x)}+6\,b^{2}e^{(4\,x)}-12\,a^{2}e^{(2\,x)}+\cdots}{3\,a^{3}\left(e^{(2\,x)}+1\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] (3*a^2*b - 2*b^3)*arctan(e^x)/a^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x
 + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(5*x) - 12*a^
2*e^(4*x) + 6*b^2*e^(4*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x - 8
*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

maple [B] time = 0.10, size = 315, normalized size = 2.79

$$\frac{2\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)\,(a-b)}}-\frac{4\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)b^{2}}{a^{2}\sqrt{(a+b)\,(a-b)}}+\frac{2b^{4}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{4}\sqrt{(a+b)\,(a-b)}}-\frac{2\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{\left(\tanh\cdots\right.}{a^{2}\left(\tanh\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*cosh(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-4/a^2/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*b^2+2/a^
4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*b^4-2/
a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5+1/a^2/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5
*b+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5*b^2-20/3/a/(tanh(1/2*x)^2+1)^3*t
anh(1/2*x)^3+4/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^3*b^2-2/a/(tanh(1/2*x)^2
+1)^3*tanh(1/2*x)+2/a^3/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)*b^2-1/a^2/(tanh(1/2
*x)^2+1)^3*tanh(1/2*x)*b+3/a^2*arctan(tanh(1/2*x))*b-2/a^4*arctan(tanh(1/2*
x))*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 5.94, size = 722, normalized size = 6.39

$$\frac{8}{3a\left(3e^{2x}+3e^{4x}+e^{6x}+1\right)}-\frac{\frac{4}{a}-\frac{2be^{x}}{a^{2}}}{2e^{2x}+e^{4x}+1}+\frac{\frac{2\left(2a^{2}-b^{2}\right)}{a^{3}}-\frac{be^{x}}{a^{2}}}{e^{2x}+1}+\frac{\ln\left(\left(\left(\frac{32a^{8}-288e^{x}a^{7}b-272a^{6}b^{2}+600e^{x}a^{5}b^{3}+456a^{4}b^{4}-416e^{x}a^{3}b^{5}-2\cdots}{a^{6}b^{4}}\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*cosh(x)),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*b*exp(x))/a^2)
/(2*exp(2*x) + exp(4*x) + 1) + ((2*(2*a^2 - b^2))/a^3 - (b*exp(x))/a^2)/(ex
p(2*x) + 1) + (log((((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456*a^4*b^4 - 272*a^6
*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*exp(x) + 600*a^5*b^
3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*b - 2*b^3 +
4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) + (16*(a^2 - b^2)*(4*a^2*b - 4*b^
3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)^3*(a - b)^3)^(1/2))/a
^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2)^2*(3*a^2 - 2*b^2)*(6*
a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^9*b^3))*((a + b)^3*(a -
 b)^3)^(1/2))/a^4 - (log(- (((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456*a^4*b^4 -
 272*a^6*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*exp(x) + 60
0*a^5*b^3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*b -
2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) - (16*(a^2 - b^2)*(4*a^2*
b - 4*b^3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)^3*(a - b)^3)^
(1/2))/a^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2)^2*(3*a^2 - 2*
b^2)*(6*a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^9*b^3))*((a + b
)^3*(a - b)^3)^(1/2))/a^4 - (b*log(exp(x) - 1i)*(3*a^2 - 2*b^2)*1i)/(2*a^4)
 + (b*log(exp(x) + 1i)*(3*a^2 - 2*b^2)*1i)/(2*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*cosh(x)),x)
[Out] Integral(tanh(x)**4/(a + b*cosh(x)), x)
```

3.180 $\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\left(a^2-b^2\right)\log(\cosh(x))}{a^3}-\frac{\left(a^2-b^2\right)\log(a+b\cosh(x))}{a^3}+\frac{\operatorname{sech}^2(x)}{2a}$$

```
[Out] (a^2-b^2)*ln(cosh(x))/a^3-(a^2-b^2)*ln(a+b*cosh(x))/a^3-b*sech(x)/a^2+1/2*s
ech(x)^2/a
```

Rubi [A] time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2721, 894}

$$\frac{\left(a^2-b^2\right)\log(\cosh(x))}{a^3}-\frac{\left(a^2-b^2\right)\log(a+b\cosh(x))}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] ((a^2 - b^2)*Log[Cosh[x]])/a^3 - ((a^2 - b^2)*Log[a + b*Cosh[x]])/a^3 - (b*
Sech[x])/a^2 + Sech[x]^2/(2*a)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = -\mathrm{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\cosh(x)\right)$$
$$= -\mathrm{Subst}\left(\int \left(\frac{b^2}{ax^3} - \frac{b^2}{a^2x^2} + \frac{-a^2+b^2}{a^3x} + \frac{a^2-b^2}{a^3(a+x)}\right) dx, x, b\cosh(x)\right)$$
$$= \frac{(a^2-b^2)\log(\cosh(x))}{a^3} - \frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3} - \frac{b\mathrm{sech}(x)}{a^2} + \frac{\mathrm{sech}^2(x)}{2a}$$

Mathematica [A] time = 0.10, size = 46, normalized size = 0.81

$$\frac{2(a^2-b^2)(\log(\cosh(x)) - \log(a+b\cosh(x))) + a^2\mathrm{sech}^2(x) - 2ab\mathrm{sech}(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (2*(a^2 - b^2)*(Log[Cosh[x]] - Log[a + b*Cosh[x]]) - 2*a*b*Sech[x] + a^2*Se
ch[x]^2)/(2*a^3)
```

fricas [B] time = 0.71, size = 450, normalized size = 7.89

$$-\frac{2\,ab\cosh(x)^3 + 2\,ab\sinh(x)^3 - 2\,a^2\cosh(x)^2 + 2\,ab\cosh(x) + 2\left(3\,ab\cosh(x) - a^2\right)\sinh(x)^2 + \left(\left(a^2-b^2\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 + 2*a*b*cosh(x) + 2*(
3*a*b*cosh(x) - a^2)*sinh(x)^2 + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cos
h(x)*sinh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^
2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh
(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh
(x))) - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b
^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2
- b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(
x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^2 - 2*a^
2*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*si
nh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(
a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))
```

giac [B] time = 0.13, size = 115, normalized size = 2.02

$$\frac{\left(a^2-b^2\right)\log\left(e^{(-x)}+e^x\right)}{a^3}-\frac{\left(a^2b-b^3\right)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^3b}-\frac{3\,a^2\left(e^{(-x)}+e^x\right)^2-3\,b^2\left(e^{(-x)}+e^x\right)^2+4\,ab\left(e^{(-x)}+\right.}{2\,a^3\left(e^{(-x)}+e^x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] (a^2 - b^2)*log(e^(-x) + e^x)/a^3 - (a^2*b - b^3)*log(abs(b*(e^(-x) + e^x)
+ 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) + e^x)^2 - 3*b^2*(e^(-x) + e^x)^2 + 4*
a*b*(e^(-x) + e^x) - 4*a^2)/(a^3*(e^(-x) + e^x)^2)
```

maple [B] time = 0.10, size = 140, normalized size = 2.46

$$-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)}{a}+\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)b^2}{a^3}+\frac{2}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -1/a*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+1/a^3*ln(a*tanh(1/2*x)^2-tanh(
1/2*x)^2*b-a-b)*b^2+2/a/(tanh(1/2*x)^2+1)^2-2/a/(tanh(1/2*x)^2+1)-2/a^2/(ta
nh(1/2*x)^2+1)*b+1/a*ln(tanh(1/2*x)^2+1)-1/a^3*ln(tanh(1/2*x)^2+1)*b^2
```

maxima [A] time = 0.44, size = 96, normalized size = 1.68

$$-\frac{2\left(be^{(-x)}-ae^{(-2\,x)}+be^{(-3\,x)}\right)}{2\,a^2e^{(-2\,x)}+a^2e^{(-4\,x)}+a^2}-\frac{\left(a^2-b^2\right)\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^3}+\frac{\left(a^2-b^2\right)\log\left(e^{(-2\,x)}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-x) - a*e^(-2*x) + b*e^(-3*x))/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^
2) - (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/a^3 + (a^2 - b^2)*log(e^(
-2*x) + 1)/a^3
```

mupad [B] time = 1.61, size = 1221, normalized size = 21.42

$$\frac{\frac{2}{a}-\frac{2\,b\,\mathrm{e}^x}{a^2}}{\mathrm{e}^{2\,x}+1}-\frac{2}{a\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}+\frac{\left(2\,\mathrm{atan}\left(\left(\left(4\,a^4\,b^3\,\left(a^2-b^2\right)^2\,\sqrt{-a^6}-4\,a^6\,b\,\left(a^2-b^2\right)^2\,\sqrt{-a^6}\right)\left(\mathrm{e}^x\left(\frac{1}{16\,a^4\,b^2\,\left(a^2-b^2\right)^3\sqrt{}}\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*cosh(x)),x)
```

```
[Out] (2/a - (2*b*exp(x))/a^2)/(exp(2*x) + 1) - 2/(a*(2*exp(2*x) + exp(4*x) + 1))
 + ((2*atan((4*a^4*b^3*(a^2 - b^2)^2*(-a^6)^(1/2) - 4*a^6*b*(a^2 - b^2)^2*(
-a^6)^(1/2))*(exp(x)*(1/(16*a^4*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2)) -
(a^2 - 2*b^2)^2/(16*a^8*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2))) + 1/(8*a^
5*b*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2)) + (a^2 - 2*b^2)/(8*a^7*b*(a^2 - b^
2)^3*((a^2 - b^2)^2)^(1/2)))) + 2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2
*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(2*a^3*(a^2
 - b^2)^2) + ((a^7 - a^5*b^2)*(-a^6)^(1/2))/(2*a^6*(a^2 - b^2)*((a^2 - b^2)
^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2*a^2*b^2)^
(1/2))/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) - (2*(a^2 - 2*b^2)*(a^2
*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4
 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*(a^2 - b^2)^2
*(-a^6)^(1/2)))*(-a^6)^(1/2))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)) - (a^6*b^2*
exp(x)*(-a^6)^(1/2)*((8*(a^4 + b^4 - 2*a^2*b^2))/(a^8*b*(a^2 - b^2)^2) - (4
*(2*a^6*b - 2*a^4*b^3)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^12*b^2*(a^2 - b^2)
*((a^2 - b^2)^2)^(1/2)) - (2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))
/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) + (2*(a^2 - 2*b^2)*(a^2*(-a^6
)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a
^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*(a^2 - b^2)^2*(-a^6
)^(1/2))))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(2*x)*(-a^6)^(1/
2)*((4*(a^2 - 2*b^2)*(a^4 + b^4 - 2*a^2*b^2))/(a^9*b^2*(a^2 - b^2)^2) + (4*
(a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 +
 b^4 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^9*b^2*(a^2 - b^2
)^2*(-a^6)^(1/2)) + (2*(2*a^6*b - 2*a^4*b^3)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))
/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) + (4*(a^7 - a^5*b^2)*(a^4 + b
^4 - 2*a^2*b^2)^(1/2))/(a^12*b^2*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)))))/(8*(a
^4 + b^4 - 2*a^2*b^2)^(1/2))))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(-a^6)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*cosh(x)), x)
```

3.181 $\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2}-\frac{\tanh(x)}{a}$$

```
[Out] b*arctan(sinh(x))/a^2+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^
(1/2)*(a+b)^(1/2)/a^2-tanh(x)/a
```

Rubi [A] time = 0.23, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2723, 3056, 3001, 3770, 2659, 208}

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/a^2 + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*T
anh[x/2])/Sqrt[a + b]])/a^2 - Tanh[x]/a
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx &= -\int \frac{\left(1-\cosh^2(x)\right)\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx\\
&= -\frac{\tanh(x)}{a}-\frac{\int \frac{(-b-a\cosh(x))\operatorname{sech}(x)}{a+b\cosh(x)}\,dx}{a}\\
&= -\frac{\tanh(x)}{a}+\frac{b\int \operatorname{sech}(x)\,dx}{a^2}-\frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\cosh(x)}\,dx}{a^2}\\
&= \frac{b\tan^{-1}(\sinh(x))}{a^2}-\frac{\tanh(x)}{a}-\frac{\left(2\left(-a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tan^{-1}(\sinh(x))}{a^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{\tanh(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 61, normalized size = 1.00

$$\frac{2\sqrt{b^2-a^2}\ \tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)-a\tanh(x)+2b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*b*ArcTan[Tanh[x/2]] + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt
[-a^2 + b^2]] - a*Tanh[x])/a^2
```

fricas [B] time = 0.63, size = 326, normalized size = 5.34

$$\left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2-b^2+2\left(b^2\cosh(x)+ab\right)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\,\mathrm{co}}\right.}{a^2\cosh(x)^2+2a^2\,\mathrm{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) +
a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*(b*cosh(x
)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) + 2*
a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2), -2*(sqrt(
-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(-sqrt(-a
^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) - a)/(a^2*cosh(x)
^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)]
```

giac [A] time = 0.13, size = 67, normalized size = 1.10

$$\frac{2\,b\arctan\left(e^x\right)}{a^2}+\frac{2\left(a^2-b^2\right)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a^2}+\frac{2}{a\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*b*arctan(e^x)/a^2 + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(s
qrt(-a^2 + b^2)*a^2) + 2/(a*(e^(2*x) + 1))
```

maple [B] time = 0.09, size = 108, normalized size = 1.77

$$\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)\,(a-b)}}-\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)b^{2}}{a^{2}\sqrt{(a+b)\,(a-b)}}-\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)b}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*cosh(x)),x)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2/a^2/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*b^2-2/a*
tanh(1/2*x)/(tanh(1/2*x)^2+1)+2/a^2*arctan(tanh(1/2*x))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.53, size = 285, normalized size = 4.67

$$\frac{2}{a+a\,\mathrm{e}^{2\,x}}+\frac{\ln\Big(64\,a^3\,b-64\,a\,b^3-32\,b^3\,\sqrt{a^2-b^2}\,+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x+64\,a^2\,b\,\sqrt{a^2-b^2}\,+128\,a^3\,\mathrm{e}^x\,\sqrt{a^2-b^2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*cosh(x)),x)
```

```
[Out] 2/(a + a*exp(2*x)) + (log(64*a^3*b - 64*a*b^3 - 32*b^3*(a^2 - b^2)^(1/2) +
128*a^4*exp(x) + 32*b^4*exp(x) + 64*a^2*b*(a^2 - b^2)^(1/2) + 128*a^3*exp(x
)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) - 96*a*b^2*exp(x)*(a^2 - b^2)^(1/2
))*(a^2 - b^2)^(1/2))/a^2 - (log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^2)^(
1/2) + 128*a^4*exp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^
3*exp(x)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^
2)^(1/2))*(a^2 - b^2)^(1/2))/a^2 - (b*(log(exp(x) - 1i)*1i - log(exp(x) + 1
i)*1i))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*cosh(x)), x)
```

3.182 $\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\log(\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

[Out] ln(cosh(x))/a-ln(a+b*cosh(x))/a

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(a + b*Cosh[x]),x]

[Out] Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\cosh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\cosh(x)\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a}$$
$$= \frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

Mathematica [**A**] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

fricas [**A**] time = 1.43, size = 40, normalized size = 2.00

$$-\frac{\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right) - \log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x) - sin
h(x))))/a
```

giac [**A**] time = 0.12, size = 33, normalized size = 1.65

$$\frac{\log\left(e^{(-x)} + e^{x}\right)}{a} - \frac{\log\left(\left|b\left(e^{(-x)} + e^{x}\right) + 2\,a\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - log(abs(b*(e^(-x) + e^x) + 2*a))/a
```

maple [**A**] time = 0.07, size = 21, normalized size = 1.05

$$\frac{\ln(\cosh(x))}{a} - \frac{\ln(a+b\cosh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x)),x)
```

```
[Out] ln(cosh(x))/a-ln(a+b*cosh(x))/a
```

maxima [A] time = 0.45, size = 33, normalized size = 1.65

$$-\frac{\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -log(2*a*e^(-x) + b*e^(-2*x) + b)/a + log(e^(-2*x) + 1)/a
```

mupad [B] time = 0.43, size = 201, normalized size = 10.05

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2}+b\,\mathrm{e}^x\,\sqrt{-a^2}+2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^2}+b\,\mathrm{e}^{3\,x}\,\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}}-\frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,\sqrt{-a^2}-4\,a^2\,b^3\,\sqrt{-a^2}\right)\left(\mathrm{e}^x\left(\frac{1}{16\,b^2\left(a^2-b^2\right)^2}-\frac{\left(a^2-\right.}{16\,a^4\,b^2}\right.\right.\right.}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) + 2*a*exp(2*x)*(-a^2)^(1/2)
 + b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^(1
/2) - 4*a^2*b^3*(-a^2)^(1/2))*(exp(x)*(1/(16*b^2*(a^2 - b^2)^2) - (a^2 - 2*
b^2)^2/(16*a^4*b^2*(a^2 - b^2)^2)) + 1/(8*a*b*(a^2 - b^2)^2) + (a^2 - 2*b^2
)/(8*a^3*b*(a^2 - b^2)^2))))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*cosh(x)), x)
```

3.183 $\int \frac{\coth(x)}{a+b\cosh(x)} dx$

Optimal. Leaf size=54

$$-\frac{a\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] 1/2*ln(1-cosh(x))/(a+b)+1/2*ln(1+cosh(x))/(a-b)-a*ln(a+b*cosh(x))/(a^2-b^2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2721, 801}

$$-\frac{a\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Cosh[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) + Log[1 + Cosh[x]]/(2*(a - b)) - (a*Log[a + b*
Cosh[x]])/(a^2 - b^2)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = -\operatorname{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)(b+x)}\right)dx, x, b\cosh(x)\right)$$
$$= \frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(1+\cosh(x))}{2(a-b)}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 0.70

$$-\frac{a\log(a+b\cosh(x))-a\log(\sinh(x))+b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Cosh[x]),x]
```

```
[Out] -((a*Log[a + b*Cosh[x]] - a*Log[Sinh[x]] + b*Log[Tanh[x/2]])/(a^2 - b^2))
```

fricas [A] time = 0.56, size = 60, normalized size = 1.11

$$-\frac{a\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh(x)+\sinh(x)+1\right)-(a-b)\log\left(\cosh(x)+\sinh(x)-1\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] -(a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh
(x) + 1) - (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

giac [A] time = 0.12, size = 67, normalized size = 1.24

$$-\frac{ab\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^2b-b^3}+\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) + 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

maple [A] time = 0.09, size = 53, normalized size = 0.98

$$-\frac{a \ln \left(a \left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right) b-a-b\right)}{(a+b)(a-b)}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*cosh(x)),x)
```

```
[Out] -a/(a+b)/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+1/(a+b)*ln(tanh(1/2*
x))
```

maxima [A] time = 0.30, size = 59, normalized size = 1.09

$$-\frac{a \log \left(2 a e^{(-x)}+b e^{(-2 x)}+b\right)}{a^{2}-b^{2}}+\frac{\log \left(e^{(-x)}+1\right)}{a-b}+\frac{\log \left(e^{(-x)}-1\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -a*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) + log(e^(-x) + 1)/(a - b) +
 log(e^(-x) - 1)/(a + b)
```

mupad [B] time = 0.43, size = 148, normalized size = 2.74

$$\frac{\ln \left(128 a b-128 a^{2}-32 b^{2}+128 a^{2} \mathrm{e}^{x}+32 b^{2} \mathrm{e}^{x}-128 a b \mathrm{e}^{x}\right)}{a+b}+\frac{\ln \left(-128 a b-128 a^{2}-32 b^{2}-128 a^{2} \mathrm{e}^{x}-32 b^{2} \mathrm{e}\right.}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*cosh(x)),x)
```

```
[Out] log(128*a*b - 128*a^2 - 32*b^2 + 128*a^2*exp(x) + 32*b^2*exp(x) - 128*a*b*e
xp(x))/(a + b) + log(- 128*a*b - 128*a^2 - 32*b^2 - 128*a^2*exp(x) - 32*b^2
*exp(x) - 128*a*b*exp(x))/(a - b) - (a*log(16*a^2*b - 4*b^3*exp(2*x) - 4*b^
3 + 32*a^3*exp(x) - 8*a*b^2*exp(x) + 16*a^2*b*exp(2*x)))/(a^2 - b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{a+b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*cosh(x)), x)
```

3.184 $\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] 2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)-
a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2727, 3767, 8, 2606, 2659, 208}

$$-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)
^(3/2)) - (a*Coth[x])/(a^2 - b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx &= \frac{a\int \operatorname{csch}^2(x)\,dx}{a^2-b^2}+\frac{a^2\int \frac{1}{a+b\cosh(x)}\,dx}{a^2-b^2}-\frac{b\int \coth(x)\operatorname{csch}(x)\,dx}{a^2-b^2}\\ &= -\frac{(ia)\,\mathrm{Subst}(\int 1\,dx,x,-i\coth(x))}{a^2-b^2}+\frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2}+\frac{(ib)\,\mathrm{Subs}}{}\\ &= \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\operatorname{csch}(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.21, size = 77, normalized size = 1.00

$$\frac{2a^2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - C
oth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

fricas [B] time = 0.41, size = 470, normalized size = 6.10

$$\left[\frac{2\,a^3 - 2\,ab^2 + \left(a^2\cosh(x)^2 + 2\,a^2\cosh(x)\sinh(x) + a^2\sinh(x)^2 - a^2\right)\sqrt{a^2-b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh}{b\cosh(x)^2}\right.}{a^4 - 2\,a^2b^2 + b^4 - \left(a^4 - 2\,a^2b^2 + b^4\right)\cosh(x)^2 - 2\left(a^4 - 2\,a^2b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
- a^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(a^
4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a*b
^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x
) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

giac [A] time = 0.14, size = 76, normalized size = 0.99

$$\frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}} + \frac{2\,(be^x-a)}{\left(a^2-b^2\right)\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2)) +
 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

maple [A] time = 0.09, size = 78, normalized size = 1.01

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2\,(a-b)} - \frac{1}{2\,(a+b)\tanh\left(\frac{x}{2}\right)} + \frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*cosh(x)),x)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a+b)/(a-b)*a^2/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.34, size = 337, normalized size = 4.38

$$-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}}{\mathrm{e}^{2\,x}-1}-\frac{2\,\mathrm{atan}\left(\left(\mathrm{e}^x\left(\frac{2\,a^2}{b^2\,{\left(a^2-b^2\right)}^2\,\sqrt{a^4}}+\frac{2\,\left(a^3\,\sqrt{a^4}-a\,b^2\,\sqrt{a^4}\right)}{a\,b^2\,\left(a^2-b^2\right)\,\sqrt{-{\left(a^2-b^2\right)}^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)-\frac{2\,\left(b^3\,\sqrt{a^4}-a^2\,b\right.}{a\,b^2\,\left(a^2-b^2\right)\,\sqrt{-{\left(a^2-b^2\right)}^3}\,\sqrt{-a^6}}\right.\right.}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*cosh(x)),x)
```

```
[Out] - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan((
exp(x)*((2*a^2)/(b^2*(a^2 - b^2)^2*(a^4)^(1/2)) + (2*(a^3*(a^4)^(1/2) - a*b
^2*(a^4)^(1/2)))/(a*b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2))) - (2*(b^3*(a^4)^(1/2) - a^2*b*(a^4)^(1/2)))/(a*
b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^
(1/2)))*((b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(b^6 -
a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2))*(a^4)^(1/2)))/(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*cosh(x)),x)
[Out] Integral(coth(x)**2/(a + b*cosh(x)), x)
```

3.185 $\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{\operatorname{csch}^2(x)(a-b\cosh(x))}{2(a^2-b^2)}-\frac{a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(\cosh(x)+1)}{4(a-b)^2}$$

```
[Out] -1/2*(a-b*cosh(x))*csch(x)^2/(a^2-b^2)+1/4*(2*a+b)*ln(1-cosh(x))/(a+b)^2+1/
4*(2*a-b)*ln(1+cosh(x))/(a-b)^2-a^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

Rubi [A] time = 0.20, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 1647, 801}

$$-\frac{a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}-\frac{\operatorname{csch}^2(x)(a-b\cosh(x))}{2(a^2-b^2)}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(\cosh(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] -((a - b*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) + ((2*a + b)*Log[1 - Cosh[x]])
/(4*(a + b)^2) + ((2*a - b)*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^3*Log[a +
b*Cosh[x]])/(a^2 - b^2)^2
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cosh(x)\right)$$
$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cosh(x)\right)}{2b^2}$$
$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right) dx, x\right.}{2b^2}$$
$$= -\frac{(a-b\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} + \frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(2a-b)\log(1+\cosh(x))}{4(a-b)^2} - \frac{a^3 \,\text{l}}{}$$

Mathematica [A] time = 0.22, size = 101, normalized size = 1.07

$$\frac{-8a^3\log(a+b\cosh(x)) + 8a^3\log(\sinh(x)) - 12a^2b\log\left(\tanh\left(\frac{x}{2}\right)\right) - (a-b)^2(a+b)\text{csch}^2\left(\frac{x}{2}\right) + (a-b)(a+b)^2\text{s}}{8(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Cosh[x]),x]
```

```
[Out] (-((a - b)^2*(a + b)*Csch[x/2]^2) - 8*a^3*Log[a + b*Cosh[x]] + 8*a^3*Log[Si
nh[x]] - 12*a^2*b*Log[Tanh[x/2]] + 4*b^3*Log[Tanh[x/2]] + (a - b)*(a + b)^2
*Sech[x/2]^2)/(8*(a - b)^2*(a + b)^2)
```

fricas [B] time = 0.44, size = 839, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^2*b - b^3)*cosh(x)^3 + 2*(a^2*b - b^3)*sinh(x)^3 - 4*(a^3 - a*b^2
)*cosh(x)^2 - 2*(2*a^3 - 2*a*b^2 - 3*(a^2*b - b^3)*cosh(x))*sinh(x)^2 + 2*(
a^2*b - b^3)*cosh(x) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sin
h(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 + 4*(a
^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(
x))) + ((2*a^3 + 3*a^2*b - b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)*cosh(
x)*sinh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b - b^3 -
2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*(2*a^3 +
 3*a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*cosh(x)^3
 - (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) + (
(2*a^3 - 3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*cosh(x)*sinh(
x)^3 + (2*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^3 - 2*(2*a^3
 - 3*a^2*b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*a^3 - 3*a^2*b
 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cosh(x)^3 - (2*a^
3 - 3*a^2*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^2*b
- b^3 + 3*(a^2*b - b^3)*cosh(x)^2 - 4*(a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3
 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a
^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 - (a^4 - 2*
a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

giac [A] time = 0.13, size = 178, normalized size = 1.89

$$-\frac{a^3 b \log\left(\left|b\left(e^{(-x)} + e^x\right) + 2\,a\right|\right)}{a^4 b - 2\,a^2 b^3 + b^5} + \frac{(2\,a - b)\log\left(e^{(-x)} + e^x + 2\right)}{4\left(a^2 - 2\,ab + b^2\right)} + \frac{(2\,a + b)\log\left(e^{(-x)} + e^x - 2\right)}{4\left(a^2 + 2\,ab + b^2\right)} - \frac{a^3\left(e^{(-x)} + e^x\right)^2 - 2\,a^2}{2\left(a^4 - 2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(2*
a - b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + b)*log(e^(-x)
 + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a^2*b*(e^(-
x) + e^x) + 2*b^3*(e^(-x) + e^x) - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-
x) + e^x)^2 - 4))
```

maple [A] time = 0.10, size = 97, normalized size = 1.03

$$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8\,(a - b)} - \frac{a^3 \ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right) - \left(\tanh^2\left(\frac{x}{2}\right)\right) b - a - b\right)}{(a + b)^2\,(a - b)^2} - \frac{1}{8\,(a + b)\tanh\left(\frac{x}{2}\right)^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right) a}{(a + b)^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right) b}{2\,(a + b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*cosh(x)),x)
```

```
[Out] -1/8*tanh(1/2*x)^2/(a-b)-a^3/(a+b)^2/(a-b)^2*ln(a*tanh(1/2*x)^2-tanh(1/2*x)
^2*b-a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/(a+b)^2*ln(tanh(1/2*x))*a+1/2/(a+b)^2*l
n(tanh(1/2*x))*b
```

maxima [A] time = 0.32, size = 156, normalized size = 1.66

$$-\frac{a^3 \log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{a^4 - 2\,a^2b^2 + b^4} + \frac{(2\,a - b)\log\left(e^{(-x)} + 1\right)}{2\left(a^2 - 2\,ab + b^2\right)} + \frac{(2\,a + b)\log\left(e^{(-x)} - 1\right)}{2\left(a^2 + 2\,ab + b^2\right)} + \frac{be^{(-x)} - 2\,ae^{(-2\,x)} + b}{a^2 - b^2 - 2\left(a^2 - b^2\right)e^{(-2\,x)} +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(2*a -
b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + b)*log(e^(-x) - 1)/(a^2
 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 - 2*(a^
2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

mupad [B] time = 1.52, size = 291, normalized size = 3.10

$$\frac{\frac{2\left(a\,b^2-a^3\right)}{\left(a^2-b^2\right)^2}+\frac{e^x\left(a^2\,b-b^3\right)}{\left(a^2-b^2\right)^2}}{e^{2\,x}-1}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1}+\frac{\ln\left(e^x+1\right)\,\left(2\,a-b\right)}{2\,a^2-4\,a\,b+2\,b^2}-\frac{a^3\,\ln\left(b^7\,e^{2\,x}-16\,a^6\,b+b^7-6\,a^2\,b^5+9\,a^4\,b^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*cosh(x)),x)
```

```
[Out] ((2*(a*b^2 - a^3))/(a^2 - b^2)^2 + (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(e
xp(2*x) - 1) - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) - 2
*exp(2*x) + 1) + (log(exp(x) + 1)*(2*a - b))/(2*a^2 - 4*a*b + 2*b^2) - (a^3
*log(b^7*exp(2*x) - 16*a^6*b + b^7 - 6*a^2*b^5 + 9*a^4*b^3 - 32*a^7*exp(x)
- 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) - 16*a^6*b*exp(2
*x) - 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 - 2*a^2*b^2) + (lo
g(exp(x) - 1)*(2*a + b))/(4*a*b + 2*a^2 + 2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*cosh(x)), x)
```

3.186 $\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=137

$$\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}+\frac{b\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\mathrm{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\mathrm{csch}(x)}{a^2-b^2}-\frac{a^3\coth(x)}{\left(a^2-b^2\right)^2}$$

```
[Out] 2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)-
a^3*coth(x)/(a^2-b^2)^2-1/3*a*coth(x)^3/(a^2-b^2)+a^2*b*csch(x)/(a^2-b^2)^2
+b*csch(x)/(a^2-b^2)+1/3*b*csch(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2659, 208}

$$-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{a^3\coth(x)}{\left(a^2-b^2\right)^2}+\frac{b\mathrm{csch}^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\mathrm{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\mathrm{csch}(x)}{a^2-b^2}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] (2*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)
^(5/2)) - (a^3*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x]^3)/(3*(a^2 - b^2)) + (a^
2*b*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2 - b^2) + (b*Csch[x]^3)/(3*(a^
2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{a\int \coth^2(x)\operatorname{csch}^2(x)\,dx}{a^2-b^2} + \frac{a^2\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx}{a^2-b^2} - \frac{b\int \coth^3(x)\operatorname{csch}(x)\,dx}{a^2-b^2}$$

$$= \frac{a^3\int \operatorname{csch}^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\cosh(x)}\,dx}{\left(a^2-b^2\right)^2} - \frac{\left(a^2b\right)\int \coth(x)\operatorname{csch}(x)\,dx}{\left(a^2-b^2\right)^2} - \frac{(ia)\operatorname{Subst}\left(\int x^2\,dx, x\right.}{a^2-b^2}$$

$$= -\frac{a\coth^3(x)}{3\left(a^2-b^2\right)} + \frac{b\operatorname{csch}(x)}{a^2-b^2} + \frac{b\operatorname{csch}^3(x)}{3\left(a^2-b^2\right)} - \frac{\left(ia^3\right)\operatorname{Subst}(\int 1\,dx, x, -i\coth(x))}{\left(a^2-b^2\right)^2} + \frac{\left(2a^4\right)\operatorname{Subst}}{}$$

$$= \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{a^3\coth(x)}{\left(a^2-b^2\right)^2} - \frac{a\coth^3(x)}{3\left(a^2-b^2\right)} + \frac{a^2b\operatorname{csch}(x)}{\left(a^2-b^2\right)^2} + \frac{b\operatorname{csch}(x)}{a^2-b^2} + \frac{b\operatorname{csch}^3(}{3\left(a^2-\right.}$$

Mathematica [A] time = 0.54, size = 131, normalized size = 0.96

$$\frac{1}{24}\left(-\frac{48a^4\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{2(5b-8a)\tanh\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{2(8a+5b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} - \frac{\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right)}{2(a+b)} + \frac{8\sinh^4\left(\frac{x}{2}\right)\operatorname{cs}}{a-b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Cosh[x]),x]
```

```
[Out] ((-48*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
- (2*(8*a + 5*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) -
 (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (2*(-8*a + 5*b)*Tanh[x/2])/(a - b)^2)/
24
```

fricas [B] time = 0.64, size = 2417, normalized size = 17.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [1/3*(6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b - 3*a^2*b^3 + b^
5)*sinh(x)^5 - 8*a^5 + 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4)
*cosh(x)^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5)*c
osh(x))*sinh(x)^4 - 4*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 4*(4*a^4*b -
```

```
5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5 - 3*a
^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2 + 12*(a^5
 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 + 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh
(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) + b)) + 6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*a^4*b - 3*a^2*b^
3 + b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 - 3*a^3*b^2 +
a*b^4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^2 + 4*(a^5 - a^3*b
^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
- 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 +
3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 +
6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh
(x)), 2/3*(3*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 3*(2*a^4*b - 3*a^2*b^3
 + b^5)*sinh(x)^5 - 4*a^5 + 5*a^3*b^2 - a*b^4 - 3*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^4 - 3*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5)
*cosh(x))*sinh(x)^4 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 2*(4*a^4*b
- 5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5 - 3
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 6*(a^5 - a^3*b^2)*cosh(x)^2 + 6*(a^5
 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 - 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh
(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2
- b^2)) + 3*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 3*(2*a^4*b - 3*a^2*b^3 +
b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*
cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*
```

```
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x))
]
```

giac [A] time = 0.15, size = 172, normalized size = 1.26

$$\frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{2\left(6\,a^2be^{(5\,x)}-3\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+3\,ab^2e^{(4\,x)}-8\,a^2be^{(3\,x)}+2\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+\cdots\right)}{3\left(a^4-2\,a^2b^2+b^4\right)\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a
^2 + b^2)) + 2/3*(6*a^2*b*e^(5*x) - 3*b^3*e^(5*x) - 6*a^3*e^(4*x) + 3*a*b^2
*e^(4*x) - 8*a^2*b*e^(3*x) + 2*b^3*e^(3*x) + 6*a^3*e^(2*x) + 6*a^2*b*e^x -
3*b^3*e^x - 4*a^3 + a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

maple [A] time = 0.11, size = 127, normalized size = 0.93

$$-\frac{\frac{a\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)b}{3}+5a\tanh\left(\frac{x}{2}\right)-3\tanh\left(\frac{x}{2}\right)b}{8\left(a-b\right)^2}+\frac{2a^4\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2\left(a+b\right)^2\sqrt{(a+b)\left(a-b\right)}}-\frac{1}{24\left(a+b\right)\tanh\left(\frac{x}{2}\right)^3}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*cosh(x)),x)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*tanh(1/2*x)^3*b+5*a*tanh(1/2*x)-3*tan
h(1/2*x)*b)+2/(a-b)^2/(a+b)^2*a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3-1/8*(5*a+3*b)/(a+b)^2/ta
nh(1/2*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.81, size = 666, normalized size = 4.86

$$\frac{\frac{4\left(a\,b^2-a^3\right)}{\left(a^2-b^2\right)^2}+\frac{8\,e^x\left(a^2\,b-b^3\right)}{3\left(a^2-b^2\right)^2}}{e^{4x}-2\,e^{2x}+1}-\frac{\frac{8\,a}{3\left(a^2-b^2\right)}-\frac{8\,b\,e^x}{3\left(a^2-b^2\right)}}{3\,e^{2x}-3\,e^{4x}+e^{6x}-1}-\frac{\frac{2\,a\left(2\,a^2-b^2\right)}{\left(a^2-b^2\right)^2}-\frac{2\,b\,e^x\left(2\,a^2-b^2\right)}{\left(a^2-b^2\right)^2}}{e^{2x}-1}+\frac{2\,\mathrm{atan}\Bigg(\Bigg(\Bigg(e^x\Bigg(\frac{2\,a^4}{b^2\left(a^2-b^2\right)^2\sqrt{a^8}\,\left(a^4-2\,a^2\,b^2+\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*cosh(x)),x)
```

```
[Out] ((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^
2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*
(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((2*a*(2*a^2 - b^2
))/(a^2 - b^2)^2 - (2*b*exp(x)*(2*a^2 - b^2))/(a^2 - b^2)^2)/(exp(2*x) - 1)
 + (2*atan((exp(x)*((2*a^4)/(b^2*(a^2 - b^2)^2*(a^8)^(1/2)*(a^4 + b^4 - 2*a
^2*b^2)) + (2*(a^5*(a^8)^(1/2) - 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^(1/2))
)/(a^3*b^2*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*
a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))) + (2*(b^5*(a^8)^(1/2
) - 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 - b^2)^5)^(
1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6
*b^4 + 5*a^8*b^2)^(1/2)))*((b^5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*
a^6*b^4 + 5*a^8*b^2)^(1/2))/2 - a^2*b^3*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b
^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + (a^4*b*(b^10 - a^10 - 5*a^2*b^8 + 10*a
^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2))*(a^8)^(1/2))/(b^10 - a^10 - 5*a
^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*cosh(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*cosh(x)), x)
```

3.187 $\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^5(x)}{5a}+\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)\text{sech}(x)}{4a}-\frac{3\tanh(x)\text{sech}(x)}{8a}$$

```
[Out] 3/8*arctan(sinh(x))/a-3/8*sech(x)*tanh(x)/a-1/4*sech(x)*tanh(x)^3/a-1/5*tan
h(x)^5/a
```

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\tanh^5(x)}{5a}+\frac{3\tan^{-1}(\sinh(x))}{8a}-\frac{\tanh^3(x)\text{sech}(x)}{4a}-\frac{3\tanh(x)\text{sech}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) - (3*Sech[x]*Tanh[x])/(8*a) - (Sech[x]*Tanh[x]^3)
/(4*a) - Tanh[x]^5/(5*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^4(x)\,dx}{a} - \frac{\int \operatorname{sech}^2(x)\tanh^4(x)\,dx}{a}\\
&= -\frac{\operatorname{sech}(x)\tanh^3(x)}{4a} + \frac{i\,\mathrm{Subst}\left(\int x^4\,dx, x, i\tanh(x)\right)}{a} + \frac{3\int \operatorname{sech}(x)\tanh^2(x)\,dx}{4a}\\
&= -\frac{3\operatorname{sech}(x)\tanh(x)}{8a} - \frac{\operatorname{sech}(x)\tanh^3(x)}{4a} - \frac{\tanh^5(x)}{5a} + \frac{3\int \operatorname{sech}(x)\,dx}{8a}\\
&= \frac{3\tan^{-1}(\sinh(x))}{8a} - \frac{3\operatorname{sech}(x)\tanh(x)}{8a} - \frac{\operatorname{sech}(x)\tanh^3(x)}{4a} - \frac{\tanh^5(x)}{5a}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 58, normalized size = 1.26

$$
\frac{\cosh^2\left(\frac{x}{2}\right)\left(30\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\left(-8\operatorname{sech}^4(x)+10\operatorname{sech}^3(x)+16\operatorname{sech}^2(x)-25\operatorname{sech}(x)-8\right)\right)}{20a(\cosh(x)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(30*ArcTan[Tanh[x/2]] + (-8 - 25*Sech[x] + 16*Sech[x]^2 + 10*S
ech[x]^3 - 8*Sech[x]^4)*Tanh[x]))/(20*a*(1 + Cosh[x]))
```

fricas [B] time = 0.52, size = 750, normalized size = 16.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/20*(25*cosh(x)^9 + 5*(45*cosh(x) - 8)*sinh(x)^8 + 25*sinh(x)^9 - 40*cosh
(x)^8 + 10*(90*cosh(x)^2 - 32*cosh(x) + 1)*sinh(x)^7 + 10*cosh(x)^7 + 70*(3
0*cosh(x)^3 - 16*cosh(x)^2 + cosh(x))*sinh(x)^6 + 70*(45*cosh(x)^4 - 32*cos
h(x)^3 + 3*cosh(x)^2)*sinh(x)^5 + 10*(315*cosh(x)^5 - 280*cosh(x)^4 + 35*co
sh(x)^3 - 8)*sinh(x)^4 - 80*cosh(x)^4 + 10*(210*cosh(x)^6 - 224*cosh(x)^5 +
 35*cosh(x)^4 - 32*cosh(x) - 1)*sinh(x)^3 - 10*cosh(x)^3 + 10*(90*cosh(x)^7
 - 112*cosh(x)^6 + 21*cosh(x)^5 - 48*cosh(x)^2 - 3*cosh(x))*sinh(x)^2 - 15*
(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 1)*sinh(
x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21*cosh(x)^
4 + 14*cosh(x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5 + 70*cosh(
x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh(x)
^2 + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x
)^3 + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 1
2*cosh(x)^2 + 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*cosh(x)^7 + 6*
cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) +
 5*(45*cosh(x)^8 - 64*cosh(x)^7 + 14*cosh(x)^6 - 64*cosh(x)^3 - 6*cosh(x)^2
 - 5)*sinh(x) - 25*cosh(x) - 8)/(a*cosh(x)^10 + 10*a*cosh(x)*sinh(x)^9 + a*
sinh(x)^10 + 5*a*cosh(x)^8 + 5*(9*a*cosh(x)^2 + a)*sinh(x)^8 + 40*(3*a*cosh
(x)^3 + a*cosh(x))*sinh(x)^7 + 10*a*cosh(x)^6 + 10*(21*a*cosh(x)^4 + 14*a*c
osh(x)^2 + a)*sinh(x)^6 + 4*(63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 15*a*cosh(x)
)*sinh(x)^5 + 10*a*cosh(x)^4 + 10*(21*a*cosh(x)^6 + 35*a*cosh(x)^4 + 15*a*c
osh(x)^2 + a)*sinh(x)^4 + 40*(3*a*cosh(x)^7 + 7*a*cosh(x)^5 + 5*a*cosh(x)^3
 + a*cosh(x))*sinh(x)^3 + 5*a*cosh(x)^2 + 5*(9*a*cosh(x)^8 + 28*a*cosh(x)^6
 + 30*a*cosh(x)^4 + 12*a*cosh(x)^2 + a)*sinh(x)^2 + 10*(a*cosh(x)^9 + 4*a*c
osh(x)^7 + 6*a*cosh(x)^5 + 4*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

giac [A] time = 0.15, size = 58, normalized size = 1.26

$$\frac{3 \arctan\left(e^{x}\right)}{4 a} - \frac{25 e^{(9 x)} - 40 e^{(8 x)} + 10 e^{(7 x)} - 80 e^{(4 x)} - 10 e^{(3 x)} - 25 e^{x} - 8}{20 a\left(e^{(2 x)} + 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 3/4*arctan(e^x)/a - 1/20*(25*e^(9*x) - 40*e^(8*x) + 10*e^(7*x) - 80*e^(4*x)
 - 10*e^(3*x) - 25*e^x - 8)/(a*(e^(2*x) + 1)^5)
```

maple [B] time = 0.13, size = 115, normalized size = 2.50

$$\frac{3\left(\tanh^{9}\left(\frac{x}{2}\right)\right)}{4a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{7\left(\tanh^{7}\left(\frac{x}{2}\right)\right)}{2a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{32\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{5a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{7\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{2a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{3\tanh\left(\frac{x}{2}\right)}{4a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{3\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+a*cosh(x)),x)
```

```
[Out] 3/4/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^9+7/2/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*
x)^7-32/5/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)^5-7/2/a/(tanh(1/2*x)^2+1)^5*tan
h(1/2*x)^3-3/4/a/(tanh(1/2*x)^2+1)^5*tanh(1/2*x)+3/4/a*arctan(tanh(1/2*x))
```

maxima [B] time = 0.50, size = 89, normalized size = 1.93

$$-\frac{25\,e^{(-x)} + 10\,e^{(-3\,x)} + 80\,e^{(-4\,x)} - 10\,e^{(-7\,x)} + 40\,e^{(-8\,x)} - 25\,e^{(-9\,x)} + 8}{20\left(5\,ae^{(-2\,x)} + 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} + 5\,ae^{(-8\,x)} + ae^{(-10\,x)} + a\right)} - \frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/20*(25*e^(-x) + 10*e^(-3*x) + 80*e^(-4*x) - 10*e^(-7*x) + 40*e^(-8*x) -
25*e^(-9*x) + 8)/(5*a*e^(-2*x) + 10*a*e^(-4*x) + 10*a*e^(-6*x) + 5*a*e^(-8*
x) + a*e^(-10*x) + a) - 3/4*arctan(e^(-x))/a
```

mupad [B] time = 1.06, size = 183, normalized size = 3.98

$$\frac{\frac{16}{a} - \frac{6\,e^{x}}{a}}{3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1} - \frac{\frac{8}{a} - \frac{9\,e^{x}}{2\,a}}{2\,e^{2\,x} + e^{4\,x} + 1} + \frac{32}{5\,a\,\left(5\,e^{2\,x} + 10\,e^{4\,x} + 10\,e^{6\,x} + 5\,e^{8\,x} + e^{10\,x} + 1\right)} - \frac{\frac{16}{a} - \frac{4\,e^{x}}{a}}{4\,e^{2\,x} + 6\,e^{4\,x} + 4\,e^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a + a*cosh(x)),x)
```

```
[Out] (16/a - (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (8/a - (9*
exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + 32/(5*a*(5*exp(2*x) + 10*exp(4
*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (16/a - (4*exp(x))/a)/(4
*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) + (2/a - (5*exp(x))/(4*
a))/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{6}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**6/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**6/(cosh(x) + 1), x)/a
```

3.188 $\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^4(x)}{4a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\operatorname{sech}(x)}{a}$$

```
[Out] -sech(x)/a+1/3*sech(x)^3/a-1/4*tanh(x)^4/a
```

Rubi [A] time = 0.09, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\tanh^4(x)}{4a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] -(Sech[x]/a) + Sech[x]^3/(3*a) - Tanh[x]^4/(4*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^3(x)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^3\,dx,x,i\tanh(x)\right)}{a}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\operatorname{sech}(x)\right)}{a}\\ &= -\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\tanh^4(x)}{4a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.83

$$\frac{2\sinh^6\left(\frac{x}{2}\right)(5\cosh(x)+3)\operatorname{sech}^4(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5/(a + a*Cosh[x]),x]
```

```
[Out] (2*(3 + 5*Cosh[x])*Sech[x]^4*Sinh[x/2]^6)/(3*a)
```

fricas [B] time = 0.46, size = 174, normalized size = 5.80

$$-\frac{2\left(3\cosh(x)^4+3\,(4\cosh(x)-1)\sinh(x)^3+3\sinh(x)^4-3\cosh(x)^3+\left(18\cosh(x)^2-9\cosh(x)+8\right)\sinh(}{3\left(a\cosh(x)^5+5\,a\cosh(x)\sinh(x)^4+a\sinh(x)^5+5\,a\cosh(x)^3+\left(10\,a\cosh(x)^2+3\,a\right)\sinh(x)^3+5\left(2\,a\cosh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x)^4 + 3*(4*cosh(x) - 1)*sinh(x)^3 + 3*sinh(x)^4 - 3*cosh(x)^3
 + (18*cosh(x)^2 - 9*cosh(x) + 8)*sinh(x)^2 + 8*cosh(x)^2 + (12*cosh(x)^3 -
 9*cosh(x)^2 + 4*cosh(x) + 3)*sinh(x) - 3*cosh(x) + 5)/(a*cosh(x)^5 + 5*a*c
osh(x)*sinh(x)^4 + a*sinh(x)^5 + 5*a*cosh(x)^3 + (10*a*cosh(x)^2 + 3*a)*sin
h(x)^3 + 5*(2*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^2 + 10*a*cosh(x) + (5*a*co
sh(x)^4 + 9*a*cosh(x)^2 + 2*a)*sinh(x))
```

giac [A] time = 0.15, size = 48, normalized size = 1.60

$$-\frac{2\left(3\left(e^{(-x)}+e^x\right)^3-3\left(e^{(-x)}+e^x\right)^2-4\,e^{(-x)}-4\,e^x+6\right)}{3\,a\left(e^{(-x)}+e^x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(e^(-x) + e^x)^3 - 3*(e^(-x) + e^x)^2 - 4*e^(-x) - 4*e^x + 6)/(a*(e
^(-x) + e^x)^4)
```

maple [A] time = 0.11, size = 30, normalized size = 1.00

$$\frac{\frac{1}{3\cosh(x)^3}-\frac{1}{4\cosh(x)^4}-\frac{1}{\cosh(x)}+\frac{1}{2\cosh(x)^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+a*cosh(x)),x)
```

```
[Out] 1/a*(1/3/cosh(x)^3-1/4/cosh(x)^4-1/cosh(x)+1/2/cosh(x)^2)
```

maxima [B] time = 0.37, size = 223, normalized size = 7.43

$$-\frac{2\,e^{(-x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}+\frac{2\,e^{(-2\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}-\frac{}{3\left(4\,ae^{(-2\,x)}+6\,ae^{(-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) + 2
*e^(-2*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 1
0/3*e^(-3*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a)
- 10/3*e^(-5*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) + 2*e^(-6*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) - 2*e^(-7*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a)
```

mupad [B] time = 1.01, size = 117, normalized size = 3.90

$$\frac{\frac{8}{a}-\frac{8\,e^x}{3\,a}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{6}{a}-\frac{8\,e^x}{3\,a}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{2}{a}-\frac{2\,e^x}{a}}{e^{2\,x}+1}-\frac{4}{a\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a + a*cosh(x)),x)
```

```
[Out] (8/a - (8*exp(x))/(3*a))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (6/a -
(8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1) + (2/a - (2*exp(x))/a)/(exp(2
*x) + 1) - 4/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^5(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**5/(cosh(x) + 1), x)/a
```

3.189 $\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^3(x)}{3a}+\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\text{sech}(x)}{2a}$$

```
[Out] 1/2*arctan(sinh(x))/a-1/2*sech(x)*tanh(x)/a-1/3*tanh(x)^3/a
```

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\tanh^3(x)}{3a}+\frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\tanh(x)\text{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) - (Sech[x]*Tanh[x])/(2*a) - Tanh[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx &= \frac{\int \operatorname{sech}(x)\tanh^2(x)\,dx}{a}-\frac{\int \operatorname{sech}^2(x)\tanh^2(x)\,dx}{a}\\ &= -\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{i\,\text{Subst}\left(\int x^2\,dx,x,i\tanh(x)\right)}{a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\tanh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 1.39

$$\frac{\cosh^2\left(\frac{x}{2}\right)\left(6\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\left(2\operatorname{sech}^2(x)-3\operatorname{sech}(x)-2\right)\right)}{3a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] (Cosh[x/2]^2*(6*ArcTan[Tanh[x/2]] + (-2 - 3*Sech[x] + 2*Sech[x]^2)*Tanh[x])
)/(3*a*(1 + Cosh[x]))
```

fricas [B] time = 0.48, size = 315, normalized size = 9.55

$$-\frac{3\cosh(x)^5+3\,(5\cosh(x)-2)\sinh(x)^4+3\sinh(x)^5-6\cosh(x)^4+6\left(5\cosh(x)^2-4\cosh(x)\right)\sinh(x)^3+6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*cosh(x)^5 + 3*(5*cosh(x) - 2)*sinh(x)^4 + 3*sinh(x)^5 - 6*cosh(x)^4
 + 6*(5*cosh(x)^2 - 4*cosh(x))*sinh(x)^3 + 6*(5*cosh(x)^3 - 6*cosh(x)^2)*si
nh(x)^2 - 3*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 +
```

```
 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*
cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cos
h(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(5*cosh(x)^4 -
 8*cosh(x)^3 - 1)*sinh(x) - 3*cosh(x) - 2)/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(
x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5
*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 +
6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*
sinh(x) + a)
```

giac [**A**] time = 0.13, size = 39, normalized size = 1.18

$$\frac{\arctan\left(e^{x}\right)}{a} - \frac{3\,e^{(5\,x)} - 6\,e^{(4\,x)} - 3\,e^{x} - 2}{3\,a\left(e^{(2\,x)} + 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] arctan(e^x)/a - 1/3*(3*e^(5*x) - 6*e^(4*x) - 3*e^x - 2)/(a*(e^(2*x) + 1)^3)
```

maple [**B**] time = 0.10, size = 71, normalized size = 2.15

$$\frac{\tanh^{5}\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}} - \frac{8\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}} - \frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}} + \frac{\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)^5-8/3/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)
^3-1/a/(tanh(1/2*x)^2+1)^3*tanh(1/2*x)+1/a*arctan(tanh(1/2*x))
```

maxima [**B**] time = 0.42, size = 57, normalized size = 1.73

$$-\frac{3\,e^{(-x)} + 6\,e^{(-4\,x)} - 3\,e^{(-5\,x)} + 2}{3\left(3\,ae^{(-2\,x)} + 3\,ae^{(-4\,x)} + ae^{(-6\,x)} + a\right)} - \frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*e^(-x) + 6*e^(-4*x) - 3*e^(-5*x) + 2)/(3*a*e^(-2*x) + 3*a*e^(-4*x)
+ a*e^(-6*x) + a) - arctan(e^(-x))/a
```

mupad [B] time = 0.96, size = 95, normalized size = 2.88

$$\frac{8}{3\,a\,\left(3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1\right)}-\frac{\frac{4}{a}-\frac{2\,{\mathrm{e}}^x}{a}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}+\frac{\frac{2}{a}-\frac{{\mathrm{e}}^x}{a}}{{\mathrm{e}}^{2\,x}+1}+\frac{\mathrm{atan}\left(\frac{{\mathrm{e}}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + a*cosh(x)),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2*
exp(2*x) + exp(4*x) + 1) + (2/a - exp(x)/a)/(exp(2*x) + 1) + atan((exp(x)*(
a^2)^(1/2))/a)/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{4}{\left(x \right)}}{\cosh{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**4/(cosh(x) + 1), x)/a
```

3.190 $\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\text{sech}^2(x)}{2a}-\frac{\text{sech}(x)}{a}$$

```
[Out] -sech(x)/a+1/2*sech(x)^2/a
```

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2606, 30, 8}

$$\frac{\text{sech}^2(x)}{2a}-\frac{\text{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] -(Sech[x]/a) + Sech[x]^2/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx = \frac{\int \operatorname{sech}(x)\tanh(x)\,dx}{a} - \frac{\int \operatorname{sech}^2(x)\tanh(x)\,dx}{a}$$
$$= -\frac{\mathrm{Subst}(\int 1\,dx, x, \operatorname{sech}(x))}{a} + \frac{\mathrm{Subst}(\int x\,dx, x, \operatorname{sech}(x))}{a}$$
$$= -\frac{\operatorname{sech}(x)}{a} + \frac{\operatorname{sech}^2(x)}{2a}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 0.89

$$\frac{2\sinh^4\left(\frac{x}{2}\right)\operatorname{sech}^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + a*Cosh[x]),x]
[Out] (2*Sech[x]^2*Sinh[x/2]^4)/a
```

fricas [B] time = 0.64, size = 66, normalized size = 3.47

$$-\frac{2\left(\cosh(x)^2 + (2\cosh(x)-1)\sinh(x) + \sinh(x)^2 - \cosh(x) + 1\right)}{a\cosh(x)^3 + 3a\cosh(x)\sinh(x)^2 + a\sinh(x)^3 + 3a\cosh(x) + \left(3a\cosh(x)^2 + a\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
[Out] -2*(cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(a*cosh(
x)^3 + 3*a*cosh(x)*sinh(x)^2 + a*sinh(x)^3 + 3*a*cosh(x) + (3*a*cosh(x)^2 +
 a)*sinh(x))
```

giac [A] time = 0.13, size = 22, normalized size = 1.16

$$-\frac{2\left(e^{(-x)} + e^x - 1\right)}{a\left(e^{(-x)} + e^x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
[Out] -2*(e^(-x) + e^x - 1)/(a*(e^(-x) + e^x)^2)
```

maple [A] time = 0.10, size = 18, normalized size = 0.95

$$\frac{-\frac{1}{\cosh(x)} + \frac{1}{2\cosh(x)^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+a*cosh(x)),x)
```

```
[Out] 1/a*(-1/cosh(x)+1/2/cosh(x)^2)
```

maxima [B] time = 0.34, size = 70, normalized size = 3.68

$$-\frac{2\,e^{(-x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} + \frac{2\,e^{(-2\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} - \frac{2\,e^{(-3\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*e^(-x)/(2*a*e^(-2*x) + a*e^(-4*x) + a) + 2*e^(-2*x)/(2*a*e^(-2*x) + a*e^
(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
```

mupad [B] time = 0.92, size = 25, normalized size = 1.32

$$-\frac{2\,\mathrm{e}^{x}\,\left(\mathrm{e}^{2\,x}-\mathrm{e}^{x}+1\right)}{a\,{\left(\mathrm{e}^{2\,x}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + a*cosh(x)),x)
```

```
[Out] -(2*exp(x)*(exp(2*x) - exp(x) + 1))/(a*(exp(2*x) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh^{3}{(x)}}{\cosh{(x)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(cosh(x) + 1), x)/a
```

3.191 $\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

```
[Out] arctan(sinh(x))/a-tanh(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Tanh[x]/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx = \frac{\int \operatorname{sech}(x)\,dx}{a} - \frac{\int \operatorname{sech}^2(x)\,dx}{a}$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{i\,\mathrm{Subst}(\int 1\,dx, x, -i\tanh(x))}{a}$$
$$= \frac{\tan^{-1}(\sinh(x))}{a} - \frac{\tanh(x)}{a}$$

Mathematica [A] time = 0.05, size = 18, normalized size = 1.20

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) - \tanh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] (2*ArcTan[Tanh[x/2]] - Tanh[x])/a
```

fricas [B] time = 0.54, size = 50, normalized size = 3.33

$$\frac{2\left(\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\arctan\left(\cosh(x) + \sinh(x)\right) + 1\right)}{a\cosh(x)^2 + 2\,a\cosh(x)\sinh(x) + a\sinh(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(cosh(x) + sinh(x)
) + 1)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

giac [A] time = 0.14, size = 22, normalized size = 1.47

$$\frac{2\arctan\left(e^x\right)}{a} + \frac{2}{a\left(e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a + 2/(a*(e^(2*x) + 1))
```

maple [A] time = 0.08, size = 31, normalized size = 2.07

$$-\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+a*cosh(x)),x)
[Out] -2/a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+2/a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.43, size = 23, normalized size = 1.53

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}-\frac{2}{ae^{(-2x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-2*x) + a)
```

mupad [B] time = 0.92, size = 33, normalized size = 2.20

$$\frac{2}{a\left(e^{2x}+1\right)}+\frac{2\,\mathrm{atan}\left(\frac{e^{x}\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + a*cosh(x)),x)
[Out] 2/(a*(exp(2*x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tanh^{2}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+a*cosh(x)),x)
[Out] Integral(tanh(x)**2/(cosh(x) + 1), x)/a
```

3.192 $\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(\cosh(x))}{a} - \frac{\log(\cosh(x)+1)}{a}$$

```
[Out] ln(cosh(x))/a-ln(1+cosh(x))/a
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\cosh(x))}{a} - \frac{\log(\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + a*Cosh[x]),x]
```

```
[Out] Log[Cosh[x]]/a - Log[1 + Cosh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx = \mathrm{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\cosh(x)\right)$$
$$= \frac{\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, a\cosh(x)\right)}{a} - \frac{\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cosh(x)\right)}{a}$$
$$= \frac{\log(\cosh(x))}{a} - \frac{\log(1+\cosh(x))}{a}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 0.67

$$-\frac{2\tanh^{-1}(2\cosh(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + a*Cosh[x]),x]
```

```
[Out] (-2*ArcTanh[1 + 2*Cosh[x]])/a
```

fricas [A] time = 0.52, size = 28, normalized size = 1.56

$$\frac{\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) - 2\log(\cosh(x)+\sinh(x)+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] (log(2*cosh(x)/(cosh(x) - sinh(x))) - 2*log(cosh(x) + sinh(x) + 1))/a
```

giac [A] time = 0.14, size = 22, normalized size = 1.22

$$\frac{\log\left(e^{(2x)}+1\right)}{a} - \frac{2\log(e^{x}+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] log(e^(2*x) + 1)/a - 2*log(e^x + 1)/a
```

maple [A] time = 0.08, size = 19, normalized size = 1.06

$$\frac{\ln(\cosh(x))}{a} - \frac{\ln(1+\cosh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+a*cosh(x)),x)
```

```
[Out] ln(cosh(x))/a-ln(1+cosh(x))/a
```

maxima [A] time = 0.33, size = 24, normalized size = 1.33

$$-\frac{2 \log\left(e^{(-x)} + 1\right)}{a} + \frac{\log\left(e^{(-2x)} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2*log(e^(-x) + 1)/a + log(e^(-2*x) + 1)/a
```

mupad [B] time = 0.08, size = 26, normalized size = 1.44

$$-\frac{2 \ln\left(36 e^{x} + 36\right) - \ln\left(3 e^{2x} + 3\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + a*cosh(x)),x)
```

```
[Out] -(2*log(36*exp(x) + 36) - log(3*exp(2*x) + 3))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tanh(x)}{\cosh(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(tanh(x)/(cosh(x) + 1), x)/a
```

3.193 $\int \frac{\coth(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\text{csch}(x)}{2a}$$

```
[Out] -1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

Rubi [A] time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2706, 2606, 30, 2611, 3770}

$$\frac{\text{csch}^2(x)}{2a}-\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\text{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + a*Cosh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/(2*a) - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^2(x)\text{csch}(x)\,dx}{a}-\frac{\int \coth(x)\text{csch}^2(x)\,dx}{a}\\ &= -\frac{\coth(x)\text{csch}(x)}{2a}+\frac{\int \text{csch}(x)\,dx}{2a}-\frac{\text{Subst}\left(\int x\,dx,x,-i\text{csch}(x)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{2a}-\frac{\coth(x)\text{csch}(x)}{2a}+\frac{\text{csch}^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.27

$$-\frac{2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+1}{2a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + a*Cosh[x]),x]
```

```
[Out] -1/2*(1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(a*(1 + Cosh[x])
)
```

fricas [B] time = 0.53, size = 103, normalized size = 3.12

$$-\frac{\left(\cosh(x)^2+2\left(\cosh(x)+1\right)\sinh(x)+\sinh(x)^2+2\cosh(x)+1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(\cosh(x)^2+2\,(\cdots\right.}{2\left(a\cosh(x)^2+a\sinh(x)^2+2\,a\cosh(x)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

giac [A] time = 0.13, size = 52, normalized size = 1.58

$$-\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{4\,a}+\frac{e^{(-x)}+e^{x}-2}{4\,a\left(e^{(-x)}+e^{x}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x - 2)/(a*(e^(-x) + e^x + 2))
```

maple [A] time = 0.10, size = 23, normalized size = 0.70

$$\frac{\tanh^{2}\left(\frac{x}{2}\right)}{4a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+a*cosh(x)),x)
```

```
[Out] 1/4/a*tanh(1/2*x)^2+1/2/a*ln(tanh(1/2*x))
```

maxima [A] time = 0.33, size = 48, normalized size = 1.45

$$-\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

mupad [B] time = 0.92, size = 51, normalized size = 1.55

$$\frac{1}{a\,\left(\mathrm{e}^{2\,x}+2\,\mathrm{e}^{x}+1\right)}-\frac{1}{a\,\left(\mathrm{e}^{x}+1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + a*cosh(x)),x)
```

```
[Out] 1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)^
(1/2))/a)/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth(x)}{\cosh(x)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)/(cosh(x) + 1), x)/a
```

3.194 $\int \frac{\coth^2(x)}{a+a\cosh(x)} \, dx$

Optimal. Leaf size=30

$$\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

```
[Out] 1/3*coth(x)^3/a-csch(x)/a-1/3*csch(x)^3/a
```

Rubi [A] time = 0.08, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + a*Cosh[x]),x]
```

```
[Out] Coth[x]^3/(3*a) - Csch[x]/a - Csch[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^3(x)\mathrm{csch}(x)\,dx}{a}-\frac{\int \coth^2(x)\mathrm{csch}^2(x)\,dx}{a}\\ &= \frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)}{a}+\frac{i\,\mathrm{Subst}\left(\int \left(-1+x^2\right)\,dx,x,-i\mathrm{csch}(x)\right)}{a}\\ &= \frac{\coth^3(x)}{3a}-\frac{\mathrm{csch}(x)}{a}-\frac{\mathrm{csch}^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.05, size = 25, normalized size = 0.83

$$\frac{(-4\cosh(x)+\cosh(2x)-3)\mathrm{csch}(x)}{6a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + a*Cosh[x]),x]
[Out] ((-3 - 4*Cosh[x] + Cosh[2*x])*Csch[x])/(6*a*(1 + Cosh[x]))
```

fricas [B] time = 0.45, size = 91, normalized size = 3.03

$$-\frac{2\left(3\,\cosh(x)^2+2\,(3\,\cosh(x)+2)\sinh(x)+3\,\sinh(x)^2+2\,\cosh(x)+1\right)}{3\left(a\cosh(x)^3+a\sinh(x)^3+2\,a\cosh(x)^2+(3\,a\cosh(x)+2\,a)\sinh(x)^2-a\cosh(x)+\left(3\,a\cosh(x)^2+4\,a\cosh(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
[Out] -2/3*(3*cosh(x)^2 + 2*(3*cosh(x) + 2)*sinh(x) + 3*sinh(x)^2 + 2*cosh(x) + 1
)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 + (3*a*cosh(x) + 2*a)*sinh(x)^
2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) - 2*a)
```

giac [A] time = 0.15, size = 35, normalized size = 1.17

$$-\frac{1}{2\,a(e^x-1)}-\frac{9\,e^{(2\,x)}+12\,e^x+7}{6\,a(e^x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(a*(e^x - 1)) - 1/6*(9*e^(2*x) + 12*e^x + 7)/(a*(e^x + 1)^3)
```

maple [A] time = 0.09, size = 29, normalized size = 0.97

$$\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{3} + 2\tanh\left(\frac{x}{2}\right) - \frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+a*cosh(x)),x)
```

```
[Out] 1/4/a*(1/3*tanh(1/2*x)^3+2*tanh(1/2*x)-1/tanh(1/2*x))
```

maxima [B] time = 0.33, size = 121, normalized size = 4.03

$$-\frac{2\,e^{(-x)}}{3\left(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a\right)} - \frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} - \frac{2\,e^{(-3\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} + \frac{}{3\left(2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -2/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a*
e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-
3*x) - a*e^(-4*x) + a) + 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

mupad [B] time = 0.93, size = 92, normalized size = 3.07

$$-\frac{\frac{e^{2\,x}}{2\,a} + \frac{1}{2\,a} + \frac{e^x}{3\,a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^x + 1} - \frac{\frac{1}{6\,a} + \frac{e^x}{2\,a}}{e^{2\,x} + 2\,e^x + 1} - \frac{1}{2\,a\,\left(e^x - 1\right)} - \frac{1}{2\,a\,\left(e^x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + a*cosh(x)),x)
```

```
[Out] - (exp(2*x)/(2*a) + 1/(2*a) + exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp(
x) + 1) - (1/(6*a) + exp(x)/(2*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(
x) - 1)) - 1/(2*a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^2(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**2/(cosh(x) + 1), x)/a
```

3.195 $\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{\coth^4(x)}{4a}-\frac{3\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}$$

```
[Out] -3/8*arctanh(cosh(x))/a+1/4*coth(x)^4/a-3/8*coth(x)*csch(x)/a-1/4*coth(x)^3
*csch(x)/a
```

Rubi [A] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\coth^4(x)}{4a}-\frac{3\tanh^{-1}(\cosh(x))}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + a*Cosh[x]),x]
```

```
[Out] (-3*ArcTanh[Cosh[x]])/(8*a) + Coth[x]^4/(4*a) - (3*Coth[x]*Csch[x])/(8*a) -
 (Coth[x]^3*Csch[x])/(4*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^4(x)\operatorname{csch}(x)\,dx}{a}-\frac{\int \coth^3(x)\operatorname{csch}^2(x)\,dx}{a}\\ &= -\frac{\coth^3(x)\operatorname{csch}(x)}{4a}+\frac{3\int \coth^2(x)\operatorname{csch}(x)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^3\,dx,x,i\coth(x)\right)}{a}\\ &= \frac{\coth^4(x)}{4a}-\frac{3\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth^3(x)\operatorname{csch}(x)}{4a}+\frac{3\int \operatorname{csch}(x)\,dx}{8a}\\ &= -\frac{3\tanh^{-1}(\cosh(x))}{8a}+\frac{\coth^4(x)}{4a}-\frac{3\coth(x)\operatorname{csch}(x)}{8a}-\frac{\coth^3(x)\operatorname{csch}(x)}{4a}\end{aligned}$$

Mathematica [A] time = 0.13, size = 60, normalized size = 1.30

$$\frac{-2\coth^2\left(\frac{x}{2}\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)-8}{16a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + a*Cosh[x]),x]
[Out] (-8 - 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]) + Se
ch[x/2]^2)/(16*a*(1 + Cosh[x]))
```

fricas [B] time = 0.53, size = 631, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(10*cosh(x)^5 + 2*(25*cosh(x) + 2)*sinh(x)^4 + 10*sinh(x)^5 + 4*cosh(x
)^4 + 4*(25*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x)^3 + 4*cosh(x)^3 + 4*(25*cosh
(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 + 3*(cosh(x)^6
 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 +
10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh
(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)
^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 -
 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh
(x) + sinh(x) + 1) - 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6
 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*
(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cos
h(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2
 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*
sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(25*cosh(x)^4 + 8*c
osh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x) + 10*cosh(x))/(a*cosh(x)^6
+ a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4
 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*c
osh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*c
osh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2
*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)
^2 - a*cosh(x) + a)*sinh(x) + a)
```

giac [B] time = 0.15, size = 94, normalized size = 2.04

$$-\frac{3\,\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a}+\frac{3\,\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a}-\frac{3\,e^{(-x)}+3\,e^{x}-2}{16\,a\left(e^{(-x)}+e^{x}-2\right)}+\frac{9\left(e^{(-x)}+e^{x}\right)^{2}+4\,e^{(-x)}+4\,e^{x}-12}{32\,a\left(e^{(-x)}+e^{x}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -3/16*log(e^(-x) + e^x + 2)/a + 3/16*log(e^(-x) + e^x - 2)/a - 1/16*(3*e^(-
x) + 3*e^x - 2)/(a*(e^(-x) + e^x - 2)) + 1/32*(9*(e^(-x) + e^x)^2 + 4*e^(-x
) + 4*e^x - 12)/(a*(e^(-x) + e^x + 2)^2)
```

maple [A] time = 0.10, size = 45, normalized size = 0.98

$$\frac{\tanh^{4}\left(\frac{x}{2}\right)}{32a}+\frac{3\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{16a}-\frac{1}{16a\tanh\left(\frac{x}{2}\right)^{2}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+a*cosh(x)),x)
```

```
[Out] 1/32/a*tanh(1/2*x)^4+3/16/a*tanh(1/2*x)^2-1/16/a/tanh(1/2*x)^2+3/8/a*ln(tan
h(1/2*x))
```

maxima [B] time = 0.34, size = 103, normalized size = 2.24

$$-\frac{5\,e^{(-x)} + 2\,e^{(-2\,x)} + 2\,e^{(-3\,x)} + 2\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{4\left(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a\right)} - \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a} + \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(5*e^(-x) + 2*e^(-2*x) + 2*e^(-3*x) + 2*e^(-4*x) + 5*e^(-5*x))/(2*a*e^
(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) +
 a) - 3/8*log(e^(-x) + 1)/a + 3/8*log(e^(-x) - 1)/a
```

mupad [B] time = 0.96, size = 132, normalized size = 2.87

$$\frac{3}{2\,a\,\left(e^{2\,x}+2\,e^{x}+1\right)}-\frac{1}{4\,a\,\left(e^{2\,x}-2\,e^{x}+1\right)}+\frac{1}{2\,a\,\left(6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1\right)}-\frac{1}{4\,a\,\left(e^{x}-1\right)}-\frac{1}{a\,\left(e^{x}+1\right)}-\frac{3\,\mathrm{atan}}{4\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + a*cosh(x)),x)
```

```
[Out] 3/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(
2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) -
 1)) - 1/(a*(exp(x) + 1)) - (3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/
2)) - 1/(a*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^{3}(x)}{\cosh(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**3/(cosh(x) + 1), x)/a
```

3.196 $\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx$

Optimal. Leaf size=41

$$\frac{\coth^5(x)}{5a}-\frac{\operatorname{csch}^5(x)}{5a}-\frac{2\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

```
[Out] 1/5*coth(x)^5/a-csch(x)/a-2/3*csch(x)^3/a-1/5*csch(x)^5/a
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\coth^5(x)}{5a}-\frac{\operatorname{csch}^5(x)}{5a}-\frac{2\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] Coth[x]^5/(5*a) - Csch[x]/a - (2*Csch[x]^3)/(3*a) - Csch[x]^5/(5*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx &= \frac{\int \coth^5(x)\operatorname{csch}(x)\,dx}{a}-\frac{\int \coth^4(x)\operatorname{csch}^2(x)\,dx}{a}\\ &= -\frac{i\operatorname{Subst}\left(\int x^4\,dx,x,i\coth(x)\right)}{a}-\frac{i\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\ &= \frac{\coth^5(x)}{5a}-\frac{i\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-i\operatorname{csch}(x)\right)}{a}\\ &= \frac{\coth^5(x)}{5a}-\frac{\operatorname{csch}(x)}{a}-\frac{2\operatorname{csch}^3(x)}{3a}-\frac{\operatorname{csch}^5(x)}{5a}\end{aligned}$$

Mathematica [A] time = 0.08, size = 41, normalized size = 1.00

$$-\frac{(8\cosh(x)+36\cosh(2x)+24\cosh(3x)-3\cosh(4x)-25)\operatorname{csch}^3(x)}{120a(\cosh(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + a*Cosh[x]),x]
```

```
[Out] -1/120*((-25 + 8*Cosh[x] + 36*Cosh[2*x] + 24*Cosh[3*x] - 3*Cosh[4*x])*Csch[
x]^3)/(a*(1 + Cosh[x]))
```

fricas [B] time = 0.44, size = 224, normalized size = 5.46

$$-\frac{2\left(15\ \cosh(x)^4+6\left(10\ \cosh(x)+3\right)\sinh(x)^3+15\ \sinh(x)^4+12\ \cdots\right.}{15\left(a\cosh(x)^5+a\sinh(x)^5+2\,a\cosh(x)^4+\left(5\,a\cosh(x)+2\,a\right)\sinh(x)^4-3\,a\cosh(x)^3+\left(10\,a\cosh(x)^2+8\,a\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

```
[Out] -2/15*(15*cosh(x)^4 + 6*(10*cosh(x) + 3)*sinh(x)^3 + 15*sinh(x)^4 + 12*cosh
(x)^3 + 2*(45*cosh(x)^2 + 18*cosh(x) + 2)*sinh(x)^2 + 4*cosh(x)^2 + 2*(30*c
osh(x)^3 + 27*cosh(x)^2 - 14*cosh(x) - 23)*sinh(x) - 4*cosh(x) + 13)/(a*cos
h(x)^5 + a*sinh(x)^5 + 2*a*cosh(x)^4 + (5*a*cosh(x) + 2*a)*sinh(x)^4 - 3*a*
cosh(x)^3 + (10*a*cosh(x)^2 + 8*a*cosh(x) - a)*sinh(x)^3 - 8*a*cosh(x)^2 +
(10*a*cosh(x)^3 + 12*a*cosh(x)^2 - 9*a*cosh(x) - 8*a)*sinh(x)^2 + 2*a*cosh(
x) + (5*a*cosh(x)^4 + 8*a*cosh(x)^3 - 3*a*cosh(x)^2 - 8*a*cosh(x) - 2*a)*si
nh(x) + 6*a)
```

giac [A] time = 0.16, size = 59, normalized size = 1.44

$$-\frac{15\,e^{(2\,x)} - 24\,e^{x} + 13}{24\,a(e^{x} - 1)^{3}} - \frac{165\,e^{(4\,x)} + 480\,e^{(3\,x)} + 650\,e^{(2\,x)} + 400\,e^{x} + 113}{120\,a(e^{x} + 1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

```
[Out] -1/24*(15*e^(2*x) - 24*e^x + 13)/(a*(e^x - 1)^3) - 1/120*(165*e^(4*x) + 480
*e^(3*x) + 650*e^(2*x) + 400*e^x + 113)/(a*(e^x + 1)^5)
```

maple [A] time = 0.10, size = 45, normalized size = 1.10

$$\frac{\frac{\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{5} + \frac{4\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3} + 6\tanh\left(\frac{x}{2}\right) - \frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}} - \frac{4}{\tanh\left(\frac{x}{2}\right)}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+a*cosh(x)),x)
```

```
[Out] 1/16/a*(1/5*tanh(1/2*x)^5+4/3*tanh(1/2*x)^3+6*tanh(1/2*x)-1/3/tanh(1/2*x)^3
-4/tanh(1/2*x))
```

maxima [B] time = 0.35, size = 469, normalized size = 11.44

$$-\frac{6\,e^{(-x)}}{5\left(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a\right)} - \frac{}{5\left(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(\cdot}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

```
[Out] -6/5*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*
e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 14/5*e^(-2*x)/(2*a*e^(-x) - 2*a
*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e
```

```
^(-8*x) + a) - 26/15*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6
*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 10/3*e^(-4*x)
/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) -
2*a*e^(-7*x) - a*e^(-8*x) + a) + 2/3*e^(-5*x)/(2*a*e^(-x) - 2*a*e^(-2*x) -
6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
 - 2*e^(-6*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*
a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 2*e^(-7*x)/(2*a*e^(-x) - 2*a*
e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^
(-8*x) + a) + 2/5/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x)
+ 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

mupad [B] time = 1.03, size = 263, normalized size = 6.41

$$\frac{1}{6\,a\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}-\frac{\frac{3\,e^{2\,x}}{8\,a}+\frac{11\,e^{3\,x}}{40\,a}+\frac{1}{8\,a}+\frac{17\,e^{x}}{40\,a}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}-\frac{\frac{11\,e^{2\,x}}{40\,a}+\frac{17}{120\,a}+\frac{e^{x}}{4\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{\frac{1}{8\,a}+\frac{11\,e^{x}}{40\,a}}{e^{2\,x}+2\,e^{x}+1}-\frac{\frac{17\,e^{2\,x}}{20\,a}+\cdots}{10\,e^{2\,x}+10\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + a*cosh(x)),x)
```

```
[Out] 1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (11*
exp(3*x))/(40*a) + 1/(8*a) + (17*exp(x))/(40*a))/(6*exp(2*x) + 4*exp(3*x) +
 exp(4*x) + 4*exp(x) + 1) - ((11*exp(2*x))/(40*a) + 17/(120*a) + exp(x)/(4*
a))/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (11*exp(x))/(40*a))
/(exp(2*x) + 2*exp(x) + 1) - ((17*exp(2*x))/(20*a) + exp(3*x)/(2*a) + (11*e
xp(4*x))/(40*a) + 11/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*
exp(4*x) + exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) - 5
/(8*a*(exp(x) - 1)) - 11/(40*a*(exp(x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\coth^{4}{\left(x \right)}}{\cosh{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+a*cosh(x)),x)
```

```
[Out] Integral(coth(x)**4/(cosh(x) + 1), x)/a
```

3.197 $\int \sqrt{a+b\cosh(x)}\,\tanh(x)\,dx$

Optimal. Leaf size=37

$$2\sqrt{a+b\cosh(x)}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*cosh(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2721, 50, 63, 207}

$$2\sqrt{a+b\cosh(x)}-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cosh(x)}\,\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\cosh(x)\right)\\ &= 2\sqrt{a+b\cosh(x)} + a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cosh(x)\right)\\ &= 2\sqrt{a+b\cosh(x)} + (2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cosh(x)}\right)\\ &= -2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right) + 2\sqrt{a+b\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$2\sqrt{a+b\cosh(x)} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

fricas [B] time = 0.72, size = 376, normalized size = 10.16

$$\left[\frac{1}{2}\sqrt{a}\,\log\left(-\frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 + 16\,ab\cosh(x)^3 + 4\left(b^2\cosh(x) + 4\,ab\right)\sinh(x)^3 + 16\,ab\cosh(x) + 2\left(16\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="fricas")
[Out] [1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^
2*cosh(x) + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2
+ 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh
```

```
(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cos
h(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(
a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cos
h(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*
sqrt(b*cosh(x) + a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*c
osh(x) + 2*(b*cosh(x) + 2*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b
*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*si
nh(x))) + 2*sqrt(b*cosh(x) + a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x) + a}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

maple [A] time = 0.06, size = 30, normalized size = 0.81

$$-2\,\mathrm{arctanh}\left(\frac{\sqrt{a + b\cosh(x)}}{\sqrt{a}}\right)\sqrt{a} + 2\sqrt{a + b\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(x))^(1/2)*tanh(x),x)
```

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*cosh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cosh(x) + a}\,\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \tanh(x)\,\sqrt{a + b\cosh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)*(a + b*cosh(x))^(1/2),x)
```

```
[Out] int(tanh(x)*(a + b*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cosh(x)}\tanh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(x))**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*cosh(x))*tanh(x), x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cosh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cosh(x)}\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

fricas [B] time = 0.65, size = 356, normalized size = 14.83

$$\left[\frac{\log\left(\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\left(b^2\cosh(x)+4\,ab\right)\sinh(x)^3+16\,ab\cosh(x)+2\left(16\,a^2+b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+24\,ab\cosh(x)+\cdots\right.}{\cosh(x)^4+4\,\cos\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x)
 + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh(x)^3 + b
*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cosh(x) + (3
*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(a) + b^2
+ 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cosh(x))*sin
h(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*si
nh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))/sqrt(a), sqrt
(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2
*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^
2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

maple [A] time = 0.07, size = 19, normalized size = 0.79

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*cosh(x))^(1/2),x)
```

```
[Out] -2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b\cosh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*cosh(x))^(1/2),x)
```

```
[Out] int(tanh(x)/(a + b*cosh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh{\left(x \right)}}{\sqrt{a+b\cosh{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*cosh(x))**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt(a + b*cosh(x)), x)
```

3.199 $\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=56

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

```
[Out] B*ln(a+b*cosh(x))/b+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^
(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4401, 2659, 208, 2668, 31}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[a + b*Cosh[x]])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\sinh(x)}{a+b\cosh(x)}\right)dx\\ &= A\int \frac{1}{a+b\cosh(x)}\,dx+B\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx\\ &= (2A)\operatorname{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{B\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cosh(x)\right)}{b}\\ &= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 55, normalized size = 0.98

$$\frac{B\log(a+b\cosh(x))}{b}-\frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*L
og[a + b*Cosh[x]])/b
```

fricas [B] time = 0.55, size = 291, normalized size = 5.20

$$\left[\frac{\sqrt{a^2-b^2}\,Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-\left(Ba^2-\right.}{a^2b-b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x + (B*a^2 - B*b^2)*log(2*(b*cosh(x) +
a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*A*b*arctan(-sq
rt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^2 - B*b^2)*x
 - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3
)]
```

giac [A] time = 0.13, size = 60, normalized size = 1.07

$$\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}-\frac{Bx}{b}+\frac{B\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*x/b + B*log(b
*e^(2*x) + 2*a*e^x + b)/b
```

maple [B] time = 0.07, size = 137, normalized size = 2.45

$$-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\tanh^{2}\left(\frac{x}{2}\right)\right)b-a-b\right)aB}{b\left(a-b\right)}-\frac{\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-\left(\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] -B/b*ln(tanh(1/2*x)-1)-B/b*ln(tanh(1/2*x)+1)+1/b/(a-b)*ln(a*tanh(1/2*x)^2-t
anh(1/2*x)^2*b-a-b)*a*B-1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*B+2
*A/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.98, size = 197, normalized size = 3.52

$$\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^x\,\sqrt{b^2-a^2}}{\left(A\,b^3-A\,a^2\,b\right)\,\sqrt{A^2}}+\frac{A^2\,a\,b\,\sqrt{b^2-a^2}}{\left(A\,b^3-A\,a^2\,b\right)\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-a^2}}-\frac{B\,x}{b}+\frac{B\,b^3\,\ln\left(4\,A^2\,b+8\,A^2\,a\,\mathrm{e}^x+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{b^4-a^2\,b^2}-\frac{B\,a^2\,b\,\ln\left(4\,A^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*cosh(x)),x)
```

```
[Out] (2*atan((A^2*b^2*exp(x)*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2))
+ (A^2*a*b*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2)))*(A^2)^(1/2))
/(b^2 - a^2)^(1/2) - (B*x)/b + (B*b^3*log(4*A^2*b + 8*A^2*a*exp(x) + 4*A^2*
b*exp(2*x)))/(b^4 - a^2*b^2) - (B*a^2*b*log(4*A^2*b + 8*A^2*a*exp(x) + 4*A^
2*b*exp(2*x)))/(b^4 - a^2*b^2)
```

sympy [A] time = 27.72, size = 741, normalized size = 13.23

$$\begin{cases} \tilde{\infty}\left(2A\,\mathrm{atan}\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh^2\left(\frac{x}{2}\right)+1\right)\right) \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)}+\frac{Bx}{b}-\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} \\ \frac{Ax+B\cosh(x)}{a} \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}-\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b} \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ab\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Bax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\right.}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(
tanh(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-A/(b*tanh(x/2)) + B*x/b - 2*B*l
og(tanh(x/2) + 1)/b + 2*B*log(tanh(x/2))/b, Eq(a, -b)), ((A*x + B*cosh(x))/
a, Eq(b, 0)), (A*tanh(x/2)/b + B*x/b - 2*B*log(tanh(x/2) + 1)/b, Eq(a, b)),
 (-A*b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
```

```
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b
/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a -
b) + b/(a - b))) + B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b) + b/(a
- b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b
/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b) + b/(a -
 b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(
a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*B*a*sqrt(a/(a - b) + b/(a -
 b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a -
 b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) +
 b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(a/(a - b) + b/(a
 - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) +
b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(a/(a - b) + b/(a
- b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/
(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + 2*B*b*sqrt(a/(a - b) + b/(a
- b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a
- b) + b/(a - b))), True))
```

3.200 $\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx$

Optimal. Leaf size=18

$$\frac{A\sinh(x)}{\cosh(x)+1}+B\log(\cosh(x)+1)$$

```
[Out] B*ln(1+cosh(x))+A*sinh(x)/(1+cosh(x))
```

Rubi [A] time = 0.08, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4401, 2648, 2667, 31}

$$\frac{A\sinh(x)}{\cosh(x)+1}+B\log(\cosh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] B*Log[1 + Cosh[x]] + (A*Sinh[x])/(1 + Cosh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx &= \int \left(\frac{A}{1+\cosh(x)}+\frac{B\sinh(x)}{1+\cosh(x)}\right)dx\\ &= A\int \frac{1}{1+\cosh(x)}\,dx+B\int \frac{\sinh(x)}{1+\cosh(x)}\,dx\\ &= \frac{A\sinh(x)}{1+\cosh(x)}+B\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\cosh(x)\right)\\ &= B\log(1+\cosh(x))+\frac{A\sinh(x)}{1+\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 19, normalized size = 1.06

$$A\tanh\left(\frac{x}{2}\right)+2B\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] 2*B*Log[Cosh[x/2]] + A*Tanh[x/2]
```

fricas [B] time = 0.49, size = 46, normalized size = 2.56

$$-\frac{Bx\cosh(x)+Bx\sinh(x)+Bx-2\left(B\cosh(x)+B\sinh(x)+B\right)\log\left(\cosh(x)+\sinh(x)+1\right)+2\,A}{\cosh(x)+\sinh(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*cosh(x) + B*x*sinh(x) + B*x - 2*(B*cosh(x) + B*sinh(x) + B)*log(cosh(
x) + sinh(x) + 1) + 2*A)/(cosh(x) + sinh(x) + 1)
```

giac [A] time = 0.12, size = 22, normalized size = 1.22

$$-Bx+2\,B\log\left(e^{x}+1\right)-\frac{2\,A}{e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*B*log(e^x + 1) - 2*A/(e^x + 1)
```

maple [A] time = 0.05, size = 28, normalized size = 1.56

$$A\tanh\left(\frac{x}{2}\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(1+cosh(x)),x)
[Out] A*tanh(1/2*x)-B*ln(tanh(1/2*x)-1)-B*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.37, size = 19, normalized size = 1.06

$$B\log\left(\cosh(x)+1\right)+\frac{2A}{e^{(-x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="maxima")
[Out] B*log(cosh(x) + 1) + 2*A/(e^(-x) + 1)
```

mupad [B] time = 0.06, size = 22, normalized size = 1.22

$$2B\ \ln\left(e^x+1\right)-\frac{2A}{e^x+1}-Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(cosh(x) + 1),x)
[Out] 2*B*log(exp(x) + 1) - (2*A)/(exp(x) + 1) - B*x
```

sympy [A] time = 0.34, size = 20, normalized size = 1.11

$$A\tanh\left(\frac{x}{2}\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1+cosh(x)),x)
[Out] A*tanh(x/2) + B*x - 2*B*log(tanh(x/2) + 1)
```

3.201 $\int \frac{A+B \sinh(x)}{1-\cosh(x)}\, dx$

Optimal. Leaf size=24

$$-\frac{A \sinh(x)}{1-\cosh(x)} - B \log(1-\cosh(x))$$

```
[Out] -B*ln(1-cosh(x))-A*sinh(x)/(1-cosh(x))
```

Rubi [A] time = 0.09, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4401, 2648, 2667, 31}

$$-\frac{A \sinh(x)}{1-\cosh(x)} - B \log(1-\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

```
[Out] -(B*Log[1 - Cosh[x]]) - (A*Sinh[x])/(1 - Cosh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx &= \int \left(-\frac{A}{-1+\cosh(x)}-\frac{B\sinh(x)}{-1+\cosh(x)}\right)dx\\ &= -\left(A\int \frac{1}{-1+\cosh(x)}\,dx\right)-B\int \frac{\sinh(x)}{-1+\cosh(x)}\,dx\\ &= -\frac{A\sinh(x)}{1-\cosh(x)}-B\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\cosh(x)\right)\\ &= -B\log(1-\cosh(x))-\frac{A\sinh(x)}{1-\cosh(x)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 19, normalized size = 0.79

$$A\coth\left(\frac{x}{2}\right)-2B\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(1 - Cosh[x]),x]
[Out] A*Coth[x/2] - 2*B*Log[Sinh[x/2]]
```

fricas [B] time = 0.48, size = 48, normalized size = 2.00

$$\frac{Bx\cosh(x)+Bx\sinh(x)-Bx-2\,(B\cosh(x)+B\sinh(x)-B)\log\left(\cosh(x)+\sinh(x)-1\right)+2\,A}{\cosh(x)+\sinh(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="fricas")
[Out] (B*x*cosh(x) + B*x*sinh(x) - B*x - 2*(B*cosh(x) + B*sinh(x) - B)*log(cosh(x
) + sinh(x) - 1) + 2*A)/(cosh(x) + sinh(x) - 1)
```

giac [A] time = 0.12, size = 22, normalized size = 0.92

$$Bx-2\,B\log\left(|e^x-1|\right)+\frac{2\,A}{e^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="giac")
[Out] B*x - 2*B*log(abs(e^x - 1)) + 2*A/(e^x - 1)
```

maple [A] time = 0.07, size = 36, normalized size = 1.50

$$B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{A}{\tanh\left(\frac{x}{2}\right)}-2B\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(1-cosh(x)),x)
[Out] B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/2*x)+1)+A/tanh(1/2*x)-2*B*ln(tanh(1/2*x))
```

maxima [A] time = 0.37, size = 20, normalized size = 0.83

$$-B\log\left(\cosh(x)-1\right)-\frac{2A}{e^{(-x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="maxima")
[Out] -B*log(cosh(x) - 1) - 2*A/(e^(-x) - 1)
```

mupad [B] time = 0.91, size = 21, normalized size = 0.88

$$B\,x+\frac{2\,A}{\mathrm{e}^x-1}-2\,B\,\ln\left(\mathrm{e}^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*sinh(x))/(cosh(x) - 1),x)
[Out] B*x + (2*A)/(exp(x) - 1) - 2*B*log(exp(x) - 1)
```

sympy [A] time = 0.51, size = 31, normalized size = 1.29

$$\frac{A}{\tanh\left(\frac{x}{2}\right)}-Bx+2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)-2B\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(1-cosh(x)),x)
[Out] A/tanh(x/2) - B*x + 2*B*log(tanh(x/2) + 1) - 2*B*log(tanh(x/2))
```

3.202 $\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=65

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}-\frac{B\log(a+b\cosh(x))}{a}+\frac{B\log(\cosh(x))}{a}$$

```
[Out] B*ln(cosh(x))/a-B*ln(a+b*cosh(x))/a+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+
b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {4401, 2659, 208, 2721, 36, 29, 31}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}-\frac{B\log(a+b\cosh(x))}{a}+\frac{B\log(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[Cosh[x]])/a - (B*Log[a + b*Cosh[x]])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\tanh(x)}{a+b\cosh(x)}\right)dx\\
&= A\int \frac{1}{a+b\cosh(x)}\,dx+B\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx\\
&= (2A)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+B\,\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx,x,b\cosh(x
\\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,b\cosh(x)\right)}{a}-\frac{B\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cos
\\
&= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(\cosh(x))}{a}-\frac{B\log(a+b\cosh(x))}{a}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 61, normalized size = 0.94

$$
\frac{B(\log(\cosh(x))-\log(a+b\cosh(x)))}{a}-\frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
Log[Cosh[x]] - Log[a + b*Cosh[x]]))/a
```

fricas [B] time = 0.58, size = 315, normalized size = 4.85

$$\left[\frac{\sqrt{a^2-b^2}\,Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-\left(Ba^2-\right.}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) + b)) - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x
))) + (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2), -(
2*sqrt(-a^2 + b^2)*A*a*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)
/(a^2 - b^2)) + (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x)))
- (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)]
```

giac [A] time = 0.12, size = 66, normalized size = 1.02

$$\frac{2\,A\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{B\log\left(be^{(2\,x)}+2\,ae^x+b\right)}{a}+\frac{B\log\left(e^{(2\,x)}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(b*e^(2*x)
 + 2*a*e^x + b)/a + B*log(e^(2*x) + 1)/a
```

maple [B] time = 0.10, size = 125, normalized size = 1.92

$$-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)B}{a-b}+\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)Bb}{a\,(a-b)}+\frac{2A\,\mathrm{arctanh}\left(\frac{(a-b)\,\mathrm{t}}{\sqrt{(a+}}\right.}{\sqrt{(a+b)\,(a-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tanh(x))/(a+b*cosh(x)),x)
```

```
[Out] -1/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)*B+1/a/(a-b)*ln(a*tanh(1/2*
x)^2-tanh(1/2*x)^2*b-a-b)*B*b+2*A/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/
2*x)/((a+b)*(a-b))^(1/2))+B/a*ln(tanh(1/2*x)^2+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 12.14, size = 160, normalized size = 2.46

$$\frac{B\,\ln\left(16\,B^2\,b^2-16\,B^2\,a^2-16\,B^2\,a^2\,\mathrm{e}^{2\,x}+16\,B^2\,b^2\,\mathrm{e}^{2\,x}\right)}{a}-\frac{B\,\ln\left(16\,B^2\,b+32\,B^2\,a\,\mathrm{e}^{x}+16\,B^2\,b\,\mathrm{e}^{2\,x}\right)}{a}-\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tanh(x))/(a + b*cosh(x)),x)
```

```
[Out] (B*log(16*B^2*b^2 - 16*B^2*a^2 - 16*B^2*a^2*exp(2*x) + 16*B^2*b^2*exp(2*x))
)/a - (B*log(16*B^2*b + 32*B^2*a*exp(x) + 16*B^2*b*exp(2*x)))/a - (2*atan((
A^2*b^2*exp(x)*(b^2 - a^2)^(1/2) + A^2*a*b*(b^2 - a^2)^(1/2))/(A*b*(a^2 - b
^2)*(A^2)^(1/2)))*(A^2)^(1/2))/(b^2 - a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tanh{\left(x \right)}}{a + b\cosh{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*tanh(x))/(a + b*cosh(x)), x)
```

3.203 $\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=100

$$-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}+\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] 1/2*B*ln(1-cosh(x))/(a+b)+1/2*B*ln(1+cosh(x))/(a-b)-a*B*ln(a+b*cosh(x))/(a^
2-b^2)+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4401, 2659, 208, 2721, 801}

$$-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}+\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) + (B*Log[1 + Cosh[x]])/(2*(a - b)) - (
a*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx &= \int \left(\frac{A}{a+b\cosh(x)}+\frac{B\coth(x)}{a+b\cosh(x)}\right)dx\\ &= A\int \frac{1}{a+b\cosh(x)}\,dx+B\int \frac{\coth(x)}{a+b\cosh(x)}\,dx\\ &= (2A)\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)-B\,\mathrm{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx,x,\right.\\ &= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}-B\,\mathrm{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{}{2(a-}\right.\right.\\ &= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}+\frac{B\log(1+\cosh(x))}{2(a-b)}-\frac{aB\log(a+b\,\mathrm{co}}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.26, size = 81, normalized size = 0.81

$$\frac{B\left(a\log(a+b\cosh(x))-a\log(\sinh(x))+b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{b^2-a^2}-\frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
a*Log[a + b*Cosh[x]] - a*Log[Sinh[x]] + b*Log[Tanh[x/2]]))/(-a^2 + b^2)
```

fricas [A] time = 2.75, size = 303, normalized size = 3.03

$$\left[-\frac{Ba\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\sqrt{a^2-b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cos}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.}{a^2-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - sqrt(a^2 - b^2)*A*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a +
B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x) - 1))/(
a^2 - b^2), -(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 2*sqrt(-a^2
+ b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2))
- (B*a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x
) - 1))/(a^2 - b^2)]
```

giac [A] time = 0.13, size = 90, normalized size = 0.90

$$-\frac{Ba\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{a^2-b^2}+\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{B\log\left(e^{x}+1\right)}{a-b}+\frac{B\log\left(|e^{x}-1|\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] -B*a*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/sqrt(-a^2 + b^2) + B*log(e^x + 1)/(a - b) + B*log(abs(e^x - 1
))/(a + b)
```

maple [A] time = 0.11, size = 139, normalized size = 1.39

$$-\frac{aB\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)}{(a+b)\,(a-b)}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)Aa}{(a+b)\,\sqrt{(a+b)\,(a-b)}}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)Ab}{(a+b)\,\sqrt{(a+b)\,(a-b)}}+\frac{B\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*coth(x))/(a+b*cosh(x)),x)
```

```
[Out] -1/(a+b)*a*B/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+2/(a+b)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*a+2/(a+b)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*b+B/(a+b)*
ln(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.68, size = 974, normalized size = 9.74

$$\frac{B\,\ln\left(e^x+1\right)}{a-b}+\frac{\ln\left(\frac{\left(\frac{32\left(A^2\,a^2\,b+2\,e^x\,A^2\,a\,b^2+A^2\,b^3+8\,e^x\,A\,B\,a^3+4\,A\,B\,a^2\,b-2\,e^x\,A\,B\,a\,b^2+4\,e^x\,B^2\,a^3+3\,B^2\,a^2\,b+5\,e^x\,B^2\,a\,b^2+B^2\,b^3\right)}{b^5}+\frac{\left(A\,\sqrt{{\left(a+b\right)}^3\,{\left(a-b\right)}^3}-B\,a^3+B\,a\right.}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*coth(x))/(a + b*cosh(x)),x)
```

```
[Out] (B*log(exp(x) + 1))/(a - b) + (log((((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b + 3
*B^2*a^2*b + 4*B^2*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*a^
3*exp(x) + 2*A^2*a*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 + ((A*((a + b)^3*(
a - b)^3)^(1/2) - B*a^3 + B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B) + a*
b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) - a^3*b
^3*(128*A - 256*B) - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A -
3*B) + 128*A*a^3*exp(x)*((a^2 - b^2)^3)^(1/2) + 96*A*a^2*b*((a^2 - b^2)^3)^
(1/2) - 32*A*a*b^2*exp(x)*((a^2 - b^2)^3)^(1/2)))/((b^7 - a^2*b^5)*(a^2 - b
^2)^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*a^3 + B*a*b^2))/(a^2 - b^2)^2 -
(32*B*(A^2*b^2*exp(x) + 4*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*A*B*a^2*ex
p(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5)*(A*((a + b)^3*(a - b)^3)^(1/2) - B
*a^3 + B*a*b^2))/(a^4 + b^4 - 2*a^2*b^2) - (log(- (32*B*(A^2*b^2*exp(x) + 4
*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2
```

```
*A*B*a*b))/b^5 - (((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b + 3*B^2*a^2*b + 4*B^2
*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*a^3*exp(x) + 2*A^2*a
*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 + A*((a + b)^3*(a - b)^3)^
(1/2) - B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B) + a*b^5*(64*A - 128*B)
 + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) - a^3*b^3*(128*A - 256*B)
 - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A - 3*B) - 128*A*a^3*e
xp(x)*((a^2 - b^2)^3)^(1/2) - 96*A*a^2*b*((a^2 - b^2)^3)^(1/2) + 32*A*a*b^2
*exp(x)*((a^2 - b^2)^3)^(1/2)))/((b^7 - a^2*b^5)*(a^2 - b^2)^2))*(B*a^3 + A
*((a + b)^3*(a - b)^3)^(1/2) - B*a*b^2))/(a^2 - b^2)^2)*(B*a^3 + A*((a + b)
^3*(a - b)^3)^(1/2) - B*a*b^2))/(a^4 + b^4 - 2*a^2*b^2) + (B*log(exp(x) - 1
))/(a + b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\coth(x)}{a + b\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*coth(x))/(a + b*cosh(x)), x)
```

3.204 $\int \frac{A+B\,\mathbf{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=62

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\tan^{-1}(\sinh(x))}{a}$$

```
[Out] B*arctan(sinh(x))/a+2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2)
)/a/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2828, 3001, 3770, 2659, 208}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

```
[Out] (B*ArcTan[Sinh[x]])/a + (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt
[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx &= \int \frac{(B+A\cosh(x))\text{sech}(x)}{a+b\cosh(x)}\,dx \\
&= \frac{B\int \text{sech}(x)\,dx}{a} + \frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}\,dx}{a} \\
&= \frac{B\tan^{-1}(\sinh(x))}{a} + \frac{(2(aA-bB))\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a} \\
&= \frac{B\tan^{-1}(\sinh(x))}{a} + \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 63, normalized size = 1.02

$$\frac{2\left(\frac{(bB-aA)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + B\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*(B*ArcTan[Tanh[x/2]] + ((-(a*A) + b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-
a^2 + b^2]])/Sqrt[-a^2 + b^2]))/a
```

fricas [A] time = 0.98, size = 249, normalized size = 4.02

$$\left[-\frac{(Aa-Bb)\sqrt{a^2-b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2-b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{a^3-ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [-((A*a - B*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) - 2*(B*a^2 - B*b^2)*arctan(cosh(x) + sinh(x)))/(
a^3 - a*b^2), -2*((A*a - B*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (B*a^2 - B*b^2)*arctan(cosh(x) + si
nh(x)))/(a^3 - a*b^2)]
```

giac [A] time = 0.13, size = 53, normalized size = 0.85

$$\frac{2\,B\arctan\left(e^{x}\right)}{a}+\frac{2\,(Aa-Bb)\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*B*arctan(e^x)/a + 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqr
t(-a^2 + b^2)*a)
```

maple [A] time = 0.10, size = 89, normalized size = 1.44

$$\frac{2A\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)\,(a-b)}}-\frac{2Bb\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)\,(a-b)}}+\frac{2B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sech(x))/(a+b*cosh(x)),x)
```

```
[Out] 2*A/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-2/a/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*B*b+2*B/
a*arctan(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 6.41, size = 636, normalized size = 10.26

$$\frac{\ln\left(\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,\mathrm{e}^x\,B^2\,a^3-2\,B^2\,a^2\,b+3\,\mathrm{e}^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}+\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,a^2\,\left(2\,B\,b^2-4\,A\,a^2\,\mathrm{e}^x+A\,b^2\,\mathrm{e}^x-2\,A\,a\,b+3\,B\,a\,b\,\mathrm{e}\right.}{b^5}\right.}{a\,b^2-a^3}\right.}{a\,b^2-a^3}\right.}{a\,b^2-a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cosh(x))/(a + b*cosh(x)),x)
```

```
[Out] (B*log(exp(x) + 1i)*1i)/a - (B*log(exp(x) - 1i)*1i)/a + (log((((a + b)*(a -
 b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*a^2*b - 4*B^2*a^
3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 + (((a + b)*(a - b))^(1/2
)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*exp(x) - 2*A*a*b +
 3*B*a*b*exp(x)))/b^5 - (32*a^2*((a + b)*(a - b))^(1/2)*(A*a - B*b)*(3*a^2*
b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2 - a^3))))/(a*b^2 -
a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*exp(x) + 4*B*a*exp(x)
))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 - a^3) - (log(- (((a +
b)*(a - b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*a^2*b - 4
*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 - (((a + b)*(a - b
))^(1/2)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*exp(x) - 2*
A*a*b + 3*B*a*b*exp(x)))/b^5 + (32*a^2*((a + b)*(a - b))^(1/2)*(A*a - B*b)*
(3*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2 - a^3))))/(a
*b^2 - a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*exp(x) + 4*B*a
*exp(x)))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 - a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\,\mathrm{sech}\,(x)}{a + b\cosh\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*sech(x))/(a + b*cosh(x)), x)
```

3.205 $\int \frac{A+B\mathbf{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=99

$$\frac{bB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}-\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

```
[Out] 1/2*B*ln(1-cosh(x))/(a+b)-1/2*B*ln(1+cosh(x))/(a-b)+b*B*ln(a+b*cosh(x))/(a^
2-b^2)+2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(
1/2)
```

Rubi [A] time = 0.31, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4225, 4401, 2659, 208, 2668, 706, 31, 633}

$$\frac{bB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}-\frac{B\log(\cosh(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
) + (B*Log[1 - Cosh[x]])/(2*(a + b)) - (B*Log[1 + Cosh[x]])/(2*(a - b)) + (
b*B*Log[a + b*Cosh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx = -\left(i\int \frac{\text{csch}(x)(iB+iA\sinh(x))}{a+b\cosh(x)}\,dx\right)$$
$$= \int \left(\frac{A}{a+b\cosh(x)} + \frac{B\text{csch}(x)}{a+b\cosh(x)}\right)dx$$
$$= A\int \frac{1}{a+b\cosh(x)}\,dx + B\int \frac{\text{csch}(x)}{a+b\cosh(x)}\,dx$$
$$= (2A)\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - (bB)\,\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x,\right.$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{(bB)\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(x)\right)}{a^2-b^2} + \frac{(bB)\,\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx,\right.}{a^2-b^2}$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{bB\log(a+b\cosh(x))}{a^2-b^2} + \frac{B\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\cosh(x)\right)}{2(a-b)} -$$
$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(1-\cosh(x))}{2(a+b)} - \frac{B\log(1+\cosh(x))}{2(a-b)} + \frac{bB\log(a+b\cosh}{a^2-b^2}$$

Mathematica [A] time = 0.20, size = 81, normalized size = 0.82

$$\frac{B\left(b\log(a+b\cosh(x))+a\log\left(\tanh\left(\frac{x}{2}\right)\right)-b\log(\sinh(x))\right)}{a^2-b^2} - \frac{2A\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

```
[Out] (-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*(
b*Log[a + b*Cosh[x]] - b*Log[Sinh[x]] + a*Log[Tanh[x/2]]))/(a^2 - b^2)
```

fricas [A] time = 3.25, size = 298, normalized size = 3.01

$$\left[\frac{Bb\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right) + \sqrt{a^2-b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}\,(b\cosh(}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.}{a^2-b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

```
[Out] [(B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + sqrt(a^2 - b^2)*A*log((b
^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x)
 + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)
^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a + B
*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x) - 1))/(a
^2 - b^2), (B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - 2*sqrt(-a^2 +
b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) -
(B*a + B*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x)
- 1))/(a^2 - b^2)]
```

giac [A] time = 0.15, size = 90, normalized size = 0.91

$$\frac{Bb\log\left(be^{(2x)}+2\,ae^{x}+b\right)}{a^2-b^2}+\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{B\log\left(e^{x}+1\right)}{a-b}+\frac{B\log\left(|e^{x}-1|\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] B*b*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt(
-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(e^x + 1)/(a - b) + B*log(abs(e^x - 1)
)/(a + b)
```

maple [A] time = 0.12, size = 138, normalized size = 1.39

$$\frac{Bb\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-\left(\tanh^2\left(\frac{x}{2}\right)\right)b-a-b\right)}{(a+b)(a-b)}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)Aa}{(a+b)\sqrt{(a+b)(a-b)}}+\frac{2\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)Ab}{(a+b)\sqrt{(a+b)(a-b)}}+\frac{B\ln\left(\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*csch(x))/(a+b*cosh(x)),x)
```

```
[Out] 1/(a+b)*B*b/(a-b)*ln(a*tanh(1/2*x)^2-tanh(1/2*x)^2*b-a-b)+2/(a+b)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*a+2/(a+b)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))*A*b+B/(a+b)*l
n(tanh(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.33, size = 983, normalized size = 9.93

$$\ln\left(\frac{\left(\frac{32\left(A^2 a^2 b+2 e^x A^2 a b^2+A^2 b^3-8 e^x A B a^2 b-4 A B a b^2+2 e^x A B b^3+4 e^x B^2 a^3+2 B^2 a^2 b+3 e^x B^2 a b^2\right)}{b^5}+\frac{\left(\frac{32\left(2 B b^4+B a^2 b^2-4 A a^4 e^x-A b^4 e^x+2 A a b^3-2 A a^3 b+6 B a\right.}{b^5}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/sinh(x))/(a + b*cosh(x)),x)
```

```
[Out] (log((((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b + 4*B^2*a^3*exp(x) + 3*B^2*a*
b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x) + 2*A^2*a*b^2*exp(x) - 8*A*B*a^
2*b*exp(x)))/b^5 + (((32*(2*B*b^4 + B*a^2*b^2 - 4*A*a^4*exp(x) - A*b^4*exp(
x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp(x) - 3*B*a^3*b*exp(x) + 5*A*a^2*
b^2*exp(x)))/b^5 - (32*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b)*(3
*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^4*exp(x) - 5*a^3*b^2*exp(x)))/(b^5*
(a^4 + b^4 - 2*a^2*b^2)))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b)
)/(a^4 + b^4 - 2*a^2*b^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b
))/(a^4 + b^4 - 2*a^2*b^2) - (32*(2*B^3*b^2 + A^2*B*b^2 - 2*A*B^2*a*b + 4*B
^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*exp(x) + A^2*B*a*b*exp(x)))/
b^5)*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b))/(a^4 + b^4 - 2*a^2*
b^2) - (B*log(exp(x) + 1))/(a - b) - (log(- (32*(2*B^3*b^2 + A^2*B*b^2 - 2*
A*B^2*a*b + 4*B^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*exp(x) + A^2*
B*a*b*exp(x)))/b^5 - (((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b + 4*B^2*a^3*e
xp(x) + 3*B^2*a*b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x) + 2*A^2*a*b^2*e
xp(x) - 8*A*B*a^2*b*exp(x)))/b^5 - (((32*(2*B*b^4 + B*a^2*b^2 - 4*A*a^4*exp
(x) - A*b^4*exp(x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp(x) - 3*B*a^3*b*e
xp(x) + 5*A*a^2*b^2*exp(x)))/b^5 + (32*(B*b^3 + A*((a + b)^3*(a - b)^3)^(1/
2) - B*a^2*b)*(3*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^4*exp(x) - 5*a^3*b^
2*exp(x)))/(b^5*(a^4 + b^4 - 2*a^2*b^2)))*(B*b^3 + A*((a + b)^3*(a - b)^3)^
```

```
(1/2) - B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2))*(B*b^3 + A*((a + b)^3*(a - b)^3)
^(1/2) - B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2))*(B*b^3 + A*((a + b)^3*(a - b)^3
)^(1/2) - B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2) + (B*log(exp(x) - 1))/(a + b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \operatorname{csch}(x)}{a + b \cosh(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*cosh(x)),x)
```

```
[Out] Integral((A + B*csch(x))/(a + b*cosh(x)), x)
```

3.206 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx$

Optimal. Leaf size=86

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\,\sqrt{a+b}}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

```
[Out] B*x/b+C*ln(a+b*cosh(e*x+d))/b/e+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*e*
x+1/2*d)/(a+b)^(1/2))/b/e/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4377, 2735, 2659, 205, 2668, 31}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\,\sqrt{a+b}}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b
]])/(Sqrt[a - b]*b*Sqrt[a + b]*e) + (C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = C\int \frac{\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx+\int \frac{A+B\cosh(d+ex)}{a+b\cosh(d+ex)}\,dx$$

$$= \frac{Bx}{b}-\frac{(-Ab+aB)\int \frac{1}{a+b\cosh(d+ex)}\,dx}{b}+\frac{C\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cos\ldots\right)}{be}$$

$$= \frac{Bx}{b}+\frac{C\log(a+b\cosh(d+ex))}{be}-\frac{(2i(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a\ldots}\ldots\right)}{be}$$

$$= \frac{Bx}{b}+\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,e}+\frac{C\log(a+b\cosh\ldots}{be}$$

Mathematica [A] time = 0.26, size = 81, normalized size = 0.94

$$\frac{\frac{2(aB-Ab)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+C\log(a+b\cosh(d+ex))+B(d+ex)}{be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

```
[Out] (B*(d + e*x) + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2] + C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

fricas [A] time = 0.47, size = 405, normalized size = 4.71

$$\left[\frac{\left((B-C)a^2-(B-C)b^2\right)ex-(Ba-Ab)\sqrt{a^2-b^2}\,\log\left(\frac{b^2\cosh(ex+d)^2+b^2\sinh(ex+d)^2+2\,ab\cosh(ex+d)+2\,a^2-b^2+2\left(b^2\cosh(ex+d)\right.}{b\cosh(ex+d)^2+b\sinh(ex+d)^2+2\,a\cosh(ex+d)+2}\right.}{\left(a^2b-b^3\right)e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="f
ricas")
```

```
[Out] [(((B - C)*a^2 - (B - C)*b^2)*e*x - (B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*co
sh(e*x + d)^2 + b^2*sinh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2
*(b^2*cosh(e*x + d) + a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x +
d) + b*sinh(e*x + d) + a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cos
h(e*x + d) + 2*(b*cosh(e*x + d) + a)*sinh(e*x + d) + b)) + (C*a^2 - C*b^2)*
log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*b - b^3
)*e), (((B - C)*a^2 - (B - C)*b^2)*e*x + 2*(B*a - A*b)*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2))
+ (C*a^2 - C*b^2)*log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d
))))/((a^2*b - b^3)*e)]
```

giac [A] time = 0.16, size = 97, normalized size = 1.13

$$\left(\frac{(xe+d)(B-C)}{b}+\frac{C\log\left(be^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}+b\right)}{b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{be^{(xe+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="g
iac")
```

```
[Out] ((x*e + d)*(B - C)/b + C*log(b*e^(2*x*e + 2*d) + 2*a*e^(x*e + d) + b)/b - 2
*(B*a - A*b)*arctan((b*e^(x*e + d) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)
*b))*e^(-1)
```

maple [B] time = 0.17, size = 276, normalized size = 3.21

$$-\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)B}{eb}-\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)C}{eb}+\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)B}{eb}-\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)C}{eb}+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

```
[Out] -1/e/b*ln(tanh(1/2*e*x+1/2*d)-1)*B-1/e/b*ln(tanh(1/2*e*x+1/2*d)-1)*C+1/e/b*
ln(tanh(1/2*e*x+1/2*d)+1)*B-1/e/b*ln(tanh(1/2*e*x+1/2*d)+1)*C+1/e/b/(a-b)*l
n(a*tanh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)*a*C-1/e/(a-b)*ln(a*t
anh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)*C+2/e/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2))*A-2/e/b/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2))*a*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.19, size = 653, normalized size = 7.59

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b-A\,e\,a^2\,b^2-B\,e\,a\,b^3+A\,e\,b^4}+\frac{a^2\,b^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b^4-A\,e\,a^2\,b^5-B\,e\,a\,b^6+A\,e\,b^7}+\frac{A\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}{b\,e\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}-\frac{}{b^{}}\right.}{\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x)),x)
```

```
[Out] (2*atan((a*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1
/2))/(A*b^4*e - B*a*b^3*e + B*a^3*b*e - A*a^2*b^2*e) + (a^2*b^2*exp(e*x)*ex
p(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(
A*b^7*e - B*a*b^6*e - A*a^2*b^5*e + B*a^3*b^4*e) + (A*exp(e*x)*exp(d)*(b^4*
```

```
e^2 - a^2*b^2*e^2)^(1/2))/(b*e*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2)) - (B*
a*exp(e*x)*exp(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2))/(b^2*e*(A^2*b^2 + B^2*a^2
- 2*A*B*a*b)^(1/2)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4*e^2 - a^2*
b^2*e^2)^(1/2) + (B*x)/b - (C*x)/b + (C*b^3*e*log(4*A^2*b^3 + 4*B^2*a^2*b -
 8*A*B*a*b^2 + 8*B^2*a^3*exp(e*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(2*e*x) +
8*A^2*a*b^2*exp(e*x)*exp(d) + 4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 16*A*B*a^2*
b*exp(e*x)*exp(d) - 8*A*B*a*b^2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 - a^2*b^2*e^
2) - (C*a^2*b*e*log(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2 + 8*B^2*a^3*exp(e
*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(2*e*x) + 8*A^2*a*b^2*exp(e*x)*exp(d) +
4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 16*A*B*a^2*b*exp(e*x)*exp(d) - 8*A*B*a*b^
2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 - a^2*b^2*e^2)
```

sympy [A] time = 31.24, size = 695, normalized size = 8.08

$$\begin{cases} \frac{\tilde{\infty} x(A+B\cosh(d)+C\sinh(d))}{\cosh(d)} \\ -\frac{A}{be\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}+\frac{Bx}{b}-\frac{B}{be\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}+\frac{Cx}{b}-\frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{be}+\frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{be} \\ \frac{Ax+\frac{B\sinh(d+ex)}{e}+\frac{C\cosh(d+ex)}{e}}{a} \\ \frac{x(A+B\cosh(d)+C\sinh(d))}{a+b\cosh(d)} \\ \frac{A\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{be}+\frac{Bx}{b}-\frac{B\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)}{be}+\frac{Cx}{b}-\frac{2C\log\left(\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{be} \\ -\frac{Ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e}+\frac{Ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{abe+b^2e}+\frac{Baex}{abe+b^2e}+\frac{Ba\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}\log\left(-\sqrt{\frac{a}{a-b}+\cdots\right.}{abe+b^2e} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/cosh(d), Eq(a, 0) & Eq(b, 0) &
 Eq(e, 0)), (-A/(b*e*tanh(d/2 + e*x/2)) + B*x/b - B/(b*e*tanh(d/2 + e*x/2))
 + C*x/b - 2*C*log(tanh(d/2 + e*x/2) + 1)/(b*e) + 2*C*log(tanh(d/2 + e*x/2)
)/(b*e), Eq(a, -b)), ((A*x + B*sinh(d + e*x)/e + C*cosh(d + e*x)/e)/a, Eq(b
, 0)), (x*(A + B*cosh(d) + C*sinh(d))/(a + b*cosh(d)), Eq(e, 0)), (A*tanh(d
/2 + e*x/2)/(b*e) + B*x/b - B*tanh(d/2 + e*x/2)/(b*e) + C*x/b - 2*C*log(tan
h(d/2 + e*x/2) + 1)/(b*e), Eq(a, b)), (-A*b*sqrt(a/(a - b) + b/(a - b))*log
(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e) + A*b*s
qrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x
/2))/(a*b*e + b**2*e) + B*a*e*x/(a*b*e + b**2*e) + B*a*sqrt(a/(a - b) + b/(
a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2
```

```
*e) - B*a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tan
h(d/2 + e*x/2))/(a*b*e + b**2*e) + B*b*e*x/(a*b*e + b**2*e) + C*a*e*x/(a*b*
e + b**2*e) + C*a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*
b*e + b**2*e) + C*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a
*b*e + b**2*e) - 2*C*a*log(tanh(d/2 + e*x/2) + 1)/(a*b*e + b**2*e) + C*b*e*
x/(a*b*e + b**2*e) + C*b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/
2))/(a*b*e + b**2*e) + C*b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x
/2))/(a*b*e + b**2*e) - 2*C*b*log(tanh(d/2 + e*x/2) + 1)/(a*b*e + b**2*e),
True))
```

3.207 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{3/2}(a+b)^{3/2}}-\frac{C}{be(a+b\cosh(d+ex))}$$

```
[Out] 2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2))/(a-b)^(3/2
)/(a+b)^(3/2)/e-C/b/e/(a+b*cosh(e*x+d))-(A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e/(
a+b*cosh(e*x+d))
```

Rubi [A] time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4377, 2754, 12, 2659, 205, 2668, 32}

$$-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}+\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{3/2}(a+b)^{3/2}}-\frac{C}{be(a+b\cosh(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/((a -
b)^(3/2)*(a + b)^(3/2)*e) - C/(b*e*(a + b*Cosh[d + e*x])) - ((A*b - a*B)*Si
nh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{\left(a^2-b^2\right)e(a+b\cosh(d+ex))} + \frac{\int \frac{-aA+bB}{a+b\cosh(d+ex)}\,dx}{-a^2+b^2} + \frac{C\,\mathrm{Subst}\big(\int$$

$$= -\frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{\left(a^2-b^2\right)e(a+b\cosh(d+ex))} + \frac{(aA-$$

$$= -\frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{\left(a^2-b^2\right)e(a+b\cosh(d+ex))} - \frac{(2i(aA$$

$$= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}e} - \frac{C}{be(a+b\cosh(d+ex))} -$$

Mathematica [A] time = 0.45, size = 115, normalized size = 0.95

$$\frac{\frac{C\left(b^2-a^2\right)-b(Ab-aB)\sinh(d+ex)}{b(a-b)(a+b)(a+b\cosh(d+ex))} + \frac{2(aA-bB)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x
]
```

```
[Out] ((2*(a*A - b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2
 + b^2)^(3/2) + ((-a^2 + b^2)*C - b*(A*b - a*B)*Sinh[d + e*x])/((a - b)*b*(
a + b)*(a + b*Cosh[d + e*x])))/e
```

fricas [B] time = 0.75, size = 1044, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"fricas")
```

```
[Out] [-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a*
b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)*sinh(e*x + d)^2 + 2*(A*a^2
*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh
(e*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^2*si
nh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x + d) +
a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) +
 a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(b*cosh
(e*x + d) + a)*sinh(e*x + d) + b)) + 2*((B + C)*a^4 - A*a^3*b - (B + 2*C)*a
^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x + d) + 2*((B + C)*a^4 - A*a^3*b - (B + 2
*C)*a^2*b^2 + A*a*b^3 + C*b^4)*sinh(e*x + d))/((a^4*b^2 - 2*a^2*b^4 + b^6)*
e*cosh(e*x + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*e*sinh(e*x + d)^2 + 2*(a^5*
b - 2*a^3*b^3 + a*b^5)*e*cosh(e*x + d) + (a^4*b^2 - 2*a^2*b^4 + b^6)*e + 2*
((a^4*b^2 - 2*a^2*b^4 + b^6)*e*cosh(e*x + d) + (a^5*b - 2*a^3*b^3 + a*b^5)*
e)*sinh(e*x + d)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4 + (A*a*b^2 - B
*b^3 + (A*a*b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)*sinh(e*x + d)^
2 + 2*(A*a^2*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 -
 B*b^3)*cosh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2)) + ((B + C)*a^4 -
A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x + d) + ((B + C)*a^4
 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*sinh(e*x + d))/((a^4*b^2
- 2*a^2*b^4 + b^6)*e*cosh(e*x + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*e*sinh(e
*x + d)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*cosh(e*x + d) + (a^4*b^2 - 2*a^
2*b^4 + b^6)*e + 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*e*cosh(e*x + d) + (a^5*b -
2*a^3*b^3 + a*b^5)*e)*sinh(e*x + d))]
```

giac [A] time = 0.18, size = 161, normalized size = 1.33

$$2\left(\frac{(Aa-Bb)\arctan\left(\frac{be^{(xe+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{Ba^2e^{(xe+d)}+Ca^2e^{(xe+d)}-Aabe^{(xe+d)}-Cb^2e^{(xe+d)}+Bab-Ab^2}{\left(a^2b-b^3\right)\left(be^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}+b\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"giac")
```

```
[Out] 2*((A*a - B*b)*arctan((b*e^(x*e + d) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sq
rt(-a^2 + b^2)) - (B*a^2*e^(x*e + d) + C*a^2*e^(x*e + d) - A*a*b*e^(x*e + d
) - C*b^2*e^(x*e + d) + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2*x*e + 2*d) +
2*a*e^(x*e + d) + b)))*e^(-1)
```

maple [A] time = 0.17, size = 144, normalized size = 1.19

$$\frac{-\frac{2\left(-\frac{(Ab-aB)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^2-b^2}+\frac{C}{a-b}\right)}{a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)b-a-b}+\frac{2(Aa-Bb)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x)
```

```
[Out] 1/e*(-2*(-(A*b-B*a)/(a^2-b^2)*tanh(1/2*e*x+1/2*d)+C/(a-b))/(a*tanh(1/2*e*x+
1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)+2*(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.59, size = 301, normalized size = 2.49

$$\frac{\frac{2\left(A\,b^3-B\,a\,b^2\right)}{b\,e\left(a^2\,b-b^3\right)}+\frac{2\,\mathrm{e}^{d+e\,x}\left(C\,b^4-B\,a^2\,b^2-C\,a^2\,b^2+A\,a\,b^3\right)}{b^2\,e\left(a^2\,b-b^3\right)}}{b+2\,a\,\mathrm{e}^{d+e\,x}+b\,\mathrm{e}^{2\,d+2\,e\,x}}+\frac{\ln\left(-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a-B\,b)}{b\left(a^2-b^2\right)}-\frac{2\,(A\,a-B\,b)\left(b+a\,\mathrm{e}^{d+e\,x}\right)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)(A\,a-B\,b)}{e\,(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{\ln\left(\frac{2\,(A\,a-B\,b)}{b\,(a+b)^{3/}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^2,x)
```

```
[Out] ((2*(A*b^3 - B*a*b^2))/(b*e*(a^2*b - b^3)) + (2*exp(d + e*x)*(C*b^4 - B*a^2
*b^2 - C*a^2*b^2 + A*a*b^3))/(b^2*e*(a^2*b - b^3)))/(b + 2*a*exp(d + e*x) +
 b*exp(2*d + 2*e*x)) + (log(- (2*exp(d + e*x)*(A*a - B*b))/(b*(a^2 - b^2))
- (2*(A*a - B*b)*(b + a*exp(d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)))*(A*
a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(A*a - B*b)*(b + a*exp(
d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(d + e*x)*(A*a - B*b))/(
b*(a^2 - b^2)))*(A*a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**2,x)
[Out] Timed out
```

3.208 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$

Optimal. Leaf size=187

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{2e\left(a^2-b^2\right)^2(a+b\cosh(d+ex))}-\frac{(Ab-aB)\sinh(d}{2e\left(a^2-b^2\right)(a+b\cos}$$

```
[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2)
)/(a-b)^(5/2)/(a+b)^(5/2)/e-1/2*C/b/e/(a+b*cosh(e*x+d))^2-1/2*(A*b-B*a)*sin
h(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*sinh(e
*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))
```

Rubi [A] time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4377, 2754, 12, 2659, 205, 2668, 32}

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{2e\left(a^2-b^2\right)^2(a+b\cosh(d+ex))}-\frac{(Ab-aB)\sinh(d}{2e\left(a^2-b^2\right)(a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*e) - C/(2*b*e*(a + b*Cosh[d + e*x])^2)
 - ((A*b - a*B)*Sinh[d + e*x])/(2*(a^2 - b^2)*e*(a + b*Cosh[d + e*x])^2) -
((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[d + e*x])/(2*(a^2 - b^2)^2*e*(a + b*Cosh[
d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}}{2\left(a^2-b^2\right)}$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} -$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} -$$

$$= -\frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} -$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}e} - \frac{C}{2be(a+b\cosh}$$

Mathematica [A] time = 0.85, size = 175, normalized size = 0.94

$$\frac{\frac{C\left(b^2-a^2\right)-b(Ab-aB)\sinh(d+ex)}{b(a-b)(a+b)(a+b\cosh(d+ex))^2} - \frac{2\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{\left(a^2B-3aAb+2b^2B\right)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x
]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[d + e*x
])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])) + ((-a^2 + b^2)*C - b*(A*b -
 a*B)*Sinh[d + e*x])/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^2))/(2*e)
```

fricas [B] time = 0.66, size = 3636, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"fricas")
```

```
[Out] [-1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 + 2*B*a^2*b^4 + 6*A*a*b^5 - 4*B*b^6 - 2*(2
*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d)^3 -
 2*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(e*x + d
)^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*
(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d)^2 + 2*(2*(B +
C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^
4 + 3*A*a*b^5 - 2*(B + C)*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 +
3*B*a*b^5 - A*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 - (2*A*a^2*b^3 - 3*B*a*b^
4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^4 + (2*A*a^2*b^
3 - 3*B*a*b^4 + A*b^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a
*b^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b
^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*a^4*b - 6*B
*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^2 + 2*(4*A*a^4*b
- 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*a*b^
4 + A*b^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(e
*x + d))*sinh(e*x + d)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(e*x
 + d) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b^4 +
 A*b^5)*cosh(e*x + d)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(e*x
+ d)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e
*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^2*sinh
(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x + d) + a*
b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a
))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(b*cosh(e
*x + d) + a)*sinh(e*x + d) + b)) + 2*(4*B*a^5*b - 10*A*a^4*b^2 + B*a^3*b^3
+ 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(e*x + d) + 2*(4*B*a^5*b - 10*A*a^4
*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4*b^2 - 3*B*
a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d)^2 + 2*(2*(B + C)*a^6
 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^2*b^4 + 3*
A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d))*sinh(e*x + d))/((a^6*b^3 - 3*a^4*b^
5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^4 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 -
 b^9)*e*sinh(e*x + d)^4 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cos
h(e*x + d)^3 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*e*cosh(e
*x + d)^2 + 4*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d) + (a
^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e)*sinh(e*x + d)^3 + 4*(a^7*b^2 - 3
*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d) + 2*(3*(a^6*b^3 - 3*a^4*b^5 +
 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 -
a*b^8)*e*cosh(e*x + d) + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^9)*
e)*sinh(e*x + d)^2 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e + 4*((a^6*b^
3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^3 + 3*(a^7*b^2 - 3*a^5*b^4
 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d)^2 + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5
```

```
+ a^2*b^7 - b^9)*e*cosh(e*x + d) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
)*e)*sinh(e*x + d)), -(B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*
B*b^6 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*
x + d)^3 - (2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh
(e*x + d)^3 + (2*(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^
3 - 3*(B - 2*C)*a^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d)^2 + (2*(
B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a^
2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6 - 3*(2*A*a^4*b^2 - 3*B*a^3*b^3 - A*a^2*b^
4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 + (2*A*a^2*b^3 - 3*B*
a*b^4 + A*b^5 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^4 + (2*A*a^
2*b^3 - 3*B*a*b^4 + A*b^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 +
 A*a*b^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a
^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*a^4*b -
 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(e*x + d)^2 + 2*(4*A*a^
4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 + 3*(2*A*a^2*b^3 - 3*B*
a*b^4 + A*b^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*co
sh(e*x + d))*sinh(e*x + d)^2 + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh
(e*x + d) + 4*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + (2*A*a^2*b^3 - 3*B*a*b
^4 + A*b^5)*cosh(e*x + d)^3 + 3*(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*cosh(
e*x + d)^2 + (4*A*a^4*b - 6*B*a^3*b^2 + 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*co
sh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
sh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2)) + (4*B*a^5*b - 10*A*a^4*b^2
 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6)*cosh(e*x + d) + (4*B*a^5*b
 - 10*A*a^4*b^2 + B*a^3*b^3 + 11*A*a^2*b^4 - 5*B*a*b^5 - A*b^6 - 3*(2*A*a^4
*b^2 - 3*B*a^3*b^3 - A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(e*x + d)^2 + 2*(2*
(B + C)*a^6 - 6*A*a^5*b + 3*(B - 2*C)*a^4*b^2 + 3*A*a^3*b^3 - 3*(B - 2*C)*a
^2*b^4 + 3*A*a*b^5 - 2*(B + C)*b^6)*cosh(e*x + d))*sinh(e*x + d))/((a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^4 + (a^6*b^3 - 3*a^4*b^5 +
3*a^2*b^7 - b^9)*e*sinh(e*x + d)^4 + 4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a
*b^8)*e*cosh(e*x + d)^3 + 2*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*b^7 - b^
9)*e*cosh(e*x + d)^2 + 4*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*
x + d) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e)*sinh(e*x + d)^3 + 4*(
a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d) + 2*(3*(a^6*b^3 -
3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^2 + 6*(a^7*b^2 - 3*a^5*b^4 + 3
*a^3*b^6 - a*b^8)*e*cosh(e*x + d) + (2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 + a^2*
b^7 - b^9)*e)*sinh(e*x + d)^2 + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e +
 4*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*e*cosh(e*x + d)^3 + 3*(a^7*b^2
- 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*e*cosh(e*x + d)^2 + (2*a^8*b - 5*a^6*b^3 +
 3*a^4*b^5 + a^2*b^7 - b^9)*e*cosh(e*x + d) + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*
b^6 - a*b^8)*e)*sinh(e*x + d))]
```

giac [B] time = 0.21, size = 387, normalized size = 2.07

$$\left(\frac{\left(2\,Aa^2-3\,Bab+Ab^2\right)\arctan\left(\frac{be^{(xe+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{2\,Aa^2b^2e^{(3\,xe+3\,d)}-3\,Bab^3e^{(3\,xe+3\,d)}+Ab^4e^{(3\,xe+3\,d)}-2\,Ba^4e^{(2\,xe+2\,d)}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"giac")
```

```
[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^(x*e + d) + a)/sqrt(-a^2 + b^2))/(
(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x*e + 3*d) -
3*B*a*b^3*e^(3*x*e + 3*d) + A*b^4*e^(3*x*e + 3*d) - 2*B*a^4*e^(2*x*e + 2*d)
 - 2*C*a^4*e^(2*x*e + 2*d) + 6*A*a^3*b*e^(2*x*e + 2*d) - 5*B*a^2*b^2*e^(2*x
*e + 2*d) + 4*C*a^2*b^2*e^(2*x*e + 2*d) + 3*A*a*b^3*e^(2*x*e + 2*d) - 2*B*b
^4*e^(2*x*e + 2*d) - 2*C*b^4*e^(2*x*e + 2*d) - 4*B*a^3*b*e^(x*e + d) + 10*A
*a^2*b^2*e^(x*e + d) - 5*B*a*b^3*e^(x*e + d) - A*b^4*e^(x*e + d) - B*a^2*b^
2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b - 2*a^2*b^3 + b^5)*(b*e^(2*x*e + 2*d) + 2*
a*e^(x*e + d) + b)^2))*e^(-1)
```

maple [A] time = 0.17, size = 273, normalized size = 1.46

$$-\frac{\frac{2\left(-\frac{\left(4Aab+A\,b^2-2B\,a^2-bBa-2B\,b^2\right)\left(\tanh^3\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{C\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{a-b}+\frac{\left(4Aab-A\,b^2-2B\,a^2+bBa-2B\,b^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}-\frac{aC}{a^2-2ab+b^2}\right)}{\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)b-a-b\right)^2}+\frac{\left(2a^2A+A\,b^2-3bBa\right)\operatorname{arcta}}{\left(a^4-2a^2b^2+b^4\right)\cdot}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x)
```

```
[Out] 1/e*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*t
anh(1/2*e*x+1/2*d)^3+C/(a-b)*tanh(1/2*e*x+1/2*d)^2+1/2*(4*A*a*b-A*b^2-2*B*a
^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)-a*C/(a^2-2*a*b+
b^2))/(a*tanh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)^2+(2*A*a^2+A*b^
2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e
*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cosh\left(d + e\,x\right) + C\sinh\left(d + e\,x\right)}{{\left(a + b\cosh\left(d + e\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3,x)
```

```
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.209 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

Optimal. Leaf size=260

$$-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sinh(d+ex)}{6e\left(a^2-b^2\right)^2(a+b\cosh(d+ex))^2}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}+\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan}{e(a-b)^{7/2}(a+b}$$

```
[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)
/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/e-1/3*C/b/e/(a+b*cosh(e*x+d))^3-1/3*(
A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^3-1/6*(5*A*a*b-2*B*a^2-3
*B*b^2)*sinh(e*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))^2-1/6*(11*A*a^2*b+4*A*b
^3-2*B*a^3-13*B*a*b^2)*sinh(e*x+d)/(a^2-b^2)^3/e/(a+b*cosh(e*x+d))
```

Rubi [A] time = 0.45, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4377, 2754, 12, 2659, 205, 2668, 32}

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\sinh(d+ex)}{6e\left(a^2-b^2\right)^3(a+b\cosh(d+ex))}-\frac{(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e
*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*e) - C/(3*b*e*(a + b*Cos
h[d + e*x])^3) - ((A*b - a*B)*Sinh[d + e*x])/(3*(a^2 - b^2)*e*(a + b*Cosh[d
 + e*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^2*B)*Sinh[d + e*x])/(6*(a^2 - b^2)^
2*e*(a + b*Cosh[d + e*x])^2) - ((11*a^2*A*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*
B)*Sinh[d + e*x])/(6*(a^2 - b^2)^3*e*(a + b*Cosh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx + \int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$$

$$= -\frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} - \frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cosh(d+e}{(a+b\cosh(d+ex))^3}}{3\left(a^2-b^2\right)}$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} -$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} -$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} -$$

$$= -\frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3\left(a^2-b^2\right)e(a+b\cosh(d+ex))^3} -$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}e} - \frac{}{3be}$$

Mathematica [A] time = 2.57, size = 245, normalized size = 0.94

$$\frac{\frac{2C\left(b^2-a^2\right)-2b(Ab-aB)\sinh(d+ex)}{b(a-b)(a+b)(a+b\cosh(d+ex))^3} + \frac{\left(2a^2B-5aAb+3b^2B\right)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))^2} + \frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} + \frac{\left(2a^3B-11a^2\right.}{(a-b)^3}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x
]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[(d + e*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + ((-5*a*A*b + 2*a^2*B + 3*b^2*
B)*Sinh[d + e*x])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])^2) + ((-11*a^2
*A*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sinh[d + e*x])/((a - b)^3*(a + b)^3*
```

```
(a + b*Cosh[d + e*x])) + (2*(-a^2 + b^2)*C - 2*b*(A*b - a*B)*Sinh[d + e*x])
/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^3))/(6*e)
```

fricas [B] time = 0.69, size = 8531, normalized size = 32.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"fricas")
```

```
[Out] [-1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 + 22*B*a^3*b^5 + 14*A*a^2*b^6 - 26*B*a*b^
7 + 8*A*b^8 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*
a*b^7 + B*b^8)*cosh(e*x + d)^5 - 6*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 +
 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*sinh(e*x + d)^5 - 30*(2*A*a^6*b^2 - 4*B*a
^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^4 -
 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*
a*b^7 + (2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 +
B*b^8)*cosh(e*x + d))*sinh(e*x + d)^4 + 4*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(
7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (
39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d)^3 + 4*(4*(B + C)
*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4
*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8 - 15*(2*
A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh
(e*x + d)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A
*a^2*b^6 + B*a*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 12*(4*B*a^7*b - 17*A*a
^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b
^7 + 2*A*b^8)*cosh(e*x + d)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3
 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*(2*A
*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(
e*x + d)^3 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*
a^2*b^6 + B*a*b^7)*cosh(e*x + d)^2 + (4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B -
 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B
+ 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d))*sinh(e*x + d)^2 - 3*
(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 - 4*B*a^2*b^5
 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*
b^6 - B*b^7)*sinh(e*x + d)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 -
 B*a*b^6)*cosh(e*x + d)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*
a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh
(e*x + d)^5 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 +
3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a
^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3
*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2
*b^5 - B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2
 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^3 +
```

4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 20*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d) + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^3 + 2*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^2 + (8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^2*sinh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x + d) + a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(b*cosh(e*x + d) + a)*sinh(e*x + d) + b)) + 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(e*x + d) + 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d)^2 + 4*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(e*x + d))*sinh(e*x + d))/((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*sinh(e*x + d)^6 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^5 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^4 + 6*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d) + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d)^5 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d)^3 + 3*(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^2 + 10*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d) + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e)*sinh(e*x + d)^4 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^2 + 4*(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b

^10 + b^12)*e*cosh(e*x + d)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3
*b^9 + a*b^11)*e*cosh(e*x + d)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6
- 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 -
10*a^3*b^9 + 3*a*b^11)*e)*sinh(e*x + d)^3 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5
*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d) + 3*(5*(a^8*b^4 - 4*a^6*b^6 + 6*
a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^4 + 20*(a^9*b^3 - 4*a^7*b^5 +
6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^3 + 6*(4*a^10*b^2 - 15*a^8*
b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^2 + 4*(2*a^11*b - 5*a
^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d) + (4*a^10*b^2
- 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e)*sinh(e*x + d)^2 + (a^8*b^
4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e + 6*((a^8*b^4 - 4*a^6*b^6
+ 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^5 + 5*(a^9*b^3 - 4*a^7*b^5
+ 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^4 + 2*(4*a^10*b^2 - 15*a
^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^3 + 2*(2*a^11*b -
5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d)^2 + (4*a^10
*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (a^9*
b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d)), -1/3*(
2*B*a^5*b^3 - 11*A*a^4*b^4 + 11*B*a^3*b^5 + 7*A*a^2*b^6 - 13*B*a*b^7 + 4*A*
b^8 - 3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 +
B*b^8)*cosh(e*x + d)^5 - 3*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2
*b^6 - 3*A*a*b^7 + B*b^8)*sinh(e*x + d)^5 - 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 +
A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^4 - 15*(2*A
*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7 +
(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*c
osh(e*x + d))*sinh(e*x + d)^4 + 2*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*
C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 1
6*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 - 2
2*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 2
9*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8 - 15*(2*A*a^5*b^
3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d
)^2 - 30*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6
+ B*a*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 6*(4*B*a^7*b - 17*A*a^6*b^2 +
13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*
b^8)*cosh(e*x + d)^2 + 6*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^
4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8 - 5*(2*A*a^5*b^3 -
4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d)^3
- 15*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 +
B*a*b^7)*cosh(e*x + d)^2 + (4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*
b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^
2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d))*sinh(e*x + d)^2 + 3*(2*A*a^3*b
^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b
^6 - B*b^7)*cosh(e*x + d)^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^
7)*sinh(e*x + d)^6 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*
cosh(e*x + d)^5 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2
*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d)^

5 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 10*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^4 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^3 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^3 + 15*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 3*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7 + 5*(2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^4 + 20*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^3 + 6*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^2 + 4*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d))*sinh(e*x + d)^2 + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d) + 6*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 - 4*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^5 + 5*(2*A*a^4*b^3 - 4*B*a^3*b^4 + 3*A*a^2*b^5 - B*a*b^6)*cosh(e*x + d)^4 + 2*(8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d)^3 + 2*(4*A*a^6*b - 8*B*a^5*b^2 + 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 - 3*B*a*b^6)*cosh(e*x + d)^2 + (8*A*a^5*b^2 - 16*B*a^4*b^3 + 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 - B*b^7)*cosh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2)) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8)*cosh(e*x + d) + 3*(4*B*a^6*b^2 - 20*A*a^5*b^3 + 18*B*a^4*b^4 + 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 + B*b^8 - 5*(2*A*a^5*b^3 - 4*B*a^4*b^4 + A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 + B*b^8)*cosh(e*x + d)^4 - 20*(2*A*a^6*b^2 - 4*B*a^5*b^3 + A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 + B*a*b^7)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 - 22*A*a^7*b + 4*(7*B - 4*C)*a^6*b^2 - 19*A*a^5*b^3 + (7*B + 24*C)*a^4*b^4 + 29*A*a^3*b^5 - (39*B + 16*C)*a^2*b^6 + 12*A*a*b^7 + 4*C*b^8)*cosh(e*x + d)^2 + 4*(4*B*a^7*b - 17*A*a^6*b^2 + 13*B*a^5*b^3 + 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 - 4*B*a*b^7 + 2*A*b^8)*cosh(e*x + d))*sinh(e*x + d))/((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^6 + (a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*sinh(e*x + d)^6 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^5 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^4 + 6*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d) + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d)^5 + 4*(2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d)^3 + 3*(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^2 + 10*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*

```
e*cosh(e*x + d) + (4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12
)*e)*sinh(e*x + d)^4 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8
 + b^12)*e*cosh(e*x + d)^2 + 4*(5*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*
b^10 + b^12)*e*cosh(e*x + d)^3 + 15*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^
3*b^9 + a*b^11)*e*cosh(e*x + d)^2 + 3*(4*a^10*b^2 - 15*a^8*b^4 + 20*a^6*b^6
 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (2*a^11*b - 5*a^9*b^3 + 10*a^5*b^7
- 10*a^3*b^9 + 3*a*b^11)*e)*sinh(e*x + d)^3 + 6*(a^9*b^3 - 4*a^7*b^5 + 6*a^
5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d) + 3*(5*(a^8*b^4 - 4*a^6*b^6 + 6
*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^4 + 20*(a^9*b^3 - 4*a^7*b^5 +
 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^3 + 6*(4*a^10*b^2 - 15*a^8
*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^2 + 4*(2*a^11*b - 5*
a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d) + (4*a^10*b^2
 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e)*sinh(e*x + d)^2 + (a^8*b
^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e + 6*((a^8*b^4 - 4*a^6*b^6
 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*e*cosh(e*x + d)^5 + 5*(a^9*b^3 - 4*a^7*b^
5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e*cosh(e*x + d)^4 + 2*(4*a^10*b^2 - 15*
a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d)^3 + 2*(2*a^11*b -
 5*a^9*b^3 + 10*a^5*b^7 - 10*a^3*b^9 + 3*a*b^11)*e*cosh(e*x + d)^2 + (4*a^1
0*b^2 - 15*a^8*b^4 + 20*a^6*b^6 - 10*a^4*b^8 + b^12)*e*cosh(e*x + d) + (a^9
*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*e)*sinh(e*x + d))]
```

giac [B] time = 0.25, size = 688, normalized size = 2.65

$$\frac{1}{3}\left(\frac{3\left(2\,Aa^3-4\,Ba^2b+3\,Aab^2-Bb^3\right)\arctan\left(\frac{be^{(xe+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{6\,Aa^3b^3e^{(5\,xe+5\,d)}-12\,Ba^2b^4e^{(5\,xe+5\,d)}+9\,Aab^5e^{(5\,xe+5\,d}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"giac")
```

```
[Out] 1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^(x*e + d) + a)
/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) +
 (6*A*a^3*b^3*e^(5*x*e + 5*d) - 12*B*a^2*b^4*e^(5*x*e + 5*d) + 9*A*a*b^5*e^
(5*x*e + 5*d) - 3*B*b^6*e^(5*x*e + 5*d) + 30*A*a^4*b^2*e^(4*x*e + 4*d) - 60
*B*a^3*b^3*e^(4*x*e + 4*d) + 45*A*a^2*b^4*e^(4*x*e + 4*d) - 15*B*a*b^5*e^(4
*x*e + 4*d) - 8*B*a^6*e^(3*x*e + 3*d) - 8*C*a^6*e^(3*x*e + 3*d) + 44*A*a^5*
b*e^(3*x*e + 3*d) - 64*B*a^4*b^2*e^(3*x*e + 3*d) + 24*C*a^4*b^2*e^(3*x*e +
3*d) + 82*A*a^3*b^3*e^(3*x*e + 3*d) - 78*B*a^2*b^4*e^(3*x*e + 3*d) - 24*C*a
^2*b^4*e^(3*x*e + 3*d) + 24*A*a*b^5*e^(3*x*e + 3*d) + 8*C*b^6*e^(3*x*e + 3*
d) - 24*B*a^5*b*e^(2*x*e + 2*d) + 102*A*a^4*b^2*e^(2*x*e + 2*d) - 102*B*a^3
*b^3*e^(2*x*e + 2*d) + 36*A*a^2*b^4*e^(2*x*e + 2*d) - 24*B*a*b^5*e^(2*x*e +
 2*d) + 12*A*b^6*e^(2*x*e + 2*d) - 12*B*a^4*b^2*e^(x*e + d) + 60*A*a^3*b^3*
e^(x*e + d) - 66*B*a^2*b^4*e^(x*e + d) + 15*A*a*b^5*e^(x*e + d) + 3*B*b^6*e
```

```
^(x*e + d) - 2*B*a^3*b^3 + 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*(b*e^(2*x*e + 2*d) + 2*a*e^(x*e + d) + b)^3))*e
^(-1)
```

maple [A] time = 0.18, size = 459, normalized size = 1.77

$$-\frac{2\left(-\frac{\left(6A\,a^2b+3Aa\,b^2+2A\,b^3-2a^3B-2B\,a^2b-6Ba\,b^2-B\,b^3\right)\left(\tanh^5\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{C\left(\tanh^4\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{a-b}+\frac{2\left(9A\,a^2b+A\,b^3-3a^3B-7Ba\,b^2\right)\left(\tanh^3\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{2aC\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{a^2-2ab+b^2}-\frac{(6A}{}\right.}{\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)b-a-b\right)^3}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x)
```

```
[Out] 1/e*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^
3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*e*x+1/2*d)^5+C/(a-b)*tanh(1/2*e
*x+1/2*d)^4+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*
a*b+b^2)*tanh(1/2*e*x+1/2*d)^3-2*a*C/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)^2-
1/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*e*x+1/2*d)+1/3*C*(3*a^2+b^2)/(a^3-3*a^2*b
+3*a*b^2-b^3))/(a*tanh(1/2*e*x+1/2*d)^2-tanh(1/2*e*x+1/2*d)^2*b-a-b)^3+(2*A
*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cosh\left(d+e\,x\right)+C\sinh\left(d+e\,x\right)}{{\left(a+b\cosh\left(d+e\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4,x)
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**4,x)
[Out] Timed out
```

3.210 $\int \frac{x}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=191

$$\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\,\sqrt{a}+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}-\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\,\sqrt{a}+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\,\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\,\sqrt{a+b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\,\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\,\sqrt{a+b}}$$

```
[Out] 1/2*x*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-1/
2*x*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+1/4*
polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-1/
4*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5630, 3320, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\,\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\,\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\,\sqrt{a+b}+2a+b}+1\right)}{2\sqrt{a}\,\sqrt{a+b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\,\sqrt{a+b}+2a\cdots}\right.}{2\sqrt{a}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cosh[x]^2),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqrt[
a + b]) - (x*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqr
t[a]*Sqrt[a + b]) + PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a +
b]))]/(4*Sqrt[a]*Sqrt[a + b]) - PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[
a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5630

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rubi steps

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = 2\int \frac{x}{2a+b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\int\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)}\right) \ldots}{2\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)}\right)}{x} \ldots\right.}{4\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \frac{\text{Li}_2\left(- \ldots\right.}{4 \ldots}$$

Mathematica [C] time = 0.63, size = 536, normalized size = 2.81

$$-\frac{i\left(\text{Li}_2\left(\frac{\left(2a+b-2i\sqrt{-a(a+b)}\right)\left(a+b-i\sqrt{-a(a+b)}\tanh(x)\right)}{b\left(a+b+i\sqrt{-a(a+b)}\tanh(x)\right)}\right)-\text{Li}_2\left(\frac{\left(2a+b+2i\sqrt{-a(a+b)}\right)\left(a+b-i\sqrt{-a(a+b)}\tanh(x)\right)}{b\left(a+b+i\sqrt{-a(a+b)}\tanh(x)\right)}\right)\right)+4x\tan^{-1}\left(\frac{(a+b)\,\text{co}\ldots}{\sqrt{-a(a\ldots}}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cosh[x]^2),x]
```

```
[Out] -1/4*(4*x*ArcTan[((a + b)*Coth[x])/Sqrt[-(a*(a + b))]] + (2*I)*ArcCos[-1 -
(2*a)/b]*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]] + (ArcCos[-1 - (2*a)/b] + 2
*ArcTan[((a + b)*Coth[x])/Sqrt[-(a*(a + b))]] - 2*ArcTan[(a*Tanh[x])/Sqrt[-
(a*(a + b))]])*Log[(Sqrt[2]*Sqrt[-(a*(a + b))])/(Sqrt[b]*E^x*Sqrt[2*a + b +
 b*Cosh[2*x]])] + (ArcCos[-1 - (2*a)/b] - 2*ArcTan[((a + b)*Coth[x])/Sqrt[-
(a*(a + b))]] + 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log[(Sqrt[2]*Sqrt
[-(a*(a + b))]*E^x)/(Sqrt[b]*Sqrt[2*a + b + b*Cosh[2*x]])] - (ArcCos[-1 - (
2*a)/b] - 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log[(2*(a + b)*(a + I*S
qrt[-(a*(a + b))])*(-1 + Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b))]*Tanh[x]
))] - (ArcCos[-1 - (2*a)/b] + 2*ArcTan[(a*Tanh[x])/Sqrt[-(a*(a + b))]])*Log
[((2*I)*(a + b)*(I*a + Sqrt[-(a*(a + b))])*(1 + Tanh[x]))/(b*(a + b + I*Sqr
t[-(a*(a + b))]*Tanh[x]))] + I*(PolyLog[2, ((2*a + b - (2*I)*Sqrt[-(a*(a +
```

```
b))])*(a + b - I*Sqrt[-(a*(a + b))]*Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b
))]*Tanh[x]))] - PolyLog[2, ((2*a + b + (2*I)*Sqrt[-(a*(a + b))])*(a + b -
I*Sqrt[-(a*(a + b))]*Tanh[x]))/(b*(a + b + I*Sqrt[-(a*(a + b))]*Tanh[x]))])
)/Sqrt[-(a*(a + b))]
```

fricas [B] time = 0.56, size = 780, normalized size = 4.08

$$-\frac{bx\sqrt{\frac{a^2+ab}{b^2}}\log\left(\frac{\left((2a+b)\cosh(x)+(2a+b)\sinh(x)-2(b\cosh(x)+b\sinh(x))\sqrt{\frac{a^2+ab}{b^2}}\right)\sqrt{-\frac{2b\sqrt{\frac{a^2+ab}{b^2}}+2a+b}{b}}+b}{b}\right)+bx\sqrt{\frac{a^2+ab}{b^2}}\log\left(-\frac{\left((2a+b\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b
)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x*sqrt((a^
2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b
)/b) + b)/b) - b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) + (2*a +
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqr
t((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + b*sqrt((a^2 + a*b)/b^2)*dilog(-(
((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^
2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) + b)/b + 1) +
 b*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/
b^2) + 2*a + b)/b) - b)/b + 1) - b*sqrt((a^2 + a*b)/b^2)*dilog(-(((2*a + b)
*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b
^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)/b + 1) - b*sqrt((a^
2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x)
+ b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a -
 b)/b) - b)/b + 1))/(a^2 + a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cosh(x)^2 + a), x)
```

maple [B] time = 0.13, size = 487, normalized size = 2.55

$$\frac{x\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{2\sqrt{a(a+b)}-2a-b}\right)}{2\sqrt{a\,(a+b)}}-\frac{x^2}{2\sqrt{a\,(a+b)}}+\frac{\mathrm{polylog}\left(2,\frac{b\,\mathrm{e}^{2x}}{2\sqrt{a(a+b)}-2a-b}\right)}{4\sqrt{a\,(a+b)}}+\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)x}{-2\sqrt{a\,(a+b)}-2a-b}+\frac{\ln\left(1-\frac{}{-2\sqrt{a}}\right.}{\sqrt{a\,(a+b)}\,\left(-2\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cosh(x)^2),x)
```

```
[Out] 1/2/(a*(a+b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/2/(a*(a+
b))^(1/2)*x^2+1/4/(a*(a+b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2
*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a
-b))*x+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*
(a+b))^(1/2)-2*a-b))*a*x+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(
1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x-1/(-2*(a*(a+b))^(1/2)-2*a-b)*x
^2-1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^2-1/2/(a*(a+b))^(1/2)/(
-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2+1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*
exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2
)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+1/4/(a*(a+b))^(
1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*
a-b))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\cosh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cosh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{b\,{\cosh(x)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*cosh(x)^2),x)
```

```
[Out] int(x/(a + b*cosh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(x/(a + b*cosh(x)**2), x)
```

3.211 $\int \frac{x^2}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=291

$$\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{x^2\log\left(\frac{be^{2}}{-2\sqrt{a}\sqrt{a+}}\right.}{2\sqrt{a}\sqrt{a}}$$

```
[Out] 1/2*x^2*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-
1/2*x^2*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+
1/2*x*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1
/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b
)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a
+b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/
(a+b)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5630, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\text{PolyLog}\left(3,\right.}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cosh[x]^2),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sq
rt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
b + 2*Sqrt[a]*Sqrt[a + b]))])/(2*Sqrt[a]*Sqrt[a + b]) - PolyLog[3, -((b*E^(
2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))]/(4*Sqrt[a]*Sqrt[a + b
])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5630

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{a+b\cosh^2(x)}\,dx &= 2\int \frac{x^2}{2a+b+b\cosh(2x)}\,dx \\
&= 4\int \frac{e^{2x}x^2}{b+2(2a+b)e^{2x}+be^{4x}}\,dx \\
&= \frac{(2b)\int \frac{e^{2x}x^2}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x^2}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} \\
&= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\int x\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a+b}+2(2a+\cdots}\right.}{\sqrt{a}\sqrt{a+b}} \cdots \\
&= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\mathrm{L}\cdots}{\cdots} \\
&= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\mathrm{L}\cdots}{\cdots} \\
&= \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{x\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\mathrm{L}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.77, size = 221, normalized size = 0.76

$$
\frac{2x\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right) - 2x\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right) - \mathrm{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right) + \mathrm{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right) + 2x^2\log\left(\frac{\cdots}{-2\cdots}\right)\cdots}{4\sqrt{a}\sqrt{a+b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cosh[x]^2),x]
```

```
[Out] (2*x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])] - 2*x^2*Log[1
 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])] + 2*x*PolyLog[2, -((b*E^(
2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))] - 2*x*PolyLog[2, -((b*E^(2*x))/(2
*a + b + 2*Sqrt[a]*Sqrt[a + b]))] - PolyLog[3, -((b*E^(2*x))/(2*a + b - 2*S
qrt[a]*Sqrt[a + b]))] + PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[
a + b]))])/(4*Sqrt[a]*Sqrt[a + b])
```

fricas [C] time = 1.38, size = 1162, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
 + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x^2*sq
rt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2
*a - b)/b) + b)/b) - b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 2*b*x*sqrt((a^2 + a*b)/b
^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(
x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) +
 b)/b + 1) + 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a +
 b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*s
qrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2
)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x)
)*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)
/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)
*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt(
(a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 2*b*sqrt((a^2 + a*b)/b^2)*poly
log(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*s
qrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) - 2
*b*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 +
a*b)/b^2) + 2*a + b)/b)/b) + 2*b*sqrt((a^2 + a*b)/b^2)*polylog(3, ((2*a + b
)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) + 2*b*sqrt((a^2 + a*
b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) +
 b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a -
b)/b)/b))/(a^2 + a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cosh(x)^2 + a), x)
```

maple [B] time = 0.12, size = 686, normalized size = 2.36

$$-\frac{2x^3}{3\left(-2\sqrt{a(a+b)}-2a-b\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)}{-2\sqrt{a(a+b)}-2a-b}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)}{-2\sqrt{a(a+b)}-2a-b}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2}\right.}{2\left(-2\sqrt{a(a+b)}-2a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cosh(x)^2),x)
```

```
[Out] -2/3/(-2*(a*(a+b))^(1/2)-2*a-b)*x^3+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2*ln(1-b
*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x*polylo
g(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*p
olylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-2/3/(a*(a+b))^(1/2)/(-2*(a*
(a+b))^(1/2)-2*a-b)*a*x^3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^
2*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(a*(a+b))^(1/2)/(-2*(a*(a+b
))^(1/2)-2*a-b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/2/(a
*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*polylog(3,b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))-1/3/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^3+1/2/(
a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2*ln(1-b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x*polylog
(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))
^(1/2)-2*a-b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/3/(a*(a+
b))^(1/2)*x^3+1/2/(a*(a+b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*
a-b))+1/2/(a*(a+b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
-1/4/(a*(a+b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\cosh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cosh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{b\,{\cosh\left(x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*cosh(x)^2),x)
```

```
[Out] int(x^2/(a + b*cosh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(x**2/(a + b*cosh(x)**2), x)
```

3.212 $\int \frac{x^3}{a+b\cosh^2(x)}\,dx$

Optimal. Leaf size=391

$$\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3\text{Li}_4\left(\cdots\right)}{8\cdots}$$

```
[Out] 1/2*x^3*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-
1/2*x^3*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+
3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^
(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/
(a+b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1
/2)/(a+b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/
a^(1/2)/(a+b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2))
)/a^(1/2)/(a+b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2
)))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5630, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{3x\text{Po}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cosh[x]^2),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*Sqrt[a]*Sqr
t[a + b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])])/(2
*Sqrt[a]*Sqrt[a + b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a
]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a + b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))])/(4*Sqrt[a]*Sqrt[a +
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))])/(
4*Sqrt[a]*Sqrt[a + b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*S
qrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 b + 2*Sqrt[a]*Sqrt[a + b]))])/(8*Sqrt[a]*Sqrt[a + b])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5630

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = 2\int \frac{x^3}{2a+b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^3}{b+2(2a+b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{-4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}} - \frac{(2b)\int \frac{e^{2x}x^3}{4\sqrt{a}\sqrt{a+b}+2(2a+b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{3\int x^2\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a+b}+2(2\ldots}\right.}{2\sqrt{a}\sqrt{a+b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \ldots$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \ldots$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \ldots$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \ldots$$

Mathematica [A] time = 0.69, size = 295, normalized size = 0.75

$$\frac{6x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)-6x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)-6x\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)+6x\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)+3\text{Li}_4\left(\right.}{8\sqrt{a}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cosh[x]^2),x]
```

```
[Out] (4*x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])] - 4*x^3*Log[1
 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b])] + 6*x^2*PolyLog[2, -((b*E
^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))] - 6*x^2*PolyLog[2, -((b*E^(2*x)
)/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))] - 6*x*PolyLog[3, -((b*E^(2*x))/(2*a +
 b - 2*Sqrt[a]*Sqrt[a + b]))] + 6*x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*S
qrt[a]*Sqrt[a + b]))] + 3*PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqr
t[a + b]))] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b]))
])/(8*Sqrt[a]*Sqrt[a + b])
```

fricas [C] time = 0.57, size = 1542, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
 + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x^3*sq
rt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2
*a - b)/b) + b)/b) - b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 3*b*x^2*sqrt((a^2 + a*b)
/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sin
h(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)
 + b)/b + 1) + 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2
*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2
*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*
b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*s
inh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b
) + b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (
2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2
```

```
*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 6*b*x*sqrt((a^2 + a*b)
/b^2)*polylog(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*
sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)
/b)/b) - 6*b*x*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a
+ b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*
sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) + 6*b*x*sqrt((a^2 + a*b)/b^2)*polylo
g(3, ((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqr
t((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) + 6*b*
x*sqrt((a^2 + a*b)/b^2)*polylog(3, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x)
+ 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*
b)/b^2) - 2*a - b)/b)/b) + 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4, ((2*a + b)*
cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b)/b) + 6*b*sqrt((a^2 + a*b
)/b^2)*polylog(4, -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a +
b)/b)/b) - 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4, ((2*a + b)*cosh(x) + (2*a +
 b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sq
rt((a^2 + a*b)/b^2) - 2*a - b)/b)/b) - 6*b*sqrt((a^2 + a*b)/b^2)*polylog(4,
 -((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((
a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)/b))/(a^2 + a
*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cosh(x)^2 + a), x)
```

maple [B] time = 0.13, size = 889, normalized size = 2.27

$$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)x^3}{-2\sqrt{a\,(a+b)}-2a-b}+\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)a\,x^3}{\sqrt{a\,(a+b)}\,\left(-2\sqrt{a\,(a+b)}-2a-b\right)}+\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)b\,x^3}{2\sqrt{a\,(a+b)}\,\left(-2\sqrt{a\,(a+b)}-2a-b\right)}-\frac{}{2\left(-2\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cosh(x)^2),x)
```

```
[Out] 1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x^
3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b)
)^(1/2)-2*a-b))*a*x^3+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b
```

```
*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x^3-1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*
x^4-1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^4-1/4/(a*(a+b))^(1/2
)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^4+3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2
,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x^2+3/2/(a*(a+b))^(1/2)/(-2*(a*(a+b
))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x^2+3/4/
(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b
))^(1/2)-2*a-b))*b*x^2-3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/
(-2*(a*(a+b))^(1/2)-2*a-b))*x-3/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b
)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x-3/4/(a*(a+b))^(1/2)/
(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))
*b*x+3/4/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2
)-2*a-b))+3/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*
x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+3/8/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*
a-b)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b+1/2/(a*(a+b))^(1/2)
*x^3*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)*x^4+3/4
/(a*(a+b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-3/4/(a
*(a+b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))+3/8/(a*(a+b
))^(1/2)*polylog(4,b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*cosh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{b\cosh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*cosh(x)^2),x)
```

```
[Out] int(x^3/(a + b*cosh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\cosh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cosh(x)**2),x)
```

```
[Out] Integral(x**3/(a + b*cosh(x)**2), x)
```

3.213 $\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] -3/4*Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Chi(3*(-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a
```

Rubi [A] time = 0.12, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3301}

$$-\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CoshIntegral[(3*Sqrt
[1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right) dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$
$$= -\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.95

$$\frac{-3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - CoshIntegral[(3*Sqrt[1 - a*
x])/Sqrt[1 + a*x]])/(4*a)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{3}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^{3}}{a^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

```
[Out] -int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh^{3}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

3.214 $\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3301}

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$
$$= -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.98

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(ax+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a - Log[1 - a*x]/(4*a) +
 Log[1 + a*x]/(4*a)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(ax+1\right)}{4\,a}-\frac{\log\left(ax-1\right)}{4\,a}-\frac{1}{4}\int\frac{e^{\left(\frac{2\,\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^2x^2-1}\,dx-\frac{1}{4}\int\frac{e^{\left(-\frac{2\,\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] 1/4*log(a*x + 1)/a - 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1)
/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sqrt
(a*x + 1))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

```
[Out] -int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

3.215 $\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] -Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3301}

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 1.00

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
```

```
[Out] integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
```

```
[Out] integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] -int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

3.216 $\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=40

$$\operatorname{Int}\left(\frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

[Out] Unintegrable(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]

[Out] -(Defer[Subst][Defer[Int][Sech[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rubi steps

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 6.34, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]

[Out] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^2x^2-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cosh(sqrt(-a*x
+ 1)/sqrt(a*x + 1))), x)
```

3.217 $\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\operatorname{Int}\left(\frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

```
[Out] Unintegrable(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sech[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/
a)
```

Rubi steps

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{sech}^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 26.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^{2}x^{2}-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^{2}x^{2}-1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{ax+1}}{\sqrt{-ax+1}\,ae^{\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}+\sqrt{-ax+1}\,a}+2\int \frac{\sqrt{ax+1}}{\left(a^{2}x^{2}-1\right)\sqrt{-ax+1}\,e^{\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}+\left(a^{2}x^{2}-1\right)\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

```
[Out] 2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + sqrt
(-a*x + 1)*a) + 2*integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(
2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] -Integral(1/(a**2*x**2*cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - cosh(sqrt(-a
*x + 1)/sqrt(a*x + 1))**2), x)
```

3.218 $\int \frac{x \sinh(x)}{(a+b\cosh(x))^2}\,dx$

Optimal. Leaf size=60

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

```
[Out] -x/b/(a+b*cosh(x))+2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(
1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5465, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]
) - x/(b*(a + b*Cosh[x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 5465

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*(a + b*Cosh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = -\frac{x}{b(a+b\cosh(x))}+\frac{\int \frac{1}{a+b\cosh(x)}\,dx}{b}$$
$$= -\frac{x}{b(a+b\cosh(x))}+\frac{2\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

Mathematica [A] time = 0.14, size = 59, normalized size = 0.98

$$-\frac{2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b\sqrt{b^2-a^2}}-\frac{x}{b(a+b\cosh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

```
[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) - x/
(b*(a + b*Cosh[x]))
```

fricas [B] time = 0.50, size = 480, normalized size = 8.00

$$\left[-\frac{2\left(a^2-b^2\right)x\cosh(x)+2\left(a^2-b^2\right)x\sinh(x)-\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x\right.}{a^2b^2-b^4+\left(a^2b^2-b^4\right)\cosh(x)^2+\left(a^2b^2-b^4\right)\sinh(x)^2+2\left(a^3b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="fricas")
```

```
[Out] [-(2*(a^2 - b^2)*x*cosh(x) + 2*(a^2 - b^2)*x*sinh(x) - (b*cosh(x)^2 + b*sin
h(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(a^2 - b^2)*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
```

```
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)))/(a^2*b^2
 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b -
 a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x)), -2*
((a^2 - b^2)*x*cosh(x) + (a^2 - b^2)*x*sinh(x) + (b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a^2*b^2 - b^4 + (
a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b - a*b^3)*co
sh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh(x)}{(b\cosh(x)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(x)/(b*cosh(x) + a)^2, x)
```

maple [B] time = 0.18, size = 138, normalized size = 2.30

$$-\frac{2x\,e^{x}}{b\left(b\,e^{2x}+2a\,e^{x}+b\right)}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{\sqrt{a^{2}-b^{2}}\,b}\right)}{\sqrt{a^{2}-b^{2}}\,b}-\frac{\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{\sqrt{a^{2}-b^{2}}\,b}\right)}{\sqrt{a^{2}-b^{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x)/(a+b*cosh(x))^2,x)
```

```
[Out] -2*x/b*exp(x)/(b*exp(2*x)+2*a*exp(x)+b)+1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*(a
^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)-1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*
(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.09, size = 110, normalized size = 1.83

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(b^4-a^2\,b^2\right)+a\,b^3+a^2\,b^2\,\mathrm{e}^x}{b^2\,\sqrt{b^4-a^2\,b^2}}\right)}{\sqrt{b^4-a^2\,b^2}}-\frac{2\,\mathrm{e}^x\,\left(a^2\,x-b^2\,x\right)}{\left(a^2\,b-b^3\right)\,\left(b+2\,a\,\mathrm{e}^x+b\,\mathrm{e}^{2\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(x))/(a + b*cosh(x))^2,x)
```

```
[Out] (2*atan((exp(x)*(b^4 - a^2*b^2) + a*b^3 + a^2*b^2*exp(x))/(b^2*(b^4 - a^2*b
^2)^(1/2))))/(b^4 - a^2*b^2)^(1/2) - (2*exp(x)*(a^2*x - b^2*x))/((a^2*b - b
^3)*(b + 2*a*exp(x) + b*exp(2*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))**2,x)
```

```
[Out] Timed out
```

3.219 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx$

Optimal. Leaf size=87

$$-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}-\frac{x}{2b(a+b\cosh(x))^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/b/(a+b)^(3/2)-1/
2*x/b/(a+b*cosh(x))^2-1/2*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5465, 2664, 12, 2659, 208}

$$-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}-\frac{x}{2b(a+b\cosh(x))^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] (a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2)) - x/(2*b*(a + b*Cosh[x])^2) - Sinh[x]/(2*(a^2 - b^2)*(a + b*Cosh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 5465

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f*x)^m*(a + b*Cosh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = -\frac{x}{2b(a+b\cosh(x))^2}+\frac{\int \frac{1}{(a+b\cosh(x))^2}\,dx}{2b}$$

$$= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{\int \frac{a}{a+b\cosh(x)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{a\int \frac{1}{a+b\cosh(x)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx,x,\tanh\left(\right.\right.}{b\left(a^2-b^2\right)}$$

$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}-\frac{x}{2b(a+b\cosh(x))^2}-\frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}$$

Mathematica [A] time = 0.26, size = 87, normalized size = 1.00

$$\frac{1}{2}\left(\frac{\frac{2a\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{x}{(a+b\cosh(x))^2}}{b}-\frac{\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

```
[Out] (((2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - x
/(a + b*Cosh[x])^2)/b - Sinh[x]/((a - b)*(a + b)*(a + b*Cosh[x])))/2
```

fricas [B] time = 0.52, size = 1692, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*b^2 - 2*b^4 + 2*(a^3*b - a*b^3)*cosh(x)^3 + 2*(a^3*b - a*b^3)*s
inh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x)^
2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b - a*b
^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*cosh(
x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cosh(x) + a^2*b)*sinh(x)^3 + a*b^2 + 2*(2
*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + 6*a^2*b*cosh(x) + 2*a^3 +
a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + 3*a^2*b*cosh(x)^2 + a^2*b + (2*a^3
+ a*b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)
^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a
^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cos
h(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(a^3*b - a*b^3)*cosh(x) + 2*(3*a
^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2
*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^4*b^3 - 2*a^2*b^5 + b^7 +
(a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 - 2*a^2*b^5 + b^7)*sinh(x)
^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2 - 2*a^3*b^4 + a
*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b - 3*a^4*
b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*(a^4*b^3 - 2*a^2*b^
5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^2 + 4
*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (
a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh
(x)^2 + (2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x))*sinh(x)), (a^2*b^2 - b^4 + (a^
3*b - a*b^3)*cosh(x)^3 + (a^3*b - a*b^3)*sinh(x)^3 + (2*a^4 - a^2*b^2 - b^4
 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x)^2 + (2*a^4 - a^2*b^2 - b^4 - 2*(a^4
 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(
x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*cosh(x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cos
h(x) + a^2*b)*sinh(x)^3 + a*b^2 + 2*(2*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*
cosh(x)^2 + 6*a^2*b*cosh(x) + 2*a^3 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3
 + 3*a^2*b*cosh(x)^2 + a^2*b + (2*a^3 + a*b^2)*cosh(x))*sinh(x))*sqrt(-a^2
+ b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) +
3*(a^3*b - a*b^3)*cosh(x) + (3*a^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^
2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x
```

```
))/(a^4*b^3 - 2*a^2*b^5 + b^7 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^
4*b^3 - 2*a^2*b^5 + b^7)*sinh(x)^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x
)^3 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))
*sinh(x)^3 + 2*(2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b
^3 + b^7 + 3*(a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4
 + a*b^6)*cosh(x))*sinh(x)^2 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*
(a^5*b^2 - 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a
^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^2 + (2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)
)*sinh(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh(x)}{(b\cosh(x) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(x)/(b*cosh(x) + a)^3, x)
```

maple [B] time = 0.23, size = 231, normalized size = 2.66

$$-\frac{2a^2x\,e^{2x} - ab\,e^{3x} - 2b^2x\,e^{2x} - 2a^2e^{2x} - b^2e^{2x} - 3ba\,e^{x} - b^2}{b\left(b\,e^{2x} + 2a\,e^{x} + b\right)^2\left(a^2 - b^2\right)} + \frac{a\ln\left(e^{x} + \frac{a\sqrt{a^2-b^2}-a^2+b^2}{\sqrt{a^2-b^2}\,b}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)\,b} - \frac{a\ln\left(e^{x} + \frac{a\sqrt{a^2-b^2}+a^2\cdots}{\sqrt{a^2-b^2}\,b}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x)/(a+b*cosh(x))^3,x)
```

```
[Out] -1/b*(2*a^2*x*exp(2*x)-a*b*exp(3*x)-2*b^2*x*exp(2*x)-2*a^2*exp(2*x)-b^2*exp
(2*x)-3*b*a*exp(x)-b^2)/(b*exp(2*x)+2*a*exp(x)+b)^2/(a^2-b^2)+1/2/(a^2-b^2)
^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2
)/b)-1/2/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)+a^2-b
^2)/(a^2-b^2)^(1/2)/b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sinh(x)}{(a + b\cosh(x))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(x))/(a + b*cosh(x))^3,x)
```

```
[Out] int((x*sinh(x))/(a + b*cosh(x))^3, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x)/(a+b*cosh(x))**3,x)
```

```
[Out] Timed out
```

3.220 $\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx$

Optimal. Leaf size=47

$$\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

```
[Out] 9/4*cosh(a)*Shi(b*x)+1/4*cosh(3*a)*Shi(3*b*x)+9/4*Chi(b*x)*sinh(a)+1/4*Chi(
3*b*x)*sinh(3*a)
```

Rubi [A] time = 0.46, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6742, 3303, 3298, 3301, 5448}

$$\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

```
[Out] (9*CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (9*Co
sh[a]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(2+\cosh^2(a+bx)\right)\sinh(a+bx)}{x}\,dx &= \int \left(\frac{2\sinh(a+bx)}{x}+\frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\right)dx\\ &= 2\int \frac{\sinh(a+bx)}{x}\,dx+\int \frac{\cosh^2(a+bx)\sinh(a+bx)}{x}\,dx\\ &= (2\cosh(a))\int \frac{\sinh(bx)}{x}\,dx+(2\sinh(a))\int \frac{\cosh(bx)}{x}\,dx+\int \Bigg(\frac{\sinh(}{4}\\ &= 2\text{Chi}(bx)\sinh(a)+2\cosh(a)\text{Shi}(bx)+\frac{1}{4}\int \frac{\sinh(a+bx)}{x}\,dx+\frac{1}{4}\int\\ &= 2\text{Chi}(bx)\sinh(a)+2\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(a)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}\\ &= \frac{9}{4}\text{Chi}(bx)\sinh(a)+\frac{1}{4}\text{Chi}(3bx)\sinh(3a)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cos\end{aligned}$$

Mathematica [A] time = 0.11, size = 41, normalized size = 0.87

$$\frac{1}{4}(9\sinh(a)\text{Chi}(bx)+\sinh(3a)\text{Chi}(3bx)+9\cosh(a)\text{Shi}(bx)+\cosh(3a)\text{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

```
[Out] (9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + 9*Cosh[a]*Si
nhIntegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

fricas [A] time = 0.50, size = 67, normalized size = 1.43

$$\frac{1}{8}\left(\text{Ei}\left(3\,bx\right)-\text{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)+\frac{9}{8}\left(\text{Ei}\left(bx\right)-\text{Ei}\left(-bx\right)\right)\cosh(a)+\frac{1}{8}\left(\text{Ei}\left(3\,bx\right)+\text{Ei}\left(-3\,bx\right)\right)\sinh\left(3\,a\right)+\frac{9}{8}\left(\text{Ei}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9/8*(Ei(b*x) - Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 9/8*(Ei(b*x) + Ei(-b*x))*sinh(a)
```

giac [A] time = 0.14, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{9}{8}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}-\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+\frac{9}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/8
*Ei(b*x)*e^a
```

maple [A] time = 0.29, size = 47, normalized size = 1.00

$$\frac{\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{8}+\frac{9\,\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{8}-\frac{9\,\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{8}-\frac{\mathrm{e}^{3a}\,\mathrm{Ei}\left(1,-3bx\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x)
```

```
[Out] 1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)-9/8*exp(a)*Ei(1,-b*x)-1/8*e
xp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.43, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{(3\,a)}-\frac{9}{8}\,\mathrm{Ei}\left(-bx\right)e^{(-a)}-\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{(-3\,a)}+\frac{9}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/8
*Ei(b*x)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(a+b\,x\right)\,\left({\cosh\left(a+b\,x\right)}^2+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x,x)
```

```
[Out] int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cosh^2\left(a+bx\right)+2\right)\sinh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+cosh(b*x+a)**2)*sinh(b*x+a)/x,x)
```

```
[Out] Integral((cosh(a + b*x)**2 + 2)*sinh(a + b*x)/x, x)
```

3.221 $\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}, x\right)$$

```
[Out] Unintegrable(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
[Out] Defer[Int][(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)} dx = \int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)} dx$$

Mathematica [A] time = 5.53, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
[Out] Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

fricas [A] time = 2.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m \sinh(dx+c)}{b\cosh(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(dx+c)}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xe^{\left(2\,dx+m\log(x)+2\,c\right)}}{b(m+1)e^{(2\,dx+2\,c)}+2\,a(m+1)e^{(dx+c)}+b(m+1)}-\frac{1}{2}\int\frac{2\left(2\,adxe^{(3\,dx+3\,c)}+2\,a(m+1)e^{(dx+c)}+b(\right.}{b^2(m+1)e^{(4\,dx+4\,c)}+4\,ab(m+1)e^{(3\,dx+3\,c)}+4\,ab(m+1)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d*
x + c) + b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) + 2*a*(m + 1
)*e^(d*x + c) + b*(m + 1) + (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d*x)
)*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) + 4*a*b*
(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) + b^2*(m + 1)*
e^(2*c))*e^(2*d*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sinh(c+d\,x)}{a+b\cosh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
[Out] int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(c + dx)}{a + b\cosh(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**m*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.222 $\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)} dx$

Optimal. Leaf size=327

$$\frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^4}+\frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}-\frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\text{Li}_2\left(-\right.}{b}$$

```
[Out] -1/4*x^4/b+x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^3*ln(1+b*exp(d*
x+c)/(a+(a^2-b^2)^(1/2)))/b/d+3*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1
/2)))/b/d^2+3*x^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-6*x*po
lylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-6*x*polylog(3,-b*exp(d*x+c
)/(a+(a^2-b^2)^(1/2)))/b/d^3+6*polylog(4,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))
/b/d^4+6*polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^4
```

Rubi [A] time = 0.48, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5562, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}-\frac{6x\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{6x\text{PolyLog}\left(3,-\frac{be^{c+d}}{\sqrt{a^2-b^2}}\right.}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^4/(4*b) + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, -((b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2]))])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b*d^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^4}{4b} + \int \frac{e^{c+dx}x^3}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx + \int \frac{e^{c+dx}x^3}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} - \frac{3\int x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right) d}{bd}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2}{}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2}{}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2}{}$$

$$= -\frac{x^4}{4b} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2}{}$$

Mathematica [A] time = 0.04, size = 326, normalized size = 1.00

$$\frac{6\text{Li}_4\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)}{bd^4} + \frac{6\text{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4} - \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3} - \frac{6x\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3} + \frac{3x^2\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2\text{Li}_2\left(-\right.}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))/(-a +
 Sqrt[a^2 - b^2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))])/(b*d^4)
```

fricas [C] time = 0.57, size = 624, normalized size = 1.91

$$-\frac{d^4x^4 - 12\,d^2x^2\text{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right) - 12\,d^2x^2\text{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(d^4*x^4 - 12*d^2*x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 12*d^2*
x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 4*c^3*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*c^3*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 24*d*x*pol
ylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*d*x*polylog(3, -(a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/
b) - 4*(d^3*x^3 + c^3)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(d^3*x^3 + c^3)*
log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 - b^2)/b^2) + b)/b) - 24*polylog(4, -(a*cosh(d*x + c) + a*sinh(
d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) -
24*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/(b*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh(dx+c)}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^4}{4\,b} - \frac{1}{2}\int \frac{4\left(ax^3e^{(dx+c)} + bx^3\right)}{b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/b - 1/2*integrate(4*(a*x^3*e^(d*x + c) + b*x^3)/(b^2*e^(2*d*x + 2*c
) + 2*a*b*e^(d*x + c) + b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sinh (c + d\,x)}{a + b\cosh (c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh (c + dx)}{a + b\cosh (c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**3*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.223 $\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\right.}{}$$

```
[Out] -1/3*x^3/b+x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^2*ln(1+b*exp(d*
x+c)/(a+(a^2-b^2)^(1/2)))/b/d+2*x*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2
)))/b/d^2+2*x*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-2*polylog(
3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-2*polylog(3,-b*exp(d*x+c)/(a+(a^
2-b^2)^(1/2)))/b/d^3
```

Rubi [A] time = 0.39, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5562, 2190, 2531, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}-\frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{2\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2]))])/(b*d^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^3}{3b}+\int \frac{e^{c+dx}x^2}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx+\int \frac{e^{c+dx}x^2}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{2\int x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\mathrm{Li}}{}$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\mathrm{Li}}{}$$

$$= -\frac{x^3}{3b}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{2x\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\mathrm{Li}}{}$$

Mathematica [A] time = 0.02, size = 244, normalized size = 1.00

$$-\frac{2\text{Li}_3\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)}{bd^3}-\frac{2\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{2x\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (
x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[2,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 - b^2])])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/(a +
Sqrt[a^2 - b^2]))])/(b*d^3)
```

fricas [C] time = 0.65, size = 497, normalized size = 2.03

$$-\frac{d^3x^3-6\,dx\text{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)-6\,dx\text{Li}_2\left(-\frac{a\cosh(dx+c)+a\sinh(dx+c)-(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(d^3*x^3 - 6*d*x*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*d*x*dilog(
-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*c^2*log(2*b*cosh(d*x + c) + 2*b*sinh(d
*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*c^2*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*(d^2*x^2 - c^2)*lo
g((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 - b^2)/b^2) + b)/b) - 3*(d^2*x^2 - c^2)*log((a*cosh(d*x + c) + a*
sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) +
 b)/b) + 6*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 6*polylog(3, -(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2))/b))/(b*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sinh(dx+c)}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh(dx+c)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3}{3\,b} - \frac{1}{2}\int \frac{4\left(ax^2e^{(dx+c)} + bx^2\right)}{b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/b - 1/2*integrate(4*(a*x^2*e^(d*x + c) + b*x^2)/(b^2*e^(2*d*x + 2*c
) + 2*a*b*e^(d*x + c) + b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sinh(c+d\,x)}{a+b\cosh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**2*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.224 $\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

```
[Out] -1/2*x^2/b+x*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x*ln(1+b*exp(d*x+c)
/(a+(a^2-b^2)^(1/2)))/b/d+polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^
2+polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2
```

Rubi [A] time = 0.24, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5562, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -x^2/(2*b) + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(
c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a
 + Sqrt[a^2 - b^2]))]/(b*d^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx &= -\frac{x^2}{2b}+\int \frac{e^{c+dx}x}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx+\int \frac{e^{c+dx}x}{a+\sqrt{a^2-b^2}+be^{c+dx}}\,dx\\
&= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{\int \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{bd}-\frac{\int
\\
&= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}-\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a-\sqrt{a^2-b^2}}\right)}{x}\,dx,x
}{bd^2}\\
&= -\frac{x^2}{2b}+\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{Li}_2\left(-\frac{b
}{a+
}\right)}{bd^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 160, normalized size = 0.99

$$\frac{\text{Li}_2\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)}{bd^2}+\frac{\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, (b*E^(c
+ d*x))/(-a + Sqrt[a^2 - b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 - b^2]))]/(b*d^2)
```

fricas [B] time = 1.63, size = 354, normalized size = 2.20

$$-\frac{d^2x^2 + 2\,c\log\left(2\,b\cosh\left(dx+c\right) + 2\,b\sinh\left(dx+c\right) + 2\,b\sqrt{\frac{a^2-b^2}{b^2}} + 2\,a\right) + 2\,c\log\left(2\,b\cosh\left(dx+c\right) + 2\,b\sinh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x^2 + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a
^2 - b^2)/b^2) + 2*a) + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b
*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d
*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b)
 - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 2*dilog(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^
2) + b)/b + 1) - 2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1))/(b*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

maple [B] time = 0.21, size = 368, normalized size = 2.29

$$-\frac{x^2}{2b}+\frac{\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{db}+\frac{\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)c}{d^2b}+\frac{\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{db}+\frac{\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)c}{d^2b}+\frac{\mathrm{dilog}\left(\frac{b\,\mathrm{e}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/2*x^2/b+1/d/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))
*x+1/d^2/b*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c+1/d
/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x+1/d^2/b*ln((b
*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d^2/b*dilog((b*exp(
```

```
d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))+1/d^2/b*dilog((-b*exp(d*x+c)
+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-2/d/b*c*x-1/d^2/b*c^2-1/d^2/b*c*l
n(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)+2/d^2/b*c*ln(exp(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{2\,b} - \frac{1}{2}\int \frac{4\left(axe^{(dx+c)} + bx\right)}{b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*integrate(4*(a*x*e^(d*x + c) + b*x)/(b^2*e^(2*d*x + 2*c) +
2*a*b*e^(d*x + c) + b^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sinh\left(c+d\,x\right)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

3.225 $\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

```
[Out] ln(a+b*cosh(d*x+c))/b/d
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2668, 31}

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Log[a + b*Cosh[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cosh(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\cosh(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
[Out] Log[a + b*Cosh[c + d*x]]/(b*d)
```

fricas [B] time = 1.00, size = 44, normalized size = 2.44

$$-\frac{dx - \log\left(\frac{2\,(b\cosh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
[Out] -(d*x - log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d)
```

giac [A] time = 0.14, size = 31, normalized size = 1.72

$$\frac{\log\left(\left|b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 2\,a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/(b*d)
```

maple [A] time = 0.03, size = 19, normalized size = 1.06

$$\frac{\ln\left(a + b\cosh\left(dx + c\right)\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
[Out] ln(a+b*cosh(d*x+c))/b/d
```

maxima [A] time = 0.32, size = 18, normalized size = 1.00

$$\frac{\log\left(b\cosh\left(dx + c\right) + a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

[Out] log(b*cosh(d*x + c) + a)/(b*d)

mupad [B] time = 0.07, size = 18, normalized size = 1.00

$$\frac{\ln\left(a+b\cosh\left(c+d\,x\right)\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)/(a + b*cosh(c + d*x)),x)

[Out] log(a + b*cosh(c + d*x))/(b*d)

sympy [A] time = 0.97, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x\sinh\left(c\right)}{a} & \text{for}\ b=0\wedge d=0\\ \frac{x\sinh\left(c\right)}{a+b\cosh\left(c\right)} & \text{for}\ d=0\\ \frac{\cosh\left(c+dx\right)}{ad} & \text{for}\ b=0\\ \frac{\log\left(\frac{a}{b}+\cosh\left(c+dx\right)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)

[Out] Piecewise((x*sinh(c)/a, Eq(b, 0) & Eq(d, 0)), (x*sinh(c)/(a + b*cosh(c)), E
q(d, 0)), (cosh(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + cosh(c + d*x))/(b*d),
 True))

3.226 $\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
[Out] Defer[Int][Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Mathematica [A] time = 14.21, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
[Out] Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sinh(dx+c)}{bx\cosh(dx+c)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)/(b*x*cosh(d*x + c) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)}{(b\cosh(dx+c)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/((b*cosh(d*x + c) + a)*x), x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)}{x(a+b\cosh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log(x)}{b} - \frac{1}{2}\int \frac{4\left(ae^{(dx+c)}+b\right)}{b^2xe^{(2\,dx+2\,c)}+2\,abxe^{(dx+c)}+b^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(x)/b - 1/2*integrate(4*(a*e^(d*x + c) + b)/(b^2*x*e^(2*d*x + 2*c) + 2*a
*b*x*e^(d*x + c) + b^2*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sinh(c+d\,x)}{x\,\left(a+b\cosh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))),x)
```

```
[Out] int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

3.227 $\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Mathematica [A] time = 25.21, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m \sinh(dx+c)^2}{b\cosh(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
[Out] integral(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(\sinh^2(dx+c)\right)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
[Out] int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
[Out] integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sinh(c+d\,x)^2}{a+b\cosh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
[Out] int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**m*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.228 $\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=495

$$\frac{6\sqrt{a^2-b^2}\,\mathrm{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^4}-\frac{6\sqrt{a^2-b^2}\,\mathrm{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^4}-\frac{6x\sqrt{a^2-b^2}\,\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{6x\sqrt{a^2-b^2}\,\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}$$

```
[Out] -1/4*a*x^4/b^2-6*cosh(d*x+c)/b/d^4-3*x^2*cosh(d*x+c)/b/d^2+6*x*sinh(d*x+c)/
b/d^3+x^3*sinh(d*x+c)/b/d+x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b
^2)^(1/2)/b^2/d-x^3*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/
b^2/d+3*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^
2/d^2-3*x^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^
2/d^2-6*x*polylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/
d^3+6*x*polylog(3,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^
3+6*polylog(4,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^4-6*
polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^4
```

Rubi [A] time = 0.84, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5566, 30, 3296, 2638, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{3x^2\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}-\frac{6x\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -(a*x^4)/(4*b^2) - (6*Cosh[c + d*x])/(b*d^4) - (3*x^2*Cosh[c + d*x])/(b*d^2
) + (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b
^2*d) - (Sqrt[a^2 - b^2]*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])]
)/(b^2*d) + (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 - b^2]))])/(b^2*d^2) - (3*Sqrt[a^2 - b^2]*x^2*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 - b^2]*x*PolyLog[3, -((
b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 - b^2]*x*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^3) + (6*Sqrt[a^2
 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^4) - (
6*Sqrt[a^2 - b^2]*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^
2*d^4) + (6*x*Sinh[c + d*x])/(b*d^3) + (x^3*Sinh[c + d*x])/(b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^3\,dx}{b^2} + \frac{\int x^3\cosh(c+dx)\,dx}{b} + \frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^4}{4b^2} + \frac{x^3\sinh(c+dx)}{bd} + \frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{c+dx}x^3}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2} - \frac{3\int x^2\sinh(c+dx)\,d}{bd}$$

$$= -\frac{ax^4}{4b^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{x^3\sinh(c+dx)}{bd} + \frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x^3}{2a-2\sqrt{a^2-b^2}+2be^{c+dx}}\,dx}{b} -$$

$$= -\frac{ax^4}{4b^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{}{a+}\right.}{b^2d}$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2}}{}$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2}}{}$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2}}{}$$

$$= -\frac{ax^4}{4b^2} - \frac{6\cosh(c+dx)}{bd^4} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{\sqrt{a^2-b^2}\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d} - \frac{\sqrt{a^2}}{}$$

Mathematica [A] time = 1.51, size = 386, normalized size = 0.78

$$\frac{4\sqrt{a^2-b^2}\left(d^3x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right) - d^3x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right) + 3d^2x^2\mathrm{Li}_2\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right) - 3d^2x^2\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right) - 6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*d^4*x^4) + 4*Sqrt[a^2 - b^2]*(d^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 3
*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 3*d^2*x^2*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 6*d*x*PolyLog[3, (b*E^(
c + d*x))/(-a + Sqrt[a^2 - b^2])] + 6*d*x*PolyLog[3, -((b*E^(c + d*x))/(a +
 Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])]
- 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 4*b*Cosh[d*x]*(
```

```
-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c]) + 4*b*(d*x*(6 + d^2*x
^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/(4*b^2*d^4)
```

fricas [C] time = 0.69, size = 1174, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d^4*x^4*cosh(d*x + c) + 2*b*d^3*x^3 + 6*b*d^2*x^2 + 12*b*d*x - 2*(b
*d^3*x^3 - 3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c)^2 - 2*(b*d^3*x^3 - 3*
b*d^2*x^2 + 6*b*d*x - 6*b)*sinh(d*x + c)^2 - 12*(b*d^2*x^2*cosh(d*x + c) +
b*d^2*x^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) +
 b)/b + 1) + 12*(b*d^2*x^2*cosh(d*x + c) + b*d^2*x^2*sinh(d*x + c))*sqrt((a
^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
 + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 4*(b*c^3*cosh(d*x +
 c) + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*
b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*(b*c^3*cosh(d*x + c)
 + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*s
inh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 4*((b*d^3*x^3 + b*c^3)*co
sh(d*x + c) + (b*d^3*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2) + b)/b) + 4*((b*d^3*x^3 + b*c^3)*cosh(d*x + c) + (b*d^3
*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a
*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)
+ b)/b) - 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*poly
log(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cos
h(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*d*x*cosh(d*
x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2))/b) - 24*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + (a*d^4*x^4 - 4*(b*d^3*x^3 -
3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c))*sinh(d*x + c) + 12*b)/(b^2*d^4*
cosh(d*x + c) + b^2*d^4*sinh(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\sinh^2 (dx + c)\right)}{a + b \cosh (dx + c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is a-b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sinh (c + d\,x)^2}{a + b \cosh (c + d\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh^2 (c + dx)}{a + b \cosh (c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**3*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.229 $\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\, dx$

Optimal. Leaf size=370

$$-\frac{2\sqrt{a^2-b^2}\,\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{2\sqrt{a^2-b^2}\,\mathrm{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{2x\sqrt{a^2-b^2}\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2x\sqrt{a^2-b^2}\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}$$

```
[Out] -1/3*a*x^3/b^2-2*x*cosh(d*x+c)/b/d^2+2*sinh(d*x+c)/b/d^3+x^2*sinh(d*x+c)/b/
d+x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-x^2*ln(1
+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d+2*x*polylog(2,-b*e
xp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-2*x*polylog(2,-b*exp
(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^2-2*polylog(3,-b*exp(d*x
+c)/(a-(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3+2*polylog(3,-b*exp(d*x+c)/
(a+(a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d^3
```

Rubi [A] time = 0.70, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5566, 30, 3296, 2637, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{2x\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{2x\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}-\frac{2\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -(a*x^3)/(3*b^2) - (2*x*Cosh[c + d*x])/(b*d^2) + (Sqrt[a^2 - b^2]*x^2*Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x^2*L
og[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (2*Sqrt[a^2 - b^2]
*x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (2*Sqr
t[a^2 - b^2]*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d
^2) - (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])
)])/(b^2*d^3) + (2*Sqrt[a^2 - b^2]*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/(b^2*d^3) + (2*Sinh[c + d*x])/(b*d^3) + (x^2*Sinh[c + d*x])/(
b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
```

```
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^2\,dx}{b^2}+\frac{\int x^2\cosh(c+dx)\,dx}{b}+\frac{\left(a^2-b^2\right)\int \frac{x^2}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^3}{3b^2}+\frac{x^2\sinh(c+dx)}{bd}+\frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{c+dx}x^2}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2}-\frac{2\int x\sinh(c+dx)\,d\cdots}{bd}$$

$$= -\frac{ax^3}{3b^2}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{x^2\sinh(c+dx)}{bd}+\frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x^2}{2a-2\sqrt{a^2-b^2}+2be^{c+dx}}\,dx}{b}-\cdots$$

$$= -\frac{ax^3}{3b^2}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\cdots\right)}{b^2d}$$

$$= -\frac{ax^3}{3b^2}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\cdots\right)}{b^2d}$$

$$= -\frac{ax^3}{3b^2}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\cdots\right)}{b^2d}$$

$$= -\frac{ax^3}{3b^2}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x^2\log\left(1+\cdots\right)}{b^2d}$$

Mathematica [A] time = 1.30, size = 293, normalized size = 0.79

$$3\sqrt{a^2-b^2}\left(d^2x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)-d^2x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)+2dx\text{Li}_2\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)-2dx\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)-2\text{Li}_3$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*d^3*x^3) + 3*Sqrt[a^2 - b^2]*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 2
*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 2*d*x*PolyLog[2,
-((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, (b*E^(c + d*x))/(-
a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]
))]) + 3*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]) + 3*b*((2 + d
^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/(3*b^2*d^3)
```

fricas [C] time = 1.70, size = 937, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*d^3*x^3*cosh(d*x + c) + 3*b*d^2*x^2 + 6*b*d*x - 3*(b*d^2*x^2 - 2*
b*d*x + 2*b)*cosh(d*x + c)^2 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*sinh(d*x + c)^
2 - 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*di
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh
(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 6
*(b*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 6*(b
*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cos
h(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 6*((b*d
^2*x^2 - b*c^2)*cosh(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 6*((b*d^2*x^2 - b*c^2)*cosh
(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 - b^2)/b^2) + b)/b) + 12*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 12*(b*cosh(d*x + c) + b
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh
(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) +
```

```
 2*(a*d^3*x^3 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*cosh(d*x + c))*sinh(d*x + c)
+ 6*b)/(b^2*d^3*cosh(d*x + c) + b^2*d^3*sinh(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^2*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(\sinh^2(dx+c)\right)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
[Out] int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is a-b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\sinh\left(c+d\,x\right)}^2}{a+b\,\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh^2(c + dx)}{a + b \cosh(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**2*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.230 $\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{a^2-b^2}\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{\sqrt{a^2-b^2}\,\mathrm{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}+\frac{x\sqrt{a^2-b^2}\,\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^2d}-\frac{x\sqrt{a^2-b^2}\,\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right.}{b^2d}$$

```
[Out] -1/2*a*x^2/b^2-cosh(d*x+c)/b/d^2+x*sinh(d*x+c)/b/d+x*ln(1+b*exp(d*x+c)/(a-(
a^2-b^2)^(1/2)))*(a^2-b^2)^(1/2)/b^2/d-x*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/
2)))*(a^2-b^2)^(1/2)/b^2/d+polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))*(a^
2-b^2)^(1/2)/b^2/d^2-polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))*(a^2-b^2)
^(1/2)/b^2/d^2
```

Rubi [A] time = 0.42, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5566, 30, 3296, 2638, 3320, 2264, 2190, 2279, 2391}

$$\frac{\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^2d^2}+\frac{x\sqrt{a^2-b^2}\,\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^2d}-\frac{x\sqrt{a^2-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -(a*x^2)/(2*b^2) - Cosh[c + d*x]/(b*d^2) + (Sqrt[a^2 - b^2]*x*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^2*d) - (Sqrt[a^2 - b^2]*x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^2*d) + (Sqrt[a^2 - b^2]*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^2*d^2) - (Sqrt[a^2 - b^2]*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^2*d^2) + (x*Sinh[c +
d*x])/(b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x\,dx}{b^2}+\frac{\int x\cosh(c+dx)\,dx}{b}+\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\cosh(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax^2}{2b^2}+\frac{x\sinh(c+dx)}{bd}+\frac{\left(2\left(a^2-b^2\right)\right)\int \frac{e^{c+dx}x}{b+2ae^{c+dx}+be^{2(c+dx)}}\,dx}{b^2}-\frac{\int \sinh(c+dx)\,dx}{bd}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}+\frac{\left(2\sqrt{a^2-b^2}\right)\int \frac{e^{c+dx}x}{2a-2\sqrt{a^2-b^2}+2be^{c+dx}}\,dx}{b}-\frac{(2\cdots}{}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-\cdots}}\right.}{b^2d}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-\cdots}}\right.}{b^2d}$$

$$= -\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-\cdots}}\right.}{b^2d}$$

Mathematica [A] time = 1.03, size = 187, normalized size = 0.77

$$\frac{2\sqrt{a^2-b^2}\left(\text{Li}_2\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}-a}\right)-\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)+dx\left(\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)-\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)\right)\right)+a(c-dx)(c+\cdots}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (a*(c - d*x)*(c + d*x) - 2*b*Cosh[c + d*x] + 2*Sqrt[a^2 - b^2]*(d*x*(Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 - b^2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 2*b*d*x*Sinh[c + d*x])/
(2*b^2*d^2)
```

fricas [B] time = 0.73, size = 669, normalized size = 2.74

$$-\frac{ad^2x^2\cosh(dx+c) + bdx - (bdx-b)\cosh(dx+c)^2 - (bdx-b)\sinh(dx+c)^2 - 2\left(b\cosh(dx+c) + b\sinh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d^2*x^2*cosh(d*x + c) + b*d*x - (b*d*x - b)*cosh(d*x + c)^2 - (b*d*
x - b)*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 -
b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*(b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x +
c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1)
- 2*(b*c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 2*(b*
c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*((b*d*x +
b*c)*cosh(d*x + c) + (b*d*x + b*c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log
((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2) + b)/b) + 2*((b*d*x + b*c)*cosh(d*x + c) + (b*d*x + b*
c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x +
 c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (
a*d^2*x^2 - 2*(b*d*x - b)*cosh(d*x + c))*sinh(d*x + c) + b)/(b^2*d^2*cosh(d
*x + c) + b^2*d^2*sinh(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh(dx+c)^2}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

maple [B] time = 0.29, size = 862, normalized size = 3.53

$$-\frac{a\,x^2}{2b^2}+\frac{(dx-1)\,e^{dx+c}}{2b\,d^2}-\frac{(dx+1)\,e^{-dx-c}}{2b\,d^2}+\frac{\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x\,a^2}{b^2d\sqrt{a^2-b^2}}-\frac{\ln\left(\frac{-b\,e^{dx+c}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{d\sqrt{a^2-b^2}}-\frac{\ln\left(\frac{b\,e^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{b^2d\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/2*a*x^2/b^2+1/2*(d*x-1)/b/d^2*exp(d*x+c)-1/2*(d*x+1)/b/d^2*exp(-d*x-c)+1
/b^2/d/(a^2-b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(
1/2)))*x*a^2-1/d/(a^2-b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(
a^2-b^2)^(1/2)))*x-1/b^2/d/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)
+a)/(a+(a^2-b^2)^(1/2)))*x*a^2+1/d/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^
2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x+1/b^2/d^2/(a^2-b^2)^(1/2)*ln((-b*exp(d*x
+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c*a^2-1/d^2/(a^2-b^2)^(1/2)*ln
((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-1/b^2/d^2/(a^2-b
^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c*a^2+1/
d^2/(a^2-b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2))
)*c+1/b^2/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(
a^2-b^2)^(1/2)))*a^2-1/d^2/(a^2-b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2-b^2)^(
1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/b^2/d^2/(a^2-b^2)^(1/2)*dilog((b*exp(d*x+c)
+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2+1/d^2/(a^2-b^2)^(1/2)*dilog((b
*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/b^2/d^2*c/(-a^2+b^2)^
(1/2)*arctan(1/2*(2*b*exp(d*x+c)+2*a)/(-a^2+b^2)^(1/2))*a^2+2/d^2*c/(-a^2+b
^2)^(1/2)*arctan(1/2*(2*b*exp(d*x+c)+2*a)/(-a^2+b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is a-b positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sinh\left(c+d\,x\right)^2}{a+b\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] -a*x/b^2+sinh(d*x+c)/b/d+2*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1
/2))*(a-b)^(1/2)*(a+b)^(1/2)/b^2/d
```

Rubi [A] time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2695, 2735, 2659, 205}

$$\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*
x)/2])/Sqrt[a + b]])/(b^2*d) + Sinh[c + d*x]/(b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{\sinh(c+dx)}{bd}+\frac{\int \frac{-b-a\cosh(c+dx)}{a+b\cosh(c+dx)}\,dx}{b}\\ &= -\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}-\left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cosh(c+dx)}\,dx\\ &= -\frac{ax}{b^2}+\frac{\sinh(c+dx)}{bd}+\frac{\left(2i\left(1-\frac{a^2}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\ &= -\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{\sinh(c+dx)}{bd}\end{aligned}$$

Mathematica [A] time = 0.18, size = 69, normalized size = 0.95

$$\frac{2\sqrt{b^2-a^2}\,\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)-a(c+dx)+b\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqr
t[-a^2 + b^2]] + b*Sinh[c + d*x])/(b^2*d)
```

fricas [B] time = 0.50, size = 415, normalized size = 5.68

$$\left[-\frac{2\,adx\cosh(dx+c)-b\cosh(dx+c)^2-b\sinh(dx+c)^2-2\sqrt{a^2-b^2}\,(\cosh(dx+c)+\sinh(dx+c))\log\left(\frac{b^2\cos\cdots}{\cdots}\right.}{2\left(b\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*sq
rt(a^2 - b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c
) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x +
c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*
cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d
*x + c) + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c)), -1/2*(2*a*d*x*cos
h(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 + 4*sqrt(-a^2 + b^2)*(co
sh(d*x + c) + sinh(d*x + c))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c) + b*
sinh(d*x + c) + a)/(a^2 - b^2)) + 2*(a*d*x - b*cosh(d*x + c))*sinh(d*x + c)
 + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))]
```

giac [A] time = 0.14, size = 89, normalized size = 1.22

$$-\frac{\frac{2\,(dx+c)a}{b^2}-\frac{e^{(dx+c)}}{b}+\frac{e^{(-dx-c)}}{b}-\frac{4\left(a^2-b^2\right)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*a/b^2 - e^(d*x + c)/b + e^(-d*x - c)/b - 4*(a^2 - b^2)*ar
ctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2))/d
```

maple [B] time = 0.08, size = 177, normalized size = 2.42

$$-\frac{1}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}-\frac{1}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,b^2}+\frac{2\,\mathrm{arcta}}{d\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/b/(t
anh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+2/d/b^2/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))*a^2-2/d/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.12, size = 176, normalized size = 2.41

$$\frac{e^{c+d\,x}}{2\,b\,d}-\frac{e^{-c-d\,x}}{2\,b\,d}-\frac{a\,x}{b^2}+\frac{\ln\left(-\frac{2\,e^{c+d\,x}\,\left(a^2-b^2\right)}{b^3}-\frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,\left(b+a\,e^{c+d\,x}\right)}{b^3}\right)\,\sqrt{a+b}\,\sqrt{a-b}}{b^2\,d}-\frac{\ln\left(\frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,\left(b+a\,e^{c+d\,x}\right)}{b^3}-\frac{2\,e^{c+d}}{\quad}\right.}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*cosh(c + d*x)),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - exp(- c - d*x)/(2*b*d) - (a*x)/b^2 + (log(- (2*exp(c
 + d*x)*(a^2 - b^2))/b^3 - (2*(a + b)^(1/2)*(a - b)^(1/2)*(b + a*exp(c + d*
x)))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/(b^2*d) - (log((2*(a + b)^(1/2)*(a -
 b)^(1/2)*(b + a*exp(c + d*x)))/b^3 - (2*exp(c + d*x)*(a^2 - b^2))/b^3)*(a
+ b)^(1/2)*(a - b)^(1/2))/(b^2*d)
```

sympy [A] time = 122.50, size = 1122, normalized size = 15.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*sinh(c)**2/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (d*x*
tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) - d*x/(b*d*tanh(c/2 +
 d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2)**2 - b*d), E
q(a, -b)), ((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*co
sh(c + d*x)/(2*d))/a, Eq(b, 0)), (x*sinh(c)**2/(a + b*cosh(c)), Eq(d, 0)),
(-d*x*tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) + d*x/(b*d*tanh
(c/2 + d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2)**2 - b
*d), Eq(a, b)), (-a*d*x*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2/(b
**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a -
 b) + b/(a - b))) + a*d*x*sqrt(a/(a - b) + b/(a - b))/(b**2*d*sqrt(a/(a - b
) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) -
 a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)*
*2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a
/(a - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*
```

```
x/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqr
t(a/(a - b) + b/(a - b))) + a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 +
d*x/2))*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 +
 d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - a*log(sqrt(a/(a - b) + b
/(a - b)) + tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2
 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - 2*b*sqrt(a/(a - b) + b
/(a - b))*tanh(c/2 + d*x/2)/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 +
d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - b*log(-sqrt(a/(a - b) + b
/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b)
+ b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) + b
*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a -
b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b)))
+ b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)*
*2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a
/(a - b) + b/(a - b))) - b*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x
/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt
(a/(a - b) + b/(a - b))), True))
```

3.232 $\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Mathematica [A] time = 116.73, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(dx+c)^2}{bx\cosh(dx+c)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")

[Out] integral(sinh(d*x + c)^2/(b*x*cosh(d*x + c) + a*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^2}{(b\cosh(dx+c)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="giac")

[Out] integrate(sinh(d*x + c)^2/((b*cosh(d*x + c) + a)*x), x)

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(dx+c)}{x\,(a+b\cosh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)

[Out] int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$2\left(a^2e^c - b^2e^c\right)\int \frac{e^{(dx)}}{b^3xe^{(2\,dx+2\,c)} + 2\,ab^2xe^{(dx+c)} + b^3x}\,dx + \frac{\text{Ei}(-dx)\,e^{(-c)}}{2\,b} + \frac{\text{Ei}(dx)\,e^c}{2\,b} - \frac{a\log(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")

[Out] 2*(a^2*e^c - b^2*e^c)*integrate(e^(d*x)/(b^3*x*e^(2*d*x + 2*c) + 2*a*b^2*x*e^(d*x + c) + b^3*x), x) + 1/2*Ei(-d*x)*e^(-c)/b + 1/2*Ei(d*x)*e^c/b - a*log(x)/b^2

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sinh(c+d\,x)^2}{x\,\left(a+b\cosh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))),x)

```
[Out] int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{x\,(a+b\cosh(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/x/(a+b*cosh(d*x+c)),x)
[Out] Integral(sinh(c + d*x)**2/(x*(a + b*cosh(c + d*x))), x)
```

3.233 $\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)},x\right)$$

```
[Out] Unintegrable(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Defer[Int][(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

Rubi steps

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Mathematica [A] time = 32.31, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m \sinh\left(dx+c\right)^3}{b\cosh\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
[Out] integral(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
[Out] integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

maple [A] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^m \left(\sinh^3(dx+c)\right)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
[Out] int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
[Out] integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m \sinh(c+d\,x)^3}{a+b\cosh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
[Out] int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**m*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.234 $\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=586

$$\frac{6(a^2-b^2)\operatorname{Li}_4\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^4}+\frac{6(a^2-b^2)\operatorname{Li}_4\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^4}-\frac{6x(a^2-b^2)\operatorname{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{6x(a^2-b^2)\operatorname{Li}_3\left(-\frac{be^{c+}}{a+\sqrt{a^2}}\right.}{b^3d^3}$$

```
[Out] 3/8*x/b/d^3+1/4*x^3/b/d-1/4*(a^2-b^2)*x^4/b^3-6*a*x*cosh(d*x+c)/b^2/d^3-a*x
^3*cosh(d*x+c)/b^2/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b
^3/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+3*(a^2-b^2)
*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+3*(a^2-b^2)*x^2*p
olylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-6*(a^2-b^2)*x*polylog(3
,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^3-6*(a^2-b^2)*x*polylog(3,-b*exp(
d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^3+6*(a^2-b^2)*polylog(4,-b*exp(d*x+c)/(a-
(a^2-b^2)^(1/2)))/b^3/d^4+6*(a^2-b^2)*polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^
(1/2)))/b^3/d^4+6*a*sinh(d*x+c)/b^2/d^4+3*a*x^2*sinh(d*x+c)/b^2/d^2-3/8*cos
h(d*x+c)*sinh(d*x+c)/b/d^4-3/4*x^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+3/4*x*sinh
(d*x+c)^2/b/d^3+1/2*x^3*sinh(d*x+c)^2/b/d
```

Rubi [A] time = 0.69, antiderivative size = 586, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5566, 3296, 2637, 5372, 3311, 30, 2635, 8, 5562, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2(a^2-b^2)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{3x^2(a^2-b^2)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{6x(a^2-b^2)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-}}\right.}{b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (3*x)/(8*b*d^3) + x^3/(4*b*d) - ((a^2 - b^2)*x^4)/(4*b^3) - (6*a*x*Cosh[c +
 d*x])/(b^2*d^3) - (a*x^3*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^3*Log[1 +
 (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + (3*(a^2 - b^2)*x^2*PolyL
og[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (3*(a^2 - b^2)
*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (6*(
a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3
) - (6*(a^2 - b^2)*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/
(b^3*d^3) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2
]))])/(b^3*d^4) + (6*(a^2 - b^2)*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))])/(b^3*d^4) + (6*a*Sinh[c + d*x])/(b^2*d^4) + (3*a*x^2*Sinh[c + d
*x])/(b^2*d^2) - (3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*x^2*Cosh[c
```

```
+ d*x]*Sinh[c + d*x])/(4*b*d^2) + (3*x*Sinh[c + d*x]^2)/(4*b*d^3) + (x^3*Si
nh[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^3\sinh(c+dx)\,dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)\,dx}{b}+\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c\cdots}{a+b\cosh(\cdots}}{b^2}$$

$$=-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{x^3\sinh^2(c+dx)}{2bd}+\frac{\left(a^2-b^2\right)\int \frac{e^{c+dx}x^3}{a-\sqrt{a^2-b^2}+be^{c+dx}}\cdots}{b^2}$$

$$=-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{\left(a^2-b^2\right)x^3\,\mathrm{l}\cdots}{\cdots}$$

$$=\frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\log\left(1+\cdots\right.}{b^3d}$$

$$=\frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\,\mathrm{lo}\cdots}{\cdots}$$

$$=\frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\,\mathrm{lo}\cdots}{\cdots}$$

$$=\frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{\left(a^2-b^2\right)x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{\left(a^2-b^2\right)x^3\,\mathrm{lo}\cdots}{\cdots}$$

Mathematica [A] time = 11.57, size = 1082, normalized size = 1.85

$$-\frac{(b-a)(a+b)\cosh\left(\frac{c}{2}\right)\operatorname{sech}(c)\sinh\left(\frac{c}{2}\right)x^4}{2b^3}-\frac{a\cosh(dx)\left(d^3\cosh(c)x^3-3d^2\sinh(c)x^2+6d\cosh(c)x-6\sinh(c)\right.}{b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -1/2*((-a + b)*(a + b)*x^4*Cosh[c/2]*Sech[c]*Sinh[c/2])/b^3 - (a*Cosh[d*x]*
(6*d*x*Cosh[c] + d^3*x^3*Cosh[c] - 6*Sinh[c] - 3*d^2*x^2*Sinh[c]))/(b^2*d^4
) + (Cosh[2*d*x]*(6*d*x*Cosh[2*c] + 4*d^3*x^3*Cosh[2*c] - 3*Sinh[2*c] - 6*d
^2*x^2*Sinh[2*c]))/(16*b*d^4) - (a*(-6*Cosh[c] - 3*d^2*x^2*Cosh[c] + 6*d*x*
Sinh[c] + d^3*x^3*Sinh[c])*Sinh[d*x])/(b^2*d^4) + ((-3*Cosh[2*c] - 6*d^2*x^
2*Cosh[2*c] + 6*d*x*Sinh[2*c] + 4*d^3*x^3*Sinh[2*c])*Sinh[2*d*x])/(16*b*d^4
) + ((-a^2 + b^2)*(-x^4 + (2*b^2*(d^3*x^3*Log[1 + ((a - Sqrt[a^2 - b^2])*(C
osh[c + d*x] - Sinh[c + d*x]))/b] - 3*d^2*x^2*PolyLog[2, ((-a + Sqrt[a^2 -
b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 6*d*x*PolyLog[3, ((-a + Sqrt[a^
2 - b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 6*PolyLog[4, ((-a + Sqrt[a^
2 - b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b])*(1 + Cosh[2*c] + Sinh[2*c]))
/(Sqrt[a^2 - b^2]*(-a + Sqrt[a^2 - b^2])*d^4) + (2*b^2*(d^3*x^3*Log[1 + ((a
 + Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 3*d^2*x^2*PolyLog
[2, ((a + Sqrt[a^2 - b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 6*d*x*Pol
yLog[3, ((a + Sqrt[a^2 - b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 6*Pol
yLog[4, ((a + Sqrt[a^2 - b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b])*(1 + C
osh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d^4) + (2*a*(
d^3*x^3*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 - b^2])]
+ 3*d^2*x^2*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 -
 b^2])] - 6*d*x*PolyLog[3, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a
^2 - b^2])] + 6*PolyLog[4, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a
^2 - b^2])])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*d^4) - (2*a*(d^3
*x^3*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 - b^2])] + 3
*d^2*x^2*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 - b
^2]))] - 6*d*x*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a
^2 - b^2]))] + 6*PolyLog[4, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt
[a^2 - b^2]))])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*d^4))*(1 - Ta
nh[c]))/(4*b^3)
```

fricas [C] time = 0.53, size = 2025, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^
2*d*x - 3*b^2)*cosh(d*x + c)^4 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x
 - 3*b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 16*(a*b*d^3*x^3 - 3*a*b*d^2*x^2 + 6
*a*b*d*x - 6*a*b)*cosh(d*x + c)^3 - 4*(4*a*b*d^3*x^3 - 12*a*b*d^2*x^2 + 24*
a*b*d*x - 24*a*b - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^4*x^4 - 2*(a^2 - b^2)*c^4)*co
sh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^4*x^4 - 8*(a^2 - b^2)*c^4 - 3*(4*b^2*d^3
*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^2 + 24*(a*b*d^3*x^3
 - 3*a*b*d^2*x^2 + 6*a*b*d*x - 6*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*b^
2 - 16*(a*b*d^3*x^3 + 3*a*b*d^2*x^2 + 6*a*b*d*x + 6*a*b)*cosh(d*x + c) + 96
*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d^2*x^2*cosh(d*x + c)
*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)
/b^2) + b)/b + 1) + 96*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)
*d^2*x^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)
*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)
^2 + 2*(a^2 - b^2)*c^3*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^3*sinh(d
*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)
/b^2) + 2*a) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^3*cosh
(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^3*sinh(d*x + c)^2)*log(2*b*cosh(d*x
 + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 32*(((a^2 -
b^2)*d^3*x^3 + (a^2 - b^2)*c^3)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^3*x^3 +
(a^2 - b^2)*c^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^3*x^3 + (a^2
- b^2)*c^3)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 32*(((a^2 -
b^2)*d^3*x^3 + (a^2 - b^2)*c^3)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^3*x^3 +
(a^2 - b^2)*c^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^3*x^3 + (a^2
- b^2)*c^3)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 192*((a^2 -
b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*sinh(d*x + c)^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 192*((a^2 - b^2)
*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*
sinh(d*x + c)^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 192*((a^2 - b^2)*d*x
*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*d*x*sinh(d*x + c)^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 192*((a^2 -
b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(d*x + c) +
(a^2 - b^2)*d*x*sinh(d*x + c)^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 4*(4
*a*b*d^3*x^3 + 12*a*b*d^2*x^2 + 24*a*b*d*x - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2
```

```
 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^3 + 12*(a*b*d^3*x^3 - 3*a*b*d^2*x^2 + 6
*a*b*d*x - 6*a*b)*cosh(d*x + c)^2 + 24*a*b + 4*((a^2 - b^2)*d^4*x^4 - 2*(a^
2 - b^2)*c^4)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^4*cosh(d*x + c)^2 + 2*b^
3*d^4*cosh(d*x + c)*sinh(d*x + c) + b^3*d^4*sinh(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^3*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(\sinh^3(dx+c)\right)}{a+b\cosh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(8\left(a^2d^4e^{(2c)} - b^2d^4e^{(2c)}\right)x^4 + \left(4b^2d^3x^3e^{(4c)} - 6b^2d^2x^2e^{(4c)} + 6b^2dxe^{(4c)} - 3b^2e^{(4c)}\right)e^{(2dx)} - 16\left(abd^3x^3e^{(3c)} - 3ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/32*(8*(a^2*d^4*e^(2*c) - b^2*d^4*e^(2*c))*x^4 + (4*b^2*d^3*x^3*e^(4*c) -
6*b^2*d^2*x^2*e^(4*c) + 6*b^2*d*x*e^(4*c) - 3*b^2*e^(4*c))*e^(2*d*x) - 16*(
a*b*d^3*x^3*e^(3*c) - 3*a*b*d^2*x^2*e^(3*c) + 6*a*b*d*x*e^(3*c) - 6*a*b*e^(
3*c))*e^(d*x) - 16*(a*b*d^3*x^3*e^c + 3*a*b*d^2*x^2*e^c + 6*a*b*d*x*e^c + 6
*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + 6*b^2*d*x + 3*b^2)*e^
(-2*d*x))*e^(-2*c)/(b^3*d^4) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^3*
e^(d*x) + (a^2*b - b^3)*x^3)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b
^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sinh(c+d\,x)^3}{a+b\cosh(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sinh^3(c + dx)}{a + b \cosh(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x**3*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.235 $\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=432

$$-\frac{2\left(a^2-b^2\right)\text{Li}_3\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{2\left(a^2-b^2\right)\text{Li}_3\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3}+\frac{2x\left(a^2-b^2\right)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2x\left(a^2-b^2\right)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{}}\right.}{b^3d^2}$$

```
[Out] 1/4*x^2/b/d-1/3*(a^2-b^2)*x^3/b^3-2*a*cosh(d*x+c)/b^2/d^3-a*x^2*cosh(d*x+c)
/b^2/d+(a^2-b^2)*x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^2)
*x^2*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+2*(a^2-b^2)*x*polylog(2,-
b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+2*(a^2-b^2)*x*polylog(2,-b*exp(d*
x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/(a-(a
^2-b^2)^(1/2)))/b^3/d^3-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/(a+(a^2-b^2)^(1
/2)))/b^3/d^3+2*a*x*sinh(d*x+c)/b^2/d^2-1/2*x*cosh(d*x+c)*sinh(d*x+c)/b/d^2
+1/4*sinh(d*x+c)^2/b/d^3+1/2*x^2*sinh(d*x+c)^2/b/d
```

Rubi [A] time = 0.56, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5566, 3296, 2638, 5372, 3310, 30, 5562, 2190, 2531, 2282, 6589}

$$\frac{2x\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{2x\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}-\frac{2\left(a^2-b^2\right)\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] x^2/(4*b*d) - ((a^2 - b^2)*x^3)/(3*b^3) - (2*a*Cosh[c + d*x])/(b^2*d^3) - (
a*x^2*Cosh[c + d*x])/(b^2*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b^2)*x^2*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2])])/(b^3*d) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(c + d*x)
)/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + (2*(a^2 - b^2)*x*PolyLog[2, -((b*E^(
c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) - (2*(a^2 - b^2)*PolyLog[3, -(
(b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^3) - (2*(a^2 - b^2)*PolyLog
[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^3) + (2*a*x*Sinh[c +
d*x])/(b^2*d^2) - (x*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + Sinh[c + d*x]
^2/(4*b*d^3) + (x^2*Sinh[c + d*x]^2)/(2*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x^2\sinh(c+dx)\,dx}{b^2} + \frac{\int x^2\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{\left(a^2-b^2\right)\int \frac{x^2\sinh(c}{a+b\cosh(}}{b^2}$$

$$= -\frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{x^2\sinh^2(c+dx)}{2bd} + \frac{\left(a^2-b^2\right)\int \frac{e^{c+dx}x^2}{a-\sqrt{a^2-b^2}+be^{c+dx}}}{b^2}$$

$$= -\frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)x^2 1}{}$$

$$= \frac{x^2}{4bd} - \frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{}{a-}\right.}{b^3d}$$

$$= \frac{x^2}{4bd} - \frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{}{a-}\right.}{b^3d}$$

$$= \frac{x^2}{4bd} - \frac{\left(a^2-b^2\right)x^3}{3b^3} - \frac{2a\cosh(c+dx)}{b^2d^3} - \frac{ax^2\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x^2\log\left(1+\frac{}{a-}\right.}{b^3d}$$

Mathematica [A] time = 8.42, size = 831, normalized size = 1.92

$$8\left(a^2-b^2\right)\tanh(c)x^3 - \frac{24ab\cosh(dx)\left(\left(d^2x^2+2\right)\cosh(c)-2dx\sinh(c)\right)}{d^3} + \frac{3b^2\cosh(2dx)\left(\left(2d^2x^2+1\right)\cosh(2c)-2dx\sinh(2c)\right)}{d^3} - \frac{24ab\left(\left(d^2x^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] ((-24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^2*C
osh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b*(-2
*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[
2*c] + (1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3 + 4*(-a^2 + b^2)*(-2*x^3
 + (3*b^2*(d^2*x^2*Log[1 + ((a - Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh[c +
 d*x]))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh
[c + d*x]))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 - b^2])*(Cosh[c + d*x] - Sinh
[c + d*x]))/b])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]*(-a + Sqrt[a^
2 - b^2])*d^3) + (3*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 - b^2])*(Cosh[c + d
*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt[a^2 - b^2])*(-Cosh[c
```

```
 + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt[a^2 - b^2])*(-Cosh[c
 + d*x] + Sinh[c + d*x]))/b])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 - b^2]
*(a + Sqrt[a^2 - b^2])*d^3) + (3*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sin
h[c + d*x]))/(a - Sqrt[a^2 - b^2])] + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] +
Sinh[c + d*x]))/(-a + Sqrt[a^2 - b^2])] - 2*PolyLog[3, (b*(Cosh[c + d*x] +
Sinh[c + d*x]))/(-a + Sqrt[a^2 - b^2])])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt
[a^2 - b^2]*d^3) - (3*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x])
)/(a + Sqrt[a^2 - b^2])] + 2*d*x*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c +
d*x]))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c
+ d*x]))/(a + Sqrt[a^2 - b^2]))])*(1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 -
b^2]*d^3))*(1 - Tanh[c]) + 8*(a^2 - b^2)*x^3*Tanh[c])/(24*b^3)
```

fricas [C] time = 0.70, size = 1622, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*x^2 + 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^4 +
 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 24*(a*b*
d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*x^2 - 4*a*b*d*
x + 4*a*b - (2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c))*sinh(d*x + c)^
3 - 16*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*cosh(d*x + c)^2 - 2*(8*(a^
2 - b^2)*d^3*x^3 + 16*(a^2 - b^2)*c^3 - 9*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)
*cosh(d*x + c)^2 + 36*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c))*sinh
(d*x + c)^2 + 3*b^2 - 24*(a*b*d^2*x^2 + 2*a*b*d*x + 2*a*b)*cosh(d*x + c) +
96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x + c)*sinh(
d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)
/b + 1) + 96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x
+ c)*sinh(d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)
/b^2) + b)/b + 1) + 48*((a^2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2
*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cos
h(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*((a^
2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2*cosh(d*x + c)*sinh(d*x + c
) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*(((a^2 - b^2)*d^2*x^2 - (a^2 -
b^2)*c^2)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*sinh(d*x +
 c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 48*(((a^2 - b^2)*d^2*x^2 - (a^2 -
b^2)*c^2)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d^2*x^2 - (a^2 - b^2)*c^2)*sinh(d*x +
```

```
c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 96*((a^2 - b^2)*cosh(d*x + c)^2 +
2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*po
lylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 - b^2)/b^2))/b) - 96*((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^
2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*polylog
(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 - b^2)/b^2))/b) - 4*(6*a*b*d^2*x^2 + 12*a*b*d*x - 3*(2*b^2*d^2
*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^3 + 18*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a
*b)*cosh(d*x + c)^2 + 12*a*b + 8*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*
cosh(d*x + c))*sinh(d*x + c))/(b^3*d^3*cosh(d*x + c)^2 + 2*b^3*d^3*cosh(d*x
 + c)*sinh(d*x + c) + b^3*d^3*sinh(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(\sinh^3 (dx + c)\right)}{a + b \cosh (dx + c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(16 \left(a^2 d^3 e^{(2\,c)} - b^2 d^3 e^{(2\,c)}\right) x^3 + 3 \left(2\,b^2 d^2 x^2 e^{(4\,c)} - 2\,b^2 d x e^{(4\,c)} + b^2 e^{(4\,c)}\right) e^{(2\,dx)} - 24 \left(abd^2 x^2 e^{(3\,c)} - 2\,abdxe^{(3\,c)} + 2\,a\right.}{48\,b^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(a^2*d^3*e^(2*c) - b^2*d^3*e^(2*c))*x^3 + 3*(2*b^2*d^2*x^2*e^(4*c)
 - 2*b^2*d*x*e^(4*c) + b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*x^2*e^(3*c) - 2
*a*b*d*x*e^(3*c) + 2*a*b*e^(3*c))*e^(d*x) - 24*(a*b*d^2*x^2*e^c + 2*a*b*d*x
*e^c + 2*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*x^2 + 2*b^2*d*x + b^2)*e^(-2*d*x)
```

```
)*e^(-2*c)/(b^3*d^3) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^2*e^(d*x)
+ (a^2*b - b^3)*x^2)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sinh(c + d\,x)^3}{a + b\cosh(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
[Out] int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sinh^3(c + dx)}{a + b\cosh(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
[Out] Integral(x**2*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.236 $\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=288

$$\frac{\left(a^2-b^2\right)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{\left(a^2-b^2\right)\text{Li}_2\left(-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{x\left(a^2-b^2\right)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^3d}+\frac{x\left(a^2-b^2\right)\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+\ldots}\right.}{b^3d}$$

```
[Out] 1/4*x/b/d-1/2*(a^2-b^2)*x^2/b^3-a*x*cosh(d*x+c)/b^2/d+(a^2-b^2)*x*ln(1+b*ex
p(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^2)*x*ln(1+b*exp(d*x+c)/(a+(a^2-b
^2)^(1/2)))/b^3/d+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^
3/d^2+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2+a*sinh
(d*x+c)/b^2/d^2-1/4*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/2*x*sinh(d*x+c)^2/b/d
```

Rubi [A] time = 0.34, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5566, 3296, 2637, 5372, 2635, 8, 5562, 2190, 2279, 2391}

$$\frac{\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{\left(a^2-b^2\right)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}\right)}{b^3d^2}+\frac{x\left(a^2-b^2\right)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{b^3d}+\frac{x\left(a^2-\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] x/(4*b*d) - ((a^2 - b^2)*x^2)/(2*b^3) - (a*x*Cosh[c + d*x])/(b^2*d) + ((a^2
 - b^2)*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 -
 b^2)*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b^3*d) + ((a^2 - b
^2)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b^3*d^2) + ((a^2
 - b^2)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b^3*d^2) + (
a*Sinh[c + d*x])/(b^2*d^2) - (Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + (x*S
inh[c + d*x]^2)/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5566

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Sinh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 2
)*Cosh[c + d*x], x], x] + Dist[(a^2 - b^2)/b^2, Int[((e + f*x)^m*Sinh[c + d
*x]^(n - 2))/(a + b*Cosh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\int x\sinh(c+dx)\,dx}{b^2} + \frac{\int x\cosh(c+dx)\sinh(c+dx)\,dx}{b} + \frac{\left(a^2-b^2\right)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}}{b^2}$$

$$= -\frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{x\sinh^2(c+dx)}{2bd} + \frac{\left(a^2-b^2\right)\int \frac{e^{c+dx}x}{a-\sqrt{a^2-b^2}+be^{c+dx}}\,dx}{b^2}$$

$$= -\frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)x\log}{b^3}$$

$$= \frac{x}{4bd} - \frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)}{}$$

$$= \frac{x}{4bd} - \frac{\left(a^2-b^2\right)x^2}{2b^3} - \frac{ax\cosh(c+dx)}{b^2d} + \frac{\left(a^2-b^2\right)x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} + \frac{\left(a^2-b^2\right)}{}$$

Mathematica [A] time = 2.96, size = 414, normalized size = 1.44

$$\frac{4\left(a^2-b^2\right)\left(2\text{Li}_2\left(\frac{b(\cosh(c+dx)+\sinh(c+dx))}{\sqrt{a^2-b^2}-a}\right)+2\text{Li}_2\left(-\frac{b(\cosh(c+dx)+\sinh(c+dx))}{a+\sqrt{a^2-b^2}}\right)+2(c+dx)\log\left(\frac{b(\sinh(c+dx)+\cosh(c+dx))}{a-\sqrt{a^2-b^2}}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-8*a*b*d*x*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*(2*
c*(c + d*x) - (c + d*x)^2 + (4*a*Sqrt[-(a^2 - b^2)^2]*c*ArcTan[(a + b*Cosh[
c + d*x] + b*Sinh[c + d*x])/Sqrt[-a^2 + b^2]])/(a^2 - b^2)^(3/2) + (4*a*Sqr
t[-(a^2 - b^2)^2]*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^
2 - b^2]])/(-a^2 + b^2)^(3/2) - 2*c*Log[2*(a + b*Cosh[c + d*x])*(Cosh[c + d
```

```
*x] + Sinh[c + d*x])] + 2*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*
x]))/(a - Sqrt[a^2 - b^2])] + 2*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[
c + d*x]))/(a + Sqrt[a^2 - b^2])] + 2*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c
 + d*x]))/(-a + Sqrt[a^2 - b^2])] + 2*PolyLog[2, -((b*(Cosh[c + d*x] + Sinh
[c + d*x]))/(a + Sqrt[a^2 - b^2]))]) + 8*a*b*Sinh[c + d*x] - b^2*Sinh[2*(c
+ d*x)])/(8*b^3*d^2)
```

fricas [B] time = 0.53, size = 1196, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*b^2*d*x - b^2)*cosh(d*x + c)^4 + (2*b^2*d*x - b^2)*sinh(d*x + c)^4
 + 2*b^2*d*x - 8*(a*b*d*x - a*b)*cosh(d*x + c)^3 - 4*(2*a*b*d*x - 2*a*b - (
2*b^2*d*x - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^2*x^2 -
2*(a^2 - b^2)*c^2)*cosh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^2*x^2 - 8*(a^2 - b^
2)*c^2 - 3*(2*b^2*d*x - b^2)*cosh(d*x + c)^2 + 12*(a*b*d*x - a*b)*cosh(d*x
+ c))*sinh(d*x + c)^2 + b^2 - 8*(a*b*d*x + a*b)*cosh(d*x + c) + 16*((a^2 -
b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 16*((a^2 - b^
2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2
)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 16*((a^2 - b^2)
*c*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b
^2)*c*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt
((a^2 - b^2)/b^2) + 2*a) - 16*((a^2 - b^2)*c*cosh(d*x + c)^2 + 2*(a^2 - b^2
)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c*sinh(d*x + c)^2)*log(2*b*co
sh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 16*(((
a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d*x + (a^2
 - b^2)*c)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*x + (a^2 - b^2)*c)*
sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 16*(((a^2 - b^2)*d*x + (
a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*
x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*x + (a^2 - b^2)*c)*sinh(d*x + c)^2)*l
og((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))
*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(2*a*b*d*x - (2*b^2*d*x - b^2)*cosh(d*x
+ c)^3 + 6*(a*b*d*x - a*b)*cosh(d*x + c)^2 + 2*a*b + 4*((a^2 - b^2)*d^2*x^2
 - 2*(a^2 - b^2)*c^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^2*cosh(d*x + c)^
2 + 2*b^3*d^2*cosh(d*x + c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

maple [B] time = 0.30, size = 860, normalized size = 2.99

$$-\frac{\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{db}-\frac{\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)c}{d^2b}+\frac{c\ln\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}+b\right)}{d^2b}-\frac{2c\ln\left(\mathrm{e}^{dx+c}\right)}{d^2b}+\frac{2cx}{db}-\frac{\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+}{-a+}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/d/b*ln((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x-1/d^2/b*l
n((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d^2/b*c*ln(b*ex
p(2*d*x+2*c)+2*a*exp(d*x+c)+b)-2/d^2/b*c*ln(exp(d*x+c))+2/d/b*c*x-1/d/b*ln(
(-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x-1/d^2/b*ln((-b*ex
p(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-1/d^2/b^3*a^2*c^2-1/2*a
*(d*x+1)/b^2/d^2*exp(-d*x-c)-1/2*a*(d*x-1)/b^2/d^2*exp(d*x+c)-1/2*x^2/b^3*a
^2+1/16*(2*d*x+1)/b/d^2*exp(-2*d*x-2*c)+1/d^2/b^3*a^2*dilog((-b*exp(d*x+c)+
(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/d^2/b^3*a^2*dilog((b*exp(d*x+c)+
(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/d/b^3*a^2*c*x+1/d/b^3*ln((-b*exp(
d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*x+1/d^2/b^3*ln((-b*exp(
d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*c+1/d/b^3*ln((b*exp(d*x
+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2*x+1/d^2/b^3*ln((b*exp(d*x+c
)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2*c+2/d^2/b^3*c*a^2*ln(exp(d*x+
c))-1/d^2/b^3*c*a^2*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)+b)+1/d^2/b*c^2-1/d^2
/b*dilog((b*exp(d*x+c)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-1/d^2/b*dilo
g((-b*exp(d*x+c)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/16*(2*d*x-1)/b/
d^2*exp(2*d*x+2*c)+1/2*x^2/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(8\left(a^2d^2e^{(2c)}-b^2d^2e^{(2c)}\right)x^2+\left(2b^2dxe^{(4c)}-b^2e^{(4c)}\right)e^{(2dx)}-8\left(abdxe^{(3c)}-abe^{(3c)}\right)e^{(dx)}-8\left(abdxe^{c}+abe^{c}\right)e^{(-dx)}\cdots}{16b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(8*(a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2*
e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) - 8*(a*b*d*x
*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^2)
 - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x*e^(d*x) + (a^2*b - b^3)*x)/(b^
4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sinh\left(c+d\,x\right)^3}{a+b\cosh\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

```
[Out] int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sinh^3\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(x*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

3.237 $\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}$$

```
[Out] -a*cosh(d*x+c)/b^2/d+1/2*cosh(d*x+c)^2/b/d+(a^2-b^2)*ln(a+b*cosh(d*x+c))/b^
3/d
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 697}

$$\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}-\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

```
[Out] -((a*Cosh[c + d*x])/(b^2*d)) + Cosh[c + d*x]^2/(2*b*d) + ((a^2 - b^2)*Log[a
 + b*Cosh[c + d*x]])/(b^3*d)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx,x,b\cosh(c+dx)\right)}{b^3d}$$
$$= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)dx,x,b\cosh(c+dx)\right)}{b^3d}$$
$$= -\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}+\frac{\left(a^2-b^2\right)\log(a+b\cosh(c+dx))}{b^3d}$$

Mathematica [A] time = 0.11, size = 55, normalized size = 0.90

$$\frac{4\left(a^2-b^2\right)\log(a+b\cosh(c+dx))-4ab\cosh(c+dx)+b^2\cosh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

```
[Out] (-4*a*b*Cosh[c + d*x] + b^2*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*Log[a + b*Cos
h[c + d*x]])/(4*b^3*d)
```

fricas [B] time = 0.64, size = 340, normalized size = 5.57

$$b^2\cosh(dx+c)^4+b^2\sinh(dx+c)^4-8\left(a^2-b^2\right)dx\cosh(dx+c)^2-4\,ab\cosh(dx+c)^3+4\left(b^2\cosh(dx+c)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 - b^2)*d*x*cosh(d*x
 + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)
^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 - b^2)*d*x - 6
*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 - b^2)*cosh(d*x + c)^2
+ 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2)*
log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2*cosh(
d*x + c)^3 - 4*(a^2 - b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2 - a*b)
*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh(d*x + c
) + b^3*d*sinh(d*x + c)^2)
```

giac [A] time = 0.15, size = 88, normalized size = 1.44

$$\frac{\frac{b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{b^2}+\frac{8\left(a^2-b^2\right)\log\left(\left|b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((b*(e^(d*x + c) + e^(-d*x - c))^2 - 4*a*(e^(d*x + c) + e^(-d*x - c)))/
b^2 + 8*(a^2 - b^2)*log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/b^3)/d
```

maple [B] time = 0.08, size = 415, normalized size = 6.80

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^{2}}{d\,b^{3}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{db}+\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{a}{d\,b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{2db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

```
[Out] -1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2+1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/2/
d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a+1/2/d/b/(ta
nh(1/2*d*x+1/2*c)-1)+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/d/b^2/(tanh(1/2*d*
x+1/2*c)+1)*a-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b^3*ln(tanh(1/2*d*x+1/2*c
)+1)*a^2+1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)+1/d/b^3/(a-b)*ln(tanh(1/2*d*x+1/2*
c)^2*a-tanh(1/2*d*x+1/2*c)^2*b-a-b)*a^3-1/d/b^2/(a-b)*ln(tanh(1/2*d*x+1/2*c
)^2*a-tanh(1/2*d*x+1/2*c)^2*b-a-b)*a^2-1/d/b/(a-b)*ln(tanh(1/2*d*x+1/2*c)^2
*a-tanh(1/2*d*x+1/2*c)^2*b-a-b)*a+1/d/(a-b)*ln(tanh(1/2*d*x+1/2*c)^2*a-tanh
(1/2*d*x+1/2*c)^2*b-a-b)
```

maxima [B] time = 0.33, size = 130, normalized size = 2.13

$$-\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^{2}d}+\frac{\left(a^{2}-b^{2}\right)(dx+c)}{b^{3}d}-\frac{4\,ae^{(-dx-c)}-be^{(-2\,dx-2\,c)}}{8\,b^{2}d}+\frac{\left(a^{2}-b^{2}\right)\log\left(2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}+\right.}{b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 - b^2)*(d*x + c)
/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 - b^2
)*log(2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) + b)/(b^3*d)
```

mupad [B] time = 1.06, size = 122, normalized size = 2.00

$$\frac{e^{-2\,c-2\,d\,x}}{8\,b\,d}+\frac{e^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{x\,\left(a^{2}-b^{2}\right)}{b^{3}}+\frac{\ln\left(b+2\,a\,e^{d\,x}\,e^{c}+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(a^{2}-b^{2}\right)}{b^{3}\,d}-\frac{a\,e^{-c-d\,x}}{2\,b^{2}\,d}-\frac{a\,e^{c+d\,x}}{2\,b^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*cosh(c + d*x)),x)
```

```
[Out] exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c + 2*d*x)/(8*b*d) - (x*(a^2 - b^2))/b^3
 + (log(b + 2*a*exp(d*x)*exp(c) + b*exp(2*c)*exp(2*d*x))*(a^2 - b^2))/(b^3*
d) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

```
[Out] Unintegrable(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] Defer[Int][Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))} dx = \int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))} dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

```
[Out] $Aborted
```

fricas [A] time = 1.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(dx+c)^3}{bx\cosh(dx+c)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^3/(b*x*cosh(d*x + c) + a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^3}{(b\cosh(dx+c)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/((b*cosh(d*x + c) + a)*x), x)
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(dx+c)}{x(a+b\cosh(dx+c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\mathrm{Ei}(2\,dx)\,e^{(2\,c)}}{4\,b}+\frac{a\mathrm{Ei}(-dx)\,e^{(-c)}}{2\,b^2}-\frac{\mathrm{Ei}(-2\,dx)\,e^{(-2\,c)}}{4\,b}-\frac{a\mathrm{Ei}(dx)\,e^{c}}{2\,b^2}+\frac{\left(a^2-b^2\right)\log(x)}{b^3}-\frac{1}{8}\int \frac{16\left(a^2b-b^3+\left(a^3e^c-ab^2e^c\right.\right.}{b^4xe^{(2\,dx+2\,c)}+2\,ab^3xe^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*Ei(2*d*x)*e^(2*c)/b + 1/2*a*Ei(-d*x)*e^(-c)/b^2 - 1/4*Ei(-2*d*x)*e^(-2*
c)/b - 1/2*a*Ei(d*x)*e^c/b^2 + (a^2 - b^2)*log(x)/b^3 - 1/8*integrate(16*(a
^2*b - b^3 + (a^3*e^c - a*b^2*e^c)*e^(d*x))/(b^4*x*e^(2*d*x + 2*c) + 2*a*b^
3*x*e^(d*x + c) + b^4*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sinh(c+d\,x)^3}{x\,\left(a+b\cosh(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))),x)
```

```
[Out] int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/x/(a+b*cosh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**3/(x*(a + b*cosh(c + d*x))), x)
```

3.239 $\int \cosh\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=54

$$\frac{x\cosh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}-\frac{bnx\sinh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}$$

```
[Out] x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)-b*n*x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5518}

$$\frac{x\cosh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}-\frac{bnx\sinh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/(
1 - b^2*n^2)
```

Rule 5518

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \cosh\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x\cosh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}-\frac{bnx\sinh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.76

$$\frac{x\left(bn\sinh\left(a+b\log\left(cx^n\right)\right)-\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{b^2n^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(-Cosh[a + b*Log[c*x^n]] + b*n*Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

fricas [A] time = 0.97, size = 44, normalized size = 0.81

$$\frac{bnx \sinh\left(bn\log(x) + b\log(c) + a\right) - x\cosh\left(bn\log(x) + b\log(c) + a\right)}{b^2n^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*sinh(b*n*log(x) + b*log(c) + a) - x*cosh(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 - 1)
```

giac [A] time = 0.12, size = 47, normalized size = 0.87

$$\frac{c^b x x^{bn} e^a}{2\,(bn+1)} - \frac{x e^{(-a)}}{2\,(bn-1)c^b x^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) - 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \cosh\left(a + b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n)),x)
```

```
[Out] int(cosh(a+b*ln(c*x^n)),x)
```

maxima [A] time = 0.34, size = 51, normalized size = 0.94

$$\frac{c^b x e^{\left(b\log(x^n)+a\right)}}{2\,(bn+1)} - \frac{x e^{(-a)}}{2\left(bc^b n - c^b\right)(x^n)^b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) - 1/2*x*e^(-a)/((b*c^b*n - c^b)*(x^n
)^b)
```

mupad [B] time = 1.00, size = 44, normalized size = 0.81

$$\frac{x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,b\,n+2}-\frac{x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,b\,n-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n)),x)
[Out] (x*exp(a)*(c*x^n)^b)/(2*b*n + 2) - (x*exp(-a))/((c*x^n)^b*(2*b*n - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh\left(a - \frac{\log\left(cx^n\right)}{n}\right) dx & \text{for}\ b = -\frac{1}{n} \\ \int \cosh\left(a + \frac{\log\left(cx^n\right)}{n}\right) dx & \text{for}\ b = \frac{1}{n} \\ \frac{bnx \sinh\left(a + bn\log\left(x\right) + b\log\left(c\right)\right)}{b^2n^2 - 1} - \frac{x \cosh\left(a + bn\log\left(x\right) + b\log\left(c\right)\right)}{b^2n^2 - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n)),x)
[Out] Piecewise((Integral(cosh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(co
sh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*sinh(a + b*n*log(x) + b*log(
c))/(b**2*n**2 - 1) - x*cosh(a + b*n*log(x) + b*log(c))/(b**2*n**2 - 1), Tr
ue))
```

3.240 $\int \cosh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=88

$$\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}$$

```
[Out] -2*b^2*n^2*x/(-4*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)-2*b*n*x*
cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(-4*b^2*n^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5520, 8}

$$\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (-2*b^2*n^2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^
2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n^2
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5520

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Cosh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (Dist[(b^2
*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Cosh[d*(a + b*Log[c*x^n])]^(
p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n}, x
] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \cosh^2(a+b\log(cx^n))\,dx=\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}-$$

$$=-\frac{2b^2n^2x}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+}{1-4b^2n^2}$$

Mathematica [A] time = 0.11, size = 56, normalized size = 0.64

$$\frac{x\left(2bn\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+4b^2n^2-1\right)}{8b^2n^2-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-1 + 4*b^2*n^2 - Cosh[2*(a + b*Log[c*x^n])] + 2*b*n*Sinh[2*(a + b*Log[c
*x^n])]))/(-2 + 8*b^2*n^2)
```

fricas [A] time = 0.53, size = 90, normalized size = 1.02

$$\frac{4\,bnx\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)-x\cosh\left(bn\log(x)+b\log(c)+a\right)^2-x\sin}{2\left(4\,b^2n^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) + a)
^2 + (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

giac [A] time = 0.17, size = 169, normalized size = 1.92

$$\frac{bc^{2\,b}nxx^{2\,bn}e^{(2\,a)}}{2\left(4\,b^2n^2-1\right)}+\frac{2\,b^2n^2x}{4\,b^2n^2-1}-\frac{c^{2\,b}xx^{2\,bn}e^{(2\,a)}}{4\left(4\,b^2n^2-1\right)}-\frac{bnxe^{(-2\,a)}}{2\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}-\frac{x}{2\left(4\,b^2n^2-1\right)}-\frac{xe^{(-2\,a)}}{4\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) + 2*b^2*n^2*x/(4*b^2*n^
2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-2*
a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \cosh^2\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^2,x)
```

maxima [A] time = 0.34, size = 67, normalized size = 0.76

$$\frac{c^{2b}xe^{\left(2b\log(x^n)+2a\right)}}{4(2bn+1)} + \frac{1}{2}x - \frac{xe^{(-2a)}}{4\left(2bc^{2b}n - c^{2b}\right)(x^n)^{2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) + 1/2*x - 1/4*x*e^(-2*a)/(
(2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

mupad [B] time = 1.01, size = 53, normalized size = 0.60

$$\frac{x}{2} - \frac{x\,e^{-2a}}{(c\,x^n)^{2b}\,(8\,b\,n-4)} + \frac{x\,e^{2a}\,(c\,x^n)^{2b}}{8\,b\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^2,x)
```

```
[Out] x/2 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^(2*b)
)/(8*b*n + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh^2\left(a - \frac{\log(cx^n)}{2n}\right)dx \\ \int \cosh^2\left(a + \frac{\log(cx^n)}{2n}\right)dx \\ -\frac{2b^2n^2x\sinh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2n^2-1} + \frac{2b^2n^2x\cosh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2n^2-1} + \frac{2bnx\sinh\left(a+bn\log(x)+b\log(c)\right)\cosh\left(a+bn\log(x)+b\log(\right.}{4b^2n^2-1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))), (
Integral(cosh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (-2*b**2*n**2
*x*sinh(a + b*n*log(x) + b*log(c))**2/(4*b**2*n**2 - 1) + 2*b**2*n**2*x*cos
h(a + b*n*log(x) + b*log(c))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*n*lo
g(x) + b*log(c))*cosh(a + b*n*log(x) + b*log(c))/(4*b**2*n**2 - 1) - x*cosh
(a + b*n*log(x) + b*log(c))**2/(4*b**2*n**2 - 1), True))
```

3.241 $\int \cosh^3\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=149

$$\frac{x\cosh^3\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}-\frac{3bnx\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^2\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}-\frac{6b^2n^2x\cosh\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4-10b^2n^2+1}+\frac{6b}{}$$

```
[Out] -6*b^2*n^2*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+x*cosh(a+b*ln(c*x
^n))^3/(-9*b^2*n^2+1)+6*b^3*n^3*x*sinh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2
+1)-3*b*n*x*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/(-9*b^2*n^2+1)
```

Rubi [A] time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5520, 5518}

$$\frac{6b^3n^3x\sinh\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4-10b^2n^2+1}+\frac{x\cosh^3\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}-\frac{6b^2n^2x\cosh\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4-10b^2n^2+1}-\frac{3bnx\sinh\left(a+b\log\left(cx\right.\right.}{1-}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^2*n^2*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (x*Cos
h[a + b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2) + (6*b^3*n^3*x*Sinh[a + b*Log[c*x^n]
])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh[a + b*Log[c*x^n]]^2*Sinh[a
+ b*Log[c*x^n]])/(1 - 9*b^2*n^2)
```

Rule 5518

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rule 5520

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Cosh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (Dist[(b^2
*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Cosh[d*(a + b*Log[c*x^n])]^(
p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n}, x
] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \cosh^3\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x\cosh^3\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2} - \frac{3bnx\cosh^2\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}$$
$$= -\frac{6b^2n^2x\cosh\left(a+b\log\left(cx^n\right)\right)}{1-10b^2n^2+9b^4n^4} + \frac{x\cosh^3\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2} + \frac{6b^3n^3x\sinh\left(a+\right.}{1-10b^2n^2+}$$

Mathematica [A] time = 0.56, size = 117, normalized size = 0.79

$$\frac{x\left(\left(3-27b^2n^2\right)\cosh\left(a+b\log\left(cx^n\right)\right)+\left(1-b^2n^2\right)\cosh\left(3\left(a+b\log\left(cx^n\right)\right)\right)+6bn\sinh\left(a+b\log\left(cx^n\right)\right)\left(\left(b^2n\right.\right.\right.}{36b^4n^4-40b^2n^2+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*((3 - 27*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (1 - b^2*n^2)*Cosh[3*(a + b*L
og[c*x^n])] + 6*b*n*(-1 + 5*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

fricas [A] time = 0.49, size = 199, normalized size = 1.34

$$-\frac{\left(b^2n^2-1\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^3+3\left(b^2n^2-1\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - 3*(b^3
*n^3 - b*n)*x*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(9*b^2*n^2 - 1)*x*cosh(
b*n*log(x) + b*log(c) + a) - 3*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log
(c) + a)^2 + (9*b^3*n^3 - b*n)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4*n
^4 - 10*b^2*n^2 + 1)
```

giac [B] time = 0.22, size = 665, normalized size = 4.46

$$\frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}+\frac{27\,b^3c^bn^3xx^{bn}e^a}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{27\,b^2c^bn^2xx^{bn}e^a}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{3\,bc^3}{8\left(9\,b^4n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8
*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)*n
^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^2*c^b*n^2*x*x^
(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a
)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) - 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 +
1)*c^(3*b)*x^(3*b*n)) - 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 +
 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b
^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) - 1/8*b^2*n^2*x*
e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c^b*x*x^(b*
n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) + 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c
^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n))
 + 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n))
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \cosh^3\left(a + b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^3,x)
```

maxima [A] time = 0.38, size = 115, normalized size = 0.77

$$\frac{c^{3\,b}xe^{\left(3\,b\log\left(x^n\right)+3\,a\right)}}{8\,(3\,bn+1)}+\frac{3\,c^bxe^{\left(b\log\left(x^n\right)+a\right)}}{8\,(bn+1)}-\frac{3\,xe^{\left(-b\log\left(x^n\right)-a\right)}}{8\left(bc^bn-c^b\right)}-\frac{xe^{(-3\,a)}}{8\left(3\,bc^{3\,b}n-c^{3\,b}\right)\left(x^n\right)^{3\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) + 3/8*c^b*x*e^(b*log(x^n)
+ a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) - 1/8*x*e^(-3*a)
/((3*b*c^(3*b)*n - c^(3*b))*(x^n)^(3*b))
```

mupad [B] time = 1.05, size = 94, normalized size = 0.63

$$\frac{x\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{24\,b\,n+8}-\frac{x\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(24\,b\,n-8\right)}-\frac{3\,x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(8\,b\,n-8\right)}+\frac{3\,x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^3,x)
```

```
[Out] (x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (x*exp(-3*a))/((c*x^n)^(3*b)*(24*
b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b*n - 8)) + (3*x*exp(a)*(c*x^n)^b)/
(8*b*n + 8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh^3\left(a - \frac{\log(cx^n)}{n}\right) dx \\ \int \cosh^3\left(a - \frac{\log(cx^n)}{3n}\right) dx \\ \int \cosh^3\left(a + \frac{\log(cx^n)}{3n}\right) dx \\ \int \cosh^3\left(a + \frac{\log(cx^n)}{n}\right) dx \\ -\frac{6b^3n^3x\sinh^3\left(a+bn\log(x)+b\log(c)\right)}{9b^4n^4-10b^2n^2+1} + \frac{9b^3n^3x\sinh\left(a+bn\log(x)+b\log(c)\right)\cosh^2\left(a+bn\log(x)+b\log(c)\right)}{9b^4n^4-10b^2n^2+1} + \frac{6b^2n^2x\sinh^2\left(a+bn\log(x)+b\log\right.}{9b^4n^4-} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(cosh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integral
(cosh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(cosh(a + l
og(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(cosh(a + log(c*x**n)/n
)**3, x), Eq(b, 1/n)), (-6*b**3*n**3*x*sinh(a + b*n*log(x) + b*log(c))**3/(
9*b**4*n**4 - 10*b**2*n**2 + 1) + 9*b**3*n**3*x*sinh(a + b*n*log(x) + b*log
(c))*cosh(a + b*n*log(x) + b*log(c))**2/(9*b**4*n**4 - 10*b**2*n**2 + 1) +
6*b**2*n**2*x*sinh(a + b*n*log(x) + b*log(c))**2*cosh(a + b*n*log(x) + b*lo
g(c))/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 7*b**2*n**2*x*cosh(a + b*n*log(x)
+ b*log(c))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 3*b*n*x*sinh(a + b*n*log(
x) + b*log(c))*cosh(a + b*n*log(x) + b*log(c))**2/(9*b**4*n**4 - 10*b**2*n*
*2 + 1) + x*cosh(a + b*n*log(x) + b*log(c))**3/(9*b**4*n**4 - 10*b**2*n**2
+ 1), True))
```

3.242 $\int \cosh^4\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=191

$$\frac{x\cosh^4\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}-\frac{4bnx\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^3\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}-\frac{12b^2n^2x\cosh^2\left(a+b\log\left(cx^n\right)\right)}{64b^4n^4-20b^2n^2+1}+$$

```
[Out] 24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-12*b^2*n^2*x*cosh(a+b*ln(c*x^n))^2/(
64*b^4*n^4-20*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)+24*b^3*n^3
*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)-4*b*n*
x*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/(-16*b^2*n^2+1)
```

Rubi [A] time = 0.05, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5520, 8}

$$-\frac{12b^2n^2x\cosh^2\left(a+b\log\left(cx^n\right)\right)}{64b^4n^4-20b^2n^2+1}+\frac{x\cosh^4\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}-\frac{4bnx\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^3\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (12*b^2*n^2*x*Cosh[a + b*Log
[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (x*Cosh[a + b*Log[c*x^n]]^4)/(1
 - 16*b^2*n^2) + (24*b^3*n^3*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh[a + b*Log[c*x^n]]^3*Sinh[a
 + b*Log[c*x^n]])/(1 - 16*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5520

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Cosh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (Dist[(b^2
*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Cosh[d*(a + b*Log[c*x^n])]^(
p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*Sinh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n}, x
] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \cosh^4\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x\cosh^4\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2} - \frac{4bnx\cosh^3\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}$$
$$= -\frac{12b^2n^2x\cosh^2\left(a+b\log\left(cx^n\right)\right)}{1-20b^2n^2+64b^4n^4} + \frac{x\cosh^4\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2} + \frac{24b^3n^3x\cosh\left(\right.}{}$$
$$= \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4} - \frac{12b^2n^2x\cosh^2\left(a+b\log\left(cx^n\right)\right)}{1-20b^2n^2+64b^4n^4} + \frac{x\cosh^4\left(a+b\log\left(c\right.\right.}{1-16b^2n^2}$$

Mathematica [A] time = 0.45, size = 167, normalized size = 0.87

$$\frac{x\left(128b^3n^3\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+16b^3n^3\sinh\left(4\left(a+b\log\left(cx^n\right)\right)\right)+\left(4-64b^2n^2\right)\cosh\left(2\left(a+b\log\left(cx^n\right.\right.\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (4 - 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^n
])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] - 8*b*n*Sinh[2*(a + b*Log[
c*x^n])] + 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*Log
[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2 + 64
*b^4*n^4))
```

fricas [A] time = 0.56, size = 293, normalized size = 1.53

$$-\frac{\left(4\,b^2n^2-1\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^4-16\left(4\,b^3n^3-bn\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] -1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3 -
 b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 +
 (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*(16*b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a)^
2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^3 + (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) +
a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

giac [B] time = 0.24, size = 777, normalized size = 4.07

$$\frac{b^3c^{4\,b}n^3xx^{4\,bn}e^{(4\,a)}}{64\,b^4n^4-20\,b^2n^2+1}+\frac{8\,b^3c^{2\,b}n^3xx^{2\,bn}e^{(2\,a)}}{64\,b^4n^4-20\,b^2n^2+1}+\frac{24\,b^4n^4x}{64\,b^4n^4-20\,b^2n^2+1}-\frac{b^2c^{4\,b}n^2xx^{4\,bn}e^{(4\,a)}}{4\left(64\,b^4n^4-20\,b^2n^2+1\right)}-\frac{4\,b^2c^{2\,b}n^2xx^{2\,b}}{64\,b^4n^4-20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 8*b^3*c
^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4*x
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)/(6
4*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64*b^4
*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 -
20*b^2*n^2 + 1) - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*
n^2 + 1) - 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2
*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n
)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x*x^(4*b*n
)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(
64*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(4*b)*x^(4*b*n)) + 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 +
 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*
c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*x*e^(-2*a)/(
(64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e^(-4*a)/((64*b^4
*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \cosh^4\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(cosh(a+b*ln(c*x^n))^4,x)
```

maxima [A] time = 0.40, size = 129, normalized size = 0.68

$$\frac{c^{4\,b}xe^{\left(4\,b\log\left(x^n\right)+4\,a\right)}}{16\,(4\,bn+1)}+\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+2\,a\right)}}{4\,(2\,bn+1)}+\frac{3}{8}\,x-\frac{xe^{\left(-2\,b\log\left(x^n\right)-2\,a\right)}}{4\left(2\,bc^{2\,b}n-c^{2\,b}\right)}-\frac{xe^{(-4\,a)}}{16\left(4\,bc^{4\,b}n-c^{4\,b}\right)\left(x^n\right)^{4\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) + 1/4*c^(2*b)*x*e^(2*b*lo
g(x^n) + 2*a)/(2*b*n + 1) + 3/8*x - 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^(2
*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b))

mupad [B] time = 1.04, size = 102, normalized size = 0.53

$$\frac{3\,x}{8}-\frac{x\,{\mathrm{e}}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)}+\frac{x\,{\mathrm{e}}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}-\frac{x\,{\mathrm{e}}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(64\,b\,n-16\right)}+\frac{x\,{\mathrm{e}}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{64\,b\,n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b*log(c*x^n))^4,x)

[Out] (3*x)/8 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^(
2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(4*
a)*(c*x^n)^(4*b))/(64*b*n + 16)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \cosh^4\left(a-\frac{\log(cx^n)}{2n}\right)dx \\ \int \cosh^4\left(a-\frac{\log(cx^n)}{4n}\right)dx \\ \int \cosh^4\left(a+\frac{\log(cx^n)}{4n}\right)dx \\ \int \cosh^4\left(a+\frac{\log(cx^n)}{2n}\right)dx \\ \frac{24b^4n^4x\sinh^4\left(a+bn\log(x)+b\log(c)\right)}{64b^4n^4-20b^2n^2+1}-\frac{48b^4n^4x\sinh^2\left(a+bn\log(x)+b\log(c)\right)\cosh^2\left(a+bn\log(x)+b\log(c)\right)}{64b^4n^4-20b^2n^2+1}+\frac{24b^4n^4x\cosh^4\left(a+bn\log(x)+\right.}{64b^4n^4-20b^2n^2+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(a+b*ln(c*x**n))**4,x)

[Out] Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))), (
Integral(cosh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(co
sh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(cosh(a + log(c
*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*n*log(x)
+ b*log(c))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a +
b*n*log(x) + b*log(c))**2*cosh(a + b*n*log(x) + b*log(c))**2/(64*b**4*n**4
- 20*b**2*n**2 + 1) + 24*b**4*n**4*x*cosh(a + b*n*log(x) + b*log(c))**4/(64
*b**4*n**4 - 20*b**2*n**2 + 1) - 24*b**3*n**3*x*sinh(a + b*n*log(x) + b*log
(c))**3*cosh(a + b*n*log(x) + b*log(c))/(64*b**4*n**4 - 20*b**2*n**2 + 1) +
 40*b**3*n**3*x*sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x) + b*log

```
(c))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**2*n**2*x*sinh(a + b*n*log
(x) + b*log(c))**2*cosh(a + b*n*log(x) + b*log(c))**2/(64*b**4*n**4 - 20*b*
*2*n**2 + 1) - 16*b**2*n**2*x*cosh(a + b*n*log(x) + b*log(c))**4/(64*b**4*n
**4 - 20*b**2*n**2 + 1) - 4*b*n*x*sinh(a + b*n*log(x) + b*log(c))*cosh(a +
b*n*log(x) + b*log(c))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) + x*cosh(a + b*
n*log(x) + b*log(c))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1), True))
```

3.243 $\int x^m \cosh\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=73

$$\frac{(m+1)x^{m+1}\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}$$

```
[Out] (1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)-b*n*x^(1+m)*sinh(a+b*ln
(c*x^n))/((1+m)^2-b^2*n^2)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5528}

$$\frac{(m+1)x^{m+1}\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2) - (b*n*x^(
1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Rule 5528

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m\cosh\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{(1+m)x^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-b^2n^2}-\frac{bnx^{1+m}\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-b^2n^2}$$

Mathematica [A] time = 0.14, size = 54, normalized size = 0.74

$$\frac{x^{m+1}\left((m+1)\cosh\left(a+b\log\left(cx^n\right)\right)-bn\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{(-bn+m+1)(bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]],x]
```

```
[Out] (x^(1 + m)*((1 + m)*Cosh[a + b*Log[c*x^n]] - b*n*Sinh[a + b*Log[c*x^n]]))/(
(1 + m - b*n)*(1 + m + b*n))
```

fricas [A] time = 0.47, size = 99, normalized size = 1.36

$$-\frac{(m+1)x\cosh\left(bn\log(x)+b\log(c)+a\right)\cosh\left(m\log(x)\right)+(m+1)x\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(m\log\right.}{b^2n^2-m^2-2m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -((m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m + 1)*x*cosh
(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)) - (b*n*x*cosh(m*log(x)) + b*n*x*
sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1)
```

giac [B] time = 0.16, size = 235, normalized size = 3.22

$$\frac{bc^bnxx^{bn}x^me^a}{2\left(b^2n^2-m^2-2m-1\right)}-\frac{c^bmxx^{bn}x^me^a}{2\left(b^2n^2-m^2-2m-1\right)}-\frac{c^bxx^{bn}x^me^a}{2\left(b^2n^2-m^2-2m-1\right)}-\frac{bnxx^me^{(-a)}}{2\left(b^2n^2-m^2-2m-1\right)c^bx^{bn}}-\frac{}{2\left(b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b*
n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2 -
 m^2 - 2*m - 1) - 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*
n)) - 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) - 1/2*x*x^
m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^m\cosh\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n)),x)
```

maxima [A] time = 0.36, size = 64, normalized size = 0.88

$$\frac{c^{b}xe^{\left(b\log\left(x^{n}\right)+m\log\left(x\right)+a\right)}}{2\left(bn+m+1\right)}-\frac{xe^{\left(-b\log\left(x^{n}\right)+m\log\left(x\right)-a\right)}}{2\left(bc^{b}n-c^{b}(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 1/2*x*e^(-b*log(x^n
) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

mupad [B] time = 1.05, size = 55, normalized size = 0.75

$$\frac{x\,x^{m}\,\mathrm{e}^{-a}}{{\left(c\,x^{n}\right)}^{b}\,\left(2\,m-2\,b\,n+2\right)}+\frac{x\,x^{m}\,\mathrm{e}^{a}\,{\left(c\,x^{n}\right)}^{b}}{2\,m+2\,b\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n)),x)
```

```
[Out] (x*x^m*exp(-a))/((c*x^n)^b*(2*m - 2*b*n + 2)) + (x*x^m*exp(a)*(c*x^n)^b)/(2
*m + 2*b*n + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log\left(x\right)\cosh\left(a\right) & \text{for}\ b=0\wedge m=-1 \\ \int x^{m}\cosh\left(a-\frac{m\log\left(cx^{n}\right)}{n}-\frac{\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\ b=-\frac{m+1}{n} \\ \int x^{m}\cosh\left(a+\frac{m\log\left(cx^{n}\right)}{n}+\frac{\log\left(cx^{n}\right)}{n}\right)dx & \text{for}\ b=\frac{m+1}{n} \\ \frac{bnxx^{m}\sinh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{b^{2}n^{2}-m^{2}-2m-1}-\frac{mxx^{m}\cosh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{b^{2}n^{2}-m^{2}-2m-1}-\frac{xx^{m}\cosh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{b^{2}n^{2}-m^{2}-2m-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)*cosh(a), Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(a - m
*log(c*x**n)/n - log(c*x**n)/n), x), Eq(b, -(m + 1)/n)), (Integral(x**m*cos
h(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*x**m*
sinh(a + b*n*log(x) + b*log(c))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*cos
h(a + b*n*log(x) + b*log(c))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*cosh(a +
 b*n*log(x) + b*log(c))/(b**2*n**2 - m**2 - 2*m - 1), True))
```

3.244 $\int x^m \cosh^2\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=120

$$\frac{(m+1)x^{m+1}\cosh^2\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-\right.}$$

```
[Out] -2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^
n))^2/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x
^n))/((1+m)^2-4*b^2*n^2)
```

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5530, 30}

$$\frac{(m+1)x^{m+1}\cosh^2\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 +
m)*Cosh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 4*b^2*n^2) - (2*b*n*x^(1 + m)*Cos
h[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5530

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (Dist[(b^2*d^2*n^2*p*(p - 1))/(b^2
*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2),
x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*Cosh[d*(a +
 b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{a
, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2, 0
]
```

Rubi steps

$$\int x^m \cosh^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{(1+m)x^{1+m}\cosh^2\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-4b^2n^2} - \frac{2bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(\right.}{(1+m)^2-4b^2n^2}$$
$$= -\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} + \frac{(1+m)x^{1+m}\cosh^2\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-4b^2n^2} - \frac{2bnx^{}}{}$$

Mathematica [A] time = 0.30, size = 87, normalized size = 0.72

$$\frac{x^{m+1}\left(-2b(m+1)n\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+(m+1)^2\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-4b^2n^2+m^2+2m+1\right)}{2(m+1)(-2bn+m+1)(2bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(1 + 2*m + m^2 - 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n])
] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)*(
1 + m + 2*b*n))
```

fricas [A] time = 0.62, size = 250, normalized size = 2.08

$$\frac{\left(m^2+2\,m+1\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^2\cosh\left(m\log(x)\right)-\left(4\,b^2n^2-m^2-2\,m-1\right)x\cosh\left(m\log(x)\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (
4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*log
(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2
- 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*m +
b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*
log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (4*b^2
*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2 + 3*m^2
 + 3*m + 1)
```

giac [B] time = 0.22, size = 759, normalized size = 6.32

$$\frac{bc^{2\,b}mnxx^{2\,bn}x^me^{(2\,a)}}{2\left(4\,b^2mn^2+4\,b^2n^2-m^3-3\,m^2-3\,m-1\right)}-\frac{c^{2\,b}m^2xx^{2\,bn}x^me^{(2\,a)}}{4\left(4\,b^2mn^2+4\,b^2n^2-m^3-3\,m^2-3\,m-1\right)}+\frac{bc^{2\,b}nx}{2\left(4\,b^2mn^2+4\,b^2n^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4
*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)
/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 2*b^2*n^2*x*x^m/(4*b^2
*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n)*x^m
*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*x*
x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1
/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*b*m*n*
x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x
^(2*b*n)) - m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4
*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2
*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3
*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m
^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^
3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)/((4*b^2*m*n^2
+ 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int x^{m} \left(\cosh^{2}\left(a+b\ln\left(c\,x^{n}\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^2,x)
```

maxima [A] time = 0.35, size = 87, normalized size = 0.72

$$\frac{c^{2\,b} x e^{\left(2\,b\log\left(x^{n}\right)+m\log\left(x\right)+2\,a\right)}}{4\,\left(2\,bn+m+1\right)}-\frac{x e^{\left(-2\,b\log\left(x^{n}\right)+m\log\left(x\right)-2\,a\right)}}{4\,\left(2\,bc^{2\,b}n-c^{2\,b}(m+1)\right)}+\frac{x^{m+1}}{2\,\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(
-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) + 1/2*x^(
m + 1)/(m + 1)
```

mupad [B] time = 1.09, size = 73, normalized size = 0.61

$$\frac{x\,x^m}{2\,m+2}+\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}+\frac{x\,x^m\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*x^m)/(2*m + 2) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) + (
x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log(x)\cosh^2(a) \\ \int x^m\cosh^2\left(a-\frac{m\log(cx^n)}{2n}-\frac{\log(cx^n)}{2n}\right)dx \\ \int x^m\cosh^2\left(a+\frac{m\log(cx^n)}{2n}+\frac{\log(cx^n)}{2n}\right)dx \\ \int \frac{\cosh^2\left(a+b\log(cx^n)\right)}{x}\,dx \\ -\frac{2b^2n^2xx^m\sinh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}+\frac{2b^2n^2xx^m\cosh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}+\frac{2bmnxx^m\sinh\left(a+bn\log(x)+b\log(c)\right)\cosh\left(a+bn\log\right.}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(a
- m*log(c*x**n)/(2*n) - log(c*x**n)/(2*n))**2, x), Eq(b, -(m + 1)/(2*n))),
(Integral(x**m*cosh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq
(b, (m + 1)/(2*n))), (Integral(cosh(a + b*log(c*x**n))**2/x, x), Eq(m, -1))
, (-2*b**2*n**2*x*x**m*sinh(a + b*n*log(x) + b*log(c))**2/(4*b**2*m*n**2 +
4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b**2*n**2*x*x**m*cosh(a + b*n*lo
g(x) + b*log(c))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1)
 + 2*b*m*n*x*x**m*sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x) + b*l
og(c))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x*
*m*sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x) + b*log(c))/(4*b**2*
m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - m**2*x*x**m*cosh(a + b*n*
log(x) + b*log(c))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m -
1) - 2*m*x*x**m*cosh(a + b*n*log(x) + b*log(c))**2/(4*b**2*m*n**2 + 4*b**2*
n**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*cosh(a + b*n*log(x) + b*log(c))**2
/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1), True))
```

3.245 $\int x^m \cosh^3\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=203

$$\frac{(m+1)x^{m+1}\cosh^3\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-9b^2n^2}-\frac{6b^2(m+1)n^2x^{m+1}\cosh\left(a+b\log\left(cx^n\right)\right)}{\left((m+1)^2-9b^2n^2\right)\left((m+1)^2-b^2n^2\right)}-\frac{3bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)\mathrm{c}}{(m+1)^2-9b^2}$$

```
[Out] -6*b^2*(1+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b
^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)+6*b^3*n^3*x
^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^2)-3*b*n*x^(1
+m)*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)
```

Rubi [A] time = 0.08, antiderivative size = 197, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5530, 5528}

$$\frac{6b^3n^3x^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)}{-10b^2(m+1)^2n^2+9b^4n^4+(m+1)^4}+\frac{(m+1)x^{m+1}\cosh^3\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-9b^2n^2}-\frac{6b^2(m+1)n^2x^{m+1}\cosh\left(a+b\log\left(c\right.\right.}{-10b^2(m+1)^2n^2+9b^4n^4+(m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^4 - 10*b^2*(
1 + m)^2*n^2 + 9*b^4*n^4) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3)/((
1 + m)^2 - 9*b^2*n^2) + (6*b^3*n^3*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 +
m)^4 - 10*b^2*(1 + m)^2*n^2 + 9*b^4*n^4) - (3*b*n*x^(1 + m)*Cosh[a + b*Log[
c*x^n]]^2*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 9*b^2*n^2)
```

Rule 5528

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rule 5530

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (Dist[(b^2*d^2*n^2*p*(p - 1))/(b^2
*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2),
x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*Cosh[d*(a +
 b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{a
```

```
, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2, 0
]
```

Rubi steps

$$\int x^m \cosh^3\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{(1+m)x^{1+m}\cosh^3\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-9b^2n^2} - \frac{3bnx^{1+m}\cosh^2\left(a+b\log\left(cx^n\right)\right)\sinh}{(1+m)^2-9b^2n^2}$$

$$= -\frac{6b^2(1+m)n^2x^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4} + \frac{(1+m)x^{1+m}\cosh^3\left(a+b\log(\right.}{(1+m)^2-9b^2n^2}$$

Mathematica [A] time = 1.48, size = 292, normalized size = 1.44

$$\frac{1}{4}x^{m+1}\left(\frac{3\sinh(bn\log(x))\left((m+1)\sinh\left(a+b\log\left(cx^n\right)-bn\log(x)\right)-bn\cosh\left(a+b\log\left(cx^n\right)-bn\log(x)\right)\right)}{(-bn+m+1)(bn+m+1)}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((3*Sinh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]])
 + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*
n)) + (3*Cosh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] - b*
n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) + (Si
nh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*
Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*b*n))
 + (Cosh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] - 3
*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*
b*n))))/4
```

fricas [B] time = 0.52, size = 584, normalized size = 2.88

$$\frac{\left(m^3-\left(b^2m+b^2\right)n^2+3\,m^2+3\,m+1\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^3\cosh\left(m\log(x)\right)+3\left(m^3-9\left(b^2m+b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^3*cosh(m*log(x)) + 3*(m^3 - 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
```

```
x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + 3*((b^3*n^3 - (b*m^2 + 2
*b*m + b)*n)*x*cosh(m*log(x)) + (b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*sinh(m*
log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*((m^3 - (b^2*m + b^2)*n^2 +
3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m^3 -
(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh
(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 3*(3*(b^3*n^3 - (b*m^2 + 2*
b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (9*b^3*n^3
 - (b*m^2 + 2*b*m + b)*n)*x*cosh(m*log(x)) + (3*(b^3*n^3 - (b*m^2 + 2*b*m +
 b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (9*b^3*n^3 - (b*m^2 + 2*b*m +
b)*n)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^3 - (b^2*m +
 b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(m^3 -
 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a))*
sinh(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m + b^2)*n^2
 + 6*m^2 + 4*m + 1)
```

giac [B] time = 0.29, size = 3225, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^3*c^b*n^
3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x^m*e
^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^
3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10
*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) -
 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b
)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*m^2*n*x*x^(b*n)*x^m
*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2
*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8
*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^(3*b)*m*n*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^b*m^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
+ 1) - 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2
*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m^2*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*x^m*e^(3*a)
```

/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^3*n^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/8*b^3*n^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*c^b*m^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*m*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 1/8*b^2*m*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*c^b*m*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8*c^(3*b)*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*m^2*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 27/8*b^2*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/8*b*m^2*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 1/8*b^2*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c^b*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*m^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/4*b*m*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*m^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/4*b*m*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/8*b*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/8*m^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*b*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/8*m*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2

```
- 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b
*n))
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int x^{m} \left(\cosh^{3}\left(a+b\ln\left(c\,x^{n}\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^3,x)
```

maxima [A] time = 0.92, size = 138, normalized size = 0.68

$$\frac{c^{3\,b} x e^{\left(3\,b \log\left(x^{n}\right)+m \log\left(x\right)+3\,a\right)}}{8\,(3\,bn+m+1)}+\frac{3\,c^{b} x e^{\left(b \log\left(x^{n}\right)+m \log\left(x\right)+a\right)}}{8\,(bn+m+1)}-\frac{3\,x e^{\left(-b \log\left(x^{n}\right)+m \log\left(x\right)-a\right)}}{8\,\left(bc^{b}n-c^{b}(m+1)\right)}-\frac{x e^{\left(-3\,b \log\left(x^{n}\right)+m \log\left(x\right)-3\,a\right)}}{8\,\left(3\,bc^{3\,b}n-c^{3\,b}(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) + 3/8*c^b*x
*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*log
(x) - a)/(b*c^b*n - c^b*(m + 1)) - 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3*a)
/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

mupad [B] time = 1.18, size = 117, normalized size = 0.58

$$\frac{3\,x\,x^{m}\,{\mathrm{e}}^{-a}}{{\left(c\,x^{n}\right)}^{b}\,\left(8\,m-8\,b\,n+8\right)}+\frac{x\,x^{m}\,{\mathrm{e}}^{-3\,a}}{{\left(c\,x^{n}\right)}^{3\,b}\,\left(8\,m-24\,b\,n+8\right)}+\frac{x\,x^{m}\,{\mathrm{e}}^{3\,a}\,{\left(c\,x^{n}\right)}^{3\,b}}{8\,m+24\,b\,n+8}+\frac{3\,x\,x^{m}\,{\mathrm{e}}^{a}\,{\left(c\,x^{n}\right)}^{b}}{8\,m+8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n))^3,x)
```

```
[Out] (3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) + (x*x^m*exp(-3*a))/((c*x^n
)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*n
+ 8) + (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

3.246 $\int x^m \cosh^4\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=266

$$\frac{(m+1)x^{m+1}\cosh^4\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-16b^2n^2}-\frac{12b^2(m+1)n^2x^{m+1}\cosh^2\left(a+b\log\left(cx^n\right)\right)}{\left((m+1)^2-16b^2n^2\right)\left((m+1)^2-4b^2n^2\right)}-\frac{4bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right.\right.}{(m+1)^2-}$$

```
[Out] 24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-12*b^2*(1
+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))^2/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n
^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^4/((1+m)^2-16*b^2*n^2)+24*b^3*n^3*x^(
1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((1+m)^2-
4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/((1+m)^2
-16*b^2*n^2)
```

Rubi [A] time = 0.13, antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5530, 30}

$$\frac{(m+1)x^{m+1}\cosh^4\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-16b^2n^2}-\frac{12b^2(m+1)n^2x^{m+1}\cosh^2\left(a+b\log\left(cx^n\right)\right)}{-20b^2(m+1)^2n^2+64b^4n^4+(m+1)^4}-\frac{4bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right.\right.}{(m+1)^2-}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (12*b^2*(1 + m)*n^2*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2)/((1 + m)^4
 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c
*x^n]]^4)/((1 + m)^2 - 16*b^2*n^2) + (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c
*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n
^4) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/((1
 + m)^2 - 16*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5530

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (Dist[(b^2*d^2*n^2*p*(p - 1))/(b^2
*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2),
```

```
x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]*Cosh[d*(a +
 b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{a
, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2, 0
]
```

Rubi steps

$$\int x^m \cosh^4\left(a+b\log\left(cx^n\right)\right)dx = \frac{(1+m)x^{1+m}\cosh^4\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-16b^2n^2} - \frac{4bnx^{1+m}\cosh^3\left(a+b\log\left(cx^n\right)\right)\sinh\left(\right.}{(1+m)^2-16b^2n^2}$$

$$= -\frac{12b^2(1+m)n^2x^{1+m}\cosh^2\left(a+b\log\left(cx^n\right)\right)}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} + \frac{(1+m)x^{1+m}\cosh^4\left(a+b\log\right.}{(1+m)^2-16b^2n^2}$$

$$= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4-20b^2(1+m)^2n^2+64b^4n^4\right)} - \frac{12b^2(1+m)n^2x^{1+m}\cosh^2\left(a\right.}{(1+m)^4-20b^2(1+m)^2n}$$

Mathematica [A] time = 3.44, size = 311, normalized size = 1.17

$$\frac{1}{8}x^{m+1}\left(\frac{4\sinh(2bn\log(x))\left((m+1)\sinh\left(2\left(a+b\log\left(cx^n\right)-bn\log(x)\right)\right)-2bn\cosh\left(2\left(a+b\log\left(cx^n\right)-bn\log\right.\right.\right.}{(-2bn+m+1)(2bn+m+1)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) + (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[x
] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1
+ m - 2*b*n)*(1 + m + 2*b*n)) + (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a -
b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]
))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cosh[4*(
a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*Log[c*
x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1 + m)*C
osh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[x] + b*L
og[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

fricas [B] time = 0.45, size = 1123, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*co
sh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) + 4*(m^4 + 4*m^3 - 16*(b^2*m
^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a
)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^
2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*
n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^4
+ 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m
^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b
*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^4 + 4*m^3 -
 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2*cosh(m*log(x)) + 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)
*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 - 4*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 +
2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b^3*m + b^3)*n^3 -
 (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3*cosh(
m*log(x)) + (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + (16*(b^3*m
 + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^4 + 4*m^3 - 4*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m
 + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 2
0*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sinh(m*log(x)))/(m^5
+ 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 - 20*(b^2*m^3 + 3*b^2*m^2 + 3*b^2*m
 + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

giac [B] time = 0.41, size = 6880, normalized size = 25.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b
^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*
m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*m^2*n
^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n
```

^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*
n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2
*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*
m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4
+ 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(4*b)*m*n^2*x*x^(4*b
*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^
2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) -
1/2*b*c^(2*b)*m^3*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20
*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) - 8*b^2*c^(2*b)*m*n^2*x*x^(2*b*n)*x^m*e^(2*a)/(64
*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*
n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*m^4*x*
x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) - 3/4*b*c^(4*b)*m^2*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*c^(2*b)*m
^4*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) - 3/2*b*c^(2*b)*m^2*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b
^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*
b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*m
^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
 m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4
*c^(4*b)*m^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1) - 3/4*b*c^(4*b)*m*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^
4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m
^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + c^(2*b)*m^3*x*x^(2*b*n)*x^m*
e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*b*c^
(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n
^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) - 8*b^3*m*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +

```
 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - b^3*m*n^3*x*x^m*e^(-4*a)/(
(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2
*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n))
 - 15*b^2*m*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) + 3/8*c^(4*b)*m^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*c^(2*b)*m^2*x*x^(
2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
 - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10
*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*m^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 8*b^3*n^3*x*x^m
*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m
^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)
*x^(2*b*n)) - 1/4*b^2*m^2*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - b^3*n^3*x*x^m*e^(-4*a)/((6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m
*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) +
 3/8*m^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1
5/2*b^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*
n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
+ 1/4*c^(4*b)*m*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2
*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^
3 + 10*m^2 + 5*m + 1) + c^(2*b)*m*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 6
4*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 -
20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/2*b*m^3*n*x*x^m*e^(-2*a)/((64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 8*
b^2*m*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*
b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5
*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*m^3*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b^2*m*n^2*x
*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(
4*b)*x^(4*b*n)) + 3/2*m^3*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 1/16*c^(4*b)*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4
```

*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^
2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*c^(2*b)*x*x^(2*b*n)*x^m*e^(2*a)/(6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m
*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*m^4*x*x^m*e^(-
2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2
*b*n)) + 3/2*b*m^2*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 4*b^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m
*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 +
5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*m
^4*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
*c^(4*b)*x^(4*b*n)) + 3/4*b*m^2*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*b^2*n^2*x*x^m*e^(-
4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4
*b*n)) + 9/4*m^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2
*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m
+ 1) + m^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1)*c^(2*b)*x^(2*b*n)) + 3/2*b*m*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*
b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20
*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*m^3*x*x^m*e^
(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
- 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^
(4*b*n)) + 3/4*b*m*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/2*m*x*x^m/(64*b^4*m*n^4 + 64*b^4*
n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2
*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*m^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 +
 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
- 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/2*b*n*x*x^
m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b
)*x^(2*b*n)) + 3/8*m^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*
m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3
 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 1/4*b*n*x*x^m*e^(-4*a)/((64*b^4*m
*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 +
5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x*
x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60
*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + m*x*x^m*e^(-
2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2

```
*b*n)) + 1/4*m*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
- 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^
2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 1/4*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^
4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b
^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*x^m*e^(-4*a
)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*
n))
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int x^{m} \left(\cosh^{4}\left(a + b \ln\left(c x^{n}\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

maxima [A] time = 0.48, size = 161, normalized size = 0.61

$$\frac{c^{4 b} x e^{\left(4 b \log\left(x^{n}\right)+m \log\left(x\right)+4 a\right)}}{16 \left(4 b n+m+1\right)}+\frac{c^{2 b} x e^{\left(2 b \log\left(x^{n}\right)+m \log\left(x\right)+2 a\right)}}{4 \left(2 b n+m+1\right)}-\frac{x e^{\left(-2 b \log\left(x^{n}\right)+m \log\left(x\right)-2 a\right)}}{4 \left(2 b c^{2 b} n-c^{2 b} (m+1)\right)}-\frac{x e^{\left(-4 b \log\left(x^{n}\right)+m \log\left(x\right)-4 a\right)}}{16 \left(4 b c^{4 b} n-c^{4 b} (m+1)\right)}+\frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) + 1/4*c^(2
*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(-2*b*log
(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4*b*
log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m + 1
)/(m + 1)
```

mupad [B] time = 1.19, size = 134, normalized size = 0.50

$$\frac{3 x x^{m}}{8 m+8}+\frac{x x^{m} e^{-2 a}}{\left(c x^{n}\right)^{2 b} \left(4 m-8 b n+4\right)}+\frac{x x^{m} e^{2 a} \left(c x^{n}\right)^{2 b}}{4 m+8 b n+4}+\frac{x x^{m} e^{-4 a}}{\left(c x^{n}\right)^{4 b} \left(16 m-64 b n+16\right)}+\frac{x x^{m} e^{4 a} \left(c x^{n}\right)^{4 b}}{16 m+64 b n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x*x^m)/(8*m + 8) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) +
 (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c*x
```

```
^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m + 64
*b*n + 16)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

3.247 $\int \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2637}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.06

$$\frac{\sinh(a)\cosh\left(b\log\left(cx^n\right)\right)}{bn}+\frac{\cosh(a)\sinh\left(b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] (Cosh[b*Log[c*x^n]]*Sinh[a])/(b*n) + (Cosh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

fricas [A] time = 0.48, size = 19, normalized size = 1.06

$$\frac{\sinh\left(bn\log(x)+b\log(c)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] sinh(b*n*log(x) + b*log(c) + a)/(b*n)
```

giac [B] time = 0.12, size = 42, normalized size = 2.33

$$\frac{\left(c^{2\,b}x^{bn}e^{(2\,a)}-\frac{1}{x^{bn}}\right)e^{(-a)}}{2\,bc^{b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] 1/2*(c^(2*b)*x^(b*n)*e^(2*a) - 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

maple [A] time = 0.05, size = 19, normalized size = 1.06

$$\frac{\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))/x,x)
[Out] sinh(a+b*ln(c*x^n))/b/n
```

maxima [A] time = 0.47, size = 18, normalized size = 1.00

$$\frac{\sinh\left(b\log\left(cx^{n}\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] sinh(b*log(c*x^n) + a)/(b*n)
```

mupad [B] time = 1.03, size = 18, normalized size = 1.00

$$\frac{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))/x,x)
```

```
[Out] sinh(a + b*log(c*x^n))/(b*n)
```

sympy [A] time = 0.98, size = 41, normalized size = 2.28

$$\begin{cases} \log(x)\cosh(a) & \text{for } b = 0 \wedge n = 0 \\ \log(x)\cosh\left(a + b\log(c)\right) & \text{for } n = 0 \\ \log(x)\cosh(a) & \text{for } b = 0 \\ \frac{\sinh\left(a + bn\log(x) + b\log(c)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a), Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(c))
, Eq(n, 0)), (log(x)*cosh(a), Eq(b, 0)), (sinh(a + b*n*log(x) + b*log(c))/(
b*n), True))
```

3.248 $\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=39

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log(x)}{2}$$

```
[Out] 1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Log[x]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}+\frac{\text{Subst}\left(\int 1\,dx,x,\log\left(cx^n\right)\right)}{2n}\\
&= \frac{\log(x)}{2}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.92

$$\frac{2\left(a+b\log\left(cx^{n}\right)\right)+\sinh\left(2\left(a+b\log\left(cx^{n}\right)\right)\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^2/x,x]
[Out] (2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

fricas [A] time = 0.43, size = 39, normalized size = 1.00

$$\frac{bn\log(x)+\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
[Out] 1/2*(b*n*log(x) + cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a))/(b*n)
```

giac [B] time = 0.13, size = 80, normalized size = 2.05

$$\frac{\left(4\,bc^{2\,b}ne^{(2\,a)}\log(x)+c^{4\,b}x^{2\,bn}e^{(4\,a)}-\frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
[Out] 1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) + c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)*
x^(2*b*n)*e^(2*a) + 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

maple [A] time = 0.08, size = 52, normalized size = 1.33

$$\frac{\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{2bn}+\frac{\ln\left(c\,x^{n}\right)}{2n}+\frac{a}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^2/x,x)
[Out] 1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/2*ln(c*x^n)/n+1/2/b/n*a
```

maxima [A] time = 0.31, size = 49, normalized size = 1.26

$$\frac{e^{\left(2\,b\log(cx^n)+2\,a\right)}}{8\,bn} - \frac{e^{\left(-2\,b\log(cx^n)-2\,a\right)}}{8\,bn} + \frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) +
1/2*log(x)
```

mupad [B] time = 1.05, size = 32, normalized size = 0.82

$$\frac{\ln\left(x^n\right)}{2\,n} + \frac{\sinh\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)}{4\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^2/x,x)
```

```
[Out] log(x^n)/(2*n) + sinh(2*a + 2*b*log(c*x^n))/(4*b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(cosh(a + b*log(c*x**n))**2/x, x)
```

3.249 $\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=42

$$\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] sinh(a+b*ln(c*x^n))/b/n+1/3*sinh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

fricas [A] time = 0.52, size = 53, normalized size = 1.26

$$\frac{\sinh\left(bn\log(x)+b\log(c)+a\right)^{3}+3\left(\cosh\left(bn\log(x)+b\log(c)+a\right)^{2}+3\right)\sinh\left(bn\log(x)+b\log(c)+a\right)}{12\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) + a
)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

giac [B] time = 0.16, size = 81, normalized size = 1.93

$$\frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)}+9\,c^{4\,b}x^{bn}e^{(4\,a)}-\frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) + 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x^
(2*b*n)*e^(2*a) + 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

maple [A] time = 0.21, size = 36, normalized size = 0.86

$$\frac{\left(\frac{2}{3}+\frac{\left(\cosh^{2}(a+b\ln(c\,x^{n}))\right)}{3}\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] 1/n/b*(2/3+1/3*cosh(a+b*ln(c*x^n))^2)*sinh(a+b*ln(c*x^n))
```

maxima [B] time = 0.33, size = 86, normalized size = 2.05

$$\frac{e^{\left(3\,b\log(cx^{n})+3\,a\right)}}{24\,bn}+\frac{3\,e^{\left(b\log(cx^{n})+a\right)}}{8\,bn}-\frac{3\,e^{\left(-b\log(cx^{n})-a\right)}}{8\,bn}-\frac{e^{\left(-3\,b\log(cx^{n})-3\,a\right)}}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) + 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8*
e^(-b*log(c*x^n) - a)/(b*n) - 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

mupad [B] time = 1.05, size = 35, normalized size = 0.83

$$\frac{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^3+3\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^3/x,x)
```

```
[Out] (3*sinh(a + b*log(c*x^n)) + sinh(a + b*log(c*x^n))^3)/(3*b*n)
```

sympy [A] time = 10.57, size = 87, normalized size = 2.07

$$\begin{cases} \log\left(x\right)\cosh^3\left(a\right) & \text{for } b=0\wedge n=0 \\ \log\left(x\right)\cosh^3\left(a+b\log\left(c\right)\right) & \text{for } n=0 \\ \log\left(x\right)\cosh^3\left(a\right) & \text{for } b=0 \\ -\frac{2\sinh^3\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{3bn}+\frac{\sinh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh^2\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(
c))**3, Eq(n, 0)), (log(x)*cosh(a)**3, Eq(b, 0)), (-2*sinh(a + b*n*log(x) +
 b*log(c))**3/(3*b*n) + sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x)
 + b*log(c))**2/(b*n), True))
```

3.250 $\int \frac{\cosh^4\left(a+b\log(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=73

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

```
[Out] 3/8*ln(x)+3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(c
*x^n))^3*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^3\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/(4*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{\cosh^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\mathrm{Subst}\left(\int \cosh^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{4bn} + \frac{3\,\mathrm{Subst}\left(\int \cosh^2(a+bx)\,dx, x\right.}{4n}$$
$$= \frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn} + \frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)\sinh\left(a\right.}{4bn}$$
$$= \frac{3\log(x)}{8} + \frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn} + \frac{\cosh^3\left(a+b\log\left(c\right.\right.}{}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.70

$$\frac{12\left(a+b\log\left(cx^n\right)\right)+8\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+\sinh\left(4\left(a+b\log\left(cx^n\right)\right)\right)}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) + 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[c
*x^n])])/(32*b*n)
```

fricas [A] time = 0.53, size = 84, normalized size = 1.15

$$\frac{\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^3+3\,bn\log(x)+\left(\cosh\left(bn\log(x)+b\log(c)+a\right.\right.}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 + 4*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

giac [A] time = 0.15, size = 114, normalized size = 1.56

$$\frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log(x)+c^{8\,b}x^{4\,bn}e^{(8\,a)}+8\,c^{6\,b}x^{2\,bn}e^{(6\,a)}-\frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+8\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) + 8*c^(6*b)
*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) + 8*c^(2*b)*x^(2*b*n)*e^
(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

maple [A] time = 0.23, size = 84, normalized size = 1.15

$$\frac{\left(\cosh^{3}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{4bn}+\frac{3\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{8bn}+\frac{3\ln\left(c\,x^{n}\right)}{8n}+\frac{3a}{8bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/4*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/b/n+3/8*cosh(a+b*ln(c*x^n))*s
inh(a+b*ln(c*x^n))/b/n+3/8*ln(c*x^n)/n+3/8/b/n*a
```

maxima [A] time = 0.33, size = 93, normalized size = 1.27

$$\frac{e^{\left(4\,b\log\left(cx^{n}\right)+4\,a\right)}}{64\,bn}+\frac{e^{\left(2\,b\log\left(cx^{n}\right)+2\,a\right)}}{8\,bn}-\frac{e^{\left(-2\,b\log\left(cx^{n}\right)-2\,a\right)}}{8\,bn}-\frac{e^{\left(-4\,b\log\left(cx^{n}\right)-4\,a\right)}}{64\,bn}+\frac{3}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) + 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) -
1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n)
+ 3/8*log(x)
```

mupad [B] time = 1.11, size = 50, normalized size = 0.68

$$\frac{3\,\ln\left(x^{n}\right)}{8\,n}+\frac{\frac{\sinh\left(2\,a+2\,b\,\ln\left(c\,x^{n}\right)\right)}{4}+\frac{\sinh\left(4\,a+4\,b\,\ln\left(c\,x^{n}\right)\right)}{32}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^4/x,x)
```

```
[Out] (3*log(x^n))/(8*n) + (sinh(2*a + 2*b*log(c*x^n))/4 + sinh(4*a + 4*b*log(c*x
^n))/32)/(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{4}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(cosh(a + b*log(c*x**n))**4/x, x)
```

3.251 $\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}+\frac{2\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] sinh(a+b*ln(c*x^n))/b/n+2/3*sinh(a+b*ln(c*x^n))^3/b/n+1/5*sinh(a+b*ln(c*x^n
))^5/b/n
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}+\frac{2\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \cosh^5(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{i\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-i\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\ &= \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{2\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.00

$$\frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}+\frac{2\sinh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

fricas [A] time = 0.55, size = 105, normalized size = 1.62

$$\frac{3\sinh\left(bn\log(x)+b\log(c)+a\right)^{5}+5\left(6\cosh\left(bn\log(x)+b\log(c)+a\right)^{2}+5\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*n*log(x) + b*log(c) + a)^5 + 5*(6*cosh(b*n*log(x) + b*log(c
) + a)^2 + 5)*sinh(b*n*log(x) + b*log(c) + a)^3 + 15*(cosh(b*n*log(x) + b*l
og(c) + a)^4 + 5*cosh(b*n*log(x) + b*log(c) + a)^2 + 10)*sinh(b*n*log(x) +
b*log(c) + a))/(b*n)
```

giac [A] time = 0.17, size = 116, normalized size = 1.78

$$\frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)}+25\,c^{8\,b}x^{3\,bn}e^{(8\,a)}+150\,c^{6\,b}x^{bn}e^{(6\,a)}-\frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+25\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) + 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*c
^(6*b)*x^(b*n)*e^(6*a) - (150*c^(4*b)*x^(4*b*n)*e^(4*a) + 25*c^(2*b)*x^(2*b
*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

maple [A] time = 0.22, size = 51, normalized size = 0.78

$$\frac{\left(\frac{8}{15}+\frac{\left(\cosh^{4}(a+b\ln(c\,x^{n}))\right)}{5}+\frac{4\left(\cosh^{2}(a+b\ln(c\,x^{n}))\right)}{15}\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] 1/n/b*(8/15+1/5*cosh(a+b*ln(c*x^n))^4+4/15*cosh(a+b*ln(c*x^n))^2)*sinh(a+b*
ln(c*x^n))
```

maxima [B] time = 0.32, size = 130, normalized size = 2.00

$$\frac{e^{\left(5\,b\log\left(cx^{n}\right)+5\,a\right)}}{160\,bn}+\frac{5\,e^{\left(3\,b\log\left(cx^{n}\right)+3\,a\right)}}{96\,bn}+\frac{5\,e^{\left(b\log\left(cx^{n}\right)+a\right)}}{16\,bn}-\frac{5\,e^{\left(-b\log\left(cx^{n}\right)-a\right)}}{16\,bn}-\frac{5\,e^{\left(-3\,b\log\left(cx^{n}\right)-3\,a\right)}}{96\,bn}-\frac{e^{\left(-5\,b\log\left(cx^{n}\right)-5\,a\right)}}{160\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) + 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
+ 5/16*e^(b*log(c*x^n) + a)/(b*n) - 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/96
*e^(-3*b*log(c*x^n) - 3*a)/(b*n) - 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

mupad [B] time = 1.17, size = 49, normalized size = 0.75

$$\frac{\frac{\sinh(a+b\,\ln(c\,x^n))^5}{5}+\frac{2\sinh(a+b\,\ln(c\,x^n))^3}{3}+\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^5/x,x)
```

```
[Out] (sinh(a + b*log(c*x^n)) + (2*sinh(a + b*log(c*x^n))^3)/3 + sinh(a + b*log(c
*x^n))^5/5)/(b*n)
```

sympy [A] time = 97.06, size = 128, normalized size = 1.97

$$\begin{cases} \log\left(x\right)\cosh^{5}\left(a\right) \\ \log\left(x\right)\cosh^{5}\left(a+b\log\left(c\right)\right) \\ \log\left(x\right)\cosh^{5}\left(a\right) \\ \frac{8\sinh^{5}\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{15bn}-\frac{4\sinh^{3}\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh^{2}\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{3bn}+\frac{\sinh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh^{4}\left(a+bn\log\right.}{bn} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((log(x)*cosh(a)**5, Eq(b, 0) & Eq(n, 0)), (log(x)*cosh(a + b*log(
c))**5, Eq(n, 0)), (log(x)*cosh(a)**5, Eq(b, 0)), (8*sinh(a + b*n*log(x) +
b*log(c))**5/(15*b*n) - 4*sinh(a + b*n*log(x) + b*log(c))**3*cosh(a + b*n*l
og(x) + b*log(c))**2/(3*b*n) + sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n
*log(x) + b*log(c))**4/(b*n), True))
```

3.252 $\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=67

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn}-\frac{6iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{5bn}$$

```
[Out] -6/5*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*El
lipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/5*cosh(a+b*ln(c*x^n))^
(3/2)*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2635, 2639}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn}-\frac{6iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Cosh[a + b*L
og[c*x^n]]^(3/2)*Sinh[a + b*Log[c*x^n]])/(5*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cosh^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\text{Subst}\left(\int \cosh^{\frac{5}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn} + \frac{3\,\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx,\right.}{5n}$$
$$= -\frac{6iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{5bn} + \frac{2\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 0.93

$$\frac{\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)} - 6iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] ((-6*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sqrt[Cosh[a + b*Log[c*x^n]
]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

[Out] integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)

maple [B] time = 0.43, size = 256, normalized size = 3.82

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\left(8\left(\cosh^7\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-16\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+}{5n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+s}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a+b*ln(c*x^n))^(5/2)/x,x)

[Out] 2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10*cosh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b*log(c*x^n))^(5/2)/x,x)

[Out] int(cosh(a + b*log(c*x^n))^(5/2)/x, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

3.253 $\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=67

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{3bn}-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*El
lipticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sinh(a+b*ln(c*x^n))*
cosh(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2635, 2641}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{3bn}-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sqrt[Cosh[a
+ b*Log[c*x^n]]]*Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\frac{\text{Subst}\left(\int\cosh^{\frac{3}{2}}(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}$$

$$=\frac{2\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{\cosh(a+bx)}}\,dx,x,\text{lo}\right.}{3n}$$

$$=-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}+\frac{2\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

Mathematica [C] time = 0.13, size = 114, normalized size = 1.70

$$\frac{2\sqrt{\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-\sinh\left(2\left(\right.\right.\right.}{3bn\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (Sinh[2*(a + b*Log[c*x^n])] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a
 + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1 + Cosh[2*(a + b*Log[
c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]])/(3*b*n*Sqrt[Cosh[a + b*Log[c*x^n]]]
)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

maple [B] time = 0.42, size = 237, normalized size = 3.54

$$\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\ \left(4\left(\cosh^5\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-6\left(\cosh^3\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+\right.}{3n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-sin
h(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2
)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^
n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/
sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] int(cosh(a + b*log(c*x^n))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(3/2)/x,x)
[Out] Integral(cosh(a + b*log(c*x**n))**(3/2)/x, x)
```

3.254 $\int \frac{\sqrt{\cosh\left(a+b\log(cx^n)\right)}}{x}\,dx$

Optimal. Leaf size=28

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2639}

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$-\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\cosh\left(b\log\left(cx^n\right)+a\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

maple [B] time = 0.33, size = 183, normalized size = 6.54

$$-\frac{2\sqrt{\left(2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1\right)\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\sqrt{-2\left(\cosh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}}{n\sqrt{2\left(\sinh^4\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)+\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\left(\cosh^2\left(\frac{a}{2}+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] -2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^
2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+1/2
*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*
x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh\left(b\log\left(cx^n\right) + a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\cosh\left(a + b\,\ln\left(c\,x^n\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] int(cosh(a + b*log(c*x^n))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh\left(a + b\log\left(cx^n\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(cosh(a + b*log(c*x**n)))/x, x)
```

3.255 $\int \frac{1}{x\sqrt{\cosh(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=28

$$-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2641}

$$-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= -\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{x\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{\cosh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

maple [B] time = 0.30, size = 183, normalized size = 6.54

$$\frac{2\sqrt{\left(2\left(\cosh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)-1\right)\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)}\,\sqrt{-\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)}\,\sqrt{-2\left(\cosh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)}}{n\sqrt{2\left(\sinh^{4}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)+\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\left(\cosh^{2}\left(\frac{a}{2}+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] 2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2
+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln(c*x
^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cosh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^(1/2)),x)
```

```
[Out] int(1/(x*cosh(a + b*log(c*x^n))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(cosh(a + b*log(c*x**n)))), x)
```

3.256 $\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=63

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}+\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

```
[Out] 2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Ellip
ticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2*sinh(a+b*ln(c*x^n))/b/n/c
osh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2636, 2639}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}+\frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] ((2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*Log[c*
x^n]])/(b*n*Sqrt[Cosh[a + b*Log[c*x^n]]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}} - \frac{\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{bn} + \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.92

$$\frac{2\left(\frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}} + iE\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (2*(I*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]/Sqrt[
Cosh[a + b*Log[c*x^n]]]))/(b*n)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

maple [A] time = 0.43, size = 141, normalized size = 2.24

$$\frac{2\sqrt{-\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)}\,\sqrt{-2\left(\sinh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right),\sqrt{2}\right)+4\cosh\left(\frac{a}{2}+\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\left(\cosh^{2}\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right)-1}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 2/n*((-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))
^2-1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2
*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2
*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cosh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\cosh\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] int(1/(x*cosh(a + b*log(c*x^n))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*cosh(a + b*log(c*x**n))**(3/2)), x)
```

3.257 $\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=67

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}$$

```
[Out] -2/3*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*El
lipticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/3*sinh(a+b*ln(c*x^n))/
b/n/cosh(a+b*ln(c*x^n))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2636, 2641}

$$\frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n) + (2*Sinh[a + b*L
og[c*x^n]])/(3*b*n*Cosh[a + b*Log[c*x^n]]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{x\cosh^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{3n}$$

$$= -\frac{2iF\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)}{3bn} + \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{3bn\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Mathematica [C] time = 0.09, size = 122, normalized size = 1.82

$$\frac{2\left(\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\cosh\left(2\left(a+\right.\right.\right.}{3bn\cosh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (2*(Sinh[a + b*Log[c*x^n]] + Cosh[a + b*Log[c*x^n]]*Hypergeometric2F1[1/4,
1/2, 5/4, -Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1
+ Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Cosh[a
+ b*Log[c*x^n]]^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

maple [B] time = 0.40, size = 295, normalized size = 4.40

$$\frac{2\left(2\sqrt{-\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)}\,\sqrt{-2\left(\sinh^2\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right),\sqrt{2}\right)\left(\sinh^2\left(\frac{a}{2}+\right.\right.\right.}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/cosh(a+b*ln(c*x^n))^(5/2),x)
```

```
[Out] 2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x
^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a+1
/2*b*ln(c*x^n))^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2
*b*ln(c*x^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*c
osh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2)*((2*cosh(1/2*a+1/
2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2*a+1/2*
b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cosh(1/2*a+1/2*b*ln(
c*x^n))^2-1)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] int(1/(x*cosh(a + b*log(c*x^n))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cosh(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

3.258 $\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$

Optimal. Leaf size=206

$$\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^2}-\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)}-\frac{5e^{-3a}x\left(cx^n\right)^{-6/n}\text{cs}}{4}$$

```
[Out] -1/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)+5/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(2*
a)/((c*x^n)^(4/n))/(1+1/exp(2*a)/((c*x^n)^(4/n)))^2+5/12*x*cosh(a+2*ln(c*x^
n)/n)^(5/2)/(1+1/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsch(exp(a)*(c*x^n)^(2/
n))*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1+1/exp(2*a)/((c*
x^n)^(4/n)))^(5/2)
```

Rubi [A] time = 0.15, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5526, 5534, 353, 349, 345, 242, 277, 215}

$$\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^2}-\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)}-\frac{5e^{-3a}x\left(cx^n\right)^{-6/n}\text{cs}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] -(x*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/4 + (5*x*Cosh[a + (2*Log[c*x^n])/n]^(
5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*Co
sh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5*x
*ArcCsch[E^a*(c*x^n)^(2/n)]*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(c
*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Rule 349

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*
(a + b*x^n)^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rule 353

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(x)}{n}\right)dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1+e^{-2a}x^{-4/n}\right)^{5/2}dx,x,cx^n\right.}{n\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{\left(5x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x\right.}{2n\left(1+e^{-2a}(cx^n)^{-}\right.}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}+\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\text{co}\right.}{}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}+\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\text{co}\right.}{}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)}-\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\text{co}\right.}{}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\cos}{12(1}$$

$$= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\cos}{12(1}$$

Mathematica [C] time = 0.49, size = 85, normalized size = 0.41

$$\frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(e^{2a}(cx^n)^{4/n}+1\right){}_2F_1\left(2,\frac{7}{2};\frac{9}{2};e^{2a}(cx^n)^{4/n}+1\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] (E^(2*a)*x*(c*x^n)^(4/n)*(1 + E^(2*a)*(c*x^n)^(4/n))*Cosh[a + (2*Log[c*x^n]
)/n]^(5/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + E^(2*a)*(c*x^n)^(4/n)])/14
```

fricas [A] time = 0.49, size = 187, normalized size = 0.91

$$\frac{\left(15\sqrt{2}x^3e^{\left(\frac{3(an+2\log(c))}{2n}\right)}\log\left(\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}-2\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}{x^2}}+2}{x^4}\right)+4\sqrt{\frac{1}{2}}\left(2x^8e^{\left(\frac{4(an+2\log(c))}{n}\right)}+14x^4e^{\left(\frac{2(an+}{}\right.}\right.\right.}{192x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(15*sqrt(2)*x^3*e^(3/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2*log
(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2
) + 2)/x^4) + 4*sqrt(1/2)*(2*x^8*e^(4*(a*n + 2*log(c))/n) + 14*x^4*e^(2*(a*
n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*
(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c))/n)/x^3
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

```
[Out] int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh\left(a + \frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + (2*log(c*x^n))/n)^(5/2),x)
```

```
[Out] int(cosh(a + (2*log(c*x^n))/n)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*ln(c*x**n)/n)**(5/2),x)
```

```
[Out] Timed out
```

3.259 $\int \sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}x\sqrt{\cosh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}-\frac{e^{-a}x\left(cx^n\right)^{-2/n}\operatorname{csch}^{-1}\left(e^a\left(cx^n\right)^{2/n}\right)\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

```
[Out] 1/2*x*cosh(a+2*ln(c*x^n)/n)^(1/2)-1/2*x*arccsch(exp(a)*(c*x^n)^(2/n))*cosh(
a+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1+1/exp(2*a)/((c*x^n)^(4/n))
)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5526, 5534, 345, 242, 277, 215}

$$\frac{1}{2}x\sqrt{\cosh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}-\frac{e^{-a}x\left(cx^n\right)^{-2/n}\operatorname{csch}^{-1}\left(e^a\left(cx^n\right)^{2/n}\right)\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2 - (x*ArcCsch[E^a*(c*x^n)^(2/n)]*Sqrt
[Cosh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 + 1/(E^(2*a)*(c*x
^n)^(4/n))])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\cosh\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sqrt{1+e^{-2a}x^{-4/n}}\,dx,x,cx^n\right)}{n\sqrt{1+e^{-2a}(cx^n)^{-4/n}}}$$

$$= \frac{\left(x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int \sqrt{1+\frac{e^{-2a}}{x^2}}\,dx,x,(cx^n)^{2/n}\right)}{2\sqrt{1+e^{-2a}(cx^n)^{-4/n}}}$$

$$= -\frac{\left(x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1+e^{-2a}x^2}}{x^2}\,dx,x,(cx^n)^{-2/n}\right)}{2\sqrt{1+e^{-2a}(cx^n)^{-4/n}}}$$

$$= \frac{1}{2}x\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{\left(e^{-2a}x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\right.}{2\sqrt{1+e^{-2a}(cx^n)^{-4/n}}}$$

$$= \frac{1}{2}x\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{e^{-a}x(cx^n)^{-2/n}\sinh^{-1}\left(e^{-a}(cx^n)^{-2/n}\right)\sqrt{\cosh\left(a+\right.}}{2\sqrt{1+e^{-2a}(cx^n)^{-4/n}}}$$

Mathematica [A] time = 0.33, size = 74, normalized size = 0.73

$$\frac{1}{2}x\left(1-\frac{\tanh^{-1}\left(\sqrt{e^{2a}(cx^n)^{4/n}+1}\right)}{\sqrt{e^{2a}(cx^n)^{4/n}+1}}\right)\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*(1 - ArcTanh[Sqrt[1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[1 + E^(2*a)*(c*x^n)^(
4/n)])*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2
```

fricas [A] time = 0.43, size = 141, normalized size = 1.38

$$\frac{1}{8}\left(4\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\left(an+2\log(c)\right)}{n}\right)}+1}{x^2}}e^{\left(\frac{an+2\log(c)}{2n}\right)}+\sqrt{2}e^{\left(\frac{an+2\log(c)}{2n}\right)}\log\left(\frac{x^4e^{\left(\frac{2\left(an+2\log(c)\right)}{n}\right)}-2\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\left(an+2\log(c)\right)}{n}\right)}}{x^2}}}{x^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(1/2*(a*n
 + 2*log(c))/n) + sqrt(2)*e^(1/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2
*log(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)
/x^2) + 2)/x^4))*e^(-(a*n + 2*log(c))/n)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh}\left(a+\frac{2\ln\left(c\,x^n\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

```
[Out] int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cosh(a + 2*log(c*x^n)/n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cosh\left(a + \frac{2\ \ln\left(c\,x^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + (2*log(c*x^n))/n)^(1/2),x)
[Out] int(cosh(a + (2*log(c*x^n))/n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cosh\left(a + \frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(a+2*ln(c*x**n)/n)**(1/2),x)
[Out] Integral(sqrt(cosh(a + 2*log(c*x**n)/n)), x)
```

3.260 $\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=42

$$-\frac{x\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/2*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5526, 5534, 264}

$$-\frac{x\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] -(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(2*Cosh[a + (2*Log[c*x^n])/n]^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(x)}{n}\right)}dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{2/n}\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{\left(1+e^{-2a}x^{-4/n}\right)^{3/2}}dx, x, cx^n\right)}{n\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] time = 0.16, size = 61, normalized size = 1.45

$$\frac{\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)-\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)}{x\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] (-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n]
)/n])/(x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])
```

fricas [A] time = 0.49, size = 68, normalized size = 1.62

$$-\frac{2\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^4 e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}{x^2}}\,e^{\left(-\frac{an+2\log(c)}{2n}\right)}}{x^4 e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n +
2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^{n})}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
[Out] int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(a+\frac{2\,\log\left(cx^{n}\right)}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + (2*log(c*x^n))/n)^(3/2),x)
[Out] int(1/cosh(a + (2*log(c*x^n))/n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a + \frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*ln(c*x**n)/n)**(3/2),x)
```

```
[Out] Integral(cosh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

3.261 $\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=101

$$-\frac{e^{-2a}x\,(cx^n)^{-4/n}\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/6*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(7/2)-1/15*x*(1
+1/exp(2*a)/((c*x^n)^(4/n)))/exp(2*a)/((c*x^n)^(4/n))/cosh(a+2*ln(c*x^n)/n)
^(7/2)
```

Rubi [A] time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5526, 5534, 271, 264}

$$-\frac{e^{-2a}x\,(cx^n)^{-4/n}\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(e^{-2a}\,(cx^n)^{-4/n}+1\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] -(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(6*Cosh[a + (2*Log[c*x^n])/n]^(7/2)) -
 (x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Cosh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5526

```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5534

```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Dist[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x\,(cx^n)^{6/n}\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{6}{n}}}{\left(1+e^{-2a}x^{-4/n}\right)^{7/2}}\,dx, x, cx^n\right)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} + \frac{\left(2e^{-2a}x\,(cx^n)^{6/n}\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1\cdots}}{\left(1+e^{-2a}\cdots\right.}\cdots\right.}{3n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)}{6\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{e^{-2a}x\,(cx^n)^{-4/n}\left(1+e^{-2a}\,(cx^n)^{-4/n}\right)}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] time = 0.27, size = 121, normalized size = 1.20

$$\frac{\left(\left(5x^4-2\right)\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)+\left(5x^4+2\right)\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)\right)\left(\sinh\left(2a+\frac{4\log(cx^n)}{n}-4\log\cdots\right.\right.}{15x^5\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] (((2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (-2 + 5*x^4)*Sinh[a -
 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n] +
Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Cosh[a + (2*Log[c*x^n])/n
]^(5/2))
```

fricas [A] time = 0.58, size = 128, normalized size = 1.27

$$-\frac{8\sqrt{\frac{1}{2}}\left(5\,x^{5}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+2\,x\right)\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}+3\,x^{8}e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) + 2*x)*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*lo
g(c))/n) + 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/n)
+ 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^{n})}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

```
[Out] int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(a + \frac{2\ln(c\,x^n)}{n}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(a + (2*log(c*x^n))/n)^(7/2),x)
```

```
[Out] int(1/cosh(a + (2*log(c*x^n))/n)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(a+2*ln(c*x**n)/n)**(7/2),x)
```

```
[Out] Timed out
```

3.262 $\int \cosh\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] (d*x+c)*cosh((b*x+a)/(d*x+c))/d-(-a*d+b*c)*cosh(b/d)*Shi((-a*d+b*c)/d/(d*x+
c))/d^2+(-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*sinh(b/d)/d^2
```

Rubi [A] time = 0.18, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5608, 3297, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*CoshIntegral[(b*c -
a*d)/(d*(c + d*x))]*Sinh[b/d])/d^2 - ((b*c - a*d)*Cosh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5608

```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Cosh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad\right.}{}$$

$$= \frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right)}{d^2} - \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [B] time = 0.37, size = 373, normalized size = 3.69

$$\frac{(bc-ad)\left(\sinh\left(\frac{b}{d}\right)-\cosh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+(bc-ad)\left(\sinh\left(\frac{b}{d}\right)+\cosh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{ad-bc}{d(c+dx)}\right)-ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(a + b*x)/(c + d*x)],x]
```

```
[Out] (2*c*d*Cosh[(a + b*x)/(c + d*x)] + 2*d^2*x*Cosh[(a + b*x)/(c + d*x)] + (b*c
 - a*d)*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*(-Cosh[b/d] + Sinh[b/d]) +
```

```
(b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/d] + Sinh[b/
d]) + b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Cosh[b
/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + b*c*Sinh[b/d]*SinhIntegral
[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/
(d*(c + d*x))] - b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + a*
d*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + b*c*Sinh[b/d]*SinhInt
egral[(b*c - a*d)/(c*d + d^2*x)] - a*d*Sinh[b/d]*SinhIntegral[(b*c - a*d)/(
c*d + d^2*x)])/(2*d^2)
```

fricas [A] time = 0.52, size = 171, normalized size = 1.69

$$\frac{2\left(d^2x+cd\right)\cosh\left(\frac{bx+a}{dx+c}\right)-\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)-(bc-ad)\mathrm{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)+\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d^2*x + c*d)*cosh((b*x + a)/(d*x + c)) - ((b*c - a*d)*Ei((b*c - a*d
)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) +
((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*d)/(d
^2*x + c*d)))*sinh(b/d))/d^2
```

giac [B] time = 5.29, size = 764, normalized size = 7.56

$$\frac{\left(b^3c^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}-2\,ab^2cd\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}-\frac{(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c}+a^2bd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}+\frac{2\,(bx+a)ab\cdots}{\cdots}\right.}{2\left(bd^2-\frac{(bx\cdots}{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-(
b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x +
 a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x
 + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d
)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*
e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^((b*x
 + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*
d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) - 1/2*(b^3*c^2*Ei((b - (
b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(
```

```
-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 2*
(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) -
(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) - a
^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b
*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [B] time = 0.12, size = 347, normalized size = 3.44

$$\frac{e^{-\frac{bx+a}{dx+c}}a}{\frac{2da}{dx+c}-\frac{2bc}{dx+c}}-\frac{e^{-\frac{bx+a}{dx+c}}cb}{2d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{e^{-\frac{b}{d}}\,\mathrm{Ei}\left(1,\frac{da-cb}{d(dx+c)}\right)a}{2d}+\frac{e^{-\frac{b}{d}}\,\mathrm{Ei}\left(1,\frac{da-cb}{d(dx+c)}\right)cb}{2d^2}+\frac{d\,e^{\frac{bx+a}{dx+c}}xa}{2da-2cb}-\frac{e^{\frac{bx+a}{dx+c}}xcb}{2\left(da-cb\right)}+\frac{e^{\frac{bx+a}{dx+c}}ca}{2da-2cb}-2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)/(d*x+c)),x)
```

```
[Out] 1/2*exp(-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/2/d*exp(-(b*x+a)/(d
*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b-1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+
c))*a+1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*c*b+1/2*d*exp((b*x+a)/(d*
x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*c*b+1/2*exp((b*x+a
)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*e
xp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*
x+c))*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cosh((b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(\frac{a+b\,x}{c+d\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((a + b*x)/(c + d*x)),x)
```

```
[Out] int(cosh((a + b*x)/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(cosh((a + b*x)/(c + d*x)), x)
```

3.263 $\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] (d*x+c)*cosh((b*x+a)/(d*x+c))^2/d-(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/d
/(d*x+c))/d^2+(-a*d+b*c)*Chi(2*(-a*d+b*c)/d/(d*x+c))*sinh(2*b/d)/d^2
```

Rubi [A] time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5608, 3313, 12, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Cosh[(a + b*x)/(c + d*x)]^2)/d + ((b*c - a*d)*CoshIntegral[(2*(b
*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5608

```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Cosh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\cosh^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2i(bc-ad))\,\text{Subst}\left(\int -\frac{i\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{((bc

Mathematica [A] time = 0.31, size = 111, normalized size = 1.04

$$\frac{2\sinh\left(\frac{2b}{d}\right)(bc-ad)\mathrm{Chi}\left(\frac{2(ad-bc)}{d(c+dx)}\right)+2\cosh\left(\frac{2b}{d}\right)(bc-ad)\mathrm{Shi}\left(\frac{2(ad-bc)}{d(c+dx)}\right)+d\left((c+dx)\cosh\left(\frac{2(a+bx)}{c+dx}\right)+dx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(d*x + (c + d*x)*Cosh[(2*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)*CoshInte
gral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] + 2*(b*c - a*d)*Cosh[(
2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

fricas [B] time = 0.64, size = 366, normalized size = 3.42

$$d^2x+\left(d^2x+cd\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2+\left(d^2x-(bc-ad)\mathrm{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2+\left((bc-ad)\mathrm{Ei}\left(-\frac{2\,(}{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*x + (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 + (d^2*x - (b*c - a*
d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*x
+ c))^2 + ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x
 + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) + ((b*c
 - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 - (b*c
 - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2 + (b*c
 - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x + a)/(
d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

giac [B] time = 15.29, size = 749, normalized size = 7.00

$$\left(2\,b^3c^2\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}-4\,ab^2cd\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}-\frac{2\,(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}}{dx+c}+2\,a^2bd^2\mathrm{Ei}\left(-\frac{2\left(b-\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*d
*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*Ei(
-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(b -
(b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b - (b*
x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*d/(d*x
+ c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(d*x + c)
)/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)*e^(
-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) + b^2*c^2*d*e^
(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x + c)) + a^2*d^3
*e^(-2*(b*x + a)/(d*x + c)) + 2*b^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)*(b*c/(
b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [B] time = 0.42, size = 358, normalized size = 3.35

$$\frac{x}{2}+\frac{e^{-\frac{2(bx+a)}{dx+c}}a}{\frac{4da}{dx+c}-\frac{4bc}{dx+c}}-\frac{e^{-\frac{2(bx+a)}{dx+c}}cb}{4d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{e^{-\frac{2b}{d}}\operatorname{Ei}\left(1,\frac{2da-2cb}{d(dx+c)}\right)a}{2d}+\frac{e^{-\frac{2b}{d}}\operatorname{Ei}\left(1,\frac{2da-2cb}{d(dx+c)}\right)cb}{2d^2}+\frac{d\,e^{\frac{2bx+2a}{dx+c}}xa}{4da-4cb}-\frac{e^{\frac{2bx+2a}{dx+c}}xcb}{4\,(da-cb)}+\frac{e^{\frac{2bx+2\cdots}{dx+\cdots}}}{4da-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((b*x+a)/(d*x+c))^2,x)
```

```
[Out] 1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/4/d*exp(-2*
(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b-1/2/d*exp(-2*b/d)*Ei(1,2*(a*
d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*c*b+1/4
*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c
)*x*c*b+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/(d*x+c
))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*
exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x+\frac{1}{4}\,\int e^{\left(\frac{2\,bc}{d^2x+cd}-\frac{2\,a}{dx+c}-\frac{2\,b}{d}\right)}\,dx+\frac{1}{4}\,\int e^{\left(-\frac{2\,bc}{d^2x+cd}+\frac{2\,a}{dx+c}+\frac{2\,b}{d}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x) +
 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh((a + b*x)/(c + d*x))^2,x)
```

```
[Out] int(cosh((a + b*x)/(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.264 $\int e^{a+bx}\cosh^4(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -1/48*exp(-3*b*x-3*a)/b-1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b+1/12*exp(3*b*x+3
*a)/b+1/80*exp(5*b*x+5*a)/b
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(48*b) - E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) + E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}+\frac{4}{x^2}+4x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= -\frac{e^{-3a-3bx}}{48b}-\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}+\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.75

$$\frac{e^{-3(a+bx)}\left(-60e^{2(a+bx)}+90e^{4(a+bx)}+20e^{6(a+bx)}+3e^{8(a+bx)}-5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

```
[Out] (-5 - 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) + 20*E^(6*(a + b*x)) + 3*E^(8
*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

fricas [A] time = 0.52, size = 113, normalized size = 1.36

$$-\frac{\cosh(bx+a)^4-16\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+10\right)\sinh(bx+a)^2+2}{120\left(b\cosh(bx+a)-b\sinh(bx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b*x + a)^2 + 20*cosh(b*x + a)^2 - 16*(c
osh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) - b*
sinh(b*x + a))
```

giac [A] time = 0.12, size = 60, normalized size = 0.72

$$-\frac{5\left(12\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-3\,e^{(5\,bx+5\,a)}-20\,e^{(3\,bx+3\,a)}-90\,e^{(bx+a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/240*(5*(12*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) - 2
0*e^(3*b*x + 3*a) - 90*e^(b*x + a))/b
```

maple [A] time = 0.22, size = 45, normalized size = 0.54

$$\frac{\frac{\left(\cosh^{5}(bx+a)\right)}{5}+\left(\frac{8}{15}+\frac{\left(\cosh^{4}(bx+a)\right)}{5}+\frac{4\left(\cosh^{2}(bx+a)\right)}{15}\right)\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^4,x)
```

```
[Out] 1/b*(1/5*cosh(b*x+a)^5+(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*x
+a))
```

maxima [A] time = 0.32, size = 68, normalized size = 0.82

$$\frac{e^{(5\,bx+5\,a)}}{80\,b}+\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{e^{(-bx-a)}}{4\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/80*e^(5*b*x + 5*a)/b + 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 1/4*e
^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

mupad [B] time = 0.50, size = 58, normalized size = 0.70

$$\frac{90\,\mathrm{e}^{a+b\,x}-60\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}+20\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^4*exp(a + b*x),x)
```

```
[Out] (90*exp(a + b*x) - 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) + 20*exp(3*a +
3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

sympy [A] time = 54.78, size = 139, normalized size = 1.67

$$\begin{cases}\frac{8e^{a}e^{bx}\sinh^{4}(a+bx)}{15b}-\frac{8e^{a}e^{bx}\sinh^{3}(a+bx)\cosh(a+bx)}{15b}-\frac{4e^{a}e^{bx}\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{5b}+\frac{4e^{a}e^{bx}\sinh(a+bx)\cosh^{3}(a+bx)}{5b}+\frac{e^{a}e^{bx}\cosh^{4}(a}{5b}\\ xe^{a}\cosh^{4}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((8*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) - 8*exp(a)*exp(b*x)*si
nh(a + b*x)**3*cosh(a + b*x)/(15*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*co
sh(a + b*x)**2/(5*b) + 4*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(5*
b) + exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*cosh(a)**
4, True))
```

3.265 $\int e^{a+bx} \cosh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[Out] -1/16*exp(-2*b*x-2*a)/b+3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 266, 43}

$$-\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^3,x]
```

```
[Out] -E^(-2*a - 2*b*x)/(16*b) + (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32
*b) + (3*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\cosh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}\\
&= -\frac{e^{-2a-2bx}}{16b}+\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.82

$$
\frac{-e^{-2(a+bx)}+3e^{2(a+bx)}+\frac{1}{2}e^{4(a+bx)}+6bx}{16b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^3,x]
[Out] (-E^(-2*(a + b*x)) + 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

fricas [B] time = 0.48, size = 95, normalized size = 1.67

$$
-\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2-3\sinh(bx+a)^3-6(2bx+1)\cosh(bx+a)+3\left(4bx-3\cos\right.}{32(b\cosh(bx+a)-b\sinh(bx+a))}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="fricas")
[Out] -1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)^
3 - 6*(2*b*x + 1)*cosh(b*x + a) + 3*(4*b*x - 3*cosh(b*x + a)^2 - 2)*sinh(b*
x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 57, normalized size = 1.00

$$\frac{12\,bx-2\left(3\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+12\,a+e^{(4\,bx+4\,a)}+6\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*x
 + 4*a) + 6*e^(2*b*x + 2*a))/b
```

maple [A] time = 0.23, size = 49, normalized size = 0.86

$$\frac{\frac{\left(\cosh^4(bx+a)\right)}{4}+\left(\frac{\left(\cosh^3(bx+a)\right)}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/4*cosh(b*x+a)^4+(1/4*cosh(b*x+a)^3+3/8*cosh(b*x+a))*sinh(b*x+a)+3/8*
b*x+3/8*a)
```

maxima [A] time = 0.32, size = 53, normalized size = 0.93

$$\frac{3\,(bx+a)}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{32\,b}+\frac{3\,e^{(2\,bx+2\,a)}}{16\,b}-\frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b + 3/16*e^(2*b*x + 2*a)/b - 1/16*e^
(-2*b*x - 2*a)/b
```

mupad [B] time = 0.26, size = 42, normalized size = 0.74

$$\frac{3\,x}{8}+\frac{\frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16}-\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*exp(a + b*x),x)
```

```
[Out] (3*x)/8 + ((3*exp(2*a + 2*b*x))/16 - exp(- 2*a - 2*b*x)/16 + exp(4*a + 4*b*
x)/32)/b
```

sympy [A] time = 15.95, size = 207, normalized size = 3.63

$$\begin{cases} \frac{3xe^a e^{bx} \sinh^3(a+bx)}{8} - \frac{3xe^a e^{bx} \sinh^2(a+bx)\cosh(a+bx)}{8} - \frac{3xe^a e^{bx} \sinh(a+bx)\cosh^2(a+bx)}{8} + \frac{3xe^a e^{bx} \cosh^3(a+bx)}{8} - \frac{5e^a e^{bx} \sinh^3(a+bx)}{8b} + \\ xe^a \cosh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*sin
h(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a +
b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - 5*exp(a)*exp(b*x)*sinh
(a + b*x)**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) +
 exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**2/b - 3*exp(a)*exp(b*x)*cosh(
a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*cosh(a)**3, True))
```

3.266 $\int e^{a+bx}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

```
[Out] -1/4*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

```
[Out] -E^(-a - b*x)/(4*b) + E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{4x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, e^{a+bx}\right)}{4b}$$
$$= -\frac{e^{-a-bx}}{4b} + \frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{12b}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{e^{-a-bx}\left(6e^{2(a+bx)}+e^{4(a+bx)}-3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x]^2,x]
[Out] (E^(-a - b*x)*(-3 + 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

fricas [A] time = 0.94, size = 54, normalized size = 1.10

$$-\frac{\cosh(bx+a)^2 - 4\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 - 3}{6\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 34, normalized size = 0.69

$$\frac{e^{(3\,bx+3\,a)} + 6\,e^{(bx+a)} - 3\,e^{(-bx-a)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) + 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

maple [A] time = 0.21, size = 35, normalized size = 0.71

$$\frac{\frac{\left(\cosh^{3}(bx+a)\right)}{3}+\left(\frac{2}{3}+\frac{\left(\cosh^{2}(bx+a)\right)}{3}\right)\sinh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a)^2,x)
[Out] 1/b*(1/3*cosh(b*x+a)^3+(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

maxima [A] time = 0.31, size = 40, normalized size = 0.82

$$\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/12*e^(3*b*x + 3*a)/b + 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

mupad [B] time = 0.97, size = 34, normalized size = 0.69

$$\frac{6\,\mathrm{e}^{a+b\,x}-3\,\mathrm{e}^{-a-b\,x}+\mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*exp(a + b*x),x)
[Out] (6*exp(a + b*x) - 3*exp(- a - b*x) + exp(3*a + 3*b*x))/(12*b)
```

sympy [A] time = 4.34, size = 78, normalized size = 1.59

$$\begin{cases} -\frac{2e^{a}e^{bx}\sinh^{2}\left(a+bx\right)}{3b}+\frac{2e^{a}e^{bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{3b}+\frac{e^{a}e^{bx}\cosh^{2}\left(a+bx\right)}{3b} & \text{for } b\neq 0 \\ xe^{a}\cosh^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a)**2,x)
[Out] Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*si
nh(a + b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), N
e(b, 0)), (x*exp(a)*cosh(a)**2, True))
```

3.267 $\int e^{a+bx} \cosh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

```
[Out] 1/4*exp(2*b*x+2*a)/b+1/2*x
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 14}

$$\frac{e^{2a+2bx}}{4b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cosh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + x/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cosh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) + x/2
```

fricas [B] time = 0.63, size = 50, normalized size = 2.17

$$\frac{(2\,bx+1)\cosh\left(bx+a\right)-(2\,bx-1)\sinh\left(bx+a\right)}{4\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*x + 1)*cosh(b*x + a) - (2*b*x - 1)*sinh(b*x + a))/(b*cosh(b*x + a
) - b*sinh(b*x + a))
```

giac [A] time = 0.11, size = 22, normalized size = 0.96

$$\frac{2\,bx+2\,a+e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*x + 2*a + e^(2*b*x + 2*a))/b
```

maple [A] time = 0.04, size = 37, normalized size = 1.61

$$\frac{\frac{\left(\cosh^{2}(bx+a)\right)}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cosh(b*x+a),x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)^2+1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)
```

maxima [A] time = 0.31, size = 24, normalized size = 1.04

$$\frac{1}{2}x+\frac{a}{2b}+\frac{e^{(2bx+2a)}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

mupad [B] time = 0.92, size = 18, normalized size = 0.78

$$\frac{x}{2}+\frac{\mathrm{e}^{2a+2bx}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(a + b*x),x)
```

```
[Out] x/2 + exp(2*a + 2*b*x)/(4*b)
```

sympy [A] time = 1.02, size = 63, normalized size = 2.74

$$\begin{cases}-\frac{xe^{a}e^{bx}\sinh(a+bx)}{2}+\frac{xe^{a}e^{bx}\cosh(a+bx)}{2}+\frac{e^{a}e^{bx}\sinh(a+bx)}{2b} & \text{for } b\neq 0\\ xe^{a}\cosh(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cosh(b*x+a),x)
```

```
[Out] Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)/2 + x*exp(a)*exp(b*x)*cosh(a +
b*x)/2 + exp(a)*exp(b*x)*sinh(a + b*x)/(2*b), Ne(b, 0)), (x*exp(a)*cosh(a),
 True))
```

3.268 $\int e^{a+bx}\operatorname{sech}(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

```
[Out] ln(1+exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 260}

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x],x]
```

```
[Out] Log[1 + E^(2*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\operatorname{sech}(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x],x]
[Out] Log[1 + E^(2*a + 2*b*x)]/b
```

fricas [A] time = 0.43, size = 30, normalized size = 1.76

$$\frac{\log\left(\frac{2\,\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="fricas")
[Out] log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

giac [A] time = 0.11, size = 16, normalized size = 0.94

$$\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="giac")
[Out] log(e^(2*b*x + 2*a) + 1)/b
```

maple [A] time = 0.03, size = 19, normalized size = 1.12

$$x+\frac{\ln\left(\cosh\left(bx+a\right)\right)}{b}+\frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a),x)
```

```
[Out] x+1/b*ln(cosh(b*x+a))+a/b
```

maxima [A] time = 0.42, size = 16, normalized size = 0.94

$$\frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] log(e^(2*b*x + 2*a) + 1)/b
```

mupad [B] time = 0.92, size = 16, normalized size = 0.94

$$\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x),x)
```

```
[Out] log(exp(2*a + 2*b*x) + 1)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a),x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x), x)
```

3.269 $\int e^{a+bx}\text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}$$

```
[Out] -2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*arctan(exp(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 288, 203}

$$\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^2,x]
```

```
[Out] (-2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))) + (2*ArcTan[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^2}{\left(1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{4\operatorname{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= -\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= -\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\tan^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 0.90

$$\frac{2\left(\tan^{-1}\left(e^{a+bx}\right)-\frac{e^{a+bx}}{e^{2(a+bx)}+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^2,x]
```

```
[Out] (2*(-(E^(a + b*x)/(1 + E^(2*(a + b*x)))) + ArcTan[E^(a + b*x)]))/b
```

fricas [B] time = 0.52, size = 105, normalized size = 2.62

$$\frac{2\left(\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1\right)\arctan\left(\cosh(bx+a)+\sinh(bx+a)\right)-\cosh(bx+a)-\sinh(bx+a)\right)}{b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*
arctan(cosh(b*x + a) + sinh(b*x + a)) - cosh(b*x + a) - sinh(b*x + a))/(b*c
osh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

giac [A] time = 0.14, size = 35, normalized size = 0.88

$$-\frac{2\left(\frac{e^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}-\arctan\left(e^{(bx+a)}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(e^(b*x + a)/(e^(2*b*x + 2*a) + 1) - arctan(e^(b*x + a)))/b
```

maple [A] time = 0.08, size = 27, normalized size = 0.68

$$-\frac{1}{b\cosh\left(bx+a\right)}+\frac{2\arctan\left(e^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^2,x)
```

```
[Out] -1/b/cosh(b*x+a)+2*arctan(exp(b*x+a))/b
```

maxima [A] time = 0.41, size = 37, normalized size = 0.92

$$\frac{2\,\arctan\left(e^{(bx+a)}\right)}{b}-\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(b*x + a))/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

mupad [B] time = 0.08, size = 48, normalized size = 1.20

$$\frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^2,x)
```

```
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(b
*(exp(2*a + 2*b*x) + 1))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$e^a \int e^{bx} \operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**2, x)
```

3.270 $\int e^{a+bx}\mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

```
[Out] 2*exp(4*b*x+4*a)/b/(1+exp(2*b*x+2*a))^2
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 264}

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^3,x]
```

```
[Out] (2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{8x^3}{(1+x^2)^3}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{8\,\text{Subst}\left(\int\frac{x^3}{(1+x^2)^3}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{2e^{4a+4bx}}{b\left(1+e^{2a+2bx}\right)^2}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^3,x]
```

```
[Out] (2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

fricas [B] time = 0.68, size = 86, normalized size = 2.97

$$-\frac{2\,(3\,\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3+3\,b\cosh(bx+a)+\left(3\,b\cosh(bx+a)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a))
```

giac [A] time = 0.12, size = 31, normalized size = 1.07

$$-\frac{2\left(2\,e^{(2\,bx+2\,a)}+1\right)}{b\left(e^{(2\,bx+2\,a)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
[Out] -2*(2*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

maple [A] time = 0.21, size = 22, normalized size = 0.76

$$\frac{-\frac{1}{2\cosh(bx+a)^2}+\tanh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^3,x)
[Out] 1/b*(-1/2/cosh(b*x+a)^2+tanh(b*x+a))
```

maxima [B] time = 0.31, size = 68, normalized size = 2.34

$$-\frac{4\,e^{(2\,bx+2\,a)}}{b\left(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1\right)}-\frac{2}{b\left(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
[Out] -4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1)) - 2/(b*(e^
(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1))
```

mupad [B] time = 0.93, size = 31, normalized size = 1.07

$$-\frac{2\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{b\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^3,x)
[Out] -(2*(2*exp(2*a + 2*b*x) + 1))/(b*(exp(2*a + 2*b*x) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**3,x)
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**3, x)
```

3.271 $\int e^{a+bx}\mathbf{sech}^4(a+bx)\,dx$

Optimal. Leaf size=95

$$\frac{e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}-\frac{8e^{3a+3bx}}{3b\left(e^{2a+2bx}+1\right)^3}+\frac{\tan^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] -8/3*exp(3*b*x+3*a)/b/(1+exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1+exp(2*b*x+2*a)
)^2+exp(b*x+a)/b/(1+exp(2*b*x+2*a))+arctan(exp(b*x+a))/b
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2282, 12, 288, 199, 203}

$$\frac{e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}-\frac{8e^{3a+3bx}}{3b\left(e^{2a+2bx}+1\right)^3}+\frac{\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^4,x]
```

```
[Out] (-8*E^(3*a + 3*b*x))/(3*b*(1 + E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1
+ E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 + E^(2*a + 2*b*x))) + ArcTan[E^(a
 + b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{16x^4}{\left(1+x^2\right)^4}\,dx,x,e^{a+bx}\right)}{b}\\
&= \frac{16\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^4}\,dx,x,e^{a+bx}\right)}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3}+\frac{8\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^3}\,dx,x,e^{a+bx}\right)}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2}+\frac{2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^{a+bx}\right)}{b}\\
&= -\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{\tan^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 64, normalized size = 0.67

$$\frac{e^{a+bx}\left(-8e^{2(a+bx)}+3e^{4(a+bx)}-3\right)}{3b\left(e^{2(a+bx)}+1\right)^3}+\frac{\tan^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^4,x]
```

```
[Out] (E^(a + b*x)*(-3 - 8*E^(2*(a + b*x)) + 3*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(
a + b*x)))^3) + ArcTan[E^(a + b*x)]/b
```

fricas [B] time = 0.65, size = 513, normalized size = 5.40

$$\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4+3\sinh(bx+a)^5+2\left(15\cosh(bx+a)^2-4\right)\sinh(bx+a)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a)
^5 + 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^3 - 8*cosh(b*x + a)^3 + 6*(5*
cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6 + 6
*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1
)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x +
 a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b
*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + co
sh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + 3*(
5*cosh(b*x + a)^4 - 8*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))
/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6
 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5
*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)
^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*
(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) +
 b)
```

giac [A] time = 0.12, size = 60, normalized size = 0.63

$$\frac{\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^3}+3\,\arctan\left(e^{(bx+a)}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*((3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2*
a) + 1)^3 + 3*arctan(e^(b*x + a)))/b
```

maple [A] time = 0.22, size = 43, normalized size = 0.45

$$-\frac{1}{3b\cosh\left(bx+a\right)^{3}}+\frac{\operatorname{sech}\left(bx+a\right)\tanh\left(bx+a\right)}{2b}+\frac{\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^4,x)
```

```
[Out] -1/3/b/cosh(b*x+a)^3+1/2/b*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x+a))/b
```

maxima [A] time = 0.41, size = 83, normalized size = 0.87

$$\frac{\arctan\left(e^{(bx+a)}\right)}{b}+\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b\left(e^{(6\,bx+6\,a)}+3\,e^{(4\,bx+4\,a)}+3\,e^{(2\,bx+2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] arctan(e^(b*x + a))/b + 1/3*(3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b
*x + a))/(b*(e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) + 1))
```

mupad [B] time = 0.96, size = 130, normalized size = 1.37

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}-\frac{8\,\mathrm{e}^{3\,a+3\,b\,x}}{3\,b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)}+\frac{\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^4,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(3*ex
p(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a + b*x)
/(b*(exp(2*a + 2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**4, x)
```

3.272 $\int e^{a+bx}\mathbf{sech}^5(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{8}{b\left(e^{2a+2bx}+1\right)^2}+\frac{32}{3b\left(e^{2a+2bx}+1\right)^3}-\frac{4}{b\left(e^{2a+2bx}+1\right)^4}$$

```
[Out] -4/b/(1+exp(2*b*x+2*a))^4+32/3/b/(1+exp(2*b*x+2*a))^3-8/b/(1+exp(2*b*x+2*a)
)^2
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 266, 43}

$$-\frac{8}{b\left(e^{2a+2bx}+1\right)^2}+\frac{32}{3b\left(e^{2a+2bx}+1\right)^3}-\frac{4}{b\left(e^{2a+2bx}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sech[a + b*x]^5,x]
```

```
[Out] -4/(b*(1 + E^(2*a + 2*b*x))^4) + 32/(3*b*(1 + E^(2*a + 2*b*x))^3) - 8/(b*(1
 + E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{32x^5}{\left(1+x^2\right)^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{32\operatorname{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{16\operatorname{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{16\operatorname{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx, x, e^{2a+2bx}\right)}{b}\\
&= -\frac{4}{b\left(1+e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{8}{b\left(1+e^{2a+2bx}\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.73

$$-\frac{4\left(4e^{2(a+bx)}+6e^{4(a+bx)}+1\right)}{3b\left(e^{2(a+bx)}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sech[a + b*x]^5,x]
```

```
[Out] (-4*(1 + 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(a + b*x)))
^4)
```

fricas [B] time = 0.48, size = 233, normalized size = 3.88

$$-\frac{}{3\left(b\cosh(bx+a)^6+6\,b\cosh(bx+a)\sinh(bx+a)^5+b\sinh(bx+a)^6+4\,b\cosh(bx+a)^4+\left(15\,b\cosh(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)^
2 + 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x
+ a)^6 + 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 4*b)*sinh(b*x + a)^4
 + 4*(5*b*cosh(b*x + a)^3 + 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*cosh(b
*x + a)^2 + (15*b*cosh(b*x + a)^4 + 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b*x +
a)^2 + 2*(3*b*cosh(b*x + a)^5 + 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x + a))*si
nh(b*x + a) + 4*b)
```

giac [A] time = 0.12, size = 42, normalized size = 0.70

$$-\frac{4\left(6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^4
)
```

maple [A] time = 0.22, size = 35, normalized size = 0.58

$$\frac{-\frac{1}{4\cosh(bx+a)^{4}}+\left(\frac{2}{3}+\frac{\mathrm{sech}(bx+a)^{2}}{3}\right)\tanh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sech(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/4/cosh(b*x+a)^4+(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

maxima [B] time = 0.33, size = 172, normalized size = 2.87

$$-\frac{8\,e^{(4\,bx+4\,a)}}{b\left(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\right)}-\frac{16\,e^{(2\,bx+2\,a)}}{3\,b\left(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4
*a) + 4*e^(2*b*x + 2*a) + 1)) - 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a) +
```

```
4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b*(e
^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a)
+ 1))
```

mupad [B] time = 0.95, size = 42, normalized size = 0.70

$$-\frac{4\left(4\,{\mathrm{e}}^{2\,a+2\,b\,x}+6\,{\mathrm{e}}^{4\,a+4\,b\,x}+1\right)}{3\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/cosh(a + b*x)^5,x)
```

```
[Out] -(4*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) +
 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{sech}^{5}{\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sech(b*x+a)**5,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*sech(a + b*x)**5, x)
```

3.273 $\int e^x \cosh^2(2x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}$$

```
[Out] -1/12/exp(3*x)+1/2*exp(x)+1/20*exp(5*x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 270}

$$-\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[2*x]^2,x]
```

```
[Out] -1/(12*E^(3*x)) + E^x/2 + E^(5*x)/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \cosh^2(2x)\,dx &= \text{Subst}\left(\int \frac{\left(1+x^4\right)^2}{4x^4}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1+x^4\right)^2}{x^4}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(2+\frac{1}{x^4}+x^4\right)dx, x, e^x\right)\\ &= -\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$-\frac{1}{12}e^{-3x}+\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[2*x]^2,x]
```

```
[Out] -1/12*1/E^(3*x) + E^x/2 + E^(5*x)/20
```

fricas [B] time = 0.44, size = 47, normalized size = 1.81

$$-\frac{\cosh(x)^4 - 16\,\cosh(x)^3\sinh(x) + 6\,\cosh(x)^2\sinh(x)^2 - 16\,\cosh(x)\sinh(x)^3 + \sinh(x)^4 - 15}{30\,(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4 - 15)/(cosh(x) - sinh(x))
```

giac [A] time = 0.14, size = 17, normalized size = 0.65

$$\frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

maple [A] time = 0.10, size = 34, normalized size = 1.31

$$\frac{\sinh(x)}{2} + \frac{\sinh(3x)}{12} + \frac{\sinh(5x)}{20} + \frac{\cosh(x)}{2} - \frac{\cosh(3x)}{12} + \frac{\cosh(5x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(2*x)^2,x)
[Out] 1/2*sinh(x)+1/12*sinh(3*x)+1/20*sinh(5*x)+1/2*cosh(x)-1/12*cosh(3*x)+1/20*c
osh(5*x)
```

maxima [A] time = 0.31, size = 17, normalized size = 0.65

$$\frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)^2,x, algorithm="maxima")
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

mupad [B] time = 0.08, size = 17, normalized size = 0.65

$$\frac{e^{5\,x}}{20} - \frac{e^{-3\,x}}{12} + \frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)^2*exp(x),x)
[Out] exp(5*x)/20 - exp(-3*x)/12 + exp(x)/2
```

sympy [B] time = 0.65, size = 42, normalized size = 1.62

$$-\frac{8e^{x}\sinh^{2}(2x)}{15} + \frac{4e^{x}\sinh(2x)\cosh(2x)}{15} + \frac{7e^{x}\cosh^{2}(2x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x)**2,x)
[Out] -8*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 + 7*exp(x)*cosh
(2*x)**2/15
```

3.274 $\int e^x \cosh(2x)\,dx$

Optimal. Leaf size=19

$$\frac{e^{3x}}{6}-\frac{e^{-x}}{2}$$

```
[Out] -1/2/exp(x)+1/6*exp(3*x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{3x}}{6}-\frac{e^{-x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[2*x],x]
```

```
[Out] -1/(2*E^x) + E^(3*x)/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \cosh(2x)\,dx = \text{Subst}\left(\int \frac{1+x^4}{2x^2}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^4}{x^2}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x^2}+x^2\right)dx, x, e^x\right)$$
$$= -\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

Mathematica [**A**] time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{6}e^{-x}\left(e^{4x}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[2*x],x]
[Out] (-3 + E^(4*x))/(6*E^x)
```

fricas [**A**] time = 0.56, size = 26, normalized size = 1.37

$$-\frac{\cosh(x)^2 - 4\cosh(x)\sinh(x) + \sinh(x)^2}{3\,(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="fricas")
[Out] -1/3*(cosh(x)^2 - 4*cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

giac [**A**] time = 0.12, size = 13, normalized size = 0.68

$$\frac{1}{6}\,e^{(3\,x)} - \frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="giac")
[Out] 1/6*e^(3*x) - 1/2*e^(-x)
```

maple [A] time = 0.07, size = 22, normalized size = 1.16

$$\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}-\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(2*x),x)
```

```
[Out] 1/2*sinh(x)+1/6*sinh(3*x)-1/2*cosh(x)+1/6*cosh(3*x)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.68

$$\frac{1}{6}e^{(3x)}-\frac{1}{2}e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*e^(3*x) - 1/2*e^(-x)
```

mupad [B] time = 0.05, size = 12, normalized size = 0.63

$$\frac{e^{-x}\left(e^{4x}-3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(2*x)*exp(x),x)
```

```
[Out] (exp(-x)*(exp(4*x) - 3))/6
```

sympy [A] time = 0.25, size = 20, normalized size = 1.05

$$\frac{2e^{x}\sinh(2x)}{3}-\frac{e^{x}\cosh(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(2*x),x)
```

```
[Out] 2*exp(x)*sinh(2*x)/3 - exp(x)*cosh(2*x)/3
```

3.275 $\int e^x \textbf{sech}(2x)\,dx$

Optimal. Leaf size=92

$$\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/2*arctan(1+exp(x)*2^(1/2))*2^(1/2)+
1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2^(1/2))
*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2282, 12, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + Log[
1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*
Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \text{sech}(2x)\,dx = \text{Subst}\left(\int \frac{2x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, e^x\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-}{-1-\sqrt{2}}\right.}{2\sqrt{}}$$
$$= \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,e^x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.26

$$\frac{2}{3}e^{3x}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x],x]
[Out] (2*E^(3*x)*Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)])/3
```

fricas [A] time = 0.59, size = 113, normalized size = 1.23

$$-\sqrt{2}\,\arctan\left(-\sqrt{2}\,e^x+\sqrt{2}\,\sqrt{\sqrt{2}\,e^x+e^{(2\,x)}+1}-1\right)-\sqrt{2}\,\arctan\left(-\sqrt{2}\,e^x+\frac{1}{2}\,\sqrt{2}\,\sqrt{-4\,\sqrt{2}\,e^x+4\,e^{(2\,x)}+4}+1\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1) - 1)
 - sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x
) + 4) + 1) - 1/4*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1/4*sqrt(2)*
log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4)
```

giac [A] time = 0.12, size = 76, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

maple [C] time = 0.14, size = 25, normalized size = 0.27

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(256\_Z^{4}+1\right)}\_R\ln\left(64\_R^{3}+\mathrm{e}^{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x),x)
```

```
[Out] 2*sum(_R*ln(64*_R^3+exp(x)),_R=RootOf(256*_Z^4+1))
```

maxima [A] time = 0.42, size = 76, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

mupad [B] time = 1.11, size = 77, normalized size = 0.84

$$\sqrt{2}\,\ln\left(4+\sqrt{2}\,\mathrm{e}^{x}\,(-2-2\mathrm{i})\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\ln\left(4+\sqrt{2}\,\mathrm{e}^{x}\,(-2+2\mathrm{i})\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\ln\left(4+\sqrt{2}\,\mathrm{e}^{x}\,(2-2\mathrm{i})\right)\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(2*x),x)
```

```
[Out] 2^(1/2)*log(4 - 2^(1/2)*exp(x)*(2 + 2i))*(1/4 + 1i/4) + 2^(1/2)*log(4 - 2^(
1/2)*exp(x)*(2 - 2i))*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 - 2i) +
4)*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 + 2i) + 4)*(1/4 + 1i/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \operatorname{sech}(2x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x),x)
```

```
[Out] Integral(exp(x)*sech(2*x), x)
```

3.276 $\int e^x \operatorname{sech}^2(2x)\,dx$

Optimal. Leaf size=111

$$-\frac{e^x}{e^{4x}+1}-\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,e^x+1\right)}{2\sqrt{2}}$$

```
[Out] -exp(x)/(1+exp(4*x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/2)+1/4*arctan(1+exp
(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+exp
(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2282, 12, 288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{e^x}{e^{4x}+1}-\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,e^x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[2*x]^2,x]
```

```
[Out] -(E^x/(1 + E^(4*x))) - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqr
t[2]*E^x]/(2*Sqrt[2]) - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) + Log[1
+ Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \text{sech}^2(2x)\,dx = \text{Subst}\left(\int \frac{4x^4}{(1+x^4)^2}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^4}{(1+x^4)^2}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$
$$= -\frac{e^x}{1+e^{4x}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{\text{S}}{}$$
$$= -\frac{e^x}{1+e^{4x}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right.}{2\sqrt{2}}$$
$$= -\frac{e^x}{1+e^{4x}} - \frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^{\cdot}\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.09, size = 106, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{8e^x}{e^{4x}+1} - \sqrt{2}\log\left(-\sqrt{2}e^x+e^{2x}+1\right) + \sqrt{2}\log\left(\sqrt{2}e^x+e^{2x}+1\right) - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}e^x\right) + 2\sqrt{2}\tan^{-1}\left(\sqrt{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[2*x]^2,x]
```

```
[Out] ((-8*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^x] + 2*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E^x + E^(2*x)] + Sqrt[2]*Log
[1 + Sqrt[2]*E^x + E^(2*x)])/8
```

fricas [B] time = 0.54, size = 162, normalized size = 1.46

$$-\frac{4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\sqrt{2}\sqrt{\sqrt{2}e^x+e^{(2x)}+1}-1\right) + 4\left(\sqrt{2}e^{(4x)}+\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\frac{1}{2}\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(4*x) + sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - (sqrt(2)*e^(
4*x) + sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*e^(4*x) + sqr
t(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 8*e^x)/(e^(4*x) + 1)
```

giac [A] time = 0.12, size = 88, normalized size = 0.79

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

maple [C] time = 0.15, size = 36, normalized size = 0.32

$$-\frac{e^{x}}{1+e^{4x}}+4\left(\sum_{\_R=\mathrm{RootOf}\left(65536\_Z^{4}+1\right)}\_R\ln\left(e^{x}+16\_R\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(2*x)^2,x)
```

```
[Out] -exp(x)/(1+exp(4*x))+4*sum(_R*ln(exp(x)+16*_R),_R=RootOf(65536*_Z^4+1))
```

maxima [A] time = 0.42, size = 88, normalized size = 0.79

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

mupad [B] time = 1.09, size = 85, normalized size = 0.77

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\left(e^x-\frac{\sqrt{2}}{2}\right)\right)}{4}-\frac{e^x}{e^{4\,x}+1}+\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\left(e^x+\frac{\sqrt{2}}{2}\right)\right)}{4}-\frac{\sqrt{2}\,\ln\left(\left(e^x-\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}\right)}{8}+\frac{\sqrt{2}\,\ln\left(\left(e^x+\frac{\sqrt{2}}{2}\right)^2\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(2*x)^2,x)
```

```
[Out] (2^(1/2)*atan(2^(1/2)*(exp(x) - 2^(1/2)/2)))/4 - exp(x)/(exp(4*x) + 1) + (2
^(1/2)*atan(2^(1/2)*(exp(x) + 2^(1/2)/2)))/4 - (2^(1/2)*log((exp(x) - 2^(1/
2)/2)^2 + 1/2))/8 + (2^(1/2)*log((exp(x) + 2^(1/2)/2)^2 + 1/2))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \operatorname{sech}^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(2*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(2*x)**2, x)
```

3.277 $\int e^x \cosh^2(3x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

[Out] -1/20/exp(5*x)+1/2*exp(x)+1/28*exp(7*x)

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 270}

$$-\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

[In] Int[E^x*Cosh[3*x]^2,x]

[Out] -1/(20*E^(5*x)) + E^x/2 + E^(7*x)/28

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \cosh^2(3x)\,dx = \text{Subst}\left(\int \frac{\left(1+x^6\right)^2}{4x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1+x^6\right)^2}{x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(2+\frac{1}{x^6}+x^6\right)dx, x, e^x\right)$$
$$= -\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$-\frac{1}{20}e^{-5x}+\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[3*x]^2,x]
```

```
[Out] -1/20*1/E^(5*x) + E^x/2 + E^(7*x)/28
```

fricas [B] time = 0.47, size = 67, normalized size = 2.58

$$-\frac{\cosh(x)^6-36\cosh(x)^5\sinh(x)+15\cosh(x)^4\sinh(x)^2-120\cosh(x)^3\sinh(x)^3+15\cosh(x)^2\sinh(x)^4-36}{70\left(\cosh(x)-\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)^6
 - 35)/(cosh(x) - sinh(x))
```

giac [A] time = 0.13, size = 17, normalized size = 0.65

$$\frac{1}{28}e^{(7x)}-\frac{1}{20}e^{(-5x)}+\frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

maple [A] time = 0.10, size = 34, normalized size = 1.31

$$\frac{\sinh(x)}{2} + \frac{\sinh(5x)}{20} + \frac{\sinh(7x)}{28} + \frac{\cosh(x)}{2} - \frac{\cosh(5x)}{20} + \frac{\cosh(7x)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(3*x)^2,x)
```

```
[Out] 1/2*sinh(x)+1/20*sinh(5*x)+1/28*sinh(7*x)+1/2*cosh(x)-1/20*cosh(5*x)+1/28*c
osh(7*x)
```

maxima [A] time = 0.31, size = 17, normalized size = 0.65

$$\frac{1}{28} e^{(7x)} - \frac{1}{20} e^{(-5x)} + \frac{1}{2} e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

mupad [B] time = 0.08, size = 17, normalized size = 0.65

$$\frac{e^{7x}}{28} - \frac{e^{-5x}}{20} + \frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)^2*exp(x),x)
```

```
[Out] exp(7*x)/28 - exp(-5*x)/20 + exp(x)/2
```

sympy [B] time = 0.64, size = 42, normalized size = 1.62

$$-\frac{18e^{x}\sinh^{2}(3x)}{35} + \frac{6e^{x}\sinh(3x)\cosh(3x)}{35} + \frac{17e^{x}\cosh^{2}(3x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x)**2,x)
```

```
[Out] -18*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 + 17*exp(x)*co
sh(3*x)**2/35
```

3.278 $\int e^x \cosh(3x)\,dx$

Optimal. Leaf size=19

$$\frac{e^{4x}}{8}-\frac{1}{4}e^{-2x}$$

```
[Out] -1/4/exp(2*x)+1/8*exp(4*x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{4x}}{8}-\frac{1}{4}e^{-2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[3*x],x]
```

```
[Out] -1/(4*E^(2*x)) + E^(4*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x\cosh(3x)\,dx = \text{Subst}\left(\int \frac{1+x^6}{2x^3}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^6}{x^3}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^3}+x^3\right)\,dx, x, e^x\right)$$
$$= -\frac{1}{4}e^{-2x} + \frac{e^{4x}}{8}$$

Mathematica [**A**] time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{8}e^{-2x}\left(e^{6x}-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[3*x],x]
```

```
[Out] (-2 + E^(6*x))/(8*E^(2*x))
```

fricas [**B**] time = 0.63, size = 38, normalized size = 2.00

$$-\frac{\cosh(x)^3 - 9\,\cosh(x)^2\sinh(x) + 3\,\cosh(x)\sinh(x)^2 - 3\,\sinh(x)^3}{8\,(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(x)^3 - 9*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 - 3*sinh(x)^3)/
(cosh(x) - sinh(x))
```

giac [**A**] time = 0.11, size = 13, normalized size = 0.68

$$\frac{1}{8}\,e^{(4\,x)} - \frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="giac")
```

```
[Out] 1/8*e^(4*x) - 1/4*e^(-2*x)
```

maple [A] time = 0.10, size = 26, normalized size = 1.37

$$\frac{\sinh(2x)}{4} + \frac{\sinh(4x)}{8} - \frac{\cosh(2x)}{4} + \frac{\cosh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(3*x),x)
[Out] 1/4*sinh(2*x)+1/8*sinh(4*x)-1/4*cosh(2*x)+1/8*cosh(4*x)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.68

$$\frac{1}{8}e^{(4x)} - \frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x, algorithm="maxima")
[Out] 1/8*e^(4*x) - 1/4*e^(-2*x)
```

mupad [B] time = 0.93, size = 12, normalized size = 0.63

$$\frac{e^{-2x}\left(e^{6x}-2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3*x)*exp(x),x)
[Out] (exp(-2*x)*(exp(6*x) - 2))/8
```

sympy [A] time = 0.25, size = 20, normalized size = 1.05

$$\frac{3e^x\sinh(3x)}{8} - \frac{e^x\cosh(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(3*x),x)
[Out] 3*exp(x)*sinh(3*x)/8 - exp(x)*cosh(3*x)/8
```

3.279 $\int e^x \mathbf{sech}(3x)\,dx$

Optimal. Leaf size=55

$$-\frac{1}{3}\log\left(e^{2x}+1\right)+\frac{1}{6}\log\left(-e^{2x}+e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*ln(exp(2*x)+1)+1/6*ln(1-exp(2*x)+exp(4*x))-1/3*arctan(1/3*(1-2*exp(2*x
))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2282, 12, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{3}\log\left(e^{2x}+1\right)+\frac{1}{6}\log\left(-e^{2x}+e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[3*x],x]
```

```
[Out] -(ArcTan[(1 - 2*E^(2*x))/Sqrt[3]]/Sqrt[3]) - Log[1 + E^(2*x)]/3 + Log[1 - E
^(2*x) + E^(4*x)]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x \operatorname{sech}(3x)\,dx &= \text{Subst}\left(\int \frac{2x^3}{1+x^6}\,dx, x, e^x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, e^{2x}\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^{2x}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, e^{2x}\right)\\
&= -\frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, e^{2x}\right)\\
&= -\frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2e^{2x}\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.44

$$\frac{1}{2}e^{4x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[3*x],x]
```

```
[Out] (E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, -E^(6*x)])/2
```

fricas [A] time = 0.53, size = 83, normalized size = 1.51

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+3\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right)+\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)-\frac{1}{3}\log\left(\frac{2}{\cosh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) - s
inh(x))) + 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

giac [A] time = 0.11, size = 44, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}-1\right)\right)+\frac{1}{6}\log\left(e^{(4x)}-e^{(2x)}+1\right)-\frac{1}{3}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1)) + 1/6*log(e^(4*x) - e^(2*x)
 + 1) - 1/3*log(e^(2*x) + 1)
```

maple [C] time = 0.14, size = 79, normalized size = 1.44

$$-\frac{\ln\left(1+e^{2x}\right)}{3}+\frac{\ln\left(e^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(e^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(e^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(e^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(3*x),x)
```

```
[Out] -1/3*ln(1+exp(2*x))+1/6*ln(exp(2*x)-1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)-1/
2+1/2*I*3^(1/2))*3^(1/2)+1/6*ln(exp(2*x)-1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*
x)-1/2-1/2*I*3^(1/2))*3^(1/2)
```

maxima [A] time = 0.43, size = 71, normalized size = 1.29

$$-\frac{1}{3}\sqrt{3}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\sqrt{3}+2e^{x}\right)+\frac{1}{6}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)+\frac{1}{6}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(sqrt(3) + 2*e^x) + 1/3*sqrt(3)*arctan(-sqrt(3) + 2*e^x)
 + 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) + 1)
 - 1/3*log(e^(2*x) + 1)
```

mupad [B] time = 1.04, size = 65, normalized size = 1.18

$$-\frac{\ln\left(8\,e^{2x}+8\right)}{3}-\ln\left(24\,e^{2x}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)+8\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)+\ln\left(8-24\,e^{2x}\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(3*x),x)
```

```
[Out] log(8 - 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6))*((3^(1/2)*1i)/6 + 1/6) - log(24
*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) + 8)*((3^(1/2)*1i)/6 - 1/6) - log(8*exp(2*
x) + 8)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x),x)
```

```
[Out] Integral(exp(x)*sech(3*x), x)
```

3.280 $\int e^x \mathbf{sech}^2(3x)\,dx$

Optimal. Leaf size=110

$$-\frac{2e^x}{3\left(e^{6x}+1\right)}-\frac{\log\left(-\sqrt{3}\,e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{\log\left(\sqrt{3}\,e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{2}{9}\tan^{-1}\left(e^x\right)-\frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right)+\frac{1}{9}\tan^{-1}\left(2e^x+\right.$$

```
[Out] -2/3*exp(x)/(1+exp(6*x))+2/9*arctan(exp(x))+1/9*arctan(2*exp(x)-3^(1/2))+1/
9*arctan(2*exp(x)+3^(1/2))-1/18*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)+1/18*
ln(1+exp(2*x)+exp(x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2282, 12, 288, 209, 634, 618, 204, 628, 203}

$$-\frac{2e^x}{3\left(e^{6x}+1\right)}-\frac{\log\left(-\sqrt{3}\,e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{\log\left(\sqrt{3}\,e^x+e^{2x}+1\right)}{6\sqrt{3}}+\frac{2}{9}\tan^{-1}\left(e^x\right)-\frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right)+\frac{1}{9}\tan^{-1}\left(2e^x+\right.$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[3*x]^2,x]
```

```
[Out] (-2*E^x)/(3*(1 + E^(6*x))) + (2*ArcTan[E^x])/9 - ArcTan[Sqrt[3] - 2*E^x]/9
+ ArcTan[Sqrt[3] + 2*E^x]/9 - Log[1 - Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3]) +
Log[1 + Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x\text{sech}^2(3x)\,dx = \text{Subst}\left(\int \frac{4x^6}{\left(1+x^6\right)^2}\,dx,x,e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^6\right)^2}\,dx,x,e^x\right)$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^6}\,dx,x,e^x\right)$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)}+\frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^x\right)+\frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx,x,e^x\right)+\frac{2}{9}\text{Sul}$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)}+\frac{2}{9}\tan^{-1}\left(e^x\right)+\frac{1}{18}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,e^x\right)+\frac{1}{18}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}}\right.$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)}+\frac{2}{9}\tan^{-1}\left(e^x\right)-\frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}}+\frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}}-\frac{1}{9}\text{Subst}\left(\int\right.$$
$$= -\frac{2e^x}{3\left(1+e^{6x}\right)}+\frac{2}{9}\tan^{-1}\left(e^x\right)-\frac{1}{9}\tan^{-1}\left(\sqrt{3}-2e^x\right)+\frac{1}{9}\tan^{-1}\left(\sqrt{3}+2e^x\right)-\frac{\log\left(1-\sqrt{3}e^x+\right.}{6\sqrt{3}}$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.31

$$\frac{2}{3}e^x\left({}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-e^{6x}\right)-\frac{1}{e^{6x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[3*x]^2,x]
[Out] (2*E^x*(-(1 + E^(6*x))^(-1) + Hypergeometric2F1[1/6, 1, 7/6, -E^(6*x)]))/3
```

fricas [A] time = 0.77, size = 154, normalized size = 1.40

$$-\frac{4\left(e^{(6x)}+1\right)\arctan\left(\sqrt{3}+\sqrt{-4\sqrt{3}e^x+4e^{(2x)}+4}-2e^x\right)+4\left(e^{(6x)}+1\right)\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}e^x+e^{(2x)}+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(4*(e^(6*x) + 1)*arctan(sqrt(3) + sqrt(-4*sqrt(3)*e^x + 4*e^(2*x) + 4
) - 2*e^x) + 4*(e^(6*x) + 1)*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*e^x + e^(2*x)
 + 1) - 2*e^x) - 4*(e^(6*x) + 1)*arctan(e^x) - (sqrt(3)*e^(6*x) + sqrt(3))*
log(4*sqrt(3)*e^x + 4*e^(2*x) + 4) + (sqrt(3)*e^(6*x) + sqrt(3))*log(-4*sqr
t(3)*e^x + 4*e^(2*x) + 4) + 12*e^x)/(e^(6*x) + 1)
```

giac [A] time = 0.12, size = 79, normalized size = 0.72

$$\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{2e^{x}}{3\left(e^{(6x)}+1\right)}+\frac{1}{9}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{9}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^x
 + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1/9
*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

maple [C] time = 0.24, size = 59, normalized size = 0.54

$$-\frac{2\,\mathrm{e}^{x}}{3\left(1+\mathrm{e}^{6x}\right)}+\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{9}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{9}+4\left(\sum_{\_R=\mathrm{RootOf}\left(1679616\_Z^{4}-1296\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{x}+36\_R\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(3*x)^2,x)
```

```
[Out] -2/3*exp(x)/(1+exp(6*x))+1/9*I*ln(exp(x)+I)-1/9*I*ln(exp(x)-I)+4*sum(_R*ln(
exp(x)+36*_R),_R=RootOf(1679616*_Z^4-1296*_Z^2+1))
```

maxima [A] time = 0.42, size = 79, normalized size = 0.72

$$\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^{x}+e^{(2x)}+1\right)-\frac{2e^{x}}{3\left(e^{(6x)}+1\right)}+\frac{1}{9}\arctan\left(\sqrt{3}+2e^{x}\right)+\frac{1}{9}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^x
 + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1/9
*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

mupad [B] time = 0.31, size = 84, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\mathrm{e}^{x}\right)}{9}+\frac{\mathrm{atan}\left(2\,\mathrm{e}^{x}+\sqrt{3}\right)}{9}+\frac{\mathrm{atan}\left(2\,\mathrm{e}^{x}-\sqrt{3}\right)}{9}-\frac{2\,\mathrm{e}^{x}}{3\,\left(\mathrm{e}^{6\,x}+1\right)}-\frac{\sqrt{3}\,\ln\left(\left(\frac{2\,\mathrm{e}^{x}}{3}-\frac{\sqrt{3}}{3}\right)^{2}+\frac{1}{9}\right)}{18}+\frac{\sqrt{3}\,\ln\left(\left(\frac{2\,\mathrm{e}^{x}}{3}+\frac{\sqrt{3}}{3}\right.\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(3*x)^2,x)
```

```
[Out] (2*atan(exp(x)))/9 + atan(2*exp(x) + 3^(1/2))/9 + atan(2*exp(x) - 3^(1/2))/
9 - (2*exp(x))/(3*(exp(6*x) + 1)) - (3^(1/2)*log(((2*exp(x))/3 - 3^(1/2)/3)
^2 + 1/9))/18 + (3^(1/2)*log(((2*exp(x))/3 + 3^(1/2)/3)^2 + 1/9))/18
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}^{2}\left(3x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(3*x)**2,x)
```

```
[Out] Integral(exp(x)*sech(3*x)**2, x)
```

3.281 $\int e^x \cosh^2(4x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}$$

```
[Out] -1/28/exp(7*x)+1/2*exp(x)+1/36*exp(9*x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 270}

$$-\frac{1}{28}e^{-7x}+\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[4*x]^2,x]
```

```
[Out] -1/(28*E^(7*x)) + E^x/2 + E^(9*x)/36
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \cosh^2(4x)\,dx = \text{Subst}\left(\int \frac{(1+x^8)^2}{4x^8}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1+x^8)^2}{x^8}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(2+\frac{1}{x^8}+x^8\right)dx, x, e^x\right)$$
$$= -\frac{1}{28}e^{-7x} + \frac{e^x}{2} + \frac{e^{9x}}{36}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$-\frac{1}{28}e^{-7x} + \frac{e^x}{2} + \frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[4*x]^2,x]
```

```
[Out] -1/28*1/E^(7*x) + E^x/2 + E^(9*x)/36
```

fricas [B] time = 0.51, size = 87, normalized size = 3.35

$$-\frac{\cosh(x)^8 - 64\cosh(x)^7\sinh(x) + 28\cosh(x)^6\sinh(x)^2 - 448\cosh(x)^5\sinh(x)^3 + 70\cosh(x)^4\sinh(x)^4 - 44}{126\,(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*co
sh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 - 63)/(cosh(x) - sinh(
x))
```

giac [A] time = 0.13, size = 17, normalized size = 0.65

$$\frac{1}{36}e^{(9x)} - \frac{1}{28}e^{(-7x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

maple [A] time = 0.10, size = 34, normalized size = 1.31

$$\frac{\sinh(x)}{2}+\frac{\sinh(7x)}{28}+\frac{\sinh(9x)}{36}+\frac{\cosh(x)}{2}-\frac{\cosh(7x)}{28}+\frac{\cosh(9x)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(4*x)^2,x)
```

```
[Out] 1/2*sinh(x)+1/28*sinh(7*x)+1/36*sinh(9*x)+1/2*cosh(x)-1/28*cosh(7*x)+1/36*c
osh(9*x)
```

maxima [A] time = 0.33, size = 17, normalized size = 0.65

$$\frac{1}{36}\,e^{(9\,x)}-\frac{1}{28}\,e^{(-7\,x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

mupad [B] time = 0.96, size = 17, normalized size = 0.65

$$\frac{e^{9\,x}}{36}-\frac{e^{-7\,x}}{28}+\frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)^2*exp(x),x)
```

```
[Out] exp(9*x)/36 - exp(-7*x)/28 + exp(x)/2
```

sympy [B] time = 0.63, size = 42, normalized size = 1.62

$$-\frac{32e^{x}\sinh^{2}(4x)}{63}+\frac{8e^{x}\sinh(4x)\cosh(4x)}{63}+\frac{31e^{x}\cosh^{2}(4x)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x)**2,x)
```

```
[Out] -32*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 + 31*exp(x)*co
sh(4*x)**2/63
```

3.282 $\int e^x \cosh(4x) \, dx$

Optimal. Leaf size=19

$$\frac{e^{5x}}{10} - \frac{1}{6}e^{-3x}$$

[Out] -1/6/exp(3*x)+1/10*exp(5*x)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{5x}}{10} - \frac{1}{6}e^{-3x}$$

Antiderivative was successfully verified.

[In] Int[E^x*Cosh[4*x],x]

[Out] -1/(6*E^(3*x)) + E^(5*x)/10

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \cosh(4x)\,dx = \text{Subst}\left(\int \frac{1+x^8}{2x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^8}{x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^4}+x^4\right)\,dx, x, e^x\right)$$
$$= -\frac{1}{6}e^{-3x} + \frac{e^{5x}}{10}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{e^{5x}}{10} - \frac{1}{6}e^{-3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[4*x],x]
```

```
[Out] -1/6*1/E^(3*x) + E^(5*x)/10
```

fricas [B] time = 1.41, size = 46, normalized size = 2.42

$$-\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4}{15\,(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="fricas")
```

```
[Out] -1/15*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

giac [A] time = 0.14, size = 13, normalized size = 0.68

$$\frac{1}{10}\,e^{(5\,x)} - \frac{1}{6}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="giac")
```

```
[Out] 1/10*e^(5*x) - 1/6*e^(-3*x)
```

maple [**A**] time = 0.09, size = 26, normalized size = 1.37

$$\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}-\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(4*x),x)
[Out] 1/6*sinh(3*x)+1/10*sinh(5*x)-1/6*cosh(3*x)+1/10*cosh(5*x)
```

maxima [**A**] time = 0.32, size = 13, normalized size = 0.68

$$\frac{1}{10}\,e^{(5\,x)}-\frac{1}{6}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x, algorithm="maxima")
[Out] 1/10*e^(5*x) - 1/6*e^(-3*x)
```

mupad [**B**] time = 0.05, size = 14, normalized size = 0.74

$$\frac{\mathrm{e}^{-3\,x}\,\left(3\,\mathrm{e}^{8\,x}-5\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(4*x)*exp(x),x)
[Out] (exp(-3*x)*(3*exp(8*x) - 5))/30
```

sympy [**A**] time = 0.25, size = 20, normalized size = 1.05

$$\frac{4e^{x}\sinh(4x)}{15}-\frac{e^{x}\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(4*x),x)
[Out] 4*exp(x)*sinh(4*x)/15 - exp(x)*cosh(4*x)/15
```

3.283 $\int e^x \mathbf{sech}(4x)\,dx$

Optimal. Leaf size=371

$$-\frac{\log\left(-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2+\sqrt{2}}\,e^x+\right.}{4\sqrt{2\left(2+\sqrt{\right.}}}$$

```
[Out] -1/2*arctan((-2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^
(1/2)+1/2*arctan((2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/
2))^(1/2)-1/4*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1
/4*ln(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/2*arctan((
-2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/2*arc
tan((2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4
*ln(1+exp(2*x)-exp(x)*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/4*ln(1+exp(2
*x)+exp(x)*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2282, 12, 299, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}+1\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2+\sqrt{2}}\,e^x+\right.}{4\sqrt{2\left(2+\sqrt{\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[4*x],x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2
])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 -
 Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt
[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/
(2*Sqrt[2*(2 - Sqrt[2])]) - Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqr
t[2*(2 - Sqrt[2])]) + Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2
 - Sqrt[2])]) + Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqr
t[2])]) - Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(4*Sqrt[2*(2 + Sqrt[2])]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^x \operatorname{sech}(4x)\,dx &= \operatorname{Subst}\left(\int \frac{2x^4}{1+x^8}\,dx, x, e^x\right)\\
&= 2\operatorname{Subst}\left(\int \frac{x^4}{1+x^8}\,dx, x, e^x\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}} - \frac{\operatorname{Subst}\left(\int \frac{x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{2\sqrt{2}} + \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{2\sqrt{2}} + \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e\right.}{2\sqrt{2}}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, e^x\right)}{4\sqrt{2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+}\right.}{4\sqrt{2}}\\
&= -\frac{\log\left(1-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{2}}\,e^x+e^{2x}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} -\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\log}{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.06

$$\frac{2}{5}e^{5x}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};-e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[4*x],x]
```

```
[Out] (2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, -E^(8*x)])/5
```

fricas [B] time = 0.61, size = 1087, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqr
t(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-s
qrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/
(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2)
- sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt
(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) +
4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqr
t(2) + 2))) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*
arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sq
rt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) + sqrt(-s
qrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sqrt(
sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*
sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x +
4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
 - sqrt(-sqrt(2) + 2))) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-s
qrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2)
+ 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(
-sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2
) + 2)*e^x + 4*e^(2*x) + 4) - 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqr
t(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqr
t(2) + 2)*e^x + 4*e^(2*x) + 4) + 1/32*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*
sqrt(-sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-
sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - 1/4*sqrt(sqrt(2) + 2)*arctan((2*sqrt(sq
rt(sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(
2) + 2)) - 1/4*sqrt(sqrt(2) + 2)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x + e^
(2*x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(-sqr
t(2) + 2)*arctan((2*sqrt(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - sqrt(-sqrt
```

```
(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 1/4*sqrt(-sqrt(2) + 2)*arctan((2*sqr
t(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt
(sqrt(2) + 2)) - 1/16*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2*x
) + 1) + 1/16*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
+ 1/16*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/16*s
qrt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

giac [A] time = 0.21, size = 249, normalized size = 0.67

$$\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}}}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2)
) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2)
 + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(sq
rt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/s
qrt(sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
 + 1/8*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/8*sq
rt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

maple [C] time = 0.14, size = 25, normalized size = 0.07

$$2\left(\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^{8}+1\right)}\_R\ln\left(-32768\_R^{5}+\mathrm{e}^{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(4*x),x)
```

```
[Out] 2*sum(_R*ln(-32768*_R^5+exp(x)),_R=RootOf(16777216*_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x, algorithm="maxima")
```

```
[Out] integrate(e^x*sech(4*x), x)
```

mupad [B] time = 4.56, size = 479, normalized size = 1.29

$$-\ln\left(32768\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)^3-512\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)+\ln\left(32768\,\mathrm{e}^x\left(\frac{\sqrt{\sqrt{2}+2}}{8}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(4*x),x)
```

```
[Out] log(32768*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 + 5
12)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(x)
*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 - 512)*((2^(1/2) +
2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(x)*(((2^(1/2) + 2)
^(1/2)*1i)/8 - (2 - 2^(1/2))^(1/2)/8)^3 - 512)*(((2^(1/2) + 2)^(1/2)*1i)/8
- (2 - 2^(1/2))^(1/2)/8) + log(32768*exp(x)*(((2^(1/2) + 2)^(1/2)*1i)/8 - (
2 - 2^(1/2))^(1/2)/8)^3 + 512)*(((2^(1/2) + 2)^(1/2)*1i)/8 - (2 - 2^(1/2))^
(1/2)/8) + 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2
))^(1/2)*1i)/8)^3*(16384 - 16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^
(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2
)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 - 16384i) + 512)*((2^(1/2)
 + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) + 2^(1/2)*log(2^(1
/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 +
16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 -
1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^
(1/2)*1i)/8)^3*(16384 + 16384i) + 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/
2))^(1/2)*1i)/8)*(1/2 - 1i/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \operatorname{sech}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x),x)
```

```
[Out] Integral(exp(x)*sech(4*x), x)
```

3.284 $\int e^x \mathbf{sech}^2(4x)\,dx$

Optimal. Leaf size=379

$$-\frac{e^x}{2\left(e^{8x}+1\right)}-\frac{1}{32}\sqrt{2-\sqrt{2}}\,\log\left(-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)+\frac{1}{32}\sqrt{2-\sqrt{2}}\,\log\left(\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)-\frac{1}{32}\sqrt{2+\sqrt{2}}$$

```
[Out] -1/2*exp(x)/(1+exp(8*x))-1/32*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))*(2-2^
(1/2))^(1/2)+1/32*ln(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)
-1/8*arctan((-2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^
(1/2)+1/8*arctan((2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/
2))^(1/2)-1/32*ln(1+exp(2*x)-exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/
32*ln(1+exp(2*x)+exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/8*arctan((-2
*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/8*arcta
n((2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2282, 12, 288, 213, 1169, 634, 618, 204, 628}

$$-\frac{e^x}{2\left(e^{8x}+1\right)}-\frac{1}{32}\sqrt{2-\sqrt{2}}\,\log\left(-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)+\frac{1}{32}\sqrt{2-\sqrt{2}}\,\log\left(\sqrt{2-\sqrt{2}}\,e^x+e^{2x}+1\right)-\frac{1}{32}\sqrt{2+\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[4*x]^2,x]
```

```
[Out] -E^x/(2*(1 + E^(8*x))) - ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2
]]]/(8*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 -
 Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/S
qrt[2 + Sqrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2
*E^x)/Sqrt[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log
[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt
[2 - Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt
[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*E^x
+ E^(2*x)])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{sech}^2(4x)\,dx = \text{Subst}\left(\int \frac{4x^8}{\left(1+x^8\right)^2}\,dx, x, e^x\right)$$

$$= 4\,\text{Subst}\left(\int \frac{x^8}{\left(1+x^8\right)^2}\,dx, x, e^x\right)$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^8}\,dx, x, e^x\right)$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}}$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, e^x\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x,\right.}{8\sqrt{2\left(2-\sqrt{2}\right)}}$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} + \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx, x, e^x\right) + \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\right.}$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} - \frac{1}{32}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,e^x+e^{2x}\right) + \frac{1}{32}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,e\right.$$

$$= -\frac{e^x}{2\left(1+e^{8x}\right)} - \frac{1}{16}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{16}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right.$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.09

$$\frac{1}{2}e^x\left({}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{8x}\right)-\frac{1}{e^{8x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[4*x]^2,x]
[Out] (E^x*(-(1 + E^(8*x))^(-1) + Hypergeometric2F1[1/8, 1, 9/8, -E^(8*x)]))/2
```

fricas [B] time = 0.54, size = 1367, normalized size = 3.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="fricas")
[Out] -1/128*(8*sqrt(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(sqrt(sqrt(2) + 2)
*e^x + e^(2*x) + 1) - sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) + 8*sq
rt(-sqrt(2) + 2)*(e^(8*x) + 1)*arctan((2*sqrt(-sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(2) + 2)) - 2*sqrt(-sqrt(2)
 + 2)*(e^(8*x) + 1)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 2*sqrt(-sqr
t(2) + 2)*(e^(8*x) + 1)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 8*(sqr
t(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(sqrt(-sqrt(2) +
2)*e^x + e^(2*x) + 1) - sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2)) + 8*
(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*arctan((2*sqrt(-sqrt(-sqrt(
2) + 2)*e^x + e^(2*x) + 1) + sqrt(-sqrt(2) + 2) - 2*e^x)/sqrt(sqrt(2) + 2))
 + 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(
-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(2*sqrt(2)*e^x - sqrt(2)
*sqrt(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x +
4*e^(2*x) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
 + sqrt(-sqrt(2) + 2))) + 4*(sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e
^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan(-(
2*sqrt(2)*e^x - sqrt(2)*sqrt(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*s
qrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2)
+ 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 4*(sqrt(2)*sqrt(sqrt(2) +
 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt
(sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(2*sqrt(2)*sqrt(sqrt(2)
+ 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + sqrt(sqrt(2)
 + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 4*(
sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(
2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*arctan((2*sqrt(2)*e^x - sqrt(2)*sqrt(-
2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2
*x) + 4) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqr
```

```
t(-sqrt(2) + 2))) - (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)*e^(8*x) +
 sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqr
t(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) - (s
qrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2
) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2
*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) +
2)*e^(8*x) - (sqrt(2)*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(
sqrt(2) + 2))*log(-2*sqrt(2)*sqrt(sqrt(2) + 2)*e^x + 2*sqrt(2)*sqrt(-sqrt(2
) + 2)*e^x + 4*e^(2*x) + 4) + (sqrt(2)*sqrt(sqrt(2) + 2)*e^(8*x) + (sqrt(2)
*e^(8*x) + sqrt(2))*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(sqrt(2) + 2))*log(-2*
sqrt(2)*sqrt(sqrt(2) + 2)*e^x - 2*sqrt(2)*sqrt(-sqrt(2) + 2)*e^x + 4*e^(2*x
) + 4) - 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2))*log(sqrt(sqrt(2)
 + 2)*e^x + e^(2*x) + 1) + 2*(sqrt(sqrt(2) + 2)*e^(8*x) + sqrt(sqrt(2) + 2)
)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1) + 64*e^x)/(e^(8*x) + 1)
```

giac [A] time = 0.14, size = 261, normalized size = 0.69

$$\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
2)) + 1/16*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqr
t(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqr
t(sqrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^
x)/sqrt(sqrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x +
e^(2*x) + 1) - 1/32*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x)
+ 1) + 1/32*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) -
1/32*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) - 1/2*e^
x/(e^(8*x) + 1)
```

maple [C] time = 0.16, size = 36, normalized size = 0.09

$$-\frac{e^{x}}{2\left(1+e^{8x}\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(281474976710656\_Z^{8}+1\right)}\_R\ln\left(e^{x}+64\_R\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(4*x)^2,x)
```

```
[Out] -1/2*exp(x)/(1+exp(8*x))+4*sum(_R*ln(exp(x)+64*_R),_R=RootOf(28147497671065
6*_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{e^{x}}{2\left(e^{(8x)}+1\right)}+4\int \frac{e^{x}}{8\left(e^{(8x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e^x/(e^(8*x) + 1) + 4*integrate(1/8*e^x/(e^(8*x) + 1), x)
```

mupad [B] time = 3.47, size = 473, normalized size = 1.25

$$-\frac{e^{x}}{2\left(e^{8x}+1\right)}+\ln\left(-\frac{e^{x}}{2}-\frac{\sqrt{\sqrt{2}+2}}{4}-\frac{\sqrt{2-\sqrt{2}}\,1i}{4}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32}+\frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)-\ln\left(\frac{\sqrt{\sqrt{2}+2}}{4}-\frac{e^{x}}{2}+\frac{\sqrt{2-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/cosh(4*x)^2,x)
```

```
[Out] log(- exp(x)/2 - (2^(1/2) + 2)^(1/2)/4 - ((2 - 2^(1/2))^(1/2)*1i)/4)*((2^(1
/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) - exp(x)/(2*(exp(8*x) + 1)
) - log((2^(1/2) + 2)^(1/2)/4 - exp(x)/2 + ((2 - 2^(1/2))^(1/2)*1i)/4)*((2^
(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) + log((2 - 2^(1/2))^(1/2
)/4 - ((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2)*(((2^(1/2) + 2)^(1/2)*1i)/32 -
 (2 - 2^(1/2))^(1/2)/32) - log(((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2 - (2 -
 2^(1/2))^(1/2)/4)*(((2^(1/2) + 2)^(1/2)*1i)/32 - (2 - 2^(1/2))^(1/2)/32) +
 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^
(1/2)*1i)/32)*(4 + 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/
32)*(1/2 + 1i/2) + 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)/32
 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2
^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*((2^(1/2) + 2)^(1/
2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i) - exp(x)/2)*((2^(1/2) + 2)^(1
/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*((
2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 + 4i) - exp(x)/2)*(
(2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 + 1i/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{sech}^{2}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(4*x)**2,x)
[Out] Integral(exp(x)*sech(4*x)**2, x)
```

3.285 $\int F^{c(a+bx)}\cosh^3(d+ex)\,dx$

Optimal. Leaf size=202

$$-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}-\frac{6bce^2\log(F)\cosh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4}$$

```
[Out] -b*c*F^(c*(b*x+a))*cosh(e*x+d)^3*ln(F)/(9*e^2-b^2*c^2*ln(F)^2)-6*b*c*e^2*F^
(c*(b*x+a))*cosh(e*x+d)*ln(F)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4
)+3*e*F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)/(9*e^2-b^2*c^2*ln(F)^2)+6*e^3
*F^(c*(b*x+a))*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^4)
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5477, 5475}

$$\frac{6e^3\sinh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}-\frac{6bce^2\log(F)\cosh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
- (6*b*c*e^2*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(9*e^4 - 10*b^2*c^2*e^2*
Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d
+ e*x])/(9*e^2 - b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sinh[d + e*x])/
(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)} + \frac{3eF^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)}{9e^2-b^2c^2\log^2(F)} + \frac{\left(6e^2\right)\int}{}$$
$$= -\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)} - \frac{6bce^2F^{c(a+bx)}\cosh(d+ex)\log(F)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a}}{}$$

Mathematica [A] time = 0.70, size = 159, normalized size = 0.79

$$\frac{F^{c(a+bx)}\left(3\cosh(d+ex)\left(b^3c^3\log^3(F)-9bce^2\log(F)\right)+\cosh(3(d+ex))\left(b^3c^3\log^3(F)-bce^2\log(F)\right)+6e\sinh(d}{4\left(b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(3*Cosh[d + e*x]*(-9*b*c*e^2*Log[F] + b^3*c^3*Log[F]^3) +
Cosh[3*(d + e*x)]*(-(b*c*e^2*Log[F]) + b^3*c^3*Log[F]^3) + 6*e*(5*e^2 - b^2
*c^2*Log[F]^2 + Cosh[2*(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2))*Sinh[d + e*x]))
/(4*(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

fricas [B] time = 0.65, size = 2218, normalized size = 10.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c^3
*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*cosh
(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e*x +
 d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*l
og(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2 - (5*
b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*c^3*cos
h(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2 + b^3*
c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) + (5*b^3*c
^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^2*e*cos
h(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)
^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3 - 3*(b^2*c^2
*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x + d)^2
```

```
- b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*cosh(e*x + d)^2
+ (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*log(F)^
3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cosh(e*x + d)^2 - b
^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e^2*cosh(e*x + d)^
2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 + 27*b*c*
e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e^2)*log(F) + 6*(3*e
^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) + (b^3*c^
3*cosh(e*x + d)^5 + 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d))*log(
F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 + 2*b^2*c^2*e*cosh(e*x + d)^3 - b^2*c^2
*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 + 18*b*c*e^2*cosh(e*x
 + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*cosh((b*c*x + a*c
)*log(F)) + (3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(
F)^3 - 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(
b^3*c^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e
^2*cosh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cos
h(e*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3
*c^3)*log(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^
2 - (5*b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2
 + b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) + (
5*b^3*c^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^
2*e*cosh(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e
*x + d)^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3 - 3*(
b^2*c^2*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x
+ d)^2 - b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*cosh(e*x
+ d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*
log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cosh(e*x + d
)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e^2*cosh(e*
x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 +
27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e^2)*log(F) +
 6*(3*e^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) +
(b^3*c^3*cosh(e*x + d)^5 + 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d
))*log(F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 + 2*b^2*c^2*e*cosh(e*x + d)^3 -
b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 + 18*b*c*e^2*c
osh(e*x + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*sinh((b*c*
x + a*c)*log(F)))/(b^4*c^4*cosh(e*x + d)^3*log(F)^4 - 10*b^2*c^2*e^2*cosh(e
*x + d)^3*log(F)^2 + 9*e^4*cosh(e*x + d)^3 + (b^4*c^4*log(F)^4 - 10*b^2*c^2
*e^2*log(F)^2 + 9*e^4)*sinh(e*x + d)^3 + 3*(b^4*c^4*cosh(e*x + d)*log(F)^4
- 10*b^2*c^2*e^2*cosh(e*x + d)*log(F)^2 + 9*e^4*cosh(e*x + d))*sinh(e*x + d
)^2 + 3*(b^4*c^4*cosh(e*x + d)^2*log(F)^4 - 10*b^2*c^2*e^2*cosh(e*x + d)^2*
log(F)^2 + 9*e^4*cosh(e*x + d)^2)*sinh(e*x + d))
```

giac [C] time = 0.27, size = 1239, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*
c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3
*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 3/4*(2*(b*
c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn
(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2)
- (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi
*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/2*I*(-6*I*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*
c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) + 6*I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)
/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*log(
abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e)*cos(-
1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b
*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*
c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c
)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(
F)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*I*(-6*I*e^(1/2*I*pi*b*c*x*sgn(F) -
 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) -
8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e) + 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1
/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8
*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F
)) - e)*x - d) + 1/4*(2*(b*c*log(abs(F)) - 3*e)*cos(-1/2*pi*b*c*x*sgn(F) +
1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2
+ 4*(b*c*log(abs(F)) - 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x
*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) -
pi*b*c)^2 + 4*(b*c*log(abs(F)) - 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs
(F)) - 3*e)*x - 3*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*
x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 1
6*b*c*log(abs(F)) - 48*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
- 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16
*b*c*log(abs(F)) - 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x -
3*d)
```

maple [A] time = 0.29, size = 326, normalized size = 1.61

$$\frac{\left(\ln(F)^3 b^3 c^3 \mathrm{e}^{6ex+6d} + 3\ln(F)^3 b^3 c^3 \mathrm{e}^{4ex+4d} - 3\ln(F)^2 b^2 c^2 e\,\mathrm{e}^{6ex+6d} + 3\ln(F)^3 b^3 c^3 \mathrm{e}^{2ex+2d} - 3\ln(F)^2 b^2 c^2 e\,\mathrm{e}^{4ex+4d} - \mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^3,x)
```

```
[Out] 1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F
)^2*b^2*c^2*e*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)-3*ln(F)^2*b^2
*c^2*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)+ln(F)^3*b^3*c^3+3*ln(F)^
2*b^2*c^2*e*exp(2*e*x+2*d)-27*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+
6*d)+3*ln(F)^2*b^2*c^2*e-27*ln(F)*b*c*e^2*exp(2*e*x+2*d)+27*e^3*exp(4*e*x+4
*d)-ln(F)*b*c*e^2-27*e^3*exp(2*e*x+2*d)-3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d
)/(b*c*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

maxima [A] time = 0.35, size = 134, normalized size = 0.66

$$\frac{F^{ac}e^{\left(bcx\log(F)+3\,ex+3\,d\right)}}{8\left(bc\log(F)+3\,e\right)}+\frac{3\,F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{8\left(bc\log(F)+e\right)}+\frac{3\,F^{ac}e^{\left(bcx\log(F)-ex\right)}}{8\left(bce^{d}\log(F)-ee^{d}\right)}+\frac{F^{ac}e^{\left(bcx\log(F)-3\,ex\right)}}{8\left(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) + 3/8*F^(a*c)
*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F)
- e*x)/(b*c*e^d*log(F) - e*e^d) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(b*c
*e^(3*d)*log(F) - 3*e*e^(3*d))
```

mupad [B] time = 1.81, size = 154, normalized size = 0.76

$$\frac{F^{a\,c+b\,c\,x}\,\left(6\,e^3\,\sinh\left(d+e\,x\right)+3\,e^3\,\cosh\left(d+e\,x\right)^2\,\sinh\left(d+e\,x\right)+b^3\,c^3\,\cosh\left(d+e\,x\right)^3\,\ln(F)^3-b\,c\,e^2\,\cosh\left(d+e\right.\right.}{b^4\,c^4\,\ln(F)^4-10\,b^2\,c^2\,e^2\,\ln(F)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x)^3,x)
```

```
[Out] (F^(a*c + b*c*x)*(6*e^3*sinh(d + e*x) + 3*e^3*cosh(d + e*x)^2*sinh(d + e*x)
 + b^3*c^3*cosh(d + e*x)^3*log(F)^3 - b*c*e^2*cosh(d + e*x)^3*log(F) - 6*b*
c*e^2*cosh(d + e*x)*log(F) - 3*b^2*c^2*e*cosh(d + e*x)^2*sinh(d + e*x)*log(
F)^2))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(F)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.286 $\int F^{c(a+bx)} \cosh^2(d+ex)\,dx$

Optimal. Leaf size=132

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

```
[Out] 2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*cos
h(e*x+d)^2*ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*cosh(e*x+d)*sinh
(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5477, 2194}

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(c
*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(c*
(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5477

```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[F
^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*Si
nh[d + e*x]*Cosh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fre
eQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1
]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} + \frac{(2e^2)\int}{4e^2-b^2}$$

$$= \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} - \frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)} + \frac{2eF^{c(a+bx)}\cosh(}{4e^2-b}$$

Mathematica [A] time = 0.24, size = 85, normalized size = 0.64

$$\frac{F^{c(a+bx)}\left(b^2c^2\log^2(F)\cosh(2(d+ex))+b^2c^2\log^2(F)-2bce\log(F)\sinh(2(d+ex))-4e^2\right)}{2b^3c^3\log^3(F)-8bce^2\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(-4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log
[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*L
og[F]^3)
```

fricas [B] time = 1.13, size = 699, normalized size = 5.30

$$\frac{\left(\left(b^2c^2\log(F)^2-2\,bce\log(F)\right)\sinh(ex+d)^4-8\,e^2\cosh(ex+d)^2+4\left(b^2c^2\cosh(ex+d)\log(F)^2-2\,bce\cosh(ex\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x
+ d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*
sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 + b^
2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x +
 d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)
^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) + 4*e^2*cosh(e*x + d
) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x +
d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(
e*x + d)^4 - 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*
b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*
b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*lo
g(F) - (3*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d
)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3
*log(F) + 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x
```

```
 + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/(b^3*c^3*cosh(e
*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^3*c^3*log(F)^3 -
 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x + d)*log(F)^3 - 4*
b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

giac [C] time = 0.24, size = 903, normalized size = 6.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi
*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
(pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)
^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi
*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b
*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x +
2*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*c
*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sg
n(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F
)) - 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I
*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) + 2
*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I
*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

maple [A] time = 0.17, size = 143, normalized size = 1.08

$$\frac{\left(\ln(F)^2 b^2 c^2 e^{4ex+4d} + 2\ln(F)^2 b^2 c^2 e^{2ex+2d} - 2\ln(F) bce\, e^{4ex+4d} + b^2 c^2 \ln(F)^2 + 2\ln(F) bce - 8e^2 e^{2ex+2d}\right) e^{-2ex-2d} F^{c(}}{4\ln(F) bc\,(bc\ln(F) - 2e)\,(bc\ln(F) + 2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d)^2,x)
```

```
[Out] 1/4*(ln(F)^2*b^2*c^2*exp(4*e*x+4*d)+2*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)-2*ln(F
)*b*c*e*exp(4*e*x+4*d)+b^2*c^2*ln(F)^2+2*ln(F)*b*c*e-8*e^2*exp(2*e*x+2*d))/
ln(F)/b/c/(b*c*ln(F)-2*e)*exp(-2*e*x-2*d)/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

maxima [A] time = 0.35, size = 94, normalized size = 0.71

$$\frac{F^{ac}e^{\left(bcx\log(F)+2\,ex+2\,d\right)}}{4\left(bc\log(F)+2\,e\right)}+\frac{F^{ac}e^{\left(bcx\log(F)-2\,ex\right)}}{4\left(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)}\right)}+\frac{F^{bcx+ac}}{2\,bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)
*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/2*F^(b*c*x
 + a*c)/(b*c*log(F))
```

mupad [B] time = 1.25, size = 100, normalized size = 0.76

$$-\frac{2\,F^{a\,c+b\,c\,x}\,e^2-F^{a\,c+b\,c\,x}\,b^2\,c^2\,\mathrm{cosh}\left(d+e\,x\right)^2\,\ln\left(F\right)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\,\mathrm{cosh}\left(d+e\,x\right)\,\mathrm{sinh}\left(d+e\,x\right)\,\ln\left(F\right)}{b^3\,c^3\,\ln\left(F\right)^3-4\,b\,c\,e^2\,\ln\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x)^2,x)
```

```
[Out] -(2*F^(a*c + b*c*x)*e^2 - F^(a*c + b*c*x)*b^2*c^2*cosh(d + e*x)^2*log(F)^2
+ 2*F^(a*c + b*c*x)*b*c*e*cosh(d + e*x)*sinh(d + e*x)*log(F))/(b^3*c^3*log(
F)^3 - 4*b*c*e^2*log(F))
```

sympy [A] time = 30.57, size = 604, normalized size = 4.58

$$\begin{cases} -\frac{x\sinh^2(d+ex)}{2} + \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ \tilde{\infty}e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex) + \tilde{\infty}e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\sinh(d+ex)\cosh(d+ex) + \tilde{\infty}e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{b} \\ \tilde{\infty}e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex) + \tilde{\infty}e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\sinh(d+ex)\cosh(d+ex) + \tilde{\infty}e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\cosh \\ F^{ac}\left(-\frac{x\sinh^2(d+ex)}{2} + \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e}\right) \\ -\frac{x\sinh^2(d+ex)}{2} + \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ \frac{F^{ac}F^{bcx}b^2c^2\log(F)^2\cosh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac}F^{bcx}bce\log(F)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} + \frac{2F^{ac}F^{bcx}e^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac}F^{bcx}e^2\cosh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d)**2,x)
```

```
[Out] Piecewise((-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cos
h(d + e*x)/(2*e), Eq(F, 1)), (zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c
))**(b*c*x)*sinh(d + e*x)**2 + zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*
c))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**2*exp(-2*e/(b*c))**(a*c)*
exp(-2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e*
*2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2 + zoo*e**
2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x)
 + zoo*e**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2,
 Eq(F, exp(2*e/(b*c)))), (F**(a*c)*(-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)
**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (-x*sinh(d + e*x)**2
/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (
F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*cosh(d + e*x)**2/(b**3*c**3*log(F)*
*3 - 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sinh(d + e*x)*
cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a*c)*F**(b*
c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F*
*(a*c)*F**(b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*l
og(F)), True))
```

3.287 $\int F^{c(a+bx)}\cosh(d+ex)\,dx$

Optimal. Leaf size=75

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

```
[Out] -b*c*F^(c*(b*x+a))*cosh(e*x+d)*ln(F)/(e^2-b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))*
sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5475}

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

```
[Out] -((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (e
*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx=-\frac{bcF^{c(a+bx)}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

Mathematica [A] time = 0.12, size = 50, normalized size = 0.67

$$\frac{F^{c(a+bx)}(e\sinh(d+ex)-bc\log(F)\cosh(d+ex))}{(e-bc\log(F))(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) + e*Sinh[d + e*x]))/((e - b*c
*Log[F])*(e + b*c*Log[F]))
```

fricas [B] time = 0.55, size = 246, normalized size = 3.28

$$-\frac{\left(e\cosh\left(ex+d\right)^{2}-\left(bc\log(F)-e\right)\sinh\left(ex+d\right)^{2}-\left(bc\cosh\left(ex+d\right)^{2}+bc\right)\log(F)-2\left(bc\cosh\left(ex+d\right)\log(F\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x
 + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh
(e*x + d) - e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*log(F
) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e
*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) - e)*sinh((b*c*x + a*c)*log
(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*log(F)
^2 - e^2)*sinh(e*x + d))
```

giac [C] time = 0.21, size = 611, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] (2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/4*I*(-
2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I
*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) + 2*I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)
/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)
) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c
*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F)
- pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c
*log(abs(F)) - e)*x - d) - 1/4*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*
b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2
```

```
*b*c*log(abs(F)) - 2*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*lo
g(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

maple [A] time = 0.07, size = 74, normalized size = 0.99

$$\frac{\left(\ln(F)bc\,e^{2ex+2d}+bc\ln(F)-e\,e^{2ex+2d}+e\right)e^{-ex-d}F^{c(bx+a)}}{2\,(bc\ln(F)-e)\,(e+bc\ln(F))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cosh(e*x+d),x)
```

```
[Out] 1/2*(ln(F)*b*c*exp(2*e*x+2*d)+b*c*ln(F)-e*exp(2*e*x+2*d)+e)/(b*c*ln(F)-e)*e
xp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

maxima [A] time = 0.34, size = 63, normalized size = 0.84

$$\frac{F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{2\left(bc\log(F)+e\right)}+\frac{F^{ac}e^{\left(bcx\log(F)-ex\right)}}{2\left(bce^{d}\log(F)-ee^{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 1/2*F^(a*c)*e^(b*
c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

mupad [B] time = 1.01, size = 74, normalized size = 0.99

$$-\frac{F^{a\,c+b\,c\,x}\,e^{-d-e\,x}\,\left(e-e\,e^{2\,d+2\,e\,x}+b\,c\,\ln(F)+b\,c\,e^{2\,d+2\,e\,x}\,\ln(F)\right)}{2\,\left(e^2-b^2\,c^2\ln(F)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*cosh(d + e*x),x)
```

```
[Out] -(F^(a*c + b*c*x)*exp(- d - e*x)*(e - e*exp(2*d + 2*e*x) + b*c*log(F) + b*c
*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

sympy [A] time = 6.47, size = 316, normalized size = 4.21

$$\begin{cases} -\frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\sinh(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\cosh(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\sinh(d+ex)}{2e} & \text{for } F=-1 \wedge b=-\frac{ie}{\pi c} \\ x\cosh(d) & \text{for } F=1 \wedge e=0 \\ \tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F=e^{-\frac{e}{bc}} \\ \tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F=e^{\frac{e}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\cosh(d+ex)}{b^2c^2\log(F)^2-e^2} - \frac{F^{ac}F^{bcx}e\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cosh(e*x+d),x)
```

```
[Out] Piecewise((-(-1)**(a*c)*(-1)**(-I*e*x/pi)*x*sinh(d + e*x)/2 + (-1)**(a*c)*(
-1)**(-I*e*x/pi)*x*cosh(d + e*x)/2 + (-1)**(a*c)*(-1)**(-I*e*x/pi)*sinh(d +
 e*x)/(2*e), Eq(F, -1) & Eq(b, -I*e/(pi*c))), (x*cosh(d), Eq(F, 1) & Eq(e,
0)), (zoo*e*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo
*e*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/
(b*c)))), (zoo*e*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) +
zoo*e*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/
(b*c)))), (F**(a*c)*F**(b*c*x)*b*c*log(F)*cosh(d + e*x)/(b**2*c**2*log(F)**
2 - e**2) - F**(a*c)*F**(b*c*x)*e*sinh(d + e*x)/(b**2*c**2*log(F)**2 - e**2
), True))
```

3.288 $\int F^{c(a+bx)}\mathrm{sech}(d+ex)\,dx$

Optimal. Leaf size=68

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

```
[Out] 2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*
ln(F)/e],-exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5492}

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x],x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
 (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\mathrm{sech}(d+ex)\,dx=\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.03

$$\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{bc\log(F)}{2e}+\frac{1}{2};\frac{bc\log(F)}{2e}+\frac{3}{2};-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x],x]
```

```
[Out] (2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 + (b*c*Log[F])/(2*e
), 3/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\operatorname{sech}\left(ex+d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\operatorname{sech}\left(ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\operatorname{sech}\left(ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-4\,F^{ac}e\int\frac{e^{\left(bcx\log(F)+ex+d\right)}}{bc\log(F)+\left(bce^{(4\,d)}\log(F)-ee^{(4\,d)}\right)e^{(4\,ex)}+2\left(bce^{(2\,d)}\log(F)-ee^{(2\,d)}\right)e^{(2\,ex)}-e}\,dx+\frac{2\,F^{ac}e^{(}}{bc\log(F)+\left(bce^{(2\,}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="maxima")
```

```
[Out] -4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*d
)*log(F) - e*e^(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2*e
*x) - e), x) + 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(2
*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x),x)
```

```
[Out] int(F^(c*(a + b*x))/cosh(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{sech}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x), x)
```

3.289 $\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$

Optimal. Leaf size=70

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

```
[Out] 4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c*
ln(F)/e],-exp(2*e*x+2*d))/(b*c*ln(F)+2*e)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5492}

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx=\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{sech}\,(ex+d)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{sech}\,(ex+d)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\,F^{ac}bce\int\frac{}{b^2c^2\log(F)^2-6\,bce\log(F)+8\,e^2+\left(b^2c^2e^{(6\,d)}\log(F)^2-6\,bcee^{(6\,d)}\log(F)+8\,e^2e^{(6\,d)}\right)e^{(6\,ex)}+3\left(b^2c^2e^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) + 8
*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d))*
e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2*e^
(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F) + 8
*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) - 4*(4*F^(a*c)*e - (F^(a*c)*b*c*e^(2*d)
*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b
*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) +
8*e^2*e^(4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*lo
g(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x)^2,x)
```

```
[Out] int(F^(c*(a + b*x))/cosh(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sech(d + e*x)**2, x)
```

3.290 $\int F^{c(a+bx)}\operatorname{sech}^3(d+ex)\,dx$

Optimal. Leaf size=124

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{e^2}+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+}{}$$

[Out] exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],-exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2+1/2*b*c*F^(c*(b*x+a))*ln(F)*sech(e*x+d)/e^2+1/2*F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)/e

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5490, 5492}

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)}{e^2}+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+}{}$$

Antiderivative was successfully verified.

[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^3,x]

[Out] (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]))/e^2 + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x])/(2*e)

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{bcF^{c(a+bx)}\log(F)\text{sech}(d+ex)}{2e^2} + \frac{F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)}{2e} + \frac{1}{2}\left(1-\frac{b^2c^2 l}{}\right.$$

$$= \frac{e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);-e^{2(d+ex)}\right)(e-bc\log(F))}{e^2} + \frac{bcF^{c(a+b}}{}$$

Mathematica [A] time = 0.26, size = 96, normalized size = 0.77

$$\frac{F^{c(a+bx)}\left(2e^{d+ex}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);-e^{2(d+ex)}\right)+\text{sech}(d+ex)(bc\log(F)+e\tanh(d+\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(2*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]) + Sech[d + e*x
]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{sech}\,(ex+d)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^3, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{sech}\,(ex+d)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$48\left(F^{ac}bcee^{d}\log(F)+F^{ac}e^{2}e^{d}\right)\int\frac{}{b^{2}c^{2}\log(F)^{2}-8\,bce\log(F)+15\,e^{2}+\left(b^{2}c^{2}e^{(8\,d)}\log(F)^{2}-8\,bcee^{(8\,d)}\log(F)+15\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F) +
 e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(F)
^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) + 4*(b^2*c^2*e^(6*d
)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^2*c^
2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x) + 4
*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*
e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) - (F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^
(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) +
15*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^
2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F)
 + 15*e^2*e^(2*d))*e^(2*e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x)^3,x)
```

```
[Out] int(F^(c*(a + b*x))/cosh(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**3,x)
```

```
[Out] Timed out
```

3.291 $\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx$

Optimal. Leaf size=133

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{3e^2}+\frac{bc\log(F)\operatorname{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh}{}$$

```
[Out] 2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*
c*ln(F)/e],-exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2+1/6*b*c*F^(c*(b*x+a))*ln(F)
*sech(e*x+d)^2/e^2+1/3*F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d)/e
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5490, 5492}

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)}{3e^2}+\frac{bc\log(F)\operatorname{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

```
[Out] (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
+ (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x]^2)/(6*e^2) + (F^(c*(a + b*x))*S
ech[d + e*x]^2*Tanh[d + e*x])/(3*e)
```

Rule 5490

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sech[d + e*x]^(n - 2))/(e^2*(n - 1)
*(n - 2)), x] + (Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n -
2)), Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*
x))*Sech[d + e*x]^(n - 1)*Sinh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b
, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
NeQ[n, 2]
```

Rule 5492

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[(2^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
+ (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e*n
 + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \frac{bcF^{c(a+bx)}\log(F)\text{sech}^2(d+ex)}{6e^2} + \frac{F^{c(a+bx)}\text{sech}^2(d+ex)\tanh(d+ex)}{3e} + \frac{1}{6}\left(4-\frac{b^2c^2\right.$$

$$= \frac{2e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};-e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2} + \frac{bcF^{c(a+}}{}$$

Mathematica [**A**] time = 0.20, size = 101, normalized size = 0.76

$$\frac{F^{c(a+bx)}\left(4e^{2(d+ex)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;-e^{2(d+ex)}\right)+\text{sech}^2(d+ex)(bc\log(F)+2e\tanh(d\right.}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(4*E^(2*(d + e*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]) + Sech[d
 + e*x]^2*(b*c*Log[F] + 2*e*Tanh[d + e*x])))/(6*e^2)
```

fricas [**F**] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{sech}\left(ex+d\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sech(e*x + d)^4, x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{sech}\left(ex+d\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sech(e*x + d)^4, x)
```

maple [**F**] time = 0.15, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{sech}\left(ex+d\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(F^(b
*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 192*
e^3 + (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*b*c
*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d)*lo
g(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e
^3*e^(8*d))*e^(8*e*x) + 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)
*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 10*(b
^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*
d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) + 5*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b
^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e
^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F^(a*c)*b^2*
c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(a*c)*e^2*e^(
4*d))*e^(4*e*x) - 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*e^2*e^(2*d))*
e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^
2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F
)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*
e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(
F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*
e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x
) + 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e
^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e^(2*e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/cosh(d + e*x)^4,x)
```

```
[Out] int(F^(c*(a + b*x))/cosh(d + e*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sech(e*x+d)**4,x)
```

```
[Out] Timed out
```

3.292 $\int e^{c(a+bx)} \cosh^2(ac + bcx)^{5/2} \, dx$

Optimal. Leaf size=250

$$-\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\cosh^2(a}{3}$$

```
[Out] -1/128*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(4*c*(b*x+a))-5/64*
sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+5/32*exp(2*c
*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+5/128*exp(4*c*(b*x+
a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/192*exp(6*c*(b*x+a))*se
ch(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+5/16*x*sech(b*c*x+a*c)*(cosh(b*
c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\cosh^2(a}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] -(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(128*b*c*E^(4*c*(a + b*x)))
- (5*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))
) + (5*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32*b
*c) + (5*E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(12
8*b*c) + (E^(6*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(1
92*b*c) + (5*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx = \left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \frac{(1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \frac{(1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc}$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \frac{(1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc}$$

$$= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \left(10+\frac{1}{x^3}+\frac{5}{x^2}+\frac{10}{x}+5x+x^2\right) d\right.}{64bc}$$

$$= -\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)}{128bc} - \frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{se}}{64bc}$$

Mathematica [A] time = 0.11, size = 106, normalized size = 0.42

$$\frac{\left(-\frac{1}{2}e^{-4c(a+bx)}-5e^{-2c(a+bx)}+10e^{2c(a+bx)}+\frac{5}{2}e^{4c(a+bx)}+\frac{1}{3}e^{6c(a+bx)}+20bcx\right)\cosh^2(c(a+bx))^{5/2}\operatorname{sech}^5(c(a+bx))}{64bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((-1/2*1/E^(4*c*(a + b*x)) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) + (
5*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 + 20*b*c*x)*(Cosh[c*(a + b*x)]
^2)^(5/2)*Sech[c*(a + b*x)]^5)/(64*b*c)
```

fricas [A] time = 0.65, size = 218, normalized size = 0.87

$$-\frac{\cosh(bcx+ac)^5+5\,\cosh(bcx+ac)\sinh(bcx+ac)^4-5\,\sinh(bcx+ac)^5-5\left(10\,\cosh(bcx+ac)^2+9\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*s
inh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 + 1
5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*sin
h(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x + a
*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*c*cos
h(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.13, size = 101, normalized size = 0.40

$$\frac{120\,bcx-3\left(30\,e^{(4\,bcx+4\,ac)}+10\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-4\,bcx-4\,ac)}+\left(2\,e^{(6\,bcx+18\,ac)}+15\,e^{(4\,bcx+16\,ac)}+60\,e^{(2\,bcx+14\,ac)}\right)e^{(-1}}{384\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(120*b*c*x - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a*c) + 1)*
e^(-4*b*c*x - 4*a*c) + (2*e^(6*b*c*x + 18*a*c) + 15*e^(4*b*c*x + 16*a*c) +
60*e^(2*b*c*x + 14*a*c))*e^(-12*a*c))/(b*c)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\left(\frac{\cosh(2bcx+2ac)}{2}+\frac{1}{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x)
```

maxima [A] time = 0.34, size = 112, normalized size = 0.45

$$\frac{5\,(bcx+ac)}{16\,bc}+\frac{e^{(6\,bcx+6\,ac)}}{192\,bc}+\frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc}+\frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc}-\frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc}-\frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*c
*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2*a
*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{e}^{c\,(a+b\,x)}\,\left(\cosh\left(a\,c+b\,c\,x\right)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.293 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=162

$$-\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\cosh^2(ac+$$

```
[Out] -1/16*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+3/16*e
xp(2*c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/32*exp(4*c*
(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+3/8*x*sech(b*c*x+a*c
)*(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\cosh^2(ac+$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(16*b*c*E^(2*c*(a + b*x))) +
 (3*E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(16*b*c)
 + (E^(4*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(32*b*c)
 + (3*x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx &= \left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh^3(ac+bcx)\,dx\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc}\\
&= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)\right)\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2c(a+bx)}\right)}{16bc}\\
&= -\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{sech}(ac+bcx)}{16bc}+\frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\text{sec}}{16bc}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 78, normalized size = 0.48

$$\frac{\left(-e^{-2c(a+bx)}+3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\cosh^2(c(a+bx))^{3/2}\text{sech}^3(c(a+bx))}{16bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((-E^(-2*c*(a + b*x)) + 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x
)*(Cosh[c*(a + b*x)]^2)^(3/2)*Sech[c*(a + b*x)]^3)/(16*b*c)
```

fricas [A] time = 0.52, size = 126, normalized size = 0.78

$$-\frac{\cosh(bcx+ac)^{3}+3\cosh(bcx+ac)\sinh(bcx+ac)^{2}-3\sinh(bcx+ac)^{3}-6(2bcx+1)\cosh(bcx+ac)+3}{32(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*si
nh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*cosh
(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

giac [A] time = 0.14, size = 73, normalized size = 0.45

$$\frac{12\,bcx-2\left(3\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-2\,bcx-2\,ac)}+\left(e^{(4\,bcx+8\,ac)}+6\,e^{(2\,bcx+6\,ac)}\right)e^{(-4\,ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 2*a*c) + (e^(4
*b*c*x + 8*a*c) + 6*e^(2*b*c*x + 6*a*c))*e^(-4*a*c))/(b*c)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\left(\frac{\cosh(2bcx+2ac)}{2}+\frac{1}{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x)
```

maxima [A] time = 0.34, size = 74, normalized size = 0.46

$$\frac{3\,(bcx+ac)}{8\,bc}+\frac{e^{(4\,bcx+4\,ac)}}{32\,bc}+\frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc}-\frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
+ 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int e^{c(a+bx)} \left(\cosh (a c + b c x)^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.294 $\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)$$

```
[Out] 1/4*exp(2*c*(b*x+a))*sech(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)/b/c+1/2*x*se
ch(b*c*x+a*c)*(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/(4*b*c) + (
x*Sqrt[Cosh[a*c + b*c*x]^2]*Sech[a*c + b*c*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx &= \left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\int e^{c(a+bx)}\cosh(ac+bcx)\,dx\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int\frac{1+x^2}{2x}\,dx,x,e^{c(a+bx)}\right)}{bc}\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int\frac{1+x^2}{x}\,dx,x,e^{c(a+bx)}\right)}{2bc}\\ &= \frac{\left(\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)\right)\mathrm{Subst}\left(\int\left(\frac{1}{x}+x\right)dx,x,e^{c(a+bx)}\right)}{2bc}\\ &= \frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\,\mathrm{sech}(ac+bc\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}+2bcx\right)\sqrt{\cosh^2(c(a+bx))}\,\mathrm{sech}(c(a+bx))}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
[Out] ((E^(2*c*(a + b*x)) + 2*b*c*x)*Sqrt[Cosh[c*(a + b*x)]^2]*Sech[c*(a + b*x)])
/(4*b*c)
```

fricas [A] time = 0.52, size = 66, normalized size = 0.89

$$\frac{(2\,bcx+1)\cosh(bcx+ac)-(2\,bcx-1)\sinh(bcx+ac)}{4\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.11, size = 23, normalized size = 0.31

$$\frac{1}{2}\,x + \frac{e^{\left(2\,bcx + 2\,ac\right)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*x + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int e^{c(bx+a)}\sqrt{\frac{\cosh\left(2bcx+2ac\right)}{2}+\frac{1}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x)
[Out] int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x)
```

maxima [A] time = 0.34, size = 29, normalized size = 0.39

$$\frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{\left(2\,bcx + 2\,ac\right)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

mupad [B] time = 0.12, size = 76, normalized size = 1.03

$$\frac{\left(x\,e^{a\,c+b\,c\,x}+\frac{e^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{e^{2\,a\,c+2\,b\,c\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(1/2),x)
[Out] ((x*exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2 +
 exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) + 1)
```

sympy [A] time = 16.61, size = 204, normalized size = 2.76

$$\begin{cases} 0 & \text{fo} \\ x & \text{fo} \\ x\sqrt{\cosh^2(ac)}\,e^{ac} & \text{fo} \\ -\frac{x\sqrt{\cosh^2(ac+bcx)}\,e^{ac}e^{bcx}\sinh(ac+bcx)}{2\cosh(ac+bcx)} + \frac{x\sqrt{\cosh^2(ac+bcx)}\,e^{ac}e^{bcx}}{2} + \frac{\sqrt{\cosh^2(ac+bcx)}\,e^{ac}e^{bcx}\sinh(ac+bcx)}{bc\cosh(ac+bcx)} - \frac{\sqrt{\cosh^2(ac+bcx)}\,e^{ac}e^{bcx}}{2bc} & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Piecewise((0, Eq(a, log(I*exp(-b*c*x))/c) | Eq(a, log(-I*exp(-b*c*x))/c)),
(x, Eq(c, 0)), (x*sqrt(cosh(a*c)**2)*exp(a*c), Eq(b, 0)), (-x*sqrt(cosh(a*c
 + b*c*x)**2)*exp(a*c)*exp(b*c*x)*sinh(a*c + b*c*x)/(2*cosh(a*c + b*c*x)) +
 x*sqrt(cosh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)/2 + sqrt(cosh(a*c + b*c*x
)**2)*exp(a*c)*exp(b*c*x)*sinh(a*c + b*c*x)/(b*c*cosh(a*c + b*c*x)) - sqrt(
cosh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)/(2*b*c), True))
```

3.295 $\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$

Optimal. Leaf size=44

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))/b/c/(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[a*c + b*c*x]^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}$$
$$= \frac{\cosh(ac+bcx)\operatorname{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$
$$= \frac{(2\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$
$$= \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

Mathematica [**A**] time = 0.06, size = 42, normalized size = 0.95

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(c(a+bx))}{bc\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[c*(a + b*x)]^
2])
```

fricas [**A**] time = 0.56, size = 42, normalized size = 0.95

$$\frac{\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

giac [A] time = 0.12, size = 20, normalized size = 0.45

$$\frac{\log\left(e^{(2\,bcx)} + e^{(-2\,ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{2\,\mathrm{e}^{c(bx+a)}}{\sqrt{2\cosh\left(2bcx+2ac\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x)
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x)
```

maxima [A] time = 0.43, size = 21, normalized size = 0.48

$$\frac{\log\left(e^{(2\,bcx+2\,ac)} + 1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{{\cosh\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(cosh(a*c + b*c*x)**2), x)
```

3.296 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] 2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+a
*c)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 264}

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx &= \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}^3(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{\cosh(ac+bcx)\,\mathrm{Subst}\left(\int \frac{8x^3}{\left(1+x^2\right)^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{(8\cosh(ac+bcx))\,\mathrm{Subst}\left(\int \frac{x^3}{\left(1+x^2\right)^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}\\ &= \frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 0.82

$$\frac{4e^{5c(a+bx)}\sqrt{\cosh^2(c(a+bx))}}{bc\left(e^{2c(a+bx)}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (4*E^(5*c*(a + b*x))*Sqrt[Cosh[c*(a + b*x)]^2])/(b*c*(1 + E^(2*c*(a + b*x))
)^3)
```

fricas [B] time = 0.49, size = 120, normalized size = 2.14

$$-\frac{2\,(3\,\cosh(bcx+ac)+\sinh(bcx+ac))}{bc\cosh(bcx+ac)^3+3\,bc\cosh(bcx+ac)\sinh(bcx+ac)^2+bc\sinh(bcx+ac)^3+3\,bc\cosh(bcx+ac)+\left(3\,bc\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*c*
cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

giac [A] time = 0.12, size = 38, normalized size = 0.68

$$-\frac{2\left(2e^{(2bcx+2ac)}+1\right)}{bc\left(e^{(2bcx+2ac)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{8\,e^{c(bx+a)}}{(2\cosh(2bcx+2ac)+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x)
```

maxima [A] time = 0.34, size = 84, normalized size = 1.50

$$-\frac{4e^{(2bcx+2ac)}}{bc\left(e^{(4bcx+4ac)}+2e^{(2bcx+2ac)}+1\right)}-\frac{2}{bc\left(e^{(4bcx+4ac)}+2e^{(2bcx+2ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 0.94, size = 76, normalized size = 1.36

$$-\frac{4\,e^{a\,c+b\,c\,x}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*((exp(a*c + b*c*x)/2 + ex
p(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int \frac{e^{bcx}}{\left(\cosh^2\left(ac + bcx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(cosh(a*c + b*c*x)**2)**(3/2), x)
```

3.297 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{8\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}-\frac{4\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+}}$$

```
[Out] -4*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)+32/
3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)-8*co
sh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}+\frac{32\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}-\frac{4\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[a*c + b*c*x
]^2]) + (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3*Sqrt[Cosh[a
*c + b*c*x]^2]) - (8*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt
[Cosh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\operatorname{sech}^5(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{\cosh(ac+bcx)\operatorname{Subst}\left(\int \frac{32x^5}{\left(1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(32\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^5}{\left(1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(16\cosh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(16\cosh(ac+bcx))\operatorname{Subst}\left(\int \left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= -\frac{4\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}} + \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}} - \frac{}{bc\,(}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 0.51

$$-\frac{4\left(4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\cosh(c(a+bx))}{3bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)])/(3*b
*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[c*(a + b*x)]^2])
```

fricas [B] time = 0.60, size = 315, normalized size = 2.23

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

giac [A] time = 0.14, size = 51, normalized size = 0.36

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) + 1)^4)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int\frac{32\,e^{c(bx+a)}}{\left(2\cosh\left(2bcx+2ac\right)+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x)
```

maxima [A] time = 0.33, size = 209, normalized size = 1.48

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 0.10, size = 89, normalized size = 0.63

$$-\frac{8\,e^{a\,c+b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}\,\left(4\,e^{2\,a\,c+2\,b\,c\,x}+6\,e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*
(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*a*c +
2*b*c*x) + 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.298 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{64\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}+\frac{48\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}-\frac{192\cosh(ac+bcx)}{5bc\left(e^{2c(a+bx)}+1\right)^5\sqrt{\cosh^2(ac+bcx)}}$$

```
[Out] 32/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^6/(cosh(b*c*x+a*c)^2)^(1/2)-1
92/5*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^5/(cosh(b*c*x+a*c)^2)^(1/2)+4
8*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)-64/3
*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{64\cosh(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\cosh^2(ac+bcx)}}+\frac{48\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\cosh^2(ac+bcx)}}-\frac{192\cosh(ac+bcx)}{5bc\left(e^{2c(a+bx)}+1\right)^5\sqrt{\cosh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (32*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[a*c + b*c
*x]^2]) - (192*Cosh[a*c + b*c*x])/(5*b*c*(1 + E^(2*c*(a + b*x)))^5*Sqrt[Cos
h[a*c + b*c*x]^2]) + (48*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*
Sqrt[Cosh[a*c + b*c*x]^2]) - (64*Cosh[a*c + b*c*x])/(3*b*c*(1 + E^(2*c*(a +
 b*x)))^3*Sqrt[Cosh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^7(ac+bcx)\,dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{\cosh(ac+bcx)\,\text{Subst}\left(\int \frac{128x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(128\cosh(ac+bcx))\,\text{Subst}\left(\int \frac{x^7}{(1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(64\cosh(ac+bcx))\,\text{Subst}\left(\int \frac{x^3}{(1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{(64\cosh(ac+bcx))\,\text{Subst}\left(\int \left(-\frac{1}{(1+x)^7}+\frac{3}{(1+x)^6}-\frac{3}{(1+x)^5}+\frac{1}{(1+x)^4}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$= \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(ac+bcx)}} - \frac{192\cosh(ac+bcx)}{5bc\left(1+e^{2c(a+bx)}\right)^5\sqrt{\cosh^2(ac+bcx)}} + \frac{}{bc}$$

Mathematica [A] time = 0.09, size = 84, normalized size = 0.44

$$-\frac{16\left(6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}+1\right)\cosh(c(a+bx))}{15bc\left(e^{2c(a+bx)}+1\right)^6\sqrt{\cosh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x))
)*Cosh[c*(a + b*x)])/(15*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[c*(a + b*x
)]^2])
```

fricas [B] time = 0.59, size = 589, normalized size = 3.08

$$-\frac{}{15\left(bc\cosh(bcx+ac)^9+9\,bc\cosh(bcx+ac)\sinh(bcx+ac)^8+bc\sinh(bcx+ac)^9+6\,bc\cosh(bcx+ac)^7+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)
+ 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*
sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)^7 +
 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*
x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a*c)^2 +
 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b
*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*x + a*c)^4
 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 21*b*c*cosh(
b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*x + a*c)^5 +
 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2
 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(
b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c))
```

giac [A] time = 0.12, size = 64, normalized size = 0.34

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{128\,\mathrm{e}^{c(bx+a)}}{(2\cosh(2bcx+2ac)+2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x)
```

maxima [B] time = 0.33, size = 386, normalized size = 2.02

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}-\frac{}{bc\left(e^{(12\,bcx-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
+ 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 0.11, size = 345, normalized size = 1.81

$$\frac{96\,e^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)}^4}-\frac{128\,e^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}-\frac{384\,e^{2\,a\,c+2\,b\,c\,x}}{5\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1
/2))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) +
 1)^4) - (128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x
)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c
+ 2*b*c*x) + 1)^3) - (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(-
 a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))
*(exp(2*a*c + 2*b*c*x) + 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x
)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

3.299 $\int e^x \cosh(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{e^x \cosh(a+bx)}{1-b^2}-\frac{be^x \sinh(a+bx)}{1-b^2}$$

```
[Out] exp(x)*cosh(b*x+a)/(-b^2+1)-b*exp(x)*sinh(b*x+a)/(-b^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5475}

$$\frac{e^x \cosh(a+bx)}{1-b^2}-\frac{be^x \sinh(a+bx)}{1-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + b*x],x]
[Out] (E^x*Cosh[a + b*x])/(1 - b^2) - (b*E^x*Sinh[a + b*x])/(1 - b^2)
```

Rule 5475

```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \cosh(a+bx)\,dx=\frac{e^x \cosh(a+bx)}{1-b^2}-\frac{be^x \sinh(a+bx)}{1-b^2}$$

Mathematica [A] time = 0.05, size = 28, normalized size = 0.68

$$\frac{e^x(b\sinh(a+bx)-\cosh(a+bx))}{b^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + b*x],x]
[Out] (E^x*(-Cosh[a + b*x] + b*Sinh[a + b*x]))/(-1 + b^2)
```

fricas [A] time = 0.61, size = 45, normalized size = 1.10

$$-\frac{\cosh\left(bx+a\right)\cosh(x)-\left(b\cosh(x)+b\sinh(x)\right)\sinh\left(bx+a\right)+\cosh\left(bx+a\right)\sinh(x)}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(cosh(b*x + a)*cosh(x) - (b*cosh(x) + b*sinh(x))*sinh(b*x + a) + cosh(b*x
+ a)*sinh(x))/(b^2 - 1)
```

giac [A] time = 0.13, size = 32, normalized size = 0.78

$$\frac{e^{(bx+a+x)}}{2\,(b+1)}-\frac{e^{(-bx-a+x)}}{2\,(b-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a + x)/(b + 1) - 1/2*e^(-b*x - a + x)/(b - 1)
```

maple [A] time = 0.09, size = 62, normalized size = 1.51

$$\frac{\sinh\left(\left(b-1\right)x+a\right)}{2b-2}+\frac{\sinh\left(\left(1+b\right)x+a\right)}{2+2b}-\frac{\cosh\left(\left(b-1\right)x+a\right)}{2\left(b-1\right)}+\frac{\cosh\left(\left(1+b\right)x+a\right)}{2+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(b*x+a),x)
```

```
[Out] 1/2/(b-1)*sinh((b-1)*x+a)+1/2/(1+b)*sinh((1+b)*x+a)-1/2*cosh((b-1)*x+a)/(b-
1)+1/2*cosh((1+b)*x+a)/(1+b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-b>0)', see `assume?` for more deta
ils)Is -b equal to -1?
```

mupad [B] time = 0.09, size = 45, normalized size = 1.10

$$-\frac{\mathrm{e}^{x-a-b\,x}\,\left(b+\mathrm{e}^{2\,a+2\,b\,x}-b\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{2\,\left(b^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(x),x)
[Out] -(exp(x - a - b*x)*(b + exp(2*a + 2*b*x) - b*exp(2*a + 2*b*x) + 1))/(2*(b^2
 - 1))
```

sympy [A] time = 0.65, size = 99, normalized size = 2.41

$$\begin{cases} \frac{xe^x\sinh(a-x)}{2}+\frac{xe^x\cosh(a-x)}{2}-\frac{e^x\sinh(a-x)}{2} & \text{for } b=-1 \\ -\frac{xe^x\sinh(a+x)}{2}+\frac{xe^x\cosh(a+x)}{2}+\frac{e^x\sinh(a+x)}{2} & \text{for } b=1 \\ \frac{be^x\sinh(a+bx)}{b^2-1}-\frac{e^x\cosh(a+bx)}{b^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(b*x+a),x)
[Out] Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 - exp(x)*sinh(a
- x)/2, Eq(b, -1)), (-x*exp(x)*sinh(a + x)/2 + x*exp(x)*cosh(a + x)/2 + exp
(x)*sinh(a + x)/2, Eq(b, 1)), (b*exp(x)*sinh(a + b*x)/(b**2 - 1) - exp(x)*c
osh(a + b*x)/(b**2 - 1), True))
```

3.300 $\int e^x \cosh\left(a+cx^2\right) dx$

Optimal. Leaf size=85

$$\frac{\sqrt{\pi}\, e^{a-\frac{1}{4c}} \operatorname{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\, e^{\frac{1}{4c}-a} \operatorname{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*exp(-a+1/4/c)*erf(1/2*(-2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*exp(a-1
/4/c)*erfi(1/2*(2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5513, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\, e^{a-\frac{1}{4c}} \operatorname{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\, e^{\frac{1}{4c}-a} \operatorname{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + c*x^2],x]
```

```
[Out] -(E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^
(a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \cosh\left(a+cx^2\right)\,dx &= \int \left(\frac{1}{2}e^{-a+x-cx^2}+\frac{1}{2}e^{a+x+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-a+x-cx^2}\,dx+\frac{1}{2}\int e^{a+x+cx^2}\,dx\\ &= \frac{1}{2}e^{a-\frac{1}{4c}}\int e^{\frac{(1+2cx)^2}{4c}}\,dx+\frac{1}{2}e^{-a+\frac{1}{4c}}\int e^{-\frac{(1-2cx)^2}{4c}}\,dx\\ &= -\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.93

$$\frac{\sqrt{\pi}\,e^{-\frac{1}{4}/c}\left(e^{\frac{1}{2}/c}(\cosh(a)-\sinh(a))\mathrm{erf}\left(\frac{2cx-1}{2\sqrt{c}}\right)+(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]) +
Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c)))
```

fricas [A] time = 0.58, size = 104, normalized size = 1.22

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)-\sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{2\,cx-1}{2\,\sqrt{c}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c))
*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) - sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)/c
) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

giac [A] time = 0.14, size = 73, normalized size = 0.86

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c}\left(2\,x+\frac{1}{c}\right)\right)e^{\left(\frac{4\,ac-1}{4\,c}\right)}}{4\,\sqrt{-c}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c}\left(2\,x-\frac{1}{c}\right)\right)e^{\left(-\frac{4\,ac-1}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c)
 - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt(c
)
```

maple [A] time = 0.23, size = 72, normalized size = 0.85

$$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(c*x^2+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*P
i^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

maxima [A] time = 0.31, size = 65, normalized size = 0.76

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\,\sqrt{-c}}\right)e^{\left(a-\frac{1}{4\,c}\right)}}{4\,\sqrt{-c}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\,\sqrt{c}}\right)e^{\left(-a+\frac{1}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) + 1/4*sq
rt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^x\cosh\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(a + c*x^2),x)
```

```
[Out] int(exp(x)*cosh(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \cosh\left(a + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*cosh(a + c*x**2), x)
```

3.301 $\int e^x \cosh\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt{\pi}\,e^{a-\frac{(b+1)^2}{4c}}\operatorname{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,e^{\frac{(1-b)^2}{4c}-a}\operatorname{erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -1/4*exp(-a+1/4*(1-b)^2/c)*erf(1/2*(-2*c*x-b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1
/4*exp(a-1/4*(1+b)^2/c)*erfi(1/2*(2*c*x+b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5513, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{a-\frac{(b+1)^2}{4c}}\operatorname{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,e^{\frac{(1-b)^2}{4c}-a}\operatorname{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cosh[a + b*x + c*x^2],x]
```

```
[Out] -(E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sq
rt[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \cosh\left(a+bx+cx^2\right)dx &= \int \left(\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(1+b)x+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-a+(1-b)x-cx^2}\,dx+\frac{1}{2}\int e^{a+(1+b)x+cx^2}\,dx\\ &= \frac{1}{2}e^{-a+\frac{(1-b)^2}{4c}}\int e^{-\frac{(1-b-2cx)^2}{4c}}\,dx+\frac{1}{2}e^{a-\frac{(1+b)^2}{4c}}\int e^{\frac{(1+b+2cx)^2}{4c}}\,dx\\ &= -\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\text{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 91, normalized size = 0.90

$$\frac{\sqrt{\pi}e^{-\frac{(b+1)^2}{4c}}\left(e^{\frac{b^2+1}{2c}}(\cosh(a)-\sinh(a))\text{erf}\left(\frac{b+2cx-1}{2\sqrt{c}}\right)+(\sinh(a)+\cosh(a))\text{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cosh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] -
 Sinh[a]) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt
[c]*E^((1 + b)^2/(4*c)))
```

fricas [A] time = 0.53, size = 130, normalized size = 1.29

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^2-4ac+2b+1}{4c}\right)+\sinh\left(-\frac{b^2-4ac+2b+1}{4c}\right)\right)\text{erf}\left(\frac{(2cx+b+1)\sqrt{-c}}{2c}\right)-\sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^2-4ac-2b+1}{4c}\right)-\text{sinh}\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*(
b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) - sqrt(pi)*s
```

```
qrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c - 2*b
 + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

giac [A] time = 0.14, size = 91, normalized size = 0.90

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c}\left(2\,x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)}}{4\,\sqrt{-c}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c}\left(2\,x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c + 2
*b + 1)/c)/sqrt(-c) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^(1
/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

maple [A] time = 0.45, size = 97, normalized size = 0.96

$$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2+2b-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1-b}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2-2b-1}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{1+b}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(c*x^2+b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)/
c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^(1/
2)*x+1/2*(1+b)/(-c)^(1/2))
```

maxima [A] time = 0.32, size = 81, normalized size = 0.80

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c}\,x-\frac{b+1}{2\,\sqrt{-c}}\right)e^{\left(a-\frac{(b+1)^2}{4\,c}\right)}}{4\,\sqrt{-c}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c}\,x+\frac{b-1}{2\,\sqrt{c}}\right)e^{\left(-a+\frac{(b-1)^2}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c)
/sqrt(-c) + 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4*(
b - 1)^2/c)/sqrt(c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^x \cosh\left(c\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cosh(a + b*x + c*x^2),x)
[Out] int(exp(x)*cosh(a + b*x + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\cosh\left(a+bx+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cosh(c*x**2+b*x+a),x)
[Out] Integral(exp(x)*cosh(a + b*x + c*x**2), x)
```

3.302 $\int e^{x^2} \cosh(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}\sqrt{\pi}\,e^{-a-\frac{b^2}{4}}\operatorname{erfi}\left(\frac{1}{2}(2x-b)\right)+\frac{1}{4}\sqrt{\pi}\,e^{a-\frac{b^2}{4}}\operatorname{erfi}\left(\frac{1}{2}(b+2x)\right)$$

```
[Out] -1/4*exp(-a-1/4*b^2)*erfi(1/2*b-x)*Pi^(1/2)+1/4*exp(a-1/4*b^2)*erfi(1/2*b+x
)*Pi^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5513, 2234, 2204}

$$\frac{1}{4}\sqrt{\pi}\,e^{-a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-b)\right)+\frac{1}{4}\sqrt{\pi}\,e^{a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(b+2x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + b*x],x]
```

```
[Out] (E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*Er
fi[(b + 2*x)/2])/4
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cosh(a+bx)\,dx &= \int \left(\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right)dx\\ &= \frac{1}{2}\int e^{-a-bx+x^2}\,dx+\frac{1}{2}\int e^{a+bx+x^2}\,dx\\ &= \frac{1}{2}e^{-a-\frac{b^2}{4}}\int e^{\frac{1}{4}(-b+2x)^2}\,dx+\frac{1}{2}e^{a-\frac{b^2}{4}}\int e^{\frac{1}{4}(b+2x)^2}\,dx\\ &= \frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(b+2x)\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 51, normalized size = 0.78

$$\frac{1}{4}\sqrt{\pi}\,e^{-\frac{b^2}{4}}\left((\sinh(a)-\cosh(a))\mathrm{erfi}\left(\frac{b}{2}-x\right)+(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{b}{2}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + b*x],x]
```

```
[Out] (Sqrt[Pi]*(Erfi[b/2 - x]*(-Cosh[a] + Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Si
nh[a])))/(4*E^(b^2/4))
```

fricas [A] time = 0.53, size = 44, normalized size = 0.68

$$\frac{1}{4}\sqrt{\pi}\left(\mathrm{erfi}\left(\frac{1}{2}b+x\right)e^{\left(\frac{1}{4}b^2+a\right)}+\mathrm{erfi}\left(-\frac{1}{2}b+x\right)e^{\left(\frac{1}{4}b^2-a\right)}\right)e^{\left(-\frac{1}{2}b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(erfi(1/2*b + x)*e^(1/4*b^2 + a) + erfi(-1/2*b + x)*e^(1/4*b^2
 - a))*e^(-1/2*b^2)
```

giac [C] time = 0.12, size = 45, normalized size = 0.69

$$\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}ib-ix\right)e^{\left(-\frac{1}{4}b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(1/
2*I*b - I*x)*e^(-1/4*b^2 - a)
```

maple [C] time = 0.26, size = 52, normalized size = 0.80

$$\frac{i\sqrt{\pi}\, \mathrm{e}^{-a-\frac{b^2}{4}} \operatorname{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\, \mathrm{e}^{a-\frac{b^2}{4}} \operatorname{erf}\left(ix+\frac{1}{2}ib\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(b*x+a),x)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*b
^2)*erf(I*x+1/2*I*b)
```

maxima [C] time = 0.32, size = 45, normalized size = 0.69

$$-\frac{1}{4} i \sqrt{\pi} \operatorname{erf}\left(\frac{1}{2} i b + i x\right) e^{\left(-\frac{1}{4} b^2 + a\right)} - \frac{1}{4} i \sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2} i b + i x\right) e^{\left(-\frac{1}{4} b^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(-1
/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cosh\left(a+b\,x\right)\,\mathrm{e}^{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*exp(x^2),x)
```

```
[Out] int(cosh(a + b*x)*exp(x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^{2}} \cosh{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + b*x), x)
```

3.303 $\int e^{x^2} \cosh\left(a + cx^2\right) dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\, e^{-a}\operatorname{erfi}\left(\sqrt{1-c}\, x\right)}{4\sqrt{1-c}} + \frac{\sqrt{\pi}\, e^{a}\operatorname{erfi}\left(\sqrt{c+1}\, x\right)}{4\sqrt{c+1}}$$

```
[Out] 1/4*erfi(x*(1-c)^(1/2))*Pi^(1/2)/exp(a)/(1-c)^(1/2)+1/4*exp(a)*erfi(x*(1+c)
^(1/2))*Pi^(1/2)/(1+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5513, 2204}

$$\frac{\sqrt{\pi}\, e^{-a}\operatorname{Erfi}\left(\sqrt{1-c}\, x\right)}{4\sqrt{1-c}} + \frac{\sqrt{\pi}\, e^{a}\operatorname{Erfi}\left(\sqrt{c+1}\, x\right)}{4\sqrt{c+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqr
t[1 + c]*x])/(4*Sqrt[1 + c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=\int\left(\frac{1}{2}e^{-a+(1-c)x^2}+\frac{1}{2}e^{a+(1+c)x^2}\right)dx$$
$$=\frac{1}{2}\int e^{-a+(1-c)x^2}\,dx+\frac{1}{2}\int e^{a+(1+c)x^2}\,dx$$
$$=\frac{e^{-a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

Mathematica [A] time = 0.10, size = 71, normalized size = 1.09

$$\frac{\sqrt{\pi}\left(\sqrt{c-1}\,(c+1)(\cosh(a)-\sinh(a))\mathrm{erf}\left(\sqrt{c-1}\,x\right)+(c-1)\sqrt{c+1}\,(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\sqrt{c+1}\,x\right)\right)}{4\left(c^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]) + (
-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)
)
```

fricas [A] time = 0.63, size = 76, normalized size = 1.17

$$\frac{\sqrt{\pi}\,((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\ \mathrm{erf}\left(\sqrt{c-1}\,x\right)-\sqrt{\pi}\,((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{-c-1}\ \mathrm{erf}}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
1)*x) - sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqrt(
-c - 1)*x))/(c^2 - 1)
```

giac [A] time = 0.14, size = 49, normalized size = 0.75

$$-\frac{\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{c-1}\,x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{-c-1}\,x\right)e^{a}}{4\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sq
rt(-c - 1)*x)*e^a/sqrt(-c - 1)
```

maple [A] time = 0.74, size = 48, normalized size = 0.74

$$\frac{\sqrt{\pi}\ \mathrm{e}^{-a} \operatorname{erf}\left(\sqrt{c-1}\ x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\ \mathrm{e}^{a} \operatorname{erf}\left(\sqrt{-1-c}\ x\right)}{4\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(c*x^2+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-1
-c)^(1/2)*erf((-1-c)^(1/2)*x)
```

maxima [A] time = 0.32, size = 47, normalized size = 0.72

$$\frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{c-1}\,x\right) e^{(-a)}}{4\,\sqrt{c-1}}+\frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-c-1}\,x\right) e^{a}}{4\,\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt(
-c - 1)*x)*e^a/sqrt(-c - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{x^2} \cosh\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(a + c*x^2),x)
```

```
[Out] int(exp(x^2)*cosh(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^{2}} \cosh\left(a+cx^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + c*x**2), x)
```

3.304 $\int e^{x^2} \cosh\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}\, e^{a-\frac{b^2}{4(c+1)}} \operatorname{erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}} - \frac{\sqrt{\pi}\, e^{-a-\frac{b^2}{4(1-c)}} \operatorname{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}$$

```
[Out] -1/4*exp(-a-1/4*b^2/(1-c))*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))*Pi^(1/2)/(1-
c)^(1/2)+1/4*exp(a-1/4*b^2/(1+c))*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2))*Pi^(1
/2)/(1+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5513, 2234, 2204}

$$\frac{\sqrt{\pi}\, e^{a-\frac{b^2}{4(c+1)}} \operatorname{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}} - \frac{\sqrt{\pi}\, e^{-a-\frac{b^2}{4(1-c)}} \operatorname{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cosh[a + b*x + c*x^2],x]
```

```
[Out] -(E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])]
)/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x
)/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx &= \int\left(\frac{1}{2}e^{-a-bx+(1-c)x^2}+\frac{1}{2}e^{a+bx+(1+c)x^2}\right)dx\\ &= \frac{1}{2}\int e^{-a-bx+(1-c)x^2}\,dx+\frac{1}{2}\int e^{a+bx+(1+c)x^2}\,dx\\ &= \frac{1}{2}e^{-a-\frac{b^2}{4(1-c)}}\int e^{\frac{(-b+2(1-c)x)^2}{4(1-c)}}\,dx+\frac{1}{2}e^{a-\frac{b^2}{4(1+c)}}\int e^{\frac{(b+2(1+c)x)^2}{4(1+c)}}\,dx\\ &= -\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}\end{aligned}$$

Mathematica [A] time = 0.37, size = 122, normalized size = 1.06

$$\frac{\sqrt{\pi}\,e^{-\frac{b^2}{4c+4}}\left(\sqrt{c-1}\,(c+1)e^{\frac{b^2c}{2(c^2-1)}}(\cosh(a)-\sinh(a))\mathrm{erf}\left(\frac{b+2(c-1)x}{2\sqrt{c-1}}\right)+(c-1)\sqrt{c+1}\,(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{b+2(c+1}{2\sqrt{c+1}}\right.\right.}{4\left(c^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cosh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1 +
 c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a]) + (-1 + c)*Sqrt[1 + c]*Erfi[(b
 + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(b^2
/(4 + 4*c)))
```

fricas [A] time = 0.55, size = 165, normalized size = 1.43

$$\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\;\mathrm{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)-\sqrt{\pi}\left((c-1)\cosh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right.\right.}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*sin
h(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b)/
sqrt(c - 1)) - sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)) + (
c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*(c +
 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

giac [A] time = 0.13, size = 101, normalized size = 0.88

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c-1}\left(2\,x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c-1}\left(2\,x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*c
 - 4*a)/(c + 1))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x + b/
(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

maple [A] time = 0.75, size = 105, normalized size = 0.91

$$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2-4a}{4(c-1)}}\operatorname{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2+4a}{4+4c}}\operatorname{erf}\left(-\sqrt{-1-c}\,x+\frac{b}{2\sqrt{-1-c}}\right)}{4\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(c*x^2+b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x+
1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-1-c)^(1/2)
*erf(-(-1-c)^(1/2)*x+1/2*b/(-1-c)^(1/2))
```

maxima [A] time = 0.33, size = 89, normalized size = 0.77

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c-1}\,x-\frac{b}{2\,\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\,\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c + 1
))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(-a
 + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{x^2}\cosh\left(c\,x^2+b\,x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cosh(a + b*x + c*x^2),x)
```

```
[Out] int(exp(x^2)*cosh(a + b*x + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2} \cosh\left(a + bx + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cosh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cosh(a + b*x + c*x**2), x)
```

3.305 $\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx$

Optimal. Leaf size=110

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\,\mathrm{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)$$

```
[Out] 1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*P
i^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^
(1/2))*Pi^(1/2)
```

Rubi [**A**] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\,\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\,\mathrm{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2],x]
```

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f])
/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi
[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\cosh\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx\\
&= \frac{1}{2}\int e^{-d-fx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+bx}\,dx\\
&= \frac{1}{2}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{2}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,dx\\
&= \frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{2fx+b\,\mathrm{l}}{2\sqrt{f}}\right.
\end{aligned}$$

Mathematica [A] time = 0.13, size = 102, normalized size = 0.93

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left(e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+(\sinh(d)+\cosh(d))\text{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2],x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi]*(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2*
Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Cosh[
d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

fricas [B] time = 0.47, size = 211, normalized size = 1.92

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)\text{erf}\left(\frac{\left(2fx+b\log(f)\right)\sqrt{-f}}{2f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)\text{erf}\left(-\frac{2fx}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*e
rf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) + sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(
f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + sqrt
(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqr
t(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

giac [A] time = 0.15, size = 106, normalized size = 0.96

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{f}\left(2\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{4\,\sqrt{f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-f}\left(2\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4
*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*l
og(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

maple [A] time = 0.16, size = 100, normalized size = 0.91

$$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{\ln(f)^2b^2-4df}{4f}}\operatorname{erf}\left(-\sqrt{f}\,x+\frac{\ln(f)b}{2\sqrt{f}}\right)}{4\sqrt{f}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{\ln(f)^2b^2-4df}{4f}}\operatorname{erf}\left(-\sqrt{-f}\,x+\frac{\ln(f)b}{2\sqrt{-f}}\right)}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2
*ln(f)*b/f^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(-f)^(1/
2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

maxima [A] time = 0.32, size = 90, normalized size = 0.82

$$\frac{1}{4}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}\,x-\frac{b\log(f)}{2\,\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4\,f}-d\right)}+\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-f}\,x-\frac{b\log(f)}{2\,\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,f}+d\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*l
og(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))*e
^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\cosh\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*cosh(d + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2), x)
```

3.306 $\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=148

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\,f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\operatorname{erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/4*(
4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*b^2*ln(f)
^2/f)*f^(-1/2+a)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5513, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\operatorname{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\,f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\operatorname{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + f^(a + b*x)/(
2*b*Log[f])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\cosh^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2d-2fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2d+2fx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{\frac{(4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{(-4fx+b\log(f))^2}{8f}}
\\
&= \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{4fx
\end{aligned}$$

Mathematica [A] time = 0.70, size = 149, normalized size = 1.01

$$\frac{1}{16}f^a\left(\frac{\sqrt{2\pi}\,e^{\frac{b^2\log^2(f)}{8f}}(\cosh(2d)-\sinh(2d))\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)}{\sqrt{f}}+\frac{\sqrt{2\pi}\,e^{-\frac{b^2\log^2(f)}{8f}}(\sinh(2d)+\cosh(2d))\mathrm{erfi}\left(\frac{b\log(f)+4f}{2\sqrt{2}\sqrt{f}}\right)}{\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4*
f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (Sq
rt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[2*d
]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

fricas [B] time = 0.62, size = 278, normalized size = 1.88

$$-\frac{\sqrt{2}\sqrt{\pi}\,b\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}\left(4\,fx+b\log(f)\right)\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\sqrt{\pi}\,b\sqrt{f}\,\cosh\left(\frac{b^2\log(f)^2+8\,a}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2)*
sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*erf(-
1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)
*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f*log(f)
 - 16*d*f)/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*sinh((b*x + a)*log(f)))/(b
*f*log(f))
```

giac [C] time = 0.18, size = 355, normalized size = 2.40

$$-\frac{\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8\,af\log(f)-16\,df}{8\,f}\right)}}{16\sqrt{f}}-\frac{\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log}{}\right.}}{16\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8*
(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)*e
rf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*
log(f) - 16*d*f)/f)/sqrt(-f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1
/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f)
- pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2
*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^
(b*x*log(abs(f)) + a*log(abs(f))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/
2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b +
```

```
4*b*log(abs(f))) + 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*
sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*
x*log(abs(f)) + a*log(abs(f)))
```

maple [A] time = 0.29, size = 126, normalized size = 0.85

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2} b^{2}-16 d f}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{f}\, x+\frac{\ln(f) b \sqrt{2}}{4 \sqrt{f}}\right)}{16 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-16 d f}{8 f}} \operatorname{erf}\left(-\sqrt{-2 f}\, x+\frac{\ln(f) b}{2 \sqrt{-2 f}}\right)}{8 \sqrt{-2 f}}+\frac{f^{a} f^{b x}}{2 \ln(f) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-16*d*f)/f)*2^(1/2)/f^(1/2)*erf(-2^(
1/2)*f^(1/2)*x+1/4*ln(f)*b*2^(1/2)/f^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/8*(ln(f
)^2*b^2-16*d*f)/f)/(-2*f)^(1/2)*erf(-(-2*f)^(1/2)*x+1/2*ln(f)*b/(-2*f)^(1/2
))+1/2*f^a/ln(f)/b*f^(b*x)
```

maxima [A] time = 0.42, size = 127, normalized size = 0.86

$$\frac{\sqrt{2}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\,\sqrt{f}\, x-\frac{\sqrt{2}\, b \log(f)}{4 \sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{8 f}-2 d\right)}}{16 \sqrt{f}}+\frac{\sqrt{2}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\,\sqrt{-f}\, x-\frac{\sqrt{2}\, b \log(f)}{4 \sqrt{-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{8 f}+2 d\right)}}{16 \sqrt{-f}}+\frac{f^{bx+}}{2\, b \log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqrt
(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf(s
qrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/f +
 2*d)/sqrt(-f) + 1/2*f^(b*x + a)/(b*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\cosh\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx} \cosh^{2}\left(d+fx^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2)**2, x)
```

3.307 $\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=239

$$\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\operatorname{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\mathrm{e}$$

```
[Out] 1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/6*(6*f*x-b*ln(f))*3^(1/2
)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf
i(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/16*exp(-d+1/4*b^2
*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)+3/16*exp(d
-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\operatorname{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\operatorname{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\mathrm{I}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 + (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)dx = \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}+\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx$$
$$=\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{d+}$$
$$=\frac{1}{8}\int e^{-3d-3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{8}\int e^{3d+3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{-d-fx^2}$$
$$=\frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{\frac{(6fx+b\log(f))^2}{12f}}\,dx+\frac{1}{8}\Bigg($$
$$=\frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erf}\Bigg(\frac{6}{}$$

Mathematica [A] time = 0.43, size = 286, normalized size = 1.20

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left(3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{3f}}(\cosh(3d)-\sinh(3d))\text{er}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt
[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqrt[
3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2
```

```
*Sqrt[f])]*(Cosh[d] - Sinh[d]) + 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[
f])]*Sinh[d] + E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*S
qrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f*x + b
*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f)))
```

fricas [B] time = 0.45, size = 443, normalized size = 1.85

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}\left(6\,fx+b\log(f)\right)\sqrt{-f}}{6\,f}\right)+\sqrt{3}\,\sqrt{\pi}\,\sqrt{f}\,\cosh\left(\frac{b^2\log(f)^2+12\,af\log(f)-36}{12\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) + sqrt(3)*sqrt(pi
)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*sqr
t(3)*(6*f*x - b*log(f))/sqrt(f)) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3
)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*
d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt
(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) + 9*sqrt(pi)*s
qrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(2*f*x +
b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + 4*a*f*l
og(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)
*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4*a*f*log(f)
 - 4*d*f)/f) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f)*s
inh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

giac [A] time = 0.17, size = 223, normalized size = 0.93

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{f}\left(6\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{f}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{-f}\left(6\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(\right.}{}\right.}}{48\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12
*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi)
*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 12*
a*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*
log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 3/16*s
qrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f
*log(f) - 4*d*f)/f)/sqrt(-f)
```

maple [A] time = 0.42, size = 207, normalized size = 0.87

$$-\frac{\sqrt{\pi}\, f^{a} e^{\frac{\ln(f)^{2} b^{2}-36 d f}{12 f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\, \sqrt{f}\, x+\frac{\ln(f) b \sqrt{3}}{6 \sqrt{f}}\right)}{48 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{\ln(f)^{2} b^{2}-36 d f}{12 f}} \operatorname{erf}\left(-\sqrt{-3 f}\, x+\frac{\ln(f) b}{2 \sqrt{-3 f}}\right)}{16 \sqrt{-3 f}}-\frac{3 \sqrt{\pi}\, f^{a} e^{\frac{\ln(f}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+d)^3,x)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(1/12*(ln(f)^2*b^2-36*d*f)/f)*3^(1/2)/f^(1/2)*erf(-3^
(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/12*(l
n(f)^2*b^2-36*d*f)/f)/(-3*f)^(1/2)*erf(-(-3*f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(
1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)
*x+1/2*ln(f)*b/f^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(
-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

maxima [A] time = 0.43, size = 200, normalized size = 0.84

$$\frac{3}{16}\,\sqrt{\pi}\, f^{a-\frac{1}{2}} \operatorname{erf}\left(\sqrt{f}\, x-\frac{b \log(f)}{2\,\sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{4\,f}-d\right)}+\frac{\sqrt{3}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{3}\,\sqrt{f}\, x-\frac{\sqrt{3}\, b \log(f)}{6\,\sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{12\,f}-3\,d\right)}}{48\,\sqrt{f}}+\frac{\sqrt{3}\,\sqrt{\pi}\, f^{a}\, e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqr
t(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt(3)
*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e^(-1
/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2
*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x} \cosh\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + f*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + f*x**2)**3, x)
```

3.308 $\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=115

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\operatorname{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

```
[Out] 1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/2
))*Pi^(1/2)+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*l
n(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*L
og[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}\,dx+\frac{1}{2}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}\,dx\\ &= \frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\int e^{\frac{(e+2fx+b\log(f))}{4f}}\\ &= \frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{er}\end{aligned}$$

Mathematica [A] time = 0.28, size = 123, normalized size = 1.07

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+e^2}{4f}}\left((\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+e^2}{2f}}\operatorname{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+(\sinh(d)+\cosh(d))\operatorname{erfi}\left(\frac{b\,\mathrm{l}}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e +
2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(e + 2*f*x + b*Lo
g[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f)))
```

fricas [B] time = 0.64, size = 251, normalized size = 2.18

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+e^2-4df+2\left(be-2af\right)\log(f)}{4f}\right)\operatorname{erf}\left(\frac{\left(2fx+b\log(f)+e\right)\sqrt{-f}}{2f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+e^2-4df-2\left(be-2a\right.}{4f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a
*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + sqrt(pi)*sqrt(f
)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(-1/
2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*lo
g(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a*f
)*log(f))/f) + sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f))*si
nh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

giac [A] time = 0.15, size = 134, normalized size = 1.17

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4af\log(f)-2be\log(f)-4df+e^2}{4f}\right)}}{4\sqrt{f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f}{}\right.}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)
^2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(f) - 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*lo
g(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

maple [A] time = 0.16, size = 126, normalized size = 1.10

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2}b^{2}-2\ln(f)be-4df+e^{2}}{4f}}\operatorname{erf}\left(-\sqrt{f}\, x+\frac{b\ln(f)-e}{2\sqrt{f}}\right)}{4\sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2}b^{2}+2\ln(f)be-4df+e^{2}}{4f}}\operatorname{erf}\left(-\sqrt{-f}\, x+\frac{e+b\ln(f)}{2\sqrt{-f}}\right)}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*er
f(-f^(1/2)*x+1/2*(b*ln(f)-e)/f^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^
2+2*ln(f)*b*e-4*d*f+e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-
f)^(1/2))
```

maxima [A] time = 0.33, size = 102, normalized size = 0.89

$$\frac{1}{4}\sqrt{\pi}\, f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}\, x-\frac{b\log(f)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{\left(b\log(f)-e\right)^{2}}{4f}\right)}+\frac{\sqrt{\pi}\, f^{a}\operatorname{erf}\left(\sqrt{-f}\, x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{\left(b\log(f)+e\right)^{2}}{4f}\right)}}{4\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
+ 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f)
 + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*cosh(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + e*x + f*x**2), x)
```

3.309 $\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=161

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\text{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\text{erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a}}{2b\log}$$

```
[Out] 1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1
/4*(2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*(
2*e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*
2^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5513, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\text{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\text{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 + f^(a + b*x)/(2*b*Log[f])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\ &= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int\exp\left(-2d-2fx^2+a\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int \cdots\\ &= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^a\right)\int e^{-\frac{(-2e-4fx+b\log(f))^2}{8f}}dx+\frac{1}{4}\left(e^{2d-\frac{(2e+b\log(f))^2}{8f}}f \cdots\right.\\ &= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\cdots\end{aligned}$$

Mathematica [A] time = 0.65, size = 220, normalized size = 1.37

$$\frac{f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+4e^2}{8f}}\left(\sqrt{\pi}\,b\log(f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2\log^2(f)+4e^2}{4f}}\text{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+4\sqrt{2}f^{b\left(\frac{e}{2f}+x\right)+\frac{1}{2}}e^{\frac{b^2\log^2(f)\cdots}{8f}}\cdots\right.}{8\sqrt{2}\,b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/2
 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*e
+ 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d]) + b
*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2
*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f])
```

fricas [B] time = 0.64, size = 334, normalized size = 2.07

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,b\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2+4\,e^2-16\,df+4\left(be-2\,af\right)\log(f)}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}\left(4\,fx+b\log(f)+2\,e\right)\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\,\sqrt{\pi}\,b\sqrt{f}\,\cosh \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqrt(
-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 -
 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f) + 2
*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f*x +
b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*sqrt(2)*
(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f - 4*(b*e - 2*a*f)*log(f))/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*sinh(
(b*x + a)*log(f)))/(b*f*log(f))
```

giac [C] time = 0.20, size = 389, normalized size = 2.42

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{f}\left(4\,x-\frac{b\log(f)-2\,e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8\,af\log(f)-4\,be\log(f)-16\,df+4\,e^2}{8\,f}\right)}}{16\,\sqrt{f}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{-f}\left(4\,x+\frac{b\log \cdots}{\cdots}\right)\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(f) - 2*e)/f))
*e^(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/f)/sq
rt(f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) +
2*e)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*log(f) + 4*b*e*log(f) - 16*d*f + 4*e
^2)/f)/sqrt(-f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f
) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) -
(pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f)
+ 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(
```

```
f)) + a*log(abs(f))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x +
1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f
))) + 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*
I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f))
 + a*log(abs(f)))
```

maple [A] time = 0.28, size = 158, normalized size = 0.98

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2} b^{2}-4\ln(f) b e-16 d f+4 e^{2}}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{f}\, x+\frac{\left(b\ln(f)-2 e\right)\sqrt{2}}{4\sqrt{f}}\right)}{16\sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4\ln(f) b e-16 d f+4 e^{2}}{8 f}} \operatorname{erf}\left(-\sqrt{-2 f}\, x+\right.}{8\sqrt{-2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)*2^(1/2
)/f^(1/2)*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))-1/8*Pi^
(1/2)*f^a*exp(-1/8*(ln(f)^2*b^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)/(-2*f)^(1/2)*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))+1/2*f^a/ln(f)/b*f^(b*x)
```

maxima [A] time = 0.43, size = 143, normalized size = 0.89

$$\frac{\sqrt{2}\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{2}\sqrt{-f}\, x-\frac{\sqrt{2}\left(b\log(f)+2 e\right)}{4\sqrt{-f}}\right) e^{\left(2 d-\frac{\left(b\log(f)+2 e\right)^{2}}{8 f}\right)}}{16\sqrt{-f}}+\frac{\sqrt{2}\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{2}\sqrt{f}\, x-\frac{\sqrt{2}\left(b\log(f)-2 e\right)}{4\sqrt{f}}\right) e^{\left(-2 d+\frac{\left(b\log(\right.}{8}\right.}}{16\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)*s
qrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f))*e
^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) + 1/2*f^(b*x + a)/(b*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\cosh\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cosh(d + e*x + f*x**2)**2, x)
```

3.310 $\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=257

$$\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}\,f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\operatorname{erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}$$

```
[Out] 1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/6*(3*e+6*f*x-b*ln(f)
)*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*(3*e+b*ln(f))^2/f)*f^
(-1/2+a)*erfi(1/6*(3*e+6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/1
6*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/2))
*Pi^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*ln
(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}\,f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\operatorname{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 + (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+b}\right.\\ &= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+fx^2\right)\right)\\ &= \frac{1}{8}\int\exp\left(-3d-3fx^2+a\log(f)-x(3e-b\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d+3\right.\\ &= \frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx+\frac{1}{8}\left(e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^a\right)\int e^{-\frac{(-3e-}{}}\\ &= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+}\end{aligned}$$

Mathematica [A] time = 0.80, size = 353, normalized size = 1.37

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+3e^2}{4f}}\left(3\sqrt{3}(\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+2e^2}{2f}}\mathrm{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+(\cosh(3d)-\sinh(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[(
e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erf
```

```
i[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 + b^2
*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d
]) + 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*Sinh[
d] + E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqr
t[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(3*e
 + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2 + b^2*L
og[f]^2)/(4*f)))
```

fricas [B] time = 0.52, size = 539, normalized size = 2.10

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2+9\,e^2-36\,df+6\,(be-2\,af)\log(f)}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}\,(6\,fx+b\log(f)+3\,e)\,\sqrt{-f}}{6\,f}\right)+\sqrt{3}\,\sqrt{\pi}\,\sqrt{f}\,\cosh\left(\frac{b^2\log(f)^2+9}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f +
 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt(-
f)/f) + sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f -
 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/sqrt(
f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sq
rt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*log(f)
)/f) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*log(f))/
f) + 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sqrt(pi)*sqr
t(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
 b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt
(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

giac [A] time = 0.16, size = 285, normalized size = 1.11

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{f}\left(6\,x-\frac{b\log(f)-3\,e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-6\,be\log(f)-36\,df+9\,e^2}{12\,f}\right)}}{48\,\sqrt{f}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{-f}\left(6\,x+\frac{b\log}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*log(f) - 3*e)/f))
*e^(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/f)/
```

```
sqrt(f) - 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f)
+ 3*e)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a*f*log(f) + 6*b*e*log(f) - 36*d*f +
 9*e^2)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/
f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sq
rt(f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(
b^2*log(f)^2 - 4*a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

maple [A] time = 0.43, size = 265, normalized size = 1.03

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2} b^{2}-6\ln(f) b e-36 d f+9 e^{2}}{12 f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\, \sqrt{f}\, x+\frac{\left(b\ln(f)-3 e\right)\sqrt{3}}{6\sqrt{f}}\right)}{48\sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6\ln(f) b e-36 d f+9 e^{2}}{12 f}} \operatorname{erf}\left(-\sqrt{-3 f}\, x\right.}{16\sqrt{-3 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(1/12*(ln(f)^2*b^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)*3^(1/
2)/f^(1/2)*erf(-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))-1/16*P
i^(1/2)*f^a*exp(-1/12*(ln(f)^2*b^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)/(-3*f)^(1/2
)*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp
(1/4*(ln(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*l
n(f)-e)/f^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*f+
e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))
```

maxima [A] time = 0.44, size = 228, normalized size = 0.89

$$\frac{\sqrt{3}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{3}\,\sqrt{-f}\, x-\frac{\sqrt{3}\left(b\log(f)+3 e\right)}{6\sqrt{-f}}\right) e^{\left(3 d-\frac{\left(b\log(f)+3 e\right)^{2}}{12 f}\right)}}{48\sqrt{-f}}+\frac{3}{16}\sqrt{\pi}\, f^{a-\frac{1}{2}} \operatorname{erf}\left(\sqrt{f}\, x-\frac{b\log(f)-e}{2\sqrt{f}}\right) e^{\left(-d+\frac{\left(b\log(f)-e\right.}{4 f}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(pi)
*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*log
(f) - e)^2/f) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqrt(
3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt(f) +
 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d - 1/4
*(b*log(f) + e)^2/f)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

3.311 $\int f^{a+cx^2} \cosh(d+ex)\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\pi}\, f^a e^{d-\frac{e^2}{4c\log(f)}} \operatorname{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}\, f^a e^{-\frac{e^2}{4c\log(f)}-d} \operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/
2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e
+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5513, 2287, 2234, 2204}

$$\frac{\sqrt{\pi}\, f^a e^{d-\frac{e^2}{4c\log(f)}} \operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}\, f^a e^{-\frac{e^2}{4c\log(f)}-d} \operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] -(E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[
Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh(d+ex)\,dx &= \int\left(\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+ex}f^{a+cx^2}\,dx\\ &= \frac{1}{2}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{2}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\ &= \frac{1}{2}\left(e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{2}\left(e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,dx\\ &= -\frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 104, normalized size = 0.78

$$\frac{\sqrt{\pi}f^ae^{-\frac{e^2}{4c\log(f)}}\left((\cosh(d)-\sinh(d))\mathrm{erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d]
- Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + S
inh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.62, size = 216, normalized size = 1.62

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)\right)\mathrm{erf}\left(\frac{(2cx\log(f)+e)\sqrt{-c\log(f)}}{2c\log(f)}\right)+}{4c\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e
^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(
c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-
c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))*
erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

giac [A] time = 0.15, size = 132, normalized size = 0.99

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^{2}+4\,cd\log(f)-e^{2}}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f}{4}\right.}}{4\,\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*
log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*
d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.16, size = 117, normalized size = 0.88

$$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{4d\ln(f)c+e^{2}}{4\ln(f)c}}\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{4d\ln(f)c-e^{2}}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((
-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(
f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f)
)^(1/2))
```

maxima [A] time = 0.32, size = 105, normalized size = 0.79

$$\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}\,x-\frac{e}{2\,\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^{2}}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}\,x+\frac{e}{2\,\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^{2}}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*
e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\cosh\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x),x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x), x)
```

3.312 $\int f^{a+cx^2} \cosh^2(d+ex)\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}-2d}\operatorname{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\operatorname{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-e^2/c/ln(f))*f^a*erfi((-e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(
1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*erfi((e+c*x*ln(f))/c^
(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^a*erfi(x*c^(1/2)*ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5513, 2204, 2287, 2234}

$$-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}-2d}\operatorname{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)}}\operatorname{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\operatorname{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[Log
[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*E
rfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = \int \left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}\right)dx$$

$$= \frac{1}{4}\int e^{-2d-2ex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d-2ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2d+2ex+a\log(f)+}$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\left(e^{2d-\frac{e^2}{c\log(f)}}f^a\right.$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}}{8\sqrt{c}\,\sqrt{\log}}$$

Mathematica [A] time = 0.23, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}}\left((\cosh(2d)-\sinh(2d))\mathrm{erfi}\left(\frac{cx\log(f)-e}{\sqrt{c}\,\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\mathrm{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)+2e^{\frac{e^2}{c\log(f)}}\mathrm{erfi}\left(\sqrt{c}\right.\right.}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-e
 + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e +
c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]*E^
(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

fricas [A] time = 0.60, size = 242, normalized size = 1.50

$$-\frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(a\log(f)\right)+\sqrt{\pi}\sinh\left(a\log(f)\right)\right)\operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)+\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{ac\log(f\ldots}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))
*erf(sqrt(-c*log(f))*x) + sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2*
c*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(f)
- e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt
(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))
+ sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf((c*x*l
og(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

giac [A] time = 0.16, size = 150, normalized size = 0.93

$$-\frac{\sqrt{\pi}f^{a}\operatorname{erf}\left(-\sqrt{-c\log(f)}\,x\right)}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^{2}+2\,cd\log(f)-e^{2}}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(\ldots\right.\right.}{8\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e^
2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e/(
c*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(
f))
```

maple [A] time = 0.25, size = 139, normalized size = 0.86

$$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{2d\ln(f)c+e^{2}}{\ln(f)c}}\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{2d\ln(f)c-e^{2}}{\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}+\frac{f^{a}\sqrt{\pi}\operatorname{erf}\left(\ldots\right.}{4\sqrt{-c\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*l
n(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln
```

```
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))+1/4*f^a
*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

maxima [A] time = 0.33, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}\,x - \frac{e}{\sqrt{-c\log(f)}}\right) e^{\left(2\,d - \frac{e^2}{c\log(f)}\right)}}{8\,\sqrt{-c\log(f)}} + \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}\,x + \frac{e}{\sqrt{-c\log(f)}}\right) e^{\left(-2\,d - \frac{e^2}{c\log(f)}\right)}}{8\,\sqrt{-c\log(f)}} + \frac{\sqrt{\pi} f^a \operatorname{er}}{4\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(c
*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sqrt
(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*e
rf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\cosh\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x)**2, x)
```

3.313 $\int f^{a+cx^2} \cosh^3(d+ex)\,dx$

Optimal. Leaf size=271

$$-\frac{3\sqrt{\pi}\, f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\, f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3\sqrt{\pi}\, f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\cdots$$

```
[Out] 3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*erfi(1
/2*(-3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/1
6*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*
Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*erfi(1/2*(3*
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5513, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}\, f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\, f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{3\sqrt{\pi}\, f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) + (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = \int\left(\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}+\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx$$
$$=\frac{1}{8}\int e^{-3d-3ex}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3d+3ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{d+ex}f^{a+}$$
$$=\frac{1}{8}\int e^{-3d-3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{8}\int e^{3d+3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int e^{-d-ex+a\log}$$
$$=\frac{1}{8}\left(e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(e^{3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\Big(3e$$
$$=-\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3e^{d-\frac{}{4c}}}{}$$

Mathematica [A] time = 0.47, size = 214, normalized size = 0.79

$$\frac{\sqrt{\pi}f^ae^{-\frac{9e^2}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e^2}{c\log(f)}}\mathrm{erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\mathrm{erf}\right.\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c*
x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e + 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Erfi[(
```

```
-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])))/(1
6*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.49, size = 426, normalized size = 1.57

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)+3\,e)\sqrt{-c\log}}{2\,c\log(f)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) - 9
*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f)))
 + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(c
*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(
-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(
f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))
*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(
sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))) + sq
rt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))))*erf(1
/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

giac [A] time = 0.17, size = 264, normalized size = 0.97

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x+\frac{3\,e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac}{}\right.}}{16\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*a
*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqr
t(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2
 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.45, size = 234, normalized size = 0.86

$$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4d\ln(f)c+3e^{2}\right)}{4\ln(f)c}} \operatorname{erf}\left(\sqrt{-c\ln(f)}\, x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3d\ln(f)c-\frac{9e^{2}}{4}}{c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}+\frac{3\sqrt{\pi}\, f^{a}\mathrm{e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(e*x+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(3/4*(4*d
*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+3/2*e/(-c
*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln
(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a
*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*
x+1/2*e/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.34, size = 211, normalized size = 0.78

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{3\,e}{2\,\sqrt{-c\log(f)}}\right) e^{\left(3\,d-\frac{9\,e^{2}}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log(f)}}+\frac{3\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{e}{2\,\sqrt{-c\log(f)}}\right) e^{\left(d-\frac{e^{2}}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log(f)}}+\frac{3\,\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d - 9
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*
x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/1
6*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e
^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,{\mathrm{cosh}\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x)**3, x)
```

3.314 $\int f^{a+cx^2}\cosh\left(d+fx^2\right)dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\pi}\,e^{-d}f^a\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,e^{d}f^a\mathrm{erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] 1/4*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/4*exp(
d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5513, 2287, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{-d}f^a\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,e^{d}f^a\mathrm{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d*
f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cosh\left(d+fx^2\right)dx &= \int \left(\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-d-fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-d+a\log(f)-x^2(f-c\log(f))}\,dx+\frac{1}{2}\int e^{d+a\log(f)+x^2(f+c\log(f))}\,dx\\
&= \frac{e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 75, normalized size = 0.93

$$\frac{1}{4}\sqrt{\pi}f^a\left(\frac{(\cosh(d)-\sinh(d))\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{\sqrt{f-c\log(f)}}+\frac{(\sinh(d)+\cosh(d))\mathrm{erfi}\left(x\sqrt{c\log(f)+f}\right)}{\sqrt{c\log(f)+f}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c*L
og[f]] + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log[f]
]))/4
```

fricas [B] time = 0.43, size = 145, normalized size = 1.79

$$-\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(a\log(f)-d\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(a\log(f)-d\right)\right)\sqrt{-c\log(f)+f}\ \mathrm{erf}\left(\sqrt{-c\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)
*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) + (sqrt
(pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*log
```

```
(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2 - f
^2)
```

giac [A] time = 0.13, size = 75, normalized size = 0.93

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)-f}\,x\right)e^{\left(a\log(f)+d\right)}}{4\sqrt{-c\log(f)-f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)+f}\,x\right)e^{\left(a\log(f)-d\right)}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) -
 f) - 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log
(f) + f)
```

maple [A] time = 0.14, size = 70, normalized size = 0.86

$$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-d}\operatorname{erf}\left(x\sqrt{f-c\ln(f)}\right)}{4\sqrt{f-c\ln(f)}}+\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{d}\operatorname{erf}\left(\sqrt{-c\ln(f)-f}\,x\right)}{4\sqrt{-c\ln(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+1/4*Pi^
(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

maxima [A] time = 0.33, size = 69, normalized size = 0.85

$$\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x\right)e^{(-d)}}{4\sqrt{-c\log(f)+f}}+\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x\right)e^{d}}{4\sqrt{-c\log(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1/
4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\cosh\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \cosh\left(d + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2), x)
```

3.315 $\int f^{a+cx^2} \cosh^2\left(d+fx^2\right) dx$

Optimal. Leaf size=128

$$\frac{\sqrt{\pi}\, e^{-2d} f^a \operatorname{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, e^{2d} f^a \operatorname{erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}\, f^a \operatorname{erfi}\left(\sqrt{c}\, x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*er
f(x*(2*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d)
*f^a*erfi(x*(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5513, 2204, 2287, 2205}

$$\frac{\sqrt{\pi}\, e^{-2d} f^a \operatorname{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, e^{2d} f^a \operatorname{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}\, f^a \operatorname{Erfi}\left(\sqrt{c}\, x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) + (
E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\ &= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d+a\log(f)-x^2(2f-c\log(f))}\,dx+\frac{1}{4}\int e^{2d+a\log(f)\cdots}\\ &= \frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\,\text{erfi}\left(\cdots\right.}{8\sqrt{2f\cdots}}\end{aligned}$$

Mathematica [A] time = 0.56, size = 179, normalized size = 1.40

$$\frac{\sqrt{\pi}f^a\left(\left(2c^2\log^2(f)-8f^2\right)\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)+\sqrt{c}\,\sqrt{\log(f)}\left(\sqrt{2f-c\log(f)}\,(c\log(f)+2f)(\sinh(2d)-\cosh\cdots\right.\right.}{8\sqrt{c}\,\sqrt{\log(f)}\left(c^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(-8*f^2 + 2*c^2*Log[f]^2) + Sqr
t[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f +
c*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f - c*
Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log
[f]]*(-4*f^2 + c^2*Log[f]^2))
```

fricas [B] time = 0.50, size = 254, normalized size = 1.98

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\cosh\left(a\log(f)-2\,d\right)+\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\sinh\left(a\log(f)-2\,d\right)\right)\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(p
i)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2*f
)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(f) +
 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) + 2*(sqrt(pi)*(c^
2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(a
*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*c*f^2*l
og(f))
```

giac [A] time = 0.14, size = 107, normalized size = 0.84

$$-\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(-\sqrt{-c \log(f)}\, x\right)}{4 \sqrt{-c \log(f)}}-\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log(f) - 2 f}\, x\right) e^{\left(a \log(f)+2 d\right)}}{8 \sqrt{-c \log(f) - 2 f}}-\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log(f) + 2 f}\, x\right) e^{\left(a \log(f)\right.}}{8 \sqrt{-c \log(f) + 2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8*
sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f) +
2*f)
```

maple [A] time = 0.27, size = 101, normalized size = 0.79

$$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-2 d} \operatorname{erf}\left(x \sqrt{2 f - c \ln(f)}\right)}{8 \sqrt{2 f - c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{2 d} \operatorname{erf}\left(\sqrt{-c \ln(f) - 2 f}\, x\right)}{8 \sqrt{-c \ln(f) - 2 f}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x\right)}{4 \sqrt{-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*d)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+1
/8*Pi^(1/2)*f^a*exp(2*d)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)+1
/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

maxima [A] time = 0.34, size = 100, normalized size = 0.78

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - 2 f}\, x\right) e^{(2 d)}}{8 \sqrt{-c \log(f) - 2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) + 2 f}\, x\right) e^{(-2 d)}}{8 \sqrt{-c \log(f) + 2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x\right)}{4 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f)
 + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f) +
2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\cosh\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2)**2, x)
```

3.316 $\int f^{a+cx^2} \cosh^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=171

$$\frac{3\sqrt{\pi}\,e^{-d}f^a\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,e^{-3d}f^a\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}\,e^{d}f^a\text{erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}\,e^{3d}f^a\text{e}\cdots}{16\cdots}$$

```
[Out] 3/16*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/16*f^
a*erf(x*(3*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*d)/(3*f-c*ln(f))^(1/2)+3/16*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d)*
f^a*erfi(x*(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5513, 2287, 2205, 2204}

$$\frac{3\sqrt{\pi}\,e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,e^{-3d}f^a\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}\,e^{d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}\,e^{3d}f^a\cdots}{16\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) + (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
+ (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)dx &= \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{8}\int e^{-3d-3fx^2}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e\\ &= \frac{1}{8}\int e^{-3d+a\log(f)-x^2(3f-c\log(f))}\,dx+\frac{1}{8}\int e^{3d+a\log(f)+x^2(3f+c\log(f))}\,dx+\frac{3}{8}\int e^{-d+a}\\ &= \frac{3e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^df^a\sqrt{\pi}}{1}\end{aligned}$$

Mathematica [A] time = 1.24, size = 270, normalized size = 1.58

$$\frac{\sqrt{\pi}f^a\left((f-c\log(f))\left(\sqrt{3f-c\log(f)}\left(c^2\log^2(f)+4cf\log(f)+3f^2\right)(\cosh(3d)-\sinh(3d))\mathrm{erf}\left(x\sqrt{3f-c\log}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*
f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*Lo
g[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[
f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x*Sqr
t[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) +
Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d]
 + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

fricas [B] time = 0.69, size = 491, normalized size = 2.87

$$-\frac{\left(\sqrt{\pi}\left(c^3\log(f)^3+3c^2f\log(f)^2-cf^2\log(f)-3f^3\right)\cosh\left(a\log(f)-3d\right)+\sqrt{\pi}\left(c^3\log(f)^3+3c^2f\log(f)^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")

[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*cosh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(f) + 3*f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*erf(sqrt(-c*log(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)

giac [A] time = 0.16, size = 155, normalized size = 0.91

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)-3\,f}\,x\right)e^{\left(a\log(f)+3\,d\right)}}{16\,\sqrt{-c\log(f)-3\,f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)-f}\,x\right)e^{\left(a\log(f)+d\right)}}{16\,\sqrt{-c\log(f)-f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)\right.}{16\,\sqrt{-c\log(}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="giac")

[Out] -1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^(a*log(f) - 3*d)/sqrt(-c*log(f) + 3*f)

maple [A] time = 0.37, size = 144, normalized size = 0.84

$$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-3d}\operatorname{erf}\left(x\sqrt{3f-c\ln(f)}\right)}{16\sqrt{3f-c\ln(f)}}+\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{3d}\operatorname{erf}\left(\sqrt{-c\ln(f)-3f}\,x\right)}{16\sqrt{-c\ln(f)-3f}}+\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}^{-d}\operatorname{erf}\left(x\sqrt{f-c\ln(f)}\right)}{16\sqrt{f-c\ln(f)}}+\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(c*x^2+a)*cosh(f*x^2+d)^3,x)

[Out] 1/16*Pi^(1/2)*f^a*exp(-3*d)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))+1/16*Pi^(1/2)*f^a*exp(3*d)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x)+3/16*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)

maxima [A] time = 0.33, size = 143, normalized size = 0.84

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - 3 f}\, x\right) e^{(3 d)}}{16 \sqrt{-c \log(f) - 3 f}} + \frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) + f}\, x\right) e^{(-d)}}{16 \sqrt{-c \log(f) + f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) + 3 f}\, x\right) e^{(-3}}{16 \sqrt{-c \log(f) + 3 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*f
) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f)
 + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f) +
 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\cosh\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + f*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \cosh^3\left(d + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + f*x**2)**3, x)
```

3.317 $\int f^{a+cx^2} \cosh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{\pi}\, f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] 1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f))
^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(
1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\, f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))/
(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*Log
[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/
(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx+\frac{1}{2}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,dx\\ &= \frac{1}{2}\left(e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{2}\left(e^{d-\frac{e^2}{4(f+c\log(f))}}\right.\\ &= \frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2x(f+c\log}{2\sqrt{f+c\log(}}\right.}{4\sqrt{f+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.67, size = 165, normalized size = 1.18

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4(c\log(f)+f)}}\left(\sqrt{c\log(f)+f}\,(\cosh(d)-\sinh(d))e^{\frac{e^2f}{2f^2-2c^2\log^2(f)}}\mathrm{erf}\left(\frac{-2cx\log(f)+e+2fx}{2\sqrt{f-c\log(f)}}\right)+\sqrt{f-c\log(f)}\,(\sinh(d)\right.}{4\sqrt{f-c\log(f)}\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*x*
Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d]) + Er
fi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]*(C
```

```
osh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sqrt[f
 + c*Log[f]])
```

fricas [B] time = 0.43, size = 321, normalized size = 2.29

$$-\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\left(cd+af\right)\log(f)}{4\left(c\log(f)-f\right)}\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\left(cd+af\right)\log(f)}{4\left(c\log(f)-f\right)}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(
c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a*
c*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*l
og(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f)
 - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f +
4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1/4*(4
*a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-
c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log
(f) + f)))/(c^2*log(f)^2 - f^2)
```

giac [A] time = 0.15, size = 172, normalized size = 1.23

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)+4\,df-e^2}{4\left(c\log(f)+f\right)}\right)}}{4\,\sqrt{-c\log(f)-f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)+f}\left(2\,x\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4
*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f
))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e
/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*
d*f - e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

maple [A] time = 0.18, size = 147, normalized size = 1.05

$$\frac{\sqrt{\pi}\;f^{a}\mathrm{e}^{-\frac{4d\ln(f)c-4df+e^2}{4\left(-f+c\ln(f)\right)}}\operatorname{erf}\left(x\sqrt{f-c\ln(f)}+\frac{e}{2\sqrt{f-c\ln(f)}}\right)}{4\sqrt{f-c\ln(f)}}-\frac{\sqrt{\pi}\;f^{a}\mathrm{e}^{\frac{4d\ln(f)c+4df-e^2}{4c\ln(f)+4f}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\;x+\frac{e}{2\sqrt{-c\ln(f)-f}}\right.}{4\sqrt{-c\ln(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))
^(1/2)*erf(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*ex
p(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f
)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

maxima [A] time = 0.34, size = 127, normalized size = 0.91

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - f}\, x - \frac{e}{2 \sqrt{-c \log(f) - f}}\right) e^{\left(d - \frac{e^{2}}{4 \left(c \log(f) + f\right)}\right)}}{4 \sqrt{-c \log(f) - f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) + f}\, x + \frac{e}{2 \sqrt{-c \log(f) + f}}\right) e^{\left(-d - \frac{e^{2}}{4 \left(c \log(f) - f\right)}\right)}}{4 \sqrt{-c \log(f) + f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(
d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*f^a*erf(sqrt
(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) -
f))/sqrt(-c*log(f) + f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a} \cosh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \cosh\left(d+ex+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2), x)
```

3.318 $\int f^{a+cx^2} \cosh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=183

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{2f-c\log(f)}-2d} \operatorname{erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)+2f}} \operatorname{erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi} f^a \operatorname{erfi}\left(\sqrt{c}\, x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-2
*d+e^2/(2*f-c*ln(f)))*f^a*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(
1/2)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*erfi((e+x*(2*f+
c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2204, 2287, 2234, 2205}

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{2f-c\log(f)}-2d} \operatorname{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)+2f}} \operatorname{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{c}\, x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sqrt
[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*Sqr
t[2*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)dx = \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+cx^2}\right)dx$$

$$= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2d-2ex+a\log(f)-x^2(2f-c\,\mathrm{lo}\right.$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+2x(-2f+}{4(-2f+c\,\mathrm{l}}\right.$$

$$= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-}}{}$$

Mathematica [A] time = 1.49, size = 258, normalized size = 1.41

$$\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{2f-c\log(f)}}\left(-\sqrt{c}\,\sqrt{\log(f)}\left((2f-c\log(f))\sqrt{c\log(f)+2f}\,(\sinh(2d)+\cosh(2d))e^{\frac{4e^2f}{c^2\log^2(f)-4f^2}}\,\mathrm{erfi}\left(\frac{cx\log(f)+e+}{\sqrt{c\log(f)+}}\right.\right.\right.}{8\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(-2*E^(e^2/(-2*f + c*Log[f]))*Erfi[S
qrt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[(
e + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c
*Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))*Er
fi[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt[2*f
 + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^2 + c
^2*Log[f]^2))
```

fricas [B] time = 0.64, size = 420, normalized size = 2.30

$$-\frac{2\left(\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\cosh\left(a\log(f)\right)+\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\sinh\left(a\log(f)\right)\right)\sqrt{-c\log(f)}\operatorname{erf}\left(\sqrt{-c\log(f)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log
(f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) + (sq
rt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(
c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f
))*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f) - 2*f
)))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + 2*f
)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((a*c*log
(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c
^2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f
)*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f) + 2*f*x
+ e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*c*f^2*log(f
))
```

giac [A] time = 0.16, size = 198, normalized size = 1.08

$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}\,x\right)}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)-2f}\left(x+\frac{e}{c\log(f)+2f}\right)\right)e^{\left(\frac{ac\log(f)^2+2cd\log(f)+2af\log(f)+4df-e^2}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}}-\frac{\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*er
f(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*d
*log(f) + 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*
f) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e^((
```

```
a*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - 2*f))
/sqrt(-c*log(f) + 2*f)
```

maple [A] time = 0.29, size = 177, normalized size = 0.97

$$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2d\ln(f)c-4df+e^{2}}{-2f+c\ln(f)}} \operatorname{erf}\left(x\sqrt{2f-c\ln(f)}+\frac{e}{\sqrt{2f-c\ln(f)}}\right)}{8\sqrt{2f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2d\ln(f)c+4df-e^{2}}{2f+c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{e}{\sqrt{-c\ln(f}}\right.}{8\sqrt{-c\ln(f)-2f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))
^(1/2)*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f
)-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x)
```

maxima [A] time = 0.34, size = 161, normalized size = 0.88

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)-2\,f}\,x-\frac{e}{\sqrt{-c\log(f)-2\,f}}\right)e^{\left(2\,d-\frac{e^{2}}{c\log(f)+2\,f}\right)}}{8\,\sqrt{-c\log(f)-2\,f}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)+2\,f}\,x+\frac{e}{\sqrt{-c\log(f)+2\,f}}\right)e}{8\,\sqrt{-c\log(f)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^(
2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(
-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\cosh\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \cosh^2\left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2)**2, x)
```

3.319 $\int f^{a+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=300

$$\frac{3\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{2x(c}{2\sqrt{}}\right.}{16\sqrt{c\log(f)+f}}$$

```
[Out] 3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))*f
^a*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(
f))^(1/2)+3/16*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+2*x*(f+c*ln(f)))/
(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9/4*e^2/(3*f+c*l
n(f)))*f^a*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(
3*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{3\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{Erfi}\left(\frac{2x(}{2}\right.}{16\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) + (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+cx}\right.\\ &= \frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+fx^2\right)\right)\\ &= \frac{1}{8}\int\exp\left(-3d-3ex+a\log(f)-x^2(3f-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d+3e\right.\\ &= \frac{1}{8}\left(3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{8}\left(e^{-3d+\frac{9}{12f-4}}\right.\\ &= \frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3e+2x(3f}{2\sqrt{3f-}}\right.}{16\sqrt{3f-c\log(f)}}\end{aligned}$$

Mathematica [A] time = 6.00, size = 478, normalized size = 1.59

$$\frac{\sqrt{\pi}\,f^a\exp\left(-\frac{1}{4}e^2\left(\frac{9}{c\log(f)+3f}+\frac{1}{c\log(f)+f}\right)\right)\left((f-c\log(f))\left(\sqrt{3f-c\log(f)}\,\left(c^2\log^2(f)+4cf\log(f)+3f^2\right)(\cosh\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3*
f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*S
qrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*
(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f + c*
Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f])/(2*
Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log
[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(3*f + c
*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f +
 c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) + E^(e^2/(4*(f + c*Log[f]))
)*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f]
)*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*((f + c*Log[
f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f
]^4))
```

fricas [B] time = 0.53, size = 847, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(f)
 - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3
)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log
(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e)*sqr
t(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*lo
g(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4
*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f
)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c
*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2*c*x*log(f
) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f
)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e
^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3
 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2
+ 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/
2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi
)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(1/4*(4*a*c*
log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)) + sqrt
(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(1/4*(4*a
*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)))*sq
```

```
rt(-c*log(f) - 3*f)*erf(1/2*(2*c*x*log(f) + 6*f*x + 3*e)*sqrt(-c*log(f) - 3
*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

giac [A] time = 0.19, size = 352, normalized size = 1.17

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{3e}{c\log(f)+3f}\right)\right)e^{\left(\frac{4ac\log(f)^2+12cd\log(f)+12af\log(f)+36df-9e^2}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f)))
*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) + 36*d*f - 9*e^2)/(
c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) +
4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4*a*c*l
og(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - f))/sqrt(-
c*log(f) + f) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x - 3*e/(c*
log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f) + 36
*d*f - 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

maple [A] time = 0.42, size = 302, normalized size = 1.01

$$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{3\left(4d\ln(f)c-12df+3e^2\right)}{4\left(-3f+c\ln(f)\right)}}\,\mathrm{erf}\left(x\sqrt{3f-c\ln(f)}+\frac{3e}{2\sqrt{3f-c\ln(f)}}\right)}{16\sqrt{3f-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{3d\ln(f)c+9df-\frac{9e^2}{4}}{3f+c\ln(f)}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{}{2\sqrt{}}\right.}{16\sqrt{-c\ln(f)-3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(-3*f+c*ln(f)))/(3*f-
c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))-1/16*Pi
^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)
^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))+3/16*Pi^(1/2
)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(
x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(4*d
*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)
*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

maxima [A] time = 0.34, size = 263, normalized size = 0.88

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)-3f}\,x-\frac{3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(3d-\frac{9e^2}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{e}{2\sqrt{-c\log(f)-f}}\right.}{16\sqrt{-c\log(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f)
)*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*
f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^2/(
c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) +
 f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqrt(-c*
log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/sqrt(-c
*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

3.320 $\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f))
/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(f)
)^2/c/ln(f))*f^a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(
1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5513, 2287, 2234, 2204}

$$\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] -(E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
 (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log
[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh(d+ex)\,dx &= \int\left(\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+ex}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int\exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)dx+\frac{1}{2}\int\exp\left(d+a\log\right.\\ &= \frac{1}{2}\left(e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}\right.\\ &= -\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+}{2\sqrt{c}\sqrt{l}}\right.}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.32, size = 134, normalized size = 0.88

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(2b\log(f)+e)}{4c\log(f)}}\left(e^{\frac{be}{c}}(\cosh(d)-\sinh(d))\mathrm{erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{\log(f)(b+2cx)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*
Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d]) + Erfi[(e + (b + 2*c*x)*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*b*L
og[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.59, size = 262, normalized size = 1.71

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{\left(b^2-4ac\right)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{\left(b^2-4ac\right)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)\right)\mathrm{erf}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(p
i)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log
(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log
(f))
```

giac [A] time = 0.15, size = 169, normalized size = 1.10

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^{2}\log(f)^{2}-4\,ac\log(f)^{2}+4\,cd\log(f)-2\,be\log(f)+e^{2}}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e
^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/
(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x +
(b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*
log(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.17, size = 156, normalized size = 1.02

$$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}-2\ln(f)be+4d\ln(f)c+e^{2}}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}+2\ln(f)be-4d\ln(f)c+e^{2}}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\right.}{4\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1/2
))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1
/2))
```

maxima [A] time = 0.35, size = 129, normalized size = 0.84

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f) + e}{2 \sqrt{-c \log(f)}}\right) e^{\left(d - \frac{(b \log(f) + e)^{2}}{4 c \log(f)}\right)}}{4 \sqrt{-c \log(f)}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f) - e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-d - \frac{(b \log(f) - e)^{2}}{4 c \log(f)}\right)}}{4 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f))
)*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(
b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c x^{2}+b x+a} \cosh(d + e x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^{2}} \cosh(d + ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x), x)
```

3.321 $\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$

Optimal. Leaf size=219

$$\frac{\sqrt{\pi}\, f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\, f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2e \ldots}{2\sqrt{c} \ldots}\right.}{8\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*e+b*ln(f)+2*c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-1/4*(2
*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)
*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5513, 2234, 2204, 2287}

$$\frac{\sqrt{\pi}\, f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\, f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+ \ldots}{2\sqrt{c} \ldots}\right.}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*
Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*Sqr
t[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx = \int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2ex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$=\frac{1}{4}\int\exp\left(-2d+a\log(f)+cx^2\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int\exp\left(2d+\right.$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+b\log(f)+}{4c\log}\right.$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+$$

Mathematica [A] time = 0.56, size = 183, normalized size = 0.84

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(b\log(f)+e)}{c\log(f)}}\left(e^{\frac{2be}{c}}(\cosh(2d)-\sinh(2d))\mathrm{erfi}\left(\frac{\log(f)(b+2cx)-2e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\mathrm{erfi}\left(\frac{\log(f)(b+2cx)+2}{2\sqrt{c}\sqrt{\log(f)}}\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b +
 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*x)
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e + (b
```

```
 + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sq
rt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.51, size = 341, normalized size = 1.56

$$-\frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)+\sqrt{-c\log(f)}\left(\sqrt{\pi}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(
pi)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/
c) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 -
4*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log
(f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*l
og(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-
1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log(f))) +
 sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f
))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))/(c*log(f
))))/(c*log(f))
```

giac [A] time = 0.15, size = 225, normalized size = 1.03

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log\cdots}{}\right.}}{8\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4
*a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*
x + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
8*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-1/4*
(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/(c*lo
g(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.28, size = 211, normalized size = 0.96

$$-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2-4\ln(f)be+8d\ln(f)c+4e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-2e}{2\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2+4\ln(f)be-8d\ln(f)c+4e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f\cdots}\right.}{8\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c+4*e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f)*b*e-8*d*ln(f)*c+4*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*l
n(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.34, size = 185, normalized size = 0.84

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f) + 2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(2 d - \frac{\left(b \log(f) + 2 e\right)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f) - 2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-2 d - \frac{\left(b \log(f) - 2 e\right)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(f
)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(
pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^(-2
*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1
/4*b^2/c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cosh\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**2, x)
```

3.322 $\int f^{a+bx+cx^2} \cosh^3(d+ex)\,dx$

Optimal. Leaf size=315

$$-\frac{3\sqrt{\pi}\,f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f}{4c\log}}}{16}$$

```
[Out] 3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(-3*d-1/4*(3*e-
b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(
1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f))*f^
a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln
(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(3*e+b*ln(
f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5513, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}\,f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}\,f^a e^{d-\frac{(b\log(}{4c\log}}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx = \int\left(\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}+\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+bx+c\ldots}\right.$$

$$=\frac{1}{8}\int e^{-3d-3ex}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3d+3ex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+bx+cx^2}\,dx+\ldots$$

$$=\frac{1}{8}\int\exp\left(-3d+a\log(f)+cx^2\log(f)-x(3e-b\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d+\ldots\right.$$

$$=\frac{1}{8}\left(3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{8}\left(e^{-3d-\frac{(3e-b\ldots}{4c\ldots}}\right.$$

$$=-\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-b\ldots}{2\ldots}\right)}{16\sqrt{c}\sqrt{\log(f)}}\ldots$$

Mathematica [A] time = 1.02, size = 262, normalized size = 0.83

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{3e(2b\log(f)+3e)}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e(b\log(f)+e)}{c\log(f)}}\mathrm{erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)\ldots\right.\right.}{16\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]

[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((e*(2*e + b*Log[f]))
/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^
```

```
((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + E^((3*b*e)/c)*Erfi[(-3*
e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])))
/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.63, size = 526, normalized size = 1.67

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{\left(b^2-4ac\right)\log(f)^2+9e^2-6(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{\left(b^2-4ac\right)\log(f)^2+9e^2-6(2cd-be)\log(f)}{4c\log(f)}\right)\right)\mathrm{erf}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
- 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*l
og(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)
*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f)))
 + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f
))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*log(f))
) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt
(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c
*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/
(c*log(f))
```

giac [A] time = 0.16, size = 343, normalized size = 1.09

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f)))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) + 9
*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*
(2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
```

```
4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b
^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f)
))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f
) + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f
) + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.39, size = 316, normalized size = 1.00

$$-\frac{\sqrt{\pi}\; f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6\ln(f) b e+12 d \ln(f) c+9 e^{2}}{4\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\; x+\frac{b\ln(f)-3e}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\; f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6\ln(f) b e-12 d \ln(f) c+9 e^{2}}{4\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\,\mathrm{l}}\right.}{16\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*ln(f)*b*e+12*d*ln(f)*c+9*e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(-c*ln(f)
)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*ln(f)*b*e-12*d*ln(f)*c+9
*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln
(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e
)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d
*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln
(f))/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.34, size = 263, normalized size = 0.83

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+3\, e}{2\, \sqrt{-c \log(f)}}\right) e^{\left(3\, d-\frac{\left(b \log(f)+3\, e\right)^{2}}{4\, c \log(f)}\right)}}{16\, \sqrt{-c \log(f)}}+\frac{3\, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+e}{2\, \sqrt{-c \log(f)}}\right) e^{\left(d-\frac{\left(b \log(f)+e\right)^{2}}{4\, c \log(f)}\right)}}{16\, \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log(
f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^(d
- 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(
sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b*log(
f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f
))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f) - 3*e)
^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\cosh\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**3, x)
```

3.323 $\int f^{a+bx+cx^2} \cosh\left(d+fx^2\right)\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

```
[Out] -1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(f
)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2
/(f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^
(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] -(E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2
*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d
- (b^2*Log[f]^2)/(4*(f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
 c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int\exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)dx+\frac{1}{2}\int\exp\left(d+a\log\right.\\ &= \frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{2}\left(e^{d-\frac{b^2\log^2}{4(f+c\log}}\right.\\ &= -\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+}{2\sqrt{f}}\right.}{4\sqrt{f+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.68, size = 185, normalized size = 1.20

$$\frac{\sqrt{\pi}f^a e^{-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\left(\sqrt{f-c\log(f)}\,(c\log(f)+f)(\cosh(d)-\sinh(d))e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\mathrm{erf}\left(\frac{2fx-\log(f)(b+2cx)}{2\sqrt{f-c\log(f)}}\right)+(f-c\log\right.}{4\left(f^2-c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x - (
b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[f]
)*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log
[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*L
og[f]^2)/(4*(f + c*Log[f])))*(f^2 - c^2*Log[f]^2))
```

fricas [B] time = 0.54, size = 324, normalized size = 2.10

$$-\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,df+4\left(cd+af\right)\log(f)}{4\left(c\log(f)-f\right)}\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,df+4\left(\right.}{4\left(c\log(f)-f\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f +
4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqr
t(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/
(c*log(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^
2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f
) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*l
og(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

giac [A] time = 0.16, size = 181, normalized size = 1.18

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\left(c\log(f)+f\right)}\right)}}{4\sqrt{-c\log(f)-f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d
*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log
(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
 + f)
```

maple [A] time = 0.18, size = 160, normalized size = 1.04

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 d \ln(f) c-4 d f}{4\left(-f+c \ln(f)\right)}} \operatorname{erf}\left(-x \sqrt{f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{f-c \ln(f)}}\right)}{4 \sqrt{f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 d \ln(f) c-4 d f}{4\left(f+c \ln(f)\right)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-f}\, x+\right.}{4 \sqrt{-c \ln(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)/(-f+c*ln(f)))/(f
-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))-1/4
*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))/(-c*ln(
f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))
```

maxima [A] time = 0.33, size = 139, normalized size = 0.90

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4\left(c \log(f)+f\right)}+d\right)}}{4 \sqrt{-c \log(f)-f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-\frac{b}{4(}\right.}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 1/4*sqrt
(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e^(-
1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\mathrm{cosh}\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^{2}} \cosh\left(d+fx^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d),x)
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2), x)
```

3.324 $\int f^{a+bx+cx^2} \cosh^2\left(d + f x^2\right) dx$

Optimal. Leaf size=225

$$-\frac{\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\,\text{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\,\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}\, f^{a-\frac{b^2}{4c}}\,\text{erfi}\left(\frac{\sqrt{\log}}{}\right.}{4\sqrt{c}\sqrt{\log(}}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f
)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1/2)+1/8*
exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(2*f+c*ln(f)
))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2234, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\,\text{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\,\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}\, f^{a-\frac{b^2}{4c}}\,\text{Erfi}\left(\frac{\sqrt{l}}{}\right.}{4\sqrt{c}\sqrt{\log}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])])/
(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]])])
/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2d+a\log(f)+bx\log(f)-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int \exp\left(2
\end{aligned}$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(}{4(-2f-}\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right.}{8\sqrt{2f-c\log(f)}}$$

Mathematica [A] time = 2.28, size = 257, normalized size = 1.14

$$\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2f^{-\frac{b^2}{4c}}\mathrm{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\left(\sqrt{2f-c\log(f)}\,(c\log(f)+2f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2f\log^2(}{4f^2-c^2\log}}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*Er
f[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Log[f
]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)*Log
[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[
2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log
[f]^2))))/8
```

fricas [B] time = 0.48, size = 466, normalized size = 2.07

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-16\,df+8\left(cd+af\right)\log(f)}{4\left(c\log(f)-2\,f\right)}\right)+\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\sinh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log
(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*d +
 a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2
*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*
log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*
d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))/(c*log(
f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f))*sqrt
(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*co
sh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4
*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

giac [A] time = 0.16, size = 239, normalized size = 1.06

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-2\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)-8\,af\log(f)-16\,df}{4\left(c\log(f)+2\,f\right)}\right)}}{8\sqrt{-c\log(f)-2\,f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2*
f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f) -
 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sq
rt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) - 2*f)
)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c)
)*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

maple [A] time = 0.36, size = 217, normalized size = 0.96

$$-\frac{\sqrt{\pi}\ f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}+8d\ln(f)c-16df}{4\left(-2f+c\ln(f)\right)}}\ \mathrm{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{2f-c\ln(f)}}\right)}{8\sqrt{2f-c\ln(f)}}-\frac{\sqrt{\pi}\ f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}-8d\ln(f)c-16df}{4\left(2f+c\ln(f)\right)}}\ \mathrm{erf}\left(-\sqrt{-c\ln(f)-\right.}{8\sqrt{-c\ln(f)-2f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*d*ln(f)*c-16*d*f)/(-2*f+c*ln(f)))
/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*d*ln(f)*c-16*d*f)/(2*f+c*ln(
f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)
-2*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.34, size = 199, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - 2 f}\, x - \frac{b \log(f)}{2 \sqrt{-c \log(f) - 2 f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4 \left(c \log(f) + 2 f\right)} + 2 d\right)}}{8 \sqrt{-c \log(f) - 2 f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) + 2 f}\, x - \frac{b \log(f)}{2 \sqrt{-c \log(f)+}}\right.}{8 \sqrt{-c \log(f) + 2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
- 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f)
+ 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) + 2*f
) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/
(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{cosh}\left(f\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2)**2, x)
```

3.325 $\int f^{a+bx+cx^2} \cosh^3\left(d+fx^2\right) dx$

Optimal. Leaf size=323

$$-\frac{3\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\operatorname{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)}}}{16}$$

```
[Out] -3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(
f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+b^2*ln(f)^
2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(
1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)+3/16*exp(d-1/4*b^2*ln(f)^2/(f+c*ln(f)))*
f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(
f))^(1/2)+1/16*exp(3*d-1/4*b^2*ln(f)^2/(3*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)
+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5513, 2287, 2234, 2205, 2204}

$$-\frac{3\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\operatorname{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)}}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) +
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)dx = \int\left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a}\right.$$

$$=\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx+cx^2}$$

$$=\frac{1}{8}\int\exp\left(-3d+a\log(f)+bx\log(f)-x^2(3f-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(3d\right.$$

$$=\frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{8}\left(e^{-3d+\ldots}\right.$$

$$=-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log\ldots}{\ldots}\right.}{16\sqrt{3f-c\log\ldots}}$$

Mathematica [B] time = 6.51, size = 2511, normalized size = 7.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]

[Out] (f^a*Sqrt[Pi]*(27*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^3*Cosh[d]*Erf[(2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]] +
 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^2*Cosh[d]*Erf[(2*f*x - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]] - 3*c^
2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]] - 3*c^3*E^
((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2*c*x*L
og[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]] + 3*E^((b^2*Log[
f]^2)/(4*(3*f - c*Log[f])))*f^3*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log
[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]] + c*E^((b^2*Log[f]^2)/(
4*(3*f - c*Log[f])))*f^2*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]] - 3*c^2*E^((b^2*Log[f]^
2)/(4*(3*f - c*Log[f])))*f*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]] - c^3*E^((b^2*Log[f
]^2)/(4*(3*f - c*Log[f])))*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]] + (27*f^3*Cosh[d]*E
rfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Lo
g[f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (27*c*f^2*Cosh[d]*Erfi[(2*f*
x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Cosh[d]*Erfi[(2*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f
]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Cosh[d]*Erfi[(2*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]])
/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*f^3*Cosh[3*d]*Erfi[(6*f*x + b*L
og[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((b
^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erfi[(6*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(6*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*L
og[f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - 27*E^((b^2*Log[f]^2)/(4*(
f - c*Log[f])))*f^3*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Sqrt[f - c*Log[f]]*Sinh[d] - 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f]
)))*f^2*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f
]*Sqrt[f - c*Log[f]]*Sinh[d] + 3*c^2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*
f*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sq
rt[f - c*Log[f]]*Sinh[d] + 3*c^3*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Erf[
(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f -
 c*Log[f]]*Sinh[d] + (27*f^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Lo
g[f]))) - (27*c*f^2*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Lo
g[f]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[
```

```
f]))) - (3*c^2*f*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f
]))) + (3*c^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])
]*Log[f]^3*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
) - 3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^3*Erf[(6*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] - c*E
^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^2*Erf[(6*f*x - b*Log[f] - 2*c*x*Lo
g[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] + 3*c
^2*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]]*Sinh[3*d] +
 c^3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d] +
 (3*f^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sq
rt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]
*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) -
(3*c^2*f*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*L
og[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f
]))) + (c^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])
]*Log[f]^3*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Lo
g[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3*f + c*Log[f
]))
```

fricas [B] time = 0.63, size = 851, normalized size = 2.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f
) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^
3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*l
og(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f))*
sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f
*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f
*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2
*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2
- 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*
```

```
log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log(f) - f)/(c*log
(f) + f)) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f
^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/(c*
log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) +
 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))
/(c*log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(
f))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log
(f)^2 + 9*f^4)
```

giac [A] time = 0.18, size = 369, normalized size = 1.14

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)-3\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)+3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-36\,df}{4\left(c\log(f)+3\,f\right)}\right)}}{16\,\sqrt{-c\log(f)-3\,f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) + 3
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f
) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/
2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) + f))/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + b*lo
g(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f)
 + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/16*sqrt(pi
)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f)))*e^(-1/4
*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 36*d*f)/(
c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

maple [A] time = 0.42, size = 326, normalized size = 1.01

$$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}+12d\ln(f)c-36df}{4\left(-3f+c\ln(f)\right)}}\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{3f-c\ln(f)}}\right)}{16\sqrt{3f-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}-12d\ln(f)c-36df}{4\left(3f+c\ln(f)\right)}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-\right.}{16\sqrt{-c\ln(f)-3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*d*ln(f)*c-36*d*f)/(-3*f+c*ln(f)
))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f-c*ln(f))
^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*d*ln(f)*c-36*d*f)/(3*f+c
*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*l
```

```
n(f)-3*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)
/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*
ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f
+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-f)^(1/2))
```

maxima [A] time = 0.35, size = 287, normalized size = 0.89

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c \log(f) - 3 f}\, x - \frac{b \log(f)}{2 \sqrt{-c \log(f) - 3 f}}\right) e^{\left(-\frac{b^2 \log(f)^2}{4 \left(c \log(f) + 3 f\right)} + 3 d\right)}}{16 \sqrt{-c \log(f) - 3 f}} + \frac{3 \sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c \log(f) - f}\, x - \frac{b \log(f)}{2 \sqrt{-c \log(f) - f}}\right.}{16 \sqrt{-c \log(f) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*f)
 + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 3/16
*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))
*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) + 1/16*sqrt(p
i)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3*f))*e^
(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\cosh\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.326 $\int f^{a+bx+cx^2} \cosh\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=161

$$\frac{\sqrt{\pi}\, f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] 1/4*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*l
n(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*(e+b*ln(
f))^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/
2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\, f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[f
] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (
E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f]
 + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int\exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx+\frac{1}{2}\int \text{ex}\\ &= \frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx+\frac{1}{2}\\ &= \frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-}{}\right.}{4\sqrt{f+c\,\text{lo}}}\end{aligned}$$

Mathematica [A] time = 1.51, size = 251, normalized size = 1.56

$$\frac{\sqrt{\pi}\,e^{-\frac{b^2\log^2(f)+e^2}{4(c\log(f)+f)}}f^{a+\frac{bef}{c^2\log^2(f)-f^2}}\left(\sqrt{f-c\log(f)}\,(c\log(f)+f)(\cosh(d)-\sinh(d))f^{\frac{be}{2(c\log(f)+f)}}\exp\left(\frac{f\left(b^2\log^2(f)+e^2\right)}{2\left(f^2-c^2\log^2(f)\right)}\right)\text{erf}\right.}{4\left(f^2-c^2\,\text{lo}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(E^((f*(e^2 + b^2*Log[f]^2)
)/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x - (
b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[f]
)*(Cosh[d] - Sinh[d]) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x + (b +
 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(
Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2 -
c^2*Log[f]^2))
```

fricas [B] time = 0.54, size = 362, normalized size = 2.25

$$-\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(-\frac{\left(b^{2}-4\,ac\right)\log(f)^{2}+e^{2}-4\,df+2\left(2\,cd-be+2\,af\right)\log(f)}{4\left(c\log(f)-f\right)}\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(-\frac{\left(b^{2}-4\,ac\right)\log(f)^{2}\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*
d*f + 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f)
+ f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a
*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x +
 b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + (sqrt(pi)*(c*log(f) -
 f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*
f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2 - 4*a*
c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + f))
)*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqrt(-c*log(
f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

giac [A] time = 0.17, size = 209, normalized size = 1.30

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)-f}\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^{2}\log(f)^{2}-4\,ac\log(f)^{2}-4\,cd\log(f)-4\,af\log(f)+2\,be\log(f)-4\,df+e^{2}}{4\left(c\log(f)+f\right)}\right)}}{4\,\sqrt{-c\log(f)-f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
+ f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f)
 + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sq
rt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f)))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 2*b*
e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

maple [A] time = 0.19, size = 186, normalized size = 1.16

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-2\ln(f) be+4d\ln(f) c-4df+e^{2}}{4\left(-f+c\ln(f)\right)}} \operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{b\ln(f)-e}{2\sqrt{f-c\ln(f)}}\right)}{4\sqrt{f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+2\ln(f) be-4d\ln(f) c-4df+e^{2}}{4\left(f+c\ln(f)\right)}} \operatorname{erf}\left(-\right.}{4\sqrt{-c\ln(f)-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/
(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f
-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(
f)*c-4*d*f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1
/2*(e+b*ln(f))/(-c*ln(f)-f)^(1/2))
```

maxima [A] time = 0.34, size = 151, normalized size = 0.94

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)-f}\, x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right) e^{\left(-\frac{\left(b\log(f)+e\right)^{2}}{4\left(c\log(f)+f\right)}+d\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)+f}\, x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right) e^{\left(-\frac{(b}{4(}\right.}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*log
(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f)
+ 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-c*l
og(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^{2}} \cosh\left(d+ex+fx^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cosh(d + e*x + f*x**2), x)
```

3.327 $\int f^{a+bx+cx^2} \cosh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=239

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\,\mathrm{lo}}{4c\,\mathrm{l}}\right.}{}$$

```
[Out] 1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2
)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(2*
e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1
/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(2*e+b*ln(f)+
2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5513, 2234, 2204, 2287, 2205}

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\,\mathrm{l}}{4c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f]))
*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*L
og[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f + 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqr
t[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)dx = \int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a\cdots}$$

$$=\frac{1}{4}\int \exp\left(-2d+a\log(f)-x(2e-b\log(f))-x^2(2f-c\log(f))\right)dx+\cdots$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\right)\int\cdots$$

$$=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2e-b\cdots}{\cdots}\right)\cdots}{8\sqrt{2f-c\log(f)}\cdots}$$

Mathematica [A] time = 6.17, size = 339, normalized size = 1.42

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\mathrm{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}e^{-\frac{b^2\log^2(f)+4e^2}{4c\log(f)+8f}}f^{a+\frac{4bef}{c^2\log^2(f)-4f^2}}\left(\sqrt{2f-c\log(f)}(c\log(f)+2f)(\cosh(2d)-\sinh(\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqrt[P
i]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f + c
*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])
]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((b*e)/
(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2*f + c
*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d])))/
(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[f]^2))
```

fricas [B] time = 0.47, size = 516, normalized size = 2.16

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2+4\,e^2-16\,df+4\left(2\,cd-be+2\,af\right)\log(f)}{4\left(c\log(f)-2\,f\right)}\right)+\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)) +
 sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-
c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*log(f)
+ 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/
4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f)
)/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f) +
2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 -
4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt
(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

giac [A] time = 0.19, size = 273, normalized size = 1.14

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-2\,f}\left(2\,x+\frac{b\log(f)+2\,e}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)-8\,af\log(f)+4\,be\log(f)-16\,df+4\,e^2}{4\left(c\log(f)+2\,f\right)}\right)}}{8\sqrt{-c\log(f)-2\,f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*log
(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*
log(f) + 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2*e)/
(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) +
8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) - 2*f))/sqrt(-c*log
(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^
2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

maple [A] time = 0.33, size = 249, normalized size = 1.04

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 \ln(f) b e+8 d \ln(f) c-16 d f+4 e^{2}}{4(-2 f+c \ln(f))}} \operatorname{erf}\left(-x \sqrt{2 f-c \ln(f)}+\frac{b \ln(f)-2 e}{2 \sqrt{2 f-c \ln(f)}}\right)}{8 \sqrt{2 f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 \ln(f) b e-8 d \ln(f) c-16 d f+4 e^{2}}{4(2 f+c \ln(f))}}}{8 \sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c-16*d*f+4*e^
2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln
(f)-2*e)/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f
)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-
c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f)-2*f)^(1/2))-1/4*Pi^(1/2)*f
^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*
ln(f))^(1/2))
```

maxima [A] time = 0.33, size = 215, normalized size = 0.90

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-2 f}\, x-\frac{b \log(f)+2 e}{2 \sqrt{-c \log(f)-2 f}}\right) e^{\left(-\frac{(b \log(f)+2 e)^{2}}{4 (c \log(f)+2 f)}+2 d\right)}}{8 \sqrt{-c \log(f)-2 f}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+2 f}\, x-\frac{b \log(f)-2}{2 \sqrt{-c \log(f)+}}\right.}{8 \sqrt{-c \log(f)+2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-c
*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*
log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*log(f
) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) - 2*f)
 - 2*d)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/
2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

3.328 $\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=344

$$\frac{\sqrt{\pi}\,f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\text{erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}\,f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\cdots$$

```
[Out] 3/16*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+(3*e-b*
ln(f))^2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*ln(f)))/(3*f
-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/
(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi
^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/(3*f+c*ln(f)))*f^
a*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3
*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5513, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\,f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\text{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}\,f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
+ (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) + (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5513

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)dx=\int\left(\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right.\right.$$

$$=\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+\right.\right.$$

$$=\frac{1}{8}\int\exp\left(-3d+a\log(f)-x(3e-b\log(f))-x^2(3f-c\log(f))\right)dx+\frac{1}{8}$$

$$=\frac{1}{8}\left(\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-3e+b\log(f)+2x(-3f+}{4(-3f+c\log(f)}\right.$$

$$=\frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f}\right.}{16}$$

Mathematica [B] time = 6.65, size = 2991, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]

[Out] (f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c*f^2*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (27*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (27*c*f^2*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*f^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (27*f^3*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*c*f^2*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[

```
f]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(
f - c*Log[f]))) + (3*c^2*f*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqr
t[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log
[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^3*Erf[(e + 2*f*x - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]]*Sinh[d]
)/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (27*f^3*Erf
i[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*
Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f])))
 - (27*c*f^2*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]
^2)/(4*(f + c*Log[f]))) - (3*c^2*f*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f
])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2
*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^3*Erfi[(e + 2*f*x +
b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]
]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*
f^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*S
qrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4
*(3*f - c*Log[f]))) - (c*f^2*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 +
 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*c^2*f*Erf[(3*e + 6
*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f
 - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f -
 c*Log[f]))) + (c^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f
 - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*
Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*f^3*Erfi[(3*e + 6*f*x + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]]*Sinh
[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*
Log[f]*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f
]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*
Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/
E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Erfi[
(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*
Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4
*(3*f + c*Log[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3
*f + c*Log[f]))
```

fricas [B] time = 0.48, size = 939, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*c
```

```
osh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*f)
*log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*
f^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*
(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(
-1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3
*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)
*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*f)*
log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2
*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d
 - b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*
x - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(p
i)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f
) + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*
log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*l
og(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi)*(c^3*log(f)^3 -
3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
+ 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + 3*f)) + sqrt
(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b
^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*
log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f) +
 3*e)*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*l
og(f)^2 + 9*f^4)
```

giac [A] time = 0.21, size = 431, normalized size = 1.25

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)+3e}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)+6be\log(f)-36df+9e^2}{4\left(c\log(f)+3f\right)}\right)}}{16\sqrt{-c\log(f)-3f}}-\frac{3\sqrt{\pi}\,\mathrm{er}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*lo
g(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a
*f*log(f) + 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) + 3*f))/sqrt(-c*log(f)
 - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/
(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*
a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f
) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log
(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*lo
g(f) - 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/
16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e)/(c*log(f
```

```
) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*
log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) +
3*f)
```

maple [A] time = 0.44, size = 384, normalized size = 1.12

$$-\frac{\sqrt{\pi}\ f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}-6\ln(f)be+12d\ln(f)c-36df+9e^{2}}{4(-3f+c\ln(f))}}\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{b\ln(f)-3e}{2\sqrt{3f-c\ln(f)}}\right)}{16\sqrt{3f-c\ln(f)}}-\frac{\sqrt{\pi}\ f^{a}\mathrm{e}^{-\frac{\ln(f)^{2}b^{2}+6\ln(f)be-12d\ln(f)c-36df+9e^{2}}{4(3f+c\ln(f))}}}{16\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*
e^2)/(-3*f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*
ln(f)-3*e)/(3*f-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*l
n(f)*b*e-12*d*ln(f)*c-36*d*f+9*e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf
(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))-3/16*Pi^(1
/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f
)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^
(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*
f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*l
n(f))/(-c*ln(f)-f)^(1/2))
```

maxima [A] time = 0.35, size = 315, normalized size = 0.92

$$\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)-3\,f}\,x-\frac{b\log(f)+3\,e}{2\,\sqrt{-c\log(f)-3\,f}}\right)e^{\left(-\frac{\left(b\log(f)+3\,e\right)^{2}}{4\left(c\log(f)+3\,f\right)}+3\,d\right)}}{16\,\sqrt{-c\log(f)-3\,f}}+\frac{3\,\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)+e}{2\,\sqrt{-c\log(f)-f}}\right.}{16\,\sqrt{-c\log(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(-
c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c
*log(f) - 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f
) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sq
rt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*lo
g(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)
/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*
(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/(c*log(f
) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**3,x)
[Out] Timed out
```

3.329 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx$

Optimal. Leaf size=20

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

```
[Out] 2*x*sinh(x)/cosh(x)^(1/2)-4*cosh(x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3315}

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] -4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx &= \int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\cosh(x)}\,dx\\ &= -4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [B] time = 0.36, size = 46, normalized size = 2.30

$$\frac{2\sinh(x)\left(x-\frac{2\sinh(x)\cosh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(\cosh(x)-1)^{3/2}\sqrt{\cosh(x)+1}}\right)}{\sqrt{\cosh(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] (2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2)
*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{3}{2}}} + x\left(\sqrt{\cosh}\,(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

mupad [B] time = 0.15, size = 39, normalized size = 1.95

$$-\frac{2\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\,\left(x+2\,e^{2\,x}-x\,e^{2\,x}+2\right)}{e^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^(1/2) + x/cosh(x)^(3/2),x)
[Out] -(2*(exp(-x)/2 + exp(x)/2)^(1/2)*(x + 2*exp(2*x) - x*exp(2*x) + 2))/(exp(2*
x) + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\cosh^{2}(x)+1\right)}{\cosh^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
[Out] Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

3.330 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx$

Optimal. Leaf size=24

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

```
[Out] 2/3*x*sinh(x)/cosh(x)^(3/2)+4/3/cosh(x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3315}

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] 4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned} \int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx &= -\left(\frac{1}{3} \int \frac{x}{\sqrt{\cosh(x)}}\, dx \right) + \int \frac{x}{\cosh^{\frac{5}{2}}(x)}\, dx \\ &= \frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)} \end{aligned}$$

Mathematica [A] time = 0.08, size = 16, normalized size = 0.67

$$\frac{2(x\tanh(x) + 2)}{3\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] (2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

fricas [B] time = 0.50, size = 109, normalized size = 4.54

$$\frac{4\left((x+2)\cosh(x)^{3}+3\,(x+2)\cosh(x)\sinh(x)^{2}+(x+2)\sinh(x)^{3}-(x-2)\cosh(x)+\left(3\,(x+2)\cosh(x)^{2}-x+\cdots\right.\right.}{3\left(\cosh(x)^{4}+4\,\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\,\cosh(x)^{2}+1\right)\sinh(x)^{2}+2\,\cosh(x)^{2}+4\left(\cosh(x)^{3}+\cos\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
(x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{3\,\sqrt{\cosh(x)}}+\frac{x}{\cosh(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{3\,\sqrt{\cosh(x)}}+\frac{x}{\cosh(x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

mupad [B] time = 0.97, size = 42, normalized size = 1.75

$$\frac{4\,e^{x}\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}\,\left(2\,e^{2\,x}-x+x\,e^{2\,x}+2\right)}{3\,{\left(e^{2\,x}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2) - x/(3*cosh(x)^(1/2)),x)
```

```
[Out] (4*exp(x)*(exp(-x)/2 + exp(x)/2)^(1/2)*(2*exp(2*x) - x + x*exp(2*x) + 2))/(
3*(exp(2*x) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x}{\cosh^{\frac{5}{2}}(x)}\right)dx+\int\frac{x}{\sqrt{\cosh(x)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/cosh(x)**(5/2), x) + Integral(x/sqrt(cosh(x)), x))/3
```

3.331 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx$

Optimal. Leaf size=47

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

```
[Out] 4/15/cosh(x)^(3/2)+2/5*x*sinh(x)/cosh(x)^(5/2)+6/5*x*sinh(x)/cosh(x)^(1/2)-
12/5*cosh(x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3315}

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] 4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/2
)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx=\frac{3}{5}\int x\sqrt{\cosh(x)}\,dx+\int \frac{x}{\cosh^{\frac{7}{2}}(x)}\,dx$$
$$=\frac{4}{15\cosh^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\frac{3}{5}\int x\sqrt{\cosh(x)}\,dx$$
$$=\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

Mathematica [A] time = 0.61, size = 64, normalized size = 1.36

$$\frac{1}{5}\sqrt{\cosh(x)}\left(6x\tanh(x)+\left(2x\tanh(x)+\frac{4}{3}\right)\text{sech}^2(x)-\frac{12\sinh^2(x)}{\sqrt{\cosh(x)-1}\,(\cosh(x)+1)^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] (Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sqr
t[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3}{5}x\sqrt{\cosh(x)}+\frac{x}{\cosh(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{7}{2}}} + \frac{3x\left(\sqrt{\cosh}\,(x)\right)}{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3}{5}\,x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

mupad [B] time = 1.11, size = 110, normalized size = 2.34

$$\frac{e^{2x}\left(\frac{8x}{5}+\frac{16}{15}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{\left(e^{2x}+1\right)^2}-\left(\frac{6x}{5}+\frac{12}{5}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}+\frac{12\,x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\left(e^{2x}+1\right)}-\frac{16\,x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\left(e^{2x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x*cosh(x)^(1/2))/5 + x/cosh(x)^(7/2),x)
```

```
[Out] (exp(2*x)*((8*x)/5 + 16/15)*(exp(-x)/2 + exp(x)/2)^(1/2))/(exp(2*x) + 1)^2
- ((6*x)/5 + 12/5)*(exp(-x)/2 + exp(x)/2)^(1/2) + (12*x*exp(2*x)*(exp(-x)/2
 + exp(x)/2)^(1/2))/(5*(exp(2*x) + 1)) - (16*x*exp(2*x)*(exp(-x)/2 + exp(x)
/2)^(1/2))/(5*(exp(2*x) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
```

```
[Out] Timed out
```

3.332 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)} \right) dx$

Optimal. Leaf size=36

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}} - 8x\sqrt{\cosh(x)} - 16iE\left(\frac{ix}{2}\middle|2\right)$$

```
[Out] -16*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))+2*
x^2*sinh(x)/cosh(x)^(1/2)-8*x*cosh(x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3316, 2639}

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}} - 8x\sqrt{\cosh(x)} - 16iE\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] -8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[Co
sh[x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3316

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[((c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[(d^2*m*(m - 1))/(b^2*f^2*(n + 1)*(n + 2)), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[(d*m*(c + d*x)^(m - 1)*(b*
Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx &= \int\frac{x^2}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\cosh(x)}\,dx\\ &= -8x\sqrt{\cosh(x)}+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}+8\int\sqrt{\cosh(x)}\,dx\\ &= -8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [C] time = 0.19, size = 76, normalized size = 2.11

$$\frac{4\sqrt{\cosh(x)}\,(\sinh(x)+\cosh(x))\left(8\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2x}\right)(\sinh(x)-\cosh(x))\sqrt{\sinh(2x)+\cosh(2x)+1}\,+x^2\sinh(x}{e^{2x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] (4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] + 8
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1 +
Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\cosh(x)}+\frac{x^2}{\cosh(x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\cosh(x)^{\frac{3}{2}}} + x^2\left(\sqrt{\cosh}\,(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
[Out] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2),x)
[Out] int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(\cosh^2(x) + 1\right)}{\cosh^{\frac{3}{2}}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
[Out] Integral(x**2*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

3.333 $\int (x + \cosh(x))^2 \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3} + \frac{x}{2} + 2x\sinh(x) - 2\cosh(x) + \frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] 1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2638, 2635, 8}

$$\frac{x^3}{3} + \frac{x}{2} + 2x\sinh(x) - 2\cosh(x) + \frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*Cosh[x] + 2*x*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\cosh(x))^2\,dx &= \int \left(x^2+2x\cosh(x)+\cosh^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\cosh(x)\,dx+\int \cosh^2(x)\,dx\\ &= \frac{x^3}{3}+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)+\frac{\int 1\,dx}{2}-2\int \sinh(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{1}{6}\left(x\left(2x^2+12\sinh(x)+3\right)+3(\sinh(x)-4)\cosh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x])^2,x]
[Out] (3*Cosh[x]*(-4 + Sinh[x]) + x*(3 + 2*x^2 + 12*Sinh[x]))/6
```

fricas [A] time = 0.67, size = 23, normalized size = 0.77

$$\frac{1}{3}\,x^3+\frac{1}{2}\,(4\,x+\cosh(x))\sinh(x)+\frac{1}{2}\,x-2\,\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*(4*x + cosh(x))*sinh(x) + 1/2*x - 2*cosh(x)
```

giac [A] time = 0.13, size = 36, normalized size = 1.20

$$\frac{1}{3}\,x^3-(x+1)e^{(-x)}+(x-1)e^x+\frac{1}{2}\,x+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^2,x, algorithm="giac")
```

[Out] 1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)

maple [A] time = 0.03, size = 25, normalized size = 0.83

$$\frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+cosh(x))^2,x)

[Out] 1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)

maxima [A] time = 0.31, size = 36, normalized size = 1.20

$$\frac{1}{3}x^3-(x+1)e^{(-x)}+(x-1)e^x+\frac{1}{2}x+\frac{1}{8}e^{(2x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cosh(x))^2,x, algorithm="maxima")

[Out] 1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)

mupad [B] time = 0.05, size = 24, normalized size = 0.80

$$\frac{x}{2}-2\cosh(x)+\frac{\cosh(x)\sinh(x)}{2}+2x\sinh(x)+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + cosh(x))^2,x)

[Out] x/2 - 2*cosh(x) + (cosh(x)*sinh(x))/2 + 2*x*sinh(x) + x^3/3

sympy [A] time = 0.19, size = 41, normalized size = 1.37

$$\frac{x^3}{3}-\frac{x\sinh^2(x)}{2}+2x\sinh(x)+\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2}-2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cosh(x))**2,x)

[Out] x**3/3 - x*sinh(x)**2/2 + 2*x*sinh(x) + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2 - 2*cosh(x)

3.334 $\int (x + \cosh(x))^3 \, dx$

Optimal. Leaf size=56

$$\frac{x^4}{4} + \frac{3x^2}{4} + 3x^2\sinh(x) + \frac{\sinh^3(x)}{3} + 7\sinh(x) - \frac{3\cosh^2(x)}{4} - 6x\cosh(x) + \frac{3}{2}x\sinh(x)\cosh(x)$$

```
[Out] 3/4*x^2+1/4*x^4-6*x*cosh(x)-3/4*cosh(x)^2+7*sinh(x)+3*x^2*sinh(x)+3/2*x*cos
h(x)*sinh(x)+1/3*sinh(x)^3
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6742, 3296, 2637, 3310, 30, 2633}

$$\frac{x^4}{4} + \frac{3x^2}{4} + 3x^2\sinh(x) + \frac{\sinh^3(x)}{3} + 7\sinh(x) - \frac{3\cosh^2(x)}{4} - 6x\cosh(x) + \frac{3}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 - 6*x*Cosh[x] - (3*Cosh[x]^2)/4 + 7*Sinh[x] + 3*x^2*Sinh[
x] + (3*x*Cosh[x]*Sinh[x])/2 + Sinh[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\cosh(x))^3\,dx &= \int \left(x^3+3x^2\cosh(x)+3x\cosh^2(x)+\cosh^3(x)\right)\,dx\\ &= \frac{x^4}{4}+3\int x^2\cosh(x)\,dx+3\int x\cosh^2(x)\,dx+\int \cosh^3(x)\,dx\\ &= \frac{x^4}{4}-\frac{3\cosh^2(x)}{4}+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(\right.\\ &= \frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+\frac{\sin}{}\\ &= \frac{3x^2}{4}+\frac{x^4}{4}-6x\cosh(x)-\frac{3\cosh^2(x)}{4}+7\sinh(x)+3x^2\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)+\frac{\text{si}}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.91

$$\frac{1}{12}\left(3x^4+9x^2+9\left(4x^2+9\right)\sinh(x)+9x\sinh(2x)+\sinh(3x)\right)-6x\cosh(x)-\frac{3}{8}\cosh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x])^3,x]
```

```
[Out] -6*x*Cosh[x] - (3*Cosh[2*x])/8 + (9*x^2 + 3*x^4 + 9*(9 + 4*x^2)*Sinh[x] + 9
*x*Sinh[2*x] + Sinh[3*x])/12
```

fricas [A] time = 0.57, size = 54, normalized size = 0.96

$$\frac{1}{4}x^4+\frac{1}{12}\sinh(x)^3+\frac{3}{4}x^2-6x\cosh(x)-\frac{3}{8}\cosh(x)^2+\frac{1}{4}\left(12x^2+6x\cosh(x)+\cosh(x)^2+27\right)\sinh(x)-\frac{3}{8}\sinh(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/12*sinh(x)^3 + 3/4*x^2 - 6*x*cosh(x) - 3/8*cosh(x)^2 + 1/4*(12*
x^2 + 6*x*cosh(x) + cosh(x)^2 + 27)*sinh(x) - 3/8*sinh(x)^2
```

giac [A] time = 0.12, size = 75, normalized size = 1.34

$$\frac{1}{4}x^4+\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}-\frac{3}{8}\left(4x^2+8x+9\right)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{8}\left(4x^2-8x+9\right)e^x+\frac{1}{24}e^{(3x)}-\frac{1}{24}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/8*(4*x^2 + 8*x + 9)*e^(-x) -
 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 9)*e^x + 1/24*e^(3*x) - 1/24*
e^(-3*x)
```

maple [A] time = 0.22, size = 52, normalized size = 0.93

$$\left(\frac{2}{3}+\frac{\left(\cosh^2(x)\right)}{3}\right)\sinh(x)+\frac{3x\cosh(x)\sinh(x)}{2}+\frac{3x^2}{4}-\frac{3\left(\cosh^2(x)\right)}{4}+3x^2\sinh(x)-6x\cosh(x)+6\sinh(x)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x))^3,x)
```

```
[Out] (2/3+1/3*cosh(x)^2)*sinh(x)+3/2*x*cosh(x)*sinh(x)+3/4*x^2-3/4*cosh(x)^2+3*x
^2*sinh(x)-6*x*cosh(x)+6*sinh(x)+1/4*x^4
```

maxima [A] time = 0.32, size = 81, normalized size = 1.45

$$\frac{1}{4}x^4+\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}-\frac{3}{2}\left(x^2+2x+2\right)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{2}\left(x^2-2x+2\right)e^x+\frac{1}{24}e^{(3x)}-\frac{3}{8}e^{(-x)}-\frac{1}{24}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/2*(x^2 + 2*x + 2)*e^(-x) - 3
/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(-x
) - 1/24*e^(-3*x) + 3/8*e^x
```

mupad [B] time = 0.07, size = 48, normalized size = 0.86

$$\frac{20\sinh(x)}{3}+3x^2\sinh(x)-\frac{3\cosh(x)^2}{4}+\frac{\cosh(x)^2\sinh(x)}{3}-6x\cosh(x)+\frac{3x^2}{4}+\frac{x^4}{4}+\frac{3x\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + cosh(x))^3,x)

[Out] (20*sinh(x))/3 + 3*x^2*sinh(x) - (3*cosh(x)^2)/4 + (cosh(x)^2*sinh(x))/3 - 6*x*cosh(x) + (3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2

sympy [A] time = 0.31, size = 85, normalized size = 1.52

$$\frac{x^4}{4}-\frac{3x^2\sinh^2(x)}{4}+3x^2\sinh(x)+\frac{3x^2\cosh^2(x)}{4}+\frac{3x\sinh(x)\cosh(x)}{2}-6x\cosh(x)-\frac{2\sinh^3(x)}{3}+\sinh(x)\cosh^2(x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cosh(x))**3,x)

[Out] x**4/4 - 3*x**2*sinh(x)**2/4 + 3*x**2*sinh(x) + 3*x**2*cosh(x)**2/4 + 3*x*s inh(x)*cosh(x)/2 - 6*x*cosh(x) - 2*sinh(x)**3/3 + sinh(x)*cosh(x)**2 + 6*si nh(x) - 3*cosh(x)**2/4

3.335 $\int \frac{\cosh(a+bx)}{c+dx^2} \, dx$

Optimal. Leaf size=213

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(\right.}{}$$

```
[Out] -1/2*Chi(b*x+b*(-c)^(1/2)/d^(1/2))*cosh(a-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/
d^(1/2)+1/2*Chi(-b*x+b*(-c)^(1/2)/d^(1/2))*cosh(a+b*(-c)^(1/2)/d^(1/2))/(-c
)^(1/2)/d^(1/2)-1/2*Shi(b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a-b*(-c)^(1/2)/d^(1/
2))/(-c)^(1/2)/d^(1/2)+1/2*Shi(b*x-b*(-c)^(1/2)/d^(1/2))*sinh(a+b*(-c)^(1/2
)/d^(1/2))/(-c)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5281, 3303, 3298, 3301}

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Cosh[a + (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(
2*Sqrt[-c]*Sqrt[d]) - (Cosh[a - (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[
-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a + (b*Sqrt[-c])/Sqrt[d]]
*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a -
 (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c
]*Sqrt[d])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int\left(\frac{\sqrt{-c}\,\cosh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\cosh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx$$

$$= -\frac{\int\frac{\cosh(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int\frac{\cosh(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int\frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int\frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\sinh\left(a-\frac{b\sqrt{\cdot}}{\sqrt{\cdot}}\right.}{}$$

$$= \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{S}}{2\sqrt{-c}}$$

Mathematica [C] time = 0.30, size = 180, normalized size = 0.85

$$\frac{i\left(\cosh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(ibx-\frac{b\sqrt{c}}{\sqrt{d}}\right)-\cosh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(ixb+\frac{\sqrt{c}\,b}{\sqrt{d}}\right)+i\left(\sinh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)+\sinh\left(a\right.\right.\right.}{2\sqrt{c}\,\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(Cosh[a - (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[-((b*Sqrt[c])/Sqrt[d])
+ I*b*x] - Cosh[a + (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[(b*Sqrt[c])/Sqrt[d]
+ I*b*x] + I*(Sinh[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
```

```
d] - I*b*x] + Sinh[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] + I*b*x])))/(Sqrt[c]*Sqrt[d])
```

fricas [B] time = 0.52, size = 316, normalized size = 1.48

$$-\frac{\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2}{d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2*c/
d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-b^2*c/
d)))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

maple [A] time = 0.12, size = 212, normalized size = 1.00

$$\frac{e^{-\frac{-b\sqrt{-cd}+da}{d}}\operatorname{Ei}\left(1,\frac{b\sqrt{-cd}+(bx+a)d-da}{d}\right)}{4\sqrt{-cd}}-\frac{e^{-\frac{b\sqrt{-cd}+da}{d}}\operatorname{Ei}\left(1,-\frac{b\sqrt{-cd}-(bx+a)d+da}{d}\right)}{4\sqrt{-cd}}-\frac{e^{\frac{b\sqrt{-cd}+da}{d}}\operatorname{Ei}\left(1,\frac{b\sqrt{-cd}-(bx+a)d+da}{d}\right)}{4\sqrt{-cd}}+\frac{e^{\frac{-b\sqrt{}}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x^2+c),x)
```

```
[Out] 1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+(b*x+a)
*d-d*a)/d)-1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(1
/2)-(b*x+a)*d+d*a)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*
(-c*d)^(1/2)-(b*x+a)*d+d*a)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+d*a)/d
)*Ei(1,-(b*(-c*d)^(1/2)+(b*x+a)*d-d*a)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)}{dx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + b\,x)}{d\,x^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a + bx)}{c + dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x**2), x)
```

3.336 $\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sinh\left(a-\frac{b(d}{}\right.}{}$$

```
[Out] Chi(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*cosh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))
/e)/(-4*c*e+d^2)^(1/2)-Chi(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*cosh(a-1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Shi(b*x+1/2*b*(d-(-4*c*e+d^2)
^(1/2))/e)*sinh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Shi(b*
x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sinh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-
4*c*e+d^2)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6728, 3303, 3298, 3301}

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sinh\left(a-\frac{b(d}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CoshIntegral[(b*(d - Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d + Sqrt[d^2 - 4
*c*e]))/(2*e)]*CoshIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[
d^2 - 4*c*e] + (Sinh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sinh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$= \frac{(2e)\int \frac{\cosh(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\cosh(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cosh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cosh}{d}}{\sqrt{d^2-4ce}}$$

$$= \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right.}{\sqrt{d^2-4ce}}$$

Mathematica [C] time = 0.50, size = 248, normalized size = 0.92

$$\frac{\cosh\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\mathrm{Ci}\left(\frac{ib\left(d+2ex-\sqrt{d^2-4ce}\right)}{2e}\right) - \cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{Ci}\left(\frac{ib\left(d+2ex+\sqrt{d^2-4ce}\right)}{2e}\right) - \sinh\left(a-\frac{b\left(\sqrt{d^2-4\right.}}{2e}\right.}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cosh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[((I/2)*b*(d - Sqr
t[d^2 - 4*c*e] + 2*e*x))/e] - Cosh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*C
osIntegral[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e] - Sinh[a - (b*(d +
Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))
/(2*e)] + I*Sinh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[((I/2)
*b*(-d + Sqrt[d^2 - 4*c*e]))/e - I*b*x])/Sqrt[d^2 - 4*c*e]
```

fricas [B] time = 0.53, size = 671, normalized size = 2.48

$$-\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^
2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*
b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a*e
+ e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2
)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b
^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c
*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e
) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^2*
d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*
e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d - 2*a*e +
e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*
Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2
*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e
)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))
/(b*d^2 - 4*b*c*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(e*x^2 + d*x + c), x)
```

maple [A] time = 0.10, size = 376, normalized size = 1.39

$$\frac{b\,\mathrm{e}^{-\frac{2ea-bd-\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{Ei}\left(1,\frac{2e(bx+a)-2ea+bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}-\frac{b\,\mathrm{e}^{-\frac{2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{Ei}\left(1,-\frac{-2e(bx+a)+2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] 1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2
*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1
/2)))*Ei(1,-1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2*
b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2
)))*Ei(1,1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/2*b/(
-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)^(1/2)))
*Ei(1,-1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c*e-d^2>0)', see `assume?` for mo
re details)Is 4*c*e-d^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(a+b\,x\right)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x + e*x^2),x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(a+bx\right)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x + e*x**2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```