Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.5-Hyperbolic-sine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [369]. This is test number [163].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (369)	% 0.00 (0)
Mathematica	% 100.00 (369)	% 0.00 (0)
Maple	% 86.18 (318)	% 13.82 (51)
Maxima	% 72.09 (266)	% 27.91 (103)
Fricas	% 82.38 (304)	% 17.62 (65)
Sympy	% 31.17 (115)	% 68.83 (254)
Giac	% 74.80 (276)	% 25.20 (93)
Mupad	% 59.89 (221)	% 40.11 (148)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

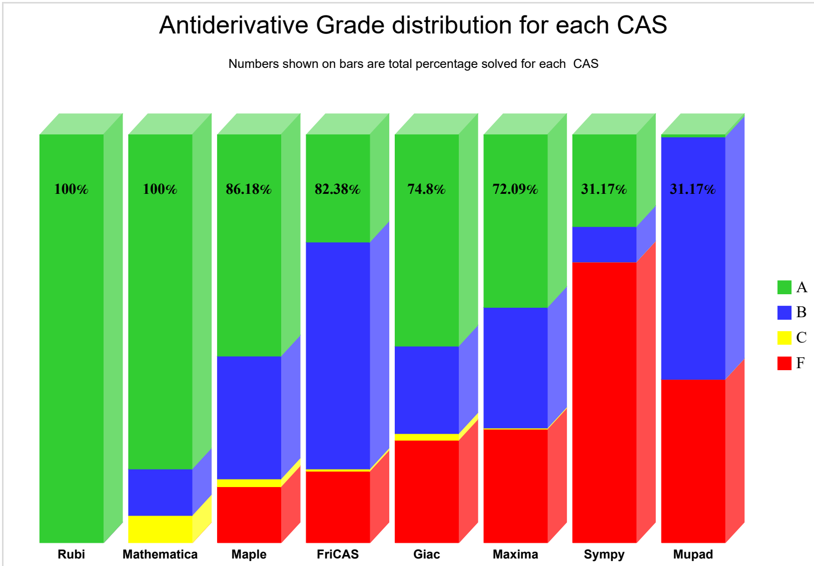
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

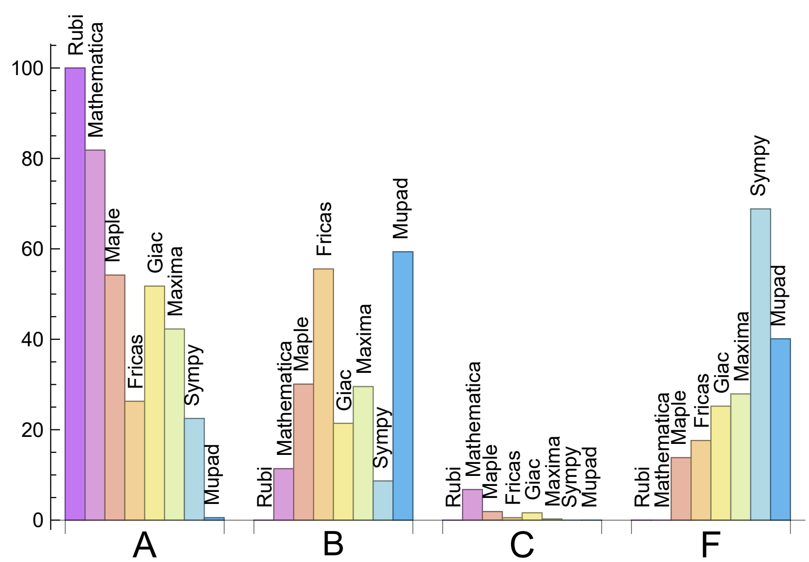
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	81.84	11.38	6.78	0.00
Maple	54.20	30.08	1.90	13.82
Maxima	42.28	29.54	0.27	27.91
Fricas	26.29	55.56	0.54	17.62
Sympy	22.49	8.67	0.00	68.83
Giac	51.76	21.41	1.63	25.20
Mupad	0.54	59.35	0.00	40.11

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	51	92.16 %	0.00 %	7.84 %
Maxima	103	98.06 %	0.00 %	1.94 %
Fricas	65	100.00 %	0.00 %	0.00 %
Sympy	254	74.80 %	23.62 %	1.57 %
Giac	93	95.70 %	4.30 %	0.00 %
Mupad	148	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

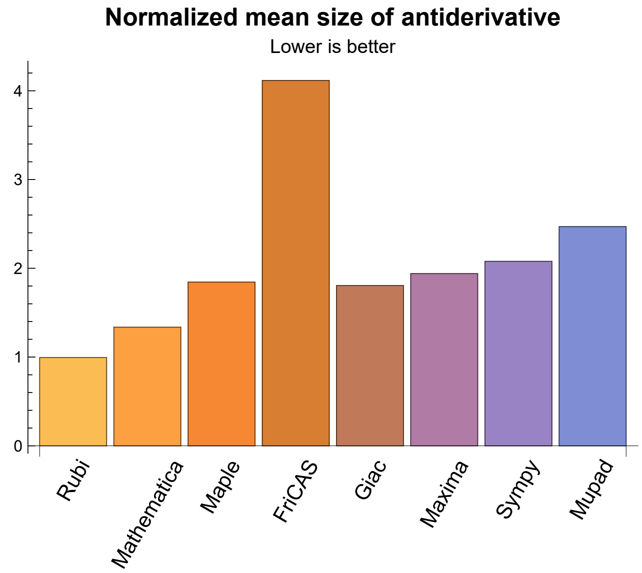
1.3 Performance

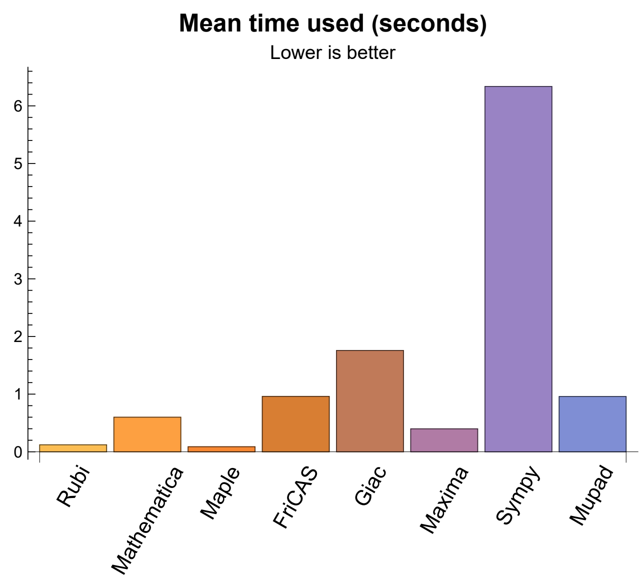
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	86.08	0.99	66.00	1.00
Mathematica	0.60	115.83	1.34	72.00	1.00
Maple	0.09	162.88	1.84	100.50	1.52
Maxima	0.40	136.45	1.94	93.50	1.49
Fricas	0.96	431.45	4.12	146.00	2.17
Sympy	6.34	100.90	2.08	58.00	1.59
Giac	1.76	188.47	1.81	78.50	1.38
Mupad	0.96	173.07	2.47	74.00	1.58

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{264, 265}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {285, 286, 300, 356, 363, 364, 368, 369}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

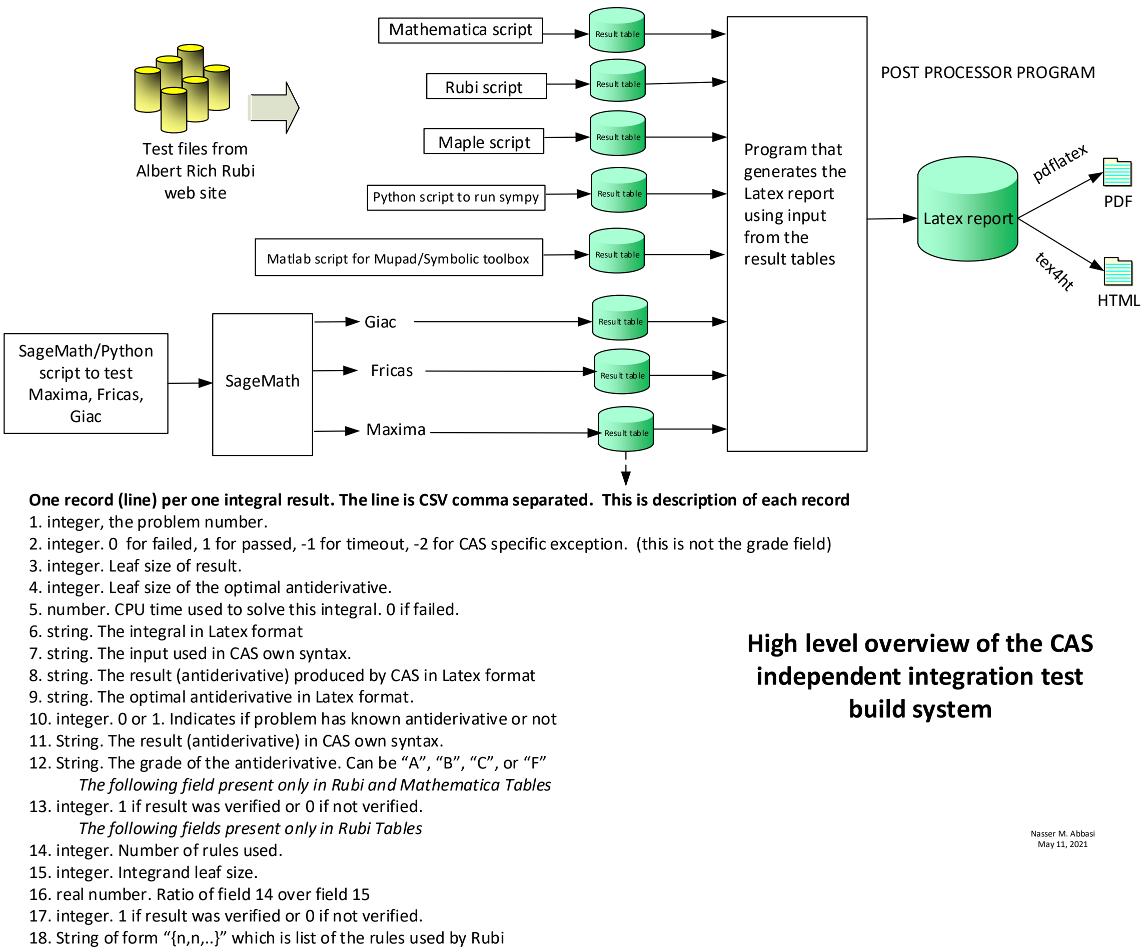
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 47, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 191, 193, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 209, 211, 212, 214, 216, 219, 221, 222, 224, 226, 228, 230, 231, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 363, 364, 366, 367 }

B grade: { 1, 40, 42, 43, 48, 52, 53, 54, 55, 68, 92, 93, 94, 95, 115, 119, 158, 160, 162, 164, 171, 175, 194, 208, 210, 213, 215, 217, 218, 220, 223, 225, 227, 248, 274, 295, 297, 298, 300, 327, 362, 365 }

C grade: { 9, 13, 17, 21, 25, 29, 148, 188, 190, 192, 200, 202, 229, 237, 239, 259, 280, 284, 285, 316, 317, 320, 321, 368, 369 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 79, 80, 81, 83, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 102, 108, 110, 111, 116, 117, 118, 119, 120, 121, 122, 130, 133, 134, 136, 137, 140, 141, 142, 145, 152, 153, 156, 157, 163, 165, 176, 183, 186, 193, 194, 195, 203, 205, 207, 212, 213, 222, 223, 224, 228, 230, 231, 232, 236, 238, 240, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 260, 264, 265, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 294, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 318, 319, 322, 323, 324, 329, 330, 331, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade: { 24, 40, 41, 42, 43, 48, 49, 50, 68, 72, 73, 82, 84, 93, 94, 95, 103, 104, 105, 106, 107, 109, 115, 126, 127, 128, 129, 131, 132, 135, 138, 139, 143, 144, 154, 155, 158, 159, 160, 161, 162, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 187, 188, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 204, 206, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 225, 226, 227, 229, 233, 234, 235, 237, 239, 241, 242, 243, 253, 255, 256, 257, 258, 259, 295, 296, 297, 298, 299, 300 }

C grade: { 312, 313, 316, 317, 320, 321, 339 }

F grade: { 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 69, 70, 71, 112, 113, 114, 123, 124, 125, 146, 147, 148, 149, 150, 151, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 285, 286, 287, 288, 325, 326, 327, 328, 332, 333, 334, 335 }

2.1.4 Maxima

A grade: { 1, 2, 4, 6, 40, 41, 43, 44, 48, 49, 50, 56, 57, 60, 61, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 84, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 119, 136, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 165, 171, 173, 175, 176, 181, 183, 184, 186, 190, 192, 193, 194, 195, 196, 202, 203, 204, 229, 230, 231, 233, 238, 239, 246, 247, 248, 249, 250, 251, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade: { 3, 5, 42, 45, 46, 47, 51, 52, 53, 54, 55, 58, 59, 62, 63, 78, 83, 85, 86, 87, 103, 104, 116, 117, 118, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 172, 174, 177, 178, 179, 180, 182, 185, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 234, 235, 236, 237, 240, 241, 242, 243, 252, 253, 254, 255, 256, 276, 278, 307, 309, 335 }

C grade: { 339 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 336, 368, 369 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 40, 43, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 88, 89, 90, 92, 93, 96, 97, 98, 99, 100, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 163, 167, 171, 173, 175, 176, 178, 194, 212, 231, 232, 247, 248, 260, 264, 265, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 285, 286, 287, 288, 289, 290, 295, 298, 301, 303, 305, 311, 316, 320, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 349, 366, 367 }

B grade: { 5, 41, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 94, 95, 101, 102, 103, 104, 123, 124, 125, 129, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 259, 272, 273, 278, 291, 292, 293, 294, 296, 297, 299, 300, 302, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 333, 334, 335, 342, 343, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

C grade: { 257, 258 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 105, 106, 107, 108, 109, 110, 111, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 261, 262, 263, 279, 280, 281, 282, 283, 284, 325, 326, 327, 328 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 40, 41, 42, 43, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 75, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 115, 116, 117, 118, 119, 120, 121, 122, 129, 133, 134, 136, 142, 171, 173, 175, 181, 184, 185, 192, 193, 201, 203, 211, 212, 220, 221, 224, 246, 247, 248, 253, 264, 265, 274, 276, 278, 301, 302, 303, 304, 311, 315, 319, 323, 324, 331, 336, 366, 367 }

B grade: { 158, 159, 160, 161, 162, 163, 164, 165, 172, 174, 176, 182, 183, 186, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 222, 223, 225, 226, 227, 310, 314, 318 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 130, 131, 132, 135, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 166, 167, 168, 169, 170, 177, 178, 179, 180, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 202, 204, 205, 206, 207, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 275, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 305, 306, 307, 308, 309, 312, 313, 316, 317, 320, 321, 322, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

2.1.7 Giac

A grade: { 2, 4, 6, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 136, 143, 144, 145, 152, 153, 154, 155, 156, 157, 167, 171, 173, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 197, 202, 203, 205, 210, 212, 220, 222, 226, 228, 230, 231, 232, 233, 236, 238, 241, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 260, 264, 265, 266, 267, 276, 277, 278, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade: { 1, 3, 5, 45, 100, 104, 131, 132, 135, 140, 141, 142, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 188, 196, 198, 199, 200, 201, 204, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 227, 229, 234, 235, 237, 239, 240, 242, 255, 256, 268, 269, 270, 271, 272, 273, 274, 275, 289, 290, 291, 295, 296, 297, 298, 299, 300, 312 }

C grade: { 322, 323, 324, 339, 343, 346 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 293, 294, 325, 326, 327, 328, 368, 369 }

2.1.8 Mupad

A grade: { 264, 265 }

B grade: { 1, 2, 3, 4, 5, 6, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 135, 136, 142, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 331, 333, 334, 335, 336, 366, 367 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 69, 70, 71, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 131, 132, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 244, 245, 255, 256, 257, 258, 259, 260, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 329, 330, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10
normalized size	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00
time (sec)	N/A	0.006	0.008	0.006	1.163	0.592	0.130	0.112	0.045

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	23	46	32	18
normalized size	1	1.00	0.92	1.08	1.28	0.92	1.84	1.28	0.72
time (sec)	N/A	0.009	0.023	0.018	1.606	0.465	0.205	0.132	0.373

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	54	38	36	54	24
normalized size	1	1.00	1.07	0.85	2.00	1.41	1.33	2.00	0.89
time (sec)	N/A	0.014	0.010	0.017	1.716	0.513	0.405	0.122	0.055

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	39	60	49	95	60	32
normalized size	1	1.00	0.72	0.85	1.30	1.07	2.07	1.30	0.70
time (sec)	N/A	0.021	0.042	0.118	2.201	0.478	0.831	0.126	0.080

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	33	82	79	58	82	31
normalized size	1	1.00	1.07	0.80	2.00	1.93	1.41	2.00	0.76
time (sec)	N/A	0.017	0.014	0.117	0.314	0.501	1.510	0.140	0.409

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	49	86	90	139	88	42
normalized size	1	1.00	0.64	0.73	1.28	1.34	2.07	1.31	0.63
time (sec)	N/A	0.033	0.039	0.115	0.309	0.576	2.851	0.124	0.133

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	75	116	0	0	0	0	-1
normalized size	1	1.00	0.73	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.139	0.090	0.000	0.626	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	164	0	0	0	0	-1
normalized size	1	1.00	0.85	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.075	0.069	0.000	0.564	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	83	100	0	0	0	0	-1
normalized size	1	1.00	1.04	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.090	0.066	0.000	0.440	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	0	0	0	-1
normalized size	1	1.00	0.93	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.105	0.056	0.000	0.396	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	0	0	0	-1
normalized size	1	1.00	0.89	1.61	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.120	0.048	0.000	0.642	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	0	0	0	-1
normalized size	1	1.00	0.75	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.053	0.062	0.000	0.698	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	101	0	0	0	0	-1
normalized size	1	1.00	1.08	1.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.078	0.065	0.000	0.472	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	73	192	0	0	0	0	-1
normalized size	1	1.00	0.71	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.170	0.068	0.000	0.593	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	76	122	0	0	0	0	-1
normalized size	1	1.00	0.66	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.278	0.069	0.000	0.565	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	170	0	0	0	0	-1
normalized size	1	1.00	0.77	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.123	0.062	0.000	0.541	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	106	0	0	0	0	-1
normalized size	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.129	0.066	0.000	0.506	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	111	0	0	0	0	-1
normalized size	1	1.00	0.93	1.98	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.046	0.060	0.000	0.604	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	89	0	0	0	0	-1
normalized size	1	1.00	0.96	1.59	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.034	0.053	0.000	0.506	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	62	159	0	0	0	0	-1
normalized size	1	1.00	0.72	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.062	0.069	0.000	0.640	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	84	114	0	0	0	0	-1
normalized size	1	1.00	0.93	1.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.100	0.066	0.000	0.489	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	79	205	0	0	0	0	-1
normalized size	1	1.00	0.67	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.166	0.073	0.000	0.557	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	65	122	0	0	0	0	-1
normalized size	1	1.00	0.71	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.166	0.073	0.000	0.632	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	169	0	0	0	0	-1
normalized size	1	1.00	0.89	2.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.059	0.068	0.000	0.596	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	94	104	0	0	0	0	-1
normalized size	1	1.00	1.52	1.68	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.138	0.067	0.000	0.805	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	91	0	0	0	0	-1
normalized size	1	1.00	0.93	3.03	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.009	0.021	0.067	0.000	0.500	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	68	0	0	0	0	-1
normalized size	1	1.00	0.93	2.27	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.009	0.024	0.055	0.000	0.557	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	159	0	0	0	0	-1
normalized size	1	1.00	0.86	2.74	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.099	0.080	0.000	1.271	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	83	113	0	0	0	0	-1
normalized size	1	1.00	1.34	1.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.062	0.070	0.000	0.478	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	80	204	0	0	0	0	-1
normalized size	1	1.00	0.88	2.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.134	0.076	0.000	0.595	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.054	0.043	0.000	0.580	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.037	0.106	0.000	0.474	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.035	0.079	0.000	0.574	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.040	0.066	0.000	0.912	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.041	0.056	0.000	0.663	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.039	0.040	0.000	0.504	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.044	0.237	0.000	0.762	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.041	0.194	0.000	0.549	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.042	0.186	0.000	0.603	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	134	138	59	64	60	50	50
normalized size	1	1.00	2.91	3.00	1.28	1.39	1.30	1.09	1.09
time (sec)	N/A	0.065	0.191	0.060	0.316	0.616	0.196	0.203	0.490

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	93	45	57	41	38	38
normalized size	1	1.00	1.14	2.58	1.25	1.58	1.14	1.06	1.06
time (sec)	N/A	0.046	0.111	0.054	0.443	0.612	0.166	0.252	0.430

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	79	52	33	38	20	26	24
normalized size	1	1.00	3.59	2.36	1.50	1.73	0.91	1.18	1.09
time (sec)	N/A	0.059	0.111	0.049	0.531	0.417	0.127	0.187	0.425

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	43	29	12	16	8	10	12
normalized size	1	1.00	3.07	2.07	0.86	1.14	0.57	0.71	0.86
time (sec)	N/A	0.027	0.057	0.039	0.593	1.020	0.088	0.193	0.407

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	21	29	33	0	24	35
normalized size	1	1.00	1.58	1.11	1.53	1.74	0.00	1.26	1.84
time (sec)	N/A	0.039	0.020	0.044	0.359	0.529	0.000	0.214	0.490

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	36	35	53	77	0	44	51
normalized size	1	1.00	1.57	1.52	2.30	3.35	0.00	1.91	2.22
time (sec)	N/A	0.058	0.042	0.046	0.361	0.490	0.000	0.306	0.636

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	49	53	79	129	0	51	70
normalized size	1	1.00	1.32	1.43	2.14	3.49	0.00	1.38	1.89
time (sec)	N/A	0.073	0.189	0.053	0.323	0.730	0.000	0.190	0.665

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	53	71	105	178	0	58	85
normalized size	1	1.00	1.13	1.51	2.23	3.79	0.00	1.23	1.81
time (sec)	N/A	0.073	0.236	0.053	0.346	1.161	0.000	0.439	0.719

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	147	116	71	91	68	50	97
normalized size	1	1.00	2.53	2.00	1.22	1.57	1.17	0.86	1.67
time (sec)	N/A	0.102	0.190	0.071	0.317	0.484	0.210	0.165	0.566

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	75	59	74	54	38	79
normalized size	1	1.00	1.02	1.70	1.34	1.68	1.23	0.86	1.80
time (sec)	N/A	0.129	0.129	0.069	0.474	0.523	0.180	0.181	0.576

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	55	52	40	50	39	22	71
normalized size	1	1.00	1.72	1.62	1.25	1.56	1.22	0.69	2.22
time (sec)	N/A	0.061	0.150	0.062	0.320	0.518	0.139	0.164	0.567

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	25	81	34	37	20	25
normalized size	1	1.00	0.71	0.81	2.61	1.10	1.19	0.65	0.81
time (sec)	N/A	0.030	0.009	0.048	0.312	0.486	0.129	0.198	0.522

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	91	44	55	82	0	34	41
normalized size	1	1.00	2.68	1.29	1.62	2.41	0.00	1.00	1.21
time (sec)	N/A	0.083	0.089	0.065	0.396	0.500	0.000	0.395	0.283

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	88	58	81	129	0	46	85
normalized size	1	1.00	2.10	1.38	1.93	3.07	0.00	1.10	2.02
time (sec)	N/A	0.117	0.395	0.069	0.321	0.473	0.000	0.206	0.709

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	131	76	105	179	0	59	79
normalized size	1	1.00	2.26	1.31	1.81	3.09	0.00	1.02	1.36
time (sec)	N/A	0.140	0.372	0.076	0.328	0.606	0.000	0.183	0.774

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	143	92	129	225	0	84	189
normalized size	1	1.00	2.23	1.44	2.02	3.52	0.00	1.31	2.95
time (sec)	N/A	0.131	1.870	0.082	0.507	0.480	0.000	0.175	1.095

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	20	20	16	17	15	17
normalized size	1	1.00	1.56	0.74	0.74	0.59	0.63	0.56	0.63
time (sec)	N/A	0.011	0.053	0.037	0.476	0.465	0.127	0.194	0.199

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	55	94	50	68	25	29
normalized size	1	1.00	1.03	0.93	1.59	0.85	1.15	0.42	0.49
time (sec)	N/A	0.026	0.103	0.079	0.347	0.522	0.201	0.344	0.530

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	81	88	211	85	114	36	40
normalized size	1	1.00	0.92	1.00	2.40	0.97	1.30	0.41	0.45
time (sec)	N/A	0.042	0.126	0.091	0.448	0.539	0.306	0.234	0.691

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	87	121	372	121	162	47	53
normalized size	1	1.00	0.74	1.03	3.18	1.03	1.38	0.40	0.45
time (sec)	N/A	0.060	0.170	0.110	0.347	0.493	0.442	0.183	0.959

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	20	20	16	17	15	17
normalized size	1	1.00	1.56	0.74	0.74	0.59	0.63	0.56	0.63
time (sec)	N/A	0.012	0.063	0.043	0.384	0.544	0.118	0.157	0.146

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	55	94	50	68	25	29
normalized size	1	1.00	1.00	0.93	1.59	0.85	1.15	0.42	0.49
time (sec)	N/A	0.027	0.075	0.068	0.319	0.688	0.202	0.235	0.536

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	81	88	211	85	114	36	40
normalized size	1	1.00	0.92	1.00	2.40	0.97	1.30	0.41	0.45
time (sec)	N/A	0.042	0.123	0.072	0.324	0.777	0.309	0.177	0.635

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	87	121	372	121	162	47	52
normalized size	1	1.00	0.74	1.03	3.18	1.03	1.38	0.40	0.44
time (sec)	N/A	0.060	0.152	0.072	0.326	0.506	0.446	0.183	0.906

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	75	0	0	76	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.073	0.411	0.000	0.623	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	0	0	76	0	0	-1
normalized size	1	1.00	1.33	0.00	0.00	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.078	0.364	0.000	0.507	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	145	0	0	101	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.461	0.149	0.000	0.849	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	0	0	63	0	0	-1
normalized size	1	1.00	1.64	0.00	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.205	0.119	0.000	0.539	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	74	89	0	28	0	0	53
normalized size	1	1.00	2.39	2.87	0.00	0.90	0.00	0.00	1.71
time (sec)	N/A	0.014	0.041	0.128	0.000	0.486	0.000	0.000	0.754

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	84	0	0	93	0	0	-1
normalized size	1	1.00	1.62	0.00	0.00	1.79	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.086	0.536	0.000	0.673	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	156	0	0	234	0	0	-1
normalized size	1	1.00	1.79	0.00	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.236	0.132	0.000	0.624	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	210	0	0	348	0	0	-1
normalized size	1	1.00	1.72	0.00	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.204	0.123	0.000	0.530	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	105	262	158	799	0	156	199
normalized size	1	1.00	0.97	2.43	1.46	7.40	0.00	1.44	1.84
time (sec)	N/A	0.315	0.436	0.042	0.432	0.857	0.000	0.189	0.757

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	174	118	459	0	117	159
normalized size	1	1.00	1.00	2.12	1.44	5.60	0.00	1.43	1.94
time (sec)	N/A	0.185	0.141	0.047	0.424	0.501	0.000	0.211	0.612

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	84	238	0	86	129
normalized size	1	1.00	1.07	1.61	1.47	4.18	0.00	1.51	2.26
time (sec)	N/A	0.116	0.097	0.038	0.498	0.583	0.000	0.191	0.542

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	63	65	134	252	67	99
normalized size	1	1.00	1.11	1.34	1.38	2.85	5.36	1.43	2.11
time (sec)	N/A	0.061	0.050	0.028	0.417	0.528	58.299	0.249	0.534

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	49	83	156	0	82	287
normalized size	1	1.00	1.16	0.98	1.66	3.12	0.00	1.64	5.74
time (sec)	N/A	0.076	0.052	0.039	0.425	0.535	0.000	0.234	0.638

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	73	100	345	0	98	292
normalized size	1	1.00	1.37	1.24	1.69	5.85	0.00	1.66	4.95
time (sec)	N/A	0.121	0.404	0.046	0.419	1.503	0.000	0.345	0.697

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	118	108	154	929	0	137	617
normalized size	1	1.00	1.46	1.33	1.90	11.47	0.00	1.69	7.62
time (sec)	N/A	0.318	0.544	0.049	0.408	1.060	0.000	0.211	0.998

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	186	158	194	1676	0	171	694
normalized size	1	1.00	1.71	1.45	1.78	15.38	0.00	1.57	6.37
time (sec)	N/A	0.491	0.945	0.054	0.429	0.562	0.000	0.176	0.879

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	118	296	256	1769	0	235	305
normalized size	1	1.00	0.73	1.83	1.58	10.92	0.00	1.45	1.88
time (sec)	N/A	0.405	0.436	0.067	0.418	0.680	0.000	0.178	0.862

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	213	208	1053	0	184	274
normalized size	1	1.00	0.83	1.85	1.81	9.16	0.00	1.60	2.38
time (sec)	N/A	0.238	0.370	0.057	0.441	0.780	0.000	0.230	0.781

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	175	149	521	0	131	228
normalized size	1	1.00	1.04	2.11	1.80	6.28	0.00	1.58	2.75
time (sec)	N/A	0.134	0.190	0.062	0.409	0.630	0.000	0.390	0.791

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	97	117	341	0	99	142
normalized size	1	1.00	1.13	1.62	1.95	5.68	0.00	1.65	2.37
time (sec)	N/A	0.074	0.107	0.037	0.417	0.553	0.000	0.201	0.701

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	91	166	162	672	0	142	1001
normalized size	1	1.00	1.07	1.95	1.91	7.91	0.00	1.67	11.78
time (sec)	N/A	0.216	0.196	0.062	0.433	0.716	0.000	0.190	2.700

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	193	251	1740	0	205	1017
normalized size	1	1.00	1.03	1.68	2.18	15.13	0.00	1.78	8.84
time (sec)	N/A	0.372	0.678	0.082	0.408	0.807	0.000	0.179	3.393

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	156	227	363	3754	0	203	977
normalized size	1	1.00	0.99	1.44	2.30	23.76	0.00	1.28	6.18
time (sec)	N/A	0.675	0.738	0.088	0.427	1.097	0.000	0.166	3.400

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	214	277	477	6430	0	236	975
normalized size	1	1.00	1.08	1.40	2.41	32.47	0.00	1.19	4.92
time (sec)	N/A	0.884	0.957	0.084	0.426	0.961	0.000	0.328	3.301

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	81	42	19	28	34	32	39
normalized size	1	1.00	1.11	0.58	0.26	0.38	0.47	0.44	0.53
time (sec)	N/A	0.028	0.034	0.046	0.416	0.512	0.309	0.578	0.351

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	142	82	79	100	87	67	106
normalized size	1	1.00	1.39	0.80	0.77	0.98	0.85	0.66	1.04
time (sec)	N/A	0.051	0.264	0.076	0.432	0.620	0.406	0.156	0.979

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	204	124	124	190	148	89	147
normalized size	1	1.00	1.56	0.95	0.95	1.45	1.13	0.68	1.12
time (sec)	N/A	0.085	0.514	0.093	0.419	0.726	0.548	0.371	1.056

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	265	164	167	280	209	111	237
normalized size	1	1.00	1.66	1.02	1.04	1.75	1.31	0.69	1.48
time (sec)	N/A	0.126	0.634	0.091	0.427	0.955	0.734	0.161	1.261

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	171	44	36	26	0	28	32
normalized size	1	1.00	4.62	1.19	0.97	0.70	0.00	0.76	0.86
time (sec)	N/A	0.013	0.032	0.047	0.413	0.482	0.000	0.240	0.615

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	183	134	64	102	0	65	102
normalized size	1	1.00	2.77	2.03	0.97	1.55	0.00	0.98	1.55
time (sec)	N/A	0.035	0.247	0.076	0.447	0.491	0.000	0.224	1.068

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	277	224	108	191	0	87	143
normalized size	1	1.00	2.92	2.36	1.14	2.01	0.00	0.92	1.51
time (sec)	N/A	0.067	0.690	0.097	0.406	0.942	0.000	0.271	1.617

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	308	314	152	281	0	109	232
normalized size	1	1.00	2.48	2.53	1.23	2.27	0.00	0.88	1.87
time (sec)	N/A	0.102	1.925	0.091	0.422	0.546	0.000	0.175	2.112

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	138	155	272	223	314	269	160
normalized size	1	1.00	0.75	0.85	1.49	1.22	1.72	1.47	0.87
time (sec)	N/A	0.273	0.651	0.041	0.323	0.490	2.174	0.419	0.603

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	108	119	182	146	240	200	114
normalized size	1	1.00	0.79	0.87	1.33	1.07	1.75	1.46	0.83
time (sec)	N/A	0.156	0.370	0.037	0.323	0.826	1.053	0.466	0.336

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	71	77	115	91	128	135	75
normalized size	1	1.00	0.77	0.84	1.25	0.99	1.39	1.47	0.82
time (sec)	N/A	0.073	0.181	0.039	0.321	0.438	0.513	0.152	0.502

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	51	55	46	78	76	41
normalized size	1	1.00	0.92	0.98	1.06	0.88	1.50	1.46	0.79
time (sec)	N/A	0.017	0.080	0.028	0.302	0.402	0.262	0.163	0.491

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	31	15
normalized size	1	1.00	1.73	1.07	1.00	1.13	1.13	2.07	1.00
time (sec)	N/A	0.010	0.008	0.004	0.325	0.662	0.130	0.256	0.428

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	52	43	67	162	187	67	55
normalized size	1	1.00	1.18	0.98	1.52	3.68	4.25	1.52	1.25
time (sec)	N/A	0.036	0.035	0.038	0.436	0.654	8.950	0.149	0.760

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	118	138	423	0	119	200
normalized size	1	1.00	1.08	1.49	1.75	5.35	0.00	1.51	2.53
time (sec)	N/A	0.063	0.263	0.068	0.418	0.511	0.000	0.427	0.876

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	117	280	315	1347	0	231	-1
normalized size	1	1.00	0.92	2.20	2.48	10.61	0.00	1.82	-0.01
time (sec)	N/A	0.122	0.278	0.092	0.427	0.615	0.000	0.386	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	159	494	551	2934	0	357	-1
normalized size	1	1.00	0.91	2.84	3.17	16.86	0.00	2.05	-0.01
time (sec)	N/A	0.220	0.724	0.101	0.447	0.604	0.000	0.232	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	178	917	0	0	0	0	-1
normalized size	1	1.00	0.99	5.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.434	0.204	0.000	0.491	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	676	0	0	0	0	-1
normalized size	1	1.00	0.93	4.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.382	0.125	0.000	0.620	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	262	0	0	0	0	-1
normalized size	1	1.00	1.08	4.37	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.188	0.119	0.000	0.831	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	125	0	0	0	0	-1
normalized size	1	1.00	1.00	2.08	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.193	0.101	0.000	0.512	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	81	456	0	0	0	0	-1
normalized size	1	1.00	0.86	4.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.156	0.136	0.000	0.779	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	166	438	0	0	0	0	-1
normalized size	1	1.00	0.84	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.644	0.234	0.000	0.841	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	101	218	0	0	0	0	-1
normalized size	1	1.00	0.79	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.363	0.122	0.000	0.523	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	100	0	0	125	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.358	0.156	0.000	0.483	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	80	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.220	0.136	0.000	0.667	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	66	0	0	48	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.074	0.184	0.000	0.403	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	53	46	26	23	19	17	21
normalized size	1	1.00	2.30	2.00	1.13	1.00	0.83	0.74	0.91
time (sec)	N/A	0.038	0.249	0.043	0.316	0.430	0.116	0.203	0.117

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	52	141	46	51	32	39
normalized size	1	1.00	0.74	1.21	3.28	1.07	1.19	0.74	0.91
time (sec)	N/A	0.042	0.030	0.041	0.331	0.460	0.203	0.176	0.607

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	91	280	71	83	46	52
normalized size	1	1.00	0.74	1.34	4.12	1.04	1.22	0.68	0.76
time (sec)	N/A	0.054	0.040	0.043	0.337	0.565	0.392	0.214	0.835

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	128	468	96	110	60	66
normalized size	1	1.00	0.74	1.41	5.14	1.05	1.21	0.66	0.73
time (sec)	N/A	0.068	0.051	0.052	0.342	0.507	0.777	0.176	1.039

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	59	46	29	24	19	18	21
normalized size	1	1.00	2.19	1.70	1.07	0.89	0.70	0.67	0.78
time (sec)	N/A	0.043	0.079	0.049	0.316	0.470	0.156	0.386	0.109

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	32	52	141	46	51	32	37
normalized size	1	1.00	0.65	1.06	2.88	0.94	1.04	0.65	0.76
time (sec)	N/A	0.044	0.029	0.048	0.336	0.465	0.222	0.175	0.595

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	92	91	281	70	82	46	52
normalized size	1	1.00	1.21	1.20	3.70	0.92	1.08	0.61	0.68
time (sec)	N/A	0.059	0.223	0.053	0.333	0.624	0.389	0.175	0.804

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	63	128	468	96	109	60	68
normalized size	1	1.00	0.62	1.27	4.63	0.95	1.08	0.59	0.67
time (sec)	N/A	0.070	0.051	0.056	0.345	0.577	0.664	0.201	1.004

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	85	0	0	186	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	2.82	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.124	0.148	0.000	0.477	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	105	0	0	267	0	0	-1
normalized size	1	1.00	1.33	0.00	0.00	3.38	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.247	0.116	0.000	0.582	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	184	0	0	356	0	0	-1
normalized size	1	1.00	1.67	0.00	0.00	3.24	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.222	0.128	0.000	0.482	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	241	1893	0	0	0	0	-1
normalized size	1	1.00	0.93	7.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.802	0.164	0.000	0.715	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	196	1037	0	0	0	0	-1
normalized size	1	1.00	0.95	5.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.568	0.314	0.000	0.508	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	151	897	0	0	0	0	-1
normalized size	1	1.00	0.92	5.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.407	0.141	0.000	0.500	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	61	101	124	147	422	75	269
normalized size	1	1.00	1.11	1.84	2.25	2.67	7.67	1.36	4.89
time (sec)	N/A	0.075	0.106	0.032	0.401	0.740	64.572	0.360	0.930

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	82	113	229	444	0	119	223
normalized size	1	1.00	1.11	1.53	3.09	6.00	0.00	1.61	3.01
time (sec)	N/A	0.083	0.153	0.044	0.415	0.603	0.000	0.174	0.879

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	131	314	537	1614	0	279	-1
normalized size	1	1.00	1.02	2.45	4.20	12.61	0.00	2.18	-0.01
time (sec)	N/A	0.175	0.289	0.055	0.431	1.003	0.000	0.246	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	189	633	982	3870	0	477	-1
normalized size	1	1.00	1.01	3.39	5.25	20.70	0.00	2.55	-0.01
time (sec)	N/A	0.325	0.487	0.063	0.460	0.843	0.000	0.226	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	66	105	128	154	340	82	331
normalized size	1	1.00	1.10	1.75	2.13	2.57	5.67	1.37	5.52
time (sec)	N/A	0.084	0.072	0.038	0.412	0.801	60.555	0.351	1.202

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	128	6	3	6	6
normalized size	1	1.00	1.00	1.17	21.33	1.00	0.50	1.00	1.00
time (sec)	N/A	0.001	0.000	0.000	0.406	0.511	0.325	0.166	0.021

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	36	230	58	0	30	49
normalized size	1	1.00	1.00	3.00	19.17	4.83	0.00	2.50	4.08
time (sec)	N/A	0.032	0.035	0.049	0.421	0.442	0.000	0.188	0.571

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	37	34	42	51	33	48
normalized size	1	1.00	0.82	1.09	1.00	1.24	1.50	0.97	1.41
time (sec)	N/A	0.037	0.088	0.030	0.408	0.866	1.711	0.189	0.589

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	109	266	0	0	0	0	-1
normalized size	1	1.00	0.80	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.525	0.142	0.000	0.895	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	159	517	0	0	0	0	-1
normalized size	1	1.00	0.90	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.654	0.240	0.000	0.565	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	236	806	0	0	0	0	-1
normalized size	1	1.00	0.94	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.825	0.349	0.000	0.467	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	53	511	0	120	-1
normalized size	1	1.00	0.68	0.60	1.00	9.64	0.00	2.26	-0.02
time (sec)	N/A	0.037	0.041	0.070	0.421	0.650	0.000	0.155	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	35	226	0	70	-1
normalized size	1	1.00	0.76	0.71	1.03	6.65	0.00	2.06	-0.03
time (sec)	N/A	0.024	0.038	0.059	0.412	0.747	0.000	0.459	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	71	19	34	21
normalized size	1	1.00	1.00	1.15	1.31	5.46	1.46	2.62	1.62
time (sec)	N/A	0.013	0.004	0.050	0.412	0.463	0.332	0.231	0.459

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	49	24	110	0	1	-1
normalized size	1	1.00	1.18	2.88	1.41	6.47	0.00	0.06	-0.06
time (sec)	N/A	0.013	0.006	0.075	0.418	0.795	0.000	0.234	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	44	71	62	327	0	37	-1
normalized size	1	1.00	1.05	1.69	1.48	7.79	0.00	0.88	-0.02
time (sec)	N/A	0.026	0.040	0.080	0.416	0.587	0.000	0.200	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	67	89	96	875	0	52	-1
normalized size	1	1.00	1.10	1.46	1.57	14.34	0.00	0.85	-0.02
time (sec)	N/A	0.037	0.101	0.099	0.422	0.573	0.000	0.224	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	67	0	0	0	0	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.185	0.114	0.000	0.455	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.064	0.092	0.000	0.468	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.086	0.097	0.000	0.827	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.029	0.098	0.000	0.559	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.086	0.088	0.000	0.943	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	69	0	0	0	0	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.204	0.093	0.000	0.420	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	53	171	100	1597	0	114	-1
normalized size	1	1.00	0.40	1.30	0.76	12.10	0.00	0.86	-0.01
time (sec)	N/A	0.049	0.162	0.203	0.426	0.576	0.000	0.301	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	38	125	63	659	0	50	-1
normalized size	1	1.00	0.49	1.60	0.81	8.45	0.00	0.64	-0.01
time (sec)	N/A	0.032	0.089	0.154	0.419	0.638	0.000	0.166	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	84	27	180	0	26	-1
normalized size	1	1.00	0.67	2.33	0.75	5.00	0.00	0.72	-0.03
time (sec)	N/A	0.015	0.037	0.183	0.416	0.539	0.000	0.299	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	50	18	122	0	13	38
normalized size	1	1.00	1.00	3.12	1.12	7.62	0.00	0.81	2.38
time (sec)	N/A	0.014	0.006	0.141	0.430	0.485	0.000	0.172	0.466

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	74	171	1163	0	27	48
normalized size	1	1.00	0.50	1.09	2.51	17.10	0.00	0.40	0.71
time (sec)	N/A	0.022	0.039	0.140	0.411	0.438	0.000	0.216	0.528

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	47	90	467	3093	0	39	256
normalized size	1	1.00	0.40	0.76	3.96	26.21	0.00	0.33	2.17
time (sec)	N/A	0.033	0.060	0.133	0.424	0.824	0.000	0.359	0.542

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	219	292	90	91	124	86	93
normalized size	1	1.00	4.38	5.84	1.80	1.82	2.48	1.72	1.86
time (sec)	N/A	0.055	0.160	0.095	0.344	0.524	0.300	0.182	0.777

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	142	75	72	100	71	77
normalized size	1	1.00	0.98	3.30	1.74	1.67	2.33	1.65	1.79
time (sec)	N/A	0.045	0.031	0.080	0.334	0.476	0.260	0.197	0.670

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	131	210	66	67	82	62	67
normalized size	1	1.00	3.45	5.53	1.74	1.76	2.16	1.63	1.76
time (sec)	N/A	0.047	0.241	0.064	0.318	0.683	0.234	0.228	0.622

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	94	51	48	63	47	51
normalized size	1	1.00	0.85	2.85	1.55	1.45	1.91	1.42	1.55
time (sec)	N/A	0.039	0.021	0.060	0.313	0.520	0.196	0.357	0.559

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	93	126	42	41	48	38	41
normalized size	1	1.00	3.58	4.85	1.62	1.58	1.85	1.46	1.58
time (sec)	N/A	0.041	0.160	0.055	0.312	0.573	0.176	0.442	0.123

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	12	13	27	22	27	23	31
normalized size	1	1.00	0.80	0.87	1.80	1.47	1.80	1.53	2.07
time (sec)	N/A	0.034	0.011	0.032	0.310	0.783	0.145	0.190	0.475

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	34	40	14	17	14	14	7
normalized size	1	1.00	4.25	5.00	1.75	2.12	1.75	1.75	0.88
time (sec)	N/A	0.031	0.047	0.046	0.314	0.610	0.118	0.154	0.462

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	5	11	15	11	10
normalized size	1	1.00	1.00	1.00	0.71	1.57	2.14	1.57	1.43
time (sec)	N/A	0.018	0.006	0.020	0.322	0.501	0.127	0.301	0.466

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	18	43	41	53	0	51	46
normalized size	1	1.00	0.75	1.79	1.71	2.21	0.00	2.12	1.92
time (sec)	N/A	0.035	0.023	0.048	0.314	0.587	0.000	0.487	0.198

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	49	53	28	0	29	63
normalized size	1	1.00	0.88	1.96	2.12	1.12	0.00	1.16	2.52
time (sec)	N/A	0.041	0.034	0.052	0.318	0.405	0.000	0.142	0.581

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	61	91	92	153	0	92	115
normalized size	1	1.00	1.17	1.75	1.77	2.94	0.00	1.77	2.21
time (sec)	N/A	0.053	0.045	0.059	0.320	0.509	0.000	0.448	0.899

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	93	205	64	0	53	231
normalized size	1	1.00	0.95	2.51	5.54	1.73	0.00	1.43	6.24
time (sec)	N/A	0.044	0.069	0.062	0.311	0.439	0.000	0.187	1.012

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	137	140	249	0	118	249
normalized size	1	1.00	1.18	1.71	1.75	3.11	0.00	1.48	3.11
time (sec)	N/A	0.067	0.072	0.061	0.356	0.540	0.000	0.376	1.911

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	121	166	54	55	65	50	54
normalized size	1	1.00	3.02	4.15	1.35	1.38	1.62	1.25	1.35
time (sec)	N/A	0.074	0.188	0.072	0.348	0.580	0.236	0.175	0.143

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	70	39	34	44	35	37
normalized size	1	1.00	1.29	5.00	2.79	2.43	3.14	2.50	2.64
time (sec)	N/A	0.034	0.020	0.068	0.327	0.463	0.186	0.357	0.095

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	82	30	31	29	26	28
normalized size	1	1.00	1.53	2.73	1.00	1.03	0.97	0.87	0.93
time (sec)	N/A	0.068	0.078	0.071	0.332	0.457	0.162	0.492	0.477

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	53	23	26	26	21	24
normalized size	1	1.00	1.00	3.79	1.64	1.86	1.86	1.50	1.71
time (sec)	N/A	0.038	0.013	0.067	0.320	0.816	0.185	0.163	0.549

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	69	29	12	16	8	10	12
normalized size	1	1.00	4.93	2.07	0.86	1.14	0.57	0.71	0.86
time (sec)	N/A	0.034	0.055	0.062	0.324	0.511	0.116	0.169	0.648

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	8	16	17	10	12
normalized size	1	1.00	1.00	1.00	0.80	1.60	1.70	1.00	1.20
time (sec)	N/A	0.020	0.010	0.027	0.314	0.565	0.120	0.141	0.578

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	70	70	104	0	70	86
normalized size	1	1.00	0.76	2.06	2.06	3.06	0.00	2.06	2.53
time (sec)	N/A	0.039	0.044	0.062	0.345	0.736	0.000	0.157	0.754

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	70	117	46	0	41	109
normalized size	1	1.00	0.84	1.89	3.16	1.24	0.00	1.11	2.95
time (sec)	N/A	0.071	0.022	0.060	0.323	0.641	0.000	0.203	0.732

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	116	120	200	0	105	198
normalized size	1	1.00	1.13	1.93	2.00	3.33	0.00	1.75	3.30
time (sec)	N/A	0.055	0.048	0.079	0.338	0.510	0.000	0.433	1.196

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	116	317	82	0	65	139
normalized size	1	1.00	0.96	2.37	6.47	1.67	0.00	1.33	2.84
time (sec)	N/A	0.078	0.040	0.086	0.338	0.586	0.000	0.227	0.533

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	45	56	33	49	31	27	41
normalized size	1	1.00	1.61	2.00	1.18	1.75	1.11	0.96	1.46
time (sec)	N/A	0.044	0.090	0.081	0.349	0.483	0.173	0.193	0.186

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	40	36	53	29	31	16	19
normalized size	1	1.00	2.00	1.80	2.65	1.45	1.55	0.80	0.95
time (sec)	N/A	0.033	0.065	0.080	0.327	0.479	0.147	0.213	0.658

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	37	12	16
normalized size	1	1.00	0.88	0.81	0.62	1.88	2.31	0.75	1.00
time (sec)	N/A	0.021	0.026	0.025	0.317	0.596	0.152	0.366	0.652

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	45	56	33	49	32	27	39
normalized size	1	1.00	1.73	2.15	1.27	1.88	1.23	1.04	1.50
time (sec)	N/A	0.040	0.085	0.084	0.323	0.481	0.152	0.468	0.157

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	36	53	29	31	16	19
normalized size	1	1.00	1.90	1.80	2.65	1.45	1.55	0.80	0.95
time (sec)	N/A	0.033	0.064	0.069	0.325	0.792	0.149	0.150	0.592

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	36	12	16
normalized size	1	1.00	0.88	0.81	0.62	1.88	2.25	0.75	1.00
time (sec)	N/A	0.022	0.027	0.029	0.329	0.580	0.148	0.192	0.200

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	121	837	308	2105	0	254	287
normalized size	1	1.00	0.88	6.07	2.23	15.25	0.00	1.84	2.08
time (sec)	N/A	0.138	0.159	0.057	0.337	0.624	0.000	0.228	1.318

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	463	674	283	1486	0	288	302
normalized size	1	1.00	3.19	4.65	1.95	10.25	0.00	1.99	2.08
time (sec)	N/A	0.414	5.831	0.056	0.426	0.675	0.000	0.470	1.241

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	447	180	865	0	139	169
normalized size	1	1.00	0.94	5.52	2.22	10.68	0.00	1.72	2.09
time (sec)	N/A	0.091	0.091	0.053	0.322	0.699	0.000	0.494	0.875

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	553	336	170	569	0	168	200
normalized size	1	1.00	5.70	3.46	1.75	5.87	0.00	1.73	2.06
time (sec)	N/A	0.241	2.245	0.050	0.412	1.346	0.000	0.230	0.856

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	185	81	221	0	61	77
normalized size	1	1.00	1.00	4.87	2.13	5.82	0.00	1.61	2.03
time (sec)	N/A	0.061	0.036	0.044	0.328	0.650	0.000	0.188	0.617

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	396	126	81	171	377	83	87
normalized size	1	1.00	7.33	2.33	1.50	3.17	6.98	1.54	1.61
time (sec)	N/A	0.109	0.768	0.040	0.412	0.772	138.906	0.222	0.632

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	22	11
normalized size	1	1.00	1.00	1.09	1.00	2.45	1.27	2.00	1.00
time (sec)	N/A	0.025	0.006	0.020	0.304	0.641	0.309	0.221	0.063

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	99	71	66	57	0	89	93
normalized size	1	1.00	2.06	1.48	1.38	1.19	0.00	1.85	1.94
time (sec)	N/A	0.060	0.096	0.043	0.413	1.020	0.000	0.178	1.352

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	67	71	89	259	0	87	321
normalized size	1	1.00	1.14	1.20	1.51	4.39	0.00	1.47	5.44
time (sec)	N/A	0.079	0.170	0.048	0.417	2.362	0.000	0.177	1.044

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	353	159	652	0	214	291
normalized size	1	1.00	0.89	4.06	1.83	7.49	0.00	2.46	3.34
time (sec)	N/A	0.122	0.129	0.056	0.418	1.482	0.000	0.195	2.237

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	182	230	1142	0	180	634
normalized size	1	1.00	1.02	1.82	2.30	11.42	0.00	1.80	6.34
time (sec)	N/A	0.213	0.343	0.063	0.441	0.613	0.000	0.470	1.607

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	1140	345	2707	0	369	548
normalized size	1	1.00	1.00	8.44	2.56	20.05	0.00	2.73	4.06
time (sec)	N/A	0.197	0.236	0.073	0.440	1.694	0.000	0.390	4.894

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	350	438	3175	0	323	1010
normalized size	1	1.00	1.00	2.40	3.00	21.75	0.00	2.21	6.92
time (sec)	N/A	0.417	0.450	0.080	0.433	2.855	0.000	0.185	2.197

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	660	290	176	833	0	178	256
normalized size	1	1.00	7.02	3.09	1.87	8.86	0.00	1.89	2.72
time (sec)	N/A	0.220	4.462	0.080	0.418	0.541	0.000	0.185	0.880

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	141	102	370	133	82	60
normalized size	1	1.00	0.92	3.52	2.55	9.25	3.32	2.05	1.50
time (sec)	N/A	0.060	0.067	0.077	0.331	0.831	1.062	0.236	0.706

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	502	119	100	362	0	97	132
normalized size	1	1.00	8.10	1.92	1.61	5.84	0.00	1.56	2.13
time (sec)	N/A	0.111	1.791	0.061	0.412	2.987	0.000	0.228	0.671

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	51	19	22	14
normalized size	1	1.00	1.00	1.08	1.00	3.92	1.46	1.69	1.08
time (sec)	N/A	0.025	0.012	0.033	0.308	0.658	0.614	0.342	0.532

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	121	201	149	383	0	186	186
normalized size	1	1.00	1.53	2.54	1.89	4.85	0.00	2.35	2.35
time (sec)	N/A	0.103	0.456	0.072	0.415	0.509	0.000	0.559	1.795

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	138	215	802	0	167	302
normalized size	1	1.00	1.01	1.48	2.31	8.62	0.00	1.80	3.25
time (sec)	N/A	0.170	0.235	0.067	0.422	1.936	0.000	0.174	0.971

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	260	548	375	2615	0	295	519
normalized size	1	1.00	1.91	4.03	2.76	19.23	0.00	2.17	3.82
time (sec)	N/A	0.165	2.438	0.099	0.415	1.706	0.000	0.183	5.073

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	137	266	490	3044	0	287	476
normalized size	1	1.00	0.95	1.85	3.40	21.14	0.00	1.99	3.31
time (sec)	N/A	0.310	0.442	0.119	0.424	1.000	0.000	0.263	1.058

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	96	93	413	88	107	53	231
normalized size	1	1.00	3.10	3.00	13.32	2.84	3.45	1.71	7.45
time (sec)	N/A	0.079	0.142	0.097	0.320	0.660	0.294	0.277	1.172

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	79	95	152	100	92	113
normalized size	1	1.00	1.17	2.19	2.64	4.22	2.78	2.56	3.14
time (sec)	N/A	0.092	0.081	0.088	0.302	0.981	0.259	0.482	0.446

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	67	47	109	40	46	29	80
normalized size	1	1.00	2.91	2.04	4.74	1.74	2.00	1.26	3.48
time (sec)	N/A	0.076	0.058	0.060	0.312	0.614	0.174	0.196	0.672

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	20	45	42	56	32	53	29
normalized size	1	1.00	0.77	1.73	1.62	2.15	1.23	2.04	1.12
time (sec)	N/A	0.055	0.021	0.059	0.307	3.172	0.209	0.175	0.171

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	28	17	15	23	24
normalized size	1	1.00	1.00	0.89	1.47	0.89	0.79	1.21	1.26
time (sec)	N/A	0.032	0.008	0.037	0.298	0.912	0.270	0.236	0.150

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	32	23	27	37	22	24	28
normalized size	1	1.00	2.67	1.92	2.25	3.08	1.83	2.00	2.33
time (sec)	N/A	0.044	0.034	0.043	0.300	0.860	0.145	0.547	0.181

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	67	33	32	24	25
normalized size	1	1.00	1.00	2.27	4.47	2.20	2.13	1.60	1.67
time (sec)	N/A	0.059	0.011	0.066	0.307	1.016	0.165	0.157	0.574

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	100	59	61	90	61	44	74
normalized size	1	1.00	3.85	2.27	2.35	3.46	2.35	1.69	2.85
time (sec)	N/A	0.078	0.038	0.074	0.872	3.279	0.278	0.177	0.307

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	33	68	205	63	70	51	44
normalized size	1	1.00	1.43	2.96	8.91	2.74	3.04	2.22	1.91
time (sec)	N/A	0.077	0.010	0.089	0.483	0.986	0.207	0.176	0.614

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	164	93	91	144	100	62	124
normalized size	1	1.00	4.56	2.58	2.53	4.00	2.78	1.72	3.44
time (sec)	N/A	0.097	0.040	0.096	0.313	1.869	0.306	0.203	0.973

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	112	116	573	106	128	65	395
normalized size	1	1.00	2.38	2.47	12.19	2.26	2.72	1.38	8.40
time (sec)	N/A	0.123	0.152	0.121	0.882	0.504	0.292	0.370	4.070

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	52	114	115	201	129	102	209
normalized size	1	1.00	0.79	1.73	1.74	3.05	1.95	1.55	3.17
time (sec)	N/A	0.065	0.102	0.108	0.356	0.872	0.305	0.180	1.503

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	84	70	197	58	66	41	139
normalized size	1	1.00	2.27	1.89	5.32	1.57	1.78	1.11	3.76
time (sec)	N/A	0.118	0.096	0.084	0.361	1.580	0.198	0.367	0.296

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	29	66	61	95	58	66	99
normalized size	1	1.00	0.81	1.83	1.69	2.64	1.61	1.83	2.75
time (sec)	N/A	0.037	0.031	0.082	0.333	1.741	0.227	0.523	0.873

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	48	56	39	33	49
normalized size	1	1.00	1.00	0.92	1.92	2.24	1.56	1.32	1.96
time (sec)	N/A	0.040	0.029	0.058	0.346	0.818	0.261	0.157	0.304

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	66	35	54	79	49	47	60
normalized size	1	1.00	2.54	1.35	2.08	3.04	1.88	1.81	2.31
time (sec)	N/A	0.068	0.146	0.077	0.356	1.596	0.220	0.170	0.746

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	51	64	72	53	53	56
normalized size	1	1.00	1.00	1.76	2.21	2.48	1.83	1.83	1.93
time (sec)	N/A	0.048	0.016	0.096	0.387	1.017	0.275	0.203	0.260

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	107	58	67	98	66	50	111
normalized size	1	1.00	3.82	2.07	2.39	3.50	2.36	1.79	3.96
time (sec)	N/A	0.094	0.056	0.076	0.336	0.795	0.303	0.158	0.208

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	68	171	59	65	38	40
normalized size	1	1.00	1.00	2.52	6.33	2.19	2.41	1.41	1.48
time (sec)	N/A	0.043	0.013	0.092	0.335	1.296	0.292	0.176	0.606

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	175	74	103	158	114	74	246
normalized size	1	1.00	3.65	1.54	2.15	3.29	2.38	1.54	5.12
time (sec)	N/A	0.091	0.061	0.108	0.357	0.895	0.340	0.188	0.770

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	108	163	241	1199	0	197	654
normalized size	1	1.00	0.87	1.31	1.94	9.67	0.00	1.59	5.27
time (sec)	N/A	0.184	0.366	0.065	0.440	1.446	0.000	0.437	1.475

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	153	357	160	655	0	211	291
normalized size	1	1.00	1.74	4.06	1.82	7.44	0.00	2.40	3.31
time (sec)	N/A	0.175	0.187	0.063	0.507	0.877	0.000	0.186	2.278

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	84	89	257	0	87	330
normalized size	1	1.00	1.00	1.22	1.29	3.72	0.00	1.26	4.78
time (sec)	N/A	0.091	0.195	0.053	0.454	0.835	0.000	0.171	0.938

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	84	66	57	0	89	95
normalized size	1	1.00	0.75	1.75	1.38	1.19	0.00	1.85	1.98
time (sec)	N/A	0.069	0.060	0.046	0.499	0.632	0.000	0.275	1.404

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	46	40	0	39	195
normalized size	1	1.00	1.00	1.05	2.30	2.00	0.00	1.95	9.75
time (sec)	N/A	0.041	0.010	0.041	0.365	1.121	0.000	0.202	0.887

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	82	107	97	228	0	95	304
normalized size	1	1.00	1.46	1.91	1.73	4.07	0.00	1.70	5.43
time (sec)	N/A	0.235	0.183	0.057	0.464	0.618	0.000	0.497	0.835

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	120	116	427	0	125	1163
normalized size	1	1.00	0.87	2.31	2.23	8.21	0.00	2.40	22.37
time (sec)	N/A	0.095	0.063	0.066	0.350	0.750	0.000	0.270	1.236

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	176	232	212	1303	0	194	778
normalized size	1	1.00	1.63	2.15	1.96	12.06	0.00	1.80	7.20
time (sec)	N/A	0.413	0.429	0.065	0.460	2.020	0.000	0.456	1.414

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	144	262	523	3534	0	292	543
normalized size	1	1.00	0.64	1.17	2.33	15.78	0.00	1.30	2.42
time (sec)	N/A	0.435	0.412	0.103	0.447	0.945	0.000	0.539	1.297

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	150	491	375	2850	0	307	501
normalized size	1	1.00	1.11	3.64	2.78	21.11	0.00	2.27	3.71
time (sec)	N/A	0.356	0.700	0.096	0.468	1.660	0.000	0.447	3.890

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	100	142	223	900	0	181	377
normalized size	1	1.00	0.69	0.99	1.55	6.25	0.00	1.26	2.62
time (sec)	N/A	0.249	0.270	0.080	0.523	0.912	0.000	0.611	1.149

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	146	248	155	423	0	199	190
normalized size	1	1.00	1.72	2.92	1.82	4.98	0.00	2.34	2.24
time (sec)	N/A	0.103	0.239	0.069	0.485	1.535	0.000	0.170	1.755

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	27	33	75	158	0	75	240
normalized size	1	1.00	0.84	1.03	2.34	4.94	0.00	2.34	7.50
time (sec)	N/A	0.053	0.051	0.054	0.329	1.158	0.000	0.145	0.974

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	102	170	165	1257	0	148	897
normalized size	1	1.00	1.28	2.12	2.06	15.71	0.00	1.85	11.21
time (sec)	N/A	0.402	0.479	0.069	0.474	1.487	0.000	0.249	1.776

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	184	202	1463	0	190	1375
normalized size	1	1.00	0.96	2.42	2.66	19.25	0.00	2.50	18.09
time (sec)	N/A	0.110	0.213	0.088	0.365	1.358	0.000	0.337	1.633

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	214	357	339	3648	0	242	1450
normalized size	1	1.00	1.35	2.25	2.13	22.94	0.00	1.52	9.12
time (sec)	N/A	0.671	0.823	0.084	0.450	1.970	0.000	0.250	1.534

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	0	356	0	0	-1
normalized size	1	1.00	1.00	0.81	0.00	9.62	0.00	0.00	-0.03
time (sec)	N/A	0.062	0.023	0.033	0.000	1.151	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	370	0	0	-1
normalized size	1	1.00	1.00	0.79	0.00	15.42	0.00	0.00	-0.04
time (sec)	N/A	0.059	0.013	0.035	0.000	1.401	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	88	68	170	745	87	198
normalized size	1	1.00	1.16	1.73	1.33	3.33	14.61	1.71	3.88
time (sec)	N/A	0.135	0.079	0.045	0.451	2.170	73.278	0.295	2.643

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	48	46	19	37	24	22	24
normalized size	1	1.00	1.92	1.84	0.76	1.48	0.96	0.88	0.96
time (sec)	N/A	0.083	0.069	0.056	0.356	1.997	0.168	0.238	0.135

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	81	44	20	36	22	21	23
normalized size	1	1.00	3.00	1.63	0.74	1.33	0.81	0.78	0.85
time (sec)	N/A	0.087	0.095	0.061	0.371	1.035	0.161	0.189	0.118

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	132	150	125	172	0	123	914
normalized size	1	1.00	1.48	1.69	1.40	1.93	0.00	1.38	10.27
time (sec)	N/A	0.177	0.368	0.054	0.466	6.493	0.000	0.210	8.673

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	73	106	183	0	102	164
normalized size	1	1.00	1.08	1.22	1.77	3.05	0.00	1.70	2.73
time (sec)	N/A	0.150	0.165	0.059	0.479	1.628	0.000	0.361	11.212

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	93	150	125	172	0	123	864
normalized size	1	1.00	1.04	1.69	1.40	1.93	0.00	1.38	9.71
time (sec)	N/A	0.252	0.250	0.058	0.535	8.215	0.000	0.512	10.644

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	86	141	172	0	90	539
normalized size	1	1.00	1.16	1.48	2.43	2.97	0.00	1.55	9.29
time (sec)	N/A	0.157	0.118	0.053	0.462	1.468	0.000	0.179	2.182

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	85	213	176	249	1318	131	656
normalized size	1	1.00	1.05	2.63	2.17	3.07	16.27	1.62	8.10
time (sec)	N/A	0.169	0.290	0.115	0.442	1.398	40.013	0.201	1.849

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	151	339	570	0	177	279
normalized size	1	1.00	1.00	1.34	3.00	5.04	0.00	1.57	2.47
time (sec)	N/A	0.169	0.519	0.184	0.449	0.682	0.000	0.297	1.196

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	170	416	726	1880	0	423	-1
normalized size	1	1.00	0.94	2.31	4.03	10.44	0.00	2.35	-0.01
time (sec)	N/A	0.270	0.685	0.208	0.467	1.278	0.000	0.353	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	235	844	1263	4350	0	717	-1
normalized size	1	1.00	0.94	3.38	5.05	17.40	0.00	2.87	-0.00
time (sec)	N/A	0.440	1.580	0.230	0.510	1.018	0.000	0.504	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	319	919	0	1655	0	0	-1
normalized size	1	1.00	0.73	2.09	0.00	3.77	0.00	0.00	-0.00
time (sec)	N/A	0.730	0.859	0.109	0.000	0.851	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	240	710	0	1247	0	0	-1
normalized size	1	1.00	0.73	2.17	0.00	3.81	0.00	0.00	-0.00
time (sec)	N/A	0.538	0.754	0.090	0.000	1.921	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	576	505	0	837	0	0	-1
normalized size	1	1.00	2.68	2.35	0.00	3.89	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.818	0.091	0.000	3.276	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	-1
normalized size	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	-0.02
time (sec)	N/A	0.456	0.103	0.214	0.455	1.078	0.000	0.337	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.112	0.081	0.228	0.000	1.451	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.037	0.157	0.000	0.796	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.031	0.105	0.000	1.084	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	13.090	0.105	0.000	1.985	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	38.063	0.102	0.000	0.940	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	0	52	44	0	47	43
normalized size	1	1.00	0.76	0.00	0.96	0.81	0.00	0.87	0.80
time (sec)	N/A	0.011	0.070	0.035	0.349	0.810	0.000	0.283	0.646

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	55	0	67	91	0	169	53
normalized size	1	1.00	0.62	0.00	0.76	1.03	0.00	1.92	0.60
time (sec)	N/A	0.021	0.115	0.200	0.356	1.050	0.000	0.264	0.646

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	0	114	200	0	665	93
normalized size	1	1.00	0.81	0.00	0.77	1.34	0.00	4.46	0.62
time (sec)	N/A	0.041	0.526	0.192	0.402	0.758	0.000	0.264	0.720

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	129	294	0	777	102
normalized size	1	1.00	0.87	0.00	0.68	1.54	0.00	4.07	0.53
time (sec)	N/A	0.051	0.442	0.249	0.394	2.892	0.000	0.297	0.689

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	0	64	98	0	235	56
normalized size	1	1.00	0.74	0.00	0.88	1.34	0.00	3.22	0.77
time (sec)	N/A	0.024	0.141	0.041	0.377	4.615	0.000	0.237	0.704

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	89	0	87	248	0	758	74
normalized size	1	1.00	0.74	0.00	0.72	2.07	0.00	6.32	0.62
time (sec)	N/A	0.048	0.302	0.187	0.363	1.377	0.000	0.227	0.754

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	197	292	0	138	585	0	3225	118
normalized size	1	0.97	1.44	0.00	0.68	2.88	0.00	15.89	0.58
time (sec)	N/A	0.086	1.346	0.191	0.397	1.926	0.000	0.357	0.849

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	260	311	0	161	1125	0	6884	136
normalized size	1	0.98	1.17	0.00	0.61	4.23	0.00	25.88	0.51
time (sec)	N/A	0.128	3.492	0.272	0.391	0.618	0.000	0.437	0.867

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	37	40	18
normalized size	1	1.00	2.06	1.06	1.00	1.06	2.06	2.22	1.00
time (sec)	N/A	0.017	0.012	0.009	0.325	0.606	0.991	0.186	0.658

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	49	40	0	81	32
normalized size	1	1.00	0.92	1.33	1.26	1.03	0.00	2.08	0.82
time (sec)	N/A	0.029	0.029	0.020	0.325	2.131	0.000	0.187	0.693

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	36	86	65	82	81	37
normalized size	1	1.00	1.05	0.84	2.00	1.51	1.91	1.88	0.86
time (sec)	N/A	0.035	0.014	0.022	0.329	1.411	10.438	0.211	0.724

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	84	93	84	0	114	51
normalized size	1	1.00	0.70	1.15	1.27	1.15	0.00	1.56	0.70
time (sec)	N/A	0.047	0.045	0.025	0.335	0.718	0.000	0.292	0.785

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	51	130	130	122	115	49
normalized size	1	1.00	1.05	0.78	2.00	2.00	1.88	1.77	0.75
time (sec)	N/A	0.042	0.019	0.031	0.337	1.085	94.651	0.209	0.839

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	96	227	0	0	0	0	-1
normalized size	1	1.00	0.86	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.090	0.089	0.000	1.219	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	114	143	0	0	0	0	-1
normalized size	1	1.00	1.03	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.138	0.020	0.000	0.673	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	0	0	0	-1
normalized size	1	1.00	0.94	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.041	0.076	0.000	0.830	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	0	0	0	-1
normalized size	1	1.00	0.92	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.045	0.061	0.000	0.490	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	0	0	0	-1
normalized size	1	1.00	0.75	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.062	0.083	0.000	0.711	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	122	144	0	0	0	0	-1
normalized size	1	1.00	1.10	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.128	0.085	0.000	0.749	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	86	0	0	162	0	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.443	0.397	0.000	1.049	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	74	0	0	117	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.287	0.273	0.000	1.399	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	0	0	68	0	0	-1
normalized size	1	1.00	1.42	0.00	0.00	1.58	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.155	0.225	0.000	0.577	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	121	0	0	128	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.273	0.227	0.000	0.554	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	48	0	102	-1
normalized size	1	1.00	1.00	1.06	0.00	1.33	0.00	2.83	-0.03
time (sec)	N/A	0.047	0.020	0.020	0.000	0.834	0.000	2.635	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	50	0	73	0	97	-1
normalized size	1	1.00	0.95	1.28	0.00	1.87	0.00	2.49	-0.03
time (sec)	N/A	0.064	0.038	0.037	0.000	1.004	0.000	3.084	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	74	0	118	0	167	-1
normalized size	1	1.00	0.92	1.25	0.00	2.00	0.00	2.83	-0.02
time (sec)	N/A	0.085	0.060	0.033	0.000	1.139	0.000	3.904	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	113	0	253	0	0	-1
normalized size	1	1.00	0.95	1.53	0.00	3.42	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.302	0.054	0.000	1.121	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	85	120	0	277	0	0	-1
normalized size	1	1.00	1.06	1.50	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.384	0.184	0.000	0.494	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	172	228	0	701	0	0	-1
normalized size	1	1.00	1.20	1.59	0.00	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.487	0.186	0.000	0.630	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	373	347	0	171	0	764	-1
normalized size	1	1.00	3.69	3.44	0.00	1.69	0.00	7.56	-0.01
time (sec)	N/A	0.179	0.713	0.051	0.000	0.434	0.000	4.233	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	112	358	0	370	0	749	-1
normalized size	1	1.00	1.05	3.35	0.00	3.46	0.00	7.00	-0.01
time (sec)	N/A	0.182	0.867	0.210	0.000	3.329	0.000	17.256	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	599	700	0	717	0	1383	-1
normalized size	1	1.00	3.09	3.61	0.00	3.70	0.00	7.13	-0.01
time (sec)	N/A	0.328	1.420	0.183	0.000	1.332	0.000	24.021	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	449	459	0	202	0	1736	-1
normalized size	1	1.00	3.71	3.79	0.00	1.67	0.00	14.35	-0.01
time (sec)	N/A	0.259	1.476	0.085	0.000	1.609	0.000	34.354	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	136	468	0	477	0	1703	-1
normalized size	1	1.00	1.05	3.63	0.00	3.70	0.00	13.20	-0.01
time (sec)	N/A	0.279	2.100	0.241	0.000	1.654	0.000	154.691	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	671	922	0	942	0	3230	-1
normalized size	1	1.00	2.97	4.08	0.00	4.17	0.00	14.29	-0.00
time (sec)	N/A	0.476	6.004	0.218	0.000	1.953	0.000	178.857	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	45	68	113	139	60	58
normalized size	1	1.00	0.75	0.54	0.82	1.36	1.67	0.72	0.70
time (sec)	N/A	0.040	0.044	0.018	0.398	0.774	59.123	0.127	0.532

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	49	53	95	182	57	42
normalized size	1	1.00	0.79	0.86	0.93	1.67	3.19	1.00	0.74
time (sec)	N/A	0.038	0.035	0.020	0.352	0.647	17.824	0.136	0.799

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	35	40	54	78	34	36
normalized size	1	1.00	0.80	0.71	0.82	1.10	1.59	0.69	0.73
time (sec)	N/A	0.031	0.021	0.018	0.379	0.804	5.154	0.136	0.615

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	24	50	80	24	18
normalized size	1	1.00	1.00	1.61	1.04	2.17	3.48	1.04	0.78
time (sec)	N/A	0.016	0.012	0.013	0.358	4.003	1.072	0.110	0.069

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	27	30	0	24	16
normalized size	1	1.00	1.00	1.00	1.42	1.58	0.00	1.26	0.84
time (sec)	N/A	0.019	0.010	0.019	0.301	2.029	0.000	0.116	0.070

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	25	52	157	0	48	52
normalized size	1	1.00	0.90	0.60	1.24	3.74	0.00	1.14	1.24
time (sec)	N/A	0.031	0.072	0.022	0.383	0.852	0.000	0.134	0.617

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	24	68	88	0	31	31
normalized size	1	1.00	0.94	0.77	2.19	2.84	0.00	1.00	1.00
time (sec)	N/A	0.028	0.019	0.034	0.296	0.542	0.000	0.126	0.582

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	37	100	705	0	75	135
normalized size	1	1.00	0.74	0.37	0.99	6.98	0.00	0.74	1.34
time (sec)	N/A	0.054	0.073	0.033	0.302	2.047	0.000	0.138	0.602

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	44	35	172	233	0	42	42
normalized size	1	1.00	0.67	0.53	2.61	3.53	0.00	0.64	0.64
time (sec)	N/A	0.056	0.037	0.122	0.361	2.728	0.000	0.130	0.601

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	47	42	17	17
normalized size	1	1.00	1.00	1.31	0.65	1.81	1.62	0.65	0.65
time (sec)	N/A	0.020	0.014	0.040	0.312	0.654	0.665	0.111	0.085

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	22	13	26	20	13	12
normalized size	1	1.00	0.84	1.16	0.68	1.37	1.05	0.68	0.63
time (sec)	N/A	0.011	0.008	0.015	0.316	0.591	0.265	0.131	0.562

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	34	18	25	0	19	26
normalized size	1	1.00	1.00	3.09	1.64	2.27	0.00	1.73	2.36
time (sec)	N/A	0.013	0.009	0.079	0.410	0.759	0.000	0.126	0.170

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	46	32	182	0	33	36
normalized size	1	1.00	1.00	1.44	1.00	5.69	0.00	1.03	1.12
time (sec)	N/A	0.024	0.037	0.086	0.398	2.302	0.000	0.114	0.715

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	67	42	17	17
normalized size	1	1.00	1.00	1.31	0.65	2.58	1.62	0.65	0.65
time (sec)	N/A	0.020	0.016	0.029	0.303	3.422	0.662	0.113	0.580

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	26	13	40	20	13	12
normalized size	1	1.00	0.84	1.37	0.68	2.11	1.05	0.68	0.63
time (sec)	N/A	0.012	0.009	0.036	0.302	1.434	0.260	0.112	0.053

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	22	79	73	83	0	43	65
normalized size	1	1.00	0.41	1.46	1.35	1.54	0.00	0.80	1.20
time (sec)	N/A	0.057	0.012	0.082	0.405	1.090	0.000	0.134	0.178

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	34	148	85	560	0	86	91
normalized size	1	1.00	0.32	1.41	0.81	5.33	0.00	0.82	0.87
time (sec)	N/A	0.139	0.026	0.099	0.405	0.855	0.000	0.126	0.401

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	87	42	17	17
normalized size	1	1.00	1.00	1.31	0.65	3.35	1.62	0.65	0.65
time (sec)	N/A	0.021	0.017	0.029	0.303	1.396	0.654	0.129	0.085

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	26	13	46	20	13	13
normalized size	1	1.00	1.00	1.37	0.68	2.42	1.05	0.68	0.68
time (sec)	N/A	0.012	0.010	0.016	0.305	1.368	0.262	0.128	0.055

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	22	56	95	132	0	96	106
normalized size	1	1.00	0.19	0.50	0.84	1.17	0.00	0.85	0.94
time (sec)	N/A	0.077	0.012	0.091	0.413	1.332	0.000	0.130	0.371

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	34	68	107	207	0	108	120
normalized size	1	1.00	0.26	0.52	0.82	1.58	0.00	0.82	0.92
time (sec)	N/A	0.088	0.025	0.099	0.413	2.978	0.000	0.135	0.921

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	157	326	134	2228	0	1239	166
normalized size	1	1.00	0.78	1.61	0.66	11.03	0.00	6.13	0.82
time (sec)	N/A	0.082	0.692	0.129	0.332	1.006	0.000	0.266	1.565

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	86	143	94	703	604	904	97
normalized size	1	1.00	0.65	1.08	0.71	5.33	4.58	6.85	0.73
time (sec)	N/A	0.053	0.206	0.071	0.323	0.516	34.104	0.221	0.916

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	77	63	244	342	612	73
normalized size	1	1.00	0.67	1.03	0.84	3.25	4.56	8.16	0.97
time (sec)	N/A	0.019	0.115	0.020	0.320	0.534	7.846	0.200	0.657

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	93	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	5.611	0.041	0.000	0.840	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	87	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	2.816	0.040	0.000	0.733	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	299	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	20.196	0.056	0.000	0.929	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	202	0	0	0	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	7.062	0.060	0.000	0.571	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	184	90	218	0	269	-1
normalized size	1	1.00	0.42	0.74	0.36	0.87	0.00	1.08	-0.00
time (sec)	N/A	0.248	0.113	0.226	0.412	0.665	0.000	0.188	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	76	152	62	126	0	195	-1
normalized size	1	1.00	0.47	0.94	0.38	0.78	0.00	1.20	-0.01
time (sec)	N/A	0.139	0.061	0.182	0.415	0.801	0.000	0.153	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	100	36	66	175	71	77
normalized size	1	1.00	0.65	1.35	0.49	0.89	2.36	0.96	1.04
time (sec)	N/A	0.112	0.037	0.200	0.411	0.686	17.302	0.124	0.632

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	39	42	0	85	-1
normalized size	1	1.00	0.96	0.00	0.85	0.91	0.00	1.85	-0.02
time (sec)	N/A	0.129	0.055	180.000	0.413	0.468	0.000	0.143	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	84	121	0	87	76
normalized size	1	1.00	0.79	0.00	1.45	2.09	0.00	1.50	1.31
time (sec)	N/A	0.143	0.071	180.000	0.414	0.812	0.000	0.188	0.588

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	72	0	209	315	0	122	89
normalized size	1	1.00	0.49	0.00	1.42	2.14	0.00	0.83	0.61
time (sec)	N/A	0.214	0.069	180.000	0.416	0.679	0.000	0.204	0.616

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	84	0	386	592	0	161	353
normalized size	1	1.00	0.42	0.00	1.94	2.97	0.00	0.81	1.77
time (sec)	N/A	0.279	0.077	180.000	0.417	0.572	0.000	0.211	0.622

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	62	0	42	99	32	45
normalized size	1	1.00	0.68	1.51	0.00	1.02	2.41	0.78	1.10
time (sec)	N/A	0.013	0.047	0.038	0.000	0.887	0.757	0.115	0.095

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	72	65	103	0	73	-1
normalized size	1	1.00	0.94	0.85	0.76	1.21	0.00	0.86	-0.01
time (sec)	N/A	0.081	0.093	0.147	0.306	0.879	0.000	0.125	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	92	97	81	129	0	91	-1
normalized size	1	1.00	0.91	0.96	0.80	1.28	0.00	0.90	-0.01
time (sec)	N/A	0.147	0.171	0.227	0.316	0.647	0.000	0.145	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	45	0	45	-1
normalized size	1	1.00	0.78	0.80	0.69	0.69	0.00	0.69	-0.02
time (sec)	N/A	0.065	0.067	0.121	0.312	0.759	0.000	0.123	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	48	47	75	0	49	-1
normalized size	1	1.00	1.11	0.74	0.72	1.15	0.00	0.75	-0.02
time (sec)	N/A	0.083	0.112	0.352	0.307	0.963	0.000	0.140	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	105	89	164	0	101	-1
normalized size	1	1.00	1.07	0.91	0.77	1.43	0.00	0.88	-0.01
time (sec)	N/A	0.168	0.399	0.392	0.319	0.480	0.000	0.145	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	103	100	90	213	0	106	-1
normalized size	1	1.00	0.94	0.91	0.82	1.94	0.00	0.96	-0.01
time (sec)	N/A	0.154	0.141	0.154	0.319	0.454	0.000	0.144	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	356	-1
normalized size	1	1.00	1.01	0.85	0.86	1.88	0.00	2.41	-0.01
time (sec)	N/A	0.194	0.765	0.220	0.412	0.708	0.000	0.184	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	287	207	200	445	0	223	-1
normalized size	1	1.00	1.20	0.87	0.84	1.86	0.00	0.93	-0.00
time (sec)	N/A	0.286	0.429	0.274	0.424	0.494	0.000	0.168	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	124	126	102	253	0	134	-1
normalized size	1	1.00	1.08	1.10	0.89	2.20	0.00	1.17	-0.01
time (sec)	N/A	0.225	0.313	0.132	0.335	0.562	0.000	0.137	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	143	334	0	390	-1
normalized size	1	1.00	1.37	0.98	0.89	2.07	0.00	2.42	-0.01
time (sec)	N/A	0.284	0.643	0.213	0.428	0.429	0.000	0.199	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	354	265	228	541	0	285	-1
normalized size	1	1.00	1.38	1.03	0.89	2.11	0.00	1.11	-0.00
time (sec)	N/A	0.477	0.777	0.257	0.425	0.518	0.000	0.186	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	217	0	132	-1
normalized size	1	1.00	0.78	0.88	0.79	1.63	0.00	0.99	-0.01
time (sec)	N/A	0.198	0.162	0.136	0.319	0.506	0.000	0.146	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	244	0	150	-1
normalized size	1	1.00	0.81	0.86	0.81	1.52	0.00	0.93	-0.01
time (sec)	N/A	0.226	0.266	0.168	0.318	0.515	0.000	0.159	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	427	0	264	-1
normalized size	1	1.00	0.79	0.86	0.78	1.58	0.00	0.97	-0.00
time (sec)	N/A	0.345	0.463	0.207	0.326	0.513	0.000	0.160	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	70	69	146	0	75	-1
normalized size	1	1.00	0.94	0.86	0.85	1.80	0.00	0.93	-0.01
time (sec)	N/A	0.170	0.351	0.117	0.321	0.433	0.000	0.155	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	-1
normalized size	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	-0.01
time (sec)	N/A	0.204	0.537	0.153	0.323	0.420	0.000	0.141	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	272	144	143	492	0	155	-1
normalized size	1	1.00	1.59	0.84	0.84	2.88	0.00	0.91	-0.01
time (sec)	N/A	0.300	1.226	0.213	0.328	0.738	0.000	0.155	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	166	147	127	322	0	172	-1
normalized size	1	1.00	1.19	1.05	0.91	2.30	0.00	1.23	-0.01
time (sec)	N/A	0.321	0.742	0.156	0.323	0.485	0.000	0.141	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	422	0	198	-1
normalized size	1	1.00	1.41	0.97	0.88	2.31	0.00	1.08	-0.01
time (sec)	N/A	0.334	1.464	0.181	0.416	0.657	0.000	0.177	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	480	302	263	848	0	352	-1
normalized size	1	1.00	1.60	1.01	0.88	2.83	0.00	1.17	-0.00
time (sec)	N/A	0.587	5.936	0.276	0.346	0.606	0.000	0.181	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	135	156	129	263	0	169	-1
normalized size	1	1.00	0.88	1.02	0.84	1.72	0.00	1.10	-0.01
time (sec)	N/A	0.301	0.339	0.145	0.338	0.676	0.000	0.150	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	211	185	343	0	225	-1
normalized size	1	1.00	0.84	0.96	0.84	1.57	0.00	1.03	-0.00
time (sec)	N/A	0.364	0.626	0.218	0.337	0.585	0.000	0.164	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	263	316	263	527	0	343	-1
normalized size	1	1.00	0.83	1.00	0.83	1.67	0.00	1.09	-0.00
time (sec)	N/A	0.462	1.020	0.240	0.373	0.646	0.000	0.181	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	179	160	139	325	0	181	-1
normalized size	1	1.00	1.16	1.04	0.90	2.11	0.00	1.18	-0.01
time (sec)	N/A	0.332	0.811	0.180	0.327	0.491	0.000	0.154	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	-1
normalized size	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	-0.00
time (sec)	N/A	0.395	2.275	0.220	0.330	0.561	0.000	0.170	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	2511	326	287	852	0	369	-1
normalized size	1	1.00	7.77	1.01	0.89	2.64	0.00	1.14	-0.00
time (sec)	N/A	0.530	6.517	0.291	0.344	1.720	0.000	0.192	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	252	186	151	363	0	209	-1
normalized size	1	1.00	1.57	1.16	0.94	2.25	0.00	1.30	-0.01
time (sec)	N/A	0.449	1.555	0.170	0.364	0.982	0.000	0.220	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	215	516	0	273	-1
normalized size	1	1.00	1.42	1.04	0.90	2.16	0.00	1.14	-0.00
time (sec)	N/A	0.546	6.234	0.217	0.360	0.496	0.000	0.202	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	315	940	0	431	-1
normalized size	1	1.00	8.69	1.12	0.92	2.73	0.00	1.25	-0.00
time (sec)	N/A	0.779	6.652	0.299	0.408	0.705	0.000	0.203	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	22	41	35	24
normalized size	1	1.00	1.00	0.83	1.17	0.73	1.37	1.17	0.80
time (sec)	N/A	0.038	0.057	0.021	0.325	0.546	0.179	0.134	0.598

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	52	81	58	85	75	46
normalized size	1	1.00	0.86	0.93	1.45	1.04	1.52	1.34	0.82
time (sec)	N/A	0.077	0.087	0.023	0.348	0.460	0.323	0.129	0.082

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	316	0	0	-1
normalized size	1	1.00	0.85	1.00	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.556	0.327	0.069	0.000	0.457	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	248	376	0	671	0	0	-1
normalized size	1	1.00	0.92	1.39	0.00	2.48	0.00	0.00	-0.00
time (sec)	N/A	0.809	0.537	0.067	0.000	0.539	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [320] had the largest ratio of [1.500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	4	3	1.00	10	0.300
8	A	3	3	1.00	10	0.300
9	A	3	3	1.00	10	0.300
10	A	2	2	1.00	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	2	2	1.00	10	0.200
12	A	3	3	1.00	10	0.300
13	A	3	3	1.00	10	0.300
14	A	4	3	1.00	10	0.300
15	A	4	3	1.00	12	0.250
16	A	3	3	1.00	12	0.250
17	A	3	3	1.00	12	0.250
18	A	2	2	1.00	12	0.167
19	A	2	2	1.00	12	0.167
20	A	3	3	1.00	12	0.250
21	A	3	3	1.00	12	0.250
22	A	4	3	1.00	12	0.250
23	A	3	2	1.00	14	0.143
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	14	0.143
26	A	1	1	1.00	14	0.071
27	A	1	1	1.00	14	0.071
28	A	2	2	1.00	14	0.143
29	A	2	2	1.00	14	0.143
30	A	3	2	1.00	14	0.143
31	A	1	1	1.00	12	0.083
32	A	1	1	1.00	12	0.083
33	A	1	1	1.00	12	0.083
34	A	1	1	1.00	12	0.083
35	A	1	1	1.00	12	0.083
36	A	1	1	1.00	12	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	1	1	1.00	10	0.100
38	A	1	1	1.00	12	0.083
39	A	1	1	1.00	12	0.083
40	A	6	5	1.00	13	0.385
41	A	2	2	1.00	13	0.154
42	A	3	3	1.00	13	0.231
43	A	2	2	1.00	11	0.182
44	A	3	3	1.00	11	0.273
45	A	5	5	1.00	13	0.385
46	A	6	6	1.00	13	0.462
47	A	6	5	1.00	13	0.385
48	A	3	3	1.00	13	0.231
49	A	6	6	1.00	13	0.462
50	A	3	3	1.00	13	0.231
51	A	2	2	1.00	11	0.182
52	A	4	4	1.00	11	0.364
53	A	6	6	1.00	13	0.462
54	A	7	7	1.00	13	0.538
55	A	7	6	1.00	13	0.462
56	A	1	1	1.00	14	0.071
57	A	2	2	1.00	14	0.143
58	A	3	2	1.00	14	0.143
59	A	4	2	1.00	14	0.143
60	A	1	1	1.00	14	0.071
61	A	2	2	1.00	14	0.143
62	A	3	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	4	2	1.00	14	0.143
64	A	3	3	1.00	16	0.188
65	A	3	3	1.00	16	0.188
66	A	3	2	1.00	17	0.118
67	A	2	2	1.00	17	0.118
68	A	1	1	1.00	17	0.059
69	A	2	2	1.00	17	0.118
70	A	3	3	1.00	17	0.176
71	A	4	3	1.00	17	0.176
72	A	7	7	1.00	13	0.538
73	A	6	6	1.00	13	0.462
74	A	6	6	1.00	13	0.462
75	A	4	4	1.00	11	0.364
76	A	5	5	1.00	11	0.454
77	A	7	7	1.00	13	0.538
78	A	7	7	1.00	13	0.538
79	A	8	7	1.00	13	0.538
80	A	7	7	1.00	13	0.538
81	A	6	6	1.00	13	0.462
82	A	5	5	1.00	13	0.385
83	A	5	5	1.00	11	0.454
84	A	6	6	1.00	11	0.546
85	A	7	7	1.00	13	0.538
86	A	8	7	1.00	13	0.538
87	A	9	7	1.00	13	0.538
88	A	4	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
89	A	6	5	1.00	14	0.357
90	A	7	6	1.00	14	0.429
91	A	8	6	1.00	14	0.429
92	A	1	1	1.00	14	0.071
93	A	3	3	1.00	14	0.214
94	A	4	4	1.00	14	0.286
95	A	5	4	1.00	14	0.286
96	A	4	3	1.00	12	0.250
97	A	3	3	1.00	12	0.250
98	A	2	2	1.00	12	0.167
99	A	1	1	1.00	12	0.083
100	A	2	1	1.00	10	0.100
101	A	3	3	1.00	12	0.250
102	A	5	5	1.00	12	0.417
103	A	6	6	1.00	12	0.500
104	A	7	6	1.00	12	0.500
105	A	7	7	1.00	10	0.700
106	A	6	6	1.00	10	0.600
107	A	2	2	1.00	10	0.200
108	A	2	2	1.00	10	0.200
109	A	4	4	1.00	10	0.400
110	A	7	7	1.00	10	0.700
111	A	5	5	1.00	13	0.385
112	A	4	3	1.00	20	0.150
113	A	3	3	1.00	20	0.150
114	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	A	2	2	1.00	15	0.133
116	A	2	2	1.00	15	0.133
117	A	3	3	1.00	15	0.200
118	A	4	3	1.00	15	0.200
119	A	2	2	1.00	17	0.118
120	A	2	2	1.00	17	0.118
121	A	3	3	1.00	17	0.176
122	A	4	3	1.00	17	0.176
123	A	3	3	1.00	20	0.150
124	A	3	3	1.00	20	0.150
125	A	4	4	1.00	20	0.200
126	A	8	6	1.00	17	0.353
127	A	7	6	1.00	17	0.353
128	A	6	6	1.00	17	0.353
129	A	4	4	1.00	15	0.267
130	A	5	5	1.00	15	0.333
131	A	6	5	1.00	15	0.333
132	A	7	5	1.00	15	0.333
133	A	4	4	1.00	20	0.200
134	A	2	2	1.00	20	0.100
135	A	2	2	1.00	16	0.125
136	A	2	2	1.00	13	0.154
137	A	5	5	1.00	17	0.294
138	A	6	6	1.00	17	0.353
139	A	7	6	1.00	17	0.353
140	A	4	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
141	A	3	3	1.00	10	0.300
142	A	2	2	1.00	10	0.200
143	A	2	2	1.00	10	0.200
144	A	3	3	1.00	10	0.300
145	A	4	3	1.00	10	0.300
146	A	7	4	1.00	10	0.400
147	A	5	4	1.00	10	0.400
148	A	4	4	1.00	10	0.400
149	A	4	4	1.00	10	0.400
150	A	5	4	1.00	10	0.400
151	A	7	4	1.00	10	0.400
152	A	7	3	1.00	10	0.300
153	A	5	3	1.00	10	0.300
154	A	3	3	1.00	10	0.300
155	A	3	3	1.00	10	0.300
156	A	3	2	1.00	10	0.200
157	A	3	2	1.00	10	0.200
158	A	5	3	1.00	13	0.231
159	A	3	2	1.00	13	0.154
160	A	4	3	1.00	13	0.231
161	A	3	2	1.00	13	0.154
162	A	3	3	1.00	13	0.231
163	A	2	1	1.00	13	0.077
164	A	2	2	1.00	13	0.154
165	A	2	2	1.00	11	0.182
166	A	4	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
167	A	3	3	1.00	13	0.231
168	A	4	3	1.00	13	0.231
169	A	3	2	1.00	13	0.154
170	A	4	3	1.00	13	0.231
171	A	4	4	1.00	13	0.308
172	A	2	2	1.00	13	0.154
173	A	3	3	1.00	13	0.231
174	A	3	2	1.00	13	0.154
175	A	2	2	1.00	13	0.154
176	A	2	2	1.00	11	0.182
177	A	4	3	1.00	11	0.273
178	A	4	3	1.00	13	0.231
179	A	4	3	1.00	13	0.231
180	A	4	2	1.00	13	0.154
181	A	3	2	1.00	15	0.133
182	A	1	1	1.00	15	0.067
183	A	2	2	1.00	13	0.154
184	A	3	2	1.00	15	0.133
185	A	1	1	1.00	15	0.067
186	A	2	2	1.00	13	0.154
187	A	3	2	1.00	13	0.154
188	A	7	6	1.00	13	0.462
189	A	3	2	1.00	13	0.154
190	A	6	6	1.00	13	0.462
191	A	3	2	1.00	13	0.154
192	A	5	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	2	2	1.00	11	0.182
194	A	6	6	1.00	11	0.546
195	A	5	5	1.00	13	0.385
196	A	7	6	1.00	13	0.462
197	A	6	6	1.00	13	0.462
198	A	8	7	1.00	13	0.538
199	A	7	6	1.00	13	0.462
200	A	6	6	1.00	13	0.462
201	A	3	2	1.00	13	0.154
202	A	5	5	1.00	13	0.385
203	A	2	2	1.00	11	0.182
204	A	7	6	1.00	11	0.546
205	A	6	6	1.00	13	0.462
206	A	7	6	1.00	13	0.462
207	A	7	6	1.00	13	0.462
208	A	6	5	1.00	13	0.385
209	A	6	5	1.00	13	0.385
210	A	5	4	1.00	13	0.308
211	A	5	5	1.00	11	0.454
212	A	4	4	1.00	11	0.364
213	A	4	4	1.00	13	0.308
214	A	5	4	1.00	13	0.308
215	A	5	5	1.00	13	0.385
216	A	5	4	1.00	13	0.308
217	A	6	5	1.00	13	0.385
218	A	10	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
219	A	4	3	1.00	13	0.231
220	A	10	5	1.00	13	0.385
221	A	4	3	1.00	11	0.273
222	A	3	2	1.00	11	0.182
223	A	7	5	1.00	13	0.385
224	A	3	2	1.00	13	0.154
225	A	9	6	1.00	13	0.462
226	A	3	2	1.00	13	0.154
227	A	11	5	1.00	13	0.385
228	A	13	9	1.00	13	0.692
229	A	7	6	1.00	13	0.462
230	A	8	7	1.00	13	0.538
231	A	6	5	1.00	11	0.454
232	A	4	4	1.00	11	0.364
233	A	7	7	1.00	13	0.538
234	A	3	2	1.00	13	0.154
235	A	7	7	1.00	13	0.538
236	A	16	9	1.00	13	0.692
237	A	7	6	1.00	13	0.462
238	A	13	9	1.00	13	0.692
239	A	6	5	1.00	11	0.454
240	A	3	2	1.00	11	0.182
241	A	8	7	1.00	13	0.538
242	A	3	2	1.00	13	0.154
243	A	8	7	1.00	13	0.538
244	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	3	3	1.00	13	0.231
246	A	7	6	1.00	15	0.400
247	A	5	4	1.00	15	0.267
248	A	5	4	1.00	17	0.235
249	A	11	9	1.00	15	0.600
250	A	9	8	1.00	15	0.533
251	A	12	11	1.00	15	0.733
252	A	6	6	1.00	15	0.400
253	A	7	7	1.00	31	0.226
254	A	8	8	1.00	31	0.258
255	A	9	8	1.00	31	0.258
256	A	10	8	1.00	31	0.258
257	A	13	8	1.00	14	0.571
258	A	11	7	1.00	14	0.500
259	A	9	6	1.00	12	0.500
260	A	13	5	1.00	20	0.250
261	A	5	3	1.00	36	0.083
262	A	4	3	1.00	36	0.083
263	A	2	2	1.00	34	0.059
264	A	0	0	0.00	0	0.000
265	A	0	0	0.00	0	0.000
266	A	1	1	1.00	11	0.091
267	A	2	2	1.00	13	0.154
268	A	2	2	1.00	13	0.154
269	A	3	2	1.00	13	0.154
270	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	2	2	1.00	17	0.118
272	A	2	2	0.97	17	0.118
273	A	3	2	0.98	17	0.118
274	A	2	1	1.00	15	0.067
275	A	3	2	1.00	17	0.118
276	A	3	1	1.00	17	0.059
277	A	4	2	1.00	17	0.118
278	A	3	1	1.00	17	0.059
279	A	4	3	1.00	19	0.158
280	A	4	3	1.00	19	0.158
281	A	3	2	1.00	19	0.105
282	A	3	2	1.00	19	0.105
283	A	4	3	1.00	19	0.158
284	A	4	3	1.00	19	0.158
285	A	8	8	1.00	18	0.444
286	A	6	6	1.00	18	0.333
287	A	3	3	1.00	18	0.167
288	A	4	4	1.00	18	0.222
289	A	4	4	1.00	10	0.400
290	A	5	5	1.00	12	0.417
291	A	6	4	1.00	12	0.333
292	A	5	5	1.00	11	0.454
293	A	6	6	1.00	13	0.462
294	A	9	5	1.00	13	0.385
295	A	5	5	1.00	14	0.357
296	A	6	6	1.00	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	9	5	1.00	16	0.312
298	A	6	6	1.00	17	0.353
299	A	7	7	1.00	19	0.368
300	A	10	6	1.00	19	0.316
301	A	4	3	1.00	16	0.188
302	A	5	4	1.00	16	0.250
303	A	4	3	1.00	16	0.188
304	A	4	3	1.00	14	0.214
305	A	3	3	1.00	14	0.214
306	A	4	4	1.00	16	0.250
307	A	3	3	1.00	16	0.188
308	A	6	5	1.00	16	0.312
309	A	5	4	1.00	16	0.250
310	A	4	3	1.00	10	0.300
311	A	4	3	1.00	8	0.375
312	A	5	5	1.00	8	0.625
313	A	6	6	1.00	10	0.600
314	A	4	3	1.00	10	0.300
315	A	4	3	1.00	8	0.375
316	A	9	9	1.00	8	1.125
317	A	13	9	1.00	10	0.900
318	A	4	3	1.00	10	0.300
319	A	4	3	1.00	8	0.375
320	A	15	12	1.00	8	1.500
321	A	16	13	1.00	10	1.300
322	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	2	2	1.00	18	0.111
324	A	1	1	1.00	16	0.062
325	A	1	1	1.00	16	0.062
326	A	1	1	1.00	18	0.056
327	A	2	2	1.00	18	0.111
328	A	2	2	1.00	18	0.111
329	A	6	5	1.00	25	0.200
330	A	6	5	1.00	25	0.200
331	A	5	4	1.00	25	0.160
332	A	4	4	1.00	25	0.160
333	A	4	4	1.00	25	0.160
334	A	6	5	1.00	25	0.200
335	A	6	5	1.00	25	0.200
336	A	1	1	1.00	10	0.100
337	A	6	4	1.00	12	0.333
338	A	6	4	1.00	15	0.267
339	A	6	3	1.00	12	0.250
340	A	4	2	1.00	14	0.143
341	A	6	3	1.00	17	0.176
342	A	8	5	1.00	16	0.312
343	A	9	6	1.00	18	0.333
344	A	14	5	1.00	18	0.278
345	A	8	5	1.00	19	0.263
346	A	9	6	1.00	21	0.286
347	A	14	5	1.00	21	0.238
348	A	8	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
349	A	9	4	1.00	18	0.222
350	A	14	4	1.00	18	0.222
351	A	6	4	1.00	18	0.222
352	A	7	4	1.00	20	0.200
353	A	10	4	1.00	20	0.200
354	A	8	5	1.00	21	0.238
355	A	9	5	1.00	23	0.217
356	A	14	5	1.00	23	0.217
357	A	8	4	1.00	19	0.210
358	A	10	4	1.00	21	0.190
359	A	14	4	1.00	21	0.190
360	A	8	5	1.00	21	0.238
361	A	10	5	1.00	23	0.217
362	A	14	5	1.00	23	0.217
363	A	8	5	1.00	24	0.208
364	A	10	5	1.00	26	0.192
365	A	14	5	1.00	26	0.192
366	A	6	5	1.00	6	0.833
367	A	9	6	1.00	6	1.000
368	A	8	4	1.00	16	0.250
369	A	8	4	1.00	19	0.210

Chapter 3

Listing of integrals

3.1 $\int \sinh(a + bx)\,dx$

Optimal. Leaf size=10

$$\frac{\cosh(a + bx)}{b}$$

```
[Out] cosh(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2638}

$$\frac{\cosh(a + bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(a + bx)\,dx = \frac{\cosh(a + bx)}{b}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.10

$$\frac{\sinh(a)\sinh(bx)}{b}+\frac{\cosh(a)\cosh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x],x]
[Out] (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

fricas [A] time = 0.59, size = 10, normalized size = 1.00

$$\frac{\cosh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="fricas")
[Out] cosh(b*x + a)/b
```

giac [B] time = 0.11, size = 26, normalized size = 2.60

$$\frac{e^{(bx+a)}}{2\,b}+\frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="giac")
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b
```

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\frac{\cosh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a),x)
[Out] cosh(b*x+a)/b
```

maxima [A] time = 1.16, size = 10, normalized size = 1.00

$$\frac{\cosh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)/b
```

mupad **[B]** time = 0.04, size = 10, normalized size = 1.00

$$\frac{\cosh\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x),x)
```

```
[Out] cosh(a + b*x)/b
```

sympy **[A]** time = 0.13, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\cosh\left(a+bx\right)}{b} & \text{for}\ b \neq 0 \\ x\sinh\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)/b, Ne(b, 0)), (x*sinh(a), True))
```

3.2 $\int \sinh^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}$$

[Out] -1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2,x]
```

```
[Out] -x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{\int 1\,dx}{2}\\ &= -\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.92

$$\frac{\sinh(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2,x]
```

```
[Out] (-2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

fricas [A] time = 0.47, size = 23, normalized size = 0.92

$$-\frac{bx - \cosh(bx + a)\sinh(bx + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - cosh(b*x + a)*sinh(b*x + a))/b
```

giac [A] time = 0.13, size = 32, normalized size = 1.28

$$-\frac{1}{2}x + \frac{e^{(2bx+2a)}}{8b} - \frac{e^{(-2bx-2a)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

maple [A] time = 0.02, size = 27, normalized size = 1.08

$$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2} - \frac{bx}{2} - \frac{a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)
```

maxima [A] time = 1.61, size = 32, normalized size = 1.28

$$-\frac{1}{2}x + \frac{e^{(2bx+2a)}}{8b} - \frac{e^{(-2bx-2a)}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

mupad [B] time = 0.37, size = 18, normalized size = 0.72

$$\frac{\sinh\left(2\,a+2\,b\,x\right)}{4\,b}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2,x)
```

```
[Out] sinh(2*a + 2*b*x)/(4*b) - x/2
```

sympy [A] time = 0.21, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sinh^2\left(a+bx\right)}{2}-\frac{x\cosh^2\left(a+bx\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b} & \text{for } b\neq 0\\ x\sinh^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh
(a + b*x)/(2*b), Ne(b, 0)), (x*sinh(a)**2, True))
```

3.3 $\int \sinh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cosh^3(a+bx)}{3b}-\frac{\cosh(a+bx)}{b}$$

```
[Out] -cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cosh^3(a+bx)}{3b}-\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3,x]
```

```
[Out] -(Cosh[a + b*x]/b) + Cosh[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh(a+bx)\right)}{b}\\ &= -\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{\cosh(3(a+bx))}{12b}-\frac{3\cosh(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3,x]
```

[Out] (-3*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b)

fricas [A] time = 0.51, size = 38, normalized size = 1.41

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-9\,\cosh\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)^3,x, algorithm="fricas")

[Out] 1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 9*cosh(b*x + a))/b

giac [B] time = 0.12, size = 54, normalized size = 2.00

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}-\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}+\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)^3,x, algorithm="giac")

[Out] 1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(-3*b*x - 3*a)/b

maple [A] time = 0.02, size = 23, normalized size = 0.85

$$\frac{\left(-\frac{2}{3}+\frac{\left(\sinh^{2}\left(bx+a\right)\right)}{3}\right)\cosh\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b*x+a)^3,x)

[Out] 1/b*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)

maxima [B] time = 1.72, size = 54, normalized size = 2.00

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}-\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}+\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)^3,x, algorithm="maxima")

[Out] 1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(-3*b*x - 3*a)/b

mupad [**B**] time = 0.06, size = 24, normalized size = 0.89

$$-\frac{3\cosh(a+bx)-\cosh(a+bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3,x)
```

```
[Out] -(3*cosh(a + b*x) - cosh(a + b*x)^3)/(3*b)
```

sympy [**A**] time = 0.40, size = 36, normalized size = 1.33

$$\begin{cases} \frac{\sinh^2(a+bx)\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b} & \text{for } b\neq 0 \\ x\sinh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)/b - 2*cosh(a + b*x)**3/(3*b), Ne(
b, 0)), (x*sinh(a)**3, True))
```

3.4 $\int \sinh^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] 3/8*x-3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)*sinh(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]*Sinh[a + b
*x]^3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sinh^4(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}-\frac{3}{4}\int \sinh^2(a+bx)\,dx\\ &= -\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 0.72

$$\frac{12(a+bx)-8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) - 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

fricas [A] time = 0.48, size = 49, normalized size = 1.07

$$\frac{\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+3\,bx+\left(\cosh\left(bx+a\right)^{3}-4\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 - 4*cosh(b*x
+ a))*sinh(b*x + a))/b
```

giac [A] time = 0.13, size = 60, normalized size = 1.30

$$\frac{3}{8}\,x+\frac{e^{(4\,bx+4\,a)}}{64\,b}-\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

maple [A] time = 0.12, size = 39, normalized size = 0.85

$$\frac{\left(\frac{\left(\sinh^{3}(bx+a)\right)}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh\left(bx+a\right)+\frac{3bx}{8}+\frac{3a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^4,x)
```

```
[Out] 1/b*((1/4*sinh(b*x+a)^3-3/8*sinh(b*x+a))*cosh(b*x+a)+3/8*b*x+3/8*a)
```

maxima [A] time = 2.20, size = 60, normalized size = 1.30

$$\frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

mupad [B] time = 0.08, size = 32, normalized size = 0.70

$$\frac{3\,x}{8} - \frac{\frac{\sinh(2\,a+2\,b\,x)}{4} - \frac{\sinh(4\,a+4\,b\,x)}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^4,x)
```

```
[Out] (3*x)/8 - (sinh(2*a + 2*b*x)/4 - sinh(4*a + 4*b*x)/32)/b
```

sympy [A] time = 0.83, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sinh^{4}(a+bx)}{8} - \frac{3x\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{4} + \frac{3x\cosh^{4}(a+bx)}{8} + \frac{5\sinh^{3}(a+bx)\cosh(a+bx)}{8b} - \frac{3\sinh(a+bx)\cosh^{3}(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sinh^{4}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/4
 + 3*x*cosh(a + b*x)**4/8 + 5*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) - 3*sinh
(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**4, True))
```

3.5 $\int \sinh^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\cosh^5(a+bx)}{5b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}$$

```
[Out] cosh(b*x+a)/b-2/3*cosh(b*x+a)^3/b+1/5*cosh(b*x+a)^5/b
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cosh^5(a+bx)}{5b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^5,x]
```

```
[Out] Cosh[a + b*x]/b - (2*Cosh[a + b*x]^3)/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.07

$$\frac{5\cosh(a+bx)}{8b}-\frac{5\cosh(3(a+bx))}{48b}+\frac{\cosh(5(a+bx))}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^5,x]
```

```
[Out] (5*Cosh[a + b*x])/(8*b) - (5*Cosh[3*(a + b*x)])/(48*b) + Cosh[5*(a + b*x)]/
(80*b)
```

fricas [B] time = 0.50, size = 79, normalized size = 1.93

$$\frac{3 \cosh(bx+a)^5 + 15 \cosh(bx+a) \sinh(bx+a)^4 - 25 \cosh(bx+a)^3 + 15 \left(2 \cosh(bx+a)^3 - 5 \cosh(bx+a)\right)}{240\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 25*cosh(b*x +
 a)^3 + 15*(2*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^2 + 150*cosh
(b*x + a))/b
```

giac [B] time = 0.14, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} + \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} + \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

maple [A] time = 0.12, size = 33, normalized size = 0.80

$$\frac{\left(\frac{8}{15} + \frac{\left(\sinh^4(bx+a)\right)}{5} - \frac{4\left(\sinh^2(bx+a)\right)}{15}\right) \cosh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^5,x)
```

```
[Out] 1/b*(8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)
```

maxima [B] time = 0.31, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} + \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} + \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="maxima")
```

[Out] 1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b

mupad [B] time = 0.41, size = 31, normalized size = 0.76

$$\frac{\frac{\cosh(a+b\,x)^5}{5}-\frac{2\cosh(a+b\,x)^3}{3}+\cosh\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(a + b*x)^5,x)

[Out] (cosh(a + b*x) - (2*cosh(a + b*x)^3)/3 + cosh(a + b*x)^5/5)/b

sympy [A] time = 1.51, size = 58, normalized size = 1.41

$$\begin{cases} \frac{\sinh^4(a+bx)\cosh(a+bx)}{b}-\frac{4\sinh^2(a+bx)\cosh^3(a+bx)}{3b}+\frac{8\cosh^5(a+bx)}{15b} & \text{for } b\neq 0 \\ x\sinh^5(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)**5,x)

[Out] Piecewise((sinh(a + b*x)**4*cosh(a + b*x)/b - 4*sinh(a + b*x)**2*cosh(a + b*x)**3/(3*b) + 8*cosh(a + b*x)**5/(15*b), Ne(b, 0)), (x*sinh(a)**5, True))

3.6 $\int \sinh^6(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{5x}{16}$$

```
[Out] -5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b-5/24*cosh(b*x+a)*sinh(b*x+a)^3/b+1/6
*cosh(b*x+a)*sinh(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (5*Cosh[a + b*x]*Sinh[
a + b*x]^3)/(24*b) + (Cosh[a + b*x]*Sinh[a + b*x]^5)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \sinh^6(a+bx)\,dx = \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b} - \frac{5}{6}\int \sinh^4(a+bx)\,dx$$
$$= -\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b} + \frac{5}{8}\int \sinh^2(a+bx)\,dx$$
$$= \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh(a+bx)\sinh^5(a+b}{6b}$$
$$= -\frac{5x}{16} + \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh(a+bx)\sinh}{6b}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.64

$$\frac{45\sinh(2(a+bx)) - 9\sinh(4(a+bx)) + \sinh(6(a+bx)) - 60a - 60bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^6,x]
```

```
[Out] (-60*a - 60*b*x + 45*Sinh[2*(a + b*x)] - 9*Sinh[4*(a + b*x)] + Sinh[6*(a +
b*x)])/(192*b)
```

fricas [A] time = 0.58, size = 90, normalized size = 1.34

$$\frac{3\cosh(bx+a)\sinh(bx+a)^5 + 2\left(5\cosh(bx+a)^3 - 9\cosh(bx+a)\right)\sinh(bx+a)^3 - 30\,bx + 3\left(\cosh(bx+a\right.}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 9*cosh(b*x +
 a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 + 15
*cosh(b*x + a))*sinh(b*x + a))/b
```

giac [A] time = 0.12, size = 88, normalized size = 1.31

$$-\frac{5}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{3\,e^{(4\,bx+4\,a)}}{128\,b} + \frac{15\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{15\,e^{(-2\,bx-2\,a)}}{128\,b} + \frac{3\,e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -5/16*x + 1/384*e^(6*b*x + 6*a)/b - 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*b
*x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b + 3/128*e^(-4*b*x - 4*a)/b - 1/384*
e^(-6*b*x - 6*a)/b
```

maple [A] time = 0.12, size = 49, normalized size = 0.73

$$\frac{\left(\frac{\left(\sinh^{5}(bx+a)\right)}{6}-\frac{5\left(\sinh^{3}(bx+a)\right)}{24}+\frac{5\sinh(bx+a)}{16}\right)\cosh\left(bx+a\right)-\frac{5bx}{16}-\frac{5a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^6,x)
```

```
[Out] 1/b*((1/6*sinh(b*x+a)^5-5/24*sinh(b*x+a)^3+5/16*sinh(b*x+a))*cosh(b*x+a)-5/
16*b*x-5/16*a)
```

maxima [A] time = 0.31, size = 86, normalized size = 1.28

$$-\frac{\left(9\,e^{(-2\,bx-2\,a)}-45\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b}-\frac{5\,(bx+a)}{16\,b}-\frac{45\,e^{(-2\,bx-2\,a)}-9\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*b*x - 2*a) - 45*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 5
/16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) - 9*e^(-4*b*x - 4*a) + e^(-6*b
*x - 6*a))/b
```

mupad [B] time = 0.13, size = 42, normalized size = 0.63

$$\frac{\frac{15\sinh(2\,a+2\,b\,x)}{64}-\frac{3\sinh(4\,a+4\,b\,x)}{64}+\frac{\sinh(6\,a+6\,b\,x)}{192}}{b}-\frac{5\,x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^6,x)
```

```
[Out] ((15*sinh(2*a + 2*b*x))/64 - (3*sinh(4*a + 4*b*x))/64 + sinh(6*a + 6*b*x)/1
92)/b - (5*x)/16
```

sympy [A] time = 2.85, size = 139, normalized size = 2.07

$$\begin{cases} \frac{5x\sinh^{6}(a+bx)}{16}-\frac{15x\sinh^{4}(a+bx)\cosh^{2}(a+bx)}{16}+\frac{15x\sinh^{2}(a+bx)\cosh^{4}(a+bx)}{16}-\frac{5x\cosh^{6}(a+bx)}{16}+\frac{11\sinh^{5}(a+bx)\cosh(a+bx)}{16b}-\frac{5\sinh^{3} \\ x\sinh^{6}(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sinh(a + b*x)**6/16 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**2
/16 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - 5*x*cosh(a + b*x)**6/16 +
 11*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a + b*x
)**3/(6*b) + 5*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)
**6, True))
```

3.7 $\int \sinh^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{10\sqrt{\sinh(a+bx)}\,\cosh(a+bx)}{21b}-\frac{10i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{\sinh(a+bx)}}$$

```
[Out] 2/7*cosh(b*x+a)*sinh(b*x+a)^(5/2)/b+10/21*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^
2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b
*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sinh(b*x+a)^(1/2)-10/21*cosh(b*x+a)*si
nh(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2635, 2642, 2641}

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{10\sqrt{\sinh(a+bx)}\,\cosh(a+bx)}{21b}-\frac{10i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b*Sqrt[Sinh[a + b*x]]) - (10*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(21*b) + (2
*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rubi steps

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} - \frac{5}{7}\int \sinh^{\frac{3}{2}}(a+bx)\,dx$$
$$= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\sinh(a+bx}}$$
$$= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} + \frac{\left(5\sqrt{i\sinh(a+bx)}\right)}{21\sqrt{\sinh(a}}$$
$$= -\frac{10iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{21b\sqrt{\sinh(a+bx)}} - \frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(}{}$$

Mathematica [A] time = 0.14, size = 75, normalized size = 0.73

$$\frac{-26\sinh(2(a+bx)) + 3\sinh(4(a+bx)) + 40i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{84b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(7/2),x]
[Out] ((40*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] -
 26*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(84*b*Sqrt[Sinh[a + b*x]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sinh\left(bx+a\right)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="fricas")
[Out] integral(sinh(b*x + a)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

maple [A] time = 0.09, size = 116, normalized size = 1.13

$$\frac{\frac{5i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{21}+\frac{2\sinh(bx+a)\left(\cosh^4(bx+a)\right)}{7}-\frac{16\left(\cosh^2(bx+a)\right)\sinh(b}{21}}{\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(7/2),x)
```

```
[Out] (5/21*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/7*sinh(b*x+a)*c
osh(b*x+a)^4-16/21*cosh(b*x+a)^2*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)
/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx+a\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a+b\,x\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(7/2),x)
```

```
[Out] int(sinh(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.8 $\int \sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}+\frac{6i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2/5*cosh(b*x+a)*sinh(b*x+a)^(3/2)/b-6/5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2635, 2640, 2639}

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}+\frac{6i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(5/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{\frac{5}{2}}(a+bx)\,dx &= \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b} - \frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx \\
&= \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b} - \frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{i\sinh(a+bx)}} \\
&= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.85

$$
\frac{\sinh(a+bx)\sinh(2(a+bx)) - 6\sqrt{i\sinh(a+bx)}\,E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{5b\sqrt{\sinh(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(5/2),x]
```

```
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Sin
h[a + b*x]*Sinh[2*(a + b*x)])/(5*b*Sqrt[Sinh[a + b*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$
\text{integral}\left(\sinh\left(bx+a\right)^{\frac{5}{2}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sinh\left(bx+a\right)^{\frac{5}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

maple [A] time = 0.07, size = 164, normalized size = 2.05

$$\frac{-\frac{6\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx}{5}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(5/2),x)
```

```
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(5/2),x)
```

```
[Out] int(sinh(a + b*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(5/2), x)
```

3.9 $\int \sinh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

```
[Out] -2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x
)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/
sinh(b*x+a)^(1/2)+2/3*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2635, 2642, 2641}

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(3/2),x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*S
qrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b} - \frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$$
$$= \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b} - \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}}$$
$$= \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}$$

Mathematica [C] time = 0.09, size = 83, normalized size = 1.04

$$\frac{\sinh(2(a+bx)) - 2\sqrt{-\sinh(2a+2bx)-\cosh(2a+2bx)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+bx))+\sinh(2(a+bx))\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(3/2),x]
[Out] (Sinh[2*(a + b*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] +
 Sinh[2*(a + b*x)]]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b*x]])/(3*b*S
qrt[Sinh[a + b*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sinh\left(bx+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(sinh(b*x + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

maple [A] time = 0.07, size = 100, normalized size = 1.25

$$\frac{-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\left(\cosh^{2}(bx+a)\right)\sinh(bx+a)}{3}}{\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(3/2),x)
```

```
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)^2
*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{sinh}\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(3/2),x)
```

```
[Out] int(sinh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{\frac{3}{2}}{\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(3/2), x)
```

3.10 $\int \sqrt{\sinh(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{2i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2640, 2639}

$$-\frac{2i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I
*Sinh[a + b*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \sqrt{\sinh(a+bx)}\,dx = \frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}}$$
$$= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

Mathematica [A] time = 0.10, size = 50, normalized size = 0.93

$$\frac{2\sqrt{i\sinh(a+bx)}\,E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|2\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh
[a + b*x]])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sinh(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

maple [A] time = 0.06, size = 108, normalized size = 2.00

$$\frac{\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)}, \frac{\sqrt{2}}{2}\right)-\text{I}\right.}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(1/2),x)
```

```
[Out] (-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sinh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(1/2),x)
```

```
[Out] int(sinh(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(a + b*x)), x)
```

3.11 $\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=54

$$-\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sin
h(b*x+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt
[Sinh[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \frac{\sqrt{i\sinh(a+bx)}\,\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{\sqrt{\sinh(a+bx)}}$$
$$= -\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

Mathematica [A] time = 0.12, size = 48, normalized size = 0.89

$$-\frac{2\sqrt{\sinh(a+bx)}\,F\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sinh[a + b*x]],x]
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\sinh\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(sinh(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sinh(b*x + a)), x)
```

maple [A] time = 0.05, size = 87, normalized size = 1.61

$$\frac{i\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(bx+a\right)+i\right)}\,\sqrt{i\sinh\left(bx+a\right)}\,\operatorname{EllipticF}\left(\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)},\frac{\sqrt{2}}{2}\right)}{\cosh\left(bx+a\right)\sqrt{\sinh\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sinh(b*x+a)^(1/2),x)

[Out] I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(sinh(b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\sinh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sinh(a + b*x)^(1/2),x)

[Out] int(1/sinh(a + b*x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sinh(b*x+a)**(1/2),x)

[Out] Integral(1/sqrt(sinh(a + b*x)), x)

3.12 $\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)+2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-3/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a - Pi/2 +
 I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\int \sqrt{\sinh(a+bx)}\,dx\\ &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}+\frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.75

$$-\frac{2\left(\cosh(a+bx)-\sqrt{i\sinh(a+bx)}\,E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-3/2),x]
[Out] (-2*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sin
h[a + b*x]]))/(b*Sqrt[Sinh[a + b*x]])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sinh(bx+a)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(sinh(b*x + a)^(-3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

maple [A] time = 0.06, size = 154, normalized size = 2.03

$$\frac{2\sqrt{1 - i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{i\sinh(bx+a)+1}\,\sqrt{i\sinh(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)}, \frac{\sqrt{2}}{2}\right) - \sqrt{1-i\,\mathrm{s}}}{\cosh(bx+a)\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(3/2),x)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(I*sinh(b*x+a)+1)^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^(3/2),x)
```

```
[Out] int(1/sinh(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-3/2), x)
```

3.13 $\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

```
[Out] -2/3*cosh(b*x+a)/b/sinh(b*x+a)^(3/2)-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2
)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*
x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sinh(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2636, 2642, 2641}

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(3*b*Sinh[a + b*x]^(3/2)) + (((2*I)/3)*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx\\ &= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}}\\ &= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.08, size = 86, normalized size = 1.08

$$-\frac{2\left(\sinh(a+bx)\sqrt{-\sinh(2a+2bx)-\cosh(2a+2bx)+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+bx))+\sinh(2(a+bx))\right)+\mathrm{co}\right.}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*(Cosh[a + b*x] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] + S
inh[2*(a + b*x)]]*Sinh[a + b*x]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b
*x]]))/(3*b*Sinh[a + b*x]^(3/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\sinh\left(bx+a\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x + a)^(-5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

maple [A] time = 0.06, size = 101, normalized size = 1.26

$$-\frac{i\sqrt{1-i\sinh\left(bx+a\right)}\,\sqrt{2}\,\sqrt{i\sinh\left(bx+a\right)+1}\,\sqrt{i\sinh\left(bx+a\right)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh\left(bx+a\right)},\frac{\sqrt{2}}{2}\right)\sinh\left(bx+\right.}{3\sinh\left(bx+a\right)^{\frac{3}{2}}\cosh\left(bx+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(5/2),x)
```

```
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(I*sinh(b*x+a)+1)
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sinh\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^(5/2),x)
```

```
[Out] int(1/sinh(a + b*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^{\frac{5}{2}}{\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-5/2), x)
```

3.14 $\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=103

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}+\frac{6i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2/5*cosh(b*x+a)/b/sinh(b*x+a)^(5/2)+6/5*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)-6/
5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}+\frac{6i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-7/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(5*b*Sinh[a + b*x]^(5/2)) + (6*Cosh[a + b*x])/(5*b*Sqrt[
Sinh[a + b*x]]) + (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh
[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}-\frac{3}{5}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx\\ &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}-\frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx\\ &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}-\frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}+\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 73, normalized size = 0.71

$$\frac{3\sinh(2(a+bx))-2\coth(a+bx)+6i(i\sinh(a+bx))^{3/2}E\left(\frac{1}{4}(-2ia-2ibx+\pi)\middle|2\right)}{5b\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-7/2),x]
[Out] (-2*Coth[a + b*x] + (6*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Si
nh[a + b*x])^(3/2) + 3*Sinh[2*(a + b*x)])/(5*b*Sinh[a + b*x]^(3/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{\sinh\left(bx+a\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="fricas")
[Out] integral(sinh(b*x + a)^(-7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

maple [A] time = 0.07, size = 192, normalized size = 1.86

$$-\frac{6\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(bx+a\right)+i\right)}\,\sqrt{i\sinh\left(bx+a\right)}\,\left(\sinh^{2}\left(bx+a\right)\right)\mathrm{EllipticE}\left(\sqrt{-i\left(\sinh\left(b\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(7/2),x)
```

```
[Out] -1/5/sinh(b*x+a)^(5/2)*(6*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x
+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticE((-I*(sinh(b*x+a
)+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*
x+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticF((-I*(sinh(b*x+
a)+I))^(1/2),1/2*2^(1/2))-6*sinh(b*x+a)^4-4*sinh(b*x+a)^2+2)/cosh(b*x+a)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sinh\left(a+b\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^(7/2),x)
```

```
[Out] int(1/sinh(a + b*x)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(7/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-7/2), x)
```

3.15 $\int (b\sinh(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$-\frac{10ib^4\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d\sqrt{b\sinh(c+dx)}}-\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d}+\frac{2b\cosh(c+dx)(b\sinh(c+d}{7d}$$

```
[Out] 2/7*b*cosh(d*x+c)*(b*sinh(d*x+c))^(5/2)/d+10/21*I*b^4*(sin(1/2*I*c+1/4*Pi+1
/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*
Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/d/(b*sinh(d*x+c))^(1/2)-10/21*
b^3*cosh(d*x+c)*(b*sinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2642, 2641}

$$-\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d}-\frac{10ib^4\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d\sqrt{b\sinh(c+dx)}}+\frac{2b\cosh(c+dx)(b\sinh(c+d}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*b^4*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]
])/(d*Sqrt[b*Sinh[c + d*x]]) - (10*b^3*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])
/(21*d) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(5/2))/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

d}, x]

Rubi steps

$$\int (b\sinh(c+dx))^{7/2}\,dx = \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} - \frac{1}{7}\left(5b^2\right)\int (b\sinh(c+dx))^{3/2}\,dx$$
$$= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} + \frac{1}{21}\left(5b^4\right.$$
$$= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} + \frac{\left(5b^4\sqrt{i\,s\right.}{}$$
$$= -\frac{10ib^4F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{21d\sqrt{b\sinh(c+dx)}} - \frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d}$$

Mathematica [A] time = 0.28, size = 76, normalized size = 0.66

$$\frac{b^3\sqrt{b\sinh(c+dx)}\left(-23\cosh(c+dx)+3\cosh(3(c+dx))-\frac{20F\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)}{\sqrt{i\sinh(c+dx)}}\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(7/2),x]
[Out] (b^3*(-23*Cosh[c + d*x] + 3*Cosh[3*(c + d*x)] - (20*EllipticF[((-2*I)*c + P
i - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]])*Sqrt[b*Sinh[c + d*x]])/(42*d)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh(dx+c)}\,b^3\sinh(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sinh(d*x + c))*b^3*sinh(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

maple [A] time = 0.07, size = 122, normalized size = 1.05

$$\frac{b^4\left(5i\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)+6\sinh\right.}{21\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(7/2),x)
```

```
[Out] 1/21*b^4*(5*I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+6*sinh(d*x+
c)*cosh(d*x+c)^4-16*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(b\sinh(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(b\sinh(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(7/2),x)
```

```
[Out] int((b*sinh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.16 $\int (b\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}+\frac{6ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}$$

```
[Out] 2/5*b*cosh(d*x+c)*(b*sinh(d*x+c))^(3/2)/d-6/5*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2
*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi
+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2640, 2639}

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}+\frac{6ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((6*I)/5)*b^2*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/
(d*Sqrt[I*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(3/2))/(5*
d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int (b\sinh(c+dx))^{5/2}\,dx = \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d} - \frac{1}{5}\left(3b^2\right)\int \sqrt{b\sinh(c+dx)}\,dx$$
$$= \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d} - \frac{\left(3b^2\sqrt{b\sinh(c+dx)}\right)\int \sqrt{i\sinh(c+dx)}\,dx}{5\sqrt{i\sinh(c+dx)}}$$
$$= \frac{6ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}$$

Mathematica [A] time = 0.12, size = 68, normalized size = 0.77

$$\frac{b^2\sqrt{b\sinh(c+dx)}\left(\sinh(2(c+dx)) - \frac{6iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)}{\sqrt{i\sinh(c+dx)}}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sinh[c + d*x]]*(((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4
, 2])/Sqrt[I*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(5*d)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh(dx+c)}\,b^2\sinh(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))*b^2*sinh(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

maple [A] time = 0.06, size = 170, normalized size = 1.93

$$-\frac{b^3\left(6\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh\left(dx+c\right)},\frac{\sqrt{2}}{2}\right)-3\sqrt{}\right.}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(5/2),x)
```

```
[Out] -1/5*b^3*(6*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-I*sinh(d
*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*Elliptic
F((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^4+2*cosh(d*x+c)^2)/cos
h(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(5/2),x)
```

```
[Out] int((b*sinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(5/2), x)
```

3.17 $\int (b\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=88

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{b\sinh(c+dx)}}$$

[Out] -2/3*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/d/(b*sinh(d*x+c))^(1/2)+2/3*b*cosh(d*x+c)*(b*sinh(d*x+c))^(1/2)/d

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2642, 2641}

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Sinh[c + d*x])^(3/2),x]

[Out] (((2*I)/3)*b^2*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt[b*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])/(3*d)

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (b\sinh(c+dx))^{3/2}\,dx = \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d} - \frac{1}{3}b^2\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx$$
$$= \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d} - \frac{\left(b^2\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3\sqrt{b\sinh(c+dx)}}$$
$$= \frac{2ib^2F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3d\sqrt{b\sinh(c+dx)}} + \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}$$

Mathematica [C] time = 0.13, size = 88, normalized size = 1.00

$$\frac{b^2\left(\sinh(2(c+dx)) - 2\sqrt{-\sinh(2c+2dx)-\cosh(2c+2dx)+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)\right)}{3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (b^2*(Sinh[2*(c + d*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cosh[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/
(3*d*Sqrt[b*Sinh[c + d*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh(dx+c)}\,b\sinh(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))*b*sinh(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

maple [A] time = 0.07, size = 106, normalized size = 1.20

$$-\frac{b^2\left(i\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh\left(dx+c\right)},\frac{\sqrt{2}}{2}\right)-2\left(\cos\right.\right.}{3\cosh\left(dx+c\right)\sqrt{b\sinh\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(3/2),x)
```

```
[Out] -1/3*b^2*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)
^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(3/2),x)
```

```
[Out] int((b*sinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(3/2), x)
```

3.18 $\int \sqrt{b \sinh(c+dx)}\, dx$

Optimal. Leaf size=56

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/d/(I*
sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2640, 2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(d*Sqrt
[I*Sinh[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \sqrt{b\sinh(c+dx)}\,dx = \frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{i\sinh(c+dx)}}$$
$$= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

Mathematica [**A**] time = 0.05, size = 52, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[b*Sinh[c + d*x]])/(
d*Sqrt[I*Sinh[c + d*x]])
```

fricas [**F**] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c)), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

maple [**A**] time = 0.06, size = 111, normalized size = 1.98

$$\frac{b\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(dx+c)+i)}\,\sqrt{i\sinh(dx+c)}\,\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\right.}{\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(1/2),x)
```

```
[Out] b*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*x+c)+I))^(1/2)*(I*sinh(d*
x+c))^(1/2)*(2*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-
I*sinh(d*x+c))^(1/2),1/2*2^(1/2)))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \sinh(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(1/2),x)
```

```
[Out] int((b*sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sinh(c + d*x)), x)
```

3.19 $\int \frac{1}{\sqrt{b \sinh(c+dx)}} \, dx$

Optimal. Leaf size=56

$$-\frac{2i\sqrt{i \sinh(c+dx)}\, F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{b \sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/d/(b*
sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{i \sinh(c+dx)}\, F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{b \sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt
[b*Sinh[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{\sqrt{b\sinh(c+dx)}}$$
$$= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.96

$$\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)}{d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sinh[c + d*x]],x]
[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt[
b*Sinh[c + d*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(dx+c\right)}}{b\sinh\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sinh(d*x + c))/(b*sinh(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

maple [A] time = 0.05, size = 89, normalized size = 1.59

$$\frac{i\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(dx+c\right)+i\right)}\,\sqrt{i\sinh\left(dx+c\right)}\;\text{EllipticF}\left(\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}, \frac{\sqrt{2}}{2}\right)}{\cosh\left(dx+c\right)\sqrt{b\sinh\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(1/2),x)
```

```
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*x+c)+I))^(1/2)*(I*sinh(d*
x+c))^(1/2)*EllipticF((-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/(
b*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sinh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sinh(c + d*x)), x)
```

3.20 $\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(1/2)+2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*
d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/b^2/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(b*d*Sqrt[b*Sinh[c + d*x]]) - ((2*I)*EllipticE[(I*c - Pi
/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(b^2*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} + \frac{\int \sqrt{b\sinh(c+dx)}\,dx}{b^2}$$
$$= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} + \frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}\,dx}{b^2\sqrt{i\sinh(c+dx)}}$$
$$= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} - \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 0.72

$$-\frac{2\left(\cosh(c+dx)-\sqrt{i\sinh(c+dx)}\,E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{bd\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sin
h[c + d*x]]))/(b*d*Sqrt[b*Sinh[c + d*x]])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh(dx+c)}}{b^2\sinh(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))/(b^2*sinh(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

maple [A] time = 0.07, size = 159, normalized size = 1.85

$$\frac{2\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\,\mathrm{si}}}{b\cosh(dx+c)\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(3/2),x)
```

```
[Out] (2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^
(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(1/2
)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh
(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/b/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\sinh(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-3/2), x)
```

3.21 $\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}+\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

```
[Out] -2/3*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(3/2)-2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*
d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/
2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/b^2/d/(b*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2642, 2641}

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}+\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(3*b*d*(b*Sinh[c + d*x])^(3/2)) + (((2*I)/3)*EllipticF[(
I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d*Sqrt[b*Sinh[c + d*x
]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx}{3b^2}$$
$$= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3b^2\sqrt{b\sinh(c+dx)}}$$
$$= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} + \frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Mathematica [C] time = 0.10, size = 84, normalized size = 0.93

$$-\frac{2\left(\sqrt{2}\sqrt{-\left(\sinh^2(c+dx)(\coth(c+dx)+1)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)+\coth(c+dx)\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d
*x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3
*b^2*d*Sqrt[b*Sinh[c + d*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh(dx+c)}}{b^3\sinh(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))/(b^3*sinh(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

maple [A] time = 0.07, size = 114, normalized size = 1.27

$$-\frac{i\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)\sinh(dx+}{3b^2\sinh(dx+c)\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(5/2),x)
```

```
[Out] -1/3/b^2/sinh(d*x+c)*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(
1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*s
inh(d*x+c)+2*cosh(d*x+c)^2)/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\sinh(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(5/2),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-5/2), x)
```

3.22 $\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}+\frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}}-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

```
[Out] -2/5*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(5/2)+6/5*cosh(d*x+c)/b^3/d/(b*sinh(d*
x+c))^(1/2)-6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*P
i+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c
))^(1/2)/b^4/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2640, 2639}

$$\frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}}+\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(5*b*d*(b*Sinh[c + d*x])^(5/2)) + (6*Cosh[c + d*x])/(5*b
^3*d*Sqrt[b*Sinh[c + d*x]]) + (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2,
2]*Sqrt[b*Sinh[c + d*x]])/(b^4*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} - \frac{3\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx}{5b^2}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{3\int \sqrt{b\sinh(c+dx)}\,dx}{5b^4}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{\left(3\sqrt{b\sinh(c+dx)}\right)\int \sqrt{i\sinh(c+}}{5b^4\sqrt{i\sinh(c+dx)}}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(}}{5b^4d\sqrt{i\sinh(c+dx)}}$$

Mathematica [A] time = 0.17, size = 79, normalized size = 0.67

$$-\frac{2\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3\sqrt{i\sinh(c+dx)}\,E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{5b^3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[((-2*I)*c
 + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(5*b^3*d*Sqrt[b*Sinh[c + d
*x]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh(dx+c)}}{b^4\sinh(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(d*x + c))/(b^4*sinh(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

maple [A] time = 0.07, size = 205, normalized size = 1.74

$$-\frac{6\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(dx+c\right)+i\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\left(\sinh^{2}\left(dx+c\right)\right)\mathrm{EllipticE}\left(\sqrt{-i\left(\sinh\left(\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(7/2),x)
```

```
[Out] -1/5/b^3/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*x
+c)+I))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c
)+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*
x+c)+I))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+
c)+I))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(b
*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-7/2), x)
```

3.23 $\int (i\sinh(c+dx))^{7/2}\,dx$

Optimal. Leaf size=91

$$-\frac{10iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d}+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}+\frac{10i\sqrt{i\sinh(c+dx)}\,\cosh(c+dx)}{21d}$$

```
[Out] 10/21*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*
x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/7*I*cosh(d*x+c)*(I*
sinh(d*x+c))^(5/2)/d+10/21*I*cosh(d*x+c)*(I*sinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 2641}

$$-\frac{10iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d}+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}+\frac{10i\sqrt{i\sinh(c+dx)}\,\cosh(c+dx)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((10*I)/21)*Cosh[c
 + d*x]*Sqrt[I*Sinh[c + d*x]])/d + (((2*I)/7)*Cosh[c + d*x]*(I*Sinh[c + d*x
])^(5/2))/d
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (i\sinh(c+dx))^{7/2}\,dx = \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d} + \frac{5}{7}\int (i\sinh(c+dx))^{3/2}\,dx$$
$$= \frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d} + \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d} + \frac{5}{21}\int \frac{1}{\sqrt{i\sinh(}$$
$$= -\frac{10iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{21d} + \frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d} + \frac{2i\cosh(c+dx)(i\sin}{7d}$$

Mathematica [A] time = 0.17, size = 65, normalized size = 0.71

$$\frac{i\left(20F\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)+\sqrt{i\sinh(c+dx)}\,(23\cosh(c+dx)-3\cosh(3(c+dx)))\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(7/2),x]
```

```
[Out] ((I/42)*(20*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + (23*Cosh[c + d*x]
 - 3*Cosh[3*(c + d*x)])*Sqrt[I*Sinh[c + d*x]]))/d
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\frac{\left(\sqrt{\frac{1}{2}}\left(-3i\,e^{(6\,dx+6\,c)}+23i\,e^{(4\,dx+4\,c)}+23i\,e^{(2\,dx+2\,c)}-3i\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+84\,de^{(3\,dx+3\,c)}\text{integral}\left(-\frac{10i\,\sqrt{\frac{1}{2}}}{}\right.\right.}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(sqrt(1/2)*(-3*I*e^(6*d*x + 6*c) + 23*I*e^(4*d*x + 4*c) + 23*I*e^(2*d*
x + 2*c) - 3*I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 84*d*e^(
3*d*x + 3*c)*integral(-10/21*I*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/
2*d*x - 1/2*c)/(d*e^(2*d*x + 2*c) - d), x))*e^(-3*d*x - 3*c)/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,\sinh(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

maple [A] time = 0.07, size = 122, normalized size = 1.34

$$-\frac{i\left(6i\sinh\left(dx+c\right)\left(\cosh^{4}\left(dx+c\right)\right)-5\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\mathrm{EllipticF}}{21\cosh\left(dx+c\right)\sqrt{i\sinh\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(7/2),x)
```

```
[Out] -1/21*I*(6*I*sinh(d*x+c)*cosh(d*x+c)^4-5*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1
+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/
2),1/2*2^(1/2))-16*I*sinh(d*x+c)*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\ \sinh\left(dx+c\right))^{\frac{7}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (\sinh\left(c+d\,x\right)\,1\mathrm{i})^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(7/2),x)
```

```
[Out] int((sinh(c + d*x)*1i)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.24 $\int (i\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=62

$$\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}-\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}$$

```
[Out] 6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c)*(I*si
nh(d*x+c))^(3/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 2639}

$$\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}-\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d
*x]*(I*Sinh[c + d*x])^(3/2))/d
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (i\sinh(c+dx))^{5/2}\,dx&=\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}+\frac{3}{5}\int\sqrt{i\sinh(c+dx)}\,dx\\&=-\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.89

$$\frac{6iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)-\sqrt{i\sinh(c+dx)}\sinh(2(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] - Sqrt[I*Sinh[c + d*x]]*
Sinh[2*(c + d*x)])/(5*d)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$-\frac{\sqrt{\frac{1}{2}}\left(e^{(5\,dx+5\,c)}-2\,e^{(4\,dx+4\,c)}-12\,e^{(3\,dx+3\,c)}-24\,e^{(2\,dx+2\,c)}-e^{(dx+c)}+2\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-10\left(de^{(3\,dx+3\,c)}\right.}{10\left(de^{(3\,dx+3\,c)}-2\,de^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(1/2)*(e^(5*d*x + 5*c) - 2*e^(4*d*x + 4*c) - 12*e^(3*d*x + 3*c)
- 24*e^(2*d*x + 2*c) - e^(d*x + c) + 2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2
*d*x - 1/2*c) - 10*(d*e^(3*d*x + 3*c) - 2*d*e^(2*d*x + 2*c))*integral(6/5*s
qrt(1/2)*(2*e^(2*d*x + 2*c) + 3*e^(d*x + c) - 2)*sqrt(I*e^(2*d*x + 2*c) - I
)*e^(-1/2*d*x - 1/2*c)/(d*e^(4*d*x + 4*c) - 4*d*e^(3*d*x + 3*c) + 3*d*e^(2*
d*x + 2*c) + 4*d*e^(d*x + c) - 4*d), x))/(d*e^(3*d*x + 3*c) - 2*d*e^(2*d*x
+ 2*c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 169, normalized size = 2.73

$$\frac{i\left(6\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh\left(dx+c\right)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\right.}{5\operatorname{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(5/2),x)
```

```
[Out] 1/5*I*(6*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*
x+c))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-I*sinh(d*x+
c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((
1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^4+2*cosh(d*x+c)^2)/cosh(d
*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i \sinh(dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\sinh(c+d\,x)\,1\mathrm{i})^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int((sinh(c + d*x)*1i)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i \sinh(c+dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(5/2), x)
```

3.25 $\int (i \sinh(c+dx))^{3/2}\, dx$

Optimal. Leaf size=62

$$\frac{2i\sqrt{i\sinh(c+dx)}\,\cosh(c+dx)}{3d}-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

```
[Out] 2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/3*I*cosh(d*x+c)*(I*si
nh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 2641}

$$\frac{2i\sqrt{i\sinh(c+dx)}\,\cosh(c+dx)}{3d}-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x]*Sqrt[I*Sinh[c + d*x]])/d
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (i\sinh(c+dx))^{3/2}\,dx = \frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d} + \frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx$$
$$= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d} + \frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}$$

Mathematica [C] time = 0.14, size = 94, normalized size = 1.52

$$-\frac{2i\sqrt{i\sinh(c+dx)}\left(\operatorname{csch}(c+dx)\sqrt{-\sinh(2c+2dx)-\cosh(2c+2dx)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[I*Sinh[c + d*x]]*(-Cosh[c + d*x] + Csch[c + d*x]*Hypergeom
etric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cos
h[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/d
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\frac{\left(\sqrt{\frac{1}{2}}\left(ie^{(2\,dx+2\,c)}+i\right)\sqrt{ie^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+3\,de^{(dx+c)}\mathrm{integral}\left(-\frac{2i\,\sqrt{\frac{1}{2}}\,\sqrt{ie^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{3\left(de^{(2\,dx+2\,c)}-d\right)},x\right)\right)e^{(-dx-c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(1/2)*(I*e^(2*d*x + 2*c) + I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*
d*x - 1/2*c) + 3*d*e^(d*x + c)*integral(-2/3*I*sqrt(1/2)*sqrt(I*e^(2*d*x +
2*c) - I)*e^(-1/2*d*x - 1/2*c)/(d*e^(2*d*x + 2*c) - d), x))*e^(-d*x - c)/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,\sinh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

maple [A] time = 0.07, size = 104, normalized size = 1.68

$$\frac{i\left(\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh\left(dx+c\right)},\frac{\sqrt{2}}{2}\right)+2i\sinh\right.}{3\cosh\left(dx+c\right)\sqrt{i\sinh\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(3/2),x)
```

```
[Out] 1/3*I*((1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+
c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+2*I*sinh(d*x+c)*co
sh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(i\,\sinh\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((sinh(c + d*x)*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(i\sinh\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(3/2), x)
```

3.26 $\int \sqrt{i \sinh (c+d x)}\, d x$

Optimal. Leaf size=30

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2639}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{i \sinh (c+dx)}\, dx = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[(Pi/2 - I*(c + d*x))/2, 2])/d
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\frac{2\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}\left(e^{(dx+c)}+2\right)e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}+\left(de^{(dx+c)}-2d\right)\text{integral}\left(\frac{2\sqrt{\frac{1}{2}}\left(2e^{(2dx+2c)}+3e^{(dx+c)}-2\right)\sqrt{ie^{(2dx+2c)}-i}e^{\left(-\frac{1}{2}dx\right.}}{de^{(4dx+4c)}-4de^{(3dx+3c)}+3de^{(2dx+2c)}+4de^{(dx+c)}-4}\right.}{de^{(dx+c)}-2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (2*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*(e^(d*x + c) + 2)*e^(-1/2*d*x - 1/
2*c) + (d*e^(d*x + c) - 2*d)*integral(2*sqrt(1/2)*(2*e^(2*d*x + 2*c) + 3*e^
(d*x + c) - 2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c)/(d*e^(4*d*x
 + 4*c) - 4*d*e^(3*d*x + 3*c) + 3*d*e^(2*d*x + 2*c) + 4*d*e^(d*x + c) - 4*d
), x))/(d*e^(d*x + c) - 2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*sinh(d*x + c)), x)
```

maple [A] time = 0.07, size = 91, normalized size = 3.03

$$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(dx+c)+i)}\,\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{1-\right.\right.}{\cosh(dx+c)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(1/2),x)
```

```
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*x+c)+I))^(1/2)*(2*Ellipti
cE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(d*x+c))^(1/2),1
/2*2^(1/2)))/cosh(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i\sinh(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*sinh(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\sinh\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(1/2),x)
[Out] int((sinh(c + d*x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i \sinh\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*sinh(c + d*x)), x)
```

3.27 $\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2641}

$$-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.93

$$\frac{2iF\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2])/d
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{de^{(2\,dx+2\,c)}-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c)/(d
*e^(2*d*x + 2*c) - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{i\,\sinh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

maple [A] time = 0.06, size = 68, normalized size = 2.27

$$\frac{i\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(dx+c\right)+i\right)}\,\operatorname{EllipticF}\left(\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)},\frac{\sqrt{2}}{2}\right)}{\cosh\left(dx+c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(1/2),x)
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*x+c)+I))^(1/2)*EllipticF(
(-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{i\,\sinh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sinh\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(1/(sinh(c + d*x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{i\sinh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*sinh(c + d*x)), x)
```

3.28 $\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*
EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2*I*cosh(d*x+c)/d/(I*sin
h(d*x+c))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2636, 2639}

$$\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + ((2*I)*Cosh[c + d*x])/(d*S
qrt[I*Sinh[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}} - \int \sqrt{i\sinh(c+dx)}\,dx$$
$$= \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d} + \frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

Mathematica [A] time = 0.10, size = 50, normalized size = 0.86

$$\frac{2\left(\sqrt{i\sinh(c+dx)}\coth(c+dx) - iE\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (2*((-I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + Coth[c + d*x]*Sqrt[I
*Sinh[c + d*x]]))/d
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\frac{4\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)} + \left(de^{(2\,dx+2\,c)}-d\right)\mathrm{integral}\left(-\frac{2\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{de^{(2\,dx+2\,c)}-d}, x\right)}{de^{(2\,dx+2\,c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (4*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(3/2*d*x + 3/2*c) + (d*e^(2*d*x
+ 2*c) - d)*integral(-2*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(1/2*d*x +
1/2*c)/(d*e^(2*d*x + 2*c) - d), x))/(d*e^(2*d*x + 2*c) - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,\sinh(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

maple [A] time = 0.08, size = 159, normalized size = 2.74

$$-\frac{i\left(2\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh\left(dx+c\right)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i}\right.}{\cosh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(3/2),x)
```

```
[Out] -I*(2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c
))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(
1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*s
inh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\ \sinh\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\sinh\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(1/(sinh(c + d*x)*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\sinh\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-3/2), x)
```

3.29 $\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=62

$$\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

```
[Out] 2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/3*I*cosh(d*x+c)/d/(I*
sinh(d*x+c))^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2636, 2641}

$$\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x])/(d*(I*Sinh[c + d*x])^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}} + \frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx$$
$$= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d} + \frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 1.34

$$\frac{2\left(\sqrt{2}\sqrt{-\left(\sinh^2(c+dx)(\coth(c+dx)+1)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)+\coth(c+dx)\right)}{3d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3*
d*Sqrt[I*Sinh[c + d*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\frac{\sqrt{\frac{1}{2}}\left(-4i\,e^{(3\,dx+3\,c)}-4i\,e^{(dx+c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+3\left(de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d\right)\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+}}}{3\left(de^{(2}\right.}\right.}{3\left(de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(1/2)*(-4*I*e^(3*d*x + 3*c) - 4*I*e^(d*x + c))*sqrt(I*e^(2*d*x + 2
*c) - I)*e^(-1/2*d*x - 1/2*c) + 3*(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c)
+ d)*integral(-2/3*I*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/
2*c)/(d*e^(2*d*x + 2*c) - d), x))/(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c)
+ d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^(-5/2), x)
```

maple [A] time = 0.07, size = 113, normalized size = 1.82

$$-\frac{i\left(-\sqrt{1-i\sinh\left(dx+c\right)}\ \sqrt{2}\ \sqrt{1+i\sinh\left(dx+c\right)}\ \sqrt{i\sinh\left(dx+c\right)}\ \mathrm{EllipticF}\left(\sqrt{1-i\sinh\left(dx+c\right)},\frac{\sqrt{2}}{2}\right)\sinh\left(d\right.}{3\sinh\left(dx+c\right)\cosh\left(dx+c\right)\sqrt{i\sinh\left(dx+c\right)}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(5/2),x)
[Out] -1/3*I/sinh(d*x+c)*(-(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2
)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*sinh
(d*x+c)+2*I*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\ \sinh\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(5/2),x)
[Out] int(1/(sinh(c + d*x)*1i)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\sinh\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(5/2),x)
[Out] Integral((I*sinh(c + d*x))**(-5/2), x)
```

3.30 $\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

```
[Out] -6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x
)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c)/d/(I
*sinh(d*x+c))^(5/2)+6/5*I*cosh(d*x+c)/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2636, 2639}

$$\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d*
x])/(d*(I*Sinh[c + d*x])^(5/2)) + (((6*I)/5)*Cosh[c + d*x])/(d*Sqrt[I*Sinh[
c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{3}{5}\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx$$
$$= \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}} - \frac{3}{5}\int \sqrt{i\sinh(c+dx)}\,dx$$
$$= \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d} + \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}$$

Mathematica [A] time = 0.13, size = 80, normalized size = 0.88

$$-\frac{2i\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3\sqrt{i\sinh(c+dx)}\,E\left(\frac{1}{4}(-2ic-2idx+\pi)\middle|2\right)\right)}{5d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((-2*I)/5)*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[(
(-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(d*Sqrt[I*Sinh[c +
 d*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\frac{4\sqrt{\frac{1}{2}}\left(3\,e^{(6\,dx+6\,c)}-8\,e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+5\left(de^{(6\,dx+6\,c)}-3\,de^{(4\,dx+4\,c)}+3\,de^{(2\,dx+2\,c)}\right.}{5\left(de^{(6\,dx+6\,c)}-3\,de^{(4\,dx+4\,c)}+3\,de^{(2\,dx+2\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(4*sqrt(1/2)*(3*e^(6*d*x + 6*c) - 8*e^(4*d*x + 4*c) + e^(2*d*x + 2*c))*
sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 5*(d*e^(6*d*x + 6*c) - 3
*d*e^(4*d*x + 4*c) + 3*d*e^(2*d*x + 2*c) - d)*integral(-6/5*sqrt(1/2)*sqrt(
I*e^(2*d*x + 2*c) - I)*e^(1/2*d*x + 1/2*c)/(d*e^(2*d*x + 2*c) - d), x))/(d*
e^(6*d*x + 6*c) - 3*d*e^(4*d*x + 4*c) + 3*d*e^(2*d*x + 2*c) - d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,\sinh(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

maple [A] time = 0.08, size = 204, normalized size = 2.24

$$-\frac{i\left(6\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(dx+c\right)+i\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\left(\sinh^{2}\left(dx+c\right)\right)\mathrm{EllipticE}\left(\sqrt{-i\left(\sinh\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(7/2),x)
[Out] -1/5*I/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*x+c
)+I))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)+
I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(-sinh(d*x+
c)+I))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+c)
+I))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(I*s
inh(d*x+c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\,\sinh\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\sinh\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(7/2),x)
[Out] int(1/(sinh(c + d*x)*1i)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\sinh\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-7/2), x)
```

3.31 $\int (b\sinh(c+dx))^{4/3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)}{7bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/7*cosh(d*x+c)*hypergeom([1/2, 7/6],[13/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(7/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)}{7bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(4/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(7/3))/(7*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{4/3}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{7/3}}{7bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)(b\sinh(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(4/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(4/3)*Tanh[c + d*x])/(7*d)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sinh(dx+c))^{\frac{1}{3}}\, b\sinh(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3)*b*sinh(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(4/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int (b\sinh(c + d\,x))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(4/3),x)
```

```
[Out] int((b*sinh(c + d*x))^(4/3), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(c + dx))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(4/3), x)
```

3.32 $\int (b\sinh(c+dx))^{2/3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)}{5bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/5*cosh(d*x+c)*hypergeom([1/2, 5/6],[11/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(5/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)}{5bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(5/3))/(5*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{2/3}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{5/3}}{5bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)(b\sinh(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(2/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(2/3)*Tanh[c + d*x])/(5*d)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sinh(dx+c))^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(2/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int (b\sinh(c+d\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(2/3),x)
[Out] int((b*sinh(c + d*x))^(2/3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(c+dx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(2/3),x)
[Out] Integral((b*sinh(c + d*x))**(2/3), x)
```

3.33 $\int \sqrt[3]{b \sinh(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)}{4bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/4*cosh(d*x+c)*hypergeom([1/2, 2/3],[5/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^
(4/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)}{4bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(4/3))/(4*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}}{4bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]
*(b*Sinh[c + d*x])^(1/3)*Tanh[c + d*x])/(4*d)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sinh(dx+c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(1/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (b \sinh (c + d\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(1/3),x)
```

```
[Out] int((b*sinh(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \sinh (c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)}{2bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/2*cosh(d*x+c)*hypergeom([1/3, 1/2],[4/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^
(2/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)}{2bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(2/3))/(2*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}}{2bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)}{2d\sqrt[3]{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(2*d*(b*Sinh[c + d*x])^(1/3))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sinh(dx+c))^{\frac{2}{3}}}{b\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3)/(b*sinh(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(1/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sinh\left(dx+c\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(1/3),x)
[Out] int(1/(b*sinh(c + d*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b \sinh{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(1/3),x)
[Out] Integral((b*sinh(c + d*x))**(-1/3), x)
```

3.35 $\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cosh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3*cosh(d*x+c)*hypergeom([1/6, 1/2],[7/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^(1
/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cosh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(1/3))/(b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}}{bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)}{d(b\sinh(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-2/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(2/3))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\sinh(dx+c))^{\frac{1}{3}}}{b\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3)/(b*sinh(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(2/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\sinh\left(c+d\,x\right)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(2/3),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-2/3), x)
```

3.36 $\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\,\sqrt[3]{b\sinh(c+dx)}}$$

```
[Out] -3*cosh(d*x+c)*hypergeom([-1/6, 1/2],[5/6],-sinh(d*x+c)^2)/b/d/(b*sinh(d*x+
c))^(1/3)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\,\sqrt[3]{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Cosh[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^2])/(b*d
*Sqrt[Cosh[c + d*x]^2]*(b*Sinh[c + d*x])^(1/3))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\,\sqrt[3]{b\sinh(c+dx)}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{d(b\sinh(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^
2]*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(4/3))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b\sinh\left(dx+c\right)\right)^{\frac{2}{3}}}{b^{2}\sinh\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3)/(b^2*sinh(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(4/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b \sinh\left(c + d\,x\right)\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(4/3),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sinh\left(c + dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-4/3), x)
```

3.37 $\int (b \sinh(c+dx))^n \, dx$

Optimal. Leaf size=70

$$\frac{\cosh(c+dx)(b\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{bd(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(b*sinh(
d*x+c))^(1+n)/b/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2643}

$$\frac{\cosh(c+dx)(b\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{bd(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^n,x]
```

```
[Out] (Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x]^
2]*(b*Sinh[c + d*x])^(1 + n))/(b*d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^n \, dx = \frac{\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{1+n}}{bd(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\tanh(c+dx)(b\sinh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(b*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sinh(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^n, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^n,x)
```

```
[Out] int((b*sinh(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^n, x)
```

<u>**mupad**</u> **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (b\sinh (c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^n,x)
```

```
[Out] int((b*sinh(c + d*x))^n, x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh (c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**n,x)
```

```
[Out] Integral((b*sinh(c + d*x))**n, x)
```

3.38 $\int (i\sinh(c+dx))^n\,dx$

Optimal. Leaf size=72

$$-\frac{i\cosh(c+dx)(i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] -I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(I*si
nh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{i\cosh(c+dx)(i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^n,x]
```

```
[Out] ((-I)*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c +
d*x]^2]*(I*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (i\sinh(c+dx))^n\,dx = -\frac{i\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)(i\sinh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(I*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{1}{2}\left(i\,e^{(2\,dx+2\,c)}-i\right)e^{(-dx-c)}\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((1/2*(I*e^(2*d*x + 2*c) - I)*e^(-d*x - c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,\sinh(dx+c))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^n, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (i\sinh(dx+c))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^n,x)
```

```
[Out] int((I*sinh(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,\sinh(dx+c))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (\sinh(c + d\,x)\ 1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^n,x)
```

```
[Out] int((sinh(c + d*x)*1i)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i \sinh(c + dx))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**n,x)
```

```
[Out] Integral((I*sinh(c + d*x))**n, x)
```

3.39 $\int (-i\sinh(c+dx))^n\,dx$

Optimal. Leaf size=72

$$\frac{i\cosh(c+dx)(-i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(-I*si
nh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{i\cosh(c+dx)(-i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-I)*Sinh[c + d*x])^n,x]
```

```
[Out] (I*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x
]^2]*((-I)*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (-i\sinh(c+dx))^n\,dx=\frac{i\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)(-i\sinh(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-I)*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*((-I)*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{1}{2}\left(-i\,e^{(2\,dx+2\,c)}+i\right)e^{(-dx-c)}\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((1/2*(-I*e^(2*d*x + 2*c) + I)*e^(-d*x - c))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-i\,\sinh(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((-I*sinh(d*x + c))^n, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int(-i\sinh(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*sinh(d*x+c))^n,x)
```

```
[Out] int((-I*sinh(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-i\,\sinh(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((-I*sinh(d*x + c))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (-\sinh(c+d\,x)\,1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sinh(c + d*x)*1i)^n,x)
```

```
[Out] int((-sinh(c + d*x)*1i)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-i\sinh(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))**n,x)
```

```
[Out] Integral((-I*sinh(c + d*x))**n, x)
```

3.40 $\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=46

$$\frac{3ix}{2}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}-\frac{3}{2}i\sinh(x)\cosh(x)$$

```
[Out] 3/2*I*x-4*cosh(x)+4/3*cosh(x)^3-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)^3/(I+
sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2767, 2748, 2635, 8, 2633}

$$\frac{3ix}{2}+\frac{4\cosh^3(x)}{3}-4\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i}-\frac{3}{2}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((3*I)/2)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 - ((3*I)/2)*Cosh[x]*Sinh[x] - (Co
sh[x]*Sinh[x]^3)/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx &= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} + \int \sinh^2(x)(-3i+4\sinh(x))\,dx \\
&= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} - 3i\int \sinh^2(x)\,dx + 4\int \sinh^3(x)\,dx \\
&= -\frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} + \frac{3}{2}i\int 1\,dx - 4\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) \\
&= \frac{3ix}{2} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [B] time = 0.19, size = 134, normalized size = 2.91

$$\frac{\cosh(x)\left(i\sinh^{-1}(\sinh(x))(\sinh(x)+i) + 2\sinh^3(x)\sqrt{\cosh^2(x)} - i\sinh^2(x)\sqrt{\cosh^2(x)} - \sinh(x)\left(7\sqrt{\cosh^2(x)}\right.\right.}{6(\sinh(x)+i)\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*((-16*I)*(ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + Sqrt[Cosh[x]^2]) -
 (16*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + 7*Sqrt[Cosh[x]^2])*Sinh[x] - I*S
qrt[Cosh[x]^2]*Sinh[x]^2 + 2*Sqrt[Cosh[x]^2]*Sinh[x]^3 + I*ArcSinh[Sinh[x]]
*(I + Sinh[x])))/(6*Sqrt[Cosh[x]^2]*(I + Sinh[x]))
```

fricas [A] time = 0.62, size = 64, normalized size = 1.39

$$\frac{(36ix-21i)e^{(4x)} - 3(12x+23)e^{(3x)} + e^{(7x)} - 2ie^{(6x)} - 18e^{(5x)} - 18ie^{(2x)} - 2e^{x} + i}{24\left(e^{(4x)} + ie^{(3x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((36*I*x - 21*I)*e^(4*x) - 3*(12*x + 23)*e^(3*x) + e^(7*x) - 2*I*e^(6*
x) - 18*e^(5*x) - 18*I*e^(2*x) - 2*e^x + I)/(e^(4*x) + I*e^(3*x))
```

giac [A] time = 0.20, size = 50, normalized size = 1.09

$$\frac{3}{2}i\,x-\frac{\left(69\,e^{(3\,x)}+18i\,e^{(2\,x)}+2\,e^{x}-i\right)e^{(-3\,x)}}{24\,(e^{x}+i)}+\frac{1}{24}\,e^{(3\,x)}-\frac{1}{8}i\,e^{(2\,x)}-\frac{7}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 3/2*I*x - 1/24*(69*e^(3*x) + 18*I*e^(2*x) + 2*e^x - I)*e^(-3*x)/(e^x + I) +
 1/24*e^(3*x) - 1/8*I*e^(2*x) - 7/8*e^x
```

maple [B] time = 0.06, size = 138, normalized size = 3.00

$$-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+sinh(x)),x)
```

```
[Out] -3/2*I*ln(tanh(1/2*x)-1)+3/2/(tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)-1/2/(tan
h(1/2*x)-1)^2-1/2*I/(tanh(1/2*x)-1)^2-1/3/(tanh(1/2*x)-1)^3+3/2*I*ln(tanh(1
/2*x)+1)-1/2/(tanh(1/2*x)+1)^2+1/2*I/(tanh(1/2*x)+1)^2-3/2/(tanh(1/2*x)+1)-
1/2*I/(tanh(1/2*x)+1)+1/3/(tanh(1/2*x)+1)^3-2*I/(tanh(1/2*x)+I)
```

maxima [A] time = 0.32, size = 59, normalized size = 1.28

$$\frac{3}{2}i\,x-\frac{4\,e^{(-x)}-36i\,e^{(-2\,x)}+138\,e^{(-3\,x)}+2i}{16\left(-3i\,e^{(-3\,x)}+3\,e^{(-4\,x)}\right)}-\frac{7}{8}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x - 1/16*(4*e^(-x) - 36*I*e^(-2*x) + 138*e^(-3*x) + 2*I)/(-3*I*e^(-3*
x) + 3*e^(-4*x)) - 7/8*e^(-x) + 1/8*I*e^(-2*x) + 1/24*e^(-3*x)
```

mupad [B] time = 0.49, size = 50, normalized size = 1.09

$$\frac{x\,3\mathrm{i}}{2}-\frac{7\,\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8}-\frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{-3\,x}}{24}+\frac{\mathrm{e}^{3\,x}}{24}-\frac{7\,\mathrm{e}^{x}}{8}-\frac{2}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] (x*3i)/2 - (7*exp(-x))/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/2
4 + exp(3*x)/24 - (7*exp(x))/8 - 2/(exp(x) + 1i)
```

sympy [A] time = 0.20, size = 60, normalized size = 1.30

$$\frac{3ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}-\frac{7e^{x}}{8}-\frac{7e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}+\frac{2}{-e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+sinh(x)),x)
```

```
[Out] 3*I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 - 7*exp(x)/8 - 7*exp(-x)/8 + I*exp(-2*
x)/8 + exp(-3*x)/24 + 2/(-exp(x) - I)
```

3.41 $\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{3x}{2}-2i\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\sinh(x)+i}+\frac{3}{2}\sinh(x)\cosh(x)$$

```
[Out] -3/2*x-2*I*cosh(x)+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(I+sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2767, 2734}

$$-\frac{3x}{2}-2i\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\sinh(x)+i}+\frac{3}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (-3*x)/2 - (2*I)*Cosh[x] + (3*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^2)/(I +
 Sinh[x])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rubi steps

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)} + \int \sinh(x)(-2i+3\sinh(x))\,dx$$
$$= -\frac{3x}{2} - 2i\cosh(x) + \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}$$

Mathematica [A] time = 0.11, size = 41, normalized size = 1.14

$$\frac{1}{2}\cosh(x)\left(-\frac{3\sinh^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}} + \frac{\sinh^2(x) - i\sinh(x) + 4}{\sinh(x) + i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*((-3*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (4 - I*Sinh[x] + Sinh[x]^
2)/(I + Sinh[x])))/2
```

fricas [B] time = 0.61, size = 57, normalized size = 1.58

$$-\frac{4(3x-1)e^{(3x)} - (-12ix - 20i)e^{(2x)} - e^{(5x)} + 3i e^{(4x)} - 3e^x + i}{8e^{(3x)} + 8i e^{(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(4*(3*x - 1)*e^(3*x) - (-12*I*x - 20*I)*e^(2*x) - e^(5*x) + 3*I*e^(4*x) -
3*e^x + I)/(8*e^(3*x) + 8*I*e^(2*x))
```

giac [A] time = 0.25, size = 38, normalized size = 1.06

$$-\frac{3}{2}x - \frac{\left(20i e^{(2x)} - 3e^x + i\right)e^{(-2x)}}{8(e^x + i)} + \frac{1}{8}e^{(2x)} - \frac{1}{2}i e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/8*(20*I*e^(2*x) - 3*e^x + I)*e^(-2*x)/(e^x + I) + 1/8*e^(2*x) -
1/2*I*e^x
```

maple [B] time = 0.05, size = 93, normalized size = 2.58

$$\frac{1}{2\tanh\left(\frac{x}{2}\right)-2}+\frac{i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+sinh(x)),x)
```

```
[Out] 1/2/(tanh(1/2*x)-1)+I/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+3/2*ln(tanh(1/2
*x)-1)+1/2/(tanh(1/2*x)+1)-I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-3/2*ln(t
anh(1/2*x)+1)+2/(tanh(1/2*x)+I)
```

maxima [A] time = 0.44, size = 45, normalized size = 1.25

$$-\frac{3}{2}x-\frac{3e^{(-x)}+20ie^{(-2x)}+i}{8\left(-ie^{(-2x)}+e^{(-3x)}\right)}-\frac{1}{2}ie^{(-x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x - 1/8*(3*e^(-x) + 20*I*e^(-2*x) + I)/(-I*e^(-2*x) + e^(-3*x)) - 1/2*
I*e^(-x) - 1/8*e^(-2*x)
```

mupad [B] time = 0.43, size = 38, normalized size = 1.06

$$\frac{e^{2x}}{8}-\frac{e^{-x}\,1i}{2}-\frac{e^{-2x}}{8}-\frac{3x}{2}-\frac{e^{x}\,1i}{2}-\frac{2i}{e^{x}+1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] exp(2*x)/8 - (exp(-x)*1i)/2 - exp(-2*x)/8 - (3*x)/2 - (exp(x)*1i)/2 - 2i/(e
xp(x) + 1i)
```

sympy [A] time = 0.17, size = 41, normalized size = 1.14

$$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{ie^{x}}{2}-\frac{ie^{-x}}{2}-\frac{e^{-2x}}{8}+\frac{2}{ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+sinh(x)),x)
```

```
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x)/2 - I*exp(-x)/2 - exp(-2*x)/8 + 2/(I*exp(x)
- 1)
```

3.42 $\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=22

$$-ix+\cosh(x)+\frac{i\cosh(x)}{\sinh(x)+i}$$

```
[Out] -I*x+cosh(x)+I*cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 2735, 2648}

$$-ix+\cosh(x)+\frac{i\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I)*x + Cosh[x] + (I*Cosh[x])/(I + Sinh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = \cosh(x) - i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx$$
$$= -ix + \cosh(x) - \int \frac{1}{i+\sinh(x)}\,dx$$
$$= -ix + \cosh(x) + \frac{i\cosh(x)}{i+\sinh(x)}$$

Mathematica [B] time = 0.11, size = 79, normalized size = 3.59

$$\frac{\cosh(x)\left(\sinh(x) + \frac{2\sinh(x)\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}} + \frac{2i\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}} + 2i\right)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*(2*I + ((2*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2]
 + Sinh[x] + (2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sinh[x])/Sqrt[Cosh[x]^2
]))/(I + Sinh[x])
```

fricas [B] time = 0.42, size = 38, normalized size = 1.73

$$\frac{(-2i\,x+i)e^{(2\,x)} + (2\,x+5)e^{x} + e^{(3\,x)} + i}{2\,e^{(2\,x)} + 2i\,e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((-2*I*x + I)*e^(2*x) + (2*x + 5)*e^x + e^(3*x) + I)/(2*e^(2*x) + 2*I*e^x)
```

giac [A] time = 0.19, size = 26, normalized size = 1.18

$$-i\,x + \frac{(5\,e^{x}+i)e^{(-x)}}{2\,(e^{x}+i)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -I*x + 1/2*(5*e^x + I)*e^(-x)/(e^x + I) + 1/2*e^x
```

maple [B] time = 0.05, size = 52, normalized size = 2.36

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+sinh(x)),x)
```

```
[Out] I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)
+2*I/(tanh(1/2*x)+I)
```

maxima [B] time = 0.53, size = 33, normalized size = 1.50

$$-i\,x+\frac{10\,e^{(-x)}-2i}{4\left(-i\,e^{(-x)}+e^{(-2\,x)}\right)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/4*(10*e^(-x) - 2*I)/(-I*e^(-x) + e^(-2*x)) + 1/2*e^(-x)
```

mupad [B] time = 0.43, size = 24, normalized size = 1.09

$$\frac{\mathrm{e}^{-x}}{2}-x\,1\mathrm{i}+\frac{\mathrm{e}^{x}}{2}+\frac{2}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(sinh(x) + 1i),x)
```

```
[Out] exp(-x)/2 - x*1i + exp(x)/2 + 2/(exp(x) + 1i)
```

sympy [A] time = 0.13, size = 20, normalized size = 0.91

$$-ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{2}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+sinh(x)),x)
```

```
[Out] -I*x + exp(x)/2 + exp(-x)/2 + 2/(exp(x) + I)
```

3.43 $\int \frac{\sinh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=14

$$x-\frac{\cosh(x)}{\sinh(x)+i}$$

```
[Out] x-cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$x-\frac{\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(I + Sinh[x]),x]
```

```
[Out] x - Cosh[x]/(I + Sinh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{i+\sinh(x)}\,dx &= x-i\int \frac{1}{i+\sinh(x)}\,dx\\ &= x-\frac{\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.06, size = 43, normalized size = 3.07

$$i\operatorname{sech}(x)\left(i\sinh(x)+2\sqrt{\cosh^2(x)}\,\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Sinh[x]),x]
```

```
[Out] I*Sech[x]*(1 + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2] + I*Si
nh[x])
```

fricas [A] time = 1.02, size = 16, normalized size = 1.14

$$\frac{xe^{x} + ix + 2i}{e^{x} + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] (x*e^x + I*x + 2*I)/(e^x + I)
```

giac [A] time = 0.19, size = 10, normalized size = 0.71

$$x + \frac{2i}{e^{x} + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] x + 2*I/(e^x + I)
```

maple [B] time = 0.04, size = 29, normalized size = 2.07

$$-\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{2}{\tanh\left(\frac{x}{2}\right) + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+sinh(x)),x)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-2/(tanh(1/2*x)+I)
```

maxima [A] time = 0.59, size = 12, normalized size = 0.86

$$x + \frac{2i}{e^{(-x)} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] x + 2*I/(e^(-x) - I)
```

mupad [B] time = 0.41, size = 12, normalized size = 0.86

$$x+\frac{2\mathrm{i}}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(sinh(x) + 1i),x)
[Out] x + 2i/(exp(x) + 1i)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.57

$$x+\frac{2}{-ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x)
[Out] x + 2/(-I*exp(x) + 1)
```

3.44 $\int \frac{\operatorname{csch}(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=19

$$\frac{\cosh(x)}{\sinh(x)+i}+i\tanh^{-1}(\cosh(x))$$

```
[Out] I*arctanh(cosh(x))+cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2747, 2648, 3770}

$$\frac{\cosh(x)}{\sinh(x)+i}+i\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Sinh[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] + Cosh[x]/(I + Sinh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = -(i\int \operatorname{csch}(x)\,dx) + i\int \frac{1}{i+\sinh(x)}\,dx$$
$$= i\tanh^{-1}(\cosh(x)) + \frac{\cosh(x)}{i+\sinh(x)}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.58

$$\operatorname{sech}(x)\left(\sinh(x) + i\sqrt{\cosh^2(x)}\,\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right) - i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Sinh[x]),x]
[Out] Sech[x]*(-I + I*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + Sinh[x])
```

fricas [B] time = 0.53, size = 33, normalized size = 1.74

$$\frac{(i e^x - 1)\log(e^x + 1) + (-i e^x + 1)\log(e^x - 1) - 2i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] ((I*e^x - 1)*log(e^x + 1) + (-I*e^x + 1)*log(e^x - 1) - 2*I)/(e^x + I)
```

giac [A] time = 0.21, size = 24, normalized size = 1.26

$$-\frac{2i}{e^x + i} + i\,\log(e^x + 1) - i\,\log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="giac")
[Out] -2*I/(e^x + I) + I*log(e^x + 1) - I*log(abs(e^x - 1))
```

maple [A] time = 0.04, size = 21, normalized size = 1.11

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{2}{\tanh\left(\frac{x}{2}\right) + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+sinh(x)),x)
```

```
[Out] -I*ln(tanh(1/2*x))+2/(tanh(1/2*x)+I)
```

maxima [A] time = 0.36, size = 29, normalized size = 1.53

$$-\frac{2i}{e^{(-x)} - i} + i \log\left(e^{(-x)} + 1\right) - i \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*I/(e^(-x) - I) + I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

mupad [B] time = 0.49, size = 35, normalized size = 1.84

$$-\ln\left(e^x\, 2i - 2i\right)\, 1i + \ln\left(e^x\, 2i + 2i\right)\, 1i - \frac{2i}{e^x + 1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i - 2i/(exp(x) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{\sinh(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)/(sinh(x) + I), x)
```

3.45 $\int \frac{\text{csch}^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$2i\coth(x) - \tanh^{-1}(\cosh(x)) + \frac{\coth(x)}{\sinh(x) + i}$$

[Out] -arctanh(cosh(x))+2*I*coth(x)+coth(x)/(I+sinh(x))

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 8, 3770}

$$2i\coth(x) - \tanh^{-1}(\cosh(x)) + \frac{\coth(x)}{\sinh(x) + i}$$

Antiderivative was successfully verified.

[In] Int[Csch[x]^2/(I + Sinh[x]),x]

[Out] -ArcTanh[Cosh[x]] + (2*I)*Coth[x] + Coth[x]/(I + Sinh[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)}{i+\sinh(x)}+\int \operatorname{csch}^2(x)(-2i+\sinh(x))\,dx\\ &= \frac{\coth(x)}{i+\sinh(x)}-2i\int \operatorname{csch}^2(x)\,dx+\int \operatorname{csch}(x)\,dx\\ &= -\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{i+\sinh(x)}-2\,\mathrm{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)\\ &= -\tanh^{-1}(\cosh(x))+2i\coth(x)+\frac{\coth(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 1.57

$$\operatorname{sech}(x)\left(2i\sinh(x)+i\operatorname{csch}(x)-\sqrt{\cosh^2(x)}\,\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Sinh[x]),x]
```

```
[Out] Sech[x]*(1 - ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + I*Csch[x] + (2*I)*S
inh[x])
```

fricas [B] time = 0.49, size = 77, normalized size = 3.35

$$-\frac{\left(e^{(3x)}+i\,e^{(2x)}-e^x-i\right)\log\left(e^x+1\right)-\left(e^{(3x)}+i\,e^{(2x)}-e^x-i\right)\log\left(e^x-1\right)-2\,e^{(2x)}-2i\,e^x+4}{e^{(3x)}+i\,e^{(2x)}-e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -((e^(3*x) + I*e^(2*x) - e^x - I)*log(e^x + 1) - (e^(3*x) + I*e^(2*x) - e^x
 - I)*log(e^x - 1) - 2*e^(2*x) - 2*I*e^x + 4)/(e^(3*x) + I*e^(2*x) - e^x -
I)
```

giac [B] time = 0.31, size = 44, normalized size = 1.91

$$\frac{2\left(e^{(2x)} + i\,e^{x} - 2\right)}{e^{(3x)} + i\,e^{(2x)} - e^{x} - i} - \log\left(e^{x} + 1\right) + \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) + I*e^x - 2)/(e^(3*x) + I*e^(2*x) - e^x - I) - log(e^x + 1) + lo
g(abs(e^x - 1))
```

maple [A] time = 0.05, size = 35, normalized size = 1.52

$$\frac{i\tanh\left(\frac{x}{2}\right)}{2} + \frac{2i}{\tanh\left(\frac{x}{2}\right) + i} + \frac{i}{2\tanh\left(\frac{x}{2}\right)} + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+sinh(x)),x)
```

```
[Out] 1/2*I*tanh(1/2*x)+2*I/(tanh(1/2*x)+I)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))
```

maxima [B] time = 0.36, size = 53, normalized size = 2.30

$$-\frac{4\left(-i\,e^{(-x)} + e^{(-2x)} - 2\right)}{2\,e^{(-x)} + 2i\,e^{(-2x)} - 2\,e^{(-3x)} - 2i} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -4*(-I*e^(-x) + e^(-2*x) - 2)/(2*e^(-x) + 2*I*e^(-2*x) - 2*e^(-3*x) - 2*I)
- log(e^(-x) + 1) + log(e^(-x) - 1)
```

mupad [B] time = 0.64, size = 51, normalized size = 2.22

$$\ln\left(2 - 2\,\mathrm{e}^{x}\right) - \ln\left(-2\,\mathrm{e}^{x} - 2\right) + \frac{2\,\mathrm{e}^{2x} - 4 + \mathrm{e}^{x}\,2\mathrm{i}}{\mathrm{e}^{2x}\,1\mathrm{i} + \mathrm{e}^{3x} - \mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(sinh(x) + 1i)),x)
```

```
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + (2*exp(2*x) + exp(x)*2i - 4)/(exp
(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{\sinh(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)**2/(sinh(x) + I), x)
```

3.46 $\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=37

$$-2\coth(x)-\frac{3}{2}i\tanh^{-1}(\cosh(x))+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{\sinh(x)+i}$$

```
[Out] -3/2*I*arctanh(cosh(x))-2*coth(x)+3/2*I*coth(x)*csch(x)+coth(x)*csch(x)/(I+
sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$-2\coth(x)-\frac{3}{2}i\tanh^{-1}(\cosh(x))+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/2)*ArcTanh[Cosh[x]] - 2*Coth[x] + ((3*I)/2)*Coth[x]*Csch[x] + (Coth
[x]*Csch[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}+\int \operatorname{csch}^3(x)(-3i+2\sinh(x))\,dx\\ &= \frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}-3i\int \operatorname{csch}^3(x)\,dx+2\int \operatorname{csch}^2(x)\,dx\\ &= \frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}+\frac{3}{2}i\int \operatorname{csch}(x)\,dx-2i\operatorname{Subst}(\int 1\,dx,x,-i\coth(x))\\ &= -\frac{3}{2}i\tanh^{-1}(\cosh(x))-2\coth(x)+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.19, size = 49, normalized size = 1.32

$$\frac{1}{2}i\tanh(x)\left(\operatorname{csch}^3(x)+2i\operatorname{csch}^2(x)+3\operatorname{csch}(x)-3\sqrt{\cosh^2(x)}\operatorname{csch}(x)\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)+4i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Sinh[x]),x]
```

```
[Out] (I/2)*(4*I + 3*Csch[x] - 3*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2]*Csch[x]
 + (2*I)*Csch[x]^2 + Csch[x]^3)*Tanh[x]
```

fricas [B] time = 0.73, size = 129, normalized size = 3.49

$$\frac{\left(-3ie^{(5x)}+3e^{(4x)}+6ie^{(3x)}-6e^{(2x)}-3ie^{x}+3\right)\log\left(e^{x}+1\right)+\left(3ie^{(5x)}-3e^{(4x)}-6ie^{(3x)}+6e^{(2x)}+3ie^{x}-3\right)\log}{2e^{(5x)}+2ie^{(4x)}-4e^{(3x)}-4ie^{(2x)}+2e^{x}+2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((-3*I*e^(5*x) + 3*e^(4*x) + 6*I*e^(3*x) - 6*e^(2*x) - 3*I*e^x + 3)*log(e^x
 + 1) + (3*I*e^(5*x) - 3*e^(4*x) - 6*I*e^(3*x) + 6*e^(2*x) + 3*I*e^x - 3)*l
og(e^x - 1) + 6*I*e^(4*x) - 6*e^(3*x) - 10*I*e^(2*x) + 2*e^x + 8*I)/(2*e^(5
*x) + 2*I*e^(4*x) - 4*e^(3*x) - 4*I*e^(2*x) + 2*e^x + 2*I)
```

giac [A] time = 0.19, size = 51, normalized size = 1.38

$$\frac{i\,e^{(3\,x)} - 2\,e^{(2\,x)} + i\,e^{x} + 2}{\left(e^{(2\,x)} - 1\right)^{2}} + \frac{2i}{e^{x} + i} - \frac{3}{2}i\,\log\left(e^{x} + 1\right) + \frac{3}{2}i\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] (I*e^(3*x) - 2*e^(2*x) + I*e^x + 2)/(e^(2*x) - 1)^2 + 2*I/(e^x + I) - 3/2*I
*log(e^x + 1) + 3/2*I*log(abs(e^x - 1))
```

maple [A] time = 0.05, size = 53, normalized size = 1.43

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{i\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8} - \frac{2}{\tanh\left(\frac{x}{2}\right) + i} + \frac{i}{8\tanh\left(\frac{x}{2}\right)^{2}} + \frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+sinh(x)),x)
```

```
[Out] -1/2*tanh(1/2*x)-1/8*I*tanh(1/2*x)^2-2/(tanh(1/2*x)+I)+1/8*I/tanh(1/2*x)^2+
3/2*I*ln(tanh(1/2*x))-1/2/tanh(1/2*x)
```

maxima [B] time = 0.32, size = 79, normalized size = 2.14

$$-\frac{8\left(e^{(-x)} + 5i\,e^{(-2\,x)} - 3\,e^{(-3\,x)} - 3i\,e^{(-4\,x)} - 4i\right)}{8\,e^{(-x)} + 16i\,e^{(-2\,x)} - 16\,e^{(-3\,x)} - 8i\,e^{(-4\,x)} + 8\,e^{(-5\,x)} - 8i} - \frac{3}{2}i\,\log\left(e^{(-x)} + 1\right) + \frac{3}{2}i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -8*(e^(-x) + 5*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) - 4*I)/(8*e^(-x) + 16
*I*e^(-2*x) - 16*e^(-3*x) - 8*I*e^(-4*x) + 8*e^(-5*x) - 8*I) - 3/2*I*log(e^
(-x) + 1) + 3/2*I*log(e^(-x) - 1)
```

mupad [B] time = 0.66, size = 70, normalized size = 1.89

$$-\frac{\ln\left(-\mathrm{e}^{x}\,3\mathrm{i}-3\mathrm{i}\right)\,3\mathrm{i}}{2}+\frac{\ln\left(-\mathrm{e}^{x}\,3\mathrm{i}+3\mathrm{i}\right)\,3\mathrm{i}}{2}+\frac{2\mathrm{i}}{\mathrm{e}^{x}+1\mathrm{i}}+\frac{\mathrm{e}^{x}\,2\mathrm{i}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}+\frac{-2+\mathrm{e}^{x}\,1\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(sinh(x) + 1i)),x)
```

```
[Out] (log(3i - exp(x)*3i)*3i)/2 - (log(- exp(x)*3i - 3i)*3i)/2 + 2i/(exp(x) + 1i
) + (exp(x)*2i)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(x)*1i - 2)/(exp(2*x) - 1
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^{3}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)**3/(sinh(x) + I), x)
```

3.47 $\int \frac{\mathrm{csch}^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=47

$$\frac{4}{3}i\coth^3(x)-4i\coth(x)+\frac{3}{2}\tanh^{-1}(\cosh(x))-\frac{3}{2}\coth(x)\mathrm{csch}(x)+\frac{\coth(x)\mathrm{csch}^2(x)}{\sinh(x)+i}$$

```
[Out] 3/2*arctanh(cosh(x))-4*I*coth(x)+4/3*I*coth(x)^3-3/2*coth(x)*csch(x)+coth(x
)*csch(x)^2/(I+sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2768, 2748, 3767, 3768, 3770}

$$\frac{4}{3}i\coth^3(x)-4i\coth(x)+\frac{3}{2}\tanh^{-1}(\cosh(x))-\frac{3}{2}\coth(x)\mathrm{csch}(x)+\frac{\coth(x)\mathrm{csch}^2(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 - (4*I)*Coth[x] + ((4*I)/3)*Coth[x]^3 - (3*Coth[x]*C
sch[x])/2 + (Coth[x]*Csch[x]^2)/(I + Sinh[x])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}+\int \operatorname{csch}^4(x)(-4i+3\sinh(x))\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}-4i\int \operatorname{csch}^4(x)\,dx+3\int \operatorname{csch}^3(x)\,dx\\ &= -\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}-\frac{3}{2}\int \operatorname{csch}(x)\,dx+4\,\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth\right.\\ &= \frac{3}{2}\tanh^{-1}(\cosh(x))-4i\coth(x)+\frac{4}{3}i\coth^3(x)-\frac{3}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.24, size = 53, normalized size = 1.13

$$\frac{1}{6}\operatorname{sech}(x)\left(-16i\sinh(x)+2i\operatorname{csch}^3(x)-3\operatorname{csch}^2(x)-8i\operatorname{csch}(x)+9\sqrt{\cosh^2(x)}\,\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)-9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Sech[x]*(-9 + 9*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] - (8*I)*Csch[x] -
 3*Csch[x]^2 + (2*I)*Csch[x]^3 - (16*I)*Sinh[x]))/6
```

fricas [B] time = 1.16, size = 178, normalized size = 3.79

$$\frac{\left(9\,e^{(7\,x)}+9i\,e^{(6\,x)}-27\,e^{(5\,x)}-27i\,e^{(4\,x)}+27\,e^{(3\,x)}+27i\,e^{(2\,x)}-9\,e^{x}-9i\right)\log\left(e^{x}+1\right)-\left(9\,e^{(7\,x)}+9i\,e^{(6\,x)}-27\,e^{(5\,x)}-\right.}{6\,e^{(7\,x)}+6i\,e^{(6\,x)}-18\,e^{(5\,x)}-18i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((9*e^(7*x) + 9*I*e^(6*x) - 27*e^(5*x) - 27*I*e^(4*x) + 27*e^(3*x) + 27*I*e
^(2*x) - 9*e^x - 9*I)*log(e^x + 1) - (9*e^(7*x) + 9*I*e^(6*x) - 27*e^(5*x)
- 27*I*e^(4*x) + 27*e^(3*x) + 27*I*e^(2*x) - 9*e^x - 9*I)*log(e^x - 1) - 18
*e^(6*x) - 18*I*e^(5*x) + 48*e^(4*x) + 48*I*e^(3*x) - 78*e^(2*x) - 14*I*e^x
 + 32)/(6*e^(7*x) + 6*I*e^(6*x) - 18*e^(5*x) - 18*I*e^(4*x) + 18*e^(3*x) +
18*I*e^(2*x) - 6*e^x - 6*I)
```

giac [A] time = 0.44, size = 58, normalized size = 1.23

$$-\frac{2}{e^{x}+i}-\frac{3\,e^{(5\,x)}+6i\,e^{(4\,x)}-24i\,e^{(2\,x)}-3\,e^{x}+10i}{3\left(e^{(2\,x)}-1\right)^{3}}+\frac{3}{2}\,\log\left(e^{x}+1\right)-\frac{3}{2}\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -2/(e^x + I) - 1/3*(3*e^(5*x) + 6*I*e^(4*x) - 24*I*e^(2*x) - 3*e^x + 10*I)/
(e^(2*x) - 1)^3 + 3/2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

maple [A] time = 0.05, size = 71, normalized size = 1.51

$$-\frac{7i\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}-\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{7i}{8\tanh\left(\frac{x}{2}\right)}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+sinh(x)),x)
```

```
[Out] -7/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2-2*I/(tanh(1/2*x)+
I)+1/24*I/tanh(1/2*x)^3-7/8*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2-3/2*ln(tanh(1/2
*x))
```

maxima [B] time = 0.35, size = 105, normalized size = 2.23

$$\frac{16\left(-7i\,e^{(-x)}+39\,e^{(-2\,x)}+24i\,e^{(-3\,x)}-24\,e^{(-4\,x)}-9i\,e^{(-5\,x)}+9\,e^{(-6\,x)}-16\right)}{48\,e^{(-x)}+144i\,e^{(-2\,x)}-144\,e^{(-3\,x)}-144i\,e^{(-4\,x)}+144\,e^{(-5\,x)}+48i\,e^{(-6\,x)}-48\,e^{(-7\,x)}-48i}+\frac{3}{2}\,\log\left(e^{(-x)}+1\right)-\frac{3}{2}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 16*(-7*I*e^(-x) + 39*e^(-2*x) + 24*I*e^(-3*x) - 24*e^(-4*x) - 9*I*e^(-5*x)
+ 9*e^(-6*x) - 16)/(48*e^(-x) + 144*I*e^(-2*x) - 144*e^(-3*x) - 144*I*e^(-4
```

```
*x) + 144*e^(-5*x) + 48*I*e^(-6*x) - 48*e^(-7*x) - 48*I) + 3/2*log(e^(-x) +
 1) - 3/2*log(e^(-x) - 1)
```

mupad [B] time = 0.72, size = 85, normalized size = 1.81

$$\frac{3\ \ln\left(3\,\mathrm{e}^x+3\right)}{2}-\frac{3\ \ln\left(3\,\mathrm{e}^x-3\right)}{2}-\frac{\mathrm{e}^x}{\mathrm{e}^{2\,x}-1}-\frac{2\,\mathrm{e}^x}{{\left(\mathrm{e}^{2\,x}-1\right)}^2}-\frac{2}{\mathrm{e}^x+1{}\mathrm{i}}-\frac{2{}\mathrm{i}}{\mathrm{e}^{2\,x}-1}+\frac{4{}\mathrm{i}}{{\left(\mathrm{e}^{2\,x}-1\right)}^2}+\frac{8{}\mathrm{i}}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(sinh(x) + 1i)),x)
```

```
[Out] (3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1) -
 (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) + 1i) - 2i/(exp(2*x) - 1) + 4i/(ex
p(2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+sinh(x)),x)
```

```
[Out] Timed out
```

3.48 $\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=58

$$-\frac{7x}{2}-\frac{16}{3}i\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}-\frac{8\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)}+\frac{7}{2}\sinh(x)\cosh(x)$$

```
[Out] -7/2*x-16/3*I*cosh(x)+7/2*cosh(x)*sinh(x)-1/3*cosh(x)*sinh(x)^3/(I+sinh(x))
^2-8/3*cosh(x)*sinh(x)^2/(I+sinh(x))
```

Rubi [A] time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2765, 2977, 2734}

$$-\frac{7x}{2}-\frac{16}{3}i\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}-\frac{8\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)}+\frac{7}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-7*x)/2 - ((16*I)/3)*Cosh[x] + (7*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^3)
/(3*(I + Sinh[x])^2) - (8*Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx &= -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int \frac{\sinh^2(x)(-3i+5\sinh(x))}{i+\sinh(x)}\,dx\\ &= -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}-\frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}-\frac{1}{3}i\int (16+21i\sinh(x))\sinh(x)\,dx\\ &= -\frac{7x}{2}-\frac{16}{3}i\cosh(x)+\frac{7}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}-\frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [B] time = 0.19, size = 147, normalized size = 2.53

$$-\frac{\sinh^3(x)\cosh(x)}{2(1-i\sinh(x))^2}-\frac{i\sqrt{2}\sqrt{1+\frac{1}{2}(-1+i\sinh(x))}\cosh(x)}{\sqrt{1+i\sinh(x)}}-\frac{31i\cosh(x)}{6(1-i\sinh(x))}+\frac{5i\cosh(x)}{6(1-i\sinh(x))^2}-\frac{7i\cosh(x)\sin^{-1}\Big(\cdots}{\sqrt{1-i\sinh(x)}\sqrt{1\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (((5*I)/6)*Cosh[x])/(1 - I*Sinh[x])^2 - (((31*I)/6)*Cosh[x])/(1 - I*Sinh[x]
) - (I*Sqrt[2]*Cosh[x]*Sqrt[1 + (-1 + I*Sinh[x])/2])/Sqrt[1 + I*Sinh[x]] -
((7*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Cosh[x])/(Sqrt[1 - I*Sinh[x]]*Sq
rt[1 + I*Sinh[x]]) - (Cosh[x]*Sinh[x]^3)/(2*(1 - I*Sinh[x])^2)
```

fricas [B] time = 0.48, size = 91, normalized size = 1.57

$$-\frac{21\,(4\,x-3)e^{(5\,x)}-(-252i\,x-147i)e^{(4\,x)}-3\,(84\,x+127)e^{(3\,x)}-(84i\,x+239i)e^{(2\,x)}-3\,e^{(7\,x)}+15i\,e^{(6\,x)}+15\,e^{x}-3\cdots}{24\,e^{(5\,x)}+72i\,e^{(4\,x)}-72\,e^{(3\,x)}-24i\,e^{(2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(21*(4*x - 3)*e^(5*x) - (-252*I*x - 147*I)*e^(4*x) - 3*(84*x + 127)*e^(3*x
) - (84*I*x + 239*I)*e^(2*x) - 3*e^(7*x) + 15*I*e^(6*x) + 15*e^x - 3*I)/(24
*e^(5*x) + 72*I*e^(4*x) - 72*e^(3*x) - 24*I*e^(2*x))
```

giac [A] time = 0.16, size = 50, normalized size = 0.86

$$-\frac{7}{2}\,x-\frac{\left(216i\,e^{(4\,x)}-405\,e^{(3\,x)}-239i\,e^{(2\,x)}+15\,e^{x}-3i\right)e^{(-2\,x)}}{24\,(e^{x}+i)^{3}}+\frac{1}{8}\,e^{(2\,x)}-i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -7/2*x - 1/24*(216*I*e^(4*x) - 405*e^(3*x) - 239*I*e^(2*x) + 15*e^x - 3*I)*
e^(-2*x)/(e^x + I)^3 + 1/8*e^(2*x) - I*e^x
```

maple [B] time = 0.07, size = 116, normalized size = 2.00

$$\frac{1}{2\tanh\left(\frac{x}{2}\right)-2}+\frac{2i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{7\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{2i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+sinh(x))^2,x)
```

```
[Out] 1/2/(tanh(1/2*x)-1)+2*I/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+7/2*ln(tanh(1
/2*x)-1)+1/2/(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-7/2*
ln(tanh(1/2*x)+1)+2*I/(tanh(1/2*x)+I)^2+4/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x
)+I)
```

maxima [A] time = 0.32, size = 71, normalized size = 1.22

$$-\frac{7}{2}\,x+\frac{30\,e^{(-x)}+478i\,e^{(-2\,x)}-810\,e^{(-3\,x)}-432i\,e^{(-4\,x)}+6i}{16\left(3i\,e^{(-2\,x)}-9\,e^{(-3\,x)}-9i\,e^{(-4\,x)}+3\,e^{(-5\,x)}\right)}-i\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -7/2*x + 1/16*(30*e^(-x) + 478*I*e^(-2*x) - 810*e^(-3*x) - 432*I*e^(-4*x) +
 6*I)/(3*I*e^(-2*x) - 9*e^(-3*x) - 9*I*e^(-4*x) + 3*e^(-5*x)) - I*e^(-x) -
1/8*e^(-2*x)
```

mupad [B] time = 0.57, size = 97, normalized size = 1.67

$$\frac{\mathrm{e}^{2\,x}}{8}-\mathrm{e}^{-x}\,1\mathrm{i}-\frac{\mathrm{e}^{-2\,x}}{8}-\frac{7\,x}{2}-\mathrm{e}^{x}\,1\mathrm{i}-\frac{-2+\frac{\mathrm{e}^{x}\,8\mathrm{i}}{3}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{4\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}+\frac{8}{3}\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}-\frac{8\mathrm{i}}{3\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (7*x)/2 - exp(x)*1i - ((exp(x)*8i)/
3 - 2)/(exp(2*x) + exp(x)*2i - 1) + (4*exp(x) - (exp(2*x)*8i)/3 + 8i/3)/(ex
p(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 8i/(3*(exp(x) + 1i))
```

sympy [A] time = 0.21, size = 68, normalized size = 1.17

$$-\frac{7x}{2} + \frac{24e^{2x} + 42ie^{x} - 22}{3ie^{3x} - 9e^{2x} - 9ie^{x} + 3} + \frac{e^{2x}}{8} - ie^{x} - ie^{-x} - \frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] -7*x/2 + (24*exp(2*x) + 42*I*exp(x) - 22)/(3*I*exp(3*x) - 9*exp(2*x) - 9*I*
exp(x) + 3) + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

3.49 $\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=44

$$-2ix+\frac{4\cosh(x)}{3}-\frac{\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)^2}+\frac{2i\cosh(x)}{\sinh(x)+i}$$

```
[Out] -2*I*x+4/3*cosh(x)-1/3*cosh(x)*sinh(x)^2/(I+sinh(x))^2+2*I*cosh(x)/(I+sinh(
x))
```

Rubi [A] time = 0.13, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2765, 2968, 3023, 12, 2735, 2648}

$$-2ix+\frac{4\cosh(x)}{3}-\frac{\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)^2}+\frac{2i\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*x + (4*Cosh[x])/3 - (Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x])^2) + ((2*I)
*Cosh[x])/(I + Sinh[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx &= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int \frac{\sinh(x)(-2i+4\sinh(x))}{i+\sinh(x)}\,dx \\
&= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-\frac{1}{3}i\int \frac{2\sinh(x)+4i\sinh^2(x)}{i+\sinh(x)}\,dx \\
&= \frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int -\frac{6i\sinh(x)}{i+\sinh(x)}\,dx \\
&= \frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-2i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx \\
&= -2ix+\frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-2\int \frac{1}{i+\sinh(x)}\,dx \\
&= -2ix+\frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{2i\cosh(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 45, normalized size = 1.02

$$\frac{1}{3}\cosh(x)\left(\frac{3\sinh^2(x)+14i\sinh(x)-10}{(\sinh(x)+i)^2}-\frac{6i\sinh^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*(((-6*I)*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (-10 + (14*I)*Sinh[x]
 + 3*Sinh[x]^2)/(I + Sinh[x])^2))/3
```

fricas [B] time = 0.52, size = 74, normalized size = 1.68

$$\frac{(-12i\,x+9i)e^{(4\,x)}+6\,(6\,x+5)e^{(3\,x)}+(36i\,x+66i)e^{(2\,x)}-(12\,x+41)e^{x}+3\,e^{(5\,x)}-3i}{6\,e^{(4\,x)}+18i\,e^{(3\,x)}-18\,e^{(2\,x)}-6i\,e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-12*I*x + 9*I)*e^(4*x) + 6*(6*x + 5)*e^(3*x) + (36*I*x + 66*I)*e^(2*x) -
(12*x + 41)*e^x + 3*e^(5*x) - 3*I)/(6*e^(4*x) + 18*I*e^(3*x) - 18*e^(2*x) -
 6*I*e^x)
```

giac [A] time = 0.18, size = 38, normalized size = 0.86

$$-2i\,x+\frac{\left(39\,e^{(3\,x)}+69i\,e^{(2\,x)}-41\,e^{x}-3i\right)e^{(-x)}}{6\,(e^{x}+i)^{3}}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*I*x + 1/6*(39*e^(3*x) + 69*I*e^(2*x) - 41*e^x - 3*I)*e^(-x)/(e^x + I)^3
+ 1/2*e^x
```

maple [B] time = 0.07, size = 75, normalized size = 1.70

$$2i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{4i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{}{(\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+sinh(x))^2,x)
```

```
[Out] 2*I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1)-2*I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x
)+1)+4/3*I/(tanh(1/2*x)+I)^3+4*I/(tanh(1/2*x)+I)-2/(tanh(1/2*x)+I)^2
```

maxima [A] time = 0.47, size = 59, normalized size = 1.34

$$-2i\,x-\frac{164\,e^{(-x)}+276i\,e^{(-2\,x)}-156\,e^{(-3\,x)}-12i}{8\left(3i\,e^{(-x)}-9\,e^{(-2\,x)}-9i\,e^{(-3\,x)}+3\,e^{(-4\,x)}\right)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*I*x - 1/8*(164*e^(-x) + 276*I*e^(-2*x) - 156*e^(-3*x) - 12*I)/(3*I*e^(-x
) - 9*e^(-2*x) - 9*I*e^(-3*x) + 3*e^(-4*x)) + 1/2*e^(-x)
```

mupad [B] time = 0.58, size = 79, normalized size = 1.80

$$\frac{\mathrm{e}^{-x}}{2}-x\,2\mathrm{i}+\frac{\mathrm{e}^{x}}{2}+\frac{2\,\mathrm{e}^{x}+\frac{4}{3}\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{2\,\mathrm{e}^{2\,x}-2+\frac{\mathrm{e}^{x}\,8\mathrm{i}}{3}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}+\frac{2}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(-x)/2 - x*2i + exp(x)/2 + (2*exp(x) + 4i/3)/(exp(2*x) + exp(x)*2i - 1)
+ (2*exp(2*x) + (exp(x)*8i)/3 - 2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)
 + 2/(exp(x) + 1i)
```

sympy [A] time = 0.18, size = 54, normalized size = 1.23

$$-2ix+\frac{-18e^{2x}-30ie^{x}+16}{-3e^{3x}-9ie^{2x}+9e^{x}+3i}+\frac{e^{x}}{2}+\frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] -2*I*x + (-18*exp(2*x) - 30*I*exp(x) + 16)/(-3*exp(3*x) - 9*I*exp(2*x) + 9*
exp(x) + 3*I) + exp(x)/2 + exp(-x)/2
```

3.50 $\int \frac{\sinh^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=32

$$x-\frac{5\cosh(x)}{3(\sinh(x)+i)}+\frac{i\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[Out] x+1/3*I*cosh(x)/(I+sinh(x))^2-5/3*cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2758, 2735, 2648}

$$x-\frac{5\cosh(x)}{3(\sinh(x)+i)}+\frac{i\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] x + ((I/3)*Cosh[x])/(I + Sinh[x])^2 - (5*Cosh[x])/(3*(I + Sinh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2} dx = \frac{i\cosh(x)}{3(i+\sinh(x))^2} + \frac{1}{3}\int \frac{-2i+3\sinh(x)}{i+\sinh(x)} dx$$
$$= x + \frac{i\cosh(x)}{3(i+\sinh(x))^2} - \frac{5}{3}i\int \frac{1}{i+\sinh(x)} dx$$
$$= x + \frac{i\cosh(x)}{3(i+\sinh(x))^2} - \frac{5\cosh(x)}{3(i+\sinh(x))}$$

Mathematica [A] time = 0.15, size = 55, normalized size = 1.72

$$-\frac{1}{3}i\cosh(x)\left(\frac{4-5i\sinh(x)}{(\sinh(x)+i)^2} - \frac{6\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (-1/3*I)*Cosh[x]*((-6*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2]
+ (4 - (5*I)*Sinh[x])/(I + Sinh[x])^2)
```

fricas [B] time = 0.52, size = 50, normalized size = 1.56

$$\frac{3\,xe^{(3\,x)} + (9i\,x + 12i)e^{(2\,x)} - 9\,(x+2)e^{x} - 3i\,x - 10i}{3\,e^{(3\,x)} + 9i\,e^{(2\,x)} - 9\,e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (3*x*e^(3*x) + (9*I*x + 12*I)*e^(2*x) - 9*(x + 2)*e^x - 3*I*x - 10*I)/(3*e^
(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

giac [A] time = 0.16, size = 22, normalized size = 0.69

$$x - \frac{-12i\,e^{(2\,x)} + 18\,e^{x} + 10i}{3\,(e^{x}+i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] x - 1/3*(-12*I*e^(2*x) + 18*e^x + 10*I)/(e^x + I)^3
```

maple [B] time = 0.06, size = 52, normalized size = 1.62

$$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+sinh(x))^2,x)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)+I)^2-4/3/(tanh(1/2*x)
+I)^3-2/(tanh(1/2*x)+I)
```

maxima [A] time = 0.32, size = 40, normalized size = 1.25

$$x-\frac{72\,e^{(-x)}+48i\,e^{(-2\,x)}-40i}{4\left(9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] x - 1/4*(72*e^(-x) + 48*I*e^(-2*x) - 40*I)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(
-3*x) - 3*I)
```

mupad [B] time = 0.57, size = 71, normalized size = 2.22

$$x+\frac{-\frac{2}{3}+\frac{e^{x}\,4i}{3}}{e^{2\,x}-1+e^{x}\,2i}-\frac{\frac{4\,e^{x}}{3}-\frac{e^{2\,x}\,4i}{3}+\frac{4}{3}i}{e^{2\,x}\,3i+e^{3\,x}-3\,e^{x}-i}+\frac{4i}{3\,\left(e^{x}+1i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] x + ((exp(x)*4i)/3 - 2/3)/(exp(2*x) + exp(x)*2i - 1) - ((4*exp(x))/3 - (exp
(2*x)*4i)/3 + 4i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 4i/(3*(exp(x
) + 1i))
```

sympy [A] time = 0.14, size = 39, normalized size = 1.22

$$x+\frac{12e^{2x}+18ie^{x}-10}{-3ie^{3x}+9e^{2x}+9ie^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] x + (12*exp(2*x) + 18*I*exp(x) - 10)/(-3*I*exp(3*x) + 9*exp(2*x) + 9*I*exp(
x) - 3)
```

3.51 $\int \frac{\sinh(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=31

$$-\frac{2i\cosh(x)}{3(\sinh(x)+i)} - \frac{\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[Out] -1/3*cosh(x)/(I+sinh(x))^2-2/3*I*cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2750, 2648}

$$-\frac{2i\cosh(x)}{3(\sinh(x)+i)} - \frac{\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -Cosh[x]/(3*(I + Sinh[x])^2) - (((2*I)/3)*Cosh[x])/(I + Sinh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh(x)}{(i+\sinh(x))^2} dx &= -\frac{\cosh(x)}{3(i+\sinh(x))^2} + \frac{2}{3}\int \frac{1}{i+\sinh(x)} dx \\ &= -\frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{2i\cosh(x)}{3(i+\sinh(x))} \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{(1-2i\sinh(x))\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Sinh[x])^2,x]
[Out] (Cosh[x]*(1 - (2*I)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

fricas [A] time = 0.49, size = 34, normalized size = 1.10

$$-\frac{6\,e^{(2\,x)}+6i\,e^{x}-4}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="fricas")
[Out] -(6*e^(2*x) + 6*I*e^x - 4)/(3*e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

giac [A] time = 0.20, size = 20, normalized size = 0.65

$$-\frac{6\,e^{(2\,x)}+6i\,e^{x}-4}{3\,(e^{x}+i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="giac")
[Out] -1/3*(6*e^(2*x) + 6*I*e^x - 4)/(e^x + I)^3
```

maple [A] time = 0.05, size = 25, normalized size = 0.81

$$\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+sinh(x))^2,x)
[Out] 2/(tanh(1/2*x)+I)^2-4/3*I/(tanh(1/2*x)+I)^3
```

maxima [B] time = 0.31, size = 81, normalized size = 2.61

$$-\frac{6i\,e^{(-x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}+\frac{6\,e^{(-2\,x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}-\frac{4}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -6*I*e^(-x)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I) + 6*e^(-2*x)/(9*e^
(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I) - 4/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^
(-3*x) - 3*I)
```

mupad [B] time = 0.52, size = 25, normalized size = 0.81

$$-\frac{2\,\left(3\,\mathrm{e}^x-\mathrm{e}^{2\,x}\,3{}\mathrm{i}+2{}\mathrm{i}\right)}{3\,{\left(-1+\mathrm{e}^x\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] -(2*(3*exp(x) - exp(2*x)*3i + 2i))/(3*(exp(x)*1i - 1)^3)
```

sympy [A] time = 0.13, size = 37, normalized size = 1.19

$$\frac{-6e^{2x}-6ie^{x}+4}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))**2,x)
```

```
[Out] (-6*exp(2*x) - 6*I*exp(x) + 4)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x) - 3*I)
```

3.52 $\int \frac{\text{csch}(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=34

$$-\frac{4i\cosh(x)}{3(\sinh(x)+i)}+\frac{\cosh(x)}{3(\sinh(x)+i)^2}+\tanh^{-1}(\cosh(x))$$

```
[Out] arctanh(cosh(x))+1/3*cosh(x)/(I+sinh(x))^2-4/3*I*cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2766, 2978, 12, 3770}

$$-\frac{4i\cosh(x)}{3(\sinh(x)+i)}+\frac{\cosh(x)}{3(\sinh(x)+i)^2}+\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Sinh[x])^2,x]
```

```
[Out] ArcTanh[Cosh[x]] + Cosh[x]/(3*(I + Sinh[x])^2) - (((4*I)/3)*Cosh[x])/(I + S
inh[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx &= \frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{1}{3}\int \frac{\operatorname{csch}(x)(3i-\sinh(x))}{i+\sinh(x)}\,dx\\
&= \frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}+\frac{1}{3}i\int 3i\operatorname{csch}(x)\,dx\\
&= \frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}-\int \operatorname{csch}(x)\,dx\\
&= \tanh^{-1}(\cosh(x))+\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}
\end{aligned}$$

Mathematica [B] time = 0.09, size = 91, normalized size = 2.68

$$\frac{\cosh\left(\frac{x}{2}\right)\left(6-9\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\cosh\left(\frac{3x}{2}\right)\left(3\log\left(\tanh\left(\frac{x}{2}\right)\right)-8\right)+6i\sinh\left(\frac{x}{2}\right)\left(2\log\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh(x)\log\right.}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x/2]*(6 - 9*Log[Tanh[x/2]]) + Cosh[(3*x)/2]*(-8 + 3*Log[Tanh[x/2]]) +
 (6*I)*(-3 + 2*Log[Tanh[x/2]] + Cosh[x]*Log[Tanh[x/2]])*Sinh[x/2])/(6*(Cosh
[x/2] - I*Sinh[x/2])^3)
```

fricas [B] time = 0.50, size = 82, normalized size = 2.41

$$\frac{\left(3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i\right)\log\left(e^{x}+1\right)-\left(3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i\right)\log\left(e^{x}-1\right)-6\,e^{(2\,x)}-18i\,e^{x}+8}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((3*e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)*log(e^x + 1) - (3*e^(3*x) + 9*I*e^
(2*x) - 9*e^x - 3*I)*log(e^x - 1) - 6*e^(2*x) - 18*I*e^x + 8)/(3*e^(3*x) +
9*I*e^(2*x) - 9*e^x - 3*I)
```

giac [A] time = 0.39, size = 34, normalized size = 1.00

$$-\frac{2\left(3\,e^{(2\,x)}+9i\,e^{x}-4\right)}{3\,(e^{x}+i)^{3}}+\log\left(e^{x}+1\right)-\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 9*I*e^x - 4)/(e^x + I)^3 + log(e^x + 1) - log(abs(e^x - 1
))
```

maple [A] time = 0.06, size = 44, normalized size = 1.29

$$\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{4i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+sinh(x))^2,x)
```

```
[Out] 4/3*I/(tanh(1/2*x)+I)^3-4*I/(tanh(1/2*x)+I)-2/(tanh(1/2*x)+I)^2-ln(tanh(1/2
*x))
```

maxima [B] time = 0.40, size = 55, normalized size = 1.62

$$\frac{2\left(-9i\,e^{(-x)}+3\,e^{(-2\,x)}-4\right)}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(-9*I*e^(-x) + 3*e^(-2*x) - 4)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*
I) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

mupad [B] time = 0.28, size = 41, normalized size = 1.21

$$\ln\left(\mathrm{e}^{x}+1\right)-\ln\left(\mathrm{e}^{x}-1\right)-\frac{2}{\mathrm{e}^{x}+1\mathrm{i}}-\frac{2\mathrm{i}}{(\mathrm{e}^{x}+1\mathrm{i})^{2}}-\frac{4}{3\,(\mathrm{e}^{x}+1\mathrm{i})^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(sinh(x) + 1i)^2),x)
```

```
[Out] log(exp(x) + 1) - log(exp(x) - 1) - 2/(exp(x) + 1i) - 2i/(exp(x) + 1i)^2 -
4/(3*(exp(x) + 1i)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))**2,x)
```

```
[Out] Integral(csch(x)/(sinh(x) + I)**2, x)
```

3.53 $\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=42

$$\frac{10\coth(x)}{3}+2i\tanh^{-1}(\cosh(x))-\frac{2i\coth(x)}{\sinh(x)+i}+\frac{\coth(x)}{3(\sinh(x)+i)^2}$$

```
[Out] 2*I*arctanh(cosh(x))+10/3*coth(x)+1/3*coth(x)/(I+sinh(x))^2-2*I*coth(x)/(I+
sinh(x))
```

Rubi [A] time = 0.12, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{10\coth(x)}{3}+2i\tanh^{-1}(\cosh(x))-\frac{2i\coth(x)}{\sinh(x)+i}+\frac{\coth(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + (10*Coth[x])/3 + Coth[x]/(3*(I + Sinh[x])^2) - ((2
*I)*Coth[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{1}{3}\int \frac{\operatorname{csch}^2(x)(4i-2\sinh(x))}{i+\sinh(x)}\,dx\\
&= \frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}+\frac{1}{3}\int \operatorname{csch}^2(x)(-10-6i\sinh(x))\,dx\\
&= \frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}-2i\int \operatorname{csch}(x)\,dx-\frac{10}{3}\int \operatorname{csch}^2(x)\,dx\\
&= 2i\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}+\frac{10}{3}i\,\mathrm{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)\\
&= 2i\tanh^{-1}(\cosh(x))+\frac{10\coth(x)}{3}+\frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [B] time = 0.39, size = 88, normalized size = 2.10

$$\frac{1}{6}\left(\frac{2}{\sinh(x)+i}+3\tanh\left(\frac{x}{2}\right)+3\coth\left(\frac{x}{2}\right)-12i\log\left(\sinh\left(\frac{x}{2}\right)\right)+12i\log\left(\cosh\left(\frac{x}{2}\right)\right)-\frac{4\sinh\left(\frac{x}{2}\right)(7\sinh(x)+8i}{\left(\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (3*Coth[x/2] + (12*I)*Log[Cosh[x/2]] - (12*I)*Log[Sinh[x/2]] + 2/(I + Sinh[
x]) - (4*Sinh[x/2]*(8*I + 7*Sinh[x]))/(I*Cosh[x/2] + Sinh[x/2])^3 + 3*Tanh[
x/2])/6
```

fricas [B] time = 0.47, size = 129, normalized size = 3.07

$$\frac{\left(6i\,e^{(5\,x)}-18\,e^{(4\,x)}-24i\,e^{(3\,x)}+24\,e^{(2\,x)}+18i\,e^{x}-6\right)\log\left(e^{x}+1\right)+\left(-6i\,e^{(5\,x)}+18\,e^{(4\,x)}+24i\,e^{(3\,x)}-24\,e^{(2\,x)}-18}{3\,e^{(5\,x)}+9i\,e^{(4\,x)}-12\,e^{(3\,x)}-12i\,e^{(2\,x)}+9\,e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((6*I*e^(5*x) - 18*e^(4*x) - 24*I*e^(3*x) + 24*e^(2*x) + 18*I*e^x - 6)*log(
e^x + 1) + (-6*I*e^(5*x) + 18*e^(4*x) + 24*I*e^(3*x) - 24*e^(2*x) - 18*I*e^
x + 6)*log(e^x - 1) - 12*I*e^(4*x) + 36*e^(3*x) + 44*I*e^(2*x) - 48*e^x - 2
0*I)/(3*e^(5*x) + 9*I*e^(4*x) - 12*e^(3*x) - 12*I*e^(2*x) + 9*e^x + 3*I)
```

giac [A] time = 0.21, size = 46, normalized size = 1.10

$$\frac{2}{e^{(2\,x)}-1}-\frac{2\left(6i\,e^{(2\,x)}-15\,e^{x}-7i\right)}{3\left(e^{x}+i\right)^{3}}+2i\,\log\left(e^{x}+1\right)-2i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/(e^(2*x) - 1) - 2/3*(6*I*e^(2*x) - 15*e^x - 7*I)/(e^x + I)^3 + 2*I*log(e^
x + 1) - 2*I*log(abs(e^x - 1))
```

maple [A] time = 0.07, size = 58, normalized size = 1.38

$$\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{6}{\tanh\left(\frac{x}{2}\right)+i}-2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+sinh(x))^2,x)
```

```
[Out] 1/2*tanh(1/2*x)-2*I/(tanh(1/2*x)+I)^2-4/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x)+
I)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)
```

maxima [B] time = 0.32, size = 81, normalized size = 1.93

$$\frac{4\left(12\,e^{(-x)} + 11i\,e^{(-2\,x)} - 9\,e^{(-3\,x)} - 3i\,e^{(-4\,x)} - 5i\right)}{9\,e^{(-x)} + 12i\,e^{(-2\,x)} - 12\,e^{(-3\,x)} - 9i\,e^{(-4\,x)} + 3\,e^{(-5\,x)} - 3i} + 2i\,\log\left(e^{(-x)} + 1\right) - 2i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 4*(12*e^(-x) + 11*I*e^(-2*x) - 9*e^(-3*x) - 3*I*e^(-4*x) - 5*I)/(9*e^(-x) +
 12*I*e^(-2*x) - 12*e^(-3*x) - 9*I*e^(-4*x) + 3*e^(-5*x) - 3*I) + 2*I*log(e
^(-x) + 1) - 2*I*log(e^(-x) - 1)
```

mupad [B] time = 0.71, size = 85, normalized size = 2.02

$$\frac{2}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} + \frac{2}{\mathrm{e}^{2\,x} - 1} - \ln\left(\mathrm{e}^{x}\,4\mathrm{i} - 4\mathrm{i}\right)\,2\mathrm{i} + \ln\left(\mathrm{e}^{x}\,4\mathrm{i} + 4\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{\mathrm{e}^{x} + 1\mathrm{i}} - \frac{4\mathrm{i}}{3\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i} + \mathrm{e}^{3\,x} - 3\,\mathrm{e}^{x} - \mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(sinh(x) + 1i)^2),x)
```

```
[Out] log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + 2/(exp(2*x) + exp(x)*2i -
 1) - 4i/(exp(x) + 1i) - 4i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)) +
2/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{2}(x)}{(\sinh(x) + i)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**2/(sinh(x) + I)**2, x)
```

3.54 $\int \frac{\text{csch}^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=58

$$\frac{16}{3}i\coth(x)-\frac{7}{2}\tanh^{-1}(\cosh(x))+\frac{7}{2}\coth(x)\text{csch}(x)-\frac{8i\coth(x)\text{csch}(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\text{csch}(x)}{3(\sinh(x)+i)^2}$$

```
[Out] -7/2*arctanh(cosh(x))+16/3*I*coth(x)+7/2*coth(x)*csch(x)+1/3*coth(x)*csch(x
)/(I+sinh(x))^2-8/3*I*coth(x)*csch(x)/(I+sinh(x))
```

Rubi [A] time = 0.14, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{16}{3}i\coth(x)-\frac{7}{2}\tanh^{-1}(\cosh(x))+\frac{7}{2}\coth(x)\text{csch}(x)-\frac{8i\coth(x)\text{csch}(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\text{csch}(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-7*ArcTanh[Cosh[x]])/2 + ((16*I)/3)*Coth[x] + (7*Coth[x]*Csch[x])/2 + (Cot
h[x]*Csch[x])/(3*(I + Sinh[x])^2) - (((8*I)/3)*Coth[x]*Csch[x])/(I + Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
```

```
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\operatorname{csch}^3(x)(5i-3\sinh(x))}{i+\sinh(x)}\,dx$$
$$= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} + \frac{1}{3}\int \operatorname{csch}^3(x)(-21-16i\sinh(x))\,dx$$
$$= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} - \frac{16}{3}i\int \operatorname{csch}^2(x)\,dx - 7\int \operatorname{csch}^3(x)\,dx$$
$$= \frac{7}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} + \frac{7}{2}\int \operatorname{csch}(x)\,dx - \frac{16}{3}\operatorname{Subst}($$
$$= -\frac{7}{2}\tanh^{-1}(\cosh(x)) + \frac{16}{3}i\coth(x) + \frac{7}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{cs}}{3(i+\sinh}$$

Mathematica [B] time = 0.37, size = 131, normalized size = 2.26

$$\frac{1}{24}\left(24i\tanh\left(\frac{x}{2}\right) + 24i\coth\left(\frac{x}{2}\right) + 3\operatorname{csch}^2\left(\frac{x}{2}\right) + 3\operatorname{sech}^2\left(\frac{x}{2}\right) + 84\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{160i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)} + \frac{}{(\mathrm{c}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] ((24*I)*Coth[x/2] + 3*Csch[x/2]^2 + 84*Log[Tanh[x/2]] + 3*Sech[x/2]^2 + 8/(
Cosh[x/2] - I*Sinh[x/2])^2 + ((160*I)*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
+ (16*Sinh[x/2])/(I*Cosh[x/2] + Sinh[x/2])^3 + (24*I)*Tanh[x/2])/24
```

fricas [B] time = 0.61, size = 179, normalized size = 3.09

$$-\frac{\left(21\,e^{(7\,x)} + 63i\,e^{(6\,x)} - 105\,e^{(5\,x)} - 147i\,e^{(4\,x)} + 147\,e^{(3\,x)} + 105i\,e^{(2\,x)} - 63\,e^{x} - 21i\right)\log\left(e^{x}+1\right) - \left(21\,e^{(7\,x)} + 63i\,e^{(}\right.}{6\,e^{(7\,x)} + 18i\,e^{(6\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((21*e^(7*x) + 63*I*e^(6*x) - 105*e^(5*x) - 147*I*e^(4*x) + 147*e^(3*x) +
105*I*e^(2*x) - 63*e^x - 21*I)*log(e^x + 1) - (21*e^(7*x) + 63*I*e^(6*x) -
105*e^(5*x) - 147*I*e^(4*x) + 147*e^(3*x) + 105*I*e^(2*x) - 63*e^x - 21*I)*
log(e^x - 1) - 42*e^(6*x) - 126*I*e^(5*x) + 196*e^(4*x) + 252*I*e^(3*x) - 1
94*e^(2*x) - 150*I*e^x + 64)/(6*e^(7*x) + 18*I*e^(6*x) - 30*e^(5*x) - 42*I*
e^(4*x) + 42*e^(3*x) + 30*I*e^(2*x) - 18*e^x - 6*I)
```

giac [A] time = 0.18, size = 59, normalized size = 1.02

$$\frac{e^{(3x)} + 4i\,e^{(2x)} + e^x - 4i}{\left(e^{(2x)} - 1\right)^2} + \frac{2\left(9\,e^{(2x)} + 21i\,e^x - 10\right)}{3\,(e^x + i)^3} - \frac{7}{2}\,\log\left(e^x + 1\right) + \frac{7}{2}\,\log\left(|e^x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] (e^(3*x) + 4*I*e^(2*x) + e^x - 4*I)/(e^(2*x) - 1)^2 + 2/3*(9*e^(2*x) + 21*I
*e^x - 10)/(e^x + I)^3 - 7/2*log(e^x + 1) + 7/2*log(abs(e^x - 1))
```

maple [A] time = 0.08, size = 76, normalized size = 1.31

$$i\tanh\left(\frac{x}{2}\right) - \frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8} + \frac{8i}{\tanh\left(\frac{x}{2}\right) + i} - \frac{4i}{3\left(\tanh\left(\frac{x}{2}\right) + i\right)^3} + \frac{2}{\left(\tanh\left(\frac{x}{2}\right) + i\right)^2} + \frac{i}{\tanh\left(\frac{x}{2}\right)} + \frac{1}{8\tanh\left(\frac{x}{2}\right)^2} + \frac{7\ln\left(\tanh\left(\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+sinh(x))^2,x)
```

```
[Out] I*tanh(1/2*x)-1/8*tanh(1/2*x)^2+8*I/(tanh(1/2*x)+I)-4/3*I/(tanh(1/2*x)+I)^3
+2/(tanh(1/2*x)+I)^2+I/tanh(1/2*x)+1/8/tanh(1/2*x)^2+7/2*ln(tanh(1/2*x))
```

maxima [B] time = 0.33, size = 105, normalized size = 1.81

$$-\frac{8\left(-75i\,e^{(-x)} + 97\,e^{(-2x)} + 126i\,e^{(-3x)} - 98\,e^{(-4x)} - 63i\,e^{(-5x)} + 21\,e^{(-6x)} - 32\right)}{72\,e^{(-x)} + 120i\,e^{(-2x)} - 168\,e^{(-3x)} - 168i\,e^{(-4x)} + 120\,e^{(-5x)} + 72i\,e^{(-6x)} - 24\,e^{(-7x)} - 24i} - \frac{7}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{7}{2}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -8*(-75*I*e^(-x) + 97*e^(-2*x) + 126*I*e^(-3*x) - 98*e^(-4*x) - 63*I*e^(-5*
x) + 21*e^(-6*x) - 32)/(72*e^(-x) + 120*I*e^(-2*x) - 168*e^(-3*x) - 168*I*e
^(-4*x) + 120*e^(-5*x) + 72*I*e^(-6*x) - 24*e^(-7*x) - 24*I) - 7/2*log(e^(-
x) + 1) + 7/2*log(e^(-x) - 1)
```

mupad [B] time = 0.77, size = 79, normalized size = 1.36

$$\frac{e^x}{e^{2x} - 1} - \frac{7\,\ln\left(e^x + 1\right)}{2} - \frac{7\,\ln\left(\frac{1}{e^x - 1}\right)}{2} + \frac{2\,e^x}{\left(e^{2x} - 1\right)^2} + \frac{6}{e^x + 1i} + \frac{2i}{(e^x + 1i)^2} + \frac{4}{3\,(e^x + 1i)^3} + \frac{4i}{e^{2x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(sinh(x) + 1i)^2),x)
```

```
[Out] exp(x)/(exp(2*x) - 1) - (7*log(exp(x) + 1))/2 - (7*log(1/(exp(x) - 1)))/2 +
 (2*exp(x))/(exp(2*x) - 1)^2 + 6/(exp(x) + 1i) + 2i/(exp(x) + 1i)^2 + 4/(3*
(exp(x) + 1i)^3) + 4i/(exp(2*x) - 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

3.55 $\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=64

$$4\coth^3(x)-12\coth(x)-5i\tanh^{-1}(\cosh(x))+5i\coth(x)\operatorname{csch}(x)-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)^2}$$

```
[Out] -5*I*arctanh(cosh(x))-12*coth(x)+4*coth(x)^3+5*I*coth(x)*csch(x)+1/3*coth(x
)*csch(x)^2/(I+sinh(x))^2-10/3*I*coth(x)*csch(x)^2/(I+sinh(x))
```

Rubi [A] time = 0.13, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$4\coth^3(x)-12\coth(x)-5i\tanh^{-1}(\cosh(x))+5i\coth(x)\operatorname{csch}(x)-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*I)*ArcTanh[Cosh[x]] - 12*Coth[x] + 4*Coth[x]^3 + (5*I)*Coth[x]*Csch[x]
+ (Coth[x]*Csch[x]^2)/(3*(I + Sinh[x])^2) - (((10*I)/3)*Coth[x]*Csch[x]^2)/
(I + Sinh[x])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{1}{3}\int \frac{\operatorname{csch}^4(x)(6i-4\sinh(x))}{i+\sinh(x)}\,dx\\
&= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}+\frac{1}{3}\int \operatorname{csch}^4(x)(-36-30i\sinh(x))\,dx\\
&= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}-10i\int \operatorname{csch}^3(x)\,dx-12\int \operatorname{csch}^4(x)\,dx\\
&= 5i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}+5i\int \operatorname{csch}(x)\,dx-12i\,\mathrm{Sub}\\
&= -5i\tanh^{-1}(\cosh(x))-12\coth(x)+4\coth^3(x)+5i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\end{aligned}$$

Mathematica [B] time = 1.87, size = 143, normalized size = 2.23

$$\frac{1}{24}\left(-44\coth\left(\frac{x}{2}\right)+6i\text{csch}^2\left(\frac{x}{2}\right)+\frac{1}{2}\sinh(x)\text{csch}^4\left(\frac{x}{2}\right)+2\left(-\frac{4}{\sinh(x)+i}-22\tanh\left(\frac{x}{2}\right)+3i\text{sech}^2\left(\frac{x}{2}\right)+60i\log\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-44*Coth[x/2] + (6*I)*Csch[x/2]^2 + (Csch[x/2]^4*Sinh[x])/2 + 2*((-60*I)*L
og[Cosh[x/2]] + (60*I)*Log[Sinh[x/2]] + (3*I)*Sech[x/2]^2 - 4*Csch[x]^3*Sin
h[x/2]^4 - 4/(I + Sinh[x]) + (8*Sinh[x/2]*(14*I + 13*Sinh[x]))/(I*Cosh[x/2]
 + Sinh[x/2])^3 - 22*Tanh[x/2]))/24
```

fricas [B] time = 0.48, size = 225, normalized size = 3.52

$$\frac{\left(-15i\,e^{(9\,x)}+45\,e^{(8\,x)}+90i\,e^{(7\,x)}-150\,e^{(6\,x)}-180i\,e^{(5\,x)}+180\,e^{(4\,x)}+150i\,e^{(3\,x)}-90\,e^{(2\,x)}-45i\,e^{x}+15\right)\log\left(e^{x}+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-15*I*e^(9*x) + 45*e^(8*x) + 90*I*e^(7*x) - 150*e^(6*x) - 180*I*e^(5*x) +
 180*e^(4*x) + 150*I*e^(3*x) - 90*e^(2*x) - 45*I*e^x + 15)*log(e^x + 1) + (
15*I*e^(9*x) - 45*e^(8*x) - 90*I*e^(7*x) + 150*e^(6*x) + 180*I*e^(5*x) - 18
0*e^(4*x) - 150*I*e^(3*x) + 90*e^(2*x) + 45*I*e^x - 15)*log(e^x - 1) + 30*I
*e^(8*x) - 90*e^(7*x) - 170*I*e^(6*x) + 270*e^(5*x) + 306*I*e^(4*x) - 310*e
^(3*x) - 198*I*e^(2*x) + 114*e^x + 48*I)/(3*e^(9*x) + 9*I*e^(8*x) - 18*e^(7
*x) - 30*I*e^(6*x) + 36*e^(5*x) + 36*I*e^(4*x) - 30*e^(3*x) - 18*I*e^(2*x)
+ 9*e^x + 3*I)
```

giac [A] time = 0.18, size = 84, normalized size = 1.31

$$-\frac{2\left(-15i\,e^{(8\,x)}+45\,e^{(7\,x)}+85i\,e^{(6\,x)}-135\,e^{(5\,x)}-153i\,e^{(4\,x)}+155\,e^{(3\,x)}+99i\,e^{(2\,x)}-57\,e^{x}-24i\right)}{3\left(e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i\right)^{3}}-5i\,\log\left(e^{x}+1\right)+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(-15*I*e^(8*x) + 45*e^(7*x) + 85*I*e^(6*x) - 135*e^(5*x) - 153*I*e^(4*
x) + 155*e^(3*x) + 99*I*e^(2*x) - 57*e^x - 24*I)/(e^(3*x) + I*e^(2*x) - e^x
 - I)^3 - 5*I*log(e^x + 1) + 5*I*log(abs(e^x - 1))
```

maple [A] time = 0.08, size = 92, normalized size = 1.44

$$-\frac{15\tanh\left(\frac{x}{2}\right)}{8}+\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}-\frac{i\left(\tanh^2\left(\frac{x}{2}\right)\right)}{4}+\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}+\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^3}-\frac{10}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{4\tanh\left(\frac{x}{2}\right)^2}+5i\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+sinh(x))^2,x)
```

```
[Out] -15/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3-1/4*I*tanh(1/2*x)^2+2*I/(tanh(1/2*x)+I
)^2+4/3/(tanh(1/2*x)+I)^3-10/(tanh(1/2*x)+I)+1/4*I/tanh(1/2*x)^2+5*I*ln(tan
h(1/2*x))+1/24/tanh(1/2*x)^3-15/8/tanh(1/2*x)
```

maxima [B] time = 0.51, size = 129, normalized size = 2.02

$$-\frac{16\left(57\,e^{(-x)}+99i\,e^{(-2\,x)}-155\,e^{(-3\,x)}-153i\,e^{(-4\,x)}+135\,e^{(-5\,x)}+85i\,e^{(-6\,x)}-45\,e^{(-7\,x)}-15i\,e^{(-8\,x)}-24i\right)}{72\,e^{(-x)}+144i\,e^{(-2\,x)}-240\,e^{(-3\,x)}-288i\,e^{(-4\,x)}+288\,e^{(-5\,x)}+240i\,e^{(-6\,x)}-144\,e^{(-7\,x)}-72i\,e^{(-8\,x)}+24\,e^{(-9\,x)}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -16*(57*e^(-x) + 99*I*e^(-2*x) - 155*e^(-3*x) - 153*I*e^(-4*x) + 135*e^(-5*
x) + 85*I*e^(-6*x) - 45*e^(-7*x) - 15*I*e^(-8*x) - 24*I)/(72*e^(-x) + 144*I
*e^(-2*x) - 240*e^(-3*x) - 288*I*e^(-4*x) + 288*e^(-5*x) + 240*I*e^(-6*x) -
 144*e^(-7*x) - 72*I*e^(-8*x) + 24*e^(-9*x) - 24*I) - 5*I*log(e^(-x) + 1) +
 5*I*log(e^(-x) - 1)
```

mupad [B] time = 1.09, size = 189, normalized size = 2.95

$$-\ln\left(-\mathrm{e}^x\,10\mathrm{i}-10\mathrm{i}\right)\,5\mathrm{i}+\ln\left(-\mathrm{e}^x\,10\mathrm{i}+10\mathrm{i}\right)\,5\mathrm{i}-\frac{\frac{16\,\mathrm{e}^x}{3}-\frac{\mathrm{e}^{2\,x}\,32\mathrm{i}}{3}+\frac{16}{3}\mathrm{i}}{12\,\mathrm{e}^{5\,x}-10\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}\,12\mathrm{i}-\mathrm{e}^{2\,x}\,6\mathrm{i}-\mathrm{e}^{6\,x}\,10\mathrm{i}-6\,\mathrm{e}^{7\,x}+\mathrm{e}^{8\,x}\,3\mathrm{i}+\mathrm{e}^{9\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(sinh(x) + 1i)^2),x)
```

```
[Out] log(10i - exp(x)*10i)*5i - log(- exp(x)*10i - 10i)*5i - ((16*exp(x))/3 - (e
xp(2*x)*32i)/3 + 16i/3)/(exp(4*x)*12i - 10*exp(3*x) - exp(2*x)*6i + 12*exp(
5*x) - exp(6*x)*10i - 6*exp(7*x) + exp(8*x)*3i + exp(9*x) + 3*exp(x) + 1i)
+ ((20*exp(2*x))/3 + (exp(x)*16i)/3 - 44/3)/(3*exp(2*x) - exp(3*x)*4i - 3*e
xp(4*x) + exp(5*x)*2i + exp(6*x) + exp(x)*2i - 1) - (10*exp(x) - exp(2*x)*1
0i + 20i/3)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{1}{1+i\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

```
[Out] I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2648}

$$\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 1.56

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

fricas [A] time = 0.46, size = 16, normalized size = 0.59

$$\frac{2i}{de^{(dx+c)} - i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I/(d*e^(d*x + c) - I*d)
```

giac [A] time = 0.19, size = 15, normalized size = 0.56

$$\frac{2i}{d\left(e^{(dx+c)} - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*I/(d*(e^(d*x + c) - I))
```

maple [A] time = 0.04, size = 20, normalized size = 0.74

$$\frac{2}{d\left(-i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c)),x)
```

```
[Out] 2/d/(-I+tanh(1/2*d*x+1/2*c))
```

maxima [A] time = 0.48, size = 20, normalized size = 0.74

$$-\frac{2}{d\left(i\,e^{(-dx-c)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/(d*(I*e^(-d*x - c) - 1))
```

mupad [B] time = 0.20, size = 17, normalized size = 0.63

$$\frac{2\mathrm{i}}{d\,\left(\mathrm{e}^{c+d\,x} - \mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1),x)
```

```
[Out] 2i/(d*(exp(c + d*x) - 1i))
```

sympy **[A]** time = 0.13, size = 17, normalized size = 0.63

$$\frac{2e^{c}}{de^{c} - ide^{-dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x)
```

```
[Out] 2*exp(c)/(d*exp(c) - I*d*exp(-d*x))
```

3.57 $\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=59

$$\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

```
[Out] 1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c
))
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c + d*x])/(
d*(1 + I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1+i\sinh(c+dx)}\,dx\\ &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.10, size = 61, normalized size = 1.03

$$\frac{-4\sinh(c+dx)+\sinh(2(c+dx))-4i\cosh(c+dx)-i\cosh(2(c+dx))+3i}{6d(\sinh(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-2),x]
```

```
[Out] (3*I - (4*I)*Cosh[c + d*x] - I*Cosh[2*(c + d*x)] - 4*Sinh[c + d*x] + Sinh[2
*(c + d*x)])/(6*d*(-I + Sinh[c + d*x])^2)
```

fricas [A] time = 0.52, size = 50, normalized size = 0.85

$$\frac{6\,e^{(dx+c)}-2i}{3\,de^{(3\,dx+3\,c)}-9i\,de^{(2\,dx+2\,c)}-9\,de^{(dx+c)}+3i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (6*e^(d*x + c) - 2*I)/(3*d*e^(3*d*x + 3*c) - 9*I*d*e^(2*d*x + 2*c) - 9*d*e^
(d*x + c) + 3*I*d)
```

giac [A] time = 0.34, size = 25, normalized size = 0.42

$$\frac{6\,e^{(dx+c)}-2i}{3\,d\left(e^{(dx+c)}-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*e^(d*x + c) - 2*I)/(d*(e^(d*x + c) - I)^3)
```

maple [A] time = 0.08, size = 55, normalized size = 0.93

$$\frac{\frac{2i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(2*I/(-I+tanh(1/2*d*x+1/2*c))^2-4/3/(-I+tanh(1/2*d*x+1/2*c))^3+2/(-I+ta
nh(1/2*d*x+1/2*c)))
```

maxima [A] time = 0.35, size = 94, normalized size = 1.59

$$\frac{6\,e^{(-dx-c)}}{d\left(9\,e^{(-dx-c)}-9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}+3i\right)}+\frac{2i}{d\left(9\,e^{(-dx-c)}-9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}+3i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 6*e^(-d*x - c)/(d*(9*e^(-d*x - c) - 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x - 3*
c) + 3*I)) + 2*I/(d*(9*e^(-d*x - c) - 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x -
3*c) + 3*I))
```

mupad [B] time = 0.53, size = 29, normalized size = 0.49

$$-\frac{\frac{2}{3}+\mathrm{e}^{c+d\,x}\,2\mathrm{i}}{d\,\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1)^2,x)
```

```
[Out] -(exp(c + d*x)*2i + 2/3)/(d*(exp(c + d*x)*1i + 1)^3)
```

sympy [A] time = 0.20, size = 68, normalized size = 1.15

$$\frac{2e^{3c}-6ie^{2c}e^{-dx}}{3de^{3c}-9ide^{2c}e^{-dx}-9de^{c}e^{-2dx}+3ide^{-3dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**2,x)
```

```
[Out] (2*exp(3*c) - 6*I*exp(2*c)*exp(-d*x))/(3*d*exp(3*c) - 9*I*d*exp(2*c)*exp(-d
*x) - 9*d*exp(c)*exp(-2*d*x) + 3*I*d*exp(-3*d*x))
```

3.58 $\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

```
[Out] 1/5*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^3+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x+
c))^2+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((I/5)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/15)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^2) + (((2*I)/15)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3} + \frac{2}{5}\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx$$
$$= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2} + \frac{2}{15}\int \frac{1}{1+i\sinh(c+dx)}\,dx$$
$$= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2} + \frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}$$

Mathematica [A] time = 0.13, size = 81, normalized size = 0.92

$$\frac{15i\sinh(c+dx) - 6i\sinh(2(c+dx)) - i\sinh(3(c+dx)) - 15\cosh(c+dx) - 6\cosh(2(c+dx)) + \cosh(3(c+dx))}{30d(\sinh(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-3),x]
```

```
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] + (15*I)*S
inh[c + d*x] - (6*I)*Sinh[2*(c + d*x)] - I*Sinh[3*(c + d*x)])/(30*d*(-I + S
inh[c + d*x])^3)
```

fricas [A] time = 0.54, size = 85, normalized size = 0.97

$$\frac{-40i\,e^{(2\,dx+2\,c)} - 20\,e^{(dx+c)} + 4i}{15\,de^{(5\,dx+5\,c)} - 75i\,de^{(4\,dx+4\,c)} - 150\,de^{(3\,dx+3\,c)} + 150i\,de^{(2\,dx+2\,c)} + 75\,de^{(dx+c)} - 15i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-40*I*e^(2*d*x + 2*c) - 20*e^(d*x + c) + 4*I)/(15*d*e^(5*d*x + 5*c) - 75*I
*d*e^(4*d*x + 4*c) - 150*d*e^(3*d*x + 3*c) + 150*I*d*e^(2*d*x + 2*c) + 75*d
*e^(d*x + c) - 15*I*d)
```

giac [A] time = 0.23, size = 36, normalized size = 0.41

$$-\frac{i\left(40\,e^{(2\,dx+2\,c)} - 20i\,e^{(dx+c)} - 4\right)}{15\,d\left(e^{(dx+c)} - i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15*I*(40*e^(2*d*x + 2*c) - 20*I*e^(d*x + c) - 4)/(d*(e^(d*x + c) - I)^5)
```

maple [A] time = 0.09, size = 88, normalized size = 1.00

$$\frac{\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{8}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(4*I/(-I+tanh(1/2*d*x+1/2*c))^2-4*I/(-I+tanh(1/2*d*x+1/2*c))^4-16/3/(-I
+tanh(1/2*d*x+1/2*c))^3+8/5/(-I+tanh(1/2*d*x+1/2*c))^5+2/(-I+tanh(1/2*d*x+1
/2*c)))
```

maxima [B] time = 0.45, size = 211, normalized size = 2.40

$$\frac{20i\,e^{(-dx-c)}}{d\left(75i\,e^{(-dx-c)}+150\,e^{(-2\,dx-2\,c)}-150i\,e^{(-3\,dx-3\,c)}-75\,e^{(-4\,dx-4\,c)}+15i\,e^{(-5\,dx-5\,c)}-15\right)}+\frac{}{d\left(75i\,e^{(-dx-c)}+150\,e^{(-2\,dx-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) + 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) - 4/(d*(75*I*e
^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) - 75*e^(-4*d*x
- 4*c) + 15*I*e^(-5*d*x - 5*c) - 15))
```

mupad [B] time = 0.69, size = 40, normalized size = 0.45

$$-\frac{\frac{4}{15}-\frac{8\,e^{2\,c+2\,d\,x}}{3}+\frac{e^{c+d\,x}\,4i}{3}}{d\,\left(1+e^{c+d\,x}\,1i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1)^3,x)
```

```
[Out] -((exp(c + d*x)*4i)/3 - (8*exp(2*c + 2*d*x))/3 + 4/15)/(d*(exp(c + d*x)*1i
+ 1)^5)
```

sympy [A] time = 0.31, size = 114, normalized size = 1.30

$$\frac{-4e^{5c}+20ie^{4c}e^{-dx}+40e^{3c}e^{-2dx}}{-15de^{5c}+75ide^{4c}e^{-dx}+150de^{3c}e^{-2dx}-150ide^{2c}e^{-3dx}-75de^{c}e^{-4dx}+15ide^{-5dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**3,x)
```

```
[Out] (-4*exp(5*c) + 20*I*exp(4*c)*exp(-d*x) + 40*exp(3*c)*exp(-2*d*x))/(-15*d*ex
p(5*c) + 75*I*d*exp(4*c)*exp(-d*x) + 150*d*exp(3*c)*exp(-2*d*x) - 150*I*d*e
xp(2*c)*exp(-3*d*x) - 75*d*exp(c)*exp(-4*d*x) + 15*I*d*exp(-5*d*x))
```

3.59 $\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=117

$$\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

```
[Out] 1/7*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^4+3/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x+
c))^3+2/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+2/35*I*cosh(d*x+c)/d/(1+I*si
nh(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((I/7)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^4) + (((3*I)/35)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x])^2) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$$
$$= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx$$
$$= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2} + \frac{2}{35}$$
$$= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2} + \frac{}{35}$$

Mathematica [A] time = 0.17, size = 87, normalized size = 0.74

$$\frac{35\sinh\left(\frac{1}{2}(c+dx)\right) - 7\sinh\left(\frac{5}{2}(c+dx)\right) + 21i\cosh\left(\frac{3}{2}(c+dx)\right) - i\cosh\left(\frac{7}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((21*I)*Cosh[(3*(c + d*x))/2] - I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x)
/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^7)
```

fricas [A] time = 0.49, size = 121, normalized size = 1.03

$$-\frac{140\,e^{(3\,dx+3\,c)} - 84i\,e^{(2\,dx+2\,c)} - 28\,e^{(dx+c)} + 4i}{35\,de^{(7\,dx+7\,c)} - 245i\,de^{(6\,dx+6\,c)} - 735\,de^{(5\,dx+5\,c)} + 1225i\,de^{(4\,dx+4\,c)} + 1225\,de^{(3\,dx+3\,c)} - 735i\,de^{(2\,dx+2\,c)} - 245\,de^{(dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -(140*e^(3*d*x + 3*c) - 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) + 4*I)/(35*d*
e^(7*d*x + 7*c) - 245*I*d*e^(6*d*x + 6*c) - 735*d*e^(5*d*x + 5*c) + 1225*I*
d*e^(4*d*x + 4*c) + 1225*d*e^(3*d*x + 3*c) - 735*I*d*e^(2*d*x + 2*c) - 245*
d*e^(d*x + c) + 35*I*d)
```

giac [A] time = 0.18, size = 47, normalized size = 0.40

$$-\frac{140\,e^{(3\,dx+3\,c)} - 84i\,e^{(2\,dx+2\,c)} - 28\,e^{(dx+c)} + 4i}{35\,d\left(e^{(dx+c)} - i\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/35*(140*e^(3*d*x + 3*c) - 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) + 4*I)/(
d*(e^(d*x + c) - I)^7)
```

maple [A] time = 0.11, size = 121, normalized size = 1.03

$$\frac{\frac{8i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{72}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{6i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{16}{7\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}-\frac{16i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{12}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+ -}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(8*I/(-I+tanh(1/2*d*x+1/2*c))^6+72/5/(-I+tanh(1/2*d*x+1/2*c))^5+6*I/(-I
+tanh(1/2*d*x+1/2*c))^2-16/7/(-I+tanh(1/2*d*x+1/2*c))^7-16*I/(-I+tanh(1/2*d
*x+1/2*c))^4-12/(-I+tanh(1/2*d*x+1/2*c))^3+2/(-I+tanh(1/2*d*x+1/2*c)))
```

maxima [B] time = 0.35, size = 372, normalized size = 3.18

$$\frac{28\,e^{(-dx-c)}}{d\left(245\,e^{(-dx-c)}-735i\,e^{(-2\,dx-2\,c)}-1225\,e^{(-3\,dx-3\,c)}+1225i\,e^{(-4\,dx-4\,c)}+735\,e^{(-5\,dx-5\,c)}-245i\,e^{(-6\,dx-6\,c)}-35\,e^{(-7\,dx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 28*e^(-d*x - c)/(d*(245*e^(-d*x - c) - 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*
d*x - 3*c) + 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) - 245*I*e^(-6*d
*x - 6*c) - 35*e^(-7*d*x - 7*c) + 35*I)) - 84*I*e^(-2*d*x - 2*c)/(d*(245*e^
(-d*x - c) - 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) + 1225*I*e^(-4*
d*x - 4*c) + 735*e^(-5*d*x - 5*c) - 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x -
 7*c) + 35*I)) - 140*e^(-3*d*x - 3*c)/(d*(245*e^(-d*x - c) - 735*I*e^(-2*d*
x - 2*c) - 1225*e^(-3*d*x - 3*c) + 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x
- 5*c) - 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x - 7*c) + 35*I)) + 4*I/(d*(24
5*e^(-d*x - c) - 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) + 1225*I*e^
(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) - 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d
*x - 7*c) + 35*I))
```

mupad [B] time = 0.96, size = 53, normalized size = 0.45

$$-\frac{\left(7\,\mathrm{e}^{c+d\,x}+\mathrm{e}^{2\,c+2\,d\,x}\,21\mathrm{i}-35\,\mathrm{e}^{3\,c+3\,d\,x}-\mathrm{i}\right)\,4\mathrm{i}}{35\,d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1)^4,x)
```

```
[Out] -((7*exp(c + d*x) + exp(2*c + 2*d*x)*21i - 35*exp(3*c + 3*d*x) - 1i)*4i)/(3
5*d*(exp(c + d*x)*1i + 1)^7)
```

sympy [A] time = 0.44, size = 162, normalized size = 1.38

$$\frac{-4e^{7c} + 28ie^{6c}e^{-dx} + 84e^{5c}e^{-2dx} - 140ie^{4c}e^{-3dx}}{-35de^{7c} + 245ide^{6c}e^{-dx} + 735de^{5c}e^{-2dx} - 1225ide^{4c}e^{-3dx} - 1225de^{3c}e^{-4dx} + 735ide^{2c}e^{-5dx} + 245de^{c}e^{-6dx} - 35ide^{-7dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**4,x)
```

```
[Out] (-4*exp(7*c) + 28*I*exp(6*c)*exp(-d*x) + 84*exp(5*c)*exp(-2*d*x) - 140*I*ex
p(4*c)*exp(-3*d*x))/(-35*d*exp(7*c) + 245*I*d*exp(6*c)*exp(-d*x) + 735*d*ex
p(5*c)*exp(-2*d*x) - 1225*I*d*exp(4*c)*exp(-3*d*x) - 1225*d*exp(3*c)*exp(-4
*d*x) + 735*I*d*exp(2*c)*exp(-5*d*x) + 245*d*exp(c)*exp(-6*d*x) - 35*I*d*ex
p(-7*d*x))
```

3.60 $\int \frac{1}{1-i\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

```
[Out] -I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2648}

$$-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((-I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx = -\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 1.56

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]))
```

fricas [A] time = 0.54, size = 16, normalized size = 0.59

$$-\frac{2i}{de^{(dx+c)} + i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*I/(d*e^(d*x + c) + I*d)
```

giac [A] time = 0.16, size = 15, normalized size = 0.56

$$-\frac{2i}{d\left(e^{(dx+c)} + i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*I/(d*(e^(d*x + c) + I))
```

maple [A] time = 0.04, size = 20, normalized size = 0.74

$$\frac{2}{d\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c)),x)
```

```
[Out] 2/d/(tanh(1/2*d*x+1/2*c)+I)
```

maxima [A] time = 0.38, size = 20, normalized size = 0.74

$$\frac{2}{d\left(i\,e^{(-dx-c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/(d*(I*e^(-d*x - c) + 1))
```

mupad [B] time = 0.15, size = 17, normalized size = 0.63

$$-\frac{2\mathrm{i}}{d\,\left(\mathrm{e}^{c+d\,x} + 1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(c + d*x)*1i - 1),x)
```

```
[Out] -2i/(d*(exp(c + d*x) + 1i))
```

sympy [A] time = 0.12, size = 17, normalized size = 0.63

$$\frac{2e^c}{de^c + ide^{-dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x)
```

```
[Out] 2*exp(c)/(d*exp(c) + I*d*exp(-d*x))
```

3.61 $\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=59

$$-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$

```
[Out] -1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x+
c))
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((-I/3)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh[c + d*x])/
(d*(1 - I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx&=-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-i\sinh(c+dx)}\,dx\\&=-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 1.00

$$-\frac{\cosh\left(\frac{3}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}{3d\left(\sinh\left(\frac{1}{2}(c+dx)\right)+i\cosh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-2),x]
```

```
[Out] -1/3*(Cosh[(3*(c + d*x))/2] + (3*I)*Sinh[(c + d*x)/2])/(d*(I*Cosh[(c + d*x)
/2] + Sinh[(c + d*x)/2])^3)
```

fricas [A] time = 0.69, size = 50, normalized size = 0.85

$$\frac{6\,e^{(dx+c)}+2i}{3\,de^{(3\,dx+3\,c)}+9i\,de^{(2\,dx+2\,c)}-9\,de^{(dx+c)}-3i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (6*e^(d*x + c) + 2*I)/(3*d*e^(3*d*x + 3*c) + 9*I*d*e^(2*d*x + 2*c) - 9*d*e^
(d*x + c) - 3*I*d)
```

giac [A] time = 0.24, size = 25, normalized size = 0.42

$$\frac{6\,e^{(dx+c)}+2i}{3\,d\left(e^{(dx+c)}+i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*e^(d*x + c) + 2*I)/(d*(e^(d*x + c) + I)^3)
```

maple [A] time = 0.07, size = 55, normalized size = 0.93

$$\frac{-\frac{2i}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i}-\frac{4}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*I/(tanh(1/2*d*x+1/2*c)+I)^2+2/(tanh(1/2*d*x+1/2*c)+I)-4/3/(tanh(1/2
*d*x+1/2*c)+I)^3)
```

maxima [A] time = 0.32, size = 94, normalized size = 1.59

$$\frac{6\,e^{(-dx-c)}}{d\left(9\,e^{(-dx-c)}+9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}-3i\right)}-\frac{2i}{d\left(9\,e^{(-dx-c)}+9i\,e^{(-2\,dx-2\,c)}-3\,e^{(-3\,dx-3\,c)}-3i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 6*e^(-d*x - c)/(d*(9*e^(-d*x - c) + 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x - 3*
c) - 3*I)) - 2*I/(d*(9*e^(-d*x - c) + 9*I*e^(-2*d*x - 2*c) - 3*e^(-3*d*x -
3*c) - 3*I))
```

mupad [B] time = 0.54, size = 29, normalized size = 0.49

$$-\frac{2\,\left(-1+\mathrm{e}^{c+d\,x}\,3\mathrm{i}\right)}{3\,d\,{\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i - 1)^2,x)
```

```
[Out] -(2*(exp(c + d*x)*3i - 1))/(3*d*(exp(c + d*x)*1i - 1)^3)
```

sympy [A] time = 0.20, size = 68, normalized size = 1.15

$$\frac{2e^{3c}+6ie^{2c}e^{-dx}}{3de^{3c}+9ide^{2c}e^{-dx}-9de^{c}e^{-2dx}-3ide^{-3dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**2,x)
```

```
[Out] (2*exp(3*c) + 6*I*exp(2*c)*exp(-d*x))/(3*d*exp(3*c) + 9*I*d*exp(2*c)*exp(-d
*x) - 9*d*exp(c)*exp(-2*d*x) - 3*I*d*exp(-3*d*x))
```

3.62 $\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

```
[Out] -1/5*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^3-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*x
+c))^2-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-I/5)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/15)*Cosh[c + d
*x])/(d*(1 - I*Sinh[c + d*x])^2) - (((2*I)/15)*Cosh[c + d*x])/(d*(1 - I*Sin
h[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3} + \frac{2}{5}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx$$
$$= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2} + \frac{2}{15}\int \frac{1}{1-i\sinh(c+dx)}\,dx$$
$$= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}$$

Mathematica [**A**] time = 0.12, size = 81, normalized size = 0.92

$$\frac{-15i\sinh(c+dx)+6i\sinh(2(c+dx))+i\sinh(3(c+dx))-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx}{30d(\sinh(c+dx)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-3),x]
```

```
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] - (15*I)*S
inh[c + d*x] + (6*I)*Sinh[2*(c + d*x)] + I*Sinh[3*(c + d*x)])/(30*d*(I + Si
nh[c + d*x])^3)
```

fricas [**A**] time = 0.78, size = 85, normalized size = 0.97

$$\frac{40i\,e^{(2\,dx+2\,c)}-20\,e^{(dx+c)}-4i}{15\,de^{(5\,dx+5\,c)}+75i\,de^{(4\,dx+4\,c)}-150\,de^{(3\,dx+3\,c)}-150i\,de^{(2\,dx+2\,c)}+75\,de^{(dx+c)}+15i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (40*I*e^(2*d*x + 2*c) - 20*e^(d*x + c) - 4*I)/(15*d*e^(5*d*x + 5*c) + 75*I*
d*e^(4*d*x + 4*c) - 150*d*e^(3*d*x + 3*c) - 150*I*d*e^(2*d*x + 2*c) + 75*d*
e^(d*x + c) + 15*I*d)
```

giac [**A**] time = 0.18, size = 36, normalized size = 0.41

$$\frac{i\left(40\,e^{(2\,dx+2\,c)}+20i\,e^{(dx+c)}-4\right)}{15\,d\left(e^{(dx+c)}+i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15*I*(40*e^(2*d*x + 2*c) + 20*I*e^(d*x + c) - 4)/(d*(e^(d*x + c) + I)^5)
```

maple [A] time = 0.07, size = 88, normalized size = 1.00

$$\frac{\frac{8}{5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{5}}+\frac{4i}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{4}}-\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}-\frac{4i}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}+\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(8/5/(tanh(1/2*d*x+1/2*c)+I)^5+4*I/(tanh(1/2*d*x+1/2*c)+I)^4-16/3/(tanh
(1/2*d*x+1/2*c)+I)^3-4*I/(tanh(1/2*d*x+1/2*c)+I)^2+2/(tanh(1/2*d*x+1/2*c)+I
))
```

maxima [B] time = 0.32, size = 211, normalized size = 2.40

$$\frac{20i\,e^{(-dx-c)}}{d\left(75i\,e^{(-dx-c)}-150\,e^{(-2\,dx-2\,c)}-150i\,e^{(-3\,dx-3\,c)}+75\,e^{(-4\,dx-4\,c)}+15i\,e^{(-5\,dx-5\,c)}+15\right)}-\frac{}{d\left(75i\,e^{(-dx-c)}-150\,e^{(-2\,dx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) - 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) + 4/(d*(75*I*e
^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) + 75*e^(-4*d*x
- 4*c) + 15*I*e^(-5*d*x - 5*c) + 15))
```

mupad [B] time = 0.63, size = 40, normalized size = 0.45

$$-\frac{4\left(10\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,5\mathrm{i}\right)}{15\,d\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(c + d*x)*1i - 1)^3,x)
```

```
[Out] -(4*(exp(c + d*x)*5i + 10*exp(2*c + 2*d*x) - 1))/(15*d*(exp(c + d*x)*1i - 1
)^5)
```

sympy [A] time = 0.31, size = 114, normalized size = 1.30

$$\frac{-4e^{5c}-20ie^{4c}e^{-dx}+40e^{3c}e^{-2dx}}{-15de^{5c}-75ide^{4c}e^{-dx}+150de^{3c}e^{-2dx}+150ide^{2c}e^{-3dx}-75de^{c}e^{-4dx}-15ide^{-5dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**3,x)
```

```
[Out] (-4*exp(5*c) - 20*I*exp(4*c)*exp(-d*x) + 40*exp(3*c)*exp(-2*d*x))/(-15*d*ex
p(5*c) - 75*I*d*exp(4*c)*exp(-d*x) + 150*d*exp(3*c)*exp(-2*d*x) + 150*I*d*e
xp(2*c)*exp(-3*d*x) - 75*d*exp(c)*exp(-4*d*x) - 15*I*d*exp(-5*d*x))
```

3.63 $\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=117

$$-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

```
[Out] -1/7*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^4-3/35*I*cosh(d*x+c)/d/(1-I*sinh(d*x
+c))^3-2/35*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-2/35*I*cosh(d*x+c)/d/(1-I*s
inh(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2648}

$$-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-I/7)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^4) - (((3*I)/35)*Cosh[c + d
*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sin
h[c + d*x])^2) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$$
$$= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx$$
$$= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2} + \frac{2}{35}\ldots$$
$$= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2} - \frac{\ldots}{35\ldots}$$

Mathematica [A] time = 0.15, size = 87, normalized size = 0.74

$$\frac{35\sinh\left(\frac{1}{2}(c+dx)\right) - 7\sinh\left(\frac{5}{2}(c+dx)\right) - 21i\cosh\left(\frac{3}{2}(c+dx)\right) + i\cosh\left(\frac{7}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right) - i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-21*I)*Cosh[(3*(c + d*x))/2] + I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x
)/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)
/2])^7)
```

fricas [A] time = 0.51, size = 121, normalized size = 1.03

$$-\frac{140\,e^{(3\,dx+3\,c)} + 84i\,e^{(2\,dx+2\,c)} - 28\,e^{(dx+c)} - 4i}{35\,de^{(7\,dx+7\,c)} + 245i\,de^{(6\,dx+6\,c)} - 735\,de^{(5\,dx+5\,c)} - 1225i\,de^{(4\,dx+4\,c)} + 1225\,de^{(3\,dx+3\,c)} + 735i\,de^{(2\,dx+2\,c)} - 245\,de^{(dx\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -(140*e^(3*d*x + 3*c) + 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) - 4*I)/(35*d*
e^(7*d*x + 7*c) + 245*I*d*e^(6*d*x + 6*c) - 735*d*e^(5*d*x + 5*c) - 1225*I*
d*e^(4*d*x + 4*c) + 1225*d*e^(3*d*x + 3*c) + 735*I*d*e^(2*d*x + 2*c) - 245*
d*e^(d*x + c) - 35*I*d)
```

giac [A] time = 0.18, size = 47, normalized size = 0.40

$$-\frac{140\,e^{(3\,dx+3\,c)} + 84i\,e^{(2\,dx+2\,c)} - 28\,e^{(dx+c)} - 4i}{35\,d\left(e^{(dx+c)} + i\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/35*(140*e^(3*d*x + 3*c) + 84*I*e^(2*d*x + 2*c) - 28*e^(d*x + c) - 4*I)/(
d*(e^(d*x + c) + I)^7)
```

maple [A] time = 0.07, size = 121, normalized size = 1.03

$$\frac{\frac{72}{5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{5}}-\frac{16}{7\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{7}}+\frac{2}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i}+\frac{16i}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{4}}-\frac{12}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}-\frac{8i}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{6}}-\frac{6i}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(72/5/(tanh(1/2*d*x+1/2*c)+I)^5-16/7/(tanh(1/2*d*x+1/2*c)+I)^7+2/(tanh(
1/2*d*x+1/2*c)+I)+16*I/(tanh(1/2*d*x+1/2*c)+I)^4-12/(tanh(1/2*d*x+1/2*c)+I)
^3-8*I/(tanh(1/2*d*x+1/2*c)+I)^6-6*I/(tanh(1/2*d*x+1/2*c)+I)^2)
```

maxima [B] time = 0.33, size = 372, normalized size = 3.18

$$\frac{28\,e^{(-dx-c)}}{d\left(245\,e^{(-dx-c)}+735i\,e^{(-2\,dx-2\,c)}-1225\,e^{(-3\,dx-3\,c)}-1225i\,e^{(-4\,dx-4\,c)}+735\,e^{(-5\,dx-5\,c)}+245i\,e^{(-6\,dx-6\,c)}-35\,e^{(-7\,dx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 28*e^(-d*x - c)/(d*(245*e^(-d*x - c) + 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*
d*x - 3*c) - 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) + 245*I*e^(-6*d
*x - 6*c) - 35*e^(-7*d*x - 7*c) - 35*I)) + 84*I*e^(-2*d*x - 2*c)/(d*(245*e^
(-d*x - c) + 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) - 1225*I*e^(-4*
d*x - 4*c) + 735*e^(-5*d*x - 5*c) + 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x -
 7*c) - 35*I)) - 140*e^(-3*d*x - 3*c)/(d*(245*e^(-d*x - c) + 735*I*e^(-2*d*
x - 2*c) - 1225*e^(-3*d*x - 3*c) - 1225*I*e^(-4*d*x - 4*c) + 735*e^(-5*d*x
- 5*c) + 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d*x - 7*c) - 35*I)) - 4*I/(d*(24
5*e^(-d*x - c) + 735*I*e^(-2*d*x - 2*c) - 1225*e^(-3*d*x - 3*c) - 1225*I*e^
(-4*d*x - 4*c) + 735*e^(-5*d*x - 5*c) + 245*I*e^(-6*d*x - 6*c) - 35*e^(-7*d
*x - 7*c) - 35*I))
```

mupad [B] time = 0.91, size = 52, normalized size = 0.44

$$-\frac{4\,\left(21\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,7\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}\,35\mathrm{i}\right)}{35\,d\,{\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i - 1)^4,x)
```

```
[Out] -(4*(exp(c + d*x)*7i + 21*exp(2*c + 2*d*x) - exp(3*c + 3*d*x)*35i - 1))/(35
*d*(exp(c + d*x)*1i - 1)^7)
```

sympy [A] time = 0.45, size = 162, normalized size = 1.38

$$\frac{-4e^{7c} - 28ie^{6c}e^{-dx} + 84e^{5c}e^{-2dx} + 140ie^{4c}e^{-3dx}}{-35de^{7c} - 245ide^{6c}e^{-dx} + 735de^{5c}e^{-2dx} + 1225ide^{4c}e^{-3dx} - 1225de^{3c}e^{-4dx} - 735ide^{2c}e^{-5dx} + 245de^{c}e^{-6dx} + 35ide^{-7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**4,x)
```

```
[Out] (-4*exp(7*c) - 28*I*exp(6*c)*exp(-d*x) + 84*exp(5*c)*exp(-2*d*x) + 140*I*ex
p(4*c)*exp(-3*d*x))/(-35*d*exp(7*c) - 245*I*d*exp(6*c)*exp(-d*x) + 735*d*ex
p(5*c)*exp(-2*d*x) + 1225*I*d*exp(4*c)*exp(-3*d*x) - 1225*d*exp(3*c)*exp(-4
*d*x) - 735*I*d*exp(2*c)*exp(-5*d*x) + 245*d*exp(c)*exp(-6*d*x) + 35*I*d*ex
p(-7*d*x))
```

3.64 $\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))*2^(1/2)/a^(1/2)
+2*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2751, 2649, 206}

$$\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}} + i\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx$$
$$= \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}} - 2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)$$
$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}} + \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 1.32

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(-i\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)+(1+i)\sqrt[4]{-1}\,\tan^{-1}\left(\frac{\tanh\left(\frac{x}{4}\right)+i}{\sqrt{2}}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (2*((1 + I)*(-1)^(1/4)*ArcTan[(I + Tanh[x/4])/Sqrt[2]] + Cosh[x/2] - I*Sinh
[x/2])*(Cosh[x/2] + I*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

fricas [A] time = 0.62, size = 76, normalized size = 1.33

$$-\frac{\sqrt{2}\,\sqrt{a}\,\log\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)+2\,\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,e^{x}-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) - sqrt(2)
*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) + 2*sqrt(1/2*I*a*
e^(-x))*(I*e^x - 1))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{a + ia\sinh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+I*a*sinh(x))^(1/2),x)
```

```
[Out] int(sinh(x)/(a+I*a*sinh(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{i\,a\sinh(x) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh(x)}{\sqrt{a + a\sinh(x)\,1\mathrm{i}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + a*sinh(x)*1i)^(1/2),x)
```

```
[Out] int(sinh(x)/(a + a*sinh(x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{\left(x \right)}}{\sqrt{ia\left(\sinh{\left(x \right)} - i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(I*a*(sinh(x) - I)), x)
```

3.65 $\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}$$

[Out] -arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a-I*a*sinh(x))^(1/2))*2^(1/2)/a^(1/2)
+2*cosh(x)/(a-I*a*sinh(x))^(1/2)

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2751, 2649, 206}

$$\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]

[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a - I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a - I*a*Sinh[x]]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx = \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}} - i\int \frac{1}{\sqrt{a-ia\sinh(x)}}\,dx$$
$$= \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}} + 2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right)$$
$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}} + \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}$$

Mathematica [A] time = 0.08, size = 76, normalized size = 1.33

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\left(\sinh\left(\frac{x}{2}\right)+(1+i)(-1)^{3/4}\tan^{-1}\left(\frac{\tanh\left(\frac{x}{4}\right)-i}{\sqrt{2}}\right)\right)\right)}{\sqrt{a-ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

```
[Out] (2*(Cosh[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*((1 + I)*(-1)^(3/4)*ArcTan[(-I
+ Tanh[x/4])/Sqrt[2]] + Sinh[x/2])))/Sqrt[a - I*a*Sinh[x]]
```

fricas [A] time = 0.51, size = 76, normalized size = 1.33

$$-\frac{\sqrt{2}\,\sqrt{a}\,\log\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)+2\,\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\,(-i\,e^{x}-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) - sqrt(2
)*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) + 2*sqrt(-1/2*I
*a*e^(-x))*(-I*e^x - 1))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a-I*a*sinh(x))^(1/2),x)
[Out] int(sinh(x)/(a-I*a*sinh(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh(x)}{\sqrt{a - a\sinh(x)\,1\mathrm{i}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a - a*sinh(x)*1i)^(1/2),x)
[Out] int(sinh(x)/(a - a*sinh(x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{\left(x \right)}}{\sqrt{- ia \left(\sinh{\left(x \right)} + i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))**(1/2),x)
[Out] Integral(sinh(x)/sqrt(-I*a*(sinh(x) + I)), x)
```

3.66 $\int (a + ia\sinh(c + dx))^{5/2}\,dx$

Optimal. Leaf size=104

$$\frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

```
[Out] 2/5*I*a*cosh(d*x+c)*(a+I*a*sinh(d*x+c))^(3/2)/d+64/15*I*a^3*cosh(d*x+c)/d/(
a+I*a*sinh(d*x+c))^(1/2)+16/15*I*a^2*cosh(d*x+c)*(a+I*a*sinh(d*x+c))^(1/2)/
d
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2647, 2646}

$$\frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/15)*a^3*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((16*I)/
15)*a^2*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d + (((2*I)/5)*a*Cosh[c
+ d*x]*(a + I*a*Sinh[c + d*x])^(3/2))/d
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx = \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d} + \frac{1}{5}(8a)\int (a+ia\sinh(c+dx))^{3/2}\,dx$$
$$= \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d} + \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/}}{5d}$$
$$= \frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}} + \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d} + \frac{2ia\cosh(c}{}$$

Mathematica [A] time = 0.46, size = 145, normalized size = 1.39

$$\frac{a^2(\sinh(c+dx)-i)^2\sqrt{a+ia\sinh(c+dx)}\left(-150\sinh\left(\frac{1}{2}(c+dx)\right)+25\sinh\left(\frac{3}{2}(c+dx)\right)+3\sinh\left(\frac{5}{2}(c+dx)\right)-1\right.}{30d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*((-150*I)*Cosh[(c +
 d*x)/2] - (25*I)*Cosh[(3*(c + d*x))/2] + (3*I)*Cosh[(5*(c + d*x))/2] - 150
*Sinh[(c + d*x)/2] + 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(
30*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

fricas [A] time = 0.85, size = 101, normalized size = 0.97

$$-\frac{\left(3\,a^2e^{(5\,dx+5\,c)}-25i\,a^2e^{(4\,dx+4\,c)}-150\,a^2e^{(3\,dx+3\,c)}-150i\,a^2e^{(2\,dx+2\,c)}-25\,a^2e^{(dx+c)}+3i\,a^2\right)\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\,e^{(-2\,dx-2\,c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(3*a^2*e^(5*d*x + 5*c) - 25*I*a^2*e^(4*d*x + 4*c) - 150*a^2*e^(3*d*x
+ 3*c) - 150*I*a^2*e^(2*d*x + 2*c) - 25*a^2*e^(d*x + c) + 3*I*a^2)*sqrt(1/2
*I*a*e^(-d*x - c))*e^(-2*d*x - 2*c)/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,a\sinh(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (a + ia \sinh (dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2),x)
[Out] int((a+I*a*sinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\, a \sinh (dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sinh (c + d\, x)\ 1\mathrm{i})^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(5/2),x)
[Out] int((a + a*sinh(c + d*x)*1i)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia \sinh (c + dx) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2),x)
[Out] Integral((I*a*sinh(c + d*x) + a)**(5/2), x)
```

3.67 $\int (a + ia\sinh(c + dx))^{3/2} dx$

Optimal. Leaf size=69

$$\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}} + \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

```
[Out] 8/3*I*a^2*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)+2/3*I*a*cosh(d*x+c)*(a+I*
a*sinh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2647, 2646}

$$\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}} + \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^2*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((2*I)/3)*
a*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned} \int (a+ia\sinh(c+dx))^{3/2}\,dx &= \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d} + \frac{1}{3}(4a)\int \sqrt{a+ia\sinh(c+dx)}\,dx \\ &= \frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}} + \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d} \end{aligned}$$

Mathematica [A] time = 0.21, size = 113, normalized size = 1.64

$$-\frac{a(\sinh(c+dx)-i)\sqrt{a+ia\sinh(c+dx)}\left(-9i\sinh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{3}{2}(c+dx)\right)+9\cosh\left(\frac{1}{2}(c+dx)\right)+\cos\right.}{3d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(a*(-I + Sinh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]]*(9*Cosh[(c + d*x)/
2] + Cosh[(3*(c + d*x))/2] - (9*I)*Sinh[(c + d*x)/2] + I*Sinh[(3*(c + d*x))
/2]))/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^3)
```

fricas [A] time = 0.54, size = 63, normalized size = 0.91

$$\frac{\left(i\,ae^{(3\,dx+3\,c)}+9\,ae^{(2\,dx+2\,c)}+9i\,ae^{(dx+c)}+a\right)\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\,e^{(-dx-c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(I*a*e^(3*d*x + 3*c) + 9*a*e^(2*d*x + 2*c) + 9*I*a*e^(d*x + c) + a)*sqr
t(1/2*I*a*e^(-d*x - c))*e^(-d*x - c)/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,a\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (a+ia\sinh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (i\,a\sinh(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\sinh(c+d\,x)\,1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(3/2),x)
[Out] int((a + a*sinh(c + d*x)*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\sinh(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(3/2),x)
[Out] Integral((I*a*sinh(c + d*x) + a)**(3/2), x)
```

3.68 $\int \sqrt{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

```
[Out] 2*I*a*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2646}

$$\frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*a*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx=\frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Mathematica [B] time = 0.04, size = 74, normalized size = 2.39

$$\frac{2\sqrt{a+ia\sinh(c+dx)}\left(\sinh\left(\frac{1}{2}(c+dx)\right)+i\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] (2*(I*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*Sqrt[a + I*a*Sinh[c + d*x]])/(
d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

fricas [A] time = 0.49, size = 28, normalized size = 0.90

$$\frac{\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\left(2\,e^{(dx+c)}+2i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] sqrt(1/2*I*a*e^(-d*x - c))*(2*e^(d*x + c) + 2*I)/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i\,a\sinh\left(dx+c\right)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

maple [B] time = 0.13, size = 89, normalized size = 2.87

$$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2dx+2c}-i+2\,\mathrm{e}^{dx+c}\right)\mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}+i\right)\left(\mathrm{e}^{dx+c}-i\right)}{\left(i\mathrm{e}^{2dx+2c}-i+2\,\mathrm{e}^{dx+c}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(1/2),x)
[Out] I*2^(1/2)*(a*(I*exp(2*d*x+2*c)-I+2*exp(d*x+c))*exp(-d*x-c))^(1/2)/(I*exp(2*
d*x+2*c)-I+2*exp(d*x+c))*(exp(d*x+c)+I)*(exp(d*x+c)-I)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{i\,a\sinh\left(dx+c\right)+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

mupad [B] time = 0.75, size = 53, normalized size = 1.71

$$\frac{\sqrt{2}\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-c-d\,x}\,{\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}}{d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*(exp(c + d*x) + 1i)*(a*exp(- c - d*x)*(exp(c + d*x) - 1i)^2*1i)^(1
/2))/(d*(exp(c + d*x) - 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\sinh{(c+dx)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*sinh(c + d*x) + a), x)
```

3.69 $\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx$

Optimal. Leaf size=52

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] I*arctanh(1/2*cosh(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2649, 206}

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d
*x]])])/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{d}$$
$$= \frac{i\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{2}\,\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.09, size = 84, normalized size = 1.62

$$\frac{(2+2i)\sqrt[4]{-1}\,\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\sinh\left(\frac{1}{2}(c+dx)\right)-i\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] ((2 + 2*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4
])]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]))/(d*Sqrt[a + I*a*Sinh[c +
d*x]])
```

fricas [B] time = 0.67, size = 93, normalized size = 1.79

$$i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{1}{2}\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)-i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{1}{2}\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] I*sqrt(2)*sqrt(1/(a*d^2))*log(1/2*sqrt(2)*a*d*sqrt(1/(a*d^2)) + sqrt(1/2*I*
a*e^(-d*x - c))) - I*sqrt(2)*sqrt(1/(a*d^2))*log(-1/2*sqrt(2)*a*d*sqrt(1/(a
*d^2)) + sqrt(1/2*I*a*e^(-d*x - c)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{i\,a\sinh(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + ia \sinh (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(1/2),x)
[Out] int(1/(a+I*a*sinh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{i\, a \sinh (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a + a \sinh (c + d\,x)\; 1{\rm i}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(1/2),x)
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia \sinh (c + dx) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(I*a*sinh(c + d*x) + a), x)
```

3.70 $\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

```
[Out] 1/2*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(3/2)+1/4*I*arctanh(1/2*cosh(d*x+c)
*a^(1/2)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2650, 2649, 206}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d*x]]
)])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])^
(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{4a}$$
$$= \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}} + \frac{i\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{2ad}$$
$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

Mathematica [A] time = 0.24, size = 156, normalized size = 1.79

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\left(\sinh\left(\frac{1}{2}(c+dx)\right)+(1-i)\sqrt[4]{-1}\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\right.\right.\right.}{2ad(\sinh(c+dx)-i)\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2] - I*((1 - I)*
(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + Sinh[(c + d*x)/2])))/(2*a*d*(-I + Si
nh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]])
```

fricas [B] time = 0.62, size = 234, normalized size = 2.69

$$\frac{\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2\,dx+2\,c)}+2\,a^2de^{(dx+c)}-i\,a^2d\right)\sqrt{\frac{1}{a^3d^2}}\,\log\left(\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)+\sqrt{\frac{1}{2}}\left(-i\,a^2de^{(2\,dx+2\,c)}-2\,a^2\right.}{2\,a^2de^{(2\,dx+2\,c)}-4i\,a^2de^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*(I*a^2*d*e^(2*d*x + 2*c) + 2*a^2*d*e^(d*x + c) - I*a^2*d)*sqrt(1
/(a^3*d^2))*log(sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/2*I*a*e^(-d*x -
c))) + sqrt(1/2)*(-I*a^2*d*e^(2*d*x + 2*c) - 2*a^2*d*e^(d*x + c) + I*a^2*d)
*sqrt(1/(a^3*d^2))*log(-sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/2*I*a*e^
(-d*x - c))) + sqrt(1/2*I*a*e^(-d*x - c))*(-2*I*e^(2*d*x + 2*c) + 2*e^(d*x
+ c)))/(2*a^2*d*e^(2*d*x + 2*c) - 4*I*a^2*d*e^(d*x + c) - 2*a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+ia\sinh{(dx+c)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)
[Out] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,a\sinh{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(3/2),x)
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\sinh{(c+dx)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*sinh(c + d*x) + a)**(-3/2), x)
```

3.71 $\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

```
[Out] 1/4*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(5/2)+3/16*I*cosh(d*x+c)/a/d/(a+I*a
*sinh(d*x+c))^(3/2)+3/32*I*arctanh(1/2*cosh(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*s
inh(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2650, 2649, 206}

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d
*x])^(5/2)) + (((3*I)/16)*Cosh[c + d*x])/(a*d*(a + I*a*Sinh[c + d*x])^(3/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{32a^2}$$

$$= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,d\right.}{16a}$$

$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+}{16ad(a+ia\sinh(c}$$

Mathematica [A] time = 0.20, size = 210, normalized size = 1.72

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4\sinh\left(\frac{1}{2}(c+dx)\right)+4i\cosh\left(\frac{1}{2}(c+dx)\right)+6\sinh\left(\frac{1}{2}(c+dx)\right)\left(\cosh\left(\frac{1}{2}(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((4*I)*Cosh[(c + d*x)/2] + (3 -
3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 4*Sinh[(c + d*x)/2] + 6*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2] + 3*((-I)*Cosh[(c +
d*x)/2] + Sinh[(c + d*x)/2])^3))/(16*d*(a + I*a*Sinh[c + d*x])^(5/2))
```

fricas [B] time = 0.53, size = 348, normalized size = 2.85

$$\frac{\sqrt{\frac{1}{2}}\left(12i\,a^3de^{(4\,dx+4\,c)}+48\,a^3de^{(3\,dx+3\,c)}-72i\,a^3de^{(2\,dx+2\,c)}-48\,a^3de^{(dx+c)}+12i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\,\log\left(\sqrt{\frac{1}{2}}\,a^3d\sqrt{\frac{1}{a^5d^2}}+\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(1/2)*(12*I*a^3*d*e^(4*d*x + 4*c) + 48*a^3*d*e^(3*d*x + 3*c) - 72*
I*a^3*d*e^(2*d*x + 2*c) - 48*a^3*d*e^(d*x + c) + 12*I*a^3*d)*sqrt(1/(a^5*d^
2))*log(sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) + s
qrt(1/2)*(-12*I*a^3*d*e^(4*d*x + 4*c) - 48*a^3*d*e^(3*d*x + 3*c) + 72*I*a^3
*d*e^(2*d*x + 2*c) + 48*a^3*d*e^(d*x + c) - 12*I*a^3*d)*sqrt(1/(a^5*d^2))*l
og(-sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) + 8*sqr
t(1/2*I*a*e^(-d*x - c))*(-3*I*e^(4*d*x + 4*c) - 11*e^(3*d*x + 3*c) - 11*I*e
^(2*d*x + 2*c) - 3*e^(d*x + c)))/(8*a^3*d*e^(4*d*x + 4*c) - 32*I*a^3*d*e^(3
*d*x + 3*c) - 48*a^3*d*e^(2*d*x + 2*c) + 32*I*a^3*d*e^(d*x + c) + 8*a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+ia\sinh\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\sinh(c+dx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*sinh(c + d*x) + a)**(-5/2), x)
```

3.72 $\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b}-\frac{ax\left(2a^2-b^2\right)}{2b^4}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh^2(x)\cosh(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2-b^2)*x/b^4-1/3*(2-3*a^2/b^2)*cosh(x)/b-1/2*a*cosh(x)*sinh(x)/
b^2+1/3*cosh(x)*sinh(x)^2/b-2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^4/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2793, 3049, 3023, 2735, 2660, 618, 206}

$$-\frac{ax\left(2a^2-b^2\right)}{2b^4}-\frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh^2(x)\cosh(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -(a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^4*Sqrt[a^2 + b^2]) - ((2 - (3*a^2)/b^2)*Cosh[x])/(3*b) - (a*Cosh[
x]*Sinh[x])/(2*b^2) + (Cosh[x]*Sinh[x]^2)/(3*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{\sinh(x)\left(2a+2b\sinh(x)+3a\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{3b}$$

$$= -\frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{-3a^2+ab\sinh(x)-2\left(3a^2-2b^2\right)\sinh^2(x)}{a+b\sinh(x)}\,dx}{6b^2}$$

$$= \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{i\int \frac{3ia^2b-3ia\left(2a^2-b^2\right)\sinh(x)}{a+b\sinh(x)}\,d}{6b^3}$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} + \frac{a^4\int \frac{}{a}}{}$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} + \frac{\left(2a^4\right)S}{}$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\left(4a^4\right)S}{}$$

$$= -\frac{a\left(2a^2-b^2\right)x}{2b^4} - \frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}} + \frac{\left(3a^2-2b^2\right)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} +$$

Mathematica [A] time = 0.44, size = 105, normalized size = 0.97

$$\frac{3b\left(4a^2-3b^2\right)\cosh(x)+3a\left(-4a^2x+\frac{8a^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+2b^2x-b^2\sinh(2x)\right)+b^3\cosh(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (3*b*(4*a^2 - 3*b^2)*Cosh[x] + b^3*Cosh[3*x] + 3*a*(-4*a^2*x + 2*b^2*x + (8
*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - b^2*Sin
h[2*x]))/(12*b^4)
```

fricas [B] time = 0.86, size = 799, normalized size = 7.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^2*b^3 + b^5)*cosh(x)^6 + (a^2*b^3 + b^5)*sinh(x)^6 - 3*(a^3*b^2 +
a*b^4)*cosh(x)^5 - 3*(a^3*b^2 + a*b^4 - 2*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^
5 + a^2*b^3 + b^5 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*b +
 a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b
^5)*cosh(x)^2 - 5*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^3 + b
^5)*cosh(x)^3 - 15*(a^3*b^2 + a*b^4)*cosh(x)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4
)*x + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(4*a^4*b + a^2*b
^3 - 3*b^5)*cosh(x)^2 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b^5)*co
sh(x)^4 - 10*(a^3*b^2 + a*b^4)*cosh(x)^3 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*c
osh(x) + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^4*cosh(
x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 +
 b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sin
h(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sin
h(x) - b)) + 3*(a^3*b^2 + a*b^4)*cosh(x) + 3*(2*(a^2*b^3 + b^5)*cosh(x)^5 +
 a^3*b^2 + a*b^4 - 5*(a^3*b^2 + a*b^4)*cosh(x)^4 - 12*(2*a^5 + a^3*b^2 - a*
b^4)*x*cosh(x)^2 + 4*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^3 + 2*(4*a^4*b + a
^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^2*b^4 +
 b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 + b^6)*cosh(x)*sinh(x)^2 + (a^2*b^4 +
b^6)*sinh(x)^3)
```

giac [A] time = 0.19, size = 156, normalized size = 1.44

$$\frac{a^4 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{b^2e^{(3\,x)}-3\,abe^{(2\,x)}+12\,a^2e^x-9\,b^2e^x}{24\,b^3}-\frac{\left(2\,a^3-ab^2\right)x}{2\,b^4}+\frac{\left(3\,ab^2e^x+b^3+3\left(4\,a^2b-3\,b^3\right)e\right.}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*
a^2*e^x - 9*b^2*e^x)/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x +
b^3 + 3*(4*a^2*b - 3*b^3)*e^(2*x))*e^(-3*x)/b^4
```

maple [B] time = 0.04, size = 262, normalized size = 2.43

$$\frac{2a^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{a^2}{b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sinh(x)),x)
```

```
[Out] 2*a^4/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)
)-1/3/b/(tanh(1/2*x)-1)^3-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)
^2-1/b^3/(tanh(1/2*x)-1)*a^2-1/2/b^2/(tanh(1/2*x)-1)*a+1/2/b/(tanh(1/2*x)-1
)+a^3/b^4*ln(tanh(1/2*x)-1)-1/2*a/b^2*ln(tanh(1/2*x)-1)+1/3/b/(tanh(1/2*x)+
1)^3+1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2/b/(tanh(1/2*x)+1)^2+1/b^3/(tanh(1/2*x)
+1)*a^2-1/2/b^2/(tanh(1/2*x)+1)*a-1/2/b/(tanh(1/2*x)+1)-a^3/b^4*ln(tanh(1/2
*x)+1)+1/2*a/b^2*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.43, size = 158, normalized size = 1.46

$$\frac{a^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^4}-\frac{\left(3\,abe^{(-x)}-b^2-3\left(4\,a^2-3\,b^2\right)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,b^3}+\frac{3\,abe^{(-2\,x)}+b^2e^{(-3\,x)}+3\left(4\,a^2-3\,b^2\right)e^{(-x)}}{24\,b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^4) - 1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 - 3*b^2)*e^(-2*
x))*e^(3*x)/b^3 + 1/24*(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 - 3*b^2)*e
^(-x))/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4
```

mupad [B] time = 0.76, size = 199, normalized size = 1.84

$$\frac{e^{-3\,x}}{24\,b}+\frac{e^{3\,x}}{24\,b}+\frac{x\,\left(a\,b^2-2\,a^3\right)}{2\,b^4}+\frac{e^x\,\left(4\,a^2-3\,b^2\right)}{8\,b^3}+\frac{a\,e^{-2\,x}}{8\,b^2}-\frac{a\,e^{2\,x}}{8\,b^2}+\frac{e^{-x}\,\left(4\,a^2-3\,b^2\right)}{8\,b^3}-\frac{a^4\,\ln\left(-\frac{2\,a^4\,e^x}{b^5}-\frac{2\,a^4\,(b-a\,e^x)}{b^5\,\sqrt{a^2+b^2}}\right)}{b^4\,\sqrt{a^2+b^2}}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*b) + exp(3*x)/(24*b) + (x*(a*b^2 - 2*a^3))/(2*b^4) + (exp(x)*
(4*a^2 - 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) + (
exp(-x)*(4*a^2 - 3*b^2))/(8*b^3) - (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^4*(
b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(b^4*(a^2 + b^2)^(1/2)) + (a^4*log
((2*a^4*(b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2)) - (2*a^4*exp(x))/b^5))/(b^4
*(a^2 + b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.73 $\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=82

$$\frac{x(2a^2-b^2)}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

```
[Out] 1/2*(2*a^2-b^2)*x/b^3-a*cosh(x)/b^2+1/2*cosh(x)*sinh(x)/b+2*a^3*arctanh((b-
a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^3/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2793, 3023, 2735, 2660, 618, 206}

$$\frac{x(2a^2-b^2)}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2
]])/(b^3*Sqrt[a^2 + b^2]) - (a*Cosh[x])/b^2 + (Cosh[x]*Sinh[x])/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \frac{\cosh(x)\sinh(x)}{2b} - \frac{\int \frac{a+b\sinh(x)+2a\sinh^2(x)}{a+b\sinh(x)}\,dx}{2b}$$
$$= -\frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{i\int \frac{-iab+i(2a^2-b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^2}$$
$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{a^3\int \frac{1}{a+b\sinh(x)}\,dx}{b^3}$$
$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3}$$
$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} + \frac{(4a^3)\,\mathrm{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\,\mathrm{ta}$$
$$= \frac{(2a^2-b^2)\,x}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 1.00

$$\frac{4a^2x - \frac{8a^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\cosh(x) - 2b^2x + b^2\sinh(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (4*a^2*x - 2*b^2*x - (8*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqr
t[-a^2 - b^2] - 4*a*b*Cosh[x] + b^2*Sinh[2*x])/(4*b^3)
```

fricas [B] time = 0.50, size = 459, normalized size = 5.60

$$\frac{(a^2b^2+b^4)\cosh(x)^4 + (a^2b^2+b^4)\sinh(x)^4 - a^2b^2 - b^4 + 4\left(2a^4+a^2b^2-b^4\right)x\cosh(x)^2 - 4\left(a^3b+ab^3\right)\cosh(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^2 + b^4)*sinh(x)^4 - a^2*b^2 - b^4
+ 4*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 + b^4)*c
osh(x)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/
(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x)^3
- 2*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^2*b^3 + b^5)*cosh(x)*sinh(x) + (a^2*b^
3 + b^5)*sinh(x)^2)
```

giac [A] time = 0.21, size = 117, normalized size = 1.43

$$-\frac{a^{3} \log\left(\frac{{\left| 2 \, b e^{x} + 2 \, a - 2 \, \sqrt{a^{2} + b^{2}} \right|}}{{\left| 2 \, b e^{x} + 2 \, a + 2 \, \sqrt{a^{2} + b^{2}} \right|}}\right)}{\sqrt{a^{2} + b^{2}} b^{3}} + \frac{b e^{\left(2 \, x\right)} - 4 \, a e^{x}}{8 \, b^{2}} + \frac{{\left(2 \, a^{2} - b^{2}\right)} x}{2 \, b^{3}} - \frac{{\left(4 \, a b e^{x} + b^{2}\right)} e^{\left(-2 \, x\right)}}{8 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) + 1/8*(b*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2
*a^2 - b^2)*x/b^3 - 1/8*(4*a*b*e^x + b^2)*e^(-2*x)/b^3
```

maple [B] time = 0.05, size = 174, normalized size = 2.12

$$-\frac{2 a^{3} \operatorname{arctanh}\left(\frac{2 a \tanh\left(\frac{x}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{b^{3} \sqrt{a^{2}+b^{2}}}+\frac{1}{2 b \left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{2 b \left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a}{b^{2} \left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) a^{2}}{b^{3}}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x)),x)
```

```
[Out] -2*a^3/b^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2
))+1/2/b/(tanh(1/2*x)-1)^2+1/2/b/(tanh(1/2*x)-1)+1/b^2/(tanh(1/2*x)-1)*a-1/
b^3*ln(tanh(1/2*x)-1)*a^2+1/2/b*ln(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^2+1
/2/b/(tanh(1/2*x)+1)-1/b^2/(tanh(1/2*x)+1)*a+1/b^3*ln(tanh(1/2*x)+1)*a^2-1/
2/b*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.42, size = 118, normalized size = 1.44

$$-\frac{a^3 \log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^3} - \frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2} - \frac{4\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^2} + \frac{\left(2\,a^2-b^2\right)x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^3) - 1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x)
 + b*e^(-2*x))/b^2 + 1/2*(2*a^2 - b^2)*x/b^3
```

mupad [B] time = 0.61, size = 159, normalized size = 1.94

$$\frac{e^{2\,x}}{8\,b} - \frac{e^{-2\,x}}{8\,b} + \frac{x\,\left(2\,a^2-b^2\right)}{2\,b^3} - \frac{a\,e^x}{2\,b^2} - \frac{a\,e^{-x}}{2\,b^2} - \frac{a^3\,\ln\left(\frac{2\,a^3\,e^x}{b^4} - \frac{2\,a^3\,(b-a\,e^x)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,\sqrt{a^2+b^2}} + \frac{a^3\,\ln\left(\frac{2\,a^3\,e^x}{b^4} + \frac{2\,a^3\,(b-a\,e^x)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] exp(2*x)/(8*b) - exp(-2*x)/(8*b) + (x*(2*a^2 - b^2))/(2*b^3) - (a*exp(x))/(
2*b^2) - (a*exp(-x))/(2*b^2) - (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b - a*
exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(a^2 + b^2)^(1/2)) + (a^3*log((2*a^
3*exp(x))/b^4 + (2*a^3*(b - a*exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(a^2
+ b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.74 $\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

```
[Out] -a*x/b^2+cosh(x)/b-2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^
2+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2746, 12, 2735, 2660, 618, 206}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt
[a^2 + b^2]) + Cosh[x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)}{b}-\frac{\int \frac{a\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{\cosh(x)}{b}-\frac{a\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}-\frac{\left(4a^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 61, normalized size = 1.07

$$\frac{a\left(\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-x\right)+b\cosh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (a*(-x + (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + b*Cosh[x])/b^2
```

fricas [B] time = 0.58, size = 238, normalized size = 4.18

$$\frac{a^2b+b^3-2\left(a^3+ab^2\right)x\cosh(x)+\left(a^2b+b^3\right)\cosh(x)^2+\left(a^2b+b^3\right)\sinh(x)^2+2\left(a^2\cosh(x)+a^2\sinh(x)\right)\sqrt{a^2+}}{2\left(\left(a^2b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b + b^3 - 2*(a^3 + a*b^2)*x*cosh(x) + (a^2*b + b^3)*cosh(x)^2 + (a
^2*b + b^3)*sinh(x)^2 + 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 + b^2)*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*((a^3
 + a*b^2)*x - (a^2*b + b^3)*cosh(x))*sinh(x))/((a^2*b^2 + b^4)*cosh(x) + (a
^2*b^2 + b^4)*sinh(x))
```

giac [A] time = 0.19, size = 86, normalized size = 1.51

$$\frac{a^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{ax}{b^2}+\frac{e^{(-x)}}{2\,b}+\frac{e^x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

maple [A] time = 0.04, size = 92, normalized size = 1.61

$$\frac{2a^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(x)),x)
```

```
[Out] 2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)
)-1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)+1/b/(tanh(1/2*x)+1)-a/b^2*ln(
tanh(1/2*x)+1)
```

maxima [A] time = 0.50, size = 84, normalized size = 1.47

$$\frac{a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{ax}{b^2}+\frac{e^{(-x)}}{2\,b}+\frac{e^x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

mupad [B] time = 0.54, size = 129, normalized size = 2.26

$$\frac{e^{-x}}{2\,b}+\frac{e^x}{2\,b}-\frac{a\,x}{b^2}-\frac{a^2\,\ln\left(-\frac{2\,a^2\,e^x}{b^3}-\frac{2\,a^2\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}+\frac{a^2\,\ln\left(\frac{2\,a^2\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}-\frac{2\,a^2\,e^x}{b^3}\right)}{b^2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] exp(-x)/(2*b) + exp(x)/(2*b) - (a*x)/b^2 - (a^2*log(- (2*a^2*exp(x))/b^3 -
(2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2)) +
(a^2*log((2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2)) - (2*a^2*exp(x))/b^
3))/(b^2*(a^2 + b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x)),x)
[Out] Timed out
```

3.75 $\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=47

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b}$$

```
[Out] x/b+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2735, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x]),x]
```

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b}\\ &= \frac{x}{b}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{x}{b}+\frac{(4a)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{x}{b}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 1.11

$$\frac{x-\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x]),x]
[Out] (x - (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])/b
```

fricas [B] time = 0.53, size = 134, normalized size = 2.85

$$\frac{\sqrt{a^2+b^2}\,a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+\left(a^2+b^2\right)\cdots}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) + (a^2 + b^2)*x)/(a^2*b + b^3)
```

giac [A] time = 0.25, size = 67, normalized size = 1.43

$$-\frac{a \log\left(\frac{\left|2\, be^{x}+2\, a-2\, \sqrt{a^{2}+b^{2}}\right|}{\left|2\, be^{x}+2\, a+2\, \sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\, b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) + x/b
```

maple [A] time = 0.03, size = 63, normalized size = 1.34

$$-\frac{2a \operatorname{arctanh}\left(\frac{2a \tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b\sqrt{a^{2}+b^{2}}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x)),x)
```

```
[Out] -2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1
/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.42, size = 65, normalized size = 1.38

$$-\frac{a \log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\, b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b) + x/b
```

mupad [B] time = 0.53, size = 99, normalized size = 2.11

$$\frac{x}{b}-\frac{a\, \ln\left(\frac{2\, a\, e^{x}}{b^{2}}-\frac{2\, a\, (b-a\, e^{x})}{b^{2}\, \sqrt{a^{2}+b^{2}}}\right)}{b\, \sqrt{a^{2}+b^{2}}}+\frac{a\, \ln\left(\frac{2\, a\, e^{x}}{b^{2}}+\frac{2\, a\, (b-a\, e^{x})}{b^{2}\, \sqrt{a^{2}+b^{2}}}\right)}{b\, \sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*sinh(x)),x)
```

```
[Out] x/b - (a*log((2*a*exp(x))/b^2 - (2*a*(b - a*exp(x)))/(b^2*(a^2 + b^2)^(1/2)
)))/(b*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(x))/b^2 + (2*a*(b - a*exp(x)))/
(b^2*(a^2 + b^2)^(1/2))))/(b*(a^2 + b^2)^(1/2))
```

sympy [A] time = 58.30, size = 252, normalized size = 5.36

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{\cosh(x)}{a} & \text{for } b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{bx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) - ib\sqrt{b^2}} - \frac{2b}{b^2\tanh\left(\frac{x}{2}\right) - ib\sqrt{b^2}} - \frac{ix\sqrt{b^2}}{b^2\tanh\left(\frac{x}{2}\right) - ib\sqrt{b^2}} & \text{for } a = -\sqrt{-b^2} \\ \frac{bx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) + ib\sqrt{b^2}} - \frac{2b}{b^2\tanh\left(\frac{x}{2}\right) + ib\sqrt{b^2}} + \frac{ix\sqrt{b^2}}{b^2\tanh\left(\frac{x}{2}\right) + ib\sqrt{b^2}} & \text{for } a = \sqrt{-b^2} \\ \frac{a\log\left(\tanh\left(\frac{x}{2}\right) - \frac{b}{a} - \frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{a\log\left(\tanh\left(\frac{x}{2}\right) - \frac{b}{a} + \frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (cosh(x)/a, Eq(b, 0)), (x/b, Eq(a,
0)), (b*x*tanh(x/2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)) - 2*b/(b**2*tanh(x/2)
 - I*b*sqrt(b**2)) - I*x*sqrt(b**2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)), Eq(a
, -sqrt(-b**2))), (b*x*tanh(x/2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)) - 2*b/(b
**2*tanh(x/2) + I*b*sqrt(b**2)) + I*x*sqrt(b**2)/(b**2*tanh(x/2) + I*b*sqrt
(b**2)), Eq(a, sqrt(-b**2))), (a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)
/(b*sqrt(a**2 + b**2)) - a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*sq
rt(a**2 + b**2)) + x/b, True))
```

3.76 $\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=50

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}$$

```
[Out] -arctanh(cosh(x))/a+2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+b
^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2747, 3770, 2660, 618, 206}

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sinh[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a) + (2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a
*Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx &= \frac{\int \operatorname{csch}(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}-\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}+\frac{(4b)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}+\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 1.16

$$\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sinh[x]),x]
```

```
[Out] ((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[T
anh[x/2]])/a
```

fricas [B] time = 0.53, size = 156, normalized size = 3.12

$$\frac{\sqrt{a^2+b^2}\, b \log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-\left(a^2+b^2\right.}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

giac [A] time = 0.23, size = 82, normalized size = 1.64

$$-\frac{b\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a}-\frac{\log\left(e^x+1\right)}{a}+\frac{\log\left(\left|e^x-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a) - log(e^x + 1)/a + log(abs(e^x - 1))/a
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$-\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sinh(x)),x)
```

```
[Out] -2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+1
/a*ln(tanh(1/2*x))
```

maxima [A] time = 0.43, size = 83, normalized size = 1.66

$$-\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}-\frac{\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-x)}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*a) - log(e^(-x) + 1)/a + log(e^(-x) - 1)/a
```

mupad [B] time = 0.64, size = 287, normalized size = 5.74

$$\frac{\ln(32\,a - 32\,a\,e^x)}{a} - \frac{\ln(32\,a + 32\,a\,e^x)}{a} + \frac{b\,\ln\left(128\,a^5\,e^x - 64\,a^2\,b^3 - 64\,a^4\,b - 128\,a^4\,e^x\,\sqrt{a^2+b^2} + 32\,a\,b^4\,e^x + 16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*sinh(x))),x)
```

```
[Out] log(32*a - 32*a*exp(x))/a - log(32*a + 32*a*exp(x))/a + (b*log(128*a^5*exp(
x) - 64*a^2*b^3 - 64*a^4*b - 128*a^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a*b^4*ex
p(x) + 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^
2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b^2 +
 a^3) - (b*log(64*a^4*b + 64*a^2*b^3 - 128*a^5*exp(x) - 128*a^4*exp(x)*(a^2
 + b^2)^(1/2) - 32*a*b^4*exp(x) - 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2 + b^2)
^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*
(a^2 + b^2)^(1/2))/(a*b^2 + a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*sinh(x)), x)
```

3.77 $\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] b*arctanh(cosh(x))/a^2-coth(x)/a-2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^
(1/2))/a^2/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 12, 2747, 3770, 2660, 618, 206}

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(a^2*Sqrt[a^2 + b^2]) - Coth[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx &= -\frac{\coth(x)}{a}-\frac{\int \frac{b\operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{\coth(x)}{a}-\frac{b\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{\coth(x)}{a}-\frac{b\int \operatorname{csch}(x)\,dx}{a^2}+\frac{b^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}-\frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.40, size = 81, normalized size = 1.37

$$
-\frac{2b\left(\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+a\tanh\left(\frac{x}{2}\right)+a\coth\left(\frac{x}{2}\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(a*Coth[x/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] + Log[Tanh[x/2]]) + a*Tanh[x/2])/a^2
```

fricas [B] time = 1.50, size = 345, normalized size = 5.85

$$
\frac{2\,a^3+2\,ab^2-\left(b^2\cosh(x)^2+2\,b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2-b^2\right)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(\ldots}{b\cosh(x)^2+\ldots}\right.}{\ldots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 -
 b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3
)*cosh(x)*sinh(x) - (a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3)*cosh(x)*sinh(x) - (
a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^4 + a^2*b^2 - (a^4 +
 a^2*b^2)*cosh(x)^2 - 2*(a^4 + a^2*b^2)*cosh(x)*sinh(x) - (a^4 + a^2*b^2)*s
inh(x)^2)
```

giac [A] time = 0.35, size = 98, normalized size = 1.66

$$\frac{b^{2}\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}+\frac{b\log\left(e^{x}+1\right)}{a^{2}}-\frac{b\log\left(\left|e^{x}-1\right|\right)}{a^{2}}-\frac{2}{a\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1)
)/a^2 - 2/(a*(e^(2*x) - 1))
```

maple [A] time = 0.05, size = 73, normalized size = 1.24

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2a}+\frac{2b^{2}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{2}\sqrt{a^{2}+b^{2}}}-\frac{1}{2a\tanh\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sinh(x)),x)
```

```
[Out] -1/2/a*tanh(1/2*x)+2*b^2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2
*b)/(a^2+b^2)^(1/2))-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))
```

maxima [A] time = 0.42, size = 100, normalized size = 1.69

$$\frac{b^{2}\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,a^{2}}+\frac{b\log\left(e^{(-x)}+1\right)}{a^{2}}-\frac{b\log\left(e^{(-x)}-1\right)}{a^{2}}+\frac{2}{ae^{(-2\,x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^2) + b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + 2/(
a*e^(-2*x) - a)
```

mupad [B] time = 0.70, size = 292, normalized size = 4.95

$$\frac{2}{a - a\,\mathrm{e}^{2\,x}} - \frac{b\,\ln\left(32\,\mathrm{e}^x - 32\right)}{a^2} + \frac{b\,\ln\left(32\,\mathrm{e}^x + 32\right)}{a^2} + \frac{b^2\,\ln\Big(128\,a^4\,\mathrm{e}^x - 64\,a\,b^3 - 64\,a^3\,b - 32\,b^3\,\sqrt{a^2+b^2} + 32\,b^4\,\mathrm{e}^x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*sinh(x))),x)
```

```
[Out] 2/(a - a*exp(2*x)) - (b*log(32*exp(x) - 32))/a^2 + (b*log(32*exp(x) + 32))/
a^2 + (b^2*log(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1
/2) + 32*b^4*exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x)
 - 64*a^2*b*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b
^2)^(1/2))/(a^4 + a^2*b^2) - (b^2*log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3 -
 64*a^3*b + 128*a^4*exp(x) + 32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^2)^(1/
2) + 160*a^2*b^2*exp(x) + 64*a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(x)*(a^2
 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x)), x)
```

3.78 $\int \frac{\text{csch}^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{b\coth(x)}{a^2}+\frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{\coth(x)\text{csch}(x)}{2a}$$

```
[Out] 1/2*(a^2-2*b^2)*arctanh(cosh(x))/a^3+b*coth(x)/a^2-1/2*coth(x)*csch(x)/a+2*
b^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\text{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^3) + (2*b^3*ArcTanh[(b - a*Tanh[x/2])
/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) + (b*Coth[x])/a^2 - (Coth[x]*Csch[
x])/(2*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = -\frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{i\int \frac{\operatorname{csch}^2(x)\left(2ib+ia\sinh(x)+ib\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{2a}$$

$$= \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} - \frac{\int \frac{\operatorname{csch}(x)\left(a^2-2b^2+ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{2a^2}$$

$$= \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} - \frac{b^3\int \frac{1}{a+b\sinh(x)}\,dx}{a^3} - \frac{\left(a^2-2b^2\right)\int \operatorname{csch}(x)\,dx}{2a^3}$$

$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^3} + \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} - \frac{\left(2b^3\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,\right.}{a^3}$$

$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^3} + \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{\left(4b^3\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx\right.}{a^3}$$

$$= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^3} + \frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} + \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Mathematica [A] time = 0.54, size = 118, normalized size = 1.46

$$-\frac{4\left(a^2-2b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{16b^3\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + a^2\operatorname{csch}^2\left(\frac{x}{2}\right) + a^2\operatorname{sech}^2\left(\frac{x}{2}\right) - 4ab\tanh\left(\frac{x}{2}\right) - 4ab\coth\left(\frac{x}{2}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -1/8*((16*b^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- 4*a*b*Coth[x/2] + a^2*Csch[x/2]^2 + 4*(a^2 - 2*b^2)*Log[Tanh[x/2]] + a^2*
Sech[x/2]^2 - 4*a*b*Tanh[x/2])/a^3
```

fricas [B] time = 1.06, size = 929, normalized size = 11.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^
2*b^2)*cosh(x))*sinh(x)^2 - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^
3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 +
 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^
2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(a^4 + a^2*b^2)*c
osh(x) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cos
h(x)*sinh(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4
- 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 -
 a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)
^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) +
 ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)*sin
h(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4
 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2
 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^3 - (a^
4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^4
+ a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^5 + a^3*b^2 + (a^5 + a^3*b^2)*cosh(x)^4 + 4*(a^5 + a^3*b^2)*cosh(x)*s
inh(x)^3 + (a^5 + a^3*b^2)*sinh(x)^4 - 2*(a^5 + a^3*b^2)*cosh(x)^2 - 2*(a^5
 + a^3*b^2 - 3*(a^5 + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + a^3*b^2)*co
sh(x)^3 - (a^5 + a^3*b^2)*cosh(x))*sinh(x))
```

giac [A] time = 0.21, size = 137, normalized size = 1.69

$$-\frac{b^3\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3}+\frac{\left(a^2-2\,b^2\right)\log\left(e^x+1\right)}{2\,a^3}-\frac{\left(a^2-2\,b^2\right)\log\left(\left|e^x-1\right|\right)}{2\,a^3}-\frac{ae^{(3\,x)}-2\,be^{(2\,x)}+ae^x+2\,b}{a^2\left(e^{(2\,x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/2*(a^2 - 2*b^2)*log(e^x + 1)/a^3 - 1
/2*(a^2 - 2*b^2)*log(abs(e^x - 1))/a^3 - (a*e^(3*x) - 2*b*e^(2*x) + a*e^x +
 2*b)/(a^2*(e^(2*x) - 1)^2)
```

maple [A] time = 0.05, size = 108, normalized size = 1.33

$$\frac{\tanh^2\left(\frac{x}{2}\right)}{8a}+\frac{\tanh\left(\frac{x}{2}\right)b}{2a^2}-\frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{1}{8a\tanh\left(\frac{x}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)b^2}{a^3}+\frac{b}{2a^2\tanh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sinh(x)),x)
```

```
[Out] 1/8/a*tanh(1/2*x)^2+1/2/a^2*tanh(1/2*x)*b-2/a^3*b^3/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/8/a/tanh(1/2*x)^2-1/2/a*ln(ta
nh(1/2*x))+1/a^3*ln(tanh(1/2*x))*b^2+1/2*b/a^2/tanh(1/2*x)
```

maxima [B] time = 0.41, size = 154, normalized size = 1.90

$$-\frac{b^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3}+\frac{ae^{(-x)}+2\,be^{(-2\,x)}+ae^{(-3\,x)}-2\,b}{2\,a^2e^{(-2\,x)}-a^2e^{(-4\,x)}-a^2}+\frac{\left(a^2-2\,b^2\right)\log\left(e^{(-x)}+1\right)}{2\,a^3}-\frac{\left(a^2-2\,b^2\right)\log\left(e^{(-x)}-1\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^3) + (a*e^(-x) + 2*b*e^(-2*x) + a*e^(-3*x) - 2*b)/(2*a^
2*e^(-2*x) - a^2*e^(-4*x) - a^2) + 1/2*(a^2 - 2*b^2)*log(e^(-x) + 1)/a^3 -
1/2*(a^2 - 2*b^2)*log(e^(-x) - 1)/a^3
```

mupad [B] time = 1.00, size = 617, normalized size = 7.62

$$\frac{e^x}{a-a\,e^{2\,x}}-\frac{2\,e^x}{a-2\,a\,e^{2\,x}+a\,e^{4\,x}}-\frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{2\,a}+\frac{\ln\left(4\,a^4+24\,b^4-20\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*sinh(x))),x)
```

```
[Out] exp(x)/(a - a*exp(2*x)) - (2*exp(x))/(a - 2*a*exp(2*x) + a*exp(4*x)) - log(
4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp
(x))/(2*a) + log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x)
 - 20*a^2*b^2*exp(x))/(2*a) + (2*b)/(a^2*exp(2*x) - a^2) + (b^2*log(4*a^4 +
 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/a
^3 - (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) -
20*a^2*b^2*exp(x)))/a^3 - (b^3*log(16*a^5*b - 48*a*b^5 - 24*b^5*(a^2 + b^2)
^(1/2) - 32*a^3*b^3 - 32*a^6*exp(x) + 24*b^6*exp(x) - 40*a^2*b^3*(a^2 + b^2
)^(1/2) - 32*a^5*exp(x)*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 56*a^4*b^2
*exp(x) + 16*a^4*b*(a^2 + b^2)^(1/2) + 72*a*b^4*exp(x)*(a^2 + b^2)^(1/2) +
72*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) + (
b^3*log(24*b^5*(a^2 + b^2)^(1/2) - 48*a*b^5 + 16*a^5*b - 32*a^3*b^3 - 32*a^
6*exp(x) + 24*b^6*exp(x) + 40*a^2*b^3*(a^2 + b^2)^(1/2) + 32*a^5*exp(x)*(a^
2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 16*a^4*b*(a^2 + b
```

```
^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^2*exp(x)*(a^2 + b^
2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sinh(x)),x)
[Out] Integral(csch(x)**3/(a + b*sinh(x)), x)
```

3.79 $\int \frac{\text{csch}^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=109

$$\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

```
[Out] -1/2*b*(a^2-2*b^2)*arctanh(cosh(x))/a^4+1/3*(2*a^2-3*b^2)*coth(x)/a^3+1/2*b
*coth(x)*csch(x)/a^2-1/3*coth(x)*csch(x)^2/a-2*b^4*arctanh((b-a*tanh(1/2*x)
)/(a^2+b^2)^(1/2))/a^4/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3}-\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{b\coth(x)\text{csch}(x)}{2a^2}-\frac{\coth(x)\text{csch}^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -(b*(a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*b^4*ArcTanh[(b - a*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a^4*Sqrt[a^2 + b^2]) + ((2*a^2 - 3*b^2)*Coth[x])/(3*
a^3) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = -\frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{i\int \frac{\operatorname{csch}^3(x)\left(3ib+2ia\sinh(x)+2ib\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{3a}$$

$$= \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} - \frac{\int \frac{\operatorname{csch}^2(x)\left(2\left(2a^2-3b^2\right)+ab\sinh(x)-3b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{6a^2}$$

$$= \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} - \frac{i\int \frac{\operatorname{csch}(x)\left(3ib\left(a^2-2b^2\right)+3iab^2\sinh\right.}{a+b\sinh(x)}}{6a^3}$$

$$= \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{a^4} + \frac{\left(b\left(a^2-\right.\right.}{}$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(}{3a}$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(}{3a}$$

$$= -\frac{b\left(a^2-2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} + \frac{\left(2a^2-3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(}{}$$

Mathematica [A] time = 0.94, size = 186, normalized size = 1.71

$$\frac{8a^3\tanh\left(\frac{x}{2}\right) - \frac{1}{2}a^3\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right) + 8a^3\sinh^4\left(\frac{x}{2}\right)\operatorname{csch}^3(x) + 4a\left(2a^2-3b^2\right)\coth\left(\frac{x}{2}\right) + \frac{48b^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 3}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((48*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 4*a
*(2*a^2 - 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 + 12*a^2*b*Log[Tanh[x/2]]
- 24*b^3*Log[Tanh[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4
 - (a^3*Csch[x/2]^4*Sinh[x])/2 + 8*a^3*Tanh[x/2] - 12*a*b^2*Tanh[x/2])/(24*
a^4)
```

fricas [B] time = 0.56, size = 1676, normalized size = 15.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 8*a^5
- 4*a^3*b^2 - 12*a*b^4 - 12*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(2*a^3*b^2 + 2*
a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 12*(5*(a^4*b + a^2*b^3)*co
sh(x)^2 - 4*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 24*(a^5 - a*b^4)*cosh(x)
^2 - 12*(2*a^5 - 2*a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 + a*b
^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4
*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*
sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*
cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cos
h(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b + a^2*b^3)*cosh(x) -
3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2*b^5
 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 - 5*(
a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 - 2*b
^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b -
 a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a^2*b
^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 +
6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1
) + 3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2
*b^5 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 -
 5*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 -
 2*b^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4
*b - a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a
^2*b^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^
2 + 6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 - 1) - 6*(a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 8*(a^3*b^2 + a
*b^4)*cosh(x)^3 + 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x
)^6 + 6*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 + a^4*b^2)*sinh(x)^6 - a^6
 - a^4*b^2 - 3*(a^6 + a^4*b^2)*cosh(x)^4 - 3*(a^6 + a^4*b^2 - 5*(a^6 + a^4*
b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + a^4*b^2)*cosh(x)^3 - 3*(a^6 + a^4*b
^2)*cosh(x))*sinh(x)^3 + 3*(a^6 + a^4*b^2)*cosh(x)^2 + 3*(a^6 + a^4*b^2 + 5
*(a^6 + a^4*b^2)*cosh(x)^4 - 6*(a^6 + a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a
^6 + a^4*b^2)*cosh(x)^5 - 2*(a^6 + a^4*b^2)*cosh(x)^3 + (a^6 + a^4*b^2)*cos
h(x))*sinh(x))
```

giac [A] time = 0.18, size = 171, normalized size = 1.57

$$\frac{b^4 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^4} - \frac{\left(a^2b-2\,b^3\right)\log\left(e^x+1\right)}{2\,a^4} + \frac{\left(a^2b-2\,b^3\right)\log\left(\left|e^x-1\right|\right)}{2\,a^4} + \frac{3\,abe^{(5\,x)}-6\,b^2e^{(4\,x)}-12\,a^2e^{(2\,x)}}{3\,a^3\left(e^{(2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) - 1/2*(a^2*b - 2*b^3)*log(e^x + 1)/a^4 +
1/2*(a^2*b - 2*b^3)*log(abs(e^x - 1))/a^4 + 1/3*(3*a*b*e^(5*x) - 6*b^2*e^(4
*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 - 6*b^2)/(a^3*(e^
(2*x) - 1)^3)
```

maple [A] time = 0.05, size = 158, normalized size = 1.45

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{24a} - \frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8a^2} + \frac{3\tanh\left(\frac{x}{2}\right)}{8a} - \frac{b^2\tanh\left(\frac{x}{2}\right)}{2a^3} + \frac{2b^4\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{1}{24a\tanh\left(\frac{x}{2}\right)^3} + \frac{3}{8a\tanh\left(\frac{x}{2}\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sinh(x)),x)
```

```
[Out] -1/24/a*tanh(1/2*x)^3-1/8/a^2*b*tanh(1/2*x)^2+3/8/a*tanh(1/2*x)-1/2/a^3*b^2
*tanh(1/2*x)+2/a^4*b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a
^2+b^2)^(1/2))-1/24/a/tanh(1/2*x)^3+3/8/a/tanh(1/2*x)-1/2/a^3/tanh(1/2*x)*b
^2+1/8/a^2*b/tanh(1/2*x)^2+1/2/a^2*b*ln(tanh(1/2*x))-1/a^4*b^3*ln(tanh(1/2*
x))
```

maxima [A] time = 0.43, size = 194, normalized size = 1.78

$$\frac{b^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^4} - \frac{3\,abe^{(-x)}-6\,b^2e^{(-4\,x)}-3\,abe^{(-5\,x)}+4\,a^2-6\,b^2-12\left(a^2-b^2\right)e^{(-2\,x)}}{3\left(3\,a^3e^{(-2\,x)}-3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}-a^3\right)} - \frac{\left(a^2b-2\,b^3\right)\log\left(e^{(-}\right.}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(-x) - 6*b^2*e^(-4*x) - 3*a*b*e^(-5*x)
```

```
 + 4*a^2 - 6*b^2 - 12*(a^2 - b^2)*e^(-2*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x
) + a^3*e^(-6*x) - a^3) - 1/2*(a^2*b - 2*b^3)*log(e^(-x) + 1)/a^4 + 1/2*(a^
2*b - 2*b^3)*log(e^(-x) - 1)/a^4
```

mupad [B] time = 0.88, size = 694, normalized size = 6.37

$$\frac{8}{3\left(a-3\,a\,e^{2\,x}+3\,a\,e^{4\,x}-a\,e^{6\,x}\right)}-\frac{4}{a-2\,a\,e^{2\,x}+a\,e^{4\,x}}-\frac{2\,b^2}{a^3\,e^{2\,x}-a^3}+\frac{b\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\right.}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*sinh(x))),x)
```

```
[Out] 8/(3*(a - 3*a*exp(2*x) + 3*a*exp(4*x) - a*exp(6*x))) - 4/(a - 2*a*exp(2*x)
+ a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) - a^3) + (b*log(4*a^4 + 24*b^4 - 20*a
^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/(2*a^2) - (b*lo
g(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b^2*e
xp(x)))/(2*a^2) - (b^3*log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*
b^4*exp(x) + 20*a^2*b^2*exp(x)))/a^4 + (b^3*log(4*a^4 + 24*b^4 - 20*a^2*b^2
 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b^2*exp(x)))/a^4 + (2*b*exp(x))/(a
^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) + (b*exp(x))/(a^2*exp(2*x) - a^2) + (b^
4*log(16*a^5*b^2 - 48*a*b^6 - 32*a^3*b^4 - 24*b^6*(a^2 + b^2)^(1/2) + 24*b^
7*exp(x) - 40*a^2*b^4*(a^2 + b^2)^(1/2) + 16*a^4*b^2*(a^2 + b^2)^(1/2) - 32
*a^6*b*exp(x) + 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) + 72*a*b^5*exp(x)*(a
^2 + b^2)^(1/2) - 32*a^5*b*exp(x)*(a^2 + b^2)^(1/2) + 72*a^3*b^3*exp(x)*(a^
2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^6 + a^4*b^2) - (b^4*log(24*b^6*(a^2 +
 b^2)^(1/2) - 48*a*b^6 - 32*a^3*b^4 + 16*a^5*b^2 + 24*b^7*exp(x) + 40*a^2*b
^4*(a^2 + b^2)^(1/2) - 16*a^4*b^2*(a^2 + b^2)^(1/2) - 32*a^6*b*exp(x) + 112
*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a*b^5*exp(x)*(a^2 + b^2)^(1/2) + 3
2*a^5*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^3*exp(x)*(a^2 + b^2)^(1/2))*(a^
2 + b^2)^(1/2))/(a^6 + a^4*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*sinh(x)), x)
```

3.80 $\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=162

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(3a^2+b^2\right)\sinh(x)\cosh(x)}{2b^2\left(a^2+b^2\right)}+\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)}+\frac{2a^3\left(3a^2+4b^2\right)\text{t}}{b^4\left(a^2\right.}$$

```
[Out] 1/2*(6*a^2-b^2)*x/b^4+2*a^3*(3*a^2+4*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^
2)^(1/2))/b^4/(a^2+b^2)^(3/2)-a*(3*a^2+2*b^2)*cosh(x)/b^3/(a^2+b^2)+1/2*(3*
a^2+b^2)*cosh(x)*sinh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)^2/b/(a^2+b^2)/(a
+b*sinh(x))
```

Rubi [A] time = 0.41, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2792, 3049, 3023, 2735, 2660, 618, 206}

$$\frac{x\left(6a^2-b^2\right)}{2b^4}-\frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)}+\frac{2a^3\left(3a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}}-\frac{a^2\sinh^2(x)\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(3a^2+b^2\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^2 - b^2)*x)/(2*b^4) + (2*a^3*(3*a^2 + 4*b^2)*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^4*(a^2 + b^2)^(3/2)) - (a*(3*a^2 + 2*b^2)*Cosh[x])/(
b^3*(a^2 + b^2)) + ((3*a^2 + b^2)*Cosh[x]*Sinh[x])/(2*b^2*(a^2 + b^2)) - (a
^2*Cosh[x]*Sinh[x]^2)/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\sinh(x)\left(2a^2-ab\sinh(x)+\left(3a^2+b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{-a\left(3a^2+b^2\right)+b\left(a^2-b^2\right)\sinh(x)-2a(}{a+b\sinh(x)}}{2b^2\left(a^2+b^2\right.}$$

$$= -\frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int}{}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh}{b\left(a^2+b^2\right)(a+b\,s}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh}{b\left(a^2+b^2\right)(a+b\,s}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b^2\right)\cosh(x)\sinh(x)}{2b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh}{b\left(a^2+b^2\right)(a+b\,s}$$

$$= \frac{\left(6a^2-b^2\right)x}{2b^4} + \frac{2a^3\left(3a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\left(a^2+b^2\right)^{3/2}} - \frac{a\left(3a^2+2b^2\right)\cosh(x)}{b^3\left(a^2+b^2\right)} + \frac{\left(3a^2+b\right.}{2b}$$

Mathematica [A] time = 0.44, size = 118, normalized size = 0.73

$$\frac{-2x\left(b^2-6a^2\right) - \frac{4a^4b\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{8a^3\left(3a^2+4b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} - 8ab\cosh(x) + b^2\sinh(2x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*(-6*a^2 + b^2)*x + (8*a^3*(3*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) - 8*a*b*Cosh[x] - (4*a^4*b*Cosh[x])/((a^2
 + b^2)*(a + b*Sinh[x])) + b^2*Sinh[2*x])/(4*b^4)
```

fricas [B] time = 0.68, size = 1769, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^4*b^3 + 2*a^2*b^5 + b^7 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^6 + (a
^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^6 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(
x)^5 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6 - (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)
)*sinh(x)^5 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a
^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^4 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^
5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^2 - 4*(6*a^6*b + 11*a^4*b^
3 + 4*a^2*b^5 - b^7)*x + 30*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^
4 + 8*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh
(x)^3 + 4*(4*a^7 + 4*a^5*b^2 + 5*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^3 - 15
*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^2 + 2*(6*a^7 + 11*a^5*b^2 + 4*a^3*b^
4 - a*b^6)*x - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*
a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - (32*a^6*b + 49*a^4*b^3 +
 18*a^2*b^5 + b^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^2
 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x)^4 + 60*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 6*(16*a^6*b + 3
3*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x
)*cosh(x)^2 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x - 24*(2*a^7 + 2*
a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh(x))*sinh(x)^2 +
8*((3*a^5*b + 4*a^3*b^3)*cosh(x)^4 + (3*a^5*b + 4*a^3*b^3)*sinh(x)^4 + 2*(3
*a^6 + 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 + 4*a^4*b^2 + 2*(3*a^5*b + 4*a^3*b^3
)*cosh(x))*sinh(x)^3 - (3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - (3*a^5*b + 4*a^3*b
^3 - 6*(3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - 6*(3*a^6 + 4*a^4*b^2)*cosh(x))*sin
h(x)^2 + 2*(2*(3*a^5*b + 4*a^3*b^3)*cosh(x)^3 + 3*(3*a^6 + 4*a^4*b^2)*cosh(
x)^2 - (3*a^5*b + 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cos
h(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b
)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 6*(a^5*b^2 + 2
*a^3*b^4 + a*b^6)*cosh(x) + 2*(3*a^5*b^2 + 6*a^3*b^4 + 3*a*b^6 + 3*(a^4*b^3
 + 2*a^2*b^5 + b^7)*cosh(x)^5 - 15*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^4
- 2*(16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4
*a^2*b^5 - b^7)*x)*cosh(x)^3 + 12*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2
+ 4*a^3*b^4 - a*b^6)*x)*cosh(x)^2 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b
^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x))/((a^4*
b^5 + 2*a^2*b^7 + b^9)*cosh(x)^4 + (a^4*b^5 + 2*a^2*b^7 + b^9)*sinh(x)^4 +
```

```
2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x)^3 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8
+ 2*(a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 - (a^4*b^5 + 2*a^2*b^7 +
 b^9)*cosh(x)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9 - 6*(a^4*b^5 + 2*a^2*b^7 + b^9
)*cosh(x)^2 - 6*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 + 2*(2*(a^
4*b^5 + 2*a^2*b^7 + b^9)*cosh(x)^3 + 3*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x
)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

giac [A] time = 0.18, size = 235, normalized size = 1.45

$$-\frac{\left(3\,a^{5}+4\,a^{3}b^{2}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}+\frac{\left(6\,a^{2}-b^{2}\right)x}{2\,b^{4}}+\frac{b^{2}e^{(2\,x)}-8\,abe^{x}}{8\,b^{4}}+\frac{\left(a^{2}b^{3}+b^{5}+8\left(2\,a^{5}-a^{3}b^{2}-ab^{4}\right)e^{(3\,x)}-\cdots}{8\left(a^{2}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(3*a^5 + 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/2*(6*a^2
 - b^2)*x/b^4 + 1/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(a^2*b^3 + b^5 + 8*
(2*a^5 - a^3*b^2 - a*b^4)*e^(3*x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(2*x) +
 6*(a^3*b^2 + a*b^4)*e^x)*e^(-2*x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b
^4)
```

maple [A] time = 0.07, size = 296, normalized size = 1.83

$$\frac{2a^{3}\tanh\left(\frac{x}{2}\right)}{b^{2}\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^{2}+b^{2}\right)}+\frac{2a^{4}}{b^{3}\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^{2}+b^{2}\right)}-\frac{6a^{5}\operatorname{arctanh}\left(\frac{2a\,\cdots}{2\,\cdots}\right.}{b^{4}\left(a^{2}+b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] 2/b^2*a^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)+2/b^3*a
^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)-6/b^4*a^5/(a^2+b^2)^(3/2)*
arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-8/b^2*a^3/(a^2+b^2)^(3/2
)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+1/2/b^2/(tanh(1/2*x)-1
)^2+1/2/b^2/(tanh(1/2*x)-1)+2/b^3/(tanh(1/2*x)-1)*a-3/b^4*ln(tanh(1/2*x)-1)
*a^2+1/2/b^2*ln(tanh(1/2*x)-1)-1/2/b^2/(tanh(1/2*x)+1)^2+1/2/b^2/(tanh(1/2*
x)+1)-2/b^3/(tanh(1/2*x)+1)*a+3/b^4*ln(tanh(1/2*x)+1)*a^2-1/2/b^2*ln(tanh(1
/2*x)+1)
```

maxima [A] time = 0.42, size = 256, normalized size = 1.58

$$-\frac{\left(3\,a^2+4\,b^2\right)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{a^2b^3+b^5-6\left(a^3b^2+ab^4\right)e^{(-x)}-\left(32\,a^4b+17\,a^2b^3+b^5\right)e^{(-2\,x)}-8\left(2\,a^5\right.}{8\left(\left(a^2b^5+b^7\right)e^{(-2\,x)}+2\left(a^3b^4+ab^6\right)e^{(-3\,x)}-\left(a^2b^5+b^7\right)e^{(-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(3*a^2 + 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/8*(a^2*b^3 + b^5 - 6
*(a^3*b^2 + a*b^4)*e^(-x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(-2*x) - 8*(2*a
^5 - a^3*b^2 - a*b^4)*e^(-3*x))/((a^2*b^5 + b^7)*e^(-2*x) + 2*(a^3*b^4 + a*
b^6)*e^(-3*x) - (a^2*b^5 + b^7)*e^(-4*x)) - 1/8*(8*a*e^(-x) + b*e^(-2*x))/b
^3 + 1/2*(6*a^2 - b^2)*x/b^4
```

mupad [B] time = 0.86, size = 305, normalized size = 1.88

$$\frac{e^{2\,x}}{8\,b^2}-\frac{e^{-2\,x}}{8\,b^2}-\frac{\frac{2\,a^4}{b^2\left(a^2\,b+b^3\right)}-\frac{2\,a^5\,e^x}{b^3\left(a^2\,b+b^3\right)}}{2\,a\,e^x-b+b\,e^{2\,x}}+\frac{x\,\left(6\,a^2-b^2\right)}{2\,b^4}-\frac{a\,e^x}{b^3}-\frac{a\,e^{-x}}{b^3}-\frac{a^3\,\ln\left(\frac{2\,e^x\,\left(3\,a^5+4\,a^3\,b^2\right)}{a^2\,b^5+b^7}-\frac{2\,a^3\,\left(b-a\,e^x\right)\left(3\,a^2+4\,b^2\right)}{b^5\,\left(a^2+b^2\right)^{3/2}}\right)\left(3\,a\right.}{b^4\,\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*a^4)/(b^2*(a^2*b + b^3)) - (2*a^
5*exp(x))/(b^3*(a^2*b + b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2 -
b^2))/(2*b^4) - (a*exp(x))/b^3 - (a*exp(-x))/b^3 - (a^3*log((2*exp(x)*(3*a^
5 + 4*a^3*b^2))/(b^7 + a^2*b^5) - (2*a^3*(b - a*exp(x))*(3*a^2 + 4*b^2))/(b
^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b^2)^(3/2)) + (a^3*log(
(2*exp(x)*(3*a^5 + 4*a^3*b^2))/(b^7 + a^2*b^5) + (2*a^3*(b - a*exp(x))*(3*a
^2 + 4*b^2))/(b^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b^2)^(3/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.81 $\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=115

$$\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{a^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{2a^2(2a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3(a^2+b^2)^{3/2}}-\frac{2ax}{b^3}$$

```
[Out] -2*a*x/b^3-2*a^2*(2*a^2+3*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b
^3/(a^2+b^2)^(3/2)+(2*a^2+b^2)*cosh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)/b/
(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.24, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2792, 3023, 2735, 2660, 618, 206}

$$\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{2a^2(2a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3(a^2+b^2)^{3/2}}-\frac{a^2\sinh(x)\cosh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*x)/b^3 - (2*a^2*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^3*(a^2 + b^2)^(3/2)) + ((2*a^2 + b^2)*Cosh[x])/(b^2*(a^2 + b^2))
- (a^2*Cosh[x]*Sinh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{a^2-ab\sinh(x)+\left(2a^2+b^2\right)\sinh^2(x)}{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int \frac{-ia^2b+2ia\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{b^2\left(a^2+b^2\right)}$$

$$= -\frac{2ax}{b^3} + \frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\left(a^2\left(2a^2+3b^2\right)\right)\int \frac{1}{a+b\sinh(x)}}{b^3\left(a^2+b^2\right)}$$

$$= -\frac{2ax}{b^3} + \frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\left(2a^2\left(2a^2+3b^2\right)\right)\text{Subst}\left(\int\right.}{b^3\left(a^2+\right.}$$

$$= -\frac{2ax}{b^3} + \frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(4a^2\left(2a^2+3b^2\right)\right)\text{Subst}\left(\int\right.}{b^3}$$

$$= -\frac{2ax}{b^3} - \frac{2a^2\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\left(a^2+b^2\right)^{3/2}} + \frac{\left(2a^2+b^2\right)\cosh(x)}{b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh}{b\left(a^2+b^2\right)(a+b\,\text{s}}$$

Mathematica [A] time = 0.37, size = 95, normalized size = 0.83

$$\frac{-\frac{2a^2\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} + \cosh(x)\left(\frac{a^3b}{\left(a^2+b^2\right)(a+b\sinh(x))}+b\right) - 2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*x - (2*a^2*(2*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]]
)/(-a^2 - b^2)^(3/2) + Cosh[x]*(b + (a^3*b)/((a^2 + b^2)*(a + b*Sinh[x]))))
/b^3
```

fricas [B] time = 0.78, size = 1053, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 - (
a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^
5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^3 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*x + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x)^2 + 2*(2*
a^6 + 2*a^4*b^2 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 + 4*(a^6 + 2*a^4*
b^2 + a^2*b^4)*x - 3*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^5*b + 2*a^3*b^3 + a*
b^5)*x)*cosh(x))*sinh(x)^2 - 2*((2*a^4*b + 3*a^2*b^3)*cosh(x)^3 + (2*a^4*b
+ 3*a^2*b^3)*sinh(x)^3 + 2*(2*a^5 + 3*a^3*b^2)*cosh(x)^2 + (4*a^5 + 6*a^3*b
^2 + 3*(2*a^4*b + 3*a^2*b^3)*cosh(x))*sinh(x)^2 - (2*a^4*b + 3*a^2*b^3)*cos
h(x) - (2*a^4*b + 3*a^2*b^3 - 3*(2*a^4*b + 3*a^2*b^3)*cosh(x)^2 - 4*(2*a^5
+ 3*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^5
 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^
5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^5*b + 2*a^3*b^3 + a*b^5
- 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5
)*x - 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x))*sinh(x))
/((a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x)^3 + (a^4*b^4 + 2*a^2*b^6 + b^8)*sinh(
x)^3 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x)^2 + (2*a^5*b^3 + 4*a^3*b^5 +
 2*a*b^7 + 3*(a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x))*sinh(x)^2 - (a^4*b^4 + 2*
a^2*b^6 + b^8)*cosh(x) - (a^4*b^4 + 2*a^2*b^6 + b^8 - 3*(a^4*b^4 + 2*a^2*b^
6 + b^8)*cosh(x)^2 - 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x))*sinh(x))
```

giac [A] time = 0.23, size = 184, normalized size = 1.60

$$\frac{\left(2\,a^4+3\,a^2b^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2b^3+b^5\right)\sqrt{a^2+b^2}}-\frac{2\,ax}{b^3}+\frac{e^x}{2\,b^2}-\frac{\left(a^2b^2+b^4+\left(4\,a^4-a^2b^2-b^4\right)e^{(2\,x)}-2\left(3\,a^3b+ab^3\right)e^x\right)e^{(-x)}}{2\left(a^2+b^2\right)\left(be^{(2\,x)}+2\,ae^x-b\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^4 + 3*a^2*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) - 2*a*x/b^3 +
 1/2*e^x/b^2 - 1/2*(a^2*b^2 + b^4 + (4*a^4 - a^2*b^2 - b^4)*e^(2*x) - 2*(3*
a^3*b + a*b^3)*e^x)*e^(-x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b^3)
```

maple [A] time = 0.06, size = 213, normalized size = 1.85

$$-\frac{2a^2\tanh\left(\frac{x}{2}\right)}{b\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}-\frac{2a^3}{b^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}+\frac{4a^4\operatorname{arctanh}\left(\frac{2a}{}\right.}{b^3\left(a^2+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] -2/b*a^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)-2/b^2*a^
3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)+4/b^3*a^4/(a^2+b^2)^(3/2)*a
rctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+6/b*a^2/(a^2+b^2)^(3/2)*a
rctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/b^2/(tanh(1/2*x)-1)+2*a
/b^3*ln(tanh(1/2*x)-1)+1/b^2/(tanh(1/2*x)+1)-2*a/b^3*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.44, size = 208, normalized size = 1.81

$$\frac{\left(2\,a^2+3\,b^2\right)a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2b^3+b^5\right)\sqrt{a^2+b^2}}+\frac{a^2b^2+b^4+2\left(3\,a^3b+ab^3\right)e^{(-x)}+\left(4\,a^4-a^2b^2-b^4\right)e^{(-2\,x)}}{2\left(\left(a^2b^4+b^6\right)e^{(-x)}+2\left(a^3b^3+ab^5\right)e^{(-2\,x)}-\left(a^2b^4+b^6\right)e^{(-3\,x)}\right)}-\frac{2\,ax}{b^3}+\frac{e^{(-x)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2 + 3*b^2)*a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) + 1/2*(a^2*b^2 + b^4 + 2*
(3*a^3*b + a*b^3)*e^(-x) + (4*a^4 - a^2*b^2 - b^4)*e^(-2*x))/((a^2*b^4 + b^
6)*e^(-x) + 2*(a^3*b^3 + a*b^5)*e^(-2*x) - (a^2*b^4 + b^6)*e^(-3*x)) - 2*a*
x/b^3 + 1/2*e^(-x)/b^2
```

mupad [B] time = 0.78, size = 274, normalized size = 2.38

$$\frac{e^{-x}}{2\,b^2}+\frac{\frac{2\,a^3}{b\left(a^2\,b+b^3\right)}-\frac{2\,a^4\,e^x}{b^2\left(a^2\,b+b^3\right)}}{2\,a\,e^x-b+b\,e^{2\,x}}+\frac{e^x}{2\,b^2}-\frac{2\,a\,x}{b^3}-\frac{a^2\,\ln\left(-\frac{2\,e^x\left(2\,a^4+3\,a^2\,b^2\right)}{a^2\,b^4+b^6}-\frac{2\,a^2\left(b-a\,e^x\right)\left(2\,a^2+3\,b^2\right)}{b^4\left(a^2+b^2\right)^{3/2}}\right)\left(2\,a^2+3\,b^2\right)}{b^3\left(a^2+b^2\right)^{3/2}}+\frac{a^2\,\ln\left(\frac{2\,a^2\,(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] exp(-x)/(2*b^2) + ((2*a^3)/(b*(a^2*b + b^3)) - (2*a^4*exp(x))/(b^2*(a^2*b +
 b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + exp(x)/(2*b^2) - (2*a*x)/b^3 - (a^2
```

```
*log(- (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4) - (2*a^2*(b - a*exp(x
))*(2*a^2 + 3*b^2))/(b^4*(a^2 + b^2)^(3/2)))*(2*a^2 + 3*b^2))/(b^3*(a^2 + b
^2)^(3/2)) + (a^2*log((2*a^2*(b - a*exp(x))*(2*a^2 + 3*b^2))/(b^4*(a^2 + b^
2)^(3/2)) - (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4))*(2*a^2 + 3*b^2)
)/(b^3*(a^2 + b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.82 $\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=83

$$\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2+2*a*(a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^2+b
^2)^(3/2)-a^2*cosh(x)/b/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2790, 2735, 2660, 618, 206}

$$\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2
*(a^2 + b^2)^(3/2)) - (a^2*Cosh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{i\int \frac{-iab+i\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}$$

$$= \frac{x}{b^2} - \frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(a\left(a^2+2b^2\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b^2\left(a^2+b^2\right)}$$

$$= \frac{x}{b^2} - \frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(2a\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2\left(a^2+b^2\right)}$$

$$= \frac{x}{b^2} - \frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\left(4a\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\,\text{ta}\right.}{b^2\left(a^2+b^2\right)}$$

$$= \frac{x}{b^2} + \frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}$$

Mathematica [A] time = 0.19, size = 86, normalized size = 1.04

$$\frac{\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (x + (2*a*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 -
 b^2)^(3/2) - (a^2*b*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/b^2
```

fricas [B] time = 0.63, size = 521, normalized size = 6.28

$$2\,a^4b+2\,a^2b^3-\left(a^4b+2\,a^2b^3+b^5\right)x\cosh(x)^2-\left(a^4b+2\,a^2b^3+b^5\right)x\sinh(x)^2+\left(a^3b+2\,ab^3-\left(a^3b+2\,ab^3\right)\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^4*b + 2*a^2*b^3 - (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^2 - (a^4*b + 2*a
^2*b^3 + b^5)*x*sinh(x)^2 + (a^3*b + 2*a*b^3 - (a^3*b + 2*a*b^3)*cosh(x)^2
- (a^3*b + 2*a*b^3)*sinh(x)^2 - 2*(a^4 + 2*a^2*b^2)*cosh(x) - 2*(a^4 + 2*a^
2*b^2 + (a^3*b + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x
)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*s
inh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*si
nh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (a^4*b + 2*a^2*b^
3 + b^5)*x - 2*(a^5 + a^3*b^2 + (a^5 + 2*a^3*b^2 + a*b^4)*x)*cosh(x) - 2*(a
^5 + a^3*b^2 + (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x) + (a^5 + 2*a^3*b^2 + a*b
^4)*x)*sinh(x))/(a^4*b^3 + 2*a^2*b^5 + b^7 - (a^4*b^3 + 2*a^2*b^5 + b^7)*co
sh(x)^2 - (a^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^2 - 2*(a^5*b^2 + 2*a^3*b^4 +
a*b^6)*cosh(x) - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6 + (a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x))*sinh(x))
```

giac [A] time = 0.39, size = 131, normalized size = 1.58

$$-\frac{\left(a^3+2\,ab^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(a^3e^x-a^2b\right)}{\left(a^2b^2+b^4\right)\left(be^{(2x)}+2\,ae^x-b\right)}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*e^x - a
^2*b)/((a^2*b^2 + b^4)*(b*e^(2*x) + 2*a*e^x - b)) + x/b^2
```

maple [B] time = 0.06, size = 175, normalized size = 2.11

$$\frac{2a\tanh\left(\frac{x}{2}\right)}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}+\frac{2a^2}{b\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}-\frac{2a^3\operatorname{arctanh}\left(\frac{2a\tan}{2\sqrt{}}\right.}{b^2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] 2*a/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)+2/b*a^2/(a*ta
nh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)-2/b^2*a^3/(a^2+b^2)^(3/2)*arctanh(
1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-4*a/(a^2+b^2)^(3/2)*arctanh(1/2*
(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tan
h(1/2*x)+1)
```

maxima [A] time = 0.41, size = 149, normalized size = 1.80

$$-\frac{\left(a^2+2\,b^2\right)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(a^3e^{(-x)}+a^2b\right)}{a^2b^3+b^5+2\left(a^3b^2+ab^4\right)e^{(-x)}-\left(a^2b^3+b^5\right)e^{(-2\,x)}}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(a^2 + 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(
a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^3*e^(-x) + a^2*b)/(a^
2*b^3 + b^5 + 2*(a^3*b^2 + a*b^4)*e^(-x) - (a^2*b^3 + b^5)*e^(-2*x)) + x/b^
2
```

mupad [B] time = 0.79, size = 228, normalized size = 2.75

$$\frac{x}{b^2}-\frac{\frac{2\,a^2}{a^2\,b+b^3}-\frac{2\,a^3\,\mathrm{e}^x}{b\left(a^2\,b+b^3\right)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}}-\frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\left(a^3+2\,a\,b^2\right)}{b^3\left(a^2+b^2\right)}-\frac{2\,a\left(a^2+2\,b^2\right)\left(b-a\,\mathrm{e}^x\right)}{b^3\left(a^2+b^2\right)^{3/2}}\right)\left(a^2+2\,b^2\right)}{b^2\left(a^2+b^2\right)^{3/2}}+\frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\left(a^3+2\,a\,b^2\right)}{b^3\left(a^2+b^2\right)}+\frac{2\,a\left(a^2+2\,b^2\right)\left(b-a\,\mathrm{e}\right.}{b^3\left(a^2+b^2\right)^{3/2}}\right.}{b^2\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] x/b^2 - ((2*a^2)/(a^2*b + b^3) - (2*a^3*exp(x))/(b*(a^2*b + b^3)))/(2*a*exp
(x) - b + b*exp(2*x)) - (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^2 + b^2))
 - (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a^2 + 2*b^2
))/(b^2*(a^2 + b^2)^(3/2)) + (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^2 +
b^2)) + (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a^2 +
2*b^2))/(b^2*(a^2 + b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.83 $\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=60

$$\frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+a*cosh(x)/(
a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2754, 12, 2660, 618, 206}

$$\frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (a*Co
sh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx &= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int \frac{b}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{(4b)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 68, normalized size = 1.13

$$
\frac{a\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{2b\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (a*C
osh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

fricas [B] time = 0.55, size = 341, normalized size = 5.68

$$-\frac{2\,a^3b+2\,ab^3+\left(b^3\cosh(x)^2+b^3\sinh(x)^2+2\,ab^2\cosh(x)-b^3+2\left(b^3\cosh(x)+ab^2\right)\sinh(x)\right)\sqrt{a^2+b^2}\,\log\left(\right.}{a^4b^2+2\,a^2b^4+b^6-\left(a^4b^2+2\,a^2b^4+b^6\right)\cosh(x)^2-\left(a^4b^2+2\,a^2b^4+b^6\right)\sinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b + 2*a*b^3 + (b^3*cosh(x)^2 + b^3*sinh(x)^2 + 2*a*b^2*cosh(x) - b^
3 + 2*(b^3*cosh(x) + a*b^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b
^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x)
- 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4 + a^2*b^2)*cosh(x
) - 2*(a^4 + a^2*b^2)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^
2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2 + 2*a
^2*b^4 + b^6)*cosh(x))*sinh(x))
```

giac [A] time = 0.20, size = 99, normalized size = 1.65

$$\frac{b\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^2e^x-ab\right)}{\left(a^2b+b^3\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^2*e^x - a*b)/((a^2*b + b^3)*(b*e^(2*x) +
 2*a*e^x - b))
```

maple [A] time = 0.04, size = 97, normalized size = 1.62

$$\frac{8\tanh\left(\frac{x}{2}\right)b+8a}{\left(-4a^2-4b^2\right)\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}-\frac{8b\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(-4a^2-4b^2\right)\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x))^2,x)
```

```
[Out] 4*(2*tanh(1/2*x)*b+2*a)/(-4*a^2-4*b^2)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-
8*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2))
```

maxima [B] time = 0.42, size = 117, normalized size = 1.95

$$\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(a^2e^{(-x)}+ab\right)}{a^2b^2+b^4+2\left(a^3b+ab^3\right)e^{(-x)}-\left(a^2b^2+b^4\right)e^{(-2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(a
^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)/(a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*e
^(-x) - (a^2*b^2 + b^4)*e^(-2*x))
```

mupad [B] time = 0.70, size = 142, normalized size = 2.37

$$\frac{\frac{2ab}{a^2b+b^3}-\frac{2a^2e^x}{a^2b+b^3}}{2ae^x-b+be^{2x}}-\frac{b\ln\left(-\frac{2e^x}{a^2+b^2}-\frac{2(b-ae^x)}{\left(a^2+b^2\right)^{3/2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{b\ln\left(\frac{2(b-ae^x)}{\left(a^2+b^2\right)^{3/2}}-\frac{2e^x}{a^2+b^2}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*a*b)/(a^2*b + b^3) - (2*a^2*exp(x))/(a^2*b + b^3))/(2*a*exp(x) - b + b*
exp(2*x)) - (b*log(- (2*exp(x))/(a^2 + b^2) - (2*(b - a*exp(x)))/(a^2 + b^2
)^(3/2)))/(a^2 + b^2)^(3/2) + (b*log((2*(b - a*exp(x)))/(a^2 + b^2)^(3/2) -
 (2*exp(x))/(a^2 + b^2)))/(a^2 + b^2)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.84 $\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\tanh^{-1}(\cosh(x))}{a^2}$$

```
[Out] -arctanh(cosh(x))/a^2+2*b*(2*a^2+b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(
1/2))/a^2/(a^2+b^2)^(3/2)+b^2*cosh(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.22, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2802, 3001, 3770, 2660, 618, 206}

$$\frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\tanh^{-1}(\cosh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a^2) + (2*b*(2*a^2 + b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)) + (b^2*Cosh[x])/(a*(a^2 + b^2)*(a + b
*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\text{csch}(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\text{csch}(x)\left(a^2+b^2-ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \text{csch}(x)\,dx}{a^2} - \frac{\left(b\left(2a^2+b^2\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{\tanh^{-1}(\cosh(x))}{a^2} + \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(2b\left(2a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,d\right.}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{\tanh^{-1}(\cosh(x))}{a^2} + \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\left(4b\left(2a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\right.}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{\tanh^{-1}(\cosh(x))}{a^2} + \frac{2b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}} + \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}$$

Mathematica [A] time = 0.20, size = 91, normalized size = 1.07

$$\frac{\frac{2b\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} + \frac{ab^2\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \log\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*b*(2*a^2 + b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2
)^(3/2) + Log[Tanh[x/2]] + (a*b^2*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/a
^2
```

fricas [B] time = 0.72, size = 672, normalized size = 7.91

$$-\frac{2\,a^3b^2 + 2\,ab^4 - \left(2\,a^2b^2+b^4-\left(2\,a^2b^2+b^4\right)\cosh(x)^2-\left(2\,a^2b^2+b^4\right)\sinh(x)^2-2\left(2\,a^3b+ab^3\right)\cosh(x)-2\,(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b^2 + 2*a*b^4 - (2*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cosh(x)^2 - (2
*a^2*b^2 + b^4)*sinh(x)^2 - 2*(2*a^3*b + a*b^3)*cosh(x) - 2*(2*a^3*b + a*b^
3 + (2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
+ b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x
)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4*b + a^2*b^3)*c
osh(x) + (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (
a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) -
2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))*lo
g(cosh(x) + sinh(x) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 +
b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 +
 a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*co
sh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(a^4*b + a^2*b^3)*sinh(x))/(
a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x)^2 - (a^
6*b + 2*a^4*b^3 + a^2*b^5)*sinh(x)^2 - 2*(a^7 + 2*a^5*b^2 + a^3*b^4)*cosh(x
) - 2*(a^7 + 2*a^5*b^2 + a^3*b^4 + (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x))*s
inh(x))
```

giac [A] time = 0.19, size = 142, normalized size = 1.67

$$-\frac{\left(2\,a^{2}b+b^{3}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+a^{2}b^{2}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(abe^{x}-b^{2}\right)}{\left(a^{3}+ab^{2}\right)\left(be^{(2\,x)}+2\,ae^{x}-b\right)}-\frac{\log\left(e^{x}+1\right)}{a^{2}}+\frac{\log\left(\left|e^{x}-1\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*b + b^3)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) - 2*(a*b*e^x - b
^2)/((a^3 + a*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - log(e^x + 1)/a^2 + log(abs(
e^x - 1))/a^2
```

maple [B] time = 0.06, size = 166, normalized size = 1.95

$$-\frac{2b^{3}\tanh\left(\frac{x}{2}\right)}{a^{2}\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^{2}+b^{2}\right)}-\frac{2b^{2}}{a\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^{2}+b^{2}\right)}-\frac{4b\operatorname{arctanh}\left(\frac{2a\tanh\ldots}{2\ldots}\right)}{\left(a^{2}+b^{2}\right)\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sinh(x))^2,x)
```

```
[Out] -2/a^2*b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)-2/a*b^
2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)-4*b/(a^2+b^2)^(3/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-2/a^2*b^3/(a^2+b^2)^(3/2)*arcta
nh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+1/a^2*ln(tanh(1/2*x))
```

maxima [A] time = 0.43, size = 162, normalized size = 1.91

$$-\frac{\left(2\,a^2b+b^3\right)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^4+a^2b^2\right)\sqrt{a^2+b^2}}+\frac{2\left(abe^{(-x)}+b^2\right)}{a^3b+ab^3+2\left(a^4+a^2b^2\right)e^{(-x)}-\left(a^3b+ab^3\right)e^{(-2\,x)}}-\frac{\log\left(e^{(-x)}+1\right)}{a^2}+\frac{\log\left(e^{(-\cdots\right.}}{a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2*b + b^3)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(
a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) + 2*(a*b*e^(-x) + b^2)/(a^3*
b + a*b^3 + 2*(a^4 + a^2*b^2)*e^(-x) - (a^3*b + a*b^3)*e^(-2*x)) - log(e^(-
x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

mupad [B] time = 2.70, size = 1001, normalized size = 11.78

$$b\,\ln\left(\frac{32\left(-8\,e^x\,a^5+4\,a^4\,b-10\,e^x\,a^3\,b^2+6\,a^2\,b^3-3\,e^x\,a\,b^4+2\,b^5\right)}{a\left(a^4\,b^3+a^2\,b^5\right)\left(a^2\,b+b^3\right)}+\frac{b\left(\frac{32\left(-4\,e^x\,a^7+2\,a^6\,b\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\frac{2\,b^5}{a\left(a^2\,b^3+b^5\right)}-\frac{2\,b^4\,e^x}{a^2\,b^3+b^5}}{2\,a\,e^x-b+b\,e^{2\,x}}+\frac{\ln\left(e^x-1\right)}{a^2}-\frac{\ln\left(e^x+1\right)}{a^2}-\frac{\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*sinh(x))^2),x)
```

```
[Out] ((2*b^5)/(a*(b^5 + a^2*b^3)) - (2*b^4*exp(x))/(b^5 + a^2*b^3))/(2*a*exp(x)
- b + b*exp(2*x)) + log(exp(x) - 1)/a^2 - log(exp(x) + 1)/a^2 - (b*log((32*
(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*exp(x) - 10*a^3*b^2*e
xp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) + (b*((32*(2*a^6*b + 2*b^7 +
8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x) - 10*a^3*b^4*exp(x) -
 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2)) - (b*(2*a^2 + b^2)*(
(a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^4*exp(x) - 4*a^2*b^2*exp
```

```
(x)))/(b*(b^5 + a^2*b^3)) + (32*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)*(3*a^4*
b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^4*(a^8 + a^2*b^6 + 3*a
^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a^2 + b^
2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a^2 +
 b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2) + (b*l
og((32*(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*exp(x) - 10*a^
3*b^2*exp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) - (b*((32*(2*a^6*b + 2
*b^7 + 8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x) - 10*a^3*b^4*e
xp(x) - 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2)) + (b*(2*a^2 +
 b^2)*((a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^4*exp(x) - 4*a^2*
b^2*exp(x)))/(b*(b^5 + a^2*b^3)) - (32*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)*
(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^4*(a^8 + a^2*b^
6 + 3*a^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a
^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(
2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)/(a + b*sinh(x))**2, x)
```

3.85 $\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=115

$$\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)}+\frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}-\frac{2b^2(3a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}}$$

```
[Out] 2*b*arctanh(cosh(x))/a^3-2*b^2*(3*a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2
+b^2)^(1/2))/a^3/(a^2+b^2)^(3/2)-(a^2+2*b^2)*coth(x)/a^2/(a^2+b^2)+b^2*coth
(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.37, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2b^2(3a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}}-\frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)}+\frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}+\frac{2b\tanh^{-1}(\cosh(x))}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*b^2*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2
])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)) - ((a^2 + 2*b^2)*Coth[x])/(a^2
*(a^2 + b^2)) + (b^2*Coth[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\text{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\text{csch}^2(x)\left(a^2+2b^2-ab\sinh(x)+b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int \frac{\text{csch}(x)\left(2ib\left(a^2+b^2\right)-iab^2\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{(2b)\int \text{csch}(x)\,dx}{a^3} + \frac{\left(b^2\left(3a^2+2b\right.\right.}{a^3\left(\right.}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\left(2b^2\left(3a^2+\right.\right.}{}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(4b^2\left(3a^2+\right.\right.}{}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2b^2\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}} - \frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)} + \frac{}{a\left(\right.}$$

Mathematica [A] time = 0.68, size = 118, normalized size = 1.03

$$-\frac{\frac{4b^2\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} + \frac{2ab^3\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + a\tanh\left(\frac{x}{2}\right) + a\coth\left(\frac{x}{2}\right) + 4b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*((4*b^2*(3*a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-
a^2 - b^2)^(3/2) + a*Coth[x/2] + 4*b*Log[Tanh[x/2]] + (2*a*b^3*Cosh[x])/((a
^2 + b^2)*(a + b*Sinh[x])) + a*Tanh[x/2])/a^3
```

fricas [B] time = 0.81, size = 1740, normalized size = 15.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^5*b + 6*a^3*b^3 + 4*a*b^5 + 2*(a^4*b^2 + a^2*b^4)*cosh(x)^3 + 2*(a^4*b
^2 + a^2*b^4)*sinh(x)^3 - 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 - 2*(a^
5*b + 3*a^3*b^3 + 2*a*b^5 - 3*(a^4*b^2 + a^2*b^4)*cosh(x))*sinh(x)^2 + (3*a
^2*b^3 + 2*b^5 + (3*a^2*b^3 + 2*b^5)*cosh(x)^4 + (3*a^2*b^3 + 2*b^5)*sinh(x
)^4 + 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 2*(3*a^3*b^2 + 2*a*b^4 + 2*(3*a^2
*b^3 + 2*b^5)*cosh(x))*sinh(x)^3 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 2*(3*a
^2*b^3 + 2*b^5 - 3*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3*(3*a^3*b^2 + 2*a*b^4)*
cosh(x))*sinh(x)^2 - 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x) - 2*(3*a^3*b^2 + 2*a*b
^4 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 +
2*(3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
+ b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x
)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(2*a^6 + 5*a^4*b^2
+ 3*a^2*b^4)*cosh(x) + 2*(a^4*b^2 + 2*a^2*b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4
+ b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2
*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 -
2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*
(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b
^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*
cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*(a^4*b^2 + 2*a^2*
b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 +
b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^
2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(
x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5
 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*
cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) - 2*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 3*(a^4*b^2 + a^2*b^4)*cosh
(x)^2 + 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x))*sinh(x))/(a^7*b + 2*a^5*b^
3 + a^3*b^5 + (a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^4 + (a^7*b + 2*a^5*b^3
+ a^3*b^5)*sinh(x)^4 + 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x)^3 + 2*(a^8 + 2
*a^6*b^2 + a^4*b^4 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x)^3 - 2
*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5 -
 3*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 3*(a^8 + 2*a^6*b^2 + a^4*b^4)*
cosh(x))*sinh(x)^2 - 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x) - 2*(a^8 + 2*a^6
*b^2 + a^4*b^4 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^3 - 3*(a^8 + 2*a^6
*b^2 + a^4*b^4)*cosh(x)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x
))
```

giac [A] time = 0.18, size = 205, normalized size = 1.78

$$\frac{\left(3\,a^2b^2+2\,b^4\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^5+a^3b^2\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^2e^{(3\,x)}-a^2be^{(2\,x)}-2\,b^3e^{(2\,x)}-2\,a^3e^x-3\,ab^2e^x+a^2b+2\,b^3\right)}{\left(a^4+a^2b^2\right)\left(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^x+b\right)}+\frac{2\,b\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^2*b^2 + 2*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^
(3*x) - a^2*b*e^(2*x) - 2*b^3*e^(2*x) - 2*a^3*e^x - 3*a*b^2*e^x + a^2*b + 2
*b^3)/((a^4 + a^2*b^2)*(b*e^(4*x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b
)) + 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3
```

maple [A] time = 0.08, size = 193, normalized size = 1.68

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2a^2}+\frac{2b^4\tanh\left(\frac{x}{2}\right)}{a^3\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}+\frac{2b^3}{a^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}+\frac{6b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] -1/2/a^2*tanh(1/2*x)+2/a^3*b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2
)*tanh(1/2*x)+2/a^2*b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)+6/a*b
^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+4/a^3
*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/2
/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))
```

maxima [B] time = 0.41, size = 251, normalized size = 2.18

$$\frac{\left(3\,a^2b^2+2\,b^4\right)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^5+a^3b^2\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^2e^{(-3\,x)}-a^2b-2\,b^3-\left(2\,a^3+3\,ab^2\right)e^{(-x)}+\left(a^2b+2\,b^3\right)\right.}{a^4b+a^2b^3+2\left(a^5+a^3b^2\right)e^{(-x)}-2\left(a^4b+a^2b^3\right)e^{(-2\,x)}-2\left(a^5+a^3b^2\right)e^{(-3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (3*a^2*b^2 + 2*b^4)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^(-3*x) - a^2
```

```
*b - 2*b^3 - (2*a^3 + 3*a*b^2)*e^(-x) + (a^2*b + 2*b^3)*e^(-2*x))/(a^4*b +
a^2*b^3 + 2*(a^5 + a^3*b^2)*e^(-x) - 2*(a^4*b + a^2*b^3)*e^(-2*x) - 2*(a^5
+ a^3*b^2)*e^(-3*x) + (a^4*b + a^2*b^3)*e^(-4*x)) + 2*b*log(e^(-x) + 1)/a^3
 - 2*b*log(e^(-x) - 1)/a^3
```

mupad [B] time = 3.39, size = 1017, normalized size = 8.84

$$\frac{\frac{2\left(25\,a^8\,b^6+90\,a^6\,b^8+96\,a^4\,b^{10}+32\,a^2\,b^{12}\right)}{a^4\,b^2\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)}-\frac{2\,\mathrm{e}^x\left(50\,a^9\,b^6+155\,a^7\,b^8+152\,a^5\,b^{10}+48\,a^3\,b^{12}\right)}{a^4\,b^3\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)}-\frac{2\,\mathrm{e}^{2\,x}\left(25\,a^8\,b^6+90\,a^6\,b^8+96\,a^4\,b^{10}+32\,a^2\,b^{12}\right)}{a^4\,b^2\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)}+\frac{2\,\mathrm{e}\cdots}{a^4\,b^3\cdots}}{b-2\,a\,\mathrm{e}^x+2\,a\,\mathrm{e}^{3\,x}-2\,b\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{4\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*sinh(x))^2),x)
```

```
[Out] ((2*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^8 + 25*a^8*b^6))/(a^4*b^2*(16*b^9
 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (2*exp(x)*(48*a^3*b^12 + 152*a^
5*b^10 + 155*a^7*b^8 + 50*a^9*b^6))/(a^4*b^3*(16*b^9 + 56*a^2*b^7 + 65*a^4*
b^5 + 25*a^6*b^3)) - (2*exp(2*x)*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^8 +
25*a^8*b^6))/(a^4*b^2*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) + (2
*exp(3*x)*(16*a^3*b^12 + 40*a^5*b^10 + 25*a^7*b^8))/(a^4*b^3*(16*b^9 + 56*a
^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) + 2*a*exp(3*x) - 2*b*ex
p(2*x) + b*exp(4*x)) - (2*b*log(exp(x) - 1))/a^3 + (2*b*log(exp(x) + 1))/a^
3 + (b^2*log(- (64*(3*a^2 + 2*b^2)*(4*a^2*b + 4*b^3 - 8*a^3*exp(x) - 7*a*b^
2*exp(x)))/(a^6*b*(a^2 + b^2)^2) - (32*(3*a^2 + 2*b^2)*(8*a^9*b - 8*b^7*((a
^2 + b^2)^3)^(1/2) + 3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^10*exp(x) -
 24*a^2*b^5*((a^2 + b^2)^3)^(1/2) - 18*a^4*b^3*((a^2 + b^2)^3)^(1/2) - 9*a^
4*b^6*exp(x) - 33*a^6*b^4*exp(x) - 40*a^8*b^2*exp(x) + 41*a^3*b^4*exp(x)*((
a^2 + b^2)^3)^(1/2) + 30*a^5*b^2*exp(x)*((a^2 + b^2)^3)^(1/2) + 14*a*b^6*ex
p(x)*((a^2 + b^2)^3)^(1/2)))/(a^6*b*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4))*(
(a^2 + b^2)^3)^(1/2)*(3*a^2 + 2*b^2))/(a^9 + a^3*b^6 + 3*a^5*b^4 + 3*a^7*b^
2) - (b^2*log((32*(3*a^2 + 2*b^2)*(8*a^9*b + 8*b^7*((a^2 + b^2)^3)^(1/2) +
3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^10*exp(x) + 24*a^2*b^5*((a^2 + b
^2)^3)^(1/2) + 18*a^4*b^3*((a^2 + b^2)^3)^(1/2) - 9*a^4*b^6*exp(x) - 33*a^6
*b^4*exp(x) - 40*a^8*b^2*exp(x) - 41*a^3*b^4*exp(x)*((a^2 + b^2)^3)^(1/2) -
 30*a^5*b^2*exp(x)*((a^2 + b^2)^3)^(1/2) - 14*a*b^6*exp(x)*((a^2 + b^2)^3)^
(1/2)))/(a^6*b*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4) - (64*(3*a^2 + 2*b^2)*(
4*a^2*b + 4*b^3 - 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^6*b*(a^2 + b^2)^2))*((
a^2 + b^2)^3)^(1/2)*(3*a^2 + 2*b^2))/(a^9 + a^3*b^6 + 3*a^5*b^4 + 3*a^7*b^2
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x))**2, x)
```

3.86 $\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=158

$$-\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{2b^3\left(4a^2+3b^2\right)\tanh^{-1}\left(\right.}{a^4\left(a^2+b^2\right)^{3/2}}$$

```
[Out] 1/2*(a^2-6*b^2)*arctanh(cosh(x))/a^4+2*b^3*(4*a^2+3*b^2)*arctanh((b-a*tanh(
1/2*x))/(a^2+b^2)^(1/2))/a^4/(a^2+b^2)^(3/2)+b*(2*a^2+3*b^2)*coth(x)/a^3/(a
^2+b^2)-1/2*(a^2+3*b^2)*coth(x)*csch(x)/a^2/(a^2+b^2)+b^2*coth(x)*csch(x)/a
/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.68, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$\frac{2b^3\left(4a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\left(a^2+b^2\right)^{3/2}}+\frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)}+\frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{\left(a^2+3b^2\right)\coth(x)\mathrm{cscl}}{2a^2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - 6*b^2)*ArcTanh[Cosh[x]])/(2*a^4) + (2*b^3*(4*a^2 + 3*b^2)*ArcTanh[(
b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^4*(a^2 + b^2)^(3/2)) + (b*(2*a^2 + 3*
b^2)*Coth[x])/(a^3*(a^2 + b^2)) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*a^2*(a
^2 + b^2)) + (b^2*Coth[x]*Csch[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}^3(x)\left(a^2+3b^2-ab\sinh(x)+2b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int \frac{\operatorname{csch}^2(x)\left(2ib\left(2a^2+3b^2\right)+ia\left(a^2-b\right.\right.}{a+b\,\mathrm{s}}}{2a^2\left(a\right.}$$

$$= \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\int \frac{\mathrm{csc}}{}}{}$$

$$= \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(a^2-\right.}{}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{}{a\left(a\right.}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)} + \frac{}{a\left(a\right.}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{2b^3\left(4a^2+3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\left(a^2+b^2\right)^{3/2}} + \frac{b\left(2a^2+3b^2\right)\mathrm{coth}}{a^3\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.74, size = 156, normalized size = 0.99

$$\frac{-4\left(a^2-6b^2\right)\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{8ab^4\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{16b^3\left(4a^2+3b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} - a^2\operatorname{csch}^2\left(\frac{x}{2}\right) - a^2\operatorname{sech}^2\left(\frac{x}{2}\right) + 8ab}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((16*b^3*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
- b^2)^(3/2) + 8*a*b*Coth[x/2] - a^2*Csch[x/2]^2 - 4*(a^2 - 6*b^2)*Log[Tanh
```

```
[x/2]] - a^2*Sech[x/2]^2 + (8*a*b^4*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
+ 8*a*b*Tanh[x/2])/(8*a^4)
```

fricas [B] time = 1.10, size = 3754, normalized size = 23.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a^5*b^2 + 20*a^3*b^4 + 12*a*b^6 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)
*cosh(x)^5 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*sinh(x)^5 - 4*(a^7 - 4*a^3*b
^4 - 3*a*b^6)*cosh(x)^4 - 2*(2*a^7 - 8*a^3*b^4 - 6*a*b^6 + 5*(a^6*b + 4*a^4
*b^3 + 3*a^2*b^5)*cosh(x))*sinh(x)^4 + 8*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*
cosh(x)^3 + 4*(4*a^6*b + 10*a^4*b^3 + 6*a^2*b^5 - 5*(a^6*b + 4*a^4*b^3 + 3*
a^2*b^5)*cosh(x)^2 - 4*(a^7 - 4*a^3*b^4 - 3*a*b^6)*cosh(x))*sinh(x)^3 - 4*(
a^7 + 6*a^5*b^2 + 11*a^3*b^4 + 6*a*b^6)*cosh(x)^2 - 4*(a^7 + 6*a^5*b^2 + 11
*a^3*b^4 + 6*a*b^6 + 5*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*cosh(x)^3 + 6*(a^7 -
 4*a^3*b^4 - 3*a*b^6)*cosh(x)^2 - 6*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cosh(
x))*sinh(x)^2 + 2*((4*a^2*b^4 + 3*b^6)*cosh(x)^6 + (4*a^2*b^4 + 3*b^6)*sinh
(x)^6 - 4*a^2*b^4 - 3*b^6 + 2*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 2*(4*a^3*b^
3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^5 - 3*(4*a^2*b^4 + 3*b
^6)*cosh(x)^4 - (12*a^2*b^4 + 9*b^6 - 15*(4*a^2*b^4 + 3*b^6)*cosh(x)^2 - 10
*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^4 - 4*(4*a^3*b^3 + 3*a*b^5)*cosh(x)
^3 - 4*(4*a^3*b^3 + 3*a*b^5 - 5*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 5*(4*a^3*b^
3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^3 + 3*(4*a^
2*b^4 + 3*b^6)*cosh(x)^2 + (12*a^2*b^4 + 9*b^6 + 15*(4*a^2*b^4 + 3*b^6)*cos
h(x)^4 + 20*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^3 - 18*(4*a^2*b^4 + 3*b^6)*cosh(x
)^2 - 12*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^2 + 2*(4*a^3*b^3 + 3*a*b^5)
*cosh(x) + 2*(4*a^3*b^3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x)^5 + 5*(4*
a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 6*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 6*(4*a^3*b
^3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x))*sqrt(a^2
+ b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2
*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a
))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b
)) - 2*(7*a^6*b + 16*a^4*b^3 + 9*a^2*b^5)*cosh(x) - (a^6*b - 4*a^4*b^3 - 11
*a^2*b^5 - 6*b^7 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^
6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x)^5 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3
*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4
*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b
^5 - 18*b^7 - 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a
^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*
b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6
*a*b^6 - 5*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*
```

a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b
^5 - 6*b^7)*cosh(x))*sinh(x)^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)
*cosh(x)^2 - (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^
4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^3 - 18*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2
- 12*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 -
4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^
4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7
- 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*
a^2*b^5 - 6*b^7)*cosh(x)^3 - 6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cos
h(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(c
osh(x) + sinh(x) + 1) + (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7 - (a^6*b -
4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5
- 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^5
- 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2
*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^
7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 - 15*(a^6*b - 4*
a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a^7 - 4*a^5*b^2 - 11*a^3*b^4
- 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*
cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 - 5*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b
^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)
^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - (3*a^6*b - 12*a
^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*
cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 - 18*(a^6
*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 12*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*
a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a
^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 -
6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(7
*a^6*b + 16*a^4*b^3 + 9*a^2*b^5 + 5*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*cosh(x)
^4 + 8*(a^7 - 4*a^3*b^4 - 3*a*b^6)*cosh(x)^3 - 12*(2*a^6*b + 5*a^4*b^3 + 3*
a^2*b^5)*cosh(x)^2 + 4*(a^7 + 6*a^5*b^2 + 11*a^3*b^4 + 6*a*b^6)*cosh(x))*si
nh(x))/(a^8*b + 2*a^6*b^3 + a^4*b^5 - (a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)
^6 - (a^8*b + 2*a^6*b^3 + a^4*b^5)*sinh(x)^6 - 2*(a^9 + 2*a^7*b^2 + a^5*b^4
)*cosh(x)^5 - 2*(a^9 + 2*a^7*b^2 + a^5*b^4 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5
)*cosh(x))*sinh(x)^5 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^4 + (3*a^8*b
+ 6*a^6*b^3 + 3*a^4*b^5 - 15*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^2 - 10*
(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x))*sinh(x)^4 + 4*(a^9 + 2*a^7*b^2 + a^5*b
^4)*cosh(x)^3 + 4*(a^9 + 2*a^7*b^2 + a^5*b^4 - 5*(a^8*b + 2*a^6*b^3 + a^4*b
^5)*cosh(x)^3 - 5*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x)^2 + 3*(a^8*b + 2*a^6*
b^3 + a^4*b^5)*cosh(x))*sinh(x)^3 - 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)
^2 - (3*a^8*b + 6*a^6*b^3 + 3*a^4*b^5 + 15*(a^8*b + 2*a^6*b^3 + a^4*b^5)*co

```
sh(x)^4 + 20*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x)^3 - 18*(a^8*b + 2*a^6*b^3
+ a^4*b^5)*cosh(x)^2 - 12*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x))*sinh(x)^2 -
2*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x) - 2*(a^9 + 2*a^7*b^2 + a^5*b^4 + 3*(a
^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^5 + 5*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(
x)^4 - 6*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^3 - 6*(a^9 + 2*a^7*b^2 + a^5
*b^4)*cosh(x)^2 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x))*sinh(x))
```

giac [A] time = 0.17, size = 203, normalized size = 1.28

$$-\frac{\left(4\,a^2b^3+3\,b^5\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+a^4b^2\right)\sqrt{a^2+b^2}}-\frac{2\left(ab^3e^x-b^4\right)}{\left(a^5+a^3b^2\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}+\frac{\left(a^2-6\,b^2\right)\log\left(e^x+1\right)}{2\,a^4}-\frac{\left(a^2-6\,b^2\right)\mathrm{lo}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(4*a^2*b^3 + 3*b^5)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) - 2*(a*b^3*e
^x - b^4)/((a^5 + a^3*b^2)*(b*e^(2*x) + 2*a*e^x - b)) + 1/2*(a^2 - 6*b^2)*l
og(e^x + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(abs(e^x - 1))/a^4 - (a*e^(3*x) - 4*
b*e^(2*x) + a*e^x + 4*b)/(a^3*(e^(2*x) - 1)^2)
```

maple [A] time = 0.09, size = 227, normalized size = 1.44

$$\frac{\tanh^2\left(\frac{x}{2}\right)}{8a^2}+\frac{\tanh\left(\frac{x}{2}\right)b}{a^3}-\frac{2b^5\tanh\left(\frac{x}{2}\right)}{a^4\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}-\frac{2b^4}{a^3\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] 1/8/a^2*tanh(1/2*x)^2+1/a^3*tanh(1/2*x)*b-2/a^4*b^5/(a*tanh(1/2*x)^2-2*tanh
(1/2*x)*b-a)/(a^2+b^2)*tanh(1/2*x)-2/a^3*b^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)
*b-a)/(a^2+b^2)-8/a^2*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2))-6/a^4*b^5/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*
b)/(a^2+b^2)^(1/2))-1/8/a^2/tanh(1/2*x)^2-1/2/a^2*ln(tanh(1/2*x))+3/a^4*ln(
tanh(1/2*x))*b^2+b/a^3/tanh(1/2*x)
```

maxima [B] time = 0.43, size = 363, normalized size = 2.30

$$-\frac{\left(4\,a^2b^3+3\,b^5\right)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^6+a^4b^2\right)\sqrt{a^2+b^2}}+\frac{4\,a^2b^2+6\,b^4+\left(7\,a^3b+9\,ab^3\right)e^{(-x)}-2\left(a^4+5\,a^2b^2+6\,b^4\right)e^{(-2\,x)}-4\left(2\,a\right.}{a^5b+a^3b^3+2\left(a^6+a^4b^2\right)e^{(-x)}-3\left(a^5b+a^3b^3\right)e^{(-2\,x)}-4\left(a^6+a^4b^2\right)e^{(-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(4*a^2*b^3 + 3*b^5)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) + (4*a^2*b^2 + 6*b^4 + (
7*a^3*b + 9*a*b^3)*e^(-x) - 2*(a^4 + 5*a^2*b^2 + 6*b^4)*e^(-2*x) - 4*(2*a^3
*b + 3*a*b^3)*e^(-3*x) - 2*(a^4 - a^2*b^2 - 3*b^4)*e^(-4*x) + (a^3*b + 3*a*
b^3)*e^(-5*x))/(a^5*b + a^3*b^3 + 2*(a^6 + a^4*b^2)*e^(-x) - 3*(a^5*b + a^3
*b^3)*e^(-2*x) - 4*(a^6 + a^4*b^2)*e^(-3*x) + 3*(a^5*b + a^3*b^3)*e^(-4*x)
+ 2*(a^6 + a^4*b^2)*e^(-5*x) - (a^5*b + a^3*b^3)*e^(-6*x)) + 1/2*(a^2 - 6*b
^2)*log(e^(-x) + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(e^(-x) - 1)/a^4
```

mupad [B] time = 3.40, size = 977, normalized size = 6.18

$$\frac{\frac{4\,b}{a^3}-\frac{e^x}{a^2}}{e^{2\,x}-1}+\frac{\frac{2\,b^7}{a^3\,\left(a^2\,b^3+b^5\right)}-\frac{2\,b^6\,e^x}{a^2\,\left(a^2\,b^3+b^5\right)}}{2\,a\,e^x-b+b\,e^{2\,x}}-\frac{\ln\left(e^x-1\right)\,\left(a^2-6\,b^2\right)}{2\,a^4}+\frac{\ln\left(e^x+1\right)\,\left(a^2-6\,b^2\right)}{2\,a^4}-\frac{2\,e^x}{a^2\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}+\frac{b^3\,\ln\left(8\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*sinh(x))^2),x)
```

```
[Out] ((4*b)/a^3 - exp(x)/a^2)/(exp(2*x) - 1) + ((2*b^7)/(a^3*(b^5 + a^2*b^3)) -
(2*b^6*exp(x))/(a^2*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2*x)) - (log(
exp(x) - 1)*(a^2 - 6*b^2))/(2*a^4) + (log(exp(x) + 1)*(a^2 - 6*b^2))/(2*a^4
) - (2*exp(x))/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) + (b^3*log((8*(4*a^2 + 3*b
^2)*(20*a^9*b^5 - 72*b^11*((a^2 + b^2)^3)^(1/2) - 9*a^3*b^11 - 30*a^5*b^9 -
 18*a^7*b^7 - 2*a^13*b + 15*a^11*b^3 + 4*a^14*exp(x) - 192*a^2*b^9*((a^2 +
b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^2)^3)^(1/2) + 27*a^4*b^10*exp(x) + 72
*a^6*b^8*exp(x) + 30*a^8*b^6*exp(x) - 48*a^10*b^4*exp(x) - 29*a^12*b^2*exp(
x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 206*a^5*b^6*exp(x)*((a^2 +
b^2)^3)^(1/2) + 8*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) + 118*a*b^10*exp(x)*((
a^2 + b^2)^3)^(1/2)))/(a^9*b^2*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4) - (8*(1
8*b^4 - 4*a^4 + 21*a^2*b^2)*(2*a^4*b - 12*b^5 - 10*a^2*b^3 - 4*a^5*exp(x) +
 21*a*b^4*exp(x) + 19*a^3*b^2*exp(x)))/(a^9*b^2*(a^2 + b^2)^2))*((a^2 + b^2
)^3)^(1/2)*(4*a^2 + 3*b^2))/(a^10 + a^4*b^6 + 3*a^6*b^4 + 3*a^8*b^2) - (b^3
*log((8*(4*a^2 + 3*b^2)*(2*a^13*b - 72*b^11*((a^2 + b^2)^3)^(1/2) + 9*a^3*b
^11 + 30*a^5*b^9 + 18*a^7*b^7 - 20*a^9*b^5 - 15*a^11*b^3 - 4*a^14*exp(x) -
192*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^2)^3)^(1/2) - 27*
a^4*b^10*exp(x) - 72*a^6*b^8*exp(x) - 30*a^8*b^6*exp(x) + 48*a^10*b^4*exp(x
) + 29*a^12*b^2*exp(x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 206*a^5
*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 8*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) +
118*a*b^10*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^9*b^2*((a^2 + b^2)^3)^(1/2)*(a
```

```
^2 + b^2)^4) - (8*(18*b^4 - 4*a^4 + 21*a^2*b^2)*(2*a^4*b - 12*b^5 - 10*a^2*
b^3 - 4*a^5*exp(x) + 21*a*b^4*exp(x) + 19*a^3*b^2*exp(x)))/(a^9*b^2*(a^2 +
b^2)^2))*((a^2 + b^2)^3)^(1/2)*(4*a^2 + 3*b^2))/(a^10 + a^4*b^6 + 3*a^6*b^4
 + 3*a^8*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**3/(a + b*sinh(x))**2, x)
```

3.87 $\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=198

$$-\frac{\left(a^2+4b^2\right)\coth(x)\operatorname{csch}^2(x)}{3a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5}-\frac{2b^4\left(5a^2+4b^2\right)\tanh^{-1}}{a^5\left(a^2+b^2\right)^3}$$

```
[Out] -b*(a^2-4*b^2)*arctanh(cosh(x))/a^5-2*b^4*(5*a^2+4*b^2)*arctanh((b-a*tanh(1
/2*x))/(a^2+b^2)^(1/2))/a^5/(a^2+b^2)^(3/2)+1/3*(2*a^4-7*a^2*b^2-12*b^4)*co
th(x)/a^4/(a^2+b^2)+b*(a^2+2*b^2)*coth(x)*csch(x)/a^3/(a^2+b^2)-1/3*(a^2+4*
b^2)*coth(x)*csch(x)^2/a^2/(a^2+b^2)+b^2*coth(x)*csch(x)^2/a/(a^2+b^2)/(a+b
*sinh(x))
```

Rubi [A] time = 0.88, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2b^4\left(5a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5\left(a^2+b^2\right)^{3/2}}+\frac{\left(-7a^2b^2+2a^4-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)}-\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5}-\frac{\left(a^2+4b^2\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] -((b*(a^2 - 4*b^2)*ArcTanh[Cosh[x]])/a^5) - (2*b^4*(5*a^2 + 4*b^2)*ArcTanh[
(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^5*(a^2 + b^2)^(3/2)) + ((2*a^4 - 7*a
^2*b^2 - 12*b^4)*Coth[x])/(3*a^4*(a^2 + b^2)) + (b*(a^2 + 2*b^2)*Coth[x]*Cs
ch[x])/(a^3*(a^2 + b^2)) - ((a^2 + 4*b^2)*Coth[x]*Csch[x]^2)/(3*a^2*(a^2 +
b^2)) + (b^2*Coth[x]*Csch[x]^2)/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\text{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\int \frac{\text{csch}^4(x)\left(a^2+4b^2-ab\sinh(x)+3b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{i\int \frac{\text{csch}^3(x)\left(6ib\left(a^2+2b^2\right)+ia\left(2a^2\cdots\right.\right.}{a+b\cdots}}{3a^2\left(\cdots\right.}$$

$$= \frac{b\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2(x)}{3a^2\left(a^2+b^2\right)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}$$

$$= \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2\cdots}{3a^2\left(a^2+b^2\right)}$$

$$= \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a^3\left(a^2+b^2\right)} - \frac{\left(a^2+4b^2\right)\coth(x)\text{csch}^2\cdots}{3a^2\left(a^2+b^2\right)}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} + \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\text{cs}\cdots}{a^3\left(a^2+b^2\right)}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} + \frac{\left(2a^4-7a^2b^2-12b^4\right)\coth(x)}{3a^4\left(a^2+b^2\right)} + \frac{b\left(a^2+2b^2\right)\coth(x)\text{cs}\cdots}{a^3\left(a^2+b^2\right)}$$

$$= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{2b^4\left(5a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5\left(a^2+b^2\right)^{3/2}} + \frac{\left(2a^4-7a^2b^2-\cdots\right.}{3a^4\left(a^2\cdots\right.}$$

Mathematica [A] time = 0.96, size = 214, normalized size = 1.08

$$\frac{-\frac{1}{2}a^3\sinh(x)\text{csch}^4\left(\frac{x}{2}\right)+8a^3\sinh^4\left(\frac{x}{2}\right)\text{csch}^3(x)+4a\left(2a^2-9b^2\right)\tanh\left(\frac{x}{2}\right)+4a\left(2a^2-9b^2\right)\coth\left(\frac{x}{2}\right)-\frac{24ab^5\text{co}\cdots}{\left(a^2+b^2\right)(a+b\cdots}}{24a^5\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((-48*b^4*(5*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
 - b^2)^(3/2) + 4*a*(2*a^2 - 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 + 24*(a
 - 2*b)*b*(a + 2*b)*Log[Tanh[x/2]] + 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*
Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - (24*a*b^5*Cosh[x])/((a^2 + b^2)
*(a + b*Sinh[x])) + 4*a*(2*a^2 - 9*b^2)*Tanh[x/2])/(24*a^5)
```

fricas [B] time = 0.96, size = 6430, normalized size = 32.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*a^7*b - 10*a^5*b^3 - 38*a^3*b^5 - 24*a*b^7 - 6*(a^6*b^2 + 3*a^4*b^4
 + 2*a^2*b^6)*cosh(x)^7 - 6*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*sinh(x)^7 - 6
*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^6 - 6*(2*a^7*b + a^5*b^3
 - 5*a^3*b^5 - 4*a*b^7 + 7*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x))*sinh(
x)^6 + 6*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)^5 + 6*(7*a^6*b^2 + 1
7*a^4*b^4 + 10*a^2*b^6 - 21*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^2 - 6
*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^5 + 6*(2*a^7*b
- 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x)^4 + 6*(2*a^7*b - 5*a^5*b^3 - 1
9*a^3*b^5 - 12*a*b^7 - 35*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^3 - 15*
(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 5*(7*a^6*b^2 + 17*a^4
*b^4 + 10*a^2*b^6)*cosh(x))*sinh(x)^4 + 6*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 -
 14*a^2*b^6)*cosh(x)^3 + 6*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2*b^6 - 3
5*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^4 - 20*(2*a^7*b + a^5*b^3 - 5*a
^3*b^5 - 4*a*b^7)*cosh(x)^3 + 10*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh
(x)^2 + 4*(2*a^7*b - 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x))*sinh(x)^3
- 2*(2*a^7*b - 23*a^5*b^3 - 61*a^3*b^5 - 36*a*b^7)*cosh(x)^2 - 2*(2*a^7*b -
 23*a^5*b^3 - 61*a^3*b^5 - 36*a*b^7 + 63*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*
cosh(x)^5 + 45*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 30*(7*
a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)^3 - 18*(2*a^7*b - 5*a^5*b^3 - 19
*a^3*b^5 - 12*a*b^7)*cosh(x)^2 - 9*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2
*b^6)*cosh(x))*sinh(x)^2 - 3*((5*a^2*b^5 + 4*b^7)*cosh(x)^8 + (5*a^2*b^5 +
4*b^7)*sinh(x)^8 + 2*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^7 + 2*(5*a^3*b^4 + 4*a*b
^6 + 4*(5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^7 + 5*a^2*b^5 + 4*b^7 - 4*(5*a^
2*b^5 + 4*b^7)*cosh(x)^6 - 2*(10*a^2*b^5 + 8*b^7 - 14*(5*a^2*b^5 + 4*b^7)*c
osh(x)^2 - 7*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)^6 - 6*(5*a^3*b^4 + 4*a*
b^6)*cosh(x)^5 - 2*(15*a^3*b^4 + 12*a*b^6 - 28*(5*a^2*b^5 + 4*b^7)*cosh(x)^
3 - 21*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 + 12*(5*a^2*b^5 + 4*b^7)*cosh(x))*si
nh(x)^5 + 6*(5*a^2*b^5 + 4*b^7)*cosh(x)^4 + 2*(15*a^2*b^5 + 12*b^7 + 35*(5*
a^2*b^5 + 4*b^7)*cosh(x)^4 + 35*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 - 30*(5*a^2
*b^5 + 4*b^7)*cosh(x)^2 - 15*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)^4 + 6*(
```

5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 + 2*(15*a^3*b^4 + 12*a*b^6 + 28*(5*a^2*b^5 + 4*b^7)*cosh(x)^5 + 35*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^4 - 40*(5*a^2*b^5 + 4*b^7)*cosh(x)^3 - 30*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 + 12*(5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^3 - 4*(5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 2*(10*a^2*b^5 + 8*b^7 - 14*(5*a^2*b^5 + 4*b^7)*cosh(x)^6 - 21*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^5 + 30*(5*a^2*b^5 + 4*b^7)*cosh(x)^4 + 30*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 - 18*(5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 9*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)^2 - 2*(5*a^3*b^4 + 4*a*b^6)*cosh(x) + 2*(4*(5*a^2*b^5 + 4*b^7)*cosh(x)^7 - 5*a^3*b^4 - 4*a*b^6 + 7*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^6 - 12*(5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 15*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^4 + 12*(5*a^2*b^5 + 4*b^7)*cosh(x)^3 + 9*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 - 4*(5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(4*a^8 - 7*a^6*b^2 - 29*a^4*b^4 - 18*a^2*b^6)*cosh(x) + 3*((a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^8 + (a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*sinh(x)^8 + a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8 + 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^7 + 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x)^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^6 - 2*(2*a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^6 - 6*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^5 - 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b^5 - 12*a*b^7 - 28*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x)^5 + 6*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 2*(3*a^6*b^2 - 6*a^4*b^4 - 21*a^2*b^6 - 12*b^8 + 35*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 - 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 15*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^4 + 6*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 + 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b^5 - 12*a*b^7 + 28*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 40*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 30*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x)^3 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 2*(2*a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^6 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^5 + 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 30*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 - 18*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^2 - 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x) - 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^7 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^6 +

12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 15*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^8 + (a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*sinh(x)^8 + a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8 + 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^7 + 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x)^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^6 - 2*(2*a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^6 - 6*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^5 - 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b^5 - 12*a*b^7 - 28*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x)^5 + 6*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 2*(3*a^6*b^2 - 6*a^4*b^4 - 21*a^2*b^6 - 12*b^8 + 35*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 - 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 15*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^4 + 6*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 + 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b^5 - 12*a*b^7 + 28*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 40*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 30*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x)^3 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 2*(2*a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^6 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^5 + 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 30*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 - 18*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^2 - 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x) - 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^7 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^6 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 15*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 9*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(4*a^8 - 7*a^6*b^2 - 29*a^4*b^4 - 18*a^2*b^6 + 21*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^6 + 18*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^5 - 15*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)^4 - 12*(2*a^7*b - 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x)^3 - 9*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2*b^6)*cosh(x)^2 + 2*(2*a^7*b - 23*a^5*b^3 - 61*a^3*b^5 - 36*a*b^7)*cosh(x))*sinh(x))/(a^9*b + 2*a^7*b^3 + a^5*b^5 + (a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^8 + (a^9*b + 2*a^7*b^3 + a^5*b^5)*sinh

```
(x)^8 + 2*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^7 + 2*(a^10 + 2*a^8*b^2 + a^
6*b^4 + 4*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*sinh(x)^7 - 4*(a^9*b + 2*a
^7*b^3 + a^5*b^5)*cosh(x)^6 - 2*(2*a^9*b + 4*a^7*b^3 + 2*a^5*b^5 - 14*(a^9*
b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 - 7*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)
)*sinh(x)^6 - 6*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^5 - 2*(3*a^10 + 6*a^8*
b^2 + 3*a^6*b^4 - 28*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^3 - 21*(a^10 + 2
*a^8*b^2 + a^6*b^4)*cosh(x)^2 + 12*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*s
inh(x)^5 + 6*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^4 + 2*(3*a^9*b + 6*a^7*b
^3 + 3*a^5*b^5 + 35*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^4 + 35*(a^10 + 2*
a^8*b^2 + a^6*b^4)*cosh(x)^3 - 30*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 -
 15*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x))*sinh(x)^4 + 6*(a^10 + 2*a^8*b^2 +
 a^6*b^4)*cosh(x)^3 + 2*(3*a^10 + 6*a^8*b^2 + 3*a^6*b^4 + 28*(a^9*b + 2*a^7
*b^3 + a^5*b^5)*cosh(x)^5 + 35*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^4 - 40*
(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^3 - 30*(a^10 + 2*a^8*b^2 + a^6*b^4)*c
osh(x)^2 + 12*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*sinh(x)^3 - 4*(a^9*b +
 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 - 2*(2*a^9*b + 4*a^7*b^3 + 2*a^5*b^5 - 14*(
a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^6 - 21*(a^10 + 2*a^8*b^2 + a^6*b^4)*co
sh(x)^5 + 30*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^4 + 30*(a^10 + 2*a^8*b^2
 + a^6*b^4)*cosh(x)^3 - 18*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^2 - 9*(a^1
0 + 2*a^8*b^2 + a^6*b^4)*cosh(x))*sinh(x)^2 - 2*(a^10 + 2*a^8*b^2 + a^6*b^4
)*cosh(x) - 2*(a^10 + 2*a^8*b^2 + a^6*b^4 - 4*(a^9*b + 2*a^7*b^3 + a^5*b^5)
*cosh(x)^7 - 7*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^6 + 12*(a^9*b + 2*a^7*b
^3 + a^5*b^5)*cosh(x)^5 + 15*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh(x)^4 - 12*(a
^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x)^3 - 9*(a^10 + 2*a^8*b^2 + a^6*b^4)*cosh
(x)^2 + 4*(a^9*b + 2*a^7*b^3 + a^5*b^5)*cosh(x))*sinh(x))
```

giac [A] time = 0.33, size = 236, normalized size = 1.19

$$\frac{\left(5\,a^2b^4+4\,b^6\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^7+a^5b^2\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^4e^x-b^5\right)}{\left(a^6+a^4b^2\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}-\frac{\left(a^2b-4\,b^3\right)\log\left(e^x+1\right)}{a^5}+\frac{\left(a^2b-4\,b^3\right)\mathrm{lo}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (5*a^2*b^4 + 4*b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^4*e^
x - b^5)/((a^6 + a^4*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - (a^2*b - 4*b^3)*log(
e^x + 1)/a^5 + (a^2*b - 4*b^3)*log(abs(e^x - 1))/a^5 + 2/3*(3*a*b*e^(5*x) -
 9*b^2*e^(4*x) - 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b*e^x + 2*a^2 - 9*b^2
)/(a^4*(e^(2*x) - 1)^3)
```

maple [A] time = 0.08, size = 277, normalized size = 1.40

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{24a^2}-\frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{4a^3}+\frac{3\tanh\left(\frac{x}{2}\right)}{8a^2}-\frac{3b^2\tanh\left(\frac{x}{2}\right)}{2a^4}+\frac{2b^6\tanh\left(\frac{x}{2}\right)}{a^5\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)\left(a^2+b^2\right)}+\frac{}{a^4\left(a\left(\mathrm{t}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -1/24/a^2*tanh(1/2*x)^3-1/4/a^3*b*tanh(1/2*x)^2+3/8/a^2*tanh(1/2*x)-3/2/a^4
*b^2*tanh(1/2*x)+2/a^5*b^6/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)*ta
nh(1/2*x)+2/a^4*b^5/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/(a^2+b^2)+10/a^3*b^
4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+8/a^5*
b^6/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/24
/a^2/tanh(1/2*x)^3+3/8/a^2/tanh(1/2*x)-3/2/a^4/tanh(1/2*x)*b^2+1/4/a^3*b/ta
nh(1/2*x)^2+1/a^3*b*ln(tanh(1/2*x))-4/a^5*b^3*ln(tanh(1/2*x))
```

maxima [B] time = 0.43, size = 477, normalized size = 2.41

$$\frac{\left(5\,a^2b^4+4\,b^6\right)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^7+a^5b^2\right)\sqrt{a^2+b^2}}+\frac{2\left(2\,a^4b-7\,a^2b^3-12\,b^5+\left(4\,a^5-11\,a^3b^2-18\,ab^4\right)e^{(-x)}-\left(2\,a^4b-25\,a^2b^3\right.\right.}{3\left(a^6b+a^4b^3+2\left(a^7+a^5b^2\right)e^{(-x)}-4\left(a^6b+a^4b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (5*a^2*b^4 + 4*b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2/3*(2*a^4*b - 7*a^2*b^
3 - 12*b^5 + (4*a^5 - 11*a^3*b^2 - 18*a*b^4)*e^(-x) - (2*a^4*b - 25*a^2*b^3
 - 36*b^5)*e^(-2*x) - 3*(4*a^5 - 7*a^3*b^2 - 14*a*b^4)*e^(-3*x) + 3*(2*a^4*
b - 7*a^2*b^3 - 12*b^5)*e^(-4*x) - 3*(7*a^3*b^2 + 10*a*b^4)*e^(-5*x) - 3*(2
*a^4*b - a^2*b^3 - 4*b^5)*e^(-6*x) + 3*(a^3*b^2 + 2*a*b^4)*e^(-7*x))/(a^6*b
 + a^4*b^3 + 2*(a^7 + a^5*b^2)*e^(-x) - 4*(a^6*b + a^4*b^3)*e^(-2*x) - 6*(a
^7 + a^5*b^2)*e^(-3*x) + 6*(a^6*b + a^4*b^3)*e^(-4*x) + 6*(a^7 + a^5*b^2)*e
^(-5*x) - 4*(a^6*b + a^4*b^3)*e^(-6*x) - 2*(a^7 + a^5*b^2)*e^(-7*x) + (a^6*
b + a^4*b^3)*e^(-8*x)) - (a^2*b - 4*b^3)*log(e^(-x) + 1)/a^5 + (a^2*b - 4*b
^3)*log(e^(-x) - 1)/a^5
```

mupad [B] time = 3.30, size = 975, normalized size = 4.92

$$\frac{\ln\left(e^x-1\right)\left(a^2\,b-4\,b^3\right)}{a^5}-\frac{8}{3\,a^2\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)}-\frac{\frac{4}{a^2}-\frac{4\,b\,e^x}{a^3}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{6\,b^2}{a^4}-\frac{2\,b\,e^x}{a^3}}{e^{2\,x}-1}-\frac{\frac{2\,b^8}{a^4\,\left(a^2\,b^3+b^5\right)}-\frac{2\,b^7\,e^x}{a^3\,\left(a^2\,b^3+b^5\right)}}{2\,a\,e^x-b+b\,e^{2\,x}}-\mathrm{l}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*sinh(x))^2),x)
```

```
[Out] (log(exp(x) - 1)*(a^2*b - 4*b^3))/a^5 - 8/(3*a^2*(3*exp(2*x) - 3*exp(4*x) +
 exp(6*x) - 1)) - (4/a^2 - (4*b*exp(x))/a^3)/(exp(4*x) - 2*exp(2*x) + 1) -
((6*b^2)/a^4 - (2*b*exp(x))/a^3)/(exp(2*x) - 1) - ((2*b^8)/(a^4*(b^5 + a^2*
b^3)) - (2*b^7*exp(x))/(a^3*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2*x))
 - (log(exp(x) + 1)*(a^2*b - 4*b^3))/a^5 - (b^4*log((32*b*(16*b^4 - 5*a^4 +
 16*a^2*b^2)*(2*a^4*b - 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*exp(x)
+ 11*a^3*b^2*exp(x)))/(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(5*a^5*b
^9 - 32*b^11*((a^2 + b^2)^3)^(1/2) - 2*a^13*b + 20*a^7*b^7 + 24*a^9*b^5 + 7
*a^11*b^3 + 4*a^14*exp(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*a^4*b^7*(
(a^2 + b^2)^3)^(1/2) - 15*a^6*b^8*exp(x) - 50*a^8*b^6*exp(x) - 52*a^10*b^4*
exp(x) - 13*a^12*b^2*exp(x) + 127*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 79
*a^5*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 5*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2
) + 51*a*b^10*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^12*((a^2 + b^2)^3)^(1/2)*(a
^2 + b^2)^4))*((a^2 + b^2)^3)^(1/2)*(5*a^2 + 4*b^2))/(a^11 + a^5*b^6 + 3*a^
7*b^4 + 3*a^9*b^2) + (b^4*log((32*b*(16*b^4 - 5*a^4 + 16*a^2*b^2)*(2*a^4*b
- 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*exp(x) + 11*a^3*b^2*exp(x)))/
(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(2*a^13*b - 32*b^11*((a^2 + b^
2)^3)^(1/2) - 5*a^5*b^9 - 20*a^7*b^7 - 24*a^9*b^5 - 7*a^11*b^3 - 4*a^14*exp
(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*a^4*b^7*((a^2 + b^2)^3)^(1/2) +
 15*a^6*b^8*exp(x) + 50*a^8*b^6*exp(x) + 52*a^10*b^4*exp(x) + 13*a^12*b^2*e
xp(x) + 127*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 79*a^5*b^6*exp(x)*((a^2
+ b^2)^3)^(1/2) + 5*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) + 51*a*b^10*exp(x)*(
(a^2 + b^2)^3)^(1/2)))/(a^12*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4))*((a^2 +
b^2)^3)^(1/2)*(5*a^2 + 4*b^2))/(a^11 + a^5*b^6 + 3*a^7*b^4 + 3*a^9*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**4/(a + b*sinh(x))**2, x)
```

3.88 $\int \frac{1}{3+5i\sinh(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] 1/4*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d-1/4*I*ln(cosh(1/2*d
*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2660, 616, 31}

$$\frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((I/4)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - ((I/4)*Log[Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= -\frac{(3i)\,\text{Subst}\left(\int \frac{1}{1+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{4d} + \frac{(3i)\,\text{Subst}\left(\int \frac{1}{9+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+i\right.\right.}{4d}$$
$$= \frac{i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.11

$$\frac{\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log(4-5\cosh(c+dx))}{8d} + \frac{i\log(5\cosh(c+dx)+4)}{8d} + \frac{\tan^{-1}\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ArcTan[3*Coth[(c + d*x)/2]]/(4*d) + ArcTan[3*Tanh[(c + d*x)/2]]/(4*d) - ((I
/8)*Log[4 - 5*Cosh[c + d*x]])/d + ((I/8)*Log[4 + 5*Cosh[c + d*x]])/d
```

fricas [A] time = 0.51, size = 28, normalized size = 0.38

$$\frac{i\log\left(e^{(dx+c)} - \frac{3}{5}i + \frac{4}{5}\right) - i\log\left(e^{(dx+c)} - \frac{3}{5}i - \frac{4}{5}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*log(e^(d*x + c) - 3/5*I + 4/5) - I*log(e^(d*x + c) - 3/5*I - 4/5))/d
```

giac [A] time = 0.58, size = 32, normalized size = 0.44

$$-\frac{-i\log\left(-(i-2)\,e^{(dx+c)} - 2i + 1\right) + i\log\left(-(2i-1)\,e^{(dx+c)} + i - 2\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(-I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) + I*log(-(2*I - 1)*e^(d*x + c)
 + I - 2))/d
```

maple [A] time = 0.05, size = 42, normalized size = 0.58

$$\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{4d}-\frac{i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c)),x)
```

```
[Out] 1/4*I/d*ln(tanh(1/2*d*x+1/2*c)-3*I)-1/4*I/d*ln(3*tanh(1/2*d*x+1/2*c)-I)
```

maxima [A] time = 0.42, size = 19, normalized size = 0.26

$$\frac{\arctan\left(\frac{5}{4}i\,e^{(-dx-c)}-\frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(5/4*I*e^(-d*x - c) - 3/4)/d
```

mupad [B] time = 0.35, size = 39, normalized size = 0.53

$$-\frac{\ln\left(-\frac{5}{2}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2-\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}+\frac{\ln\left(\frac{5}{2}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2+\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3),x)
```

```
[Out] (log(exp(d*x)*exp(c)*(2 + 3i/2) + 5/2)*1i)/(4*d) - (log(exp(d*x)*exp(c)*(2
- 3i/2) - 5/2)*1i)/(4*d)
```

sympy [A] time = 0.31, size = 34, normalized size = 0.47

$$\frac{\operatorname{RootSum}\left(400z^{2}+1,\left(i\mapsto i\log\left(16iie^{c}+\frac{3ie^{c}}{5}+e^{-dx}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x)
```

```
[Out] RootSum(400*_z**2 + 1, Lambda(_i, _i*log(16*_i*I*exp(c) + 3*I*exp(c)/5 + ex
p(-d*x))))/d
```

3.89 $\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=102

$$\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}\right.\right.}{64d}$$

```
[Out] -3/64*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d+3/64*I*ln(cosh(1/
2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/16*I*cosh(d*x+c)/d/(3+5*I*sinh(d*
x+c))
```

Rubi [A] time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2664, 12, 2660, 616, 31}

$$\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (((-3*I)/64)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((3*I)/64
)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/16)*Cosh[c
+ d*x])/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{1}{16}\int -\frac{3}{3+5i\sinh(c+dx)}\,dx$$
$$= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} - \frac{3}{16}\int \frac{1}{3+5i\sinh(c+dx)}\,dx$$
$$= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{8d}$$
$$= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{(9i)\,\mathrm{Subst}\left(\int \frac{1}{1+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{64d} - \frac{(9i)\,\mathrm{Sul}}{}$$
$$= -\frac{3i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5i\cos}{16d(3+5i}$$

Mathematica [A] time = 0.26, size = 142, normalized size = 1.39

$$\frac{40\sinh\left(\frac{1}{2}(c+dx)\right)\left(\frac{3}{\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)} + \frac{1}{3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)}\right) - 9\left(2\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-9*(2*ArcTan[3*Coth[(c + d*x)/2]] + 2*ArcTan[3*Tanh[(c + d*x)/2]] - I*Log[
4 - 5*Cosh[c + d*x]] + I*Log[4 + 5*Cosh[c + d*x]]) + 40*((3*Cosh[(c + d*x)/
2] + I*Sinh[(c + d*x)/2])^(-1) + 3/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x
)/2]))*Sinh[(c + d*x)/2])/(384*d)
```

fricas [A] time = 0.62, size = 100, normalized size = 0.98

$$\frac{\left(-15i\,e^{(2\,dx+2\,c)}-18\,e^{(dx+c)}+15i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+\left(15i\,e^{(2\,dx+2\,c)}+18\,e^{(dx+c)}-15i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i-\frac{4}{5}\right)+2\cdots}{320\,de^{(2\,dx+2\,c)}-384i\,de^{(dx+c)}-320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((-15*I*e^(2*d*x + 2*c) - 18*e^(d*x + c) + 15*I)*log(e^(d*x + c) - 3/5*I +
4/5) + (15*I*e^(2*d*x + 2*c) + 18*e^(d*x + c) - 15*I)*log(e^(d*x + c) - 3/5
*I - 4/5) + 24*I*e^(d*x + c) + 40)/(320*d*e^(2*d*x + 2*c) - 384*I*d*e^(d*x
+ c) - 320*d)
```

giac [A] time = 0.16, size = 67, normalized size = 0.66

$$-\frac{\frac{8\left(-3i\,e^{(dx+c)}-5\right)}{5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5}+3i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)-3i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*(-3*I*e^(d*x + c) - 5)/(5*e^(2*d*x + 2*c) - 6*I*e^(d*x + c) - 5) +
 3*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) - 3*I*log(-(2*I - 1)*e^(d*x + c) +
 I - 2))/d
```

maple [A] time = 0.08, size = 82, normalized size = 0.80

$$-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{64d}+\frac{5}{16d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}+\frac{3i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{64d}+\frac{5}{48d\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^2,x)
```

```
[Out] -3/64*I/d*ln(tanh(1/2*d*x+1/2*c)-3*I)+5/16/d/(tanh(1/2*d*x+1/2*c)-3*I)+3/64
*I/d*ln(3*tanh(1/2*d*x+1/2*c)-I)+5/48/d/(3*tanh(1/2*d*x+1/2*c)-I)
```

maxima [A] time = 0.43, size = 79, normalized size = 0.77

$$\frac{3i\,\log\left(\frac{10\,e^{(-dx-c)}+6i-8}{10\,e^{(-dx-c)}+6i+8}\right)}{64\,d}+\frac{3i\,e^{(-dx-c)}-5}{-8\,d\left(-6i\,e^{(-dx-c)}-5\,e^{(-2\,dx-2\,c)}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/64*I*log((10*e^(-d*x - c) + 6*I - 8)/(10*e^(-d*x - c) + 6*I + 8))/d + (3*
I*e^(-d*x - c) - 5)/(d*(48*I*e^(-d*x - c) + 40*e^(-2*d*x - 2*c) - 40))
```

mupad [B] time = 0.98, size = 106, normalized size = 1.04

$$-\frac{5}{8\,\left(5\,d-5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{c+d\,x}\,6\mathrm{i}\right)}-\frac{\ln\left(-\frac{15}{4}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(-3-\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d}+\frac{\ln\left(\frac{15}{4}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(-3+\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d}-\frac{}{8\,\left(5\,d-5\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3)^2,x)
```

```
[Out] (log(15/4 - exp(d*x)*exp(c)*(3 - 9i/4))*3i)/(64*d) - (log(- exp(d*x)*exp(c)
*(3 + 9i/4) - 15/4)*3i)/(64*d) - 5/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(2*
c + 2*d*x))) - (exp(c + d*x)*3i)/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(2*c
+ 2*d*x)))
```

sympy [A] time = 0.41, size = 87, normalized size = 0.85

$$\frac{-5e^{2c}+3ie^{c}e^{-dx}}{-40de^{2c}+48ide^{c}e^{-dx}+40de^{-2dx}}+\frac{\operatorname{RootSum}\left(2560000z^2+9,\left(i\mapsto i\log\left(-\frac{1280iie^{c}}{3}+\frac{3ie^{c}}{5}+e^{-dx}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**2,x)
```

```
[Out] (-5*exp(2*c) + 3*I*exp(c)*exp(-d*x))/(-40*d*exp(2*c) + 48*I*d*exp(c)*exp(-d
*x) + 40*d*exp(-2*d*x)) + RootSum(2560000*_z**2 + 9, Lambda(_i, _i*log(-128
0*_i*I*exp(c)/3 + 3*I*exp(c)/5 + exp(-d*x))))/d
```

3.90 $\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\,\mathrm{lo}}{}$$

```
[Out] 43/2048*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d-43/2048*I*ln(co
sh(1/2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/32*I*cosh(d*x+c)/d/(3+5*I*si
nh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2664, 2754, 12, 2660, 616, 31}

$$-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (((43*I)/2048)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - (((43*I)
/2048)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/32)*Co
sh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) - (((45*I)/512)*Cosh[c + d*x])
/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} + \frac{1}{32}\int \frac{-6+5i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,dx$$

$$= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))} + \frac{1}{512}\int \frac{43}{3+5i\sinh(c+$$

$$= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))} + \frac{43}{512}\int \frac{1}{3+5i\sinh(c+$$

$$= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))} - \frac{(43i)\,\mathrm{Subst}\left(\int \frac{1}{3+10x+3x}\right.}{2}$$

$$= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))} - \frac{(129i)\,\mathrm{Subst}\left(\int \frac{1}{1+3x}\,dx\right.}{204}$$

$$= \frac{43i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5i\,\mathrm{co}}{32d(3+5}$$

Mathematica [A] time = 0.51, size = 204, normalized size = 1.56

$$\frac{86\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)-43i\log(4-5\cosh(c+dx))+43i\log(5\cosh(c+dx)+4)+\sinh\left(\frac{1}{2}(c+dx)\right)\left(-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (86*ArcTan[3*Coth[(c + d*x)/2]] + 86*ArcTan[3*Tanh[(c + d*x)/2]] - (43*I)*L
og[4 - 5*Cosh[c + d*x]] + (43*I)*Log[4 + 5*Cosh[c + d*x]] - (80*I)/(3*Cosh[
(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + (80*I)/(Cosh[(c + d*x)/2] + (3*I)*S
inh[(c + d*x)/2])^2 + (-120/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) - 3
60/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(4096*
d)
```

fricas [A] time = 0.73, size = 190, normalized size = 1.45

$$\frac{\left(1075i\,e^{(4\,dx+4\,c)}+2580\,e^{(3\,dx+3\,c)}-3698i\,e^{(2\,dx+2\,c)}-2580\,e^{(dx+c)}+1075i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+\left(-1075i\,e^{(4\,dx+4\ldots}\right.}{51200\,de^{(4\,dx+4\,c)}-122880i\,de^{(3\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((1075*I*e^(4*d*x + 4*c) + 2580*e^(3*d*x + 3*c) - 3698*I*e^(2*d*x + 2*c) -
2580*e^(d*x + c) + 1075*I)*log(e^(d*x + c) - 3/5*I + 4/5) + (-1075*I*e^(4*d
*x + 4*c) - 2580*e^(3*d*x + 3*c) + 3698*I*e^(2*d*x + 2*c) + 2580*e^(d*x + c
) - 1075*I)*log(e^(d*x + c) - 3/5*I - 4/5) - 1720*I*e^(3*d*x + 3*c) - 3096*
e^(2*d*x + 2*c) + 2600*I*e^(d*x + c) + 1800)/(51200*d*e^(4*d*x + 4*c) - 122
880*I*d*e^(3*d*x + 3*c) - 176128*d*e^(2*d*x + 2*c) + 122880*I*d*e^(d*x + c)
 + 51200*d)
```

giac [A] time = 0.37, size = 89, normalized size = 0.68

$$-\frac{\frac{8\left(-215i\,e^{(3\,dx+3\,c)}-387\,e^{(2\,dx+2\,c)}+325i\,e^{(dx+c)}+225\right)}{\left(-5i\,e^{(2\,dx+2\,c)}-6\,e^{(dx+c)}+5i\right)^{2}}-43i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)+43i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)\ldots}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(8*(-215*I*e^(3*d*x + 3*c) - 387*e^(2*d*x + 2*c) + 325*I*e^(d*x + c
) + 225)/(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I)^2 - 43*I*log(-(I - 2)
*e^(d*x + c) - 2*I + 1) + 43*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d
```

maple [A] time = 0.09, size = 124, normalized size = 0.95

$$\frac{25i}{128d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)^2}+\frac{43i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{2048d}+\frac{15}{512d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}-\frac{43i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{2048d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^3,x)
```

```
[Out] 25/128*I/d/(tanh(1/2*d*x+1/2*c)-3*I)^2+43/2048*I/d*ln(tanh(1/2*d*x+1/2*c)-3
*I)+15/512/d/(tanh(1/2*d*x+1/2*c)-3*I)-43/2048*I/d*ln(3*tanh(1/2*d*x+1/2*c)
-I)-25/1152*I/d/(3*tanh(1/2*d*x+1/2*c)-I)^2-155/4608/d/(3*tanh(1/2*d*x+1/2*
c)-I)
```

maxima [A] time = 0.42, size = 124, normalized size = 0.95

$$-\frac{43i\log\left(\frac{10e^{(-dx-c)}+6i-8}{10e^{(-dx-c)}+6i+8}\right)}{2048d}-\frac{-325ie^{(-dx-c)}-387e^{(-2dx-2c)}+215ie^{(-3dx-3c)}+225}{d\left(-15360ie^{(-dx-c)}-22016e^{(-2dx-2c)}+15360ie^{(-3dx-3c)}+6400e^{(-4dx-4c)}+6400\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -43/2048*I*log((10*e^(-d*x - c) + 6*I - 8)/(10*e^(-d*x - c) + 6*I + 8))/d -
 (-325*I*e^(-d*x - c) - 387*e^(-2*d*x - 2*c) + 215*I*e^(-3*d*x - 3*c) + 225
)/(d*(-15360*I*e^(-d*x - c) - 22016*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x -
3*c) + 6400*e^(-4*d*x - 4*c) + 6400))
```

mupad [B] time = 1.06, size = 147, normalized size = 1.12

$$\frac{\frac{129}{6400d}+\frac{e^{c+dx}\,43i}{1280d}}{1-e^{2c+2dx}+\frac{e^{c+dx}\,6i}{5}}-\frac{\ln\left(-\frac{215}{4}+e^{c+dx}\left(43-\frac{129}{4}i\right)\right)43i}{2048d}+\frac{\ln\left(\frac{215}{4}+e^{c+dx}\left(43+\frac{129}{4}i\right)\right)43i}{2048d}-\frac{-\frac{}{20}}{e^{4c+4dx}-\frac{86e^{2c+2d}}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3)^3,x)
```

```
[Out] ((exp(c + d*x)*43i)/(1280*d) + 129/(6400*d))/((exp(c + d*x)*6i)/5 - exp(2*c
 + 2*d*x) + 1) - (log(exp(c + d*x)*(43 - 129i/4) - 215/4)*43i)/(2048*d) + (
log(exp(c + d*x)*(43 + 129i/4) + 215/4)*43i)/(2048*d) - ((exp(c + d*x)*7i)/
(1000*d) - 3/(200*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/25 - (e
xp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)
```

sympy [A] time = 0.55, size = 148, normalized size = 1.13

$$\frac{225e^{4c} - 325ie^{3c}e^{-dx} - 387e^{2c}e^{-2dx} + 215ie^{c}e^{-3dx}}{-6400de^{4c} + 15360ide^{3c}e^{-dx} + 22016de^{2c}e^{-2dx} - 15360ide^{c}e^{-3dx} - 6400de^{-4dx}} + \frac{\operatorname{RootSum}\left(65536000000z^{2} + 184\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**3,x)
```

```
[Out] (225*exp(4*c) - 325*I*exp(3*c)*exp(-d*x) - 387*exp(2*c)*exp(-2*d*x) + 215*I
*exp(c)*exp(-3*d*x))/(-6400*d*exp(4*c) + 15360*I*d*exp(3*c)*exp(-d*x) + 220
16*d*exp(2*c)*exp(-2*d*x) - 15360*I*d*exp(c)*exp(-3*d*x) - 6400*d*exp(-4*d*
x)) + RootSum(65536000000*_z**2 + 1849, Lambda(_i, _i*log(204800*_i*I*exp(c
)/43 + 3*I*exp(c)/5 + exp(-d*x))))/d
```

3.91 $\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=160

$$\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+d\right.\right.}{327}$$

[Out] -279/32768*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d+279/32768*I*ln(cosh(1/2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/48*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))^2+995/24576*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2664, 2754, 12, 2660, 616, 31}

$$\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+d\right.\right.}{327}$$

Antiderivative was successfully verified.

[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-4),x]

[Out] (((-279*I)/32768)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((279*I)/32768)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/48)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^3) - (((25*I)/512)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) + (((995*I)/24576)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} + \frac{1}{48}\int \frac{-9+10i\sinh(c+dx)}{(3+5i\sinh(c+dx))^3}\,dx$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{\int \frac{154-75i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,dx}{1536}$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx}{24576d(3+5i\sinh(c}$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx}{24576d(3+5i\sinh(c}$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx}{24576d(3+5i\sinh(c}$$

$$= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx}{24576d(3+5i\sinh(c}$$

$$= -\frac{279i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{}{48d(3}$$

Mathematica [A] time = 0.63, size = 265, normalized size = 1.66

$$\frac{-5022\tan^{-1}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 2511i\log(4-5\cosh(c+dx)) - 2511i\log(5\cosh(c+dx)+4) + 40\sinh\left(\frac{1}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (-5022*ArcTan[3*Coth[(c + d*x)/2]] - 5022*ArcTan[3*Tanh[(c + d*x)/2]] + (25
11*I)*Log[4 - 5*Cosh[c + d*x]] - (2511*I)*Log[4 + 5*Cosh[c + d*x]] + (4640*
I)/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (1440*I)/(Cosh[(c + d*x)
/2] + (3*I)*Sinh[(c + d*x)/2])^2 + 40*(80/(3*Cosh[(c + d*x)/2] + I*Sinh[(c
+ d*x)/2])^3 + 199/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + 240/(Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2])^3 + 597/(Cosh[(c + d*x)/2] + (3*I)*
Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(589824*d)
```

fricas [B] time = 0.95, size = 280, normalized size = 1.75

$$\frac{\left(-104625i\,e^{(6\,dx+6\,c)} - 376650\,e^{(5\,dx+5\,c)} + 765855i\,e^{(4\,dx+4\,c)} + 934092\,e^{(3\,dx+3\,c)} - 765855i\,e^{(2\,dx+2\,c)} - 376650\,e^{(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((-104625*I*e^(6*d*x + 6*c) - 376650*e^(5*d*x + 5*c) + 765855*I*e^(4*d*x +
4*c) + 934092*e^(3*d*x + 3*c) - 765855*I*e^(2*d*x + 2*c) - 376650*e^(d*x +
c) + 104625*I)*log(e^(d*x + c) - 3/5*I + 4/5) + (104625*I*e^(6*d*x + 6*c) +
 376650*e^(5*d*x + 5*c) - 765855*I*e^(4*d*x + 4*c) - 934092*e^(3*d*x + 3*c)
 + 765855*I*e^(2*d*x + 2*c) + 376650*e^(d*x + c) - 104625*I)*log(e^(d*x + c
) - 3/5*I - 4/5) + 167400*I*e^(5*d*x + 5*c) + 502200*e^(4*d*x + 4*c) - 8883
36*I*e^(3*d*x + 3*c) - 954480*e^(2*d*x + 2*c) + 549000*I*e^(d*x + c) + 1990
00)/(12288000*d*e^(6*d*x + 6*c) - 44236800*I*d*e^(5*d*x + 5*c) - 89948160*d
*e^(4*d*x + 4*c) + 109707264*I*d*e^(3*d*x + 3*c) + 89948160*d*e^(2*d*x + 2*
c) - 44236800*I*d*e^(d*x + c) - 12288000*d)
```

giac [A] time = 0.16, size = 111, normalized size = 0.69

$$\frac{\frac{8\left(20925i\,e^{(5\,dx+5\,c)}+62775\,e^{(4\,dx+4\,c)}-111042i\,e^{(3\,dx+3\,c)}-119310\,e^{(2\,dx+2\,c)}+68625i\,e^{(dx+c)}+24875\right)}{\left(5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5\right)^{3}}-837i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)+}{98304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/98304*(8*(20925*I*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) - 111042*I*e^(3
*d*x + 3*c) - 119310*e^(2*d*x + 2*c) + 68625*I*e^(d*x + c) + 24875)/(5*e^(2
*d*x + 2*c) - 6*I*e^(d*x + c) - 5)^3 - 837*I*log(-(I - 2)*e^(d*x + c) - 2*I
 + 1) + 837*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d
```

maple [A] time = 0.09, size = 164, normalized size = 1.02

$$-\frac{279i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{32768d}+\frac{75i}{1024d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)^{2}}-\frac{125}{768d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)^{3}}+\frac{345}{8192d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^4,x)
```

```
[Out] -279/32768*I/d*ln(tanh(1/2*d*x+1/2*c)-3*I)+75/1024*I/d/(tanh(1/2*d*x+1/2*c)
-3*I)^2-125/768/d/(tanh(1/2*d*x+1/2*c)-3*I)^3+345/8192/d/(tanh(1/2*d*x+1/2*
c)-3*I)+275/27648*I/d/(3*tanh(1/2*d*x+1/2*c)-I)^2+279/32768*I/d*ln(3*tanh(1
/2*d*x+1/2*c)-I)-125/20736/d/(3*tanh(1/2*d*x+1/2*c)-I)^3+3505/221184/d/(3*t
anh(1/2*d*x+1/2*c)-I)
```

maxima [A] time = 0.43, size = 167, normalized size = 1.04

$$\frac{279i\log\left(\frac{10e^{(-dx-c)}+6i-8}{10e^{(-dx-c)}+6i+8}\right)}{32768d}+\frac{68625ie^{(-dx-c)}+119310e^{(-2dx-2c)}-111042ie^{(-3dx-3c)}-62775e\cdots}{d\left(5529600ie^{(-dx-c)}+11243520e^{(-2dx-2c)}-13713408ie^{(-3dx-3c)}-11243520e^{(-4dx-4\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 279/32768*I*log((10*e^(-d*x - c) + 6*I - 8)/(10*e^(-d*x - c) + 6*I + 8))/d
+ (68625*I*e^(-d*x - c) + 119310*e^(-2*d*x - 2*c) - 111042*I*e^(-3*d*x - 3*
c) - 62775*e^(-4*d*x - 4*c) + 20925*I*e^(-5*d*x - 5*c) - 24875)/(d*(5529600
*I*e^(-d*x - c) + 11243520*e^(-2*d*x - 2*c) - 13713408*I*e^(-3*d*x - 3*c) -
 11243520*e^(-4*d*x - 4*c) + 5529600*I*e^(-5*d*x - 5*c) + 1536000*e^(-6*d*x
 - 6*c) - 1536000))
```

mupad [B] time = 1.26, size = 237, normalized size = 1.48

$$-\frac{\frac{837}{102400\,d}+\frac{e^{c+d\,x}\,279i}{20480\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,6i}{5}}+\frac{\frac{7}{3750\,d}+\frac{e^{c+d\,x}\,39i}{6250\,d}}{\frac{183\,e^{4\,c+4\,d\,x}}{25}-\frac{183\,e^{2\,c+2\,d\,x}}{25}-e^{6\,c+6\,d\,x}+1+\frac{e^{c+d\,x}\,18i}{5}-\frac{e^{3\,c+3\,d\,x}\,1116i}{125}+\frac{e^{5\,c+5\,d\,x}\,18i}{5}}-\frac{\ln\left(-\frac{1395}{4}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3)^4,x)
```

```
[Out] ((exp(c + d*x)*39i)/(6250*d) + 7/(3750*d))/((exp(c + d*x)*18i)/5 - (183*exp
(2*c + 2*d*x))/25 - (exp(3*c + 3*d*x)*1116i)/125 + (183*exp(4*c + 4*d*x))/2
5 + (exp(5*c + 5*d*x)*18i)/5 - exp(6*c + 6*d*x) + 1) - ((exp(c + d*x)*279i)
/(20480*d) + 837/(102400*d))/((exp(c + d*x)*6i)/5 - exp(2*c + 2*d*x) + 1) -
 (log(- exp(c + d*x)*(279 + 837i/4) - 1395/4)*279i)/(32768*d) + (log(1395/4
 - exp(c + d*x)*(279 - 837i/4))*279i)/(32768*d) - ((exp(c + d*x)*93i)/(1600
0*d) + 791/(80000*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/25 - (e
xp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)
```

sympy [A] time = 0.73, size = 209, normalized size = 1.31

$$\frac{24875e^{6c}-68625ie^{5c}e^{-dx}-119310e^{4c}e^{-2dx}+111042ie^{3c}e^{-3dx}+62775e^{2c}e^{-4dx}-20925ie^{c}e^{-\cdots}}{1536000de^{6c}-5529600ide^{5c}e^{-dx}-11243520de^{4c}e^{-2dx}+13713408ide^{3c}e^{-3dx}+11243520de^{2c}e^{-4dx}-5529600ide\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**4,x)
```

```
[Out] (24875*exp(6*c) - 68625*I*exp(5*c)*exp(-d*x) - 119310*exp(4*c)*exp(-2*d*x)
+ 111042*I*exp(3*c)*exp(-3*d*x) + 62775*exp(2*c)*exp(-4*d*x) - 20925*I*exp(
c)*exp(-5*d*x))/(1536000*d*exp(6*c) - 5529600*I*d*exp(5*c)*exp(-d*x) - 1124
3520*d*exp(4*c)*exp(-2*d*x) + 13713408*I*d*exp(3*c)*exp(-3*d*x) + 11243520*
d*exp(2*c)*exp(-4*d*x) - 5529600*I*d*exp(c)*exp(-5*d*x) - 1536000*d*exp(-6*
d*x)) + RootSum(419430400000000*_z**2 + 77841, Lambda(_i, _i*log(-16384000*
_i*I*exp(c)/279 + 3*I*exp(c)/5 + exp(-d*x))))/d
```

3.92 $\int \frac{1}{5+3i\sinh(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{4}-\frac{i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

```
[Out] 1/4*x-1/2*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2657}

$$\frac{x}{4}-\frac{i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx=\frac{x}{4}-\frac{i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

Mathematica [B] time = 0.03, size = 171, normalized size = 4.62

$$-\frac{\log(5\cosh(c+dx)-4\sinh(c+dx))}{8d}+\frac{\log(4\sinh(c+dx)+5\cosh(c+dx))}{8d}-\frac{i\tan^{-1}\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx\right.}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx\right.}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((-1/4*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/
2] - 2*Sinh[(c + d*x)/2])])/d + ((I/4)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])])/d - Log[5*Cosh[c +
 d*x] - 4*Sinh[c + d*x]]/(8*d) + Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]]/(8*
d)
```

fricas [A] time = 0.48, size = 26, normalized size = 0.70

$$\frac{\log\left(e^{(dx+c)} - \frac{1}{3}i\right) - \log\left(e^{(dx+c)} - 3i\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(e^(d*x + c) - 1/3*I) - log(e^(d*x + c) - 3*I))/d
```

giac [A] time = 0.24, size = 28, normalized size = 0.76

$$\frac{\log\left(3\,e^{(dx+c)} - i\right) - \log\left(e^{(dx+c)} - 3i\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(3*e^(d*x + c) - I) - log(e^(d*x + c) - 3*I))/d
```

maple [A] time = 0.05, size = 44, normalized size = 1.19

$$-\frac{\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{4d}+\frac{\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c)),x)
```

```
[Out] -1/4/d*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)+1/4/d*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I
)
```

maxima [A] time = 0.41, size = 36, normalized size = 0.97

$$\frac{\log\left(-\frac{6\left(-i\,e^{(-dx-c)}+3\right)}{6i\,e^{(-dx-c)}-2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*log(-6*(-I*e^(-d*x - c) + 3)/(6*I*e^(-d*x - c) - 2))/d
```

mupad [B] time = 0.61, size = 32, normalized size = 0.86

$$-\frac{\ln\left(-\frac{e^{d\,x}\,e^{c}}{2}+\frac{3}{2}\mathrm{i}\right)-\ln\left(\frac{9\,e^{d\,x}\,e^{c}}{2}-\frac{3}{2}\mathrm{i}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5),x)
```

```
[Out] -(log(3i/2 - (exp(d*x)*exp(c))/2) - log((9*exp(d*x)*exp(c))/2 - 3i/2))/(4*d
)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotInvertible

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x)
```

```
[Out] Exception raised: NotInvertible
```

3.93 $\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}-\frac{5i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}+\frac{5x}{64}$$

```
[Out] 5/64*x-5/32*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/16*I*cosh(d*x+c)/d/
(5+3*I*sinh(d*x+c))
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.214, Rules used = {2664, 12, 2657}

$$-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}-\frac{5i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}+\frac{5x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (((5*I)/32)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d - (((
3*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))} - \frac{1}{16}\int -\frac{5}{5+3i\sinh(c+dx)}\,dx$$
$$= -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))} + \frac{5}{16}\int \frac{1}{5+3i\sinh(c+dx)}\,dx$$
$$= \frac{5x}{64} - \frac{5i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d} - \frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

Mathematica [B] time = 0.25, size = 183, normalized size = 2.77

$$\frac{-\frac{120\cosh(c+dx)}{3\sinh(c+dx)-5i} - 25\log(5\cosh(c+dx)-4\sinh(c+dx)) + 25\log(4\sinh(c+dx)+5\cosh(c+dx)) - 50i\tan^{-1}}{640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (24*I - (50*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c +
d*x)/2] - 2*Sinh[(c + d*x)/2])] + (50*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh
[(c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 25*Log[5*Cosh[c
 + d*x] - 4*Sinh[c + d*x]] + 25*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] - (1
20*Cosh[c + d*x])/(-5*I + 3*Sinh[c + d*x]))/(640*d)
```

fricas [A] time = 0.49, size = 102, normalized size = 1.55

$$\frac{5\left(3e^{(2dx+2c)} - 10ie^{(dx+c)} - 3\right)\log\left(e^{(dx+c)} - \frac{1}{3}i\right) - 5\left(3e^{(2dx+2c)} - 10ie^{(dx+c)} - 3\right)\log\left(e^{(dx+c)} - 3i\right) - 40ie^{(dx+c)} -}{192de^{(2dx+2c)} - 640ide^{(dx+c)} - 192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 1/3*I) - 5*
(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 3*I) - 40*I*e^
(d*x + c) - 24)/(192*d*e^(2*d*x + 2*c) - 640*I*d*e^(d*x + c) - 192*d)
```

giac [A] time = 0.22, size = 65, normalized size = 0.98

$$-\frac{\frac{8\left(5ie^{(dx+c)}+3\right)}{3e^{(2dx+2c)}-10ie^{(dx+c)}-3} - 5\log\left(3e^{(dx+c)} - i\right) + 5\log\left(e^{(dx+c)} - 3i\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*(5*I*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3) -
 5*log(3*e^(d*x + c) - I) + 5*log(e^(d*x + c) - 3*I))/d
```

maple [B] time = 0.08, size = 134, normalized size = 2.03

$$-\frac{9}{80d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}+\frac{3i}{20d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}-\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{64d}-\frac{}{80d\left(5\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^2,x)
```

```
[Out] -9/80/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)+3/20*I/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I
)-5/64/d*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)-9/80/d/(5*tanh(1/2*d*x+1/2*c)+4-3*
I)-3/20*I/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)+5/64/d*ln(5*tanh(1/2*d*x+1/2*c)+4
-3*I)
```

maxima [A] time = 0.45, size = 64, normalized size = 0.97

$$-\frac{5i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{32\,d}-\frac{5i\,e^{(-dx-c)}-3}{-8\,d\left(-10i\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -5/32*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (5*I*e^(-d*x - c) - 3)/(d*(80*
I*e^(-d*x - c) + 24*e^(-2*d*x - 2*c) - 24))
```

mupad [B] time = 1.07, size = 102, normalized size = 1.55

$$\frac{3}{8\left(3\,d-3\,d\,e^{2\,c+2\,d\,x}+d\,e^{c+d\,x}\,10i\right)}-\frac{5\,\ln\left(-\frac{5\,e^{d\,x}\,e^{c}}{4}+\frac{15}{4}i\right)}{64\,d}+\frac{5\,\ln\left(\frac{45\,e^{d\,x}\,e^{c}}{4}-\frac{15}{4}i\right)}{64\,d}+\frac{e^{c+d\,x}\,5i}{8\left(3\,d-3\,d\,e^{2\,c+2\,d\,x}+d\,e^{c+d\,x}\,10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5)^2,x)
```

```
[Out] 3/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*x))) - (5*log(15i/4 - (5
*exp(d*x)*exp(c))/4))/(64*d) + (5*log((45*exp(d*x)*exp(c))/4 - 15i/4))/(64*
```

```
d) + (exp(c + d*x)*5i)/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*x))
)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotInvertible

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**2,x)
```

```
[Out] Exception raised: NotInvertible
```

3.94 $\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{59i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}+\frac{59x}{2048}$$

```
[Out] 59/2048*x-59/1024*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/32*I*cosh(d*x
+c)/d/(5+3*I*sinh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2664, 2754, 12, 2657}

$$-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{59i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}+\frac{59x}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (((59*I)/1024)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
 - (((3*I)/32)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((45*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{1}{32}\int \frac{-10+3i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}\,dx$$

$$= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))} + \frac{1}{512}\int \frac{59}{5+3i\sinh(c\cdots}$$

$$= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))} + \frac{59}{512}\int \frac{1}{5+3i\sinh(c\cdots}$$

$$= \frac{59x}{2048} - \frac{59i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d} - \frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c\cdots}{512d(5+3i\sinh\cdots}$$

Mathematica [B] time = 0.69, size = 277, normalized size = 2.92

$$\frac{-\frac{144\sinh\left(\frac{1}{2}(c+dx)\right)\left(5\sinh\left(\frac{1}{2}(c+dx)\right)-3i\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{3\sinh(c+dx)-5i} + \frac{48}{\left((1+2i)\cosh\left(\frac{1}{2}(c+dx)\right)-(2+i)\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{48}{\left((1+2i)\sinh\left(\frac{1}{2}(c+dx)\right)+(2+i)\cos\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-118*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/
2] - 2*Sinh[(c + d*x)/2])] + (118*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(c
+ d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 59*Log[5*Cosh[c + d
*x] - 4*Sinh[c + d*x]] + 59*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] + 48/((1
 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2 + 48/((2 + I)*Cosh
[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^2 - (144*Sinh[(c + d*x)/2]*((-
3*I)*Cosh[(c + d*x)/2] + 5*Sinh[(c + d*x)/2]))/(-5*I + 3*Sinh[c + d*x]))/(4
096*d)
```

fricas [B] time = 0.94, size = 191, normalized size = 2.01

$$\frac{\left(531\,e^{(4\,dx+4\,c)}-3540i\,e^{(3\,dx+3\,c)}-6962\,e^{(2\,dx+2\,c)}+3540i\,e^{(dx+c)}+531\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-\left(531\,e^{(4\,dx+4\,c)}-3540i\,e\right.}{18432\,de^{(4\,dx+4\,c)}-122880i\,de^{(3\,dx+3\,c)}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((531*e^(4*d*x + 4*c) - 3540*I*e^(3*d*x + 3*c) - 6962*e^(2*d*x + 2*c) + 354
0*I*e^(d*x + c) + 531)*log(e^(d*x + c) - 1/3*I) - (531*e^(4*d*x + 4*c) - 35
40*I*e^(3*d*x + 3*c) - 6962*e^(2*d*x + 2*c) + 3540*I*e^(d*x + c) + 531)*log
(e^(d*x + c) - 3*I) - 1416*I*e^(3*d*x + 3*c) - 7080*e^(2*d*x + 2*c) + 5784*
I*e^(d*x + c) + 1080)/(18432*d*e^(4*d*x + 4*c) - 122880*I*d*e^(3*d*x + 3*c)
 - 241664*d*e^(2*d*x + 2*c) + 122880*I*d*e^(d*x + c) + 18432*d)
```

giac [A] time = 0.27, size = 87, normalized size = 0.92

$$-\frac{\frac{8\left(-177i\,e^{(3\,dx+3\,c)}-885\,e^{(2\,dx+2\,c)}+723i\,e^{(dx+c)}+135\right)}{\left(-3i\,e^{(2\,dx+2\,c)}-10\,e^{(dx+c)}+3i\right)^2}-59\,\log\left(3\,e^{(dx+c)}-i\right)+59\,\log\left(e^{(dx+c)}-3i\right)}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(8*(-177*I*e^(3*d*x + 3*c) - 885*e^(2*d*x + 2*c) + 723*I*e^(d*x + c
) + 135)/(-3*I*e^(2*d*x + 2*c) - 10*e^(d*x + c) + 3*I)^2 - 59*log(3*e^(d*x
+ c) - I) + 59*log(e^(d*x + c) - 3*I))/d
```

maple [B] time = 0.10, size = 224, normalized size = 2.36

$$-\frac{63}{3200d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^2}-\frac{27i}{400d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^2}-\frac{963}{12800d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}+\frac{}{1600}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^3,x)
```

```
[Out] -63/3200/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^2-27/400*I/d/(5*tanh(1/2*d*x+1/2*c
)-4-3*I)^2-963/12800/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)+123/1600*I/d/(5*tanh(1
/2*d*x+1/2*c)-4-3*I)-59/2048/d*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)+63/3200/d/(5
*tanh(1/2*d*x+1/2*c)+4-3*I)^2-27/400*I/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^2-96
```

```
3/12800/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)-123/1600*I/d/(5*tanh(1/2*d*x+1/2*c)
+4-3*I)+59/2048/d*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I)
```

maxima [A] time = 0.41, size = 108, normalized size = 1.14

$$-\frac{59i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{1024d}-\frac{-723ie^{(-dx-c)}-885e^{(-2dx-2c)}+177ie^{(-3dx-3c)}+135}{d\left(-15360ie^{(-dx-c)}-30208e^{(-2dx-2c)}+15360ie^{(-3dx-3c)}+2304e^{(-4dx-4c)}+230\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -59/1024*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (-723*I*e^(-d*x - c) - 885*
e^(-2*d*x - 2*c) + 177*I*e^(-3*d*x - 3*c) + 135)/(d*(-15360*I*e^(-d*x - c)
- 30208*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x - 3*c) + 2304*e^(-4*d*x - 4*c)
 + 2304))
```

mupad [B] time = 1.62, size = 143, normalized size = 1.51

$$\frac{\frac{295}{2304d}+\frac{e^{c+dx}59i}{768d}}{1-e^{2c+2dx}+\frac{e^{c+dx}10i}{3}}-\frac{59\ln\left(-\frac{59e^{c+dx}}{4}+\frac{177}{4}i\right)}{2048d}+\frac{59\ln\left(\frac{531e^{c+dx}}{4}-\frac{177}{4}i\right)}{2048d}-\frac{\frac{5}{72d}+\frac{e^{c+dx}41i}{216d}}{e^{4c+4dx}-\frac{118e^{2c+2dx}}{9}+1+\frac{e^{c+dx}20i}{3}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5)^3,x)
```

```
[Out] ((exp(c + d*x)*59i)/(768*d) + 295/(2304*d))/((exp(c + d*x)*10i)/3 - exp(2*c
 + 2*d*x) + 1) - (59*log(177i/4 - (59*exp(c + d*x))/4))/(2048*d) + (59*log(
(531*exp(c + d*x))/4 - 177i/4))/(2048*d) - ((exp(c + d*x)*41i)/(216*d) + 5/
(72*d))/((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3*d*x
)*20i)/3 + exp(4*c + 4*d*x) + 1)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotInvertible

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**3,x)
```

```
[Out] Exception raised: NotInvertible
```

3.95 $\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=124

$$-\frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{385i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d}+$$

```
[Out] 385/32768*x-385/16384*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-1/16*I*cosh
(d*x+c)/d/(5+3*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))^
2-311/8192*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2664, 2754, 12, 2657}

$$-\frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{385i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d}+$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (((385*I)/16384)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])
])/d - ((I/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^3) - (((25*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((311*I)/8192)*Cosh[c
+ d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
```

```
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{1}{48}\int \frac{-15+6i\sinh(c+dx)}{(5+3i\sinh(c+dx))^3}\,dx$$

$$= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} + \frac{\int \frac{186-75i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}\,dx}{1536}$$

$$= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+d}{8192d(5+3i\sinh(c}$$

$$= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+d}{8192d(5+3i\sinh(c}$$

$$= \frac{385x}{32768} - \frac{385i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d} - \frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(}{512d(5+3i\sin}$$

Mathematica [B] time = 1.93, size = 308, normalized size = 2.48

$$\frac{\frac{2(-298563i\sinh(c+dx)+89364i\sinh(2(c+dx))+8397i\sinh(3(c+dx))+166615\cosh(c+dx)+82530\cosh(2(c+dx))-13995\cosh(3(c+dx))-235150)}{(3\sinh(c+dx)-5i)^3} + \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-3850*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)
/2] - 2*Sinh[(c + d*x)/2])] + (3850*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 1925*Log[5*Cosh[c
```

```
 + d*x] - 4*Sinh[c + d*x]] + 1925*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] +
(2656 - 192*I)/((1 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2
+ (2656 + 192*I)/((2 + I)*Cosh[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^
2 + (2*(-235150 + 166615*Cosh[c + d*x] + 82530*Cosh[2*(c + d*x)] - 13995*Co
sh[3*(c + d*x)] - (298563*I)*Sinh[c + d*x] + (89364*I)*Sinh[2*(c + d*x)] +
(8397*I)*Sinh[3*(c + d*x)]))/(-5*I + 3*Sinh[c + d*x])^3)/(327680*d)
```

fricas [B] time = 0.55, size = 281, normalized size = 2.27

$$\frac{\left(31185\,e^{(6\,dx+6\,c)} - 311850i\,e^{(5\,dx+5\,c)} - 1133055\,e^{(4\,dx+4\,c)} + 1778700i\,e^{(3\,dx+3\,c)} + 1133055\,e^{(2\,dx+2\,c)} - 311850i\,e^{(dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((31185*e^(6*d*x + 6*c) - 311850*I*e^(5*d*x + 5*c) - 1133055*e^(4*d*x + 4*c
) + 1778700*I*e^(3*d*x + 3*c) + 1133055*e^(2*d*x + 2*c) - 311850*I*e^(d*x +
 c) - 31185)*log(e^(d*x + c) - 1/3*I) - (31185*e^(6*d*x + 6*c) - 311850*I*e
^(5*d*x + 5*c) - 1133055*e^(4*d*x + 4*c) + 1778700*I*e^(3*d*x + 3*c) + 1133
055*e^(2*d*x + 2*c) - 311850*I*e^(d*x + c) - 31185)*log(e^(d*x + c) - 3*I)
- 83160*I*e^(5*d*x + 5*c) - 693000*e^(4*d*x + 4*c) + 1915760*I*e^(3*d*x + 3
*c) + 1747728*e^(2*d*x + 2*c) - 588600*I*e^(d*x + c) - 67176)/(2654208*d*e^
(6*d*x + 6*c) - 26542080*I*d*e^(5*d*x + 5*c) - 96436224*d*e^(4*d*x + 4*c) +
 151388160*I*d*e^(3*d*x + 3*c) + 96436224*d*e^(2*d*x + 2*c) - 26542080*I*d*
e^(d*x + c) - 2654208*d)
```

giac [A] time = 0.17, size = 109, normalized size = 0.88

$$-\frac{\frac{8\left(10395i\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}-239470i\,e^{(3\,dx+3\,c)}-218466\,e^{(2\,dx+2\,c)}+73575i\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)^{3}} - 1155\,\log\left(3\,e^{(dx+c)}-i\right) + 1155\,\log\left(\right.}{98304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(8*(10395*I*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) - 239470*I*e^(
3*d*x + 3*c) - 218466*e^(2*d*x + 2*c) + 73575*I*e^(d*x + c) + 8397)/(3*e^(2
*d*x + 2*c) - 10*I*e^(d*x + c) - 3)^3 - 1155*log(3*e^(d*x + c) - I) + 1155*
log(e^(d*x + c) - 3*I))/d
```

maple [B] time = 0.09, size = 314, normalized size = 2.53

$$\frac{1053}{32000d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^{3}}+\frac{99i}{8000d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^{3}}-\frac{783}{128000d\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^{2}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^4,x)
```

```
[Out] 1053/32000/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^3+99/8000*I/d/(5*tanh(1/2*d*x+1/
2*c)-4-3*I)^3-783/128000/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^2-3753/64000*I/d/(
5*tanh(1/2*d*x+1/2*c)-4-3*I)^2-39933/1024000/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I
)+8361/256000*I/d/(5*tanh(1/2*d*x+1/2*c)-4-3*I)-385/32768/d*ln(5*tanh(1/2*d
*x+1/2*c)-4-3*I)+1053/32000/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^3-99/8000*I/d/(
5*tanh(1/2*d*x+1/2*c)+4-3*I)^3+783/128000/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^2
-3753/64000*I/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^2-39933/1024000/d/(5*tanh(1/2
*d*x+1/2*c)+4-3*I)-8361/256000*I/d/(5*tanh(1/2*d*x+1/2*c)+4-3*I)+385/32768/
d*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I)
```

maxima [A] time = 0.42, size = 152, normalized size = 1.23

$$-\frac{385i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{16384\,d}-\frac{73575i\,e^{(-dx-c)}+218466\,e^{(-2\,dx-2\,c)}-239470i\,e^{(-3\,dx-3\,c)}-86}{d\left(3317760i\,e^{(-dx-c)}+12054528\,e^{(-2\,dx-2\,c)}-18923520i\,e^{(-3\,dx-3\,c)}-12054528\,e^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -385/16384*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (73575*I*e^(-d*x - c) + 2
18466*e^(-2*d*x - 2*c) - 239470*I*e^(-3*d*x - 3*c) - 86625*e^(-4*d*x - 4*c)
 + 10395*I*e^(-5*d*x - 5*c) - 8397)/(d*(3317760*I*e^(-d*x - c) + 12054528*e
^(-2*d*x - 2*c) - 18923520*I*e^(-3*d*x - 3*c) - 12054528*e^(-4*d*x - 4*c) +
 3317760*I*e^(-5*d*x - 5*c) + 331776*e^(-6*d*x - 6*c) - 331776))
```

mupad [B] time = 2.11, size = 232, normalized size = 1.87

$$\frac{\frac{1925}{36864\,d}+\frac{e^{c+d\,x}\,385i}{12288\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,10i}{3}}+\frac{\frac{41}{486\,d}+\frac{e^{c+d\,x}\,365i}{1458\,d}}{\frac{109\,e^{4\,c+4\,d\,x}}{3}-\frac{109\,e^{2\,c+2\,d\,x}}{3}-e^{6\,c+6\,d\,x}+1+e^{c+d\,x}\,10i-\frac{e^{3\,c+3\,d\,x}\,1540i}{27}+e^{5\,c+5\,d\,x}\,10i}-\frac{385\ln\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5)^4,x)
```

```
[Out] ((exp(c + d*x)*385i)/(12288*d) + 1925/(36864*d))/((exp(c + d*x)*10i)/3 - ex
p(2*c + 2*d*x) + 1) + ((exp(c + d*x)*365i)/(1458*d) + 41/(486*d))/(exp(c +
d*x)*10i - (109*exp(2*c + 2*d*x))/3 - (exp(3*c + 3*d*x)*1540i)/27 + (109*ex
p(4*c + 4*d*x))/3 + exp(5*c + 5*d*x)*10i - exp(6*c + 6*d*x) + 1) - (385*log
(1155i/4 - (385*exp(c + d*x))/4))/(32768*d) + (385*log((3465*exp(c + d*x))/
4 - 1155i/4))/(32768*d) - ((exp(c + d*x)*385i)/(10368*d) + 3461/(31104*d))/
```

```
((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3*d*x)*20i)/3
 + exp(4*c + 4*d*x) + 1)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotInvertible

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**4,x)
```

```
[Out] Exception raised: NotInvertible
```

3.96 $\int (a + b\sinh(c + dx))^5 \, dx$

Optimal. Leaf size=183

$$\frac{b(47a^2 - 16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} + \frac{7ab^2(22a^2 - 23b^2)\sinh(c+dx)\cosh(c+dx)}{120d} + \frac{b(107a^4 - 1}{}$$

```
[Out] 1/8*a*(8*a^4-40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4-192*a^2*b^2+16*b^4)*cosh(
d*x+c)/d+7/120*a*b^2*(22*a^2-23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47*a
^2-16*b^2)*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d+9/20*a*b*cosh(d*x+c)*(a+b*sinh
(d*x+c))^3/d+1/5*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^4/d
```

Rubi [A] time = 0.27, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2656, 2753, 2734}

$$\frac{b(-192a^2b^2 + 107a^4 + 16b^4)\cosh(c+dx)}{30d} + \frac{b(47a^2 - 16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} + \frac{7ab^2(22a^2 - 2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^5,x]
```

```
[Out] (a*(8*a^4 - 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 - 192*a^2*b^2 + 16*b^4)
*Cosh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 - 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 - 16*b^2)*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)
/(60*d) + (9*a*b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^3)/(20*d) + (b*Cosh[c
+ d*x]*(a + b*Sinh[c + d*x])^4)/(5*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^5\,dx = \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} + \frac{1}{5}\int (a+b\sinh(c+dx))^3\left(5a^2-4b^2+9ab\,\text{si}\right.$$
$$= \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} + \frac{1}{20}$$
$$= \frac{b\left(47a^2-16b^2\right)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} + \frac{9ab\cosh(c+dx)(a+b\sinh(c}{20d}$$
$$= \frac{1}{8}a\left(8a^4-40a^2b^2+15b^4\right)x + \frac{b\left(107a^4-192a^2b^2+16b^4\right)\cosh(c+dx)}{30d} + \frac{7ab^2\left(22a\right.}{}$$

Mathematica [A] time = 0.65, size = 138, normalized size = 0.75

$$\frac{50\left(8a^2b^3-b^5\right)\cosh(3(c+dx))+15a\left(40\left(2a^2b^2-b^4\right)\sinh(2(c+dx))+4\left(8a^4-40a^2b^2+15b^4\right)(c+dx)+5b^4\,\text{s}\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^5,x]
```

```
[Out] (300*b*(8*a^4 - 12*a^2*b^2 + b^4)*Cosh[c + d*x] + 50*(8*a^2*b^3 - b^5)*Cosh
[3*(c + d*x)] + 6*b^5*Cosh[5*(c + d*x)] + 15*a*(4*(8*a^4 - 40*a^2*b^2 + 15*
b^4)*(c + d*x) + 40*(2*a^2*b^2 - b^4)*Sinh[2*(c + d*x)] + 5*b^4*Sinh[4*(c +
 d*x)]))/(480*d)
```

fricas [A] time = 0.49, size = 223, normalized size = 1.22

$$\frac{3\,b^5\cosh(dx+c)^5+15\,b^5\cosh(dx+c)\sinh(dx+c)^4+150\,ab^4\cosh(dx+c)\sinh(dx+c)^3+25\left(8\,a^2b^3-b^5\right)\text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^5*cosh(d*x + c)^5 + 15*b^5*cosh(d*x + c)*sinh(d*x + c)^4 + 150*a
*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 25*(8*a^2*b^3 - b^5)*cosh(d*x + c)^3 +
 30*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*d*x + 15*(2*b^5*cosh(d*x + c)^3 + 5*(8*
a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + 150*(8*a^4*b - 12*a^2*b^3 +
 b^5)*cosh(d*x + c) + 150*(a*b^4*cosh(d*x + c)^3 + 4*(2*a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.42, size = 269, normalized size = 1.47

$$\frac{b^5 e^{(5\,dx+5\,c)}}{160\,d}+\frac{5\,ab^4 e^{(4\,dx+4\,c)}}{64\,d}-\frac{5\,ab^4 e^{(-4\,dx-4\,c)}}{64\,d}+\frac{b^5 e^{(-5\,dx-5\,c)}}{160\,d}+\frac{1}{8}\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)x+\frac{5\left(8\,a^2b^3-b^5\right)e^{(3\,dx+3\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^(
-4*d*x - 4*c)/d + 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 - 40*a^3*b^2 +
15*a*b^4)*x + 5/96*(8*a^2*b^3 - b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2 - a
*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(d*x + c)/d +
 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 - a*b^4)
*e^(-2*d*x - 2*c)/d + 5/96*(8*a^2*b^3 - b^5)*e^(-3*d*x - 3*c)/d
```

maple [A] time = 0.04, size = 155, normalized size = 0.85

$$\frac{b^5\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cosh\left(dx+c\right)+5a\,b^4\left(\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+5*a*b^4*((
1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+10*a^2*b^3*(-
2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+10*a^3*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-
1/2*d*x-1/2*c)+5*a^4*b*cosh(d*x+c)+a^5*(d*x+c))
```

maxima [A] time = 0.32, size = 272, normalized size = 1.49

$$\frac{5}{64}\,ab^4\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{5}{4}\,a^3b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^5x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 5/4*a^3*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2
*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c
)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^
(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 5*a^4*b*cosh(d*x + c)/d
```

mupad [B] time = 0.60, size = 160, normalized size = 0.87

$$\frac{75\,b^5\cosh\left(c+d\,x\right)-\frac{25\,b^5\cosh(3\,c+3\,d\,x)}{2}+\frac{3\,b^5\cosh(5\,c+5\,d\,x)}{2}-900\,a^2\,b^3\cosh\left(c+d\,x\right)-150\,a\,b^4\sinh\left(2\,c+2\,d\,x\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x))^5,x)
```

```
[Out] (75*b^5*cosh(c + d*x) - (25*b^5*cosh(3*c + 3*d*x))/2 + (3*b^5*cosh(5*c + 5*
d*x))/2 - 900*a^2*b^3*cosh(c + d*x) - 150*a*b^4*sinh(2*c + 2*d*x) + (75*a*b
^4*sinh(4*c + 4*d*x))/4 + 100*a^2*b^3*cosh(3*c + 3*d*x) + 300*a^3*b^2*sinh(
2*c + 2*d*x) + 600*a^4*b*cosh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x - 600*
a^3*b^2*d*x)/(120*d)
```

sympy [A] time = 2.17, size = 314, normalized size = 1.72

$$\begin{cases} a^5x+\frac{5a^4b\cosh\left(c+dx\right)}{d}+5a^3b^2x\sinh^2\left(c+dx\right)-5a^3b^2x\cosh^2\left(c+dx\right)+\frac{5a^3b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{d}+\frac{10a^2b^3\sinh^2\left(c+dx\right)\cdots}{d} \\ x\left(a+b\sinh\left(c\right)\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*cosh(c + d*x)/d + 5*a**3*b**2*x*sinh(c + d*x)*
*2 - 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 10*a**2*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 20*a**2*b**3*cosh(c
+ d*x)**3/(3*d) + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 + 25*a*b**4*sinh(c
 + d*x)**3*cosh(c + d*x)/(8*d) - 15*a*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(
8*d) + b**5*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b**5*sinh(c + d*x)**2*cosh
(c + d*x)**3/(3*d) + 8*b**5*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*s
inh(c))**5, True))
```

3.97 $\int (a+b\sinh(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{ab\left(19a^2-16b^2\right)\cosh(c+dx)}{6d}+\frac{b^2\left(26a^2-9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(8a^4-24a^2b^2+3b^4\right)+\frac{b\cosh(c$$

```
[Out] 1/8*(8*a^4-24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2-16*b^2)*cosh(d*x+c)/d+1/24*b
^2*(26*a^2-9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*cosh(d*x+c)*(a+b*sinh(
d*x+c))^2/d+1/4*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^3/d
```

Rubi [**A**] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2656, 2753, 2734}

$$\frac{ab\left(19a^2-16b^2\right)\cosh(c+dx)}{6d}+\frac{b^2\left(26a^2-9b^2\right)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(-24a^2b^2+8a^4+3b^4\right)+\frac{b\cosh($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] ((8*a^4 - 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 - 16*b^2)*Cosh[c + d*x])/
(6*d) + (b^2*(26*a^2 - 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)/(12*d) + (b*Cosh[c + d*x]*(a + b*Sin
h[c + d*x])^3)/(4*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+\frac{1}{4}\int (a+b\sinh(c+dx))^2\left(4a^2-3b^2+7ab\,\text{si}\right.$$
$$=\frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+\frac{1}{12}\int$$
$$=\frac{1}{8}\left(8a^4-24a^2b^2+3b^4\right)x+\frac{ab\left(19a^2-16b^2\right)\cosh(c+dx)}{6d}+\frac{b^2\left(26a^2-9b^2\right)\cosh(c}{24d}$$

Mathematica [A] time = 0.37, size = 108, normalized size = 0.79

$$\frac{96ab\left(4a^2-3b^2\right)\cosh(c+dx)+3\left(8\left(6a^2b^2-b^4\right)\sinh(2(c+dx))+4\left(8a^4-24a^2b^2+3b^4\right)(c+dx)+b^4\sinh(4(\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] (96*a*b*(4*a^2 - 3*b^2)*Cosh[c + d*x] + 32*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(
8*a^4 - 24*a^2*b^2 + 3*b^4)*(c + d*x) + 8*(6*a^2*b^2 - b^4)*Sinh[2*(c + d*x
)] + b^4*Sinh[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.83, size = 146, normalized size = 1.07

$$\frac{3\,b^4\cosh(dx+c)\sinh(dx+c)^3+8\,ab^3\cosh(dx+c)^3+24\,ab^3\cosh(dx+c)\sinh(dx+c)^2+3\left(8\,a^4-24\,a^2b^2+\right.}{24\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^3 + 24*a*
b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*d*x + 24
*(4*a^3*b - 3*a*b^3)*cosh(d*x + c) + 3*(b^4*cosh(d*x + c)^3 + 4*(6*a^2*b^2
- b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.47, size = 200, normalized size = 1.46

$$\frac{b^4e^{(4\,dx+4\,c)}}{64\,d}+\frac{ab^3e^{(3\,dx+3\,c)}}{6\,d}+\frac{ab^3e^{(-3\,dx-3\,c)}}{6\,d}-\frac{b^4e^{(-4\,dx-4\,c)}}{64\,d}+\frac{1}{8}\left(8\,a^4-24\,a^2b^2+3\,b^4\right)x+\frac{\left(6\,a^2b^2-b^4\right)e^{(2\,dx+2\,c)}}{8\,d}+\frac{\left(4\,a^3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d + 1/6*a*b^3*e^(-3*
d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 - 24*a^2*b^2 + 3*b^
4)*x + 1/8*(6*a^2*b^2 - b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b - 3*a*b^3)*e^
(d*x + c)/d + 1/2*(4*a^3*b - 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 - b^4
)*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.04, size = 119, normalized size = 0.87

$$\frac{b^4\left(\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4a\,b^3\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh(dx+c)+6a^2b^2\left(\frac{\cosh(}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+4*
a*b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+6*a^2*b^2*(1/2*cosh(d*x+c)*sinh(
d*x+c)-1/2*d*x-1/2*c)+4*a^3*b*cosh(d*x+c)+a^4*(d*x+c))
```

maxima [A] time = 0.32, size = 182, normalized size = 1.33

$$\frac{1}{64}b^4\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{3}{4}a^2b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^4x+\frac{1}{6}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/64*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 3/4*a^2*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 4*a^3*b*cosh(d*x + c)/d
```

mupad [B] time = 0.34, size = 114, normalized size = 0.83

$$\frac{\frac{3\,b^4\sinh(4\,c+4\,d\,x)}{4}-6\,b^4\sinh(2\,c+2\,d\,x)+8\,a\,b^3\cosh(3\,c+3\,d\,x)+36\,a^2\,b^2\sinh(2\,c+2\,d\,x)-72\,a\,b^3\cosh(c+}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x))^4,x)
```

```
[Out] ((3*b^4*sinh(4*c + 4*d*x))/4 - 6*b^4*sinh(2*c + 2*d*x) + 8*a*b^3*cosh(3*c +
 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) - 72*a*b^3*cosh(c + d*x) + 96*a^3*b*
cosh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x - 72*a^2*b^2*d*x)/(24*d)
```

sympy [A] time = 1.05, size = 240, normalized size = 1.75

$$\begin{cases} a^4x + \frac{4a^3b\cosh(c+dx)}{d} + 3a^2b^2x\sinh^2(c+dx) - 3a^2b^2x\cosh^2(c+dx) + \frac{3a^2b^2\sinh(c+dx)\cosh(c+dx)}{d} + \frac{4ab^3\sinh^2(c+dx)\cosh(c+dx)}{d} \\ x(a+b\sinh(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*cosh(c + d*x)/d + 3*a**2*b**2*x*sinh(c + d*x)*
*2 - 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 4*a*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 8*a*b**3*cosh(c + d*x)**
3/(3*d) + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 + 5*b**4*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) - 3*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a
+ b*sinh(c))**4, True))
```

3.98 $\int (a+b\sinh(c+dx))^3\,dx$

Optimal. Leaf size=92

$$\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d}+\frac{1}{2}ax(2a^2-3b^2)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+d$$

```
[Out] 1/2*a*(2*a^2-3*b^2)*x+2/3*b*(4*a^2-b^2)*cosh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c)
*sinh(d*x+c)/d+1/3*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d}+\frac{1}{2}ax(2a^2-3b^2)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 3*b^2)*x)/2 + (2*b*(4*a^2 - b^2)*Cosh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*Cosh[c + d*x]*(a + b*Sinh[c + d*x
])^2)/(3*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^3\,dx = \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d} + \frac{1}{3}\int (a+b\sinh(c+dx))\left(3a^2-2b^2+5ab\sin\right.$$
$$= \frac{1}{2}a\left(2a^2-3b^2\right)x + \frac{2b\left(4a^2-b^2\right)\cosh(c+dx)}{3d} + \frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d} + $$

Mathematica [A] time = 0.18, size = 71, normalized size = 0.77

$$\frac{6a\left(2a^2-3b^2\right)(c+dx)-9b\left(b^2-4a^2\right)\cosh(c+dx)+9ab^2\sinh(2(c+dx))+b^3\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (6*a*(2*a^2 - 3*b^2)*(c + d*x) - 9*b*(-4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cos
h[3*(c + d*x)] + 9*a*b^2*Sinh[2*(c + d*x)])/(12*d)
```

fricas [A] time = 0.44, size = 91, normalized size = 0.99

$$\frac{b^3\cosh\left(dx+c\right)^3+3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+18\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+6\left(2\,a^3-3\,ab^2\right)dx+9}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 18*a*b^2*
cosh(d*x + c)*sinh(d*x + c) + 6*(2*a^3 - 3*a*b^2)*d*x + 9*(4*a^2*b - b^3)*c
osh(d*x + c))/d
```

giac [A] time = 0.15, size = 135, normalized size = 1.47

$$\frac{b^3e^{(3\,dx+3\,c)}}{24\,d}+\frac{3\,ab^2e^{(2\,dx+2\,c)}}{8\,d}-\frac{3\,ab^2e^{(-2\,dx-2\,c)}}{8\,d}+\frac{b^3e^{(-3\,dx-3\,c)}}{24\,d}+\frac{1}{2}\left(2\,a^3-3\,ab^2\right)x+\frac{3\left(4\,a^2b-b^3\right)e^{(dx+c)}}{8\,d}+\frac{3\left(4\,a^2b-b^3\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2*
d*x - 2*c)/d + 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 - 3*a*b^2)*x + 3/8*
(4*a^2*b - b^3)*e^(d*x + c)/d + 3/8*(4*a^2*b - b^3)*e^(-d*x - c)/d
```

maple [A] time = 0.04, size = 77, normalized size = 0.84

$$\frac{b^3\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)+3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\cosh\left(dx+c\right)+a^3\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh
(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*cosh(d*x+c)+a^3*(d*x+c))
```

maxima [A] time = 0.32, size = 115, normalized size = 1.25

$$-\frac{3}{8}\,ab^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x+\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3\,a^2b\cosh\left(dx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^
3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c
)/d) + 3*a^2*b*cosh(d*x + c)/d
```

mupad [B] time = 0.50, size = 75, normalized size = 0.82

$$\frac{6\,d\,x\,a^3+18\,a^2\,b\cosh\left(c+d\,x\right)+9\sinh\left(c+d\,x\right)\,a\,b^2\cosh\left(c+d\,x\right)-9\,d\,x\,a\,b^2+2\,b^3\cosh\left(c+d\,x\right)^3-6\,b^3\cos}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x))^3,x)
```

```
[Out] (2*b^3*cosh(c + d*x)^3 - 6*b^3*cosh(c + d*x) + 18*a^2*b*cosh(c + d*x) + 6*a
^3*d*x + 9*a*b^2*cosh(c + d*x)*sinh(c + d*x) - 9*a*b^2*d*x)/(6*d)
```

sympy [A] time = 0.51, size = 128, normalized size = 1.39

$$\begin{cases} a^3x+\frac{3a^2b\cosh\left(c+dx\right)}{d}+\frac{3ab^2x\sinh^2\left(c+dx\right)}{2}-\frac{3ab^2x\cosh^2\left(c+dx\right)}{2}+\frac{3ab^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d}+\frac{b^3\sinh^2\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b^3c}{} \\ x\left(a+b\sinh\left(c\right)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*cosh(c + d*x)/d + 3*a*b**2*x*sinh(c + d*x)**2/
2 - 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/(2
*d) + b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b**3*cosh(c + d*x)**3/(3*d)
, Ne(d, 0)), (x*(a + b*sinh(c))**3, True))
```

3.99 $\int (a+b\sinh(c+dx))^2\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}x\left(2a^2-b^2\right)+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2-b^2)*x+2*a*b*cosh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2-b^2\right)+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*x)/2 + (2*a*b*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^2\,dx=\frac{1}{2}\left(2a^2-b^2\right)x+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.92

$$\frac{2\left(2a^2-b^2\right)(c+dx)+8ab\cosh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*(2*a^2 - b^2)*(c + d*x) + 8*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(
4*d)
```

fricas [A] time = 0.40, size = 46, normalized size = 0.88

$$\frac{b^2\cosh(dx+c)\sinh(dx+c)+\left(2a^2-b^2\right)dx+4ab\cosh(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)*sinh(d*x + c) + (2*a^2 - b^2)*d*x + 4*a*b*cosh(d*x +
 c))/d
```

giac [A] time = 0.16, size = 76, normalized size = 1.46

$$\frac{1}{2}\left(2a^2-b^2\right)x+\frac{b^2e^{(2dx+2c)}}{8d}+\frac{abe^{(dx+c)}}{d}+\frac{abe^{(-dx-c)}}{d}-\frac{b^2e^{(-2dx-2c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 - b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d + a*b*e
^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

maple [A] time = 0.03, size = 51, normalized size = 0.98

$$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\cosh(dx+c)+a^2(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*cosh(d*x+c)+a^2*
(d*x+c))
```

maxima [A] time = 0.30, size = 55, normalized size = 1.06

$$-\frac{1}{8}b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^2x+\frac{2ab\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*cos
h(d*x + c)/d
```

mupad [B] time = 0.49, size = 41, normalized size = 0.79

$$a^2\,x-\frac{b^2\,x}{2}+\frac{\frac{\sinh(2\,c+2\,d\,x)\,b^2}{4}+2\,a\cosh\left(c+d\,x\right)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x))^2,x)
```

```
[Out] a^2*x - (b^2*x)/2 + ((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*cosh(c + d*x))/d
```

sympy [A] time = 0.26, size = 78, normalized size = 1.50

$$\begin{cases} a^2x+\frac{2ab\cosh\left(c+dx\right)}{d}+\frac{b^2x\sinh^2\left(c+dx\right)}{2}-\frac{b^2x\cosh^2\left(c+dx\right)}{2}+\frac{b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x\left(a+b\sinh\left(c\right)\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*cosh(c + d*x)/d + b**2*x*sinh(c + d*x)**2/2 - b**
2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a + b*sinh(c))**2, True))
```

3.100 $\int (a + b\sinh(c + dx))\,dx$

Optimal. Leaf size=15

$$ax + \frac{b\cosh(c + dx)}{d}$$

[Out] a*x+b*cosh(d*x+c)/d

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2638}

$$ax + \frac{b\cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sinh[c + d*x],x]

[Out] a*x + (b*Cosh[c + d*x])/d

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + b\sinh(c + dx))\,dx &= ax + b\int \sinh(c + dx)\,dx\\ &= ax + \frac{b\cosh(c + dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.73

$$ax + \frac{b\sinh(c)\sinh(dx)}{d} + \frac{b\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sinh[c + d*x],x]

[Out] a*x + (b*Cosh[c]*Cosh[d*x])/d + (b*Sinh[c]*Sinh[d*x])/d

fricas [A] time = 0.66, size = 17, normalized size = 1.13

$$\frac{adx + b\cosh\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="fricas")
[Out] (a*d*x + b*cosh(d*x + c))/d
```

giac [B] time = 0.26, size = 31, normalized size = 2.07

$$ax + \frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="giac")
[Out] a*x + 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d)
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$ax + \frac{b\cosh\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c),x)
[Out] a*x+b*cosh(d*x+c)/d
```

maxima [A] time = 0.33, size = 15, normalized size = 1.00

$$ax + \frac{b\cosh\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="maxima")
[Out] a*x + b*cosh(d*x + c)/d
```

mupad [B] time = 0.43, size = 15, normalized size = 1.00

$$a\,x + \frac{b\cosh\left(c + d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x),x)
[Out] a*x + (b*cosh(c + d*x))/d
```

<u>**sympy**</u> **[A]** time = 0.13, size = 17, normalized size = 1.13

$$ax + b\left(\begin{cases} \frac{\cosh(c+dx)}{d} & \text{for } d \neq 0 \\ x\sinh(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

3.101 $\int \frac{1}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[Out] -2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2660, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = -\frac{(2i)\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$

$$= \frac{(4i)\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 1.18

$$\frac{2\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{d\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d)
```

fricas [B] time = 0.65, size = 162, normalized size = 3.68

$$\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)-2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^
2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*
cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))/(sqrt(a
^2 + b^2)*d)
```

giac [A] time = 0.15, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

maple [A] time = 0.04, size = 43, normalized size = 0.98

$$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{d\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

maxima [A] time = 0.44, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-dx-c)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 +
 b^2)))/(sqrt(a^2 + b^2)*d)
```

mupad [B] time = 0.76, size = 55, normalized size = 1.25

$$\frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{\sqrt{-a^{2}\,d^{2}-b^{2}\,d^{2}}}\right)}{\sqrt{-a^{2}\,d^{2}-b^{2}\,d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)),x)
```

```
[Out] (2*atan((a*d + b*d*exp(d*x)*exp(c))/(- a^2*d^2 - b^2*d^2)^(1/2)))/(- a^2*d^
2 - b^2*d^2)^(1/2)
```

sympy [A] time = 8.95, size = 187, normalized size = 4.25

$$\begin{cases} \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd} & \text{for } a=0 \\ \frac{2i\sqrt{b^2}}{b^2 d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-ibd\sqrt{b^2}} & \text{for } a=-\sqrt{-b^2} \\ -\frac{2i\sqrt{b^2}}{b^2 d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd\sqrt{b^2}} & \text{for } a=\sqrt{-b^2} \\ \frac{x}{a+b\sinh(c)} & \text{for } d=0 \\ -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((log(tanh(c/2 + d*x/2))/(b*d), Eq(a, 0)), (2*I*sqrt(b**2)/(b**2*d
*tanh(c/2 + d*x/2) - I*b*d*sqrt(b**2)), Eq(a, -sqrt(-b**2))), (-2*I*sqrt(b*
*2)/(b**2*d*tanh(c/2 + d*x/2) + I*b*d*sqrt(b**2)), Eq(a, sqrt(-b**2))), (x/
(a + b*sinh(c)), Eq(d, 0)), (-log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**
2)/a)/(d*sqrt(a**2 + b**2)) + log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**
2)/a)/(d*sqrt(a**2 + b**2)), True))
```

3.102 $\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

```
[Out] -2*a*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-b
*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 12, 2660, 618, 204}

$$-\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2
)*d) - (b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx &= -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}+\frac{\int \frac{a}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}+\frac{a\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}-\frac{(2ia)\,\mathrm{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right.}{\left(a^2+b^2\right)d}\\
&= -\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}+\frac{(4ia)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2+b^2\right)-x^2}\,dx,x,-2ib+2a\tan\right.}{\left(a^2+b^2\right)d}\\
&= -\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 85, normalized size = 1.08

$$-\frac{\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}}+\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)(a+b\sinh(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] -(((2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3
/2) + (b*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])))/d)
```

fricas [B] time = 0.51, size = 423, normalized size = 5.35

$$-\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+ab\sinh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)-ab+2\left(ab\cosh\left(dx+c\right)+a^2\right)\sinh}{\left(a^4b+2\,a^2b^3+b^5\right)d\cosh\left(dx+c\right)^2+\left(a^4b+2\,a^2b^3+b^5\right)d\sinh\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cosh
(d*x + c) - a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 + b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(
a^3 + a*b^2)*cosh(d*x + c) - 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4*b + 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2
 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) - (a^4*b + 2*a^2*b^3 + b^5)*
d + 2*((a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4
)*d)*sinh(d*x + c))
```

giac [A] time = 0.43, size = 119, normalized size = 1.51

$$\frac{\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(ae^{(dx+c)}-b\right)}{\left(a^2+b^2\right)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +
 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*e^(d*x + c) - b)/((a^2
+ b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)))/d
```

maple [A] time = 0.07, size = 118, normalized size = 1.49

$$\frac{-\frac{2\left(-\frac{b^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^{2}+b^{2}\right)}-\frac{b}{a^{2}+b^{2}}\right)}{\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*(-b^2/a/(a^2+b^2)*tanh(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tanh(1/2*d*x+1/
2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [A] time = 0.42, size = 138, normalized size = 1.75

$$\frac{a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-dx-c)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}-\frac{2\left(ae^{(-dx-c)}+b\right)}{\left(a^{2}b+b^{3}+2\left(a^{3}+ab^{2}\right)e^{(-dx-c)}-\left(a^{2}b+b^{3}\right)e^{(-2\,dx-2\,c)}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(a*e^(-d*x - c) + b)/((a^2*b + b^3 + 2*
(a^3 + a*b^2)*e^(-d*x - c) - (a^2*b + b^3)*e^(-2*d*x - 2*c))*d)
```

mupad [B] time = 0.88, size = 200, normalized size = 2.53

$$\frac{a\,\ln\left(\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b\,\left(a^2+b^2\right)^{3/2}}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,\left(a^2+b^2\right)}\right)}{d\,\left(a^2+b^2\right)^{3/2}}-\frac{a\,\ln\left(-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,\left(a^2+b^2\right)}-\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b\,\left(a^2+b^2\right)^{3/2}}\right)}{d\,\left(a^2+b^2\right)^{3/2}}-\frac{\frac{2\,b^2}{d\,\left(a^2\,b+b^3\right)}-\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(a^2\,b+b^3\right)}}{2\,a\,\mathrm{e}^{c+d\,x}-b+b\,\mathrm{e}^{2\,c+2\,d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x))^2,x)
```

```
[Out] (a*log((2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2)) - (2*a*exp(c + d*x)
)/(b*(a^2 + b^2))))/(d*(a^2 + b^2)^(3/2)) - (a*log(- (2*a*exp(c + d*x))/(b*
(a^2 + b^2)) - (2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2))))/(d*(a^2 +
 b^2)^(3/2)) - ((2*b^2)/(d*(a^2*b + b^3)) - (2*a*b*exp(c + d*x))/(d*(a^2*b
+ b^3)))/(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.103 $\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{3ab\cosh(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

```
[Out] -(2*a^2-b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(
5/2)/d-1/2*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^2-3/2*a*b*cosh(d*x+c
)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$-\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{3ab\cosh(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))}-\frac{b\cosh(c+dx)}{2d\left(a^2+b^2\right)(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-3),x]
```

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) - (b*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x])
^2) - (3*a*b*Cosh[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{\int \frac{-2a+b\sinh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{\int \frac{2a^2 \cdots}{a+b\sin \cdots}}{2\left(a^2 \cdots\right.}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{\left(2a^2-b \cdots\right.}{\cdots}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} - \frac{\left(i\left(2a^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))} + \frac{\left(2i\left(2a^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} - \frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2} - \frac{\cdots}{2\left(a^2 \cdots\right.}$$

<u>Mathematica</u> [A] time = 0.28, size = 117, normalized size = 0.92

$$\frac{\frac{2\left(2a^2-b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{b\cosh(c+dx)\left(4a^2+3ab\sinh(c+dx)+b^2\right)}{(a+b\sinh(c+dx))^2}}{2d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[
-a^2 - b^2] - (b*Cosh[c + d*x]*(4*a^2 + b^2 + 3*a*b*Sinh[c + d*x]))/(a + b*
Sinh[c + d*x])^2)/(2*(a^2 + b^2)^2*d)
```

<u>fricas</u> [B] time = 0.61, size = 1347, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*b^2 + 6*a*b^4 + 2*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2*
(2*a^4*b + a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 + a^3*b^2 - a*b^4)*cos
h(d*x + c)^2 + 6*(2*a^5 + a^3*b^2 - a*b^4 + (2*a^4*b + a^2*b^3 - b^5)*cosh(
d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^2 - b^4)*cosh(d*x + c)^4 + (2*a^2*b^2
 - b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 - b^4 + 4*(2*a^3*b - a*b^3)*cosh(d*x +
c)^3 + 4*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cosh(d*x + c))*sinh(d*x + c)^
3 + 2*(4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 - 4*a^2*b^2 + b^
4 + 3*(2*a^2*b^2 - b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^2 - 4*(2*a^3*b - a*b^3)*cosh(d*x + c) - 4*(2*a^3*b - a*b^3
- (2*a^2*b^2 - b^4)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 -
 (4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*lo
g((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
+ b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*co
sh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(10*a
^4*b + 11*a^2*b^3 + b^5)*cosh(d*x + c) - 2*(10*a^4*b + 11*a^2*b^3 + b^5 - 3
*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^4 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*sinh(d*x + c)^4 + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 + 5*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 + 3*a^4
*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c) + (a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d)*sinh(d*x + c)^3 - 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos
h(d*x + c) + 2*(3*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c)^2
 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c) + (2*a^8 + 5*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 + 3*a^4*b
^4 + 3*a^2*b^6 + b^8)*d + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^3 + 3*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 +
 (2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c) - (a^7*b +
 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d)*sinh(d*x + c))
```

giac [A] time = 0.39, size = 231, normalized size = 1.82

$$\frac{\frac{\left(2\,a^2-b^2\right)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(2\,a^2be^{(3\,dx+3\,c)}-b^3e^{(3\,dx+3\,c)}+6\,a^3e^{(2\,dx+2\,c)}-3\,ab^2e^{(2\,dx+2\,c)}-10\,a^2be^{(dx+c)}-b^3e^{(dx+c)}+3\,ab^2\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 - b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2
*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^
2 + b^2)) + 2*(2*a^2*b*e^(3*d*x + 3*c) - b^3*e^(3*d*x + 3*c) + 6*a^3*e^(2*d
*x + 2*c) - 3*a*b^2*e^(2*d*x + 2*c) - 10*a^2*b*e^(d*x + c) - b^3*e^(d*x + c
) + 3*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
- b)^2))/d
```

maple [B] time = 0.09, size = 280, normalized size = 2.20

$$\frac{-\frac{2\left(-\frac{b^{2}\left(5a^{2}+2b^{2}\right)\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{b\left(4a^{4}-7a^{2}b^{2}-2b^{4}\right)\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(11a^{2}+2b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)a}+\frac{b\left(4a^{2}+b^{2}\right)}{2a^{4}+4a^{2}b^{2}+2b^{4}}\right)}{\left(\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b-a\right)^{2}}+\frac{\left(2a^{2}-b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3
-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tanh(1/2*d*x+1/2*c)^
2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tanh(1/2*d*x+1/2*c)+1/2*b*(4
*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*
c)*b-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [B] time = 0.43, size = 315, normalized size = 2.48

$$\frac{\left(2\,a^{2}-b^{2}\right)\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-dx-c)}-a+\sqrt{a^{2}+b^{2}}}\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}\,d}-\frac{3\,ab^{2}+\left(10\,a^{2}b+b^{3}\right)e^{(-dx-c)}+3\left(2\,a^{3}-\right.}{\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}+4\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)e^{(-dx-c)}+2\left(2\,a^{6}+3\,a^{4}b^{2}-b^{6}\right)e^{(-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2 - b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c
) - a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d) - (3*
a*b^2 + (10*a^2*b + b^3)*e^(-d*x - c) + 3*(2*a^3 - a*b^2)*e^(-2*d*x - 2*c)
- (2*a^2*b - b^3)*e^(-3*d*x - 3*c))/((a^4*b^2 + 2*a^2*b^4 + b^6 + 4*(a^5*b
+ 2*a^3*b^3 + a*b^5)*e^(-d*x - c) + 2*(2*a^6 + 3*a^4*b^2 - b^6)*e^(-2*d*x -
 2*c) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*d*x - 3*c) + (a^4*b^2 + 2*a^2*b
^4 + b^6)*e^(-4*d*x - 4*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\sinh\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.104 $\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=174

$$-\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{6d\left(a^2+b^2\right)^3(a+b\sinh(c+dx))}-\frac{5ab\cosh(c+dx)}{6d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))^2}-$$

```
[Out] -a*(2*a^2-3*b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(7/2)/d-1/3*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^3-5/6*a*b*cosh(d
*x+c)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))^2-1/6*b*(11*a^2-4*b^2)*cosh(d*x+c)/(a
^2+b^2)^3/d/(a+b*sinh(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2664, 2754, 12, 2660, 618, 204}

$$-\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}-\frac{b\left(11a^2-4b^2\right)\cosh(c+dx)}{6d\left(a^2+b^2\right)^3(a+b\sinh(c+dx))}-\frac{5ab\cosh(c+dx)}{6d\left(a^2+b^2\right)^2(a+b\sinh(c+dx))^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((
a^2 + b^2)^(7/2)*d)) - (b*Cosh[c + d*x])/(3*(a^2 + b^2)*d*(a + b*Sinh[c + d
*x])^3) - (5*a*b*Cosh[c + d*x])/(6*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x])^2)
 - (b*(11*a^2 - 4*b^2)*Cosh[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*Sinh[c + d*
x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{\int \frac{-3a+2b\sinh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} + \frac{\int \frac{2(3a^2\cdots}{(\cdots}}{\cdots}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(1\cdots\right.}{6\left(a^2+\cdots\right.}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(1\cdots\right.}{6\left(a^2+\cdots\right.}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(1\cdots\right.}{6\left(a^2+\cdots\right.}$$

$$= -\frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6\left(a^2+b^2\right)^2 d(a+b\sinh(c+dx))^2} - \frac{b\left(1\cdots\right.}{6\left(a^2+\cdots\right.}$$

$$= -\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}d} - \frac{b\cosh(c+dx)}{3\left(a^2+b^2\right)d(a+b\sinh(c+dx))^3} - \frac{\cdots}{6\left(\cdots\right.}$$

Mathematica [A] time = 0.72, size = 159, normalized size = 0.91

$$\frac{\frac{6a\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{b\cosh(c+dx)\left(-18a^4+3ab\left(b^2-9a^2\right)\sinh(c+dx)-5a^2b^2+\left(4b^4-11a^2b^2\right)\sinh^2(c+dx)-2b^4\right)}{(a+b\sinh(c+dx))^3}}{6d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 - 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2] + (b*Cosh[c + d*x]*(-18*a^4 - 5*a^2*b^2 - 2*b^4 + 3*a*b*(-9
*a^2 + b^2)*Sinh[c + d*x] + (-11*a^2*b^2 + 4*b^4)*Sinh[c + d*x]^2))/(a + b*
Sinh[c + d*x])^3)/(6*(a^2 + b^2)^3*d)
```

fricas [B] time = 0.60, size = 2934, normalized size = 16.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*c
osh(d*x + c)^5 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 - 30*(2*
a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 - 30*(2*a^6*b - a^4*b^3 - 3*a^
2*b^5 + (2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*
(22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^3 - 4*(22*a^7 -
 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6 + 15*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*co
sh(d*x + c)^2 + 30*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 12*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c)^2 + 1
2*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7 - 5*(2*a^5*b^2 - a^3*b^4 - 3*a
*b^6)*cosh(d*x + c)^3 - 15*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2
- (22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c
)^2 + 3*((2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 - 3*a*b^5)*sinh
(d*x + c)^6 - 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)
^5 + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c))*sinh(d
*x + c)^5 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b
 - 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 - 12*a^4*b^2
+ 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3
*b^3 - 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^
2 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 - 3*(8*a^5*b - 14*a^3*b^3 +
 3*a*b^5 - 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^4 - 20*(2*a^4*b^2 - 3*a^2*
b^4)*cosh(d*x + c)^3 - 6*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 -
 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 -
 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^4 + 2*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 2*(4*a^6 - 12*a^4*b^2 + 9
*a^2*b^4)*cosh(d*x + c)^2 - (8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c
)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c)
+ a)*sinh(d*x + c) - b)) - 30*(4*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(d*x + c)
 - 6*(20*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6 + 5*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)
*cosh(d*x + c)^4 + 20*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(
22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^2 - 4*(17*a^6*b
+ 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c))*sinh(d*x + c))/((a^8*b^3 +
```

```
 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^6 + (a^8*b^3 + 4
*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 + 4
*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b
+ 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x + c)^4 + 6*((a^8*
b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c) + (a^9*b^2
+ 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^1
1 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(
5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^2 +
10*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) +
 (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d)*sinh(d*x + c)^
4 - 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x +
 c)^2 + 4*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*
x + c)^3 + 15*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh
(d*x + c)^2 + 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*
cosh(d*x + c) + (2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d
)*sinh(d*x + c)^3 + 6*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10
)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^1
1)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*
b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^
7 - b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b
^8 - 3*a*b^10)*d*cosh(d*x + c) - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a
^4*b^7 - b^11)*d)*sinh(d*x + c)^2 - (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^
2*b^9 + b^11)*d + 6*((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d
*cosh(d*x + c)^5 + 5*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)
*d*cosh(d*x + c)^4 + 2*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b
^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 -
3*a*b^10)*d*cosh(d*x + c)^2 - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*
b^7 - b^11)*d*cosh(d*x + c) + (a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8
+ a*b^10)*d)*sinh(d*x + c))
```

giac [B] time = 0.23, size = 357, normalized size = 2.05

$$\frac{\frac{3\left(2\,a^{3}-3\,ab^{2}\right)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}+\frac{2\left(6\,a^{3}b^{2}e^{(5\,dx+5\,c)}-9\,ab^{4}e^{(5\,dx+5\,c)}+30\,a^{4}be^{(4\,dx+4\,c)}-45\,a^{2}b^{3}e^{(4\,dx+4\,c)}+44\,a^{5}e^{(3\,dx+3\,c)}-82\,a^{3}b^{2}e^{(3\,dx+3\,c}\cdots\right.}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}\cdots\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(6*a^3*b^2*e^(5*d*x + 5*c) - 9*a*b^4*e^(5*d
*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) - 45*a^2*b^3*e^(4*d*x + 4*c) + 44*a^5*
```

```
e^(3*d*x + 3*c) - 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a*b^4*e^(3*d*x + 3*c) - 1
02*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2*c) - 12*b^5*e^(2*d*x + 2
*c) + 60*a^3*b^2*e^(d*x + c) - 15*a*b^4*e^(d*x + c) - 11*a^2*b^3 + 4*b^5)/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) -
b)^3))/d
```

maple [B] time = 0.10, size = 494, normalized size = 2.84

$$-\frac{2\left(-\frac{b^2(9a^4+6a^2b^2+2b^4)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{b(6a^6-27a^4b^2-12a^2b^4-4b^6)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b^2(54a^6-21a^4b^2-4a^2b^4-4b^6)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^3(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b(6a^6-20a^4b^2-3a^2b^4-2b^6)(\mathrm{ta}}{a^2(a^6+3a^4b^2+3a^2b^4}\right.}{\left(\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b-a\right)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
anh(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*a
^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2-
4*a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^3+1/a^2*
b*(6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*d*x+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b
^6)*tanh(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6))/(tanh(1/2*d*x+1/2*c)^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)^3+a*(2*a^2-3
*b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

maxima [B] time = 0.45, size = 551, normalized size = 3.17

$$\frac{(2a^2-3b^2)a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{a^2+b^2}\,d}-\frac{1}{3\left(a^6b^3+3a^4b^5+3a^2b^7+b^9+6\left(a^7b^2+3a^5b^4+3a^3b^6+ab^8\right)e^{(-dx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2 - 3*b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
 - c) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)*d) - 1/3*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-d*x - c)
+ 6*(17*a^4*b - 6*a^2*b^3 + 2*b^5)*e^(-2*d*x - 2*c) + 2*(22*a^5 - 41*a^3*b^
2 + 12*a*b^4)*e^(-3*d*x - 3*c) - 15*(2*a^4*b - 3*a^2*b^3)*e^(-4*d*x - 4*c)
+ 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*d*x - 5*c))/((a^6*b^3 + 3*a^4*b^5 + 3*a^2*b
^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-d*x - c) + 3*(4*
```

```
a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-2*d*x - 2*c) + 4*(2*a^9
 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*d*x - 3*c) - 3*(4*a^8
*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*d*x - 4*c) + 6*(a^7*b^2
+ 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*d*x - 5*c) - (a^6*b^3 + 3*a^4*b^5 +
3*a^2*b^7 + b^9)*e^(-6*d*x - 6*c))*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\sinh(c + d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.105 $\int (a+b\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=179

$$-\frac{16ia\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{a+b\sinh(x)}}+\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{5}b\cosh(x)(a+b\sin$$

```
[Out] 2/5*b*cosh(x)*(a+b*sinh(x))^(3/2)+16/15*a*b*cosh(x)*(a+b*sinh(x))^(1/2)+2/1
5*I*(23*a^2-9*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ellipt
icE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/((a+
b*sinh(x))/(a-I*b))^(1/2)-16/15*I*a*(a^2+b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)
/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2
))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{16ia\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{a+b\sinh(x)}}+\frac{2i\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{5}b\cosh(x)(a+b\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (16*a*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*b*Cosh[x]*(a + b*Sinh[x])^(3/2
))/5 + (((2*I)/15)*(23*a^2 - 9*b^2)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a +
b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (((16*I)/15)*a*(
a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(x))^{5/2}\,dx = \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2}{5}\int \sqrt{a+b\sinh(x)}\left(\frac{1}{2}\left(5a^2-3b^2\right)+4ab\sinh(x)\right)dx$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{4}{15}\int \frac{\frac{1}{4}a\left(15a^2-17b^2\right)\ldots}{\sqrt{a\ldots}}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{1}{15}\left(23a^2-9b^2\right)\int \sqrt{a\ldots}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{\left(\left(23a^2-9b^2\right)\sqrt{a+b\,\mathrm{s}\ldots}\right.}{15\sqrt{\ldots}}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i\left(23a^2-9b^2\right)E\left(\frac{\pi}{4}-\ldots\right.}{15\sqrt{\frac{a+\ldots}{\ldots}}}$$

Mathematica [A] time = 0.43, size = 178, normalized size = 0.99

$$\frac{b\cosh(x)\left(22a^2+28ab\sinh(x)+3b^2\cosh(2x)-3b^2\right)-16ia\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+2\left(23\ldots\right.}{15\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (2*((23*I)*a^3 + 23*a^2*b - (9*I)*a*b^2 - 9*b^3)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (16*I)*a*(a^2 + b
^2)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)] + b*Cosh[x]*(22*a^2 - 3*b^2 + 3*b^2*Cosh[2*x] + 28*a*b*Sinh[x]))
/(15*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\sinh(x)^2+2\,ab\sinh(x)+a^2\right)\sqrt{b\sinh(x)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sinh(x)^2 + 2*a*b*sinh(x) + a^2)*sqrt(b*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

maple [B] time = 0.20, size = 917, normalized size = 5.12

$$\frac{\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^3b}{15}+\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{a}{}}\right.}{15}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(5/2),x)
```

```
[Out] 2/15*(8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a
)/(I*b+a))^(1/2))*a^3*b+8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-
a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^3+15*(-(a+b*sinh(x))/(I*b-a))^(1/2)
*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a
+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4+6*(-(a+b*sinh(x))/
(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*
EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2-
9*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))
*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*b^4-23*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/
2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*a^4-14*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))
*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(
I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2+9*(-(a+b*sinh(x))/(I*b-a))^
(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE
((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^4+3*b^4*sinh(x)
^4+14*a*b^3*sinh(x)^3+11*a^2*b^2*sinh(x)^2+3*b^4*sinh(x)^2+14*a*b^3*sinh(x)
+11*a^2*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\sinh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x))^(5/2),x)
```

```
[Out] int((a + b*sinh(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sinh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(5/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(5/2), x)
```

3.106 $\int (a+b\sinh(x))^{3/2}\,dx$

Optimal. Leaf size=150

$$-\frac{2i(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\sqrt{a+b\sinh(x)}}+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] 2/3*b*cosh(x)*(a+b*sinh(x))^(1/2)+8/3*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin
(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(
a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(a^2+b^2)*(sin(1/4*P
i+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/
2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2656, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2i(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\sqrt{a+b\sinh(x)}}+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x
, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (
((2*I)/3)*(a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
b*Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sinh(x))^{3/2}\,dx = \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2-b^2\right)+2ab\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

$$= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)} + \frac{1}{3}(4a)\int \sqrt{a+b\sinh(x)}\,dx + \frac{1}{3}\left(-a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sinl}}$$

$$= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)} + \frac{\left(4a\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{\left(\left(-a^2-b^2\right)\right.}{}$$

$$= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)} + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i\left(a^2+b^2\right)F\left(\frac{\pi}{4}-\frac{ix}{2}\right.}{3\sqrt{a+b\,s}}$$

Mathematica [A] time = 0.38, size = 139, normalized size = 0.93

$$\frac{-2i\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+2b\cosh(x)(a+b\sinh(x))+8a(b+ia)\sqrt{\frac{a+b\sinh(x)}{a-ib}}E\left(\frac{1}{4}(\pi-2ix\right.}{3\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*(a + b*Sinh[x]) + 8*a*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((
-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*Ell
ipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*
b)])/(3*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((b\sinh(x)+a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(x) + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

maple [B] time = 0.12, size = 676, normalized size = 4.51

$$\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2b}{3}+\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a-}{}}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(3/2),x)
```

```
[Out] 2/3*(I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sin
h(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a^2*b+I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a
))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(
1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sin
h(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(
x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3+3*(-(a+b*sinh(x))/(I*b-a))
^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Elliptic
F((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2-4*(-(a+b*s
inh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a)
)^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*
a^3-4*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I
*b+a))^(1/2))*a*b^2+b^3*sinh(x)^3+a*b^2*sinh(x)^2+b^3*sinh(x)+a*b^2)/b/cosh
(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {(a+b\,\mathrm{sinh}(x))}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x))^(3/2),x)
```

```
[Out] int((a + b*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sinh (x))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(3/2), x)
```

3.107 $\int \sqrt{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I
*b))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2655, 2653}

$$\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt
[(a + b*Sinh[x])/(a - I*b)]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\sinh(x)}\,dx = \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$
$$= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Mathematica [A] time = 0.19, size = 65, normalized size = 1.08

$$\frac{2(b+ia)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sinh(x)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

maple [B] time = 0.12, size = 262, normalized size = 4.37

$$\frac{2\,(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(}{ib-a}}\right.\right.}{b\cosh(x)\sqrt{a+b\sinh(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(1/2),x)
```

```
[Out] 2/b*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((
I+sinh(x))*b/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*
b+a))^(1/2))*a-a*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a)
)^(1/2)))/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh(x) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + b \sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x))^(1/2),x)
```

```
[Out] int((a + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)), x)
```

3.108 $\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh
(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2663, 2661}

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a -
 I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \frac{\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{\sqrt{a+b\sinh(x)}}$$
$$= \frac{2iF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

Mathematica [A] time = 0.19, size = 60, normalized size = 1.00

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[x]],x]
[Out] ((2*I)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh(x)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

maple [A] time = 0.10, size = 125, normalized size = 2.08

$$-\frac{2\left(ib-a\right)\sqrt{-\frac{a+b\sinh\left(x\right)}{ib-a}}\,\sqrt{\frac{\left(i-\sinh\left(x\right)\right)b}{ib+a}}\,\sqrt{\frac{\left(i+\sinh\left(x\right)\right)b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh\left(x\right)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)}{b\cosh\left(x\right)\sqrt{a+b\sinh\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(1/2),x)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh\left(x\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x))^(1/2),x)
```

```
[Out] int(1/(a + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(x)), x)
```

3.109 $\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] -2*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2
)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/
2))*(a+b*sinh(x))^(1/2)/(a^2+b^2)/((a+b*sinh(x))/(a-I*b))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2664, 21, 2655, 2653}

$$-\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*EllipticE[Pi/4 -
(I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)*Sqrt[(a + b*Sin
h[x])/(a - I*b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx &= -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} - \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2}\\
&= -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{\int \sqrt{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\
&= -\frac{2b\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 81, normalized size = 0.86

$$\frac{-2b\cosh(x)+2(b+ia)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x] + 2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b
)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh(x)+a}}{b^2\sinh(x)^2+2\,ab\sinh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x) + a)/(b^2*sinh(x)^2 + 2*a*b*sinh(x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

maple [B] time = 0.14, size = 456, normalized size = 4.85

$$\frac{2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2+2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(3/2),x)
```

```
[Out] 2*((-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x)
)*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a^2+(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)
*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I
*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-s
inh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sin
h(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^2-b^2*sinh(x)^2-b^2)/(a^2+
b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x))^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(-3/2), x)
```

3.110 $\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{8ia\sqrt{a+b\sinh(x)}\,\ldots}{3\left(a^2+b^2\right)^2\sqrt{\ldots}}$$

```
[Out] -2/3*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-8/3*a*b*cosh(x)/(a^2+b^2)^2/(a
+b*sinh(x))^(1/2)+8/3*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)
*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/
2)/(a^2+b^2)^2/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(sin(1/4*Pi+1/2*I*x)^2)^
(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))
^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a^2+b^2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{8ia\sqrt{a+b\sinh(x)}\,\ldots}{3\left(a^2+b^2\right)^2\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*b*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (8*a*b*Cosh[x])/(3*(
a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x,
(2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/3)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a
+ b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2-b^2\right)+ab\sinh(x}{\sqrt{a+b\sinh(x)}}}{3\left(a^2+b^2\right)^2}$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{(4a)\int\sqrt{a+b\sinh(x}}{3\left(a^2+b^2\right)^2}$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{\left(4a\sqrt{a+b\sinh(x)}\right)}{3\left(a^2+b^2\right)^2}$$

$$= -\frac{2b\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{}}{3\left(a^2+b^2\right)^2\sqrt{\frac{a}{}}}$$

Mathematica [A] time = 0.64, size = 166, normalized size = 0.84

$$\frac{-2b\cosh(x)\left(5a^2+4ab\sinh(x)+b^2\right)-2i\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}(a+b\sinh(x))F\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)+\frac{8ia(a+b\sin}{}}{3\left(a^2+b^2\right)^2(a+b\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (((8*I)*a*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])
^2)/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*EllipticF[(Pi - (2*
I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*
b)] - 2*b*Cosh[x]*(5*a^2 + b^2 + 4*a*b*Sinh[x]))/(3*(a^2 + b^2)^2*(a + b*Si
nh[x])^(3/2))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh(x)+a}}{b^3\sinh(x)^3+3\,ab^2\sinh(x)^2+3\,a^2b\sinh(x)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x) + a)/(b^3*sinh(x)^3 + 3*a*b^2*sinh(x)^2 + 3*a^2*b*s
inh(x) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

maple [A] time = 0.23, size = 438, normalized size = 2.22

$$\frac{\sqrt{(a+b\sinh(x))\left(\cosh^2(x)\right)}\left(-\frac{2\sqrt{(a+b\sinh(x))\left(\cosh^2(x)\right)}}{3b\left(a^2+b^2\right)\left(\sinh(x)+\frac{a}{b}\right)^2}-\frac{8b\left(\cosh^2(x)\right)a}{3\left(a^2+b^2\right)^2\sqrt{(a+b\sinh(x))\left(\cosh^2(x)\right)}}+\frac{2\left(3a^2-b^2\right)\left(\frac{a}{b}-i\right)\sqrt{\frac{-b\sinh(x)-a}{ib-a}}\ \sqrt{\cdots}}{\left(3a^4+6a^2\cdots\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(5/2),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*(-2/3/b/(a^2+b^2)*((a+b*sinh(x))*cosh(x)^2)
^(1/2)/(sinh(x)+a/b)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/((a+b*sinh(x))*cosh(x)
^2)^(1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-b*sinh(x)-a)/(I*
b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a
+b*sinh(x))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I
*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(
x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a
-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(
I*b+a))^(1/2))))/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x))^(5/2),x)
[Out] int(1/(a + b*sinh(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(5/2),x)
[Out] Integral((a + b*sinh(x))**(-5/2), x)
```

3.111 $\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=128

$$\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a
-I*b))^(1/2)-2*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ellipt
icF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^
(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*S
qrt[(a + b*Sinh[x])/(a - I*b)]) - ((2*I)*a*EllipticF[Pi/4 - (I/2)*x, (2*b)/
(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx &= \frac{\int \sqrt{a+b\sinh(x)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}\\
&= \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{\left(a\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{b\sqrt{a+b\sinh(x)}}\\
&= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2iaF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 101, normalized size = 0.79

$$\frac{2\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left((b+ia)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)-iaF\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*((I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - I*a*Ellip
ticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[x])/(a - I*b
)])/(b*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(x)}{\sqrt{b\sinh(x)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(x)/sqrt(b*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

maple [A] time = 0.12, size = 218, normalized size = 1.70

$$\frac{2\,(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(}{ib-a}}\right.\right.}{b^2\cosh(x)\sqrt{a+b\sinh(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x))^(1/2),x)
```

```
[Out] 2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*sinh(x))^(1/2),x)
[Out] int(sinh(x)/(a + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{(x)}}{\sqrt{a+b\sinh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))**(1/2),x)
[Out] Integral(sinh(x)/sqrt(a + b*sinh(x)), x)
```

3.112 $\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=112

$$\frac{64a^3(5B+7iA)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(5B+7iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(5B+7iA)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B$$

```
[Out] 2/35*a*(7*I*A+5*B)*cosh(x)*(a+I*a*sinh(x))^(3/2)+2/7*B*cosh(x)*(a+I*a*sinh(
x))^(5/2)+64/105*a^3*(7*I*A+5*B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+16/105*a^2*(
7*I*A+5*B)*cosh(x)*(a+I*a*sinh(x))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2751, 2647, 2646}

$$\frac{64a^3(5B+7iA)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(5B+7iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(5B+7iA)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (64*a^3*((7*I)*A + 5*B)*Cosh[x])/(105*Sqrt[a + I*a*Sinh[x]]) + (16*a^2*((7*
I)*A + 5*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/105 + (2*a*((7*I)*A + 5*B)*Cosh[
x]*(a + I*a*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(5/2))/7
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}+\frac{1}{7}(7A-5iB)\int (a+ia\sinh(x))^{5/2}\,dx$$
$$= \frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+ia\sinh($$
$$= \frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(7iA+5B)\cosh(x)($$
$$= \frac{64a^3(7iA+5B)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)}$$

Mathematica [A] time = 0.36, size = 100, normalized size = 0.89

$$\frac{a^2\sqrt{a+ia\sinh(x)}\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)((-392A+505iB)\sinh(x)+(-120B-42iA)\cosh(2x)+1246iA-15}{210\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (a^2*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((1246*I)*A + 1040*B +
 ((-42*I)*A - 120*B)*Cosh[2*x] + (-392*A + (505*I)*B)*Sinh[x] - (15*I)*B*Si
nh[3*x]))/(210*(Cosh[x/2] + I*Sinh[x/2]))
```

fricas [A] time = 0.48, size = 125, normalized size = 1.12

$$-\frac{1}{420}\left(15\,Ba^2e^{(7\,x)}+(42\,A-105i\,B)a^2e^{(6\,x)}+35\,(-10i\,A-11\,B)a^2e^{(5\,x)}-(2100\,A-1575i\,B)a^2e^{(4\,x)}+525\,(-4i\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/420*(15*B*a^2*e^(7*x) + (42*A - 105*I*B)*a^2*e^(6*x) + 35*(-10*I*A - 11*
B)*a^2*e^(5*x) - (2100*A - 1575*I*B)*a^2*e^(4*x) + 525*(-4*I*A - 3*B)*a^2*e
^(3*x) - (350*A - 385*I*B)*a^2*e^(2*x) + 21*(2*I*A + 5*B)*a^2*e^x - 15*I*B*
a^2)*sqrt(1/2*I*a*e^(-x))*e^(-3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x)+A)(i\,a\sinh(x)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (a + ia\sinh(x))^{\frac{5}{2}} (A + B\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

```
[Out] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x) + A)(i\,a\sinh(x) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\sinh(x))\, (a + a\sinh(x)\,1\mathrm{i})^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2),x)
```

```
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(5/2)*(A+B*sinh(x)),x)
```

```
[Out] Timed out
```

3.113 $\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=81

$$\frac{8a^2(3B+5iA)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(3B+5iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

```
[Out] 2/5*B*cosh(x)*(a+I*a*sinh(x))^(3/2)+8/15*a^2*(5*I*A+3*B)*cosh(x)/(a+I*a*sin
h(x))^(1/2)+2/15*a*(5*I*A+3*B)*cosh(x)*(a+I*a*sinh(x))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2751, 2647, 2646}

$$\frac{8a^2(3B+5iA)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(3B+5iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (8*a^2*((5*I)*A + 3*B)*Cosh[x])/(15*Sqrt[a + I*a*Sinh[x]]) + (2*a*((5*I)*A
+ 3*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/15 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(
3/2))/5
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{1}{5}(5A-3iB)\int (a+ia\sinh(x))^{3/2}\,dx$$
$$= \frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/}$$
$$= \frac{8a^2(5iA+3B)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}$$

Mathematica [A] time = 0.22, size = 83, normalized size = 1.02

$$-\frac{a\sqrt{a+ia\sinh(x)}\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)(2(5A-9iB)\sinh(x)-50iA+3B\cosh(2x)-39B)}{15\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] -1/15*(a*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((-50*I)*A - 39*B
+ 3*B*Cosh[2*x] + 2*(5*A - (9*I)*B)*Sinh[x]))/(Cosh[x/2] + I*Sinh[x/2])
```

fricas [A] time = 0.67, size = 80, normalized size = 0.99

$$\frac{1}{30}\left(3iBae^{(5x)}-5(-2iA-3B)ae^{(4x)}+(90A-60iB)ae^{(3x)}-30(-3iA-2B)ae^{(2x)}+(10A-15iB)ae^{x}-3Ba\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(3*I*B*a*e^(5*x) - 5*(-2*I*A - 3*B)*a*e^(4*x) + (90*A - 60*I*B)*a*e^(3
*x) - 30*(-3*I*A - 2*B)*a*e^(2*x) + (10*A - 15*I*B)*a*e^x - 3*B*a)*sqrt(1/2
*I*a*e^(-x))*e^(-2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x)+A)(i\,a\sinh(x)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (a + ia\sinh(x))^{\frac{3}{2}} (A + B\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
[Out] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x) + A)(i\,a\sinh(x) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\sinh(x))\,(a + a\sinh(x)\,1\mathrm{i})^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2),x)
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\,(\sinh(x) - i))^{\frac{3}{2}} (A + B\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(3/2)*(A+B*sinh(x)),x)
[Out] Integral((I*a*(sinh(x) - I))**(3/2)*(A + B*sinh(x)), x)
```

3.114 $\int \sqrt{a+ia\sinh(x)}\,(A+B\sinh(x))\,dx$

Optimal. Leaf size=48

$$\frac{2a(B+3iA)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

```
[Out] 2/3*a*(3*I*A+B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+2/3*B*cosh(x)*(a+I*a*sinh(x))
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2751, 2646}

$$\frac{2a(B+3iA)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*a*((3*I)*A + B)*Cosh[x])/(3*Sqrt[a + I*a*Sinh[x]]) + (2*B*Cosh[x]*Sqrt[a
 + I*a*Sinh[x]])/3
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ia\sinh(x)}\,(A+B\sinh(x))\,dx &= \frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{1}{3}(3A-iB)\int \sqrt{a+ia\sinh(x)}\,dx\\ &= \frac{2a(3iA+B)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 1.38

$$\frac{2\sqrt{a+ia\sinh(x)}\left(\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)(3A+B\sinh(x)-2iB)}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*(I*Cosh[x/2] + Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*(3*A - (2*I)*B + B*Sinh[
x]))/(3*(Cosh[x/2] + I*Sinh[x/2]))
```

fricas [A] time = 0.40, size = 48, normalized size = 1.00

$$\frac{1}{3}\left(Be^{(3x)}+(6A-3iB)e^{(2x)}-3(-2iA-B)e^{x}-iB\right)\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(B*e^(3*x) + (6*A - 3*I*B)*e^(2*x) - 3*(-2*I*A - B)*e^x - I*B)*sqrt(1/2
*I*a*e^(-x))*e^(-x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(B\sinh(x)+A)\sqrt{i\,a\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int\sqrt{a+ia\sinh(x)}\,(A+B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

```
[Out] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(B\sinh(x)+A)\sqrt{i\,a\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (A + B\sinh(x))\,\sqrt{a + a\sinh(x)\,1\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2),x)
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\sinh\left(x\right) - i\right)}\,\left(A + B\sinh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(1/2)*(A+B*sinh(x)),x)
[Out] Integral(sqrt(I*a*(sinh(x) - I))*(A + B*sinh(x)), x)
```

3.115 $\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$Bx-\frac{(B+iA)\cosh(x)}{\sinh(x)+i}$$

[Out] B*x-(I*A+B)*cosh(x)/(I+sinh(x))

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2735, 2648}

$$Bx-\frac{(B+iA)\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sinh[x])/(I + Sinh[x]),x]

[Out] B*x - ((I*A + B)*Cosh[x])/(I + Sinh[x])

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx &= Bx-(-A+iB)\int \frac{1}{i+\sinh(x)}\,dx\\ &= Bx-\frac{(iA+B)\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.25, size = 53, normalized size = 2.30

$$\cosh(x)\left(\frac{2iB\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}-\frac{B+iA}{\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

```
[Out] Cosh[x]*(((2*I)*B*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2] - (I
*A + B)/(I + Sinh[x]))
```

fricas [A] time = 0.43, size = 23, normalized size = 1.00

$$\frac{Bxe^x + iBx - 2A + 2iB}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*e^x + I*B*x - 2*A + 2*I*B)/(e^x + I)
```

giac [A] time = 0.20, size = 17, normalized size = 0.74

$$Bx - \frac{2(A - iB)}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*(A - I*B)/(e^x + I)
```

maple [B] time = 0.04, size = 46, normalized size = 2.00

$$-\frac{2B}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x)),x)
```

```
[Out] -2/(tanh(1/2*x)+I)*B-2*I/(tanh(1/2*x)+I)*A-B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/
2*x)+1)
```

maxima [**A**] time = 0.32, size = 26, normalized size = 1.13

$$B\left(x+\frac{2i}{e^{(-x)}-i}\right)-\frac{2A}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] B*(x + 2*I/(e^(-x) - I)) - 2*A/(e^(-x) - I)
```

mupad [**B**] time = 0.12, size = 21, normalized size = 0.91

$$B\,x-\frac{2\,A-B\,2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i),x)
```

```
[Out] B*x - (2*A - B*2i)/(exp(x) + 1i)
```

sympy [**A**] time = 0.12, size = 19, normalized size = 0.83

$$Bx+\frac{-2iA-2B}{ie^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x)
```

```
[Out] B*x + (-2*I*A - 2*B)/(I*exp(x) - 1)
```

3.116 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=43

$$-\frac{(A+2iB)\cosh(x)}{3(\sinh(x)+i)}-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[Out] -1/3*(I*A+B)*cosh(x)/(I+sinh(x))^2-1/3*(A+2*I*B)*cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2750, 2648}

$$-\frac{(A+2iB)\cosh(x)}{3(\sinh(x)+i)}-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

```
[Out] -((I*A + B)*Cosh[x])/(3*(I + Sinh[x])^2) - ((A + (2*I)*B)*Cosh[x])/(3*(I +
Sinh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx &= -\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}+\frac{1}{3}(-iA+2B)\int \frac{1}{i+\sinh(x)}\,dx\\ &= -\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}-\frac{(A+2iB)\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.74

$$\frac{\cosh(x)(-(A+2iB)\sinh(x)-2iA+B)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*((-2*I)*A + B - (A + (2*I)*B)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

fricas [A] time = 0.46, size = 46, normalized size = 1.07

$$-\frac{6\,Be^{(2\,x)}+(6\,A+6i\,B)e^{x}+2i\,A-4\,B}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(6*B*e^(2*x) + (6*A + 6*I*B)*e^x + 2*I*A - 4*B)/(3*e^(3*x) + 9*I*e^(2*x) -
 9*e^x - 3*I)
```

giac [A] time = 0.18, size = 32, normalized size = 0.74

$$-\frac{6\,Be^{(2\,x)}+6\,Ae^{x}+6i\,Be^{x}+2i\,A-4\,B}{3\,(e^{x}+i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*B*e^(2*x) + 6*A*e^x + 6*I*B*e^x + 2*I*A - 4*B)/(e^x + I)^3
```

maple [A] time = 0.04, size = 52, normalized size = 1.21

$$-\frac{-2iA-2B}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2A}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2\,(2iB-2A)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^2,x)
```

```
[Out] -(-2*I*A-2*B)/(tanh(1/2*x)+I)^2-2*A/(tanh(1/2*x)+I)-2/3*(2*I*B-2*A)/(tanh(1
/2*x)+I)^3
```

maxima [**B**] time = 0.33, size = 141, normalized size = 3.28

$$-2\,A\left(\frac{3\,e^{(-x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}-\frac{i}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}\right)+\frac{1}{2}\,B\left(-\frac{12i\,e^{(-x)}}{9\,e^{(-x)}+9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*A*(3*e^(-x)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I) - I/(9*e^(-x) +
 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I)) + 1/2*B*(-12*I*e^(-x)/(9*e^(-x) + 9*I*e^
(-2*x) - 3*e^(-3*x) - 3*I) + 12*e^(-2*x)/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3
*x) - 3*I) - 8/(9*e^(-x) + 9*I*e^(-2*x) - 3*e^(-3*x) - 3*I))
```

mupad [**B**] time = 0.61, size = 39, normalized size = 0.91

$$-\frac{\frac{2\,A}{3}+\frac{B\,4\mathrm{i}}{3}-\mathrm{e}^x\,\left(-2\,B+A\,2\mathrm{i}\right)-B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{{\left(-1+\mathrm{e}^x\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^2,x)
```

```
[Out] -((2*A)/3 + (B*4i)/3 - exp(x)*(A*2i - 2*B) - B*exp(2*x)*2i)/(exp(x)*1i - 1)
^3
```

sympy [**A**] time = 0.20, size = 51, normalized size = 1.19

$$\frac{2iA+6Be^{2x}-4B+(6A+6iB)\,e^{x}}{-3e^{3x}-9ie^{2x}+9e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**2,x)
```

```
[Out] (2*I*A + 6*B*exp(2*x) - 4*B + (6*A + 6*I*B)*exp(x))/(-3*exp(3*x) - 9*I*exp(
2*x) + 9*exp(x) + 3*I)
```

3.117 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx$

Optimal. Leaf size=68

$$\frac{(-3B+2iA)\cosh(x)}{15(\sinh(x)+i)}-\frac{(2A+3iB)\cosh(x)}{15(\sinh(x)+i)^2}-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

```
[Out] -1/5*(I*A+B)*cosh(x)/(I+sinh(x))^3-1/15*(2*A+3*I*B)*cosh(x)/(I+sinh(x))^2+1
/15*(2*I*A-3*B)*cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2750, 2650, 2648}

$$\frac{(-3B+2iA)\cosh(x)}{15(\sinh(x)+i)}-\frac{(2A+3iB)\cosh(x)}{15(\sinh(x)+i)^2}-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

```
[Out] -((I*A + B)*Cosh[x])/(5*(I + Sinh[x])^3) - ((2*A + (3*I)*B)*Cosh[x])/(15*(I
 + Sinh[x])^2) + (((2*I)*A - 3*B)*Cosh[x])/(15*(I + Sinh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx &= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}+\frac{1}{5}(-2iA+3B)\int \frac{1}{(i+\sinh(x))^2}\,dx\\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}-\frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2}+\frac{1}{15}(-2A-3iB)\int \frac{1}{i+\sinh(x)}\,dx\\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}-\frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2}+\frac{(2iA-3B)\cosh(x)}{15(i+\sinh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.74

$$\frac{\cosh(x)\left((-3B+2iA)\sinh^2(x)-3(2A+3iB)\sinh(x)-7iA+3B\right)}{15(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

```
[Out] (Cosh[x]*((-7*I)*A + 3*B - 3*(2*A + (3*I)*B)*Sinh[x] + ((2*I)*A - 3*B)*Sinh
[x]^2))/(15*(I + Sinh[x])^3)
```

fricas [A] time = 0.57, size = 71, normalized size = 1.04

$$-\frac{30\,Be^{(3\,x)}+(40\,A+30i\,B)e^{(2\,x)}+10\,(2i\,A-3\,B)e^{x}-4\,A-6i\,B}{15\,e^{(5\,x)}+75i\,e^{(4\,x)}-150\,e^{(3\,x)}-150i\,e^{(2\,x)}+75\,e^{x}+15i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -(30*B*e^(3*x) + (40*A + 30*I*B)*e^(2*x) + 10*(2*I*A - 3*B)*e^x - 4*A - 6*I
*B)/(15*e^(5*x) + 75*I*e^(4*x) - 150*e^(3*x) - 150*I*e^(2*x) + 75*e^x + 15*
I)
```

giac [A] time = 0.21, size = 46, normalized size = 0.68

$$-\frac{30\,Be^{(3\,x)}+40\,Ae^{(2\,x)}+30i\,Be^{(2\,x)}+20i\,Ae^{x}-30\,Be^{x}-4\,A-6i\,B}{15\,(e^{x}+i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/15*(30*B*e^(3*x) + 40*A*e^(2*x) + 30*I*B*e^(2*x) + 20*I*A*e^x - 30*B*e^x
 - 4*A - 6*I*B)/(e^x + I)^5
```

maple [A] time = 0.04, size = 91, normalized size = 1.34

$$\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}-\frac{-8iB+8A}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{2iB-4A}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2\left(-4iA-4B\right)}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2\left(8iA+6B\right)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^3,x)
```

```
[Out] 2*I*A/(tanh(1/2*x)+I)-1/2*(8*A-8*I*B)/(tanh(1/2*x)+I)^4-(-4*A+2*I*B)/(tanh(
1/2*x)+I)^2-2/5*(-4*I*A-4*B)/(tanh(1/2*x)+I)^5-2/3*(8*I*A+6*B)/(tanh(1/2*x)
+I)^3
```

maxima [B] time = 0.34, size = 280, normalized size = 4.12

$$A\left(\frac{20i\,e^{(-x)}}{75\,e^{(-x)}+150i\,e^{(-2\,x)}-150\,e^{(-3\,x)}-75i\,e^{(-4\,x)}+15\,e^{(-5\,x)}-15i}-\frac{40\,e^{(-2\,x)}}{75\,e^{(-x)}+150i\,e^{(-2\,x)}-150\,e^{(-3\,x)}-75i\,e^{(-4\,x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] A*(20*I*e^(-x)/(75*e^(-x) + 150*I*e^(-2*x) - 150*e^(-3*x) - 75*I*e^(-4*x) +
 15*e^(-5*x) - 15*I) - 40*e^(-2*x)/(75*e^(-x) + 150*I*e^(-2*x) - 150*e^(-3*
x) - 75*I*e^(-4*x) + 15*e^(-5*x) - 15*I) + 4/(75*e^(-x) + 150*I*e^(-2*x) -
150*e^(-3*x) - 75*I*e^(-4*x) + 15*e^(-5*x) - 15*I)) - 1/2*B*(20*e^(-x)/(25*
e^(-x) + 50*I*e^(-2*x) - 50*e^(-3*x) - 25*I*e^(-4*x) + 5*e^(-5*x) - 5*I) +
20*I*e^(-2*x)/(25*e^(-x) + 50*I*e^(-2*x) - 50*e^(-3*x) - 25*I*e^(-4*x) + 5*
e^(-5*x) - 5*I) - 20*e^(-3*x)/(25*e^(-x) + 50*I*e^(-2*x) - 50*e^(-3*x) - 25
*I*e^(-4*x) + 5*e^(-5*x) - 5*I) - 4*I/(25*e^(-x) + 50*I*e^(-2*x) - 50*e^(-3
*x) - 25*I*e^(-4*x) + 5*e^(-5*x) - 5*I))
```

mupad [B] time = 0.83, size = 52, normalized size = 0.76

$$\frac{\frac{A\,4\mathrm{i}}{15}-\frac{2\,B}{5}-\frac{A\,\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}+\mathrm{e}^{x}\,\left(\frac{4\,A}{3}+B\,2\mathrm{i}\right)+2\,B\,\mathrm{e}^{2\,x}-B\,\mathrm{e}^{3\,x}\,2\mathrm{i}}{{\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^3,x)
```

```
[Out] ((A*4i)/15 - (2*B)/5 - (A*exp(2*x)*8i)/3 + exp(x)*((4*A)/3 + B*2i) + 2*B*ex
p(2*x) - B*exp(3*x)*2i)/(exp(x)*1i - 1)^5
```

sympy [A] time = 0.39, size = 83, normalized size = 1.22

$$\frac{-4iA + 30iBe^{3x} + 6B + (-20A - 30iB)\,e^{x} + (40iA - 30B)\,e^{2x}}{-15ie^{5x} + 75e^{4x} + 150ie^{3x} - 150e^{2x} - 75ie^{x} + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**3,x)
```

```
[Out] (-4*I*A + 30*I*B*exp(3*x) + 6*B + (-20*A - 30*I*B)*exp(x) + (40*I*A - 30*B)
*exp(2*x))/(-15*I*exp(5*x) + 75*exp(4*x) + 150*I*exp(3*x) - 150*exp(2*x) -
75*I*exp(x) + 15)
```

3.118 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx$

Optimal. Leaf size=91

$$\frac{2(3A+4iB)\cosh(x)}{105(\sinh(x)+i)}+\frac{2(-4B+3iA)\cosh(x)}{105(\sinh(x)+i)^2}-\frac{(3A+4iB)\cosh(x)}{35(\sinh(x)+i)^3}-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

```
[Out] -1/7*(I*A+B)*cosh(x)/(I+sinh(x))^4-1/35*(3*A+4*I*B)*cosh(x)/(I+sinh(x))^3+2
/105*(3*I*A-4*B)*cosh(x)/(I+sinh(x))^2+2/105*(3*A+4*I*B)*cosh(x)/(I+sinh(x)
)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2750, 2650, 2648}

$$\frac{2(3A+4iB)\cosh(x)}{105(\sinh(x)+i)}+\frac{2(-4B+3iA)\cosh(x)}{105(\sinh(x)+i)^2}-\frac{(3A+4iB)\cosh(x)}{35(\sinh(x)+i)^3}-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

```
[Out] -((I*A + B)*Cosh[x])/(7*(I + Sinh[x])^4) - ((3*A + (4*I)*B)*Cosh[x])/(35*(I
 + Sinh[x])^3) + (2*((3*I)*A - 4*B)*Cosh[x])/(105*(I + Sinh[x])^2) + (2*(3*
A + (4*I)*B)*Cosh[x])/(105*(I + Sinh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}+\frac{1}{7}(-3iA+4B)\int \frac{1}{(i+\sinh(x))^3}\,dx$$
$$= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}-\frac{1}{35}(2(3A+4iB))\int \frac{1}{(i+\sinh(x))^2}\,dx$$
$$= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{1}{105}(2(3iA-4B))\int$$
$$= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{2(3A+4iB)\cosh(x)}{105(i+\sinh(x))}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.74

$$\frac{\cosh(x)\left((6A+8iB)\sinh^3(x)+8i(3A+4iB)\sinh^2(x)-13(3A+4iB)\sinh(x)-36iA+13B\right)}{105(\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

```
[Out] (Cosh[x]*((-36*I)*A + 13*B - 13*(3*A + (4*I)*B)*Sinh[x] + (8*I)*(3*A + (4*I
)*B)*Sinh[x]^2 + (6*A + (8*I)*B)*Sinh[x]^3))/(105*(I + Sinh[x])^4)
```

fricas [A] time = 0.51, size = 96, normalized size = 1.05

$$-\frac{280\,Be^{(4\,x)}+(420\,A+280i\,B)e^{(3\,x)}+84\,(3i\,A-4\,B)e^{(2\,x)}-(84\,A+112i\,B)e^{x}-12i\,A+16\,B}{105\,e^{(7\,x)}+735i\,e^{(6\,x)}-2205\,e^{(5\,x)}-3675i\,e^{(4\,x)}+3675\,e^{(3\,x)}+2205i\,e^{(2\,x)}-735\,e^{x}-105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -(280*B*e^(4*x) + (420*A + 280*I*B)*e^(3*x) + 84*(3*I*A - 4*B)*e^(2*x) - (8
4*A + 112*I*B)*e^x - 12*I*A + 16*B)/(105*e^(7*x) + 735*I*e^(6*x) - 2205*e^(
5*x) - 3675*I*e^(4*x) + 3675*e^(3*x) + 2205*I*e^(2*x) - 735*e^x - 105*I)
```

giac [A] time = 0.18, size = 60, normalized size = 0.66

$$-\frac{280\,Be^{(4\,x)}+420\,Ae^{(3\,x)}+280i\,Be^{(3\,x)}+252i\,Ae^{(2\,x)}-336\,Be^{(2\,x)}-84\,Ae^{x}-112i\,Be^{x}-12i\,A+16\,B}{105\,(e^{x}+i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="giac")
```

```
[Out] -1/105*(280*B*e^(4*x) + 420*A*e^(3*x) + 280*I*B*e^(3*x) + 252*I*A*e^(2*x) -
 336*B*e^(2*x) - 84*A*e^x - 112*I*B*e^x - 12*I*A + 16*B)/(e^x + I)^7
```

maple [A] time = 0.05, size = 128, normalized size = 1.41

$$-\frac{24iA+24B}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}+\frac{2A}{\tanh\left(\frac{x}{2}\right)+i}-\frac{-32iA-24B}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{6iA+2B}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2\left(32iB-36A\right)}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2\left(-10iB+18A\right)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2}{7(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^4,x)
```

```
[Out] -1/3*(24*I*A+24*B)/(tanh(1/2*x)+I)^6+2*A/(tanh(1/2*x)+I)-1/2*(-32*I*A-24*B)
/(tanh(1/2*x)+I)^4-(6*I*A+2*B)/(tanh(1/2*x)+I)^2-2/5*(-36*A+32*I*B)/(tanh(1
/2*x)+I)^5-2/3*(18*A-10*I*B)/(tanh(1/2*x)+I)^3-2/7*(8*A-8*I*B)/(tanh(1/2*x)
+I)^7
```

maxima [B] time = 0.34, size = 468, normalized size = 5.14

$$\frac{1}{2}\,B\left(\frac{224i\,e^{(-x)}}{735\,e^{(-x)}+2205i\,e^{(-2\,x)}-3675\,e^{(-3\,x)}-3675i\,e^{(-4\,x)}+2205\,e^{(-5\,x)}+735i\,e^{(-6\,x)}-105\,e^{(-7\,x)}-105i}-\frac{}{735\,e^{(-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 1/2*B*(224*I*e^(-x)/(735*e^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x) - 3675*I*
e^(-4*x) + 2205*e^(-5*x) + 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I) - 672*e^(
-2*x)/(735*e^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x) - 3675*I*e^(-4*x) + 220
5*e^(-5*x) + 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I) - 560*I*e^(-3*x)/(735*e
^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x) - 3675*I*e^(-4*x) + 2205*e^(-5*x) +
 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I) + 560*e^(-4*x)/(735*e^(-x) + 2205*I
*e^(-2*x) - 3675*e^(-3*x) - 3675*I*e^(-4*x) + 2205*e^(-5*x) + 735*I*e^(-6*x
) - 105*e^(-7*x) - 105*I) + 32/(735*e^(-x) + 2205*I*e^(-2*x) - 3675*e^(-3*x
) - 3675*I*e^(-4*x) + 2205*e^(-5*x) + 735*I*e^(-6*x) - 105*e^(-7*x) - 105*I
)) + A*(28*e^(-x)/(245*e^(-x) + 735*I*e^(-2*x) - 1225*e^(-3*x) - 1225*I*e^(
-4*x) + 735*e^(-5*x) + 245*I*e^(-6*x) - 35*e^(-7*x) - 35*I) + 84*I*e^(-2*x)
/(245*e^(-x) + 735*I*e^(-2*x) - 1225*e^(-3*x) - 1225*I*e^(-4*x) + 735*e^(-5
*x) + 245*I*e^(-6*x) - 35*e^(-7*x) - 35*I) - 140*e^(-3*x)/(245*e^(-x) + 735
*I*e^(-2*x) - 1225*e^(-3*x) - 1225*I*e^(-4*x) + 735*e^(-5*x) + 245*I*e^(-6*
x) - 35*e^(-7*x) - 35*I) - 4*I/(245*e^(-x) + 735*I*e^(-2*x) - 1225*e^(-3*x)
 - 1225*I*e^(-4*x) + 735*e^(-5*x) + 245*I*e^(-6*x) - 35*e^(-7*x) - 35*I))
```

mupad [B] time = 1.04, size = 66, normalized size = 0.73

$$-\frac{\frac{16\,B}{105}+4\,A\,\mathrm{e}^{3\,x}-\mathrm{e}^{x}\,\left(\frac{4\,A}{5}+\frac{B\,16\mathrm{i}}{15}\right)-\frac{16\,B\,\mathrm{e}^{2\,x}}{5}+\frac{8\,B\,\mathrm{e}^{4\,x}}{3}-\frac{A\,4\mathrm{i}}{35}+\frac{A\,\mathrm{e}^{2\,x}\,12\mathrm{i}}{5}+\frac{B\,\mathrm{e}^{3\,x}\,8\mathrm{i}}{3}}{{\left(\mathrm{e}^{x}+1\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^4,x)
```

```
[Out] -((16*B)/105 - (A*4i)/35 + (A*exp(2*x)*12i)/5 + 4*A*exp(3*x) - exp(x)*((4*A
)/5 + (B*16i)/15) - (16*B*exp(2*x))/5 + (B*exp(3*x)*8i)/3 + (8*B*exp(4*x))/
3)/(exp(x) + 1i)^7
```

sympy [A] time = 0.78, size = 110, normalized size = 1.21

$$\frac{12iA-280Be^{4x}-16B+(-420A-280iB)\,e^{3x}+(84A+112iB)\,e^{x}+(-252iA+336B)\,e^{2x}}{105e^{7x}+735ie^{6x}-2205e^{5x}-3675ie^{4x}+3675e^{3x}+2205ie^{2x}-735e^{x}-105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**4,x)
```

```
[Out] (12*I*A - 280*B*exp(4*x) - 16*B + (-420*A - 280*I*B)*exp(3*x) + (84*A + 112
*I*B)*exp(x) + (-252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) + 735*I*exp(6*x)
- 2205*exp(5*x) - 3675*I*exp(4*x) + 3675*exp(3*x) + 2205*I*exp(2*x) - 735*e
xp(x) - 105*I)
```

3.119 $\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx$

Optimal. Leaf size=27

$$-Bx+\frac{(-B+iA)\cosh(x)}{-\sinh(x)+i}$$

```
[Out] -B*x+(I*A-B)*cosh(x)/(I-sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 2648}

$$-Bx+\frac{(-B+iA)\cosh(x)}{-\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

```
[Out] -(B*x) + ((I*A - B)*Cosh[x])/(I - Sinh[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx &= -Bx+(A+iB)\int \frac{1}{i-\sinh(x)}\,dx\\ &= -Bx+\frac{(iA-B)\cosh(x)}{i-\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.08, size = 59, normalized size = 2.19

$$\frac{\left(-\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)\left(Bx\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)(2A+B(x+2i))\right)}{\sinh(x)-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

```
[Out] ((I*Cosh[x/2] - Sinh[x/2])*(B*x*Cosh[x/2] + I*(2*A + B*(2*I + x))*Sinh[x/2]
))/(-I + Sinh[x])
```

fricas [A] time = 0.47, size = 24, normalized size = 0.89

$$-\frac{Bxe^{x} - i\,Bx - 2\,A - 2i\,B}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*e^x - I*B*x - 2*A - 2*I*B)/(e^x - I)
```

giac [A] time = 0.39, size = 18, normalized size = 0.67

$$-Bx + \frac{2\,(A + i\,B)}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*(A + I*B)/(e^x - I)
```

maple [A] time = 0.05, size = 46, normalized size = 1.70

$$B\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \frac{2B}{\tanh\left(\frac{x}{2}\right) - i} - \frac{2iA}{\tanh\left(\frac{x}{2}\right) - i} - B\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x)),x)
```

```
[Out] B*ln(tanh(1/2*x)-1)+2/(tanh(1/2*x)-I)*B-2*I/(tanh(1/2*x)-I)*A-B*ln(tanh(1/2
*x)+1)
```

maxima [A] time = 0.32, size = 29, normalized size = 1.07

$$-\frac{1}{2}\,B\left(2\,x - \frac{4i}{e^{(-x)} + i}\right) + \frac{2\,A}{e^{(-x)} + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="maxima")
[Out] -1/2*B*(2*x - 4*I/(e^(-x) + I)) + 2*A/(e^(-x) + I)
```

mupad [B] time = 0.11, size = 21, normalized size = 0.78

$$-B\,x+\frac{2\,A+B\,2\mathrm{i}}{\mathrm{e}^x-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*sinh(x))/(sinh(x) - 1i),x)
[Out] (2*A + B*2i)/(exp(x) - 1i) - B*x
```

sympy [A] time = 0.16, size = 19, normalized size = 0.70

$$-Bx+\frac{-2iA+2B}{-ie^x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x)
[Out] -B*x + (-2*I*A + 2*B)/(-I*exp(x) - 1)
```

3.120 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx$

Optimal. Leaf size=49

$$\frac{(A-2iB)\cosh(x)}{3(-\sinh(x)+i)}+\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}$$

```
[Out] 1/3*(I*A-B)*cosh(x)/(I-sinh(x))^2+1/3*(A-2*I*B)*cosh(x)/(I-sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2750, 2648}

$$\frac{(A-2iB)\cosh(x)}{3(-\sinh(x)+i)}+\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(3*(I - Sinh[x])^2) + ((A - (2*I)*B)*Cosh[x])/(3*(I - S
inh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx &= \frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{1}{3}(-iA-2B)\int \frac{1}{i-\sinh(x)}\,dx\\ &= \frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{(A-2iB)\cosh(x)}{3(i-\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.65

$$\frac{\cosh(x)(-(A-2iB)\sinh(x)+2iA+B)}{3(\sinh(x)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*((2*I)*A + B - (A - (2*I)*B)*Sinh[x]))/(3*(-I + Sinh[x])^2)
```

fricas [A] time = 0.47, size = 46, normalized size = 0.94

$$-\frac{6\,Be^{(2\,x)}+(6\,A-6i\,B)e^{x}-2i\,A-4\,B}{3\,e^{(3\,x)}-9i\,e^{(2\,x)}-9\,e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(6*B*e^(2*x) + (6*A - 6*I*B)*e^x - 2*I*A - 4*B)/(3*e^(3*x) - 9*I*e^(2*x) -
 9*e^x + 3*I)
```

giac [A] time = 0.18, size = 32, normalized size = 0.65

$$-\frac{6\,Be^{(2\,x)}+6\,Ae^{x}-6i\,Be^{x}-2i\,A-4\,B}{3\,(e^{x}-i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*B*e^(2*x) + 6*A*e^x - 6*I*B*e^x - 2*I*A - 4*B)/(e^x - I)^3
```

maple [A] time = 0.05, size = 52, normalized size = 1.06

$$-\frac{2A}{\tanh\left(\frac{x}{2}\right)-i}-\frac{2iA-2B}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{2\,(-2iB-2A)}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^2,x)
```

```
[Out] -2*A/(tanh(1/2*x)-I)-(2*I*A-2*B)/(tanh(1/2*x)-I)^2-2/3*(-2*I*B-2*A)/(tanh(1
/2*x)-I)^3
```

maxima [B] time = 0.34, size = 141, normalized size = 2.88

$$-A\left(\frac{6\,e^{(-x)}}{9\,e^{(-x)}-9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}+3i}+\frac{2i}{9\,e^{(-x)}-9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}+3i}\right)+\frac{1}{2}\,B\left(\frac{12i\,e^{(-x)}}{9\,e^{(-x)}-9i\,e^{(-2\,x)}-3\,e^{(-3\,x)}+3i}+\dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -A*(6*e^(-x)/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*x) + 3*I) + 2*I/(9*e^(-x) -
 9*I*e^(-2*x) - 3*e^(-3*x) + 3*I)) + 1/2*B*(12*I*e^(-x)/(9*e^(-x) - 9*I*e^(
-2*x) - 3*e^(-3*x) + 3*I) + 12*e^(-2*x)/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*
x) + 3*I) - 8/(9*e^(-x) - 9*I*e^(-2*x) - 3*e^(-3*x) + 3*I))
```

mupad [B] time = 0.60, size = 37, normalized size = 0.76

$$\frac{\frac{2\,A}{3}-\frac{B\,4\mathrm{i}}{3}+\mathrm{e}^{x}\,\left(2\,B+A\,2\mathrm{i}\right)+B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) - 1i)^2,x)
```

```
[Out] ((2*A)/3 - (B*4i)/3 + exp(x)*(A*2i + 2*B) + B*exp(2*x)*2i)/(exp(x)*1i + 1)^
3
```

sympy [A] time = 0.22, size = 51, normalized size = 1.04

$$\frac{-2iA+6Be^{2x}-4B+(6A-6iB)\,e^{x}}{-3e^{3x}+9ie^{2x}+9e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**2,x)
```

```
[Out] (-2*I*A + 6*B*exp(2*x) - 4*B + (6*A - 6*I*B)*exp(x))/(-3*exp(3*x) + 9*I*exp
(2*x) + 9*exp(x) - 3*I)
```

3.121 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3} dx$

Optimal. Leaf size=76

$$-\frac{(3B+2iA)\cosh(x)}{15(-\sinh(x)+i)}+\frac{(2A-3iB)\cosh(x)}{15(-\sinh(x)+i)^2}+\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}$$

```
[Out] 1/5*(I*A-B)*cosh(x)/(I-sinh(x))^3+1/15*(2*A-3*I*B)*cosh(x)/(I-sinh(x))^2-1/
15*(2*I*A+3*B)*cosh(x)/(I-sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2750, 2650, 2648}

$$-\frac{(3B+2iA)\cosh(x)}{15(-\sinh(x)+i)}+\frac{(2A-3iB)\cosh(x)}{15(-\sinh(x)+i)^2}+\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(5*(I - Sinh[x])^3) + ((2*A - (3*I)*B)*Cosh[x])/(15*(I
- Sinh[x])^2) - (((2*I)*A + 3*B)*Cosh[x])/(15*(I - Sinh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx &= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{1}{5}(-2iA-3B)\int \frac{1}{(i-\sinh(x))^2}\,dx\\ &= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}+\frac{1}{15}(-2A+3iB)\int \frac{1}{i-\sinh(x)}\,dx\\ &= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}-\frac{(2iA+3B)\cosh(x)}{15(i-\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.22, size = 92, normalized size = 1.21

$$-\frac{5(2A-3iB)\cosh\left(\frac{3x}{2}\right)-20iA\sinh\left(\frac{x}{2}\right)+2iA\sinh\left(\frac{5x}{2}\right)-15B\sinh\left(\frac{x}{2}\right)+3B\sinh\left(\frac{5x}{2}\right)+15iB\cosh\left(\frac{x}{2}\right)}{30\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

```
[Out] -1/30*((15*I)*B*Cosh[x/2] + 5*(2*A - (3*I)*B)*Cosh[(3*x)/2] - (20*I)*A*Sinh
[x/2] - 15*B*Sinh[x/2] + (2*I)*A*Sinh[(5*x)/2] + 3*B*Sinh[(5*x)/2])/(Cosh[x
/2] + I*Sinh[x/2])^5
```

fricas [A] time = 0.62, size = 70, normalized size = 0.92

$$\frac{30\,Be^{(3\,x)}+(40\,A-30i\,B)e^{(2\,x)}-10\,(2i\,A+3\,B)e^{x}-4\,A+6i\,B}{15\,e^{(5\,x)}-75i\,e^{(4\,x)}-150\,e^{(3\,x)}+150i\,e^{(2\,x)}+75\,e^{x}-15i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (30*B*e^(3*x) + (40*A - 30*I*B)*e^(2*x) - 10*(2*I*A + 3*B)*e^x - 4*A + 6*I*
B)/(15*e^(5*x) - 75*I*e^(4*x) - 150*e^(3*x) + 150*I*e^(2*x) + 75*e^x - 15*I
)
```

giac [A] time = 0.18, size = 46, normalized size = 0.61

$$\frac{30\,Be^{(3\,x)}+40\,Ae^{(2\,x)}-30i\,Be^{(2\,x)}-20i\,Ae^{x}-30\,Be^{x}-4\,A+6i\,B}{15\,(e^{x}-i)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/15*(30*B*e^(3*x) + 40*A*e^(2*x) - 30*I*B*e^(2*x) - 20*I*A*e^x - 30*B*e^x
- 4*A + 6*I*B)/(e^x - I)^5
```

maple [A] time = 0.05, size = 91, normalized size = 1.20

$$-\frac{2iB+4A}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}+\frac{2iA}{\tanh\left(\frac{x}{2}\right)-i}-\frac{2\left(-4iA+4B\right)}{5\left(\tanh\left(\frac{x}{2}\right)-i\right)^{5}}-\frac{2\left(8iA-6B\right)}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{-8iB-8A}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^3,x)
```

```
[Out] -(4*A+2*I*B)/(tanh(1/2*x)-I)^2+2*I*A/(tanh(1/2*x)-I)-2/5*(-4*I*A+4*B)/(tanh
(1/2*x)-I)^5-2/3*(8*I*A-6*B)/(tanh(1/2*x)-I)^3-1/2*(-8*A-8*I*B)/(tanh(1/2*x
)-I)^4
```

maxima [B] time = 0.33, size = 281, normalized size = 3.70

$$-A\left(-\frac{20i\,e^{(-x)}}{75\,e^{(-x)}-150i\,e^{(-2\,x)}-150\,e^{(-3\,x)}+75i\,e^{(-4\,x)}+15\,e^{(-5\,x)}+15i}-\frac{40\,e^{(-2\,x)}}{75\,e^{(-x)}-150i\,e^{(-2\,x)}-150\,e^{(-3\,x)}+75i\,e^{(-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -A*(-20*I*e^(-x)/(75*e^(-x) - 150*I*e^(-2*x) - 150*e^(-3*x) + 75*I*e^(-4*x)
 + 15*e^(-5*x) + 15*I) - 40*e^(-2*x)/(75*e^(-x) - 150*I*e^(-2*x) - 150*e^(-
3*x) + 75*I*e^(-4*x) + 15*e^(-5*x) + 15*I) + 4/(75*e^(-x) - 150*I*e^(-2*x)
- 150*e^(-3*x) + 75*I*e^(-4*x) + 15*e^(-5*x) + 15*I)) + 1/2*B*(20*e^(-x)/(2
5*e^(-x) - 50*I*e^(-2*x) - 50*e^(-3*x) + 25*I*e^(-4*x) + 5*e^(-5*x) + 5*I)
- 20*I*e^(-2*x)/(25*e^(-x) - 50*I*e^(-2*x) - 50*e^(-3*x) + 25*I*e^(-4*x) +
5*e^(-5*x) + 5*I) - 20*e^(-3*x)/(25*e^(-x) - 50*I*e^(-2*x) - 50*e^(-3*x) +
25*I*e^(-4*x) + 5*e^(-5*x) + 5*I) + 4*I/(25*e^(-x) - 50*I*e^(-2*x) - 50*e^(
-3*x) + 25*I*e^(-4*x) + 5*e^(-5*x) + 5*I))
```

mupad [B] time = 0.80, size = 52, normalized size = 0.68

$$\frac{2\,B\,{\mathrm{e}}^{2\,x}-\frac{2\,B}{5}+\frac{A\,{\mathrm{e}}^{2\,x}\,8{}\mathrm{i}}{3}+{\mathrm{e}}^x\,\left(\frac{4\,A}{3}-B\,2{}\mathrm{i}\right)-\frac{A\,4{}\mathrm{i}}{15}+B\,{\mathrm{e}}^{3\,x}\,2{}\mathrm{i}}{{\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*sinh(x))/(sinh(x) - 1i)^3,x)
```

```
[Out] ((A*exp(2*x)*8i)/3 - (2*B)/5 - (A*4i)/15 + exp(x)*((4*A)/3 - B*2i) + 2*B*ex
p(2*x) + B*exp(3*x)*2i)/(exp(x)*1i + 1)^5
```

sympy [A] time = 0.39, size = 82, normalized size = 1.08

$$\frac{-4iA + 30iBe^{3x} - 6B + (20A - 30iB)\,e^{x} + (40iA + 30B)\,e^{2x}}{15ie^{5x} + 75e^{4x} - 150ie^{3x} - 150e^{2x} + 75ie^{x} + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**3,x)
```

```
[Out] (-4*I*A + 30*I*B*exp(3*x) - 6*B + (20*A - 30*I*B)*exp(x) + (40*I*A + 30*B)*
exp(2*x))/(15*I*exp(5*x) + 75*exp(4*x) - 150*I*exp(3*x) - 150*exp(2*x) + 75
*I*exp(x) + 15)
```

3.122 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx$

Optimal. Leaf size=101

$$-\frac{2(3A-4iB)\cosh(x)}{105(-\sinh(x)+i)}-\frac{2(4B+3iA)\cosh(x)}{105(-\sinh(x)+i)^2}+\frac{(3A-4iB)\cosh(x)}{35(-\sinh(x)+i)^3}+\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}$$

```
[Out] 1/7*(I*A-B)*cosh(x)/(I-sinh(x))^4+1/35*(3*A-4*I*B)*cosh(x)/(I-sinh(x))^3-2/
105*(3*I*A+4*B)*cosh(x)/(I-sinh(x))^2-2/105*(3*A-4*I*B)*cosh(x)/(I-sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2750, 2650, 2648}

$$-\frac{2(3A-4iB)\cosh(x)}{105(-\sinh(x)+i)}-\frac{2(4B+3iA)\cosh(x)}{105(-\sinh(x)+i)^2}+\frac{(3A-4iB)\cosh(x)}{35(-\sinh(x)+i)^3}+\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(7*(I - Sinh[x])^4) + ((3*A - (4*I)*B)*Cosh[x])/(35*(I
- Sinh[x])^3) - (2*((3*I)*A + 4*B)*Cosh[x])/(105*(I - Sinh[x])^2) - (2*(3*A
 - (4*I)*B)*Cosh[x])/(105*(I - Sinh[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{1}{7}(-3iA-4B)\int \frac{1}{(i-\sinh(x))^3}\,dx\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{1}{35}(2(3A-4iB))\int \frac{1}{(i-\sinh(x))^2}\,dx\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}+\frac{1}{105}(2(3iA+4B))\int \frac{}{i}\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}-\frac{2(3A-4iB)\cosh(x)}{105(i-\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.62

$$\frac{\cosh(x)\left((6A-8iB)\sinh^3(x)+(-32B-24iA)\sinh^2(x)+(-39A+52iB)\sinh(x)+36iA+13B\right)}{105(\sinh(x)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

```
[Out] (Cosh[x]*((36*I)*A + 13*B + (-39*A + (52*I)*B)*Sinh[x] + ((-24*I)*A - 32*B)
*Sinh[x]^2 + (6*A - (8*I)*B)*Sinh[x]^3))/(105*(-I + Sinh[x])^4)
```

fricas [A] time = 0.58, size = 96, normalized size = 0.95

$$-\frac{280\,Be^{(4\,x)}+(420\,A-280i\,B)e^{(3\,x)}+84\,(-3i\,A-4\,B)e^{(2\,x)}-(84\,A-112i\,B)e^{x}+12i\,A+16\,B}{105\,e^{(7\,x)}-735i\,e^{(6\,x)}-2205\,e^{(5\,x)}+3675i\,e^{(4\,x)}+3675\,e^{(3\,x)}-2205i\,e^{(2\,x)}-735\,e^{x}+105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -(280*B*e^(4*x) + (420*A - 280*I*B)*e^(3*x) + 84*(-3*I*A - 4*B)*e^(2*x) - (
84*A - 112*I*B)*e^x + 12*I*A + 16*B)/(105*e^(7*x) - 735*I*e^(6*x) - 2205*e^
(5*x) + 3675*I*e^(4*x) + 3675*e^(3*x) - 2205*I*e^(2*x) - 735*e^x + 105*I)
```

giac [A] time = 0.20, size = 60, normalized size = 0.59

$$-\frac{280\,Be^{(4\,x)}+420\,Ae^{(3\,x)}-280i\,Be^{(3\,x)}-252i\,Ae^{(2\,x)}-336\,Be^{(2\,x)}-84\,Ae^{x}+112i\,Be^{x}+12i\,A+16\,B}{105\,(e^{x}-i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="giac")
```

```
[Out] -1/105*(280*B*e^(4*x) + 420*A*e^(3*x) - 280*I*B*e^(3*x) - 252*I*A*e^(2*x) -
 336*B*e^(2*x) - 84*A*e^x + 112*I*B*e^x + 12*I*A + 16*B)/(e^x - I)^7
```

maple [A] time = 0.06, size = 128, normalized size = 1.27

$$-\frac{32iA-24B}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}+\frac{2A}{\tanh\left(\frac{x}{2}\right)-i}-\frac{2\left(-32iB-36A\right)}{5\left(\tanh\left(\frac{x}{2}\right)-i\right)^{5}}-\frac{-6iA+2B}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{2\left(10iB+18A\right)}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{2\left(8iB+8A\right)}{7\left(\tanh\left(\frac{x}{2}\right)-i\right)^{7}}-\frac{-}{3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^4,x)
```

```
[Out] -1/2*(32*I*A-24*B)/(tanh(1/2*x)-I)^4+2*A/(tanh(1/2*x)-I)-2/5*(-36*A-32*I*B)
/(tanh(1/2*x)-I)^5-(-6*I*A+2*B)/(tanh(1/2*x)-I)^2-2/3*(18*A+10*I*B)/(tanh(1
/2*x)-I)^3-2/7*(8*A+8*I*B)/(tanh(1/2*x)-I)^7-1/3*(-24*I*A+24*B)/(tanh(1/2*x
)-I)^6
```

maxima [B] time = 0.35, size = 468, normalized size = 4.63

$$\frac{1}{2}B\left(-\frac{224i\,e^{(-x)}}{735\,e^{(-x)}-2205i\,e^{(-2\,x)}-3675\,e^{(-3\,x)}+3675i\,e^{(-4\,x)}+2205\,e^{(-5\,x)}-735i\,e^{(-6\,x)}-105\,e^{(-7\,x)}+105i}-\frac{}{735\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 1/2*B*(-224*I*e^(-x)/(735*e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*x) + 3675*I
*e^(-4*x) + 2205*e^(-5*x) - 735*I*e^(-6*x) - 105*e^(-7*x) + 105*I) - 672*e^
(-2*x)/(735*e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*x) + 3675*I*e^(-4*x) + 22
05*e^(-5*x) - 735*I*e^(-6*x) - 105*e^(-7*x) + 105*I) + 560*I*e^(-3*x)/(735*
e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*x) + 3675*I*e^(-4*x) + 2205*e^(-5*x)
- 735*I*e^(-6*x) - 105*e^(-7*x) + 105*I) + 560*e^(-4*x)/(735*e^(-x) - 2205*
I*e^(-2*x) - 3675*e^(-3*x) + 3675*I*e^(-4*x) + 2205*e^(-5*x) - 735*I*e^(-6*
x) - 105*e^(-7*x) + 105*I) + 32/(735*e^(-x) - 2205*I*e^(-2*x) - 3675*e^(-3*
x) + 3675*I*e^(-4*x) + 2205*e^(-5*x) - 735*I*e^(-6*x) - 105*e^(-7*x) + 105*
I)) + A*(28*e^(-x)/(245*e^(-x) - 735*I*e^(-2*x) - 1225*e^(-3*x) + 1225*I*e^
(-4*x) + 735*e^(-5*x) - 245*I*e^(-6*x) - 35*e^(-7*x) + 35*I) - 84*I*e^(-2*x
)/(245*e^(-x) - 735*I*e^(-2*x) - 1225*e^(-3*x) + 1225*I*e^(-4*x) + 735*e^(-
5*x) - 245*I*e^(-6*x) - 35*e^(-7*x) + 35*I) - 140*e^(-3*x)/(245*e^(-x) - 73
5*I*e^(-2*x) - 1225*e^(-3*x) + 1225*I*e^(-4*x) + 735*e^(-5*x) - 245*I*e^(-6
*x) - 35*e^(-7*x) + 35*I) + 4*I/(245*e^(-x) - 735*I*e^(-2*x) - 1225*e^(-3*x
) + 1225*I*e^(-4*x) + 735*e^(-5*x) - 245*I*e^(-6*x) - 35*e^(-7*x) + 35*I))
```

mupad [B] time = 1.00, size = 68, normalized size = 0.67

$$\frac{\frac{12\,A\,\mathrm{e}^{2\,x}}{5}+\frac{B\,16\mathrm{i}}{105}-\frac{4\,A}{35}+A\,\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,\left(\frac{16\,B}{15}+\frac{A\,4\mathrm{i}}{5}\right)-\frac{B\,\mathrm{e}^{2\,x}\,16\mathrm{i}}{5}+\frac{8\,B\,\mathrm{e}^{3\,x}}{3}+\frac{B\,\mathrm{e}^{4\,x}\,8\mathrm{i}}{3}}{{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) - 1i)^4,x)
```

```
[Out] ((B*16i)/105 - (4*A)/35 + (12*A*exp(2*x))/5 + A*exp(3*x)*4i - exp(x)*((A*4i
)/5 + (16*B)/15) - (B*exp(2*x)*16i)/5 + (8*B*exp(3*x))/3 + (B*exp(4*x)*8i)/
3)/(exp(x)*1i + 1)^7
```

sympy [A] time = 0.66, size = 109, normalized size = 1.08

$$\frac{-12iA-280Be^{4x}-16B+(-420A+280iB)\,e^{3x}+(84A-112iB)\,e^{x}+(252iA+336B)\,e^{2x}}{105e^{7x}-735ie^{6x}-2205e^{5x}+3675ie^{4x}+3675e^{3x}-2205ie^{2x}-735e^{x}+105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**4,x)
```

```
[Out] (-12*I*A - 280*B*exp(4*x) - 16*B + (-420*A + 280*I*B)*exp(3*x) + (84*A - 11
2*I*B)*exp(x) + (252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) - 735*I*exp(6*x)
- 2205*exp(5*x) + 3675*I*exp(4*x) + 3675*exp(3*x) - 2205*I*exp(2*x) - 735*e
xp(x) + 105*I)
```

3.123 $\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{2}\,(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

```
[Out] (I*A-B)*arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))*2^(1/2)/
a^(1/2)+2*B*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}\,(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (Sqrt[2]*(I*A - B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]]
)])/Sqrt[a] + (2*B*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx &= \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}+(A+iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx\\ &= \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}+(2(iA-B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)\\ &= \frac{\sqrt{2}\,(iA-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 85, normalized size = 1.29

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left((1+i)\sqrt[4]{-1}\,(B-iA)\tan^{-1}\left(\frac{\tanh\left(\frac{x}{4}\right)+i}{\sqrt{2}}\right)-iB\sinh\left(\frac{x}{2}\right)+B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (2*(Cosh[x/2] + I*Sinh[x/2])*((1 + I)*(-1)^(1/4)*((-I)*A + B)*ArcTan[(I + T
anh[x/4])/Sqrt[2]] + B*Cosh[x/2] - I*B*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

fricas [B] time = 0.48, size = 186, normalized size = 2.82

$$\frac{a\sqrt{-\frac{8\,A^2+16i\,AB-8\,B^2}{a}}\,\log\left(-\frac{4\,\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,A-B)+a\sqrt{-\frac{8\,A^2+16i\,AB-8\,B^2}{a}}}{-4i\,A+4\,B}\right)-a\sqrt{-\frac{8\,A^2+16i\,AB-8\,B^2}{a}}\,\log\left(-\frac{4\,\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,A-B)-a\sqrt{-\frac{8\,A}{}}}{-4i\,A+4\,B}\right.}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*sqrt(-(8*A^2 + 16*I*A*B - 8*B^2)/a)*log(-(4*sqrt(1/2*I*a*e^(-x))*(I*
A - B) + a*sqrt(-(8*A^2 + 16*I*A*B - 8*B^2)/a))/(-4*I*A + 4*B)) - a*sqrt(-(
8*A^2 + 16*I*A*B - 8*B^2)/a)*log(-(4*sqrt(1/2*I*a*e^(-x))*(I*A - B) - a*sqr
t(-(8*A^2 + 16*I*A*B - 8*B^2)/a))/(-4*I*A + 4*B)) - 4*sqrt(1/2*I*a*e^(-x))*
(I*B*e^x - B))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x)+A}{\sqrt{i\,a\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{A + B\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x)
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x) + A}{\sqrt{i\,a\sinh(x) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A + B\sinh(x)}{\sqrt{a + a\sinh(x)\,1i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sinh{(x)}}{\sqrt{ia\left(\sinh{(x)} - i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(1/2),x)
[Out] Integral((A + B*sinh(x))/sqrt(I*a*(sinh(x) - I)), x)
```

3.124 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{(3B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

```
[Out] 1/2*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(3/2)+1/4*(I*A+3*B)*arctanh(1/2*cosh(x)
*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2750, 2649, 206}

$$\frac{(3B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

```
[Out] ((I*A + 3*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/(2
*Sqrt[2]*a^(3/2)) + ((I*A - B)*Cosh[x])/(2*(a + I*a*Sinh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(A-3iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx}{4a}$$
$$= \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(iA+3B)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)}{2a}$$
$$= \frac{(iA+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}\,a^{3/2}} + \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

Mathematica [A] time = 0.25, size = 105, normalized size = 1.33

$$\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left((A+iB)\sinh\left(\frac{x}{2}\right)+i(A+iB)\cosh\left(\frac{x}{2}\right)+(1+i)\sqrt[4]{-1}\,(A-3iB)(\sinh(x)-i)\tan^{-1}\left(\frac{\tanh}{\sqrt{}}\right.\right.}{2(a+ia\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

```
[Out] ((Cosh[x/2] + I*Sinh[x/2])*(I*(A + I*B)*Cosh[x/2] + (A + I*B)*Sinh[x/2] + (
1 + I)*(-1)^(1/4)*(A - (3*I)*B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(-I + Sinh[
x])))/(2*(a + I*a*Sinh[x])^(3/2))
```

fricas [B] time = 0.58, size = 267, normalized size = 3.38

$$\frac{\sqrt{\frac{1}{2}}\left(a^2e^{(2x)}-2i\,a^2e^x-a^2\right)\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}}\,\log\left(\frac{\sqrt{\frac{1}{2}}\,a^2\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,A+3\,B)}{i\,A+3\,B}\right)-\sqrt{\frac{1}{2}}\left(a^2e^{(2x)}-2i\,a^2e^x-\right.}{2\left(a^2e^{(2x)}-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*e^x - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B
^2)/a^3)*log((sqrt(1/2)*a^2*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3) + sqrt(1/2*I
*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)) - sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*e^x
 - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3)*log(-(sqrt(1/2)*a^2*sqrt(-(A^2 -
 6*I*A*B - 9*B^2)/a^3) - sqrt(1/2*I*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)) - (
```

```
2*(I*A - B)*e^(2*x) - (2*A + 2*I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/(a^2*e^(2*x)
 - 2*I*a^2*e^x - a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x) + A}{(i\,a\sinh(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x) + A}{(i\,a\sinh(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\sinh(x)}{(a + a\sinh(x)\,1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sinh(x)}{(ia(\sinh(x) - i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(3/2),x)
```

```
[Out] Integral((A + B*sinh(x))/(I*a*(sinh(x) - I))**(3/2), x)
```

3.125 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{(5B+3iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(5B+3iA)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

```
[Out] 1/4*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(5/2)+1/16*(3*I*A+5*B)*cosh(x)/a/(a+I*a
*sinh(x))^(3/2)+1/32*(3*I*A+5*B)*arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a
*sinh(x))^(1/2))/a^(5/2)*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2750, 2650, 2649, 206}

$$\frac{(5B+3iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(5B+3iA)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

```
[Out] (((3*I)*A + 5*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])]
)/(16*Sqrt[2]*a^(5/2)) + ((I*A - B)*Cosh[x])/(4*(a + I*a*Sinh[x])^(5/2)) +
(((3*I)*A + 5*B)*Cosh[x])/(16*a*(a + I*a*Sinh[x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3A-5iB)\int \frac{1}{(a+ia\sinh(x))^{3/2}}\,dx}{8a}$$

$$= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3A-5iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx}{32a^2}$$

$$= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3iA+5B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a c}{\sqrt{a+}}\right.}{16a^2}$$

$$= \frac{(3iA+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}\,a^{5/2}} + \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}$$

Mathematica [A] time = 0.22, size = 184, normalized size = 1.67

$$\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(8(A+iB)\sinh\left(\frac{x}{2}\right)+2(5B+3iA)\sinh\left(\frac{x}{2}\right)(\sinh(x)-i)+(5B+3iA)\left(\cosh\left(\frac{x}{2}\right)+i\sin\right.\right.}{16(a+ia}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

```
[Out] ((Cosh[x/2] + I*Sinh[x/2])*((4*I)*(A + I*B)*(Cosh[x/2] + I*Sinh[x/2]) + ((3
*I)*A + 5*B)*(Cosh[x/2] + I*Sinh[x/2])^3 + (1 - I)*(-1)^(1/4)*(3*A - (5*I)*
B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(Cosh[x/2] + I*Sinh[x/2])^4 + 8*(A + I*B
)*Sinh[x/2] + 2*((3*I)*A + 5*B)*Sinh[x/2]*(-I + Sinh[x]))/(16*(a + I*a*Sin
h[x])^(5/2))
```

fricas [B] time = 0.48, size = 356, normalized size = 3.24

$$\frac{\sqrt{\frac{1}{2}}\left(4\,a^3e^{(4\,x)}-16i\,a^3e^{(3\,x)}-24\,a^3e^{(2\,x)}+16i\,a^3e^x+4\,a^3\right)\sqrt{-\frac{9\,A^2-30i\,AB-25\,B^2}{a^5}}\,\log\left(\frac{\sqrt{\frac{1}{2}}\,a^3\sqrt{-\frac{9\,A^2-30i\,AB-25\,B^2}{a^5}}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(}{3i\,A+5\,B}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(1/2)*(4*a^3*e^(4*x) - 16*I*a^3*e^(3*x) - 24*a^3*e^(2*x) + 16*I*a^
3*e^x + 4*a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log((sqrt(1/2)*a^3*sq
rt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5) + sqrt(1/2*I*a*e^(-x))*(3*I*A + 5*B))/
(3*I*A + 5*B)) - sqrt(1/2)*(4*a^3*e^(4*x) - 16*I*a^3*e^(3*x) - 24*a^3*e^(2*
x) + 16*I*a^3*e^x + 4*a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log(-(sqr
t(1/2)*a^3*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5) - sqrt(1/2*I*a*e^(-x))*(3
*I*A + 5*B))/(3*I*A + 5*B)) + 8*((-3*I*A - 5*B)*e^(4*x) - (11*A + 3*I*B)*e^
(3*x) + (-11*I*A + 3*B)*e^(2*x) - (3*A - 5*I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/
(8*a^3*e^(4*x) - 32*I*a^3*e^(3*x) - 48*a^3*e^(2*x) + 32*I*a^3*e^x + 8*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x)+A}{(i\,a\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

```
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x)+A}{(i\,a\sinh(x)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sinh(x)}{(a + a \sinh(x) \, 1i)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(5/2),x)
[Out] Timed out
```

3.126 $\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=259

$$\frac{2}{105}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}-\frac{2i\left(a^2+b^2\right)\left(15a^2B+56aAb-25b^2B\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}\right.}{105b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2/35*(7*A*b+5*B*a)*cosh(x)*(a+b*sinh(x))^(3/2)+2/7*B*cosh(x)*(a+b*sinh(x))^
(5/2)+2/105*(56*A*a*b+15*B*a^2-25*B*b^2)*cosh(x)*(a+b*sinh(x))^(1/2)+2/105*
I*(161*A*a^2*b-63*A*b^3+15*B*a^3-145*B*a*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)
/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2
))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/105*I*(a^2+b^2)*(5
6*A*a*b+15*B*a^2-25*B*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x
)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(
a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2}{105}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}-\frac{2i\left(a^2+b^2\right)\left(15a^2B+56aAb-25b^2B\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{\pi}{4}\right.}{105b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*(56*a*A*b + 15*a^2*B - 25*b^2*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/105 + (2*(
7*A*b + 5*a*B)*Cosh[x]*(a + b*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + b*Sinh
[x])^(5/2))/7 + (((2*I)/105)*(161*a^2*A*b - 63*A*b^3 + 15*a^3*B - 145*a*b^2
*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt
[(a + b*Sinh[x])/(a - I*b)]) - (((2*I)/105)*(a^2 + b^2)*(56*a*A*b + 15*a^2*
B - 25*b^2*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} + \frac{2}{7}\int (a+b\sinh(x))^{3/2}\left(\frac{1}{2}(7aA-5bB)+\right.$$

$$= \frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5}$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{35}(7Ab+5a$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{35}(7Ab+5a$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{35}(7Ab+5a$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{35}(7Ab+5a$$

Mathematica [A] time = 0.80, size = 241, normalized size = 0.93

$$\frac{\cosh(x)(a+b\sinh(x))\left(90a^2B+6b\sinh(x)(15aB+7Ab)+154aAb+15b^2B\cosh(2x)-65b^2B\right)+\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left(b(\right.}{}}{105\sqrt{a+b\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (((2*I)*(b*(105*a^3*A - 119*a*A*b^2 - 135*a^2*b*B + 25*b^3*B)*EllipticF[(Pi
 - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + (161*a^2*A*b - 63*A*b^3 + 15*a^3*B -
 145*a*b^2*B)*((a - I*b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]
- a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x]
)/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x])*(154*a*A*b + 90*a^2*B - 65*b^2*B
+ 15*b^2*B*Cosh[2*x] + 6*b*(7*A*b + 15*a*B)*Sinh[x]))/(105*Sqrt[a + b*Sinh[
x]])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb^2\sinh(x)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\sinh(x)^2 + \left(Ba^2 + 2\,Aab\right)\sinh(x)\right)\sqrt{b\sinh(x)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*sinh(x)^3 + A*a^2 + (2*B*a*b + A*b^2)*sinh(x)^2 + (B*a^2 +
2*A*a*b)*sinh(x))*sqrt(b*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

maple [B] time = 0.16, size = 1893, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

```
[Out] 2/105*(-63*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(
(I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*b^5+63*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b
/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*
b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^5-15*B*(-(a+b*sinh(x))/(I*b-a))^(1/
2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-
(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^5+77*A*a^2*b^3*sin
h(x)^2+45*B*a^3*b^2*sinh(x)^2+35*B*a*b^4*sinh(x)^2+98*A*a*b^4*sinh(x)+90*B*
a^2*b^3*sinh(x)+60*B*a*b^4*sinh(x)^4+98*A*a*b^4*sinh(x)^3+90*B*a^2*b^3*sinh
(x)^3+56*I*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(
(I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*a*b^4+15*I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x
))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))
/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4*b+105*A*(-(a+b*sinh(x))/(I*b-
a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Ellip
ticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4*b-161*A*(
-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/
(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^
(1/2))*a^4*b-98*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1
/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),
(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^3-120*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-
sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*si
nh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3*b^2-120*B*(-(a+b*sinh(x
```

```
))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/
2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^4
+130*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sin
h(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a^3*b^2+15*B*b^5*sinh(x)^5+21*A*b^5*sinh(x)^4-10*B*b^5*sinh(
x)^3+21*A*b^5*sinh(x)^2-25*B*b^5*sinh(x)+77*A*a^2*b^3+45*B*a^3*b^2-25*B*a*b
^4+56*I*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a
)/(I*b+a))^(1/2))*a^3*b^2-10*I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x)
)*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/
(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^3+145*B*(-(a+b*sinh(x))/(I*b
-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Elli
pticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^4-25*I*B
*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*
b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a)
)^(1/2))*b^5+42*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1
/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),
(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^3)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\sinh(x))\,(a + b\sinh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(5/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sinh(x))\,(a + b\sinh(x))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(5/2)*(A+B*sinh(x)),x)
[Out] Integral((A + B*sinh(x))*(a + b*sinh(x))**(5/2), x)
```

3.127 $\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=207

$$-\frac{2i\left(a^2+b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15b\sqrt{a+b\sinh(x)}}+\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] 2/5*B*cosh(x)*(a+b*sinh(x))^(3/2)+2/15*(5*A*b+3*B*a)*cosh(x)*(a+b*sinh(x))^
(1/2)+2/15*I*(20*A*a*b+3*B*a^2-9*B*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1
/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+
b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/15*I*(a^2+b^2)*(5*A*b+3*
B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi
+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*s
inh(x))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2i\left(a^2+b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15b\sqrt{a+b\sinh(x)}}+\frac{2i\left(3a^2B+20aAb-9b^2B\right)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*(5*A*b + 3*a*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*B*Cosh[x]*(a + b*Si
nh[x])^(3/2))/5 + (((2*I)/15)*(20*a*A*b + 3*a^2*B - 9*b^2*B)*EllipticE[Pi/4
 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/15)*(a^2 + b^2)*(5*A*b + 3*a*B)*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2}{5}\int \sqrt{a+b\sinh(x)}\left(\frac{1}{2}(5aA-3bB) + \frac{1}{2}\right.$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

Mathematica [A] time = 0.57, size = 196, normalized size = 0.95

$$\frac{2\left(\cosh(x)(a+b\sinh(x))(6aB+5Ab+3bB\sinh(x)) + \frac{i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left(b\left(15a^2A-12abB-5Ab^2\right)F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+\left(3a^2B+20aAb-9\right.\right.}{b}\right.}{15\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*((I*(b*(15*a^2*A - 5*A*b^2 - 12*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*
I)*b)/(a - I*b)] + (20*a*A*b + 3*a^2*B - 9*b^2*B)*((a - I*b)*EllipticE[(Pi
- (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)
*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x])/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x
])*(5*A*b + 6*a*B + 3*b*B*Sinh[x])))/(15*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\sinh(x)^2 + Aa + (Ba+Ab)\sinh(x)\right)\sqrt{b\sinh(x)+a}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*b*sinh(x)^2 + A*a + (B*a + A*b)*sinh(x))*sqrt(b*sinh(x) + a), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

maple [B] time = 0.31, size = 1037, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*(B*b^2*(2/5/b*sinh(x)*((a+b*sinh(x))*cosh(x
)^2)^(1/2)-8/15*a/b^2*((a+b*sinh(x))*cosh(x)^2)^(1/2)-4/15*a/b*(a/b-I)*((-b
*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*
b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-
a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*(-3/5+8/15*a^2/b^2)*(a/b-I)*((-b*sinh(
x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^
(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(
I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))
^(1/2),((a-I*b)/(I*b+a))^(1/2))))+(A*b^2+2*B*a*b)*(2/3/b*((a+b*sinh(x))*cos
h(x)^2)^(1/2)-2/3*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*
b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*E
llipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))-4/3*a/b*(a
/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE((
(-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sin
h(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))))+2*(2*A*a*b+B*a^2)*(a/b-I)
*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*
b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*EllipticE(((-b*s
inh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-
a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))+2*a^2*A*(a/b-I)*((-b*sinh(x)-a)
/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)
/((a+b*sinh(x))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),(
(a-I*b)/(I*b+a))^(1/2)))/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x) + A)(b\sinh(x) + a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\sinh(x))\,(a + b\sinh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(3/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sinh(x))\,(a + b\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(3/2)*(A+B*sinh(x)),x)
[Out] Integral((A + B*sinh(x))*(a + b*sinh(x))**(3/2), x)
```

3.128 $\int \sqrt{a+b\sinh(x)}\,(A+B\sinh(x))\,dx$

Optimal. Leaf size=164

$$-\frac{2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{a+b\sinh(x)}}+\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{3}B\cosh(x)\sqrt{a+b\sin}$$

```
[Out] 2/3*B*cosh(x)*(a+b*sinh(x))^(1/2)+2/3*I*(3*A*b+B*a)*(sin(1/4*Pi+1/2*I*x)^2)
^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b)
)^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(a^2+b^2
)*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*si
nh(x))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{a+b\sinh(x)}}+\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{3}B\cosh(x)\sqrt{a+b\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*B*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((2*I)/3)*(3*A*b + a*B)*EllipticE[Pi
/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])
/(a - I*b)]) - (((2*I)/3)*(a^2 + b^2)*B*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*
a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{a+b\sinh(x)}\,(A+B\sinh(x))\,dx = \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA-bB)+\frac{1}{2}(3Ab+aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} - \frac{\left(\left(a^2+b^2\right)B\right)\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{3b} + \frac{(3Ab+}{}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{\left((3Ab+aB)\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2i(3Ab+aB)E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Mathematica [A] time = 0.41, size = 151, normalized size = 0.92

$$\frac{-2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+2(b+ia)(aB+3Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+2bB}{3b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*b*B*Cosh[x]*(a + b*Sinh[x]) + 2*(I*a + b)*(3*A*b + a*B)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(
a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/(3*b*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((B\sinh(x)+A)\sqrt{b\sinh(x)+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x)+A)\sqrt{b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

maple [B] time = 0.14, size = 897, normalized size = 5.47

$$\frac{\frac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^{2}b}{3}+\frac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\right.}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

```
[Out] 2/3*(I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+s
inh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
/(I*b+a))^(1/2))*a^2*b+I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I
*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*(
(I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b
*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b+3*A*(-(a+b*sinh(x)
)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2
)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3-3*
A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))
*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*a^2*b-3*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^
(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2
),(-(I*b-a)/(I*b+a))^(1/2))*b^3-B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x
))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))
/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3-B*(-(a+b*sinh(x))/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((
-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2+B*b^3*sinh(x)
^3+B*a*b^2*sinh(x)^2+B*b^3*sinh(x)+B*a*b^2)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sinh(x)+A)\sqrt{b\sinh(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

<u>**mupad**</u> **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\sinh(x))\,\sqrt{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(1/2),x)
```

```
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(1/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sinh(x))\,\sqrt{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(1/2)*(A+B*sinh(x)),x)
```

```
[Out] Integral((A + B*sinh(x))*sqrt(a + b*sinh(x)), x)
```

3.129 $\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=55

$$\frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] B*x/b-2*(A*b-B*a)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2735, 2660, 618, 206}

$$\frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b - (2*(A*b - a*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqr
t[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{Bx}{b}-\frac{(i(iAb-iaB))\int \frac{1}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{Bx}{b}-\frac{(2i(iAb-iaB))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{Bx}{b}+\frac{(4i(iAb-iaB))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 61, normalized size = 1.11

$$\frac{\frac{2(Ab-aB)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
[Out] (B*x + (2*(A*b - a*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2])/b
```

fricas [B] time = 0.74, size = 147, normalized size = 2.67

$$-\frac{(Ba-Ab)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((B*a - A*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x)/(a^2*b + b^3)
```

giac [**A**] time = 0.36, size = 75, normalized size = 1.36

$$\frac{Bx}{b} - \frac{(Ba - Ab)\log\left(\frac{\left|2\,be^{x} + 2\,a - 2\,\sqrt{a^2 + b^2}\right|}{\left|2\,be^{x} + 2\,a + 2\,\sqrt{a^2 + b^2}\right|}\right)}{\sqrt{a^2 + b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

maple [**B**] time = 0.03, size = 101, normalized size = 1.84

$$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right) - 2b}{2\sqrt{a^2 + b^2}}\right)A}{\sqrt{a^2 + b^2}} - \frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right) - 2b}{2\sqrt{a^2 + b^2}}\right)aB}{b\sqrt{a^2 + b^2}} - \frac{B\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b} + \frac{B\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A-2/b/
(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a*B-B/b*
ln(tanh(1/2*x)-1)+B/b*ln(tanh(1/2*x)+1)
```

maxima [**B**] time = 0.40, size = 124, normalized size = 2.25

$$-B\left(\frac{a\log\left(\frac{be^{(-x)} - a - \sqrt{a^2 + b^2}}{be^{(-x)} - a + \sqrt{a^2 + b^2}}\right)}{\sqrt{a^2 + b^2}\,b} - \frac{x}{b}\right) + \frac{A\log\left(\frac{be^{(-x)} - a - \sqrt{a^2 + b^2}}{be^{(-x)} - a + \sqrt{a^2 + b^2}}\right)}{\sqrt{a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^
(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

mupad [B] time = 0.93, size = 269, normalized size = 4.89

$$\frac{Bx}{b}-\frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^x\,\sqrt{-a^2\,b^2-b^4}\,\left(\frac{2\left(A\,b\,\sqrt{-a^2\,b^2-b^4}-B\,a\,\sqrt{-a^2\,b^2-b^4}\right)}{b^4\,\sqrt{-a^2\,b^2-b^4}\,\sqrt{(A\,b-B\,a)^2}}+\frac{2\,a^2\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{b^2\,\sqrt{-b^2\,(a^2+b^2)}\,\sqrt{-a^2\,b^2-b^4}\,(A\,b-B\,a)}\right)}{2}-\frac{a\,b\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{-b^2\,(a^2+b^2)}\,(A\,b-B\,a)}\right)\,\sqrt{A^2}}{\sqrt{-a^2\,b^2-b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*x)/b - (2*atan((b^2*exp(x)*(- b^4 - a^2*b^2)^(1/2)*((2*(A*b*(- b^4 - a^2
*b^2)^(1/2) - B*a*(- b^4 - a^2*b^2)^(1/2)))/(b^4*(- b^4 - a^2*b^2)^(1/2)*((
A*b - B*a)^2)^(1/2)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^2*(
-b^2*(a^2 + b^2))^(1/2)*(- b^4 - a^2*b^2)^(1/2)*(A*b - B*a))))/2 - (a*b*(A^
2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((-b^2*(a^2 + b^2))^(1/2)*(A*b - B*a)))
*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(- b^4 - a^2*b^2)^(1/2)
```

sympy [A] time = 64.57, size = 422, normalized size = 7.67

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx\right) \\ \frac{Ax+B\cosh(x)}{a} \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx}{b} \\ -\frac{2Ab}{b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}+\frac{Bbx}{b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}+\frac{iBx\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}-\frac{2iB\sqrt{b^2}}{b^2+ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} \\ -\frac{2Ab}{b^2-ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}+\frac{Bbx}{b^2-ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}-\frac{iBx\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}{b^2-ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)}+\frac{2iB\sqrt{b^2}}{b^2-ib\sqrt{b^2}\tanh\left(\frac{x}{2}\right)} \\ -\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}-\frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}+\frac{Bx}{b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), ((A*x + B*co
sh(x))/a, Eq(b, 0)), ((A*log(tanh(x/2)) + B*x)/b, Eq(a, 0)), (-2*A*b/(b**2
+ I*b*sqrt(b**2)*tanh(x/2)) + B*b*x/(b**2 + I*b*sqrt(b**2)*tanh(x/2)) + I*B
*x*sqrt(b**2)*tanh(x/2)/(b**2 + I*b*sqrt(b**2)*tanh(x/2)) - 2*I*B*sqrt(b**2
```

```
)/(b**2 + I*b*sqrt(b**2)*tanh(x/2)), Eq(a, -sqrt(-b**2))), (-2*A*b/(b**2 -
I*b*sqrt(b**2)*tanh(x/2)) + B*b*x/(b**2 - I*b*sqrt(b**2)*tanh(x/2)) - I*B*x
*sqrt(b**2)*tanh(x/2)/(b**2 - I*b*sqrt(b**2)*tanh(x/2)) + 2*I*B*sqrt(b**2)/
(b**2 - I*b*sqrt(b**2)*tanh(x/2)), Eq(a, sqrt(-b**2))), (-A*log(tanh(x/2) -
 b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + A*log(tanh(x/2) - b/a + sqr
t(a**2 + b**2)/a)/sqrt(a**2 + b**2) + B*a*log(tanh(x/2) - b/a - sqrt(a**2 +
 b**2)/a)/(b*sqrt(a**2 + b**2)) - B*a*log(tanh(x/2) - b/a + sqrt(a**2 + b**
2)/a)/(b*sqrt(a**2 + b**2)) + B*x/b, True))
```

3.130 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)(a+b\sinh(x))}$$

```
[Out] -2*(A*a+B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-(A*
b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 206}

$$-\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/
2) - ((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\int \frac{-aA-bB}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{(aA+bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{(2(aA+bB))\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{(4(aA+bB))\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 82, normalized size = 1.11

$$
\frac{\frac{2(aA+bB)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\cosh(x)(aB-Ab)}{a+b\sinh(x)}}{a^2+b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*(a*A + b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + ((-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x]))/(a^2 + b^2)
```

fricas [B] time = 0.60, size = 444, normalized size = 6.00

$$-\frac{2\,Ba^3b-2\,Aa^2b^2+2\,Bab^3-2\,Ab^4-\left(Aab^2+Bb^3-\left(Aab^2+Bb^3\right)\cosh(x)^2-\left(Aab^2+Bb^3\right)\sinh(x)^2-2\left(Aa\cdots\right.\right.}{a^4b^2+2\,a^2b^4+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*B*a^3*b - 2*A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4 - (A*a*b^2 + B*b^3 - (A*a*b
^2 + B*b^3)*cosh(x)^2 - (A*a*b^2 + B*b^3)*sinh(x)^2 - 2*(A*a^2*b + B*a*b^2)
*cosh(x) - 2*(A*a^2*b + B*a*b^2 + (A*a*b^2 + B*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 2*(B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*cosh(x) - 2*(B*a^4 - A*a
^3*b + B*a^2*b^2 - A*a*b^3)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2
 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

giac [A] time = 0.17, size = 119, normalized size = 1.61

$$\frac{(Aa+Bb)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(Ba^2e^x-Aabe^x-Bab+Ab^2\right)}{\left(a^2b+b^3\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (A*a + B*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a +
2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(B*a^2*e^x - A*a*b*e^x - B*a*b +
A*b^2)/((a^2*b + b^3)*(b*e^(2*x) + 2*a*e^x - b))
```

maple [A] time = 0.04, size = 113, normalized size = 1.53

$$-\frac{2\left(-\frac{b(Ab-aB)\tanh\left(\frac{x}{2}\right)}{a\left(a^2+b^2\right)}-\frac{Ab-aB}{a^2+b^2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a}+\frac{2\left(Aa+bB\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^2,x)
```

```
[Out] -2*(-b*(A*b-B*a)/a/(a^2+b^2)*tanh(1/2*x)-(A*b-B*a)/(a^2+b^2))/(a*tanh(1/2*x
)^2-2*tanh(1/2*x)*b-a)+2*(A*a+B*b)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/
2*x)-2*b)/(a^2+b^2)^(1/2))
```

maxima [B] time = 0.42, size = 229, normalized size = 3.09

$$A\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(ae^{(-x)}+b\right)}{a^2b+b^3+2\left(a^3+ab^2\right)e^{(-x)}-\left(a^2b+b^3\right)e^{(-2x)}}\right)+B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{}{a^2b^2+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] A*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) - 2*(a*e^(-x) + b)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x)
 - (a^2*b + b^3)*e^(-2*x))) + B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)/(a^
2*b^2 + b^4 + 2*(a^3*b + a*b^3)*e^(-x) - (a^2*b^2 + b^4)*e^(-2*x)))
```

mupad [B] time = 0.88, size = 223, normalized size = 3.01

$$\frac{\ln\left(\frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,\left(a^2+b^2\right)^{3/2}}-\frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3}\right)(A\,a+B\,b)}{\left(a^2+b^2\right)^{3/2}}-\frac{\ln\left(-\frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3}-\frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,\left(a^2+b^2\right)^{3/2}}\right)(A\,a+B\,b)}{\left(a^2+b^2\right)^{3/2}}-\frac{\frac{2\left(A\,b^3-B\,a\,b^2\right)}{b\left(a^2\,b+b^3\right)}+\frac{2}{}}{2\,a\,e^x-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^2,x)
```

```
[Out] (log((2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)^(3/2)) - (2*exp(x)*(A*a
+ B*b))/(a^2*b + b^3))*(A*a + B*b))/(a^2 + b^2)^(3/2) - (log(- (2*exp(x)*(A
*a + B*b))/(a^2*b + b^3) - (2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)^(3
```

```
/2)))*(A*a + B*b))/(a^2 + b^2)^(3/2) - ((2*(A*b^3 - B*a*b^2))/(b*(a^2*b + b
^3)) + (2*exp(x)*(B*a^2*b^2 - A*a*b^3))/(b^2*(a^2*b + b^3)))/(2*a*exp(x) -
b + b*exp(2*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.131 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx$

Optimal. Leaf size=128

$$-\frac{\left(2a^2A+3abB-Ab^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\cosh(x)\left(a^2(-B)+3aAb+2b^2B\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\cosh(x)(Ab-aB)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2}$$

```
[Out] -(2*A*a^2-A*b^2+3*B*a*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(5/2)-1/2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^2-1/2*(3*A*a*b-B*a^2
+2*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))
```

Rubi [A] time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 206}

$$-\frac{\left(2a^2A+3abB-Ab^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\cosh(x)\left(a^2(-B)+3aAb+2b^2B\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\cosh(x)(Ab-aB)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A - A*b^2 + 3*a*b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/
(a^2 + b^2)^(5/2)) - ((A*b - a*B)*Cosh[x])/(2*(a^2 + b^2)*(a + b*Sinh[x])^2
) - ((3*a*A*b - a^2*B + 2*b^2*B)*Cosh[x])/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\int \frac{-2(aA+bB)+(Ab-aB)\sinh(x)}{(a+b\sinh(x))^2}\,dx}{2\left(a^2+b^2\right)}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\int \frac{2a^2A-Ab^2+3abB}{a+b\sinh(x)}\,dx}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\left(2a^2A-Ab^2+3abB\right)}{2\left(a^2+b\right.}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\left(2a^2A-Ab^2+3abB\right)}{}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} - \frac{\left(2\left(2a^2A-Ab^2+3al\right.\right.}{}$$

$$= -\frac{\left(2a^2A-Ab^2+3abB\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-\right.}{2\left(a^2+\right.}$$

Mathematica [A] time = 0.29, size = 131, normalized size = 1.02

$$\frac{\frac{2(2a^2A+3abB-Ab^2)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{\cosh(x)(a^2B-3aAb-2b^2B)}{a+b\sinh(x)}+\frac{(a^2+b^2)\cosh(x)(aB-Ab)}{(a+b\sinh(x))^2}}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

```
[Out] ((2*(2*a^2*A - A*b^2 + 3*a*b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])
/Sqrt[-a^2 - b^2] + ((a^2 + b^2)*(-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x])^2
+ ((-3*a*A*b + a^2*B - 2*b^2*B)*Cosh[x])/(a + b*Sinh[x]))/(2*(a^2 + b^2)^2)
```

fricas [B] time = 1.00, size = 1614, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 - 2*B*a^2*b^4 - 6*A*a*b^5 - 4*B*b^6 - 2*(2*
A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2*A
*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2*B*
a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B
*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*
a^2*b^4 + 3*A*a*b^5 + 2*B*b^6 - 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 +
3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5 +
(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*
b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^3 + 4*(2*A
*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x
))*sinh(x)^3 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x)^2 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
+ 3*(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b^2 + 3*B*a^2*
b^3 - A*a*b^4)*cosh(x))*sinh(x)^2 - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)
*cosh(x) - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 - (2*A*a^2*b^3 + 3*B*a*b^
4 - A*b^5)*cosh(x)^3 - 3*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^2 -
(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x
))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*a^2*b^4 - 5
*B*a*b^5 - A*b^6)*cosh(x) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*
a^2*b^4 - 5*B*a*b^5 - A*b^6 + 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*
```

```
B*a*b^5 - A*b^6)*cosh(x)^2 - 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3
*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B*b^6)*cosh(x))*sinh(x))/(a^6*b^3 + 3*a^
4*b^5 + 3*a^2*b^7 + b^9 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^4
 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*sinh(x)^4 + 4*(a^7*b^2 + 3*a^5*b
^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^3 + 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*
b^8 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 + 2*(2*a^8
*b + 5*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x)^2 + 2*(2*a^8*b + 5*a^6*
b^3 + 3*a^4*b^5 - a^2*b^7 - b^9 + 3*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)
*cosh(x)^2 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2
 - 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x) - 4*(a^7*b^2 + 3*a^5
*b^4 + 3*a^3*b^6 + a*b^8 - (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^
3 - 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^2 - (2*a^8*b + 5*a^
6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x))*sinh(x))
```

giac [B] time = 0.25, size = 279, normalized size = 2.18

$$-\frac{\left(2\,Aa^2+3\,Bab-Ab^2\right)\log\left(\frac{\left|-2\,be^x-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^x-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\,Aa^2b^2e^{(3\,x)}+3\,Bab^3e^{(3\,x)}-Ab^4e^{(3\,x)}-2\,Ba^4e^{(2\,x)}+6\,Aa^3be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*A*a^2 + 3*B*a*b - A*b^2)*log(abs(-2*b*e^x - 2*a - 2*sqrt(a^2 + b^2)
)/abs(-2*b*e^x - 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^
2 + b^2)) + (2*A*a^2*b^2*e^(3*x) + 3*B*a*b^3*e^(3*x) - A*b^4*e^(3*x) - 2*B*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 3*A*a*b^3*e^(2*x) -
 2*B*b^4*e^(2*x) + 4*B*a^3*b*e^x - 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^x - A*b^4
*e^x - B*a^2*b^2 + 3*A*a*b^3 + 2*B*b^4)/((a^4*b + 2*a^2*b^3 + b^5)*(b*e^(2*
x) + 2*a*e^x - b)^2)
```

maple [B] time = 0.06, size = 314, normalized size = 2.45

$$-\frac{2\left(-\frac{b\left(5A\,a^2b+2A\,b^3-3a^3B\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(4A\,a^4b-7A\,a^2b^3-2A\,b^5-2B\,a^5+5B\,a^3b^2-2Ba\,b^4\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b\left(11A\,a^2b+2A\,b^3-5a^3B+4Ba\,b^2\right)\tan}{2\left(a^4+2a^2b^2+b^4\right)a}}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^3,x)
```

```
[Out] -2*(-1/2*b*(5*A*a^2*b+2*A*b^3-3*B*a^3)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*x)^3-
1/2*(4*A*a^4*b-7*A*a^2*b^3-2*A*b^5-2*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4+2*a^
2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/2*b*(11*A*a^2*b+2*A*b^3-5*B*a^3+4*B*a*b^2)/(
```

```
a^4+2*a^2*b^2+b^4)/a*tanh(1/2*x)+1/2*(4*A*a^2*b+A*b^3-2*B*a^3+B*a*b^2)/(a^4
+2*a^2*b^2+b^4))/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)^2+(2*A*a^2-A*b^2+3*B*a
*b)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2))
```

maxima [B] time = 0.43, size = 537, normalized size = 4.20

$$\frac{1}{2}\left(\frac{3\,ab\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,ab^3e^{(-3\,x)}+a^2b^2-2\,b^4+\left(4\,a^3b-5\,ab^3\right)e^{(-x)}\right.}{a^4b^3+2\,a^2b^5+b^7+4\left(a^5b^2+2\,a^3b^4+ab^6\right)e^{(-x)}+2\left(2\,a^6b+3\,a^4b^3-b^7\right)e^{(-2\,x)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 +
b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^3*e^(-3*x) + a^
2*b^2 - 2*b^4 + (4*a^3*b - 5*a*b^3)*e^(-x) + (2*a^4 - 5*a^2*b^2 + 2*b^4)*e^
(-2*x))/(a^4*b^3 + 2*a^2*b^5 + b^7 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*e^(-x)
 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*e^(-2*x) - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)
*e^(-3*x) + (a^4*b^3 + 2*a^2*b^5 + b^7)*e^(-4*x)))*B + 1/2*A*((2*a^2 - b^2)
*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a
^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a*b^2 + (10*a^2*b + b^3)*e^(-
x) + 3*(2*a^3 - a*b^2)*e^(-2*x) - (2*a^2*b - b^3)*e^(-3*x))/(a^4*b^2 + 2*a^
2*b^4 + b^6 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-x) + 2*(2*a^6 + 3*a^4*b^2 -
 b^6)*e^(-2*x) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*x) + (a^4*b^2 + 2*a^2*
b^4 + b^6)*e^(-4*x)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^3,x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

3.132 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$

Optimal. Leaf size=187

$$-\frac{\cosh(x)\left(-2a^2B+5aAb+3b^2B\right)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2}-\frac{\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}-\frac{\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{b-a\tan}{\sqrt{a^2}}\right.}{\left(a^2+b^2\right)^{7/2}}$$

```
[Out] -(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1
/2))/(a^2+b^2)^(7/2)-1/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^3-1/6*(5
*A*a*b-2*B*a^2+3*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^2-1/6*(11*A*a^2*b
-4*A*b^3-2*B*a^3+13*B*a*b^2)*cosh(x)/(a^2+b^2)^3/(a+b*sinh(x))
```

Rubi [A] time = 0.33, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 206}

$$-\frac{\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{\cosh(x)\left(11a^2Ab-2a^3B+13ab^2B-4Ab^3\right)}{6\left(a^2+b^2\right)^3(a+b\sinh(x))}-\frac{\cosh(x)\left(-\right.}{6\left(a^2+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2 + b^2)^(7/2)) - ((A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a
+ b*Sinh[x])^3) - ((5*a*A*b - 2*a^2*B + 3*b^2*B)*Cosh[x])/(6*(a^2 + b^2)^2*
(a + b*Sinh[x])^2) - ((11*a^2*A*b - 4*A*b^3 - 2*a^3*B + 13*a*b^2*B)*Cosh[x]
)/(6*(a^2 + b^2)^3*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\int \frac{-3(aA+bB)+2(Ab-aB)\sinh(x)}{(a+b\sinh(x))^3}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} + \frac{\int \frac{2\left(3a^2A-2Ab^2+5abB\right)-}{(a+b}}{6\left(a\right.}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2\right.}{6\left(a^2+b^2\right)^3}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2\right.}{6\left(a^2+b^2\right)^3}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2\right.}{6\left(a^2+b^2\right)^3}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3-2\right.}{6\left(a^2+b^2\right)^3}$$

$$= -\frac{\left(2a^3A-3aAb^2+4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} -$$

Mathematica [A] time = 0.49, size = 189, normalized size = 1.01

$$\frac{\frac{2\left(a^2+b^2\right)^2\cosh(x)(aB-Ab)}{(a+b\sinh(x))^3} + \frac{\left(a^2+b^2\right)\cosh(x)\left(2a^2B-5aAb-3b^2B\right)}{(a+b\sinh(x))^2} + \frac{6\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{\cosh(x)\left(2a^3B-11a^2Ab\right.}{a+b\sinh}}{6\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*(a^2 + b^2)^2*(-(A*b) + a*B)*Cosh[x])/
(a + b*Sinh[x])^3 + ((a^2 + b^2)*(-5*a*A*b + 2*a^2*B - 3*b^2*B)*Cosh[x])/(a
```

```
 + b*Sinh[x])^2 + ((-11*a^2*A*b + 4*A*b^3 + 2*a^3*B - 13*a*b^2*B)*Cosh[x])/
(a + b*Sinh[x]))/(6*(a^2 + b^2)^3)
```

fricas [B] time = 0.84, size = 3870, normalized size = 20.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 - 22*B*a^3*b^5 - 14*A*a^2*b^6 - 26*B*a*b^7
 + 8*A*b^8 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 - B*b^8)*cosh(x)^5 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^
2*b^6 - 3*A*a*b^7 - B*b^8)*sinh(x)^5 + 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^
4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^4 + 30*(2*A*a^6*b^2 +
4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7 + (2*A*a^5*b^
3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*cosh(x))*sin
h(x)^4 - 4*(4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^
4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x)^3 - 4*(4*B*a^8 - 22*A
*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^
2*b^6 - 12*A*a*b^7 - 15*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^
6 - 3*A*a*b^7 - B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^
4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x))*sinh(x)^3 + 12*(4*B*a^7*b
 - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6
+ 4*B*a*b^7 - 2*A*b^8)*cosh(x)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*
b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8 + 5*(
2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*co
sh(x)^3 + 15*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2
*b^6 - B*a*b^7)*cosh(x)^2 - (4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5
*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x))*sin
h(x)^2 + 3*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7 - (2*A*a^3*b^4 +
4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^6 - (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3
*A*a*b^6 - B*b^7)*sinh(x)^6 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 -
B*a*b^6)*cosh(x)^5 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 +
 (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 - 3*(8*
A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*
cosh(x)^4 - 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*
A*a*b^6 + B*b^7 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)
^2 + 10*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(x
)^4 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^
5 + 3*B*a*b^6)*cosh(x)^3 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B
*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b
^6 - B*b^7)*cosh(x)^3 + 15*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b
^6)*cosh(x)^2 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5
+ 3*A*a*b^6 + B*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 1
```

```
4*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2 +
 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7 - 5*(2*A*a^3
*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^4 - 20*(2*A*a^4*b^3 + 4*B*a
^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^3 - 6*(8*A*a^5*b^2 + 16*B*a^4*b^3 -
 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 - 4*(4*A*a^6*b +
 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(
x))*sinh(x)^2 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(
x) - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 +
4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 + 4*B*a^3*b^4 -
 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^
3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b + 8*B*a^5
*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(x)^2 - (
8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7
)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*
a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^
2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 - 18*B*a^4*
b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 - B*b^8)*cosh(x) - 6*(4*B*a^6
*b^2 - 20*A*a^5*b^3 - 18*B*a^4*b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^
7 - B*b^8 - 5*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*
b^7 - B*b^8)*cosh(x)^4 - 20*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^
3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b - 28*B*a
^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*
b^7)*cosh(x)^2 - 4*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4
- 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8)*cosh(x))*sinh(x))/(a^8*
b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12 - (a^8*b^4 + 4*a^6*b^6 + 6*
a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^6 - (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 +
 4*a^2*b^10 + b^12)*sinh(x)^6 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*
b^9 + a*b^11)*cosh(x)^5 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
a*b^11 + (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x))*sin
h(x)^5 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(
x)^4 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12 + 5*(a^8
*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^2 + 10*(a^9*b^3 +
 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x))*sinh(x)^4 - 4*(2*a^11
*b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11)*cosh(x)^3 - 4*(2*a^11*
b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11 + 5*(a^8*b^4 + 4*a^6*b^6
 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^3 + 15*(a^9*b^3 + 4*a^7*b^5 + 6*a
^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x)^2 + 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^
6*b^6 + 10*a^4*b^8 - b^12)*cosh(x))*sinh(x)^3 + 3*(4*a^10*b^2 + 15*a^8*b^4
+ 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x)^2 + 3*(4*a^10*b^2 + 15*a^8*b^4 +
20*a^6*b^6 + 10*a^4*b^8 - b^12 - 5*(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2
*b^10 + b^12)*cosh(x)^4 - 20*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
 a*b^11)*cosh(x)^3 - 6*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 -
 b^12)*cosh(x)^2 - 4*(2*a^11*b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*
b^11)*cosh(x))*sinh(x)^2 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
```

```
 a*b^11)*cosh(x) - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11
+ (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^5 + 5*(a^9*
b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x)^4 + 2*(4*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x)^3 + 2*(2*a^11*b + 5
*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11)*cosh(x)^2 - (4*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x))*sinh(x))
```

giac [B] time = 0.23, size = 477, normalized size = 2.55

$$\frac{\left(2\,Aa^3 + 4\,Ba^2b - 3\,Aab^2 - Bb^3\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)\sqrt{a^2 + b^2}} + \frac{6\,Aa^3b^3e^{(5\,x)} + 12\,Ba^2b^4e^{(5\,x)} - 9\,Aab^5e^{(5\,x)} - 3\,Bb^6e^{(5\,x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a^3 + 4*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(6*A*a^3*b^3*e^(5*x) + 12*B*a^2*b^4*
e^(5*x) - 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 30*A*a^4*b^2*e^(4*x) + 60*B
*a^3*b^3*e^(4*x) - 45*A*a^2*b^4*e^(4*x) - 15*B*a*b^5*e^(4*x) - 8*B*a^6*e^(3
*x) + 44*A*a^5*b*e^(3*x) + 64*B*a^4*b^2*e^(3*x) - 82*A*a^3*b^3*e^(3*x) - 78
*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) + 24*B*a^5*b*e^(2*x) - 102*A*a^4*b^
2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*e^(2*x) + 24*B*a*b^5*e^(2*
x) - 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a^3*b^3*e^x + 66*B*a^2*b^4*
e^x - 15*A*a*b^5*e^x + 3*B*b^6*e^x + 2*B*a^3*b^3 - 11*A*a^2*b^4 - 13*B*a*b^
5 + 4*A*b^6)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*e^(2*x) + 2*a*e^x -
b)^3)
```

maple [B] time = 0.06, size = 633, normalized size = 3.39

$$-\frac{2\left(-\frac{b\left(9A\,a^4b+6A\,a^2b^3+2A\,b^5-4B\,a^5+B\,a^3b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)} - \frac{\left(6A\,a^6b-27A\,a^4b^3-12A\,a^2b^5-4A\,b^7-2B\,a^7+14B\,a^5b^2-11B\,a^3b^4-2Ba\,b^6\right)\left(\tanh^4\left(\frac{x}{2}\right)\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)a^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^4,x)
```

```
[Out] -2*(-1/2*b*(9*A*a^4*b+6*A*a^2*b^3+2*A*b^5-4*B*a^5+B*a^3*b^2)/a/(a^6+3*a^4*b
^2+3*a^2*b^4+b^6)*tanh(1/2*x)^5-1/2*(6*A*a^6*b-27*A*a^4*b^3-12*A*a^2*b^5-4*
A*b^7-2*B*a^7+14*B*a^5*b^2-11*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)/a^2*tanh(1/2*x)^4+1/3/a^3*b*(54*A*a^6*b-21*A*a^4*b^3-4*A*a^2*b^5-4*A*
b^7-18*B*a^7+42*B*a^5*b^2-17*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+
```

```
b^6)*tanh(1/2*x)^3+1/a^2*(6*A*a^6*b-20*A*a^4*b^3-3*A*a^2*b^5-2*A*b^7-2*B*a^
7+10*B*a^5*b^2-14*B*a^3*b^4-B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*x)^2-1/2/a*b*(27*A*a^4*b+4*A*a^2*b^3+2*A*b^5-8*B*a^5+19*B*a^3*b^2+2*B*a*b^
4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*x)-1/6*(18*A*a^4*b+5*A*a^2*b^3+2*
A*b^5-6*B*a^5+10*B*a^3*b^2+B*a*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6))/(a*tanh(
1/2*x)^2-2*tanh(1/2*x)*b-a)^3+(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)/(a^6+3*a^
4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))
```

maxima [B] time = 0.46, size = 982, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*a^2 - 3*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a
 + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2))
- 2*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-x) + 6*(17*a^4*b - 6*a
^2*b^3 + 2*b^5)*e^(-2*x) + 2*(22*a^5 - 41*a^3*b^2 + 12*a*b^4)*e^(-3*x) - 15
*(2*a^4*b - 3*a^2*b^3)*e^(-4*x) + 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*x))/(a^6*b^
3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^
8)*e^(-x) + 3*(4*a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-2*x) +
 4*(2*a^9 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*x) - 3*(4*a^
8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*x) + 6*(a^7*b^2 + 3*a^5
*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*x) - (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9
)*e^(-6*x)))*A + 1/6*B*(3*(4*a^2*b - b^3)*log((b*e^(-x) - a - sqrt(a^2 + b^
2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
sqrt(a^2 + b^2)) + 2*(2*a^3*b^3 - 13*a*b^5 + 3*(4*a^4*b^2 - 22*a^2*b^4 - b^
6)*e^(-x) + 6*(4*a^5*b - 17*a^3*b^3 + 4*a*b^5)*e^(-2*x) + 2*(4*a^6 - 32*a^4
*b^2 + 39*a^2*b^4)*e^(-3*x) + 15*(4*a^3*b^3 - a*b^5)*e^(-4*x) - 3*(4*a^2*b^
4 - b^6)*e^(-5*x))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10 + 6*(a^7*b^3 + 3
*a^5*b^5 + 3*a^3*b^7 + a*b^9)*e^(-x) + 3*(4*a^8*b^2 + 11*a^6*b^4 + 9*a^4*b^
6 + a^2*b^8 - b^10)*e^(-2*x) + 4*(2*a^9*b + 3*a^7*b^3 - 3*a^5*b^5 - 7*a^3*b
^7 - 3*a*b^9)*e^(-3*x) - 3*(4*a^8*b^2 + 11*a^6*b^4 + 9*a^4*b^6 + a^2*b^8 -
b^10)*e^(-4*x) + 6*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*e^(-5*x) - (a^
6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*e^(-6*x)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sinh(x)}{(a + b \sinh(x))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^4,x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**4,x)
```

```
[Out] Timed out
```

3.133 $\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2B\left(a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{Bx}{b}$$

```
[Out] B*x/b+2*(a^2-b^2)*B*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/b/(a^2+b^2
)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2735, 2660, 618, 206}

$$\frac{2B\left(a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b + (2*(a^2 - b^2)*B*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b
*Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{Bx}{b}-\frac{\left(i\left(-iaB+\frac{ib^2B}{a}\right)\right)\int\frac{1}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{Bx}{b}-\frac{\left(2i\left(-iaB+\frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int\frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{Bx}{b}+\frac{\left(4i\left(-iaB+\frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int\frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{Bx}{b}+\frac{2\left(a^2-b^2\right)B\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 1.10

$$\frac{B\left(ax-\frac{2\left(a^2-b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*(a*x - (2*(a^2 - b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2]))/(a*b)
```

fricas [B] time = 0.80, size = 154, normalized size = 2.57

$$-\frac{\left(Ba^2-Bb^2\right)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)}{a^3b+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((B*a^2 - B*b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*
b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)
*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*
(b*cosh(x) + a)*sinh(x) - b)) - (B*a^3 + B*a*b^2)*x)/(a^3*b + a*b^3)
```

giac [A] time = 0.35, size = 82, normalized size = 1.37

$$\frac{Bx}{b} - \frac{\left(Ba^2 - Bb^2\right)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a^2 - B*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*
e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b)
```

maple [A] time = 0.04, size = 105, normalized size = 1.75

$$-\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)aB}{b\sqrt{a^2+b^2}} + \frac{2Bb\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}} - \frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b} + \frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] -2/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a*B
+2*B/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
-B/b*ln(tanh(1/2*x)-1)+B/b*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.41, size = 128, normalized size = 2.13

$$-B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b} - \frac{x}{b}\right) + \frac{Bb\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + B*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a)
```

mupad [B] time = 1.20, size = 331, normalized size = 5.52

$$\frac{2\,\mathrm{atan}\left(\frac{a\,b^2\,\mathrm{e}^x\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(\frac{2\left(B\,a^2\,\sqrt{-a^4\,b^2-a^2\,b^4}-B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\right)}{a^2\,b^4\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\sqrt{B^2\,{\left(a^2-b^2\right)}^2}}+\frac{2\,a^2\,\sqrt{B^2\,a^4-2\,B^2\,a^2\,b^2+B^2\,b^4}}{B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(a^2-b^2\right)\,\sqrt{-a^2\,b^2\,\left(a^2+b^2\right)}}\right)}{2}-\frac{a^2\,b\,\sqrt{B^2\,a^4-2\,B^2\,a^2\,b^2+B^2\,b^4}}{B\,\left(a^2-b^2\right)\,\sqrt{-a^2\,b^2\,\left(a^2+b^2\right)}}\right.}{\sqrt{-a^4\,b^2-a^2\,b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sinh(x) + (B*b)/a)/(a + b*sinh(x)),x)
```

```
[Out] (2*atan((a*b^2*exp(x)*(- a^2*b^4 - a^4*b^2)^(1/2)*((2*(B*a^2*(- a^2*b^4 - a
^4*b^2)^(1/2) - B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)))/(a^2*b^4*(- a^2*b^4 - a
^4*b^2)^(1/2)*(B^2*(a^2 - b^2)^2)^(1/2)) + (2*a^2*(B^2*a^4 + B^2*b^4 - 2*B^
2*a^2*b^2)^(1/2))/(B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)*(a^2 - b^2)*(-a^2*b^2*
(a^2 + b^2))^(1/2))))/2 - (a^2*b*(B^2*a^4 + B^2*b^4 - 2*B^2*a^2*b^2)^(1/2))
/(B*(a^2 - b^2)*(-a^2*b^2*(a^2 + b^2))^(1/2)))*(B^2*a^4 + B^2*b^4 - 2*B^2*a
^2*b^2)^(1/2))/(- a^2*b^4 - a^4*b^2)^(1/2) + (B*x)/b
```

sympy [A] time = 60.55, size = 340, normalized size = 5.67

$$\begin{cases} \text{NaN} & \text{fo} \\ \frac{B\cosh(x)}{a} & \text{fo} \\ \frac{Bbx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right)-ib\sqrt{b^2}} - \frac{4Bb}{b^2\tanh\left(\frac{x}{2}\right)-ib\sqrt{b^2}} - \frac{iBx\sqrt{b^2}}{b^2\tanh\left(\frac{x}{2}\right)-ib\sqrt{b^2}} & \text{fo} \\ \frac{Bbx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right)+ib\sqrt{b^2}} - \frac{4Bb}{b^2\tanh\left(\frac{x}{2}\right)+ib\sqrt{b^2}} + \frac{iBx\sqrt{b^2}}{b^2\tanh\left(\frac{x}{2}\right)+ib\sqrt{b^2}} & \text{fo} \\ \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{Bx}{b} - \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} + \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*cosh(x)/a, Eq(b, 0)), (B*b*x*tanh(
x/2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)) - 4*B*b/(b**2*tanh(x/2) - I*b*sqrt(b
```

```
**2)) - I*B*x*sqrt(b**2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)), Eq(a, -sqrt(-b*
*2))), (B*b*x*tanh(x/2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)) - 4*B*b/(b**2*tan
h(x/2) + I*b*sqrt(b**2)) + I*B*x*sqrt(b**2)/(b**2*tanh(x/2) + I*b*sqrt(b**2
)), Eq(a, sqrt(-b**2))), (B*a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b
*sqrt(a**2 + b**2)) - B*a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*sqr
t(a**2 + b**2)) + B*x/b - B*b*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(a
*sqrt(a**2 + b**2)) + B*b*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a*sqr
t(a**2 + b**2)), True))
```

3.134 $\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] B*x/b
```

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b
```

fricas [A] time = 0.51, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

giac [A] time = 0.17, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] B*x/b
```

maxima [B] time = 0.41, size = 128, normalized size = 21.33

$$-B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}-\frac{x}{b}\right)+\frac{Ba\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + B*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

mupad [B] time = 0.02, size = 6, normalized size = 1.00

$$\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sinh(x) + (B*a)/b)/(a + b*sinh(x)),x)
```

```
[Out] (B*x)/b
```

sympy [A] time = 0.33, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] B*x/b
```

3.135 $\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

```
[Out] -cosh(x)/(b+a*sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2754, 8}

$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

```
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{b+a\sinh(x)}-\frac{\int 0\,dx}{a^2+b^2}\\ &= -\frac{\cosh(x)}{b+a\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 12, normalized size = 1.00

$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

```
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

fricas [B] time = 0.44, size = 58, normalized size = 4.83

$$\frac{2\,(b\cosh(x)+b\sinh(x)-a)}{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\left(a^2\cosh(x)+ab\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*(b*cosh(x) + b*sinh(x) - a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x
) - a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

giac [B] time = 0.19, size = 30, normalized size = 2.50

$$\frac{2\,(be^x-a)}{\left(ae^{(2\,x)}+2\,be^x-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(b*e^x - a)/((a*e^(2*x) + 2*b*e^x - a)*a)
```

maple [B] time = 0.05, size = 36, normalized size = 3.00

$$-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{b}-1\right)}{\left(\tanh^2\left(\frac{x}{2}\right)\right)b-2a\tanh\left(\frac{x}{2}\right)-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*sinh(x))/(b+a*sinh(x))^2,x)
```

```
[Out] -2*(-a/b*tanh(1/2*x)-1)/(tanh(1/2*x)^2*b-2*a*tanh(1/2*x)-b)
```

maxima [B] time = 0.42, size = 230, normalized size = 19.17

$$-b\left(\frac{a\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(b^2e^{(-x)}+ab\right)}{a^4+a^2b^2+2\left(a^3b+ab^3\right)e^{(-x)}-\left(a^4+a^2b^2\right)e^{(-2\,x)}}\right)+a\left(\frac{b\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{}{a^3+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -b*(a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/(a^2 + b^2)^(3/2) + 2*(b^2*e^(-x) + a*b)/(a^4 + a^2*b^2 + 2*(a^3*b + a*b^
3)*e^(-x) - (a^4 + a^2*b^2)*e^(-2*x))) + a*(b*log((a*e^(-x) - b - sqrt(a^2
+ b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) +
 a)/(a^3 + a*b^2 + 2*(a^2*b + b^3)*e^(-x) - (a^3 + a*b^2)*e^(-2*x)))
```

mupad [B] time = 0.57, size = 49, normalized size = 4.08

$$\frac{\frac{2\,e^x\,\left(a^3\,b+a\,b^3\right)}{a\,\left(a^3+a\,b^2\right)}-2}{2\,b\,e^x-a+a\,e^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*sinh(x))/(b + a*sinh(x))^2,x)
```

```
[Out] ((2*exp(x)*(a*b^3 + a^3*b))/(a*(a*b^2 + a^3)) - 2)/(2*b*exp(x) - a + a*exp(
2*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.136 $\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx$

Optimal. Leaf size=34

$$\frac{4x}{\sqrt{5}}-x-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{\sinh(x)+\sqrt{5}+2}\right)}{\sqrt{5}}$$

```
[Out] -x+4/5*x*5^(1/2)-8/5*arctanh(cosh(x)/(2+sinh(x)+5^(1/2)))*5^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2735, 2657}

$$\frac{4x}{\sqrt{5}}-x-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{\sinh(x)+\sqrt{5}+2}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

```
[Out] -x + (4*x)/Sqrt[5] - (8*ArcTanh[Cosh[x]/(2 + Sqrt[5] + Sinh[x])])/Sqrt[5]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx &= -x+4\int \frac{1}{2+\sinh(x)}\,dx\\ &= -x+\frac{4x}{\sqrt{5}}-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{2+\sqrt{5}+\sinh(x)}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 28, normalized size = 0.82

$$-x-\frac{8\tanh^{-1}\left(\frac{1-2\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sinh[x])/(2 + Sinh[x]),x]
[Out] -x - (8*ArcTanh[(1 - 2*Tanh[x/2])/Sqrt[5]])/Sqrt[5]
```

fricas [A] time = 0.87, size = 42, normalized size = 1.24

$$\frac{4}{5}\sqrt{5}\log\left(-\frac{\left(2\sqrt{5}-5\right)\cosh(x)-2\left(\sqrt{5}-2\right)\sinh(x)+\sqrt{5}-2}{\sinh(x)+2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="fricas")
[Out] 4/5*sqrt(5)*log(-((2*sqrt(5) - 5)*cosh(x) - 2*(sqrt(5) - 2)*sinh(x) + sqrt(
5) - 2)/(sinh(x) + 2)) - x
```

giac [A] time = 0.19, size = 33, normalized size = 0.97

$$\frac{4}{5}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+2e^{x}+4\right|}{2\left(\sqrt{5}+e^{x}+2\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="giac")
[Out] 4/5*sqrt(5)*log(1/2*abs(-2*sqrt(5) + 2*e^x + 4)/(sqrt(5) + e^x + 2)) - x
```

maple [A] time = 0.03, size = 37, normalized size = 1.09

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8\sqrt{5}\ \mathrm{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-1\right)\sqrt{5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-sinh(x))/(2+sinh(x)),x)
```

```
[Out] ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)+8/5*5^(1/2)*arctanh(1/5*(2*tanh(1/2*x)-
1)*5^(1/2))
```

maxima [A] time = 0.41, size = 34, normalized size = 1.00

$$\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-e^{(-x)}+2}{\sqrt{5}+e^{(-x)}-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="maxima")
```

```
[Out] 4/5*sqrt(5)*log(-(sqrt(5) - e^(-x) + 2)/(sqrt(5) + e^(-x) - 2)) - x
```

mupad [B] time = 0.59, size = 48, normalized size = 1.41

$$\frac{4\sqrt{5}\,\ln\left(-8\,e^x-\frac{4\sqrt{5}\,(4\,e^x-2)}{5}\right)}{5}-x-\frac{4\sqrt{5}\,\ln\left(\frac{4\sqrt{5}\,(4\,e^x-2)}{5}-8\,e^x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sinh(x) - 2)/(sinh(x) + 2),x)
```

```
[Out] (4*5^(1/2)*log(- 8*exp(x) - (4*5^(1/2)*(4*exp(x) - 2))/5))/5 - x - (4*5^(1/
2)*log((4*5^(1/2)*(4*exp(x) - 2))/5 - 8*exp(x)))/5
```

sympy [A] time = 1.71, size = 51, normalized size = 1.50

$$-x+\frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{5}-\frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x)
```

```
[Out] -x + 4*sqrt(5)*log(tanh(x/2) - 1/2 + sqrt(5)/2)/5 - 4*sqrt(5)*log(tanh(x/2)
 - sqrt(5)/2 - 1/2)/5
```

3.137 $\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=136

$$\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] 2*I*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*P
i+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/
(a-I*b))^(1/2)+2*I*(A*b-B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I
*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))
/(a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*B*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b
*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*(A*b - a*B)*EllipticF[Pi/4 - (I/
2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[
x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx &= \frac{B\int\sqrt{a+b\sinh(x)}\,dx}{b}+\frac{(i(-iAb+iaB))\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}\\
&= \frac{\left(B\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{\left(i(-iAb+iaB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}}{b\sqrt{a+b\sinh(x)}}\\
&= \frac{2iBE\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2i(Ab-aB)F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 109, normalized size = 0.80

$$\frac{2\sqrt{\frac{a+b\sinh(x)}{a-ib}}\left(i(Ab-aB)F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+B(b+ia)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*((I*a + b)*B*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + I*(A*b
- a*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[
x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\sinh(x)+A}{\sqrt{b\sinh(x)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x)+A}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

maple [A] time = 0.14, size = 266, normalized size = 1.96

$$-\frac{2\,(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\;\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\;\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\;\left(-iB\,\text{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b+iB\,\text{EllipticF}\left(\sqrt{-\right.\right.}{b^{2}\cosh(x)\sqrt{a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*(-I*B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*b+I*B*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b+A*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*b-B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x)+A}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(1/2),x)
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(1/2),x)
[Out] Integral((A + B*sinh(x))/sqrt(a + b*sinh(x)), x)
```

3.138 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2iB\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] -2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*(A*b-B*a)*(sin(1/4*P
i+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/
2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/(a^2+b^2)/((a+b*sinh(x))/(a-I*b
))^(1/2)+2*I*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(
cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2
)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2iB\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*(A*b -
a*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*(a^
2 + b^2)*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*B*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} - \frac{2\int \frac{\frac{1}{2}(-aA-bB)-\frac{1}{2}(Ab-aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{B\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{\left((Ab-aB)\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{(B}{}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{2i(Ab-aB)E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2iBF\left(\frac{\pi}{4}\right.}{b}$$

Mathematica [A] time = 0.65, size = 159, normalized size = 0.90

$$\frac{2iB\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+2b\cosh(x)(aB-Ab)+\frac{2i(Ab-aB)(a+b\sinh(x))E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}}{b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*(-(A*b) + a*B)*Cosh[x] + ((2*I)*(A*b - a*B)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x]))/Sqrt[(a + b*Sinh[x])/(a - I*b)] +
(2*I)*(a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[
(a + b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\sinh(x)+A)\sqrt{b\sinh(x)+a}}{b^2\sinh(x)^2+2\,ab\sinh(x)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)*sqrt(b*sinh(x) + a)/(b^2*sinh(x)^2 + 2*a*b*sinh(x)
 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

maple [B] time = 0.24, size = 517, normalized size = 2.94

$$\frac{\sqrt{(a + b\sinh(x))\left(\cosh^2(x)\right)}\left(\frac{2B\left(\frac{a}{b}-i\right)\sqrt{\frac{-b\sinh(x)-a}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-b\sinh(x)-a}{ib-a}},\sqrt{\frac{-ib+a}{ib+a}}\right)}{b\sqrt{(a+b\sinh(x))\left(\cosh^2(x)\right)}} + \frac{(Ab-aB)\left(-\frac{\ }{(a^2+b^2)\ }\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*(2*B/b*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/
2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x
))*cosh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+
a))^(1/2))+(A*b-B*a)/b*(-2*b*cosh(x)^2/(a^2+b^2)/((a+b*sinh(x))*cosh(x)^2)^
(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)
*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*b/(a^2
+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((-a/b-I)*Ell
ipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(
((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))))/cosh(x)/(a+b*sin
h(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(3/2),x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

3.139 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i(\cdots}{\cdots}$$

```
[Out] -2/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-2/3*(4*A*a*b-B*a^2+3*B
*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^(1/2)+2/3*I*(4*A*a*b-B*a^2+3*B*b^2)
*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2
*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/(a^2+b^2)^2/((a+b*si
nh(x))/(a-I*b))^(1/2)-2/3*I*(A*b-B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4
*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b
*sinh(x))/(a-I*b))^(1/2)/b/(a^2+b^2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (2*(4*a*A*
b - a^2*B + 3*b^2*B)*Cosh[x])/(3*(a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((2*
I)/3)*(4*a*A*b - a^2*B + 3*b^2*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)
]*Sqrt[a + b*Sinh[x]])/(b*(a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(a - I*b)]) -
(((2*I)/3)*(A*b - a*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
 b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA+bB)+\frac{1}{2}(Ab-aB)\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2A-Ab^2+4ab\right.}{}}{3}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} - \frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\,\mathrm{s}}}}{3b\left(a^2+b^2\right.}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{\left(\left(4aAb-a^2B+3\right.\right.}{}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} + \frac{2i\left(4aAb-a^2B+\right.}{3b\left(\right.}$$

Mathematica [A] time = 0.82, size = 236, normalized size = 0.94

$$\frac{2i\left(ib\cosh(x)\left(-\left(a^2B-4aAb-3b^2B\right)(a+b\sinh(x))-\left(\left(a^2+b^2\right)(aB-Ab)\right)\right)+\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,(a+b\sinh(x))\left(b\left(3\right.\right.}{3b\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (((2*I)/3)*((b*(3*a^2*A - A*b^2 + 4*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2
*I)*b)/(a - I*b)] + (4*a*A*b - a^2*B + 3*b^2*B)*((a - I*b)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b
)/(a - I*b)]))*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*b)] + I*b*Cosh[x
]*(-((a^2 + b^2)*(-(A*b) + a*B)) - (-4*a*A*b + a^2*B - 3*b^2*B)*(a + b*Sinh
[x]))))/(b*(a^2 + b^2)^2*(a + b*Sinh[x])^(3/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\sinh(x)+A)\sqrt{b\sinh(x)+a}}{b^3\sinh(x)^3+3\,ab^2\sinh(x)^2+3\,a^2b\sinh(x)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*sinh(x) + A)*sqrt(b*sinh(x) + a)/(b^3*sinh(x)^3 + 3*a*b^2*sinh(
x)^2 + 3*a^2*b*sinh(x) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

maple [B] time = 0.35, size = 806, normalized size = 3.21

$$\frac{\sqrt{(a+b\sinh(x))\left(\cosh^2(x)\right)}\left(B\left(-\frac{2b\left(\cosh^2(x)\right)}{\left(a^2+b^2\right)\sqrt{(a+b\sinh(x))\left(\cosh^2(x)\right)}}+\frac{2a\left(\frac{a}{b}-i\right)\sqrt{\frac{-b\sinh(x)-a}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-b\sinh(x)-a}{ib-a}}\right.}{\left(a^2+b^2\right)\sqrt{(a+b\sinh(x))\left(\cosh^2(x)\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x)
```

```
[Out] ((a+b*sinh(x))*cosh(x)^2)^(1/2)*(B/b*(-2*b*cosh(x)^2/(a^2+b^2)/((a+b*sinh(x
))*cosh(x)^2)^(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((
I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*co
sh(x)^2)^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(
1/2))+2*b/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)
*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)
)+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))))+(A*
b-B*a)/b*(-2/3/b/(a^2+b^2)*((a+b*sinh(x))*cosh(x)^2)^(1/2)/(sinh(x)+a/b)^2-
8/3*b*cosh(x)^2/(a^2+b^2)^2*a/((a+b*sinh(x))*cosh(x)^2)^(1/2)+2*(3*a^2-b^2)
/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x)
)*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^
(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+8/3
*a*b/(a^2+b^2)^2*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b
+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/((a+b*sinh(x))*cosh(x)^2)^(1/2)*((
-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I
```

```
*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))))/cosh(
x)/(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(5/2),x)
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(5/2),x)
[Out] Timed out
```

3.140 $\int \left(a \sinh^2(x)\right)^{5/2} dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2 \coth(x)\sqrt{a \sinh^2(x)} + \frac{1}{5}\coth(x)\left(a \sinh^2(x)\right)^{5/2} - \frac{4}{15}a \coth(x)\left(a \sinh^2(x)\right)^{3/2}$$

[Out] -4/15*a*coth(x)*(a*sinh(x)^2)^(3/2)+1/5*coth(x)*(a*sinh(x)^2)^(5/2)+8/15*a^2*coth(x)*(a*sinh(x)^2)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3203, 3207, 2638}

$$\frac{8}{15}a^2 \coth(x)\sqrt{a \sinh^2(x)} + \frac{1}{5}\coth(x)\left(a \sinh^2(x)\right)^{5/2} - \frac{4}{15}a \coth(x)\left(a \sinh^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a*Sinh[x]^2)^(5/2),x]

[Out] (8*a^2*Coth[x]*Sqrt[a*Sinh[x]^2])/15 - (4*a*Coth[x]*(a*Sinh[x]^2)^(3/2))/15 + (Coth[x]*(a*Sinh[x]^2)^(5/2))/5

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(a\sinh^2(x)\right)^{5/2}\,dx &= \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{1}{5}(4a)\int \left(a\sinh^2(x)\right)^{3/2}\,dx\\
&= -\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\sinh^2(x)}\,dx\\
&= -\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\text{csch}(x)\sqrt{a\sinh^2(x)}\right)\\
&= \frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)}-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\cosh(x)-25\cosh(3x)+3\cosh(5x))\text{csch}(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(5/2),x]
```

```
[Out] (a^2*(150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/
240
```

fricas [B] time = 0.65, size = 511, normalized size = 9.64

$$\frac{\left(30\,a^2\cosh(x)e^x\sinh(x)^9+3\,a^2e^x\sinh(x)^{10}+5\left(27\,a^2\cosh(x)^2-5\,a^2\right)e^x\sinh(x)^8+40\left(9\,a^2\cosh(x)^3-5\,a^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cosh
(x)^2 - 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 - 5*a^2*cosh(x))*e^x*sin
h(x)^7 + 10*(63*a^2*cosh(x)^4 - 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6 +
4*(189*a^2*cosh(x)^5 - 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)^5 +
 10*(63*a^2*cosh(x)^6 - 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 + 15*a^2)*e^x
*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 - 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^3 + 15
*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 - 140*a^2*cosh(x)^6 + 450
*a^2*cosh(x)^4 + 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a^2*cosh(
x)^9 - 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 + 60*a^2*cosh(x)^3 - 5*a^2*cosh(
x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 - 25*a^2*cosh(x)^8 + 150*a^2*cosh(x)^6
```

```
+ 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 + 3*a^2)*e^x)*sqrt(a*e^(4*x) - 2*a*e
^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) - 1)*sinh(x)^5 - cosh(x)^5
 + 5*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(2*x) - cosh(x
)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) - cosh(x)^3)*sinh(x)^2 + 5*(cosh(x)^
4*e^(2*x) - cosh(x)^4)*sinh(x))
```

giac [B] time = 0.15, size = 120, normalized size = 2.26

$$\frac{1}{480}\left(3\,a^2e^{(5\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^x\right)-25\,a^2e^{(3\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^x\right)+150\,a^2e^x\mathrm{sgn}\left(e^{(3\,x)}-e^x\right)+\left(150\,a^2e^{(4\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*e^(5*x)*sgn(e^(3*x) - e^x) - 25*a^2*e^(3*x)*sgn(e^(3*x) - e^x)
 + 150*a^2*e^x*sgn(e^(3*x) - e^x) + (150*a^2*e^(4*x)*sgn(e^(3*x) - e^x) - 2
5*a^2*e^(2*x)*sgn(e^(3*x) - e^x) + 3*a^2*sgn(e^(3*x) - e^x))*e^(-5*x))*sqrt
(a)
```

maple [A] time = 0.07, size = 32, normalized size = 0.60

$$\frac{a^3\sinh(x)\cosh(x)\left(3\left(\sinh^4(x)\right)-4\left(\sinh^2(x)\right)+8\right)}{15\sqrt{a\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(5/2),x)
```

```
[Out] 1/15*a^3*sinh(x)*cosh(x)*(3*sinh(x)^4-4*sinh(x)^2+8)/(a*sinh(x)^2)^(1/2)
```

maxima [A] time = 0.42, size = 53, normalized size = 1.00

$$-\frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)}+\frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)}-\frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)}+\frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)}-\frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)}-\frac{5}{16}\,a^{\frac{5}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) + 5/96*
a^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) - 5/16*a^(5/2)*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a\sinh(x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(5/2),x)
```

```
[Out] int((a*sinh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^2(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(5/2), x)
```

3.141 $\int \left(a \sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=34

$$\frac{1}{3} \coth(x) \left(a \sinh^2(x)\right)^{3/2} - \frac{2}{3} a \coth(x) \sqrt{a \sinh^2(x)}$$

```
[Out] 1/3*coth(x)*(a*sinh(x)^2)^(3/2)-2/3*a*coth(x)*(a*sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3203, 3207, 2638}

$$\frac{1}{3} \coth(x) \left(a \sinh^2(x)\right)^{3/2} - \frac{2}{3} a \coth(x) \sqrt{a \sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*a*Coth[x]*Sqrt[a*Sinh[x]^2])/3 + (Coth[x]*(a*Sinh[x]^2)^(3/2))/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\sinh^2(x)\right)^{3/2}\,dx &= \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2} - \frac{1}{3}(2a)\int \sqrt{a\sinh^2(x)}\,dx\\ &= \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2} - \frac{1}{3}\left(2a\operatorname{csch}(x)\sqrt{a\sinh^2(x)}\right)\int \sinh(x)\,dx\\ &= -\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)} + \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 0.76

$$\frac{1}{12}a(\cosh(3x) - 9\cosh(x))\operatorname{csch}(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(3/2),x]
```

```
[Out] (a*(-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/12
```

fricas [B] time = 0.75, size = 226, normalized size = 6.65

$$\frac{\left(6\,a\cosh(x)e^x\sinh(x)^5 + ae^x\sinh(x)^6 + 3\left(5\,a\cosh(x)^2 - 3\,a\right)e^x\sinh(x)^4 + 4\left(5\,a\cosh(x)^3 - 9\,a\cosh(x)\right)e^x\sinh}{24\left(\cosh(x)^3e^{(2\,x)} + \left(e^{(2\,x)} -\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 - 3*a)
*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*cos
h(x)^4 - 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 - 6*a*cosh(x)
^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 - 9*a*cosh(x)^4 - 9*a*cosh(x)^
2 + a)*e^x)*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x) + (
e^(2*x) - 1)*sinh(x)^3 - cosh(x)^3 + 3*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^
2 + 3*(cosh(x)^2*e^(2*x) - cosh(x)^2)*sinh(x))
```

giac [B] time = 0.46, size = 70, normalized size = 2.06

$$-\frac{1}{24}\left(\left(9\,e^{(2\,x)}\operatorname{sgn}\left(e^{(3\,x)} - e^x\right) - \operatorname{sgn}\left(e^{(3\,x)} - e^x\right)\right)e^{(-3\,x)} - e^{(3\,x)}\operatorname{sgn}\left(e^{(3\,x)} - e^x\right) + 9\,e^x\operatorname{sgn}\left(e^{(3\,x)} - e^x\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="giac")
```

[Out] -1/24*((9*e^(2*x)*sgn(e^(3*x) - e^x) - sgn(e^(3*x) - e^x))*e^(-3*x) - e^(3*x)*sgn(e^(3*x) - e^x) + 9*e^x*sgn(e^(3*x) - e^x))*a^(3/2)

maple [A] time = 0.06, size = 24, normalized size = 0.71

$$\frac{a^2 \sinh(x)\cosh(x)\left(\sinh^2(x)-2\right)}{3\sqrt{a\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sinh(x)^2)^(3/2),x)

[Out] 1/3*a^2*sinh(x)*cosh(x)*(sinh(x)^2-2)/(a*sinh(x)^2)^(1/2)

maxima [A] time = 0.41, size = 35, normalized size = 1.03

$$-\frac{1}{24}\,a^{\frac{3}{2}}e^{(3\,x)}+\frac{3}{8}\,a^{\frac{3}{2}}e^{(-x)}-\frac{1}{24}\,a^{\frac{3}{2}}e^{(-3\,x)}+\frac{3}{8}\,a^{\frac{3}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="maxima")

[Out] -1/24*a^(3/2)*e^(3*x) + 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^(3/2)*e^x

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(a\,\sinh(x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sinh(x)^2)^(3/2),x)

[Out] int((a*sinh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^{2}{\left(x\right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sinh(x)**2)**(3/2),x)

[Out] Integral((a*sinh(x)**2)**(3/2), x)

3.142 $\int \sqrt{a \sinh^2(x)}\, dx$

Optimal. Leaf size=13

$$\coth(x)\sqrt{a \sinh^2(x)}$$

[Out] coth(x)*(a*sinh(x)^2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 2638}

$$\coth(x)\sqrt{a \sinh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*Sinh[x]^2],x]

[Out] Coth[x]*Sqrt[a*Sinh[x]^2]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a \sinh^2(x)}\, dx &= \left(\operatorname{csch}(x)\sqrt{a \sinh^2(x)}\right)\int \sinh(x)\, dx\\
&= \coth(x)\sqrt{a \sinh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\coth(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

fricas [B] time = 0.46, size = 71, normalized size = 5.46

$$\frac{\left(2\cosh(x)e^{x}\sinh(x) + e^{x}\sinh(x)^{2} + \left(\cosh(x)^{2} + 1\right)e^{x}\right)\sqrt{ae^{(4x)} - 2ae^{(2x)} + a}\,e^{(-x)}}{2\left(\cosh(x)e^{(2x)} + \left(e^{(2x)} - 1\right)\sinh(x) - \cosh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)*sqrt(a*e^
(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) - 1)*sinh(x) -
cosh(x))
```

giac [B] time = 0.23, size = 34, normalized size = 2.62

$$\frac{1}{2}\left(e^{(-x)}\mathrm{sgn}\left(e^{(3x)} - e^{x}\right) + e^{x}\mathrm{sgn}\left(e^{(3x)} - e^{x}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x)*sgn(e^(3*x) - e^x) + e^x*sgn(e^(3*x) - e^x))*sqrt(a)
```

maple [A] time = 0.05, size = 15, normalized size = 1.15

$$\frac{a\sinh(x)\cosh(x)}{\sqrt{a\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(1/2),x)
```

```
[Out] 1/(a*sinh(x)^2)^(1/2)*a*sinh(x)*cosh(x)
```

maxima [A] time = 0.41, size = 17, normalized size = 1.31

$$-\frac{1}{2}\sqrt{a}\,e^{(-x)} - \frac{1}{2}\sqrt{a}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*e^(-x) - 1/2*sqrt(a)*e^x
```

mupad [B] time = 0.46, size = 21, normalized size = 1.62

$$\sqrt{a}\,\coth(x)\sqrt{\left(\frac{e^{-x}}{2} - \frac{e^{x}}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(1/2),x)
```

```
[Out] a^(1/2)*coth(x)*((exp(-x)/2 - exp(x)/2)^2)^(1/2)
```

sympy [A] time = 0.33, size = 19, normalized size = 1.46

$$\frac{\sqrt{a}\sqrt{\sinh^{2}(x)}\cosh(x)}{\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**2)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(sinh(x)**2)*cosh(x)/sinh(x)
```

3.143 $\int \frac{1}{\sqrt{a \sinh^2(x)}} \, dx$

Optimal. Leaf size=17

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{a\sinh^2(x)}}$$

```
[Out] -arctanh(cosh(x))*sinh(x)/(a*sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3770}

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^2],x]
```

```
[Out] -((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[a*Sinh[x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{\sqrt{a\sinh^2(x)}}$$
$$= -\frac{\tanh^{-1}(\cosh(x))\sinh(x)}{\sqrt{a\sinh^2(x)}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$\frac{\sinh(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^2],x]
```

```
[Out] (Log[Tanh[x/2]]*Sinh[x])/Sqrt[a*Sinh[x]^2]
```

fricas [B] time = 0.80, size = 110, normalized size = 6.47

$$\left[\frac{\sqrt{ae^{(4x)} - 2\,ae^{(2x)} + a}\,\log\left(\frac{\cosh(x)+\sinh(x)-1}{\cosh(x)+\sinh(x)+1}\right)}{ae^{(2x)} - a}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{ae^{(4x)}-2\,ae^{(2x)}+a}\,\sqrt{-a}}{a\cosh(x)e^{(2x)}-a\cosh(x)+\left(ae^{(2x)}-a\right)\sinh(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*log((cosh(x) + sinh(x) - 1)/(cosh(x) + s
inh(x) + 1))/(a*e^(2*x) - a), 2*sqrt(-a)*arctan(sqrt(a*e^(4*x) - 2*a*e^(2*x
) + a)*sqrt(-a)/(a*cosh(x)*e^(2*x) - a*cosh(x) + (a*e^(2*x) - a)*sinh(x)))/
a]
```

giac [A] time = 0.23, size = 1, normalized size = 0.06

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 0
```

maple [B] time = 0.08, size = 49, normalized size = 2.88

$$-\frac{\sinh(x)\sqrt{a\left(\cosh^{2}(x)\right)}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cosh^{2}(x)\right)}+2a}{\sinh(x)}\right)}{\sqrt{a}\,\cosh(x)\sqrt{a\left(\sinh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(1/2),x)
```

```
[Out] -sinh(x)*(a*cosh(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2)+a)/s
inh(x))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

maxima [A] time = 0.42, size = 24, normalized size = 1.41

$$\frac{\log\left(e^{(-x)}+1\right)}{\sqrt{a}}-\frac{\log\left(e^{(-x)}-1\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(-x) + 1)/sqrt(a) - log(e^(-x) - 1)/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a\,\sinh(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(1/2),x)
```

```
[Out] int(1/(a*sinh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sinh^{2}{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**2), x)
```

3.144 $\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

```
[Out] -1/2*coth(x)/a/(a*sinh(x)^2)^(1/2)+1/2*arctanh(cosh(x))*sinh(x)/a/(a*sinh(x
)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3204, 3207, 3770}

$$\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(-3/2),x]
```

```
[Out] -Coth[x]/(2*a*Sqrt[a*Sinh[x]^2]) + (ArcTanh[Cosh[x]]*Sinh[x])/(2*a*Sqrt[a*S
inh[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{2a}$$
$$= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{2a\sqrt{a\sinh^2(x)}}$$
$$= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} + \frac{\tanh^{-1}(\cosh(x))\sinh(x)}{2a\sqrt{a\sinh^2(x)}}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 1.05

$$-\frac{\sinh^3(x)\left(\operatorname{csch}^2\left(\frac{x}{2}\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)+4\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{8\left(a\sinh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(-3/2),x]
```

```
[Out] -1/8*((Csch[x/2]^2 + 4*Log[Tanh[x/2]] + Sech[x/2]^2)*Sinh[x]^3)/(a*Sinh[x]^
2)^(3/2)
```

fricas [B] time = 0.59, size = 327, normalized size = 7.79

$$\frac{\left(6\cosh(x)e^x\sinh(x)^2+2e^x\sinh(x)^3+2\left(3\cosh(x)^2+1\right)e^x\sinh(x)+2\left(\cosh(x)^3+\cosh(x)\right)e^x-\left(4\cosh(x)e\right.}{2\left(a^2\cosh(x)^4-\left(a^2e^{(2x)}-a^2\right)\sinh(x)^4-2a^2\cosh(x)^2-4\left(a^2\cosh(x)e^{(2x)}-a^2\cosh(x)\right)\sinh(x)^3+2\left(3a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(x)*e^x*sinh(x)^2 + 2*e^x*sinh(x)^3 + 2*(3*cosh(x)^2 + 1)*e^x*si
nh(x) + 2*(cosh(x)^3 + cosh(x))*e^x - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x
)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 - cosh(x))*e^x*sinh(
x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)*log((cosh(x) + sinh(x) + 1)/(cosh(x
```

```
) + sinh(x) - 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4
- (a^2*e^(2*x) - a^2)*sinh(x)^4 - 2*a^2*cosh(x)^2 - 4*(a^2*cosh(x)*e^(2*x)
- a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*cosh(x)^2 - a^2 - (3*a^2*cosh(x)^2 - a^
2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^4 - 2*a^2*cosh(x)^2 + a^2)*e^(2*
x) + 4*(a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x)
)*sinh(x))
```

giac [A] time = 0.20, size = 37, normalized size = 0.88

$$-\frac{e^{(-x)} + e^{x}}{\left(\left(e^{(-x)} + e^{x}\right)^{2} - 4\right)a^{\frac{3}{2}}\operatorname{sgn}\left(e^{(3x)} - e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(e^(-x) + e^x)/(((e^(-x) + e^x)^2 - 4)*a^(3/2)*sgn(e^(3*x) - e^x))
```

maple [B] time = 0.08, size = 71, normalized size = 1.69

$$-\frac{\sqrt{a\left(\cosh^{2}(x)\right)}\left(-\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\cosh^{2}(x)\right)}+2a}{\sinh(x)}\right)a\left(\sinh^{2}(x)\right)+\sqrt{a}\sqrt{a\left(\cosh^{2}(x)\right)}\right)}{2a^{\frac{5}{2}}\sinh(x)\cosh(x)\sqrt{a\left(\sinh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(3/2),x)
```

```
[Out] -1/2/a^(5/2)/sinh(x)*(a*cosh(x)^2)^(1/2)*(-ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2
)+a)/sinh(x))*a*sinh(x)^2+a^(1/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)^2
)^(1/2)
```

maxima [A] time = 0.42, size = 62, normalized size = 1.48

$$-\frac{e^{(-x)} + e^{(-3x)}}{2a^{\frac{3}{2}}e^{(-2x)} - a^{\frac{3}{2}}e^{(-4x)} - a^{\frac{3}{2}}} - \frac{\log\left(e^{(-x)} + 1\right)}{2a^{\frac{3}{2}}} + \frac{\log\left(e^{(-x)} - 1\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + e^(-3*x))/(2*a^(3/2)*e^(-2*x) - a^(3/2)*e^(-4*x) - a^(3/2)) - 1/
2*log(e^(-x) + 1)/a^(3/2) + 1/2*log(e^(-x) - 1)/a^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\sinh(x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(3/2),x)
```

```
[Out] int(1/(a*sinh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(-3/2), x)
```

3.145 $\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\sinh(x)\tanh^{-1}(\cosh(x))}{8a^2\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

```
[Out] -1/4*coth(x)/a/(a*sinh(x)^2)^(3/2)+3/8*coth(x)/a^2/(a*sinh(x)^2)^(1/2)-3/8*
arctanh(cosh(x))*sinh(x)/a^2/(a*sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3204, 3207, 3770}

$$\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\sinh(x)\tanh^{-1}(\cosh(x))}{8a^2\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -Coth[x]/(4*a*(a*Sinh[x]^2)^(3/2)) + (3*Coth[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
- (3*ArcTanh[Cosh[x]]*Sinh[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx &= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} - \frac{3\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx}{4a} \\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{8a^2} \\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} + \frac{(3\sinh(x))\int \operatorname{csch}(x)\,dx}{8a^2\sqrt{a\sinh^2(x)}} \\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} - \frac{3\tanh^{-1}(\cosh(x))\sinh(x)}{8a^2\sqrt{a\sinh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 1.10

$$-\frac{\operatorname{csch}(x)\sqrt{a\sinh^2(x)}\left(\operatorname{csch}^4\left(\frac{x}{2}\right)-6\operatorname{csch}^2\left(\frac{x}{2}\right)-\operatorname{sech}^4\left(\frac{x}{2}\right)-6\operatorname{sech}^2\left(\frac{x}{2}\right)-24\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{64a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -1/64*(Csch[x]*(-6*Csch[x/2]^2 + Csch[x/2]^4 - 24*Log[Tanh[x/2]] - 6*Sech[x
/2]^2 - Sech[x/2]^4)*Sqrt[a*Sinh[x]^2])/a^3
```

fricas [B] time = 0.57, size = 875, normalized size = 14.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(42*cosh(x)*e^x*sinh(x)^6 + 6*e^x*sinh(x)^7 + 2*(63*cosh(x)^2 - 11)*e^
x*sinh(x)^5 + 10*(21*cosh(x)^3 - 11*cosh(x))*e^x*sinh(x)^4 + 2*(105*cosh(x)
```

```
^4 - 110*cosh(x)^2 - 11)*e^x*sinh(x)^3 + 2*(63*cosh(x)^5 - 110*cosh(x)^3 -
33*cosh(x))*e^x*sinh(x)^2 + 2*(21*cosh(x)^6 - 55*cosh(x)^4 - 33*cosh(x)^2 +
 3)*e^x*sinh(x) + 2*(3*cosh(x)^7 - 11*cosh(x)^5 - 11*cosh(x)^3 + 3*cosh(x))
*e^x + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x
*sinh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 -
30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x)
)*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh
(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (
cosh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)*log((cosh(x)
+ sinh(x) - 1)/(cosh(x) + sinh(x) + 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*
e^(-x)/(a^3*cosh(x)^8 - 4*a^3*cosh(x)^6 - (a^3*e^(2*x) - a^3)*sinh(x)^8 - 8
*(a^3*cosh(x)*e^(2*x) - a^3*cosh(x))*sinh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3
*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*co
sh(x)^3 - 3*a^3*cosh(x) - (7*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x
)^5 - 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3 - (3
5*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*c
osh(x)^5 - 10*a^3*cosh(x)^3 + 3*a^3*cosh(x) - (7*a^3*cosh(x)^5 - 10*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 - 15*
a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^6 - 15*a^3*cosh(x)^4
 + 9*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^2 - (a^3*cosh(x)^8 - 4*a^3*cosh(
x)^6 + 6*a^3*cosh(x)^4 - 4*a^3*cosh(x)^2 + a^3)*e^(2*x) + 8*(a^3*cosh(x)^7
- 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x) - (a^3*cosh(x)^7 - 3*a^3*
cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh(x))
```

giac [A] time = 0.22, size = 52, normalized size = 0.85

$$\frac{3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}}{4\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}a^{\frac{5}{2}}\operatorname{sgn}\left(e^{(3\,x)}-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/(((e^(-x) + e^x)^2 - 4)^2*a^(
5/2)*sgn(e^(3*x) - e^x))
```

maple [A] time = 0.10, size = 89, normalized size = 1.46

$$\frac{\sqrt{a\left(\cosh^{2}(x)\right)}\left(-3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cosh^{2}(x)\right)}+2a}{\sinh(x)}\right)a\left(\sinh^{4}(x)\right)+3\sqrt{a\left(\cosh^{2}(x)\right)}\,\left(\sinh^{2}(x)\right)\sqrt{a}\,-2\sqrt{a}\,\sqrt{a\left(\cosh^{2}(x)\right)}}{8a^{\frac{7}{2}}\sinh(x)^{3}\cosh(x)\sqrt{a\left(\sinh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(5/2),x)
```

```
[Out] 1/8/a^(7/2)/sinh(x)^3*(a*cosh(x)^2)^(1/2)*(-3*ln(2*(a^(1/2)*(a*cosh(x)^2)^(
1/2)+a)/sinh(x))*a*sinh(x)^4+3*(a*cosh(x)^2)^(1/2)*sinh(x)^2*a^(1/2)-2*a^(1
/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

maxima [A] time = 0.42, size = 96, normalized size = 1.57

$$\frac{3\,e^{(-x)} - 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} + 3\,e^{(-7\,x)}}{4\left(4\,a^{\frac{5}{2}}e^{(-2\,x)} - 6\,a^{\frac{5}{2}}e^{(-4\,x)} + 4\,a^{\frac{5}{2}}e^{(-6\,x)} - a^{\frac{5}{2}}e^{(-8\,x)} - a^{\frac{5}{2}}\right)} + \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a^{\frac{5}{2}}} - \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*a^(5/2)*e^(-2*x)
 - 6*a^(5/2)*e^(-4*x) + 4*a^(5/2)*e^(-6*x) - a^(5/2)*e^(-8*x) - a^(5/2)) +
3/8*log(e^(-x) + 1)/a^(5/2) - 3/8*log(e^(-x) - 1)/a^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\,\mathrm{sinh}(x)^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*sinh(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^{2}{\left(x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(-5/2), x)
```

3.146 $\int \left(a \sinh^3(x)\right)^{5/2} \, dx$

Optimal. Leaf size=135

$$-\frac{26}{165}a^2 \sinh^3(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{2}{15}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^3(x)}-$$

```
[Out] -26/77*a^2*coth(x)*(a*sinh(x)^3)^(1/2)+78/385*a^2*cosh(x)*sinh(x)*(a*sinh(x
)^3)^(1/2)-26/165*a^2*cosh(x)*sinh(x)^3*(a*sinh(x)^3)^(1/2)+2/15*a^2*cosh(x
)*sinh(x)^5*(a*sinh(x)^3)^(1/2)+26/77*I*a^2*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^
2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh
(x))^(1/2)*(a*sinh(x)^3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3207, 2635, 2642, 2641}

$$\frac{2}{15}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{26}{165}a^2\sinh^3(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{2}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(5/2),x]
```

```
[Out] (-26*a^2*Coth[x]*Sqrt[a*Sinh[x]^3])/77 + ((26*I)/77)*a^2*Csch[x]^2*Elliptic
F[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3] + (78*a^2*Cosh[x]*Si
nh[x]*Sqrt[a*Sinh[x]^3])/385 - (26*a^2*Cosh[x]*Sinh[x]^3*Sqrt[a*Sinh[x]^3])
/165 + (2*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]^3])/15
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx = \frac{\left(a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{15}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} - \frac{\left(13a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{11}{2}}(x)\,dx}{15\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} + \frac{\left(39a^2\sqrt{\right.}}{}$$

$$= \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{26}{77}ia^2\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)} + \frac{7}{38}$$

Mathematica [A] time = 0.19, size = 67, normalized size = 0.50

$$\frac{a^2\text{csch}(x)\sqrt{a\sinh^3(x)}\left(-15465\cosh(x)+3657\cosh(3x)-749\cosh(5x)+77\cosh(7x)-\frac{12480F\left(\frac{1}{4}(\pi-2ix)\middle|2\right)}{\sqrt{i\sinh(x)}}\right)}{36960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(5/2),x]
```

```
[Out] (a^2*Csch[x]*(-15465*Cosh[x] + 3657*Cosh[3*x] - 749*Cosh[5*x] + 77*Cosh[7*x
] - (12480*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]])*Sqrt[a*Sinh[x]^
3])/36960
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sinh(x)^3}\,a^2\sinh(x)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3)*a^2*sinh(x)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sinh(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\left(a\left(\sinh^3(x)\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(5/2),x)
```

```
[Out] int((a*sinh(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sinh(x)^3\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(5/2),x)
[Out] int((a*sinh(x)^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(5/2),x)
[Out] Timed out
```

3.147 $\int \left(a\sinh^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=83

$$-\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)}+\frac{2}{9}a\sinh^2(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{14ia\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}$$

```
[Out] -14/45*a*cosh(x)*(a*sinh(x)^3)^(1/2)+2/9*a*cosh(x)*sinh(x)^2*(a*sinh(x)^3)^
(1/2)+14/15*I*a*csch(x)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*E
llipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*(a*sinh(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3207, 2635, 2640, 2639}

$$\frac{2}{9}a\sinh^2(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)}+\frac{14ia\operatorname{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(3/2),x]
```

```
[Out] (-14*a*Cosh[x]*Sqrt[a*Sinh[x]^3])/45 + (((14*I)/15)*a*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2]*Sqrt[a*Sinh[x]^3])/Sqrt[I*Sinh[x]] + (2*a*Cosh[x]*Sinh[x]^2
*Sqrt[a*Sinh[x]^3])/9
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{9}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} - \frac{\left(7a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{5}{2}}(x)\,dx}{9\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} + \frac{\left(7a\sqrt{a\sinh^3(x)}\right)\int \sqrt{}}{15\sinh^{\frac{3}{2}}(x}$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} + \frac{\left(7a\text{csch}(x)\sqrt{a\sinh^3(}\right.}{15\sqrt{i\sin}}$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{14ia\text{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}} + \frac{2}{9}a\cosh(x)\sinh$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.69

$$\frac{1}{180}a\text{csch}(x)\sqrt{a\sinh^3(x)}\left(-38\sinh(2x)+5\sinh(4x)+168\sqrt{i\sinh(x)}\,\text{csch}(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(3/2),x]
```

```
[Out] (a*Csch[x]*Sqrt[a*Sinh[x]^3]*(168*Csch[x]*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]] - 38*Sinh[2*x] + 5*Sinh[4*x]))/180
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sinh(x)^3}\,a\sinh(x)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sinh(x)^3)*a*sinh(x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a\left(\sinh^3(x)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(3/2),x)
[Out] int((a*sinh(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\mathrm{sinh}\left(x\right)}^3\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(3/2),x)
```

```
[Out] int((a*sinh(x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^3(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**3)**(3/2), x)
```

3.148 $\int \sqrt{a \sinh^3(x)}\, dx$

Optimal. Leaf size=62

$$\frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\sqrt{i\sinh(x)}\,\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}$$

```
[Out] 2/3*coth(x)*(a*sinh(x)^3)^(1/2)-2/3*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(1/
2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))^(
1/2)*(a*sinh(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3207, 2635, 2642, 2641}

$$\frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\sqrt{i\sinh(x)}\,\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (2*Coth[x]*Sqrt[a*Sinh[x]^3])/3 - ((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (I/2
)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\sinh^3(x)}\,dx &= \frac{\sqrt{a\sinh^3(x)}\int \sinh^{\frac{3}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}\\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{\sqrt{a\sinh^3(x)}\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{3\sinh^{\frac{3}{2}}(x)}\\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{1}{3}\left(\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx\\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 60, normalized size = 0.97

$$\frac{2}{3}\sqrt{a\sinh^3(x)}\left(\coth(x)-\sqrt{2}\operatorname{csch}^2(x)\sqrt{-\sinh(x)(\sinh(x)+\cosh(x))}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2x)+\sinh(2x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (2*Sqrt[a*Sinh[x]^3]*(Coth[x] - Sqrt[2]*Csch[x]^2*Hypergeometric2F1[1/4, 1/
2, 5/4, Cosh[2*x] + Sinh[2*x]]*Sqrt[-(Sinh[x]*(Cosh[x] + Sinh[x]))]))/3
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sinh(x)^3}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sinh(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sinh^3(x)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(1/2),x)
[Out] int((a*sinh(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sinh(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a \sinh(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(1/2),x)
[Out] int((a*sinh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sinh^3(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(1/2),x)
[Out] Integral(sqrt(a*sinh(x)**3), x)
```

3.149 $\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx$

Optimal. Leaf size=60

$$-\frac{2\sinh(x)\cosh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

```
[Out] -2*cosh(x)*sinh(x)/(a*sinh(x)^3)^(1/2)+2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/si
n(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2/(I*sinh(
x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3207, 2636, 2640, 2639}

$$-\frac{2\sinh(x)\cosh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^3] + ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2
]*Sinh[x]^2)/(Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx &= \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{\sinh^{\frac{3}{2}}(x)\int\sqrt{\sinh(x)}\,dx}{\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{\sinh^2(x)\int\sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.70

$$-\frac{2\sinh(x)\left(\cosh(x)-\sqrt{i\sinh(x)}\,E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\right)}{\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/Sqr
t[a*Sinh[x]^3]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sinh(x)^{3}}}{a\sinh(x)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sinh(x)^3)/(a*sinh(x)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sinh(x)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sinh^{3}(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(1/2),x)
[Out] int(1/(a*sinh(x)^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sinh(x)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a\,{\sinh(x)}^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sinh^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**3), x)
```

3.150 $\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}+\frac{10i\sqrt{i\sinh(x)}\,\sinh(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}$$

```
[Out] 10/21*cosh(x)/a/(a*sinh(x)^3)^(1/2)-2/7*coth(x)*csch(x)/a/(a*sinh(x)^3)^(1/
2)+10/21*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(
1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)*(I*sinh(x))^(1/2)/a/(a*sinh(x)^3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3207, 2636, 2642, 2641}

$$\frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}+\frac{10i\sqrt{i\sinh(x)}\,\sinh(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (10*Cosh[x])/(21*a*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x])/(7*a*Sqrt[a*Sin
h[x]^3]) + (((10*I)/21)*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sinh[x
])/(a*Sqrt[a*Sinh[x]^3])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sinh^3(x)}}$$

$$= -\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} - \frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\sinh^3(x)}}$$

$$= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}$$

$$= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{\left(5\sqrt{i\sinh(x)}\,\sinh(x)\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}$$

$$= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{10iF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\,\sinh(x)}{21a\sqrt{a\sinh^3(x)}}$$

Mathematica [A] time = 0.09, size = 53, normalized size = 0.61

$$\frac{2\left(5\cosh(x)-3\coth(x)\text{csch}(x)+5(i\sinh(x))^{3/2}F\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\right)}{21a\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (2*(5*Cosh[x] - 3*Coth[x]*Csch[x] + 5*EllipticF[(Pi - (2*I)*x)/4, 2]*(I*Sin
h[x])^(3/2)))/(21*a*Sqrt[a*Sinh[x]^3])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sinh(x)^3}}{a^2\sinh(x)^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sinh(x)^3)/(a^2*sinh(x)^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sinh^3(x)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\sinh(x)^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(3/2),x)
[Out] int(1/(a*sinh(x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^{3}(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(3/2),x)
[Out] Integral((a*sinh(x)**3)**(-3/2), x)
```

3.151 $\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\sinh^3(x)}}-\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}}-\frac{154i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}}+\frac{22\coth(x)\text{csch}^2(}{117a^2\sqrt{a\sinh^3(}}$$

```
[Out] -154/585*coth(x)/a^2/(a*sinh(x)^3)^(1/2)+22/117*coth(x)*csch(x)^2/a^2/(a*si
nh(x)^3)^(1/2)-2/13*coth(x)*csch(x)^4/a^2/(a*sinh(x)^3)^(1/2)+154/195*cosh(
x)*sinh(x)/a^2/(a*sinh(x)^3)^(1/2)-154/195*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/
sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2/a^2/(I
*sinh(x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3207, 2636, 2640, 2639}

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\sinh^3(x)}}-\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}}-\frac{154i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}}+\frac{22\coth(x)\text{csch}^2(}{117a^2\sqrt{a\sinh^3(}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(-5/2),x]
```

```
[Out] (-154*Coth[x])/(585*a^2*Sqrt[a*Sinh[x]^3]) + (22*Coth[x]*Csch[x]^2)/(117*a^
2*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x]^4)/(13*a^2*Sqrt[a*Sinh[x]^3]) + (
154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Sinh[x]^3]) - (((154*I)/195)*EllipticE
[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/(a^2*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(11\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\sinh^3(x)}}$$

$$= \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int -}{195a^2\sqrt{a\sinh^3(}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}}$$

Mathematica [A] time = 0.20, size = 69, normalized size = 0.51

$$\frac{462\sinh(x)\cosh(x) - 2\coth(x)\left(45\operatorname{csch}^4(x) - 55\operatorname{csch}^2(x) + 77\right) + 462i(i\sinh(x))^{3/2}E\left(\frac{1}{4}(\pi - 2ix)\middle|2\right)}{585a^2\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(-5/2),x]

[Out] (-2*Coth[x]*(77 - 55*Csch[x]^2 + 45*Csch[x]^4) + (462*I)*EllipticE[(Pi - (2
*I)*x)/4, 2]*(I*Sinh[x])^(3/2) + 462*Cosh[x]*Sinh[x])/(585*a^2*Sqrt[a*Sinh[
x]^3])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sinh(x)^3}}{a^3\sinh(x)^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sinh(x)^3)/(a^3*sinh(x)^9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\left(\sinh^3(x)\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(5/2),x)
[Out] int(1/(a*sinh(x)^3)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\sinh(x)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**3)**(-5/2), x)
```

3.152 $\int \left(a \sinh^4(x)\right)^{5/2} \, dx$

Optimal. Leaf size=132

$$-\frac{21}{128}a^2 \sinh(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{1}{10}a^2\sinh^7(x)\cosh(x)\sqrt{a\sinh^4(x)}-\frac{9}{80}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^4(x)}+$$

```
[Out] 63/256*a^2*coth(x)*(a*sinh(x)^4)^(1/2)-63/256*a^2*x*csch(x)^2*(a*sinh(x)^4)
^(1/2)-21/128*a^2*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+21/160*a^2*cosh(x)*si
nh(x)^3*(a*sinh(x)^4)^(1/2)-9/80*a^2*cosh(x)*sinh(x)^5*(a*sinh(x)^4)^(1/2)+
1/10*a^2*cosh(x)*sinh(x)^7*(a*sinh(x)^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$\frac{1}{10}a^2\sinh^7(x)\cosh(x)\sqrt{a\sinh^4(x)}-\frac{9}{80}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{21}{160}a^2\sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*Coth[x]*Sqrt[a*Sinh[x]^4])/256 - (63*a^2*x*Csch[x]^2*Sqrt[a*Sinh[x]
^4])/256 - (21*a^2*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/128 + (21*a^2*Cosh[x]
*Sinh[x]^3*Sqrt[a*Sinh[x]^4])/160 - (9*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]
^4])/80 + (a^2*Cosh[x]*Sinh[x]^7*Sqrt[a*Sinh[x]^4])/10
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx = \left(a^2\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} - \frac{1}{10}\left(9a^2\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^8(x)\,dx$$
$$= -\frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} + \frac{1}{80}\left(63a^2\mathrm{c}\right.$$
$$= \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\operatorname{cosh}$$
$$= -\frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\operatorname{cos}$$
$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\operatorname{sinh}$$
$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{63}{256}a^2x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a}$$

Mathematica [A] time = 0.16, size = 53, normalized size = 0.40

$$\frac{a(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) - 25\sinh(8x) + 2\sinh(10x))\operatorname{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(5/2),x]
```

```
[Out] (a*Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x]
+ 150*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

fricas [B] time = 0.58, size = 1597, normalized size = 12.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*(
76*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 - 15*a^
```

2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 - 255*a^2*cosh(x)^2 + 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 - 425*a^2*cosh(x)^3 + 50*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 - 1275*a^2*cosh(x)^4 + 300*a^2*cosh(x)^2 - 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2*cosh(x)^7 - 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^13 + 60*(4199*a^2*cosh(x)^8 - 7735*a^2*cosh(x)^6 + 4550*a^2*cosh(x)^4 - 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*cosh(x)^9 - 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 - 2730*a^2*cosh(x)^3 + 315*a^2*cosh(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 - 546975*a^2*cosh(x)^8 + 600600*a^2*cosh(x)^6 - 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(x)^2 - 2520*a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 - 60775*a^2*cosh(x)^9 + 85800*a^2*cosh(x)^7 - 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(x)^3 - 2520*a^2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 - 36465*a^2*cosh(x)^10 + 64350*a^2*cosh(x)^8 - 60060*a^2*cosh(x)^6 + 34650*a^2*cosh(x)^4 - 7560*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*a^2*cosh(x)^13 - 3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 - 8580*a^2*cosh(x)^7 + 6930*a^2*cosh(x)^5 - 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^7 + 60*(1292*a^2*cosh(x)^14 - 7735*a^2*cosh(x)^12 + 20020*a^2*cosh(x)^10 - 30030*a^2*cosh(x)^8 + 32340*a^2*cosh(x)^6 - 17640*a^2*x*cosh(x)^4 - 980*a^2*cosh(x)^2 + 10*a^2)*e^(2*x)*sinh(x)^6 + 24*(1292*a^2*cosh(x)^15 - 8925*a^2*cosh(x)^13 + 27300*a^2*cosh(x)^11 - 50050*a^2*cosh(x)^9 + 69300*a^2*cosh(x)^7 - 52920*a^2*x*cosh(x)^5 - 4900*a^2*cosh(x)^3 + 150*a^2*cosh(x))*e^(2*x)*sinh(x)^5 + 30*(323*a^2*cosh(x)^16 - 2550*a^2*cosh(x)^14 + 9100*a^2*cosh(x)^12 - 20020*a^2*cosh(x)^10 + 34650*a^2*cosh(x)^8 - 35280*a^2*x*cosh(x)^6 - 4900*a^2*cosh(x)^4 + 300*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^4 + 120*(19*a^2*cosh(x)^17 - 170*a^2*cosh(x)^15 + 700*a^2*cosh(x)^13 - 1820*a^2*cosh(x)^11 + 3850*a^2*cosh(x)^9 - 5040*a^2*x*cosh(x)^7 - 980*a^2*cosh(x)^5 + 100*a^2*cosh(x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x)^3 + 5*(76*a^2*cosh(x)^18 - 765*a^2*cosh(x)^16 + 3600*a^2*cosh(x)^14 - 10920*a^2*cosh(x)^12 + 27720*a^2*cosh(x)^10 - 45360*a^2*x*cosh(x)^8 - 11760*a^2*cosh(x)^6 + 1800*a^2*cosh(x)^4 - 180*a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^2 + 10*(4*a^2*cosh(x)^19 - 45*a^2*cosh(x)^17 + 240*a^2*cosh(x)^15 - 840*a^2*cosh(x)^13 + 2520*a^2*cosh(x)^11 - 5040*a^2*x*cosh(x)^9 - 1680*a^2*cosh(x)^7 + 360*a^2*cosh(x)^5 - 60*a^2*cosh(x)^3 + 5*a^2*cosh(x))*e^(2*x)*sinh(x) + (2*a^2*cosh(x)^20 - 25*a^2*cosh(x)^18 + 150*a^2*cosh(x)^16 - 600*a^2*cosh(x)^14 + 2100*a^2*cosh(x)^12 - 5040*a^2*x*cosh(x)^10 - 2100*a^2*cosh(x)^8 + 600*a^2*cosh(x)^6 - 150*a^2*cosh(x)^4 + 25*a^2*cosh(x)^2 - 2*a^2)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^10*e^(4*x) - 2*cosh(x)^10*e^(2*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^10 + cosh(x)^10 + 10*(cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 + 45*(cosh(x)^2*e^(4*x) - 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^8 + 120*(cosh(x)^3*e^(4*x) - 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^7 + 210*(cosh(x)^4*e^(4*x) - 2*cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^6 + 252*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5*e^(2*x) + cosh(x)^5)*sinh(x)^5 + 210*(cosh(x)^6*e^(4*x) - 2*cosh(x)^6*e^(2*x) + cosh(x)^6)*sinh(x)^4 + 120*(cosh(x)^7*e^(4*x) - 2*cosh(x)^7*e^(2*x) + cosh(x)^

```
7)*sinh(x)^3 + 45*(cosh(x)^8*e^(4*x) - 2*cosh(x)^8*e^(2*x) + cosh(x)^8)*sin
h(x)^2 + 10*(cosh(x)^9*e^(4*x) - 2*cosh(x)^9*e^(2*x) + cosh(x)^9)*sinh(x))
```

giac [A] time = 0.30, size = 114, normalized size = 0.86

$$-\frac{1}{20480}\left(5040\,a^2x - 2\,a^2e^{(10\,x)} + 25\,a^2e^{(8\,x)} - 150\,a^2e^{(6\,x)} + 600\,a^2e^{(4\,x)} - 2100\,a^2e^{(2\,x)} - \left(5754\,a^2e^{(10\,x)} - 2100\,a^2e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*(5040*a^2*x - 2*a^2*e^(10*x) + 25*a^2*e^(8*x) - 150*a^2*e^(6*x) +
600*a^2*e^(4*x) - 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) - 2100*a^2*e^(8*x)
+ 600*a^2*e^(6*x) - 150*a^2*e^(4*x) + 25*a^2*e^(2*x) - 2*a^2)*e^(-10*x))*sq
rt(a)
```

maple [A] time = 0.20, size = 171, normalized size = 1.30

$$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}\,a^{\frac{3}{2}}\left(8\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a}\,\left(\sinh^4(2x)\right) - 50\sqrt{a\left(\sinh^2(2x\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(5/2),x)
```

```
[Out] 1/2560*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*a^(3/2)*(8*(a*
sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4-50*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*cos
h(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2-325*cosh(2
*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-315*ln(
a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/((-1+cosh(2*x))^2*a)^
(1/2)
```

maxima [A] time = 0.43, size = 100, normalized size = 0.76

$$-\frac{63}{256}\,a^{\frac{5}{2}}x - \frac{1}{20480}\left(25\,a^{\frac{5}{2}}e^{(-2\,x)} - 150\,a^{\frac{5}{2}}e^{(-4\,x)} + 600\,a^{\frac{5}{2}}e^{(-6\,x)} - 2100\,a^{\frac{5}{2}}e^{(-8\,x)} + 2100\,a^{\frac{5}{2}}e^{(-12\,x)} - 600\,a^{\frac{5}{2}}e^{(-14\,x)} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -63/256*a^(5/2)*x - 1/20480*(25*a^(5/2)*e^(-2*x) - 150*a^(5/2)*e^(-4*x) + 6
00*a^(5/2)*e^(-6*x) - 2100*a^(5/2)*e^(-8*x) + 2100*a^(5/2)*e^(-12*x) - 600*
a^(5/2)*e^(-14*x) + 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) + 2*a^(5/2
)*e^(-20*x) - 2*a^(5/2))*e^(10*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sinh(x)^4\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(5/2),x)
```

```
[Out] int((a*sinh(x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^4(x)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(5/2), x)
```

3.153 $\int \left(a\sinh^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{1}{6}a\sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)}-\frac{5}{16}ax\text{csch}^2(x)$$

```
[Out] 5/16*a*coth(x)*(a*sinh(x)^4)^(1/2)-5/16*a*x*csch(x)^2*(a*sinh(x)^4)^(1/2)-5
/24*a*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+1/6*a*cosh(x)*sinh(x)^3*(a*sinh(x
)^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$\frac{1}{6}a\sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)}-\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)}-\frac{5}{16}ax\text{csch}^2(x)\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(3/2),x]
```

```
[Out] (5*a*Coth[x]*Sqrt[a*Sinh[x]^4])/16 - (5*a*x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/16
 - (5*a*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/24 + (a*Cosh[x]*Sinh[x]^3*Sqrt[a
*Sinh[x]^4])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \left(a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^6(x)\,dx$$
$$= \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{1}{6}\left(5a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^4(x)\,dx$$
$$= -\frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} + \frac{1}{8}\left(5a\text{csch}^2(x)\sqrt{\right.$$
$$= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\,\text{s}}$$
$$= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x}$$

Mathematica [A] time = 0.09, size = 38, normalized size = 0.49

$$\frac{1}{192}(-60x + 45\sinh(2x) - 9\sinh(4x) + \sinh(6x))\text{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(3/2),x]
```

```
[Out] (Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6
*x]))/192
```

fricas [B] time = 0.64, size = 659, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*cos
h(x)^2 - 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 - 9*a*cosh(x))*e^(2*x
)*sinh(x)^9 + 45*(11*a*cosh(x)^4 - 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8 + 7
2*(11*a*cosh(x)^5 - 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6*(15
4*a*cosh(x)^6 - 315*a*cosh(x)^4 + 210*a*cosh(x)^2 - 20*a*x)*e^(2*x)*sinh(x)
^6 + 36*(22*a*cosh(x)^7 - 63*a*cosh(x)^5 + 70*a*cosh(x)^3 - 20*a*x*cosh(x))
*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 - 42*a*cosh(x)^6 + 70*a*cosh(x)^4 -
 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 - 54*a*cosh(x
)^7 + 126*a*cosh(x)^5 - 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*sinh(x)^3
```

```
+ 3*(22*a*cosh(x)^10 - 135*a*cosh(x)^8 + 420*a*cosh(x)^6 - 600*a*x*cosh(x)^
4 - 90*a*cosh(x)^2 + 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11 - 15*a*cosh
(x)^9 + 60*a*cosh(x)^7 - 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 + 3*a*cosh(x))*
e^(2*x)*sinh(x) + (a*cosh(x)^12 - 9*a*cosh(x)^10 + 45*a*cosh(x)^8 - 120*a*x
*cosh(x)^6 - 45*a*cosh(x)^4 + 9*a*cosh(x)^2 - a)*e^(2*x))*sqrt(a*e^(8*x) -
4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^6*e^(4*x) -
2*cosh(x)^6*e^(2*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(
cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 + 15*(cosh(x)^2*e^
(4*x) - 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(cosh(x)^3*e^(4*x)
- 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(x)^4*e^(4*x) - 2*co
sh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5
*e^(2*x) + cosh(x)^5)*sinh(x))
```

giac [A] time = 0.17, size = 50, normalized size = 0.64

$$\frac{1}{384}\left(\left(110\,e^{(6\,x)}-45\,e^{(4\,x)}+9\,e^{(2\,x)}-1\right)e^{(-6\,x)}-120\,x+e^{(6\,x)}-9\,e^{(4\,x)}+45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))*a^(3/2)
```

maple [A] time = 0.15, size = 125, normalized size = 1.60

$$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}\,\sqrt{a}\,\left(2\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a}\,\left(\sinh^2(2x)\right)-9\cosh(2x)\sqrt{a}\right.}{96\sinh(2x)\sqrt{(-1+\cosh(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(3/2),x)
```

```
[Out] 1/96*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*a^(1/2)*(2*(a*si
nh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2-9*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1
/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-15*ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2
)^(1/2))*a)/sinh(2*x)/((-1+cosh(2*x))^2*a)^(1/2)
```

maxima [A] time = 0.42, size = 63, normalized size = 0.81

$$-\frac{5}{16}\,a^{\frac{3}{2}}x-\frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)}-45\,a^{\frac{3}{2}}e^{(-4\,x)}+45\,a^{\frac{3}{2}}e^{(-8\,x)}-9\,a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}e^{(-12\,x)}-a^{\frac{3}{2}}\right)e^{(6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -5/16*a^(3/2)*x - 1/384*(9*a^(3/2)*e^(-2*x) - 45*a^(3/2)*e^(-4*x) + 45*a^(3
/2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) + a^(3/2)*e^(-12*x) - a^(3/2))*e^(6*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sinh(x)^4\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(3/2),x)
```

```
[Out] int((a*sinh(x)^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^4(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(3/2), x)
```

3.154 $\int \sqrt{a \sinh^4(x)}\, dx$

Optimal. Leaf size=36

$$\frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)} - \frac{1}{2}x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}$$

```
[Out] 1/2*coth(x)*(a*sinh(x)^4)^(1/2)-1/2*x*csch(x)^2*(a*sinh(x)^4)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$\frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)} - \frac{1}{2}x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^4],x]
```

```
[Out] (Coth[x]*Sqrt[a*Sinh[x]^4])/2 - (x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a\sinh^4(x)}\,dx = \left(\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^2(x)\,dx$$
$$= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)} - \frac{1}{2}\left(\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int 1\,dx$$
$$= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)} - \frac{1}{2}x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}$$

Mathematica [**A**] time = 0.04, size = 24, normalized size = 0.67

$$\frac{1}{2}\sqrt{a\sinh^4(x)}\left(\coth(x) - x\operatorname{csch}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^4],x]
```

```
[Out] ((Coth[x] - x*Csch[x]^2)*Sqrt[a*Sinh[x]^4])/2
```

fricas [**B**] time = 0.54, size = 180, normalized size = 5.00

$$\frac{\left(4\cosh(x)e^{(2x)}\sinh(x)^3 + e^{(2x)}\sinh(x)^4 + 2\left(3\cosh(x)^2 - 2x\right)e^{(2x)}\sinh(x)^2 + 4\left(\cosh(x)^3 - 2x\cosh(x)\right)e^{(2x)}\text{si}}{8\left(\cosh(x)^2e^{(4x)} - 2\cosh(x)^2e^{(2x)} + \left(e^{(4x)} - 2e^{(2x)} + 1\right)\sinh(x)^2 + \cosh(x)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 - 2*x
)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh(x
)^4 - 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x
) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) - 2*cosh(x)^2*e^(2*x) + (e
^(4*x) - 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) - 2*cosh
(x)*e^(2*x) + cosh(x))*sinh(x))
```

giac [**A**] time = 0.30, size = 26, normalized size = 0.72

$$\frac{1}{8}\left(\left(2e^{(2x)} - 1\right)e^{(-2x)} - 4x + e^{(2x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))*sqrt(a)
```

maple [B] time = 0.18, size = 84, normalized size = 2.33

$$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}\left(\sqrt{a\left(\sinh^2(2x)\right)}\sqrt{a}-\ln\left(\sqrt{a}\cosh(2x)+\sqrt{a\left(\sinh^2(2\right.}\right.\right.}{4\sqrt{a}\sinh(2x)\sqrt{(-1+\cosh(2x))^2 a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(1/2),x)
```

```
[Out] 1/4*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*((a*sinh(2*x)^2)^
(1/2)*a^(1/2)-ln(a^(1/2)*cosh(2*x)+(a*sinh(2*x)^2)^(1/2))*a)/a^(1/2)/sinh(2
*x)/((-1+cosh(2*x))^2*a)^(1/2)
```

maxima [A] time = 0.42, size = 27, normalized size = 0.75

$$-\frac{1}{8}\left(\sqrt{a}\,e^{(-4\,x)}-\sqrt{a}\right)e^{(2\,x)}-\frac{1}{2}\sqrt{a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) - 1/2*sqrt(a)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{a\sinh(x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(1/2),x)
```

```
[Out] int((a*sinh(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sinh^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sinh(x)**4), x)
```

3.155 $\int \frac{1}{\sqrt{a \sinh^4(x)}} \, dx$

Optimal. Leaf size=16

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

```
[Out] -cosh(x)*sinh(x)/(a*sinh(x)^4)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 3767, 8}

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^4],x]
```

```
[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = \frac{\sinh^2(x)\int \operatorname{csch}^2(x)\,dx}{\sqrt{a\sinh^4(x)}}$$
$$= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}(\int 1\,dx, x, -i\coth(x))}{\sqrt{a\sinh^4(x)}}$$
$$= -\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^4],x]
```

```
[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

fricas [B] time = 0.49, size = 122, normalized size = 7.62

$$-\frac{2\sqrt{ae^{(8x)} - 4\,ae^{(6x)} + 6\,ae^{(4x)} - 4\,ae^{(2x)} + a}}{a\cosh(x)^2 + \left(ae^{(4x)} - 2\,ae^{(2x)} + a\right)\sinh(x)^2 + \left(a\cosh(x)^2 - a\right)e^{(4x)} - 2\left(a\cosh(x)^2 - a\right)e^{(2x)} + 2\left(a\cosh(x)e^{(4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(a*cosh(x)
^2 + (a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 - a)*e^(4*x) -
2*(a*cosh(x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) +
a*cosh(x))*sinh(x) - a)
```

giac [A] time = 0.17, size = 13, normalized size = 0.81

$$-\frac{2}{\sqrt{a}\left(e^{(2x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(a)*(e^(2*x) - 1))
```

maple [B] time = 0.14, size = 50, normalized size = 3.12

$$-\frac{\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(\cosh\left(2x\right)+1\right)}\,\sqrt{a\left(\sinh^{2}\left(2x\right)\right)}}{a\sinh\left(2x\right)\sqrt{\left(-1+\cosh\left(2x\right)\right)^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(1/2),x)
[Out] -(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/a*(a*sinh(2*x)^2)^(1/2)/sinh(2*x)/(
(-1+cosh(2*x))^2*a)^(1/2)
```

maxima [A] time = 0.43, size = 18, normalized size = 1.12

$$\frac{2}{\sqrt{a}e^{(-2x)}-\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="maxima")
[Out] 2/(sqrt(a)*e^(-2*x) - sqrt(a))
```

mupad [B] time = 0.47, size = 38, normalized size = 2.38

$$\frac{e^{-x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^{4}}}{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(1/2),x)
[Out] (exp(-x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 - exp(x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\sinh^{4}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(1/2),x)
[Out] Integral(1/sqrt(a*sinh(x)**4), x)
```

3.156 $\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sinh(x)\cosh(x)}{a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}+\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}$$

```
[Out] 2/3*cosh(x)^2*coth(x)/a/(a*sinh(x)^4)^(1/2)-1/5*cosh(x)^2*coth(x)^3/a/(a*si
nh(x)^4)^(1/2)-cosh(x)*sinh(x)/a/(a*sinh(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3767}

$$-\frac{\sinh(x)\cosh(x)}{a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}+\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(-3/2),x]
```

```
[Out] (2*Cosh[x]^2*Coth[x])/(3*a*Sqrt[a*Sinh[x]^4]) - (Cosh[x]^2*Coth[x]^3)/(5*a*
Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a*Sqrt[a*Sinh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = \frac{\sinh^2(x)\int \operatorname{csch}^6(x)\,dx}{a\sqrt{a\sinh^4(x)}}$$

$$= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\coth(x)\right)}{a\sqrt{a\sinh^4(x)}}$$

$$= \frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}} - \frac{\cosh(x)\sinh(x)}{a\sqrt{a\sinh^4(x)}}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.50

$$-\frac{\sinh^5(x)\cosh(x)\left(3\operatorname{csch}^4(x)-4\operatorname{csch}^2(x)+8\right)}{15\left(a\sinh^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(-3/2),x]
```

```
[Out] -1/15*(Cosh[x]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x]^5)/(a*Sinh[x]^4)^(3/
2)
```

fricas [B] time = 0.44, size = 1163, normalized size = 17.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x)
^2 - 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (1
0*cosh(x)^4 - 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*
e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) - 2*a^2*
e^(2*x) + a^2)*sinh(x)^10 - 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x) - 2*a
^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 - a^2 + (9
*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(9*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^
8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^3
 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x
)^7 + 10*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^4 - 1
4*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a
^2)*e^(2*x))*sinh(x)^6 - 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5 - 70*a^2*co
```

```
sh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*co
sh(x))*e^(4*x) - 2*(63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e
^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x
)^2 - a^2 + (21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(4*x) - 2*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^
5 + 5*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5 + 5*
a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5
 + 5*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(9*a^2*cosh(x)^8 -
 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2 + (9*a^2*cosh
(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2)*e^(4*
x) - 2*(9*a^2*cosh(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh
(x)^2 + a^2)*e^(2*x))*sinh(x)^2 - a^2 + (a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 +
 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(
a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 + 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*
a^2*cosh(x)^2 - a^2)*e^(2*x) + 10*(a^2*cosh(x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*
cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^9 - 4*a^2*cosh(x)^
7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) - 2*(a^2*cosh(
x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(2*x))*sinh(x))
```

giac [A] time = 0.22, size = 27, normalized size = 0.40

$$-\frac{16\left(10\,e^{(4\,x)}-5\,e^{(2\,x)}+1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) - 1)^5)
```

maple [A] time = 0.14, size = 74, normalized size = 1.09

$$-\frac{4\left(2\left(\cosh^{2}(2x)\right)-6\cosh(2x)+7\right)\sqrt{a\left(\sinh^{2}(2x)\right)}\,\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}}{15a^{2}(-1+\cosh(2x))^{2}\sinh(2x)\sqrt{(-1+\cosh(2x))^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(3/2),x)
```

```
[Out] -4/15/a^2*(2*cosh(2*x)^2-6*cosh(2*x)+7)*(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2
*x))*(cosh(2*x)+1))^(1/2)/(-1+cosh(2*x))^2/sinh(2*x)/((-1+cosh(2*x))^2*a)^(
1/2)
```

maxima [B] time = 0.41, size = 171, normalized size = 2.51

$$-\frac{16\,e^{(-2\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}-10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}-5\,a^{\frac{3}{2}}e^{(-8\,x)}+a^{\frac{3}{2}}e^{(-10\,x)}-a^{\frac{3}{2}}\right)}+\frac{32\,e^{(-4\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)}-10\,a^{\frac{3}{2}}e^{(-4\,x)}+10\,a^{\frac{3}{2}}e^{(-6\,x)}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6
*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 32/3*e^(-4*x)/(5*
a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^
(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) - 10*a^(3
/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x)
 - a^(3/2))
```

mupad [B] time = 0.53, size = 48, normalized size = 0.71

$$-\frac{64\,\mathrm{e}^{2\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}\,\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\,a^2\,{\left(\mathrm{e}^{2\,x}-1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(3/2),x)
```

```
[Out] -(64*exp(2*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*x)
+ 1))/(15*a^2*(exp(2*x) - 1)^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(-3/2), x)
```

3.157 $\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}}-\frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}$$

```
[Out] 4/3*cosh(x)^2*coth(x)/a^2/(a*sinh(x)^4)^(1/2)-6/5*cosh(x)^2*coth(x)^3/a^2/(
a*sinh(x)^4)^(1/2)+4/7*cosh(x)^2*coth(x)^5/a^2/(a*sinh(x)^4)^(1/2)-1/9*cosh
(x)^2*coth(x)^7/a^2/(a*sinh(x)^4)^(1/2)-cosh(x)*sinh(x)/a^2/(a*sinh(x)^4)^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3767}

$$-\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}}-\frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] (4*Cosh[x]^2*Coth[x])/(3*a^2*Sqrt[a*Sinh[x]^4]) - (6*Cosh[x]^2*Coth[x]^3)/(
5*a^2*Sqrt[a*Sinh[x]^4]) + (4*Cosh[x]^2*Coth[x]^5)/(7*a^2*Sqrt[a*Sinh[x]^4]
) - (Cosh[x]^2*Coth[x]^7)/(9*a^2*Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a^
2*Sqrt[a*Sinh[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \frac{\sinh^2(x)\int \text{csch}^{10}(x)\,dx}{a^2\sqrt{a\sinh^4(x)}}$$
$$= -\frac{\left(i\sinh^2(x)\right)\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)dx, x, -i\coth(x)\right)}{a^2\sqrt{a\sinh^4(x)}}$$
$$= \frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}} - \frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}} + \frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}} -$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.40

$$-\frac{\sinh(x)\cosh(x)\left(35\text{csch}^8(x)-40\text{csch}^6(x)+48\text{csch}^4(x)-64\text{csch}^2(x)+128\right)}{315a^2\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] -1/315*(Cosh[x]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 - 40*Csch[x]^6 + 35*Csch
[x]^8)*Sinh[x])/(a^2*Sqrt[a*Sinh[x]^4])
```

fricas [B] time = 0.82, size = 3093, normalized size = 26.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*c
osh(x)^2 - 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 - cosh(x))*e^(2*x)*sinh
(x)^5 + 36*(245*cosh(x)^4 - 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(147*c
osh(x)^5 - 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)^6 -
 140*cosh(x)^4 + 24*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^7 - 2
8*cosh(x)^5 + 8*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8 - 84*
cosh(x)^6 + 36*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e
^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 - 9*a^3*co
sh(x)^16 + (a^3*e^(4*x) - 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^3*cosh(x)
*e^(4*x) - 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a^3*cosh(x)
```

```
^14 + 9*(17*a^3*cosh(x)^2 - a^3 + (17*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(17*
a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3 - 3*a^3*cos
h(x) + (17*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^3 - 3
*a^3*cosh(x))*e^(2*x))*sinh(x)^15 - 84*a^3*cosh(x)^12 + 36*(85*a^3*cosh(x)^
4 - 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^
(4*x) - 2*(85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^14 +
 504*(17*a^3*cosh(x)^5 - 10*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^5
 - 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^5 - 10*a^3*c
osh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*cosh(x)^10 + 84*(221*
a^3*cosh(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3 + (221*a^3*cosh(
x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(221*a^3*cos
h(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^12 +
144*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh(
x) + (221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3
- 7*a^3*cosh(x))*e^(2*x))*sinh(x)^11 - 126*a^3*cosh(x)^8 + 18*(2431*a^3*cos
h(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^
3 + (2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3
*cosh(x)^2 + 7*a^3)*e^(4*x) - 2*(2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 +
2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^10 + 4*(12
155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*co
sh(x)^3 + 315*a^3*cosh(x) + (12155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18
018*a^3*cosh(x)^5 - 4620*a^3*cosh(x)^3 + 315*a^3*cosh(x))*e^(4*x) - 2*(1215
5*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*cosh
(x)^3 + 315*a^3*cosh(x))*e^(2*x))*sinh(x)^9 + 84*a^3*cosh(x)^6 + 18*(2431*a
^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^
4 + 315*a^3*cosh(x)^2 - 7*a^3 + (2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 +
 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(4*
x) - 2*(2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 231
0*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(2*x))*sinh(x)^8 + 144*(221*
a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5
+ 105*a^3*cosh(x)^3 - 7*a^3*cosh(x) + (221*a^3*cosh(x)^11 - 715*a^3*cosh(x)
^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^
7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh(x))*e^(2*x))*sinh(x)
^7 - 36*a^3*cosh(x)^4 + 84*(221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*
a^3*cosh(x)^8 - 924*a^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 +
a^3 + (221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a
^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(221
*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a^3*cosh(x)
^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 504*(
17*a^3*cosh(x)^13 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)
^7 + 63*a^3*cosh(x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^13
 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(
x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^13 - 78*
```

```
a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(x)^5 -
 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 9*a^3*cosh(x)^2 + 36*
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3 +
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3)*e
^(4*x) - 2*(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 -
1155*a^3*cosh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)
^2 - a^3)*e^(2*x))*sinh(x)^4 + 48*(17*a^3*cosh(x)^15 - 105*a^3*cosh(x)^13 +
 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 - 147*a^3*cosh(
x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (17*a^3*cosh(x)^15 - 105*a^3*cosh
(x)^13 + 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 - 147*a
^3*cosh(x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x
)^15 - 105*a^3*cosh(x)^13 + 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^
3*cosh(x)^7 - 147*a^3*cosh(x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x)
)*sinh(x)^3 - a^3 + 9*(17*a^3*cosh(x)^16 - 120*a^3*cosh(x)^14 + 364*a^3*cos
h(x)^12 - 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 - 392*a^3*cosh(x)^6 + 140*
a^3*cosh(x)^4 - 24*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^16 - 120*a^3*cosh(
x)^14 + 364*a^3*cosh(x)^12 - 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 - 392*a
^3*cosh(x)^6 + 140*a^3*cosh(x)^4 - 24*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(17*
a^3*cosh(x)^16 - 120*a^3*cosh(x)^14 + 364*a^3*cosh(x)^12 - 616*a^3*cosh(x)^
10 + 630*a^3*cosh(x)^8 - 392*a^3*cosh(x)^6 + 140*a^3*cosh(x)^4 - 24*a^3*cos
h(x)^2 + a^3)*e^(2*x))*sinh(x)^2 + (a^3*cosh(x)^18 - 9*a^3*cosh(x)^16 + 36*
a^3*cosh(x)^14 - 84*a^3*cosh(x)^12 + 126*a^3*cosh(x)^10 - 126*a^3*cosh(x)^8
 + 84*a^3*cosh(x)^6 - 36*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2
*(a^3*cosh(x)^18 - 9*a^3*cosh(x)^16 + 36*a^3*cosh(x)^14 - 84*a^3*cosh(x)^12
 + 126*a^3*cosh(x)^10 - 126*a^3*cosh(x)^8 + 84*a^3*cosh(x)^6 - 36*a^3*cosh(
x)^4 + 9*a^3*cosh(x)^2 - a^3)*e^(2*x) + 18*(a^3*cosh(x)^17 - 8*a^3*cosh(x)^
15 + 28*a^3*cosh(x)^13 - 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)^9 - 56*a^3*cosh
(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3*cosh(x)^3 + a^3*cosh(x) + (a^3*cosh(x)^17
- 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 - 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)
^9 - 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3*cosh(x)^3 + a^3*cosh(x))*e
^(4*x) - 2*(a^3*cosh(x)^17 - 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 - 56*a^3*
cosh(x)^11 + 70*a^3*cosh(x)^9 - 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3
*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x))
```

giac [A] time = 0.36, size = 39, normalized size = 0.33

$$-\frac{256\left(126\,e^{(8\,x)}-84\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\,a^{\frac{5}{2}}\left(e^{(2\,x)}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

[Out] -256/315*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(a^(5/2)*(e^(2*x) - 1)^9)

maple [A] time = 0.13, size = 90, normalized size = 0.76

$$-\frac{16\left(8\left(\cosh^{4}(2x)\right)-40\left(\cosh^{3}(2x)\right)+84\left(\cosh^{2}(2x)\right)-100\cosh(2x)+83\right)\sqrt{a\left(\sinh^{2}(2x)\right)}\,\sqrt{a(-1+\cosh(2}{315a^{3}(-1+\cosh(2x))^{4}\sinh(2x)\sqrt{(-1+\cosh(2x))^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*sinh(x)^4)^(5/2),x)

[Out] -16/315/a^3*(8*cosh(2*x)^4-40*cosh(2*x)^3+84*cosh(2*x)^2-100*cosh(2*x)+83)*(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/(-1+cosh(2*x))^4/sinh(2*x)/((-1+cosh(2*x))^2*a)^(1/2)

maxima [B] time = 0.42, size = 467, normalized size = 3.96

$$-\frac{256\,e^{(-2\,x)}}{35\left(9\,a^{\frac{5}{2}}e^{(-2\,x)}-36\,a^{\frac{5}{2}}e^{(-4\,x)}+84\,a^{\frac{5}{2}}e^{(-6\,x)}-126\,a^{\frac{5}{2}}e^{(-8\,x)}+126\,a^{\frac{5}{2}}e^{(-10\,x)}-84\,a^{\frac{5}{2}}e^{(-12\,x)}+36\,a^{\frac{5}{2}}e^{(-14\,x)}-9\,a^{\frac{5}{2}}e^{(-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="maxima")

[Out] -256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2)) + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2)) - 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2))

mupad [B] time = 0.54, size = 256, normalized size = 2.17

$$-\frac{2048\,\mathrm{e}^{4\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}}{5\,a^3\,{\left(\mathrm{e}^{2\,x}-1\right)}^5\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}-\frac{4096\,\mathrm{e}^{4\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}}{3\,a^3\,{\left(\mathrm{e}^{2\,x}-1\right)}^6\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}-\frac{12288\,\mathrm{e}^{4\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}}{7\,a^3\,{\left(\mathrm{e}^{2\,x}-1\right)}^7\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(5/2),x)
```

```
[Out] - (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(5*a^3*(exp(2*x) - 1)^
5*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (4096*exp(4*x)*(a*(exp(-x)/2 - exp(
x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
 - (12288*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x) - 1
)^7*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*exp(4*x)*(a*(exp(-x)/2 - ex
p(x)/2)^4)^(1/2))/(a^3*(exp(2*x) - 1)^8*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
 - (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*x) - 1)
^9*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^4\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(-5/2), x)
```

3.158 $\int \frac{\cosh^8(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=50

$$-\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{1}{6}i\sinh(x)\cosh^5(x)-\frac{5}{24}i\sinh(x)\cosh^3(x)-\frac{5}{16}i\sinh(x)\cosh(x)$$

```
[Out] -5/16*I*x+1/7*cosh(x)^7-5/16*I*cosh(x)*sinh(x)-5/24*I*cosh(x)^3*sinh(x)-1/6
*I*cosh(x)^5*sinh(x)
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{1}{6}i\sinh(x)\cosh^5(x)-\frac{5}{24}i\sinh(x)\cosh^3(x)-\frac{5}{16}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*x + Cosh[x]^7/7 - ((5*I)/16)*Cosh[x]*Sinh[x] - ((5*I)/24)*Cosh[
x]^3*Sinh[x] - (I/6)*Cosh[x]^5*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = \frac{\cosh^7(x)}{7} - i\int \cosh^6(x)\,dx$$
$$= \frac{\cosh^7(x)}{7} - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{6}i\int \cosh^4(x)\,dx$$
$$= \frac{\cosh^7(x)}{7} - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{8}i\int \cosh^2(x)\,dx$$
$$= \frac{\cosh^7(x)}{7} - \frac{5}{16}i\cosh(x)\sinh(x) - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{16}i\int 1\,dx$$
$$= -\frac{5ix}{16} + \frac{\cosh^7(x)}{7} - \frac{5}{16}i\cosh(x)\sinh(x) - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x)$$

Mathematica [B] time = 0.16, size = 219, normalized size = 4.38

$$\frac{\cosh^9(x)\left(48\sqrt{1+i\sinh(x)}\,\sinh^7(x) - 8i\sqrt{1+i\sinh(x)}\,\sinh^6(x) + 200\sqrt{1+i\sinh(x)}\,\sinh^5(x) - 38i\sqrt{1+i\sin\dots}\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]^9*((6*I)*(35*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x
]] + 8*Sqrt[1 + I*Sinh[x]]) + 279*Sqrt[1 + I*Sinh[x]]*Sinh[x] - (87*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^2 + 326*Sqrt[1 + I*Sinh[x]]*Sinh[x]^3 - (38*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^4 + 200*Sqrt[1 + I*Sinh[x]]*Sinh[x]^5 - (8*I)*Sqrt[
1 + I*Sinh[x]]*Sinh[x]^6 + 48*Sqrt[1 + I*Sinh[x]]*Sinh[x]^7))/(336*Sqrt[1 +
 I*Sinh[x]]*(-I + Sinh[x])^4*(I + Sinh[x])^5)
```

fricas [B] time = 0.52, size = 91, normalized size = 1.82

$$\frac{1}{2688}\left(-840i\,xe^{(7\,x)} + 3\,e^{(14\,x)} - 7i\,e^{(13\,x)} + 21\,e^{(12\,x)} - 63i\,e^{(11\,x)} + 63\,e^{(10\,x)} - 315i\,e^{(9\,x)} + 105\,e^{(8\,x)} + 105\,e^{(6\,x)} + 31\dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2688*(-840*I*x*e^(7*x) + 3*e^(14*x) - 7*I*e^(13*x) + 21*e^(12*x) - 63*I*e
^(11*x) + 63*e^(10*x) - 315*I*e^(9*x) + 105*e^(8*x) + 105*e^(6*x) + 315*I*e
^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x) + 7*I*e^x + 3)*e^(-7*x)
```

giac [B] time = 0.18, size = 86, normalized size = 1.72

$$\frac{1}{2688}\left(105\,e^{(6\,x)} + 315i\,e^{(5\,x)} + 63\,e^{(4\,x)} + 63i\,e^{(3\,x)} + 21\,e^{(2\,x)} + 7i\,e^{x} + 3\right)e^{(-7\,x)} - \frac{5}{16}i\,x + \frac{1}{896}\,e^{(7\,x)} - \frac{1}{384}i\,e^{(6\,x)} + \frac{1}{128}\,e\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2688*(105*e^(6*x) + 315*I*e^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x
) + 7*I*e^x + 3)*e^(-7*x) - 5/16*I*x + 1/896*e^(7*x) - 1/384*I*e^(6*x) + 1/
128*e^(5*x) - 3/128*I*e^(4*x) + 3/128*e^(3*x) - 15/128*I*e^(2*x) + 5/128*e^
x
```

maple [B] time = 0.10, size = 292, normalized size = 5.84

$$\frac{5i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{16}-\frac{11i}{16\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{5i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{16}-\frac{11i}{16\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{7\left(\tanh\left(\frac{x}{2}\right)-1\right)^{7}}+\frac{1}{7\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^8/(I+sinh(x)),x)
```

```
[Out] -1/7/(tanh(1/2*x)-1)^7+1/7/(tanh(1/2*x)+1)^7-5/4/(tanh(1/2*x)+1)^4-5/4/(tan
h(1/2*x)-1)^4-1/2/(tanh(1/2*x)-1)^6-1/(tanh(1/2*x)-1)^5+1/(tanh(1/2*x)+1)^5
-1/2/(tanh(1/2*x)+1)^6-9/8/(tanh(1/2*x)-1)^3+9/8/(tanh(1/2*x)+1)^3-5/16/(ta
nh(1/2*x)-1)-11/16/(tanh(1/2*x)-1)^2-11/16/(tanh(1/2*x)+1)^2+5/16/(tanh(1/2
*x)+1)-1/2*I/(tanh(1/2*x)-1)^5-11/16*I/(tanh(1/2*x)-1)-1/6*I/(tanh(1/2*x)-1
)^6-19/16*I/(tanh(1/2*x)-1)^2-11/16*I/(tanh(1/2*x)+1)-7/6*I/(tanh(1/2*x)-1)
^3+19/16*I/(tanh(1/2*x)+1)^2-1/2*I/(tanh(1/2*x)+1)^5+5/16*I*ln(tanh(1/2*x)-
1)-5/16*I*ln(tanh(1/2*x)+1)-7/6*I/(tanh(1/2*x)+1)^3-I/(tanh(1/2*x)-1)^4+I/(
tanh(1/2*x)+1)^4+1/6*I/(tanh(1/2*x)+1)^6
```

maxima [B] time = 0.34, size = 90, normalized size = 1.80

$$-\frac{1}{5376}\left(14i\,e^{(-x)}-42\,e^{(-2\,x)}+126i\,e^{(-3\,x)}-126\,e^{(-4\,x)}+630i\,e^{(-5\,x)}-210\,e^{(-6\,x)}-6\right)e^{(7\,x)}-\frac{5}{16}i\,x+\frac{5}{128}\,e^{(-x)}+\frac{15}{128}i\,e^{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/5376*(14*I*e^(-x) - 42*e^(-2*x) + 126*I*e^(-3*x) - 126*e^(-4*x) + 630*I*
e^(-5*x) - 210*e^(-6*x) - 6)*e^(7*x) - 5/16*I*x + 5/128*e^(-x) + 15/128*I*e
^(-2*x) + 3/128*e^(-3*x) + 3/128*I*e^(-4*x) + 1/128*e^(-5*x) + 1/384*I*e^(-
6*x) + 1/896*e^(-7*x)
```

mupad [B] time = 0.78, size = 93, normalized size = 1.86

$$\frac{5\,\mathrm{e}^{-x}}{128}+\frac{3\,\mathrm{e}^{-3\,x}}{128}+\frac{3\,\mathrm{e}^{3\,x}}{128}+\frac{\mathrm{e}^{-5\,x}}{128}+\frac{\mathrm{e}^{5\,x}}{128}+\frac{\mathrm{e}^{-7\,x}}{896}+\frac{\mathrm{e}^{7\,x}}{896}+\frac{5\,\mathrm{e}^{x}}{128}-\frac{x\,5\mathrm{i}}{16}+\frac{\mathrm{e}^{-2\,x}\,15\mathrm{i}}{128}-\frac{\mathrm{e}^{2\,x}\,15\mathrm{i}}{128}+\frac{\mathrm{e}^{-4\,x}\,3\mathrm{i}}{128}-\frac{\mathrm{e}^{4\,x}\,3\mathrm{i}}{128}+\frac{\mathrm{e}^{-6\,x}\,1\mathrm{i}}{384}-\frac{\mathrm{e}^{6\,x}\,1\mathrm{i}}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^8/(sinh(x) + 1i),x)
```

```
[Out] (5*exp(-x))/128 - (x*5i)/16 + (exp(-2*x)*15i)/128 - (exp(2*x)*15i)/128 + (3
*exp(-3*x))/128 + (3*exp(3*x))/128 + (exp(-4*x)*3i)/128 - (exp(4*x)*3i)/128
 + exp(-5*x)/128 + exp(5*x)/128 + (exp(-6*x)*1i)/384 - (exp(6*x)*1i)/384 +
exp(-7*x)/896 + exp(7*x)/896 + (5*exp(x))/128
```

sympy [B] time = 0.30, size = 124, normalized size = 2.48

$$-\frac{5ix}{16}+\frac{e^{7x}}{896}-\frac{ie^{6x}}{384}+\frac{e^{5x}}{128}-\frac{3ie^{4x}}{128}+\frac{3e^{3x}}{128}-\frac{15ie^{2x}}{128}+\frac{5e^{x}}{128}+\frac{5e^{-x}}{128}+\frac{15ie^{-2x}}{128}+\frac{3e^{-3x}}{128}+\frac{3ie^{-4x}}{128}+\frac{e^{-5x}}{128}+\frac{ie^{-6x}}{384}+\frac{e^{-7x}}{896}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**8/(I+sinh(x)),x)
```

```
[Out] -5*I*x/16 + exp(7*x)/896 - I*exp(6*x)/384 + exp(5*x)/128 - 3*I*exp(4*x)/128
 + 3*exp(3*x)/128 - 15*I*exp(2*x)/128 + 5*exp(x)/128 + 5*exp(-x)/128 + 15*I
*exp(-2*x)/128 + 3*exp(-3*x)/128 + 3*I*exp(-4*x)/128 + exp(-5*x)/128 + I*ex
p(-6*x)/384 + exp(-7*x)/896
```

3.159 $\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{6}(-\sinh(x)+i)^6-\frac{4}{5}i(-\sinh(x)+i)^5-(-\sinh(x)+i)^4$$

```
[Out] -(I-sinh(x))^4-4/5*I*(I-sinh(x))^5+1/6*(I-sinh(x))^6
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{1}{6}(-\sinh(x)+i)^6-\frac{4}{5}i(-\sinh(x)+i)^5-(-\sinh(x)+i)^4$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^7/(I + Sinh[x]),x]
```

```
[Out] -(I - Sinh[x])^4 - ((4*I)/5)*(I - Sinh[x])^5 + (I - Sinh[x])^6/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = -\operatorname{Subst}\left(\int (i-x)^3(i+x)^2\,dx, x, \sinh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-4(i-x)^3-4i(i-x)^4+(i-x)^5\right)\,dx, x, \sinh(x)\right)$$
$$= -(i-\sinh(x))^4-\frac{4}{5}i(i-\sinh(x))^5+\frac{1}{6}(i-\sinh(x))^6$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.98

$$\frac{1}{30}\sinh(x)\left(5\sinh^5(x)-6i\sinh^4(x)+15\sinh^3(x)-20i\sinh^2(x)+15\sinh(x)-30i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^7/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-30*I + 15*Sinh[x] - (20*I)*Sinh[x]^2 + 15*Sinh[x]^3 - (6*I)*Sinh
[x]^4 + 5*Sinh[x]^5))/30
```

fricas [B] time = 0.48, size = 72, normalized size = 1.67

$$\frac{1}{1920}\left(5e^{(12x)}-12ie^{(11x)}+30e^{(10x)}-100ie^{(9x)}+75e^{(8x)}-600ie^{(7x)}+600ie^{(5x)}+75e^{(4x)}+100ie^{(3x)}+30e^{(2x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*e^(12*x) - 12*I*e^(11*x) + 30*e^(10*x) - 100*I*e^(9*x) + 75*e^(8*
x) - 600*I*e^(7*x) + 600*I*e^(5*x) + 75*e^(4*x) + 100*I*e^(3*x) + 30*e^(2*x
) + 12*I*e^x + 5)*e^(-6*x)
```

giac [B] time = 0.20, size = 71, normalized size = 1.65

$$-\frac{1}{1920}\left(-600ie^{(5x)}-75e^{(4x)}-100ie^{(3x)}-30e^{(2x)}-12ie^{x}-5\right)e^{(-6x)}+\frac{1}{384}e^{(6x)}-\frac{1}{160}ie^{(5x)}+\frac{1}{64}e^{(4x)}-\frac{5}{96}ie^{(3x)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/1920*(-600*I*e^(5*x) - 75*e^(4*x) - 100*I*e^(3*x) - 30*e^(2*x) - 12*I*e^
x - 5)*e^(-6*x) + 1/384*e^(6*x) - 1/160*I*e^(5*x) + 1/64*e^(4*x) - 5/96*I*e
^(3*x) + 5/128*e^(2*x) - 5/16*I*e^x
```

maple [B] time = 0.08, size = 142, normalized size = 3.30

$$\frac{\frac{11}{12}+\frac{11i}{12}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{\frac{11}{16}+\frac{7i}{8}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{\frac{7}{8}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{\frac{1}{2}+\frac{i}{5}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}+\frac{\frac{5}{16}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{6}}+\frac{\frac{11}{16}-}{\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(I+sinh(x)),x)
```

```
[Out] (11/12+11/12*I)/(tanh(1/2*x)-1)^3+(11/16+7/8*I)/(tanh(1/2*x)-1)^2+(7/8+1/2*
I)/(tanh(1/2*x)-1)^4+(1/2+1/5*I)/(tanh(1/2*x)-1)^5+(5/16+I)/(tanh(1/2*x)-1)
+1/6/(tanh(1/2*x)-1)^6+(11/16-7/8*I)/(tanh(1/2*x)+1)^2+(7/8-1/2*I)/(tanh(1/
2*x)+1)^4+(-1/2+1/5*I)/(tanh(1/2*x)+1)^5+(-5/16+I)/(tanh(1/2*x)+1)+(-11/12+
11/12*I)/(tanh(1/2*x)+1)^3+1/6/(tanh(1/2*x)+1)^6
```

maxima [B] time = 0.33, size = 75, normalized size = 1.74

$$-\frac{1}{1920}\left(12i\,e^{(-x)}-30\,e^{(-2\,x)}+100i\,e^{(-3\,x)}-75\,e^{(-4\,x)}+600i\,e^{(-5\,x)}-5\right)e^{(6\,x)}+\frac{5}{16}i\,e^{(-x)}+\frac{5}{128}\,e^{(-2\,x)}+\frac{5}{96}i\,e^{(-3\,x)}+\frac{1}{64}\,e^{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*I*e^(-x) - 30*e^(-2*x) + 100*I*e^(-3*x) - 75*e^(-4*x) + 600*I*e
^(-5*x) - 5)*e^(6*x) + 5/16*I*e^(-x) + 5/128*e^(-2*x) + 5/96*I*e^(-3*x) + 1
/64*e^(-4*x) + 1/160*I*e^(-5*x) + 1/384*e^(-6*x)
```

mupad [B] time = 0.67, size = 77, normalized size = 1.79

$$\frac{\mathrm{e}^{-x}\,5\mathrm{i}}{16}+\frac{5\,\mathrm{e}^{-2\,x}}{128}+\frac{5\,\mathrm{e}^{2\,x}}{128}+\frac{\mathrm{e}^{-3\,x}\,5\mathrm{i}}{96}-\frac{\mathrm{e}^{3\,x}\,5\mathrm{i}}{96}+\frac{\mathrm{e}^{-4\,x}}{64}+\frac{\mathrm{e}^{4\,x}}{64}+\frac{\mathrm{e}^{-5\,x}\,1\mathrm{i}}{160}-\frac{\mathrm{e}^{5\,x}\,1\mathrm{i}}{160}+\frac{\mathrm{e}^{-6\,x}}{384}+\frac{\mathrm{e}^{6\,x}}{384}-\frac{\mathrm{e}^{x}\,5\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*5i)/16 + (5*exp(-2*x))/128 + (5*exp(2*x))/128 + (exp(-3*x)*5i)/96
- (exp(3*x)*5i)/96 + exp(-4*x)/64 + exp(4*x)/64 + (exp(-5*x)*1i)/160 - (exp
(5*x)*1i)/160 + exp(-6*x)/384 + exp(6*x)/384 - (exp(x)*5i)/16
```

sympy [B] time = 0.26, size = 100, normalized size = 2.33

$$\frac{e^{6x}}{384}-\frac{ie^{5x}}{160}+\frac{e^{4x}}{64}-\frac{5ie^{3x}}{96}+\frac{5e^{2x}}{128}-\frac{5ie^{x}}{16}+\frac{5ie^{-x}}{16}+\frac{5e^{-2x}}{128}+\frac{5ie^{-3x}}{96}+\frac{e^{-4x}}{64}+\frac{ie^{-5x}}{160}+\frac{e^{-6x}}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**7/(I+sinh(x)),x)
```

```
[Out] exp(6*x)/384 - I*exp(5*x)/160 + exp(4*x)/64 - 5*I*exp(3*x)/96 + 5*exp(2*x)/
128 - 5*I*exp(x)/16 + 5*I*exp(-x)/16 + 5*exp(-2*x)/128 + 5*I*exp(-3*x)/96 +
 exp(-4*x)/64 + I*exp(-5*x)/160 + exp(-6*x)/384
```

3.160 $\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{1}{4}i\sinh(x)\cosh^3(x)-\frac{3}{8}i\sinh(x)\cosh(x)$$

```
[Out] -3/8*I*x+1/5*cosh(x)^5-3/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{1}{4}i\sinh(x)\cosh^3(x)-\frac{3}{8}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/8)*x + Cosh[x]^5/5 - ((3*I)/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Si
nh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx &= \frac{\cosh^5(x)}{5} - i\int \cosh^4(x)\,dx \\ &= \frac{\cosh^5(x)}{5} - \frac{1}{4}i\cosh^3(x)\sinh(x) - \frac{3}{4}i\int \cosh^2(x)\,dx \\ &= \frac{\cosh^5(x)}{5} - \frac{3}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x) - \frac{3}{8}i\int 1\,dx \\ &= -\frac{3ix}{8} + \frac{\cosh^5(x)}{5} - \frac{3}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x)\end{aligned}$$

Mathematica [B] time = 0.24, size = 131, normalized size = 3.45

$$-\frac{i\cosh^7(x)\left(8\sinh^5(x) - 2i\sinh^4(x) + 26\sinh^3(x) - 9i\sinh^2(x) + 33\sinh(x) + \frac{30i\sqrt{1-i\sinh(x)}\,\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}} + \right.}{40\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x]),x]
```

```
[Out] ((-1/40*I)*Cosh[x]^7*(8*I + ((30*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqr
t[1 - I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] + 33*Sinh[x] - (9*I)*Sinh[x]^2 + 26*S
inh[x]^3 - (2*I)*Sinh[x]^4 + 8*Sinh[x]^5))/((Cosh[x/2] - I*Sinh[x/2])^8*(Co
sh[x/2] + I*Sinh[x/2])^6)
```

fricas [B] time = 0.68, size = 67, normalized size = 1.76

$$\frac{1}{320}\left(-120i\,xe^{(5\,x)} + 2\,e^{(10\,x)} - 5i\,e^{(9\,x)} + 10\,e^{(8\,x)} - 40i\,e^{(7\,x)} + 20\,e^{(6\,x)} + 20\,e^{(4\,x)} + 40i\,e^{(3\,x)} + 10\,e^{(2\,x)} + 5i\,e^{x} + 2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/320*(-120*I*x*e^(5*x) + 2*e^(10*x) - 5*I*e^(9*x) + 10*e^(8*x) - 40*I*e^(7
*x) + 20*e^(6*x) + 20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^
(-5*x)
```

giac [B] time = 0.23, size = 62, normalized size = 1.63

$$\frac{1}{320}\left(20\,e^{(4\,x)} + 40i\,e^{(3\,x)} + 10\,e^{(2\,x)} + 5i\,e^{x} + 2\right)e^{(-5\,x)} - \frac{3}{8}i\,x + \frac{1}{160}\,e^{(5\,x)} - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{32}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{16}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/320*(20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^(-5*x) - 3/8
*I*x + 1/160*e^(5*x) - 1/64*I*e^(4*x) + 1/32*e^(3*x) - 1/8*I*e^(2*x) + 1/16
*e^x
```

maple [B] time = 0.06, size = 210, normalized size = 5.53

$$-\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}-\frac{5}{8\left(\tanh\left(\frac{x}{2}\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(I+sinh(x)),x)
```

```
[Out] -1/4*I/(tanh(1/2*x)-1)^4-1/2/(tanh(1/2*x)-1)^4-1/2*I/(tanh(1/2*x)+1)^3-3/8/
(tanh(1/2*x)-1)-3/8*I*ln(tanh(1/2*x)+1)-5/8/(tanh(1/2*x)-1)^2-7/8*I/(tanh(1
/2*x)-1)^2-3/4/(tanh(1/2*x)-1)^3-5/8*I/(tanh(1/2*x)+1)-1/5/(tanh(1/2*x)-1)^
5+7/8*I/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*x)+1)^4+3/8*I*ln(tanh(1/2*x)-1)+3/4
/(tanh(1/2*x)+1)^3+1/4*I/(tanh(1/2*x)+1)^4+3/8/(tanh(1/2*x)+1)-1/2*I/(tanh(
1/2*x)-1)^3-5/8/(tanh(1/2*x)+1)^2-5/8*I/(tanh(1/2*x)-1)+1/5/(tanh(1/2*x)+1)
^5
```

maxima [B] time = 0.32, size = 66, normalized size = 1.74

$$-\frac{1}{320}\left(5i\,e^{(-x)}-10\,e^{(-2\,x)}+40i\,e^{(-3\,x)}-20\,e^{(-4\,x)}-2\right)e^{(5\,x)}-\frac{3}{8}i\,x+\frac{1}{16}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{32}\,e^{(-3\,x)}+\frac{1}{64}i\,e^{(-4\,x)}+\frac{1}{160}\,e^{(-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/320*(5*I*e^(-x) - 10*e^(-2*x) + 40*I*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)
 - 3/8*I*x + 1/16*e^(-x) + 1/8*I*e^(-2*x) + 1/32*e^(-3*x) + 1/64*I*e^(-4*x)
 + 1/160*e^(-5*x)
```

mupad [B] time = 0.62, size = 67, normalized size = 1.76

$$\frac{\mathrm{e}^{-x}}{16}+\frac{\mathrm{e}^{-3\,x}}{32}+\frac{\mathrm{e}^{3\,x}}{32}+\frac{\mathrm{e}^{-5\,x}}{160}+\frac{\mathrm{e}^{5\,x}}{160}+\frac{\mathrm{e}^{x}}{16}-\frac{x\,3\mathrm{i}}{8}+\frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8}-\frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{-4\,x}\,1\mathrm{i}}{64}-\frac{\mathrm{e}^{4\,x}\,1\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(sinh(x) + 1i),x)
```

```
[Out] exp(-x)/16 - (x*3i)/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/32 +
 exp(3*x)/32 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 + exp(-5*x)/160 + exp(5
*x)/160 + exp(x)/16
```

sympy [B] time = 0.23, size = 82, normalized size = 2.16

$$-\frac{3ix}{8} + \frac{e^{5x}}{160} - \frac{ie^{4x}}{64} + \frac{e^{3x}}{32} - \frac{ie^{2x}}{8} + \frac{e^{x}}{16} + \frac{e^{-x}}{16} + \frac{ie^{-2x}}{8} + \frac{e^{-3x}}{32} + \frac{ie^{-4x}}{64} + \frac{e^{-5x}}{160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(I+sinh(x)),x)
```

```
[Out] -3*I*x/8 + exp(5*x)/160 - I*exp(4*x)/64 + exp(3*x)/32 - I*exp(2*x)/8 + exp(
x)/16 + exp(-x)/16 + I*exp(-2*x)/8 + exp(-3*x)/32 + I*exp(-4*x)/64 + exp(-5
*x)/160
```

3.161 $\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\sinh^4(x)}{4}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^2(x)}{2}-i\sinh(x)$$

```
[Out] -I*sinh(x)+1/2*sinh(x)^2-1/3*I*sinh(x)^3+1/4*sinh(x)^4
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\frac{\sinh^4(x)}{4}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^2(x)}{2}-i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(I + Sinh[x]),x]
```

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2 - (I/3)*Sinh[x]^3 + Sinh[x]^4/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \text{Subst}\left(\int (i-x)^2(i+x)\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-i+x-ix^2+x^3\right)\,dx, x, \sinh(x)\right)$$
$$= -i\sinh(x)+\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^4(x)}{4}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.85

$$\frac{1}{12}\sinh(x)\left(3\sinh^3(x)-4i\sinh^2(x)+6\sinh(x)-12i\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-12*I + 6*Sinh[x] - (4*I)*Sinh[x]^2 + 3*Sinh[x]^3))/12
```

fricas [B] time = 0.52, size = 48, normalized size = 1.45

$$\frac{1}{192}\left(3\,e^{(8\,x)}-8i\,e^{(7\,x)}+12\,e^{(6\,x)}-72i\,e^{(5\,x)}+72i\,e^{(3\,x)}+12\,e^{(2\,x)}+8i\,e^{x}+3\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*e^(8*x) - 8*I*e^(7*x) + 12*e^(6*x) - 72*I*e^(5*x) + 72*I*e^(3*x) +
 12*e^(2*x) + 8*I*e^x + 3)*e^(-4*x)
```

giac [B] time = 0.36, size = 47, normalized size = 1.42

$$-\frac{1}{192}\left(-72i\,e^{(3\,x)}-12\,e^{(2\,x)}-8i\,e^{x}-3\right)e^{(-4\,x)}+\frac{1}{64}\,e^{(4\,x)}-\frac{1}{24}i\,e^{(3\,x)}+\frac{1}{16}\,e^{(2\,x)}-\frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/192*(-72*I*e^(3*x) - 12*e^(2*x) - 8*I*e^x - 3)*e^(-4*x) + 1/64*e^(4*x) -
 1/24*I*e^(3*x) + 1/16*e^(2*x) - 3/8*I*e^x
```

maple [B] time = 0.06, size = 94, normalized size = 2.85

$$\frac{\frac{5}{8}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{\frac{1}{2}+\frac{i}{3}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{\frac{3}{8}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{\frac{5}{8}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{-\frac{1}{2}+\frac{i}{3}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{-\frac{3}{8}\cdots}{\tanh\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(I+sinh(x)),x)
```

```
[Out] (5/8+1/2*I)/(tanh(1/2*x)-1)^2+(1/2+1/3*I)/(tanh(1/2*x)-1)^3+(3/8+I)/(tanh(1
/2*x)-1)+1/4/(tanh(1/2*x)-1)^4+(5/8-1/2*I)/(tanh(1/2*x)+1)^2+(-1/2+1/3*I)/(
tanh(1/2*x)+1)^3+(-3/8+I)/(tanh(1/2*x)+1)+1/4/(tanh(1/2*x)+1)^4
```

maxima [B] time = 0.31, size = 51, normalized size = 1.55

$$-\frac{1}{192}\left(8i\,e^{(-x)}-12\,e^{(-2\,x)}+72i\,e^{(-3\,x)}-3\right)e^{(4\,x)}+\frac{3}{8}i\,e^{(-x)}+\frac{1}{16}\,e^{(-2\,x)}+\frac{1}{24}i\,e^{(-3\,x)}+\frac{1}{64}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*I*e^(-x) - 12*e^(-2*x) + 72*I*e^(-3*x) - 3)*e^(4*x) + 3/8*I*e^(-x
) + 1/16*e^(-2*x) + 1/24*I*e^(-3*x) + 1/64*e^(-4*x)
```

mupad [B] time = 0.56, size = 51, normalized size = 1.55

$$\frac{\mathrm{e}^{-x}\,3\mathrm{i}}{8}+\frac{\mathrm{e}^{-2\,x}}{16}+\frac{\mathrm{e}^{2\,x}}{16}+\frac{\mathrm{e}^{-3\,x}\,1\mathrm{i}}{24}-\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{24}+\frac{\mathrm{e}^{-4\,x}}{64}+\frac{\mathrm{e}^{4\,x}}{64}-\frac{\mathrm{e}^{x}\,3\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*3i)/8 + exp(-2*x)/16 + exp(2*x)/16 + (exp(-3*x)*1i)/24 - (exp(3*x)
*1i)/24 + exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*3i)/8
```

sympy [B] time = 0.20, size = 63, normalized size = 1.91

$$\frac{e^{4x}}{64}-\frac{ie^{3x}}{24}+\frac{e^{2x}}{16}-\frac{3ie^{x}}{8}+\frac{3ie^{-x}}{8}+\frac{e^{-2x}}{16}+\frac{ie^{-3x}}{24}+\frac{e^{-4x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(I+sinh(x)),x)
```

```
[Out] exp(4*x)/64 - I*exp(3*x)/24 + exp(2*x)/16 - 3*I*exp(x)/8 + 3*I*exp(-x)/8 +
exp(-2*x)/16 + I*exp(-3*x)/24 + exp(-4*x)/64
```

3.162 $\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\sinh(x)\cosh(x)$$

```
[Out] -1/2*I*x+1/3*cosh(x)^3-1/2*I*cosh(x)*sinh(x)
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2682, 2635, 8}

$$-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (-I/2)*x + Cosh[x]^3/3 - (I/2)*Cosh[x]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = \frac{\cosh^3(x)}{3} - i\int \cosh^2(x)\,dx$$
$$= \frac{\cosh^3(x)}{3} - \frac{1}{2}i\cosh(x)\sinh(x) - \frac{1}{2}i\int 1\,dx$$
$$= -\frac{ix}{2} + \frac{\cosh^3(x)}{3} - \frac{1}{2}i\cosh(x)\sinh(x)$$

Mathematica [B] time = 0.16, size = 93, normalized size = 3.58

$$\frac{\cosh^5(x)\left(2\sinh^3(x) - i\sinh^2(x) + 5\sinh(x) + \frac{6i\sqrt{1-i\sinh(x)}\,\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}} + 2i\right)}{6(\sinh(x)-i)^2(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]^5*(2*I + ((6*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sin
h[x]])/Sqrt[1 + I*Sinh[x]] + 5*Sinh[x] - I*Sinh[x]^2 + 2*Sinh[x]^3))/(6*(-I
 + Sinh[x])^2*(I + Sinh[x])^3)
```

fricas [B] time = 0.57, size = 41, normalized size = 1.58

$$\frac{1}{24}\left(-12i\,xe^{(3\,x)} + e^{(6\,x)} - 3i\,e^{(5\,x)} + 3\,e^{(4\,x)} + 3\,e^{(2\,x)} + 3i\,e^{x} + 1\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-12*I*x*e^(3*x) + e^(6*x) - 3*I*e^(5*x) + 3*e^(4*x) + 3*e^(2*x) + 3*I
*e^x + 1)*e^(-3*x)
```

giac [B] time = 0.44, size = 38, normalized size = 1.46

$$\frac{1}{24}\left(3\,e^{(2\,x)} + 3i\,e^{x} + 1\right)e^{(-3\,x)} - \frac{1}{2}i\,x + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) + 3*I*e^x + 1)*e^(-3*x) - 1/2*I*x + 1/24*e^(3*x) - 1/8*I*e^
(2*x) + 1/8*e^x
```

maple [B] time = 0.06, size = 126, normalized size = 4.85

$$\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+sinh(x)),x)
```

```
[Out] 1/2*I*ln(tanh(1/2*x)-1)-1/2/(tanh(1/2*x)-1)^2-1/2*I/(tanh(1/2*x)-1)^2-1/2/(
tanh(1/2*x)-1)-1/2*I/(tanh(1/2*x)-1)-1/3/(tanh(1/2*x)-1)^3-1/2*I*ln(tanh(1/
2*x)+1)+1/2/(tanh(1/2*x)+1)-1/2*I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2+1/2
*I/(tanh(1/2*x)+1)^2+1/3/(tanh(1/2*x)+1)^3
```

maxima [B] time = 0.31, size = 42, normalized size = 1.62

$$-\frac{1}{48}\left(6i\,e^{(-x)}-6\,e^{(-2\,x)}-2\right)e^{(3\,x)}-\frac{1}{2}i\,x+\frac{1}{8}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/48*(6*I*e^(-x) - 6*e^(-2*x) - 2)*e^(3*x) - 1/2*I*x + 1/8*e^(-x) + 1/8*I*
e^(-2*x) + 1/24*e^(-3*x)
```

mupad [B] time = 0.12, size = 41, normalized size = 1.58

$$\frac{\mathrm{e}^{-x}}{8}+\frac{\mathrm{e}^{-3\,x}}{24}+\frac{\mathrm{e}^{3\,x}}{24}+\frac{\mathrm{e}^{x}}{8}-\frac{x\,1\mathrm{i}}{2}+\frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8}-\frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] exp(-x)/8 - (x*1i)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/24 +
exp(3*x)/24 + exp(x)/8
```

sympy [B] time = 0.18, size = 48, normalized size = 1.85

$$-\frac{ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}+\frac{e^{x}}{8}+\frac{e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+sinh(x)),x)
```

```
[Out] -I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 + exp(x)/8 + exp(-x)/8 + I*exp(-2*x)/8
+ exp(-3*x)/24
```

3.163 $\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^2(x)}{2} - i\sinh(x)$$

[Out] -I*sinh(x)+1/2*sinh(x)^2

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2667}

$$\frac{\sinh^2(x)}{2} - i\sinh(x)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^3/(I + Sinh[x]),x]

[Out] (-I)*Sinh[x] + Sinh[x]^2/2

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx &= -\text{Subst}\left(\int (i-x)\,dx, x, \sinh(x)\right)\\ &= -i\sinh(x) + \frac{\sinh^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.80

$$\frac{1}{2}\sinh(x)(\sinh(x) - 2i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x]),x]
[Out] (Sinh[x]*(-2*I + Sinh[x]))/2
```

fricas [A] time = 0.78, size = 22, normalized size = 1.47

$$\frac{1}{8}\left(e^{(4x)} - 4i\,e^{(3x)} + 4i\,e^{x} + 1\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/8*(e^(4*x) - 4*I*e^(3*x) + 4*I*e^x + 1)*e^(-2*x)
```

giac [B] time = 0.19, size = 23, normalized size = 1.53

$$-\frac{1}{8}\left(-4i\,e^{x} - 1\right)e^{(-2x)} + \frac{1}{8}\,e^{(2x)} - \frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="giac")
[Out] -1/8*(-4*I*e^x - 1)*e^(-2*x) + 1/8*e^(2*x) - 1/2*I*e^x
```

maple [A] time = 0.03, size = 13, normalized size = 0.87

$$-i\sinh(x) + \frac{\left(\sinh^{2}(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+sinh(x)),x)
[Out] -I*sinh(x)+1/2*sinh(x)^2
```

maxima [B] time = 0.31, size = 27, normalized size = 1.80

$$\frac{1}{8}\left(-4i\,e^{(-x)} + 1\right)e^{(2x)} + \frac{1}{2}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="maxima")
[Out] 1/8*(-4*I*e^(-x) + 1)*e^(2*x) + 1/2*I*e^(-x) + 1/8*e^(-2*x)
```

mupad [B] time = 0.47, size = 31, normalized size = 2.07

$$\frac{e^{-2x}\left(e^{4x}+1\right)}{8}-\frac{e^{-2x}\left(4\,e^{3x}-4\,e^{x}\right)1i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] (exp(-2*x)*(exp(4*x) + 1))/8 - (exp(-2*x)*(4*exp(3*x) - 4*exp(x))*1i)/8
```

sympy [B] time = 0.15, size = 27, normalized size = 1.80

$$\frac{e^{2x}}{8}-\frac{ie^{x}}{2}+\frac{ie^{-x}}{2}+\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+sinh(x)),x)
```

```
[Out] exp(2*x)/8 - I*exp(x)/2 + I*exp(-x)/2 + exp(-2*x)/8
```

3.164 $\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=8

$$\cosh(x) - ix$$

```
[Out] -I*x+cosh(x)
```

Rubi [A] time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2682, 8}

$$\cosh(x) - ix$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I)*x + Cosh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx &= \cosh(x) - i\int 1\,dx \\ &= -ix + \cosh(x) \end{aligned}$$

Mathematica [B] time = 0.05, size = 34, normalized size = 4.25

$$\cosh(x) + 2\sqrt{\cosh^2(x)}\,\text{sech}(x)\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Sinh[x]),x]
[Out] Cosh[x] + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x]
```

fricas [B] time = 0.61, size = 17, normalized size = 2.12

$$\frac{1}{2}\left(-2i\,xe^{x}+e^{(2\,x)}+1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/2*(-2*I*x*e^x + e^(2*x) + 1)*e^(-x)
```

giac [B] time = 0.15, size = 14, normalized size = 1.75

$$-i\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

maple [B] time = 0.05, size = 40, normalized size = 5.00

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+sinh(x)),x)
[Out] I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)
```

maxima [B] time = 0.31, size = 14, normalized size = 1.75

$$-i\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="maxima")
[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

mupad [B] time = 0.46, size = 7, normalized size = 0.88

$$\cosh(x)-x\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(sinh(x) + 1i),x)
```

```
[Out] cosh(x) - x*1i
```

sympy [B] time = 0.12, size = 14, normalized size = 1.75

$$-ix + \frac{e^{x}}{2} + \frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+sinh(x)),x)
```

```
[Out] -I*x + exp(x)/2 + exp(-x)/2
```

3.165 $\int \frac{\cosh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=7

$$\log(\sinh(x)+i)$$

[Out] ln(I+sinh(x))

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 31}

$$\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(I + Sinh[x]),x]

[Out] Log[I + Sinh[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{i+\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right)\\ &= \log(i+\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Sinh[x]),x]
[Out] Log[I + Sinh[x]]
```

fricas [B] time = 0.50, size = 11, normalized size = 1.57

$$-x + 2 \log(e^x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] -x + 2*log(e^x + I)
```

giac [B] time = 0.30, size = 11, normalized size = 1.57

$$-x + 2 \log(e^x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="giac")
[Out] -x + 2*log(e^x + I)
```

maple [A] time = 0.02, size = 7, normalized size = 1.00

$$\ln(i + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+sinh(x)),x)
[Out] ln(I+sinh(x))
```

maxima [A] time = 0.32, size = 5, normalized size = 0.71

$$\log(\sinh(x) + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] log(sinh(x) + I)
```

mupad [B] time = 0.47, size = 10, normalized size = 1.43

$$\ln(\cosh(x)) - \mathrm{atan}(\sinh(x))\, 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(sinh(x) + 1i),x)
```

```
[Out] log(cosh(x)) - atan(sinh(x))*1i
```

sympy [B] time = 0.13, size = 15, normalized size = 2.14

$$x(1+2i)-2i\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x)
```

```
[Out] x*(1 + 2*I) - 2*I*log(exp(x) + I)
```

3.166 $\int \frac{\text{sech}(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{i}{2(\sinh(x)+i)}-\frac{1}{2}i\tan^{-1}(\sinh(x))$$

```
[Out] -1/2*I*arctan(sinh(x))-1/2*I/(I+sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2667, 44, 203}

$$-\frac{i}{2(\sinh(x)+i)}-\frac{1}{2}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Sinh[x]),x]
```

```
[Out] (-I/2)*ArcTan[Sinh[x]] - (I/2)/(I + Sinh[x])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{(i-x)(i+x)^2}\,dx, x, \sinh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{i}{2(i+x)^2}+\frac{i}{2\left(1+x^2\right)}\right)dx, x, \sinh(x)\right)$$
$$= -\frac{i}{2(i+\sinh(x))}-\frac{1}{2}i\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{1}{2}i\tan^{-1}(\sinh(x))-\frac{i}{2(i+\sinh(x))}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.75

$$-\frac{1}{2}i\left(\tan^{-1}(\sinh(x))+\frac{1}{\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Sinh[x]),x]
[Out] (-1/2*I)*(ArcTan[Sinh[x]] + (I + Sinh[x])^(-1))
```

fricas [B] time = 0.59, size = 53, normalized size = 2.21

$$\frac{\left(e^{(2x)}+2ie^{x}-1\right)\log\left(e^{x}+i\right)-\left(e^{(2x)}+2ie^{x}-1\right)\log\left(e^{x}-i\right)-2ie^{x}}{2e^{(2x)}+4ie^{x}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] ((e^(2*x) + 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) + 2*I*e^x - 1)*log(e^x - I
) - 2*I*e^x)/(2*e^(2*x) + 4*I*e^x - 2)
```

giac [B] time = 0.49, size = 51, normalized size = 2.12

$$-\frac{e^{(-x)}-e^{x}-6i}{4\left(e^{(-x)}-e^{x}-2i\right)}+\frac{1}{4}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{4}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/4*(e^(-x) - e^x - 6*I)/(e^(-x) - e^x - 2*I) + 1/4*log(-e^(-x) + e^x + 2*
I) - 1/4*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.05, size = 43, normalized size = 1.79

$$\frac{i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+sinh(x)),x)
```

```
[Out] I/(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^2+1/2*ln(tanh(1/2*x)+I)-1/2*ln(tanh(1/2
*x)-I)
```

maxima [B] time = 0.31, size = 41, normalized size = 1.71

$$\frac{2i\,e^{(-x)}}{-4i\,e^{(-x)}+2\,e^{(-2\,x)}-2}-\frac{1}{2}\,\log\left(e^{(-x)}+i\right)+\frac{1}{2}\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*I*e^(-x)/(-4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

mupad [B] time = 0.20, size = 46, normalized size = 1.92

$$\frac{\ln\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}{2}-\frac{1}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{1\mathrm{i}}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 - 1/(exp(2*x) + exp(x)*2i - 1)
- 1i/(exp(x) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)/(sinh(x) + I), x)
```

3.167 $\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{2}{3}i\tanh(x)-\frac{i\operatorname{sech}(x)}{3(\sinh(x)+i)}$$

```
[Out] -1/3*I*sech(x)/(I+sinh(x))-2/3*I*tanh(x)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 3767, 8}

$$-\frac{2}{3}i\tanh(x)-\frac{i\operatorname{sech}(x)}{3(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Sinh[x]),x]
```

```
[Out] ((-I/3)*Sech[x])/(I + Sinh[x]) - ((2*I)/3)*Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))}-\frac{2}{3}i\int \operatorname{sech}^2(x)\,dx\\ &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))}+\frac{2}{3}\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)\\ &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))}-\frac{2}{3}i\tanh(x)\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 22, normalized size = 0.88

$$-\frac{1}{3}i\left(2\tanh(x)+\frac{\operatorname{sech}(x)}{\sinh(x)+i}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Sinh[x]),x]
[Out] (-1/3*I)*(Sech[x]/(I + Sinh[x]) + 2*Tanh[x])
```

fricas [**A**] time = 0.40, size = 28, normalized size = 1.12

$$-\frac{8\,e^{x}+4i}{3\,e^{(4\,x)}+6i\,e^{(3\,x)}+6i\,e^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] -(8*e^x + 4*I)/(3*e^(4*x) + 6*I*e^(3*x) + 6*I*e^x - 3)
```

giac [**A**] time = 0.14, size = 29, normalized size = 1.16

$$\frac{1}{2\,(e^{x}-i)}-\frac{3\,e^{(2\,x)}+12i\,e^{x}-5}{6\,(e^{x}+i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] 1/2/(e^x - I) - 1/6*(3*e^(2*x) + 12*I*e^x - 5)/(e^x + I)^3
```

maple [**B**] time = 0.05, size = 49, normalized size = 1.96

$$-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{3i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+sinh(x)),x)
```

```
[Out] -1/(tanh(1/2*x)+I)^2+2/3*I/(tanh(1/2*x)+I)^3-3/2*I/(tanh(1/2*x)+I)-1/2*I/(t
anh(1/2*x)-I)
```

maxima [B] time = 0.32, size = 53, normalized size = 2.12

$$-\frac{8\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}+\frac{4i}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -8*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 4*I/(-6*I*e^(-x)
- 6*I*e^(-3*x) + 3*e^(-4*x) - 3)
```

mupad [B] time = 0.58, size = 63, normalized size = 2.52

$$-\frac{8\,\mathrm{e}^x}{3\left(\mathrm{e}^{2\,x}+1\right)^3}-\frac{8\,\mathrm{e}^x\,\left(\mathrm{e}^{2\,x}-1\right)}{3\left(\mathrm{e}^{2\,x}+1\right)^3}+\frac{\mathrm{e}^{2\,x}\,16\mathrm{i}}{3\left(\mathrm{e}^{2\,x}+1\right)^3}-\frac{\left(\mathrm{e}^{2\,x}-1\right)\,4\mathrm{i}}{3\left(\mathrm{e}^{2\,x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(sinh(x) + 1i)),x)
```

```
[Out] (exp(2*x)*16i)/(3*(exp(2*x) + 1)^3) - (8*exp(x))/(3*(exp(2*x) + 1)^3) - ((e
xp(2*x) - 1)*4i)/(3*(exp(2*x) + 1)^3) - (8*exp(x)*(exp(2*x) - 1))/(3*(exp(2
*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**2/(sinh(x) + I), x)
```

3.168 $\int \frac{\text{sech}^3(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=52

$$\frac{i}{8(-\sinh(x)+i)} - \frac{i}{4(\sinh(x)+i)} + \frac{1}{8(\sinh(x)+i)^2} - \frac{3}{8} i \tan^{-1}(\sinh(x))$$

```
[Out] -3/8*I*arctan(sinh(x))+1/8*I/(I-sinh(x))+1/8/(I+sinh(x))^2-1/4*I/(I+sinh(x)
)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 203}

$$\frac{i}{8(-\sinh(x)+i)} - \frac{i}{4(\sinh(x)+i)} + \frac{1}{8(\sinh(x)+i)^2} - \frac{3}{8} i \tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/8)*ArcTan[Sinh[x]] + (I/8)/(I - Sinh[x]) + 1/(8*(I + Sinh[x])^2) -
(I/4)/(I + Sinh[x])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \operatorname{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^3}\,dx, x, \sinh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(\frac{i}{8(-i+x)^2} - \frac{1}{4(i+x)^3} + \frac{i}{4(i+x)^2} - \frac{3i}{8\left(1+x^2\right)}\right) dx, x, \sinh(x)\right)$$
$$= \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))} - \frac{3}{8} i \operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{3}{8} i \tan^{-1}(\sinh(x)) + \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.17

$$-\frac{i \operatorname{sech}^2(x)\left(3\sinh^3(x)\tan^{-1}(\sinh(x)) + \sinh^2(x)\left(3+3i\tan^{-1}(\sinh(x))\right) + 3\sinh(x)\left(\tan^{-1}(\sinh(x)) + i\right) + 3i\right.}{8(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-1/8*I)*Sech[x]^2*(2 + (3*I)*ArcTan[Sinh[x]] + 3*(I + ArcTan[Sinh[x]])*Si
nh[x] + (3 + (3*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 3*ArcTan[Sinh[x]]*Sinh[x]^3
))/(I + Sinh[x])
```

fricas [B] time = 0.51, size = 153, normalized size = 2.94

$$\frac{\left(3 e^{(6x)} + 6i e^{(5x)} + 3 e^{(4x)} + 12i e^{(3x)} - 3 e^{(2x)} + 6i e^{x} - 3\right)\log\left(e^{x} + i\right) - \left(3 e^{(6x)} + 6i e^{(5x)} + 3 e^{(4x)} + 12i e^{(3x)} - 3\right.}{8 e^{(6x)} + 16i e^{(5x)} + 8 e^{(4x)} + 32i e^{(3x)} - 8 e^{(2x)} +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((3*e^(6*x) + 6*I*e^(5*x) + 3*e^(4*x) + 12*I*e^(3*x) - 3*e^(2*x) + 6*I*e^x
- 3)*log(e^x + I) - (3*e^(6*x) + 6*I*e^(5*x) + 3*e^(4*x) + 12*I*e^(3*x) - 3
*e^(2*x) + 6*I*e^x - 3)*log(e^x - I) - 6*I*e^(5*x) + 12*e^(4*x) - 4*I*e^(3*
x) - 12*e^(2*x) - 6*I*e^x)/(8*e^(6*x) + 16*I*e^(5*x) + 8*e^(4*x) + 32*I*e^(
3*x) - 8*e^(2*x) + 16*I*e^x - 8)
```

giac [B] time = 0.45, size = 92, normalized size = 1.77

$$\frac{3 e^{(-x)} - 3 e^{x} + 10i}{16\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{9\left(e^{(-x)} - e^{x}\right)^2 - 52i e^{(-x)} + 52i e^{x} - 84}{32\left(e^{(-x)} - e^{x} - 2i\right)^2} + \frac{3}{16}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{3}{16}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*e^(-x) - 3*e^x + 10*I)/(e^(-x) - e^x + 2*I) - 1/32*(9*(e^(-x) - e^x
)^2 - 52*I*e^(-x) + 52*I*e^x - 84)/(e^(-x) - e^x - 2*I)^2 + 3/16*log(-e^(-x
) + e^x + 2*I) - 3/16*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.06, size = 91, normalized size = 1.75

$$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8}+\frac{i}{4\tanh\left(\frac{x}{2}\right)-4i}-\frac{}{4\left(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+sinh(x)),x)
```

```
[Out] -1/2/(tanh(1/2*x)+I)^4+I/(tanh(1/2*x)+I)-I/(tanh(1/2*x)+I)^3+3/2/(tanh(1/2*
x)+I)^2+3/8*ln(tanh(1/2*x)+I)+1/4*I/(tanh(1/2*x)-I)-1/4/(tanh(1/2*x)-I)^2-3
/8*ln(tanh(1/2*x)-I)
```

maxima [B] time = 0.32, size = 92, normalized size = 1.77

$$\frac{8\left(3i\,e^{(-x)}-6\,e^{(-2\,x)}+2i\,e^{(-3\,x)}+6\,e^{(-4\,x)}+3i\,e^{(-5\,x)}\right)}{-64i\,e^{(-x)}-32\,e^{(-2\,x)}-128i\,e^{(-3\,x)}+32\,e^{(-4\,x)}-64i\,e^{(-5\,x)}+32\,e^{(-6\,x)}-32}-\frac{3}{8}\log\left(e^{(-x)}+i\right)+\frac{3}{8}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 8*(3*I*e^(-x) - 6*e^(-2*x) + 2*I*e^(-3*x) + 6*e^(-4*x) + 3*I*e^(-5*x))/(-64
*I*e^(-x) - 32*e^(-2*x) - 128*I*e^(-3*x) + 32*e^(-4*x) - 64*I*e^(-5*x) + 32
*e^(-6*x) - 32) - 3/8*log(e^(-x) + I) + 3/8*log(e^(-x) - I)
```

mupad [B] time = 0.90, size = 115, normalized size = 2.21

$$\frac{3\,\ln\left(-\frac{3}{4}+\frac{\mathrm{e}^{x}\,3\mathrm{i}}{4}\right)}{8}-\frac{3\,\ln\left(\frac{3}{4}+\frac{\mathrm{e}^{x}\,3\mathrm{i}}{4}\right)}{8}-\frac{1}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}\right)}-\frac{1}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)}-\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{x}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(sinh(x) + 1i)),x)
```

```
[Out] (3*log((exp(x)*3i)/4 - 3/4))/8 - (3*log((exp(x)*3i)/4 + 3/4))/8 - 1/(2*(exp
(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - 1/(4*(exp(x)*2i - exp(
```

```
2*x) + 1)) - 1i/(4*(exp(x) - 1i)) - 1i/(2*(exp(x) + 1i)) - 1i/(exp(2*x)*3i
+ exp(3*x) - 3*exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{\sinh(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**3/(sinh(x) + I), x)
```

3.169 $\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=37

$$\frac{4}{15}i\tanh^3(x)-\frac{4}{5}i\tanh(x)-\frac{i\operatorname{sech}^3(x)}{5(\sinh(x)+i)}$$

```
[Out] -1/5*I*sech(x)^3/(I+sinh(x))-4/5*I*tanh(x)+4/15*I*tanh(x)^3
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2672, 3767}

$$\frac{4}{15}i\tanh^3(x)-\frac{4}{5}i\tanh(x)-\frac{i\operatorname{sech}^3(x)}{5(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((-I/5)*Sech[x]^3)/(I + Sinh[x]) - ((4*I)/5)*Tanh[x] + ((4*I)/15)*Tanh[x]^3
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} - \frac{4}{5}i\int \operatorname{sech}^4(x)\,dx$$
$$= -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} + \frac{4}{5}\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(x)\right)$$
$$= -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} - \frac{4}{5}i\tanh(x) + \frac{4}{15}i\tanh^3(x)$$

Mathematica [A] time = 0.07, size = 35, normalized size = 0.95

$$-\frac{1}{15}i\left(8\tanh^3(x) + \frac{3\operatorname{sech}^3(x)}{\sinh(x)+i} + 12\tanh(x)\operatorname{sech}^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Sinh[x]),x]
```

```
[Out] (-1/15*I)*((3*Sech[x]^3)/(I + Sinh[x]) + 12*Sech[x]^2*Tanh[x] + 8*Tanh[x]^3
)
```

fricas [B] time = 0.44, size = 64, normalized size = 1.73

$$-\frac{96\,e^{(3\,x)} + 32i\,e^{(2\,x)} + 32\,e^{x} + 16i}{15\,e^{(8\,x)} + 30i\,e^{(7\,x)} + 30\,e^{(6\,x)} + 90i\,e^{(5\,x)} + 90i\,e^{(3\,x)} - 30\,e^{(2\,x)} + 30i\,e^{x} - 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(96*e^(3*x) + 32*I*e^(2*x) + 32*e^x + 16*I)/(15*e^(8*x) + 30*I*e^(7*x) + 3
0*e^(6*x) + 90*I*e^(5*x) + 90*I*e^(3*x) - 30*e^(2*x) + 30*I*e^x - 15)
```

giac [B] time = 0.19, size = 53, normalized size = 1.43

$$\frac{9\,e^{(2\,x)} - 24i\,e^{x} - 11}{24\,(e^{x} - i)^3} - \frac{45\,e^{(4\,x)} + 240i\,e^{(3\,x)} - 490\,e^{(2\,x)} - 320i\,e^{x} + 73}{120\,(e^{x} + i)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(9*e^(2*x) - 24*I*e^x - 11)/(e^x - I)^3 - 1/120*(45*e^(4*x) + 240*I*e^
(3*x) - 490*e^(2*x) - 320*I*e^x + 73)/(e^x + I)^5
```

maple [B] time = 0.06, size = 93, normalized size = 2.51

$$-\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{5i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{11i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+sinh(x)),x)
```

```
[Out] -2/5*I/(tanh(1/2*x)+I)^5+5/3*I/(tanh(1/2*x)+I)^3-11/8*I/(tanh(1/2*x)+I)+1/(
tanh(1/2*x)+I)^4-3/2/(tanh(1/2*x)+I)^2+1/6*I/(tanh(1/2*x)-I)^3-5/8*I/(tanh(
1/2*x)-I)+1/4/(tanh(1/2*x)-I)^2
```

maxima [B] time = 0.31, size = 205, normalized size = 5.54

$$-\frac{32\,e^{(-x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}+\frac{}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -32*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 32*I*e^(-2*x)/(-30*I*e^(-x)
 - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x
) + 15*e^(-8*x) - 15) - 96*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 16
*I/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x
) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15)
```

mupad [B] time = 1.01, size = 231, normalized size = 6.24

$$-\frac{1}{6\left(e^{2\,x}\,3i-e^{3\,x}+3\,e^{x}-i\right)}-\frac{\frac{3\,e^{x}}{40}+\frac{1}{8}i}{e^{2\,x}-1+e^{x}\,2i}-\frac{\frac{3\,e^{2\,x}}{40}-\frac{5}{24}+\frac{e^{x}\,1i}{4}}{e^{2\,x}\,3i+e^{3\,x}-3\,e^{x}-i}+\frac{1i}{4\left(1-e^{2\,x}+e^{x}\,2i\right)}+\frac{3}{8\,\left(e^{x}-i\right)}-\frac{3}{40\,\left(e^{x}+1i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(sinh(x) + 1i)),x)
```

```
[Out] 1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((3*exp(x))/40 + 1i/8)/(exp(2*x) + exp(
x)*2i - 1) - ((3*exp(2*x))/40 + (exp(x)*1i)/4 - 5/24)/(exp(2*x)*3i + exp(3*
x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 3/(8
*(exp(x) - 1i)) - 3/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (3*exp(3*x))/40
 - (5*exp(x))/8 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i +
1) - ((exp(3*x)*1i)/2 - (5*exp(2*x))/4 + (3*exp(4*x))/40 - (exp(x)*1i)/2 +
3/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{\sinh(x) + i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**4/(sinh(x) + I), x)
```

3.170 $\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=80

$$\frac{i}{8(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{32(-\sinh(x)+i)^2}+\frac{3}{32(\sinh(x)+i)^2}+\frac{i}{24(\sinh(x)+i)^3}-\frac{5}{16}i\tan^{-1}(\sinh(x))$$

```
[Out] -5/16*I*arctan(sinh(x))-1/32/(I-sinh(x))^2+1/8*I/(I-sinh(x))+1/24*I/(I+sinh
(x))^3+3/32/(I+sinh(x))^2-3/16*I/(I+sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 203}

$$\frac{i}{8(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{32(-\sinh(x)+i)^2}+\frac{3}{32(\sinh(x)+i)^2}+\frac{i}{24(\sinh(x)+i)^3}-\frac{5}{16}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*ArcTan[Sinh[x]] - 1/(32*(I - Sinh[x])^2) + (I/8)/(I - Sinh[x])
+ (I/24)/(I + Sinh[x])^3 + 3/(32*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x]
)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

])

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{(i-x)^3(i+x)^4}\,dx, x, \sinh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{1}{16(-i+x)^3} - \frac{i}{8(-i+x)^2} + \frac{i}{8(i+x)^4} + \frac{3}{16(i+x)^3} - \frac{3i}{16(i+x)^2} + \frac{5i}{16\left(1+x^2\right.}\right.\right.$$
$$= -\frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{3}{32(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x)}$$
$$= -\frac{5}{16}i\tan^{-1}(\sinh(x)) - \frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{3}{32(i+\sinh}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 1.18

$$-\frac{i\operatorname{sech}^4(x)\left(15\sinh^5(x)\tan^{-1}(\sinh(x)) + 15\sinh^4(x)\left(1+i\tan^{-1}(\sinh(x))\right) + 15\sinh^3(x)\left(2\tan^{-1}(\sinh(x)) + i\right.}{48(\sinh(x) +}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-1/48*I)*Sech[x]^4*(8 + (15*I)*ArcTan[Sinh[x]] + 5*(5*I + 3*ArcTan[Sinh[x
]])*Sinh[x] + 5*(5 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 15*(I + 2*ArcTan[Si
nh[x]])*Sinh[x]^3 + 15*(1 + I*ArcTan[Sinh[x]])*Sinh[x]^4 + 15*ArcTan[Sinh[x
]]*Sinh[x]^5))/(I + Sinh[x])
```

fricas [B] time = 0.54, size = 249, normalized size = 3.11

$$\frac{\left(15\,e^{(10\,x)} + 30i\,e^{(9\,x)} + 45\,e^{(8\,x)} + 120i\,e^{(7\,x)} + 30\,e^{(6\,x)} + 180i\,e^{(5\,x)} - 30\,e^{(4\,x)} + 120i\,e^{(3\,x)} - 45\,e^{(2\,x)} + 30i\,e^{x} - 15\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((15*e^(10*x) + 30*I*e^(9*x) + 45*e^(8*x) + 120*I*e^(7*x) + 30*e^(6*x) + 18
0*I*e^(5*x) - 30*e^(4*x) + 120*I*e^(3*x) - 45*e^(2*x) + 30*I*e^x - 15)*log(
e^x + I) - (15*e^(10*x) + 30*I*e^(9*x) + 45*e^(8*x) + 120*I*e^(7*x) + 30*e^
(6*x) + 180*I*e^(5*x) - 30*e^(4*x) + 120*I*e^(3*x) - 45*e^(2*x) + 30*I*e^x
- 15)*log(e^x - I) - 30*I*e^(9*x) + 60*e^(8*x) - 80*I*e^(7*x) + 220*e^(6*x)
```

```
 - 36*I*e^(5*x) - 220*e^(4*x) - 80*I*e^(3*x) - 60*e^(2*x) - 30*I*e^x)/(48*e
^(10*x) + 96*I*e^(9*x) + 144*e^(8*x) + 384*I*e^(7*x) + 96*e^(6*x) + 576*I*e
^(5*x) - 96*e^(4*x) + 384*I*e^(3*x) - 144*e^(2*x) + 96*I*e^x - 48)
```

giac [B] time = 0.38, size = 118, normalized size = 1.48

$$\frac{15\left(e^{(-x)}-e^{x}\right)^{2}+76i\,e^{(-x)}-76i\,e^{x}-100}{64\left(e^{(-x)}-e^{x}+2i\right)^{2}}-\frac{55\left(e^{(-x)}-e^{x}\right)^{3}-402i\left(e^{(-x)}-e^{x}\right)^{2}-1020\,e^{(-x)}+1020\,e^{x}+936i}{192\left(e^{(-x)}-e^{x}-2i\right)^{3}}+\frac{5}{32}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/64*(15*(e^(-x) - e^x)^2 + 76*I*e^(-x) - 76*I*e^x - 100)/(e^(-x) - e^x + 2
*I)^2 - 1/192*(55*(e^(-x) - e^x)^3 - 402*I*(e^(-x) - e^x)^2 - 1020*e^(-x) +
 1020*e^x + 936*I)/(e^(-x) - e^x - 2*I)^3 + 5/32*log(-e^(-x) + e^x + 2*I) -
 5/32*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.06, size = 137, normalized size = 1.71

$$\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{25i}{12\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{15}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{15}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{5\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(I+sinh(x)),x)
```

```
[Out] I/(tanh(1/2*x)+I)^5+I/(tanh(1/2*x)+I)-25/12*I/(tanh(1/2*x)+I)^3+1/3/(tanh(1
/2*x)+I)^6-15/8/(tanh(1/2*x)+I)^4+15/8/(tanh(1/2*x)+I)^2+5/16*ln(tanh(1/2*x
)+I)+3/8*I/(tanh(1/2*x)-I)-1/4*I/(tanh(1/2*x)-I)^3+1/8/(tanh(1/2*x)-I)^4-1/
2/(tanh(1/2*x)-I)^2-5/16*ln(tanh(1/2*x)-I)
```

maxima [B] time = 0.36, size = 140, normalized size = 1.75

$$\frac{32\left(15i\,e^{(-x)}-30\,e^{(-2x)}+40i\,e^{(-3x)}-110\,e^{(-4x)}+18i\,e^{(-5x)}+110\,e^{(-6x)}+40i\,e^{(-7x)}+30\,e^{(-8x)}\right.}{-1536i\,e^{(-x)}-2304\,e^{(-2x)}-6144i\,e^{(-3x)}-1536\,e^{(-4x)}-9216i\,e^{(-5x)}+1536\,e^{(-6x)}-6144i\,e^{(-7x)}+2304\,e^{(-8x)}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 32*(15*I*e^(-x) - 30*e^(-2*x) + 40*I*e^(-3*x) - 110*e^(-4*x) + 18*I*e^(-5*x
) + 110*e^(-6*x) + 40*I*e^(-7*x) + 30*e^(-8*x) + 15*I*e^(-9*x))/(-1536*I*e^
(-x) - 2304*e^(-2*x) - 6144*I*e^(-3*x) - 1536*e^(-4*x) - 9216*I*e^(-5*x) +
```

```
1536*e^(-6*x) - 6144*I*e^(-7*x) + 2304*e^(-8*x) - 1536*I*e^(-9*x) + 768*e^(
-10*x) - 768) - 5/16*log(e^(-x) + I) + 5/16*log(e^(-x) - I)
```

mupad [B] time = 1.91, size = 249, normalized size = 3.11

$$\frac{5\ln\left(-\frac{5}{8}+\frac{e^{x}\,5i}{8}\right)}{16}-\frac{5\ln\left(\frac{5}{8}+\frac{e^{x}\,5i}{8}\right)}{16}-\frac{1i}{e^{5x}-10\,e^{3x}+e^{4x}\,5i-e^{2x}\,10i+5\,e^{x}+1i}+\frac{1i}{4\,\left(e^{2x}\,3i-e^{3x}+3\,e^{x}-i\right)}+\frac{}{8\,\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5*(sinh(x) + 1i)),x)
```

```
[Out] (5*log((exp(x)*5i)/8 - 5/8))/16 - (5*log((exp(x)*5i)/8 + 5/8))/16 - 1i/(exp
(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) + 1i/(4*(
exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 1/(8*(exp(4*x) - exp(3*x)*4i - 6
*exp(2*x) + exp(x)*4i + 1)) + 5/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - e
xp(x)*4i + 1)) - 1/(8*(exp(x)*2i - exp(2*x) + 1)) - 1i/(4*(exp(x) - 1i)) -
3i/(8*(exp(x) + 1i)) - 1/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp
(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 5i/(12*(exp(2*x)*3i + exp(3*x) - 3*
exp(x) - 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{5}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**5/(sinh(x) + I), x)
```

3.171 $\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=40

$$-\frac{5x}{8}-\frac{5}{12}i\cosh^3(x)+\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{8}\sinh(x)\cosh(x)$$

```
[Out] -5/8*x-5/12*I*cosh(x)^3-5/8*cosh(x)*sinh(x)+1/4*cosh(x)^5/(I+sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2679, 2682, 2635, 8}

$$-\frac{5x}{8}-\frac{5}{12}i\cosh^3(x)+\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{8}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-5*x)/8 - ((5*I)/12)*Cosh[x]^3 - (5*Cosh[x]*Sinh[x])/8 + Cosh[x]^5/(4*(I +
 Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx &= \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5}{4}i\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx \\
&= -\frac{5}{12}i\cosh^3(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5}{4}\int \cosh^2(x)\,dx \\
&= -\frac{5}{12}i\cosh^3(x) - \frac{5}{8}\cosh(x)\sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5\int 1\,dx}{8} \\
&= -\frac{5x}{8} - \frac{5}{12}i\cosh^3(x) - \frac{5}{8}\cosh(x)\sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))}
\end{aligned}$$

Mathematica [B] time = 0.19, size = 121, normalized size = 3.02

$$-\frac{i\cosh^7(x)\left(6\sinh^4(x) - 10i\sinh^3(x) + 7\sinh^2(x) - 25i\sinh(x) + \frac{30\sqrt{1-i\sinh(x)}\,\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}} + 16\right)}{24\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/24*I)*Cosh[x]^7*(16 + (30*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 -
 I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] - (25*I)*Sinh[x] + 7*Sinh[x]^2 - (10*I)*Si
nh[x]^3 + 6*Sinh[x]^4))/((Cosh[x/2] - I*Sinh[x/2])^8*(Cosh[x/2] + I*Sinh[x/
2])^6)
```

fricas [A] time = 0.58, size = 55, normalized size = 1.38

$$-\frac{1}{192}\left(120\,xe^{(4\,x)} - 3\,e^{(8\,x)} + 16i\,e^{(7\,x)} + 24\,e^{(6\,x)} + 48i\,e^{(5\,x)} + 48i\,e^{(3\,x)} - 24\,e^{(2\,x)} + 16i\,e^{x} + 3\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/192*(120*x*e^(4*x) - 3*e^(8*x) + 16*I*e^(7*x) + 24*e^(6*x) + 48*I*e^(5*x
) + 48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x)
```

giac [A] time = 0.18, size = 50, normalized size = 1.25

$$-\frac{1}{192}\left(48i\,e^{(3\,x)} - 24\,e^{(2\,x)} + 16i\,e^{x} + 3\right)e^{(-4\,x)} - \frac{5}{8}\,x + \frac{1}{64}\,e^{(4\,x)} - \frac{1}{12}i\,e^{(3\,x)} - \frac{1}{8}\,e^{(2\,x)} - \frac{1}{4}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/192*(48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x) - 5/8*x + 1/64*e
^(4*x) - 1/12*I*e^(3*x) - 1/8*e^(2*x) - 1/4*I*e^x
```

maple [B] time = 0.07, size = 166, normalized size = 4.15

$$\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{}{4(\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(I+sinh(x))^2,x)
```

```
[Out] 1/2/(tanh(1/2*x)-1)^3-I/(tanh(1/2*x)+1)-1/8/(tanh(1/2*x)-1)^2+I/(tanh(1/2*x
)+1)^2-3/8/(tanh(1/2*x)-1)-2/3*I/(tanh(1/2*x)+1)^3+1/4/(tanh(1/2*x)-1)^4+5/
8*ln(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)+1)^3+I/(tanh(1/2*x)-1)^2+1/8/(tanh(1/2
*x)+1)^2+2/3*I/(tanh(1/2*x)-1)^3-3/8/(tanh(1/2*x)+1)+I/(tanh(1/2*x)-1)-1/4/
(tanh(1/2*x)+1)^4-5/8*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.35, size = 54, normalized size = 1.35

$$-\frac{1}{192}\left(16i\,e^{(-x)} + 24\,e^{(-2\,x)} + 48i\,e^{(-3\,x)} - 3\right)e^{(4\,x)} - \frac{5}{8}\,x - \frac{1}{4}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{12}i\,e^{(-3\,x)} - \frac{1}{64}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/192*(16*I*e^(-x) + 24*e^(-2*x) + 48*I*e^(-3*x) - 3)*e^(4*x) - 5/8*x - 1/
4*I*e^(-x) + 1/8*e^(-2*x) - 1/12*I*e^(-3*x) - 1/64*e^(-4*x)
```

mupad [B] time = 0.14, size = 54, normalized size = 1.35

$$\frac{\mathrm{e}^{-2\,x}}{8}-\frac{\mathrm{e}^{-x}\,1{}\mathrm{i}}{4}-\frac{5\,x}{8}-\frac{\mathrm{e}^{2\,x}}{8}-\frac{\mathrm{e}^{-3\,x}\,1{}\mathrm{i}}{12}-\frac{\mathrm{e}^{3\,x}\,1{}\mathrm{i}}{12}-\frac{\mathrm{e}^{-4\,x}}{64}+\frac{\mathrm{e}^{4\,x}}{64}-\frac{\mathrm{e}^{x}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(-2*x)/8 - (exp(-x)*1i)/4 - (5*x)/8 - exp(2*x)/8 - (exp(-3*x)*1i)/12 - (
exp(3*x)*1i)/12 - exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*1i)/4
```

sympy [A] time = 0.24, size = 65, normalized size = 1.62

$$-\frac{5x}{8}+\frac{e^{4x}}{64}-\frac{ie^{3x}}{12}-\frac{e^{2x}}{8}-\frac{ie^{x}}{4}-\frac{ie^{-x}}{4}+\frac{e^{-2x}}{8}-\frac{ie^{-3x}}{12}-\frac{e^{-4x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(I+sinh(x))**2,x)
```

```
[Out] -5*x/8 + exp(4*x)/64 - I*exp(3*x)/12 - exp(2*x)/8 - I*exp(x)/4 - I*exp(-x)/
4 + exp(-2*x)/8 - I*exp(-3*x)/12 - exp(-4*x)/64
```

3.172 $\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{3}(-\sinh(x)+i)^3$$

```
[Out] -1/3*(I-sinh(x))^3
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 32}

$$-\frac{1}{3}(-\sinh(x)+i)^3$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -(I - Sinh[x])^3/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} \, dx &= \text{Subst}\left(\int (i-x)^2 \, dx, x, \sinh(x)\right)\\ &= -\frac{1}{3}(i-\sinh(x))^3\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.29

$$\frac{1}{6}\sinh(x)(-6i\sinh(x)+\cosh(2x)-7)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] ((-7 + Cosh[2*x] - (6*I)*Sinh[x])*Sinh[x])/6
```

fricas [B] time = 0.46, size = 34, normalized size = 2.43

$$\frac{1}{24}\left(e^{(6x)} - 6i\,e^{(5x)} - 15\,e^{(4x)} + 15\,e^{(2x)} - 6i\,e^{x} - 1\right)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(e^(6*x) - 6*I*e^(5*x) - 15*e^(4*x) + 15*e^(2*x) - 6*I*e^x - 1)*e^(-3*
x)
```

giac [B] time = 0.36, size = 35, normalized size = 2.50

$$\frac{1}{24}\left(15\,e^{(2x)} - 6i\,e^{x} - 1\right)e^{(-3x)} + \frac{1}{24}\,e^{(3x)} - \frac{1}{4}i\,e^{(2x)} - \frac{5}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*e^(2*x) - 6*I*e^x - 1)*e^(-3*x) + 1/24*e^(3*x) - 1/4*I*e^(2*x) - 5
/8*e^x
```

maple [B] time = 0.07, size = 70, normalized size = 5.00

$$\frac{1-i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{-\frac{1}{2}-i}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{\frac{1}{2}-i}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1+i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(I+sinh(x))^2,x)
```

```
[Out] (1-I)/(tanh(1/2*x)-1)-(1/2+I)/(tanh(1/2*x)-1)^2-1/3/(tanh(1/2*x)-1)^3+(1/2-
I)/(tanh(1/2*x)+1)^2+(1+I)/(tanh(1/2*x)+1)-1/3/(tanh(1/2*x)+1)^3
```

maxima [B] time = 0.33, size = 39, normalized size = 2.79

$$-\frac{1}{96}\left(24i\,e^{(-x)} + 60\,e^{(-2x)} - 4\right)e^{(3x)} + \frac{5}{8}\,e^{(-x)} - \frac{1}{4}i\,e^{(-2x)} - \frac{1}{24}\,e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(24*I*e^(-x) + 60*e^(-2*x) - 4)*e^(3*x) + 5/8*e^(-x) - 1/4*I*e^(-2*x)
 - 1/24*e^(-3*x)
```

mupad [B] time = 0.09, size = 37, normalized size = 2.64

$$\frac{5\,e^{-x}}{8}-\frac{e^{-3\,x}}{24}+\frac{e^{3\,x}}{24}-\frac{5\,e^{x}}{8}-\frac{e^{-2\,x}\,1i}{4}-\frac{e^{2\,x}\,1i}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(sinh(x) + 1i)^2,x)
```

```
[Out] (5*exp(-x))/8 - (exp(-2*x)*1i)/4 - (exp(2*x)*1i)/4 - exp(-3*x)/24 + exp(3*x
)/24 - (5*exp(x))/8
```

sympy [B] time = 0.19, size = 44, normalized size = 3.14

$$\frac{e^{3x}}{24}-\frac{ie^{2x}}{4}-\frac{5e^{x}}{8}+\frac{5e^{-x}}{8}-\frac{ie^{-2x}}{4}-\frac{e^{-3x}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(I+sinh(x))**2,x)
```

```
[Out] exp(3*x)/24 - I*exp(2*x)/4 - 5*exp(x)/8 + 5*exp(-x)/8 - I*exp(-2*x)/4 - exp
(-3*x)/24
```

3.173 $\int \frac{\cosh^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=30

$$-\frac{3x}{2}-\frac{3}{2}i\cosh(x)+\frac{\cosh^3(x)}{2(\sinh(x)+i)}$$

```
[Out] -3/2*x-3/2*I*cosh(x)+1/2*cosh(x)^3/(I+sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2679, 2682, 8}

$$-\frac{3x}{2}-\frac{3}{2}i\cosh(x)+\frac{\cosh^3(x)}{2(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-3*x)/2 - ((3*I)/2)*Cosh[x] + Cosh[x]^3/(2*(I + Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3}{2}i\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx$$
$$= -\frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3\int 1\,dx}{2}$$
$$= -\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 1.53

$$\frac{1}{2}(\sinh(x)-4i)\cosh(x) - 3i\sqrt{\cosh^2(x)}\,\mathrm{sech}(x)\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-3*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x] + (Cosh[
x]*(-4*I + Sinh[x]))/2
```

fricas [A] time = 0.46, size = 31, normalized size = 1.03

$$-\frac{1}{8}\left(12\,xe^{(2\,x)} - e^{(4\,x)} + 8i\,e^{(3\,x)} + 8i\,e^{x} + 1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*x*e^(2*x) - e^(4*x) + 8*I*e^(3*x) + 8*I*e^x + 1)*e^(-2*x)
```

giac [A] time = 0.49, size = 26, normalized size = 0.87

$$-\frac{1}{8}\,(8i\,e^{x}+1)e^{(-2\,x)} - \frac{3}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/8*(8*I*e^x + 1)*e^(-2*x) - 3/2*x + 1/8*e^(2*x) - I*e^x
```

maple [B] time = 0.07, size = 82, normalized size = 2.73

$$\frac{1}{2\tanh\left(\frac{x}{2}\right)-2} + \frac{2i}{\tanh\left(\frac{x}{2}\right)-1} + \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2} + \frac{1}{2\tanh\left(\frac{x}{2}\right)+2} - \frac{2i}{\tanh\left(\frac{x}{2}\right)+1} - \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+sinh(x))^2,x)
```

```
[Out] 1/2/(tanh(1/2*x)-1)+2*I/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+3/2*ln(tanh(1
/2*x)-1)+1/2/(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-3/2*
ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.33, size = 30, normalized size = 1.00

$$-\frac{1}{8}\left(8i\,e^{(-x)}-1\right)e^{(2\,x)}-\frac{3}{2}\,x-i\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(8*I*e^(-x) - 1)*e^(2*x) - 3/2*x - I*e^(-x) - 1/8*e^(-2*x)
```

mupad [B] time = 0.48, size = 28, normalized size = 0.93

$$\frac{e^{2\,x}}{8}-e^{-x}\,1\mathrm{i}-\frac{e^{-2\,x}}{8}-\frac{3\,x}{2}-e^{x}\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (3*x)/2 - exp(x)*1i
```

sympy [A] time = 0.16, size = 29, normalized size = 0.97

$$-\frac{3x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

3.174 $\int \frac{\cosh^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=14

$$\sinh(x) - 2i\log(\sinh(x) + i)$$

```
[Out] -2*I*ln(I+sinh(x))+sinh(x)
```

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 43}

$$\sinh(x) - 2i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{(i+\sinh(x))^2} \, dx &= -\text{Subst}\left(\int \frac{i-x}{i+x} \, dx, x, \sinh(x)\right) \\
&= -\text{Subst}\left(\int \left(-1+\frac{2i}{i+x}\right) \, dx, x, \sinh(x)\right) \\
&= -2i\log(i+\sinh(x)) + \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\sinh(x) - 2i\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x])^2,x]
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

fricas [B] time = 0.82, size = 26, normalized size = 1.86

$$\frac{1}{2}\left(4i\,xe^{x} - 8i\,e^{x}\log\left(e^{x} + i\right) + e^{(2\,x)} - 1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
[Out] 1/2*(4*I*x*e^x - 8*I*e^x*log(e^x + I) + e^(2*x) - 1)*e^(-x)
```

giac [B] time = 0.16, size = 21, normalized size = 1.50

$$2i\,x - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4i\,\log\left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
[Out] 2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^x + I)
```

maple [B] time = 0.07, size = 53, normalized size = 3.79

$$2i\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - \frac{1}{\tanh\left(\frac{x}{2}\right) - 1} + 2i\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{1}{\tanh\left(\frac{x}{2}\right) + 1} - 4i\ln\left(\tanh\left(\frac{x}{2}\right) + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+sinh(x))^2,x)
[Out] 2*I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1)+2*I*ln(tanh(1/2*x)+1)-1/(tanh(1/2*x
)+1)-4*I*ln(tanh(1/2*x)+I)
```

maxima [B] time = 0.32, size = 23, normalized size = 1.64

$$-2i\,x - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4i\,\log\left(e^{(-x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^(-x) - I)
```

mupad [B] time = 0.55, size = 24, normalized size = 1.71

$$\frac{e^x}{2} - \frac{e^{-x}}{2} + x\,2i - \ln\left(e^x + 1i\right)\,4i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] x*2i - exp(-x)/2 - log(exp(x) + 1i)*4i + exp(x)/2
```

sympy [B] time = 0.18, size = 26, normalized size = 1.86

$$2ix + \frac{e^x}{2} - 4i\log\left(e^x + i\right) - \frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] 2*I*x + exp(x)/2 - 4*I*log(exp(x) + I) - exp(-x)/2
```

3.175 $\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=14

$$x-\frac{2\cosh(x)}{\sinh(x)+i}$$

```
[Out] x-2*cosh(x)/(I+sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2680, 8}

$$x-\frac{2\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] x - (2*Cosh[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx &= -\frac{2\cosh(x)}{i+\sinh(x)}+\int 1\,dx\\ &= x-\frac{2\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] time = 0.05, size = 69, normalized size = 4.93

$$\frac{2\cosh^3(x)\left(-1-\frac{\sqrt{1-i\sinh(x)}\,\sin^{-1}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{1+i\sinh(x)}}\right)}{(\sinh(x)-i)(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2/(I + Sinh[x])^2,x]

[Out] (2*Cosh[x]^3*(-1 - (ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x]])/Sqrt[1 + I*Sinh[x]]))/((-I + Sinh[x])*(I + Sinh[x])^2)

fricas [A] time = 0.51, size = 16, normalized size = 1.14

$$\frac{xe^x + ix + 4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")

[Out] (x*e^x + I*x + 4*I)/(e^x + I)

giac [A] time = 0.17, size = 10, normalized size = 0.71

$$x + \frac{4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="giac")

[Out] x + 4*I/(e^x + I)

maple [B] time = 0.06, size = 29, normalized size = 2.07

$$-\frac{4}{\tanh\left(\frac{x}{2}\right)+i} - \ln\left(\tanh\left(\frac{x}{2}\right)-1\right) + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^2/(I+sinh(x))^2,x)

[Out] -4/(tanh(1/2*x)+I)-ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)

maxima [A] time = 0.32, size = 12, normalized size = 0.86

$$x + \frac{4i}{e^{(-x)} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
[Out] x + 4*I/(e^(-x) - I)
```

mupad [B] time = 0.65, size = 12, normalized size = 0.86

$$x + \frac{4\mathrm{i}}{\mathrm{e}^{x} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(sinh(x) + 1i)^2,x)
[Out] x + 4i/(exp(x) + 1i)
```

sympy [A] time = 0.12, size = 8, normalized size = 0.57

$$x + \frac{4}{-ie^{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+sinh(x))**2,x)
[Out] x + 4/(-I*exp(x) + 1)
```

3.176 $\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{\sinh(x)+i}$$

```
[Out] -1/(I+sinh(x))
```

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2667, 32}

$$-\frac{1}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -(I + Sinh[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{(i+x)^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$-\frac{1}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -(I + Sinh[x])^(-1)
```

fricas [A] time = 0.56, size = 16, normalized size = 1.60

$$-\frac{2\,e^{x}}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*e^x/(e^(2*x) + 2*I*e^x - 1)
```

giac [A] time = 0.14, size = 10, normalized size = 1.00

$$-\frac{2\,e^{x}}{(e^{x}+i)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x + I)^2
```

maple [A] time = 0.03, size = 10, normalized size = 1.00

$$-\frac{1}{i+\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+sinh(x))^2,x)
```

```
[Out] -1/(I+sinh(x))
```

maxima [A] time = 0.31, size = 8, normalized size = 0.80

$$-\frac{1}{\sinh(x)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(sinh(x) + I)
```

mupad [B] time = 0.58, size = 12, normalized size = 1.20

$$-\frac{1i}{-1+\sinh(x)\,1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] -1i/(sinh(x)*1i - 1)
```

sympy [B] time = 0.12, size = 17, normalized size = 1.70

$$\frac{2e^{x}}{-e^{2x}-2ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))**2,x)
```

```
[Out] 2*exp(x)/(-exp(2*x) - 2*I*exp(x) + 1)
```

3.177 $\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{4(\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)^2}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

```
[Out] -1/4*arctan(sinh(x))-1/4*I/(I+sinh(x))^2-1/4/(I+sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2667, 44, 203}

$$-\frac{1}{4(\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)^2}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Sinh[x])^2,x]
```

```
[Out] -ArcTan[Sinh[x]]/4 - (I/4)/(I + Sinh[x])^2 - 1/(4*(I + Sinh[x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{(i-x)(i+x)^3}\,dx, x, \sinh(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(-\frac{i}{2(i+x)^3}-\frac{1}{4(i+x)^2}+\frac{1}{4\left(1+x^2\right)}\right)dx, x, \sinh(x)\right)\\ &= -\frac{i}{4(i+\sinh(x))^2}-\frac{1}{4(i+\sinh(x))}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{4}\tan^{-1}(\sinh(x))-\frac{i}{4(i+\sinh(x))^2}-\frac{1}{4(i+\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 0.76

$$\frac{1}{4}\left(-\tan^{-1}(\sinh(x))-\frac{\sinh(x)+2i}{(\sinh(x)+i)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Sinh[x])^2,x]
```

```
[Out] (-ArcTan[Sinh[x]] - (2*I + Sinh[x])/(I + Sinh[x])^2)/4
```

fricas [B] time = 0.74, size = 104, normalized size = 3.06

$$\frac{\left(-i\,e^{(4\,x)}+4\,e^{(3\,x)}+6i\,e^{(2\,x)}-4\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(i\,e^{(4\,x)}-4\,e^{(3\,x)}-6i\,e^{(2\,x)}+4\,e^{x}+i\right)\log\left(e^{x}-i\right)-2\,e^{(3\,x)}-8i\,e}{4\,e^{(4\,x)}+16i\,e^{(3\,x)}-24\,e^{(2\,x)}-16i\,e^{x}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) + (I*e^(4*
x) - 4*e^(3*x) - 6*I*e^(2*x) + 4*e^x + I)*log(e^x - I) - 2*e^(3*x) - 8*I*e^
(2*x) + 2*e^x)/(4*e^(4*x) + 16*I*e^(3*x) - 24*e^(2*x) - 16*I*e^x + 4)
```

giac [B] time = 0.16, size = 70, normalized size = 2.06

$$\frac{3i\left(e^{(-x)}-e^{x}\right)^2+20\,e^{(-x)}-20\,e^{x}-44i}{16\left(e^{(-x)}-e^{x}-2i\right)^2}-\frac{1}{8}i\,\log\left(i\,e^{(-x)}-i\,e^{x}+2\right)+\frac{1}{8}i\,\log\left(i\,e^{(-x)}-i\,e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(3*I*(e^(-x) - e^x)^2 + 20*e^(-x) - 20*e^x - 44*I)/(e^(-x) - e^x - 2*I
)^2 - 1/8*I*log(I*e^(-x) - I*e^x + 2) + 1/8*I*log(I*e^(-x) - I*e^x - 2)
```

maple [B] time = 0.06, size = 70, normalized size = 2.06

$$\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{4}-\frac{5i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+sinh(x))^2,x)
```

```
[Out] I/(tanh(1/2*x)+I)^4-1/4*I*ln(tanh(1/2*x)+I)-5/2*I/(tanh(1/2*x)+I)^2-2/(tanh
(1/2*x)+I)^3+3/2/(tanh(1/2*x)+I)+1/4*I*ln(tanh(1/2*x)-I)
```

maxima [B] time = 0.35, size = 70, normalized size = 2.06

$$-\frac{2\left(e^{(-x)}+4i\,e^{(-2\,x)}-e^{(-3\,x)}\right)}{16i\,e^{(-x)}-24\,e^{(-2\,x)}-16i\,e^{(-3\,x)}+4\,e^{(-4\,x)}+4}-\frac{1}{4}i\,\log\left(i\,e^{(-x)}+1\right)+\frac{1}{4}i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(e^(-x) + 4*I*e^(-2*x) - e^(-3*x))/(16*I*e^(-x) - 24*e^(-2*x) - 16*I*e^(
-3*x) + 4*e^(-4*x) + 4) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

mupad [B] time = 0.75, size = 86, normalized size = 2.53

$$-\frac{\mathrm{atan}\left(\mathrm{e}^{x}\right)}{2}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}}-\frac{1}{2\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}-\frac{2}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(sinh(x) + 1i)^2),x)
```

```
[Out] 1i/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 1i/(2*(exp(2*x)
+ exp(x)*2i - 1)) - atan(exp(x))/2 - 1/(2*(exp(x) + 1i)) - 2/(exp(2*x)*3i +
 exp(3*x) - 3*exp(x) - 1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(sinh(x) + I)**2, x)
```

3.178 $\int \frac{\text{sech}^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=37

$$-\frac{2\tanh(x)}{5}-\frac{\text{sech}(x)}{5(\sinh(x)+i)}-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

```
[Out] -1/5*I*sech(x)/(I+sinh(x))^2-1/5*sech(x)/(I+sinh(x))-2/5*tanh(x)
```

Rubi [A] time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 3767, 8}

$$-\frac{2\tanh(x)}{5}-\frac{\text{sech}(x)}{5(\sinh(x)+i)}-\frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((-I/5)*Sech[x])/(I + Sinh[x])^2 - Sech[x]/(5*(I + Sinh[x])) - (2*Tanh[x])/
5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx &= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{3}{5}i\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx \\
&= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{\operatorname{sech}(x)}{5(i+\sinh(x))} - \frac{2}{5}\int \operatorname{sech}^2(x)\,dx \\
&= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{\operatorname{sech}(x)}{5(i+\sinh(x))} - \frac{2}{5}i\,\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right) \\
&= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{\operatorname{sech}(x)}{5(i+\sinh(x))} - \frac{2\tanh(x)}{5}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.84

$$
-\frac{\operatorname{sech}(x)(-5\sinh(x)+\sinh(3x)+4i\cosh(2x))}{10(\sinh(x)+i)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] -1/10*(Sech[x]*((4*I)*Cosh[2*x] - 5*Sinh[x] + Sinh[3*x]))/(I + Sinh[x])^2
```

fricas [A] time = 0.64, size = 46, normalized size = 1.24

$$
-\frac{20\,e^{(2\,x)}+16i\,e^{x}-4}{5\,e^{(6\,x)}+20i\,e^{(5\,x)}-25\,e^{(4\,x)}-25\,e^{(2\,x)}-20i\,e^{x}+5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(20*e^(2*x) + 16*I*e^x - 4)/(5*e^(6*x) + 20*I*e^(5*x) - 25*e^(4*x) - 25*e^
(2*x) - 20*I*e^x + 5)
```

giac [A] time = 0.20, size = 41, normalized size = 1.11

$$
-\frac{i}{4\,(e^{x}-i)}-\frac{-5i\,e^{(4\,x)}+30\,e^{(3\,x)}+80i\,e^{(2\,x)}-50\,e^{x}-11i}{20\,(e^{x}+i)^{5}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*I/(e^x - I) - 1/20*(-5*I*e^(4*x) + 30*e^(3*x) + 80*I*e^(2*x) - 50*e^x
- 11*I)/(e^x + I)^5
```

maple [B] time = 0.06, size = 70, normalized size = 1.89

$$-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{5i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{3}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{7}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+sinh(x))^2,x)
```

```
[Out] -2*I/(tanh(1/2*x)+I)^4+5/2*I/(tanh(1/2*x)+I)^2-4/5/(tanh(1/2*x)+I)^5+3/(tan
h(1/2*x)+I)^3-7/4/(tanh(1/2*x)+I)-1/4/(tanh(1/2*x)-I)
```

maxima [B] time = 0.32, size = 117, normalized size = 3.16

$$-\frac{16i\,e^{(-x)}}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+5}+\frac{20\,e^{(-2\,x)}}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -16*I*e^(-x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*e^(-5*x) + 5*e
^(-6*x) + 5) + 20*e^(-2*x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*
e^(-5*x) + 5*e^(-6*x) + 5) - 4/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 2
0*I*e^(-5*x) + 5*e^(-6*x) + 5)
```

mupad [B] time = 0.73, size = 109, normalized size = 2.95

$$-\frac{16\,\mathrm{e}^{x}\,\left(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^{x}\right)}{5\,\left(\mathrm{e}^{2\,x}+1\right)^{5}}-\frac{\left(4\,\mathrm{e}^{2\,x}-\frac{4}{5}\right)\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1\right)}{\left(\mathrm{e}^{2\,x}+1\right)^{5}}-\frac{\mathrm{e}^{x}\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1\right)\,16\mathrm{i}}{5\,\left(\mathrm{e}^{2\,x}+1\right)^{5}}+\frac{\left(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^{x}\right)\,\left(4\,\mathrm{e}^{2\,x}-\frac{4}{5}\right)\,1\mathrm{i}}{\left(\mathrm{e}^{2\,x}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(sinh(x) + 1i)^2),x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*(4*exp(2*x) - 4/5)*1i)/(exp(2*x) + 1)^5 - (exp(x)*
(exp(4*x) - 6*exp(2*x) + 1)*16i)/(5*(exp(2*x) + 1)^5) - (16*exp(x)*(4*exp(3
*x) - 4*exp(x)))/(5*(exp(2*x) + 1)^5) - ((4*exp(2*x) - 4/5)*(exp(4*x) - 6*e
xp(2*x) + 1))/(exp(2*x) + 1)^5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{2}(x)}{(\sinh(x)+i)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**2/(sinh(x) + I)**2, x)
```

3.179 $\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=60

$$\frac{1}{16(-\sinh(x)+i)}-\frac{3}{16(\sinh(x)+i)}-\frac{i}{8(\sinh(x)+i)^2}+\frac{1}{12(\sinh(x)+i)^3}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

```
[Out] -1/4*arctan(sinh(x))+1/16/(I-sinh(x))+1/12/(I+sinh(x))^3-1/8*I/(I+sinh(x))^
2-3/16/(I+sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2667, 44, 203}

$$\frac{1}{16(-\sinh(x)+i)}-\frac{3}{16(\sinh(x)+i)}-\frac{i}{8(\sinh(x)+i)^2}+\frac{1}{12(\sinh(x)+i)^3}-\frac{1}{4}\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -ArcTan[Sinh[x]]/4 + 1/(16*(I - Sinh[x])) + 1/(12*(I + Sinh[x])^3) - (I/8)/
(I + Sinh[x])^2 - 3/(16*(I + Sinh[x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = \operatorname{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^4}\,dx, x, \sinh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(\frac{1}{16(-i+x)^2} - \frac{1}{4(i+x)^4} + \frac{i}{4(i+x)^3} + \frac{3}{16(i+x)^2} - \frac{1}{4\left(1+x^2\right)}\right)dx, x, \sinh(x)\right)$$
$$= \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{3}{16(i+\sinh(x))} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1 \ldots$$
$$= -\frac{1}{4}\tan^{-1}(\sinh(x)) + \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{3}{16(i+\sinh \ldots$$

Mathematica [A] time = 0.05, size = 68, normalized size = 1.13

$$-\frac{\operatorname{sech}^2(x)\left(6i\sinh^2(x) + 3\sinh^4(x)\tan^{-1}(\sinh(x)) + \sinh^3(x)\left(3+6i\tan^{-1}(\sinh(x))\right) + \sinh(x)\left(-1+6i\tan^{-1} \ldots\right.\right.}{12(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -1/12*(Sech[x]^2*(4*I - 3*ArcTan[Sinh[x]] + (-1 + (6*I)*ArcTan[Sinh[x]])*Si
nh[x] + (6*I)*Sinh[x]^2 + (3 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^3 + 3*ArcTan[
Sinh[x]]*Sinh[x]^4))/(I + Sinh[x])^2
```

fricas [B] time = 0.51, size = 200, normalized size = 3.33

$$\frac{\left(-3i e^{(8x)} + 12e^{(7x)} + 12i e^{(6x)} + 12e^{(5x)} + 30i e^{(4x)} - 12e^{(3x)} + 12i e^{(2x)} - 12e^{x} - 3i\right)\log\left(e^{x}+i\right) + \left(3i e^{(8x)} - 12 \ldots\right.}{12e^{(8x)} + 48i e^{(7x)} - 48 \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((-3*I*e^(8*x) + 12*e^(7*x) + 12*I*e^(6*x) + 12*e^(5*x) + 30*I*e^(4*x) - 12
*e^(3*x) + 12*I*e^(2*x) - 12*e^x - 3*I)*log(e^x + I) + (3*I*e^(8*x) - 12*e^
(7*x) - 12*I*e^(6*x) - 12*e^(5*x) - 30*I*e^(4*x) + 12*e^(3*x) - 12*I*e^(2*x
) + 12*e^x + 3*I)*log(e^x - I) - 6*e^(7*x) - 24*I*e^(6*x) + 26*e^(5*x) - 16
*I*e^(4*x) - 26*e^(3*x) - 24*I*e^(2*x) + 6*e^x)/(12*e^(8*x) + 48*I*e^(7*x)
- 48*e^(6*x) + 48*I*e^(5*x) - 120*e^(4*x) - 48*I*e^(3*x) - 48*e^(2*x) - 48*
I*e^x + 12)
```

giac [B] time = 0.43, size = 105, normalized size = 1.75

$$\frac{-i\,e^{(-x)}+i\,e^{x}+3}{8\left(e^{(-x)}-e^{x}+2i\right)}+\frac{11i\left(e^{(-x)}-e^{x}\right)^{3}+84\left(e^{(-x)}-e^{x}\right)^{2}-228i\,e^{(-x)}+228i\,e^{x}-240}{48\left(e^{(-x)}-e^{x}-2i\right)^{3}}-\frac{1}{8}i\,\log\left(-e^{(-x)}+e^{x}+2i\right)+\frac{1}{8}i\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*(-I*e^(-x) + I*e^x + 3)/(e^(-x) - e^x + 2*I) + 1/48*(11*I*(e^(-x) - e^x
)^3 + 84*(e^(-x) - e^x)^2 - 228*I*e^(-x) + 228*I*e^x - 240)/(e^(-x) - e^x -
 2*I)^3 - 1/8*I*log(-e^(-x) + e^x + 2*I) + 1/8*I*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.08, size = 116, normalized size = 1.93

$$\frac{7i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{4}-\frac{23i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{11}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+sinh(x))^2,x)
```

```
[Out] 7/2*I/(tanh(1/2*x)+I)^4-2/3*I/(tanh(1/2*x)+I)^6-1/4*I*ln(tanh(1/2*x)+I)-23/
8*I/(tanh(1/2*x)+I)^2+2/(tanh(1/2*x)+I)^5-11/3/(tanh(1/2*x)+I)^3+11/8/(tanh
(1/2*x)+I)+1/8*I/(tanh(1/2*x)-I)^2+1/4*I*ln(tanh(1/2*x)-I)+1/8/(tanh(1/2*x)
-I)
```

maxima [B] time = 0.34, size = 120, normalized size = 2.00

$$-\frac{8\left(3\,e^{(-x)}+12i\,e^{(-2x)}-13\,e^{(-3x)}+8i\,e^{(-4x)}+13\,e^{(-5x)}+12i\,e^{(-6x)}-3\,e^{(-7x)}\right)}{192i\,e^{(-x)}-192\,e^{(-2x)}+192i\,e^{(-3x)}-480\,e^{(-4x)}-192i\,e^{(-5x)}-192\,e^{(-6x)}-192i\,e^{(-7x)}+48\,e^{(-8x)}+48}-\frac{1}{4}i\,\log\left(i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -8*(3*e^(-x) + 12*I*e^(-2*x) - 13*e^(-3*x) + 8*I*e^(-4*x) + 13*e^(-5*x) + 1
2*I*e^(-6*x) - 3*e^(-7*x))/(192*I*e^(-x) - 192*e^(-2*x) + 192*I*e^(-3*x) -
480*e^(-4*x) - 192*I*e^(-5*x) - 192*e^(-6*x) - 192*I*e^(-7*x) + 48*e^(-8*x)
 + 48) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

mupad [B] time = 1.20, size = 198, normalized size = 3.30

$$-\frac{\mathrm{atan}\left(\mathrm{e}^{x}\right)}{2}-\frac{2}{\mathrm{e}^{5x}-10\,\mathrm{e}^{3x}+\mathrm{e}^{4x}\,5\mathrm{i}-\mathrm{e}^{2x}\,10\mathrm{i}+5\,\mathrm{e}^{x}+1\mathrm{i}}-\frac{1\mathrm{i}}{8\left(\mathrm{e}^{2x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}-\frac{3\mathrm{i}}{2\left(\mathrm{e}^{4x}-6\,\mathrm{e}^{2x}+1+\mathrm{e}^{3x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(sinh(x) + 1i)^2),x)
```

```
[Out] 1i/(8*(exp(x)*2i - exp(2*x) + 1)) - 2/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)
*10i + exp(5*x) + 5*exp(x) + 1i) - 1i/(8*(exp(2*x) + exp(x)*2i - 1)) - 3i/(
2*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - atan(exp(x))/2 -
 1/(8*(exp(x) - 1i)) - 3/(8*(exp(x) + 1i)) + 2i/(3*(15*exp(2*x) - exp(3*x)*
20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 1/(3*(exp(2*
x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**3/(sinh(x) + I)**2, x)
```

3.180 $\int \frac{\text{sech}^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=49

$$\frac{4\tanh^3(x)}{21}-\frac{4\tanh(x)}{7}-\frac{\text{sech}^3(x)}{7(\sinh(x)+i)}-\frac{i\,\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

```
[Out] -1/7*I*sech(x)^3/(I+sinh(x))^2-1/7*sech(x)^3/(I+sinh(x))-4/7*tanh(x)+4/21*t
anh(x)^3
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2672, 3767}

$$\frac{4\tanh^3(x)}{21}-\frac{4\tanh(x)}{7}-\frac{\text{sech}^3(x)}{7(\sinh(x)+i)}-\frac{i\,\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((-I/7)*Sech[x]^3)/(I + Sinh[x])^2 - Sech[x]^3/(7*(I + Sinh[x])) - (4*Tanh[
x])/7 + (4*Tanh[x]^3)/21
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\text{sech}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{5}{7}i\int \frac{\text{sech}^4(x)}{i+\sinh(x)}\,dx$$
$$= -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\text{sech}^3(x)}{7(i+\sinh(x))} - \frac{4}{7}\int \text{sech}^4(x)\,dx$$
$$= -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\text{sech}^3(x)}{7(i+\sinh(x))} - \frac{4}{7}i\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(x)\right)$$
$$= -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\text{sech}^3(x)}{7(i+\sinh(x))} - \frac{4\tanh(x)}{7} + \frac{4\tanh^3(x)}{21}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.96

$$-\frac{\text{sech}^3(x)(-14\sinh(x) - 3\sinh(3x) + \sinh(5x) + 8i\cosh(2x) + 4i\cosh(4x))}{42(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] -1/42*(Sech[x]^3*((8*I)*Cosh[2*x] + (4*I)*Cosh[4*x] - 14*Sinh[x] - 3*Sinh[3
*x] + Sinh[5*x]))/(I + Sinh[x])^2
```

fricas [B] time = 0.59, size = 82, normalized size = 1.67

$$-\frac{224\,e^{(4\,x)} + 128i\,e^{(3\,x)} + 48\,e^{(2\,x)} + 64i\,e^{x} - 16}{21\,e^{(10\,x)} + 84i\,e^{(9\,x)} - 63\,e^{(8\,x)} + 168i\,e^{(7\,x)} - 294\,e^{(6\,x)} - 294\,e^{(4\,x)} - 168i\,e^{(3\,x)} - 63\,e^{(2\,x)} - 84i\,e^{x} + 21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(224*e^(4*x) + 128*I*e^(3*x) + 48*e^(2*x) + 64*I*e^x - 16)/(21*e^(10*x) +
84*I*e^(9*x) - 63*e^(8*x) + 168*I*e^(7*x) - 294*e^(6*x) - 294*e^(4*x) - 168
*I*e^(3*x) - 63*e^(2*x) - 84*I*e^x + 21)
```

giac [A] time = 0.23, size = 65, normalized size = 1.33

$$-\frac{6i\,e^{(2\,x)} + 15\,e^{x} - 7i}{24\,(e^{x}-i)^3} - \frac{-42i\,e^{(6\,x)} + 315\,e^{(5\,x)} + 1015i\,e^{(4\,x)} - 1750\,e^{(3\,x)} - 1344i\,e^{(2\,x)} + 511\,e^{x} + 79i}{168\,(e^{x}+i)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/24*(6*I*e^(2*x) + 15*e^x - 7*I)/(e^x - I)^3 - 1/168*(-42*I*e^(6*x) + 315
*e^(5*x) + 1015*I*e^(4*x) - 1750*e^(3*x) - 1344*I*e^(2*x) + 511*e^x + 79*I)
/(e^x + I)^7
```

maple [B] time = 0.09, size = 116, normalized size = 2.37

$$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{5i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{23i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{7\left(\tanh\left(\frac{x}{2}\right)+i\right)^{7}}-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{55}{12\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{}{8(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+sinh(x))^2,x)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^6-5*I/(tanh(1/2*x)+I)^4+23/8*I/(tanh(1/2*x)+I)^2+4/7/(t
anh(1/2*x)+I)^7-4/(tanh(1/2*x)+I)^5+55/12/(tanh(1/2*x)+I)^3-13/8/(tanh(1/2*
x)+I)-1/8*I/(tanh(1/2*x)-I)^2+1/12/(tanh(1/2*x)-I)^3-3/8/(tanh(1/2*x)-I)
```

maxima [B] time = 0.34, size = 317, normalized size = 6.47

$$-\frac{64i\,e^{(-x)}}{84i\,e^{(-x)}-63\,e^{(-2\,x)}+168i\,e^{(-3\,x)}-294\,e^{(-4\,x)}-294\,e^{(-6\,x)}-168i\,e^{(-7\,x)}-63\,e^{(-8\,x)}-84i\,e^{(-9\,x)}+21\,e^{(-10\,x)}+21}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -64*I*e^(-x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*x) - 2
94*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-10*x) +
 21) + 48*e^(-2*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*
x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-1
0*x) + 21) - 128*I*e^(-3*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 2
94*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) +
 21*e^(-10*x) + 21) + 224*e^(-4*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3
*x) - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(
-9*x) + 21*e^(-10*x) + 21) - 16/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x)
 - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*
x) + 21*e^(-10*x) + 21)
```

mupad [B] time = 0.53, size = 139, normalized size = 2.84

$$\frac{\left(4\,{\mathrm{e}}^{3\,x}-4\,{\mathrm{e}}^{x}\right)\,\left(\frac{16\,{\mathrm{e}}^{2\,x}}{7}+\frac{32\,{\mathrm{e}}^{4\,x}}{3}-\frac{16}{21}\right)\,1{}\mathrm{i}}{{\left({\mathrm{e}}^{2\,x}+1\right)}^7}-\frac{\left({\mathrm{e}}^{4\,x}-6\,{\mathrm{e}}^{2\,x}+1\right)\,\left(\frac{16\,{\mathrm{e}}^{2\,x}}{7}+\frac{32\,{\mathrm{e}}^{4\,x}}{3}-\frac{16}{21}\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^7}-\frac{\left(4\,{\mathrm{e}}^{3\,x}-4\,{\mathrm{e}}^{x}\right)\,\left(\frac{128\,{\mathrm{e}}^{3\,x}}{21}+\frac{64\,{\mathrm{e}}^{x}}{21}\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^7}-\frac{(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(sinh(x) + 1i)^2),x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*((16*exp(2*x))/7 + (32*exp(4*x))/3 - 16/21)*1i)/(e
xp(2*x) + 1)^7 - ((exp(4*x) - 6*exp(2*x) + 1)*((16*exp(2*x))/7 + (32*exp(4*
x))/3 - 16/21))/(exp(2*x) + 1)^7 - ((4*exp(3*x) - 4*exp(x))*((128*exp(3*x))
/21 + (64*exp(x))/21))/(exp(2*x) + 1)^7 - (((128*exp(3*x))/21 + (64*exp(x))
/21)*(exp(4*x) - 6*exp(2*x) + 1)*1i)/(exp(2*x) + 1)^7
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{(\sinh(x) + i)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**4/(sinh(x) + I)**2, x)
```

3.181 $\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx$

Optimal. Leaf size=28

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

```
[Out] I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2667, 43}

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -\left(i\,\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, i\sinh(x)\right)\right)$$
$$= -\left(i\,\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, i\sinh(x)\right)\right)$$
$$= i\log(i-\sinh(x)) + \frac{2i}{1+i\sinh(x)}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 1.61

$$\frac{2i\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))+\sinh(x)\left(-2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x))\right)+2}{\sinh(x)-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

```
[Out] (2 + (2*I)*ArcTan[Tanh[x/2]] + Log[Cosh[x]] + (-2*ArcTan[Tanh[x/2]] + I*Log
[Cosh[x]])*Sinh[x])/(-I + Sinh[x])
```

fricas [B] time = 0.48, size = 49, normalized size = 1.75

$$\frac{-i\,xe^{(2\,x)}-2\,(x-2)e^{x}+\left(2i\,e^{(2\,x)}+4\,e^{x}-2i\right)\log\left(e^{x}-i\right)+i\,x}{e^{(2\,x)}-2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x + (2*I*e^(2*x) + 4*e^x - 2*I)*log(e^x - I) +
I*x)/(e^(2*x) - 2*I*e^x - 1)
```

giac [A] time = 0.19, size = 27, normalized size = 0.96

$$\frac{4\,e^{x}}{\left(e^{x}-i\right)^{2}}-i\,\log\left(i\,e^{x}\right)+2i\,\log\left(-i\,e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x - I)^2 - I*log(I*e^x) + 2*I*log(-I*e^x - 1)
```

maple [B] time = 0.08, size = 56, normalized size = 2.00

$$-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{4}{\tanh\left(\frac{x}{2}\right)-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1+I*sinh(x))^3,x)
```

```
[Out] -I*ln(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)-I)-4*I/(tanh(1/
2*x)-I)^2-4/(tanh(1/2*x)-I)
```

maxima [A] time = 0.35, size = 33, normalized size = 1.18

$$i\,x-\frac{4\,e^{(-x)}}{2i\,e^{(-x)}+e^{(-2\,x)}-1}+2i\,\log\left(e^{(-x)}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

mupad [B] time = 0.19, size = 41, normalized size = 1.46

$$-x\,1\mathrm{i}+\ln\left(\mathrm{e}^{x}-\mathrm{i}\right)\,2\mathrm{i}-\frac{4\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{4}{\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(sinh(x)*1i + 1)^3,x)
```

```
[Out] log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1i
)
```

sympy [A] time = 0.17, size = 31, normalized size = 1.11

$$x\left(-2+i\right)+2\log\left(e^{x}-i\right)-\frac{4e^{x}}{-e^{2x}+2ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1+I*sinh(x))**3,x)
```

```
[Out] x*(-2 + I) + 2*log(exp(x) - I) - 4*exp(x)/(-exp(2*x) + 2*I*exp(x) + 1)
```

3.182 $\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx$

Optimal. Leaf size=20

$$\frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

```
[Out] 1/3*I*cosh(x)^3/(1+I*sinh(x))^3
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2671}

$$\frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
```

```
[Out] ((I/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 2.00

$$-\frac{i\left(\cosh\left(\frac{3x}{2}\right)-3\cosh\left(\frac{x}{2}\right)\right)}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
```

```
[Out] ((-1/3*I)*(-3*Cosh[x/2] + Cosh[(3*x)/2]))/(Cosh[x/2] + I*Sinh[x/2])^3
```

fricas [B] time = 0.48, size = 29, normalized size = 1.45

$$\frac{-6i\,e^{(2\,x)}+2i}{3\,e^{(3\,x)}-9i\,e^{(2\,x)}-9\,e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="fricas")
[Out] (-6*I*e^(2*x) + 2*I)/(3*e^(3*x) - 9*I*e^(2*x) - 9*e^x + 3*I)
```

giac [A] time = 0.21, size = 16, normalized size = 0.80

$$-\frac{6i\,e^{(2\,x)}-2i}{3\,(e^{x}-i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="giac")
[Out] -1/3*(6*I*e^(2*x) - 2*I)/(e^x - I)^3
```

maple [B] time = 0.08, size = 36, normalized size = 1.80

$$\frac{2}{\tanh\left(\frac{x}{2}\right)-i}-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+I*sinh(x))^3,x)
[Out] 2/(tanh(1/2*x)-I)-8/3/(tanh(1/2*x)-I)^3+4*I/(tanh(1/2*x)-I)^2
```

maxima [B] time = 0.33, size = 53, normalized size = 2.65

$$\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}-\frac{2}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="maxima")
[Out] 6*e^(-2*x)/(-9*I*e^(-x) - 9*e^(-2*x) + 3*I*e^(-3*x) + 3) - 2/(-9*I*e^(-x) -
 9*e^(-2*x) + 3*I*e^(-3*x) + 3)
```

mupad [B] time = 0.66, size = 19, normalized size = 0.95

$$-\frac{2\,e^{2\,x}-\frac{2}{3}}{{\left(1+e^x\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(sinh(x)*1i + 1)^3,x)
[Out] -(2*exp(2*x) - 2/3)/(exp(x)*1i + 1)^3
```

sympy [B] time = 0.15, size = 31, normalized size = 1.55

$$\frac{6e^{2x}-2}{3ie^{3x}+9e^{2x}-9ie^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+I*sinh(x))**3,x)
[Out] (6*exp(2*x) - 2)/(3*I*exp(3*x) + 9*exp(2*x) - 9*I*exp(x) - 3)
```

3.183 $\int \frac{\cosh(x)}{(1+i\sinh(x))^3} dx$

Optimal. Leaf size=16

$$\frac{i}{2(1+i\sinh(x))^2}$$

```
[Out] 1/2*I/(1+I*sinh(x))^2
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 32}

$$\frac{i}{2(1+i\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 + I*Sinh[x])^3,x]
```

```
[Out] (I/2)/(1 + I*Sinh[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(1+i\sinh(x))^3} dx &= -\left(i \operatorname{Subst}\left(\int \frac{1}{(1+x)^3} dx, x, i\sinh(x)\right)\right)\\ &= \frac{i}{2(1+i\sinh(x))^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 0.88

$$-\frac{i}{2(\sinh(x)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 + I*Sinh[x])^3,x]
```

```
[Out] (-1/2*I)/(-I + Sinh[x])^2
```

fricas [B] time = 0.60, size = 30, normalized size = 1.88

$$-\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}-4i\,e^{(3\,x)}-6\,e^{(2\,x)}+4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*I*e^(2*x)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

giac [A] time = 0.37, size = 12, normalized size = 0.75

$$-\frac{2i\,e^{(2\,x)}}{(e^{x}-i)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*I*e^(2*x)/(e^x - I)^4
```

maple [A] time = 0.02, size = 13, normalized size = 0.81

$$\frac{i}{2\,(1+i\sinh(x))^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1+I*sinh(x))^3,x)
```

```
[Out] 1/2*I/(1+I*sinh(x))^2
```

maxima [A] time = 0.32, size = 10, normalized size = 0.62

$$\frac{i}{2\,(i\,\sinh(x)+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*I/(I*sinh(x) + 1)^2
```

mupad [B] time = 0.65, size = 16, normalized size = 1.00

$$-\frac{e^{2x}\,2i}{(1+e^{x}\,1i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(sinh(x)*1i + 1)^3,x)
```

```
[Out] -(exp(2*x)*2i)/(exp(x)*1i + 1)^4
```

sympy [B] time = 0.15, size = 37, normalized size = 2.31

$$-\frac{2ie^{2x}}{e^{4x}-4ie^{3x}-6e^{2x}+4ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))**3,x)
```

```
[Out] -2*I*exp(2*x)/(exp(4*x) - 4*I*exp(3*x) - 6*exp(2*x) + 4*I*exp(x) + 1)
```

3.184 $\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

```
[Out] -I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2667, 43}

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = i\,\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -i\sinh(x)\right)$$
$$= i\,\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)dx, x, -i\sinh(x)\right)$$
$$= -i\log(i+\sinh(x)) - \frac{2i}{1-i\sinh(x)}$$

Mathematica [A] time = 0.08, size = 45, normalized size = 1.73

$$\frac{-2i\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))-2\sinh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)-i\sinh(x)\log(\cosh(x))+2}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

```
[Out] (2 - (2*I)*ArcTan[Tanh[x/2]] + Log[Cosh[x]] - 2*ArcTan[Tanh[x/2]]*Sinh[x] -
 I*Log[Cosh[x]]*Sinh[x])/(I + Sinh[x])
```

fricas [B] time = 0.48, size = 49, normalized size = 1.88

$$\frac{i\,xe^{(2\,x)}-2\,(x-2)e^{x}+\left(-2i\,e^{(2\,x)}+4\,e^{x}+2i\right)\log\left(e^{x}+i\right)-i\,x}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x + (-2*I*e^(2*x) + 4*e^x + 2*I)*log(e^x + I) -
I*x)/(e^(2*x) + 2*I*e^x - 1)
```

giac [A] time = 0.47, size = 27, normalized size = 1.04

$$\frac{4\,e^{x}}{\left(e^{x}+i\right)^{2}}+i\,\log\left(-i\,e^{x}\right)-2i\,\log\left(i\,e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x + I)^2 + I*log(-I*e^x) - 2*I*log(I*e^x - 1)
```

maple [B] time = 0.08, size = 56, normalized size = 2.15

$$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1-I*sinh(x))^3,x)
```

```
[Out] I*ln(tanh(1/2*x)-1)+I*ln(tanh(1/2*x)+1)+4*I/(tanh(1/2*x)+I)^2-2*I*ln(tanh(1
/2*x)+I)-4/(tanh(1/2*x)+I)
```

maxima [A] time = 0.32, size = 33, normalized size = 1.27

$$-i\,x-\frac{4\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-2i\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

mupad [B] time = 0.16, size = 39, normalized size = 1.50

$$x\,1\mathrm{i}-\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)\,2\mathrm{i}-\frac{4\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}+\frac{4}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^3/(sinh(x)*1i - 1)^3,x)
```

```
[Out] x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1i
)
```

sympy [A] time = 0.15, size = 32, normalized size = 1.23

$$x\left(-2-i\right)+2\log\left(e^{x}+i\right)-\frac{4e^{x}}{-e^{2x}-2ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1-I*sinh(x))**3,x)
```

```
[Out] x*(-2 - I) + 2*log(exp(x) + I) - 4*exp(x)/(-exp(2*x) - 2*I*exp(x) + 1)
```

3.185 $\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx$

Optimal. Leaf size=20

$$-\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

```
[Out] -1/3*I*cosh(x)^3/(1-I*sinh(x))^3
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2671}

$$-\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

```
[Out] ((-I/3)*Cosh[x]^3)/(1 - I*Sinh[x])^3
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

Mathematica [A] time = 0.06, size = 38, normalized size = 1.90

$$\frac{\cosh\left(\frac{3x}{2}\right)-3\cosh\left(\frac{x}{2}\right)}{3\left(\sinh\left(\frac{x}{2}\right)+i\cosh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-3*Cosh[x/2] + Cosh[(3*x)/2])/(3*(I*Cosh[x/2] + Sinh[x/2])^3)
```

fricas [B] time = 0.79, size = 29, normalized size = 1.45

$$\frac{6i\,e^{(2\,x)}-2i}{3\,e^{(3\,x)}+9i\,e^{(2\,x)}-9\,e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (6*I*e^(2*x) - 2*I)/(3*e^(3*x) + 9*I*e^(2*x) - 9*e^x - 3*I)
```

giac [A] time = 0.15, size = 16, normalized size = 0.80

$$-\frac{-6i\,e^{(2\,x)}+2i}{3\,(e^{x}+i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/3*(-6*I*e^(2*x) + 2*I)/(e^x + I)^3
```

maple [B] time = 0.07, size = 36, normalized size = 1.80

$$-\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{2}{\tanh\left(\frac{x}{2}\right)+i}-\frac{8}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1-I*sinh(x))^3,x)
```

```
[Out] -4*I/(tanh(1/2*x)+I)^2+2/(tanh(1/2*x)+I)-8/3/(tanh(1/2*x)+I)^3
```

maxima [B] time = 0.33, size = 53, normalized size = 2.65

$$-\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}+9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}-3}+\frac{2}{-9i\,e^{(-x)}+9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -6*e^(-2*x)/(-9*I*e^(-x) + 9*e^(-2*x) + 3*I*e^(-3*x) - 3) + 2/(-9*I*e^(-x)
+ 9*e^(-2*x) + 3*I*e^(-3*x) - 3)
```

mupad [B] time = 0.59, size = 19, normalized size = 0.95

$$\frac{2\left(3\,\mathrm{e}^{2\,x}-1\right)}{3\,\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^2/(sinh(x)*1i - 1)^3,x)
```

```
[Out] (2*(3*exp(2*x) - 1))/(3*(exp(x)*1i - 1)^3)
```

sympy [A] time = 0.15, size = 31, normalized size = 1.55

$$\frac{6e^{2x}-2}{-3ie^{3x}+9e^{2x}+9ie^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1-I*sinh(x))**3,x)
```

```
[Out] (6*exp(2*x) - 2)/(-3*I*exp(3*x) + 9*exp(2*x) + 9*I*exp(x) - 3)
```

3.186 $\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx$

Optimal. Leaf size=16

$$-\frac{i}{2(1-i\sinh(x))^2}$$

```
[Out] -1/2*I/(1-I*sinh(x))^2
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2667, 32}

$$-\frac{i}{2(1-i\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-I/2)/(1 - I*Sinh[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx &= i\,\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, -i\sinh(x)\right)\\ &= -\frac{i}{2(1-i\sinh(x))^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 0.88

$$\frac{i}{2(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 - I*Sinh[x])^3,x]
[Out] (I/2)/(I + Sinh[x])^2
```

fricas [B] time = 0.58, size = 30, normalized size = 1.88

$$\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="fricas")
[Out] 2*I*e^(2*x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

giac [A] time = 0.19, size = 12, normalized size = 0.75

$$\frac{2i\,e^{(2\,x)}}{(e^{x}+i)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="giac")
[Out] 2*I*e^(2*x)/(e^x + I)^4
```

maple [A] time = 0.03, size = 13, normalized size = 0.81

$$-\frac{i}{2\,(1-i\sinh(x))^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1-I*sinh(x))^3,x)
[Out] -1/2*I/(1-I*sinh(x))^2
```

maxima [A] time = 0.33, size = 10, normalized size = 0.62

$$-\frac{i}{2\,(-i\,\sinh(x)+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*I/(-I*sinh(x) + 1)^2
```

mupad [B] time = 0.20, size = 16, normalized size = 1.00

$$\frac{e^{2x}\,2i}{\left(-1+e^{x}\,1i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)/(sinh(x)*1i - 1)^3,x)
```

```
[Out] (exp(2*x)*2i)/(exp(x)*1i - 1)^4
```

sympy [B] time = 0.15, size = 36, normalized size = 2.25

$$\frac{2ie^{2x}}{e^{4x}+4ie^{3x}-6e^{2x}-4ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))**3,x)
```

```
[Out] 2*I*exp(2*x)/(exp(4*x) + 4*I*exp(3*x) - 6*exp(2*x) - 4*I*exp(x) + 1)
```

3.187 $\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=138

$$\frac{\left(a^2+b^2\right)^3\log(a+b\sinh(x))}{b^7}-\frac{a\left(a^2+3b^2\right)\sinh^3(x)}{3b^4}+\frac{\left(a^2+3b^2\right)\sinh^4(x)}{4b^3}-\frac{a\left(a^4+3a^2b^2+3b^4\right)\sinh(x)}{b^6}+\frac{\left(a^4+3\right.}{}$$

```
[Out] (a^2+b^2)^3*ln(a+b*sinh(x))/b^7-a*(a^4+3*a^2*b^2+3*b^4)*sinh(x)/b^6+1/2*(a^
4+3*a^2*b^2+3*b^4)*sinh(x)^2/b^5-1/3*a*(a^2+3*b^2)*sinh(x)^3/b^4+1/4*(a^2+3
*b^2)*sinh(x)^4/b^3-1/5*a*sinh(x)^5/b^2+1/6*sinh(x)^6/b
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{\left(a^2+3b^2\right)\sinh^4(x)}{4b^3}-\frac{a\left(a^2+3b^2\right)\sinh^3(x)}{3b^4}+\frac{\left(3a^2b^2+a^4+3b^4\right)\sinh^2(x)}{2b^5}-\frac{a\left(3a^2b^2+a^4+3b^4\right)\sinh(x)}{b^6}+\frac{\left(a^2+b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^3*Log[a + b*Sinh[x]])/b^7 - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[
x])/b^6 + ((a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x]^2)/(2*b^5) - (a*(a^2 + 3*b^2)*
Sinh[x]^3)/(3*b^4) + ((a^2 + 3*b^2)*Sinh[x]^4)/(4*b^3) - (a*Sinh[x]^5)/(5*b
^2) + Sinh[x]^6/(6*b)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^3}{a+x}\,dx, x, b\sinh(x)\right)}{b^7}$$

$$= -\frac{\text{Subst}\left(\int \left(a^5\left(1+\frac{3b^2\left(a^2+b^2\right)}{a^4}\right)-\left(a^4+3a^2b^2+3b^4\right)x+a\left(a^2+3b^2\right)x^2-\left(a^2+3b^2\right)x^3+\right.\right.}{b^7}$$

$$= \frac{\left(a^2+b^2\right)^3\log(a+b\sinh(x))}{b^7} - \frac{a\left(a^4+3a^2b^2+3b^4\right)\sinh(x)}{b^6} + \frac{\left(a^4+3a^2b^2+3b^4\right)\sinh^2}{2b^5}$$

Mathematica [A] time = 0.16, size = 121, normalized size = 0.88

$$\frac{30b^2\left(a^2+b^2\right)^2\sinh^2(x)+60\left(a^2+b^2\right)^3\log(a+b\sinh(x))+15b^4\left(a^2+b^2\right)\cosh^4(x)-20ab^3\left(a^2+3b^2\right)\sinh^3(}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] (15*b^4*(a^2 + b^2)*Cosh[x]^4 + 10*b^6*Cosh[x]^6 + 60*(a^2 + b^2)^3*Log[a +
 b*Sinh[x]] - 60*a*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x] + 30*b^2*(a^2 + b^2)
^2*Sinh[x]^2 - 20*a*b^3*(a^2 + 3*b^2)*Sinh[x]^3 - 12*a*b^5*Sinh[x]^5)/(60*b
^7)
```

fricas [B] time = 0.62, size = 2105, normalized size = 15.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*b
^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^10 + 6*(55*b^
6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 + 10*b^6)*sinh(x)^10 - 20*(4*a^3
*b^3 + 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 - 4*a
^3*b^3 - 9*a*b^5 + 15*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^4*b^2
 + 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5*cosh(
x)^3 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 + 2*b^6)*cosh(x)^2 -
12*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*x*cosh(x)^6 - 120*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^7 +
 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b - 22*a^3*b^3 - 19*a*b
```

```
^5 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + (
16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 + 12*a*b^5*cosh(x) + 1
2*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 + 2*b^6)*cosh(x)^
4 - 140*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*
b^6)*cosh(x)^2 - 160*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 70*(8*a^5*b +
22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 + 120*(8*a^5*b + 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b^5*cosh(x)^6 + 40
*a^5*b + 110*a^3*b^3 + 95*a*b^5 + 315*(a^2*b^4 + 2*b^6)*cosh(x)^5 - 105*(4*
a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x
)^3 - 480*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x) - 105*(8*a^5*b + 22
*a^3*b^3 + 19*a*b^5)*cosh(x)^2)*sinh(x)^5 + 15*(16*a^4*b^2 + 40*a^2*b^4 + 2
9*b^6)*cosh(x)^4 + 15*(165*b^6*cosh(x)^8 - 264*a*b^5*cosh(x)^7 + 420*(a^2*b
^4 + 2*b^6)*cosh(x)^6 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 - 168*(4*a^3*b^3 +
 9*a*b^5)*cosh(x)^5 + 70*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^4 - 192
0*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 280*(8*a^5*b + 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^3 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x))*si
nh(x)^4 + 20*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 20*(55*b^6*cosh(x)^9 - 99*a*
b^5*cosh(x)^8 + 180*(a^2*b^4 + 2*b^6)*cosh(x)^7 - 84*(4*a^3*b^3 + 9*a*b^5)*
cosh(x)^6 + 4*a^3*b^3 + 9*a*b^5 + 42*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cos
h(x)^5 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 210*(8*a^5*
b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 + 60*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)
*cosh(x)^2 + 3*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^3 + 30*(
a^2*b^4 + 2*b^6)*cosh(x)^2 + 30*(11*b^6*cosh(x)^10 - 22*a*b^5*cosh(x)^9 + 4
5*(a^2*b^4 + 2*b^6)*cosh(x)^8 - 24*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^7 + 14*(16
*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^6 + a^2*b^4 + 2*b^6 - 960*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^4 - 84*(8*a^5*b + 22*a^3*b^3 + 19*a*b^
5)*cosh(x)^5 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 + 3*(16*a^4*b
^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^2 + 2*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh
(x)^2 + 1920*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5*sinh(x) + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^4*sinh(x)^2 + 20*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(
x)^3*sinh(x)^3 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2*sinh(x)^4
 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*sinh(x)^6)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)))
 + 12*(5*b^6*cosh(x)^11 - 11*a*b^5*cosh(x)^10 + 25*(a^2*b^4 + 2*b^6)*cosh(x
)^9 - 15*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^8 + 10*(16*a^4*b^2 + 40*a^2*b^4 + 29
*b^6)*cosh(x)^7 - 960*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 70*
(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^6 + a*b^5 + 50*(8*a^5*b + 22*a^3*
b^3 + 19*a*b^5)*cosh(x)^4 + 5*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^3
+ 5*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + 5*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x))
/(b^7*cosh(x)^6 + 6*b^7*cosh(x)^5*sinh(x) + 15*b^7*cosh(x)^4*sinh(x)^2 + 20
*b^7*cosh(x)^3*sinh(x)^3 + 15*b^7*cosh(x)^2*sinh(x)^4 + 6*b^7*cosh(x)*sinh(
x)^5 + b^7*sinh(x)^6)
```

giac [A] time = 0.23, size = 254, normalized size = 1.84

$$\frac{5\,b^5\left(e^{(-x)}-e^x\right)^6+12\,ab^4\left(e^{(-x)}-e^x\right)^5+30\,a^2b^3\left(e^{(-x)}-e^x\right)^4+90\,b^5\left(e^{(-x)}-e^x\right)^4+80\,a^3b^2\left(e^{(-x)}-e^x\right)^3+240\,ab^4(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^5*(e^(-x) - e^x)^6 + 12*a*b^4*(e^(-x) - e^x)^5 + 30*a^2*b^3*(e^
(-x) - e^x)^4 + 90*b^5*(e^(-x) - e^x)^4 + 80*a^3*b^2*(e^(-x) - e^x)^3 + 240
*a*b^4*(e^(-x) - e^x)^3 + 240*a^4*b*(e^(-x) - e^x)^2 + 720*a^2*b^3*(e^(-x)
- e^x)^2 + 720*b^5*(e^(-x) - e^x)^2 + 960*a^5*(e^(-x) - e^x) + 2880*a^3*b^2
*(e^(-x) - e^x) + 2880*a*b^4*(e^(-x) - e^x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^7
```

maple [B] time = 0.06, size = 837, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(a+b*sinh(x)),x)
```

```
[Out] 1/6/b/(tanh(1/2*x)-1)^6+1/6/b/(tanh(1/2*x)+1)^6+1/b*ln(a*tanh(1/2*x)^2-2*ta
nh(1/2*x)*b-a)+1/2/b/(tanh(1/2*x)-1)^5+9/8/b/(tanh(1/2*x)-1)^4-1/2/b/(tanh(
1/2*x)+1)^5+9/8/b/(tanh(1/2*x)+1)^4+17/12/b/(tanh(1/2*x)-1)^3+29/16/b/(tanh
(1/2*x)-1)^2+19/16/b/(tanh(1/2*x)-1)-17/12/b/(tanh(1/2*x)+1)^3+29/16/b/(tan
h(1/2*x)+1)^2-19/16/b/(tanh(1/2*x)+1)-1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2
*x)+1)-3/b^3*ln(tanh(1/2*x)-1)*a^2-3/b^3*ln(tanh(1/2*x)+1)*a^2+1/b^6/(tanh(
1/2*x)+1)*a^5-1/2/b^5/(tanh(1/2*x)+1)*a^4+3/b^4/(tanh(1/2*x)+1)*a^3+1/5/b^2
/(tanh(1/2*x)+1)^5*a+1/4/b^3/(tanh(1/2*x)+1)^4*a^2-1/2/b^2/(tanh(1/2*x)+1)^
4*a+1/3/b^4/(tanh(1/2*x)+1)^3*a^3-1/2/b^3/(tanh(1/2*x)+1)^3*a^2+5/4/b^2/(ta
nh(1/2*x)+1)^3*a-1/b^7*ln(tanh(1/2*x)+1)*a^6-3/b^5*ln(tanh(1/2*x)+1)*a^4+1/
2/b^5/(tanh(1/2*x)+1)^2*a^4-1/2/b^4/(tanh(1/2*x)+1)^2*a^3+13/8/b^3/(tanh(1/
2*x)+1)^2*a^2+1/3/b^4/(tanh(1/2*x)-1)^3*a^3+1/2/b^3/(tanh(1/2*x)-1)^3*a^2+5
/4/b^2/(tanh(1/2*x)-1)^3*a-1/b^7*ln(tanh(1/2*x)-1)*a^6-3/b^5*ln(tanh(1/2*x)
-1)*a^4+1/2/b^5/(tanh(1/2*x)-1)^2*a^4+1/2/b^4/(tanh(1/2*x)-1)^2*a^3+13/8/b^
3/(tanh(1/2*x)-1)^2*a^2+1/b^6/(tanh(1/2*x)-1)*a^5+1/2/b^5/(tanh(1/2*x)-1)*a
^4+3/b^4/(tanh(1/2*x)-1)*a^3+1/b^7*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^
6+3/b^5*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^4+3/b^3*ln(a*tanh(1/2*x)^2-
2*tanh(1/2*x)*b-a)*a^2+1/5/b^2/(tanh(1/2*x)-1)^5*a+1/4/b^3/(tanh(1/2*x)-1)^
4*a^2+1/2/b^2/(tanh(1/2*x)-1)^4*a+3/b^2/(tanh(1/2*x)+1)*a+11/8/b^2/(tanh(1/
2*x)-1)^2*a+11/8/b^3/(tanh(1/2*x)-1)*a^2+3/b^2/(tanh(1/2*x)-1)*a-11/8/b^2/(
tanh(1/2*x)+1)^2*a-11/8/b^3/(tanh(1/2*x)+1)*a^2
```

maxima [B] time = 0.34, size = 308, normalized size = 2.23

$$-\frac{\left(12\,ab^4e^{(-x)}-5\,b^5-30\,\left(a^2b^3+2\,b^5\right)e^{(-2\,x)}+20\,\left(4\,a^3b^2+9\,ab^4\right)e^{(-3\,x)}-15\,\left(16\,a^4b+40\,a^2b^3+29\,b^5\right)e^{(-4\,x)}+12}{1920\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 + 2*b^5)*e^(-2*x) + 20*(4*a^
3*b^2 + 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-4*x) +
120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 + 1/1920*(12*a*b^
4*e^(-5*x) + 5*b^5*e^(-6*x) + 120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-x) +
15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 + 9*a*b^4)*e^(
-3*x) + 30*(a^2*b^3 + 2*b^5)*e^(-4*x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*x/b^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*a*e^(-x) + b*e^(-2
*x) - b)/b^7
```

mupad [B] time = 1.32, size = 287, normalized size = 2.08

$$\frac{e^{-6\,x}}{384\,b}+\frac{e^{6\,x}}{384\,b}+\frac{e^{-x}\,\left(8\,a^5+22\,a^3\,b^2+19\,a\,b^4\right)}{16\,b^6}+\frac{e^{-3\,x}\,\left(4\,a^3+9\,a\,b^2\right)}{96\,b^4}-\frac{e^{3\,x}\,\left(4\,a^3+9\,a\,b^2\right)}{96\,b^4}+\frac{e^{-4\,x}\,\left(a^2+2\,b^2\right)}{64\,b^3}+\frac{e^{4\,x}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(a + b*sinh(x)),x)
```

```
[Out] exp(-6*x)/(384*b) + exp(6*x)/(384*b) + (exp(-x)*(19*a*b^4 + 8*a^5 + 22*a^3*
b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 + 4*a^3))/(96*b^4) - (exp(3*x)*(9*a*b^
2 + 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 + 2*b^2))/(64*b^3) + (exp(4*x)*(a^2
+ 2*b^2))/(64*b^3) + (a*exp(-5*x))/(160*b^2) - (a*exp(5*x))/(160*b^2) - (x*
(a^2 + b^2)^3)/b^7 + (exp(-2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b^5) +
 (exp(2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*b^4 +
8*a^5 + 22*a^3*b^2))/(16*b^6) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2))/b^7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**7/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.188 $\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=145

$$-\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right)\sinh(x)\right)}{8b^5}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3\right.}{12b^3}$$

```
[Out] -1/8*a*(8*a^4+20*a^2*b^2+15*b^4)*x/b^6-2*(a^2+b^2)^(5/2)*arctanh((b-a*tanh(
1/2*x))/(a^2+b^2)^(1/2))/b^6+1/5*cosh(x)^5/b+1/12*cosh(x)^3*(4*a^2+4*b^2-3*
a*b*sinh(x))/b^3+1/8*cosh(x)*(8*(a^2+b^2)^2-a*b*(4*a^2+7*b^2)*sinh(x))/b^5
```

Rubi [A] time = 0.41, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2695, 2865, 2735, 2660, 618, 206}

$$-\frac{ax\left(20a^2b^2+8a^4+15b^4\right)}{8b^6}-\frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] -(a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*x)/(8*b^6) - (2*(a^2 + b^2)^(5/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/b^6 + Cosh[x]^5/(5*b) + (Cosh[x]^3*(4*
(a^2 + b^2) - 3*a*b*Sinh[x]))/(12*b^3) + (Cosh[x]*(8*(a^2 + b^2)^2 - a*b*(4
*a^2 + 7*b^2)*Sinh[x]))/(8*b^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \frac{\cosh^5(x)}{5b} + \frac{i\int \frac{\cosh^4(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3} - \frac{i\int \frac{\cosh^2(x)\left(ib\left(a^2+4b^2\right)-ia\left(4a^2+7b^2\right)\sinh(x)\right)}{a+b\sinh(x)}\,d}{4b^3}$$

$$= \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh(x)\left(8\left(a^2+b^2\right)^2-ab\left(4a^2+7b^2\right.\right.}{8b^5}$$

$$= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh(x}{}$$

$$= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh(x}{}$$

$$= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4\left(a^2+b^2\right)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh(x}{}$$

$$= -\frac{a\left(8a^4+20a^2b^2+15b^4\right)x}{8b^6} - \frac{2\left(a^2+b^2\right)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)}{}$$

Mathematica [C] time = 5.83, size = 463, normalized size = 3.19

$$\frac{\cosh(x)\left(8b^2\left(5a^2+11b^2\right)\sinh^2(x)-15ab\left(4a^2+9b^2\right)\sinh(x)-\frac{240\left(a^2+b^2\right)^2\tanh^{-1}\left(\frac{\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}}{\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}}\right)}{\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}}+8\left(15a^4+35a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(8*(15*a^4 + 35*a^2*b^2 + 23*b^4) - 15*a*b*(4*a^2 + 9*b^2)*Sinh[x]
 + 8*b^2*(5*a^2 + 11*b^2)*Sinh[x]^2 - 30*a*b^3*Sinh[x]^3 + 24*b^4*Sinh[x]^4
 - (30*(-1)^(3/4)*Sqrt[b]*(8*a^4 - (4*I)*a^3*b + 16*a^2*b^2 - (7*I)*a*b^3 +
 8*b^4)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b
))])/Sqrt[b]])/(Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))]) - (240*(a^2 + b^2)^2*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/(a - I*b
```

```
))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]])/(Sqrt[-((b*(-I + Sinh[x]))/(a +
 I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]) + (240*(a - I*b)^(5/2)*(a + I
*b)^(3/2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqr
t[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])])/(Sqrt[-((b*(-I + Sinh[x
]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(120*b^5)
```

fricas [B] time = 0.67, size = 1486, normalized size = 10.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^5
*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 + 7*b^5)*cosh(x)^8 + 5*(54*b^5*
cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 + 14*b^5)*sinh(x)^8 - 120*(a^3*b^2
 + 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*b
^2 - 12*a*b^4 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b + 54*a^2*b^3 + 3
3*b^5 + 14*(4*a^2*b^3 + 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 + 2*a*b^4)*cosh(x))*
sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4 +
 280*(4*a^2*b^3 + 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 6
0*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*c
osh(x))*sinh(x)^5 + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^4 +
10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 + 48*a^4*b + 108*a^2*b^3 + 66*b
^5 + 70*(4*a^2*b^3 + 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 -
 60*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b + 18*a^2*b^3 +
11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 20*(36*b
^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 + 7*b^5)*cosh(x)^5 + 6*a^
3*b^2 + 12*a*b^4 - 210*(a^3*b^2 + 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 + 20*a^3*b
^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 +
 12*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 + 10*(4*a^2*b^3 + 7*
b^5)*cosh(x)^2 + 10*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3
+ 7*b^5)*cosh(x)^6 - 252*(a^3*b^2 + 2*a*b^4)*cosh(x)^5 + 4*a^2*b^3 + 7*b^5
- 120*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b + 18*a^2*b^
3 + 11*b^5)*cosh(x)^4 + 36*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*(
a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x)^2 + 960*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^5 + 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)^3*sinh(x)^2 + 10*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 +
5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^5)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + 5*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b^
```

```
3 + 7*b^5)*cosh(x)^7 - 168*(a^3*b^2 + 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^4 + 72*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^5 + 3*a*b^4 + 48*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2
+ 2*a*b^4)*cosh(x)^2 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x)
^5 + 5*b^6*cosh(x)^4*sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)^
2*sinh(x)^3 + 5*b^6*cosh(x)*sinh(x)^4 + b^6*sinh(x)^5)
```

giac [B] time = 0.47, size = 288, normalized size = 1.99

$$\frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} + 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} - 240\,ab^3e^{(2\,x)} + 480\,a^4e^x + 1080\,a^2b^2e^x + 660\,b^4}{960\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) + 70*b^4*e^(3*
x) - 120*a^3*b*e^(2*x) - 240*a*b^3*e^(2*x) + 480*a^4*e^x + 1080*a^2*b^2*e^x
 + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*(15
*a*b^4*e^x + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(a^3*
b^2 + 2*a*b^4)*e^(3*x) + 10*(4*a^2*b^3 + 7*b^5)*e^(2*x))*e^(-5*x)/b^6 + (a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

maple [B] time = 0.06, size = 674, normalized size = 4.65

$$\frac{2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)-1\right)^5} - \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^4} + \frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)+1\right)^5} - \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} + \frac{6a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(a+b*sinh(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+6*a^2/
b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/5/
b/(tanh(1/2*x)-1)^5-1/2/b/(tanh(1/2*x)-1)^4+1/5/b/(tanh(1/2*x)+1)^5-1/2/b/(
tanh(1/2*x)+1)^4+2/b^6/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a
^2+b^2)^(1/2))*a^6-13/12/b/(tanh(1/2*x)-1)^3-9/8/b/(tanh(1/2*x)-1)^2-15/8/b
/(tanh(1/2*x)-1)+13/12/b/(tanh(1/2*x)+1)^3-9/8/b/(tanh(1/2*x)+1)^2+15/8/b/(
tanh(1/2*x)+1)+6*a^4/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/
(a^2+b^2)^(1/2))+1/b^5/(tanh(1/2*x)+1)*a^4-1/2/b^4/(tanh(1/2*x)+1)*a^3+1/4/
b^2/(tanh(1/2*x)+1)^4*a+1/3/b^3/(tanh(1/2*x)+1)^3*a^2-1/2/b^2/(tanh(1/2*x)+
1)^3*a+1/2/b^4/(tanh(1/2*x)+1)^2*a^3-1/2/b^3/(tanh(1/2*x)+1)^2*a^2-1/3/b^3/
```

```
(tanh(1/2*x)-1)^3*a^2-1/2/b^2/(tanh(1/2*x)-1)^3*a-1/2/b^4/(tanh(1/2*x)-1)^2
*a^3-1/2/b^3/(tanh(1/2*x)-1)^2*a^2-1/b^5/(tanh(1/2*x)-1)*a^4-1/2/b^4/(tanh(
1/2*x)-1)*a^3-1/4/b^2/(tanh(1/2*x)-1)^4*a-a^5/b^6*ln(tanh(1/2*x)+1)+a^5/b^6
*ln(tanh(1/2*x)-1)-9/8/b^2/(tanh(1/2*x)+1)*a-5/2*a^3/b^4*ln(tanh(1/2*x)+1)-
15/8*a/b^2*ln(tanh(1/2*x)+1)-11/8/b^2/(tanh(1/2*x)-1)^2*a-5/2/b^3/(tanh(1/2
*x)-1)*a^2-9/8/b^2/(tanh(1/2*x)-1)*a+5/2*a^3/b^4*ln(tanh(1/2*x)-1)+15/8*a/b
^2*ln(tanh(1/2*x)-1)+11/8/b^2/(tanh(1/2*x)+1)^2*a+5/2/b^3/(tanh(1/2*x)+1)*a
^2
```

maxima [B] time = 0.43, size = 283, normalized size = 1.95

$$-\frac{\left(15\,ab^3e^{(-x)}-6\,b^4-10\left(4\,a^2b^2+7\,b^4\right)e^{(-2\,x)}+120\left(a^3b+2\,ab^3\right)e^{(-3\,x)}-60\left(8\,a^4+18\,a^2b^2+11\,b^4\right)e^{(-4\,x)}\right)e^{(5\,x)}}{960\,b^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/960*(15*a*b^3*e^(-x) - 6*b^4 - 10*(4*a^2*b^2 + 7*b^4)*e^(-2*x) + 120*(a^
3*b + 2*a*b^3)*e^(-3*x) - 60*(8*a^4 + 18*a^2*b^2 + 11*b^4)*e^(-4*x))*e^(5*x
)/b^5 + 1/960*(15*a*b^3*e^(-4*x) + 6*b^4*e^(-5*x) + 60*(8*a^4 + 18*a^2*b^2
+ 11*b^4)*e^(-x) + 120*(a^3*b + 2*a*b^3)*e^(-2*x) + 10*(4*a^2*b^2 + 7*b^4)*
e^(-3*x))/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + (a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

mupad [B] time = 1.24, size = 302, normalized size = 2.08

$$\frac{e^{-5\,x}}{160\,b}+\frac{e^{5\,x}}{160\,b}-\frac{\ln\left(-\frac{2\,e^x\,\left(a^2+b^2\right)^3}{b^7}-\frac{2\,\left(b-a\,e^x\right)\,\left(a^2+b^2\right)^{5/2}}{b^7}\right)\,\left(a^2+b^2\right)^{5/2}}{b^6}+\frac{\ln\left(\frac{2\,\left(b-a\,e^x\right)\,\left(a^2+b^2\right)^{5/2}}{b^7}-\frac{2\,e^x\,\left(a^2+b^2\right)^3}{b^7}\right)\,\left(a^2+b^2\right)^{5/2}}{b^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(a + b*sinh(x)),x)
```

```
[Out] exp(-5*x)/(160*b) + exp(5*x)/(160*b) - (log(- (2*exp(x)*(a^2 + b^2)^3)/b^7
- (2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7)*(a^2 + b^2)^(5/2))/b^6 + (log((
2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7 - (2*exp(x)*(a^2 + b^2)^3)/b^7)*(a^
2 + b^2)^(5/2))/b^6 - (x*(15*a*b^4 + 8*a^5 + 20*a^3*b^2))/(8*b^6) + (exp(x)
*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x))/(64*b^2) - (a*exp(
4*x))/(64*b^2) + (exp(-x)*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (exp(-3
*x)*(4*a^2 + 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 + 7*b^2))/(96*b^3) + (exp(
-2*x)*(2*a*b^2 + a^3))/(8*b^4) - (exp(2*x)*(2*a*b^2 + a^3))/(8*b^4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.189 $\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a\sinh^3(x)}{3b^2}+\frac{\sinh^4(x)}{4b}$$

```
[Out] (a^2+b^2)^2*ln(a+b*sinh(x))/b^5-a*(a^2+2*b^2)*sinh(x)/b^4+1/2*(a^2+2*b^2)*s
inh(x)^2/b^3-1/3*a*sinh(x)^3/b^2+1/4*sinh(x)^4/b
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a\sinh^3(x)}{3b^2}+\frac{\sinh^4(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^2*Log[a + b*Sinh[x]])/b^5 - (a*(a^2 + 2*b^2)*Sinh[x])/b^4 + ((
a^2 + 2*b^2)*Sinh[x]^2)/(2*b^3) - (a*Sinh[x]^3)/(3*b^2) + Sinh[x]^4/(4*b)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^2}{a+x}\,dx, x, b\sinh(x)\right)}{b^5}$$
$$= \frac{\text{Subst}\left(\int \left(-a\left(a^2+2b^2\right)+\left(a^2+2b^2\right)x-ax^2+x^3+\frac{\left(a^2+b^2\right)^2}{a+x}\right)dx, x, b\sinh(x)\right)}{b^5}$$
$$= \frac{\left(a^2+b^2\right)^2\log(a+b\sinh(x))}{b^5}-\frac{a\left(a^2+2b^2\right)\sinh(x)}{b^4}+\frac{\left(a^2+2b^2\right)\sinh^2(x)}{2b^3}-\frac{a\sinh^3(x)}{3b^2}$$

Mathematica [A] time = 0.09, size = 76, normalized size = 0.94

$$\frac{6b^2\left(a^2+b^2\right)\sinh^2(x)-12ab\left(a^2+2b^2\right)\sinh(x)+12\left(a^2+b^2\right)^2\log(a+b\sinh(x))-4ab^3\sinh^3(x)+3b^4\cosh^4}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (3*b^4*Cosh[x]^4 + 12*(a^2 + b^2)^2*Log[a + b*Sinh[x]] - 12*a*b*(a^2 + 2*b^
2)*Sinh[x] + 6*b^2*(a^2 + b^2)*Sinh[x]^2 - 4*a*b^3*Sinh[x]^3)/(12*b^5)
```

fricas [B] time = 0.70, size = 865, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*cos
h(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^6 + 4*(21*b^4*cosh
(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 + 9*b^4)*sinh(x)^6 - 192*(a^4 + 2*a^2*
b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b + 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*cosh(
x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b - 7*a*b^3 + 3*(2*a^2*b^2 + 3*b^4)*cosh(x
))*sinh(x)^5 + 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cosh(x)^3
 + 90*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 - 96*(a^4 + 2*a^2*b^2 + b^4)*x - 60*(4*
a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 + 24*(4*a^3*b + 7*a*b^3)*cosh(x
)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 + 12*a^3*b + 21*a*b^3 + 30*(
2*a^2*b^2 + 3*b^4)*cosh(x)^3 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x) - 30*(4
*a^3*b + 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 +
 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 + 3*b^4)*cosh(x)^
```

```
4 + 2*a^2*b^2 + 3*b^4 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 20*(4*a^3*
b + 7*a*b^3)*cosh(x)^3 + 6*(4*a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3*sinh(
x) + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^4)*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3*cosh(x)^6 + 9*(2*
a^2*b^2 + 3*b^4)*cosh(x)^5 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^3 - 15*(4
*a^3*b + 7*a*b^3)*cosh(x)^4 + a*b^3 + 9*(4*a^3*b + 7*a*b^3)*cosh(x)^2 + 3*(
2*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(
x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)
```

giac [A] time = 0.49, size = 139, normalized size = 1.72

$$\frac{3\,b^3\left(e^{(-x)}-e^x\right)^4+8\,ab^2\left(e^{(-x)}-e^x\right)^3+24\,a^2b\left(e^{(-x)}-e^x\right)^2+48\,b^3\left(e^{(-x)}-e^x\right)^2+96\,a^3\left(e^{(-x)}-e^x\right)+192\,ab^2\left(e^{(-x)}-\right.}{192\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b^3*(e^(-x) - e^x)^4 + 8*a*b^2*(e^(-x) - e^x)^3 + 24*a^2*b*(e^(-x)
 - e^x)^2 + 48*b^3*(e^(-x) - e^x)^2 + 96*a^3*(e^(-x) - e^x) + 192*a*b^2*(e^
(-x) - e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(-b*(e^(-x) - e^x) + 2*a)
)/b^5
```

maple [B] time = 0.05, size = 447, normalized size = 5.52

$$\frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{9}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{7}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+b*sinh(x)),x)
```

```
[Out] 1/b*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+1/4/b/(tanh(1/2*x)-1)^4+1/4/b/(ta
nh(1/2*x)+1)^4+1/2/b/(tanh(1/2*x)-1)^3+9/8/b/(tanh(1/2*x)-1)^2+7/8/b/(tanh(
1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^3+9/8/b/(tanh(1/2*x)+1)^2-7/8/b/(tanh(1/2*x
)+1)-1/b*ln(tanh(1/2*x)-1)-1/b*ln(tanh(1/2*x)+1)-2/b^3*ln(tanh(1/2*x)-1)*a^
2-2/b^3*ln(tanh(1/2*x)+1)*a^2+1/b^4/(tanh(1/2*x)+1)*a^3+1/3/b^2/(tanh(1/2*x
)+1)^3*a-1/b^5*ln(tanh(1/2*x)+1)*a^4+1/2/b^3/(tanh(1/2*x)+1)^2*a^2+1/3/b^2/
(tanh(1/2*x)-1)^3*a-1/b^5*ln(tanh(1/2*x)-1)*a^4+1/2/b^3/(tanh(1/2*x)-1)^2*a
^2+1/b^4/(tanh(1/2*x)-1)*a^3+1/b^5*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^
4+2/b^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2+2/b^2/(tanh(1/2*x)+1)*a+1
/2/b^2/(tanh(1/2*x)-1)^2*a+1/2/b^3/(tanh(1/2*x)-1)*a^2+2/b^2/(tanh(1/2*x)-1
)*a-1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2/b^3/(tanh(1/2*x)+1)*a^2
```

maxima [B] time = 0.32, size = 180, normalized size = 2.22

$$-\frac{\left(8\,ab^2e^{(-x)}-3\,b^3-12\,\left(2\,a^2b+3\,b^3\right)e^{(-2\,x)}+24\,\left(4\,a^3+7\,ab^2\right)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4}+\frac{8\,ab^2e^{(-3\,x)}+3\,b^3e^{(-4\,x)}+24\,\left(4\,a^3+\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b + 3*b^3)*e^(-2*x) + 24*(4*a^3
+ 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 + 1/192*(8*a*b^2*e^(-3*x) + 3*b^3*e^(-4*x)
 + 24*(4*a^3 + 7*a*b^2)*e^(-x) + 12*(2*a^2*b + 3*b^3)*e^(-2*x))/b^4 + (a^4
+ 2*a^2*b^2 + b^4)*x/b^5 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*a*e^(-x) + b*e^(-
2*x) - b)/b^5
```

mupad [B] time = 0.88, size = 169, normalized size = 2.09

$$\frac{e^{-4\,x}}{64\,b}+\frac{e^{4\,x}}{64\,b}+\frac{\ln\left(2\,a\,e^x-b+b\,e^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5}+\frac{e^{-x}\,\left(4\,a^3+7\,a\,b^2\right)}{8\,b^4}+\frac{a\,e^{-3\,x}}{24\,b^2}-\frac{a\,e^{3\,x}}{24\,b^2}-\frac{x\,\left(a^2+b^2\right)^2}{b^5}+\frac{e^{-2\,x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a + b*sinh(x)),x)
```

```
[Out] exp(-4*x)/(64*b) + exp(4*x)/(64*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^4
 + b^4 + 2*a^2*b^2))/b^5 + (exp(-x)*(7*a*b^2 + 4*a^3))/(8*b^4) + (a*exp(-3*
x))/(24*b^2) - (a*exp(3*x))/(24*b^2) - (x*(a^2 + b^2)^2)/b^5 + (exp(-2*x)*(
2*a^2 + 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 + 3*b^2))/(16*b^3) - (exp(x)*(7
*a*b^2 + 4*a^3))/(8*b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.190 $\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=97

$$-\frac{ax\left(2a^2+3b^2\right)}{2b^4}-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}+\frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3}+\frac{\cosh^3(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2+3*b^2)*x/b^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*tanh(1/2*x))/(a^2
+b^2)^(1/2))/b^4+1/3*cosh(x)^3/b+1/2*cosh(x)*(2*a^2+2*b^2-a*b*sinh(x))/b^3
```

Rubi [A] time = 0.24, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2695, 2865, 2735, 2660, 618, 206}

$$-\frac{ax\left(2a^2+3b^2\right)}{2b^4}-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}+\frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3}+\frac{\cosh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -(a*(2*a^2 + 3*b^2)*x)/(2*b^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tanh[x
/2])/Sqrt[a^2 + b^2]])/b^4 + Cosh[x]^3/(3*b) + (Cosh[x]*(2*(a^2 + b^2) - a*
b*Sinh[x]))/(2*b^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{\cosh^3(x)}{3b} + \frac{i\int \frac{\cosh^2(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}$$
$$= \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} - \frac{i\int \frac{ib\left(a^2+2b^2\right)-ia\left(2a^2+3b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}$$
$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} + \frac{\left(a^2+b^2\right)^2\int \frac{1}{a+b\sinh(x}$$
$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} + \frac{\left(2\left(a^2+b^2\right)^2\right)\text{Subst}}{}$$
$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-ab\sinh(x)\right)}{2b^3} - \frac{\left(4\left(a^2+b^2\right)^2\right)\text{Subst}}{}$$
$$= -\frac{a\left(2a^2+3b^2\right)x}{2b^4} - \frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2\left(a^2+b^2\right)-\right.}{2b^3}$$

Mathematica [C] time = 2.25, size = 553, normalized size = 5.70

$$\frac{\cosh^3(x)\left(\sqrt{a+ib}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(2\sqrt{a-ib}\left(3a^2+4b^2\right)\sqrt{1+i\sinh(x)}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}+(3-3i)\sqrt{2}\sqrt{b}\left(2a^2-iab\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]^3*(-12*Sqrt[a - I*b]*Sqrt[a + I*b]*(a^2 + b^2)*ArcTanh[Sqrt[-((b*(
I + Sinh[x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I
*Sinh[x]] + 12*(a - I*b)^2*(a + I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I +
Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]
)]*Sqrt[1 + I*Sinh[x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))
]*((3 - 3*I)*Sqrt[2]*Sqrt[b]*(2*a^2 - I*a*b + 2*b^2)*ArcSin[((1/2 + I/2)*Sq
rt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*Sqrt[a - I*b
]*(3*a^2 + 4*b^2)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]
- 3*a*Sqrt[a - I*b]*b*Sqrt[1 + I*Sinh[x]]*Sinh[x]*Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))] + 2*Sqrt[a - I*b]*b^2*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(
I + Sinh[x]))/(a - I*b))])))/(6*(a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sqrt[1 +
```

```
I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b)))^(3/2)*(-((b*(I + Sinh[x]))/(a
- I*b)))^(3/2))
```

fricas [B] time = 1.35, size = 569, normalized size = 5.87

$$\frac{b^3\cosh(x)^6 + b^3\sinh(x)^6 - 3\,ab^2\cosh(x)^5 + 3\left(2\,b^3\cosh(x) - ab^2\right)\sinh(x)^5 - 12\left(2\,a^3 + 3\,ab^2\right)x\cosh(x)^3 + 3\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
- a*b^2)*sinh(x)^5 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b + 5*b^3)
*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b + 5*b^3)*sinh(x
)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2*a^3
 + 3*a*b^2)*x + 6*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x)^3 + b^3 + 3*(4*a^2*b +
 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 + 4*a^2*b + 5*b
^3 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b + 5*b^3)*cosh(x)^2)*sinh(x
)^2 + 24*((a^2 + b^2)*cosh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2*sinh(x) + 3*(a^2
+ b^2)*cosh(x)*sinh(x)^2 + (a^2 + b^2)*sinh(x)^3)*sqrt(a^2 + b^2)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) +
a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 3*(2*b^3*co
sh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*b
 + 5*b^3)*cosh(x)^3 + a*b^2 + 2*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x))/(b^4*co
sh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3
)
```

giac [A] time = 0.23, size = 168, normalized size = 1.73

$$\frac{b^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^2e^x + 15\,b^2e^x}{24\,b^3} - \frac{\left(2\,a^3 + 3\,ab^2\right)x}{2\,b^4} + \frac{\left(3\,ab^2e^x + b^3 + 3\left(4\,a^2b + 5\,b^3\right)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4} + \frac{\left(a^4 + 2\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 15*b^2*e^x)/b^3 - 1/2*(2*a
^3 + 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x + b^3 + 3*(4*a^2*b + 5*b^3)*e^(2*x)
)*e^(-3*x)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2
 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

maple [B] time = 0.05, size = 336, normalized size = 3.46

$$\frac{2a^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}+\frac{4a^2 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{}{2b^2\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sinh(x)),x)
```

```
[Out] 2*a^4/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)
)+4*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/
2))+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/
3/b/(tanh(1/2*x)-1)^3-1/2/b^2/(tanh(1/2*x)-1)^2*a-1/2/b/(tanh(1/2*x)-1)^2-1
/b^3/(tanh(1/2*x)-1)*a^2-1/2/b^2/(tanh(1/2*x)-1)*a-3/2/b/(tanh(1/2*x)-1)+a^
3/b^4*ln(tanh(1/2*x)-1)+3/2*a/b^2*ln(tanh(1/2*x)-1)+1/3/b/(tanh(1/2*x)+1)^3
+1/2/b^2/(tanh(1/2*x)+1)^2*a-1/2/b/(tanh(1/2*x)+1)^2+1/b^3/(tanh(1/2*x)+1)*
a^2-1/2/b^2/(tanh(1/2*x)+1)*a+3/2/b/(tanh(1/2*x)+1)-a^3/b^4*ln(tanh(1/2*x)+
1)-3/2*a/b^2*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.41, size = 170, normalized size = 1.75

$$-\frac{\left(3\,abe^{(-x)}-b^2-3\left(4\,a^2+5\,b^2\right)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,b^3}+\frac{3\,abe^{(-2\,x)}+b^2e^{(-3\,x)}+3\left(4\,a^2+5\,b^2\right)e^{(-x)}}{24\,b^3}-\frac{\left(2\,a^3+3\,ab^2\right)x}{2\,b^4}+\frac{\left(a^4+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 + 5*b^2)*e^(-2*x))*e^(3*x)/b^3 + 1/24*
(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 + 5*b^2)*e^(-x))/b^3 - 1/2*(2*a^3
 + 3*a*b^2)*x/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - sqrt(a^2 +
b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

mupad [B] time = 0.86, size = 200, normalized size = 2.06

$$\frac{e^{-3\,x}}{24\,b}+\frac{e^{3\,x}}{24\,b}-\frac{\ln\left(-\frac{2\,e^x\left(a^2+b^2\right)^2}{b^5}-\frac{2\,(b-a\,e^x)\left(a^2+b^2\right)^{3/2}}{b^5}\right)\left(a^2+b^2\right)^{3/2}}{b^4}+\frac{\ln\left(\frac{2\,(b-a\,e^x)\left(a^2+b^2\right)^{3/2}}{b^5}-\frac{2\,e^x\left(a^2+b^2\right)^2}{b^5}\right)\left(a^2+b^2\right)^{3/2}}{b^4}-\frac{x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*b) + exp(3*x)/(24*b) - (log(- (2*exp(x)*(a^2 + b^2)^2)/b^5 -
(2*(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5)*(a^2 + b^2)^(3/2))/b^4 + (log((2*
(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5 - (2*exp(x)*(a^2 + b^2)^2)/b^5)*(a^2
+ b^2)^(3/2))/b^4 - (x*(3*a*b^2 + 2*a^3))/(2*b^4) + (exp(x)*(4*a^2 + 5*b^2)
)/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) + (exp(-x)*(4*a^2
+ 5*b^2))/(8*b^3)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.191 $\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=38

$$\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

```
[Out] (a^2+b^2)*ln(a+b*sinh(x))/b^3-a*sinh(x)/b^2+1/2*sinh(x)^2/b
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[x]])/b^3 - (a*Sinh[x])/b^2 + Sinh[x]^2/(2*b)
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{a+x}\,dx, x, b\sinh(x)\right)}{b^3}$$
$$= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2-b^2}{a+x}\right)dx, x, b\sinh(x)\right)}{b^3}$$
$$= \frac{\left(a^2+b^2\right)\log(a+b\sinh(x))}{b^3} - \frac{a\sinh(x)}{b^2} + \frac{\sinh^2(x)}{2b}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$-\frac{-\left(a^2+b^2\right)\log(a+b\sinh(x)) + ab\sinh(x) - \frac{1}{2}b^2\sinh^2(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -((-((a^2 + b^2)*Log[a + b*Sinh[x]]) + a*b*Sinh[x] - (b^2*Sinh[x]^2)/2)/b^3
)
```

fricas [B] time = 0.65, size = 221, normalized size = 5.82

$$\frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 - 4\,ab\cosh(x)^3 - 8\left(a^2+b^2\right)x\cosh(x)^2 + 4\left(b^2\cosh(x) - ab\right)\sinh(x)^3 + 4\,ab\cosh(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 + b^2)*x*cosh
(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*b^2*cosh(x)^
2 - 6*a*b*cosh(x) - 4*(a^2 + b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 + b^2)*cosh(
x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2)*log(2*(b*sinh
(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4*(a^2
 + b^2)*x*cosh(x) + a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) +
b^3*sinh(x)^2)
```

giac [A] time = 0.19, size = 61, normalized size = 1.61

$$\frac{b\left(e^{(-x)} - e^x\right)^2 + 4\,a\left(e^{(-x)} - e^x\right)}{8\,b^2} + \frac{\left(a^2+b^2\right)\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*(e^(-x) - e^x)^2 + 4*a*(e^(-x) - e^x))/b^2 + (a^2 + b^2)*log(abs(-b*
(e^(-x) - e^x) + 2*a))/b^3
```

maple [B] time = 0.04, size = 185, normalized size = 4.87

$$\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)a^2}{b^3}+\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{b}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sinh(x)),x)
```

```
[Out] 1/b^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2+1/b*ln(a*tanh(1/2*x)^2-2*ta
nh(1/2*x)*b-a)+1/2/b/(tanh(1/2*x)-1)^2+1/2/b/(tanh(1/2*x)-1)+1/b^2/(tanh(1/
2*x)-1)*a-1/b^3*ln(tanh(1/2*x)-1)*a^2-1/b*ln(tanh(1/2*x)-1)+1/2/b/(tanh(1/2
*x)+1)^2-1/2/b/(tanh(1/2*x)+1)+1/b^2/(tanh(1/2*x)+1)*a-1/b^3*ln(tanh(1/2*x)
+1)*a^2-1/b*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.33, size = 81, normalized size = 2.13

$$-\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2}+\frac{4\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^2}+\frac{\left(a^2+b^2\right)x}{b^3}+\frac{\left(a^2+b^2\right)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 + 1/8*(4*a*e^(-x) + b*e^(-2*x))/b^2 + (a^
2 + b^2)*x/b^3 + (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

mupad [B] time = 0.62, size = 77, normalized size = 2.03

$$\frac{\mathrm{e}^{-2\,x}}{8\,b}+\frac{\mathrm{e}^{2\,x}}{8\,b}+\frac{\ln\left(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}\right)\,\left(a^2+b^2\right)}{b^3}-\frac{a\,\mathrm{e}^x}{2\,b^2}-\frac{x\,\left(a^2+b^2\right)}{b^3}+\frac{a\,\mathrm{e}^{-x}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^2 +
 b^2))/b^3 - (a*exp(x))/(2*b^2) - (x*(a^2 + b^2))/b^3 + (a*exp(-x))/(2*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

3.192 $\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

```
[Out] -a*x/b^2+cosh(x)/b-2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(
1/2)/b^2
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2695, 2735, 2660, 618, 206}

$$-\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/b^2 + Cosh[x]/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)}{b}+\frac{i\int \frac{-ib+ia\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{\left(2\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}-\frac{\left(4\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}+\frac{\cosh(x)}{b}
\end{aligned}
$$

Mathematica [C] time = 0.77, size = 396, normalized size = 7.33

$$
\frac{\cosh(x)\left(\sqrt{a+ib}\,\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(\sqrt{a-ib}\,\sqrt{1+i\sinh(x)}\,\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}\,-2(-1)^{3/4}\sqrt{b}\,\sin^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\,\sqrt{-\frac{b(\sinh(x)+\cdots}{a-ib}}}{\sqrt{b}}\right.\right.\right.\cdots}{b\sqrt{a-ib}\,\sqrt{a+ib}\,\sqrt{1+i\sinh\cdots}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(-2*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/(
a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]] + 2*(
a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt
[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + Sqr
t[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b]*Arc
Sin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b
]] + Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]
)))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh
[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])
```

fricas [B] time = 0.77, size = 171, normalized size = 3.17

$$-\frac{2\,ax\cosh(x) - b\cosh(x)^2 - b\sinh(x)^2 - 2\sqrt{a^2+b^2}\,(\cosh(x)+\sinh(x))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+}{b\cosh(x)^2+b\sinh(x)}\right.}{2\left(b^2\cosh(x)+b^2\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 + b^2)*(cosh(x
) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b
^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(
x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(
x) - b)) + 2*(a*x - b*cosh(x))*sinh(x) - b)/(b^2*cosh(x) + b^2*sinh(x))
```

giac [A] time = 0.22, size = 83, normalized size = 1.54

$$-\frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{\sqrt{a^2+b^2}\,\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log(abs(2*b*e^x + 2*a
 - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/b^2
```

maple [B] time = 0.04, size = 126, normalized size = 2.33

$$\frac{2a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2} + \frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(x)),x)
```

```
[Out] 2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)
)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/b/
(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)+1/b/(tanh(1/2*x)+1)-a/b^2*ln(tanh(1
/2*x)+1)
```

maxima [A] time = 0.41, size = 81, normalized size = 1.50

$$-\frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{\sqrt{a^2+b^2}\,\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log((b*e^(-x) - a - s
qrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/b^2
```

mupad [B] time = 0.63, size = 87, normalized size = 1.61

$$\frac{e^{-x}}{2\,b} + \frac{e^x}{2\,b} - \frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^4}}{b^2\,\sqrt{a^2+b^2}} + \frac{e^x\,\sqrt{-b^4}}{b\,\sqrt{a^2+b^2}}\right)\sqrt{a^2+b^2}}{\sqrt{-b^4}} - \frac{a\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] exp(-x)/(2*b) + exp(x)/(2*b) - (2*atan((a*(-b^4)^(1/2))/(b^2*(a^2 + b^2)^(1
/2)) + (exp(x)*(-b^4)^(1/2))/(b*(a^2 + b^2)^(1/2)))*(a^2 + b^2)^(1/2))/(-b^
4)^(1/2) - (a*x)/b^2
```

sympy [A] time = 138.91, size = 377, normalized size = 6.98

$$\begin{cases} \tilde{\infty}\left(\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{2}{\tanh^2\left(\frac{x}{2}\right)-1}\right) \\ \frac{\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{2}{\tanh^2\left(\frac{x}{2}\right)-1}}{b} \\ \frac{-\frac{x\sinh^2(x)}{2}+\frac{x\cosh^2(x)}{2}+\frac{\sinh(x)\cosh(x)}{2}}{a} \\ -\frac{ax\tanh^2\left(\frac{x}{2}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} + \frac{ax}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} - \frac{2b}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} - \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} + \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b}}{a}\right.}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x
/2))/(tanh(x/2)**2 - 1) - 2/(tanh(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), ((lo
g(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x/2))/(tanh(x/2)**2
 - 1) - 2/(tanh(x/2)**2 - 1))/b, Eq(a, 0)), ((-x*sinh(x)**2/2 + x*cosh(x)**
2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), (-a*x*tanh(x/2)**2/(b**2*tanh(x/2)**
2 - b**2) + a*x/(b**2*tanh(x/2)**2 - b**2) - 2*b/(b**2*tanh(x/2)**2 - b**2)
 - sqrt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)*tanh(x/2)**
2/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt
(a**2 + b**2)/a)/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/
2) - b/a + sqrt(a**2 + b**2)/a)*tanh(x/2)**2/(b**2*tanh(x/2)**2 - b**2) - s
qrt(a**2 + b**2)*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b**2*tanh(x/2)
**2 - b**2), True))
```

3.193 $\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sinh(x))}{b}$$

[Out] ln(a+b*sinh(x))/b

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 31}

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + b*Sinh[x]),x]

[Out] Log[a + b*Sinh[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}\\ &= \frac{\log(a+b\sinh(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[a + b*Sinh[x]]/b
```

fricas [B] time = 0.64, size = 27, normalized size = 2.45

$$-\frac{x-\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

giac [A] time = 0.22, size = 22, normalized size = 2.00

$$\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$\frac{\ln\left(a+b\sinh(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sinh(x)),x)
```

```
[Out] ln(a+b*sinh(x))/b
```

maxima [A] time = 0.30, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sinh(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] log(b*sinh(x) + a)/b
```

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\sinh(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*sinh(x)),x)
[Out] log(a + b*sinh(x))/b
```

sympy [A] time = 0.31, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log\left(\frac{a}{b}+\sinh\left(x\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{\sinh\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x)
[Out] Piecewise((log(a/b + sinh(x))/b, Ne(b, 0)), (sinh(x)/a, True))
```

3.194 $\int \frac{\text{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=48

$$\frac{b\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}$$

```
[Out] a*arctan(sinh(x))/(a^2+b^2)-b*ln(cosh(x))/(a^2+b^2)+b*ln(a+b*sinh(x))/(a^2+
b^2)
```

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2668, 706, 31, 635, 204, 260}

$$\frac{b\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sinh[x]),x]
```

```
[Out] (a*ArcTan[Sinh[x]])/(a^2 + b^2) - (b*Log[Cosh[x]])/(a^2 + b^2) + (b*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx &= -\left(b\,\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\right)\\
&= \frac{b\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{a^2+b^2}+\frac{b\,\text{Subst}\left(\int \frac{-a+x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{b\log(a+b\sinh(x))}{a^2+b^2}+\frac{b\,\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{-b^2-x^2}\,dx, x, b\,\text{s}\right.}{a^2+b^2}\\
&= \frac{a\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}+\frac{b\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [B] time = 0.10, size = 99, normalized size = 2.06

$$-\frac{b\left(\left(\sqrt{-b^2}-a\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{2\sqrt{-b^2}\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a
 + b*Sinh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/(Sqrt[-b^2]*
(a^2 + b^2))
```

fricas [A] time = 1.02, size = 57, normalized size = 1.19

$$\frac{2\,a\arctan\left(\cosh(x)+\sinh(x)\right)+b\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(x) + sinh(x)) + b*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) - b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

giac [A] time = 0.18, size = 89, normalized size = 1.85

$$\frac{b^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^2b+b^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^2+b^2\right)}-\frac{b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) - 1/2*b*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

maple [A] time = 0.04, size = 71, normalized size = 1.48

$$\frac{b\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{a^2+b^2}-\frac{b\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{a^2+b^2}+\frac{2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sinh(x)),x)
```

```
[Out] b/(a^2+b^2)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-1/(a^2+b^2)*b*ln(tanh(1/2
*x)^2+1)+2/(a^2+b^2)*a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.41, size = 66, normalized size = 1.38

$$-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) + b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) - b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

mupad [B] time = 1.35, size = 93, normalized size = 1.94

$$\frac{b\,\ln\left(4\,b^3\,e^{2\,x}-a^2\,b-4\,b^3+2\,a^3\,e^x+8\,a\,b^2\,e^x+a^2\,b\,e^{2\,x}\right)}{a^2+b^2}-\frac{\ln\left(e^x+1i\right)}{b+a\,1i}-\frac{\ln\left(1+e^x\,1i\right)\,1i}{a+b\,1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*sinh(x))),x)
```

```
[Out] (b*log(4*b^3*exp(2*x) - a^2*b - 4*b^3 + 2*a^3*exp(x) + 8*a*b^2*exp(x) + a^2
*b*exp(2*x)))/(a^2 + b^2) - log(exp(x) + 1i)/(a*1i + b) - (log(exp(x)*1i +
1)*1i)/(a + b*1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*sinh(x)), x)
```

3.195 $\int \frac{\text{sech}^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=59

$$\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+sech(x)*(
b+a*sinh(x))/(a^2+b^2)
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2696, 12, 2660, 618, 206}

$$\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (Se
ch[x]*(b + a*Sinh[x]))/(a^2 + b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx &= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{\int \frac{b^2}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{b^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}-\frac{\left(4b^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 67, normalized size = 1.14

$$\frac{\frac{2b^2\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+a\tanh(x)+b\operatorname{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] ((2*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + b*Se
ch[x] + a*Tanh[x])/(a^2 + b^2)
```

fricas [B] time = 2.36, size = 259, normalized size = 4.39

$$-\frac{2\,a^{3}+2\,ab^{2}-\left(b^{2}\cosh(x)^{2}+2\,b^{2}\cosh(x)\sinh(x)+b^{2}\sinh(x)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\,\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh}{b\cosh(x)^{2}}\right.}{a^{4}+2\,a^{2}b^{2}+b^{4}+\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\cosh(x)^{2}+2\left(a^{4}+2\,a^{2}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
+ b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

giac [A] time = 0.18, size = 87, normalized size = 1.47

$$\frac{b^{2}\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{2\,(be^{x}-a)}{\left(a^{2}+b^{2}\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

maple [A] time = 0.05, size = 71, normalized size = 1.20

$$\frac{2b^{2}\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{x}{2}\right)-b\right)}{\left(a^{2}+b^{2}\right)\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sinh(x)),x)
```

```
[Out] 2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-2/
(a^2+b^2)*(-a*tanh(1/2*x)-b)/(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.42, size = 89, normalized size = 1.51

$$\frac{b^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(be^{(-x)}+a\right)}{a^2+b^2+\left(a^2+b^2\right)e^{(-2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(a^2 + b^2)^(3/2) + 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

mupad [B] time = 1.04, size = 321, normalized size = 5.44

$$-\frac{\frac{2a}{a^2+b^2}-\frac{2b\,e^x}{a^2+b^2}}{e^{2x}+1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2}{\sqrt{b^4}\,\left(a^2+b^2\right)^2}+\frac{2a\left(a^3\sqrt{b^4}+a\,b^2\sqrt{b^4}\right)}{b^4\sqrt{-\left(a^2+b^2\right)^3}\,\left(a^2+b^2\right)\sqrt{-a^6-3a^4b^2-3a^2b^4-b^6}}\right)-\frac{2a\left(b^3\sqrt{b^4}+a^2b\sqrt{b^4}\right.}{b^4\sqrt{-\left(a^2+b^2\right)^3}\,\left(a^2+b^2\right)\sqrt{-a^6-3}}\right.\right.\right.}{\sqrt{-a^6-3a^4b^2-3a^2b^4-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*sinh(x))),x)
```

```
[Out] - ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan((
exp(x)*(2/((b^4)^(1/2)*(a^2 + b^2)^2) + (2*a*(a^3*(b^4)^(1/2) + a*b^2*(b^4)
^(1/2)))/(b^4*(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 -
 3*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) + a^2*b*(b^4)^(1/2)))/(b^4*(-(a
^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))
)*((b^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^6 - b^
6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(- a^6 - b^6 - 3*a^2*b^4
 - 3*a^4*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*sinh(x)), x)
```

3.196 $\int \frac{\mathrm{sech}^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=87

$$\frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{\mathrm{sech}^2(x)(a\sinh(x)+b)}{2\left(a^2+b^2\right)}+\frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] 1/2*a*(a^2+3*b^2)*arctan(sinh(x))/(a^2+b^2)^2-b^3*ln(cosh(x))/(a^2+b^2)^2+b
^3*ln(a+b*sinh(x))/(a^2+b^2)^2+1/2*sech(x)^2*(b+a*sinh(x))/(a^2+b^2)
```

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2668, 741, 801, 635, 203, 260}

$$\frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}-\frac{b^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}+\frac{\mathrm{sech}^2(x)(a\sinh(x)+b)}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) - (b^3*Log[Cosh[x]])/(a
^2 + b^2)^2 + (b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(b + a*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = b^3\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\operatorname{Subst}\left(\int \frac{a^2+2b^2+ax}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\operatorname{Subst}\left(\int \left(-\frac{2b^2}{\left(a^2+b^2\right)(a+x)} + \frac{-a^3-3ab^2+2b^2x}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}$$

$$= \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\operatorname{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)^2}$$

$$= \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b^3\operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^2} + \frac{(a}{}$$

$$= \frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2} - \frac{b^3\log(\cosh(x))}{\left(a^2+b^2\right)^2} + \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}^2(x)(b+a\sinh}{2\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.13, size = 77, normalized size = 0.89

$$\frac{b\left(a^2+b^2\right)\operatorname{sech}^2(x) + 2a\left(a^2+3b^2\right)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + a\left(a^2+b^2\right)\tanh(x)\operatorname{sech}(x) + 2b^3(\log(a+b\sinh(x)) - \log}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*(a^2 + 3*b^2)*ArcTan[Tanh[x/2]] + 2*b^3*(-Log[Cosh[x]] + Log[a + b*Sin
h[x]]) + b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 +
 b^2)^2)
```

fricas [B] time = 1.48, size = 652, normalized size = 7.49

$$\frac{\left(a^3+ab^2\right)\cosh(x)^3 + \left(a^3+ab^2\right)\sinh(x)^3 + 2\left(a^2b+b^3\right)\cosh(x)^2 + \left(2\,a^2b + 2\,b^3 + 3\left(a^3+ab^2\right)\cosh(x)\right)\sinh(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 + 2*(a^2*b + b^3)*cosh(x
)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 + 3*a*b
^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(
x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(
a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 +
 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(
x) + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(
x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*c
osh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (b^3*cosh(x)^
4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*
b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*l
og(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(x)^
2 - 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4
 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

giac [B] time = 0.19, size = 214, normalized size = 2.46

$$\frac{b^4 \log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^4 b + 2\,a^2 b^3 + b^5} - \frac{b^3 \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2\,a^2 b^2 + b^4\right)} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(a^3 + 3\,ab^2\right)}{4\left(a^4 + 2\,a^2 b^2 + b^4\right)} + \frac{b^3\left(e^{(\cdot}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*b^3*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/2*(
e^(2*x) - 1)*e^(-x)))*(a^3 + 3*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(b^3*(e
^(-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 4*a^2*b +
8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

maple [B] time = 0.06, size = 353, normalized size = 4.06

$$\frac{b^3 \ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right)b - a\right)}{a^4 + 2a^2b^2 + b^4} - \frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)a^3}{\left(a^4 + 2a^2b^2 + b^4\right)\left(\tanh^2\left(\frac{x}{2}\right) + 1\right)^2} - \frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)a\,b^2}{\left(a^4 + 2a^2b^2 + b^4\right)\left(\tanh^2\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sinh(x)),x)
```

```
[Out] b^3/(a^4+2*a^2*b^2+b^4)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-1/(a^4+2*a^2*
b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^3-1/(a^4+2*a^2*b^2+b^4)/(tanh(
1/2*x)^2+1)^2*tanh(1/2*x)^3*a*b^2-2/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2
*tanh(1/2*x)^2*a^2*b-2/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^
2*b^3+1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^3+1/(a^4+2*a^
2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a*b^2-1/(a^4+2*a^2*b^2+b^4)*b^3*
ln(tanh(1/2*x)^2+1)+1/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*a^3+3/(a^4+2*
a^2*b^2+b^4)*arctan(tanh(1/2*x))*a*b^2
```

maxima [A] time = 0.42, size = 159, normalized size = 1.83

$$\frac{b^3 \log\left(-2\, ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{b^3 \log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{\left(a^3 + 3\,ab^2\right)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{ae^{(-x)} + 2\,be^{(-2\,x)} - ae^{(-3}}{a^2 + b^2 + 2\left(a^2 + b^2\right)e^{(-2\,x)} + \left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(e^(
-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 + 3*a*b^2)*arctan(e^(-x))/(a^4 +
2*a^2*b^2 + b^4) + (a*e^(-x) + 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2*(a
^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

mupad [B] time = 2.24, size = 291, normalized size = 3.34

$$\frac{\frac{2\left(a^2\,b+b^3\right)}{\left(a^2+b^2\right)^2} + \frac{e^x\left(a^3+a\,b^2\right)}{\left(a^2+b^2\right)^2}}{e^{2\,x}+1} - \frac{\frac{2\,b}{a^2+b^2} + \frac{2\,a\,e^x}{a^2+b^2}}{2\,e^{2\,x}+e^{4\,x}+1} - \frac{\ln\left(1+e^x\,1i\right)\,\left(a+b\,2i\right)}{2\,\left(-a^2\,1i+2\,a\,b+b^2\,1i\right)} + \frac{b^3\,\ln\left(16\,b^7\,e^{2\,x}-a^6\,b-16\,b^7-9\,a^2\,b^5-6\,a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*sinh(x))),x)
```

```
[Out] ((2*(a^2*b + b^3))/(a^2 + b^2)^2 + (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(e
xp(2*x) + 1) - ((2*b)/(a^2 + b^2) + (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x) +
 exp(4*x) + 1) - (log(exp(x)*1i + 1)*(a + b*2i))/(2*(2*a*b - a^2*1i + b^2*1
i)) + (b^3*log(16*b^7*exp(2*x) - a^6*b - 16*b^7 - 9*a^2*b^5 - 6*a^4*b^3 + 2
*a^7*exp(x) + 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a*b^6*exp(x) + a
^6*b*exp(2*x) + 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^4 + b^4 + 2*a^2*
b^2) - (log(exp(x) + 1i)*(a*1i + 2*b))/(2*(a*b*2i - a^2 + b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*sinh(x)), x)
```

3.197 $\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=100

$$\frac{\operatorname{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\operatorname{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{3\left(a^2+b^2\right)^2}$$

```
[Out] -2*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/3*sech(
x)^3*(b+a*sinh(x))/(a^2+b^2)+1/3*sech(x)*(3*b^3+a*(2*a^2+5*b^2)*sinh(x))/(a
^2+b^2)^2
```

Rubi [A] time = 0.21, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2696, 2866, 12, 2660, 618, 206}

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\operatorname{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}+\frac{\operatorname{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (Se
ch[x]^3*(b + a*Sinh[x]))/(3*(a^2 + b^2)) + (Sech[x]*(3*b^3 + a*(2*a^2 + 5*b
^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} - \frac{\int \frac{\operatorname{sech}^2(x)\left(-2a^2-3b^2-2ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{3(a^2+b^2)}$$

$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\int \frac{3b^4}{a+b\sinh(x)}\,dx}{3(a^2+b^2)^2}$$

$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}$$

$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{(2b^4)\operatorname{Subst}\left(\int \frac{1}{a+2bx-a}\right.}{(a^2+b}$$

$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3(a^2+b^2)^2} - \frac{(4b^4)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)}\right.}{(a^2}$$

$$= -\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(\right.}{3(a^2+b^2)^2}$$

Mathematica [A] time = 0.34, size = 102, normalized size = 1.02

$$\frac{a\left(2a^2+5b^2\right)\tanh(x)+\left(a^2+b^2\right)\operatorname{sech}^3(x)(a\sinh(x)+b)+\frac{6b^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+3b^3\operatorname{sech}(x)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 3*b^
3*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) + a*(2*a^2 + 5*b^2)*Tanh[
x])/(3*(a^2 + b^2)^2)
```

fricas [B] time = 0.61, size = 1142, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*b^3 + b^5)*cosh(x)^5 + 6*(a^2*b^3 + b^5)*sinh(x)^5 - 4*a^5 - 14
*a^3*b^2 - 10*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4 -
5*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh
(x)^3 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 + b^5)*cosh(x)^2 - 6*(
a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x
)^2 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 + b^5)*cosh(x)^3 + 3*(a^3*
b^2 + a*b^4)*cosh(x)^2 - (2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2 +
 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 + 3*b^4*cosh(x)
^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*sinh(x)^4 + b^4 + 4*(5*b^4
*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^
2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh
(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) + 6*(a^2*b^3 + b^5)*cosh(x) + 6*(a^2*b^3 + b^5 + 5*(a^2*b^
3 + b^5)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 + 2*(2*a^4*b + 7*a^2*b^3
 + 5*b^5)*cosh(x)^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^6 +
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 3*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

giac [A] time = 0.47, size = 180, normalized size = 1.80

$$\frac{b^4 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,b^3e^{(5\,x)}-3\,ab^2e^{(4\,x)}+4\,a^2be^{(3\,x)}+10\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}-12\,ab^2e^{(2\,x)}+3\,b^3e^x-\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(5*x)
- 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) + 10*b^3*e^(3*x) - 6*a^3*e^(2*x) - 12*a
*b^2*e^(2*x) + 3*b^3*e^x - 2*a^3 - 5*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*
x) + 1)^3)
```

maple [B] time = 0.06, size = 182, normalized size = 1.82

$$\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}} - \frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tanh^4\left(\frac{x}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tanh^3\left(\cdots\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(\tanh^2\left(\frac{x}{2}\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sinh(x)),x)
```

```
[Out] 2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+(-a^2
*b-2*b^3)*tanh(1/2*x)^4+(-2/3*a^3-8/3*a*b^2)*tanh(1/2*x)^3-2*b^3*tanh(1/2*x
)^2+(-a^3-2*a*b^2)*tanh(1/2*x)-1/3*a^2*b-4/3*b^3)/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.44, size = 230, normalized size = 2.30

$$\frac{b^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}} + \frac{2\left(3\,b^3e^{(-x)}+3\,ab^2e^{(-4\,x)}+3\,b^3e^{(-5\,x)}+2\,a^3+5\,ab^2+6\left(a^3+2\,ab^2\right)e^{(-2\,x)}+2\left(2\,a\cdots\right.\right.}{3\left(a^4+2\,a^2b^2+b^4+3\left(a^4+2\,a^2b^2+b^4\right)e^{(-2\,x)}+3\left(a^4+2\,a^2b^2+b^4\right)e^{(-4\,x)}+\left(a^4+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(-x) + 3*a*b^2*e^(
-4*x) + 3*b^3*e^(-5*x) + 2*a^3 + 5*a*b^2 + 6*(a^3 + 2*a*b^2)*e^(-2*x) + 2*(
2*a^2*b + 5*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^
4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*
e^(-6*x))
```

mupad [B] time = 1.61, size = 634, normalized size = 6.34

$$\frac{\frac{2\,b^3\,e^x}{\left(a^2+b^2\right)^2}-\frac{2\,a\,b^2}{\left(a^2+b^2\right)^2}}{e^{2\,x}+1}-\frac{\frac{4\left(a^3+a\,b^2\right)}{\left(a^2+b^2\right)^2}-\frac{8\,e^x\left(a^2\,b+b^3\right)}{3\left(a^2+b^2\right)^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{8\,a}{3\left(a^2+b^2\right)}-\frac{8\,b\,e^x}{3\left(a^2+b^2\right)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,b^2}{\sqrt{b^8}\left(a^2+b^2\right)^2\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\cdots}{b^6\sqrt{\cdots}}\right.\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*sinh(x))),x)
```

```
[Out] ((2*b^3*exp(x))/(a^2 + b^2)^2 - (2*a*b^2)/(a^2 + b^2)^2)/(exp(2*x) + 1) - (
(4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2
```

```
))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) - (8*b*exp(x))/(3*(
a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (2*atan((exp(x)*((2
*b^2)/((b^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(a^5*(b^8)
^(1/2) + 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 + b^2)^5)^
(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*
a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*a*(b^5*(b^8)^(1/2) + 2*a^2*b^3*(b^8)^(1/2
) + a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)
*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*
((b^5*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/
2))/2 + (a^4*b*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8
*b^2)^(1/2))/2 + a^2*b^3*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b
^4 - 5*a^8*b^2)^(1/2)))*(b^8)^(1/2))/(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^
6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x)), x)
```

3.198 $\int \frac{\text{sech}^5(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=135

$$\frac{\text{sech}^4(x)(a\sinh(x)+b)}{4\left(a^2+b^2\right)}+\frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{b^5\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{\text{sech}^2(x)\left(a\left(3a^2+7b^2\right)\sinh(x)+4b^3\right)}{8\left(a^2+b^2\right)^2}+\frac{a\left(3a^4\right.}{}$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*arctan(sinh(x))/(a^2+b^2)^3-b^5*ln(cosh(x))
/(a^2+b^2)^3+b^5*ln(a+b*sinh(x))/(a^2+b^2)^3+1/4*sech(x)^4*(b+a*sinh(x))/(a
^2+b^2)+1/8*sech(x)^2*(4*b^3+a*(3*a^2+7*b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A] time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2668, 741, 823, 801, 635, 203, 260}

$$\frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{a\left(10a^2b^2+3a^4+15b^4\right)\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)^3}-\frac{b^5\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{\text{sech}^4(x)(a\sinh(x)+b)}{4\left(a^2+b^2\right)}+\text{sech}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/(8*(a^2 + b^2)^3) - (b^5*
Log[Cosh[x]])/(a^2 + b^2)^3 + (b^5*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (Sec
h[x]^4*(b + a*Sinh[x]))/(4*(a^2 + b^2)) + (Sech[x]^2*(4*b^3 + a*(3*a^2 + 7*
b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = -\left(b^5\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^3}\,dx,x,b\sinh(x)\right)\right)$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{b^3\operatorname{Subst}\left(\int \frac{3a^2+4b^2+3ax}{(a+x)\left(-b^2-x^2\right)^2}\,dx,x,b\sinh(x)\right)}{4\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} + \frac{b\operatorname{Subst}\left(\int \frac{-3a^4-7a^2b^2-}{(a+x}\right.}{8}$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} + \frac{b\operatorname{Subst}\left(\int\left(\frac{8b^4}{\left(a^2+b^2\right)(a+}\right.\right.}{}$$

$$= \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}$$

$$= \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a\left(3a^2+7b^2\right)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}$$

$$= \frac{a\left(3a^4+10a^2b^2+15b^4\right)\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)^3} - \frac{b^5\log(\cosh(x))}{\left(a^2+b^2\right)^3} + \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \operatorname{sech}$$

Mathematica [A] time = 0.24, size = 135, normalized size = 1.00

$$\frac{2b\left(a^2+b^2\right)^2\operatorname{sech}^4(x)+2a\left(a^2+b^2\right)^2\tanh(x)\operatorname{sech}^3(x)+4b^3\left(a^2+b^2\right)\operatorname{sech}^2(x)+\left(6a^5+20a^3b^2+30ab^4\right)\tan^{-1}}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] ((6*a^5 + 20*a^3*b^2 + 30*a*b^4)*ArcTan[Tanh[x/2]] + 8*b^5*(-Log[Cosh[x]] +
 Log[a + b*Sinh[x]]) + 4*b^3*(a^2 + b^2)*Sech[x]^2 + 2*b*(a^2 + b^2)^2*Sech
[x]^4 + a*(3*a^4 + 10*a^2*b^2 + 7*b^4)*Sech[x]*Tanh[x] + 2*a*(a^2 + b^2)^2*
Sech[x]^3*Tanh[x])/(8*(a^2 + b^2)^3)
```

fricas [B] time = 1.69, size = 2707, normalized size = 20.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + (3*a^5 + 10*a^3*b^2 + 7*a*b
^4)*sinh(x)^7 + 8*(a^2*b^3 + b^5)*cosh(x)^6 + (8*a^2*b^3 + 8*b^5 + 7*(3*a^5
 + 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 26*a^3*b^2 + 15*a*b
^4)*cosh(x)^5 + (11*a^5 + 26*a^3*b^2 + 15*a*b^4 + 21*(3*a^5 + 10*a^3*b^2 +
7*a*b^4)*cosh(x)^2 + 48*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^5 + 16*(a^4*b + 3*
a^2*b^3 + 2*b^5)*cosh(x)^4 + (16*a^4*b + 48*a^2*b^3 + 32*b^5 + 35*(3*a^5 +
10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 + b^5)*cosh(x)^2 + 5*(11*a^5
 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 26*a^3*b^2 + 15*a*
b^4)*cosh(x)^3 - (11*a^5 + 26*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 +
 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 + b^5)*cosh(x)^3 - 10*(11*a^5 + 26*a^3*b
^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^
3 + 8*(a^2*b^3 + b^5)*cosh(x)^2 + (21*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x
)^5 + 8*a^2*b^3 + 8*b^5 + 120*(a^2*b^3 + b^5)*cosh(x)^4 + 10*(11*a^5 + 26*a
^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3
*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^8 + 8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^
7 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 + 10*a^3*b^2 + 15*
a*b^4)*cosh(x)^6 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 10
*a^3*b^2 + 15*a*b^4 + 6*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(9*a^
5 + 30*a^3*b^2 + 45*a*b^4 + 35*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 +
30*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 10*
a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 10*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 10*a^
3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
6 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 15*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^4 + 9*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5
+ 10*a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^5 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (3*a^5 + 10*a^3*b^2 +
 15*a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (3*a^5 + 10*a^3*b^
2 + 7*a*b^4)*cosh(x) + 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sin
h(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cos
h(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b
^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*c
osh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6
 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 +
```

```
3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) - 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b
^5*sinh(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b
^5*cosh(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)
^5 + b^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7
*b^5*cosh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cos
h(x)^6 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(
x)^7 + 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*cosh
(x)/(cosh(x) - sinh(x))) + (7*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^6 + 48
*(a^2*b^3 + b^5)*cosh(x)^5 - 3*a^5 - 10*a^3*b^2 - 7*a*b^4 + 5*(11*a^5 + 26*
a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^3 -
3*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 16*(a^2*b^3 + b^5)*cosh(x))*
sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2
)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*
sinh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*
a^4*b^2 + 9*a^2*b^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
)^4 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
(x)^7 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
))*sinh(x))
```

giac [B] time = 0.39, size = 369, normalized size = 2.73

$$\frac{b^6\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{b^5\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(3\,a^5+10\,a^3b^2+\right.}{16\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
 - 1/2*b^5*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b
^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(3*b^5*(e^(-x) - e^x)^4 - 3*a
```

```
^5*(e^(-x) - e^x)^3 - 10*a^3*b^2*(e^(-x) - e^x)^3 - 7*a*b^4*(e^(-x) - e^x)^
3 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 32*b^5*(e^(-x) - e^x)^2 - 20*a^5*(e^(-x) -
 e^x) - 56*a^3*b^2*(e^(-x) - e^x) - 36*a*b^4*(e^(-x) - e^x) + 16*a^4*b + 64
*a^2*b^3 + 96*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2 +
 4)^2)
```

maple [B] time = 0.07, size = 1140, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*sinh(x)),x)
```

```
[Out] -5/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7*a^5-4/
(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^6*b^5+3/4/(a^
6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5*a^5-4/(a^6+3*a
^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^4*b^5-3/4/(a^6+3*a^4*
b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3*a^5-4/(a^6+3*a^4*b^2+3
*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^2*b^5+5/4/(a^6+3*a^4*b^2+3*a^
2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)*a^5+5/2/(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)*arctan(tanh(1/2*x))*a^3*b^2+15/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*arctan
(tanh(1/2*x))*a*b^4-7/2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*t
anh(1/2*x)^7*a^3*b^2-9/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*
tanh(1/2*x)^7*a*b^4-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tan
h(1/2*x)^6*a^4*b-6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1
/2*x)^6*a^2*b^3+1/2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(
1/2*x)^5*a^3*b^2-1/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh
(1/2*x)^5*a*b^4-4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/
2*x)^4*a^2*b^3-1/2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1
/2*x)^3*a^3*b^2+1/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(
1/2*x)^3*a*b^4-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2
*x)^2*a^4*b-6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)
^2*a^2*b^3+7/2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x
)*a^3*b^2+9/4/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)
*a*b^4-1/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*b^5*ln(tanh(1/2*x)^2+1)+3/4/(a^6+3*a
^4*b^2+3*a^2*b^4+b^6)*arctan(tanh(1/2*x))*a^5+b^5/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)
```

maxima [B] time = 0.44, size = 345, normalized size = 2.56

$$\frac{b^5 \log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{b^5 \log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{\left(3\,a^5 + 10\,a^3b^2 + 15\,ab^4\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{8\,b^3e^{(-2\,x)} + \cdots}{4\left(a^4 + 2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^5*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 b^5*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^5 + 1
0*a^3*b^2 + 15*a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 + 7*a*b^2)*e^(-x) + (11*a^3 +
 15*a*b^2)*e^(-3*x) + 16*(a^2*b + 2*b^3)*e^(-4*x) - (11*a^3 + 15*a*b^2)*e^(
-5*x) - (3*a^3 + 7*a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2
*b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*
b^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

mupad [B] time = 4.89, size = 548, normalized size = 4.06

$$\frac{\frac{2\left(2a^2b+b^3\right)}{\left(a^2+b^2\right)^2}-\frac{e^x\left(3ab^2-a^3\right)}{2\left(a^2+b^2\right)^2}}{2e^{2x}+e^{4x}+1}-\frac{\frac{8\left(a^2b+b^3\right)}{\left(a^2+b^2\right)^2}+\frac{6e^x\left(a^3+ab^2\right)}{\left(a^2+b^2\right)^2}}{3e^{2x}+3e^{4x}+e^{6x}+1}+\frac{\frac{4b}{a^2+b^2}+\frac{4ae^x}{a^2+b^2}}{4e^{2x}+6e^{4x}+4e^{6x}+e^{8x}+1}+\frac{\frac{2\left(a^2b^3+b^5\right)}{\left(a^2+b^2\right)^3}+\frac{e^x\left(3a^5+10a^3b^2+7a\right.}{4\left(a^2+b^2\right)^3}}{e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5*(a + b*sinh(x))),x)
```

```
[Out] ((2*(2*a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(3*a*b^2 - a^3))/(2*(a^2 + b^2
)^2))/(2*exp(2*x) + exp(4*x) + 1) - ((8*(a^2*b + b^3))/(a^2 + b^2)^2 + (6*e
xp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1
) + ((4*b)/(a^2 + b^2) + (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) + ((2*(b^5 + a^2*b^3))/(a^2 + b^2)^3 + (exp(x
)*(7*a*b^4 + 3*a^5 + 10*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) + (b^5*
log(256*b^11*exp(2*x) - 9*a^10*b - 256*b^11 - 225*a^2*b^9 - 300*a^4*b^7 - 1
90*a^6*b^5 - 60*a^8*b^3 + 18*a^11*exp(x) + 225*a^2*b^9*exp(2*x) + 300*a^4*b
^7*exp(2*x) + 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b^10*exp(x
) + 9*a^10*b*exp(2*x) + 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) + 380*a^7*b
^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (log
(exp(x)*1i + 1)*(9*a*b - a^2*3i + b^2*8i))/(8*(3*a*b^2 - a^2*b*3i - a^3 + b
^3*1i)) - (log(exp(x) + 1i)*(a*b*9i - 3*a^2 + 8*b^2))/(8*(a*b^2*3i - 3*a^2*
b - a^3*1i + b^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*sinh(x)), x)
```

3.199 $\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=146

$$\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}-\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{15\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}(x)\left(a\left(8a^4+2\right.\right.}{1}$$

```
[Out] -2*b^6*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+1/5*sech(
x)^5*(b+a*sinh(x))/(a^2+b^2)+1/15*sech(x)^3*(5*b^3+a*(4*a^2+9*b^2)*sinh(x))
/(a^2+b^2)^2+1/15*sech(x)*(15*b^5+a*(8*a^4+26*a^2*b^2+33*b^4)*sinh(x))/(a^2
+b^2)^3
```

Rubi [A] time = 0.42, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2696, 2866, 12, 2660, 618, 206}

$$-\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}+\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{15\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}(x)\left(a\left(26a^2b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^6*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (Se
ch[x]^5*(b + a*Sinh[x]))/(5*(a^2 + b^2)) + (Sech[x]^3*(5*b^3 + a*(4*a^2 + 9
*b^2)*Sinh[x]))/(15*(a^2 + b^2)^2) + (Sech[x]*(15*b^5 + a*(8*a^4 + 26*a^2*b
^2 + 33*b^4)*Sinh[x]))/(15*(a^2 + b^2)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)} - \frac{\int \frac{\operatorname{sech}^4(x)\left(-4a^2-5b^2-4ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{5\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15\left(a^2+b^2\right)^2} + \frac{\int \frac{\operatorname{sech}^2(x)\left(8a^4+18a^2b^2+\right.}{a+}}{15\left(\right.}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8\right.\right.}{15}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8\right.\right.}{15}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8\right.\right.}{15}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a\left(8\right.\right.}{15}$$

$$= -\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} + \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh\right.}{15\left(a^2+b^2\right)^2}$$

Mathematica [A] time = 0.45, size = 146, normalized size = 1.00

$$\frac{3\left(a^2+b^2\right)^2\operatorname{sech}^5(x)(a\sinh(x)+b) + \frac{30b^6\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \left(a^2+b^2\right)\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right) + }{15\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] ((30*b^6*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 15*
b^5*Sech[x] + 3*(a^2 + b^2)^2*Sech[x]^5*(b + a*Sinh[x]) + (a^2 + b^2)*Sech[
x]^3*(5*b^3 + a*(4*a^2 + 9*b^2)*Sinh[x]) + a*(8*a^4 + 26*a^2*b^2 + 33*b^4)*
Tanh[x])/(15*(a^2 + b^2)^3)
```

fricas [B] time = 2.85, size = 3175, normalized size = 21.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^2*b^5 + b^7)*cosh(x)^9 + 30*(a^2*b^5 + b^7)*sinh(x)^9 - 30*(a^3
*b^4 + a*b^6)*cosh(x)^8 - 30*(a^3*b^4 + a*b^6 - 9*(a^2*b^5 + b^7)*cosh(x))*
sinh(x)^8 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^7 + 40*(a^4*b^3 + 5*a^
2*b^5 + 4*b^7 + 27*(a^2*b^5 + b^7)*cosh(x)^2 - 6*(a^3*b^4 + a*b^6)*cosh(x))
*sinh(x)^7 - 16*a^7 - 68*a^5*b^2 - 118*a^3*b^4 - 66*a*b^6 - 60*(a^5*b^2 + 4
*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 20*(3*a^5*b^2 + 12*a^3*b^4 + 9*a*b^6 - 126*
(a^2*b^5 + b^7)*cosh(x)^3 + 42*(a^3*b^4 + a*b^6)*cosh(x)^2 - 14*(a^4*b^3 +
5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*
b^5 + 89*b^7)*cosh(x)^5 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7 +
 945*(a^2*b^5 + b^7)*cosh(x)^4 - 420*(a^3*b^4 + a*b^6)*cosh(x)^3 + 210*(a^4
*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*co
sh(x))*sinh(x)^5 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^
4 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6 - 189*(a^2*b^5 + b^7)*co
sh(x)^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^4 - 70*(a^4*b^3 + 5*a^2*b^5 + 4*b^7
)*cosh(x)^3 + 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^2 - (24*a^6*b + 92
*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x))*sinh(x)^4 + 40*(a^4*b^3 + 5*a^2*b
^5 + 4*b^7)*cosh(x)^3 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7 + 63*(a^2*b^5 + b^7
)*cosh(x)^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^5 + 35*(a^4*b^3 + 5*a^2*b^5 + 4*
b^7)*cosh(x)^4 - 30*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^3 + (24*a^6*b +
 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^2 - 2*(8*a^7 + 31*a^5*b^2 + 47*
a^3*b^4 + 24*a*b^6)*cosh(x))*sinh(x)^3 - 20*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^
4 + 15*a*b^6)*cosh(x)^2 + 20*(54*(a^2*b^5 + b^7)*cosh(x)^7 - 4*a^7 - 17*a^5
*b^2 - 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^6 + 42*(a^4*b^3
 + 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x
)^4 + 2*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^3 - 6*(8*a^7
 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^2 + 6*(a^4*b^3 + 5*a^2*b^5 +
 4*b^7)*cosh(x))*sinh(x)^2 + 15*(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9
+ b^6*sinh(x)^10 + 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 + 10*b^6*cosh(x)^4 +
5*(9*b^6*cosh(x)^2 + b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^3
 + b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 + 14*b^6*cosh(x)^2 + b^6)*
sinh(x)^6 + b^6 + 4*(63*b^6*cosh(x)^5 + 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))*
sinh(x)^5 + 10*(21*b^6*cosh(x)^6 + 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 + b^
6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 + 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 + b^
6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 + 28*b^6*cosh(x)^6 + 30*b^6*cosh(
x)^4 + 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 + 4*b^6*cosh(x
)^7 + 6*b^6*cosh(x)^5 + 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x))*sqrt(a^2 +
b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(
```

```
b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))
/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b))
 + 30*(a^2*b^5 + b^7)*cosh(x) + 10*(27*(a^2*b^5 + b^7)*cosh(x)^8 - 24*(a^3*
b^4 + a*b^6)*cosh(x)^7 + 3*a^2*b^5 + 3*b^7 + 28*(a^4*b^3 + 5*a^2*b^5 + 4*b^
7)*cosh(x)^6 - 36*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^5 + 2*(24*a^6*b +
 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^4 - 8*(8*a^7 + 31*a^5*b^2 + 47*
a^3*b^4 + 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2
- 4*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^4 + 15*a*b^6)*cosh(x))*sinh(x))/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*(a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*sinh(x)^10 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*cosh(x)^8 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 9
*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^8 + a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 40*(3*(a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*cosh(x)^6 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21*(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 14*(a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 70*(a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*cosh(x))*sinh(x)^5 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*cosh(x)^4 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21
*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^6 + 35*(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 15*(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^7 + 7*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x))*sinh(
x)^3 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2 + 5*(9*(
a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^8 + a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 28*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8)*cosh(x)^6 + 30*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh
(x)^4 + 12*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(
x)^2 + 10*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^9 + 4*(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^7 + 6*(a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 4*(a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*cosh(x))*sinh(x))
```

giac [B] time = 0.19, size = 323, normalized size = 2.21

$$\frac{b^6 \log\left(\frac{\left|2\, be^x + 2\, a - 2\, \sqrt{a^2+b^2}\right|}{\left|2\, be^x + 2\, a + 2\, \sqrt{a^2+b^2}\right|}\right)}{\left(a^6 + 3\, a^4 b^2 + 3\, a^2 b^4 + b^6\right)\sqrt{a^2+b^2}} + \frac{2\left(15\, b^5 e^{(9\, x)} - 15\, a b^4 e^{(8\, x)} + 20\, a^2 b^3 e^{(7\, x)} + 80\, b^5 e^{(7\, x)} - 30\, a^3 b^2 e^{(6\, x)} - 90\, a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(1
5*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b^3*e^(7*x) + 80*b^5*e^(7*x) - 30
*a^3*b^2*e^(6*x) - 90*a*b^4*e^(6*x) + 48*a^4*b*e^(5*x) + 136*a^2*b^3*e^(5*x
) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) - 230*a^3*b^2*e^(4*x) - 240*a*b^4*e^(4
*x) + 20*a^2*b^3*e^(3*x) + 80*b^5*e^(3*x) - 40*a^5*e^(2*x) - 130*a^3*b^2*e^
(2*x) - 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*a^5 - 26*a^3*b^2 - 33*a*b^4)/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^5)
```

maple [B] time = 0.08, size = 350, normalized size = 2.40

$$\frac{2b^6 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^5-3a^3b^2-3a\,b^4\right)\left(\tanh^9\left(\frac{x}{2}\right)\right)+\left(-a^4b-3a^2b^3-3b^5\right)\left(\tanh^8\left(\frac{x}{2}\right)\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^6/(a+b*sinh(x)),x)
```

```
[Out] 2*b^6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*x)-2*b)/(a^2+b^2)^(1/2))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^5-3*a^3*b^
2-3*a*b^4)*tanh(1/2*x)^9+(-a^4*b-3*a^2*b^3-3*b^5)*tanh(1/2*x)^8+(-4/3*a^5-1
6/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^7+(-2*a^2*b^3-6*b^5)*tanh(1/2*x)^6+(-58/15
*a^5-166/15*a^3*b^2-66/5*a*b^4)*tanh(1/2*x)^5+(-2*a^4*b-16/3*a^2*b^3-28/3*b
^5)*tanh(1/2*x)^4+(-4/3*a^5-16/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^3+(-2/3*a^2*b
^3-14/3*b^5)*tanh(1/2*x)^2+(-a^5-3*a^3*b^2-3*a*b^4)*tanh(1/2*x)-1/5*a^4*b-1
1/15*a^2*b^3-23/15*b^5)/(tanh(1/2*x)^2+1)^5
```

maxima [B] time = 0.43, size = 438, normalized size = 3.00

$$\frac{b^6\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,b^5e^{(-x)}+15\,ab^4e^{(-8\,x)}+15\,b^5e^{(-9\,x)}+8\,a^5+26\,a^3b^2+33\,ab^4+10\left(4\,a\right.\right.}{15\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6+5\left(a^6+3\,a^4b^2+3\,a^2b^4+b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^6*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(15*b^5*e^(-x)
 + 15*a*b^4*e^(-8*x) + 15*b^5*e^(-9*x) + 8*a^5 + 26*a^3*b^2 + 33*a*b^4 + 10
*(4*a^5 + 13*a^3*b^2 + 15*a*b^4)*e^(-2*x) + 20*(a^2*b^3 + 4*b^5)*e^(-3*x) +
```

```
 10*(8*a^5 + 23*a^3*b^2 + 24*a*b^4)*e^(-4*x) + 2*(24*a^4*b + 68*a^2*b^3 + 8
9*b^5)*e^(-5*x) + 30*(a^3*b^2 + 3*a*b^4)*e^(-6*x) + 20*(a^2*b^3 + 4*b^5)*e^
(-7*x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*e^(-2*x) + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-4*x) + 10*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-6*x) + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*e^(-8*x) + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-10*x))
```

mupad [B] time = 2.20, size = 1010, normalized size = 6.92

$$\frac{\frac{2\,b^5\,e^x}{{\left(a^2+b^2\right)}^3}-\frac{2\,a\,b^4}{{\left(a^2+b^2\right)}^3}}{e^{2\,x}+1}-\frac{\frac{8\,\left(4\,a^3+3\,a\,b^2\right)}{3\,{\left(a^2+b^2\right)}^2}-\frac{8\,e^x\,\left(12\,a^2\,b+7\,b^3\right)}{15\,{\left(a^2+b^2\right)}^2}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{4\,\left(a^3\,b^2+a\,b^4\right)}{{\left(a^2+b^2\right)}^3}-\frac{8\,e^x\,\left(a^2\,b^3+b^5\right)}{3\,{\left(a^2+b^2\right)}^3}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{\frac{32\,a}{5\,\left(a^2+b^2\right)}-\frac{32\,b\,e^x}{5\,\left(a^2+b^2\right)}}{5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^6*(a + b*sinh(x))),x)
```

```
[Out] ((2*b^5*exp(x))/(a^2 + b^2)^3 - (2*a*b^4)/(a^2 + b^2)^3)/(exp(2*x) + 1) - (
(8*(3*a*b^2 + 4*a^3))/(3*(a^2 + b^2)^2) - (8*exp(x)*(12*a^2*b + 7*b^3))/(15
*(a^2 + b^2)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a*b^4 + a^
3*b^2))/(a^2 + b^2)^3 - (8*exp(x)*(b^5 + a^2*b^3))/(3*(a^2 + b^2)^3))/(2*ex
p(2*x) + exp(4*x) + 1) - ((32*a)/(5*(a^2 + b^2)) - (32*b*exp(x))/(5*(a^2 +
b^2)))/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1
) + ((16*(a*b^2 + a^3))/(a^2 + b^2)^2 - (64*exp(x)*(a^2*b + b^3))/(5*(a^2 +
 b^2)^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) - (2*atan((
exp(x)*((2*b^4)/((b^12)^(1/2)*(a^2 + b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) + 3*a^5*b^2*(b^12)^
(1/2) + a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 + b^2)^7)^(1/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 -
35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^12)^(1/2) + 3
*a^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) + a^6*b*(b^12)^(1/2)))/(b^8*
(-(a^2 + b^2)^7)^(1/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(- a^14 - b^14 -
 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*
b^2)^(1/2)))*((b^7*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 -
 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (3*a^2*b^5*(- a^14 - b^1
4 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^
12*b^2)^(1/2))/2 + (3*a^4*b^3*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 3
5*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (a^6*b*(- a^1
4 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4
 - 7*a^12*b^2)^(1/2))/2))*(b^12)^(1/2))/(- a^14 - b^14 - 7*a^2*b^12 - 21*a^
4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**6/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**6/(a + b*sinh(x)), x)
```

3.200 $\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=94

$$\frac{3x\left(2a^2+b^2\right)}{2b^4}+\frac{6a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

```
[Out] 3/2*(2*a^2+b^2)*x/b^4-3/2*cosh(x)*(2*a-b*sinh(x))/b^3-cosh(x)^3/b/(a+b*sinh
(x))+6*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^4
```

Rubi [A] time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2693, 2865, 2735, 2660, 618, 206}

$$\frac{3x\left(2a^2+b^2\right)}{2b^4}+\frac{6a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (3*(2*a^2 + b^2)*x)/(2*b^4) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/b^4 - (3*Cosh[x]*(2*a - b*Sinh[x]))/(2*b^3) - Cosh[x]^3
/(b*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{3\int \frac{\cosh^2(x)\sinh(x)}{a+b\sinh(x)}\,dx}{b}$$

$$= -\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{(3i)\int \frac{iab-i(2a^2+b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} - \frac{(3a(a^2+b^2))\int \frac{1}{a+b\,\mathrm{si}}}{b^4}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} - \frac{(6a(a^2+b^2))\,\mathrm{Subst}\,(}{}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{(12a(a^2+b^2))\,\mathrm{Subst}}{}$$

$$= \frac{3(2a^2+b^2)x}{2b^4} + \frac{6a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\mathrm{co}}{b(a+}$$

Mathematica [C] time = 4.46, size = 660, normalized size = 7.02

$$\frac{\cosh^3(x)\left(-12a(a^2+b^2)\sqrt{1+i\sinh(x)}\,(a+b\sinh(x))\tanh^{-1}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}}{\sqrt{a+ib}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}}\right)+\sqrt{a+ib}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(6(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (Cosh[x]^3*(12*a*Sqrt[a - I*b]*(a + I*b)^(3/2)*ArcTanh[Sqrt[-((b*(I + Sinh[
x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]
*(a + b*Sinh[x]) - 12*a*(a^2 + b^2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + S
inh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])
]*Sqrt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[
x]))/(a + I*b))]*(6*(-1)^(3/4)*a*Sqrt[b]*(2*a^2 + I*a*b + b^2)*ArcSin[((1/2
 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 6*(-
1)^(3/4)*b^(3/2)*(2*a^2 + I*a*b + b^2)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sq
rt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]]*Sinh[x] - 2*Sqrt[a - I*b]*(3*a
^3 + (3*I)*a^2*b + a*b^2 + I*b^3)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x
```

```
]))/(a - I*b))] - 3*a*Sqrt[a - I*b]*(a + I*b)*b*Sqrt[1 + I*Sinh[x]]*Sinh[x]
*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + Sqrt[a - I*b]*(a + I*b)*b^2*Sqrt[1
+ I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(2*(a - I*b)
^(3/2)*(a + I*b)^(5/2)*b*Sqrt[1 + I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b
)))^(3/2)*(-((b*(I + Sinh[x]))/(a - I*b)))^(3/2)*(a + b*Sinh[x]))
```

fricas [B] time = 0.54, size = 833, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 6*a*b^2*cosh(x)^5 + 6*(b^3*cosh(x) - a
*b^2)*sinh(x)^5 - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^4 + (15*b
^3*cosh(x)^2 - 30*a*b^2*cosh(x) - 16*a^2*b - b^3 + 12*(2*a^2*b + b^3)*x)*si
nh(x)^4 + 6*a*b^2*cosh(x) + 8*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(
x)^3 + 4*(5*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 + 4*a^3 + 4*a*b^2 + 6*(2*a^3
 + a*b^2)*x - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x)^3 +
b^3 - (32*a^2*b + 17*b^3 + 12*(2*a^2*b + b^3)*x)*cosh(x)^2 + (15*b^3*cosh(x
)^4 - 60*a*b^2*cosh(x)^3 - 32*a^2*b - 17*b^3 - 6*(16*a^2*b + b^3 - 12*(2*a^
2*b + b^3)*x)*cosh(x)^2 - 12*(2*a^2*b + b^3)*x + 24*(2*a^3 + 2*a*b^2 + 3*(2
*a^3 + a*b^2)*x)*cosh(x))*sinh(x)^2 + 24*(a*b*cosh(x)^4 + a*b*sinh(x)^4 + 2
*a^2*cosh(x)^3 - a*b*cosh(x)^2 + 2*(2*a*b*cosh(x) + a^2)*sinh(x)^3 + (6*a*b
*cosh(x)^2 + 6*a^2*cosh(x) - a*b)*sinh(x)^2 + 2*(2*a*b*cosh(x)^3 + 3*a^2*co
sh(x)^2 - a*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*si
nh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*
a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(3*b^3*cosh(x)^5 - 15*a*b^2
*cosh(x)^4 - 2*(16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^3 + 3*a*b^2
+ 12*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(x)^2 - (32*a^2*b + 17*b^3
 + 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)^4 +
 2*a*b^4*cosh(x)^3 - b^5*cosh(x)^2 + 2*(2*b^5*cosh(x) + a*b^4)*sinh(x)^3 +
(6*b^5*cosh(x)^2 + 6*a*b^4*cosh(x) - b^5)*sinh(x)^2 + 2*(2*b^5*cosh(x)^3 +
3*a*b^4*cosh(x)^2 - b^5*cosh(x))*sinh(x))
```

giac [B] time = 0.18, size = 178, normalized size = 1.89

$$\frac{3\left(2\,a^2+b^2\right)x}{2\,b^4}-\frac{3\left(a^3+ab^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{b^2e^{(2\,x)}-8\,abe^x}{8\,b^4}+\frac{\left(6\,ab^2e^x+b^3+8\left(2\,a^3+ab^2\right)e^{(3\,x)}-\left(32\,a\right.\right.}{8\left(be^{(2\,x)}+2\,ae^x-b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*(2*a^2 + b^2)*x/b^4 - 3*(a^3 + a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^
2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1
/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(6*a*b^2*e^x + b^3 + 8*(2*a^3 + a*b^
2)*e^(3*x) - (32*a^2*b + 17*b^3)*e^(2*x))*e^(-2*x)/((b*e^(2*x) + 2*a*e^x -
b)*b^4)
```

maple [B] time = 0.08, size = 290, normalized size = 3.09

$$\frac{2a\tanh\left(\frac{x}{2}\right)}{b^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2\tanh\left(\frac{x}{2}\right)}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)a}+\frac{2a^2}{b^3\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] 2/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a*tanh(1/2*x)+2/(a*tanh(1/2*x)^2-
2*tanh(1/2*x)*b-a)/a*tanh(1/2*x)+2/b^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*
a^2+2/b/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-6/b^4*a*(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+1/2/b^2/(tanh(1/2*x)-1)^2+1/2/b
^2/(tanh(1/2*x)-1)+2/b^3/(tanh(1/2*x)-1)*a-3/b^4*ln(tanh(1/2*x)-1)*a^2-3/2/
b^2*ln(tanh(1/2*x)-1)-1/2/b^2/(tanh(1/2*x)+1)^2+1/2/b^2/(tanh(1/2*x)+1)-2/b
^3/(tanh(1/2*x)+1)*a+3/b^4*ln(tanh(1/2*x)+1)*a^2+3/2/b^2*ln(tanh(1/2*x)+1)
```

maxima [B] time = 0.42, size = 176, normalized size = 1.87

$$-\frac{6\,ab^2e^{(-x)}-b^3+\left(32\,a^2b+17\,b^3\right)e^{(-2\,x)}+8\left(2\,a^3+ab^2\right)e^{(-3\,x)}}{8\left(b^5e^{(-2\,x)}+2\,ab^4e^{(-3\,x)}-b^5e^{(-4\,x)}\right)}-\frac{3\,\sqrt{a^2+b^2}\,a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^4}-\frac{8\,ae^{(-x)}+be}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(6*a*b^2*e^(-x) - b^3 + (32*a^2*b + 17*b^3)*e^(-2*x) + 8*(2*a^3 + a*b^
2)*e^(-3*x))/(b^5*e^(-2*x) + 2*a*b^4*e^(-3*x) - b^5*e^(-4*x)) - 3*sqrt(a^2
+ b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^
2)))/b^4 - 1/8*(8*a*e^(-x) + b*e^(-2*x))/b^3 + 3/2*(2*a^2 + b^2)*x/b^4
```

mupad [B] time = 0.88, size = 256, normalized size = 2.72

$$\frac{\mathrm{e}^{2\,x}}{8\,b^2}-\frac{\mathrm{e}^{-2\,x}}{8\,b^2}-\frac{\frac{2\left(a^4\,b^2+2\,a^2\,b^4+b^6\right)}{b^4\left(a^2\,b+b^3\right)}-\frac{2\,\mathrm{e}^x\left(a^5\,b^2+2\,a^3\,b^4+a\,b^6\right)}{b^5\left(a^2\,b+b^3\right)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}}+\frac{x\left(6\,a^2+3\,b^2\right)}{2\,b^4}-\frac{a\,\mathrm{e}^x}{b^3}-\frac{a\,\mathrm{e}^{-x}}{b^3}-\frac{3\,a\,\ln\left(\frac{6\,a\,\mathrm{e}^x\left(a^2+b^2\right)}{b^5}-\frac{6\,a\,(b-a\,\mathrm{e}^x)}{b^5}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*(b^6 + 2*a^2*b^4 + a^4*b^2))/(b^
4*(a^2*b + b^3)) - (2*exp(x)*(a*b^6 + 2*a^3*b^4 + a^5*b^2))/(b^5*(a^2*b + b
^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2 + 3*b^2))/(2*b^4) - (a*exp(
x))/b^3 - (a*exp(-x))/b^3 - (3*a*log((6*a*exp(x)*(a^2 + b^2))/b^5 - (6*a*(b
 - a*exp(x))*(a^2 + b^2)^(1/2))/b^5)*(a^2 + b^2)^(1/2))/b^4 + (3*a*log((6*a
*(b - a*exp(x))*(a^2 + b^2)^(1/2))/b^5 + (6*a*exp(x)*(a^2 + b^2))/b^5)*(a^2
 + b^2)^(1/2))/b^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.201 $\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=40

$$-\frac{a^2+b^2}{b^3(a+b\sinh(x))}-\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}$$

```
[Out] -2*a*ln(a+b*sinh(x))/b^3+sinh(x)/b^2+(-a^2-b^2)/b^3/(a+b*sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2668, 697}

$$-\frac{a^2+b^2}{b^3(a+b\sinh(x))}-\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Sinh[x]])/b^3 + Sinh[x]/b^2 - (a^2 + b^2)/(b^3*(a + b*Sinh[
x]))
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{(a+x)^2}\,dx, x, b\sinh(x)\right)}{b^3}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{-a^2-b^2}{(a+x)^2}+\frac{2a}{a+x}\right)dx, x, b\sinh(x)\right)}{b^3}$$
$$= -\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}$$

Mathematica [**A**] time = 0.07, size = 37, normalized size = 0.92

$$-\frac{\frac{a^2+b^2}{a+b\sinh(x)}+2a\log(a+b\sinh(x))-b\sinh(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] -((2*a*Log[a + b*Sinh[x]] - b*Sinh[x] + (a^2 + b^2)/(a + b*Sinh[x]))/b^3)
```

fricas [**B**] time = 0.83, size = 370, normalized size = 9.25

$$b^2\cosh(x)^4 + b^2\sinh(x)^4 + 2\,(2\,abx + ab)\cosh(x)^3 + 2\left(2\,abx + 2\,b^2\cosh(x) + ab\right)\sinh(x)^3 + 2\left(4\,a^2x - 2\,a^2 - 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 2*(2*a*b*x + a*b)*cosh(x)^3 + 2*(2*a*b
*x + 2*b^2*cosh(x) + a*b)*sinh(x)^3 + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x)^2
 + 2*(3*b^2*cosh(x)^2 + 4*a^2*x - 2*a^2 - 3*b^2 + 3*(2*a*b*x + a*b)*cosh(x)
)*sinh(x)^2 + b^2 - 2*(2*a*b*x + a*b)*cosh(x) - 4*(a*b*cosh(x)^3 + a*b*sinh
(x)^3 + 2*a^2*cosh(x)^2 - a*b*cosh(x) + (3*a*b*cosh(x) + 2*a^2)*sinh(x)^2 +
 (3*a*b*cosh(x)^2 + 4*a^2*cosh(x) - a*b)*sinh(x))*log(2*(b*sinh(x) + a)/(co
sh(x) - sinh(x))) + 2*(2*b^2*cosh(x)^3 - 2*a*b*x + 3*(2*a*b*x + a*b)*cosh(x
)^2 - a*b + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x))*sinh(x))/(b^4*cosh(x)^3 +
b^4*sinh(x)^3 + 2*a*b^3*cosh(x)^2 - b^4*cosh(x) + (3*b^4*cosh(x) + 2*a*b^3)
*sinh(x)^2 + (3*b^4*cosh(x)^2 + 4*a*b^3*cosh(x) - b^4)*sinh(x))
```

giac [**B**] time = 0.24, size = 82, normalized size = 2.05

$$-\frac{e^{(-x)}-e^x}{2\,b^2}-\frac{2\,a\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^3}+\frac{2\left(ab\left(e^{(-x)}-e^x\right)-a^2+b^2\right)}{\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/b^2 - 2*a*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 + 2*(a*
b*(e^(-x) - e^x) - a^2 + b^2)/((b*(e^(-x) - e^x) - 2*a)*b^3)
```

maple [B] time = 0.08, size = 141, normalized size = 3.52

$$-\frac{2a\tanh\left(\frac{x}{2}\right)}{b^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}-\frac{2\tanh\left(\frac{x}{2}\right)}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)a}-\frac{2a\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b \ldots}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] -2/b^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a*tanh(1/2*x)-2/(a*tanh(1/2*x)^2
-2*tanh(1/2*x)*b-a)/a*tanh(1/2*x)-2/b^3*a*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*
b-a)+2*a/b^3*ln(tanh(1/2*x)-1)-1/b^2/(tanh(1/2*x)-1)+2*a/b^3*ln(tanh(1/2*x)
+1)-1/b^2/(tanh(1/2*x)+1)
```

maxima [B] time = 0.33, size = 102, normalized size = 2.55

$$\frac{2\,abe^{(-x)}+b^2-\left(4\,a^2+5\,b^2\right)e^{(-2\,x)}}{2\left(b^4e^{(-x)}+2\,ab^3e^{(-2\,x)}-b^4e^{(-3\,x)}\right)}-\frac{2\,ax}{b^3}-\frac{e^{(-x)}}{2\,b^2}-\frac{2\,a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*e^(-x) + b^2 - (4*a^2 + 5*b^2)*e^(-2*x))/(b^4*e^(-x) + 2*a*b^3*e
^(-2*x) - b^4*e^(-3*x)) - 2*a*x/b^3 - 1/2*e^(-x)/b^2 - 2*a*log(-2*a*e^(-x)
+ b*e^(-2*x) - b)/b^3
```

mupad [B] time = 0.71, size = 60, normalized size = 1.50

$$\frac{\frac{\cosh(x)^2}{b}-\frac{2\sinh(x)^3}{a}+\frac{2\cosh(x)^2\sinh(x)}{a}+\frac{2\,a\sinh(x)}{b^2}}{a+b\sinh(x)}-\frac{2\,a\,\ln\left(a+b\sinh(x)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] (cosh(x)^2/b - (2*sinh(x)^3)/a + (2*cosh(x)^2*sinh(x))/a + (2*a*sinh(x))/b^
2)/(a + b*sinh(x)) - (2*a*log(a + b*sinh(x)))/b^3
```

sympy [A] time = 1.06, size = 133, normalized size = 3.32

$$\begin{cases} -\frac{2a^2\log\left(\frac{a}{b}+\sinh(x)\right)}{ab^3+b^4\sinh(x)} - \frac{2a^2}{ab^3+b^4\sinh(x)} - \frac{2ab\log\left(\frac{a}{b}+\sinh(x)\right)\sinh(x)}{ab^3+b^4\sinh(x)} + \frac{2b^2\sinh^2(x)}{ab^3+b^4\sinh(x)} - \frac{b^2\cosh^2(x)}{ab^3+b^4\sinh(x)} & \text{for } b \neq 0 \\ \frac{-\frac{2\sinh^3(x)}{3}+\sinh(x)\cosh^2(x)}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((-2*a**2*log(a/b + sinh(x))/(a*b**3 + b**4*sinh(x)) - 2*a**2/(a*b
**3 + b**4*sinh(x)) - 2*a*b*log(a/b + sinh(x))*sinh(x)/(a*b**3 + b**4*sinh(
x)) + 2*b**2*sinh(x)**2/(a*b**3 + b**4*sinh(x)) - b**2*cosh(x)**2/(a*b**3 +
 b**4*sinh(x)), Ne(b, 0)), ((-2*sinh(x)**3/3 + sinh(x)*cosh(x)**2)/a**2, Tr
ue))
```

3.202 $\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=62

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[Out] x/b^2-cosh(x)/b/(a+b*sinh(x))+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^2/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2693, 2735, 2660, 618, 206}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}
\end{aligned}$$

Mathematica [C] time = 1.79, size = 502, normalized size = 8.10

$$\frac{\cosh(x)\left(\sqrt{a+ib}\sqrt{-\frac{b(\sinh(x)-i)}{a+ib}}\left(-\sqrt{a-ib}\left(a^2+b^2\right)\sqrt{1+i\sinh(x)}\sqrt{-\frac{b(\sinh(x)+i)}{a-ib}}+2\sqrt[4]{-1}\,b^{3/2}(b+ia)\sinh(x)\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*(2*a*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]*(a +
 b*Sinh[x]) - 2*a*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))
/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1
 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a
+ I*b))]*(2*(-1)^(1/4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b
]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*(-1)^(1/4)*b^(3/2)*(I*
a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b)
)])/Sqrt[b]]*Sinh[x] - Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x]]*Sqrt[-
((b*(I + Sinh[x]))/(a - I*b))])))/((a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sqrt[1
 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x])
)/(a - I*b))]*(a + b*Sinh[x]))
```

fricas [B] time = 2.99, size = 362, normalized size = 5.84

$$-\frac{\left(a^2b+b^3\right)x\cosh(x)^2+\left(a^2b+b^3\right)x\sinh(x)^2-2\,a^2b-2\,b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2\,a^2\cosh(x)-ab+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a*b^2)*
x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*x)*si
nh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)
^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cos
h(x))*sinh(x))
```

giac [A] time = 0.23, size = 97, normalized size = 1.56

$$-\frac{a\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{x}{b^2}+\frac{2\,(ae^{x}-b)}{\left(be^{(2\,x)}+2\,ae^{x}-b\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*a*
e^x - b)*b^2)
```

maple [B] time = 0.06, size = 119, normalized size = 1.92

$$\frac{2\tanh\left(\frac{x}{2}\right)}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)a}+\frac{2}{b\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\ln\left(\tanh\left(\right.\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] 2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)/a*tanh(1/2*x)+2/b/(a*tanh(1/2*x)^2-2*
tanh(1/2*x)*b-a)-2/b^2*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/
(a^2+b^2)^(1/2))-1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.41, size = 100, normalized size = 1.61

$$-\frac{2\left(ae^{(-x)}+b\right)}{2\,ab^2e^{(-x)}-b^3e^{(-2\,x)}+b^3}-\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(-x) + b)/(2*a*b^2*e^(-x) - b^3*e^(-2*x) + b^3) - a*log((b*e^(-x) -
 a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^
2) + x/b^2
```

mupad [B] time = 0.67, size = 132, normalized size = 2.13

$$\frac{x}{b^2}-\frac{\frac{2}{b}-\frac{2\,a\,\mathrm{e}^x}{b^2}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^3}-\frac{2\,a\,(b-a\,\mathrm{e}^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^x}{b^3}+\frac{2\,a\,(b-a\,\mathrm{e}^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] x/b^2 - (2/b - (2*a*exp(x))/b^2)/(2*a*exp(x) - b + b*exp(2*x)) - (a*log((2*
a*exp(x))/b^3 - (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 +
b^2)^(1/2)) + (a*log((2*a*exp(x))/b^3 + (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^
2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

3.203 $\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{b(a+b\sinh(x))}$$

```
[Out] -1/b/(a+b*sinh(x))
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2668, 32}

$$-\frac{1}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(1/(b*(a + b*Sinh[x])))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\sinh(x)\right)}{b}\\ &= -\frac{1}{b(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(1/(b*(a + b*Sinh[x])))
```

fricas [B] time = 0.66, size = 51, normalized size = 3.92

$$-\frac{2\left(\cosh(x)+\sinh(x)\right)}{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)-b^{2}+2\left(b^{2}\cosh(x)+ab\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) - b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

giac [A] time = 0.34, size = 22, normalized size = 1.69

$$\frac{2}{\left(b\left(e^{(-x)}-e^{x}\right)-2\,a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/((b*(e^(-x) - e^x) - 2*a)*b)
```

maple [A] time = 0.03, size = 14, normalized size = 1.08

$$-\frac{1}{b\left(a+b\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sinh(x))^2,x)
```

```
[Out] -1/b/(a+b*sinh(x))
```

maxima [A] time = 0.31, size = 13, normalized size = 1.00

$$-\frac{1}{(b\sinh(x)+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sinh(x) + a)*b)
```

mupad [B] time = 0.53, size = 14, normalized size = 1.08

$$\frac{\sinh(x)}{a\ (a + b\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] sinh(x)/(a*(a + b*sinh(x)))
```

sympy [A] time = 0.61, size = 19, normalized size = 1.46

$$\begin{cases} -\frac{1}{ab+b^2\sinh{(x)}} & \text{for } b \neq 0 \\ \frac{\sinh{(x)}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((-1/(a*b + b**2*sinh(x)), Ne(b, 0)), (sinh(x)/a**2, True))
```

3.204 $\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cosh(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] (a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^2-2*a*b*ln(cosh(x))/(a^2+b^2)^2+2*a*b*l
n(a+b*sinh(x))/(a^2+b^2)^2-b/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2668, 710, 801, 635, 203, 260}

$$-\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cosh(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^2 - (2*a*b*Log[Cosh[x]])/(a^2 + b
^2)^2 + (2*a*b*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 - b/((a^2 + b^2)*(a + b*Si
nh[x]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx &= -\left(b\operatorname{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\right)\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\operatorname{Subst}\left(\int \frac{a-x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\operatorname{Subst}\left(\int \left(-\frac{2a}{\left(a^2+b^2\right)(a+x)}+\frac{-a^2+b^2+2ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\operatorname{Subst}\left(\int \frac{-a^2+b^2+2ax}{b^2+x^2}\,dx, x, b\sinh(\right.}{\left(a^2+b^2\right)^2}\\
&= \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{(2ab)\operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x\right.}{\left(a^2+b^2\right)^2}\\
&= \frac{\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{2ab\log(\cosh(x))}{\left(a^2+b^2\right)^2} + \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)(a+}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 121, normalized size = 1.53

$$-\frac{b\left(\frac{2(a^2+b^2)}{a+b\sinh(x)}+\left(\frac{b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)+\left(\frac{a^2-b^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)-4a\log(a+b\sin}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*(b*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a*
Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh
[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x])))/(a^2 + b^2)^2
```

fricas [B] time = 0.51, size = 383, normalized size = 4.85

$$2\left(\left(a^2b-b^3-\left(a^2b-b^3\right)\cosh(x)^2-\left(a^2b-b^3\right)\sinh(x)^2-2\left(a^3-ab^2\right)\cosh(x)-2\left(a^3-ab^2+\left(a^2b-b^3\right)\cosh(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^
3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*arcta
n(cosh(x) + sinh(x)) + (a^2*b + b^3)*cosh(x) - (a*b^2*cosh(x)^2 + a*b^2*sin
h(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2
*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2
+ 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (a^2*b + b^3)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 -
(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 -
 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b
+ 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

giac [B] time = 0.56, size = 186, normalized size = 2.35

$$\frac{2\,ab^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^2-b^2\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{2}{\left(a^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(pi + 2*arctan(1/2*(
```

```
e^(2*x) - 1)*e^(-x)))*(a^2 - b^2)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a*b^2*(e^(-x
) - e^x) - 3*a^2*b - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x) - 2*a)
)
```

maple [B] time = 0.07, size = 201, normalized size = 2.54

$$-\frac{2b^2 a \tanh\left(\frac{x}{2}\right)}{\left(a^2+b^2\right)^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}-\frac{2b^4\tanh\left(\frac{x}{2}\right)}{\left(a^2+b^2\right)^2 a\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2ba\ln\left(a\left(\tanh^2\right.\right.}{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sinh(x))^2,x)
```

```
[Out] -2*b^2/(a^2+b^2)^2*a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2*b^4/
(a^2+b^2)^2/a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2*b/(a^2+b^2)
^2*a*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2/(a^4+2*a^2*b^2+b^4)*a*b*ln(tan
h(1/2*x)^2+1)+2/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*a^2-2/(a^4+2*a^2*b^
2+b^4)*arctan(tanh(1/2*x))*b^2
```

maxima [A] time = 0.41, size = 149, normalized size = 1.89

$$\frac{2\,ab\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(a^2-b^2\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,be^{(-x)}}{a^2b+b^3+2\left(a^3+ab^2\right)e^{(-x)}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - 2*a*b*log
(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^2 - b^2)*arctan(e^(-x))/(a^4
+ 2*a^2*b^2 + b^4) - 2*b*e^(-x)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x) - (a^
2*b + b^3)*e^(-2*x))
```

mupad [B] time = 1.79, size = 186, normalized size = 2.35

$$\frac{2\,a\,b\,\ln\left(b^5\,e^{2\,x}-a^4\,b-b^5-14\,a^2\,b^3+2\,a^5\,e^x+14\,a^2\,b^3\,e^{2\,x}+2\,a\,b^4\,e^x+a^4\,b\,e^{2\,x}+28\,a^3\,b^2\,e^x\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,b^2}{\left(a^2\,b+b^3\right)\,\left(2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*sinh(x))^2),x)
```

```
[Out] (2*a*b*log(b^5*exp(2*x) - a^4*b - b^5 - 14*a^2*b^3 + 2*a^5*exp(x) + 14*a^2*
b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) + 28*a^3*b^2*exp(x)))/(a^4 +
 b^4 + 2*a^2*b^2) - (log(exp(x) + 1i)*1i)/(a*b*2i - a^2 + b^2) - (2*b^2*exp
```

```
(x))/((a^2*b + b^3)*(2*a*exp(x) - b + b*exp(2*x))) - log(exp(x)*1i + 1)/(2*
a*b - a^2*1i + b^2*1i)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(a + b*sinh(x))**2, x)
```

3.205 $\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\operatorname{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] -6*a*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-b*sech(
x)/(a^2+b^2)/(a+b*sinh(x))+sech(x)*(3*a*b+(a^2-2*b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A] time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2694, 2866, 12, 2660, 618, 206}

$$-\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\operatorname{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-6*a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
b*Sech[x])/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]*(3*a*b + (a^2 - 2*b^2)*
Sinh[x]))/(a^2 + b^2)^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} - \frac{\int \frac{\operatorname{sech}^2(x)(-a+2b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{\int \frac{3ab^2}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}$$

$$= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(3ab^2)\int \frac{1}{a+b\sinh(x)}\,d}{(a^2+b^2)^2}$$

$$= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(6ab^2)\operatorname{Subst}\left(\int \frac{}{a+2b}\right.}{(a^2}$$

$$= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} - \frac{(12ab^2)\operatorname{Subst}\left(\int \frac{}{4(a}\right.}{}$$

$$= -\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh\right.}{(a^2+b^2)^2}$$

Mathematica [A] time = 0.23, size = 94, normalized size = 1.01

$$\frac{\frac{6ab^2\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + a^2\tanh(x) - \frac{b^3\cosh(x)}{a+b\sinh(x)} + 2ab\operatorname{sech}(x) - b^2\tanh(x)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 2*
a*b*Sech[x] - (b^3*Cosh[x])/(a + b*Sinh[x]) + a^2*Tanh[x] - b^2*Tanh[x])/(a
^2 + b^2)^2
```

fricas [B] time = 1.94, size = 802, normalized size = 8.62

$$-\frac{2a^4b - 2a^2b^3 - 4b^5 + 6\left(a^3b^2+ab^4\right)\cosh(x)^3 + 6\left(a^3b^2+ab^4\right)\sinh(x)^3 + 6\left(a^4b+a^2b^3\right)\cosh(x)^2 + 6\left(a^4b+a\right.}{a^6b + 3a^4b^3 + 3a^2b^5 + b^7 - \left(a^6b+3a^4b^3+3a^2b^5+b^7\right)\cosh(x)^4 -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 6*(a^3*b^2 + a*b^4)*cosh(x)^3 + 6*(a^3*b^2
+ a*b^4)*sinh(x)^3 + 6*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b + a^2*b^3 + 3
*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^2 + 3*(a*b^3*cosh(x)^4 + a*b^3*sinh(x)^
4 + 2*a^2*b^2*cosh(x)^3 + 2*a^2*b^2*cosh(x) - a*b^3 + 2*(2*a*b^3*cosh(x) +
a^2*b^2)*sinh(x)^3 + 6*(a*b^3*cosh(x)^2 + a^2*b^2*cosh(x))*sinh(x)^2 + 2*(2
*a*b^3*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2 + a^2*b^2)*sinh(x))*sqrt(a^2 + b^2)*
log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*c
osh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*
(2*a^5 + a^3*b^2 - a*b^4)*cosh(x) - 2*(2*a^5 + a^3*b^2 - a*b^4 - 9*(a^3*b^2
 + a*b^4)*cosh(x)^2 - 6*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*
b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (
a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7 + 3*a^5*b^2 + 3*a^3
*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*((a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x
))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) - 2*(a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x))
```

giac [A] time = 0.17, size = 167, normalized size = 1.80

$$\frac{3\,ab^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,ab^2e^{(3\,x)}+3\,a^2be^{(2\,x)}-2\,a^3e^x+ab^2e^x+a^2b-2\,b^3\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3*a*b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^2*e^(3
*x) + 3*a^2*b*e^(2*x) - 2*a^3*e^x + a*b^2*e^x + a^2*b - 2*b^3)/((a^4 + 2*a^
2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b))
```

maple [A] time = 0.07, size = 138, normalized size = 1.48

$$-\frac{2b^2\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tanh\left(\frac{x}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] -2*b^2/(a^2+b^2)^2*((-b^2/a*tanh(1/2*x)-b)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b
-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(tanh(1/2*x)^2+1)
```

maxima [B] time = 0.42, size = 215, normalized size = 2.31

$$\frac{3\,ab^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,a^2be^{(-2\,x)}-3\,ab^2e^{(-3\,x)}+a^2b-2\,b^3+\left(2\,a^3-ab^2\right)e^{(-x)}\right)}{a^4b+2\,a^2b^3+b^5+2\left(a^5+2\,a^3b^2+ab^4\right)e^{(-x)}+2\left(a^5+2\,a^3b^2+ab^4\right)e^{(-3\,x)}-\left(a^4b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2
)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*e^(-2*x) - 3*a*b
^2*e^(-3*x) + a^2*b - 2*b^3 + (2*a^3 - a*b^2)*e^(-x))/(a^4*b + 2*a^2*b^3 +
b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-
3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

mupad [B] time = 0.97, size = 302, normalized size = 3.25

$$\frac{\frac{6\,a^4\,b^4\,\mathrm{e}^{2\,x}}{\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}-\frac{2\left(2\,a^2\,b^6-a^4\,b^4\right)}{\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}+\frac{6\,a^3\,b^5\,\mathrm{e}^{3\,x}}{\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}+\frac{2\,a\,\mathrm{e}^x\left(a^2\,b^6-2\,a^4\,b^4\right)}{b\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}}{2\,a\,\mathrm{e}^x-b+2\,a\,\mathrm{e}^{3\,x}+b\,\mathrm{e}^{4\,x}}-\frac{3\,a\,b^2\,\ln\left(-\frac{6\,a\,b\,\mathrm{e}^x}{\left(a^2+b^2\right)^2}-\frac{6\,a\,b\,\left(b-a\,\mathrm{e}^x\right)}{\left(a^2+b^2\right)^{5/2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*sinh(x))^2),x)
```

```
[Out] ((6*a^4*b^4*exp(2*x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*(2*a^2*b^6 - a
^4*b^4))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^3*b^5*exp(3*x))/((a*b^2 +
 a^3)*(a*b^5 + a^3*b^3)) + (2*a*exp(x)*(a^2*b^6 - 2*a^4*b^4))/(b*(a*b^2 + a
^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*exp(3*x) + b*exp(4*x)) - (3*a
*b^2*log(- (6*a*b*exp(x))/(a^2 + b^2)^2 - (6*a*b*(b - a*exp(x)))/(a^2 + b^2
)^(5/2)))/(a^2 + b^2)^(5/2) + (3*a*b^2*log((6*a*b*(b - a*exp(x)))/(a^2 + b^
2)^(5/2) - (6*a*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**2/(a + b*sinh(x))**2, x)
```

3.206 $\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=136

$$\frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{\operatorname{sech}^2(x)(a\sinh(x)+b)}{2\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}-\frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{\left(a^4+6a^2\right.}{}$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(sinh(x))/(a^2+b^2)^3-4*a*b^3*ln(cosh(x))/(
a^2+b^2)^3+4*a*b^3*ln(a+b*sinh(x))/(a^2+b^2)^3+1/2*b*(a^2-3*b^2)/(a^2+b^2)^
2/(a+b*sinh(x))+1/2*sech(x)^2*(b+a*sinh(x))/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2668, 741, 801, 635, 203, 260}

$$\frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{\left(6a^2b^2+a^4-3b^4\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^3}-\frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{\text{s}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^3) - (4*a*b^3*Lo
g[Cosh[x]])/(a^2 + b^2)^3 + (4*a*b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (b
*(a^2 - 3*b^2))/(2*(a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(b + a*Sinh[
x]))/(2*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = b^3\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\text{Subst}\left(\int \frac{a^2+3b^2+2ax}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\,\text{Subst}\left(\int\left(\frac{a^2-3b^2}{\left(a^2+b^2\right)(a+x)^2} - \frac{8ab^2}{\left(a^2+b^2\right)^2(a+x)} + \frac{-a^4-6a^2b^2+3b^4+8ab^2x}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right.\right.}{2\left(a^2+b^2\right)}$$

$$= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b}{\cdots}$$

$$= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{(\cdots}{\cdots}$$

$$= \frac{\left(a^4+6a^2b^2-3b^4\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^3} - \frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3} + \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\cdots}{2(\cdots}$$

Mathematica [A] time = 2.44, size = 260, normalized size = 1.91

$$-\frac{\frac{b\left(\frac{2a\left(a^2+b^2\right)\left(\left(\sqrt{-b^2}-a\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}}+\left(3b^2-a^2\right)\left(\frac{2\left(a^2+b^2\right)}{a+b\sinh(x)}+\left(\frac{b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{-b^2}-b\sinh(\cdots\right.\right.}{\left(a^2+b^2\right)^2}}{4\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/4*((-2*Sech[x]^2*(b + a*Sinh[x]))/(a + b*Sinh[x]) + (b*((2*a*(a^2 + b^2)
*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a + b*Si
nh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (-a^2
+ 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a
*Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sin
h[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x]))))/(a^2 + b^2)^2)/(a^2 + b^2)
```

fricas [B] time = 1.71, size = 2615, normalized size = 19.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")

[Out] -((a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^5 + (a^4*b - 2*a^2*b^3 - 3*b^5)*sinh(x)^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + (2*a^5 + 4*a^3*b^2 + 2*a*b^4 + 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^4 + 2*(3*a^4*b + 2*a^2*b^3 - b^5)*cosh(x)^3 + 2*(3*a^4*b + 2*a^2*b^3 - b^5 + 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^2 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 - 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^3 - 6*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 3*(3*a^4*b + 2*a^2*b^3 - b^5)*cosh(x))*sinh(x)^2 + ((a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^6 + (a^4*b + 6*a^2*b^3 - 3*b^5)*sinh(x)^6 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^5 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 3*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^5 - a^4*b - 6*a^2*b^3 + 3*b^5 + (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^4 + (a^4*b + 6*a^2*b^3 - 3*b^5 + 15*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^2 + 10*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^4 + 4*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 + 4*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^2 + (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5 - 15*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^4 - 20*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 - 6*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^2 - 12*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^2 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x) + 2*(3*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^5 + a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^4 + 2*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 6*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x) + 4*(a*b^4*cosh(x)^6 + a*b^4*sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3 - a*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh(x)^5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*sinh(x)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 + 12*a^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*cosh(x)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3)*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - 4*(a*b^4*cosh(x)^6 + a*b^4*sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3 - a*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh(x)^5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*sinh(x)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 + 12*a^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*cosh(x)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3)*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^4*b - 2*a^2*b^3 - 3*b^5 + 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^4 + 8*(a^5 + 2*a^3*b^2 + a*b^4)*cos

```
h(x)^3 + 6*(3*a^4*b + 2*a^2*b^3 - b^5)*cosh(x)^2 - 4*(a^5 + 2*a^3*b^2 + a*b
^4)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4
*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*s
inh(x)^6 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x
))*sinh(x)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3
*a^4*b^3 + 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x)^2 + 10*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*
a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b
^7)*cosh(x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 +
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*cosh(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2
+ 3*a^3*b^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x))*sinh(x))
```

giac [B] time = 0.18, size = 295, normalized size = 2.17

$$\frac{4\,ab^4\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{2\,ab^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^4+6\,a^2b^2-3\right.}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 4*a*b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 2*a*b^3*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^4 + 6*a^2*b^2 - 3*b^4)
/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^2*b*(e^(-x) - e^x)^2 - 3*b^3*(e^(
-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 8*a^2*b - 8*
b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-x) - e^x)^2 +
4*b*(e^(-x) - e^x) - 8*a))
```

maple [B] time = 0.10, size = 548, normalized size = 4.03

$$-\frac{2b^4a\tanh\left(\frac{x}{2}\right)}{\left(a^2+b^2\right)^3\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}-\frac{2b^6\tanh\left(\frac{x}{2}\right)}{\left(a^2+b^2\right)^3a\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{4b^3a\ln\left(a\left(\tanh\right.\right.}{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] -2*b^4/(a^2+b^2)^3*a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-2*b^6/
(a^2+b^2)^3/a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+4*b^3/(a^2+b^
2)^3*a*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2
)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^4+1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(ta
nh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b^4-4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/
2*x)^2+1)^2*tanh(1/2*x)^2*a^3*b-4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x
)^2+1)^2*tanh(1/2*x)^2*a*b^3+1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2
+1)^2*tanh(1/2*x)*a^4-1/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)/(tanh(1/2*x)^2+1)^2*t
anh(1/2*x)*b^4-4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*a*b^3*ln(tanh(1/2*x)^2+1)+1/
(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arctan(tanh(1/2*x))*a^4+6/(a^4+2*a^2*b^2+b^4)
/(a^2+b^2)*arctan(tanh(1/2*x))*a^2*b^2-3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*arct
an(tanh(1/2*x))*b^4
```

maxima [B] time = 0.42, size = 375, normalized size = 2.76

$$\frac{4\,ab^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\cdots}{a^4b+2\,a^2b^3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 4*a*b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) - 4*a*b^3*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 +
 6*a^2*b^2 - 3*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((
a^2*b - 3*b^3)*e^(-x) + 2*(a^3 + a*b^2)*e^(-2*x) + 2*(3*a^2*b - b^3)*e^(-3*
x) - 2*(a^3 + a*b^2)*e^(-4*x) + (a^2*b - 3*b^3)*e^(-5*x))/(a^4*b + 2*a^2*b^
3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^
(-2*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e
^(-4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*
e^(-6*x))
```

mupad [B] time = 5.07, size = 519, normalized size = 3.82

$$\frac{\frac{4\left(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7\right)}{\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)^2}+\frac{e^x\left(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8\right)}{\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)^2}}{e^{2\,x}+1}-\frac{\frac{4\,a\,b}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,e^x\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\ln\left(e^x+1\mathrm{i}\right)\,\left(a-b\,3\mathrm{i}\right)}{2\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}+\frac{\cdots}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*sinh(x))^2),x)
```

```
[Out] ((4*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a^
2*b^2)^2) + (exp(x)*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^4
+ b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((4*a*b)/(a^4 + b^4 + 2*a^2*b^2) +
(2*exp(x)*(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x) + 1)
 + (log(exp(x) + 1i)*(a - b*3i))/(2*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) +
(log(exp(x)*1i + 1)*(a*1i - 3*b))/(2*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) +
 (4*a*b^3*log(9*b^9*exp(2*x) - a^8*b - 9*b^9 - 220*a^2*b^7 - 30*a^4*b^5 - 1
2*a^6*b^3 + 2*a^9*exp(x) + 220*a^2*b^7*exp(2*x) + 30*a^4*b^5*exp(2*x) + 12*
a^6*b^3*exp(2*x) + 18*a*b^8*exp(x) + a^8*b*exp(2*x) + 440*a^3*b^6*exp(x) +
60*a^5*b^4*exp(x) + 24*a^7*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
 - (2*exp(x)*(b^10 + 2*a^2*b^8 + a^4*b^6))/(b^2*(a^2*b + b^3)*(a^2 + b^2)*(
2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**3/(a + b*sinh(x))**2, x)
```

3.207 $\int \frac{\text{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=144

$$-\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))}+\frac{\text{sech}^3(x)\left((a^2-4b^2)\sinh(x)+5ab\right)}{3(a^2+b^2)^2}-\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{\text{sech}(x)\left((2a^4+9\right.}{}$$

[Out] -10*a*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-b*sech(x)^3/(a^2+b^2)/(a+b*sinh(x))+1/3*sech(x)^3*(5*a*b+(a^2-4*b^2)*sinh(x))/(a^2+b^2)^2+1/3*sech(x)*(15*a*b^3+(2*a^4+9*a^2*b^2-8*b^4)*sinh(x))/(a^2+b^2)^3

Rubi [A] time = 0.31, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2694, 2866, 12, 2660, 618, 206}

$$-\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))}+\frac{\text{sech}^3(x)\left((a^2-4b^2)\sinh(x)+5ab\right)}{3(a^2+b^2)^2}+\frac{\text{sech}(x)\left((9a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^4/(a + b*Sinh[x])^2,x]

[Out] (-10*a*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) - (b*Sech[x]^3)/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]^3*(5*a*b + (a^2 - 4*b^2)*Sinh[x]))/(3*(a^2 + b^2)^2) + (Sech[x]*(15*a*b^3 + (2*a^4 + 9*a^2*b^2 - 8*b^4)*Sinh[x]))/(3*(a^2 + b^2)^3)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\int \frac{\operatorname{sech}^4(x)(-a+4b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\int \frac{\operatorname{sech}^2(x)\left(a\left(2a^2+7b^2\right.\right.}{a+b\,s}}{3\left(a^2\right.}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\right.}{3}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\right.}{3}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\right.}{3}$$

$$= -\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2} + \frac{\operatorname{sech}(x)\left(15ab^3+\right.}{3}$$

$$= -\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)(a+b\sinh(x))} + \frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\mathrm{s}\right.}{3\left(a^2+b^2\right)^2}$$

Mathematica [A] time = 0.44, size = 137, normalized size = 0.95

$$\frac{\left(a^2+b^2\right)\operatorname{sech}^3(x)\left(\left(a^2-b^2\right)\sinh(x)+2ab\right) + \frac{30ab^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \left(2a^4+9a^2b^2-5b^4\right)\tanh(x) - \frac{3b^5\cosh(x)}{a+b\sinh(x)} +}{3\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((30*a*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 1
2*a*b^3*Sech[x] - (3*b^5*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2)*Sech[x]^3*(
2*a*b + (a^2 - b^2)*Sinh[x]) + (2*a^4 + 9*a^2*b^2 - 5*b^4)*Tanh[x])/(3*(a^2
 + b^2)^3)
```

fricas [B] time = 1.00, size = 3044, normalized size = 21.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(30*(a^3*b^4 + a*b^6)*cosh(x)^7 + 30*(a^3*b^4 + a*b^6)*sinh(x)^7 + 4*a
^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 30*(a^4*b^3 + a^2*b^5)*cosh(x)^6 +
 30*(a^4*b^3 + a^2*b^5 + 7*(a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^6 - 10*(2*a^5
*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^5 - 10*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6
 - 63*(a^3*b^4 + a*b^6)*cosh(x)^2 - 18*(a^4*b^3 + a^2*b^5)*cosh(x))*sinh(x)
^5 + 10*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^4 + 10*(2*a^6*b + 13*a^
4*b^3 + 11*a^2*b^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^3 + 45*(a^4*b^3 + a^2*b^
5)*cosh(x)^2 - 5*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^4 - 2*(
12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 - 13*a*b^6)*cosh(x)^3 - 2*(12*a^7 + 56*a^5
*b^2 + 31*a^3*b^4 - 13*a*b^6 - 525*(a^3*b^4 + a*b^6)*cosh(x)^4 - 300*(a^4*b
^3 + a^2*b^5)*cosh(x)^3 + 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^2 -
20*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x))*sinh(x)^3 + 2*(4*a^6*b + 37
*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*cosh(x)^2 + 2*(4*a^6*b + 37*a^4*b^3 + 17*a^
2*b^5 - 16*b^7 + 315*(a^3*b^4 + a*b^6)*cosh(x)^5 + 225*(a^4*b^3 + a^2*b^5)*
cosh(x)^4 - 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^3 + 30*(2*a^6*b +
13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^2 - 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 -
13*a*b^6)*cosh(x))*sinh(x)^2 + 15*(a*b^5*cosh(x)^8 + a*b^5*sinh(x)^8 + 2*a^
2*b^4*cosh(x)^7 + 2*a*b^5*cosh(x)^6 + 6*a^2*b^4*cosh(x)^5 + 6*a^2*b^4*cosh(
x)^3 - 2*a*b^5*cosh(x)^2 + 2*(4*a*b^5*cosh(x) + a^2*b^4)*sinh(x)^7 + 2*a^2*
b^4*cosh(x) + 2*(14*a*b^5*cosh(x)^2 + 7*a^2*b^4*cosh(x) + a*b^5)*sinh(x)^6
- a*b^5 + 2*(28*a*b^5*cosh(x)^3 + 21*a^2*b^4*cosh(x)^2 + 6*a*b^5*cosh(x) +
3*a^2*b^4)*sinh(x)^5 + 10*(7*a*b^5*cosh(x)^4 + 7*a^2*b^4*cosh(x)^3 + 3*a*b^
5*cosh(x)^2 + 3*a^2*b^4*cosh(x))*sinh(x)^4 + 2*(28*a*b^5*cosh(x)^5 + 35*a^2
*b^4*cosh(x)^4 + 20*a*b^5*cosh(x)^3 + 30*a^2*b^4*cosh(x)^2 + 3*a^2*b^4)*sin
h(x)^3 + 2*(14*a*b^5*cosh(x)^6 + 21*a^2*b^4*cosh(x)^5 + 15*a*b^5*cosh(x)^4
+ 30*a^2*b^4*cosh(x)^3 + 9*a^2*b^4*cosh(x) - a*b^5)*sinh(x)^2 + 2*(4*a*b^5*
cosh(x)^7 + 7*a^2*b^4*cosh(x)^6 + 6*a*b^5*cosh(x)^5 + 15*a^2*b^4*cosh(x)^4
+ 9*a^2*b^4*cosh(x)^2 - 2*a*b^5*cosh(x) + a^2*b^4)*sinh(x))*sqrt(a^2 + b^2)
*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2
*(4*a^7 + 22*a^5*b^2 + 17*a^3*b^4 - a*b^6)*cosh(x) - 2*(4*a^7 + 22*a^5*b^2
+ 17*a^3*b^4 - a*b^6 - 105*(a^3*b^4 + a*b^6)*cosh(x)^6 - 90*(a^4*b^3 + a^2*
b^5)*cosh(x)^5 + 25*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^4 - 20*(2*a^6
*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^3 + 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b
^4 - 13*a*b^6)*cosh(x)^2 - 2*(4*a^6*b + 37*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*c
osh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 - (a^8*b
```

```
+ 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^8 - (a^8*b + 4*a^6*b^3 +
 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 +
 a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(
x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^6 - 2*(a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 + 14*(a^8*b + 4*a^6*b^3 + 6*
a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a
^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 +
 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 +
 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 + 6*(a^8*b
+ 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(x)^5 - 10*(7*(a^8*
b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 + 7*(a^9 + 4*a^7*b^2
 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b
^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
 + a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 + 3*a
*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^5 + 35*
(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 + 20*(a^8*b + 4
*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 + 30*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2)*sinh(x)^3 + 2*(a^8*b + 4*a^6*b^3 +
 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5
+ 4*a^2*b^7 + b^9 - 14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*co
sh(x)^6 - 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 -
15*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 - 30*(a^9 +
4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 9*(a^9 + 4*a^7*b^2 +
 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 - 2*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x) - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(
x)^7 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^6 + 6*(a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^5 + 15*(a^9 + 4*a^7
*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 + 9*(a^9 + 4*a^7*b^2 + 6*a^
5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4
*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

giac [B] time = 0.26, size = 287, normalized size = 1.99

$$\frac{5\,ab^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^4e^x-b^5\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}+\frac{2\left(12\,ab^3e^{(5\,x)}-9\,a^2b^2e^{(4\,}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 5*a*b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(
a*b^4*e^x - b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e^x
- b)) + 2/3*(12*a*b^3*e^(5*x) - 9*a^2*b^2*e^(4*x) + 3*b^4*e^(4*x) + 8*a^3*b
*e^(3*x) + 32*a*b^3*e^(3*x) - 6*a^4*e^(2*x) - 18*a^2*b^2*e^(2*x) + 12*b^4*e
^(2*x) + 12*a*b^3*e^x - 2*a^4 - 9*a^2*b^2 + 5*b^4)/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

maple [A] time = 0.12, size = 266, normalized size = 1.85

$$-\frac{2b^4\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a}-\frac{5a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(-2a^3b-6a\,b^3\right)\left(\tanh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tanh(1/2*x)-b)/(a*tanh(1/2*x)
^2-2*tanh(1/2*x)*b-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2)))-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*
tanh(1/2*x)^5+(-2*a^3*b-6*a*b^3)*tanh(1/2*x)^4+(-2/3*a^4-6*a^2*b^2+8/3*b^4)
*tanh(1/2*x)^3-8*a*b^3*tanh(1/2*x)^2+(-a^4-3*a^2*b^2+2*b^4)*tanh(1/2*x)-2/3
*a^3*b-14/3*a*b^3)/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.42, size = 490, normalized size = 3.40

$$\frac{5\,ab^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,a^2b^3e^{(-6\,x)}-15\,ab^4e^{(-7\,x)}\right.}{3\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)e^{(-x)}+2\left(a^6b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2
)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/3*(15*a^2*b^3
*e^(-6*x) - 15*a*b^4*e^(-7*x) + 2*a^4*b + 9*a^2*b^3 - 8*b^5 + (4*a^5 + 18*a
^3*b^2 - a*b^4)*e^(-x) + (4*a^4*b + 33*a^2*b^3 - 16*b^5)*e^(-2*x) + (12*a^5
 + 44*a^3*b^2 - 13*a*b^4)*e^(-3*x) + 5*(2*a^4*b + 11*a^2*b^3)*e^(-4*x) + 5*
(2*a^3*b^2 - 7*a*b^4)*e^(-5*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 + 2*(a
^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7)*e^(-2*x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*x) + 6*(
```

```
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b^3 + 3*a^
2*b^5 + b^7)*e^(-6*x) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-7*x) -
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-8*x))
```

mupad [B] time = 1.06, size = 476, normalized size = 3.31

$$\frac{\frac{8\left(a^2-b^2\right)}{3\left(a^4+2a^2b^2+b^4\right)}-\frac{16\,a\,b\,e^x}{3\left(a^4+2a^2b^2+b^4\right)}}{3e^{2x}+3e^{4x}+e^{6x}+1}-\frac{\frac{4\left(a^6+a^4b^2-a^2b^4-b^6\right)}{\left(a^4+2a^2b^2+b^4\right)^2}-\frac{16\,e^x\left(a^5b+2a^3b^3+ab^5\right)}{3\left(a^4+2a^2b^2+b^4\right)^2}}{2e^{2x}+e^{4x}+1}-\frac{\frac{2\left(3a^4b^2+2a^2b^4-b^6\right)}{\left(a^4+2a^2b^2+b^4\right)^2}-\frac{8\,e^x\left(a^3b^3+ab^5\right)}{\left(a^4+2a^2b^2+b^4\right)^2}}{e^{2x}+1}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*sinh(x))^2),x)
```

```
[Out] ((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b^
4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^6
- a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^5*b
 + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) -
 ((2*(2*a^2*b^4 - b^6 + 3*a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (8*exp(x)*(
a*b^5 + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*x) + 1) - ((2*(b^11 + a
^2*b^9))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x)*(a*b^11 + a^3*b^9))/
(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b + b*exp(2*x)) - (5*a*b^4
*log(- (10*a*b^3*(b - a*exp(x)))/(a^2 + b^2)^(7/2) - (10*a*b^3*exp(x))/(a^2
 + b^2)^3))/(a^2 + b^2)^(7/2) + (5*a*b^4*log((10*a*b^3*(b - a*exp(x)))/(a^2
 + b^2)^(7/2) - (10*a*b^3*exp(x))/(a^2 + b^2)^3))/(a^2 + b^2)^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x))**2, x)
```

3.208 $\int \frac{\tanh^4(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{5}i\tanh^5(x) - \frac{1}{5}\text{sech}^5(x) + \frac{2\text{sech}^3(x)}{3} - \text{sech}(x)$$

```
[Out] -sech(x)+2/3*sech(x)^3-1/5*sech(x)^5-1/5*I*tanh(x)^5
```

Rubi [A] time = 0.08, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{1}{5}i\tanh^5(x) - \frac{1}{5}\text{sech}^5(x) + \frac{2\text{sech}^3(x)}{3} - \text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Sinh[x]),x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5 - (I/5)*Tanh[x]^5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh^4(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^5(x)\,dx\\ &= -\operatorname{Subst}\left(\int x^4\,dx, x, i\tanh(x)\right)-\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \operatorname{sech}(x)\right)\\ &= -\frac{1}{5}i\tanh^5(x)-\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \operatorname{sech}(x)\right)\\ &= -\operatorname{sech}(x)+\frac{2\operatorname{sech}^3(x)}{3}-\frac{\operatorname{sech}^5(x)}{5}-\frac{1}{5}i\tanh^5(x)\end{aligned}$$

Mathematica [B] time = 0.14, size = 96, normalized size = 3.10

$$-\frac{64i\sinh(x)+178i\sinh(2x)-192i\sinh(3x)+89i\sinh(4x)-534\cosh(x)+288\cosh(2x)-178\cosh(3x)+2}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x]),x]
```

```
[Out] -1/960*(200 - 534*Cosh[x] + 288*Cosh[2*x] - 178*Cosh[3*x] + 24*Cosh[4*x] +
(64*I)*Sinh[x] + (178*I)*Sinh[2*x] - (192*I)*Sinh[3*x] + (89*I)*Sinh[4*x])/
((Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] + I*Sinh[x/2])^3)
```

fricas [B] time = 0.66, size = 88, normalized size = 2.84

$$-\frac{30\,e^{(7\,x)}+30i\,e^{(6\,x)}+10\,e^{(5\,x)}+50i\,e^{(4\,x)}+26\,e^{(3\,x)}+42i\,e^{(2\,x)}-18\,e^{x}+6i}{15\,e^{(8\,x)}+30i\,e^{(7\,x)}+30\,e^{(6\,x)}+90i\,e^{(5\,x)}+90i\,e^{(3\,x)}-30\,e^{(2\,x)}+30i\,e^{x}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(30*e^(7*x) + 30*I*e^(6*x) + 10*e^(5*x) + 50*I*e^(4*x) + 26*e^(3*x) + 42*I
*e^(2*x) - 18*e^x + 6*I)/(15*e^(8*x) + 30*I*e^(7*x) + 30*e^(6*x) + 90*I*e^(
5*x) + 90*I*e^(3*x) - 30*e^(2*x) + 30*I*e^x - 15)
```

giac [B] time = 0.28, size = 53, normalized size = 1.71

$$-\frac{15\,e^{(2\,x)}-24i\,e^{x}-13}{24\,(e^{x}-i)^{3}}-\frac{165\,e^{(4\,x)}+480i\,e^{(3\,x)}-650\,e^{(2\,x)}-400i\,e^{x}+113}{120\,(e^{x}+i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/24*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - 1/120*(165*e^(4*x) + 480*I
*e^(3*x) - 650*e^(2*x) - 400*I*e^x + 113)/(e^x + I)^5
```

maple [B] time = 0.10, size = 93, normalized size = 3.00

$$\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{}{6(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+sinh(x)),x)
```

```
[Out] 1/3*I/(tanh(1/2*x)+I)^3-2/5*I/(tanh(1/2*x)+I)^5-3/8*I/(tanh(1/2*x)+I)+1/(ta
nh(1/2*x)+I)^4+1/2/(tanh(1/2*x)+I)^2+3/8*I/(tanh(1/2*x)-I)+1/6*I/(tanh(1/2*
x)-I)^3+1/4/(tanh(1/2*x)-I)^2
```

maxima [B] time = 0.32, size = 413, normalized size = 13.32

$$\frac{18\,e^{(-x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}+\frac{}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 18*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 42*I*e^(-2*x)/(-30*I*e^(-x)
- 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
 + 15*e^(-8*x) - 15) - 26*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3
*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 50*
I*e^(-4*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 10*e^(-5*x)/(-30*I*e^(-x) -
 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
```

```
+ 15*e^(-8*x) - 15) + 30*I*e^(-6*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 30
*e^(-7*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 6*I/(-30*I*e^(-x) - 30*e^(-2
*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-
8*x) - 15)
```

mupad [B] time = 1.17, size = 231, normalized size = 7.45

$$-\frac{1}{6\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{\frac{11\,\mathrm{e}^{x}}{40}+\frac{1}{8}\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{\frac{11\,\mathrm{e}^{2\,x}}{40}-\frac{17}{120}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}+\frac{1\mathrm{i}}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)}-\frac{5}{8\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{11}{40\,\left(\mathrm{e}^{x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] 1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((11*exp(x))/40 + 1i/8)/(exp(2*x) + exp
(x)*2i - 1) - ((11*exp(2*x))/40 + (exp(x)*1i)/4 - 17/120)/(exp(2*x)*3i + ex
p(3*x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) -
5/(8*(exp(x) - 1i)) - 11/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (11*exp(3*
x))/40 - (17*exp(x))/40 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(
x)*4i + 1) - ((exp(3*x)*1i)/2 - (17*exp(2*x))/20 + (11*exp(4*x))/40 - (exp(
x)*1i)/2 + 11/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*
exp(x) + 1i)
```

sympy [B] time = 0.29, size = 107, normalized size = 3.45

$$\frac{30e^{7x}+30ie^{6x}+10e^{5x}+50ie^{4x}+26e^{3x}+42ie^{2x}-18e^{x}+6i}{-15e^{8x}-30ie^{7x}-30e^{6x}-90ie^{5x}-90ie^{3x}+30e^{2x}-30ie^{x}+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+sinh(x)),x)
```

```
[Out] (30*exp(7*x) + 30*I*exp(6*x) + 10*exp(5*x) + 50*I*exp(4*x) + 26*exp(3*x) +
42*I*exp(2*x) - 18*exp(x) + 6*I)/(-15*exp(8*x) - 30*I*exp(7*x) - 30*exp(6*x
) - 90*I*exp(5*x) - 90*I*exp(3*x) + 30*exp(2*x) - 30*I*exp(x) + 15)
```

3.209 $\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\tan^{-1}(\sinh(x))-\frac{1}{4}\tanh^3(x)\text{sech}(x)-\frac{3}{8}\tanh(x)\text{sech}(x)$$

```
[Out] 3/8*arctan(sinh(x))-3/8*sech(x)*tanh(x)-1/4*sech(x)*tanh(x)^3-1/4*I*tanh(x)
^4
```

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\tan^{-1}(\sinh(x))-\frac{1}{4}\tanh^3(x)\text{sech}(x)-\frac{3}{8}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 - (3*Sech[x]*Tanh[x])/8 - (Sech[x]*Tanh[x]^3)/4 - (I/
4)*Tanh[x]^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh^3(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^4(x)\,dx\\
&= -\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-i\,\mathrm{Subst}\left(\int x^3\,dx,x,i\tanh(x)\right)+\frac{3}{4}\int \operatorname{sech}(x)\tanh^2(x)\,dx\\
&= -\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\int \operatorname{sech}(x)\,dx\\
&= \frac{3}{8}\tan^{-1}(\sinh(x))-\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 42, normalized size = 1.17

$$
\frac{1}{8}\left(3\tan^{-1}(\sinh(x))-\frac{5\sinh^2(x)+i\sinh(x)+2}{(\sinh(x)-i)(\sinh(x)+i)^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]] - (2 + I*Sinh[x] + 5*Sinh[x]^2)/((-I + Sinh[x])*(I + Sin
h[x])^2))/8
```

fricas [B] time = 0.98, size = 152, normalized size = 4.22

$$
\frac{\left(3i\,e^{(6\,x)}-6\,e^{(5\,x)}+3i\,e^{(4\,x)}-12\,e^{(3\,x)}-3i\,e^{(2\,x)}-6\,e^{x}-3i\right)\log\left(e^{x}+i\right)+\left(-3i\,e^{(6\,x)}+6\,e^{(5\,x)}-3i\,e^{(4\,x)}+12\,e^{(3\,x)}+\cdots\right.}{8\,e^{(6\,x)}+16i\,e^{(5\,x)}+8\,e^{(4\,x)}+32i\,e^{(3\,x)}-8\,e^{(2\,x)}\cdots}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((3*I*e^(6*x) - 6*e^(5*x) + 3*I*e^(4*x) - 12*e^(3*x) - 3*I*e^(2*x) - 6*e^x
- 3*I)*log(e^x + I) + (-3*I*e^(6*x) + 6*e^(5*x) - 3*I*e^(4*x) + 12*e^(3*x)
+ 3*I*e^(2*x) + 6*e^x + 3*I)*log(e^x - I) - 10*e^(5*x) - 4*I*e^(4*x) + 4*e^
(3*x) + 4*I*e^(2*x) - 10*e^x)/(8*e^(6*x) + 16*I*e^(5*x) + 8*e^(4*x) + 32*I*
e^(3*x) - 8*e^(2*x) + 16*I*e^x - 8)
```

giac [B] time = 0.48, size = 92, normalized size = 2.56

$$\frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{16\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{9i\left(e^{(-x)} - e^{x}\right)^{2} + 4\,e^{(-x)} - 4\,e^{x} + 12i}{32\left(e^{(-x)} - e^{x} - 2i\right)^{2}} + \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/32*(9*I*(e^(-x) -
e^x)^2 + 4*e^(-x) - 4*e^x + 12*I)/(e^(-x) - e^x - 2*I)^2 + 3/16*I*log(-e^(-
x) + e^x + 2*I) - 3/16*I*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.09, size = 79, normalized size = 2.19

$$-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right) + i\right)^{4}} + \frac{3i\ln\left(\tanh\left(\frac{x}{2}\right) + i\right)}{8} + \frac{1}{\left(\tanh\left(\frac{x}{2}\right) + i\right)^{3}} + \frac{1}{2\tanh\left(\frac{x}{2}\right) + 2i} - \frac{3i\ln\left(\tanh\left(\frac{x}{2}\right) - i\right)}{8} + \frac{i}{4\left(\tanh\left(\frac{x}{2}\right) - i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+sinh(x)),x)
```

```
[Out] -1/2*I/(tanh(1/2*x)+I)^4+3/8*I*ln(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^3+1/2/(t
anh(1/2*x)+I)-3/8*I*ln(tanh(1/2*x)-I)+1/4*I/(tanh(1/2*x)-I)^2+1/4/(tanh(1/2
*x)-I)
```

maxima [B] time = 0.30, size = 95, normalized size = 2.64

$$\frac{5\,e^{(-x)} + 2i\,e^{(-2\,x)} - 2\,e^{(-3\,x)} - 2i\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{-8i\,e^{(-x)} - 4\,e^{(-2\,x)} - 16i\,e^{(-3\,x)} + 4\,e^{(-4\,x)} - 8i\,e^{(-5\,x)} + 4\,e^{(-6\,x)} - 4} + \frac{3}{8}i\,\log\left(i\,e^{(-x)} + 1\right) - \frac{3}{8}i\,\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] (5*e^(-x) + 2*I*e^(-2*x) - 2*e^(-3*x) - 2*I*e^(-4*x) + 5*e^(-5*x))/(-8*I*e^
(-x) - 4*e^(-2*x) - 16*I*e^(-3*x) + 4*e^(-4*x) - 8*I*e^(-5*x) + 4*e^(-6*x)
- 4) + 3/8*I*log(I*e^(-x) + 1) - 3/8*I*log(I*e^(-x) - 1)
```

mupad [B] time = 0.45, size = 113, normalized size = 3.14

$$\frac{3\,\mathrm{atan}\left(\mathrm{e}^x\right)}{4}+\frac{3\mathrm{i}}{2\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}\right)}+\frac{1\mathrm{i}}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{1}{4\,\left(\mathrm{e}^x-\mathrm{i}\right)}-\frac{1}{\mathrm{e}^x+1\mathrm{i}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] (3*atan(exp(x)))/4 + 3i/(2*(exp(2*x) + exp(x)*2i - 1)) - 1i/(2*(exp(3*x)*4i
 - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) + 1i/(4*(exp(x)*2i - exp(2*x) +
1)) - 1/(4*(exp(x) - 1i)) - 1/(exp(x) + 1i) + 1/(exp(2*x)*3i + exp(3*x) - 3
*exp(x) - 1i)
```

sympy [B] time = 0.26, size = 100, normalized size = 2.78

$$\frac{5e^{5x}+2ie^{4x}-2e^{3x}-2ie^{2x}+5e^{x}}{-4e^{6x}-8ie^{5x}-4e^{4x}-16ie^{3x}+4e^{2x}-8ie^{x}+4}+\frac{3\log\left(e^{x}-i\right)}{8}-\frac{3\log\left(e^{x}+i\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+sinh(x)),x)
```

```
[Out] (5*exp(5*x) + 2*I*exp(4*x) - 2*exp(3*x) - 2*I*exp(2*x) + 5*exp(x))/(-4*exp(
6*x) - 8*I*exp(5*x) - 4*exp(4*x) - 16*I*exp(3*x) + 4*exp(2*x) - 8*I*exp(x)
+ 4) + 3*log(exp(x) - I)/8 - 3*log(exp(x) + I)/8
```

3.210 $\int \frac{\tanh^2(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{3}i\tanh^3(x)+\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

```
[Out] -sech(x)+1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$-\frac{1}{3}i\tanh^3(x)+\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Sinh[x]),x]
```

```
[Out] -Sech[x] + Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^3(x)\,dx\\ &= \operatorname{Subst}\left(\int x^2\,dx, x, i\tanh(x)\right)+\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \operatorname{sech}(x)\right)\\ &= -\operatorname{sech}(x)+\frac{\operatorname{sech}^3(x)}{3}-\frac{1}{3}i\tanh^3(x)\end{aligned}$$

Mathematica [B] time = 0.06, size = 67, normalized size = 2.91

$$\frac{4i\sinh(x)-\cosh(2x)+(5-5i\sinh(x))\cosh(x)-3}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x]),x]
[Out] (-3 - Cosh[2*x] + Cosh[x]*(5 - (5*I)*Sinh[x]) + (4*I)*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2]))
```

fricas [B] time = 0.61, size = 40, normalized size = 1.74

$$-\frac{6\,e^{(3\,x)}+6i\,e^{(2\,x)}-2\,e^{x}+2i}{3\,e^{(4\,x)}+6i\,e^{(3\,x)}+6i\,e^{x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] -(6*e^(3*x) + 6*I*e^(2*x) - 2*e^x + 2*I)/(3*e^(4*x) + 6*I*e^(3*x) + 6*I*e^x
 - 3)
```

giac [A] time = 0.20, size = 29, normalized size = 1.26

$$-\frac{1}{2\,(e^{x}-i)}-\frac{9\,e^{(2\,x)}+12i\,e^{x}-7}{6\,(e^{x}+i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(e^x - I) - 1/6*(9*e^(2*x) + 12*I*e^x - 7)/(e^x + I)^3
```

maple [B] time = 0.06, size = 47, normalized size = 2.04

$$-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{i}{2\tanh\left(\frac{x}{2}\right)-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+sinh(x)),x)
```

```
[Out] -2/3*I/(tanh(1/2*x)+I)^3-1/2*I/(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^2+1/2*I/(t
anh(1/2*x)-I)
```

maxima [B] time = 0.31, size = 109, normalized size = 4.74

$$\frac{2\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}+\frac{6i\,e^{(-2\,x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}-\frac{6\,e^{(-3\,x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}+\frac{}{-6i\,e^{(-x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 6*I*e^(-2*x)/(-6*I
*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) - 6*e^(-3*x)/(-6*I*e^(-x) - 6*I*e^
(-3*x) + 3*e^(-4*x) - 3) + 2*I/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3
)
```

mupad [B] time = 0.67, size = 80, normalized size = 3.48

$$-\frac{\frac{e^{x}}{2}+\frac{1}{6}\mathrm{i}}{e^{2\,x}-1+e^{x}\,2\mathrm{i}}-\frac{\frac{e^{2\,x}}{2}-\frac{1}{2}+\frac{e^{x}\,1\mathrm{i}}{3}}{e^{2\,x}\,3\mathrm{i}+e^{3\,x}-3\,e^{x}-\mathrm{i}}-\frac{1}{2\,\left(e^{x}-\mathrm{i}\right)}-\frac{1}{2\,\left(e^{x}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(sinh(x) + 1i),x)
```

```
[Out] - (exp(x)/2 + 1i/6)/(exp(2*x) + exp(x)*2i - 1) - (exp(2*x)/2 + (exp(x)*1i)/
3 - 1/2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 1/(2*(exp(x) - 1i)) - 1
/(2*(exp(x) + 1i))
```

sympy [B] time = 0.17, size = 46, normalized size = 2.00

$$\frac{6e^{3x}+6ie^{2x}-2e^{x}+2i}{-3e^{4x}-6ie^{3x}-6ie^{x}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+sinh(x)),x)
```

```
[Out] (6*exp(3*x) + 6*I*exp(2*x) - 2*exp(x) + 2*I)/(-3*exp(4*x) - 6*I*exp(3*x) -
6*I*exp(x) + 3)
```

3.211 $\int \frac{\tanh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}i\text{sech}^2(x)+\frac{1}{2}\tan^{-1}(\sinh(x))-\frac{1}{2}\tanh(x)\text{sech}(x)$$

```
[Out] 1/2*arctan(sinh(x))+1/2*I*sech(x)^2-1/2*sech(x)*tanh(x)
```

Rubi [A] time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2706, 2606, 30, 2611, 3770}

$$\frac{1}{2}i\text{sech}^2(x)+\frac{1}{2}\tan^{-1}(\sinh(x))-\frac{1}{2}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/2 + (I/2)*Sech[x]^2 - (Sech[x]*Tanh[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^2(x)\,dx\\ &= -\frac{1}{2}\operatorname{sech}(x)\tanh(x)+i\,\text{Subst}\left(\int x\,dx,x,\operatorname{sech}(x)\right)+\frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\tan^{-1}(\sinh(x))+\frac{1}{2}i\operatorname{sech}^2(x)-\frac{1}{2}\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.77

$$\frac{1}{2}\tan^{-1}(\sinh(x))-\frac{1}{2(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/2 - 1/(2*(I + Sinh[x]))
```

fricas [B] time = 3.17, size = 56, normalized size = 2.15

$$\frac{\left(i\,e^{(2\,x)}-2\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(-i\,e^{(2\,x)}+2\,e^{x}+i\right)\log\left(e^{x}-i\right)-2\,e^{x}}{2\,e^{(2\,x)}+4i\,e^{x}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((I*e^(2*x) - 2*e^x - I)*log(e^x + I) + (-I*e^(2*x) + 2*e^x + I)*log(e^x -
I) - 2*e^x)/(2*e^(2*x) + 4*I*e^x - 2)
```

giac [B] time = 0.17, size = 53, normalized size = 2.04

$$\frac{-i\,e^{(-x)}+i\,e^{x}+2}{4\left(e^{(-x)}-e^{x}-2i\right)}+\frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(-I*e^(-x) + I*e^x + 2)/(e^(-x) - e^x - 2*I) + 1/4*I*log(-e^(-x) + e^x
+ 2*I) - 1/4*I*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.06, size = 45, normalized size = 1.73

$$-\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}+\frac{1}{\tanh\left(\frac{x}{2}\right)+i}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+sinh(x)),x)
```

```
[Out] -I/(tanh(1/2*x)+I)^2+1/2*I*ln(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)-1/2*I*ln(tan
h(1/2*x)-I)
```

maxima [B] time = 0.31, size = 42, normalized size = 1.62

$$\frac{e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}+\frac{1}{2}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{1}{2}i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) + 1/2*I*log(I*e^(-x) + 1) - 1/2*I*log(I
*e^(-x) - 1)
```

mupad [B] time = 0.17, size = 29, normalized size = 1.12

$$\operatorname{atan}\left(\mathrm{e}^{x}\right)+\frac{1\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{1}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(sinh(x) + 1i),x)
```

```
[Out] atan(exp(x)) + 1i/(exp(2*x) + exp(x)*2i - 1) - 1/(exp(x) + 1i)
```

sympy [A] time = 0.21, size = 32, normalized size = 1.23

$$\frac{\log\left(e^{x}-i\right)}{2}-\frac{\log\left(e^{x}+i\right)}{2}+\frac{e^{x}}{-e^{2x}-2ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x)
[Out] log(exp(x) - I)/2 - log(exp(x) + I)/2 + exp(x)/(-exp(2*x) - 2*I*exp(x) + 1)
```

3.212 $\int \frac{\coth(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=19

$$i\log(\sinh(x)+i) - i\log(\sinh(x))$$

[Out] -I*ln(sinh(x))+I*ln(I+sinh(x))

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2707, 36, 29, 31}

$$i\log(\sinh(x)+i) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(I + Sinh[x]),x]

[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = \text{Subst}\left(\int \frac{1}{x(i+x)}\,dx, x, \sinh(x)\right)$$
$$= -\left(i\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh(x)\right)\right) + i\,\text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right)$$
$$= -i\log(\sinh(x)) + i\log(i+\sinh(x))$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$i\log(\sinh(x)+i) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Sinh[x]),x]
[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]
```

fricas [A] time = 0.91, size = 17, normalized size = 0.89

$$-i\,\log\left(e^{(2\,x)}-1\right) + 2i\,\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] -I*log(e^(2*x) - 1) + 2*I*log(e^x + I)
```

giac [A] time = 0.24, size = 23, normalized size = 1.21

$$-i\,\log\left(e^{x}+1\right) + 2i\,\log\left(e^{x}+i\right) - i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="giac")
[Out] -I*log(e^x + 1) + 2*I*log(e^x + I) - I*log(abs(e^x - 1))
```

maple [A] time = 0.04, size = 17, normalized size = 0.89

$$-i\ln\left(\sinh(x)\right) + i\ln\left(i+\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+sinh(x)),x)
```

```
[Out] -I*ln(sinh(x))+I*ln(I+sinh(x))
```

maxima [B] time = 0.30, size = 28, normalized size = 1.47

$$-i\log\left(e^{(-x)}+1\right)+2i\log\left(e^{(-x)}-i\right)-i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] -I*log(e^(-x) + 1) + 2*I*log(e^(-x) - I) - I*log(e^(-x) - 1)
```

mupad [B] time = 0.15, size = 24, normalized size = 1.26

$$\ln\left(-36\,\mathrm{e}^{x}-36\mathrm{i}\right)\,2\mathrm{i}-\ln\left(3-3\,\mathrm{e}^{2\,x}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(sinh(x) + 1i),x)
[Out] log(- 36*exp(x) - 36i)*2i - log(3 - 3*exp(2*x))*1i
```

sympy [A] time = 0.27, size = 15, normalized size = 0.79

$$-2\log\left(e^{x}+i\right)+\log\left(e^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x)
[Out] -2*log(exp(x) + I) + log(exp(2*x) - 1)
```

3.213 $\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=12

$$-\tanh^{-1}(\cosh(x)) + i\coth(x)$$

```
[Out] -arctanh(cosh(x))+I*coth(x)
```

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 3767, 8, 3770}

$$-\tanh^{-1}(\cosh(x)) + i\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Sinh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + I*Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = -\left(i\int \operatorname{csch}^2(x)\,dx\right) + \int \operatorname{csch}(x)\,dx$$
$$= -\tanh^{-1}(\cosh(x)) - \text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)$$
$$= -\tanh^{-1}(\cosh(x)) + i\coth(x)$$

Mathematica [B] time = 0.03, size = 32, normalized size = 2.67

$$\frac{1}{2}i\tanh\left(\frac{x}{2}\right) + \frac{1}{2}i\coth\left(\frac{x}{2}\right) + \log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Sinh[x]),x]
[Out] (I/2)*Coth[x/2] + Log[Tanh[x/2]] + (I/2)*Tanh[x/2]
```

fricas [B] time = 0.86, size = 37, normalized size = 3.08

$$-\frac{\left(e^{(2x)}-1\right)\log\left(e^{x}+1\right) - \left(e^{(2x)}-1\right)\log\left(e^{x}-1\right) - 2i}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] -((e^(2*x) - 1)*log(e^x + 1) - (e^(2*x) - 1)*log(e^x - 1) - 2*I)/(e^(2*x) -
 1)
```

giac [B] time = 0.55, size = 24, normalized size = 2.00

$$\frac{2i}{e^{(2x)}-1} - \log\left(e^{x}+1\right) + \log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] 2*I/(e^(2*x) - 1) - log(e^x + 1) + log(abs(e^x - 1))
```

maple [A] time = 0.04, size = 23, normalized size = 1.92

$$\frac{i\tanh\left(\frac{x}{2}\right)}{2} + \frac{i}{2\tanh\left(\frac{x}{2}\right)} + \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+sinh(x)),x)
[Out] 1/2*I*tanh(1/2*x)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))
```

maxima [B] time = 0.30, size = 27, normalized size = 2.25

$$-\frac{2i}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="maxima")
[Out] -2*I/(e^(-2*x) - 1) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

mupad [B] time = 0.18, size = 28, normalized size = 2.33

$$\ln\left(2-2\,\mathrm{e}^{x}\right)-\ln\left(-2\,\mathrm{e}^{x}-2\right)+\frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(sinh(x) + 1i),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2i/(exp(2*x) - 1)
```

sympy [B] time = 0.14, size = 22, normalized size = 1.83

$$\log\left(e^{x}-1\right)-\log\left(e^{x}+1\right)+\frac{2}{-ie^{2x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+sinh(x)),x)
[Out] log(exp(x) - 1) - log(exp(x) + 1) + 2/(-I*exp(2*x) + I)
```

3.214 $\int \frac{\coth^3(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=15

$$-\operatorname{csch}(x)+\frac{1}{2} i \operatorname{csch}^2(x)$$

```
[Out] -csch(x)+1/2*I*csch(x)^2
```

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2606, 30, 8}

$$-\operatorname{csch}(x)+\frac{1}{2} i \operatorname{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = -\left(i\int \coth(x)\operatorname{csch}^2(x)\,dx\right) + \int \coth(x)\operatorname{csch}(x)\,dx$$
$$= -(i\,\mathrm{Subst}(\int 1\,dx, x, -i\operatorname{csch}(x))) - i\,\mathrm{Subst}(\int x\,dx, x, -i\operatorname{csch}(x))$$
$$= -\operatorname{csch}(x) + \frac{1}{2}i\operatorname{csch}^2(x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\operatorname{csch}(x) + \frac{1}{2}i\operatorname{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Sinh[x]),x]
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

fricas [B] time = 1.02, size = 33, normalized size = 2.20

$$-\frac{2\,e^{(3\,x)} - 2i\,e^{(2\,x)} - 2\,e^{x}}{e^{(4\,x)} - 2\,e^{(2\,x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="fricas")
[Out] -(2*e^(3*x) - 2*I*e^(2*x) - 2*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

giac [B] time = 0.16, size = 24, normalized size = 1.60

$$\frac{2\,e^{(-x)} - 2\,e^{x} + 2i}{\left(e^{(-x)} - e^{x}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="giac")
[Out] (2*e^(-x) - 2*e^x + 2*I)/(e^(-x) - e^x)^2
```

maple [B] time = 0.07, size = 34, normalized size = 2.27

$$\frac{\tanh\left(\frac{x}{2}\right)}{2} + \frac{i\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8} - \frac{1}{2\tanh\left(\frac{x}{2}\right)} + \frac{i}{8\tanh\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+sinh(x)),x)
```

```
[Out] 1/2*tanh(1/2*x)+1/8*I*tanh(1/2*x)^2-1/2/tanh(1/2*x)+1/8*I/tanh(1/2*x)^2
```

maxima [B] time = 0.31, size = 67, normalized size = 4.47

$$\frac{2\,e^{(-x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}-\frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}-\frac{2\,e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*I*e^(-2*x)/(2*e^(-2*x) - e^(-4*x)
- 1) - 2*e^(-3*x)/(2*e^(-2*x) - e^(-4*x) - 1)
```

mupad [B] time = 0.57, size = 25, normalized size = 1.67

$$\frac{2\,\mathrm{e}^x\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,1\mathrm{i}\right)}{{\left(\mathrm{e}^{2\,x}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(sinh(x) + 1i),x)
```

```
[Out] (2*exp(x)*(exp(x)*1i - exp(2*x) + 1))/(exp(2*x) - 1)^2
```

sympy [B] time = 0.16, size = 32, normalized size = 2.13

$$\frac{-2e^{3x}+2ie^{2x}+2e^{x}}{e^{4x}-2e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+sinh(x)),x)
```

```
[Out] (-2*exp(3*x) + 2*I*exp(2*x) + 2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

3.215 $\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{3} i \coth^3(x) - \frac{1}{2}\tanh^{-1}(\cosh(x)) - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

```
[Out] -1/2*arctanh(cosh(x))+1/3*I*coth(x)^3-1/2*coth(x)*csch(x)
```

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{1}{3} i \coth^3(x) - \frac{1}{2}\tanh^{-1}(\cosh(x)) - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Sinh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]]/2 + (I/3)*Coth[x]^3 - (Coth[x]*Csch[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth^2(x)\text{csch}^2(x)\,dx\right)+\int \coth^2(x)\text{csch}(x)\,dx\\ &= -\frac{1}{2}\coth(x)\text{csch}(x)+\frac{1}{2}\int \text{csch}(x)\,dx-\text{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cosh(x))+\frac{1}{3}i\coth^3(x)-\frac{1}{2}\coth(x)\text{csch}(x)\end{aligned}$$

Mathematica [B] time = 0.04, size = 100, normalized size = 3.85

$$\frac{1}{6}i\tanh\left(\frac{x}{2}\right)+\frac{1}{6}i\coth\left(\frac{x}{2}\right)-\frac{1}{8}\text{csch}^2\left(\frac{x}{2}\right)-\frac{1}{8}\text{sech}^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{24}i\coth\left(\frac{x}{2}\right)\text{csch}^2\left(\frac{x}{2}\right)-\frac{1}{24}i\tanh\left(\frac{x}{2}\right)\text{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Sinh[x]),x]
```

```
[Out] (I/6)*Coth[x/2] - Csch[x/2]^2/8 + (I/24)*Coth[x/2]*Csch[x/2]^2 + Log[Tanh[x
/2]]/2 - Sech[x/2]^2/8 + (I/6)*Tanh[x/2] - (I/24)*Sech[x/2]^2*Tanh[x/2]
```

fricas [B] time = 3.28, size = 90, normalized size = 3.46

$$-\frac{3\left(e^{(6x)}-3e^{(4x)}+3e^{(2x)}-1\right)\log\left(e^x+1\right)-3\left(e^{(6x)}-3e^{(4x)}+3e^{(2x)}-1\right)\log\left(e^x-1\right)+6e^{(5x)}-12ie^{(4x)}-6e^{x}}{6\left(e^{(6x)}-3e^{(4x)}+3e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)*log(e^x + 1) - 3*(e^(6*x) - 3
*e^(4*x) + 3*e^(2*x) - 1)*log(e^x - 1) + 6*e^(5*x) - 12*I*e^(4*x) - 6*e^x -
 4*I)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

giac [B] time = 0.18, size = 44, normalized size = 1.69

$$-\frac{3\,e^{(5\,x)} - 6i\,e^{(4\,x)} - 3\,e^{x} - 2i}{3\,\left(e^{(2\,x)} - 1\right)^{3}} - \frac{1}{2}\,\log\left(e^{x} + 1\right) + \frac{1}{2}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*e^(5*x) - 6*I*e^(4*x) - 3*e^x - 2*I)/(e^(2*x) - 1)^3 - 1/2*log(e^x
+ 1) + 1/2*log(abs(e^x - 1))
```

maple [B] time = 0.07, size = 59, normalized size = 2.27

$$\frac{i\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{24\tanh\left(\frac{x}{2}\right)^{3}}+\frac{i}{8\tanh\left(\frac{x}{2}\right)}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+sinh(x)),x)
```

```
[Out] 1/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2+1/24*I/tanh(1/2*x)
^3+1/8*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2+1/2*ln(tanh(1/2*x))
```

maxima [B] time = 0.87, size = 61, normalized size = 2.35

$$\frac{3\,e^{(-x)} - 6i\,e^{(-4\,x)} - 3\,e^{(-5\,x)} - 2i}{3\,\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*e^(-x) - 6*I*e^(-4*x) - 3*e^(-5*x) - 2*I)/(3*e^(-2*x) - 3*e^(-4*x) +
 e^(-6*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

mupad [B] time = 0.31, size = 74, normalized size = 2.85

$$\frac{\ln\left(1-\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{2}-\frac{\mathrm{e}^{x}}{\mathrm{e}^{2\,x}-1}-\frac{2\,\mathrm{e}^{x}}{{\left(\mathrm{e}^{2\,x}-1\right)}^2}+\frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1}+\frac{4\mathrm{i}}{{\left(\mathrm{e}^{2\,x}-1\right)}^2}+\frac{8\mathrm{i}}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(sinh(x) + 1i),x)
```

```
[Out] log(1 - exp(x))/2 - log(exp(x) + 1)/2 - exp(x)/(exp(2*x) - 1) - (2*exp(x))/
(exp(2*x) - 1)^2 + 2i/(exp(2*x) - 1) + 4i/(exp(2*x) - 1)^2 + 8i/(3*(exp(2*x
) - 1)^3)
```

sympy [B] time = 0.28, size = 61, normalized size = 2.35

$$\frac{3e^{5x} - 6ie^{4x} - 3e^{x} - 2i}{-3e^{6x} + 9e^{4x} - 9e^{2x} + 3} + \frac{\log\left(e^{x} - 1\right)}{2} - \frac{\log\left(e^{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+sinh(x)),x)
```

```
[Out] (3*exp(5*x) - 6*I*exp(4*x) - 3*exp(x) - 2*I)/(-3*exp(6*x) + 9*exp(4*x) - 9*
exp(2*x) + 3) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

3.216 $\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}i\coth^4(x)-\frac{\operatorname{csch}^3(x)}{3}-\operatorname{csch}(x)$$

```
[Out] 1/4*I*coth(x)^4-csch(x)-1/3*csch(x)^3
```

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$\frac{1}{4}i\coth^4(x)-\frac{\operatorname{csch}^3(x)}{3}-\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(I + Sinh[x]),x]
```

```
[Out] (I/4)*Coth[x]^4 - Csch[x] - Csch[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth^3(x)\operatorname{csch}^2(x)\,dx\right)+\int \coth^3(x)\operatorname{csch}(x)\,dx\\ &= i\,\text{Subst}\left(\int x^3\,dx, x, i\coth(x)\right)+i\,\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, -i\operatorname{csch}(x)\right)\\ &= \frac{1}{4}i\coth^4(x)-\operatorname{csch}(x)-\frac{\operatorname{csch}^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.43

$$\frac{1}{4}i\operatorname{csch}^4(x)-\frac{\operatorname{csch}^3(x)}{3}+\frac{1}{2}i\operatorname{csch}^2(x)-\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Sinh[x]),x]
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 + (I/4)*Csch[x]^4
```

fricas [B] time = 0.99, size = 63, normalized size = 2.74

$$-\frac{6\,e^{(7\,x)}-6i\,e^{(6\,x)}-10\,e^{(5\,x)}+10\,e^{(3\,x)}-6i\,e^{(2\,x)}-6\,e^{x}}{3\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="fricas")
[Out] -1/3*(6*e^(7*x) - 6*I*e^(6*x) - 10*e^(5*x) + 10*e^(3*x) - 6*I*e^(2*x) - 6*e
^x)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

giac [B] time = 0.18, size = 51, normalized size = 2.22

$$\frac{6\left(e^{(-x)}-e^{x}\right)^3+6i\left(e^{(-x)}-e^{x}\right)^2+8\,e^{(-x)}-8\,e^{x}+12i}{3\left(e^{(-x)}-e^{x}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/3*(6*(e^(-x) - e^x)^3 + 6*I*(e^(-x) - e^x)^2 + 8*e^(-x) - 8*e^x + 12*I)/(
e^(-x) - e^x)^4
```

maple [B] time = 0.09, size = 68, normalized size = 2.96

$$\frac{3\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\left(\tanh^4\left(\frac{x}{2}\right)\right)}{64}+\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}+\frac{i\left(\tanh^2\left(\frac{x}{2}\right)\right)}{16}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^3}+\frac{i}{64\tanh\left(\frac{x}{2}\right)^4}-\frac{3}{8\tanh\left(\frac{x}{2}\right)}+\frac{i}{16\tanh\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+sinh(x)),x)
```

```
[Out] 3/8*tanh(1/2*x)+1/64*I*tanh(1/2*x)^4+1/24*tanh(1/2*x)^3+1/16*I*tanh(1/2*x)^
2-1/24/tanh(1/2*x)^3+1/64*I/tanh(1/2*x)^4-3/8/tanh(1/2*x)+1/16*I/tanh(1/2*x
)^2
```

maxima [B] time = 0.48, size = 205, normalized size = 8.91

$$\frac{2\,e^{(-x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{2i\,e^{(-2\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{10\,e^{(-3\,x)}}{3\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-2*
x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 10/3*e^(-3*x)/(4
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 10/3*e^(-5*x)/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-6*x)/(4*e^(-2*x) -
 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*e^(-7*x)/(4*e^(-2*x) - 6*e^(-4
*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

mupad [B] time = 0.61, size = 44, normalized size = 1.91

$$\frac{2\,\mathrm{e}^{x}\,\left(5\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{6\,x}+3+\mathrm{e}^{5\,x}\,3\mathrm{i}+\mathrm{e}^{x}\,3\mathrm{i}\right)}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(sinh(x) + 1i),x)
```

```
[Out] (2*exp(x)*(5*exp(4*x) - 5*exp(2*x) + exp(5*x)*3i - 3*exp(6*x) + exp(x)*3i +
 3))/(3*(exp(2*x) - 1)^4)
```

sympy [B] time = 0.21, size = 70, normalized size = 3.04

$$\frac{-6e^{7x} + 6ie^{6x} + 10e^{5x} - 10e^{3x} + 6ie^{2x} + 6e^{x}}{3e^{8x} - 12e^{6x} + 18e^{4x} - 12e^{2x} + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+sinh(x)),x)
```

```
[Out] (-6*exp(7*x) + 6*I*exp(6*x) + 10*exp(5*x) - 10*exp(3*x) + 6*I*exp(2*x) + 6*
exp(x))/(3*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

3.217 $\int \frac{\coth^6(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=36

$$\frac{1}{5} i \coth^5(x) - \frac{3}{8} \tanh^{-1}(\cosh(x)) - \frac{1}{4} \coth^3(x) \operatorname{csch}(x) - \frac{3}{8} \coth(x) \operatorname{csch}(x)$$

```
[Out] -3/8*arctanh(cosh(x))+1/5*I*coth(x)^5-3/8*coth(x)*csch(x)-1/4*coth(x)^3*csc
h(x)
```

Rubi [A] time = 0.10, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{1}{5} i \coth^5(x) - \frac{3}{8} \tanh^{-1}(\cosh(x)) - \frac{1}{4} \coth^3(x) \operatorname{csch}(x) - \frac{3}{8} \coth(x) \operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Sinh[x]),x]
```

```
[Out] (-3*ArcTanh[Cosh[x]])/8 + (I/5)*Coth[x]^5 - (3*Coth[x]*Csch[x])/8 - (Coth[x
]^3*Csch[x])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth^4(x)\text{csch}^2(x)\,dx\right)+\int \coth^4(x)\text{csch}(x)\,dx\\ &= -\frac{1}{4}\coth^3(x)\text{csch}(x)+\frac{3}{4}\int \coth^2(x)\text{csch}(x)\,dx+\text{Subst}\left(\int x^4\,dx,x,i\coth(x)\right)\\ &= \frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\text{csch}(x)-\frac{1}{4}\coth^3(x)\text{csch}(x)+\frac{3}{8}\int \text{csch}(x)\,dx\\ &= -\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\text{csch}(x)-\frac{1}{4}\coth^3(x)\text{csch}(x)\end{aligned}$$

Mathematica [B] time = 0.04, size = 164, normalized size = 4.56

$$\frac{1}{10}i\tanh\left(\frac{x}{2}\right)+\frac{1}{10}i\coth\left(\frac{x}{2}\right)-\frac{1}{64}\text{csch}^4\left(\frac{x}{2}\right)-\frac{5}{32}\text{csch}^2\left(\frac{x}{2}\right)+\frac{1}{64}\text{sech}^4\left(\frac{x}{2}\right)-\frac{5}{32}\text{sech}^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{160}i$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Sinh[x]),x]
```

```
[Out] (I/10)*Coth[x/2] - (5*Csch[x/2]^2)/32 + ((7*I)/160)*Coth[x/2]*Csch[x/2]^2 -
 Csch[x/2]^4/64 + (I/160)*Coth[x/2]*Csch[x/2]^4 + (3*Log[Tanh[x/2]])/8 - (5
*Sech[x/2]^2)/32 + Sech[x/2]^4/64 + (I/10)*Tanh[x/2] - ((7*I)/160)*Sech[x/2
]^2*Tanh[x/2] + (I/160)*Sech[x/2]^4*Tanh[x/2]
```

fricas [B] time = 1.87, size = 144, normalized size = 4.00

$$-\frac{15\left(e^{(10x)}-5e^{(8x)}+10e^{(6x)}-10e^{(4x)}+5e^{(2x)}-1\right)\log\left(e^x+1\right)-15\left(e^{(10x)}-5e^{(8x)}+10e^{(6x)}-10e^{(4x)}+5e^{(2x}\right.}{40\left(e^{(10x)}-5e^{(8x)}+10e^{(6x)}-10e^{(4x)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/40*(15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x) - 1)*
log(e^x + 1) - 15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*
x) - 1)*log(e^x - 1) + 50*e^(9*x) - 80*I*e^(8*x) - 20*e^(7*x) - 160*I*e^(4*
x) + 20*e^(3*x) - 50*e^x - 16*I)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^
(4*x) + 5*e^(2*x) - 1)
```

giac [B] time = 0.20, size = 62, normalized size = 1.72

$$-\frac{25\,e^{(9\,x)} - 40i\,e^{(8\,x)} - 10\,e^{(7\,x)} - 80i\,e^{(4\,x)} + 10\,e^{(3\,x)} - 25\,e^{x} - 8i}{20\left(e^{(2\,x)} - 1\right)^{5}} - \frac{3}{8}\log\left(e^{x} + 1\right) + \frac{3}{8}\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/20*(25*e^(9*x) - 40*I*e^(8*x) - 10*e^(7*x) - 80*I*e^(4*x) + 10*e^(3*x) -
 25*e^x - 8*I)/(e^(2*x) - 1)^5 - 3/8*log(e^x + 1) + 3/8*log(abs(e^x - 1))
```

maple [B] time = 0.10, size = 93, normalized size = 2.58

$$\frac{i\tanh\left(\frac{x}{2}\right)}{16} + \frac{i\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{160} + \frac{\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{64} + \frac{i\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{32} + \frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8} + \frac{i}{160\tanh\left(\frac{x}{2}\right)^{5}} + \frac{i}{32\tanh\left(\frac{x}{2}\right)^{3}} - \frac{1}{64\tanh\left(\frac{x}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+sinh(x)),x)
```

```
[Out] 1/16*I*tanh(1/2*x)+1/160*I*tanh(1/2*x)^5+1/64*tanh(1/2*x)^4+1/32*I*tanh(1/2
*x)^3+1/8*tanh(1/2*x)^2+1/160*I/tanh(1/2*x)^5+1/32*I/tanh(1/2*x)^3-1/64/tan
h(1/2*x)^4+1/16*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2+3/8*ln(tanh(1/2*x))
```

maxima [B] time = 0.31, size = 91, normalized size = 2.53

$$\frac{25\,e^{(-x)} - 10\,e^{(-3\,x)} - 80i\,e^{(-4\,x)} + 10\,e^{(-7\,x)} - 40i\,e^{(-8\,x)} - 25\,e^{(-9\,x)} - 8i}{20\left(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1\right)} - \frac{3}{8}\log\left(e^{(-x)} + 1\right) + \frac{3}{8}\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/20*(25*e^(-x) - 10*e^(-3*x) - 80*I*e^(-4*x) + 10*e^(-7*x) - 40*I*e^(-8*x)
 - 25*e^(-9*x) - 8*I)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
+ e^(-10*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

mupad [B] time = 0.97, size = 124, normalized size = 3.44

$$\frac{3\ln\left(\frac{3}{4}-\frac{3\,\mathrm{e}^x}{4}\right)}{8}-\frac{3\ln\left(\frac{3\,\mathrm{e}^x}{4}+\frac{3}{4}\right)}{8}-\frac{5\,\mathrm{e}^x}{4\,\left(\mathrm{e}^{2\,x}-1\right)}-\frac{9\,\mathrm{e}^x}{2\left(\mathrm{e}^{2\,x}-1\right)^2}-\frac{6\,\mathrm{e}^x}{\left(\mathrm{e}^{2\,x}-1\right)^3}-\frac{4\,\mathrm{e}^x}{\left(\mathrm{e}^{2\,x}-1\right)^4}+\frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1}+\frac{8\mathrm{i}}{\left(\mathrm{e}^{2\,x}-1\right)^2}+\frac{16\mathrm{i}}{\left(\mathrm{e}^{2\,x}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(sinh(x) + 1i),x)
```

```
[Out] (3*log(3/4 - (3*exp(x))/4))/8 - (3*log((3*exp(x))/4 + 3/4))/8 - (5*exp(x))/
(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/(exp(2*x)
 - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 2i/(exp(2*x) - 1) + 8i/(exp(2*x) -
1)^2 + 16i/(exp(2*x) - 1)^3 + 16i/(exp(2*x) - 1)^4 + 32i/(5*(exp(2*x) - 1)^
5)
```

sympy [B] time = 0.31, size = 100, normalized size = 2.78

$$\frac{3\log\left(e^x-1\right)}{8}-\frac{3\log\left(e^x+1\right)}{8}+\frac{25e^{9x}-40ie^{8x}-10e^{7x}-80ie^{4x}+10e^{3x}-25e^x-8i}{-20e^{10x}+100e^{8x}-200e^{6x}+200e^{4x}-100e^{2x}+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+sinh(x)),x)
```

```
[Out] 3*log(exp(x) - 1)/8 - 3*log(exp(x) + 1)/8 + (25*exp(9*x) - 40*I*exp(8*x) -
10*exp(7*x) - 80*I*exp(4*x) + 10*exp(3*x) - 25*exp(x) - 8*I)/(-20*exp(10*x)
 + 100*exp(8*x) - 200*exp(6*x) + 200*exp(4*x) - 100*exp(2*x) + 20)
```

3.218 $\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=47

$$\frac{2\tanh^7(x)}{7} - \frac{\tanh^5(x)}{5} + \frac{2}{7}i\operatorname{sech}^7(x) - \frac{4}{5}i\operatorname{sech}^5(x) + \frac{2}{3}i\operatorname{sech}^3(x)$$

```
[Out] 2/3*I*sech(x)^3-4/5*I*sech(x)^5+2/7*I*sech(x)^7-1/5*tanh(x)^5+2/7*tanh(x)^7
```

Rubi [A] time = 0.12, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2711, 2607, 14, 2606, 270, 30}

$$\frac{2\tanh^7(x)}{7} - \frac{\tanh^5(x)}{5} + \frac{2}{7}i\operatorname{sech}^7(x) - \frac{4}{5}i\operatorname{sech}^5(x) + \frac{2}{3}i\operatorname{sech}^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((4*I)/5)*Sech[x]^5 + ((2*I)/7)*Sech[x]^7 - Tanh[x]^5
/5 + (2*Tanh[x]^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \int \left(-\operatorname{sech}^4(x)\tanh^4(x) - 2i\operatorname{sech}^3(x)\tanh^5(x) + \operatorname{sech}^2(x)\tanh^6(x)\right)dx$$

$$= -\left(2i\int \operatorname{sech}^3(x)\tanh^5(x)\,dx\right) - \int \operatorname{sech}^4(x)\tanh^4(x)\,dx + \int \operatorname{sech}^2(x)\tanh^6(x)\,dx$$

$$= i\,\mathrm{Subst}\left(\int x^6\,dx, x, i\tanh(x)\right) + i\,\mathrm{Subst}\left(\int x^4\left(1+x^2\right)dx, x, i\tanh(x)\right) + 2i\,\mathrm{Subst}\left(\int x^2$$

$$= \frac{\tanh^7(x)}{7} + i\,\mathrm{Subst}\left(\int \left(x^4+x^6\right)dx, x, i\tanh(x)\right) + 2i\,\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \mathrm{se}$$

$$= \frac{2}{3}i\operatorname{sech}^3(x) - \frac{4}{5}i\operatorname{sech}^5(x) + \frac{2}{7}i\operatorname{sech}^7(x) - \frac{\tanh^5(x)}{5} + \frac{2\tanh^7(x)}{7}$$

Mathematica [B] time = 0.15, size = 112, normalized size = 2.38

$$-\frac{1232\sinh(x) + 824\sinh(2x) - 1896\sinh(3x) + 412\sinh(4x) + 72\sinh(5x) + 1442i\cosh(x) - 1664i\cosh(2x) -}{13440\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^7\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] -1/13440*(-672*I + (1442*I)*Cosh[x] - (1664*I)*Cosh[2*x] + (309*I)*Cosh[3*x
] + (288*I)*Cosh[4*x] - (103*I)*Cosh[5*x] + 1232*Sinh[x] + 824*Sinh[2*x] -
```

```
1896*Sinh[3*x] + 412*Sinh[4*x] + 72*Sinh[5*x])/((Cosh[x/2] - I*Sinh[x/2])^7
*(Cosh[x/2] + I*Sinh[x/2])^3)
```

fricas [B] time = 0.50, size = 106, normalized size = 2.26

$$-\frac{210\,e^{(8\,x)} + 280i\,e^{(7\,x)} - 280\,e^{(6\,x)} + 168i\,e^{(5\,x)} + 28\,e^{(4\,x)} + 136i\,e^{(3\,x)} - 264\,e^{(2\,x)} - 72i\,e^{x} + 18}{105\,e^{(10\,x)} + 420i\,e^{(9\,x)} - 315\,e^{(8\,x)} + 840i\,e^{(7\,x)} - 1470\,e^{(6\,x)} - 1470\,e^{(4\,x)} - 840i\,e^{(3\,x)} - 315\,e^{(2\,x)} - 420i\,e^{x} + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(210*e^(8*x) + 280*I*e^(7*x) - 280*e^(6*x) + 168*I*e^(5*x) + 28*e^(4*x) +
136*I*e^(3*x) - 264*e^(2*x) - 72*I*e^x + 18)/(105*e^(10*x) + 420*I*e^(9*x)
- 315*e^(8*x) + 840*I*e^(7*x) - 1470*e^(6*x) - 1470*e^(4*x) - 840*I*e^(3*x)
 - 315*e^(2*x) - 420*I*e^x + 105)
```

giac [B] time = 0.37, size = 65, normalized size = 1.38

$$-\frac{-6i\,e^{(2\,x)} - 9\,e^{x} + 5i}{24\,(e^{x} - i)^{3}} - \frac{210i\,e^{(6\,x)} - 105\,e^{(5\,x)} + 175i\,e^{(4\,x)} - 910\,e^{(3\,x)} - 756i\,e^{(2\,x)} + 427\,e^{x} + 31i}{840\,(e^{x} + i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/24*(-6*I*e^(2*x) - 9*e^x + 5*I)/(e^x - I)^3 - 1/840*(210*I*e^(6*x) - 105
*e^(5*x) + 175*I*e^(4*x) - 910*e^(3*x) - 756*I*e^(2*x) + 427*e^x + 31*I)/(e
^x + I)^7
```

maple [B] time = 0.12, size = 116, normalized size = 2.47

$$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}} - \frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}} - \frac{i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}} + \frac{4}{7\left(\tanh\left(\frac{x}{2}\right)+i\right)^{7}} - \frac{12}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}} - \frac{1}{12\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+sinh(x))^2,x)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^6-I/(tanh(1/2*x)+I)^4-1/8*I/(tanh(1/2*x)+I)^2+4/7/(tanh
(1/2*x)+I)^7-12/5/(tanh(1/2*x)+I)^5-1/12/(tanh(1/2*x)+I)^3-1/8/(tanh(1/2*x)
+I)-1/8*I/(tanh(1/2*x)-I)^2+1/12/(tanh(1/2*x)-I)^3+1/8/(tanh(1/2*x)-I)
```

maxima [B] time = 0.88, size = 573, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 72*I*e^(-x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) -
 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-1
0*x) + 105) - 264*e^(-2*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) -
1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9
*x) + 105*e^(-10*x) + 105) - 136*I*e^(-3*x)/(420*I*e^(-x) - 315*e^(-2*x) +
840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8
*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 28*e^(-4*x)/(420*I*e^(-x) - 3
15*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*
x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) - 168*I*e^(-5*x)/
(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*
x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
- 280*e^(-6*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x
) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^
(-10*x) + 105) - 280*I*e^(-7*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*
x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*
e^(-9*x) + 105*e^(-10*x) + 105) + 210*e^(-8*x)/(420*I*e^(-x) - 315*e^(-2*x)
 + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^
(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 18/(420*I*e^(-x) - 315*e^(
-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 3
15*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
```

mupad [B] time = 4.07, size = 395, normalized size = 8.40

$$-\frac{\frac{25\,e^{4x}}{168}-\frac{e^{2x}}{4}+\frac{5}{168}+\frac{e^{3x}\,5i}{42}+\frac{e^{5x}\,1i}{28}-\frac{e^{x}\,5i}{84}}{15\,e^{2x}-15\,e^{4x}+e^{6x}-1-e^{3x}\,20i+e^{5x}\,6i+e^{x}\,6i}+\frac{1i}{12\left(e^{2x}\,3i-e^{3x}+3\,e^{x}-i\right)}-\frac{\frac{5}{168}+\frac{e^{x}\,1i}{28}}{e^{2x}-1+e^{x}\,2i}-\frac{\frac{5\,e^{5x}}{28}\cdots}{e^{2x}\,21i+35\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] 1i/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) - ((exp(3*x)*5i)/42 - exp(
2*x)/4 + (25*exp(4*x))/168 + (exp(5*x)*1i)/28 - (exp(x)*5i)/84 + 5/168)/(15
*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)*6i
 - 1) - ((exp(x)*1i)/28 + 5/168)/(exp(2*x) + exp(x)*2i - 1) - ((exp(4*x)*5i
)/28 - exp(3*x)/2 - (exp(2*x)*5i)/28 + (5*exp(5*x))/28 + (exp(6*x)*1i)/28 +
 (5*exp(x))/28 - 1i/28)/(exp(2*x)*21i + 35*exp(3*x) - exp(4*x)*35i - 21*exp
(5*x) + exp(6*x)*7i + exp(7*x) - 7*exp(x) - 1i) - ((exp(2*x)*1i)/28 + (5*ex
p(x))/84 + 1i/84)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 1/(8*(exp(x)*2
i - exp(2*x) + 1)) + 1i/(4*(exp(x) - 1i)) - 1i/(28*(exp(x) + 1i)) - ((5*exp
(2*x))/56 + (exp(3*x)*1i)/28 + (exp(x)*1i)/28 - 1/40)/(exp(3*x)*4i - 6*exp(
2*x) + exp(4*x) - exp(x)*4i + 1) - ((exp(2*x)*1i)/14 + (5*exp(3*x))/42 + (e
```

```
xp(4*x)*1i)/28 - exp(x)/10 - 1i/84)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*1
0i + exp(5*x) + 5*exp(x) + 1i)
```

sympy [B] time = 0.29, size = 128, normalized size = 2.72

$$\frac{-210e^{8x} - 280ie^{7x} + 280e^{6x} - 168ie^{5x} - 28e^{4x} - 136ie^{3x} + 264e^{2x} + 72ie^{x} - 18}{105e^{10x} + 420ie^{9x} - 315e^{8x} + 840ie^{7x} - 1470e^{6x} - 1470e^{4x} - 840ie^{3x} - 315e^{2x} - 420ie^{x} + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] (-210*exp(8*x) - 280*I*exp(7*x) + 280*exp(6*x) - 168*I*exp(5*x) - 28*exp(4*
x) - 136*I*exp(3*x) + 264*exp(2*x) + 72*I*exp(x) - 18)/(105*exp(10*x) + 420
*I*exp(9*x) - 315*exp(8*x) + 840*I*exp(7*x) - 1470*exp(6*x) - 1470*exp(4*x)
 - 840*I*exp(3*x) - 315*exp(2*x) - 420*I*exp(x) + 105)
```

3.219 $\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=66

$$-\frac{i}{16(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}+\frac{i}{12(\sinh(x)+i)^3}-\frac{1}{8}i\tan^{-1}(\sinh(x))$$

```
[Out] -1/8*I*arctan(sinh(x))-1/16*I/(I-sinh(x))+1/12*I/(I+sinh(x))^3-1/4/(I+sinh(
x))^2-3/16*I/(I+sinh(x))
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2707, 88, 203}

$$-\frac{i}{16(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}+\frac{i}{12(\sinh(x)+i)^3}-\frac{1}{8}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-I/8)*ArcTan[Sinh[x]] - (I/16)/(I - Sinh[x]) + (I/12)/(I + Sinh[x])^3 - 1/
(4*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x])
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{x^3}{(i-x)^2(i+x)^4}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{i}{16(-i+x)^2} - \frac{i}{4(i+x)^4} + \frac{1}{2(i+x)^3} + \frac{3i}{16(i+x)^2} - \frac{i}{8\left(1+x^2\right)}\right) dx, x, \sinh(x$$
$$= -\frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))} - \frac{1}{8}i\,\text{Subst}\Big(\int$$
$$= -\frac{1}{8}i\tan^{-1}(\sinh(x)) - \frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.79

$$\frac{1}{48}\left(\frac{2\left(-3i\sinh^3(x) - 6\sinh^2(x) - 7i\sinh(x) + 2\right)}{(\sinh(x)-i)(\sinh(x)+i)^3} - 6i\tan^{-1}(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] ((-6*I)*ArcTan[Sinh[x]] + (2*(2 - (7*I)*Sinh[x] - 6*Sinh[x]^2 - (3*I)*Sinh[
x]^3))/((-I + Sinh[x])*(I + Sinh[x])^3))/48
```

fricas [B] time = 0.87, size = 201, normalized size = 3.05

$$\frac{\left(3\,e^{(8\,x)} + 12i\,e^{(7\,x)} - 12\,e^{(6\,x)} + 12i\,e^{(5\,x)} - 30\,e^{(4\,x)} - 12i\,e^{(3\,x)} - 12\,e^{(2\,x)} - 12i\,e^{x} + 3\right)\log\left(e^{x}+i\right) - \left(3\,e^{(8\,x)} + 12i\,e^{(7}\right.}{24\,e^{(8\,x)} + 96i\,e^{(7\,x)} - 96\,e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((3*e^(8*x) + 12*I*e^(7*x) - 12*e^(6*x) + 12*I*e^(5*x) - 30*e^(4*x) - 12*I*
e^(3*x) - 12*e^(2*x) - 12*I*e^x + 3)*log(e^x + I) - (3*e^(8*x) + 12*I*e^(7*
x) - 12*e^(6*x) + 12*I*e^(5*x) - 30*e^(4*x) - 12*I*e^(3*x) - 12*e^(2*x) - 1
2*I*e^x + 3)*log(e^x - I) - 6*I*e^(7*x) - 24*e^(6*x) - 38*I*e^(5*x) + 80*e^
(4*x) + 38*I*e^(3*x) - 24*e^(2*x) + 6*I*e^x)/(24*e^(8*x) + 96*I*e^(7*x) - 9
6*e^(6*x) + 96*I*e^(5*x) - 240*e^(4*x) - 96*I*e^(3*x) - 96*e^(2*x) - 96*I*e
^x + 24)
```

giac [B] time = 0.18, size = 102, normalized size = 1.55

$$\frac{e^{(-x)} - e^{x}}{16\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{11\left(e^{(-x)} - e^{x}\right)^3 - 102i\left(e^{(-x)} - e^{x}\right)^2 - 180\,e^{(-x)} + 180\,e^{x} + 104i}{96\left(e^{(-x)} - e^{x} - 2i\right)^3} + \frac{1}{16}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(e^(-x) - e^x)/(e^(-x) - e^x + 2*I) - 1/96*(11*(e^(-x) - e^x)^3 - 102*
I*(e^(-x) - e^x)^2 - 180*e^(-x) + 180*e^x + 104*I)/(e^(-x) - e^x - 2*I)^3 +
 1/16*log(-e^(-x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.11, size = 114, normalized size = 1.73

$$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^5}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^3}-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^6}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^4}-\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+sinh(x))^2,x)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^5-2/3*I/(tanh(1/2*x)+I)^3-1/8*I/(tanh(1/2*x)+I)+2/3/(ta
nh(1/2*x)+I)^6-2/(tanh(1/2*x)+I)^4-1/8/(tanh(1/2*x)+I)^2+1/8*ln(tanh(1/2*x)
+I)-1/8*I/(tanh(1/2*x)-I)+1/8/(tanh(1/2*x)-I)^2-1/8*ln(tanh(1/2*x)-I)
```

maxima [B] time = 0.36, size = 115, normalized size = 1.74

$$\frac{-3i\,e^{(-x)}-12\,e^{(-2\,x)}-19i\,e^{(-3\,x)}+40\,e^{(-4\,x)}+19i\,e^{(-5\,x)}-12\,e^{(-6\,x)}+3i\,e^{(-7\,x)}}{48i\,e^{(-x)}-48\,e^{(-2\,x)}+48i\,e^{(-3\,x)}-120\,e^{(-4\,x)}-48i\,e^{(-5\,x)}-48\,e^{(-6\,x)}-48i\,e^{(-7\,x)}+12\,e^{(-8\,x)}+12}-\frac{1}{8}\log\left(e^{(-x)}+i\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-3*I*e^(-x) - 12*e^(-2*x) - 19*I*e^(-3*x) + 40*e^(-4*x) + 19*I*e^(-5*x) -
12*e^(-6*x) + 3*I*e^(-7*x))/(48*I*e^(-x) - 48*e^(-2*x) + 48*I*e^(-3*x) - 12
0*e^(-4*x) - 48*I*e^(-5*x) - 48*e^(-6*x) - 48*I*e^(-7*x) + 12*e^(-8*x) + 12
) - 1/8*log(e^(-x) + I) + 1/8*log(e^(-x) - I)
```

mupad [B] time = 1.50, size = 209, normalized size = 3.17

$$\frac{\ln\left(-\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8}-\frac{\ln\left(\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8}-\frac{2i}{e^{5\,x}-10\,e^{3\,x}+e^{4\,x}\,5i-e^{2\,x}\,10i+5\,e^x+1i}-\frac{11}{8\,\left(e^{2\,x}-1+e^x\,2i\right)}+\frac{3}{e^{4\,x}-6\,e^{2\,x}+1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 2i/(exp(4*x)*5i -
 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) - 11/(8*(exp(2*x) +
```

```
 exp(x)*2i - 1)) + 3/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)
+ 1/(8*(exp(x)*2i - exp(2*x) + 1)) + 1i/(8*(exp(x) - 1i)) - 3i/(8*(exp(x) +
 1i)) - 2/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(
6*x) + exp(x)*6i - 1)) + 8i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

sympy [B] time = 0.30, size = 129, normalized size = 1.95

$$\frac{-3ie^{7x}-12e^{6x}-19ie^{5x}+40e^{4x}+19ie^{3x}-12e^{2x}+3ie^{x}}{12e^{8x}+48ie^{7x}-48e^{6x}+48ie^{5x}-120e^{4x}-48ie^{3x}-48e^{2x}-48ie^{x}+12}+\text{RootSum}\left(64z^{2}+1,\left(i\mapsto i\log\left(8i+e^{x}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (-3*I*exp(7*x) - 12*exp(6*x) - 19*I*exp(5*x) + 40*exp(4*x) + 19*I*exp(3*x)
- 12*exp(2*x) + 3*I*exp(x))/(12*exp(8*x) + 48*I*exp(7*x) - 48*exp(6*x) + 48
*I*exp(5*x) - 120*exp(4*x) - 48*I*exp(3*x) - 48*exp(2*x) - 48*I*exp(x) + 12
) + RootSum(64*_z**2 + 1, Lambda(_i, _i*log(8*_i + exp(x))))
```

3.220 $\int \frac{\tanh^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=37

$$\frac{2\tanh^5(x)}{5} - \frac{\tanh^3(x)}{3} - \frac{2}{5}i\operatorname{sech}^5(x) + \frac{2}{3}i\operatorname{sech}^3(x)$$

```
[Out] 2/3*I*sech(x)^3-2/5*I*sech(x)^5-1/3*tanh(x)^3+2/5*tanh(x)^5
```

Rubi [A] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2711, 2607, 14, 2606, 30}

$$\frac{2\tanh^5(x)}{5} - \frac{\tanh^3(x)}{3} - \frac{2}{5}i\operatorname{sech}^5(x) + \frac{2}{3}i\operatorname{sech}^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((2*I)/5)*Sech[x]^5 - Tanh[x]^3/3 + (2*Tanh[x]^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx &= -\int \left(\operatorname{sech}^4(x)\tanh^2(x) + 2i\operatorname{sech}^3(x)\tanh^3(x) - \operatorname{sech}^2(x)\tanh^4(x)\right) dx \\
&= -\left(2i\int \operatorname{sech}^3(x)\tanh^3(x)\,dx\right) - \int \operatorname{sech}^4(x)\tanh^2(x)\,dx + \int \operatorname{sech}^2(x)\tanh^4(x)\,dx \\
&= -\left(i\,\text{Subst}\left(\int x^4\,dx, x, i\tanh(x)\right)\right) - i\,\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, i\tanh(x)\right) - 2i\,\text{Subst}\Big(\\
&= \frac{\tanh^5(x)}{5} - i\,\text{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, i\tanh(x)\right) - 2i\,\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \operatorname{sech}(\right. \\
&= \frac{2}{3}i\operatorname{sech}^3(x) - \frac{2}{5}i\operatorname{sech}^5(x) - \frac{\tanh^3(x)}{3} + \frac{2\tanh^5(x)}{5}
\end{aligned}
$$

Mathematica [B] time = 0.10, size = 84, normalized size = 2.27

$$
\frac{140\sinh(x) - 44\sinh(2x) - 4\sinh(3x) - 55i\cosh(x) - 16i\cosh(2x) + 11i\cosh(3x) + 80i}{240\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (80*I - (55*I)*Cosh[x] - (16*I)*Cosh[2*x] + (11*I)*Cosh[3*x] + 140*Sinh[x]
- 44*Sinh[2*x] - 4*Sinh[3*x])/(240*(Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] +
 I*Sinh[x/2]))
```

fricas [B] time = 1.58, size = 58, normalized size = 1.57

$$
-\frac{30\,e^{(4\,x)} + 40i\,e^{(3\,x)} - 40\,e^{(2\,x)} - 8i\,e^{x} + 2}{15\,e^{(6\,x)} + 60i\,e^{(5\,x)} - 75\,e^{(4\,x)} - 75\,e^{(2\,x)} - 60i\,e^{x} + 15}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(30*e^(4*x) + 40*I*e^(3*x) - 40*e^(2*x) - 8*I*e^x + 2)/(15*e^(6*x) + 60*I*
e^(5*x) - 75*e^(4*x) - 75*e^(2*x) - 60*I*e^x + 15)
```

giac [A] time = 0.37, size = 41, normalized size = 1.11

$$\frac{i}{4\left(e^{x}-i\right)}-\frac{15 i e^{(4 x)}+30 e^{(3 x)}+40 i e^{(2 x)}-50 e^{x}-7 i}{60\left(e^{x}+i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*I/(e^x - I) - 1/60*(15*I*e^(4*x) + 30*e^(3*x) + 40*I*e^(2*x) - 50*e^x -
 7*I)/(e^x + I)^5
```

maple [B] time = 0.08, size = 70, normalized size = 1.89

$$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{5}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+sinh(x))^2,x)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^4-1/2*I/(tanh(1/2*x)+I)^2+4/5/(tanh(1/2*x)+I)^5-5/3/(ta
nh(1/2*x)+I)^3-1/4/(tanh(1/2*x)+I)+1/4/(tanh(1/2*x)-I)
```

maxima [B] time = 0.36, size = 197, normalized size = 5.32

$$\frac{8 i e^{(-x)}}{60 i e^{(-x)}-75 e^{(-2 x)}-75 e^{(-4 x)}-60 i e^{(-5 x)}+15 e^{(-6 x)}+15}-\frac{40 e^{(-2 x)}}{60 i e^{(-x)}-75 e^{(-2 x)}-75 e^{(-4 x)}-60 i e^{(-5 x)}+15 e^{(-6 x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 8*I*e^(-x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^
(-6*x) + 15) - 40*e^(-2*x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*
e^(-5*x) + 15*e^(-6*x) + 15) - 40*I*e^(-3*x)/(60*I*e^(-x) - 75*e^(-2*x) - 7
5*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 30*e^(-4*x)/(60*I*e^(-x) -
 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 2/(60*I*e^
(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15)
```

mupad [B] time = 0.30, size = 139, normalized size = 3.76

$$\frac{\left(4\,{\mathrm{e}}^{3\,x}-4\,{\mathrm{e}}^{x}\right)\,\left(2\,{\mathrm{e}}^{4\,x}-\frac{8\,{\mathrm{e}}^{2\,x}}{3}+\frac{2}{15}\right)\,1{}\mathrm{i}}{{\left({\mathrm{e}}^{2\,x}+1\right)}^5}-\frac{\left({\mathrm{e}}^{4\,x}-6\,{\mathrm{e}}^{2\,x}+1\right)\,\left(2\,{\mathrm{e}}^{4\,x}-\frac{8\,{\mathrm{e}}^{2\,x}}{3}+\frac{2}{15}\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^5}-\frac{\left(4\,{\mathrm{e}}^{3\,x}-4\,{\mathrm{e}}^{x}\right)\,\left(\frac{8\,{\mathrm{e}}^{3\,x}}{3}-\frac{8\,{\mathrm{e}}^{x}}{15}\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^5}-\frac{\left(\frac{8\,{\mathrm{e}}^{3\,x}}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*(2*exp(4*x) - (8*exp(2*x))/3 + 2/15)*1i)/(exp(2*x)
 + 1)^5 - ((exp(4*x) - 6*exp(2*x) + 1)*(2*exp(4*x) - (8*exp(2*x))/3 + 2/15)
)/(exp(2*x) + 1)^5 - ((4*exp(3*x) - 4*exp(x))*((8*exp(3*x))/3 - (8*exp(x))/
15))/(exp(2*x) + 1)^5 - (((8*exp(3*x))/3 - (8*exp(x))/15)*(exp(4*x) - 6*exp
(2*x) + 1)*1i)/(exp(2*x) + 1)^5
```

sympy [A] time = 0.20, size = 66, normalized size = 1.78

$$\frac{-30e^{4x} - 40ie^{3x} + 40e^{2x} + 8ie^{x} - 2}{15e^{6x} + 60ie^{5x} - 75e^{4x} - 75e^{2x} - 60ie^{x} + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (-30*exp(4*x) - 40*I*exp(3*x) + 40*exp(2*x) + 8*I*exp(x) - 2)/(15*exp(6*x)
+ 60*I*exp(5*x) - 75*exp(4*x) - 75*exp(2*x) - 60*I*exp(x) + 15)
```

3.221 $\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{i}{4(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}-\frac{1}{4}i\tan^{-1}(\sinh(x))$$

```
[Out] -1/4*I*arctan(sinh(x))-1/4/(I+sinh(x))^2-1/4*I/(I+sinh(x))
```

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2707, 77, 203}

$$-\frac{i}{4(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}-\frac{1}{4}i\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Sinh[x])^2,x]
```

```
[Out] (-I/4)*ArcTan[Sinh[x]] - 1/(4*(I + Sinh[x])^2) - (I/4)/(I + Sinh[x])
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = -\text{Subst}\left(\int \frac{x}{(i-x)(i+x)^3}\,dx, x, \sinh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{1}{2(i+x)^3} - \frac{i}{4(i+x)^2} + \frac{i}{4\left(1+x^2\right)}\right) dx, x, \sinh(x)\right)$$
$$= -\frac{1}{4(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))} - \frac{1}{4} i\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{1}{4} i \tan^{-1}(\sinh(x)) - \frac{1}{4(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.81

$$-\frac{i\left(\sinh(x) + (\sinh(x)+i)^2 \tan^{-1}(\sinh(x))\right)}{4(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/4*I)*(Sinh[x] + ArcTan[Sinh[x]]*(I + Sinh[x])^2))/(I + Sinh[x])^2
```

fricas [B] time = 1.74, size = 95, normalized size = 2.64

$$\frac{\left(e^{(4\,x)} + 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} - 4i\,e^{x} + 1\right)\log\left(e^{x} + i\right) - \left(e^{(4\,x)} + 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} - 4i\,e^{x} + 1\right)\log\left(e^{x} - i\right) - 2i\,e^{(3\,x)} + 2i\,e}{4\,e^{(4\,x)} + 16i\,e^{(3\,x)} - 24\,e^{(2\,x)} - 16i\,e^{x} + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - (e^(4*x)
+ 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x - I) - 2*I*e^(3*x) + 2*I*e
^x)/(4*e^(4*x) + 16*I*e^(3*x) - 24*e^(2*x) - 16*I*e^x + 4)
```

giac [B] time = 0.52, size = 66, normalized size = 1.83

$$-\frac{3\left(e^{(-x)} - e^{x}\right)^2 - 20i\,e^{(-x)} + 20i\,e^{x} - 12}{16\left(e^{(-x)} - e^{x} - 2i\right)^2} + \frac{1}{8}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{8}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/16*(3*(e^(-x) - e^x)^2 - 20*I*e^(-x) + 20*I*e^x - 12)/(e^(-x) - e^x - 2*
I)^2 + 1/8*log(-e^(-x) + e^x + 2*I) - 1/8*log(-e^(-x) + e^x - 2*I)
```

maple [B] time = 0.08, size = 66, normalized size = 1.83

$$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+sinh(x))^2,x)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^3-1/2*I/(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^4-3/2/(tanh(1
/2*x)+I)^2+1/4*ln(tanh(1/2*x)+I)-1/4*ln(tanh(1/2*x)-I)
```

maxima [B] time = 0.33, size = 61, normalized size = 1.69

$$\frac{-i\,e^{(-x)}+i\,e^{(-3\,x)}}{8i\,e^{(-x)}-12\,e^{(-2\,x)}-8i\,e^{(-3\,x)}+2\,e^{(-4\,x)}+2}-\frac{1}{4}\,\log\left(e^{(-x)}+i\right)+\frac{1}{4}\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-I*e^(-x) + I*e^(-3*x))/(8*I*e^(-x) - 12*e^(-2*x) - 8*I*e^(-3*x) + 2*e^(-4
*x) + 2) - 1/4*log(e^(-x) + I) + 1/4*log(e^(-x) - I)
```

mupad [B] time = 0.87, size = 99, normalized size = 2.75

$$\frac{\ln\left(-\frac{1}{2}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}\right)}{4}-\frac{\ln\left(\frac{1}{2}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}\right)}{4}-\frac{3}{2\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}+\frac{2\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] log((exp(x)*1i)/2 - 1/2)/4 - log((exp(x)*1i)/2 + 1/2)/4 - 3/(2*(exp(2*x) +
exp(x)*2i - 1)) + 1/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) -
 1i/(2*(exp(x) + 1i)) + 2i/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)
```

sympy [A] time = 0.23, size = 58, normalized size = 1.61

$$\frac{-ie^{3x}+ie^{x}}{2e^{4x}+8ie^{3x}-12e^{2x}-8ie^{x}+2}+\operatorname{RootSum}\left(16z^{2}+1,\left(i\mapsto i\log\left(4i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))**2,x)
```

```
[Out] (-I*exp(3*x) + I*exp(x))/(2*exp(4*x) + 8*I*exp(3*x) - 12*exp(2*x) - 8*I*exp
(x) + 2) + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))
```

3.222 $\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=25

$$-\frac{i}{\sinh(x)+i}-\log(\sinh(x))+\log(\sinh(x)+i)$$

[Out] -ln(sinh(x))+ln(I+sinh(x))-I/(I+sinh(x))

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2707, 44}

$$-\frac{i}{\sinh(x)+i}-\log(\sinh(x))+\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(I + Sinh[x])^2,x]

[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{1}{x(i+x)^2}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{x} + \frac{i}{(i+x)^2} + \frac{1}{i+x}\right)dx, x, \sinh(x)\right)$$
$$= -\log(\sinh(x)) + \log(i+\sinh(x)) - \frac{i}{i+\sinh(x)}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 1.00

$$-\frac{i}{\sinh(x)+i} - \log(\sinh(x)) + \log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Sinh[x])^2,x]
```

```
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

fricas [B] time = 0.82, size = 56, normalized size = 2.24

$$-\frac{\left(e^{(2x)} + 2i\,e^x - 1\right)\log\left(e^{(2x)} - 1\right) - \left(2\,e^{(2x)} + 4i\,e^x - 2\right)\log\left(e^x + i\right) + 2i\,e^x}{e^{(2x)} + 2i\,e^x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((e^(2*x) + 2*I*e^x - 1)*log(e^(2*x) - 1) - (2*e^(2*x) + 4*I*e^x - 2)*log(
e^x + I) + 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

giac [A] time = 0.16, size = 33, normalized size = 1.32

$$-\frac{2i\,e^x}{(e^x+i)^2} - \log\left(e^x+1\right) + 2\,\log\left(e^x+i\right) - \log\left(|e^x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*I*e^x/(e^x + I)^2 - log(e^x + 1) + 2*log(e^x + I) - log(abs(e^x - 1))
```

maple [A] time = 0.06, size = 23, normalized size = 0.92

$$-\ln\left(\sinh(x)\right) + \ln\left(i+\sinh(x)\right) - \frac{i}{i+\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+sinh(x))^2,x)
```

```
[Out] -ln(sinh(x))+ln(I+sinh(x))-I/(I+sinh(x))
```

maxima [B] time = 0.35, size = 48, normalized size = 1.92

$$\frac{2i\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-\log\left(e^{(-x)}+1\right)+2\,\log\left(e^{(-x)}-i\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*I*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - log(e^(-x) + 1) + 2*log(e^(-x) -
I) - log(e^(-x) - 1)
```

mupad [B] time = 0.30, size = 49, normalized size = 1.96

$$2\,\ln\left(36\,\mathrm{e}^x+36\mathrm{i}\right)-\ln\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-3\mathrm{i}\right)-\frac{2}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}}-\frac{2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] 2*log(36*exp(x) + 36i) - log(exp(2*x)*3i - 3i) - 2/(exp(2*x) + exp(x)*2i -
1) - 2i/(exp(x) + 1i)
```

sympy [B] time = 0.26, size = 39, normalized size = 1.56

$$2i\log\left(e^{x}+i\right)-i\log\left(e^{2x}-1\right)-\frac{2ie^{x}}{e^{2x}+2ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))**2,x)
```

```
[Out] 2*I*log(exp(x) + I) - I*log(exp(2*x) - 1) - 2*I*exp(x)/(exp(2*x) + 2*I*exp(
x) - 1)
```

3.223 $\int \frac{\coth^2(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=26

$$\coth(x) + 2i\tanh^{-1}(\cosh(x)) + \frac{2i\coth(x)}{-\operatorname{csch}(x) + i}$$

```
[Out] 2*I*arctanh(cosh(x))+3*coth(x)-2*I*coth(x)/(I+sinh(x))
```

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2709, 3770, 3767, 8, 3777}

$$\coth(x) + 2i\tanh^{-1}(\cosh(x)) + \frac{2i\coth(x)}{-\operatorname{csch}(x) + i}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + Coth[x] + ((2*I)*Coth[x])/(I - Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx &= \int \left(2-2i\operatorname{csch}(x)-\operatorname{csch}^2(x)+\frac{2i}{-i+\operatorname{csch}(x)}\right)dx\\ &= 2x-2i\int \operatorname{csch}(x)\,dx+2i\int \frac{1}{-i+\operatorname{csch}(x)}\,dx-\int \operatorname{csch}^2(x)\,dx\\ &= 2x+2i\tanh^{-1}(\cosh(x))+\frac{2i\coth(x)}{i-\operatorname{csch}(x)}+i\,\mathrm{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)+2i\int i\,dx\\ &= 2i\tanh^{-1}(\cosh(x))+\coth(x)+\frac{2i\coth(x)}{i-\operatorname{csch}(x)}\end{aligned}$$

Mathematica [B] time = 0.15, size = 66, normalized size = 2.54

$$\frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+\coth\left(\frac{x}{2}\right)-4i\log\left(\sinh\left(\frac{x}{2}\right)\right)+4i\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (Coth[x/2] + (4*I)*Log[Cosh[x/2]] - (4*I)*Log[Sinh[x/2]] + (8*Sinh[x/2])/(C
osh[x/2] - I*Sinh[x/2]) + Tanh[x/2])/2
```

fricas [B] time = 1.60, size = 79, normalized size = 3.04

$$\frac{\left(2i\,e^{(3\,x)}-2\,e^{(2\,x)}-2i\,e^{x}+2\right)\log\left(e^{x}+1\right)+\left(-2i\,e^{(3\,x)}+2\,e^{(2\,x)}+2i\,e^{x}-2\right)\log\left(e^{x}-1\right)-4i\,e^{(2\,x)}+2\,e^{x}+6i}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] ((2*I*e^(3*x) - 2*e^(2*x) - 2*I*e^x + 2)*log(e^x + 1) + (-2*I*e^(3*x) + 2*e
^(2*x) + 2*I*e^x - 2)*log(e^x - 1) - 4*I*e^(2*x) + 2*e^x + 6*I)/(e^(3*x) +
I*e^(2*x) - e^x - I)
```

giac [B] time = 0.17, size = 47, normalized size = 1.81

$$\frac{-4i\,e^{(2\,x)}+2\,e^{x}+6i}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i}+2i\,\log\left(e^{x}+1\right)-2i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] (-4*I*e^(2*x) + 2*e^x + 6*I)/(e^(3*x) + I*e^(2*x) - e^x - I) + 2*I*log(e^x
+ 1) - 2*I*log(abs(e^x - 1))
```

maple [A] time = 0.08, size = 35, normalized size = 1.35

$$\frac{\tanh\left(\frac{x}{2}\right)}{2}+\frac{4}{\tanh\left(\frac{x}{2}\right)+i}-2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+sinh(x))^2,x)
```

```
[Out] 1/2*tanh(1/2*x)+4/(tanh(1/2*x)+I)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)
```

maxima [B] time = 0.36, size = 54, normalized size = 2.08

$$\frac{2\,e^{(-x)}+4i\,e^{(-2\,x)}-6i}{e^{(-x)}+i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+2i\,\log\left(e^{(-x)}+1\right)-2i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (2*e^(-x) + 4*I*e^(-2*x) - 6*I)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) + 2*I*
log(e^(-x) + 1) - 2*I*log(e^(-x) - 1)
```

mupad [B] time = 0.75, size = 60, normalized size = 2.31

$$-\ln\left(\mathrm{e}^{x}\,4\mathrm{i}-4\mathrm{i}\right)\,2\mathrm{i}+\ln\left(\mathrm{e}^{x}\,4\mathrm{i}+4\mathrm{i}\right)\,2\mathrm{i}+\frac{2\,\mathrm{e}^{x}-\mathrm{e}^{2\,x}\,4\mathrm{i}+6\mathrm{i}}{\mathrm{e}^{2\,x}\,1\mathrm{i}+\mathrm{e}^{3\,x}-\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + (2*exp(x) - exp(2*x)*4i +
 6i)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

sympy [B] time = 0.22, size = 49, normalized size = 1.88

$$\frac{4e^{2x} + 2ie^{x} - 6}{ie^{3x} - e^{2x} - ie^{x} + 1} + 2\log\left(e^{x} - 1\right) - 2\log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (4*exp(2*x) + 2*I*exp(x) - 6)/(I*exp(3*x) - exp(2*x) - I*exp(x) + 1) + 2*lo
g(exp(x) - 1) - 2*log(exp(x) + 1)
```

3.224 $\int \frac{\coth^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=29

$$\frac{\operatorname{csch}^2(x)}{2} + 2i\operatorname{csch}(x) + 2\log(\sinh(x)) - 2\log(\sinh(x) + i)$$

```
[Out] 2*I*csch(x)+1/2*csch(x)^2+2*ln(sinh(x))-2*ln(I+sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2707, 77}

$$\frac{\operatorname{csch}^2(x)}{2} + 2i\operatorname{csch}(x) + 2\log(\sinh(x)) - 2\log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = -\text{Subst}\left(\int \frac{i-x}{x^3(i+x)}\,dx, x, \sinh(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2i}{x^2}-\frac{2}{x}+\frac{2}{i+x}\right)dx, x, \sinh(x)\right)$$
$$= 2i\text{csch}(x)+\frac{\text{csch}^2(x)}{2}+2\log(\sinh(x))-2\log(i+\sinh(x))$$

Mathematica [**A**] time = 0.02, size = 29, normalized size = 1.00

$$\frac{\text{csch}^2(x)}{2}+2i\text{csch}(x)+2\log(\sinh(x))-2\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

fricas [**B**] time = 1.02, size = 72, normalized size = 2.48

$$\frac{2\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^{(2x)}-1\right)-4\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^{x}+i\right)+4ie^{(3x)}+2e^{(2x)}-4ie^{x}}{e^{(4x)}-2e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) - 4*(e^(4*x) - 2*e^(2*x) + 1)
*log(e^x + I) + 4*I*e^(3*x) + 2*e^(2*x) - 4*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1
)
```

giac [**B**] time = 0.20, size = 53, normalized size = 1.83

$$\frac{4ie^{(3x)}+2e^{(2x)}-4ie^{x}}{\left(e^{x}+1\right)^2\left(e^{x}-1\right)^2}+2\log\left(e^{x}+1\right)-4\log\left(e^{x}+i\right)+2\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] (4*I*e^(3*x) + 2*e^(2*x) - 4*I*e^x)/((e^x + 1)^2*(e^x - 1)^2) + 2*log(e^x +
 1) - 4*log(e^x + I) + 2*log(abs(e^x - 1))
```

maple [A] time = 0.10, size = 51, normalized size = 1.76

$$-i\tanh\left(\frac{x}{2}\right)+\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}-4\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+\frac{i}{\tanh\left(\frac{x}{2}\right)}+\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+sinh(x))^2,x)
```

```
[Out] -I*tanh(1/2*x)+1/8*tanh(1/2*x)^2-4*ln(tanh(1/2*x)+I)+I/tanh(1/2*x)+1/8/tanh
(1/2*x)^2+2*ln(tanh(1/2*x))
```

maxima [B] time = 0.39, size = 64, normalized size = 2.21

$$\frac{-4i\,e^{(-x)}-2\,e^{(-2\,x)}+4i\,e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+2\,\log\left(e^{(-x)}+1\right)-4\,\log\left(e^{(-x)}-i\right)+2\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-4*I*e^(-x) - 2*e^(-2*x) + 4*I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*l
og(e^(-x) + 1) - 4*log(e^(-x) - I) + 2*log(e^(-x) - 1)
```

mupad [B] time = 0.26, size = 56, normalized size = 1.93

$$\frac{2}{e^{4\,x}-2\,e^{2\,x}+1}+2\,\ln\left(-e^{2\,x}\,6i+6i\right)-4\,\ln\left(144\,e^{x}+144i\right)+\frac{2+e^{x}\,4i}{e^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] 2*log(6i - exp(2*x)*6i) - 4*log(144*exp(x) + 144i) + 2/(exp(4*x) - 2*exp(2*
x) + 1) + (exp(x)*4i + 2)/(exp(2*x) - 1)
```

sympy [A] time = 0.27, size = 53, normalized size = 1.83

$$\frac{4ie^{3x}+2e^{2x}-4ie^{x}}{e^{4x}-2e^{2x}+1}-4\log\left(e^{x}+i\right)+2\log\left(e^{2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (4*I*exp(3*x) + 2*exp(2*x) - 4*I*exp(x))/(exp(4*x) - 2*exp(2*x) + 1) - 4*lo
g(exp(x) + I) + 2*log(exp(2*x) - 1)
```

3.225 $\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=28

$$\frac{\coth^3(x)}{3} - 2\coth(x) - i\tanh^{-1}(\cosh(x)) + i\coth(x)\operatorname{csch}(x)$$

```
[Out] -I*arctanh(cosh(x))-2*coth(x)+1/3*coth(x)^3+I*coth(x)*csch(x)
```

Rubi [A] time = 0.09, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2708, 2757, 3767, 8, 3768, 3770}

$$\frac{\coth^3(x)}{3} - 2\coth(x) - i\tanh^{-1}(\cosh(x)) + i\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-I)*ArcTanh[Cosh[x]] - 2*Coth[x] + Coth[x]^3/3 + I*Coth[x]*Csch[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx &= \int \operatorname{csch}^4(x)(i-\sinh(x))^2\,dx\\ &= \int \left(\operatorname{csch}^2(x)-2i\operatorname{csch}^3(x)-\operatorname{csch}^4(x)\right)\,dx\\ &= -\left(2i\int \operatorname{csch}^3(x)\,dx\right)+\int \operatorname{csch}^2(x)\,dx-\int \operatorname{csch}^4(x)\,dx\\ &= i\coth(x)\operatorname{csch}(x)+i\int \operatorname{csch}(x)\,dx-i\,\text{Subst}(\int 1\,dx,x,-i\coth(x))-i\,\text{Subst}\left(\int \left(1+x^2\right)\right.\\ &= -i\tanh^{-1}(\cosh(x))-2\coth(x)+\frac{\coth^3(x)}{3}+i\coth(x)\operatorname{csch}(x)\end{aligned}$$

Mathematica [B] time = 0.06, size = 107, normalized size = 3.82

$$-\frac{5}{6}\tanh\left(\frac{x}{2}\right)-\frac{5}{6}\coth\left(\frac{x}{2}\right)+\frac{1}{4}i\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{4}i\operatorname{sech}^2\left(\frac{x}{2}\right)+i\log\left(\sinh\left(\frac{x}{2}\right)\right)-i\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{24}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*Coth[x/2])/6 + (I/4)*Csch[x/2]^2 + (Coth[x/2]*Csch[x/2]^2)/24 - I*Log[C
osh[x/2]] + I*Log[Sinh[x/2]] + (I/4)*Sech[x/2]^2 - (5*Tanh[x/2])/6 - (Sech[
x/2]^2*Tanh[x/2])/24
```

fricas [B] time = 0.79, size = 98, normalized size = 3.50

$$\frac{\left(-3i\,e^{(6\,x)}+9i\,e^{(4\,x)}-9i\,e^{(2\,x)}+3i\right)\log\left(e^{x}+1\right)+\left(3i\,e^{(6\,x)}-9i\,e^{(4\,x)}+9i\,e^{(2\,x)}-3i\right)\log\left(e^{x}-1\right)+6i\,e^{(5\,x)}-6\,e^{(4\,x)}}{3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((-3*I*e^(6*x) + 9*I*e^(4*x) - 9*I*e^(2*x) + 3*I)*log(e^x + 1) + (3*I*e
^(6*x) - 9*I*e^(4*x) + 9*I*e^(2*x) - 3*I)*log(e^x - 1) + 6*I*e^(5*x) - 6*e^
(4*x) + 24*e^(2*x) - 6*I*e^x - 10)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

giac [B] time = 0.16, size = 50, normalized size = 1.79

$$-\frac{-6i\,e^{(5\,x)} + 6\,e^{(4\,x)} - 24\,e^{(2\,x)} + 6i\,e^{x} + 10}{3\,\left(e^{(2\,x)} - 1\right)^{3}} - i\,\log\left(e^{x} + 1\right) + i\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(-6*I*e^(5*x) + 6*e^(4*x) - 24*e^(2*x) + 6*I*e^x + 10)/(e^(2*x) - 1)^3
 - I*log(e^x + 1) + I*log(abs(e^x - 1))
```

maple [B] time = 0.08, size = 58, normalized size = 2.07

$$-\frac{7\tanh\left(\frac{x}{2}\right)}{8}+\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24}-\frac{i\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{4}+\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{7}{8\tanh\left(\frac{x}{2}\right)}+\frac{i}{4\tanh\left(\frac{x}{2}\right)^{2}}+i\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+sinh(x))^2,x)
```

```
[Out] -7/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3-1/4*I*tanh(1/2*x)^2+1/24/tanh(1/2*x)^3-
7/8/tanh(1/2*x)+1/4*I/tanh(1/2*x)^2+I*ln(tanh(1/2*x))
```

maxima [B] time = 0.34, size = 67, normalized size = 2.39

$$\frac{-6i\,e^{(-x)} - 24\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 6i\,e^{(-5\,x)} + 10}{3\,\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - i\,\log\left(e^{(-x)} + 1\right) + i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(-6*I*e^(-x) - 24*e^(-2*x) + 6*e^(-4*x) + 6*I*e^(-5*x) + 10)/(3*e^(-2*x
) - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

mupad [B] time = 0.21, size = 111, normalized size = 3.96

$$-\ln\left(-\mathrm{e}^{x}\,2\mathrm{i}-2\mathrm{i}\right)\,1\mathrm{i}+\ln\left(-\mathrm{e}^{x}\,2\mathrm{i}+2\mathrm{i}\right)\,1\mathrm{i}-\frac{\frac{2\,\mathrm{e}^{4\,x}}{3}-4\,\mathrm{e}^{2\,x}+\frac{2}{3}-\frac{\mathrm{e}^{3\,x}\,8\mathrm{i}}{3}+\frac{\mathrm{e}^{x}\,8\mathrm{i}}{3}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1}+\frac{\frac{4}{3}+\frac{\mathrm{e}^{x}\,4\mathrm{i}}{3}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}+\frac{-\frac{4}{3}+\mathrm{e}^{x}\,2\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] log(2i - exp(x)*2i)*1i - log(- exp(x)*2i - 2i)*1i - ((2*exp(4*x))/3 - (exp(
3*x)*8i)/3 - 4*exp(2*x) + (exp(x)*8i)/3 + 2/3)/(3*exp(2*x) - 3*exp(4*x) + e
xp(6*x) - 1) + ((exp(x)*4i)/3 + 4/3)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(x)*
2i - 4/3)/(exp(2*x) - 1)
```

sympy [B] time = 0.30, size = 66, normalized size = 2.36

$$\operatorname{RootSum}\left(z^{2}+1,\left(i\mapsto i\log\left(ii+e^{x}\right)\right)\right)+\frac{6ie^{5x}-6e^{4x}+24e^{2x}-6ie^{x}-10}{3e^{6x}-9e^{4x}+9e^{2x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+sinh(x))**2,x)
```

```
[Out] RootSum(_z**2 + 1, Lambda(_i, _i*log(_i*I + exp(x)))) + (6*I*exp(5*x) - 6*e
xp(4*x) + 24*exp(2*x) - 6*I*exp(x) - 10)/(3*exp(6*x) - 9*exp(4*x) + 9*exp(2
*x) - 3)
```

3.226 $\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=27

$$\frac{\operatorname{csch}^4(x)}{4}+\frac{2}{3}i\operatorname{csch}^3(x)-\frac{\operatorname{csch}^2(x)}{2}$$

[Out] -1/2*csch(x)^2+2/3*I*csch(x)^3+1/4*csch(x)^4

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2707, 43}

$$\frac{\operatorname{csch}^4(x)}{4}+\frac{2}{3}i\operatorname{csch}^3(x)-\frac{\operatorname{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^5/(I + Sinh[x])^2,x]

[Out] -Csch[x]^2/2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{(i-x)^2}{x^5}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{x^5} - \frac{2i}{x^4} + \frac{1}{x^3}\right)dx, x, \sinh(x)\right)$$
$$= -\frac{1}{2}\text{csch}^2(x) + \frac{2}{3}i\text{csch}^3(x) + \frac{\text{csch}^4(x)}{4}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\text{csch}^4(x)}{4} + \frac{2}{3}i\text{csch}^3(x) - \frac{\text{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -1/2*Csch[x]^2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

fricas [B] time = 1.30, size = 59, normalized size = 2.19

$$-\frac{6\,e^{(6\,x)} - 16i\,e^{(5\,x)} - 24\,e^{(4\,x)} + 16i\,e^{(3\,x)} + 6\,e^{(2\,x)}}{3\left(e^{(8\,x)} - 4\,e^{(6\,x)} + 6\,e^{(4\,x)} - 4\,e^{(2\,x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*e^(6*x) - 16*I*e^(5*x) - 24*e^(4*x) + 16*I*e^(3*x) + 6*e^(2*x))/(e^
(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

giac [A] time = 0.18, size = 38, normalized size = 1.41

$$-\frac{6\left(e^{(-x)} - e^{x}\right)^2 + 16i\,e^{(-x)} - 16i\,e^{x} - 12}{3\left(e^{(-x)} - e^{x}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*(e^(-x) - e^x)^2 + 16*I*e^(-x) - 16*I*e^x - 12)/(e^(-x) - e^x)^4
```

maple [B] time = 0.09, size = 68, normalized size = 2.52

$$\frac{i\tanh\left(\frac{x}{2}\right)}{4}+\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{64}-\frac{i\left(\tanh^3\left(\frac{x}{2}\right)\right)}{12}-\frac{3\left(\tanh^2\left(\frac{x}{2}\right)\right)}{16}+\frac{i}{12\tanh\left(\frac{x}{2}\right)^3}+\frac{1}{64\tanh\left(\frac{x}{2}\right)^4}-\frac{i}{4\tanh\left(\frac{x}{2}\right)}-\frac{3}{16\tanh\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+sinh(x))^2,x)
```

```
[Out] 1/4*I*tanh(1/2*x)+1/64*tanh(1/2*x)^4-1/12*I*tanh(1/2*x)^3-3/16*tanh(1/2*x)^
2+1/12*I/tanh(1/2*x)^3+1/64/tanh(1/2*x)^4-1/4*I/tanh(1/2*x)-3/16/tanh(1/2*x
)^2
```

maxima [B] time = 0.33, size = 171, normalized size = 6.33

$$\frac{2\,e^{(-2\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{16i\,e^{(-3\,x)}}{3\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}-\frac{8\,e^{(-4\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 16/3*I*e
^(-3*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 8*e^(-4*x)/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 16/3*I*e^(-5*x)/(4*
e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 2*e^(-6*x)/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

mupad [B] time = 0.61, size = 40, normalized size = 1.48

$$-\frac{2\,\mathrm{e}^{2\,x}\,\left(3\,\mathrm{e}^{4\,x}-12\,\mathrm{e}^{2\,x}+3-\mathrm{e}^{3\,x}\,8\mathrm{i}+\mathrm{e}^{x}\,8\mathrm{i}\right)}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(sinh(x) + 1i)^2,x)
```

```
[Out] -(2*exp(2*x)*(3*exp(4*x) - exp(3*x)*8i - 12*exp(2*x) + exp(x)*8i + 3))/(3*(
exp(2*x) - 1)^4)
```

sympy [B] time = 0.29, size = 65, normalized size = 2.41

$$\frac{-6e^{6x}+16ie^{5x}+24e^{4x}-16ie^{3x}-6e^{2x}}{3e^{8x}-12e^{6x}+18e^{4x}-12e^{2x}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+sinh(x))**2,x)
```

```
[Out] (-6*exp(6*x) + 16*I*exp(5*x) + 24*exp(4*x) - 16*I*exp(3*x) - 6*exp(2*x))/(3
*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

3.227 $\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=48

$$\frac{\coth^5(x)}{5}-\frac{2\coth^3(x)}{3}-\frac{1}{4}i\tanh^{-1}(\cosh(x))+\frac{1}{2}i\coth(x)\text{csch}^3(x)+\frac{1}{4}i\coth(x)\text{csch}(x)$$

```
[Out] -1/4*I*arctanh(cosh(x))-2/3*coth(x)^3+1/5*coth(x)^5+1/4*I*coth(x)*csch(x)+1
/2*I*coth(x)*csch(x)^3
```

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2709, 3767, 8, 3768, 3770}

$$\frac{\coth^5(x)}{5}-\frac{2\coth^3(x)}{3}-\frac{1}{4}i\tanh^{-1}(\cosh(x))+\frac{1}{2}i\coth(x)\text{csch}^3(x)+\frac{1}{4}i\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-I/4)*ArcTanh[Cosh[x]] - (2*Coth[x]^3)/3 + Coth[x]^5/5 + (I/4)*Coth[x]*Csc
h[x] + (I/2)*Coth[x]*Csch[x]^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx &= \int \left(\operatorname{csch}^2(x) - 2i\operatorname{csch}^3(x) - 2i\operatorname{csch}^5(x) - \operatorname{csch}^6(x)\right)\,dx\\
&= -\left(2i\int \operatorname{csch}^3(x)\,dx\right) - 2i\int \operatorname{csch}^5(x)\,dx + \int \operatorname{csch}^2(x)\,dx - \int \operatorname{csch}^6(x)\,dx\\
&= i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3(x) + i\int \operatorname{csch}(x)\,dx - i\,\mathrm{Subst}(\int 1\,dx, x, -i\coth(x))\\
&= -i\tanh^{-1}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3(x)\\
&= -\frac{1}{4}i\tanh^{-1}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3(x)\end{aligned}$$

Mathematica [B] time = 0.06, size = 175, normalized size = 3.65

$$-\frac{7}{30}\tanh\left(\frac{x}{2}\right)-\frac{7}{30}\coth\left(\frac{x}{2}\right)+\frac{1}{32}i\operatorname{csch}^4\left(\frac{x}{2}\right)+\frac{1}{16}i\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{1}{32}i\operatorname{sech}^4\left(\frac{x}{2}\right)+\frac{1}{16}i\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{1}{4}i\log\left(\sinh\left(\frac{x}{2}\right)\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-7*Coth[x/2])/30 + (I/16)*Csch[x/2]^2 - (19*Coth[x/2]*Csch[x/2]^2)/480 + (
I/32)*Csch[x/2]^4 + (Coth[x/2]*Csch[x/2]^4)/160 - (I/4)*Log[Cosh[x/2]] + (I
/4)*Log[Sinh[x/2]] + (I/16)*Sech[x/2]^2 - (I/32)*Sech[x/2]^4 - (7*Tanh[x/2]
)/30 + (19*Sech[x/2]^2*Tanh[x/2])/480 + (Sech[x/2]^4*Tanh[x/2])/160
```

fricas [B] time = 0.90, size = 158, normalized size = 3.29

$$\frac{\left(-15i\,e^{(10\,x)}+75i\,e^{(8\,x)}-150i\,e^{(6\,x)}+150i\,e^{(4\,x)}-75i\,e^{(2\,x)}+15i\right)\log\left(e^{x}+1\right)+\left(15i\,e^{(10\,x)}-75i\,e^{(8\,x)}+150i\,e^{(6\,x)}\right.}{60\left(e^{(10\,x)}-5\,e^{(8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/60*((-15*I*e^(10*x) + 75*I*e^(8*x) - 150*I*e^(6*x) + 150*I*e^(4*x) - 75*I
*e^(2*x) + 15*I)*log(e^x + 1) + (15*I*e^(10*x) - 75*I*e^(8*x) + 150*I*e^(6*
x) - 150*I*e^(4*x) + 75*I*e^(2*x) - 15*I)*log(e^x - 1) + 30*I*e^(9*x) - 120
*e^(8*x) + 180*I*e^(7*x) + 480*e^(6*x) - 80*e^(4*x) - 180*I*e^(3*x) + 160*e
^(2*x) - 30*I*e^x - 56)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5
*e^(2*x) - 1)
```

giac [B] time = 0.19, size = 74, normalized size = 1.54

$$-\frac{-15i\,e^{(9\,x)}+60\,e^{(8\,x)}-90i\,e^{(7\,x)}-240\,e^{(6\,x)}+40\,e^{(4\,x)}+90i\,e^{(3\,x)}-80\,e^{(2\,x)}+15i\,e^{x}+28}{30\left(e^{(2\,x)}-1\right)^{5}}-\frac{1}{4}i\,\log\left(e^{x}+1\right)+\frac{1}{4}i\,\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/30*(-15*I*e^(9*x) + 60*e^(8*x) - 90*I*e^(7*x) - 240*e^(6*x) + 40*e^(4*x)
 + 90*I*e^(3*x) - 80*e^(2*x) + 15*I*e^x + 28)/(e^(2*x) - 1)^5 - 1/4*I*log(e
^x + 1) + 1/4*I*log(abs(e^x - 1))
```

maple [B] time = 0.11, size = 74, normalized size = 1.54

$$-\frac{3\tanh\left(\frac{x}{2}\right)}{16}+\frac{\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{160}-\frac{i\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{32}-\frac{5\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{96}-\frac{5}{96\tanh\left(\frac{x}{2}\right)^{3}}+\frac{i}{32\tanh\left(\frac{x}{2}\right)^{4}}-\frac{3}{16\tanh\left(\frac{x}{2}\right)}+\frac{i\ln\left(\tanh(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+sinh(x))^2,x)
```

```
[Out] -3/16*tanh(1/2*x)+1/160*tanh(1/2*x)^5-1/32*I*tanh(1/2*x)^4-5/96*tanh(1/2*x)
^3-5/96/tanh(1/2*x)^3+1/32*I/tanh(1/2*x)^4-3/16/tanh(1/2*x)+1/4*I*ln(tanh(1
/2*x))+1/160/tanh(1/2*x)^5
```

maxima [B] time = 0.36, size = 103, normalized size = 2.15

$$\frac{-15i\,e^{(-x)}-80\,e^{(-2\,x)}-90i\,e^{(-3\,x)}+40\,e^{(-4\,x)}-240\,e^{(-6\,x)}+90i\,e^{(-7\,x)}+60\,e^{(-8\,x)}+15i\,e^{(-9\,x)}+28}{30\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}-\frac{1}{4}i\,\log\left(e^{(-x)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(-15*I*e^(-x) - 80*e^(-2*x) - 90*I*e^(-3*x) + 40*e^(-4*x) - 240*e^(-6*
x) + 90*I*e^(-7*x) + 60*e^(-8*x) + 15*I*e^(-9*x) + 28)/(5*e^(-2*x) - 10*e^(
```

```
-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) - 1/4*I*log(e^(-x) + 1) +
 1/4*I*log(e^(-x) - 1)
```

mupad [B] time = 0.77, size = 246, normalized size = 5.12

$$-\frac{80\,e^{4\,x}-160\,e^{2\,x}-480\,e^{6\,x}+120\,e^{8\,x}+56-\ln\left(-\frac{e^{x}\,1i}{2}-\frac{1}{2}i\right)\,15i+\ln\left(-\frac{e^{x}\,1i}{2}+\frac{1}{2}i\right)\,15i+e^{3\,x}\,180i-e^{7\,x}\,180i-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(sinh(x) + 1i)^2,x)
```

```
[Out] -(log(1i/2 - (exp(x)*1i)/2)*15i - log(- (exp(x)*1i)/2 - 1i/2)*15i - 160*exp
(2*x) + exp(3*x)*180i + 80*exp(4*x) - 480*exp(6*x) - exp(7*x)*180i + 120*ex
p(8*x) - exp(9*x)*30i + exp(x)*30i + log(- (exp(x)*1i)/2 - 1i/2)*exp(2*x)*7
5i - log(1i/2 - (exp(x)*1i)/2)*exp(2*x)*75i - log(- (exp(x)*1i)/2 - 1i/2)*e
xp(4*x)*150i + log(1i/2 - (exp(x)*1i)/2)*exp(4*x)*150i + log(- (exp(x)*1i)/
2 - 1i/2)*exp(6*x)*150i - log(1i/2 - (exp(x)*1i)/2)*exp(6*x)*150i - log(- (
exp(x)*1i)/2 - 1i/2)*exp(8*x)*75i + log(1i/2 - (exp(x)*1i)/2)*exp(8*x)*75i
+ log(- (exp(x)*1i)/2 - 1i/2)*exp(10*x)*15i - log(1i/2 - (exp(x)*1i)/2)*exp
(10*x)*15i + 56)/(60*(exp(2*x) - 1)^5)
```

sympy [B] time = 0.34, size = 114, normalized size = 2.38

$$\operatorname{RootSum}\left(16z^{2}+1,\left(i\mapsto i\log\left(4ii+e^{x}\right)\right)\right)+\frac{15ie^{9x}-60e^{8x}+90ie^{7x}+240e^{6x}-40e^{4x}-90ie^{3x}+80e^{2x}-15ie^{x}-}{30e^{10x}-150e^{8x}+300e^{6x}-300e^{4x}+150e^{2x}-30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+sinh(x))**2,x)
```

```
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i*I + exp(x)))) + (15*I*exp(9*x)
 - 60*exp(8*x) + 90*I*exp(7*x) + 240*exp(6*x) - 40*exp(4*x) - 90*I*exp(3*x)
 + 80*exp(2*x) - 15*I*exp(x) - 28)/(30*exp(10*x) - 150*exp(8*x) + 300*exp(6
*x) - 300*exp(4*x) + 150*exp(2*x) - 30)
```

3.228 $\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=124

$$-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)}-\frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{a^3\tanh(x)}{(a^2+b^2)^2}$$

```
[Out] -2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-a^2*b*sec
h(x)/(a^2+b^2)^2-b*sech(x)/(a^2+b^2)+1/3*b*sech(x)^3/(a^2+b^2)-a^3*tanh(x)/
(a^2+b^2)^2-1/3*a*tanh(x)^3/(a^2+b^2)
```

Rubi [A] time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2660, 618, 206}

$$-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{a\tanh^3(x)}{3(a^2+b^2)}-\frac{a^3\tanh(x)}{(a^2+b^2)^2}+\frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)}-\frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (a^
2*b*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x])/(a^2 + b^2) + (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a^3*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x]^3)/(3*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a\int \operatorname{sech}^2(x)\tanh^2(x)\,dx}{a^2+b^2} + \frac{a^2\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx}{a^2+b^2} + \frac{b\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a^2+b^2}$$

$$= -\frac{a^3\int \operatorname{sech}^2(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{\left(a^2b\right)\int \operatorname{sech}(x)\tanh(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{(ia)\operatorname{Subst}\left(\int x^2\,dx \ldots}{a^2+b \ldots}$$

$$= -\frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} - \frac{\left(ia^3\right)\operatorname{Subst}(\int 1\,dx, x, -i\tanh(x))}{\left(a^2+b^2\right)^2} + \frac{\left(2a^4\right)\operatorname{Subst} \ldots}{\ldots}$$

$$= -\frac{a^2b\operatorname{sech}(x)}{\left(a^2+b^2\right)^2} - \frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{a^3\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh^3(x)}{3\left(a^2+b^2\right)} - \frac{\left(4a^4\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+ \ldots\right.} \ldots\right.}{\left(\ldots\right.}$$

$$= -\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{a^2b\operatorname{sech}(x)}{\left(a^2+b^2\right)^2} - \frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)} - \frac{a^3\tanh(x)}{\left(a^2+b^2\right)^2} - \frac{a\tanh^3 \ldots}{3\left(a^2+ \ldots\right.}$$

Mathematica [A] time = 0.37, size = 108, normalized size = 0.87

$$\frac{-a\left(4a^2+b^2\right)\tanh(x) - 3b\left(2a^2+b^2\right)\operatorname{sech}(x) + \left(a^2+b^2\right)\operatorname{sech}^3(x)(a\sinh(x)+b) + \frac{6a^4\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*a^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 3*b*
(2*a^2 + b^2)*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) - a*(4*a^2 +
b^2)*Tanh[x])/(3*(a^2 + b^2)^2)
```

fricas [B] time = 1.45, size = 1199, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b + 3*a^2*b^3 + b^
5)*sinh(x)^5 - 8*a^5 - 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4)
*cosh(x)^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*c
osh(x))*sinh(x)^4 + 4*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^3 + 4*(4*a^4*b +
5*a^2*b^3 + b^5 + 15*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 6*(2*a^5 + 3*a
^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + a^3*b^2)*cosh(x)^2 - 12*(a^5
 + a^3*b^2 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 + 3*(2*a^5 + 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 - 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh
(x)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
+ b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) - b)) + 6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*a^4*b + 3*a^2*b^
3 + b^5 + 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 + 3*a^3*b^2 +
a*b^4)*cosh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2 - 4*(a^5 + a^3*b
^2)*cosh(x))*sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
+ 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x))*sinh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 +
3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 +
6*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh
(x))
```

giac [A] time = 0.44, size = 197, normalized size = 1.59

$$\frac{a^4 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(6\,a^2be^{(5\,x)}+3\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}-3\,ab^2e^{(4\,x)}+8\,a^2be^{(3\,x)}+2\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}+\cdots\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(e^{(2\,x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^2*b*e^(5*x
) + 3*b^3*e^(5*x) - 6*a^3*e^(4*x) - 3*a*b^2*e^(4*x) + 8*a^2*b*e^(3*x) + 2*b
```

```
^3*e^(3*x) - 6*a^3*e^(2*x) + 6*a^2*b*e^x + 3*b^3*e^x - 4*a^3 - a*b^2)/((a^4
 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

maple [A] time = 0.06, size = 163, normalized size = 1.31

$$\frac{32a^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(16a^4+32a^2b^2+16b^4\right)\sqrt{a^2+b^2}}+\frac{-2a^3\left(\tanh^5\left(\frac{x}{2}\right)\right)-2a^2b\left(\tanh^4\left(\frac{x}{2}\right)\right)+2\left(-\frac{10}{3}a^3-\frac{4}{3}a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+2\left(-4\right.}{\left(a^2+b^2\right)^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sinh(x)),x)
```

```
[Out] 32*a^4/(16*a^4+32*a^2*b^2+16*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2
*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^2*(-a^3*tanh(1/2*x)^5-a^2*b*tanh(1/2*
x)^4+(-10/3*a^3-4/3*a*b^2)*tanh(1/2*x)^3+(-4*a^2*b-2*b^3)*tanh(1/2*x)^2-a^3
*tanh(1/2*x)-5/3*a^2*b-2/3*b^3)/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.44, size = 241, normalized size = 1.94

$$\frac{a^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(6\,a^3e^{(-2\,x)}+4\,a^3+ab^2+3\left(2\,a^2b+b^3\right)e^{(-x)}+2\left(4\,a^2b+b^3\right)e^{(-3\,x)}+3\left(2\,a^3+ab^2\right.\right.}{3\left(a^4+2\,a^2b^2+b^4+3\left(a^4+2\,a^2b^2+b^4\right)e^{(-2\,x)}+3\left(a^4+2\,a^2b^2+b^4\right)e^{(-4\,x)}+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^3*e^(-2*x) + 4*a^3 + a
*b^2 + 3*(2*a^2*b + b^3)*e^(-x) + 2*(4*a^2*b + b^3)*e^(-3*x) + 3*(2*a^3 + a
*b^2)*e^(-4*x) + 3*(2*a^2*b + b^3)*e^(-5*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^
4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 +
 2*a^2*b^2 + b^4)*e^(-6*x))
```

mupad [B] time = 1.47, size = 654, normalized size = 5.27

$$\frac{\frac{2\,a\left(2\,a^2+b^2\right)}{\left(a^2+b^2\right)^2}-\frac{2\,b\,e^x\left(2\,a^2+b^2\right)}{\left(a^2+b^2\right)^2}}{e^{2\,x}+1}-\frac{\frac{4\left(a^3+a\,b^2\right)}{\left(a^2+b^2\right)^2}-\frac{8\,e^x\left(a^2\,b+b^3\right)}{3\left(a^2+b^2\right)^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\frac{8\,a}{3\left(a^2+b^2\right)}-\frac{8\,b\,e^x}{3\left(a^2+b^2\right)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,a^4}{b^2\,\sqrt{a^8}\,\left(a^2+b^2\right)^2\left(a^4+2\,a^2\,b^2+\right.}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] ((2*a*(2*a^2 + b^2))/(a^2 + b^2)^2 - (2*b*exp(x)*(2*a^2 + b^2))/(a^2 + b^2)
^2)/(exp(2*x) + 1) - ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b +
b^3))/(3*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2
)) - (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
 - (2*atan((exp(x)*((2*a^4)/(b^2*(a^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4 + 2*a
^2*b^2)) + (2*(a^5*(a^8)^(1/2) + 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^(1/2))
)/(a^3*b^2*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 -
5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*(b^5*(a^8)^(1
/2) + 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 + b^2)^5)
^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10
*a^6*b^4 - 5*a^8*b^2)^(1/2)))*((b^5*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6
 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + (a^4*b*(- a^10 - b^10 - 5*a^2*b^8 - 1
0*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + a^2*b^3*(- a^10 - b^10 - 5*a
^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*(a^8)^(1/2))/(- a^10
- b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sinh(x)), x)
```

3.229 $\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=88

$$\frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)}-\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] 1/2*b*(3*a^2+b^2)*arctan(sinh(x))/(a^2+b^2)^2+a^3*ln(cosh(x))/(a^2+b^2)^2-a
^3*ln(a+b*sinh(x))/(a^2+b^2)^2+1/2*sech(x)^2*(a-b*sinh(x))/(a^2+b^2)
```

Rubi [A] time = 0.18, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2721, 1647, 801, 635, 203, 260}

$$-\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}+\frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) + (a^3*Log[Cosh[x]])/(a
^2 + b^2)^2 - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(a - b*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2+b^2}+\frac{b^2\left(2a^2+b^2\right)x}{a^2+b^2}}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2b^2}$$

$$= \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{\left(a^2+b^2\right)^2(a+x)} - \frac{b^2\left(3a^2b^2+b^4+2a^3x\right)}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)}{2b^2}$$

$$= -\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} + \frac{\text{Subst}\left(\int \frac{3a^2b^2+b^4+2a^3x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} + \frac{a^3\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^2} + \cdots$$

$$= \frac{b\left(3a^2+b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(a-b\,\text{si}\cdots}{2\left(a^2+b^2\right)\cdots}$$

Mathematica [C] time = 0.19, size = 153, normalized size = 1.74

$$\frac{a\,\text{sech}^2(x)}{2\left(a^2+b^2\right)} - \frac{b\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\tanh(x)\text{sech}(x)}{2\left(a^2+b^2\right)} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\left(a^3-i\left(2a^2b+b^3\right)\right)\log(-\sinh(x)+i)}{2\left(a^2+b^2\right)^2} + \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*ArcTan[Sinh[x]])/(a^2 + b^2) + ((a^3 - I*(2*a^2*b + b^3))*Log[I - S
inh[x]])/(2*(a^2 + b^2)^2) + ((a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[x]])/(
2*(a^2 + b^2)^2) - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (a*Sech[x]^2)/(
2*(a^2 + b^2)) - (b*Sech[x]*Tanh[x])/(2*(a^2 + b^2))
```

fricas [B] time = 0.88, size = 655, normalized size = 7.44

$$-\frac{\left(a^2b+b^3\right)\cosh(x)^3 + \left(a^2b+b^3\right)\sinh(x)^3 - 2\left(a^3+ab^2\right)\cosh(x)^2 - \left(2a^3+2ab^2-3\left(a^2b+b^3\right)\cosh(x)\right)\sinh(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*cosh(x)^3 + (a^2*b + b^3)*sinh(x)^3 - 2*(a^3 + a*b^2)*cosh(
x)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(x))*sinh(x)^2 - ((3*a^2*b +
b^3)*cosh(x)^4 + 4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (3*a^2*b + b^3)*sinh
(x)^4 + 3*a^2*b + b^3 + 2*(3*a^2*b + b^3)*cosh(x)^2 + 2*(3*a^2*b + b^3 + 3*
(3*a^2*b + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^2*b + b^3)*cosh(x)^3 + (3*a^
2*b + b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b + b^3)*cosh
(x) + (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh
(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*
cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^3*cosh(x)
^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3
*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*
log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(x)
^2 + 4*(a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh
(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^
4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)
)
```

giac [B] time = 0.19, size = 211, normalized size = 2.40

$$-\frac{a^3 b \log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^4 b + 2\,a^2 b^3 + b^5} + \frac{a^3 \log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2\,a^2 b^2 + b^4\right)} + \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^2 b + b^3\right)}{4\left(a^4 + 2\,a^2 b^2 + b^4\right)} - \frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^
3*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*(3*a^2*b + b^3)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^3
*(e^(-x) - e^x)^2 - 2*a^2*b*(e^(-x) - e^x) - 2*b^3*(e^(-x) - e^x) - 4*a*b^2
)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

maple [B] time = 0.06, size = 357, normalized size = 4.06

$$-\frac{8a^3 \ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right) b - a\right)}{8a^4 + 16a^2 b^2 + 8b^4} + \frac{\left(\tanh^3\left(\frac{x}{2}\right)\right) a^2 b}{\left(a^4 + 2a^2 b^2 + b^4\right)\left(\tanh^2\left(\frac{x}{2}\right) + 1\right)^2} + \frac{\left(\tanh^3\left(\frac{x}{2}\right)\right) b^3}{\left(a^4 + 2a^2 b^2 + b^4\right)\left(\tanh^2\left(\frac{x}{2}\right) + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sinh(x)),x)
```

```
[Out] -8*a^3/(8*a^4+16*a^2*b^2+8*b^4)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+1/(a^
4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^2*b+1/(a^4+2*a^2*b^2+b
^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*b^3-2/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x
)^2+1)^2*tanh(1/2*x)^2*a^3-2/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1
/2*x)^2*a*b^2-1/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^2*b-1
/(a^4+2*a^2*b^2+b^4)/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*b^3+1/(a^4+2*a^2*b^2+b
^4)*a^3*ln(tanh(1/2*x)^2+1)+3/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*a^2*b
+1/(a^4+2*a^2*b^2+b^4)*arctan(tanh(1/2*x))*b^3
```

maxima [A] time = 0.51, size = 160, normalized size = 1.82

$$-\frac{a^3 \log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^4 + 2\,a^2b^2 + b^4} + \frac{a^3 \log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{\left(3\,a^2b + b^3\right)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{be^{(-x)} - 2\,ae^{(-2\,x)} - be^{(}}{a^2 + b^2 + 2\left(a^2 + b^2\right)e^{(-2\,x)} + \left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + a^3*log(e^
(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (3*a^2*b + b^3)*arctan(e^(-x))/(a^4 +
 2*a^2*b^2 + b^4) - (b*e^(-x) - 2*a*e^(-2*x) - b*e^(-3*x))/(a^2 + b^2 + 2*(
a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

mupad [B] time = 2.28, size = 291, normalized size = 3.31

$$\frac{\frac{2\left(a^3+a\,b^2\right)}{\left(a^2+b^2\right)^2} - \frac{e^x\left(a^2\,b+b^3\right)}{\left(a^2+b^2\right)^2}}{e^{2\,x}+1} - \frac{\frac{2\,a}{a^2+b^2} - \frac{2\,b\,e^x}{a^2+b^2}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{\ln\left(1+e^x\,1i\right)\,\left(2\,a+b\,1i\right)}{2\,\left(a^2+a\,b\,2i-b^2\right)} - \frac{a^3\,\ln\left(b^7\,e^{2\,x} - 16\,a^6\,b - b^7 - 6\,a^2\,b^5 - 9\,a^4\,b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] ((2*(a*b^2 + a^3))/(a^2 + b^2)^2 - (exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2)/(e
xp(2*x) + 1) - ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(2*exp(2*x) +
 exp(4*x) + 1) + (log(exp(x)*1i + 1)*(2*a + b*1i))/(2*(a*b*2i + a^2 - b^2))
 - (a^3*log(b^7*exp(2*x) - 16*a^6*b - b^7 - 6*a^2*b^5 - 9*a^4*b^3 + 32*a^7*
exp(x) + 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) + 16*a^6*
b*exp(2*x) + 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 + 2*a^2*b^2
) + (log(exp(x) + 1i)*(a*2i + b))/(2*(2*a*b + a^2*1i - b^2*1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sinh(x)), x)
```

3.230 $\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=69

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}$$

```
[Out] -2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-b*sech(x)
/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2727, 3767, 8, 2606, 2660, 618, 206}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*
Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = -\frac{a\int \operatorname{sech}^2(x)\,dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}+\frac{b\int \operatorname{sech}(x)\tanh(x)\,dx}{a^2+b^2}$$

$$= -\frac{(ia)\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)}{a^2+b^2}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{b\operatorname{Subst}(}{}$$

$$= -\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{\left(4a^2\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$= -\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}$$

Mathematica [A] time = 0.20, size = 69, normalized size = 1.00

$$\frac{a\left(\frac{2a\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\tanh(x)\right)-b\text{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-(b*Sech[x]) + a*((2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] - Tanh[x]))/(a^2 + b^2)
```

fricas [B] time = 0.83, size = 257, normalized size = 3.72

$$\frac{2\,a^3+2\,ab^2+\left(a^2\cosh(x)^2+2\,a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+a^2\right)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x}{b\cosh(x)^2+b}\right.}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh(x)^2+2\left(a^4+2\,a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 +
 a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^4
 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2
 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

giac [A] time = 0.17, size = 87, normalized size = 1.26

$$\frac{a^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\,(be^x-a)}{\left(a^2+b^2\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

maple [A] time = 0.05, size = 84, normalized size = 1.22

$$\frac{8a^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(4a^2+4b^2\right)\sqrt{a^2+b^2}}+\frac{-2a\tanh\left(\frac{x}{2}\right)-2b}{\left(a^2+b^2\right)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sinh(x)),x)
```

```
[Out] 8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+
b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*x)-b)/(tanh(1/2*x)^2+1)
```

maxima [A] time = 0.45, size = 89, normalized size = 1.29

$$\frac{a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(be^{(-x)}+a\right)}{a^2+b^2+\left(a^2+b^2\right)e^{(-2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(a^2 + b^2)^(3/2) - 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

mupad [B] time = 0.94, size = 330, normalized size = 4.78

$$\frac{\frac{2a}{a^2+b^2}-\frac{2b\,e^x}{a^2+b^2}}{e^{2x}+1}-\frac{2\,\mathrm{atan}\left(\left(\frac{b^3\sqrt{-a^6-3a^4b^2-3a^2b^4-b^6}}{2}+\frac{a^2b\sqrt{-a^6-3a^4b^2-3a^2b^4-b^6}}{2}\right)\left(e^x\left(\frac{2a^2}{b^2\sqrt{a^4}\left(a^2+b^2\right)^2}+\frac{2\left(a^3\sqrt{\right.}{a\,b^2\sqrt{-\left(a^2+b^2\right)^3}\,\left(a^2\right.}\right.\right.\right.}{\sqrt{-a^6-3a^4b^2-3a^2b^4-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan(((b
^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^6 - b^6 - 3
*a^2*b^4 - 3*a^4*b^2)^(1/2))/2)*(exp(x)*((2*a^2)/(b^2*(a^4)^(1/2)*(a^2 + b^
2)^2) + (2*(a^3*(a^4)^(1/2) + a*b^2*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^3)^(
1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))) - (2*(b^3*(a
^4)^(1/2) + a^2*b*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(
- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))))*(a^4)^(1/2))/(- a^6 - b^6 - 3
*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sinh(x)), x)
```

3.231 $\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=48

$$-\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}$$

```
[Out] b*arctan(sinh(x))/(a^2+b^2)+a*ln(cosh(x))/(a^2+b^2)-a*ln(a+b*sinh(x))/(a^2+
b^2)
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2721, 801, 635, 203, 260}

$$-\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/(a^2 + b^2) + (a*Log[Cosh[x]])/(a^2 + b^2) - (a*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx &= -\text{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{a}{\left(a^2+b^2\right)(a+x)} + \frac{-b^2-ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2} + \frac{a\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh\right.}{a^2+b^2}\\
&= \frac{b\tan^{-1}(\sinh(x))}{a^2+b^2} + \frac{a\log(\cosh(x))}{a^2+b^2} - \frac{a\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.75

$$\frac{-a\log(a+b\sinh(x)) + a\log(\cosh(x)) + 2b\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x]),x]
```

```
[Out] (2*b*ArcTan[Tanh[x/2]] + a*Log[Cosh[x]] - a*Log[a + b*Sinh[x]])/(a^2 + b^2)
```

fricas [A] time = 0.63, size = 57, normalized size = 1.19

$$\frac{2\,b\arctan\left(\cosh(x)+\sinh(x)\right) - a\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right) + a\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*b*arctan(cosh(x) + sinh(x)) - a*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + a*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

giac [A] time = 0.27, size = 89, normalized size = 1.85

$$-\frac{ab\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2a\right|\right)}{a^{2}b+b^{3}}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)b}{2\left(a^{2}+b^{2}\right)}+\frac{a\log\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)}{2\left(a^{2}+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

maple [A] time = 0.05, size = 84, normalized size = 1.75

$$-\frac{2a\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{2a^{2}+2b^{2}}+\frac{2a\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}{2a^{2}+2b^{2}}+\frac{4b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^{2}+2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sinh(x)),x)
```

```
[Out] -2*a/(2*a^2+2*b^2)*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(2*a^2+2*b^2)*a*
ln(tanh(1/2*x)^2+1)+4/(2*a^2+2*b^2)*b*arctan(tanh(1/2*x))
```

maxima [A] time = 0.50, size = 66, normalized size = 1.38

$$-\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^{2}+b^{2}}-\frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^{2}+b^{2}}+\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^{2}+b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-x))/(a^2 + b^2) - a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2)
```

mupad [B] time = 1.40, size = 95, normalized size = 1.98

$$\frac{\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)}{a-b\,1\mathrm{i}}-\frac{a\,\ln\left(b^{3}\,\mathrm{e}^{2\,x}-4\,a^{2}\,b-b^{3}+8\,a^{3}\,\mathrm{e}^{x}+2\,a\,b^{2}\,\mathrm{e}^{x}+4\,a^{2}\,b\,\mathrm{e}^{2\,x}\right)}{a^{2}+b^{2}}+\frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)\,1\mathrm{i}}{-b+a\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*sinh(x)),x)
```

```
[Out] (log(exp(x)*1i + 1)*1i)/(a*1i - b) + log(exp(x) + 1i)/(a - b*1i) - (a*log(b
^3*exp(2*x) - 4*a^2*b - b^3 + 8*a^3*exp(x) + 2*a*b^2*exp(x) + 4*a^2*b*exp(2
*x)))/(a^2 + b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*sinh(x)), x)
```

3.232 $\int \frac{\coth(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}$$

```
[Out] ln(sinh(x))/a-ln(a+b*sinh(x))/a
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth(x)}{a+b\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x(a+x)}\,dx,x,b\sinh(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,b\sinh(x)\right)}{a}-\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{a}\\
&= \frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\log(\sinh(x))}{a}-\frac{\log(a+b\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

fricas [A] time = 1.12, size = 40, normalized size = 2.00

$$-\frac{\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(x) - sin
h(x))))/a
```

giac [A] time = 0.20, size = 39, normalized size = 1.95

$$-\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a + log(abs(-e^(-x) + e^x))/a
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

$$\frac{\ln\left(\sinh(x)\right)}{a}-\frac{\ln\left(a+b\sinh(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)),x)
```

```
[Out] ln(sinh(x))/a-ln(a+b*sinh(x))/a
```

maxima [B] time = 0.37, size = 46, normalized size = 2.30

$$-\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a}+\frac{\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-x)}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -log(-2*a*e^(-x) + b*e^(-2*x) - b)/a + log(e^(-x) + 1)/a + log(e^(-x) - 1)/
a
```

mupad [B] time = 0.89, size = 195, normalized size = 9.75

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2}+b\,\mathrm{e}^x\,\sqrt{-a^2}-2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^2}-b\,\mathrm{e}^{3\,x}\,\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}}-\frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,\sqrt{-a^2}+4\,a^2\,b^3\,\sqrt{-a^2}\right)\left(\frac{1}{8\,a\,b\,\left(a^2+b^2\right)^2}-\mathrm{e}^x\,\left(\frac{1}{16\,b^2\,\left(a^2\right.}\right.\right.\right.}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x)),x)
```

```
[Out] (2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) - 2*a*exp(2*x)*(-a^2)^(1/2)
 - b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^(1
/2) + 4*a^2*b^3*(-a^2)^(1/2))*(1/(8*a*b*(a^2 + b^2)^2) - exp(x)*(1/(16*b^2*
(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^4*b^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2
)/(8*a^3*b*(a^2 + b^2)^2))))/(-a^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*sinh(x)), x)
```

3.233 $\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] b*arctanh(cosh(x))/a^2-coth(x)/a-2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2
))*(a^2+b^2)^(1/2)/a^2
```

Rubi [A] time = 0.23, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 206}

$$-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/a^2 - Coth[x]/a
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{csch}^2(x)\left(1+\sinh^2(x)\right)}{a+b\sinh(x)}\,dx$$
$$= -\frac{\coth(x)}{a} + \frac{i\int \frac{\operatorname{csch}(x)(ib-ia\sinh(x))}{a+b\sinh(x)}\,dx}{a}$$
$$= -\frac{\coth(x)}{a} - \frac{b\int \operatorname{csch}(x)\,dx}{a^2} + \frac{\left(a^2+b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} + \frac{\left(2\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} - \frac{\left(4\left(a^2+b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2} - \frac{\coth(x)}{a}$$

Mathematica [A] time = 0.18, size = 82, normalized size = 1.46

$$-\frac{\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\left(\sinh(x)\left(2\sqrt{-a^2-b^2}\,\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+a\cosh(x)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(Csch[x/2]*Sech[x/2]*(a*Cosh[x] + (2*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Ta
nh[x/2])/Sqrt[-a^2 - b^2]] + b*Log[Tanh[x/2]])*Sinh[x]))/a^2
```

fricas [B] time = 0.62, size = 228, normalized size = 4.07

$$\sqrt{a^2+b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)s}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log((b^2*c
osh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a
```

```
*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 +
 b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) + 1) - (b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) - 1
) - 2*a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)
```

giac [A] time = 0.50, size = 95, normalized size = 1.70

$$\frac{b\log\left(e^{x}+1\right)}{a^{2}}-\frac{b\log\left(|e^{x}-1|\right)}{a^{2}}+\frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{a^{2}}-\frac{2}{a\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1))/a^2 + sqrt(a^2 + b^2)*log(abs(2*b*
e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/a^2
- 2/(a*(e^(2*x) - 1))
```

maple [B] time = 0.06, size = 107, normalized size = 1.91

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2a}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}+\frac{2b^{2}\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{a^{2}\sqrt{a^{2}+b^{2}}}-\frac{1}{2a\tanh\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sinh(x)),x)
```

```
[Out] -1/2/a*tanh(1/2*x)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))+2*b^2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a
^2+b^2)^(1/2))-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))
```

maxima [A] time = 0.46, size = 97, normalized size = 1.73

$$\frac{b\log\left(e^{(-x)}+1\right)}{a^{2}}-\frac{b\log\left(e^{(-x)}-1\right)}{a^{2}}+\frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{a^{2}}+\frac{2}{ae^{(-2\,x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + sqrt(a^2 + b^2)*log((b*e^(-
x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/a^2 + 2/(a*e^(-
2*x) - a)
```

mupad [B] time = 0.84, size = 304, normalized size = 5.43

$$\frac{2}{a-a\,e^{2\,x}}-\frac{b\,\ln\left(32\,a^2+32\,b^2-32\,a^2\,e^x-32\,b^2\,e^x\right)}{a^2}+\frac{b\,\ln\left(32\,a^2+32\,b^2+32\,a^2\,e^x+32\,b^2\,e^x\right)}{a^2}+\frac{\ln\left(128\,a^4\,e^x-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*sinh(x)),x)
```

```
[Out] 2/(a - a*exp(2*x)) - (b*log(32*a^2 + 32*b^2 - 32*a^2*exp(x) - 32*b^2*exp(x)
))/a^2 + (b*log(32*a^2 + 32*b^2 + 32*a^2*exp(x) + 32*b^2*exp(x)))/a^2 + (lo
g(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*
exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x) - 64*a^2*b*(
a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^
2 - (log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3 - 64*a^3*b + 128*a^4*exp(x) +
32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x) + 64*
a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1
/2))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sinh(x)), x)
```

3.234 $\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{b\operatorname{csch}(x)}{a^2}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}-\frac{\operatorname{csch}^2(x)}{2a}$$

```
[Out] b*csch(x)/a^2-1/2*csch(x)^2/a+(a^2+b^2)*ln(sinh(x))/a^3-(a^2+b^2)*ln(a+b*si
nh(x))/a^3
```

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2721, 894}

$$\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}+\frac{b\operatorname{csch}(x)}{a^2}-\frac{\operatorname{csch}^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*Csch[x])/a^2 - Csch[x]^2/(2*a) + ((a^2 + b^2)*Log[Sinh[x]])/a^3 - ((a^2
+ b^2)*Log[a + b*Sinh[x]])/a^3
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = -\operatorname{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)}\,dx, x, b\sinh(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-\frac{b^2}{ax^3}+\frac{b^2}{a^2x^2}+\frac{-a^2-b^2}{a^3x}+\frac{a^2+b^2}{a^3(a+x)}\right)dx, x, b\sinh(x)\right)$$
$$= \frac{b\operatorname{csch}(x)}{a^2}-\frac{\operatorname{csch}^2(x)}{2a}+\frac{\left(a^2+b^2\right)\log(\sinh(x))}{a^3}-\frac{\left(a^2+b^2\right)\log(a+b\sinh(x))}{a^3}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 0.87

$$\frac{2\left(a^2+b^2\right)(\log(\sinh(x))-\log(a+b\sinh(x)))-a^2\operatorname{csch}^2(x)+2ab\operatorname{csch}(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + b^2)*(Log[Sinh[x]] - Log[a + b*Si
nh[x]]))/(2*a^3)
```

fricas [B] time = 0.75, size = 427, normalized size = 8.21

$$\frac{2\,ab\cosh(x)^3+2\,ab\sinh(x)^3-2\,a^2\cosh(x)^2-2\,ab\cosh(x)+2\left(3\,ab\cosh(x)-a^2\right)\sinh(x)^2-\left(\left(a^2+b^2\right)\cosh(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 - 2*a*b*cosh(x) + 2*(3
*a*b*cosh(x) - a^2)*sinh(x)^2 - ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh
(x)*sinh(x)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2
 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(
x)^3 - (a^2 + b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(
x))) + ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x)^3 + (a^2 + b^
2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*cosh(x)^2 - a^2 -
 b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^2 + b^2)*cosh(x
))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^2 - 2*a^2
*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sin
h(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 + 4*(a
^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))
```

giac [B] time = 0.27, size = 125, normalized size = 2.40

$$\frac{\left(a^2+b^2\right)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^3}-\frac{\left(a^2b+b^3\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^3b}-\frac{3\,a^2\left(e^{(-x)}-e^x\right)^2+3\,b^2\left(e^{(-x)}-e^x\right)^2+4\,ab\left(e\right.}{2\,a^3\left(e^{(-x)}-e^x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(-e^(-x) + e^x))/a^3 - (a^2*b + b^3)*log(abs(-b*(e^(-x)
- e^x) + 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) - e^x)^2 + 3*b^2*(e^(-x) - e^x)
^2 + 4*a*b*(e^(-x) - e^x) + 4*a^2)/(a^3*(e^(-x) - e^x)^2)
```

maple [B] time = 0.07, size = 120, normalized size = 2.31

$$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8a}-\frac{\tanh\left(\frac{x}{2}\right)b}{2a^2}-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{a}-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)b^2}{a^3}-\frac{}{8a\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x)),x)
```

```
[Out] -1/8/a*tanh(1/2*x)^2-1/2/a^2*tanh(1/2*x)*b-1/a*ln(a*tanh(1/2*x)^2-2*tanh(1/
2*x)*b-a)-1/a^3*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2-1/8/a/tanh(1/2*x)
^2+1/a*ln(tanh(1/2*x))+1/a^3*ln(tanh(1/2*x))*b^2+1/2*b/a^2/tanh(1/2*x)
```

maxima [B] time = 0.35, size = 116, normalized size = 2.23

$$-\frac{2\left(be^{(-x)}-ae^{(-2\,x)}-be^{(-3\,x)}\right)}{2\,a^2e^{(-2\,x)}-a^2e^{(-4\,x)}-a^2}-\frac{\left(a^2+b^2\right)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^3}+\frac{\left(a^2+b^2\right)\log\left(e^{(-x)}+1\right)}{a^3}+\frac{\left(a^2+b^2\right)\log\left(\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-x) - a*e^(-2*x) - b*e^(-3*x))/(2*a^2*e^(-2*x) - a^2*e^(-4*x) - a^
2) - (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^3 + (a^2 + b^2)*log(e^
(-x) + 1)/a^3 + (a^2 + b^2)*log(e^(-x) - 1)/a^3
```

mupad [B] time = 1.24, size = 1163, normalized size = 22.37

$$\frac{\left(2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{-a^6}\,\sqrt{a^4+2\,a^2\,b^2+b^4}+2\,b^2\,\sqrt{-a^6}\,\sqrt{a^4+2\,a^2\,b^2+b^4}}{2\,a^3\,\left(a^2+b^2\right)^2}+\frac{\left(a^7+a^5\,b^2\right)\sqrt{-a^6}}{2\,a^6\,\sqrt{\left(a^2+b^2\right)^2}\,\left(a^2+b^2\right)}-\frac{a^6\,b^2\,\mathrm{e}^x\,\sqrt{-a^6}\,\left(\frac{8\left(a^4+2\,a^2\,b^2+b^4\right)}{a^8\,b\,\left(a^2+b^2\right)^2}-\frac{4\left(2\,a^6\,b+2\,a^4\,b^3\right)}{a^{12}\,b^2\,\sqrt{\left(a^2\right.}}\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3/(a + b*sinh(x)),x)

[Out] ((2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(2*a^3*(a^2 + b^2)^2) + ((a^7 + a^5*b^2)*(-a^6)^(1/2))/(2*a^6*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (a^6*b^2*exp(x)*(-a^6)^(1/2)*((8*(a^4 + b^4 + 2*a^2*b^2))/(a^8*b*(a^2 + b^2)^2) - (4*(2*a^6*b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (2*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*(-a^6)^(1/2)*(a^2 + b^2)^2)))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) - (a^6*b^2*exp(2*x)*(-a^6)^(1/2)*((4*(a^2 + 2*b^2)*(a^4 + b^4 + 2*a^2*b^2))/(a^9*b^2*(a^2 + b^2)^2) + (4*(a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^9*b^2*(-a^6)^(1/2)*(a^2 + b^2)^2) + (2*(2*a^6*b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (4*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2))))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*(-a^6)^(1/2)*(a^2 + b^2)^2))*(-a^6)^(1/2))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2))) - 2*atan((4*a^6*b*(-a^6)^(1/2)*(a^2 + b^2)^2 + 4*a^4*b^3*(-a^6)^(1/2)*(a^2 + b^2)^2)*(1/(8*a^5*b*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3) - exp(x)*(1/(16*a^4*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3) - (a^2 + 2*b^2)^2/(16*a^8*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3)) + (a^2 + 2*b^2)/(8*a^7*b*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3))))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-a^6)^(1/2) - 2/(a*(exp(4*x) - 2*exp(2*x) + 1)) - (2/a - (2*b*exp(x))/a^2)/(exp(2*x) - 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**3/(a+b*sinh(x)),x)

[Out] Integral(coth(x)**3/(a + b*sinh(x)), x)

3.235 $\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=108

$$\frac{b\coth(x)\mathrm{csch}(x)}{2a^2}-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3}-\frac{\mathrm{co}}{}$$

```
[Out] 1/2*b*(3*a^2+2*b^2)*arctanh(cosh(x))/a^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*tan
h(1/2*x))/(a^2+b^2)^(1/2))/a^4-1/3*(4*a^2+3*b^2)*coth(x)/a^3+1/2*b*coth(x)*
csch(x)/a^2-1/3*coth(x)*csch(x)^2/a
```

Rubi [A] time = 0.41, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2725, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4}-\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3}+\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{b\coth(x)\mathrm{csch}(x)}{2a^2}-\frac{\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^4 - ((4*a^2 + 3*b^2)*Coth[x])/(3*a^3
) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{\int \frac{\operatorname{csch}^2(x)\left(2\left(4a^2+3b^2\right)-ab\sinh(x)+3\left(2a^2+b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}dx}{6a^2}$$

$$= -\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{i\int \frac{\operatorname{csch}(x)\left(3ib\left(3a^2+2b^2\right)-3ia(2\ldots}{a+b\sinh(x)}}{6a^3}$$

$$= -\frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{\left(a^2+b^2\right)^2\int \frac{1}{a+b\sinh(x)}dx\ldots}{a^4}$$

$$= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}\ldots}{3a}$$

$$= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{\left(4a^2+3b^2\right)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}\ldots}{3a}$$

$$= \frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{2\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4} - \frac{\left(4a^2+3b^2\right)\coth(x\ldots}{3a^3}$$

Mathematica [A] time = 0.43, size = 176, normalized size = 1.63

$$\frac{-\frac{1}{2}a^3\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right) + 8a^3\sinh^4\left(\frac{x}{2}\right)\operatorname{csch}^3(x) - 4a\left(4a^2+3b^2\right)\tanh\left(\frac{x}{2}\right) - 4a\left(4a^2+3b^2\right)\coth\left(\frac{x}{2}\right) - 12b\left(3a^2\ldots\right.}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (48*(-a^2 - b^2)^(3/2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] - 4*a*(4*
a^2 + 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 - 12*b*(3*a^2 + 2*b^2)*Log[Tan
h[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2
]^4*Sinh[x])/2 - 4*a*(4*a^2 + 3*b^2)*Tanh[x/2])/(24*a^4)
```

fricas [B] time = 2.02, size = 1303, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^2*b*cosh(x)^5 + 6*a^2*b*sinh(x)^5 - 12*(2*a^3 + a*b^2)*cosh(x)^4 +
 6*(5*a^2*b*cosh(x) - 4*a^3 - 2*a*b^2)*sinh(x)^4 - 6*a^2*b*cosh(x) + 12*(5*
a^2*b*cosh(x)^2 - 4*(2*a^3 + a*b^2)*cosh(x))*sinh(x)^3 - 16*a^3 - 12*a*b^2
+ 24*(a^3 + a*b^2)*cosh(x)^2 + 12*(5*a^2*b*cosh(x)^3 + 2*a^3 + 2*a*b^2 - 6*
(2*a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 + b^2)*cosh(x)^6 + 6*(a^2 +
b^2)*cosh(x)*sinh(x)^5 + (a^2 + b^2)*sinh(x)^6 - 3*(a^2 + b^2)*cosh(x)^4 +
3*(5*(a^2 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 + b^2)*cosh(x
)^3 - 3*(a^2 + b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2 + 3*(5*(a^
2 + b^2)*cosh(x)^4 - 6*(a^2 + b^2)*cosh(x)^2 + a^2 + b^2)*sinh(x)^2 - a^2 -
 b^2 + 6*((a^2 + b^2)*cosh(x)^5 - 2*(a^2 + b^2)*cosh(x)^3 + (a^2 + b^2)*cos
h(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) - b)) + 3*((3*a^2*b + 2*b^3)*cosh(x)^6 + 6*(3*a^2*b +
2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh(x)^6 - 3*(3*a^2*b + 2*b^3
)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b + 2*b^3)*cosh(x))*sinh(x)^
3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b + 2*b^3
)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^2 +
6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b^3)*cosh(x)^3 + (3*a^2*b +
 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b + 2*b^3)
*cosh(x)^6 + 6*(3*a^2*b + 2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh
(x)^6 - 3*(3*a^2*b + 2*b^3)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2
*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b
+ 2*b^3)*cosh(x))*sinh(x)^3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)
^2 + 3*(5*(3*a^2*b + 2*b^3)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^
3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b
^3)*cosh(x)^3 + (3*a^2*b + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) -
 1) + 6*(5*a^2*b*cosh(x)^4 - 8*(2*a^3 + a*b^2)*cosh(x)^3 - a^2*b + 8*(a^3 +
 a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*si
nh(x)^6 - 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sin
h(x)^4 - a^4 + 4*(5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cos
h(x)^4 - 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x
)^3 + a^4*cosh(x))*sinh(x))
```

giac [B] time = 0.46, size = 194, normalized size = 1.80

$$\frac{\left(3\,a^{2}b+2\,b^{3}\right)\log\left(e^{x}+1\right)}{2\,a^{4}}-\frac{\left(3\,a^{2}b+2\,b^{3}\right)\log\left(\left|e^{x}-1\right|\right)}{2\,a^{4}}+\frac{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,a^{4}}+\frac{3\,abe^{(5\,x)}-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*b + 2*b^3)*log(e^x + 1)/a^4 - 1/2*(3*a^2*b + 2*b^3)*log(abs(e^x
- 1))/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^
2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/3*(3*
a*b*e^(5*x) - 12*a^2*e^(4*x) - 6*b^2*e^(4*x) + 12*a^2*e^(2*x) + 12*b^2*e^(2
*x) - 3*a*b*e^x - 8*a^2 - 6*b^2)/(a^3*(e^(2*x) - 1)^3)
```

maple [B] time = 0.06, size = 232, normalized size = 2.15

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{24a}-\frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8a^2}-\frac{5\tanh\left(\frac{x}{2}\right)}{8a}-\frac{b^2\tanh\left(\frac{x}{2}\right)}{2a^3}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{4b^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sinh(x)),x)
```

```
[Out] -1/24/a*tanh(1/2*x)^3-1/8/a^2*b*tanh(1/2*x)^2-5/8/a*tanh(1/2*x)-1/2/a^3*b^2
*tanh(1/2*x)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^
(1/2))+4*b^2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2
)^(1/2))+2/a^4*b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2))-1/24/a/tanh(1/2*x)^3-5/8/a/tanh(1/2*x)-1/2/a^3/tanh(1/2*x)*b^2+1
/8/a^2*b/tanh(1/2*x)^2-3/2/a^2*b*ln(tanh(1/2*x))-1/a^4*b^3*ln(tanh(1/2*x))
```

maxima [B] time = 0.46, size = 212, normalized size = 1.96

$$-\frac{3\,abe^{(-x)}-3\,abe^{(-5\,x)}-8\,a^2-6\,b^2+12\left(a^2+b^2\right)e^{(-2\,x)}-6\left(2\,a^2+b^2\right)e^{(-4\,x)}}{3\left(3\,a^3e^{(-2\,x)}-3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}-a^3\right)}+\frac{\left(3\,a^2b+2\,b^3\right)\log\left(e^{(-x)}+1\right)}{2\,a^4}-\frac{(3\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b*e^(-x) - 3*a*b*e^(-5*x) - 8*a^2 - 6*b^2 + 12*(a^2 + b^2)*e^(-2*
x) - 6*(2*a^2 + b^2)*e^(-4*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(-6
*x) - a^3) + 1/2*(3*a^2*b + 2*b^3)*log(e^(-x) + 1)/a^4 - 1/2*(3*a^2*b + 2*b
^3)*log(e^(-x) - 1)/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - sqrt(
a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4)
```

mupad [B] time = 1.41, size = 778, normalized size = 7.20

$$\ln\left(-\frac{8\left(-30\,e^x\,a^9+18\,a^8\,b-101\,e^x\,a^7\,b^2+60\,a^6\,b^3-126\,e^x\,a^5\,b^4+74\,a^4\,b^5-69\,e^x\,a^3\,b^6+40\,a^2\,b^7-14\,e^x\,a\,b^8+8\,b^9\right)}{a^9\,b^3}-\frac{\sqrt{\left(a^2+b^2\right)^3}\left(\frac{8\left(4\,a^8-36\,e^x\,a^7\,b+34\,a^6\,b\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*sinh(x)),x)
```

```
[Out] (log(- (8*(18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 30*a^9
*exp(x) - 14*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) - 101*a^
7*b^2*exp(x)))/(a^9*b^3) - (((a^2 + b^2)^3)^(1/2)*((8*(4*a^8 + 8*b^8 + 36*a
^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*exp(x) - 36*a^7*b*exp(x) - 52*a
^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^4) - (((16*(4*a^4*b + 4*b^5 + 8*
a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) - 15*a^3*b^2*exp(x)))/(a*b^5) + (32
*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(
x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a^4))/a^4)*((a^2 + b^2)^3)^(1/2))/a^
4 - ((2*(2*a^2 + b^2))/a^3 - (b*exp(x))/a^2)/(exp(2*x) - 1) - (4/a - (2*b*e
xp(x))/a^2)/(exp(4*x) - 2*exp(2*x) + 1) - (log((((a^2 + b^2)^3)^(1/2)*((8*(
4*a^8 + 8*b^8 + 36*a^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*exp(x) - 36
*a^7*b*exp(x) - 52*a^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^4) + (((16*(
4*a^4*b + 4*b^5 + 8*a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) - 15*a^3*b^2*ex
p(x)))/(a*b^5) - (32*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp
(x) - 3*a^3*b^2*exp(x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a^4))/a^4 - (8*(
18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 30*a^9*exp(x) - 1
4*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) - 101*a^7*b^2*exp(x
)))/(a^9*b^3))*((a^2 + b^2)^3)^(1/2))/a^4 - 8/(3*a*(3*exp(2*x) - 3*exp(4*x)
 + exp(6*x) - 1)) - (log(exp(x) - 1)*(3*a^2*b + 2*b^3))/(2*a^4) + (log(exp(
x) + 1)*(3*a^2*b + 2*b^3))/(2*a^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sinh(x)),x)
[Out] Integral(coth(x)**4/(a + b*sinh(x)), x)
```

3.236 $\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2} \, dx$

Optimal. Leaf size=224

$$-\frac{\left(a^2-b^2\right)\tanh^3(x)}{3\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2}+\frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2}-\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3(a+b\sinh(x))}-\frac{\left(2a^4 \ldots\right.}{\ldots}$$

```
[Out] -2*a^5*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+8*a^3*b^2
*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-4*a^3*b*sech(x)
/(a^2+b^2)^3+2/3*a*b*sech(x)^3/(a^2+b^2)^2-a^4*b*cosh(x)/(a^2+b^2)^3/(a+b*s
inh(x))+(a^2-b^2)*tanh(x)/(a^2+b^2)^2-(2*a^4-3*a^2*b^2-b^4)*tanh(x)/(a^2+b^
2)^3-1/3*(a^2-b^2)*tanh(x)^3/(a^2+b^2)^2
```

Rubi [A] time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2731, 2664, 12, 2660, 618, 206, 2669, 3767, 8}

$$-\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{\left(a^2-b^2\right)\tanh^3(x)}{3\left(a^2+b^2\right)^2}-\frac{\left(-3a^2b^2+2a^4-b^4\right)\tanh(x)}{\left(a^2+b^2\right)^3}+\frac{\left(a^2-b \ldots\right.}{\left(a^2 \ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^5*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (8*
a^3*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) - (4*
a^3*b*Sech[x])/(a^2 + b^2)^3 + (2*a*b*Sech[x]^3)/(3*(a^2 + b^2)^2) - (a^4*b
*Cosh[x])/((a^2 + b^2)^3*(a + b*Sinh[x])) + ((a^2 - b^2)*Tanh[x])/(a^2 + b^
2)^2 - ((2*a^4 - 3*a^2*b^2 - b^4)*Tanh[x])/(a^2 + b^2)^3 - ((a^2 - b^2)*Tan
h[x]^3)/(3*(a^2 + b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \int \left(\frac{a^4}{\left(a^2+b^2\right)^2 (a+b\sinh(x))^2} - \frac{4a^3b^2}{\left(a^2+b^2\right)^3 (a+b\sinh(x))} + \frac{\operatorname{sech}^4(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2a\right.}{\left(a^2+b^2\right)^2}\right.$$

$$= \frac{\int \operatorname{sech}^2(x)\left(-2a^4\left(1-\frac{3a^2b^2+b^4}{2a^4}\right)+4a^3b\sinh(x)\right)dx}{\left(a^2+b^2\right)^3} - \frac{\left(4a^3b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^3} + \frac{\int \operatorname{sech}^4}{}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3 (a+b\sinh(x))} + \frac{a^4\int \frac{a}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^3} - \frac{\left(8a^3b^2\right.}{}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3 (a+b\sinh(x))} + \frac{a^5\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(16a^3b\right.}{}$$

$$= \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3 (a+b\sinh(x))} +$$

$$= \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^4b\cosh(x)}{\left(a^2+b^2\right)^3 (a+b\sinh(x))} +$$

$$= -\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} + \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{\left(a^2+b^2\right)^3} + \frac{2ab\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)^2} -$$

Mathematica [A] time = 0.41, size = 144, normalized size = 0.64

$$\frac{-\frac{3a^4b\cosh(x)}{a+b\sinh(x)} - 12a^3b\operatorname{sech}(x) + \left(a^2+b^2\right)\operatorname{sech}^3(x)\left(\left(a^2-b^2\right)\sinh(x)+2ab\right) + \left(-4a^4+9a^2b^2+b^4\right)\tanh(x) + \frac{6a^3(a}{}}{3\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^3*(a^2 - 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2] - 12*a^3*b*Sech[x] - (3*a^4*b*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2
```

```
)*Sech[x]^3*(2*a*b + (a^2 - b^2)*Sinh[x]) + (-4*a^4 + 9*a^2*b^2 + b^4)*Tanh
[x])/(3*(a^2 + b^2)^3)
```

fricas [B] time = 0.95, size = 3534, normalized size = 15.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^7 + 6*(a^7 - 3*a^5*b^2 - 4*a^
3*b^4)*sinh(x)^7 - 14*a^6*b + 4*a^4*b^3 + 20*a^2*b^5 + 2*b^7 - 6*(7*a^6*b +
 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^6 - 6*(7*a^6*b + 10*a^4*b^3 + 4*a^2*
b^5 + b^7 - 7*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x))*sinh(x)^6 + 2*(21*a^7
- a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^5 + 2*(21*a^7 - a^5*b^2 - 20*a^3*
b^4 + 2*a*b^6 + 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^2 - 18*(7*a^6*b +
10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x))*sinh(x)^5 - 2*(41*a^6*b + 34*a^4*b^3
 - 10*a^2*b^5 - 3*b^7)*cosh(x)^4 - 2*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 -
3*b^7 - 105*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^3 + 45*(7*a^6*b + 10*a^4*
b^3 + 4*a^2*b^5 + b^7)*cosh(x)^2 - 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b
^6)*cosh(x))*sinh(x)^4 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cos
h(x)^3 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6 + 105*(a^7 - 3*a^5*b
^2 - 4*a^3*b^4)*cosh(x)^4 - 60*(7*a^6*b + 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cos
h(x)^3 + 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^2 - 4*(41*a^6
*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 - 2*(35*a^6*b + 26
*a^4*b^3 - 8*a^2*b^5 + b^7)*cosh(x)^2 - 2*(35*a^6*b + 26*a^4*b^3 - 8*a^2*b^
5 + b^7 - 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^5 + 45*(7*a^6*b + 10*a^4
*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^4 - 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a
*b^6)*cosh(x)^3 + 6*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^2
- 3*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x))*sinh(x)^2 - 3*((a
^5*b - 4*a^3*b^3)*cosh(x)^8 + (a^5*b - 4*a^3*b^3)*sinh(x)^8 + 2*(a^6 - 4*a^
4*b^2)*cosh(x)^7 + 2*(a^6 - 4*a^4*b^2 + 4*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh
(x)^7 + 2*(a^5*b - 4*a^3*b^3)*cosh(x)^6 + 2*(a^5*b - 4*a^3*b^3 + 14*(a^5*b
- 4*a^3*b^3)*cosh(x)^2 + 7*(a^6 - 4*a^4*b^2)*cosh(x))*sinh(x)^6 - a^5*b + 4
*a^3*b^3 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 2*(3*a^6 - 12*a^4*b^2 + 28*(a^5*
b - 4*a^3*b^3)*cosh(x)^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^2 + 6*(a^5*b - 4*a^
3*b^3)*cosh(x))*sinh(x)^5 + 10*(7*(a^5*b - 4*a^3*b^3)*cosh(x)^4 + 7*(a^6 -
4*a^4*b^2)*cosh(x)^3 + 3*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 3*(a^6 - 4*a^4*b^2
)*cosh(x))*sinh(x)^4 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 - 12*a^4*b^
2 + 28*(a^5*b - 4*a^3*b^3)*cosh(x)^5 + 35*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 20*
(a^5*b - 4*a^3*b^3)*cosh(x)^3 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^2)*sinh(x)^3 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 2*(14*(a^5*b - 4*a^3*b^3)*cosh(x)^6 - a^
5*b + 4*a^3*b^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 15*(a^5*b - 4*a^3*b^3)*c
osh(x)^4 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 9*(a^6 - 4*a^4*b^2)*cosh(x))*si
nh(x)^2 + 2*(a^6 - 4*a^4*b^2)*cosh(x) + 2*(4*(a^5*b - 4*a^3*b^3)*cosh(x)^7
```

```
+ 7*(a^6 - 4*a^4*b^2)*cosh(x)^6 + a^6 - 4*a^4*b^2 + 6*(a^5*b - 4*a^3*b^3)*c
osh(x)^5 + 15*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 9*(a^6 - 4*a^4*b^2)*cosh(x)^2 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(11*a^7 + 5*a^5*b^
2 - 8*a^3*b^4 - 2*a*b^6)*cosh(x) + 2*(11*a^7 + 5*a^5*b^2 - 8*a^3*b^4 - 2*a*
b^6 + 21*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^6 - 18*(7*a^6*b + 10*a^4*b^3
 + 4*a^2*b^5 + b^7)*cosh(x)^5 + 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)
*cosh(x)^4 - 4*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^3 + 3*(
21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x)^2 - 2*(35*a^6*b + 26*a^
4*b^3 - 8*a^2*b^5 + b^7)*cosh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
)^8 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9
+ 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2
 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
*b^7 + b^9)*cosh(x))*sinh(x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b
^7 + b^9)*cosh(x)^6 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 +
14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4
*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a
^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 +
 18*a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*
a^2*b^7 + b^9)*cosh(x)^3 + 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*cosh(x)^2 + 6*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
))*sinh(x)^5 - 10*(7*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh
(x)^4 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(
a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7
*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b
^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*
a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*cosh(x)^5 + 35*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)
*cosh(x)^4 + 20*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3
 + 30*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2)*sinh(x)^
3 + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 2*(a^8*
b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 - 14*(a^8*b + 4*a^6*b^3 + 6*a^4
*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^6 - 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*cosh(x)^5 - 15*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b
^9)*cosh(x)^4 - 30*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x
)^3 - 9*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^
2 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x) - 2*(a^9 +
4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^
5 + 4*a^2*b^7 + b^9)*cosh(x)^7 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
 + a*b^8)*cosh(x)^6 + 6*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*c
osh(x)^5 + 15*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 +
 9*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 - 2*(a^8*b +
```

```
4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

giac [A] time = 0.54, size = 292, normalized size = 1.30

$$\frac{\left(a^5-4\,a^3b^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(a^5e^x-a^4b\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(be^{(2\,x)}+2\,ae^x-b\right)}-\frac{2\left(12\,a^3be^{(5\,x)}-6\,a^4e^{(4\,x)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^5 - 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x +
2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b
^2)) + 2*(a^5*e^x - a^4*b)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x)
+ 2*a*e^x - b)) - 2/3*(12*a^3*b*e^(5*x) - 6*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x)
 + 3*b^4*e^(4*x) + 16*a^3*b*e^(3*x) - 8*a*b^3*e^(3*x) - 6*a^4*e^(2*x) + 18*
a^2*b^2*e^(2*x) + 12*a^3*b*e^x - 4*a^4 + 9*a^2*b^2 + b^4)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

maple [A] time = 0.10, size = 262, normalized size = 1.17

$$-\frac{2a^3\left(\frac{-b^2\tanh\left(\frac{x}{2}\right)-ab}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a}-\frac{\left(a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{2\left(-a^4+3a^2b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+2\left(-2a^3b+2a\,b^3\right)\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -2*a^3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2*tanh(1/2*x)-a*b)/(a*tanh(1/2*x)
^2-2*tanh(1/2*x)*b-a)-(a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2
*x)-2*b)/(a^2+b^2)^(1/2)))+2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4+3*a^2*b^2
)*tanh(1/2*x)^5+(-2*a^3*b+2*a*b^3)*tanh(1/2*x)^4+(-10/3*a^4+6*a^2*b^2+4/3*b
^4)*tanh(1/2*x)^3-8*a^3*b*tanh(1/2*x)^2+(-a^4+3*a^2*b^2)*tanh(1/2*x)-10/3*a
^3*b+2/3*a*b^3)/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.45, size = 523, normalized size = 2.33

$$\frac{\left(a^2-4\,b^2\right)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(7\,a^4b-9\,a^2b^3-b^5+\left(11\,a^5-6\,a^3b^2-2\,a\right.\right.}{3\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)e^{(-x)}+2\left(a^6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (a^2 - 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt
(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2/3*(
7*a^4*b - 9*a^2*b^3 - b^5 + (11*a^5 - 6*a^3*b^2 - 2*a*b^4)*e^(-x) + (35*a^4
*b - 9*a^2*b^3 + b^5)*e^(-2*x) + (21*a^5 - 32*a^3*b^2 - 8*a*b^4)*e^(-3*x) +
 (41*a^4*b - 7*a^2*b^3 - 3*b^5)*e^(-4*x) + (21*a^5 - 22*a^3*b^2 + 2*a*b^4)*
e^(-5*x) + 3*(7*a^4*b + 3*a^2*b^3 + b^5)*e^(-6*x) + 3*(a^5 - 4*a^3*b^2)*e^(
-7*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-2*x) + 6*(a
^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-6*x) + 2
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-7*x) - (a^6*b + 3*a^4*b^3 + 3*a^
2*b^5 + b^7)*e^(-8*x))
```

mupad [B] time = 1.30, size = 543, normalized size = 2.42

$$\frac{\frac{8\left(a^2-b^2\right)}{3\left(a^4+2a^2b^2+b^4\right)}-\frac{16\,a\,b\,e^x}{3\left(a^4+2a^2b^2+b^4\right)}}{3\,e^{2x}+3\,e^{4x}+e^{6x}+1}-\frac{\frac{4\left(a^6+a^4b^2-a^2b^4-b^6\right)}{\left(a^4+2a^2b^2+b^4\right)^2}-\frac{16\,e^x\left(a^5b+2a^3b^3+ab^5\right)}{3\left(a^4+2a^2b^2+b^4\right)^2}}{2\,e^{2x}+e^{4x}+1}-\frac{\frac{2\left(a^6b^5+a^4b^7\right)}{b^3\left(a^2b+b^3\right)\left(a^2+b^2\right)^3}-\frac{2\,e^x\left(a^7b^5+a^5b^7\right)}{b^4\left(a^2b+b^3\right)\left(a^2+b^2\right)^3}}{2\,a\,e^x-b+b\,e^{2x}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] ((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b^
4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^6
- a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^5*b
 + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) -
 ((2*(a^4*b^7 + a^6*b^5))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x)*(a^
5*b^7 + a^7*b^5))/(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b + b*ex
p(2*x)) - ((2*(b^6 - 2*a^6 + 4*a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^
2 + (8*exp(x)*(a^5*b + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*x) + 1)
- (log(- (2*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3) - (2*(a^5 - 4*a^3*b
^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(7/2)))*(a^5 - 4*a^3*b^2))/(a^2 + b^2)^(
7/2) + (log((2*(a^5 - 4*a^3*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(7/2)) - (2
*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3))*(a^5 - 4*a^3*b^2))/(a^2 + b^2
)^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sinh(x))**2, x)
```

3.237 $\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=135

$$-\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{ab\left(3a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{a^2\left(a^2-3b^2\right)\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{\operatorname{sech}^2(x)\left(a^2-2ab\right.}{2\left(a^2+b\right.}$$

```
[Out] a*b*(3*a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^3+a^2*(a^2-3*b^2)*ln(cosh(x))/(a^
2+b^2)^3-a^2*(a^2-3*b^2)*ln(a+b*sinh(x))/(a^2+b^2)^3+a^3/(a^2+b^2)^2/(a+b*s
inh(x))+1/2*sech(x)^2*(a^2-b^2-2*a*b*sinh(x))/(a^2+b^2)^2
```

Rubi [A] time = 0.36, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2721, 1647, 1629, 635, 203, 260}

$$\frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{ab\left(3a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{a^2\left(a^2-3b^2\right)\log(\cos}{\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*b*(3*a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^3 + (a^2*(a^2 - 3*b^2)*Log[
Cosh[x]])/(a^2 + b^2)^3 - (a^2*(a^2 - 3*b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2
)^3 + a^3/((a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(a^2 - b^2 - 2*a*b*S
inh[x]))/(2*(a^2 + b^2)^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{\left(a^2+b^2\right)^2}+\frac{2a^2b^2x}{a^2+b^2}-\frac{2ab^4x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2b^2}$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int\left(\frac{2a^3b^2}{\left(a^2+b^2\right)^2(a+x)^2}+\frac{2a^2b^2\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3(a+x)}+\frac{2ab^2\left(-b^2\left(3a^2-\right.\right.}{\left(a^2+b^2\right)}\right.\right.}{2b^2}$$

$$= -\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-2a\right.}{2\left(a^2+b^2\right)^2}$$

$$= -\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-2a\right.}{2\left(a^2+b^2\right)^2}$$

$$= \frac{ab\left(3a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^2\left(a^2-3b^2\right)\log(\cosh(x))}{\left(a^2+b^2\right)^3} - \frac{a^2\left(a^2-3b^2\right)\log(a+b\sin\right.}{\left(a^2+b^2\right)^3}$$

Mathematica [C] time = 0.70, size = 150, normalized size = 1.11

$$\frac{\left(a^4-b^4\right)\text{sech}^2(x)-2a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))-2ab\left(a^2+b^2\right)\tan^{-1}(\sinh(x))-2ab\left(a^2+b^2\right)\tanh(x)\text{sech}}{2\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^2*(a - I*b)*(a - (3*I)*b)*Log[I - S
inh[x]] + a^2*(a + I*b)*(a + (3*I)*b)*Log[I + Sinh[x]] - 2*a^2*(a^2 - 3*b^2
)*Log[a + b*Sinh[x]] + (a^4 - b^4)*Sech[x]^2 + (2*a^3*(a^2 + b^2))/(a + b*S
inh[x]) - 2*a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^2)^3)
```

fricas [B] time = 1.66, size = 2850, normalized size = 21.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")

[Out] -(2*(a^5 - a*b^4)*cosh(x)^5 + 2*(a^5 - a*b^4)*sinh(x)^5 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5 - 5*(a^5 - a*b^4)*cosh(x))*sinh(x)^4 + 8*(a^5 + a^3*b^2)*cosh(x)^3 + 4*(2*a^5 + 2*a^3*b^2 + 5*(a^5 - a*b^4)*cosh(x)^2 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5 + 10*(a^5 - a*b^4)*cosh(x)^3 - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 12*(a^5 + a^3*b^2)*cosh(x))*sinh(x)^2 + 2*((3*a^3*b^2 - a*b^4)*cosh(x)^6 + (3*a^3*b^2 - a*b^4)*sinh(x)^6 + 2*(3*a^4*b - a^2*b^3)*cosh(x)^5 + 2*(3*a^4*b - a^2*b^3 + 3*(3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^5 - 3*a^3*b^2 + a*b^4 + (3*a^3*b^2 - a*b^4)*cosh(x)^4 + (3*a^3*b^2 - a*b^4 + 15*(3*a^3*b^2 - a*b^4)*cosh(x)^2 + 10*(3*a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^4*b - a^2*b^3)*cosh(x)^3 + 4*(3*a^4*b - a^2*b^3 + 5*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 5*(3*a^4*b - a^2*b^3)*cosh(x)^2 + (3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - (3*a^3*b^2 - a*b^4)*cosh(x)^2 - (3*a^3*b^2 - a*b^4 - 15*(3*a^3*b^2 - a*b^4)*cosh(x)^4 - 20*(3*a^4*b - a^2*b^3)*cosh(x)^3 - 6*(3*a^3*b^2 - a*b^4)*cosh(x)^2 - 12*(3*a^4*b - a^2*b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^4*b - a^2*b^3)*cosh(x) + 2*(3*(3*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*a^4*b - a^2*b^3 + 5*(3*a^4*b - a^2*b^3)*cosh(x)^4 + 2*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 6*(3*a^4*b - a^2*b^3)*cosh(x)^2 - (3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^5 - a*b^4)*cosh(x) - ((a^4*b - 3*a^2*b^3)*cosh(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cosh(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 - 12*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 + a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 - (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^4*b - 3*a^2*b^3)*cosh(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cosh(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 - 12*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*

```
(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 + a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*co
sh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 -
 (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 2*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*cosh(x)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*c
osh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(
x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^6
- 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a^5*b^2 +
3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x
)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + 10
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6
 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*a^5*b^2 +
 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^6*b +
3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^6*b + 3*a
^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) -
2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 2*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh
(x))
```

giac [B] time = 0.45, size = 307, normalized size = 2.27

$$\frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)\left(3\,a^3b - ab^3\right)}{2\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} + \frac{\left(a^4 - 3\,a^2b^2\right)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} - \frac{\left(a^4b - 3\,a^2b^3\right)\log\left(\left|-b\left(e^{(-}\right.\right.\right.}{a^6b + 3\,a^4b^3 + 3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^3*b - a*b^3)/(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b^2)*log((e^(-x) - e^x)^2 + 4)/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2*b^3)*log(abs(-b*(e^(-x
) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 2*(a^3*(e^(-x) - e
^x)^2 - a*b^2*(e^(-x) - e^x)^2 + a^2*b*(e^(-x) - e^x) + b^3*(e^(-x) - e^x)
+ 6*a^3 - 2*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-
x) - e^x)^2 + 4*b*(e^(-x) - e^x) - 8*a))
```

maple [B] time = 0.10, size = 491, normalized size = 3.64

$$\frac{2a^4\tanh\left(\frac{x}{2}\right)b}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2a^2\tanh\left(\frac{x}{2}\right)b^3}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] 2*a^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2
*x)*b-a)*b+2*a^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*tanh(1/2*x)/(a*tanh(1/2*x)^2
-2*tanh(1/2*x)*b-a)*b^3-a^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(a*tanh(1/2*x)^
2-2*tanh(1/2*x)*b-a)+3*a^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*ln(a*tanh(1/2*x)^2
-2*tanh(1/2*x)*b-a)*b^2+2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a^3
*b+2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3*a*b^3-2/(a^2+b^2)^3/(tan
h(1/2*x)^2+1)^2*tanh(1/2*x)^2*a^4+2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*tanh(1/
2*x)^2*b^4-2/(a^2+b^2)^3/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a^3*b-2/(a^2+b^2)^
3/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)*a*b^3+1/(a^2+b^2)^3*ln(tanh(1/2*x)^2+1)*a
^4-3/(a^2+b^2)^3*ln(tanh(1/2*x)^2+1)*a^2*b^2+6/(a^2+b^2)^3*arctan(tanh(1/2*
x))*a^3*b-2/(a^2+b^2)^3*a*arctan(tanh(1/2*x))*b^3
```

maxima [B] time = 0.47, size = 375, normalized size = 2.78

$$-\frac{2\left(3\,a^3b-ab^3\right)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(a^4-3\,a^2b^2\right)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4-3\,a^2b^2\right)\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{}{a^4b+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(3*a^3*b - a*b^3)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
a^4 - 3*a^2*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6) + (a^4 - 3*a^2*b^2)*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6) + 2*(4*a^3*e^(-3*x) + (a^3 - a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2
*x) + (a^2*b + b^3)*e^(-4*x) + (a^3 - a*b^2)*e^(-5*x))/(a^4*b + 2*a^2*b^3 +
 b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^(-2
*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-
4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(
-6*x))
```

mupad [B] time = 3.89, size = 501, normalized size = 3.71

$$\frac{\frac{2\left(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8\right)}{\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)^2}-\frac{2\,e^x\left(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7\right)}{\left(a^2+b^2\right)\left(a^4+2\,a^2\,b^2+b^4\right)^2}}{e^{2\,x}+1}-\frac{\frac{2\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a\,b\,e^x}{a^4+2\,a^2\,b^2+b^4}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{a\,\ln\left(e^x+1\mathrm{i}\right)}{-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}}-\frac{\ln\left(15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^
2)^2) - (2*exp(x)*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(a^
4 + b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((2*(a^2 - b^2))/(a^4 + b^4 + 2*a
^2*b^2) - (4*a*b*exp(x))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x) +
1) - (a*log(exp(x) + 1i))/(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i) - (log(15*a^6
*b^3 - a^2*b^7 - 30*a^4*b^5 - 4*a^8*b + 8*a^9*exp(x) + a^2*b^7*exp(2*x) + 3
0*a^4*b^5*exp(2*x) - 15*a^6*b^3*exp(2*x) + 4*a^8*b*exp(2*x) + 2*a^3*b^6*exp
(x) + 60*a^5*b^4*exp(x) - 30*a^7*b^2*exp(x))*(a^4 - 3*a^2*b^2))/(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2) - (a*log(exp(x)*1i + 1)*1i)/(a*b^2*3i + 3*a^2*b -
a^3*1i - b^3) + (2*exp(x)*(a^3*b^6 + 2*a^5*b^4 + a^7*b^2))/(b*(a^2*b + b^3)
*(a^2 + b^2)*(2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sinh(x))**2, x)
```

3.238 $\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=144

$$\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\left(a^2-b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] -2*a^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+4*a*b^2*a
rctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-2*a*b*sech(x)/(a^
2+b^2)^2-a^2*b*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))-(a^2-b^2)*tanh(x)/(a^2+b^2
)^2
```

Rubi [A] time = 0.25, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2731, 2664, 12, 2660, 618, 206, 2669, 3767, 8}

$$-\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{\left(a^2-b^2\right)\tanh(x)}{\left(a^2+b^2\right)^2}-\frac{2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (4*
a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (2*a*
b*Sech[x])/(a^2 + b^2)^2 - (a^2*b*Cosh[x])/((a^2 + b^2)^2*(a + b*Sinh[x]))
- ((a^2 - b^2)*Tanh[x])/(a^2 + b^2)^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = -\int\left(-\frac{a^2}{\left(a^2+b^2\right)(a+b\sinh(x))^2}+\frac{2ab^2}{\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{\operatorname{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)\right.}{\left(a^2+b^2\right.}\right.$$

$$= -\frac{\int \operatorname{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)dx}{\left(a^2+b^2\right)^2}-\frac{\left(2ab^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{a^2\int \frac{1}{(a+b\sinh(x))}}{a^2+b^2}$$

$$= -\frac{2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{a^2\int \frac{a}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}-\frac{\left(4ab^2\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx}\right.}{\left(a^2+\right.}$$

$$= -\frac{2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}+\frac{a^3\int \frac{1}{a+b\sinh(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{\left(8ab^2\right)\operatorname{Subst}\left(\int \frac{}{4\left(a^2+\right.}\right.}{(}$$

$$= \frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\left(a^2-b^2\right)\tanh(}{\left(a^2+b^2\right)^2}$$

$$= \frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh(x)}{\left(a^2+b^2\right)^2(a+b\sinh(x))}-\frac{\left(a^2-b^2\right)\tanh(}{\left(a^2+b^2\right)^2}$$

$$= -\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}-\frac{a^2b\cosh}{\left(a^2+b^2\right)^2(a+}$$

Mathematica [A] time = 0.27, size = 100, normalized size = 0.69

$$\frac{\left(b^2-a^2\right)\tanh(x)+\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{a^2b\cosh(x)}{a+b\sinh(x)}-2ab\operatorname{sech}(x)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*a*(a^2 - 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
 b^2] - 2*a*b*Sech[x] - (a^2*b*Cosh[x])/(a + b*Sinh[x]) + (-a^2 + b^2)*Tanh
[x])/(a^2 + b^2)^2
```

fricas [B] time = 0.91, size = 900, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^4*b + 2*a^2*b^3 - 2*b^5 - 2*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 2*(a
^5 - a^3*b^2 - 2*a*b^4)*sinh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2
 + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - 3*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x))*sinh
(x)^2 + ((a^3*b - 2*a*b^3)*cosh(x)^4 + (a^3*b - 2*a*b^3)*sinh(x)^4 - a^3*b
+ 2*a*b^3 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 2*(a^4 - 2*a^2*b^2 + 2*(a^3*b -
 2*a*b^3)*cosh(x))*sinh(x)^3 + 6*((a^3*b - 2*a*b^3)*cosh(x)^2 + (a^4 - 2*a^
2*b^2)*cosh(x))*sinh(x)^2 + 2*(a^4 - 2*a^2*b^2)*cosh(x) + 2*(a^4 - 2*a^2*b^
2 + 2*(a^3*b - 2*a*b^3)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x))
*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*s
inh(x) - b)) - 6*(a^5 + a^3*b^2)*cosh(x) - 2*(3*a^5 + 3*a^3*b^2 + 3*(a^5 -
a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(
x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*(
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
2)*sinh(x))
```

giac [A] time = 0.61, size = 181, normalized size = 1.26

$$\frac{\left(a^{3}-2\,ab^{2}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}+\frac{2\left(a^{3}e^{(3\,x)}-2\,ab^{2}e^{(3\,x)}-4\,a^{2}be^{(2\,x)}-b^{3}e^{(2\,x)}+3\,a^{3}e^{x}-2\,a^{2}b+b^{3}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^{x}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^3 - 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*
a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*
e^(3*x) - 2*a*b^2*e^(3*x) - 4*a^2*b*e^(2*x) - b^3*e^(2*x) + 3*a^3*e^x - 2*a
^2*b + b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b
))
```

maple [A] time = 0.08, size = 142, normalized size = 0.99

$$-\frac{2a\left(\frac{-b^2\tanh\left(\frac{x}{2}\right)-ab}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}+\frac{2\left(-a^2+b^2\right)\tanh\left(\frac{x}{2}\right)-4ab}{\left(a^4+2a^2b^2+b^4\right)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] -2*a/(a^2+b^2)^2*((-b^2*tanh(1/2*x)-a*b)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a
)-(a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(
1/2)))+2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(tanh(1/2*x)^2+
1)
```

maxima [A] time = 0.52, size = 223, normalized size = 1.55

$$\frac{\left(a^2-2\,b^2\right)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^3e^{(-x)}+2\,a^2b-b^3+\left(4\,a^2b+b^3\right)e^{(-2\,x)}+\left(a^3-2\,ab^2\right)\right.}{a^4b+2\,a^2b^3+b^5+2\left(a^5+2\,a^3b^2+ab^4\right)e^{(-x)}+2\left(a^5+2\,a^3b^2+ab^4\right)e^{(-3\,x)}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (a^2 - 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*e^(-x) + 2
*a^2*b - b^3 + (4*a^2*b + b^3)*e^(-2*x) + (a^3 - 2*a*b^2)*e^(-3*x))/(a^4*b
+ 2*a^2*b^3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3*b^2
 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

mupad [B] time = 1.15, size = 377, normalized size = 2.62

$$\frac{\frac{2\left(a^2\,b^9-2\,a^4\,b^7\right)}{b^3\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}-\frac{2\,e^{2\,x}\left(4\,a^4\,b^7+a^2\,b^9\right)}{b^3\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}+\frac{6\,a^5\,b^3\,e^x}{\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}-\frac{2\,a\,e^{3\,x}\left(2\,a^2\,b^9-a^4\,b^7\right)}{b^4\left(a^3+a\,b^2\right)\left(a^3\,b^3+a\,b^5\right)}}{2\,a\,e^x-b+2\,a\,e^{3\,x}+b\,e^{4\,x}}-\frac{a\,\ln\left(\frac{2\,e^x\left(2\,a\,b^2-a^3\right)}{b\left(a^2+b^2\right)^2}-\frac{2\,a\left(a^2-\right.}{b\left(a\right.}\right.}{\left(a^2+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*(a^2*b^9 - 2*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*exp(2
*x)*(a^2*b^9 + 4*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^5*b
```

```
^3*exp(x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*a*exp(3*x)*(2*a^2*b^9 - a
^4*b^7))/(b^4*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*exp(3
*x) + b*exp(4*x)) - (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2) - (
2*a*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2))/(a^
2 + b^2)^(5/2) + (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2) + (2*a
*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2))/(a^2 +
 b^2)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sinh(x))**2, x)
```

3.239 $\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=85

$$\frac{a}{(a^2+b^2)(a+b\sinh(x))}-\frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{2ab\tan^{-1}(\sinh(x))}{(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2}$$

```
[Out] 2*a*b*arctan(sinh(x))/(a^2+b^2)^2+(a^2-b^2)*ln(cosh(x))/(a^2+b^2)^2-(a^2-b^
2)*ln(a+b*sinh(x))/(a^2+b^2)^2+a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2721, 801, 635, 203, 260}

$$\frac{a}{(a^2+b^2)(a+b\sinh(x))}-\frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{2ab\tan^{-1}(\sinh(x))}{(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*a*b*ArcTan[Sinh[x]])/(a^2 + b^2)^2 + ((a^2 - b^2)*Log[Cosh[x]])/(a^2 + b
^2)^2 - ((a^2 - b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + a/((a^2 + b^2)*(a
+ b*Sinh[x]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = -\text{Subst}\left(\int \frac{x}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)$$
$$= -\text{Subst}\left(\int\left(\frac{a}{\left(a^2+b^2\right)(a+x)^2}+\frac{a^2-b^2}{\left(a^2+b^2\right)^2(a+x)}+\frac{-2ab^2-\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)dx, x, b\sin$$
$$= -\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\text{Subst}\left(\int\frac{-2ab^2-\left(a^2-b^2\right)x}{b^2+x^2}\,dx, x\right.}{\left(a^2+b^2\right)^2}$$
$$= -\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(2ab^2\right)\text{Subst}\left(\int\frac{1}{b^2+x^2}\,dx, x,\right.}{\left(a^2+b^2\right)^2}$$
$$= \frac{2ab\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)\log(\cosh(x))}{\left(a^2+b^2\right)^2}-\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{}{\left(a^2+b^2\right.}$$

Mathematica [C] time = 0.24, size = 146, normalized size = 1.72

$$\frac{a\left(2\left(\left(b^2-a^2\right)\log(a+b\sinh(x))+a^2+b^2\right)+(a-ib)^2\log(-\sinh(x)+i)+(a+ib)^2\log(\sinh(x)+i)\right)+b\sinh(}{2\left(a^2+b^2\right)^2(a+b\sinh(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*((a - I*b)^2*Log[I - Sinh[x]] + (a + I*b)^2*Log[I + Sinh[x]] + 2*(a^2 +
b^2 + (-a^2 + b^2)*Log[a + b*Sinh[x]])) + b*((a - I*b)^2*Log[I - Sinh[x]] +
```

```
(a + I*b)^2*Log[I + Sinh[x]] + 2*(-a^2 + b^2)*Log[a + b*Sinh[x]])*Sinh[x])
/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

fricas [B] time = 1.53, size = 423, normalized size = 4.98

$$-\frac{4\left(ab^2\cosh(x)^2+ab^2\sinh(x)^2+2\,a^2b\cosh(x)-ab^2+2\left(ab^2\cosh(x)+a^2b\right)\sinh(x)\right)\arctan\left(\cosh(x)+\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(4*(a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2
*cosh(x) + a^2*b)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^3 + a*b^2)*cosh
(x) + (a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*
(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*lo
g(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^2*b - b^3 - (a^2*b - b^3)*cos
h(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2
 + (a^2*b - b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(
a^3 + a*b^2)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*
cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^
4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)
)*sinh(x))
```

giac [B] time = 0.17, size = 199, normalized size = 2.34

$$\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)ab}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{\left(a^2b-b^3\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a*b/(a^4 + 2*a^2*b^2 + b^4) + 1/2
*(a^2 - b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - (a^2*b - b
^3)*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + (a^2*b*(e
^(-x) - e^x) - b^3*(e^(-x) - e^x) - 4*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(
-x) - e^x) - 2*a))
```

maple [B] time = 0.07, size = 248, normalized size = 2.92

$$\frac{2\tanh\left(\frac{x}{2}\right)a^2b}{\left(a^2+b^2\right)^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2\tanh\left(\frac{x}{2}\right)b^3}{\left(a^2+b^2\right)^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}-\frac{\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)\right.}{\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sinh(x))^2,x)
```

```
[Out] 2/(a^2+b^2)^2*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2*b+2/(a^2+
b^2)^2*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^3-1/(a^2+b^2)^2*ln
(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*a^2+1/(a^2+b^2)^2*ln(a*tanh(1/2*x)^2-2*
tanh(1/2*x)*b-a)*b^2+2/(2*a^4+4*a^2*b^2+2*b^4)*ln(tanh(1/2*x)^2+1)*a^2-2/(2
*a^4+4*a^2*b^2+2*b^4)*ln(tanh(1/2*x)^2+1)*b^2+8/(2*a^4+4*a^2*b^2+2*b^4)*a*b
*arctan(tanh(1/2*x))
```

maxima [A] time = 0.49, size = 155, normalized size = 1.82

$$-\frac{4\,ab\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,ae^{(-x)}}{a^2b+b^3+2\left(a^3+ab^2\right)e^{(-x)}-\left(a^2b+b^3\right)e^{(-2\,x)}}-\frac{\left(a^2-b^2\right)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -4*a*b*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + 2*a*e^(-x)/(a^2*b + b^3 + 2
*(a^3 + a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2*x)) - (a^2 - b^2)*log(-2*a*e^(-
x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(e^(-2*x) + 1
)/(a^4 + 2*a^2*b^2 + b^4)
```

mupad [B] time = 1.75, size = 190, normalized size = 2.24

$$\frac{\ln\left(1+e^x\,1i\right)}{a^2+a\,b\,2i-b^2}-\frac{\ln\left(b^5\,e^{2\,x}-a^4\,b-b^5+a^2\,b^3+2\,a^5\,e^x-a^2\,b^3\,e^{2\,x}+2\,a\,b^4\,e^x+a^4\,b\,e^{2\,x}-2\,a^3\,b^2\,e^x\right)\,\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{}{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] log(exp(x)*1i + 1)/(a*b*2i + a^2 - b^2) + (log(exp(x) + 1i)*1i)/(2*a*b + a^
2*1i - b^2*1i) - (log(b^5*exp(2*x) - a^4*b - b^5 + a^2*b^3 + 2*a^5*exp(x) -
 a^2*b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) - 2*a^3*b^2*exp(x))*(a^
2 - b^2))/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b*exp(x))/((a^2*b + b^3)*(2*a*exp(
x) - b + b*exp(2*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(x\right)}{\left(a+b\sinh\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))**2,x)
[Out] Integral(tanh(x)/(a + b*sinh(x))**2, x)
```

3.240 $\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=32

$$-\frac{\log(a+b\sinh(x))}{a^2}+\frac{\log(\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}$$

```
[Out] ln(sinh(x))/a^2-ln(a+b*sinh(x))/a^2+1/a/(a+b*sinh(x))
```

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2721, 44}

$$-\frac{\log(a+b\sinh(x))}{a^2}+\frac{\log(\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[Sinh[x]]/a^2 - Log[a + b*Sinh[x]]/a^2 + 1/(a*(a + b*Sinh[x]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right)dx, x, b\sinh(x)\right)$$
$$= \frac{\log(\sinh(x))}{a^2} - \frac{\log(a+b\sinh(x))}{a^2} + \frac{1}{a(a+b\sinh(x))}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.84

$$\frac{\frac{a}{a+b\sinh(x)} - \log(a+b\sinh(x)) + \log(\sinh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (Log[Sinh[x]] - Log[a + b*Sinh[x]] + a/(a + b*Sinh[x]))/a^2
```

fricas [B] time = 1.16, size = 158, normalized size = 4.94

$$\frac{2\,a\cosh(x) - \left(b\cosh(x)^2 + b\sinh(x)^2 + 2\,a\cosh(x) + 2\,(b\cosh(x)+a)\sinh(x) - b\right)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right) + \left(b\cosh \ldots\right.}{a^2b\cosh(x)^2 + a^2b\sinh(x)^2 + 2\,a^3\cosh(x) - \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*cosh(x) - (b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 2*a*sinh(x))/(a^2*b*cosh(x)^2 + a^2*b*sinh(x)^2 + 2*a^3
*cosh(x) - a^2*b + 2*(a^2*b*cosh(x) + a^3)*sinh(x))
```

giac [B] time = 0.15, size = 75, normalized size = 2.34

$$-\frac{\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^2} + \frac{\log\left(\left|-e^{(-x)} + e^x\right|\right)}{a^2} + \frac{b\left(e^{(-x)} - e^x\right) - 4\,a}{\left(b\left(e^{(-x)} - e^x\right) - 2\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a^2 + log(abs(-e^(-x) + e^x))/a^2 + (b*(
e^(-x) - e^x) - 4*a)/((b*(e^(-x) - e^x) - 2*a)*a^2)
```

maple [A] time = 0.05, size = 33, normalized size = 1.03

$$\frac{\ln(\sinh(x))}{a^2}-\frac{\ln(a+b\sinh(x))}{a^2}+\frac{1}{a\,(a+b\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x))^2,x)
```

```
[Out] ln(sinh(x))/a^2-ln(a+b*sinh(x))/a^2+1/a/(a+b*sinh(x))
```

maxima [B] time = 0.33, size = 75, normalized size = 2.34

$$\frac{2\,e^{(-x)}}{2\,a^2e^{(-x)}-abe^{(-2\,x)}+ab}-\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2}+\frac{\log\left(e^{(-x)}+1\right)}{a^2}+\frac{\log\left(e^{(-x)}-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(2*a^2*e^(-x) - a*b*e^(-2*x) + a*b) - log(-2*a*e^(-x) + b*e^(-2*x)
 - b)/a^2 + log(e^(-x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

mupad [B] time = 0.97, size = 240, normalized size = 7.50

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^4}+b\,\mathrm{e}^x\,\sqrt{-a^4}-2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}-b\,\mathrm{e}^{3\,x}\,\sqrt{-a^4}}{a^3}\right)-2\,\mathrm{atan}\left(\left(4\,a^5\,b\,\sqrt{-a^4}+4\,a^3\,b^3\,\sqrt{-a^4}\right)\left(\frac{1}{8\,a^3\,b\,\left(a^2+b^2\right)^2}-\mathrm{e}^x\left(\frac{1}{16\,a^2\,b^2\,(\cdots}\right.\right.\right.}{\sqrt{-a^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x))^2,x)
```

```
[Out] (2*atan((a*(-a^4)^(1/2) + b*exp(x)*(-a^4)^(1/2) - 2*a*exp(2*x)*(-a^4)^(1/2)
 - b*exp(3*x)*(-a^4)^(1/2))/a^3) - 2*atan((4*a^5*b*(-a^4)^(1/2) + 4*a^3*b^3
*(-a^4)^(1/2))*(1/(8*a^3*b*(a^2 + b^2)^2) - exp(x)*(1/(16*a^2*b^2*(a^2 + b^
2)^2) - (a^2 + 2*b^2)^2/(16*a^6*b^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^5*
b*(a^2 + b^2)^2))))/(-a^4)^(1/2) + (2*b^3*exp(x)*(a^2 + b^2))/(a*(b^5 + a^2
*b^3)*(2*a*exp(x) - b + b*exp(2*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)/(a + b*sinh(x))**2, x)
```

3.241 $\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=80

$$\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\coth(x)}{a^2}-\frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{\coth(x)}{a(a+b\sinh(x))}$$

```
[Out] 2*b*arctanh(cosh(x))/a^3-2*coth(x)/a^2+coth(x)/a/(a+b*sinh(x))-2*(a^2+2*b^2
)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 206}

$$-\frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) - (2*Coth[x])/a^2 + Coth[x]/(a*(a + b*
Sinh[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^2(x)\left(1+\sinh^2(x)\right)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}^2(x)\left(2\left(a^2+b^2\right)+\left(a^2+b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{i\int \frac{\operatorname{csch}(x)\left(2ib\left(a^2+b^2\right)-ia\left(a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} - \frac{(2b)\int \operatorname{csch}(x)\,dx}{a^3} + \frac{\left(a^2+2b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{\left(2\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\right.}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} - \frac{\left(4\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x}\right.}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))}$$

Mathematica [A] time = 0.48, size = 102, normalized size = 1.28

$$-\frac{-\frac{4\left(a^2+2b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{2ab\cosh(x)}{a+b\sinh(x)} + a\tanh\left(\frac{x}{2}\right) + a\coth\left(\frac{x}{2}\right) + 4b\log\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*((-4*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] + a*Coth[x/2] + 4*b*Log[Tanh[x/2]] + (2*a*b*Cosh[x])/(a + b*Sinh[
x]) + a*Tanh[x/2])/a^3
```

fricas [B] time = 1.49, size = 1257, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*sinh(x
)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^2*b^2
)*cosh(x))*sinh(x)^2 + ((a^2*b + 2*b^3)*cosh(x)^4 + (a^2*b + 2*b^3)*sinh(x)
^4 + 2*(a^3 + 2*a*b^2)*cosh(x)^3 + 2*(a^3 + 2*a*b^2 + 2*(a^2*b + 2*b^3)*cos
h(x))*sinh(x)^3 + a^2*b + 2*b^3 - 2*(a^2*b + 2*b^3)*cosh(x)^2 - 2*(a^2*b +
2*b^3 - 3*(a^2*b + 2*b^3)*cosh(x)^2 - 3*(a^3 + 2*a*b^2)*cosh(x))*sinh(x)^2
- 2*(a^3 + 2*a*b^2)*cosh(x) + 2*(2*(a^2*b + 2*b^3)*cosh(x)^3 - a^3 - 2*a*b^
2 + 3*(a^3 + 2*a*b^2)*cosh(x)^2 - 2*(a^2*b + 2*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 6*(a^4 + a^2*b^2)*cosh(x) + 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^
2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^3*b
 + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh(x)
^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*cos
h(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*b^2
 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh(x)
)*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2
*b^2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^
3*b + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh
(x)^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*
cosh(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*
b^2 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(3*a^4 + 3*a^2*b^2 - 3*(a^4 +
a^2*b^2)*cosh(x)^2 + 4*(a^3*b + a*b^3)*cosh(x))*sinh(x))/(a^5*b + a^3*b^3 +
 (a^5*b + a^3*b^3)*cosh(x)^4 + (a^5*b + a^3*b^3)*sinh(x)^4 + 2*(a^6 + a^4*b
^2)*cosh(x)^3 + 2*(a^6 + a^4*b^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x)^3 -
 2*(a^5*b + a^3*b^3)*cosh(x)^2 - 2*(a^5*b + a^3*b^3 - 3*(a^5*b + a^3*b^3)*c
osh(x)^2 - 3*(a^6 + a^4*b^2)*cosh(x))*sinh(x)^2 - 2*(a^6 + a^4*b^2)*cosh(x)
 - 2*(a^6 + a^4*b^2 - 2*(a^5*b + a^3*b^3)*cosh(x)^3 - 3*(a^6 + a^4*b^2)*cos
h(x)^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x))
```

giac [A] time = 0.25, size = 148, normalized size = 1.85

$$\frac{2\,b\log\left(e^{x}+1\right)}{a^{3}}-\frac{2\,b\log\left(|e^{x}-1|\right)}{a^{3}}+\frac{\left(a^{2}+2\,b^{2}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,a^{3}}+\frac{2\left(ae^{(3\,x)}-2\,be^{(2\,x)}-3\,ae^{x}+2\,b\right)}{\left(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^{x}+b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3 + (a^2 + 2*b^2)*log(abs(2*
b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(s
```

```
qrt(a^2 + b^2)*a^3) + 2*(a*e^(3*x) - 2*b*e^(2*x) - 3*a*e^x + 2*b)/((b*e^(4*
x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b)*a^2)
```

maple [B] time = 0.07, size = 170, normalized size = 2.12

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2a^2}+\frac{2b^2\tanh\left(\frac{x}{2}\right)}{a^3\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2b}{a^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right.}{a\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sinh(x))^2,x)
```

```
[Out] -1/2/a^2*tanh(1/2*x)+2/a^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2*tanh(1/2
*x)+2/a^2/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b+2/a/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+4/a^3/(a^2+b^2)^(1/2)*arctanh(1
/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*b^2-1/2/a^2/tanh(1/2*x)-2/a^3*b*l
n(tanh(1/2*x))
```

maxima [B] time = 0.47, size = 165, normalized size = 2.06

$$-\frac{2\left(3\,ae^{(-x)}-2\,be^{(-2\,x)}-ae^{(-3\,x)}+2\,b\right)}{2\,a^3e^{(-x)}-2\,a^2be^{(-2\,x)}-2\,a^3e^{(-3\,x)}+a^2be^{(-4\,x)}+a^2b}+\frac{2\,b\log\left(e^{(-x)}+1\right)}{a^3}-\frac{2\,b\log\left(e^{(-x)}-1\right)}{a^3}+\frac{\left(a^2+2\,b^2\right)\log\left(\frac{be^{(-}}{be^{(-}}\right.}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(3*a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x) + 2*b)/(2*a^3*e^(-x) - 2*a^2*b*e
^(-2*x) - 2*a^3*e^(-3*x) + a^2*b*e^(-4*x) + a^2*b) + 2*b*log(e^(-x) + 1)/a^
3 - 2*b*log(e^(-x) - 1)/a^3 + (a^2 + 2*b^2)*log((b*e^(-x) - a - sqrt(a^2 +
b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

mupad [B] time = 1.78, size = 897, normalized size = 11.21

$$\frac{\frac{4\left(25\,a^8\,b^8+65\,a^6\,b^{10}+56\,a^4\,b^{12}+16\,a^2\,b^{14}\right)}{a^4\,b^4\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)}-\frac{6\,e^x\left(25\,a^9\,b^8+65\,a^7\,b^{10}+56\,a^5\,b^{12}+16\,a^3\,b^{14}\right)}{a^4\,b^5\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)}-\frac{4\,e^{2\,x}\left(25\,a^8\,b^8+65\,a^6\,b^{10}+56\,a^4\,b^{12}+16\,a^2\,b^{14}\right)}{a^4\,b^4\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)}+\frac{2\,e^3}{}}{b-2\,a\,e^x+2\,a\,e^{3\,x}-2\,b\,e^{2\,x}+b\,e^{4\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] ((4*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^10 + 25*a^8*b^8))/(a^4*b^4*(16*b^
9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (6*exp(x)*(16*a^3*b^14 + 56*a^
5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a^4*b^5*(16*b^9 + 56*a^2*b^7 + 65*a^4*
b^5 + 25*a^6*b^3)) - (4*exp(2*x)*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^10 +
 25*a^8*b^8))/(a^4*b^4*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) + (
2*exp(3*x)*(16*a^3*b^14 + 56*a^5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a^4*b^5
*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) + 2*a*ex
p(3*x) - 2*b*exp(2*x) + b*exp(4*x)) - (2*b*log(64*exp(x) - 64))/a^3 + (2*b*
log(64*exp(x) + 64))/a^3 - (log(((a^2 + 2*b^2)*((32*(a^4 + 8*b^4 + 12*a^2*b
^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))/(a^4*b^4) + ((a^2 + 2*b^2)*((32*(2
*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2*exp(x)))/b^5 - (32*(a^2 + 2*b^2)*(a
^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b
^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2))*(a^2 + b^2)^(1/2))
/(a^5 + a^3*b^2) - (64*(a^2 + 2*b^2)*(4*b - 7*a*exp(x)))/(a^6*b^3))*(a^2 +
2*b^2)*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) + (log(- ((a^2 + 2*b^2)*((32*(a^4
 + 8*b^4 + 12*a^2*b^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))/(a^4*b^4) - ((a
^2 + 2*b^2)*((32*(2*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2*exp(x)))/b^5 + (
32*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*
a^3*b^2*exp(x)))/(b^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2))
*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) - (64*(a^2 + 2*b^2)*(4*b - 7*a*exp(x)))
/(a^6*b^3))*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)**2/(a + b*sinh(x))**2, x)
```

3.242 $\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=76

$$\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{\left(a^2+3b^2\right)\log(\sinh(x))}{a^4}-\frac{\left(a^2+3b^2\right)\log(a+b\sinh(x))}{a^4}+\frac{a^2+b^2}{a^3(a+b\sinh(x))}$$

```
[Out] 2*b*csch(x)/a^3-1/2*csch(x)^2/a^2+(a^2+3*b^2)*ln(sinh(x))/a^4-(a^2+3*b^2)*l
n(a+b*sinh(x))/a^4+(a^2+b^2)/a^3/(a+b*sinh(x))
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2721, 894}

$$\frac{a^2+b^2}{a^3(a+b\sinh(x))}+\frac{\left(a^2+3b^2\right)\log(\sinh(x))}{a^4}-\frac{\left(a^2+3b^2\right)\log(a+b\sinh(x))}{a^4}+\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*Csch[x])/a^3 - Csch[x]^2/(2*a^2) + ((a^2 + 3*b^2)*Log[Sinh[x]])/a^4 -
((a^2 + 3*b^2)*Log[a + b*Sinh[x]])/a^4 + (a^2 + b^2)/(a^3*(a + b*Sinh[x]))
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = -\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)^2}\,dx, x, b\sinh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{b^2}{a^2x^3}+\frac{2b^2}{a^3x^2}+\frac{-a^2-3b^2}{a^4x}+\frac{a^2+b^2}{a^3(a+x)^2}+\frac{a^2+3b^2}{a^4(a+x)}\right)dx, x, b\sinh(x)\right)$$
$$= \frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{\left(a^2+3b^2\right)\log(\sinh(x))}{a^4}-\frac{\left(a^2+3b^2\right)\log(a+b\sinh(x))}{a^4}+\frac{}{a^3}$$

Mathematica [A] time = 0.21, size = 73, normalized size = 0.96

$$\frac{\frac{2a\left(a^2+b^2\right)}{a+b\sinh(x)}+2\left(a^2+3b^2\right)\log(\sinh(x))-2\left(a^2+3b^2\right)\log(a+b\sinh(x))-a^2\text{csch}^2(x)+4ab\text{csch}(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (4*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + 3*b^2)*Log[Sinh[x]] - 2*(a^2 + 3*
b^2)*Log[a + b*Sinh[x]] + (2*a*(a^2 + b^2))/(a + b*Sinh[x]))/(2*a^4)
```

fricas [B] time = 1.36, size = 1463, normalized size = 19.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (6*a^2*b*cosh(x)^4 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2)*sinh(x
)^5 - 6*a^2*b*cosh(x)^2 + 2*(3*a^2*b + 5*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4
 - 4*(2*a^3 + 3*a*b^2)*cosh(x)^3 + 4*(6*a^2*b*cosh(x) - 2*a^3 - 3*a*b^2 + 5
*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^3 + 2*(18*a^2*b*cosh(x)^2 + 10*(a^3 + 3
*a*b^2)*cosh(x)^3 - 3*a^2*b - 6*(2*a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a
^3 + 3*a*b^2)*cosh(x) - ((a^2*b + 3*b^3)*cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x
)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*co
sh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2
*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*
a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*cosh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3
+ 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^3 - a^2*b - 3*b^3
 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2*b + 3*b^3)*cosh(x)^4 + 20*(a^3 +
3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a
^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b
```

```
+ 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x
)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*cosh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(
x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^2*b + 3*b^3)*
cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^
3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh
(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2
)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*c
osh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3 + 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*
cosh(x))*sinh(x)^3 - a^2*b - 3*b^3 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2
*b + 3*b^3)*cosh(x)^4 + 20*(a^3 + 3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18
*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3
 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b + 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*co
sh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*c
osh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sin
h(x))) + 2*(12*a^2*b*cosh(x)^3 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*a^2*b*cosh
(x) + a^3 + 3*a*b^2 - 6*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x))/(a^4*b*cosh(x
)^6 + a^4*b*sinh(x)^6 + 2*a^5*cosh(x)^5 - 3*a^4*b*cosh(x)^4 - 4*a^5*cosh(x)
^3 + 3*a^4*b*cosh(x)^2 + 2*a^5*cosh(x) + 2*(3*a^4*b*cosh(x) + a^5)*sinh(x)^
5 - a^4*b + (15*a^4*b*cosh(x)^2 + 10*a^5*cosh(x) - 3*a^4*b)*sinh(x)^4 + 4*(
5*a^4*b*cosh(x)^3 + 5*a^5*cosh(x)^2 - 3*a^4*b*cosh(x) - a^5)*sinh(x)^3 + (1
5*a^4*b*cosh(x)^4 + 20*a^5*cosh(x)^3 - 18*a^4*b*cosh(x)^2 - 12*a^5*cosh(x)
+ 3*a^4*b)*sinh(x)^2 + 2*(3*a^4*b*cosh(x)^5 + 5*a^5*cosh(x)^4 - 6*a^4*b*cos
h(x)^3 - 6*a^5*cosh(x)^2 + 3*a^4*b*cosh(x) + a^5)*sinh(x))
```

giac [B] time = 0.34, size = 190, normalized size = 2.50

$$\frac{\left(a^2+3\,b^2\right)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^4}-\frac{\left(a^2b+3\,b^3\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b}+\frac{a^2b\left(e^{(-x)}-e^x\right)+3\,b^3\left(e^{(-x)}-e^x\right)-4\,a^3-}{\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + 3*b^2)*log(abs(-e^(-x) + e^x))/a^4 - (a^2*b + 3*b^3)*log(abs(-b*(e^(
-x) - e^x) + 2*a))/(a^4*b) + (a^2*b*(e^(-x) - e^x) + 3*b^3*(e^(-x) - e^x) -
 4*a^3 - 8*a*b^2)/((b*(e^(-x) - e^x) - 2*a)*a^4) - 1/2*(3*a^2*(e^(-x) - e^x
)^2 + 9*b^2*(e^(-x) - e^x)^2 + 8*a*b*(e^(-x) - e^x) + 4*a^2)/(a^4*(e^(-x) -
 e^x)^2)
```

maple [B] time = 0.09, size = 184, normalized size = 2.42

$$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8a^2}-\frac{\tanh\left(\frac{x}{2}\right)b}{a^3}+\frac{2\tanh\left(\frac{x}{2}\right)b}{a^2\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2\tanh\left(\frac{x}{2}\right)b^3}{a^4\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}-\frac{\ln\left(a\left(\tan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x))^2,x)
```

```
[Out] -1/8/a^2*tanh(1/2*x)^2-1/a^3*tanh(1/2*x)*b+2/a^2*tanh(1/2*x)/(a*tanh(1/2*x)
^2-2*tanh(1/2*x)*b-a)*b+2/a^4*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-
a)*b^3-1/a^2*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)-3/a^4*ln(a*tanh(1/2*x)^2
-2*tanh(1/2*x)*b-a)*b^2-1/8/a^2/tanh(1/2*x)^2+1/a^2*ln(tanh(1/2*x))+3/a^4*l
n(tanh(1/2*x))*b^2+b/a^3/tanh(1/2*x)
```

maxima [B] time = 0.36, size = 202, normalized size = 2.66

$$\frac{2\left(3\,abe^{(-2\,x)}-3\,abe^{(-4\,x)}+\left(a^{2}+3\,b^{2}\right)e^{(-x)}-2\left(2\,a^{2}+3\,b^{2}\right)e^{(-3\,x)}+\left(a^{2}+3\,b^{2}\right)e^{(-5\,x)}\right)}{2\,a^{4}e^{(-x)}-3\,a^{3}be^{(-2\,x)}-4\,a^{4}e^{(-3\,x)}+3\,a^{3}be^{(-4\,x)}+2\,a^{4}e^{(-5\,x)}-a^{3}be^{(-6\,x)}+a^{3}b}-\frac{\left(a^{2}+3\,b^{2}\right)\log\left(-2\,ae^{(-x)}\right.}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) + (a^2 + 3*b^2)*e^(-x) - 2*(2*a^2 + 3*b^
2)*e^(-3*x) + (a^2 + 3*b^2)*e^(-5*x))/(2*a^4*e^(-x) - 3*a^3*b*e^(-2*x) - 4*
a^4*e^(-3*x) + 3*a^3*b*e^(-4*x) + 2*a^4*e^(-5*x) - a^3*b*e^(-6*x) + a^3*b)
- (a^2 + 3*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^4 + (a^2 + 3*b^2)*log(e
^(-x) + 1)/a^4 + (a^2 + 3*b^2)*log(e^(-x) - 1)/a^4
```

mupad [B] time = 1.63, size = 1375, normalized size = 18.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] (2*exp(x)*(a*b^7 + 2*a^3*b^5 + a^5*b^3))/(a^4*(b^5 + a^2*b^3)*(2*a*exp(x) -
 b + b*exp(2*x))) - 2/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) - ((2*atan((4*a^9*b
*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2) + 12*a^5*b^5*((a^2 + 3*b^2)^2)^(1/2)*
(-a^8)^(1/2) + 16*a^7*b^3*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2))*(exp(x)*((a
^2 + 2*b^2)^2/(16*a^10*b^2*(a^4 + 3*b^4 + 4*a^2*b^2)^2) - 1/(16*a^6*b^2*(a^
2 + 3*b^2)^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^9*b*(a^4 + 3*b^4 + 4*a^2*
b^2)^2) + 1/(8*a^7*b*(a^2 + 3*b^2)^2*(a^2 + b^2)^2))) - 2*atan((a^2*(-a^8)^
(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6
*a^2*b^2)^(1/2))/(2*a^4*(a^4 + 3*b^4 + 4*a^2*b^2)) + ((a^8 + 3*a^6*b^2)*(-a
^8)^(1/2))/(2*a^8*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) - (a^8*b^2*exp(2*x)*
(-a^8)^(1/2)*((4*(a^2 + 2*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2))/(a^12*b^2*(a^4 +
3*b^4 + 4*a^2*b^2)) + (4*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2)
+ 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 + 9*b^4 + 6*a^2*
b^2)^(1/2))/(a^12*b^2*(-a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)) + (2*(2*a^7*b
 + 6*a^5*b^3)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2)^
(1/2)*(a^2 + b^2)) + (4*(a^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/
```

```
(a^16*b^2*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2))))/(8*(a^4 + 9*b^4 + 6*a^2*b^
2)^(1/2)) + (a^8*b^2*exp(3*x)*((2*(a^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^
2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)
*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^
4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^13*b^3*(-
a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)))*(-a^8)^(1/2))/(8*(a^4 + 9*b^4 + 6*a^
2*b^2)^(1/2)) - (a^8*b^2*exp(x)*(-a^8)^(1/2)*((8*(a^4 + 9*b^4 + 6*a^2*b^2))
/(a^11*b*(a^4 + 3*b^4 + 4*a^2*b^2)) - (4*(2*a^7*b + 6*a^5*b^3)*(a^4 + 9*b^4
 + 6*a^2*b^2)^(1/2))/(a^16*b^2*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) + (2*(a
^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2
)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*
a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 +
 9*b^4 + 6*a^2*b^2)^(1/2))/(a^13*b^3*(-a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)
)))/(8*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))))*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/
(-a^8)^(1/2) - (2/a^2 - (4*b*exp(x))/a^3)/(exp(2*x) - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x))**2, x)
```

3.243 $\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=159

$$-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{2\sqrt{a^2+b^2}\,\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5}-\frac{\left(7a^2+\right.}{}$$

```
[Out] b*(3*a^2+4*b^2)*arctanh(cosh(x))/a^5-1/3*(7*a^2+12*b^2)*coth(x)/a^4+(a^2+2*
b^2)*coth(x)*csch(x)/a^3/b-1/3*(3+4*b^2/a^2)*coth(x)*csch(x)/b/(a+b*sinh(x)
)-1/3*coth(x)*csch(x)^2/a/(a+b*sinh(x))-2*(a^2+4*b^2)*arctanh((b-a*tanh(1/2
*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^5
```

Rubi [A] time = 0.67, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 206}

$$-\frac{2\sqrt{a^2+b^2}\,\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4}+\frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5}+\frac{\left(a^2+2b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cosh[x]])/a^5 - (2*Sqrt[a^2 + b^2]*(a^2 + 4*b^2)
*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - ((7*a^2 + 12*b^2)*Coth[x
])/(3*a^4) + ((a^2 + 2*b^2)*Coth[x]*Csch[x])/(a^3*b) - ((3 + (4*b^2)/a^2)*C
oth[x]*Csch[x])/(3*b*(a + b*Sinh[x])) - (Coth[x]*Csch[x]^2)/(3*a*(a + b*Sin
h[x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} - \frac{\int \frac{\operatorname{csch}^3(x)\left(6\left(a^2+2b^2\right)-ab\sinh(x)+\left(3a^2+8b^2\right)\operatorname{s}\right.}{a+b\sinh(x)}}{3a^2b}$$

$$= \frac{\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} - \frac{i\int \frac{\operatorname{csch}^2(}{}}{}$$

$$= -\frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\operatorname{co}}{3a(}$$

$$= -\frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\operatorname{co}}{3a(}$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3b} -$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{\left(7a^2+12b^2\right)\coth(x)}{3a^4} + \frac{\left(a^2+2b^2\right)\coth(x)\operatorname{csch}(x)}{a^3b} -$$

$$= \frac{b\left(3a^2+4b^2\right)\tanh^{-1}(\cosh(x))}{a^5} - \frac{2\sqrt{a^2+b^2}\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{\left(7a^2+1\right.}{}$$

Mathematica [A] time = 0.82, size = 214, normalized size = 1.35

$$\frac{-\frac{1}{2}a^3\sinh(x)\operatorname{csch}^4\left(\frac{x}{2}\right)+8a^3\sinh^4\left(\frac{x}{2}\right)\operatorname{csch}^3(x)-4a\left(4a^2+9b^2\right)\tanh\left(\frac{x}{2}\right)-4a\left(4a^2+9b^2\right)\coth\left(\frac{x}{2}\right)-24b\left(3a^2\right.}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((48*(a^4 + 5*a^2*b^2 + 4*b^4)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] - 4*a*(4*a^2 + 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 - 24
*b*(3*a^2 + 4*b^2)*Log[Tanh[x/2]] + 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*S
inh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - (24*a*b*(a^2 + b^2)*Cosh[x])/(a
+ b*Sinh[x]) - 4*a*(4*a^2 + 9*b^2)*Tanh[x/2])/(24*a^5)
```

fricas [B] time = 1.97, size = 3648, normalized size = 22.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^4 + 2*a^2*b^2)*cosh(x)^7 + 6*(a^4 + 2*a^2*b^2)*sinh(x)^7 - 6*(a^3
*b + 4*a*b^3)*cosh(x)^6 - 6*(a^3*b + 4*a*b^3 - 7*(a^4 + 2*a^2*b^2)*cosh(x))
*sinh(x)^6 - 6*(7*a^4 + 10*a^2*b^2)*cosh(x)^5 - 6*(7*a^4 + 10*a^2*b^2 - 21*
(a^4 + 2*a^2*b^2)*cosh(x)^2 + 6*(a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^5 + 6*(7
*a^3*b + 12*a*b^3)*cosh(x)^4 + 6*(7*a^3*b + 12*a*b^3 + 35*(a^4 + 2*a^2*b^2)
*cosh(x)^3 - 15*(a^3*b + 4*a*b^3)*cosh(x)^2 - 5*(7*a^4 + 10*a^2*b^2)*cosh(x
))*sinh(x)^4 + 14*a^3*b + 24*a*b^3 + 42*(a^4 + 2*a^2*b^2)*cosh(x)^3 + 6*(35
*(a^4 + 2*a^2*b^2)*cosh(x)^4 + 7*a^4 + 14*a^2*b^2 - 20*(a^3*b + 4*a*b^3)*co
sh(x)^3 - 10*(7*a^4 + 10*a^2*b^2)*cosh(x)^2 + 4*(7*a^3*b + 12*a*b^3)*cosh(x
))*sinh(x)^3 - 2*(25*a^3*b + 36*a*b^3)*cosh(x)^2 + 2*(63*(a^4 + 2*a^2*b^2)*
cosh(x)^5 - 45*(a^3*b + 4*a*b^3)*cosh(x)^4 - 25*a^3*b - 36*a*b^3 - 30*(7*a^
4 + 10*a^2*b^2)*cosh(x)^3 + 18*(7*a^3*b + 12*a*b^3)*cosh(x)^2 + 63*(a^4 + 2
*a^2*b^2)*cosh(x))*sinh(x)^2 + 3*((a^2*b + 4*b^3)*cosh(x)^8 + (a^2*b + 4*b^
3)*sinh(x)^8 + 2*(a^3 + 4*a*b^2)*cosh(x)^7 + 2*(a^3 + 4*a*b^2 + 4*(a^2*b +
4*b^3)*cosh(x))*sinh(x)^7 - 4*(a^2*b + 4*b^3)*cosh(x)^6 - 2*(2*a^2*b + 8*b^
3 - 14*(a^2*b + 4*b^3)*cosh(x)^2 - 7*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^6 - 6
*(a^3 + 4*a*b^2)*cosh(x)^5 + 2*(28*(a^2*b + 4*b^3)*cosh(x)^3 - 3*a^3 - 12*a
*b^2 + 21*(a^3 + 4*a*b^2)*cosh(x)^2 - 12*(a^2*b + 4*b^3)*cosh(x))*sinh(x)^5
 + 6*(a^2*b + 4*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 4*b^3)*cosh(x)^4 + 35*(a^3
+ 4*a*b^2)*cosh(x)^3 + 3*a^2*b + 12*b^3 - 30*(a^2*b + 4*b^3)*cosh(x)^2 - 15
*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^4 + 6*(a^3 + 4*a*b^2)*cosh(x)^3 + 2*(28*(
a^2*b + 4*b^3)*cosh(x)^5 + 35*(a^3 + 4*a*b^2)*cosh(x)^4 - 40*(a^2*b + 4*b^3
)*cosh(x)^3 + 3*a^3 + 12*a*b^2 - 30*(a^3 + 4*a*b^2)*cosh(x)^2 + 12*(a^2*b +
 4*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*b^3 - 4*(a^2*b + 4*b^3)*cosh(x)^2 +
2*(14*(a^2*b + 4*b^3)*cosh(x)^6 + 21*(a^3 + 4*a*b^2)*cosh(x)^5 - 30*(a^2*b
+ 4*b^3)*cosh(x)^4 - 30*(a^3 + 4*a*b^2)*cosh(x)^3 - 2*a^2*b - 8*b^3 + 18*(a
^2*b + 4*b^3)*cosh(x)^2 + 9*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^2 - 2*(a^3 + 4
*a*b^2)*cosh(x) + 2*(4*(a^2*b + 4*b^3)*cosh(x)^7 + 7*(a^3 + 4*a*b^2)*cosh(x
)^6 - 12*(a^2*b + 4*b^3)*cosh(x)^5 - 15*(a^3 + 4*a*b^2)*cosh(x)^4 + 12*(a^2
*b + 4*b^3)*cosh(x)^3 - a^3 - 4*a*b^2 + 9*(a^3 + 4*a*b^2)*cosh(x)^2 - 4*(a^
2*b + 4*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(11*a^4 + 18*a^2*b^2)*cosh(x
) + 3*((3*a^2*b^2 + 4*b^4)*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3
*a^3*b + 4*a*b^3)*cosh(x)^7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*
cosh(x))*sinh(x)^7 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4
```

- 14*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x
)^6 - 6*(3*a^3*b + 4*a*b^3)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b
^2 + 4*b^4)*cosh(x)^3 - 21*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 +
4*b^4)*cosh(x))*sinh(x)^5 + 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*
b^2 + 4*b^4)*cosh(x)^4 + 9*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x
)^3 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*si
nh(x)^4 + 3*a^2*b^2 + 4*b^4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^
2*b^2 + 4*b^4)*cosh(x)^5 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*
a*b^3 - 40*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^2
 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x
)^2 + 2*(14*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 + 21*(3*a^3*b + 4*a*b^3)*cosh(x)^
5 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 - 6*a^2*b^2 - 8*b^4 - 30*(3*a^3*b + 4*
a*b^3)*cosh(x)^3 + 18*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 + 9*(3*a^3*b + 4*a*b^3)
*cosh(x))*sinh(x)^2 - 2*(3*a^3*b + 4*a*b^3)*cosh(x) + 2*(4*(3*a^2*b^2 + 4*b
^4)*cosh(x)^7 + 7*(3*a^3*b + 4*a*b^3)*cosh(x)^6 - 12*(3*a^2*b^2 + 4*b^4)*co
sh(x)^5 - 15*(3*a^3*b + 4*a*b^3)*cosh(x)^4 - 3*a^3*b - 4*a*b^3 + 12*(3*a^2*
b^2 + 4*b^4)*cosh(x)^3 + 9*(3*a^3*b + 4*a*b^3)*cosh(x)^2 - 4*(3*a^2*b^2 + 4
*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b^2 + 4*b^4)
*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3*a^3*b + 4*a*b^3)*cosh(x)^
7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^7 - 4*(3*
a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4 - 14*(3*a^2*b^2 + 4*b^4)*
cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^6 - 6*(3*a^3*b + 4*a*b^3
)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 21
*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^5
+ 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 9
*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^3 - 30*(3*a^2*b^2 + 4*b^
4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 + 3*a^2*b^2 + 4*b^
4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^2*b^2 + 4*b^4)*cosh(x)^5 +
 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*a*b^3 - 40*(3*a^2*b^2 + 4*
b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 + 4*b^4)*
cosh(x))*sinh(x)^3 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 + 2*(14*(3*a^2*b^2 + 4
*b^4)*cosh(x)^6 + 21*(3*a^3*b + 4*a*b^3)*cosh(x)^5 - 30*(3*a^2*b^2 + 4*b^4)
*cosh(x)^4 - 6*a^2*b^2 - 8*b^4 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 18*(3*a
^2*b^2 + 4*b^4)*cosh(x)^2 + 9*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 - 2*(3
*a^3*b + 4*a*b^3)*cosh(x) + 2*(4*(3*a^2*b^2 + 4*b^4)*cosh(x)^7 + 7*(3*a^3*b
 + 4*a*b^3)*cosh(x)^6 - 12*(3*a^2*b^2 + 4*b^4)*cosh(x)^5 - 15*(3*a^3*b + 4*
a*b^3)*cosh(x)^4 - 3*a^3*b - 4*a*b^3 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 + 9
*(3*a^3*b + 4*a*b^3)*cosh(x)^2 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x))*lo
g(cosh(x) + sinh(x) - 1) + 2*(21*(a^4 + 2*a^2*b^2)*cosh(x)^6 - 18*(a^3*b +
4*a*b^3)*cosh(x)^5 - 15*(7*a^4 + 10*a^2*b^2)*cosh(x)^4 - 11*a^4 - 18*a^2*b^
2 + 12*(7*a^3*b + 12*a*b^3)*cosh(x)^3 + 63*(a^4 + 2*a^2*b^2)*cosh(x)^2 - 2*
(25*a^3*b + 36*a*b^3)*cosh(x))*sinh(x))/(a^5*b*cosh(x)^8 + a^5*b*sinh(x)^8
+ 2*a^6*cosh(x)^7 - 4*a^5*b*cosh(x)^6 - 6*a^6*cosh(x)^5 + 6*a^5*b*cosh(x)^4
 + 6*a^6*cosh(x)^3 - 4*a^5*b*cosh(x)^2 + 2*(4*a^5*b*cosh(x) + a^6)*sinh(x)^
7 - 2*a^6*cosh(x) + 2*(14*a^5*b*cosh(x)^2 + 7*a^6*cosh(x) - 2*a^5*b)*sinh(x

```
)^6 + a^5*b + 2*(28*a^5*b*cosh(x)^3 + 21*a^6*cosh(x)^2 - 12*a^5*b*cosh(x) -
 3*a^6)*sinh(x)^5 + 2*(35*a^5*b*cosh(x)^4 + 35*a^6*cosh(x)^3 - 30*a^5*b*cos
h(x)^2 - 15*a^6*cosh(x) + 3*a^5*b)*sinh(x)^4 + 2*(28*a^5*b*cosh(x)^5 + 35*a
^6*cosh(x)^4 - 40*a^5*b*cosh(x)^3 - 30*a^6*cosh(x)^2 + 12*a^5*b*cosh(x) + 3
*a^6)*sinh(x)^3 + 2*(14*a^5*b*cosh(x)^6 + 21*a^6*cosh(x)^5 - 30*a^5*b*cosh(
x)^4 - 30*a^6*cosh(x)^3 + 18*a^5*b*cosh(x)^2 + 9*a^6*cosh(x) - 2*a^5*b)*sin
h(x)^2 + 2*(4*a^5*b*cosh(x)^7 + 7*a^6*cosh(x)^6 - 12*a^5*b*cosh(x)^5 - 15*a
^6*cosh(x)^4 + 12*a^5*b*cosh(x)^3 + 9*a^6*cosh(x)^2 - 4*a^5*b*cosh(x) - a^6
)*sinh(x))
```

giac [A] time = 0.25, size = 242, normalized size = 1.52

$$\frac{\left(3\,a^2b+4\,b^3\right)\log\left(e^x+1\right)}{a^5}-\frac{\left(3\,a^2b+4\,b^3\right)\log\left(\left|e^x-1\right|\right)}{a^5}+\frac{\left(a^4+5\,a^2b^2+4\,b^4\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^5}+\frac{2\left(a^3e^x+a\cdots\right.}{\left(be^{(2\,x)}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^2*b + 4*b^3)*log(e^x + 1)/a^5 - (3*a^2*b + 4*b^3)*log(abs(e^x - 1))/a^
5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/ab
s(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5) + 2*(a^3*e^x +
a*b^2*e^x - a^2*b - b^3)/((b*e^(2*x) + 2*a*e^x - b)*a^4) + 2/3*(3*a*b*e^(5*
x) - 6*a^2*e^(4*x) - 9*b^2*e^(4*x) + 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b
*e^x - 4*a^2 - 9*b^2)/(a^4*(e^(2*x) - 1)^3)
```

maple [B] time = 0.08, size = 357, normalized size = 2.25

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{24a^2}-\frac{b\left(\tanh^2\left(\frac{x}{2}\right)\right)}{4a^3}-\frac{5\tanh\left(\frac{x}{2}\right)}{8a^2}-\frac{3b^2\tanh\left(\frac{x}{2}\right)}{2a^4}+\frac{2b^2\tanh\left(\frac{x}{2}\right)}{a^3\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}+\frac{2\tanh\cdots}{a^5\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sinh(x))^2,x)
```

```
[Out] -1/24/a^2*tanh(1/2*x)^3-1/4/a^3*b*tanh(1/2*x)^2-5/8/a^2*tanh(1/2*x)-3/2/a^4
*b^2*tanh(1/2*x)+2/a^3/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^2*tanh(1/2*x)+
2/a^5/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*tanh(1/2*x)*b^4+2/a^2/(a*tanh(1/2
*x)^2-2*tanh(1/2*x)*b-a)*b+2/a^4/(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)*b^3+2/
a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+10/a^3
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*b^2+8/a
^5/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*b^4-1
```

```
/24/a^2/tanh(1/2*x)^3-5/8/a^2/tanh(1/2*x)-3/2/a^4/tanh(1/2*x)*b^2+1/4/a^3*b
/tanh(1/2*x)^2-3/a^3*b*ln(tanh(1/2*x))-4/a^5*b^3*ln(tanh(1/2*x))
```

maxima [B] time = 0.45, size = 339, normalized size = 2.13

$$-\frac{2\left(7a^{2}b+12b^{3}+\left(11a^{3}+18ab^{2}\right)e^{(-x)}-\left(25a^{2}b+36b^{3}\right)e^{(-2x)}-21\left(a^{3}+2ab^{2}\right)e^{(-3x)}+3\left(7a^{2}b+12b^{3}\right)e^{(-4x)}}{3\left(2a^{5}e^{(-x)}-4a^{4}be^{(-2x)}-6a^{5}e^{(-3x)}+6a^{4}be^{(-4x)}+6a^{5}e^{(-5x)}-4a^{4}be^{(-6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(7*a^2*b + 12*b^3 + (11*a^3 + 18*a*b^2)*e^(-x) - (25*a^2*b + 36*b^3)*e
^(-2*x) - 21*(a^3 + 2*a*b^2)*e^(-3*x) + 3*(7*a^2*b + 12*b^3)*e^(-4*x) + 3*(
7*a^3 + 10*a*b^2)*e^(-5*x) - 3*(a^2*b + 4*b^3)*e^(-6*x) - 3*(a^3 + 2*a*b^2)
*e^(-7*x))/(2*a^5*e^(-x) - 4*a^4*b*e^(-2*x) - 6*a^5*e^(-3*x) + 6*a^4*b*e^(-
4*x) + 6*a^5*e^(-5*x) - 4*a^4*b*e^(-6*x) - 2*a^5*e^(-7*x) + a^4*b*e^(-8*x)
+ a^4*b) + (3*a^2*b + 4*b^3)*log(e^(-x) + 1)/a^5 - (3*a^2*b + 4*b^3)*log(e^
(-x) - 1)/a^5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log((b*e^(-x) - a - sqrt(a^2 + b^
2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5)
```

mupad [B] time = 1.53, size = 1450, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] (3*b*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*exp(x) + 128*b^4*exp(x) +
224*a^2*b^2*exp(x)))/a^3 - 4/(a^2*exp(2*x) - a^2) - (6*b^2)/(a^4*exp(2*x) -
 a^4) - 8/(3*(3*a^2*exp(2*x) - 3*a^2*exp(4*x) + a^2*exp(6*x) - a^2)) - (4*a
^3*b^7)/(a^5*b^7*exp(2*x) - a^7*b^5 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^
6*exp(x) + 2*a^8*b^4*exp(x)) - (2*a^5*b^5)/(a^5*b^7*exp(2*x) - a^7*b^5 - a^
5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^6*exp(x) + 2*a^8*b^4*exp(x)) - (3*b*log(
96*a^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 224*a^2*b
^2*exp(x)))/a^3 - 4/(a^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) - (4*b^3*log(96*a
^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 224*a^2*b^2*e
xp(x)))/a^5 + (4*b^3*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*exp(x) + 1
28*b^4*exp(x) + 224*a^2*b^2*exp(x)))/a^5 + (log(128*a^6*exp(x) - 256*a*b^5
- 64*a^5*b - 320*a^3*b^3 - 128*b^5*(a^2 + b^2)^(1/2) + 128*b^6*exp(x) - 288
*a^2*b^3*(a^2 + b^2)^(1/2) + 128*a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4
*exp(x) + 672*a^4*b^2*exp(x) - 64*a^4*b*(a^2 + b^2)^(1/2) + 384*a*b^4*exp(x
)*(a^2 + b^2)^(1/2) + 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/
2))/a^3 - (log(128*b^5*(a^2 + b^2)^(1/2) - 256*a*b^5 - 64*a^5*b - 320*a^3*b
```

```
^3 + 128*a^6*exp(x) + 128*b^6*exp(x) + 288*a^2*b^3*(a^2 + b^2)^(1/2) - 128*
a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672*a^4*b^2*exp(x) + 64
*a^4*b*(a^2 + b^2)^(1/2) - 384*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 608*a^3*b^2
*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^3 - (2*a*b^9)/(a^5*b^7*exp(
2*x) - a^7*b^5 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^6*exp(x) + 2*a^8*b^4*
exp(x)) + (4*b*exp(x))/(a^3*exp(4*x) - 2*a^3*exp(2*x) + a^3) + (2*b*exp(x))
/(a^3*exp(2*x) - a^3) + (4*b^2*log(128*a^6*exp(x) - 256*a*b^5 - 64*a^5*b -
320*a^3*b^3 - 128*b^5*(a^2 + b^2)^(1/2) + 128*b^6*exp(x) - 288*a^2*b^3*(a^2
 + b^2)^(1/2) + 128*a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672
*a^4*b^2*exp(x) - 64*a^4*b*(a^2 + b^2)^(1/2) + 384*a*b^4*exp(x)*(a^2 + b^2)
^(1/2) + 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^5 - (4*
b^2*log(128*b^5*(a^2 + b^2)^(1/2) - 256*a*b^5 - 64*a^5*b - 320*a^3*b^3 + 12
8*a^6*exp(x) + 128*b^6*exp(x) + 288*a^2*b^3*(a^2 + b^2)^(1/2) - 128*a^5*exp
(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672*a^4*b^2*exp(x) + 64*a^4*b*
(a^2 + b^2)^(1/2) - 384*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 608*a^3*b^2*exp(x)
*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^5 + (2*a^2*b^9*exp(x))/(a^5*b^8*ex
p(2*x) - a^7*b^6 - a^5*b^8 + a^7*b^6*exp(2*x) + 2*a^6*b^7*exp(x) + 2*a^8*b^
5*exp(x)) + (4*a^4*b^7*exp(x))/(a^5*b^8*exp(2*x) - a^7*b^6 - a^5*b^8 + a^7*
b^6*exp(2*x) + 2*a^6*b^7*exp(x) + 2*a^8*b^5*exp(x)) + (2*a^6*b^5*exp(x))/(a
^5*b^8*exp(2*x) - a^7*b^6 - a^5*b^8 + a^7*b^6*exp(2*x) + 2*a^6*b^7*exp(x) +
 2*a^8*b^5*exp(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)**4/(a + b*sinh(x))**2, x)
```

3.244 $\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$

Optimal. Leaf size=37

$$2\sqrt{a+b\sinh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*sinh(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2721, 50, 63, 207}

$$2\sqrt{a+b\sinh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{a+b\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sinh(x)\right)\\ &= 2\sqrt{a+b\sinh(x)}+a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)\\ &= 2\sqrt{a+b\sinh(x)}+(2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)\\ &= -2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sinh(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$2\sqrt{a+b\sinh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

fricas [B] time = 1.15, size = 356, normalized size = 9.62

$$\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\left(b^2\cosh(x)+4\,ab\right)\sinh(x)^3-16\,ab\cosh(x)+2\left(16\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^
2*cosh(x) + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2
+ 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh
```

```
(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cos
h(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(
a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cos
h(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*
sqrt(b*sinh(x) + a), sqrt(-a)*arctan(4*sqrt(b*sinh(x) + a)*sqrt(-a)*(cosh(x
) + sinh(x))/(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2*a)
*sinh(x) - b)) + 2*sqrt(b*sinh(x) + a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh(x)+a}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

maple [A] time = 0.03, size = 30, normalized size = 0.81

$$-2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)\sqrt{a}\,+2\sqrt{a+b\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x))^(1/2),x)
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*sinh(x))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh(x)+a}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \coth(x)\,\sqrt{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*sinh(x))^(1/2),x)
[Out] int(coth(x)*(a + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh{(x)}} \coth{(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(x))*coth(x), x)
```

3.245 $\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))/a^(1/2)

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 63, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]],x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [**A**] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

fricas [**B**] time = 1.40, size = 370, normalized size = 15.42

$$\left[\frac{\log\left(\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\left(b^2\cosh(x)+4\,ab\right)\sinh(x)^3-16\,ab\cosh(x)+2\left(16\,a^2-b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+24\,ab\cosh(x)+\cdots\right.}{\cosh(x)^4+4\cos\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
[Out] [1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x)
 + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh(x)^3 + b
*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cosh(x) + (3
*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(a) + b^2
+ 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cosh(x))*sin
h(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*si
nh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1))/sqrt(a), sqrt
(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2
*a)*sinh(x) - b)*sqrt(b*sinh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^
2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

maple [A] time = 0.04, size = 19, normalized size = 0.79

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x))^(1/2),x)
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x))^(1/2),x)
[Out] int(coth(x)/(a + b*sinh(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(x\right)}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)), x)
```

3.246 $\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=51

$$\frac{B\log(a+b\sinh(x))}{b}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] B*ln(a+b*sinh(x))/b-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4401, 2660, 618, 206, 2668, 31}

$$\frac{B\log(a+b\sinh(x))}{b}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\cosh(x)}{a+b\sinh(x)}\right)dx\\
&= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\
&= (2A)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) + \frac{B\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}\\
&= \frac{B\log(a+b\sinh(x))}{b} - (4A)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{B\log(a+b\sinh(x))}{b}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 59, normalized size = 1.16

$$
\frac{2A\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{B\log(a+b\sinh(x))}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*Log[
a + b*Sinh[x]])/b
```

fricas [B] time = 2.17, size = 170, normalized size = 3.33

$$\frac{\sqrt{a^2+b^2}\,Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-\left(Ba^2+\right.}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x + (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

giac [A] time = 0.29, size = 87, normalized size = 1.71

$$\frac{A\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}-\frac{Bx}{b}+\frac{B\log\left(\left|be^{(2\,x)}+2\,ae^x-b\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) - B*x/b + B*log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

maple [A] time = 0.04, size = 88, normalized size = 1.73

$$\frac{B\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{b}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)A}{\sqrt{a^2+b^2}}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] 1/b*B*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(
2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A-B/b*ln(tanh(1/2*x)-1)-B/b*ln(tanh(1
/2*x)+1)
```

maxima [A] time = 0.45, size = 68, normalized size = 1.33

$$\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log\left(b\sinh(x)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sq
rt(a^2 + b^2) + B*log(b*sinh(x) + a)/b
```

mupad [B] time = 2.64, size = 198, normalized size = 3.88

$$\frac{B\,b^3\,\ln\left(8\,A^2\,a\,e^x-4\,A^2\,b+4\,A^2\,b\,e^{2\,x}\right)}{a^2\,b^2+b^4}-\frac{B\,x}{b}-\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,e^x\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b+A\,b^3\right)\,\sqrt{A^2}}+\frac{A^2\,a\,b\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b+A\,b^3\right)\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{-a^2-b^2}}+\frac{B\,a^2\,b\,\ln\left(8\,A^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*b^3*log(8*A^2*a*exp(x) - 4*A^2*b + 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2) -
(B*x)/b - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)*(
A^2)^(1/2)) + (A^2*a*b*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)*(A^2)^(1/2))
)*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) + (B*a^2*b*log(8*A^2*a*exp(x) - 4*A^2*b
+ 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2)
```

sympy [A] time = 73.28, size = 745, normalized size = 14.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2))), Eq(a, 0) & Eq(b, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0)), ((A*log(ta
nh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tanh(x/2)))/b, Eq(a, 0)), (
2*I*A*sqrt(b**2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)) + B*b*x*tanh(x/2)/(b**2*
tanh(x/2) - I*b*sqrt(b**2)) + 2*B*b*log(-I*b/sqrt(b**2) + tanh(x/2))*tanh(x
/2)/(b**2*tanh(x/2) - I*b*sqrt(b**2)) - 2*B*b*log(tanh(x/2) + 1)*tanh(x/2)/
(b**2*tanh(x/2) - I*b*sqrt(b**2)) - I*B*x*sqrt(b**2)/(b**2*tanh(x/2) - I*b*
sqrt(b**2)) - 2*I*B*sqrt(b**2)*log(-I*b/sqrt(b**2) + tanh(x/2))/(b**2*tanh(
x/2) - I*b*sqrt(b**2)) + 2*I*B*sqrt(b**2)*log(tanh(x/2) + 1)/(b**2*tanh(x/2
```

```
) - I*b*sqrt(b**2)), Eq(a, -sqrt(-b**2))), (-2*I*A*sqrt(b**2)/(b**2*tanh(x/
2) + I*b*sqrt(b**2)) + B*b*x*tanh(x/2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)) +
2*B*b*log(I*b/sqrt(b**2) + tanh(x/2))*tanh(x/2)/(b**2*tanh(x/2) + I*b*sqrt(
b**2)) - 2*B*b*log(tanh(x/2) + 1)*tanh(x/2)/(b**2*tanh(x/2) + I*b*sqrt(b**2
)) + I*B*x*sqrt(b**2)/(b**2*tanh(x/2) + I*b*sqrt(b**2)) + 2*I*B*sqrt(b**2)*
log(I*b/sqrt(b**2) + tanh(x/2))/(b**2*tanh(x/2) + I*b*sqrt(b**2)) - 2*I*B*s
qrt(b**2)*log(tanh(x/2) + 1)/(b**2*tanh(x/2) + I*b*sqrt(b**2)), Eq(a, sqrt(
-b**2))), (-A*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2)
+ A*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + B*x/b -
2*B*log(tanh(x/2) + 1)/b + B*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/b +
 B*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/b, True))
```

3.247 $\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=25

$$B\log(\sinh(x)+i)-\frac{A\cosh(x)}{1-i\sinh(x)}$$

```
[Out] B*ln(I+sinh(x))-A*cosh(x)/(1-I*sinh(x))
```

Rubi [A] time = 0.08, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4401, 2648, 2667, 31}

$$B\log(\sinh(x)+i)-\frac{A\cosh(x)}{1-i\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(I + Sinh[x]),x]
```

```
[Out] B*Log[I + Sinh[x]] - (A*Cosh[x])/(1 - I*Sinh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx &= \int \left(\frac{iA}{-1+i\sinh(x)}+\frac{iB\cosh(x)}{-1+i\sinh(x)}\right)dx\\
&= (iA)\int \frac{1}{-1+i\sinh(x)}\,dx+(iB)\int \frac{\cosh(x)}{-1+i\sinh(x)}\,dx\\
&= -\frac{A\cosh(x)}{1-i\sinh(x)}+B\,\mathrm{Subst}\left(\int \frac{1}{-1+x}\,dx,x,i\sinh(x)\right)\\
&= B\log(i+\sinh(x))-\frac{A\cosh(x)}{1-i\sinh(x)}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 48, normalized size = 1.92

$$
-\frac{2iA\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}-2iB\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+B\log(\cosh(x))
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(I + Sinh[x]),x]
```

```
[Out] (-2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]] - ((2*I)*A*Sinh[x/2])/(Cosh[x/2
] - I*Sinh[x/2])
```

fricas [A] time = 2.00, size = 37, normalized size = 1.48

$$
-\frac{Bxe^{x}+iBx-\left(2Be^{x}+2iB\right)\log\left(e^{x}+i\right)+2A}{e^{x}+i}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*e^x + I*B*x - (2*B*e^x + 2*I*B)*log(e^x + I) + 2*A)/(e^x + I)
```

giac [A] time = 0.24, size = 22, normalized size = 0.88

$$
-Bx+2B\log\left(e^{x}+i\right)-\frac{2A}{e^{x}+i}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*B*log(e^x + I) - 2*A/(e^x + I)
```

maple [A] time = 0.06, size = 46, normalized size = 1.84

$$-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2B\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(I+sinh(x)),x)
[Out] -B*ln(tanh(1/2*x)-1)-B*ln(tanh(1/2*x)+1)+2*B*ln(tanh(1/2*x)+I)-2*I/(tanh(1/
2*x)+I)*A
```

maxima [A] time = 0.36, size = 19, normalized size = 0.76

$$B\log\left(\sinh(x)+i\right)-\frac{2\,A}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="maxima")
[Out] B*log(sinh(x) + I) - 2*A/(e^(-x) - I)
```

mupad [B] time = 0.14, size = 24, normalized size = 0.96

$$-B\,x-\frac{2\,A}{\mathrm{e}^x+1\mathrm{i}}+2\,B\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(sinh(x) + 1i),x)
[Out] 2*B*log(exp(x) + 1i) - (2*A)/(exp(x) + 1i) - B*x
```

sympy [A] time = 0.17, size = 24, normalized size = 0.96

$$\frac{2A}{-e^{x}-i}+3Bx-2B\log\left(e^{x}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x)
[Out] 2*A/(-exp(x) - I) + 3*B*x - 2*B*log(exp(x) + I)
```

3.248 $\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx$

Optimal. Leaf size=27

$$\frac{A\cosh(x)}{1+i\sinh(x)} - B\log(-\sinh(x)+i)$$

```
[Out] -B*ln(I-sinh(x))+A*cosh(x)/(1+I*sinh(x))
```

Rubi [A] time = 0.09, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4401, 2648, 2667, 31}

$$\frac{A\cosh(x)}{1+i\sinh(x)} - B\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

```
[Out] -(B*Log[I - Sinh[x]]) + (A*Cosh[x])/(1 + I*Sinh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = \int \left(-\frac{iA}{1+i\sinh(x)}-\frac{iB\cosh(x)}{1+i\sinh(x)}\right)dx$$
$$= -\left((iA)\int \frac{1}{1+i\sinh(x)}\,dx\right)-(iB)\int \frac{\cosh(x)}{1+i\sinh(x)}\,dx$$
$$= \frac{A\cosh(x)}{1+i\sinh(x)} - B\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, i\sinh(x)\right)$$
$$= -B\log(i-\sinh(x)) + \frac{A\cosh(x)}{1+i\sinh(x)}$$

Mathematica [B] time = 0.09, size = 81, normalized size = 3.00

$$-\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(\sinh\left(\frac{x}{2}\right)\left(2A+2iB\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+B\log(\cosh(x))\right)+B\cosh\left(\frac{x}{2}\right)\left(2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right.\right.\right.\right.}{\sinh(x)-i}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

```
[Out] -(((Cosh[x/2] + I*Sinh[x/2])*(B*Cosh[x/2]*(2*ArcTan[Tanh[x/2]] - I*Log[Cosh
[x]]) + (2*A + (2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]])*Sinh[x/2]))/(-I
+ Sinh[x]))
```

fricas [A] time = 1.03, size = 36, normalized size = 1.33

$$\frac{Bxe^{x} - i\,Bx - \left(2\,Be^{x} - 2i\,B\right)\log\left(e^{x} - i\right) + 2\,A}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*e^x - I*B*x - (2*B*e^x - 2*I*B)*log(e^x - I) + 2*A)/(e^x - I)
```

giac [A] time = 0.19, size = 21, normalized size = 0.78

$$Bx - 2\,B\log\left(e^{x} - i\right) + \frac{2\,A}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*B*log(e^x - I) + 2*A/(e^x - I)
```

maple [A] time = 0.06, size = 44, normalized size = 1.63

$$B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2iA}{\tanh\left(\frac{x}{2}\right)-i}-2B\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(I-sinh(x)),x)
[Out] B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/2*x)+1)-2*I/(tanh(1/2*x)-I)*A-2*B*ln(tanh(1
/2*x)-I)
```

maxima [A] time = 0.37, size = 20, normalized size = 0.74

$$-B\log\left(\sinh(x)-i\right)+\frac{2\,A}{e^{(-x)}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="maxima")
[Out] -B*log(sinh(x) - I) + 2*A/(e^(-x) + I)
```

mupad [B] time = 0.12, size = 23, normalized size = 0.85

$$B\,x+\frac{2\,A}{\mathrm{e}^x-\mathrm{i}}-2\,B\,\ln\left(\mathrm{e}^x-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*cosh(x))/(sinh(x) - 1i),x)
[Out] B*x + (2*A)/(exp(x) - 1i) - 2*B*log(exp(x) - 1i)
```

sympy [A] time = 0.16, size = 22, normalized size = 0.81

$$-\frac{2A}{-e^{x}+i}-3Bx+2B\log\left(e^{x}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x)
[Out] -2*A/(-exp(x) + I) - 3*B*x + 2*B*log(exp(x) - I)
```

3.249 $\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}+\frac{bB\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{aB\log(\cosh(x))}{a^2+b^2}$$

```
[Out] b*B*arctan(sinh(x))/(a^2+b^2)+a*B*ln(cosh(x))/(a^2+b^2)-a*B*ln(a+b*sinh(x))
/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4401, 2660, 618, 206, 2721, 801, 635, 203, 260}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}+\frac{bB\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{aB\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (b*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] + (a*B*Log[Cosh[x]])/(a^2 + b^2) - (a*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\tanh(x)}{a+b\sinh(x)}\right)dx$$
$$= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$$
$$= (2A)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - B\,\text{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\,\text{si}\right.$$
$$= -\left((4A)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) - B\,\text{Subst}\left(\int\left(\frac{a}{\left(a^2+b^2\right)(a\right.}\right.\right.$$
$$= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2} - \frac{B\,\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}$$
$$= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2} + \frac{(aB)\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}$$
$$= \frac{bB\tan^{-1}(\sinh(x))}{a^2+b^2} - \frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{aB\log(\cosh(x))}{a^2+b^2} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2}$$

Mathematica [A] time = 0.37, size = 132, normalized size = 1.48

$$-\frac{\cosh(x)(A+B\tanh(x))\left(2A\left(a^2+b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+2bB\sqrt{-a^2-b^2}\,\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+aB\sqrt{-a^2-b^2}\,(\text{lo}\right.}{\left(-a^2-b^2\right)^{3/2}(A\cosh(x)+B\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] -((Cosh[x]*(2*b*Sqrt[-a^2 - b^2]*B*ArcTan[Tanh[x/2]] + 2*A*(a^2 + b^2)*ArcT
an[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Sqrt[-a^2 - b^2]*B*(Log[Cosh[x]]
 - Log[a + b*Sinh[x]]))*(A + B*Tanh[x]))/((-a^2 - b^2)^(3/2)*(A*Cosh[x] + B
*Sinh[x])))
```

fricas [B] time = 6.49, size = 172, normalized size = 1.93

$$\frac{2\,Bb\arctan\left(\cosh(x)+\sinh(x)\right) - Ba\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right) + Ba\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right) + \sqrt{a^2+b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2}{\right.}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*b*arctan(cosh(x) + sinh(x)) - B*a*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) + B*a*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

giac [A] time = 0.21, size = 123, normalized size = 1.38

$$\frac{2\,Bb\arctan\left(e^{x}\right)}{a^2+b^2}+\frac{Ba\log\left(e^{(2\,x)}+1\right)}{a^2+b^2}-\frac{Ba\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^2+b^2}+\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*B*b*arctan(e^x)/(a^2 + b^2) + B*a*log(e^(2*x) + 1)/(a^2 + b^2) - B*a*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

maple [A] time = 0.05, size = 150, normalized size = 1.69

$$-\frac{aB\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{a^2+b^2}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)a^2A}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)A\,b^2}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{Ba\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tanh(x))/(a+b*sinh(x)),x)
```

```
[Out] -1/(a^2+b^2)*a*B*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(a^2+b^2)^(3/2)*ar
ctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a^2*A+2/(a^2+b^2)^(3/2)*ar
ctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A*b^2+B/(a^2+b^2)*a*ln(tan
h(1/2*x)^2+1)+2*B/(a^2+b^2)*b*arctan(tanh(1/2*x))
```

maxima [A] time = 0.47, size = 125, normalized size = 1.40

$$-B\left(\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(2*b*arctan(e^(-x))/(a^2 + b^2) + a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a
^2 + b^2) - a*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt(a
^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

mupad [B] time = 8.67, size = 914, normalized size = 10.27

$$-\frac{\ln\left(\frac{32\,B\,\left(A^2\,a\,b+e^x\,A^2\,b^2-4\,e^x\,A\,B\,a^2+2\,A\,B\,a\,b-e^x\,A\,B\,b^2-4\,e^x\,B^2\,a^2+B^2\,a\,b\right)}{b^5}-\frac{\left(\frac{32\,\left(-A^2\,a^2\,b-2\,e^x\,A^2\,a\,b^2+A^2\,b^3+8\,e^x\,A\,B\,a^3-4\,A\,B\,a^2\,b+2\,e^x\,A\,B\,a\,b^2+4\,e^x}{b^5}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tanh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*log(exp(x) + 1i))/(a - b*1i) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*ex
p(x) + A^2*a*b + B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/
b^5 - (((32*(A^2*b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp(x)
 - 5*B^2*a*b^2*exp(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*exp(x)
 + 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*b^2)*
(a*b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) + a^
3*b^3*(128*A - 256*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2*b^4*ex
p(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A - 3*B) + 96*A*a^2*b*((a^2 + b^2)^3)^(
1/2) - 128*A*a^3*exp(x)*((a^2 + b^2)^3)^(1/2) - 32*A*a*b^2*exp(x)*((a^2 + b
^2)^3)^(1/2)))/(b^5*(a^2 + b^2)^3))*(B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*
b^2))/(a^2 + b^2)^2)*(B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*b^2))/(a^4 + b^
4 + 2*a^2*b^2) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*exp(x) + A^2*a*b +
B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5 - (((32*(A^2*
b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp(x) - 5*B^2*a*b^2*ex
p(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*exp(x) + 2*A*B*a*b^2*ex
p(x)))/b^5 - ((A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2)*(a*b^5*(64*A - 12
8*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) + a^3*b^3*(128*A - 25
6*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2*b^4*exp(x)*(A - 3*B) +
96*a^4*b^2*exp(x)*(A - 3*B) - 96*A*a^2*b*((a^2 + b^2)^3)^(1/2) + 128*A*a^3*
exp(x)*((a^2 + b^2)^3)^(1/2) + 32*A*a*b^2*exp(x)*((a^2 + b^2)^3)^(1/2)))/(b
^5*(a^2 + b^2)^3))*(A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2))/(a^2 + b^2)
^2)*(A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2))/(a^4 + b^4 + 2*a^2*b^2) +
(B*log(exp(x) - 1i)*1i)/(a*1i - b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\tanh(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*tanh(x))/(a + b*sinh(x)), x)
```

3.250 $\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\log(a+b\sinh(x))}{a}+\frac{B\log(\sinh(x))}{a}$$

```
[Out] B*ln(sinh(x))/a-B*ln(a+b*sinh(x))/a-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)
^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4401, 2660, 618, 206, 2721, 36, 29, 31}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\log(a+b\sinh(x))}{a}+\frac{B\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
Sinh[x]])/a - (B*Log[a + b*Sinh[x]])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\coth(x)}{a+b\sinh(x)}\right)dx$$
$$= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\coth(x)}{a+b\sinh(x)}\,dx$$
$$= (2A)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right) + B\,\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx,x,b\sinh(x)\right)$$
$$= -\left((4A)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) + \frac{B\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,b\sinh(\right.}{a}$$
$$= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{B\log(\sinh(x))}{a} - \frac{B\log(a+b\sinh(x))}{a}$$

Mathematica [**A**] time = 0.17, size = 65, normalized size = 1.08

$$\frac{2A\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{B(\log(\sinh(x))-\log(a+b\sinh(x)))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*(Log
[Sinh[x]] - Log[a + b*Sinh[x]]))/a
```

fricas [**B**] time = 1.63, size = 183, normalized size = 3.05

$$\frac{\sqrt{a^2+b^2}\,Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-\left(Ba^2+\right.}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + (B*a^2 + B*b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^3 + a*b^2)
```

giac [A] time = 0.36, size = 102, normalized size = 1.70

$$\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}+\frac{B\log\left(e^{x}+1\right)}{a}-\frac{B\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a}+\frac{B\log\left(\left|e^{x}-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) + B*log(e^x + 1)/a - B*log(abs(b*e^(2*x) + 2*a*e^
x - b))/a + B*log(abs(e^x - 1))/a
```

maple [A] time = 0.06, size = 73, normalized size = 1.22

$$-\frac{B\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{a}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)A}{\sqrt{a^2+b^2}}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*coth(x))/(a+b*sinh(x)),x)
```

```
[Out] -1/a*B*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(a^2+b^2)^(1/2)*arctanh(1/2*
(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A+B/a*ln(tanh(1/2*x))
```

maxima [A] time = 0.48, size = 106, normalized size = 1.77

$$-B\left(\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a}-\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(log(-2*a*e^(-x) + b*e^(-2*x) - b)/a - log(e^(-x) + 1)/a - log(e^(-x) -
1)/a) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b
^2)))/sqrt(a^2 + b^2)
```

mupad [B] time = 11.21, size = 164, normalized size = 2.73

$$\frac{B\ \ln\left(16\,B^2\,a^2+16\,B^2\,b^2-16\,B^2\,a^2\,\mathrm{e}^{2\,x}-16\,B^2\,b^2\,\mathrm{e}^{2\,x}\right)}{a}-\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^{x}\,\sqrt{-a^2-b^2}+A^2\,a\,b\,\sqrt{-a^2-b^2}}{A\,b\,\sqrt{A^2}\,\left(a^2+b^2\right)}\right)\,\sqrt{A^2}}{\sqrt{-a^2-b^2}}-\frac{B\ \ln\left(32\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*coth(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*log(16*B^2*a^2 + 16*B^2*b^2 - 16*B^2*a^2*exp(2*x) - 16*B^2*b^2*exp(2*x))
)/a - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2) + A^2*a*b*(- a^2 - b^2)^(
1/2))/(A*b*(A^2)^(1/2)*(a^2 + b^2)))*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) - (B*
log(32*B^2*a*exp(x) - 16*B^2*b + 16*B^2*b*exp(2*x)))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\coth(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*coth(x))/(a + b*sinh(x)), x)
```

3.251 $\int \frac{A+B\textbf{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}+\frac{aB\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{bB\log(\cosh(x))}{a^2+b^2}$$

```
[Out] a*B*arctan(sinh(x))/(a^2+b^2)-b*B*ln(cosh(x))/(a^2+b^2)+b*B*ln(a+b*sinh(x))
/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {4226, 4401, 2660, 618, 206, 2668, 706, 31, 635, 204, 260}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}+\frac{aB\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{bB\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

```
[Out] (a*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] - (b*B*Log[Cosh[x]])/(a^2 + b^2) + (b*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4226

```
Int[(u_)*((A_) + (B_.)*sec[(a_.) + (b_.)*(x_)]), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Cos[a + b*x]))/Cos[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx &= \int \frac{(B+A\cosh(x))\text{sech}(x)}{a+b\sinh(x)}\,dx \\
&= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\text{sech}(x)}{a+b\sinh(x)}\right)dx \\
&= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\text{sech}(x)}{a+b\sinh(x)}\,dx \\
&= (2A)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - (bB)\,\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx, x,\right. \\
&= -\left((4A)\,\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) + \frac{(bB)\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\right.}{a^2+b^2} \\
&= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{bB\log(a+b\sinh(x))}{a^2+b^2} + \frac{(bB)\,\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(x\right.}{a^2+b^2} \\
&= \frac{aB\tan^{-1}(\sinh(x))}{a^2+b^2} - \frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{bB\log(\cosh(x))}{a^2+b^2} + \frac{bB\log(a+b\sinh(x)}{a^2+b^2}
\end{aligned}
$$

Mathematica [A] time = 0.25, size = 93, normalized size = 1.04

$$
\frac{2A\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{2aB\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2} - \frac{bB(\log(\cosh(x))-\log(a+b\sinh(x)))}{a^2+b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*B*ArcTan[Tanh[x/2]])/(a^2 + b^2) + (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[
-a^2 - b^2]])/Sqrt[-a^2 - b^2] - (b*B*(Log[Cosh[x]] - Log[a + b*Sinh[x]]))/
(a^2 + b^2)
```

fricas [B] time = 8.22, size = 172, normalized size = 1.93

$$\frac{2\,Ba\arctan\left(\cosh(x)+\sinh(x)\right)+Bb\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-Bb\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2}{\cdots}\right.}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*a*arctan(cosh(x) + sinh(x)) + B*b*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) - B*b*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

giac [A] time = 0.51, size = 123, normalized size = 1.38

$$\frac{2\,Ba\arctan\left(e^{x}\right)}{a^2+b^2}-\frac{Bb\log\left(e^{(2\,x)}+1\right)}{a^2+b^2}+\frac{Bb\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^2+b^2}+\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(e^x)/(a^2 + b^2) - B*b*log(e^(2*x) + 1)/(a^2 + b^2) + B*b*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

maple [A] time = 0.06, size = 150, normalized size = 1.69

$$\frac{bB\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2\tanh\left(\frac{x}{2}\right)b-a\right)}{a^2+b^2}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)a^2A}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)A\,b^2}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{Bb\ln\left(\mathrm{ta}\cdots\right.}{a^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sech(x))/(a+b*sinh(x)),x)
```

```
[Out] 1/(a^2+b^2)*b*B*ln(a*tanh(1/2*x)^2-2*tanh(1/2*x)*b-a)+2/(a^2+b^2)^(3/2)*arc
tanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*a^2*A+2/(a^2+b^2)^(3/2)*arc
tanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A*b^2-B/(a^2+b^2)*b*ln(tanh
(1/2*x)^2+1)+2*B/(a^2+b^2)*a*arctan(tanh(1/2*x))
```

maxima [A] time = 0.53, size = 125, normalized size = 1.40

$$-B\left(\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}+\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a
^2 + b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt(a
^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

mupad [B] time = 10.64, size = 864, normalized size = 9.71

$$\ln\left(\frac{\left(A\,\sqrt{{\left(a^2+b^2\right)}^3}+B\,b^3+B\,a^2\,b\right)\left(b^3\,\left(32\,A^2-96\,e^x\,A\,B+64\,B^2\right)-128\,A^2\,e^x\,\sqrt{{\left(a^2+b^2\right)}^3}-a\,b^2\,\left(96\,e^x\,A^2-192\,A\,B+128\,e^x\,B^2\right)-128\,a^3\,e^x\,\left(A^2+B^2\right)+a^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (log(((A*((a^2 + b^2)^3)^(1/2) + B*b^3 + B*a^2*b)*(b^3*(32*A^2 + 64*B^2 - 9
6*A*B*exp(x)) - 128*A^2*exp(x)*((a^2 + b^2)^3)^(1/2) - a*b^2*(96*A^2*exp(x)
 + 128*B^2*exp(x) - 192*A*B) - 128*a^3*exp(x)*(A^2 + B^2) + a^2*b*(64*A^2 +
 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6*(2*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(
1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^2*
b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^3*b^3*(4*A - 19*B*e
xp(x)))/((a^2 + b^2)^3)^(1/2) + (64*A*a*b^5*(A - 4*B*exp(x)))/((a^2 + b^2)^
3)^(1/2) + (32*A*a^5*b*(2*A - 11*B*exp(x)))/((a^2 + b^2)^3)^(1/2)))/(b^5*(a
^2 + b^2)^2) - (32*B*(2*B^2*b^2 - A^2*b^2 + 4*A*B*a^2*exp(x) + A*B*b^2*exp(
x) + A^2*a*b*exp(x) - 4*B^2*a*b*exp(x) - 2*A*B*a*b))/b^5)*(A*((a^2 + b^2)^3
)^(1/2) + B*b^3 + B*a^2*b))/(a^4 + b^4 + 2*a^2*b^2) - (B*log(exp(x) + 1i))/
(a*1i + b) - (B*log(exp(x) - 1i)*1i)/(a + b*1i) + (log(- (32*B*(2*B^2*b^2 -
 A^2*b^2 + 4*A*B*a^2*exp(x) + A*B*b^2*exp(x) + A^2*a*b*exp(x) - 4*B^2*a*b*e
xp(x) - 2*A*B*a*b))/b^5 - ((B*b^3 - A*((a^2 + b^2)^3)^(1/2) + B*a^2*b)*(a*b
^2*(96*A^2*exp(x) + 128*B^2*exp(x) - 192*A*B) - 128*A^2*exp(x)*((a^2 + b^2)
^3)^(1/2) - b^3*(32*A^2 + 64*B^2 - 96*A*B*exp(x)) + 128*a^3*exp(x)*(A^2 + B
^2) - a^2*b*(64*A^2 + 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6*(2*B + 3*A*exp(x
)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(x)))/((a^2 + b^2)^
```

```
3)^(1/2) + (32*A*a^2*b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*
a^3*b^3*(4*A - 19*B*exp(x)))/((a^2 + b^2)^3)^(1/2) + (64*A*a*b^5*(A - 4*B*e
xp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^5*b*(2*A - 11*B*exp(x)))/((a^2 + b^
2)^3)^(1/2)))/(b^5*(a^2 + b^2)^2))*(B*b^3 - A*((a^2 + b^2)^3)^(1/2) + B*a^2
*b))/(a^4 + b^4 + 2*a^2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\operatorname{sech}(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*sech(x))/(a + b*sinh(x)), x)
```

3.252 $\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=58

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{B\tanh^{-1}(\cosh(x))}{a}$$

```
[Out] -B*arctanh(cosh(x))/a-2*(A*a-B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/a/(a^2+b^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2828, 3001, 3770, 2660, 618, 206}

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{B\tanh^{-1}(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

```
[Out] -((B*ArcTanh[Cosh[x]])/a) - (2*(a*A - b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a
^2 + b^2]])/(a*Sqrt[a^2 + b^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e +
 f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx &= -\left(i\int \frac{\text{csch}(x)(iB+iA\sinh(x))}{a+b\sinh(x)}\,dx\right)\\
&= \frac{B\int \text{csch}(x)\,dx}{a}+\frac{(aA-bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{B\tanh^{-1}(\cosh(x))}{a}+\frac{(2(aA-bB))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{B\tanh^{-1}(\cosh(x))}{a}-\frac{(4(aA-bB))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{B\tanh^{-1}(\cosh(x))}{a}-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 67, normalized size = 1.16

$$\frac{\frac{2(aA-bB)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

```
[Out] ((2*(a*A - b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + B*Log[Tanh[x/2]])/a
```

fricas [B] time = 1.47, size = 172, normalized size = 2.97

$$-\frac{(Aa-Bb)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((A*a - B*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) + (B*a^2 + B*b^2)*log(cosh(x) + sinh(x) + 1) - (B
*a^2 + B*b^2)*log(cosh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

giac [A] time = 0.18, size = 90, normalized size = 1.55

$$-\frac{B\log\left(e^x+1\right)}{a}+\frac{B\log\left(|e^x-1|\right)}{a}+\frac{(Aa-Bb)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*log(e^x + 1)/a + B*log(abs(e^x - 1))/a + (A*a - B*b)*log(abs(2*b*e^x + 2
*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*a)
```

maple [A] time = 0.05, size = 86, normalized size = 1.48

$$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)A}{\sqrt{a^2+b^2}}-\frac{2Bb\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*csch(x))/(a+b*sinh(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))*A-2*B/
a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+B/a*
ln(tanh(1/2*x))
```

maxima [B] time = 0.46, size = 141, normalized size = 2.43

$$-B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*a) + log(e^(-x) + 1)/a - log(e^(-x) - 1)/a) + A*log((b*e
^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b
^2)
```

mupad [B] time = 2.18, size = 539, normalized size = 9.29

$$\frac{B\,\ln\left(e^x-1\right)}{a}-\frac{B\,\ln\left(e^x+1\right)}{a}-\frac{\ln\left(\frac{(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,e^x\,B^2\,a^3+2\,B^2\,a^2\,b-3\,e^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}-\frac{(A\,a-B\,b)\left(\frac{32\,a^2\left(2\,B\,b^2+4\,A\,a^2\,e^x+A\,b^2\,e^x-\cdots}{b^5}\cdots\right.}{\cdots}\right.}{a\,\sqrt{a^2+b^2}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/sinh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*log(exp(x) - 1))/a - (B*log(exp(x) + 1))/a - (log(((A*a - B*b)*((32*(2*B
^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*a*b^2*exp(x) -
2*A*B*a*b^2))/b^5 - ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^2*exp(x) + A*b^2
*exp(x) - 2*A*a*b - 3*B*a*b*exp(x)))/b^5 + (32*a*(A*a - B*b)*(3*a^2*b + 2*b
^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(a*(a^2 + b^
2)^(1/2))))/(a*(a^2 + b^2)^(1/2)) + (32*B*(A*a - B*b)*(A*b*exp(x) - 2*B*b +
 4*B*a*exp(x)))/b^5)*(A*a - B*b)*(a^2 + b^2)^(1/2))/(a*b^2 + a^3) + (log((3
2*B*(A*a - B*b)*(A*b*exp(x) - 2*B*b + 4*B*a*exp(x)))/b^5 - ((A*a - B*b)*((3
2*(2*B^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*a*b^2*exp
(x) - 2*A*B*a*b^2))/b^5 + ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^2*exp(x) +
 A*b^2*exp(x) - 2*A*a*b - 3*B*a*b*exp(x)))/b^5 - (32*a*(A*a - B*b)*(3*a^2*b
 + 2*b^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(a*(a^
```

```
2 + b^2)^(1/2))))/(a*(a^2 + b^2)^(1/2)))*(A*a - B*b)*(a^2 + b^2)^(1/2))/(a*
b^2 + a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\operatorname{csch}(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*csch(x))/(a + b*sinh(x)), x)
```

3.253 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx$

Optimal. Leaf size=81

$$-\frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{ce\sqrt{a^2+c^2}}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

```
[Out] C*x/c+B*ln(a+c*sinh(e*x+d))/c/e-2*(A*c-C*a)*arctanh((c-a*tanh(1/2*e*x+1/2*d
))/(a^2+c^2)^(1/2))/c/e/(a^2+c^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4376, 2735, 2660, 618, 204, 2668, 31}

$$-\frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{ce\sqrt{a^2+c^2}}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

```
[Out] (C*x)/c - (2*(A*c - a*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]]
)/(c*Sqrt[a^2 + c^2]*e) + (B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = B\int \frac{\cosh(d+ex)}{a+c\sinh(d+ex)}\,dx + \int \frac{A+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx$$

$$= \frac{Cx}{c} - \frac{(i(iAc-iaC))\int \frac{1}{a+c\sinh(d+ex)}\,dx}{c} + \frac{B\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, c\sin\right.}{ce}$$

$$= \frac{Cx}{c} + \frac{B\log(a+c\sinh(d+ex))}{ce} - \frac{(2i(Ac-aC))\,\text{Subst}\left(\int \frac{1}{a-2icx+ax}\right.}{ce}$$

$$= \frac{Cx}{c} + \frac{B\log(a+c\sinh(d+ex))}{ce} + \frac{(4i(Ac-aC))\,\text{Subst}\left(\int \frac{1}{-4(a^2+c^2)}\right.}{}$$

$$= \frac{Cx}{c} - \frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}\,e} + \frac{B\log(a+c\sinh(d+}{ce}$$

Mathematica [A] time = 0.29, size = 85, normalized size = 1.05

$$\frac{\frac{2(Ac-aC)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} + B\log(a+c\sinh(d+ex)) + C(d+ex)}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

```
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 -
c^2]])/Sqrt[-a^2 - c^2] + B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

fricas [B] time = 1.40, size = 249, normalized size = 3.07

$$-\frac{\left((B-C)a^2+(B-C)c^2\right)ex+(Ca-Ac)\sqrt{a^2+c^2}\,\log\left(\frac{c^2\cosh(ex+d)^2+c^2\sinh(ex+d)^2+2\,ac\cosh(ex+d)+2\,a^2+c^2+2\left(c^2\cosh(ex+d)\right.}{c\cosh(ex+d)^2+c\sinh(ex+d)^2+2\,a\cosh(ex+d)+2}\right.}{\left(a^2c+c^3\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="f
ricas")
```

```
[Out] -(((B - C)*a^2 + (B - C)*c^2)*e*x + (C*a - A*c)*sqrt(a^2 + c^2)*log((c^2*co
sh(e*x + d)^2 + c^2*sinh(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2
```

```
*(c^2*cosh(e*x + d) + a*c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x +
d) + c*sinh(e*x + d) + a))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cos
h(e*x + d) + 2*(c*cosh(e*x + d) + a)*sinh(e*x + d) - c)) - (B*a^2 + B*c^2)*
log(2*(c*sinh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*c + c^3
)*e)
```

giac [A] time = 0.20, size = 131, normalized size = 1.62

$$-\left(\frac{(xe+d)(B-C)}{c}-\frac{B\log\left(\left|ce^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}-c\right|\right)}{c}+\frac{(Ca-Ac)\log\left(\frac{\left|2\,ce^{(xe+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(xe+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\sqrt{a^2+c^2}\,c}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="g
iac")
```

```
[Out] -((x*e + d)*(B - C)/c - B*log(abs(c*e^(2*x*e + 2*d) + 2*a*e^(x*e + d) - c))
/c + (C*a - A*c)*log(abs(2*c*e^(x*e + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c
*e^(x*e + d) + 2*a + 2*sqrt(a^2 + c^2)))/(sqrt(a^2 + c^2)*c))*e^(-1)
```

maple [B] time = 0.12, size = 213, normalized size = 2.63

$$-\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)B}{ec}-\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)C}{ec}-\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)B}{ec}+\frac{\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)C}{ec}+\frac{B\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

```
[Out] -1/e/c*ln(tanh(1/2*e*x+1/2*d)-1)*B-1/e/c*ln(tanh(1/2*e*x+1/2*d)-1)*C-1/e/c*
ln(tanh(1/2*e*x+1/2*d)+1)*B+1/e/c*ln(tanh(1/2*e*x+1/2*d)+1)*C+1/e/c*B*ln(a*
tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)+2/e/(a^2+c^2)^(1/2)*arctan
h(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2))*A-2/e/c/(a^2+c^2)^(1/2
)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2))*C*a
```

maxima [B] time = 0.44, size = 176, normalized size = 2.17

$$-C\left(\frac{a\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}\,ce}-\frac{ex+d}{ce}\right)+\frac{A\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}\,e}+\frac{B\log\left(c\sinh\left(ex+d\right)+a\right)}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="m
axima")
```

```
[Out] -C*(a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt
(a^2 + c^2)))/(sqrt(a^2 + c^2)*c*e) - (e*x + d)/(c*e)) + A*log((c*e^(-e*x -
 d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 + c^2)))/(sqrt(a^
2 + c^2)*e) + B*log(c*sinh(e*x + d) + a)/(c*e)
```

mupad [B] time = 1.85, size = 656, normalized size = 8.10

$$\frac{C\,x}{c}-\frac{B\,x}{c}-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c+A\,e\,a^2\,c^2-C\,e\,a\,c^3+A\,e\,c^4}-\frac{a^2\,c^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c^4+A\,e\,a^2\,c^5-C\,e\,a\,c^6+A\,e\,c^7}+\frac{A\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{-a^2\,c^2\,e}}{c\,e\,\sqrt{A^2\,c^2-2\,A\,C\,a}}\right.}{\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x)),x)
```

```
[Out] (C*x)/c - (B*x)/c - (2*atan((a*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 + C
^2*a^2 - 2*A*C*a*c)^(1/2))/(A*c^4*e - C*a*c^3*e - C*a^3*c*e + A*a^2*c^2*e)
- (a^2*c^2*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 + C^2*a
^2 - 2*A*C*a*c)^(1/2))/(A*c^7*e - C*a*c^6*e + A*a^2*c^5*e - C*a^3*c^4*e) +
(A*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2))/(c*e*(A^2*c^2 + C^2*a^2
 - 2*A*C*a*c)^(1/2)) - (C*a*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)
)/(c^2*e*(A^2*c^2 + C^2*a^2 - 2*A*C*a*c)^(1/2)))*(A^2*c^2 + C^2*a^2 - 2*A*C
*a*c)^(1/2))/(- c^4*e^2 - a^2*c^2*e^2)^(1/2) + (B*c^3*e*log(8*A*C*a*c^2 - 4
*C^2*a^2*c - 4*A^2*c^3 + 8*C^2*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp(2*d)*exp
(2*e*x) + 8*A^2*a*c^2*exp(e*x)*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(2*e*x) - 1
6*A*C*a^2*c*exp(e*x)*exp(d) - 8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/(c^4*e^2 +
a^2*c^2*e^2) + (B*a^2*c*e*log(8*A*C*a*c^2 - 4*C^2*a^2*c - 4*A^2*c^3 + 8*C^2
*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp(2*d)*exp(2*e*x) + 8*A^2*a*c^2*exp(e*x)
*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(2*e*x) - 16*A*C*a^2*c*exp(e*x)*exp(d) -
8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/(c^4*e^2 + a^2*c^2*e^2)
```

sympy [A] time = 40.01, size = 1318, normalized size = 16.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/sinh(d), Eq(a, 0) & Eq(c, 0) &
 Eq(e, 0)), (2*I*A/(c*e*tanh(d/2 + e*x/2) - I*c*e) + B*e*x*tanh(d/2 + e*x/2
)/(c*e*tanh(d/2 + e*x/2) - I*c*e) - I*B*e*x/(c*e*tanh(d/2 + e*x/2) - I*c*e)
 - 2*B*log(tanh(d/2 + e*x/2) + 1)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2)
```

```
- I*c*e) + 2*I*B*log(tanh(d/2 + e*x/2) + 1)/(c*e*tanh(d/2 + e*x/2) - I*c*e)
 + 2*B*log(tanh(d/2 + e*x/2) - I)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2)
- I*c*e) - 2*I*B*log(tanh(d/2 + e*x/2) - I)/(c*e*tanh(d/2 + e*x/2) - I*c*e)
 + C*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c*e) - I*C*e*x/(c*e*t
anh(d/2 + e*x/2) - I*c*e) - 2*C/(c*e*tanh(d/2 + e*x/2) - I*c*e), Eq(a, -I*c
)), (-2*I*A/(c*e*tanh(d/2 + e*x/2) + I*c*e) + B*e*x*tanh(d/2 + e*x/2)/(c*e*
tanh(d/2 + e*x/2) + I*c*e) + I*B*e*x/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*B*
log(tanh(d/2 + e*x/2) + 1)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e
) - 2*I*B*log(tanh(d/2 + e*x/2) + 1)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*B*
log(tanh(d/2 + e*x/2) + I)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e
) + 2*I*B*log(tanh(d/2 + e*x/2) + I)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + C*e*
x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + I*C*e*x/(c*e*tanh(d/2
 + e*x/2) + I*c*e) - 2*C/(c*e*tanh(d/2 + e*x/2) + I*c*e), Eq(a, I*c)), ((A*
x + B*sinh(d + e*x)/e + C*cosh(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cosh(d)
+ C*sinh(d))/(a + c*sinh(d)), Eq(e, 0)), ((A*log(tanh(d/2 + e*x/2))/e + B*x
 - 2*B*log(tanh(d/2 + e*x/2) + 1)/e + B*log(tanh(d/2 + e*x/2))/e + C*x)/c,
Eq(a, 0)), (-A*c*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2
+ c**2)/a)/(a**2*c*e + c**3*e) + A*c*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2
) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + B*a**2*e*x/(a**2*c*e +
 c**3*e) - 2*B*a**2*log(tanh(d/2 + e*x/2) + 1)/(a**2*c*e + c**3*e) + B*a**2
*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + B
*a**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e
) + B*c**2*e*x/(a**2*c*e + c**3*e) - 2*B*c**2*log(tanh(d/2 + e*x/2) + 1)/(a
**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/
a)/(a**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c
**2)/a)/(a**2*c*e + c**3*e) + C*a**2*e*x/(a**2*c*e + c**3*e) + C*a*sqrt(a**
2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c*e + c*
*3*e) - C*a*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c**
2)/a)/(a**2*c*e + c**3*e) + C*c**2*e*x/(a**2*c*e + c**3*e), True))
```

3.254 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$

Optimal. Leaf size=113

$$-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{3/2}}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

```
[Out] -2*(A*a+C*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))/(a^2+c^2)^(
3/2)/e-B/c/e/(a+c*sinh(e*x+d))-(A*c-C*a)*cosh(e*x+d)/(a^2+c^2)/e/(a+c*sinh(
e*x+d))
```

Rubi [A] time = 0.17, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{3/2}}-\frac{(Ac-aC)\cosh(d+ex)}{e\left(a^2+c^2\right)(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]])/((a^2 +
 c^2)^(3/2)*e) - B/(c*e*(a + c*Sinh[d + e*x])) - ((A*c - a*C)*Cosh[d + e*x]
)/((a^2 + c^2)*e*(a + c*Sinh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} - \frac{\int \frac{-aA-cC}{a+c\sinh(d+ex)}\,dx}{a^2+c^2} + \frac{B\,\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} + \frac{(aA+}{}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} - \frac{(2i(aA}{}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{\left(a^2+c^2\right)e(a+c\sinh(d+ex))} + \frac{(4i(aA}{}$$

$$= -\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{3/2}e} - \frac{B}{ce(a+c\sinh(d+ex))} - $$

Mathematica [A] time = 0.52, size = 113, normalized size = 1.00

$$\frac{\frac{2(aA+cC)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{B\left(a^2+c^2\right)+c(Ac-aC)\cosh(d+ex)}{c(a+c\sinh(d+ex))}}{e\left(a^2+c^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x
]
```

```
[Out] ((2*(a*A + c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 - c^2]])/Sqrt[-a
^2 - c^2] - (B*(a^2 + c^2) + c*(A*c - a*C)*Cosh[d + e*x])/(c*(a + c*Sinh[d
+ e*x])))/((a^2 + c^2)*e)
```

fricas [B] time = 0.68, size = 570, normalized size = 5.04

$$2\,Ca^3c - 2\,Aa^2c^2 + 2\,Cac^3 - 2\,Ac^4 - \left(Aac^2 + Cc^3 - \left(Aac^2 + Cc^3\right)\cosh\left(ex+d\right)^2 - \left(Aac^2 + Cc^3\right)\sinh\left(ex+d\right)^2 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"fricas")
```

```
[Out] (2*C*a^3*c - 2*A*a^2*c^2 + 2*C*a*c^3 - 2*A*c^4 - (A*a*c^2 + C*c^3 - (A*a*c^
2 + C*c^3)*cosh(e*x + d)^2 - (A*a*c^2 + C*c^3)*sinh(e*x + d)^2 - 2*(A*a^2*c
 + C*a*c^2)*cosh(e*x + d) - 2*(A*a^2*c + C*a*c^2 + (A*a*c^2 + C*c^3)*cosh(e
*x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh
(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*
c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a
))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e
*x + d) + a)*sinh(e*x + d) - c)) - 2*((B + C)*a^4 - A*a^3*c + (2*B + C)*a^2
*c^2 - A*a*c^3 + B*c^4)*cosh(e*x + d) - 2*((B + C)*a^4 - A*a^3*c + (2*B + C
)*a^2*c^2 - A*a*c^3 + B*c^4)*sinh(e*x + d))/((a^4*c^2 + 2*a^2*c^4 + c^6)*e*
cosh(e*x + d)^2 + (a^4*c^2 + 2*a^2*c^4 + c^6)*e*sinh(e*x + d)^2 + 2*(a^5*c
+ 2*a^3*c^3 + a*c^5)*e*cosh(e*x + d) - (a^4*c^2 + 2*a^2*c^4 + c^6)*e + 2*((
a^4*c^2 + 2*a^2*c^4 + c^6)*e*cosh(e*x + d) + (a^5*c + 2*a^3*c^3 + a*c^5)*e)
*sinh(e*x + d))
```

giac [A] time = 0.30, size = 177, normalized size = 1.57

$$\left(\frac{(Aa+Cc)\log\left(\frac{\left|2\,ce^{(xe+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(xe+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Ba^2e^{(xe+d)}+Ca^2e^{(xe+d)}-Aace^{(xe+d)}+Bc^2e^{(xe+d)}-Cac+Ac^2\right)}{\left(a^2c+c^3\right)\left(ce^{(2\,xe+2\,d)}+2\,ae^{(xe+d)}-c\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"giac")
```

```
[Out] ((A*a + C*c)*log(abs(2*c*e^(x*e + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(
x*e + d) + 2*a + 2*sqrt(a^2 + c^2)))/(a^2 + c^2)^(3/2) - 2*(B*a^2*e^(x*e +
d) + C*a^2*e^(x*e + d) - A*a*c*e^(x*e + d) + B*c^2*e^(x*e + d) - C*a*c + A*
c^2)/((a^2*c + c^3)*(c*e^(2*x*e + 2*d) + 2*a*e^(x*e + d) - c)))*e^(-1)
```

maple [A] time = 0.18, size = 151, normalized size = 1.34

$$\frac{-\frac{2\left(-\frac{\left(Ac^2-Ba^2-Bc^2-Cac\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^2+c^2\right)}-\frac{Ac-Ca}{a^2+c^2}\right)}{a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a}+\frac{2(Aa+Cc)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x)
```

```
[Out] 1/e*(-2*(-(A*c^2-B*a^2-B*c^2-C*a*c)/a/(a^2+c^2)*tanh(1/2*e*x+1/2*d)-(A*c-C*
a)/(a^2+c^2))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)+2*(A*a+C*
c)/(a^2+c^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2
)))
```

maxima [B] time = 0.45, size = 339, normalized size = 3.00

$$A\left(\frac{a\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}e}-\frac{2\left(ae^{(-ex-d)}+c\right)}{\left(a^2c+c^3+2\left(a^3+ac^2\right)e^{(-ex-d)}-\left(a^2c+c^3\right)e^{(-2\,ex-2\,d)}\right)e}\right)+C\left(\frac{c\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"maxima")
```

```
[Out] A*(a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(
a^2 + c^2)))/((a^2 + c^2)^(3/2)*e) - 2*(a*e^(-e*x - d) + c)/((a^2*c + c^3 +
 2*(a^3 + a*c^2)*e^(-e*x - d) - (a^2*c + c^3)*e^(-2*e*x - 2*d))*e)) + C*(c*
log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 +
 c^2)))/((a^2 + c^2)^(3/2)*e) + 2*(a^2*e^(-e*x - d) + a*c)/((a^2*c^2 + c^4
+ 2*(a^3*c + a*c^3)*e^(-e*x - d) - (a^2*c^2 + c^4)*e^(-2*e*x - 2*d))*e)) -
2*B*e^(-e*x - d)/((2*a*c*e^(-e*x - d) - c^2*e^(-2*e*x - 2*d) + c^2)*e)
```

mupad [B] time = 1.20, size = 279, normalized size = 2.47

$$\frac{\ln\left(\frac{2\,\left(A\,a+C\,c\right)\,\left(c-a\,\mathrm{e}^{d+e\,x}\right)}{c\,{\left(a^2+c^2\right)}^{3/2}}-\frac{2\,\mathrm{e}^{d+e\,x}\,\left(A\,a+C\,c\right)}{c\,\left(a^2+c^2\right)}\right)\,\left(A\,a+C\,c\right)}{e\,{\left(a^2+c^2\right)}^{3/2}}-\frac{\ln\left(-\frac{2\,\mathrm{e}^{d+e\,x}\,\left(A\,a+C\,c\right)}{c\,\left(a^2+c^2\right)}-\frac{2\,\left(A\,a+C\,c\right)\,\left(c-a\,\mathrm{e}^{d+e\,x}\right)}{c\,{\left(a^2+c^2\right)}^{3/2}}\right)\,\left(A\,a+C\,c\right)}{e\,{\left(a^2+c^2\right)}^{3/2}}-\frac{\frac{2\left(A\,c^3\ldots\right.}{c\,e\left(a^2\ldots\right.}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^2,x)
```

```
[Out] (log((2*(A*a + C*c)*(c - a*exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)) - (2*exp(d
 + e*x)*(A*a + C*c))/(c*(a^2 + c^2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)) - (
log(- (2*exp(d + e*x)*(A*a + C*c))/(c*(a^2 + c^2)) - (2*(A*a + C*c)*(c - a*
exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)) -
((2*(A*c^3 - C*a*c^2))/(c*e*(a^2*c + c^3)) + (2*exp(d + e*x)*(B*c^4 + B*a^2
*c^2 + C*a^2*c^2 - A*a*c^3))/(c^2*e*(a^2*c + c^3)))/(2*a*exp(d + e*x) - c +
 c*exp(2*d + 2*e*x))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**2,x)
```

```
[Out] Timed out
```

3.255 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(2a^2A+3acC-Ac^2\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{5/2}}-\frac{\left(a^2(-C)+3aAc+2c^2C\right)\cosh(d+ex)}{2e\left(a^2+c^2\right)^2(a+c\sinh(d+ex))}-\frac{(Ac-aC)\cosh(d}{2e\left(a^2+c^2\right)(a+c\sinh}$$

```
[Out] -(2*A*a^2-A*c^2+3*C*a*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))
/(a^2+c^2)^(5/2)/e-1/2*B/c/e/(a+c*sinh(e*x+d))^2-1/2*(A*c-C*a)*cosh(e*x+d)/
(a^2+c^2)/e/(a+c*sinh(e*x+d))^2-1/2*(3*A*a*c-C*a^2+2*C*c^2)*cosh(e*x+d)/(a^
2+c^2)^2/e/(a+c*sinh(e*x+d))
```

Rubi [A] time = 0.27, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$-\frac{\left(2a^2A+3acC-Ac^2\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{5/2}}-\frac{\left(a^2(-C)+3aAc+2c^2C\right)\cosh(d+ex)}{2e\left(a^2+c^2\right)^2(a+c\sinh(d+ex))}-\frac{(Ac-aC)\cosh(d}{2e\left(a^2+c^2\right)(a+c\sinh}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x]
```

```
[Out] -(((2*a^2*A - A*c^2 + 3*a*c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 +
 c^2]])/((a^2 + c^2)^(5/2)*e)) - B/(2*c*e*(a + c*Sinh[d + e*x])^2) - ((A*c
- a*C)*Cosh[d + e*x])/(2*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^2) - ((3*a*A*c
 - a^2*C + 2*c^2*C)*Cosh[d + e*x])/(2*(a^2 + c^2)^2*e*(a + c*Sinh[d + e*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2} - \frac{\int \frac{-2(aA+cC)+(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,d}{2\left(a^2+c^2\right)}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2} - \frac{(3}{2}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2} - \frac{(3}{2}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2} - \frac{(3}{2}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2} - \frac{(3}{2}$$

$$= -\frac{\left(2a^2A-Ac^2+3acC\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{5/2}e} - \frac{B}{2ce(a+c\sinh(}$$

Mathematica [A] time = 0.69, size = 170, normalized size = 0.94

$$\frac{-\frac{\left(a^2+c^2\right)\left(B\left(a^2+c^2\right)+c(Ac-aC)\cosh(d+ex)\right)}{c(a+c\sinh(d+ex))^2} + \frac{2\left(2a^2A+3acC-Ac^2\right)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} + \frac{\left(a^2C-3aAc-2c^2C\right)\cosh(d+ex)}{a+c\sinh(d+ex)}}{2e\left(a^2+c^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x
]
```

```
[Out] ((2*(2*a^2*A - A*c^2 + 3*a*c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2
- c^2]])/Sqrt[-a^2 - c^2] - ((a^2 + c^2)*(B*(a^2 + c^2) + c*(A*c - a*C)*Cos
h[d + e*x]))/(c*(a + c*Sinh[d + e*x])^2) + ((-3*a*A*c + a^2*C - 2*c^2*C)*Co
sh[d + e*x])/(a + c*Sinh[d + e*x]))/(2*(a^2 + c^2)^2*e)
```

fricas [B] time = 1.28, size = 1880, normalized size = 10.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*C*a^4*c^2 - 6*A*a^3*c^3 - 2*C*a^2*c^4 - 6*A*a*c^5 - 4*C*c^6 - 2*(2*
A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d)^3 -
2*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*sinh(e*x + d)
^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(
2*B - C)*a^2*c^4 + 3*A*a*c^5 + 2*(B + C)*c^6)*cosh(e*x + d)^2 + 2*(2*(B + C
)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4
 + 3*A*a*c^5 + 2*(B + C)*c^6 - 3*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3
*C*a*c^5 - A*c^6)*cosh(e*x + d))*sinh(e*x + d)^2 + (2*A*a^2*c^3 + 3*C*a*c^4
 - A*c^5 + (2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d)^4 + (2*A*a^2*c^3
 + 3*C*a*c^4 - A*c^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*
c^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 + (2*A*a^2*c^
3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*a^4*c + 6*C*
a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cosh(e*x + d)^2 + 2*(4*A*a^4*c +
 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 + 3*(2*A*a^2*c^3 + 3*C*a*c^4
 - A*c^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*
x + d))*sinh(e*x + d)^2 - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*x
+ d) - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 - (2*A*a^2*c^3 + 3*C*a*c^4 -
A*c^5)*cosh(e*x + d)^3 - 3*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*x +
 d)^2 - (4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cosh(e*
x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh(
e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*c
)*sinh(e*x + d) + 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a)
)/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e*
x + d) + a)*sinh(e*x + d) - c)) - 2*(4*C*a^5*c - 10*A*a^4*c^2 - C*a^3*c^3 -
 11*A*a^2*c^4 - 5*C*a*c^5 - A*c^6)*cosh(e*x + d) - 2*(4*C*a^5*c - 10*A*a^4*
c^2 - C*a^3*c^3 - 11*A*a^2*c^4 - 5*C*a*c^5 - A*c^6 + 3*(2*A*a^4*c^2 + 3*C*a
^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d)^2 - 2*(2*(B + C)*a^6
- 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4 + 3*A
*a*c^5 + 2*(B + C)*c^6)*cosh(e*x + d))*sinh(e*x + d))/((a^6*c^3 + 3*a^4*c^5
 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d)^4 + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 +
c^9)*e*sinh(e*x + d)^4 + 4*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e*cosh
(e*x + d)^3 + 2*(2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*e*cosh(e*
x + d)^2 + 4*((a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d) + (a^
7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e)*sinh(e*x + d)^3 - 4*(a^7*c^2 + 3*
a^5*c^4 + 3*a^3*c^6 + a*c^8)*e*cosh(e*x + d) + 2*(3*(a^6*c^3 + 3*a^4*c^5 +
3*a^2*c^7 + c^9)*e*cosh(e*x + d)^2 + 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a
```

```
*c^8)*e*cosh(e*x + d) + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*e
)*sinh(e*x + d)^2 + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e + 4*((a^6*c^3
 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d)^3 + 3*(a^7*c^2 + 3*a^5*c^4
+ 3*a^3*c^6 + a*c^8)*e*cosh(e*x + d)^2 + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 -
 a^2*c^7 - c^9)*e*cosh(e*x + d) - (a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)
*e)*sinh(e*x + d))
```

giac [B] time = 0.35, size = 423, normalized size = 2.35

$$-\frac{1}{2}\left(\frac{\left(2\,Aa^2+3\,Cac-Ac^2\right)\log\left(\frac{\left|-2\,ce^{(xe+d)}-2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|-2\,ce^{(xe+d)}-2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\left(a^4+2\,a^2c^2+c^4\right)\sqrt{a^2+c^2}}-\frac{2\left(2\,Aa^2c^2e^{(3\,xe+3\,d)}+3\,Cac^3e^{(3\,xe+3\,d)}-Ac^4e^{(3\,xe+3\,d)}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"giac")
```

```
[Out] -1/2*((2*A*a^2 + 3*C*a*c - A*c^2)*log(abs(-2*c*e^(x*e + d) - 2*a - 2*sqrt(a
^2 + c^2))/abs(-2*c*e^(x*e + d) - 2*a + 2*sqrt(a^2 + c^2)))/((a^4 + 2*a^2*c
^2 + c^4)*sqrt(a^2 + c^2)) - 2*(2*A*a^2*c^2*e^(3*x*e + 3*d) + 3*C*a*c^3*e^(
3*x*e + 3*d) - A*c^4*e^(3*x*e + 3*d) - 2*B*a^4*e^(2*x*e + 2*d) - 2*C*a^4*e^
(2*x*e + 2*d) + 6*A*a^3*c*e^(2*x*e + 2*d) - 4*B*a^2*c^2*e^(2*x*e + 2*d) + 5
*C*a^2*c^2*e^(2*x*e + 2*d) - 3*A*a*c^3*e^(2*x*e + 2*d) - 2*B*c^4*e^(2*x*e +
 2*d) - 2*C*c^4*e^(2*x*e + 2*d) + 4*C*a^3*c*e^(x*e + d) - 10*A*a^2*c^2*e^(x
*e + d) - 5*C*a*c^3*e^(x*e + d) - A*c^4*e^(x*e + d) - C*a^2*c^2 + 3*A*a*c^3
 + 2*C*c^4)/((a^4*c + 2*a^2*c^3 + c^5)*(c*e^(2*x*e + 2*d) + 2*a*e^(x*e + d)
 - c)^2))*e^(-1)
```

maple [B] time = 0.21, size = 416, normalized size = 2.31

$$-\frac{\frac{2\left(-\frac{\left(5Aa^2c^2+2Ac^4-2Ba^4-4Ba^2c^2-2Bc^4-3Ca^3c\right)\left(\tanh^3\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2a\left(a^4+2a^2c^2+c^4\right)}-\frac{\left(4Aa^4c-7Aa^2c^3-2Ac^5+2Ba^4c+4Ba^2c^3+2Bc^5-2Ca^5+5Ca^3c^2-2Cac^4\right)\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2\left(a^4+2a^2c^2+c^4\right)a^2}+\frac{\left(11Aa^2c^2\cdots\right.}{}\right.}{\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x)
```

```
[Out] 1/e*(-2*(-1/2*(5*A*a^2*c^2+2*A*c^4-2*B*a^4-4*B*a^2*c^2-2*B*c^4-3*C*a^3*c)/a
/(a^4+2*a^2*c^2+c^4)*tanh(1/2*e*x+1/2*d)^3-1/2*(4*A*a^4*c-7*A*a^2*c^3-2*A*c
^5+2*B*a^4*c+4*B*a^2*c^3+2*B*c^5-2*C*a^5+5*C*a^3*c^2-2*C*a*c^4)/(a^4+2*a^2*
c^2+c^4)/a^2*tanh(1/2*e*x+1/2*d)^2+1/2*(11*A*a^2*c^2+2*A*c^4-2*B*a^4-4*B*a^
```

```
2*c^2-2*B*c^4-5*C*a^3*c+4*C*a*c^3)/(a^4+2*a^2*c^2+c^4)/a*tanh(1/2*e*x+1/2*d
)+1/2*(4*A*a^2*c+A*c^3-2*C*a^3+C*a*c^2)/(a^4+2*a^2*c^2+c^4))/(a*tanh(1/2*e*
x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)^2+(2*A*a^2-A*c^2+3*C*a*c)/(a^4+2*a^2*
c^2+c^4)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2
)^(1/2)))
```

maxima [B] time = 0.47, size = 726, normalized size = 4.03

$$\frac{1}{2}C\left(\frac{3\,ac\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\left(a^4+2\,a^2c^2+c^4\right)\sqrt{a^2+c^2}\,e}+\frac{2\left(3\,ac^3e^{(-3\,ex-3\,d)}+a^2c^2-2\,c^4+\left(4\,a^3c-5\right.\right.}{\left(a^4c^3+2\,a^2c^5+c^7+4\left(a^5c^2+2\,a^3c^4+ac^6\right)e^{(-ex-d)}+2\left(2\,a^6c+3\,a^4c^3-c^7\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"maxima")
```

```
[Out] 1/2*C*(3*a*c*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a
 + sqrt(a^2 + c^2)))/((a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2)*e) + 2*(3*a*c
^3*e^(-3*e*x - 3*d) + a^2*c^2 - 2*c^4 + (4*a^3*c - 5*a*c^3)*e^(-e*x - d) +
(2*a^4 - 5*a^2*c^2 + 2*c^4)*e^(-2*e*x - 2*d))/((a^4*c^3 + 2*a^2*c^5 + c^7 +
 4*(a^5*c^2 + 2*a^3*c^4 + a*c^6)*e^(-e*x - d) + 2*(2*a^6*c + 3*a^4*c^3 - c^
7)*e^(-2*e*x - 2*d) - 4*(a^5*c^2 + 2*a^3*c^4 + a*c^6)*e^(-3*e*x - 3*d) + (a
^4*c^3 + 2*a^2*c^5 + c^7)*e^(-4*e*x - 4*d))*e)) + 1/2*A*((2*a^2 - c^2)*log(
(c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 + c^2
)))/((a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2)*e) - 2*(3*a*c^2 + (10*a^2*c +
c^3)*e^(-e*x - d) + 3*(2*a^3 - a*c^2)*e^(-2*e*x - 2*d) - (2*a^2*c - c^3)*e^
(-3*e*x - 3*d))/((a^4*c^2 + 2*a^2*c^4 + c^6 + 4*(a^5*c + 2*a^3*c^3 + a*c^5)
*e^(-e*x - d) + 2*(2*a^6 + 3*a^4*c^2 - c^6)*e^(-2*e*x - 2*d) - 4*(a^5*c + 2
*a^3*c^3 + a*c^5)*e^(-3*e*x - 3*d) + (a^4*c^2 + 2*a^2*c^4 + c^6)*e^(-4*e*x
- 4*d))*e)) - 2*B*e^(-2*e*x - 2*d)/((4*a*c^2*e^(-e*x - d) - 4*a*c^2*e^(-3*e
*x - 3*d) + c^3*e^(-4*e*x - 4*d) + c^3 + 2*(2*a^2*c - c^3)*e^(-2*e*x - 2*d)
)*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh\left(d+e\,x\right)+C\sinh\left(d+e\,x\right)}{{\left(a+c\sinh\left(d+e\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3,x)
```

```
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**3,x)
```

```
[Out] Timed out
```

3.256 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$

Optimal. Leaf size=250

$$-\frac{\left(-2a^2C+5aAc+3c^2C\right)\cosh(d+ex)}{6e\left(a^2+c^2\right)^2(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}-\frac{\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\tan}{e\left(a^2+c^2\right)^{7/2}}$$

```
[Out] -(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2
+c^2)^(1/2))/(a^2+c^2)^(7/2)/e-1/3*B/c/e/(a+c*sinh(e*x+d))^3-1/3*(A*c-C*a)*
cosh(e*x+d)/(a^2+c^2)/e/(a+c*sinh(e*x+d))^3-1/6*(5*A*a*c-2*C*a^2+3*C*c^2)*c
osh(e*x+d)/(a^2+c^2)^2/e/(a+c*sinh(e*x+d))^2-1/6*(11*A*a^2*c-4*A*c^3-2*C*a^
3+13*C*a*c^2)*cosh(e*x+d)/(a^2+c^2)^3/e/(a+c*sinh(e*x+d))
```

Rubi [A] time = 0.44, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$-\frac{\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{7/2}}-\frac{\left(11a^2Ac-2a^3C+13ac^2C-4Ac^3\right)\cosh(d+ex)}{6e\left(a^2+c^2\right)^3(a+c\sinh(d+ex))}-\frac{(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTanh[(c - a*Tanh[(d + e*x)/
2])/Sqrt[a^2 + c^2]])/((a^2 + c^2)^(7/2)*e)) - B/(3*c*e*(a + c*Sinh[d + e*x
])^3) - ((A*c - a*C)*Cosh[d + e*x])/(3*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^
3) - ((5*a*A*c - 2*a^2*C + 3*c^2*C)*Cosh[d + e*x])/(6*(a^2 + c^2)^2*e*(a +
c*Sinh[d + e*x])^2) - ((11*a^2*A*c - 4*A*c^3 - 2*a^3*C + 13*a*c^2*C)*Cosh[d
 + e*x])/(6*(a^2 + c^2)^3*e*(a + c*Sinh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} - \frac{\int \frac{-3(aA+cC)+2(Ac-aC)\sinh(d+ex}{(a+c\sinh(d+ex))^3}}{3\left(a^2+c^2\right)}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} -$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} -$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} -$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} -$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3} -$$

$$= -\frac{\left(2a^3A-3aAc^2+4a^2cC-c^3C\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{7/2}e} - \frac{}{3ce}$$

Mathematica [A] time = 1.58, size = 235, normalized size = 0.94

$$\frac{-\frac{2\left(a^2+c^2\right)^2\left(B\left(a^2+c^2\right)+c(Ac-aC)\cosh(d+ex)\right)}{c(a+c\sinh(d+ex))^3} + \frac{\left(a^2+c^2\right)\left(2a^2C-5aAc-3c^2C\right)\cosh(d+ex)}{(a+c\sinh(d+ex))^2} + \frac{6\left(2a^3A+4a^2cC-3aAc^2-c^3C\right)\tan^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}}{6e\left(a^2+c^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x
]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTan[(c - a*Tanh[(d + e*x)/
2])/Sqrt[-a^2 - c^2]])/Sqrt[-a^2 - c^2] - (2*(a^2 + c^2)^2*(B*(a^2 + c^2) +
 c*(A*c - a*C)*Cosh[d + e*x]))/(c*(a + c*Sinh[d + e*x])^3) + ((a^2 + c^2)*(
-5*a*A*c + 2*a^2*C - 3*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d + e*x])^2 + ((-1
1*a^2*A*c + 4*A*c^3 + 2*a^3*C - 13*a*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d +
e*x]))/(6*(a^2 + c^2)^3*e)
```

fricas [B] time = 1.02, size = 4350, normalized size = 17.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"fricas")
```

```
[Out] 1/6*(4*C*a^5*c^3 - 22*A*a^4*c^4 - 22*C*a^3*c^5 - 14*A*a^2*c^6 - 26*C*a*c^7
+ 8*A*c^8 + 6*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*
c^7 - C*c^8)*cosh(e*x + d)^5 + 6*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3
*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*sinh(e*x + d)^5 + 30*(2*A*a^6*c^2 + 4*C*a^5
*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(e*x + d)^4 + 3
0*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*
c^7 + (2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*
c^8)*cosh(e*x + d))*sinh(e*x + d)^4 - 4*(4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*
B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16
*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d)^3 - 4*(4*(B + C)*a
^8 - 22*A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c
^4 + 29*A*a^3*c^5 + (16*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8 - 15*(2*A*
a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(e
*x + d)^2 - 30*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a
^2*c^6 - C*a*c^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 12*(4*C*a^7*c - 17*A*a^6
*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6 + 4*C*a*c^7
 - 2*A*c^8)*cosh(e*x + d)^2 + 12*(4*C*a^7*c - 17*A*a^6*c^2 - 13*C*a^5*c^3 -
 11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6 + 4*C*a*c^7 - 2*A*c^8 + 5*(2*A*a
^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(e*
x + d)^3 + 15*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^
2*c^6 - C*a*c^7)*cosh(e*x + d)^2 - (4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*B - 7
*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16*B +
39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d))*sinh(e*x + d)^2 + 3*(2
*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7 - (2*A*a^3*c^4 + 4*C*a^2*c^5 -
 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^6 - (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^
6 - C*c^7)*sinh(e*x + d)^6 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C
*a*c^6)*cosh(e*x + d)^5 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*
c^6 + (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d))*sinh(e
*x + d)^5 - 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*
A*a*c^6 + C*c^7)*cosh(e*x + d)^4 - 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3
```

*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7 + 5*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A
*a*c^6 - C*c^7)*cosh(e*x + d)^2 + 10*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c
^5 - C*a*c^6)*cosh(e*x + d))*sinh(e*x + d)^4 - 4*(4*A*a^6*c + 8*C*a^5*c^2 -
12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(e*x + d)^3 - 4
*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C
*a*c^6 + 5*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^3
+ 15*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d)^2 +
3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*
c^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*
a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x + d)^2 + 3*(8*A*a^5*c^2
+ 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7 - 5*(2*A*a
^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^4 - 20*(2*A*a^4*c^3
+ 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d)^3 - 6*(8*A*a^5*c^2 +
16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x + d
)^2 - 4*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^
5 + 3*C*a*c^6)*cosh(e*x + d))*sinh(e*x + d)^2 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^
4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d) - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3
*A*a^2*c^5 - C*a*c^6 + (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh
(e*x + d)^5 + 5*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*
x + d)^4 + 2*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A
*a*c^6 + C*c^7)*cosh(e*x + d)^3 + 2*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3
- 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(e*x + d)^2 - (8*A*a^5*c^2 +
16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x +
d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh(e*x
+ d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*c)*si
nh(e*x + d) + 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a))/(c
*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e*x +
d) + a)*sinh(e*x + d) - c)) - 6*(4*C*a^6*c^2 - 20*A*a^5*c^3 - 18*C*a^4*c^4
- 15*A*a^3*c^5 - 23*C*a^2*c^6 + 5*A*a*c^7 - C*c^8)*cosh(e*x + d) - 6*(4*C*a
^6*c^2 - 20*A*a^5*c^3 - 18*C*a^4*c^4 - 15*A*a^3*c^5 - 23*C*a^2*c^6 + 5*A*a*
c^7 - C*c^8 - 5*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*
a*c^7 - C*c^8)*cosh(e*x + d)^4 - 20*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4
+ 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 -
22*A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 +
29*A*a^3*c^5 + (16*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d)
^2 - 4*(4*C*a^7*c - 17*A*a^6*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c
^5 + 4*A*a^2*c^6 + 4*C*a*c^7 - 2*A*c^8)*cosh(e*x + d))*sinh(e*x + d))/((a^8
*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^6 + (a^8*
c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*sinh(e*x + d)^6 + 6*(a^9
*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^5 + 3*(4
*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^4
+ 6*((a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)
+ (a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e)*sinh(e*x + d)^5
+ 4*(2*a^11*c + 5*a^9*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x
+ d)^3 + 3*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh

```
(e*x + d)^2 + 10*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*c
osh(e*x + d) + (4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e
)*sinh(e*x + d)^4 - 3*(4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 -
c^12)*e*cosh(e*x + d)^2 + 4*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^1
0 + c^12)*e*cosh(e*x + d)^3 + 15*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c
^9 + a*c^11)*e*cosh(e*x + d)^2 + 3*(4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 +
10*a^4*c^8 - c^12)*e*cosh(e*x + d) + (2*a^11*c + 5*a^9*c^3 - 10*a^5*c^7 - 1
0*a^3*c^9 - 3*a*c^11)*e)*sinh(e*x + d)^3 + 6*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c
^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d) + 3*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^
4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^4 + 20*(a^9*c^3 + 4*a^7*c^5 + 6*
a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^3 + 6*(4*a^10*c^2 + 15*a^8*c^
4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^2 + 4*(2*a^11*c + 5*a^9
*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x + d) - (4*a^10*c^2 +
15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e)*sinh(e*x + d)^2 - (a^8*c^4
+ 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e + 6*((a^8*c^4 + 4*a^6*c^6 +
6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^5 + 5*(a^9*c^3 + 4*a^7*c^5 +
 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^4 + 2*(4*a^10*c^2 + 15*a^8
*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^3 + 2*(2*a^11*c + 5*
a^9*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x + d)^2 - (4*a^10*c
^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d) + (a^9*c^
3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e)*sinh(e*x + d))
```

giac [B] time = 0.50, size = 717, normalized size = 2.87

$$\frac{1}{6}\left(\frac{3\left(2\,Aa^3+4\,Ca^2c-3\,Aac^2-Cc^3\right)\log\left(\frac{\left|2\,ce^{(xe+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(xe+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\left(a^6+3\,a^4c^2+3\,a^2c^4+c^6\right)\sqrt{a^2+c^2}}+\frac{2\left(6\,Aa^3c^3e^{(5\,xe+5\,d)}+12\,Ca^2c^4e^{(5\,xe+5\,d)}-9\,A\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"giac")
```

```
[Out] 1/6*(3*(2*A*a^3 + 4*C*a^2*c - 3*A*a*c^2 - C*c^3)*log(abs(2*c*e^(x*e + d) +
2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(x*e + d) + 2*a + 2*sqrt(a^2 + c^2)))/((
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)) + 2*(6*A*a^3*c^3*e^(5*x
*e + 5*d) + 12*C*a^2*c^4*e^(5*x*e + 5*d) - 9*A*a*c^5*e^(5*x*e + 5*d) - 3*C*
c^6*e^(5*x*e + 5*d) + 30*A*a^4*c^2*e^(4*x*e + 4*d) + 60*C*a^3*c^3*e^(4*x*e
+ 4*d) - 45*A*a^2*c^4*e^(4*x*e + 4*d) - 15*C*a*c^5*e^(4*x*e + 4*d) - 8*B*a^
6*e^(3*x*e + 3*d) - 8*C*a^6*e^(3*x*e + 3*d) + 44*A*a^5*c*e^(3*x*e + 3*d) -
24*B*a^4*c^2*e^(3*x*e + 3*d) + 64*C*a^4*c^2*e^(3*x*e + 3*d) - 82*A*a^3*c^3*
e^(3*x*e + 3*d) - 24*B*a^2*c^4*e^(3*x*e + 3*d) - 78*C*a^2*c^4*e^(3*x*e + 3*
d) + 24*A*a*c^5*e^(3*x*e + 3*d) - 8*B*c^6*e^(3*x*e + 3*d) + 24*C*a^5*c*e^(2
*x*e + 2*d) - 102*A*a^4*c^2*e^(2*x*e + 2*d) - 102*C*a^3*c^3*e^(2*x*e + 2*d)
```

```
 + 36*A*a^2*c^4*e^(2*x*e + 2*d) + 24*C*a*c^5*e^(2*x*e + 2*d) - 12*A*c^6*e^(
2*x*e + 2*d) - 12*C*a^4*c^2*e^(x*e + d) + 60*A*a^3*c^3*e^(x*e + d) + 66*C*a
^2*c^4*e^(x*e + d) - 15*A*a*c^5*e^(x*e + d) + 3*C*c^6*e^(x*e + d) + 2*C*a^3
*c^3 - 11*A*a^2*c^4 - 13*C*a*c^5 + 4*A*c^6)/((a^6*c + 3*a^4*c^3 + 3*a^2*c^5
 + c^7)*(c*e^(2*x*e + 2*d) + 2*a*e^(x*e + d) - c)^3))*e^(-1)
```

maple [B] time = 0.23, size = 844, normalized size = 3.38

$$-\frac{2\left(-\frac{\left(9A a^4c^2+6A a^2c^4+2A c^6-2B a^6-6B a^4c^2-6B a^2c^4-2B c^6-4C a^5c+C a^3c^3\right)\left(\tanh^5\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2a\left(a^6+3a^4c^2+3a^2c^4+c^6\right)}-\frac{\left(6A a^6c-27A a^4c^3-12A a^2c^5-4A c^7+4B a^6c+12B a^4c^3+12B a^2c^5+4B c^7-2\right.}{2\left(a^6+3a^4c^2+3a^2c^4+c^6\right)a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x)
```

```
[Out] 1/e*(-2*(-1/2*(9*A*a^4*c^2+6*A*a^2*c^4+2*A*c^6-2*B*a^6-6*B*a^4*c^2-6*B*a^2*
c^4-2*B*c^6-4*C*a^5*c+C*a^3*c^3)/a/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e
*x+1/2*d)^5-1/2*(6*A*a^6*c-27*A*a^4*c^3-12*A*a^2*c^5-4*A*c^7+4*B*a^6*c+12*B
*a^4*c^3+12*B*a^2*c^5+4*B*c^7-2*C*a^7+14*C*a^5*c^2-11*C*a^3*c^4-2*C*a*c^6)/
(a^6+3*a^4*c^2+3*a^2*c^4+c^6)/a^2*tanh(1/2*e*x+1/2*d)^4+1/3/a^3*(54*A*a^6*c
^2-21*A*a^4*c^4-4*A*a^2*c^6-4*A*c^8-6*B*a^8-14*B*a^6*c^2-6*B*a^4*c^4+6*B*a^
2*c^6+4*B*c^8-18*C*a^7*c+42*C*a^5*c^3-17*C*a^3*c^5-2*C*a*c^7)/(a^6+3*a^4*c^
2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^3+1/a^2*(6*A*a^6*c-20*A*a^4*c^3-3*A*a^
2*c^5-2*A*c^7+2*B*a^6*c+6*B*a^4*c^3+6*B*a^2*c^5+2*B*c^7-2*C*a^7+10*C*a^5*c^
2-14*C*a^3*c^4-C*a*c^6)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^2
-1/2/a*(27*A*a^4*c^2+4*A*a^2*c^4+2*A*c^6-2*B*a^6-6*B*a^4*c^2-6*B*a^2*c^4-2*
B*c^6-8*C*a^5*c+19*C*a^3*c^3+2*C*a*c^5)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(
1/2*e*x+1/2*d)-1/6*(18*A*a^4*c+5*A*a^2*c^3+2*A*c^5-6*C*a^5+10*C*a^3*c^2+C*a
*c^4)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*
e*x+1/2*d)-a)^3+(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)/(a^6+3*a^4*c^2+3*a^2*c^
4+c^6)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^
(1/2)))
```

maxima [B] time = 0.51, size = 1263, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"maxima")
```

```
[Out] 1/6*A*(3*(2*a^2 - 3*c^2)*a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^
(-e*x - d) - a + sqrt(a^2 + c^2)))/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqr
```

```
t(a^2 + c^2)*e) - 2*(11*a^2*c^3 - 4*c^5 + 15*(4*a^3*c^2 - a*c^4)*e^(-e*x -
d) + 6*(17*a^4*c - 6*a^2*c^3 + 2*c^5)*e^(-2*e*x - 2*d) + 2*(22*a^5 - 41*a^3
*c^2 + 12*a*c^4)*e^(-3*e*x - 3*d) - 15*(2*a^4*c - 3*a^2*c^3)*e^(-4*e*x - 4*
d) + 3*(2*a^3*c^2 - 3*a*c^4)*e^(-5*e*x - 5*d))/((a^6*c^3 + 3*a^4*c^5 + 3*a^
2*c^7 + c^9 + 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-e*x - d) + 3*
(4*a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-2*e*x - 2*d) + 4*(2*
a^9 + 3*a^7*c^2 - 3*a^5*c^4 - 7*a^3*c^6 - 3*a*c^8)*e^(-3*e*x - 3*d) - 3*(4*
a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-4*e*x - 4*d) + 6*(a^7*c
^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-5*e*x - 5*d) - (a^6*c^3 + 3*a^4*c^5
 + 3*a^2*c^7 + c^9)*e^(-6*e*x - 6*d))*e)) + 1/6*C*(3*(4*a^2*c - c^3)*log((c
*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 + c^2))
)/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)*e) + 2*(2*a^3*c^3 -
13*a*c^5 + 3*(4*a^4*c^2 - 22*a^2*c^4 - c^6)*e^(-e*x - d) + 6*(4*a^5*c - 17*
a^3*c^3 + 4*a*c^5)*e^(-2*e*x - 2*d) + 2*(4*a^6 - 32*a^4*c^2 + 39*a^2*c^4)*e
^(-3*e*x - 3*d) + 15*(4*a^3*c^3 - a*c^5)*e^(-4*e*x - 4*d) - 3*(4*a^2*c^4 -
c^6)*e^(-5*e*x - 5*d))/((a^6*c^4 + 3*a^4*c^6 + 3*a^2*c^8 + c^10 + 6*(a^7*c^
3 + 3*a^5*c^5 + 3*a^3*c^7 + a*c^9)*e^(-e*x - d) + 3*(4*a^8*c^2 + 11*a^6*c^4
 + 9*a^4*c^6 + a^2*c^8 - c^10)*e^(-2*e*x - 2*d) + 4*(2*a^9*c + 3*a^7*c^3 -
3*a^5*c^5 - 7*a^3*c^7 - 3*a*c^9)*e^(-3*e*x - 3*d) - 3*(4*a^8*c^2 + 11*a^6*c
^4 + 9*a^4*c^6 + a^2*c^8 - c^10)*e^(-4*e*x - 4*d) + 6*(a^7*c^3 + 3*a^5*c^5
+ 3*a^3*c^7 + a*c^9)*e^(-5*e*x - 5*d) - (a^6*c^4 + 3*a^4*c^6 + 3*a^2*c^8 +
c^10)*e^(-6*e*x - 6*d))*e)) - 8/3*B*e^(-3*e*x - 3*d)/((6*a*c^3*e^(-e*x - d)
 + 6*a*c^3*e^(-5*e*x - 5*d) - c^4*e^(-6*e*x - 6*d) + c^4 + 3*(4*a^2*c^2 - c
^4)*e^(-2*e*x - 2*d) + 4*(2*a^3*c - 3*a*c^3)*e^(-3*e*x - 3*d) - 3*(4*a^2*c^
2 - c^4)*e^(-4*e*x - 4*d))*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\cosh(d + e\,x) + C\sinh(d + e\,x)}{(a + c\sinh(d + e\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4,x)
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**4,x)
[Out] Timed out
```

3.257 $\int \frac{x^3}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=439

$$\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{3\text{Li}_4\left(-\right.}{8}$$

```
[Out] 1/2*x^3*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-
1/2*x^3*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+
3/4*x^2*polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^
(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/
(a-b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1
/2)/(a-b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/
a^(1/2)/(a-b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2))
)/a^(1/2)/(a-b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2
)))/a^(1/2)/(a-b)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5629, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{3x\text{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sq
rt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a -
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(
4*Sqrt[a]*Sqrt[a - b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[
a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5629

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = 2\int \frac{x^3}{2a-b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^3}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^3}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{3\int x^2\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a\cdots}\right.}{2\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \cdots$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \cdots$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \cdots$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \cdots$$

Mathematica [A] time = 0.86, size = 319, normalized size = 0.73

$$\frac{-6x^2\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)+6x^2\mathrm{Li}_2\left(\frac{be^{2x}}{-2a+2\sqrt{a-b}\sqrt{a}+b}\right)+6x\mathrm{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)-6x\mathrm{Li}_3\left(\frac{be^{2x}}{-2a+2\sqrt{a-b}\sqrt{a}+b}\right)-3\mathrm{Li}_4}{8\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (-4*x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)] + 4*x^3*Log[
1 - (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] - 6*x^2*PolyLog[2, -((b
*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))] + 6*x^2*PolyLog[2, (b*E^(2*x)
)/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] + 6*x*PolyLog[3, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))] - 6*x*PolyLog[3, (b*E^(2*x))/(-2*a + 2*Sqrt[a
]*Sqrt[a - b] + b)] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a -
b] - b))] + 3*PolyLog[4, (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)])/(
8*Sqrt[a]*Sqrt[a - b])
```

fricas [C] time = 0.85, size = 1655, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^3*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 3*b*x^2*sqrt((a^2 - a*b)
/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sin
h(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)
 + b)/b + 1) + 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2
*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2
*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*
b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*s
inh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b
) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (
2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2
```

```
*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 6*b*x*sqrt((a^2 - a*b)
/b^2)*polylog(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*
sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)
/b)/b) - 6*b*x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a
- b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*
sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*x*sqrt((a^2 - a*b)/b^2)*polylo
g(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqr
t((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 6*b*
x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
+ 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*
b)/b^2) - 2*a + b)/b)/b) + 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*
cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^
2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*sqrt((a^2 - a*b
)/b^2)*polylog(4, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a -
b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*cosh(x) + (2*a -
 b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sq
rt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4,
 -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((
a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b))/(a^2 - a
*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

maple [B] time = 0.11, size = 919, normalized size = 2.09

$$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)x^3}{-2\sqrt{a\,(a-b)}\,-2a+b}+\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)a\,x^3}{\sqrt{a\,(a-b)}\,\left(-2\sqrt{a\,(a-b)}\,-2a+b\right)}-\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)b\,x^3}{2\sqrt{a\,(a-b)}\,\left(-2\sqrt{a\,(a-b)}\,-2a+b\right)}-\frac{}{2\left(-2\sqrt{\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*sinh(x)^2),x)
```

```
[Out] 1/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x^
3+1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b)
)^(1/2)-2*a+b))*a*x^3-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b
```

```
*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b*x^3-1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*
x^4-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^4+1/4/(a*(a-b))^(1/2
)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^4+3/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2
,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x^2+3/2/(a*(a-b))^(1/2)/(-2*(a*(a-b
))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x^2-3/4/
(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))*b*x^2-3/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/
(-2*(a*(a-b))^(1/2)-2*a+b))*x-3/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b
)*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x+3/4/(a*(a-b))^(1/2)/
(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))
*b*x+3/4/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(4,b*exp(2*x)/(-2*(a*(a-b))^(1/2
)-2*a+b))+3/4/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(4,b*exp(2*
x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a-3/8/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*
a+b)*polylog(4,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b+1/2/(a*(a-b))^(1/2)
*x^3*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))-1/4/(a*(a-b))^(1/2)*x^4+3/4
/(a*(a-b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))-3/4/(a
*(a-b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))+3/8/(a*(a-b
))^(1/2)*polylog(4,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{b\,{\mathrm{sinh}\left(x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*sinh(x)^2),x)
```

```
[Out] int(x^3/(a + b*sinh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sinh(x)**2),x)
[Out] Integral(x**3/(a + b*sinh(x)**2), x)
```

3.258 $\int \frac{x^2}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=327

$$\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+\cdots}\right.}{2\sqrt{a}\sqrt{a-\cdots}}$$

```
[Out] 1/2*x^2*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-
1/2*x^2*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+
1/2*x*polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1
/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b
)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a
-b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/
(a-b)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5629, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{\text{PolyLog}\left(3,-\cdots\right.}{4\sqrt{a}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a
 - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
2*Sqrt[a]*Sqrt[a - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - PolyLog[3, -((b*E^(
2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b
])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5629

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = 2\int \frac{x^2}{2a-b+b\cosh(2x)}\,dx$$
$$= 4\int \frac{e^{2x}x^2}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x^2}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^2}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\int x\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)}{\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\text{Li}_2}{}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\text{Li}_2}{}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\text{Li}_2}{}$$

Mathematica [A] time = 0.75, size = 240, normalized size = 0.73

$$\frac{-2x\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)+2x\text{Li}_2\left(\frac{be^{2x}}{-2a+2\sqrt{a-b}\sqrt{a}+b}\right)+\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)-\text{Li}_3\left(\frac{be^{2x}}{-2a+2\sqrt{a-b}\sqrt{a}+b}\right)-2x^2\log\left(\frac{}{2\sqrt{a}}\right.}{4\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sinh[x]^2),x]
```

```
[Out] (-2*x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)] + 2*x^2*Log[
1 - (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] - 2*x*PolyLog[2, -((b*E
^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))] + 2*x*PolyLog[2, (b*E^(2*x))/(-
2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] + PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[
a]*Sqrt[a - b] - b))] - PolyLog[3, (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b
] + b)])/(4*Sqrt[a]*Sqrt[a - b])
```

fricas [C] time = 1.92, size = 1247, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^2*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 2*b*x*sqrt((a^2 - a*b)/b
^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(
x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) +
 b)/b + 1) + 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a -
 b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*s
qrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2
)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x)
)*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)
/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)
*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 2*b*sqrt((a^2 - a*b)/b^2)*poly
log(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*s
qrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) - 2
*b*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b)/b) + 2*b*sqrt((a^2 - a*b)/b^2)*polylog(3, ((2*a - b
)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 2*b*sqrt((a^2 - a*
b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
 b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
b)/b)/b))/(a^2 - a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

maple [B] time = 0.09, size = 710, normalized size = 2.17

$$-\frac{2x^3}{3\left(-2\sqrt{a\,(a-b)}-2a+b\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)}{-2\sqrt{a\,(a-b)}-2a+b}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)}{-2\sqrt{a\,(a-b)}-2a+b}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)}{2\left(-2\sqrt{a\,(a-b)}-2a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sinh(x)^2),x)
```

```
[Out] -2/3/(-2*(a*(a-b))^(1/2)-2*a+b)*x^3+1/(-2*(a*(a-b))^(1/2)-2*a+b)*x^2*ln(1-b
*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/(-2*(a*(a-b))^(1/2)-2*a+b)*x*polylo
g(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*p
olylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-2/3/(a*(a-b))^(1/2)/(-2*(a*
(a-b))^(1/2)-2*a+b)*a*x^3+1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^
2*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/(a*(a-b))^(1/2)/(-2*(a*(a-b
))^(1/2)-2*a+b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/2/(a
*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*polylog(3,b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))+1/3/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^3-1/2/(
a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^2*ln(1-b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x*polylog
(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/4/(a*(a-b))^(1/2)/(-2*(a*(a-b))
^(1/2)-2*a+b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/3/(a*(a-
b))^(1/2)*x^3+1/2/(a*(a-b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*
a+b))+1/2/(a*(a-b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
-1/4/(a*(a-b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{b\,{\sinh(x)}^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sinh(x)^2),x)
```

```
[Out] int(x^2/(a + b*sinh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \sinh^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x**2/(a + b*sinh(x)**2), x)
```

3.259 $\int \frac{x}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=215

$$\frac{\mathrm{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{\mathrm{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}$$

```
[Out] 1/2*x*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-1/
2*x*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+1/4*
polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-1/
4*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5629, 3320, 2264, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}\right.}{2\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sinh[x]^2),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqrt[
a - b]) - (x*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqr
t[a]*Sqrt[a - b]) + PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] -
 b))]/(4*Sqrt[a]*Sqrt[a - b]) - PolyLog[2, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*S
qrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5629

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rubi steps

$$\int \frac{x}{a+b\sinh^2(x)}\,dx = 2\int \frac{x}{2a-b+b\cosh(2x)}\,dx$$
$$= 4\int \frac{e^{2x}x}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\int \log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)}{x}\,d\right.}{4\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{\text{Li}_2\left(-\frac{}{2a}\right.}{4\sqrt{a}}$$

Mathematica [C] time = 0.82, size = 576, normalized size = 2.68

$$-\frac{-2i\cos^{-1}\left(1-\frac{2a}{b}\right)\tan^{-1}\left(\frac{\sqrt{ab-a^2}\tanh(x)}{a}\right) - \log\left(\frac{2a\left(\sqrt{a(b-a)}-ia+ib\right)(\tanh(x)-1)}{b\sqrt{a(b-a)}\tanh(x)-iab}\right)\left(2\tan^{-1}\left(\frac{\sqrt{ab-a^2}\tanh(x)}{a}\right)+\cos^{-1}\left(1-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sinh[x]^2),x]
```

```
[Out] -1/4*(4*x*ArcTan[(a*Coth[x])/Sqrt[-(a*(a - b))]] - (2*I)*ArcCos[1 - (2*a)/b
]*ArcTan[(Sqrt[-a^2 + a*b]*Tanh[x])/a] + (ArcCos[1 - (2*a)/b] + 2*(ArcTan[(
a*Coth[x])/Sqrt[-(a*(a - b))]] + ArcTan[(Sqrt[-a^2 + a*b]*Tanh[x])/a]))*Log
[(Sqrt[2]*Sqrt[a*(-a + b)])/(Sqrt[b]*E^x*Sqrt[2*a - b + b*Cosh[2*x]])] + (A
rcCos[1 - (2*a)/b] - 2*(ArcTan[(a*Coth[x])/Sqrt[-(a*(a - b))]] + ArcTan[(Sq
rt[-a^2 + a*b]*Tanh[x])/a]))*Log[(Sqrt[2]*Sqrt[a*(-a + b)]*E^x)/(Sqrt[b]*Sq
rt[2*a - b + b*Cosh[2*x]])] - (ArcCos[1 - (2*a)/b] + 2*ArcTan[(Sqrt[-a^2 +
a*b]*Tanh[x])/a])*Log[(2*a*((-I)*a + I*b + Sqrt[a*(-a + b)])*(-1 + Tanh[x])
)/((-I)*a*b + b*Sqrt[a*(-a + b)]*Tanh[x])] - (ArcCos[1 - (2*a)/b] - 2*ArcTa
n[(Sqrt[-a^2 + a*b]*Tanh[x])/a])*Log[(2*a*(I*a - I*b + Sqrt[a*(-a + b)])*(1
 + Tanh[x]))/((-I)*a*b + b*Sqrt[a*(-a + b)]*Tanh[x])] + I*(-PolyLog[2, ((-2
*a + b - (2*I)*Sqrt[a*(-a + b)])*(I*a + Sqrt[a*(-a + b)]*Tanh[x]))/((-I)*a*
```

```
b + b*Sqrt[a*(-a + b)]*Tanh[x])] + PolyLog[2, ((-2*a + b + (2*I)*Sqrt[a*(-a
 + b)])*(I*a + Sqrt[a*(-a + b)]*Tanh[x]))/((-I)*a*b + b*Sqrt[a*(-a + b)]*Ta
nh[x])]))/Sqrt[a*(-a + b)]
```

fricas [B] time = 3.28, size = 837, normalized size = 3.89

$$-\frac{bx\sqrt{\frac{a^2-ab}{b^2}}\log\left(\frac{\left((2a-b)\cosh(x)+(2a-b)\sinh(x)-2\left(b\cosh(x)+b\sinh(x)\right)\sqrt{\frac{a^2-ab}{b^2}}\right)\sqrt{-\frac{2b\sqrt{\frac{a^2-ab}{b^2}}+2a-b}{b}}+b}{b}\right)+bx\sqrt{\frac{a^2-ab}{b^2}}\log\left(-\frac{\left((2a-\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b
)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x*sqrt((a^
2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b
)/b) + b)/b) - b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) + (2*a -
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqr
t((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + b*sqrt((a^2 - a*b)/b^2)*dilog(-(
((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^
2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) + b)/b + 1) +
 b*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/
b^2) + 2*a - b)/b) - b)/b + 1) - b*sqrt((a^2 - a*b)/b^2)*dilog(-(((2*a - b)
*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b
^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)/b + 1) - b*sqrt((a^
2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x)
+ b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b) - b)/b + 1))/(a^2 - a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\sinh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

maple [B] time = 0.09, size = 505, normalized size = 2.35

$$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)x}{-2\sqrt{a\,(a-b)}-2a+b}+\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)ax}{\sqrt{a\,(a-b)}\,\left(-2\sqrt{a\,(a-b)}-2a+b\right)}-\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)bx}{2\sqrt{a\,(a-b)}\,\left(-2\sqrt{a\,(a-b)}-2a+b\right)}-\frac{}{-2\sqrt{a\,(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sinh(x)^2),x)
```

```
[Out] 1/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x+
1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^
(1/2)-2*a+b))*a*x-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp
(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b*x-1/(-2*(a*(a-b))^(1/2)-2*a+b)*x^2-1/(a
*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^2+1/2/(a*(a-b))^(1/2)/(-2*(a*(
a-b))^(1/2)-2*a+b)*b*x^2+1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x
)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b
)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a-1/4/(a*(a-b))^(1/2)/(-
2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b
+1/2/(a*(a-b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))-1/2/(a*(a
-b))^(1/2)*x^2+1/4/(a*(a-b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(1/2)-
2*a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{b\,{\sinh(x)}^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sinh(x)^2),x)
```

```
[Out] int(x/(a + b*sinh(x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x/(a + b*sinh(x)**2), x)
```

3.260 $\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{9}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

```
[Out] -9/4*Chi(b*x)*cosh(a)+1/4*Chi(3*b*x)*cosh(3*a)-9/4*Shi(b*x)*sinh(a)+1/4*Shi
(3*b*x)*sinh(3*a)
```

Rubi [A] time = 0.46, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6742, 3303, 3298, 3301, 5448}

$$-\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{9}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (9*S
inh[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx &= \int \left(-\frac{2\cosh(a+bx)}{x}+\frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\right)dx\\ &= -\left(2\int \frac{\cosh(a+bx)}{x}\,dx\right)+\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx\\ &= -\left((2\cosh(a))\int \frac{\cosh(bx)}{x}\,dx\right)-(2\sinh(a))\int \frac{\sinh(bx)}{x}\,dx+\int \Bigg(\\ &= -2\cosh(a)\text{Chi}(bx)-2\sinh(a)\text{Shi}(bx)-\frac{1}{4}\int \frac{\cosh(a+bx)}{x}\,dx+\frac{1}{4}\\ &= -2\cosh(a)\text{Chi}(bx)-2\sinh(a)\text{Shi}(bx)-\frac{1}{4}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx\\ &= -\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{9}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\end{aligned}$$

Mathematica [A] time = 0.10, size = 41, normalized size = 0.87

$$\frac{1}{4}(-9\cosh(a)\text{Chi}(bx)+\cosh(3a)\text{Chi}(3bx)-9\sinh(a)\text{Shi}(bx)+\sinh(3a)\text{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] - 9*Sinh[a]*S
inhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

fricas [A] time = 1.08, size = 67, normalized size = 1.43

$$\frac{1}{8}\left(\text{Ei}\left(3\,bx\right)+\text{Ei}\left(-3\,bx\right)\right)\cosh\left(3\,a\right)-\frac{9}{8}\left(\text{Ei}\left(bx\right)+\text{Ei}\left(-bx\right)\right)\cosh(a)+\frac{1}{8}\left(\text{Ei}\left(3\,bx\right)-\text{Ei}\left(-3\,bx\right)\right)\sinh\left(3\,a\right)-\frac{9}{8}\left(\text{Ei}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9/8*(Ei(b*x) + Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 9/8*(Ei(b*x) - Ei(-b*x))*sinh(a)
```

giac [A] time = 0.34, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)}-\frac{9}{8}\,\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)}+\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}-\frac{9}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

maple [A] time = 0.21, size = 47, normalized size = 1.00

$$-\frac{\mathrm{e}^{-3a}\,\mathrm{Ei}\left(1,3bx\right)}{8}+\frac{9\,\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,bx\right)}{8}+\frac{9\,\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-bx\right)}{8}-\frac{\mathrm{e}^{3a}\,\mathrm{Ei}\left(1,-3bx\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x)
```

```
[Out] -1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)+9/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

maxima [A] time = 0.45, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}\left(3\,bx\right)e^{\left(3\,a\right)}-\frac{9}{8}\,\mathrm{Ei}\left(-bx\right)e^{\left(-a\right)}+\frac{1}{8}\,\mathrm{Ei}\left(-3\,bx\right)e^{\left(-3\,a\right)}-\frac{9}{8}\,\mathrm{Ei}\left(bx\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh\left(a+b\,x\right)\,\left({\mathrm{sinh}\left(a+b\,x\right)}^2-2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x,x)
```

```
[Out] int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sinh^2\left(a+bx\right)-2\right)\cosh\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)**2)/x,x)
```

```
[Out] Integral((sinh(a + b*x)**2 - 2)*cosh(a + b*x)/x, x)
```

3.261 $\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[Out] 3/4*Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Shi(3*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3298}

$$\frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - SinhIntegral[(3*Sqrt[
1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{i\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x} - \frac{i\sinh(3x)}{4x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$
$$= \frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.95

$$\frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) - \text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - SinhIntegral[(3*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/(4*a)
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^{3}}{a^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

```
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh^{3}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

3.262 $\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

Optimal. Leaf size=58

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\mathrm{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6681, 3312, 3301}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\mathrm{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) + Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$
$$= -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} + \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.98

$$-\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} + \frac{\log(1-ax)}{4a} - \frac{\log(ax+1)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a + Log[1 - a*x]/(4*a) -
 Log[1 + a*x]/(4*a)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log(ax+1)}{4\,a}+\frac{\log(ax-1)}{4\,a}-\frac{1}{4}\int\frac{e^{\left(\frac{2\,\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^2x^2-1}\,dx-\frac{1}{4}\int\frac{e^{\left(-\frac{2\,\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -1/4*log(a*x + 1)/a + 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1
)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sqr
t(a*x + 1))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

```
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

3.263 $\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

[Out] -Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6681, 3298}

$$-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]

[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.00

$$-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

```
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

3.264 $\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

Optimal. Leaf size=40

$$\operatorname{Int}\left(\frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)}, x\right)$$

```
[Out] Unintegrable(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(Defer[Subst][Defer[Int][Csch[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}(x)}{x} \, dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 13.09, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

fricas [A] time = 1.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sinh(sqrt(-a*x
+ 1)/sqrt(a*x + 1))), x)
```

3.265 $\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)},x\right)$$

[Out] Unintegrable(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]

[Out] -(Defer[Subst][Defer[Int][Csch[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rubi steps

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}^2(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 38.06, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]

[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

fricas [A] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2x^2+1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{ax+1}}{\sqrt{-ax+1}\,ae^{\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}-\sqrt{-ax+1}\,a}-\int \frac{\sqrt{ax+1}}{\left(a^2x^2-1\right)\sqrt{-ax+1}\,e^{\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}+\left(a^2x^2-1\right)\sqrt{-ax+1}}\,dx+\int \frac{}{\left(a^2x^2-1\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

```
[Out] 2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - sqrt
(-a*x + 1)*a) - integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sq
rt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x) + integrate
(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sqrt(-a*x + 1)/sqrt(a*x + 1
)) - (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] -Integral(1/(a**2*x**2*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - sinh(sqrt(-a
*x + 1)/sqrt(a*x + 1))**2), x)
```

3.266 $\int \sinh\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=54

$$\frac{x\sinh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}-\frac{bnx\cosh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}$$

```
[Out] -b*n*x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5517}

$$\frac{x\sinh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}-\frac{bnx\cosh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2)) + (x*Sinh[a + b*Log[c*x^n]]
)/(1 - b^2*n^2)
```

Rule 5517

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Cosh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \sinh\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{bnx\cosh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}+\frac{x\sinh\left(a+b\log\left(cx^n\right)\right)}{1-b^2n^2}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.76

$$\frac{x\left(bn\cosh\left(a+b\log\left(cx^n\right)\right)-\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{b^2n^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(b*n*Cosh[a + b*Log[c*x^n]] - Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

fricas [A] time = 0.81, size = 44, normalized size = 0.81

$$\frac{bnx\cosh\left(bn\log(x)+b\log(c)+a\right)-x\sinh\left(bn\log(x)+b\log(c)+a\right)}{b^2n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a) - x*sinh(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 - 1)
```

giac [A] time = 0.28, size = 47, normalized size = 0.87

$$\frac{c^bxx^{bn}e^a}{2\,(bn+1)}+\frac{xe^{(-a)}}{2\,(bn-1)c^bx^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) + 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sinh\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n)),x)
```

```
[Out] int(sinh(a+b*ln(c*x^n)),x)
```

maxima [A] time = 0.35, size = 52, normalized size = 0.96

$$\frac{c^bxe^{\left(b\log(x^n)+a\right)}}{2\,(bn+1)}+\frac{xe^{\left(-b\log(x^n)-a\right)}}{2\left(bc^bn-c^b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) + 1/2*x*e^(-b*log(x^n) - a)/(b*c^b*n
 - c^b)
```

mupad [B] time = 0.65, size = 43, normalized size = 0.80

$$\frac{x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,b\,n-2\right)}+\frac{x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,b\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n)),x)
[Out] (x*exp(-a))/((c*x^n)^b*(2*b*n - 2)) + (x*exp(a)*(c*x^n)^b)/(2*b*n + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sinh{\left(a - \frac{\log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = - \frac{1}{n} \\ \int \sinh{\left(a + \frac{\log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = \frac{1}{n} \\ \frac{b n x \cosh{\left(a + b n \log{\left(x \right)} + b \log{\left(c \right)} \right)}}{b^{2} n^{2} - 1} - \frac{x \sinh{\left(a + b n \log{\left(x \right)} + b \log{\left(c \right)} \right)}}{b^{2} n^{2} - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n)),x)
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(si
nh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*cosh(a + b*n*log(x) + b*log(
c))/(b**2*n**2 - 1) - x*sinh(a + b*n*log(x) + b*log(c))/(b**2*n**2 - 1), Tr
ue))
```

3.267 $\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=88

$$\frac{x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{1-4b^2n^2}-\frac{2bnx\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}$$

```
[Out] 2*b^2*n^2*x/(-4*b^2*n^2+1)-2*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/
(-4*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5519, 8}

$$\frac{x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{1-4b^2n^2}-\frac{2bnx\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x)/(1 - 4*b^2*n^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*
Log[c*x^n]])/(1 - 4*b^2*n^2) + (x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5519

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Sinh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (-Dist[(b^
2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Sinh[d*(a + b*Log[c*x^n])]^
(p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a + b*
Log[c*x^n])]^(p - 1))/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{2bnx\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{1-4b^2n^2}+\frac{x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{1-4b^2n^2}+$$

$$=\frac{2b^2n^2x}{1-4b^2n^2}-\frac{2bnx\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{1-4b^2n^2}+\frac{x\sinh^2\left(a+b\log\right.}{1-4b^2n}$$

Mathematica [A] time = 0.12, size = 55, normalized size = 0.62

$$-\frac{x\left(-2bn\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+4b^2n^2-1\right)}{8b^2n^2-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] -((x*(-1 + 4*b^2*n^2 + Cosh[2*(a + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a + b*Log
[c*x^n])]))/(-2 + 8*b^2*n^2))
```

fricas [A] time = 1.05, size = 91, normalized size = 1.03

$$\frac{4\,bnx\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)-x\cosh\left(bn\log(x)+b\log(c)+a\right)^2-x\,\mathrm{si}}{2\left(4\,b^2n^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) + a)
^2 - (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

giac [A] time = 0.26, size = 169, normalized size = 1.92

$$\frac{bc^{2\,b}nxx^{2\,bn}e^{(2\,a)}}{2\left(4\,b^2n^2-1\right)}-\frac{2\,b^2n^2x}{4\,b^2n^2-1}-\frac{c^{2\,b}xx^{2\,bn}e^{(2\,a)}}{4\left(4\,b^2n^2-1\right)}-\frac{bnxe^{(-2\,a)}}{2\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}+\frac{x}{2\left(4\,b^2n^2-1\right)}-\frac{xe^{(-2\,a)}}{4\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 2*b^2*n^2*x/(4*b^2*n^
2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-2*
a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \sinh^2\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^2,x)
```

maxima [A] time = 0.36, size = 67, normalized size = 0.76

$$\frac{c^{2\,b}xe^{\left(2\,b\log(x^n)+2\,a\right)}}{4\,(2\,bn+1)}-\frac{1}{2}\,x-\frac{xe^{(-2\,a)}}{4\left(2\,bc^{2\,b}n-c^{2\,b}\right)(x^n)^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) - 1/2*x - 1/4*x*e^(-2*a)/(
(2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

mupad [B] time = 0.65, size = 53, normalized size = 0.60

$$\frac{x\,e^{2\,a}\,(c\,x^n)^{2\,b}}{8\,b\,n+4}-\frac{x\,e^{-2\,a}}{(c\,x^n)^{2\,b}\,(8\,b\,n-4)}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*exp(2*a)*(c*x^n)^(2*b))/(8*b*n + 4) - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*
n - 4)) - x/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sinh^2\left(a-\frac{\log(cx^n)}{2n}\right)dx \\ \int \sinh^2\left(a+\frac{\log(cx^n)}{2n}\right)dx \\ \frac{2b^2n^2x\sinh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2n^2-1}-\frac{2b^2n^2x\cosh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2n^2-1}+\frac{2bnx\sinh\left(a+bn\log(x)+b\log(c)\right)\cosh\left(a+bn\log(x)+b\log(c)\right)}{4b^2n^2-1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))), (
Integral(sinh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (2*b**2*n**2*
x*sinh(a + b*n*log(x) + b*log(c))**2/(4*b**2*n**2 - 1) - 2*b**2*n**2*x*cosh
(a + b*n*log(x) + b*log(c))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*n*log
(x) + b*log(c))*cosh(a + b*n*log(x) + b*log(c))/(4*b**2*n**2 - 1) - x*sinh(
a + b*n*log(x) + b*log(c))**2/(4*b**2*n**2 - 1), True))
```

3.268 $\int \sinh^3\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=149

$$\frac{x\sinh^3\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}-\frac{3bnx\sinh^2\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}+\frac{6b^2n^2x\sinh\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4-10b^2n^2+1}-\frac{6}{}$$

[Out] -6*b^3*n^3*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+6*b^2*n^2*x*sinh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)-3*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^2/(-9*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^3/(-9*b^2*n^2+1)

Rubi [A] time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5519, 5517}

$$\frac{x\sinh^3\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}+\frac{6b^2n^2x\sinh\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4-10b^2n^2+1}-\frac{6b^3n^3x\cosh\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4-10b^2n^2+1}-\frac{3bnx\sinh^2\left(a+b\log\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2
*n^2*x*Sinh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/(1 - 9*b^2*n^2) + (x*Sinh[a +
b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2)
```

Rule 5517

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> -Simp[(
x*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] + Simp[(b*d*n*x*Cosh[d*
(a + b*Log[c*x^n])])/(b^2*d^2*n^2 - 1), x] /; FreeQ[{a, b, c, d, n}, x] &&
NeQ[b^2*d^2*n^2 - 1, 0]
```

Rule 5519

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Sinh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (-Dist[(b^
2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Sinh[d*(a + b*Log[c*x^n])]^
(p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a + b*
Log[c*x^n])]^(p - 1))/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \sinh^3\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{3bnx\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^2\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}+\frac{x\sinh^3\left(a+b\log\left(cx^n\right)\right)}{1-9b^2n^2}$$

$$= -\frac{6b^3n^3x\cosh\left(a+b\log\left(cx^n\right)\right)}{1-10b^2n^2+9b^4n^4}+\frac{6b^2n^2x\sinh\left(a+b\log\left(cx^n\right)\right)}{1-10b^2n^2+9b^4n^4}-\frac{3bnx\cosh\left(a+\right.}{}$$

Mathematica [A] time = 0.53, size = 120, normalized size = 0.81

$$\frac{x\left(-3bn\left(9b^2n^2-1\right)\cosh\left(a+b\log\left(cx^n\right)\right)+3bn\left(b^2n^2-1\right)\cosh\left(3\left(a+b\log\left(cx^n\right)\right)\right)-2\sinh\left(a+b\log\left(cx^n\right)\right)\left(\right.}{36b^4n^4-40b^2n^2+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(-3*b*n*(-1 + 9*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + 3*b*n*(-1 + b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 2*(1 - 13*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a +
b*Log[c*x^n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

fricas [A] time = 0.76, size = 200, normalized size = 1.34

$$\frac{3\left(b^3n^3-bn\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^3+9\left(b^3n^3-bn\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*(b^3*n^3 - b
*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - (
b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^3 - 3*(9*b^3*n^3 - b*n)*x*co
sh(b*n*log(x) + b*log(c) + a) - 3*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^2 - (9*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4
- 10*b^2*n^2 + 1)
```

giac [B] time = 0.26, size = 665, normalized size = 4.46

$$\frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{27\,b^3c^bn^3xx^{bn}e^a}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}+\frac{27\,b^2c^bn^2xx^{bn}e^a}{8\left(9\,b^4n^4-10\,b^2n^2+1\right)}-\frac{3\,bc^{3\,b}}{8\left(9\,b^4n^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8
*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)*n
^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8*b^2*c^b*n^2*x*x^
(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a
)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) + 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 +
1)*c^(3*b)*x^(3*b*n)) + 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 +
 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b
^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) + 1/8*b^2*n^2*x*
e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^(b*
n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) - 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c
^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n))
 - 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n))
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \sinh^3(a + b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^3,x)
```

maxima [A] time = 0.40, size = 114, normalized size = 0.77

$$\frac{c^{3\,b}xe^{\left(3\,b\log(x^n)+3\,a\right)}}{8\,(3\,bn+1)} - \frac{3\,c^b xe^{\left(b\log(x^n)+a\right)}}{8\,(bn+1)} - \frac{3\,xe^{\left(-b\log(x^n)-a\right)}}{8\left(bc^b n - c^b\right)} + \frac{xe^{\left(-3\,b\log(x^n)-3\,a\right)}}{8\left(3\,bc^{3\,b}n - c^{3\,b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) - 3/8*c^b*x*e^(b*log(x^n)
+ a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) + 1/8*x*e^(-3*b*
log(x^n) - 3*a)/(3*b*c^(3*b)*n - c^(3*b))
```

mupad [B] time = 0.72, size = 93, normalized size = 0.62

$$\frac{x\,\mathrm{e}^{-3\,a}}{{(c\,x^n)}^{3\,b}\,(24\,b\,n-8)} - \frac{3\,x\,\mathrm{e}^{-a}}{{(c\,x^n)}^{b}\,(8\,b\,n-8)} + \frac{x\,\mathrm{e}^{3\,a}\,{(c\,x^n)}^{3\,b}}{24\,b\,n+8} - \frac{3\,x\,\mathrm{e}^{a}\,{(c\,x^n)}^{b}}{8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] (x*exp(-3*a))/((c*x^n)^(3*b)*(24*b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b*
n - 8)) + (x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (3*x*exp(a)*(c*x^n)^b)/
(8*b*n + 8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sinh^3\left(a - \frac{\log(cx^n)}{n}\right) dx \\ \int \sinh^3\left(a - \frac{\log(cx^n)}{3n}\right) dx \\ \int \sinh^3\left(a + \frac{\log(cx^n)}{3n}\right) dx \\ \int \sinh^3\left(a + \frac{\log(cx^n)}{n}\right) dx \\ \frac{9b^3n^3x\sinh^2(a+bn\log(x)+b\log(c))\cosh(a+bn\log(x)+b\log(c))}{9b^4n^4-10b^2n^2+1} - \frac{6b^3n^3x\cosh^3(a+bn\log(x)+b\log(c))}{9b^4n^4-10b^2n^2+1} - \frac{7b^2n^2x\sinh^3(a+bn\log(x)+b\log(c}{9b^4n^4-10b^2n^2+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integral
(sinh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(sinh(a + l
og(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(sinh(a + log(c*x**n)/n
)**3, x), Eq(b, 1/n)), (9*b**3*n**3*x*sinh(a + b*n*log(x) + b*log(c))**2*co
sh(a + b*n*log(x) + b*log(c))/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 6*b**3*n**
3*x*cosh(a + b*n*log(x) + b*log(c))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 7
*b**2*n**2*x*sinh(a + b*n*log(x) + b*log(c))**3/(9*b**4*n**4 - 10*b**2*n**2
 + 1) + 6*b**2*n**2*x*sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x) +
 b*log(c))**2/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 3*b*n*x*sinh(a + b*n*log(x
) + b*log(c))**2*cosh(a + b*n*log(x) + b*log(c))/(9*b**4*n**4 - 10*b**2*n**
2 + 1) + x*sinh(a + b*n*log(x) + b*log(c))**3/(9*b**4*n**4 - 10*b**2*n**2 +
 1), True))
```

3.269 $\int \sinh^4\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=191

$$\frac{x\sinh^4\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}-\frac{4bnx\sinh^3\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}+\frac{12b^2n^2x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{64b^4n^4-20b^2n^2+1}-$$

```
[Out] 24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-24*b^3*n^3*x*cosh(a+b*ln(c*x^n))*sin
h(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)+12*b^2*n^2*x*sinh(a+b*ln(c*x^n))
^2/(64*b^4*n^4-20*b^2*n^2+1)-4*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n)
)^3/(-16*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)
```

Rubi [A] time = 0.05, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5519, 8}

$$\frac{12b^2n^2x\sinh^2\left(a+b\log\left(cx^n\right)\right)}{64b^4n^4-20b^2n^2+1}+\frac{x\sinh^4\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}-\frac{4bnx\sinh^3\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*Cosh[a + b*Log
[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^
2*x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(1 - 16*b^2*n^2) + (x*Sinh[a +
 b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5519

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> -S
imp[(x*Sinh[d*(a + b*Log[c*x^n])]^p)/(b^2*d^2*n^2*p^2 - 1), x] + (-Dist[(b^
2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 - 1), Int[Sinh[d*(a + b*Log[c*x^n])]^
(p - 2), x], x] + Simp[(b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a + b*
Log[c*x^n])]^(p - 1))/(b^2*d^2*n^2*p^2 - 1), x]) /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \sinh^4\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{4bnx\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}+\frac{x\sinh^4\left(a+b\log\left(cx^n\right)\right)}{1-16b^2n^2}$$
$$= -\frac{24b^3n^3x\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{1-20b^2n^2+64b^4n^4}+\frac{12b^2n^2x\sinh^2\left(a+b\log\right.}{1-20b^2n^2+64b^4}$$
$$= \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4}-\frac{24b^3n^3x\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{1-20b^2n^2+64b^4n^4}+$$

Mathematica [A] time = 0.44, size = 167, normalized size = 0.87

$$\frac{x\left(-128b^3n^3\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+16b^3n^3\sinh\left(4\left(a+b\log\left(cx^n\right)\right)\right)+\left(64b^2n^2-4\right)\cosh\left(2\left(a+b\log\left(cx^n\right.\right.\right.}{8(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (-4 + 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] + 8*b*n*Sinh[2*(a + b*Log
[c*x^n])] - 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*Lo
g[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2 + 6
4*b^4*n^4))
```

fricas [A] time = 2.89, size = 294, normalized size = 1.54

$$-\frac{\left(4\,b^2n^2-1\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^4-16\left(4\,b^3n^3-bn\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] -1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3 -
 b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 +
 (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 - 4*(16*b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a)^
2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^3 - (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) +
a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

giac [B] time = 0.30, size = 777, normalized size = 4.07

$$\frac{b^3c^{4\,b}n^3xx^{4\,bn}e^{(4\,a)}}{64\,b^4n^4-20\,b^2n^2+1}-\frac{8\,b^3c^{2\,b}n^3xx^{2\,bn}e^{(2\,a)}}{64\,b^4n^4-20\,b^2n^2+1}+\frac{24\,b^4n^4x}{64\,b^4n^4-20\,b^2n^2+1}-\frac{b^2c^{4\,b}n^2xx^{4\,bn}e^{(4\,a)}}{4\left(64\,b^4n^4-20\,b^2n^2+1\right)}+\frac{4\,b^2c^{2\,b}n^2xx^{2}}{64\,b^4n^4-20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 8*b^3*c
^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4*x
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)/(6
4*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64*b^4
*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 -
20*b^2*n^2 + 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*
n^2 + 1) + 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2
*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n
)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x*x^(4*b*n
)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(
64*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 +
 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*
c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*x*e^(-2*a)/(
(64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e^(-4*a)/((64*b^4
*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \sinh^4\left(a+b\ln\left(c\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^4,x)
```

maxima [A] time = 0.39, size = 129, normalized size = 0.68

$$\frac{c^{4\,b}xe^{\left(4\,b\log\left(x^n\right)+4\,a\right)}}{16\,(4\,bn+1)}-\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+2\,a\right)}}{4\,(2\,bn+1)}+\frac{3}{8}\,x+\frac{xe^{\left(-2\,b\log\left(x^n\right)-2\,a\right)}}{4\left(2\,bc^{2\,b}n-c^{2\,b}\right)}-\frac{xe^{(-4\,a)}}{16\left(4\,bc^{4\,b}n-c^{4\,b}\right)\left(x^n\right)^{4\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) - 1/4*c^(2*b)*x*e^(2*b*lo
g(x^n) + 2*a)/(2*b*n + 1) + 3/8*x + 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^(2
*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b))
```

mupad [B] time = 0.69, size = 102, normalized size = 0.53

$$\frac{3\,x}{8}+\frac{x\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)}-\frac{x\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}-\frac{x\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(64\,b\,n-16\right)}+\frac{x\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{64\,b\,n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x)/8 + (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) - (x*exp(2*a)*(c*x^n)^(
2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(4*
a)*(c*x^n)^(4*b))/(64*b*n + 16)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sinh^4\left(a-\frac{\log\left(cx^n\right)}{2n}\right)dx \\ \int \sinh^4\left(a-\frac{\log\left(cx^n\right)}{4n}\right)dx \\ \int \sinh^4\left(a+\frac{\log\left(cx^n\right)}{4n}\right)dx \\ \int \sinh^4\left(a+\frac{\log\left(cx^n\right)}{2n}\right)dx \\ \frac{24b^4n^4x\sinh^4\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{64b^4n^4-20b^2n^2+1}-\frac{48b^4n^4x\sinh^2\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh^2\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{64b^4n^4-20b^2n^2+1}+\frac{24b^4n^4x\cosh^4\left(a+bn\log\left(x\right)+b\mathrm{l}\right.}{64b^4n^4-20b^2n^2+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))), (
Integral(sinh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(si
nh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(sinh(a + log(c
*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*n*log(x)
+ b*log(c))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a +
b*n*log(x) + b*log(c))**2*cosh(a + b*n*log(x) + b*log(c))**2/(64*b**4*n**4
- 20*b**2*n**2 + 1) + 24*b**4*n**4*x*cosh(a + b*n*log(x) + b*log(c))**4/(64
*b**4*n**4 - 20*b**2*n**2 + 1) + 40*b**3*n**3*x*sinh(a + b*n*log(x) + b*log
(c))**3*cosh(a + b*n*log(x) + b*log(c))/(64*b**4*n**4 - 20*b**2*n**2 + 1) -
 24*b**3*n**3*x*sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x) + b*log
```

```
(c))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 16*b**2*n**2*x*sinh(a + b*n*log
(x) + b*log(c))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**2*n**2*x*sinh(
a + b*n*log(x) + b*log(c))**2*cosh(a + b*n*log(x) + b*log(c))**2/(64*b**4*n
**4 - 20*b**2*n**2 + 1) - 4*b*n*x*sinh(a + b*n*log(x) + b*log(c))**3*cosh(a
 + b*n*log(x) + b*log(c))/(64*b**4*n**4 - 20*b**2*n**2 + 1) + x*sinh(a + b*
n*log(x) + b*log(c))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1), True))
```

3.270 $\int x^m \sinh\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=73

$$\frac{(m+1)x^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}$$

```
[Out] -b*n*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*l
n(c*x^n))/((1+m)^2-b^2*n^2)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5527}

$$\frac{(m+1)x^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}-\frac{bnx^{m+1}\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)) + ((1 + m)*
x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Rule 5527

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m\sinh\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-b^2n^2}+\frac{(1+m)x^{1+m}\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-b^2n^2}$$

Mathematica [A] time = 0.14, size = 54, normalized size = 0.74

$$\frac{x^{m+1}\left((m+1)\sinh\left(a+b\log\left(cx^n\right)\right)-bn\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{(-bn+m+1)(bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] (x^(1 + m)*(-(b*n*Cosh[a + b*Log[c*x^n]]) + (1 + m)*Sinh[a + b*Log[c*x^n]])
)/((1 + m - b*n)*(1 + m + b*n))
```

fricas [A] time = 4.62, size = 98, normalized size = 1.34

$$\frac{bnx\cosh\left(bn\log(x)+b\log(c)+a\right)\cosh\left(m\log(x)\right)+bnx\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(m\log(x)\right)-\left((\right.}{b^2n^2-m^2-2\,m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + b*n*x*cosh(b*n*log(
x) + b*log(c) + a)*sinh(m*log(x)) - ((m + 1)*x*cosh(m*log(x)) + (m + 1)*x*s
inh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1)
```

giac [B] time = 0.24, size = 235, normalized size = 3.22

$$\frac{bc^bnxx^{bn}x^me^a}{2\left(b^2n^2-m^2-2\,m-1\right)}-\frac{c^bmxx^{bn}x^me^a}{2\left(b^2n^2-m^2-2\,m-1\right)}-\frac{c^bxx^{bn}x^me^a}{2\left(b^2n^2-m^2-2\,m-1\right)}+\frac{bnxx^me^{(-a)}}{2\left(b^2n^2-m^2-2\,m-1\right)c^bx^{bn}}+\frac{}{2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b*
n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2 -
 m^2 - 2*m - 1) + 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*
n)) + 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) + 1/2*x*x^
m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{m}\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n)),x)
```

maxima [A] time = 0.38, size = 64, normalized size = 0.88

$$\frac{c^{b} x e^{\left(b \log\left(x^{n}\right) + m \log\left(x\right) + a\right)}}{2 \, {\left(b n + m + 1\right)}} + \frac{x e^{\left(-b \log\left(x^{n}\right) + m \log\left(x\right) - a\right)}}{2 \, {\left(b c^{b} n - c^{b} {\left(m + 1\right)}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) + 1/2*x*e^(-b*log(x^n
) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

mupad [B] time = 0.70, size = 56, normalized size = 0.77

$$\frac{x\,x^m\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,m+2\,b\,n+2}-\frac{x\,x^m\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,m-2\,b\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*log(c*x^n)),x)
```

```
[Out] (x*x^m*exp(a)*(c*x^n)^b)/(2*m + 2*b*n + 2) - (x*x^m*exp(-a))/((c*x^n)^b*(2*
m - 2*b*n + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log{\left(x \right)} \sinh{\left(a \right)} & \text{for}\: b = 0 \wedge m = -1 \\\int x^{m} \sinh{\left(a - \frac{m \log{\left(c x^{n} \right)}}{n} - \frac{\log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = - \frac{m + 1}{n} \\\int x^{m} \sinh{\left(a + \frac{m \log{\left(c x^{n} \right)}}{n} + \frac{\log{\left(c x^{n} \right)}}{n} \right)}\, dx & \text{for}\: b = \frac{m + 1}{n} \\\frac{b n x x^{m} \cosh{\left(a + b n \log{\left(x \right)} + b \log{\left(c \right)} \right)}}{b^{2} n^{2} - m^{2} - 2 m - 1} - \frac{m x x^{m} \sinh{\left(a + b n \log{\left(x \right)} + b \log{\left(c \right)} \right)}}{b^{2} n^{2} - m^{2} - 2 m - 1} - \frac{x x^{m} \sinh{\left(a + b n \log{\left(x \right)} + b \log{\left(c \right)} \right)}}{b^{2} n^{2} - m^{2} - 2 m - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)*sinh(a), Eq(b, 0) & Eq(m, -1)), (Integral(x**m*sinh(a - m
*log(c*x**n)/n - log(c*x**n)/n), x), Eq(b, -(m + 1)/n)), (Integral(x**m*sin
h(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*x**m*
cosh(a + b*n*log(x) + b*log(c))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*sin
h(a + b*n*log(x) + b*log(c))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*sinh(a +
 b*n*log(x) + b*log(c))/(b**2*n**2 - m**2 - 2*m - 1), True))
```

3.271 $\int x^m \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=120

$$\frac{(m+1)x^{m+1}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}+\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-\right.}$$

[Out] 2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^2/((1+m)^2-4*b^2*n^2)

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5529, 30}

$$\frac{(m+1)x^{m+1}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}-\frac{2bnx^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-4b^2n^2}+\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) - (2*b*n*x^(1 + m)*
Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2) + ((
1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 4*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5529

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (-Dist[(b^2*d^2*n^2*p*(p - 1))/(b^
2*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2),
 x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a
+ b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{
a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
0]
```

Rubi steps

$$\int x^m \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{2bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-4b^2n^2}+\frac{(1+m)x^{1+m}\sinh^2\left(\right.}{(1+m)^2-}$$

$$=\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)}-\frac{2bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(\right.\right.}{(1+m)^2-4b^2n^2}$$

Mathematica [A] time = 0.30, size = 89, normalized size = 0.74

$$\frac{x^{m+1}\left(-2b(m+1)n\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+(m+1)^2\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+4b^2n^2-m^2-2m-1\right)}{2(m+1)(-2bn+m+1)(2bn+m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(-1 - 2*m - m^2 + 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n]
)] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)*
(1 + m + 2*b*n))
```

fricas [A] time = 1.38, size = 248, normalized size = 2.07

$$\frac{\left(m^2+2\,m+1\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^2\cosh\left(m\log(x)\right)+\left(4\,b^2n^2-m^2-2\,m-1\right)x\cosh\left(m\log(x)\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (
4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*log
(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2
- 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*m +
b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*
log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (4*b^2
*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2 + 3*m^2
 + 3*m + 1)
```

giac [B] time = 0.23, size = 758, normalized size = 6.32

$$\frac{bc^{2\,b}mnxx^{2\,bn}x^m e^{(2\,a)}}{2\left(4\,b^2mn^2+4\,b^2n^2-m^3-3\,m^2-3\,m-1\right)}-\frac{c^{2\,b}m^2xx^{2\,bn}x^m e^{(2\,a)}}{4\left(4\,b^2mn^2+4\,b^2n^2-m^3-3\,m^2-3\,m-1\right)}+\frac{bc^{2\,b}nxx^{2}}{2\left(4\,b^2mn^2+4\,b^2n^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="giac")

[Out] 1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 2*b^2*n^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*b*m*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) + m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int x^{m} \left(\sinh^{2}\left(a + b\ln\left(c\,x^{n}\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*sinh(a+b*ln(c*x^n))^2,x)

[Out] int(x^m*sinh(a+b*ln(c*x^n))^2,x)

maxima [A] time = 0.36, size = 87, normalized size = 0.72

$$\frac{c^{2\,b} x e^{\left(2\,b\log\left(x^{n}\right)+m\log\left(x\right)+2\,a\right)}}{4\,(2\,bn+m+1)} - \frac{x e^{\left(-2\,b\log\left(x^{n}\right)+m\log\left(x\right)-2\,a\right)}}{4\left(2\,bc^{2\,b}n - c^{2\,b}(m+1)\right)} - \frac{x^{m+1}}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")

[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/2*x^(m + 1)/(m + 1)

mupad [B] time = 0.75, size = 74, normalized size = 0.62

$$\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)} - \frac{x\,x^m}{2\,m+2} + \frac{x\,x^m\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) - (x*x^m)/(2*m + 2) + (
x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log(x)\sinh^2(a) \\ \int x^m \sinh^2\left(a - \frac{m\log(cx^n)}{2n} - \frac{\log(cx^n)}{2n}\right)dx \\ \int x^m \sinh^2\left(a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right)dx \\ \int \frac{\sinh^2\left(a+b\log(cx^n)\right)}{x}dx \\ \frac{2b^2n^2xx^m\sinh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{2b^2n^2xx^m\cosh^2\left(a+bn\log(x)+b\log(c)\right)}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bmnxx^m\sinh\left(a+bn\log(x)+b\log(c)\right)\cosh\left(a+bn\log(x)+\right.}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*sinh(a
- m*log(c*x**n)/(2*n) - log(c*x**n)/(2*n))**2, x), Eq(b, -(m + 1)/(2*n))),
(Integral(x**m*sinh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq
(b, (m + 1)/(2*n))), (Integral(sinh(a + b*log(c*x**n))**2/x, x), Eq(m, -1))
, (2*b**2*n**2*x*x**m*sinh(a + b*n*log(x) + b*log(c))**2/(4*b**2*m*n**2 + 4
*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - 2*b**2*n**2*x*x**m*cosh(a + b*n*log
(x) + b*log(c))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1)
+ 2*b*m*n*x*x**m*sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x) + b*lo
g(c))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x**
m*sinh(a + b*n*log(x) + b*log(c))*cosh(a + b*n*log(x) + b*log(c))/(4*b**2*m
*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - m**2*x*x**m*sinh(a + b*n*l
og(x) + b*log(c))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1
) - 2*m*x*x**m*sinh(a + b*n*log(x) + b*log(c))**2/(4*b**2*m*n**2 + 4*b**2*n
**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*sinh(a + b*n*log(x) + b*log(c))**2/
(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1), True))
```

3.272 $\int x^m \sinh^3\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=203

$$\frac{(m+1)x^{m+1}\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-9b^2n^2}+\frac{6b^2(m+1)n^2x^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)}{\left((m+1)^2-9b^2n^2\right)\left((m+1)^2-b^2n^2\right)}-\frac{3bnx^{m+1}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-9b}$$

```
[Out] -6*b^3*n^3*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^2
)+6*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-
b^2*n^2)-3*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^2/((1+m)^2-9
*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)
```

Rubi [A] time = 0.09, antiderivative size = 197, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5529, 5527}

$$\frac{(m+1)x^{m+1}\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-9b^2n^2}+\frac{6b^2(m+1)n^2x^{m+1}\sinh\left(a+b\log\left(cx^n\right)\right)}{-10b^2(m+1)^2n^2+9b^4n^4+(m+1)^4}-\frac{6b^3n^3x^{m+1}\cosh\left(a+b\log\left(cx\right.\right.}{-10b^2(m+1)^2n^2+9b^4n^4+(m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^4 - 10*b^2*(1 + m)^2
*n^2 + 9*b^4*n^4) + (6*b^2*(1 + m)*n^2*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((
1 + m)^4 - 10*b^2*(1 + m)^2*n^2 + 9*b^4*n^4) - (3*b*n*x^(1 + m)*Cosh[a + b*
Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 9*b^2*n^2) + ((1 + m)*x^
(1 + m)*Sinh[a + b*Log[c*x^n]]^3)/((1 + m)^2 - 9*b^2*n^2)
```

Rule 5527

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])])/(b^2*d
^2*e*n^2 - e*(m + 1)^2), x] + Simp[(b*d*n*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c
*x^n])])/(b^2*d^2*e*n^2 - e*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n},
x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rule 5529

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (-Dist[(b^2*d^2*n^2*p*(p - 1))/(b^
2*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2),
 x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a
+ b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{
```

```
a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
0]
```

Rubi steps

$$\int x^m \sinh^3\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{3bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^2\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-9b^2n^2}+\frac{(1+m)x^{1+m}\sinh^3}{(1+m)^2-}$$
$$=-\frac{6b^3n^3x^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4}+\frac{6b^2(1+m)n^2x^{1+m}\sinh\left(a+b\log\left(cx\right.\right.}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4}$$

Mathematica [A] time = 1.35, size = 292, normalized size = 1.44

$$\frac{1}{4}x^{m+1}\left(-\frac{3\cosh(bn\log(x))\left((m+1)\sinh\left(a+b\log\left(cx^n\right)-bn\log(x)\right)-bn\cosh\left(a+b\log\left(cx^n\right)-bn\log(x)\right)\right)}{(-bn+m+1)(bn+m+1)}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((-3*Cosh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]]
) + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b
*n)) - (3*Sinh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] - b
*n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) + (C
osh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)
*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*b*n)
) + (Sinh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] -
3*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3
*b*n))))/4
```

fricas [B] time = 1.93, size = 585, normalized size = 2.88

$$3\left(b^3n^3-\left(bm^2+2\,bm+b\right)n\right)x\cosh\left(bn\log(x)+b\log(c)+a\right)^3\cosh\left(m\log(x)\right)-3\left(9\,b^3n^3-\left(bm^2+2\,bm+b\right)n\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^
3*cosh(m*log(x)) - 3*(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x)
```

```
+ b*log(c) + a)*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m +
1)*x*cosh(m*log(x)) + (m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*sinh(m*
log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 9*((b^3*n^3 - (b*m^2 + 2*b*m +
 b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n
*log(x) + b*log(c) + a)^2 + 3*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (m^3 - 9*(b^2*m + b^2)
*n^2 + 3*m^2 + 3*m + 1)*x*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^
2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (m^3 - 9*(b^2*m + b^2)*n
^2 + 3*m^2 + 3*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) +
3*((b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 -
(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a))*sinh
(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6
*m^2 + 4*m + 1)
```

giac [B] time = 0.36, size = 3225, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^3*c^b*n^
3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x^m*e
^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^
3 + 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10
*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) -
 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b
)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*m^2*n*x*x^(b*n)*x^m
*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2
*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8
*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^(3*b)*m*n*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*c^b*m^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
+ 1) + 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2
*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m^2*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*x^m*e^(3*a)
```

```
/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*
m^2 + 4*m + 1) - 27/8*b^3*n^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/
8*b^3*n^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
- 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m^2*x*
x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n
^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 -
10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)
 + 3/8*c^(3*b)*m*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b
^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*m*n^2*x*x
^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 +
4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*b^2*m*n^2*x*x^m*e^(-3*a)/((9*b^
4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 +
4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1
/8*c^(3*b)*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n
^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*m^2*n*x*x^m*e^(-a)
/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6
*m^2 + 4*m + 1)*c^b*x^(b*n)) - 27/8*b^2*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b
*x^(b*n)) - 3/8*b*m^2*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^
2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) +
1/8*b^2*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^
4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^
(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2
 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*m^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2
*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*
n)) + 3/4*b*m*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 1/8*m^3*x*x^m*e^
(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m
^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/4*b*m*n*x*x^m*e^(-3*a)/((9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4
*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*
n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n
)) + 3/8*b*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/8*m^2*x*x^m*e^(-3
*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*b*n*x*x^m*e^(-3*a)/((9*b^4*n^4
- 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m +
1)*c^(3*b)*x^(3*b*n)) + 9/8*m*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/
8*m*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*x^m*e^(-a)/((9
*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2
 + 4*m + 1)*c^b*x^(b*n)) - 1/8*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2
```

```
- 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b
*n))
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int x^m \left(\sinh^3\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

maxima [A] time = 0.40, size = 138, normalized size = 0.68

$$\frac{c^{3\,b} x e^{\left(3\,b\log\left(x^n\right)+m\log\left(x\right)+3\,a\right)}}{8\,(3\,bn+m+1)}-\frac{3\,c^b x e^{\left(b\log\left(x^n\right)+m\log\left(x\right)+a\right)}}{8\,(bn+m+1)}-\frac{3\,x e^{\left(-b\log\left(x^n\right)+m\log\left(x\right)-a\right)}}{8\left(bc^b n-c^b(m+1)\right)}+\frac{x e^{\left(-3\,b\log\left(x^n\right)+m\log\left(x\right)-3\,a\right)}}{8\left(3\,bc^{3\,b}n-c^{3\,b}(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) - 3/8*c^b*x
*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*log
(x) - a)/(b*c^b*n - c^b*(m + 1)) + 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3*a)
/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

mupad [B] time = 0.85, size = 118, normalized size = 0.58

$$\frac{3\,x\,x^m\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(8\,m-8\,b\,n+8\right)}-\frac{x\,x^m\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(8\,m-24\,b\,n+8\right)}+\frac{x\,x^m\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{8\,m+24\,b\,n+8}-\frac{3\,x\,x^m\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^b}{8\,m+8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] (3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) - (x*x^m*exp(-3*a))/((c*x^n
)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*n
+ 8) - (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

3.273 $\int x^m \sinh^4\left(a+b\log\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=266

$$\frac{(m+1)x^{m+1}\sinh^4\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-16b^2n^2}+\frac{12b^2(m+1)n^2x^{m+1}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{\left((m+1)^2-16b^2n^2\right)\left((m+1)^2-4b^2n^2\right)}-\frac{4bnx^{m+1}\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-16}$$

```
[Out] 24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-24*b^3*n^
3*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((1+
m)^2-4*b^2*n^2)+12*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))^2/((1+m)^2-16*
b^2*n^2)/((1+m)^2-4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(
c*x^n))^3/((1+m)^2-16*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^4/((1+m)^2
-16*b^2*n^2)
```

Rubi [A] time = 0.13, antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5529, 30}

$$\frac{(m+1)x^{m+1}\sinh^4\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-16b^2n^2}+\frac{12b^2(m+1)n^2x^{m+1}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{-20b^2(m+1)^2n^2+64b^4n^4+(m+1)^4}-\frac{4bnx^{m+1}\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(m+1)^2-16}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4) + (12*b^2*(1 + m)*n^2*x^
(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^
4*n^4) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/
((1 + m)^2 - 16*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^4)/((1
 + m)^2 - 16*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5529

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> -Simp[((m + 1)*(e*x)^(m + 1)*Sinh[d*(a + b*Log[c*x^n])]^p)
/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x] + (-Dist[(b^2*d^2*n^2*p*(p - 1))/(b^
2*d^2*n^2*p^2 - (m + 1)^2), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2),
```

```
 x], x] + Simp[(b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*Sinh[d*(a
+ b*Log[c*x^n])]^(p - 1))/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2), x]) /; FreeQ[{
a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2,
0]
```

Rubi steps

$$\int x^m \sinh^4\left(a+b\log\left(cx^n\right)\right)dx = -\frac{4bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-16b^2n^2}+\frac{(1+m)x^{1+m}\sinh}{(1+m)^2}$$
$$= -\frac{24b^3n^3x^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4}+\frac{12b^2(1+m)n^2x}{(1+m)^4-2}$$
$$= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4-20b^2(1+m)^2n^2+64b^4n^4\right)}-\frac{24b^3n^3x^{1+m}\cosh\left(a+b\log\right.}{(1+m)^4-20b^2(}$$

Mathematica [A] time = 3.49, size = 311, normalized size = 1.17

$$\frac{1}{8}x^{m+1}\left(-\frac{4\sinh(2bn\log(x))\left((m+1)\sinh\left(2\left(a+b\log\left(cx^n\right)-bn\log(x)\right)\right)-2bn\cosh\left(2\left(a+b\log\left(cx^n\right)-bn\right.\right.\right.}{(-2bn+m+1)(2bn+m+1)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) - (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[x
] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1
+ m - 2*b*n)*(1 + m + 2*b*n)) - (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a -
b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]
))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cosh[4*(
a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*Log[c*
x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1 + m)*C
osh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[x] + b*L
og[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

fricas [B] time = 0.62, size = 1125, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*co
sh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) - 4*(m^4 + 4*m^3 - 16*(b^2*m
^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a
)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^
2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*
n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^4
+ 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m
^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b
*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^4 + 4*m^3 -
 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2*cosh(m*log(x)) - 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)
*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 - 4*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 -
2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b^3*m + b^3)*n^3 -
 (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3*cosh(
m*log(x)) - (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 - (16*(b^3*m
 + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^4 + 4*m^3 - 4*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^4 - 4*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m
 + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 2
0*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sinh(m*log(x)))/(m^5
+ 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 - 20*(b^2*m^3 + 3*b^2*m^2 + 3*b^2*m
 + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

giac [B] time = 0.44, size = 6884, normalized size = 25.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b
^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*
m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*m^2*n
^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n
```

```
^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*
n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2
*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*
m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4
+ 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(4*b)*m*n^2*x*x^(4*b
*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^
2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) +
1/2*b*c^(2*b)*m^3*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20
*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) + 8*b^2*c^(2*b)*m*n^2*x*x^(2*b*n)*x^m*e^(2*a)/(64
*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*
n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*m^4*x*
x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) - 3/4*b*c^(4*b)*m^2*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*c^(2*b)*m
^4*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + 3/2*b*c^(2*b)*m^2*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b
^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*
b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*m
^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
 m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4
*c^(4*b)*m^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1) - 3/4*b*c^(4*b)*m*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^
4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m
^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - c^(2*b)*m^3*x*x^(2*b*n)*x^m*
e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
- 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*b*c^
(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n
^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) + 8*b^3*m*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
```

10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - b^3*m*n^3*x*x^m*e^(-4*a)/(
(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2
*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n))
 - 15*b^2*m*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) + 3/8*c^(4*b)*m^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*c^(2*b)*m^2*x*x^(
2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
 + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10
*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*m^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 8*b^3*n^3*x*x^m
*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m
^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)
*x^(2*b*n)) - 1/4*b^2*m^2*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - b^3*n^3*x*x^m*e^(-4*a)/((6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m
*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) +
 3/8*m^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1
5/2*b^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*
n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
+ 1/4*c^(4*b)*m*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2
*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^
3 + 10*m^2 + 5*m + 1) - c^(2*b)*m*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 6
4*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 -
20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b*m^3*n*x*x^m*e^(-2*a)/((64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 8*
b^2*m*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*
b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5
*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*m^3*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64
*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 2
0*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b^2*m*n^2*x
*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2
 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(
4*b)*x^(4*b*n)) + 3/2*m^3*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 1/16*c^(4*b)*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4

*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*c^(2*b)*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*m^4*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 3/2*b*m^2*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 4*b^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*m^4*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/4*b*m^2*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*b^2*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 9/4*m^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - m^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 3/2*b*m*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*m^3*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/4*b*m*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/2*m*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*m^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 3/8*m^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 1/4*b*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - m*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2

```
*b*n)) + 1/4*m*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
- 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^
2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^
4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b
^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*x^m*e^(-4*a
)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*
n))
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int x^{m} \left(\sinh^{4}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

maxima [A] time = 0.39, size = 161, normalized size = 0.61

$$\frac{c^{4\,b}xe^{\left(4\,b\log(x^{n})+m\log(x)+4\,a\right)}}{16\,(4\,bn+m+1)}-\frac{c^{2\,b}xe^{\left(2\,b\log(x^{n})+m\log(x)+2\,a\right)}}{4\,(2\,bn+m+1)}+\frac{xe^{\left(-2\,b\log(x^{n})+m\log(x)-2\,a\right)}}{4\left(2\,bc^{2\,b}n-c^{2\,b}(m+1)\right)}-\frac{xe^{\left(-4\,b\log(x^{n})+m\log(x)-4\,a\right)}}{16\left(4\,bc^{4\,b}n-c^{4\,b}(m+1)\right)}+\frac{3}{8\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) - 1/4*c^(2
*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) + 1/4*x*e^(-2*b*log
(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4*b*
log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m + 1
)/(m + 1)
```

mupad [B] time = 0.87, size = 136, normalized size = 0.51

$$\frac{3\,x\,x^{m}}{8\,m+8}-\frac{x\,x^{m}\,\mathrm{e}^{-2\,a}}{{\left(c\,x^{n}\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}-\frac{x\,x^{m}\,\mathrm{e}^{2\,a}\,{\left(c\,x^{n}\right)}^{2\,b}}{4\,m+8\,b\,n+4}+\frac{x\,x^{m}\,\mathrm{e}^{-4\,a}}{{\left(c\,x^{n}\right)}^{4\,b}\,\left(16\,m-64\,b\,n+16\right)}+\frac{x\,x^{m}\,\mathrm{e}^{4\,a}\,{\left(c\,x^{n}\right)}^{4\,b}}{16\,m+64\,b\,n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x*x^m)/(8*m + 8) - (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) -
 (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c*x
```

```
^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m + 64
*b*n + 16)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

3.274 $\int \frac{\sinh(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=18

$$\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] cosh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2638}

$$\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \sinh(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.06

$$\frac{\sinh(a)\sinh\left(b\log\left(cx^n\right)\right)}{bn}+\frac{\cosh(a)\cosh\left(b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] (Cosh[a]*Cosh[b*Log[c*x^n]])/(b*n) + (Sinh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

fricas [A] time = 0.61, size = 19, normalized size = 1.06

$$\frac{\cosh\left(bn\log(x) + b\log(c) + a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

```
[Out] cosh(b*n*log(x) + b*log(c) + a)/(b*n)
```

giac [B] time = 0.19, size = 40, normalized size = 2.22

$$\frac{\left(c^{2\,b}x^{bn}e^{(2\,a)} + \frac{1}{x^{bn}}\right)e^{(-a)}}{2\,bc^{b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*(c^(2*b)*x^(b*n)*e^(2*a) + 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

maple [A] time = 0.01, size = 19, normalized size = 1.06

$$\frac{\cosh\left(a + b\ln\left(c\,x^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))/x,x)
```

```
[Out] cosh(a+b*ln(c*x^n))/b/n
```

maxima [A] time = 0.32, size = 18, normalized size = 1.00

$$\frac{\cosh\left(b\log\left(cx^{n}\right) + a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

```
[Out] cosh(b*log(c*x^n) + a)/(b*n)
```

mupad [B] time = 0.66, size = 18, normalized size = 1.00

$$\frac{\cosh\left(a + b\,\ln\left(c\,x^{n}\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))/x,x)
```

```
[Out] cosh(a + b*log(c*x^n))/(b*n)
```

sympy **[A]** time = 0.99, size = 37, normalized size = 2.06

$$\begin{cases} \log(x)\sinh(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\sinh\left(a + b\log(c)\right) & \text{for } n = 0 \\ \frac{\cosh\left(a+bn\log(x)+b\log(c)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sinh(
a + b*log(c)), Eq(n, 0)), (cosh(a + b*n*log(x) + b*log(c))/(b*n), True))
```

3.275 $\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=39

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -Log[x]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sinh^2(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}-\frac{\text{Subst}\left(\int 1\,dx,x,\log\left(cx^n\right)\right)}{2n}\\
&= -\frac{\log(x)}{2}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.92

$$\frac{\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] (-2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

fricas [A] time = 2.13, size = 40, normalized size = 1.03

$$-\frac{bn\log(x)-\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -1/2*(b*n*log(x) - cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(
c) + a))/(b*n)
```

giac [B] time = 0.19, size = 81, normalized size = 2.08

$$-\frac{\left(4\,bc^{2\,b}ne^{(2\,a)}\log(x)-c^{4\,b}x^{2\,bn}e^{(4\,a)}-\frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) - c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)
*x^(2*b*n)*e^(2*a) - 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

maple [A] time = 0.02, size = 52, normalized size = 1.33

$$\frac{\cosh\left(a+b\ln\left(c\,x^n\right)\right)\sinh\left(a+b\ln\left(c\,x^n\right)\right)}{2bn}-\frac{\ln\left(c\,x^n\right)}{2n}-\frac{a}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^2/x,x)
```

```
[Out] 1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n-1/2*ln(c*x^n)/n-1/2/b/n*a
```

maxima [A] time = 0.33, size = 49, normalized size = 1.26

$$\frac{e^{\left(2\,b\log(cx^n)+2\,a\right)}}{8\,bn}-\frac{e^{\left(-2\,b\log(cx^n)-2\,a\right)}}{8\,bn}-\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) -
1/2*log(x)
```

mupad [B] time = 0.69, size = 32, normalized size = 0.82

$$\frac{\sinh\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)}{4\,b\,n}-\frac{\ln\left(x^n\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^2/x,x)
```

```
[Out] sinh(2*a + 2*b*log(c*x^n))/(4*b*n) - log(x^n)/(2*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**2/x, x)
```

3.276 $\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=43

$$\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] -cosh(a+b*ln(c*x^n))/b/n+1/3*cosh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -(Cosh[a + b*Log[c*x^n]]/(b*n)) + Cosh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx&=\frac{\text{Subst}\left(\int \sinh^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=-\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\&=-\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.05

$$\frac{\cosh\left(3\left(a+b\log\left(cx^n\right)\right)\right)}{12bn}-\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] (-3*Cosh[a + b*Log[c*x^n]])/(4*b*n) + Cosh[3*(a + b*Log[c*x^n])]/(12*b*n)
```

fricas [A] time = 1.41, size = 65, normalized size = 1.51

$$\frac{\cosh\left(bn\log(x) + b\log(c) + a\right)^3 + 3\cosh\left(bn\log(x) + b\log(c) + a\right)\sinh\left(bn\log(x) + b\log(c) + a\right)^2 - 9\cosh\left(\right.}{12\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^2 - 9*cosh(b*n*log(x) + b*log(c) + a))/(b*
n)
```

giac [A] time = 0.21, size = 81, normalized size = 1.88

$$\frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)} - 9\,c^{4\,b}x^{bn}e^{(4\,a)} - \frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) - 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x^
(2*b*n)*e^(2*a) - 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

maple [A] time = 0.02, size = 36, normalized size = 0.84

$$\frac{\left(-\frac{2}{3} + \frac{\left(\sinh^2(a+b\ln(c\,x^n))\right)}{3}\right)\cosh\left(a + b\ln\left(c\,x^n\right)\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] 1/n/b*(-2/3+1/3*sinh(a+b*ln(c*x^n))^2)*cosh(a+b*ln(c*x^n))
```

maxima [B] time = 0.33, size = 86, normalized size = 2.00

$$\frac{e^{\left(3\,b\log(cx^n)+3\,a\right)}}{24\,bn} - \frac{3\,e^{\left(b\log(cx^n)+a\right)}}{8\,bn} - \frac{3\,e^{\left(-b\log(cx^n)-a\right)}}{8\,bn} + \frac{e^{\left(-3\,b\log(cx^n)-3\,a\right)}}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) - 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8*
e^(-b*log(c*x^n) - a)/(b*n) + 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

mupad [B] time = 0.72, size = 37, normalized size = 0.86

$$-\frac{3\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)-\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^3/x,x)
```

```
[Out] -(3*cosh(a + b*log(c*x^n)) - cosh(a + b*log(c*x^n))^3)/(3*b*n)
```

sympy [A] time = 10.44, size = 82, normalized size = 1.91

$$\begin{cases} \log\left(x\right)\sinh^3\left(a\right) & \text{for}\ b=0\wedge\left(b=0\vee n=0\right) \\ \log\left(x\right)\sinh^3\left(a+b\log\left(c\right)\right) & \text{for}\ n=0 \\ \frac{\sinh^2\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{bn}-\frac{2\cosh^3\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{3bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*si
nh(a + b*log(c))**3, Eq(n, 0)), (sinh(a + b*n*log(x) + b*log(c))**2*cosh(a
+ b*n*log(x) + b*log(c))/(b*n) - 2*cosh(a + b*n*log(x) + b*log(c))**3/(3*b*
n), True))
```

3.277 $\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{3\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

```
[Out] 3/8*ln(x)-3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(c
*x^n))*sinh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2635, 8}

$$\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{3\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 - (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(4*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{\sinh^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\text{Subst}\left(\int \sinh^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{4bn} - \frac{3\,\text{Subst}\left(\int \sinh^2(a+bx)\,dx, x, \text{lo}\right.}{4n}$$
$$= -\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn} + \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a\right.}{4bn}$$
$$= \frac{3\log(x)}{8} - \frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn} + \frac{\cosh\left(a+b\log\left(cx^n\right)\right.}{}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.70

$$\frac{12\left(a+b\log\left(cx^n\right)\right) - 8\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right) + \sinh\left(4\left(a+b\log\left(cx^n\right)\right)\right)}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) - 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[c
*x^n])])/(32*b*n)
```

fricas [A] time = 0.72, size = 84, normalized size = 1.15

$$\frac{\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^3 + 3\,bn\log(x) + \left(\cosh\left(bn\log(x)+b\log(c)+a\right)^3\right.}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

giac [A] time = 0.29, size = 114, normalized size = 1.56

$$\frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log(x) + c^{8\,b}x^{4\,bn}e^{(8\,a)} - 8\,c^{6\,b}x^{2\,bn}e^{(6\,a)} - \frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 8\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) - 8*c^(6*b)
*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) - 8*c^(2*b)*x^(2*b*n)*e^^
(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

maple [A] time = 0.02, size = 84, normalized size = 1.15

$$\frac{\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\left(\sinh^{3}\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{4bn}-\frac{3\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{8bn}+\frac{3\ln\left(c\,x^{n}\right)}{8n}+\frac{3a}{8bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/4*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^3/b/n-3/8*cosh(a+b*ln(c*x^n))*s
inh(a+b*ln(c*x^n))/b/n+3/8*ln(c*x^n)/n+3/8/b/n*a
```

maxima [A] time = 0.33, size = 93, normalized size = 1.27

$$\frac{e^{\left(4\,b\log(cx^{n})+4\,a\right)}}{64\,bn}-\frac{e^{\left(2\,b\log(cx^{n})+2\,a\right)}}{8\,bn}+\frac{e^{\left(-2\,b\log(cx^{n})-2\,a\right)}}{8\,bn}-\frac{e^{\left(-4\,b\log(cx^{n})-4\,a\right)}}{64\,bn}+\frac{3}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) - 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) +
1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n)
+ 3/8*log(x)
```

mupad [B] time = 0.78, size = 51, normalized size = 0.70

$$\frac{3\,\ln\left(x^{n}\right)}{8\,n}-\frac{\frac{\sinh(2\,a+2\,b\,\ln(c\,x^{n}))}{4}-\frac{\sinh(4\,a+4\,b\,\ln(c\,x^{n}))}{32}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^4/x,x)
```

```
[Out] (3*log(x^n))/(8*n) - (sinh(2*a + 2*b*log(c*x^n))/4 - sinh(4*a + 4*b*log(c*x
^n))/32)/(b*n)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^{4}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**4/x, x)
```

3.278 $\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\cosh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}-\frac{2\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] cosh(a+b*ln(c*x^n))/b/n-2/3*cosh(a+b*ln(c*x^n))^3/b/n+1/5*cosh(a+b*ln(c*x^n
))^5/b/n
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2633}

$$\frac{\cosh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}-\frac{2\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n) - (2*Cosh[a + b*Log[c*x^n]]^3)/(3*b*n) + Cosh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sinh^5(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}-\frac{2\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{\cosh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.05

$$\frac{5\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn}-\frac{5\cosh\left(3\left(a+b\log\left(cx^n\right)\right)\right)}{48bn}+\frac{\cosh\left(5\left(a+b\log\left(cx^n\right)\right)\right)}{80bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (5*Cosh[a + b*Log[c*x^n]])/(8*b*n) - (5*Cosh[3*(a + b*Log[c*x^n])])/(48*b*n
) + Cosh[5*(a + b*Log[c*x^n])]/(80*b*n)
```

fricas [B] time = 1.08, size = 130, normalized size = 2.00

$$\frac{3\cosh\left(bn\log(x)+b\log(c)+a\right)^{5}+15\cosh\left(bn\log(x)+b\log(c)+a\right)\sinh\left(bn\log(x)+b\log(c)+a\right)^{4}-25\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 15*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a)^4 - 25*cosh(b*n*log(x) + b*log(c) + a)
^3 + 15*(2*cosh(b*n*log(x) + b*log(c) + a)^3 - 5*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + 150*cosh(b*n*log(x) + b*log(c) +
 a))/(b*n)
```

giac [A] time = 0.21, size = 115, normalized size = 1.77

$$\frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)}-25\,c^{8\,b}x^{3\,bn}e^{(8\,a)}+150\,c^{6\,b}x^{bn}e^{(6\,a)}+\frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-25\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) - 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*c
^(6*b)*x^(b*n)*e^(6*a) + (150*c^(4*b)*x^(4*b*n)*e^(4*a) - 25*c^(2*b)*x^(2*b
*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

maple [A] time = 0.03, size = 51, normalized size = 0.78

$$\frac{\left(\frac{8}{15}+\frac{\left(\sinh^{4}(a+b\ln(c\,x^{n}))\right)}{5}-\frac{4\left(\sinh^{2}(a+b\ln(c\,x^{n}))\right)}{15}\right)\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] 1/n/b*(8/15+1/5*sinh(a+b*ln(c*x^n))^4-4/15*sinh(a+b*ln(c*x^n))^2)*cosh(a+b*
ln(c*x^n))
```

maxima [B] time = 0.34, size = 130, normalized size = 2.00

$$\frac{e^{\left(5\,b\log(cx^n)+5\,a\right)}}{160\,bn}-\frac{5\,e^{\left(3\,b\log(cx^n)+3\,a\right)}}{96\,bn}+\frac{5\,e^{\left(b\log(cx^n)+a\right)}}{16\,bn}+\frac{5\,e^{\left(-b\log(cx^n)-a\right)}}{16\,bn}-\frac{5\,e^{\left(-3\,b\log(cx^n)-3\,a\right)}}{96\,bn}+\frac{e^{\left(-5\,b\log(cx^n)-5\,a\right)}}{160\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) - 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
+ 5/16*e^(b*log(c*x^n) + a)/(b*n) + 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/96
*e^(-3*b*log(c*x^n) - 3*a)/(b*n) + 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

mupad [B] time = 0.84, size = 49, normalized size = 0.75

$$\frac{\frac{\cosh(a+b\ \ln(c\,x^n))^5}{5}-\frac{2\cosh(a+b\ \ln(c\,x^n))^3}{3}+\cosh\left(a+b\ \ln\left(c\,x^n\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^5/x,x)
```

```
[Out] (cosh(a + b*log(c*x^n)) - (2*cosh(a + b*log(c*x^n))^3)/3 + cosh(a + b*log(c
*x^n))^5/5)/(b*n)
```

sympy [A] time = 94.65, size = 122, normalized size = 1.88

$$\begin{cases} \log\left(x\right)\sinh^5\left(a\right) \\ \log\left(x\right)\sinh^5\left(a+b\log\left(c\right)\right) \\ \frac{\sinh^4\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{bn}-\frac{4\sinh^2\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)\cosh^3\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{3bn}+\frac{8\cosh^5\left(a+bn\log\left(x\right)+b\log\left(c\right)\right)}{15bn} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*si
nh(a + b*log(c))**5, Eq(n, 0)), (sinh(a + b*n*log(x) + b*log(c))**4*cosh(a
+ b*n*log(x) + b*log(c))/(b*n) - 4*sinh(a + b*n*log(x) + b*log(c))**2*cosh(
a + b*n*log(x) + b*log(c))**3/(3*b*n) + 8*cosh(a + b*n*log(x) + b*log(c))**
5/(15*b*n), True))
```

3.279 $\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$\frac{2\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn}+\frac{6i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2/5*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^(3/2)/b/n-6/5*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c*x^n))^(1
/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2635, 2640, 2639}

$$\frac{2\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn}+\frac{6i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sinh[a + b*Log[c*x^n]]^(3/2))/(5*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \frac{\sinh^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\text{Subst}\left(\int \sinh^{\frac{5}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn} - \frac{3\,\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\;dx\right.}{5n}$$
$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn} - \frac{\left(3\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\right)\text{S}}{5n\sqrt{i\,\text{si}}}$$
$$= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}} + \frac{2\cosh\left(a+b\log\left(\right.\right.}{}$$

Mathematica [A] time = 0.09, size = 96, normalized size = 0.86

$$\frac{\sinh\left(a+b\log\left(cx^n\right)\right)\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-6\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right)\middle|2\right)}{5bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]] + Sinh[a + b*Log[c*x^n]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n*Sq
rt[Sinh[a + b*Log[c*x^n]]])
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

maple [A] time = 0.09, size = 227, normalized size = 2.05

$$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{i\sinh(a+b\ln(c\,x^n))+1}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\cdots}{\cdots}}{n\cosh\left(a+b\ln\left(c\,x^n\right)\right)\sqrt{s\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(5/2)/x,x)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n))+1)
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln
(c*x^n))+1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^(5/2)/x,x)
```

```
[Out] int(sinh(a + b*log(c*x^n))^(5/2)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

3.280 $\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$\frac{2\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1
/2*I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*
(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2/3*cosh(a+b*ln
(c*x^n))*sinh(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2635, 2642, 2641}

$$\frac{2\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(3*b*n)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\text{Subst}\left(\int \sinh^{\frac{3}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \text{l}\right.}{3n}$$

$$= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn} - \frac{\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\ \text{Su}}{3n\sqrt{\sinh}}$$

$$= \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}} + \frac{2\cosh\left(a+b\log\right.}{}$$

Mathematica [C] time = 0.14, size = 114, normalized size = 1.03

$$\frac{\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2\sqrt{-\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh\left(2\right.\right.}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a
 + b*Log[c*x^n])]]*Sqrt[1 - Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[
c*x^n])]] + Sinh[2*(a + b*Log[c*x^n])])/(3*b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]
)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

maple [A] time = 0.02, size = 143, normalized size = 1.29

$$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))}\,\sqrt{2}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))+1}\,\sqrt{i\sinh(a+b\ln(c\,x^{n}))}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^{n}))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\left(\cosh^{2}(a+b\ln(c\,x^{n}))\right)\sinh}{3}}{n\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(3/2)/x,x)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n))+
1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] int(sinh(a + b*log(c*x^n))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{\frac{3}{2}}\left(a + b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**(3/2)/x, x)
```

3.281 $\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sin
h(a+b*ln(c*x^n))^(1/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2640, 2639}

$$-\frac{2i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c
*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$
$$= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.94

$$\frac{2\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(\frac{\pi}{2}-i\left(a+b\log\left(cx^n\right)\right)\right)\middle|2\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
[Out] (2*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

maple [A] time = 0.08, size = 146, normalized size = 2.03

$$\frac{\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\,\ldots}\right.\right.}{n\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x^{n}\ldots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(1/2)/x,x)
```

```
[Out] 1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] int(sinh(a + b*log(c*x^n))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(1/2)/x,x)
[Out] Integral(sqrt(sinh(a + b*log(c*x**n)))/x, x)
```

3.282 $\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=72

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*(I*
sinh(a+b*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2642, 2641}

$$-\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log
[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= \frac{\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\ \text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

$$= -\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.92

$$-\frac{2\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] integral(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

maple [A] time = 0.06, size = 120, normalized size = 1.67

$$\frac{i\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\mathrm{EllipticF}\left(\sqrt{-i\left(\sinh\right.}\right.}{n\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(1/2),x)
```

```
[Out] I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+I)
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(1/2)),x)
```

```
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(sinh(a + b*log(c*x**n)))), x)
```

3.283 $\int \frac{1}{x\sinh^{\frac{3}{2}}\left(a+b\log(cx^n)\right)}\,dx$

Optimal. Leaf size=107

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2*I*(sin(1/2*I*a+1/4*P
i+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c*x^n))^(1/2)
/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2636, 2640, 2639}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(b
*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}} + \frac{\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}} + \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,\text{Subst}\left(\int \sqrt{i\sinh(a+}\right.}{n\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\right.}}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.75

$$-\frac{2\left(\cosh\left(a+b\log\left(cx^n\right)\right)-\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{4}\left(-2ia-2ib\log\left(cx^n\right)+\pi\right)\middle|2\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n]
)/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{x\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh\left(b \log\left(cx^{n}\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

maple [A] time = 0.08, size = 212, normalized size = 1.98

$$\frac{2\sqrt{1 - i\sinh\left(a + b\ln\left(c\,x^{n}\right)\right)}\,\sqrt{2}\,\sqrt{i\sinh\left(a + b\ln\left(c\,x^{n}\right)\right) + 1}\,\sqrt{i\sinh\left(a + b\ln\left(c\,x^{n}\right)\right)}\,\operatorname{EllipticE}\left(\sqrt{1 - i\sinh\left(a + \right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(3/2),x)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n))+1)^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(I*sinh(a+b*ln(c*x^n)
)+1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh\left(b \log\left(cx^{n}\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\mathrm{sinh}\left(a+b\,\ln\left(c\,x^n\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(3/2)),x)
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(1/(x*sinh(a + b*log(c*x**n))**(3/2)), x)
```

3.284 $\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2/3*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(3/2)-2/3*I*(sin(1/2*I*a+1
/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ell
ipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*(I*sinh(a+b*ln(c*x^n)
))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2636, 2642, 2641}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sinh[a + b*Log[c*x^n]]^(3/2)) + (((2*I)/
3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{1}{x\sinh^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{3n}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} - \frac{\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\right.}{3n\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

$$= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} + \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+\right.}}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [C] time = 0.13, size = 122, normalized size = 1.10

$$-\frac{2\left(\sinh\left(a+b\log\left(cx^n\right)\right)\sqrt{-\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh\left(2\left(a\right.\right.\right.}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a +
b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]*Sinh[a + b*Log[c*x^n]]*Sqrt[1
- Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Sinh[a
+ b*Log[c*x^n]]^(3/2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{x\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")

[Out] integral(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh\left(b \log\left(cx^{n}\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")

[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)

maple [A] time = 0.08, size = 144, normalized size = 1.30

$$-\frac{i\sqrt{1-i\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\sqrt{2}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+1}\,\sqrt{i\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh\left(a-\right.}\right.}{3n\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sinh(a+b*ln(c*x^n))^(5/2),x)

[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)
*(I*sinh(a+b*ln(c*x^n))+1)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1
-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*l
n(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh\left(b \log\left(cx^{n}\right) + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

3.285 $\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

Optimal. Leaf size=209

$$-\frac{5e^{-2a}x\,(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)^2}-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}-\frac{5e^{-3a}x\,(cx^n)^{-6/n}\,\mathrm{cs}\cdots}{4\left(\cdots\right.}$$

```
[Out] -1/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)-5/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(2*
a)/((c*x^n)^(4/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)))^2+5/12*x*sinh(a+2*ln(c*x^
n)/n)^(5/2)/(1-1/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsc(exp(a)*(c*x^n)^(2/n
))*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1-1/exp(2*a)/((c*x
^n)^(4/n)))^(5/2)
```

Rubi [A] time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5525, 5533, 353, 349, 345, 242, 277, 216}

$$-\frac{5e^{-2a}x\,(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)^2}-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}-\frac{5e^{-3a}x\,(cx^n)^{-6/n}\,\mathrm{cs}\cdots}{4\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] -(x*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/4 - (5*x*Sinh[a + (2*Log[c*x^n])/n]^(
5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*Si
nh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5*x
*ArcCsc[E^a*(c*x^n)^(2/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(c*
x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 349

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*
(a + b*x^n)^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rule 353

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(x)}{n}\right)dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1-e^{-2a}x^{-4/n}\right)^{5/2}\,dx,x,cx^n\right)}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{\left(5x\left(cx^n\right)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1}\right.}{2n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right.}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{\left(5e^{-2a}x\left(cx^n\right)^{-6/n}\sinh\right.}{}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{\left(5e^{-2a}x\left(cx^n\right)^{-6/n}\sinh\right.}{}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}+\frac{\left(5e^{-2a}x\left(cx^n\right)^{-6/n}\sinh\right.}{}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)-\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}+\frac{5x\sinh^{\frac{5}{2}}}{12\left(1-\right.}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)-\frac{5e^{-2a}x\left(cx^n\right)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}+\frac{5x\sinh^{\frac{5}{2}}}{12\left(1-\right.}$$

Mathematica [C] time = 0.44, size = 86, normalized size = 0.41

$$\frac{1}{14}e^{2a}x\left(cx^n\right)^{4/n}\left(e^{2a}\left(cx^n\right)^{4/n}-1\right){}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-e^{2a}\left(cx^n\right)^{4/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] (E^(2*a)*x*(c*x^n)^(4/n)*(-1 + E^(2*a)*(c*x^n)^(4/n))*Hypergeometric2F1[2,
7/2, 9/2, 1 - E^(2*a)*(c*x^n)^(4/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/14
```

fricas [A] time = 1.05, size = 162, normalized size = 0.78

$$\frac{\left(15\sqrt{2}\,x^3\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}\right)e^{\left(\frac{3\,(an+2\,\log(c))}{2\,n}\right)}+2\sqrt{\frac{1}{2}}\left(2\,x^8e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}-14\,x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-3\right.\right.}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log
(c))/n) - 1)/x^2))*e^(3/2*(a*n + 2*log(c))/n) + 2*sqrt(1/2)*(2*x^8*e^(4*(a*
n + 2*log(c))/n) - 14*x^4*e^(2*(a*n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n
 + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c
))/n)/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+\frac{2\,\log\left(cx^n\right)}{n}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\ln\left(c\,x^n\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

```
[Out] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+\frac{2\,\log\left(cx^n\right)}{n}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + (2*log(c*x^n))/n)^(5/2),x)
```

```
[Out] int(sinh(a + (2*log(c*x^n))/n)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(5/2),x)
```

```
[Out] Timed out
```

3.286 $\int \sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=103

$$\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}+\frac{e^{-a}x\left(cx^n\right)^{-2/n}\csc^{-1}\left(e^a\left(cx^n\right)^{2/n}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

```
[Out] 1/2*x*sinh(a+2*ln(c*x^n)/n)^(1/2)+1/2*x*arccsc(exp(a)*(c*x^n)^(2/n))*sinh(a
+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)))
^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5525, 5533, 345, 242, 277, 216}

$$\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}+\frac{e^{-a}x\left(cx^n\right)^{-2/n}\csc^{-1}\left(e^a\left(cx^n\right)^{2/n}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2 + (x*ArcCsc[E^a*(c*x^n)^(2/n)]*Sqrt[
Sinh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 - 1/(E^(2*a)*(c*x^
n)^(4/n))])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx = \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\sinh\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x\left(cx^n\right)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sqrt{1-e^{-2a}x^{-4/n}}\,dx,x,cx^n\right)}{n\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$= \frac{\left(x\left(cx^n\right)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int\sqrt{1-\frac{e^{-2a}}{x^2}}\,dx,x,\left(cx^n\right)^{2/n}\right)}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$= -\frac{\left(x\left(cx^n\right)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int\frac{\sqrt{1-e^{-2a}x^2}}{x^2}\,dx,x,\left(cx^n\right)^{-2/n}\right)}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$= \frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)} + \frac{\left(e^{-2a}x\left(cx^n\right)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\,(}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/}}}$$

$$= \frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)} + \frac{e^{-a}x\left(cx^n\right)^{-2/n}\sin^{-1}\left(e^{-a}\left(cx^n\right)^{-2/n}\right)\sqrt{\sinh\left(a+\right.}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

Mathematica [A] time = 0.29, size = 74, normalized size = 0.72

$$\frac{1}{2}x\left(1-\frac{\tan^{-1}\left(\sqrt{e^{2a}\left(cx^n\right)^{4/n}-1}\right)}{\sqrt{e^{2a}\left(cx^n\right)^{4/n}-1}}\right)\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*(1 - ArcTan[Sqrt[-1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[-1 + E^(2*a)*(c*x^n)^
(4/n)])*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2
```

fricas [A] time = 1.40, size = 117, normalized size = 1.14

$$\frac{1}{4}\left(2\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^4e^{\left(\frac{2\left(an+2\log(c)\right)}{n}\right)}-1}{x^2}}\,e^{\left(\frac{an+2\log(c)}{2n}\right)}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^4e^{\left(\frac{2\left(an+2\log(c)\right)}{n}\right)}-1}{x^2}}\right)e^{\left(\frac{an+2\log(c)}{2n}\right)}\right)e^{\left(-\frac{an+2\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(1/2*(a*n
 + 2*log(c))/n) - sqrt(2)*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n +
2*log(c))/n) - 1)/x^2))*e^(1/2*(a*n + 2*log(c))/n))*e^(-(a*n + 2*log(c))/n)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh}\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

```
[Out] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh\left(a+\frac{2\,\log\left(cx^{n}\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(a + 2*log(c*x^n)/n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sinh\left(a + \frac{2\ln\left(c\,x^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + (2*log(c*x^n))/n)^(1/2),x)
[Out] int(sinh(a + (2*log(c*x^n))/n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh\left(a + \frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(1/2),x)
[Out] Integral(sqrt(sinh(a + 2*log(c*x**n)/n)), x)
```

3.287 $\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{x\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/2*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5525, 5533, 264}

$$-\frac{x\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
[Out] -(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(2*Sinh[a + (2*Log[c*x^n])/n]^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{2/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{3/2}}\,dx,x,cx^n\right)}{n\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] time = 0.15, size = 61, normalized size = 1.42

$$\frac{\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)-\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)}{x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] (-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n]
)/n])/(x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])
```

fricas [A] time = 0.58, size = 68, normalized size = 1.58

$$-\frac{2\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}-1}{x^2}}\,e^{\left(-\frac{an+2\log(c)}{2n}\right)}}{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n +
2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(a + \frac{2\ln(c\,x^{n})}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(a + \frac{2\,\log\left(cx^{n}\right)}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\sinh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + (2*log(c*x^n))/n)^(3/2),x)
[Out] int(1/sinh(a + (2*log(c*x^n))/n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(3/2),x)
```

```
[Out] Integral(sinh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

3.288 $\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=103

$$\frac{e^{-2a}x\left(cx^n\right)^{-4/n}\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/6*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(7/2)+1/15*x*(1
-1/exp(2*a)/((c*x^n)^(4/n)))/exp(2*a)/((c*x^n)^(4/n))/sinh(a+2*ln(c*x^n)/n)
^(7/2)
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5525, 5533, 271, 264}

$$\frac{e^{-2a}x\left(cx^n\right)^{-4/n}\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] -(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(6*Sinh[a + (2*Log[c*x^n])/n]^(7/2)) +
 (x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Sinh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5525

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5533

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x\,(cx^n)^{6/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{6}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{7/2}}\,dx, x, cx^n\right)}{n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{\left(2e^{-2a}x\,(cx^n)^{6/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1}}{\left(1-e^{-2a}\right.}\right.}{3n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} + \frac{e^{-2a}x\,(cx^n)^{-4/n}\left(1-e^{-2a}\,(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] time = 0.27, size = 121, normalized size = 1.17

$$\frac{\left(\left(5x^4+2\right)\sinh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)+\left(5x^4-2\right)\cosh\left(a+\frac{2\log(cx^n)}{n}-2\log(x)\right)\right)\left(\sinh\left(2a+\frac{4\log(cx^n)}{n}-4\,\text{lo}\right.\right.}{15x^5\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] (((-2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (2 + 5*x^4)*Sinh[a -
 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n] +
Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Sinh[a + (2*Log[c*x^n])/n
]^(5/2))
```

fricas [A] time = 0.55, size = 128, normalized size = 1.24

$$-\frac{8\sqrt{\frac{1}{2}}\left(5\,x^5e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,x\right)\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}-3\,x^8e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) - 2*x)*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*lo
g(c))/n) - 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/n)
- 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

```
[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(a + \frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(a + \frac{2\ln(c\,x^n)}{n}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + (2*log(c*x^n))/n)^(7/2),x)
[Out] int(1/sinh(a + (2*log(c*x^n))/n)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(7/2),x)
[Out] Timed out
```

3.289 $\int \sinh\left(\frac{a}{c+dx}\right)\,dx$

Optimal. Leaf size=36

$$\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}-\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

```
[Out] -a*Chi(a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))/d
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5310, 5302, 3297, 3301}

$$\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}-\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)],x]
```

```
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh\left(\frac{a}{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int \sinh\left(\frac{a}{x}\right)\,dx, x, c+dx\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh(ax)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{\cosh(ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= -\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)],x]
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

fricas [A] time = 0.83, size = 48, normalized size = 1.33

$$-\frac{a\text{Ei}\left(\frac{a}{dx+c}\right) + a\text{Ei}\left(-\frac{a}{dx+c}\right) - 2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(a*Ei(a/(d*x + c)) + a*Ei(-a/(d*x + c)) - 2*(d*x + c)*sinh(a/(d*x + c)
))/d
```

giac [B] time = 2.64, size = 102, normalized size = 2.83

$$-\frac{\left(\frac{a^3\text{Ei}\left(\frac{a}{dx+c}\right)}{dx+c} - a^2e^{\left(\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2d} - \frac{\left(\frac{a^3\text{Ei}\left(-\frac{a}{dx+c}\right)}{dx+c} + a^2e^{\left(-\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a^3*Ei(a/(d*x + c))/(d*x + c) - a^2*e^(a/(d*x + c)))*(d*x + c)/(a^2*d
) - 1/2*(a^3*Ei(-a/(d*x + c))/(d*x + c) + a^2*e^(-a/(d*x + c)))*(d*x + c)/(
a^2*d)
```

maple [A] time = 0.02, size = 38, normalized size = 1.06

$$-\frac{a\left(-\frac{(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{a}+\mathrm{X}\left(\frac{a}{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c)),x)
```

```
[Out] -1/d*a*(-1/a*(d*x+c)*sinh(a/(d*x+c))+Chi(a/(d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ad\int\frac{xe^{\left(\frac{a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx+\frac{1}{2}\,ad\int\frac{xe^{\left(-\frac{a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx+\frac{1}{2}\,xe^{\left(\frac{a}{dx+c}\right)}-\frac{1}{2}\,xe^{\left(-\frac{a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*a*d
*integrate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*x*e^(a/(d
*x + c)) - 1/2*x*e^(-a/(d*x + c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\sinh\left(\frac{a}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(c + d*x)),x)
```

```
[Out] int(sinh(a/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(\frac{a}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x)
```

```
[Out] Integral(sinh(a/(c + d*x)), x)
```

3.290 $\int \sinh^2\left(\frac{a}{c+dx}\right) dx$

Optimal. Leaf size=39

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\mathrm{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

```
[Out] -a*Shi(2*a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))^2/d
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5310, 5302, 3313, 12, 3298}

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\mathrm{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2)/d - (a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
```

```
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh^2\left(\frac{a}{c+dx}\right) dx &= \frac{\text{Subst}\left(\int \sinh^2\left(\frac{a}{x}\right) dx, x, c+dx\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{\sinh^2(ax)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} + \frac{(2ia)\,\text{Subst}\left(\int \frac{i\sinh(2ax)}{2x} dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{\sinh(2ax)}{x} dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 0.95

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right) - a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)]^2,x]
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2 - a*SinhIntegral[(2*a)/(c + d*x)])/d
```

fricas [A] time = 1.00, size = 73, normalized size = 1.87

$$\frac{(dx+c)\cosh\left(\frac{a}{dx+c}\right)^2 + (dx+c)\sinh\left(\frac{a}{dx+c}\right)^2 - dx - a\text{Ei}\left(\frac{2a}{dx+c}\right) + a\text{Ei}\left(-\frac{2a}{dx+c}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((d*x + c)*cosh(a/(d*x + c))^2 + (d*x + c)*sinh(a/(d*x + c))^2 - d*x -
a*Ei(2*a/(d*x + c)) + a*Ei(-2*a/(d*x + c)))/d
```

giac [B] time = 3.08, size = 97, normalized size = 2.49

$$-\frac{\left(\frac{2\,a^3\mathrm{Ei}\left(\frac{2\,a}{dx+c}\right)}{dx+c}-\frac{2\,a^3\mathrm{Ei}\left(-\frac{2\,a}{dx+c}\right)}{dx+c}-a^2e^{\left(\frac{2\,a}{dx+c}\right)}-a^2e^{\left(-\frac{2\,a}{dx+c}\right)}+2\,a^2\right)(dx+c)}{4\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^3*Ei(2*a/(d*x + c))/(d*x + c) - 2*a^3*Ei(-2*a/(d*x + c))/(d*x + c
) - a^2*e^(2*a/(d*x + c)) - a^2*e^(-2*a/(d*x + c)) + 2*a^2)*(d*x + c)/(a^2*
d)
```

maple [A] time = 0.04, size = 50, normalized size = 1.28

$$-\frac{a\left(\frac{dx+c}{2a}-\frac{(dx+c)\cosh\left(\frac{2a}{dx+c}\right)}{2a}+\mathrm{Shi}\left(\frac{2a}{dx+c}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c))^2,x)
```

```
[Out] -1/d*a*(1/2/a*(d*x+c)-1/2/a*(d*x+c)*cosh(2*a/(d*x+c))+Shi(2*a/(d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ad\int\frac{xe^{\left(\frac{2\,a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx-\frac{1}{2}\,ad\int\frac{xe^{\left(-\frac{2\,a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx+\frac{1}{4}\,xe^{\left(\frac{2\,a}{dx+c}\right)}+\frac{1}{4}\,xe^{\left(-\frac{2\,a}{dx+c}\right)}-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*d*integrate(x*e^(2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*a
*d*integrate(x*e^(-2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*x*e^(
2*a/(d*x + c)) + 1/4*x*e^(-2*a/(d*x + c)) - 1/2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int {\sinh\left(\frac{a}{c+d\,x}\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(c + d*x))^2,x)
```

```
[Out] int(sinh(a/(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.291 $\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx$

Optimal. Leaf size=59

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d}-\frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d}+\frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

```
[Out] 3/4*a*Chi(a/(d*x+c))/d-3/4*a*Chi(3*a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))^3/d
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5310, 5302, 3313, 3301}

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d}-\frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d}+\frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)]^3,x]
```

```
[Out] (3*a*CoshIntegral[a/(c + d*x)])/(4*d) - (3*a*CoshIntegral[(3*a)/(c + d*x)])
/(4*d) + ((c + d*x)*Sinh[a/(c + d*x)]^3)/d
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3\left(\frac{a}{c+dx}\right)dx &= \frac{\text{Subst}\left(\int \sinh^3\left(\frac{a}{x}\right)dx, x, c+dx\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh^3(ax)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\,\text{Subst}\left(\int \left(\frac{\cosh(ax)}{4x} - \frac{\cosh(3ax)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\,\text{Subst}\left(\int \frac{\cosh(ax)}{x}dx, x, \frac{1}{c+dx}\right)}{4d} - \frac{(3a)\,\text{Subst}\left(\int \frac{\cosh(3ax)}{x}dx, x,\right.}{4d}\\
&= \frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d} - \frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d} + \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.92

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right) - 3a\text{Chi}\left(\frac{3a}{c+dx}\right) + 4(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)]^3,x]
```

```
[Out] (3*a*CoshIntegral[a/(c + d*x)] - 3*a*CoshIntegral[(3*a)/(c + d*x)] + 4*(c +
 d*x)*Sinh[a/(c + d*x)]^3)/(4*d)
```

fricas [B] time = 1.14, size = 118, normalized size = 2.00

$$\frac{2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)^3 - 3\,a\text{Ei}\left(\frac{3\,a}{dx+c}\right) + 3\,a\text{Ei}\left(\frac{a}{dx+c}\right) + 3\,a\text{Ei}\left(-\frac{a}{dx+c}\right) - 3\,a\text{Ei}\left(-\frac{3\,a}{dx+c}\right) + 6\left((dx+c)\cosh\left(\frac{a}{dx+c}\right)^2 -\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(d*x + c)*sinh(a/(d*x + c))^3 - 3*a*Ei(3*a/(d*x + c)) + 3*a*Ei(a/(d*
x + c)) + 3*a*Ei(-a/(d*x + c)) - 3*a*Ei(-3*a/(d*x + c)) + 6*((d*x + c)*cosh
(a/(d*x + c))^2 - d*x - c)*sinh(a/(d*x + c)))/d
```

giac [B] time = 3.90, size = 167, normalized size = 2.83

$$-\frac{\left(\frac{3\,a^3\mathrm{Ei}\left(\frac{3\,a}{dx+c}\right)}{dx+c}-\frac{3\,a^3\mathrm{Ei}\left(\frac{a}{dx+c}\right)}{dx+c}-\frac{3\,a^3\mathrm{Ei}\left(-\frac{a}{dx+c}\right)}{dx+c}+\frac{3\,a^3\mathrm{Ei}\left(-\frac{3\,a}{dx+c}\right)}{dx+c}-a^2e^{\left(\frac{3\,a}{dx+c}\right)}+3\,a^2e^{\left(\frac{a}{dx+c}\right)}-3\,a^2e^{\left(-\frac{a}{dx+c}\right)}+a^2e^{\left(-\frac{3\,a}{dx+c}\right)}\right)(dx+c}{8\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^3*Ei(3*a/(d*x + c))/(d*x + c) - 3*a^3*Ei(a/(d*x + c))/(d*x + c) -
 3*a^3*Ei(-a/(d*x + c))/(d*x + c) + 3*a^3*Ei(-3*a/(d*x + c))/(d*x + c) - a^
2*e^(3*a/(d*x + c)) + 3*a^2*e^(a/(d*x + c)) - 3*a^2*e^(-a/(d*x + c)) + a^2*
e^(-3*a/(d*x + c)))*(d*x + c)/(a^2*d)
```

maple [A] time = 0.03, size = 74, normalized size = 1.25

$$-\frac{a\left(\frac{3(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{4a}-\frac{3\mathrm{X}\left(\frac{a}{dx+c}\right)}{4}-\frac{(dx+c)\sinh\left(\frac{3a}{dx+c}\right)}{4a}+\frac{3\mathrm{X}\left(\frac{3a}{dx+c}\right)}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c))^3,x)
```

```
[Out] -1/d*a*(3/4/a*(d*x+c)*sinh(a/(d*x+c))-3/4*Chi(a/(d*x+c))-1/4/a*(d*x+c)*sinh
(3*a/(d*x+c))+3/4*Chi(3*a/(d*x+c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{8}\,ad\int\frac{xe^{\left(\frac{3\,a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx-\frac{3}{8}\,ad\int\frac{xe^{\left(\frac{a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx-\frac{3}{8}\,ad\int\frac{xe^{\left(-\frac{a}{dx+c}\right)}}{d^2x^2+2\,cdx+c^2}\,dx+\frac{3}{8}\,ad\int\frac{xe^{\left(-\frac{3\,a}{dx+c}\right)}}{d^2x^2+2\,cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*d*integrate(x*e^(3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a
*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a*d*inte
grate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 3/8*a*d*integrate(
x*e^(-3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/8*x*e^(3*a/(d*x + c)
```

```
) - 3/8*x*e^(a/(d*x + c)) + 3/8*x*e^(-a/(d*x + c)) - 1/8*x*e^(-3*a/(d*x + c
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sinh\left(\frac{a}{c+d\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(c + d*x))^3,x)
```

```
[Out] int(sinh(a/(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.292 $\int \sinh\left(\frac{bx}{c+dx}\right) dx$

Optimal. Leaf size=74

$$\frac{bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}-\frac{bc\sinh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}$$

```
[Out] b*c*Chi(b*c/d/(d*x+c))*cosh(b/d)/d^2-b*c*Shi(b*c/d/(d*x+c))*sinh(b/d)/d^2+(
d*x+c)*sinh(b*x/(d*x+c))/d
```

Rubi [A] time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5607, 3297, 3303, 3298, 3301}

$$\frac{bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}-\frac{bc\sinh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/d^2 + ((c + d*x)*Sinh[(b*
x)/(c + d*x)])/d - (b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh\left(\frac{bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{\left(bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left(bc\sinh\left(\frac{b}{d}\right)\right)\text{Subst}}{}$$

$$= \frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}$$

Mathematica [A] time = 0.30, size = 70, normalized size = 0.95

$$\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right) - bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right) + d(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))] + d*(c + d*x)*Sinh[(b*x)/(
c + d*x)] - b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

fricas [B] time = 1.12, size = 253, normalized size = 3.42

$$-\frac{bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\sinh\left(\frac{bx}{dx+c}\right)^2-\left(bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2+bc\mathrm{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)-2\left(d^2x+cd\right)\cdots}{2\left(d^2\cosh\left(\frac{bx}{dx+c}\right)^2-d^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b/d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(
-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh
(b/d) - 2*(d^2*x + c*d)*sinh(b*x/(d*x + c)) - (b*c*Ei(-b*c/(d^2*x + c*d))*c
osh(b*x/(d*x + c))^2 - b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c))^2 - b
*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(
b*x/(d*x + c))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(\frac{bx}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c)), x)
```

maple [A] time = 0.05, size = 113, normalized size = 1.53

$$-\frac{\mathrm{e}^{-\frac{bx}{dx+c}}(dx+c)}{2d}-\frac{cb\,\mathrm{e}^{-\frac{b}{d}}\,\mathrm{Ei}\left(1,-\frac{bc}{d(dx+c)}\right)}{2d^2}+\frac{\mathrm{e}^{\frac{bx}{dx+c}}x}{2}+\frac{c\,\mathrm{e}^{\frac{bx}{dx+c}}}{2d}-\frac{cb\,\mathrm{e}^{\frac{b}{d}}\,\mathrm{Ei}\left(1,\frac{bc}{d(dx+c)}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c)),x)
```

```
[Out] -1/2/d*exp(-b*x/(d*x+c))*(d*x+c)-1/2*c*b/d^2*exp(-b/d)*Ei(1,-b*c/d/(d*x+c))
+1/2*exp(b*x/(d*x+c))*x+1/2*c/d*exp(b*x/(d*x+c))-1/2*c*b/d^2*exp(b/d)*Ei(1,
b*c/d/(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}bc\int\frac{xe^{\left(\frac{bc}{d^2x+cd}\right)}}{d^2x^2e^{\frac{b}{d}}+2cdxe^{\frac{b}{d}}+c^2e^{\frac{b}{d}}}\,dx-\frac{1}{2}bc\int\frac{xe^{\left(-\frac{bc}{d^2x+cd}+\frac{b}{d}\right)}}{d^2x^2+2cdx+c^2}\,dx-\frac{1}{2}\left(xe^{\left(\frac{bc}{d^2x+cd}\right)}-xe^{\left(-\frac{bc}{d^2x+cd}+\frac{2b}{d}\right)}\right)e^{\left(-\frac{b}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="maxima")

[Out] -1/2*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/d) + c^2*e^(b/d)), x) - 1/2*b*c*integrate(x*e^(-b*c/(d^2*x + c*d) + b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*(x*e^(b*c/(d^2*x + c*d)) - x*e^(-b*c/(d^2*x + c*d) + 2*b/d))*e^(-b/d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{b\,x}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh((b*x)/(c + d*x)),x)

[Out] int(sinh((b*x)/(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(\frac{bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x/(d*x+c)),x)

[Out] Integral(sinh(b*x/(c + d*x)), x)

3.293 $\int \sinh^2\left(\frac{bx}{c+dx}\right) dx$

Optimal. Leaf size=80

$$\frac{bc\sinh\left(\frac{2b}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}-\frac{bc\cosh\left(\frac{2b}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}$$

```
[Out] -b*c*cosh(2*b/d)*Shi(2*b*c/d/(d*x+c))/d^2+b*c*Chi(2*b*c/d/(d*x+c))*sinh(2*b
/d)/d^2+(d*x+c)*sinh(b*x/(d*x+c))^2/d
```

Rubi [A] time = 0.15, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5607, 3313, 12, 3303, 3298, 3301}

$$\frac{bc\sinh\left(\frac{2b}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}-\frac{bc\cosh\left(\frac{2b}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] (b*c*CoshIntegral[(2*b*c)/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 + ((c + d*x)*Si
nh[(b*x)/(c + d*x)]^2)/d - (b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(d*(c +
d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^2\left(\frac{bx}{c+dx}\right)dx &= -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{(2ibc)\,\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} \\
&= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} \\
&= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(bc\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left(bc\sinh\left(\frac{2b}{d}\right)\right)\text{S}}{} \\
&= \frac{bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 85, normalized size = 1.06

$$\frac{2bc\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)-2bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)+d\left((c+dx)\cosh\left(\frac{2bx}{c+dx}\right)-dx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*b*x)/(c + d*x)]) + 2*b*c*CoshIntegral[(2*b*c
)/(d*(c + d*x))]*Sinh[(2*b)/d] - 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(
d*(c + d*x))])/(2*d^2)
```

fricas [B] time = 0.49, size = 277, normalized size = 3.46

$$-\frac{d^2x-\left(d^2x+cd\right)\cosh\left(\frac{bx}{dx+c}\right)^2+\left(bc\text{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)-d^2x-cd\right)\sinh\left(\frac{bx}{dx+c}\right)^2-\left(bc\text{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right)\cosh\left(\cdots\right.\right.}{2\left(d^2\cosh\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh(b*x/(d*x + c))^2 + (b*c*Ei(-2*b*c/(d^2*x +
 c*d))*cosh(2*b/d) - d^2*x - c*d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(-2*b*c/(d
^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(2*b*c/(d^2*x + c*d)))*cosh(2*b/
d) - (b*c*Ei(-2*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(-2*b*c/(d
^2*x + c*d))*sinh(b*x/(d*x + c))^2 + b*c*Ei(2*b*c/(d^2*x + c*d)))*sinh(2*b/
d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(b*x/(d*x + c))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(\frac{bx}{dx+c}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c))^2, x)
```

maple [A] time = 0.18, size = 120, normalized size = 1.50

$$-\frac{x}{2}+\frac{\mathrm{e}^{-\frac{2bx}{dx+c}}\,(dx+c)}{4d}+\frac{cb\,\mathrm{e}^{-\frac{2b}{d}}\,\text{Ei}\left(1,-\frac{2bc}{d(dx+c)}\right)}{2d^2}+\frac{\mathrm{e}^{\frac{2bx}{dx+c}}x}{4}+\frac{c\,\mathrm{e}^{\frac{2bx}{dx+c}}}{4d}-\frac{cb\,\mathrm{e}^{\frac{2b}{d}}\,\text{Ei}\left(1,\frac{2bc}{d(dx+c)}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c))^2,x)
```

```
[Out] -1/2*x+1/4/d*exp(-2*b*x/(d*x+c))*(d*x+c)+1/2*c*b/d^2*exp(-2*b/d)*Ei(1,-2*b*
c/d/(d*x+c))+1/4*exp(2*b*x/(d*x+c))*x+1/4*c/d*exp(2*b*x/(d*x+c))-1/2*c*b/d^
2*exp(2*b/d)*Ei(1,2*b*c/d/(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,bc\int\frac{xe^{\left(\frac{2\,bc}{d^2x+cd}\right)}}{d^2x^2e^{\left(\frac{2\,b}{d}\right)}+2\,cdxe^{\left(\frac{2\,b}{d}\right)}+c^2e^{\left(\frac{2\,b}{d}\right)}}\,dx-\frac{1}{2}\,bc\int\frac{xe^{\left(-\frac{2\,bc}{d^2x+cd}+\frac{2\,b}{d}\right)}}{d^2x^2+2\,cdx+c^2}\,dx+\frac{1}{4}\left(xe^{\left(\frac{2\,bc}{d^2x+cd}\right)}+xe^{\left(-\frac{2\,bc}{d^2x+cd}+\frac{4\,b}{d}\right)}\right)e^{\left(-\frac{2\,b}{d}\right)}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*integrate(x*e^(2*b*c/(d^2*x + c*d))/(d^2*x^2*e^(2*b/d) + 2*c*d*x*e^
(2*b/d) + c^2*e^(2*b/d)), x) - 1/2*b*c*integrate(x*e^(-2*b*c/(d^2*x + c*d)
+ 2*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*(x*e^(2*b*c/(d^2*x + c*d)) + x
*e^(-2*b*c/(d^2*x + c*d) + 4*b/d))*e^(-2*b/d) - 1/2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x)/(c + d*x))^2,x)
```

```
[Out] int(sinh((b*x)/(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.294 $\int \sinh^3\left(\frac{bx}{c+dx}\right) dx$

Optimal. Leaf size=143

$$-\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}-\frac{3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}$$

```
[Out] -3/4*b*c*Chi(b*c/d/(d*x+c))*cosh(b/d)/d^2+3/4*b*c*Chi(3*b*c/d/(d*x+c))*cosh
(3*b/d)/d^2+3/4*b*c*Shi(b*c/d/(d*x+c))*sinh(b/d)/d^2-3/4*b*c*Shi(3*b*c/d/(d
*x+c))*sinh(3*b/d)/d^2+(d*x+c)*sinh(b*x/(d*x+c))^3/d
```

Rubi [A] time = 0.25, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5607, 3313, 3303, 3298, 3301}

$$-\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}-\frac{3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/(4*d^2) + (3*b*c*Cosh[
(3*b)/d]*CoshIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(b*
x)/(c + d*x)]^3)/d + (3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/(4
*d^2) - (3*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{(3bc)\,\text{Subst}\left(\int \left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{4x}+\frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{(3bc)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3bc)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\right.}{x}\right.}{4d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(3bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{\left(3bc\cosh\left(\frac{3b}{d}\right)\right)}{}$$

$$= -\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} + \frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{3bc}{}$$

Mathematica [A] time = 0.49, size = 172, normalized size = 1.20

$$\frac{-3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)+3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)-3d^2x\sinh\left(\frac{bx}{c+dx}\right)+d^2x\sinh\left(\frac{3bx}{c+dx}\right)+3bc\sinh\left(\frac{b}{d}\right)\text{Sh}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))] + 3*b*c*Cosh[(3*b)/d]*C
oshIntegral[(3*b*c)/(d*(c + d*x))] - 3*c*d*Sinh[(b*x)/(c + d*x)] - 3*d^2*x*
Sinh[(b*x)/(c + d*x)] + c*d*Sinh[(3*b*x)/(c + d*x)] + d^2*x*Sinh[(3*b*x)/(c
 + d*x)] + 3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))] - 3*b*c*Sinh[(
3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```

fricas [B] time = 0.63, size = 701, normalized size = 4.90

$$3\left(bc\mathrm{Ei}\left(-\frac{3\,bc}{d^2x+cd}\right)\cosh\left(\frac{3\,b}{d}\right)-bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\right)\sinh\left(\frac{bx}{dx+c}\right)^4+2\left(d^2x+cd\right)\sinh\left(\frac{bx}{dx+c}\right)^3-6\left(bc\mathrm{Ei}\left(-\frac{3}{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(3*b/d) - b*c*Ei(-b*c/(d^2*x + c*d
))*cosh(b/d))*sinh(b*x/(d*x + c))^4 + 2*(d^2*x + c*d)*sinh(b*x/(d*x + c))^3
 - 6*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(3*b/d) - b*c*
Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(b/d))*sinh(b*x/(d*x + c))
^2 + 3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^4 + b*c*Ei(3*b*c/(
d^2*x + c*d)))*cosh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
c))^4 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh(b/d) - 6*(d^2*x - (d^2*x + c*d)*cos
h(b*x/(d*x + c))^2 + c*d)*sinh(b*x/(d*x + c)) + 3*(b*c*Ei(-3*b*c/(d^2*x + c
*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-3*b*c/(d^2*x + c*d))*sinh(b*x/(d*x +
 c))^4 - b*c*Ei(3*b*c/(d^2*x + c*d)))*sinh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x +
 c*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c
))^4 - b*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^4 - 2
*d^2*cosh(b*x/(d*x + c))^2*sinh(b*x/(d*x + c))^2 + d^2*sinh(b*x/(d*x + c))^
4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(\frac{bx}{dx+c}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c))^3, x)
```

maple [A] time = 0.19, size = 228, normalized size = 1.59

$$-\frac{e^{-\frac{3bx}{dx+c}}(dx+c)}{8d}-\frac{3cb\,e^{-\frac{3b}{d}}\,\mathrm{Ei}\left(1,-\frac{3bc}{d(dx+c)}\right)}{8d^2}+\frac{3\,e^{-\frac{bx}{dx+c}}(dx+c)}{8d}+\frac{3cb\,e^{-\frac{b}{d}}\,\mathrm{Ei}\left(1,-\frac{bc}{d(dx+c)}\right)}{8d^2}+\frac{e^{\frac{3bx}{dx+c}}x}{8}+\frac{c\,e^{\frac{3bx}{dx+c}}}{8d}-\frac{3cb\,e^{\frac{3b}{d}}\,\mathrm{Ei}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c))^3,x)
```

```
[Out] -1/8/d*exp(-3*b*x/(d*x+c))*(d*x+c)-3/8*c*b/d^2*exp(-3*b/d)*Ei(1,-3*b*c/d/(d
*x+c))+3/8/d*exp(-b*x/(d*x+c))*(d*x+c)+3/8*c*b/d^2*exp(-b/d)*Ei(1,-b*c/d/(d
*x+c))+1/8*exp(3*b*x/(d*x+c))*x+1/8*c/d*exp(3*b*x/(d*x+c))-3/8*c*b/d^2*exp(
3*b/d)*Ei(1,3*b*c/d/(d*x+c))-3/8*exp(b*x/(d*x+c))*x-3/8*c/d*exp(b*x/(d*x+c)
)+3/8*c*b/d^2*exp(b/d)*Ei(1,b*c/d/(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3}{8}\,bc\int\frac{xe^{\left(\frac{3\,bc}{d^2x+cd}\right)}}{d^2x^2e^{\left(\frac{3\,b}{d}\right)}+2\,cdxe^{\left(\frac{3\,b}{d}\right)}+c^2e^{\left(\frac{3\,b}{d}\right)}}\,dx+\frac{3}{8}\,bc\int\frac{xe^{\left(\frac{bc}{d^2x+cd}\right)}}{d^2x^2e^{\frac{b}{d}}+2\,cdxe^{\frac{b}{d}}+c^2e^{\frac{b}{d}}}\,dx+\frac{3}{8}\,bc\int\frac{xe^{\left(-\frac{bc}{d^2x+cd}+\frac{b}{d}\right)}}{d^2x^2+2\,cdx+c^2}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -3/8*b*c*integrate(x*e^(3*b*c/(d^2*x + c*d))/(d^2*x^2*e^(3*b/d) + 2*c*d*x*e
^(3*b/d) + c^2*e^(3*b/d)), x) + 3/8*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(
d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/d) + c^2*e^(b/d)), x) + 3/8*b*c*integrate(x*
e^(-b*c/(d^2*x + c*d) + b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*b*c*integr
ate(x*e^(-3*b*c/(d^2*x + c*d) + 3*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/8*
(x*e^(3*b*c/(d^2*x + c*d)) - 3*x*e^(b*c/(d^2*x + c*d) + 2*b/d) + 3*x*e^(-b*
c/(d^2*x + c*d) + 4*b/d) - x*e^(-3*b*c/(d^2*x + c*d) + 6*b/d))*e^(-3*b/d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{sinh}\left(\frac{b\,x}{c+d\,x}\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x)/(c + d*x))^3,x)
```

```
[Out] int(sinh((b*x)/(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.295 $\int \sinh\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] (-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*cosh(b/d)/d^2-(-a*d+b*c)*Shi((-a*d+b*c
)/d/(d*x+c))*sinh(b/d)/d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))/d
```

Rubi [A] time = 0.18, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5607, 3297, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c +
 d*x)*Sinh[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*Sinh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-a\right.}{}$$

$$= \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right.}{d^2}$$

Mathematica [B] time = 0.71, size = 373, normalized size = 3.69

$$\frac{(bc-ad)\left(\cosh\left(\frac{b}{d}\right)-\sinh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+(bc-ad)\left(\sinh\left(\frac{b}{d}\right)+\cosh\left(\frac{b}{d}\right)\right)\text{Chi}\left(\frac{ad-bc}{d(c+dx)}\right)+ad\sinh\left(\frac{b}{d}\right)\text{Shi}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*(Cosh[b/d] - Sinh[b/d]
) + (b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/d] + Sin
```

```
h[b/d]) + 2*c*d*Sinh[(a + b*x)/(c + d*x)] + 2*d^2*x*Sinh[(a + b*x)/(c + d*x
)] + b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Cosh[b/
d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + b*c*Sinh[b/d]*SinhIntegral[
(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(
d*(c + d*x))] + b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - a*d
*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - b*c*Sinh[b/d]*SinhInte
gral[(b*c - a*d)/(c*d + d^2*x)] + a*d*Sinh[b/d]*SinhIntegral[(b*c - a*d)/(c
*d + d^2*x)])/(2*d^2)
```

fricas [A] time = 0.43, size = 171, normalized size = 1.69

$$\frac{\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\mathrm{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)-\left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)-(bc-\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*cosh(b/d) + 2*(d^2*x + c*d)*sinh((b*x + a)/(d*x + c)) -
((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d
^2*x + c*d)))*sinh(b/d))/d^2
```

giac [B] time = 4.23, size = 764, normalized size = 7.56

$$\frac{\left(b^3c^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}-2\,ab^2cd\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}-\frac{(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c}+a^2bd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}+\frac{2\,(bx+a)abc\cdots}{\cdots}\right.}{2\left(bd^2-\frac{(bx+\cdots}{dx\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-(
b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x +
 a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x
 + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d
)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*
e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^((b*x
 + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*
d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) + 1/2*(b^3*c^2*Ei((b - (
b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(
```

```
-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 2*
(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) -
(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) - a
^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b
*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [B] time = 0.05, size = 347, normalized size = 3.44

$$-\frac{\mathrm{e}^{-\frac{bx+a}{dx+c}}a}{2\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{\mathrm{e}^{-\frac{bx+a}{dx+c}}cb}{2d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{\mathrm{e}^{-\frac{b}{d}}\,\mathrm{Ei}\left(1,\frac{da-cb}{d(dx+c)}\right)a}{2d}-\frac{\mathrm{e}^{-\frac{b}{d}}\,\mathrm{Ei}\left(1,\frac{da-cb}{d(dx+c)}\right)cb}{2d^2}+\frac{d\,\mathrm{e}^{\frac{bx+a}{dx+c}}xa}{2da-2cb}-\frac{\mathrm{e}^{\frac{bx+a}{dx+c}}xcb}{2\left(da-cb\right)}+\frac{\mathrm{e}^{\frac{bx+a}{dx+c}}}{2da-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c)),x)
```

```
[Out] -1/2*exp(-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/2/d*exp(-(b*x+a)/(
d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b+1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x
+c))*a-1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*c*b+1/2*d*exp((b*x+a)/(d
*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*c*b+1/2*exp((b*x+
a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*
exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d
*x+c))*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(\frac{bx+a}{dx+c}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)/(c + d*x)),x)
```

```
[Out] int(sinh((a + b*x)/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x)
[Out] Integral(sinh((a + b*x)/(c + d*x)), x)
```

3.296 $\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[Out] -(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/d/(d*x+c))/d^2+(-a*d+b*c)*Chi(2*(-
a*d+b*c)/d/(d*x+c))*sinh(2*b/d)/d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))^2/d
```

Rubi [A] time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5607, 3313, 12, 3303, 3298, 3301}

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2
 + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(2i(bc-ad))\,\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{((bc -}{}$$

$$= \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-}{d(c+}\right.}{d^2}$$

Mathematica [A] time = 0.87, size = 112, normalized size = 1.05

$$\frac{2\sinh\left(\frac{2b}{d}\right)(bc-ad)\mathrm{Chi}\left(\frac{2(ad-bc)}{d(c+dx)}\right)+2\cosh\left(\frac{2b}{d}\right)(bc-ad)\mathrm{Shi}\left(\frac{2(ad-bc)}{d(c+dx)}\right)+d\left((c+dx)\cosh\left(\frac{2(a+bx)}{c+dx}\right)-dx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)*CoshI
ntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] + 2*(b*c - a*d)*Cos
h[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

fricas [B] time = 3.33, size = 370, normalized size = 3.46

$$-\frac{d^2x-\left(d^2x+cd\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2-\left(d^2x-(bc-ad)\mathrm{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2-\left((bc-ad)\mathrm{Ei}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 - (d^2*x - (b*c - a
*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*x
 + c))^2 - ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*
x + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) - ((b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 - (b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2 + (b*
c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x + a)/
(d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

giac [B] time = 17.26, size = 749, normalized size = 7.00

$$\frac{\left(2\,b^3c^2\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}-4\,ab^2cd\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}-\frac{2\,(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}}{dx+c}+2\,a^2bd^2\mathrm{Ei}\left(-\frac{2\left(b\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*d
*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*Ei(
-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(b -
(b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b - (b*
x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*d/(d*x
+ c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(d*x + c)
)/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)*e^(
-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) + b^2*c^2*d*e^
(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x + c)) + a^2*d^3
*e^(-2*(b*x + a)/(d*x + c)) - 2*b^2*c^2*d + 4*a*b*c*d^2 - 2*a^2*d^3)*(b*c/(
b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [B] time = 0.21, size = 358, normalized size = 3.35

$$-\frac{x}{2}+\frac{e^{-\frac{2(bx+a)}{dx+c}} a}{\frac{4da}{dx+c}-\frac{4bc}{dx+c}}-\frac{e^{-\frac{2(bx+a)}{dx+c}} cb}{4d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{e^{-\frac{2b}{d}} \operatorname{Ei}\left(1, \frac{2da-2cb}{d(dx+c)}\right) a}{2d}+\frac{e^{-\frac{2b}{d}} \operatorname{Ei}\left(1, \frac{2da-2cb}{d(dx+c)}\right) cb}{2d^2}+\frac{d\, e^{\frac{2bx+2a}{dx+c}} xa}{4da-4cb}-\frac{e^{\frac{2bx+2a}{dx+c}} xcb}{4\,(da-cb)}+\frac{e^{\frac{2bx+2}{dx+c}}}{4da-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/4/d*exp(-2
*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b-1/2/d*exp(-2*b/d)*Ei(1,2*(a
*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*c*b+1/
4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*
c)*x*c*b+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/(d*x+
c))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1/2/d^2
*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,x+\frac{1}{4}\,\int e^{\left(\frac{2\,bc}{d^2x+cd}-\frac{2\,a}{dx+c}-\frac{2\,b}{d}\right)}\,dx+\frac{1}{4}\,\int e^{\left(-\frac{2\,bc}{d^2x+cd}+\frac{2\,a}{dx+c}+\frac{2\,b}{d}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x)
+ 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)/(c + d*x))^2,x)
[Out] int(sinh((a + b*x)/(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))**2,x)
[Out] Timed out
```

3.297 $\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=194

$$-\frac{3\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}+\frac{3\cosh\left(\frac{3b}{d}\right)(bc-ad)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\sinh}{}$$

```
[Out] -3/4*(-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*cosh(b/d)/d^2+3/4*(-a*d+b*c)*Chi(
3*(-a*d+b*c)/d/(d*x+c))*cosh(3*b/d)/d^2+3/4*(-a*d+b*c)*Shi((-a*d+b*c)/d/(d*
x+c))*sinh(b/d)/d^2-3/4*(-a*d+b*c)*Shi(3*(-a*d+b*c)/d/(d*x+c))*sinh(3*b/d)/
d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))^3/d
```

Rubi [A] time = 0.33, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5607, 3313, 3303, 3298, 3301}

$$-\frac{3\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}+\frac{3\cosh\left(\frac{3b}{d}\right)(bc-ad)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*(b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2)
+ (3*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(b*c - a*d))/(d*(c + d*x))])
/(4*d^2) + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sinh
[(3*b)/d]*SinhIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)dx = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x}+\frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{4x}\right)dx, x\right.}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3(bc-ad)}{}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left(3(bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{(3(b}{}$$

$$= -\frac{3(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sin}{d}$$

Mathematica [B] time = 1.42, size = 599, normalized size = 3.09

$$-3ad\sinh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+3bc\sinh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)+3ad\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)-3bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc-ad}{xd^2+cd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (6*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))]
 - 3*b*c*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*a*d*Cosh[b/d
]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*b*c*CoshIntegral[(b*c - a*d)/
(c*d + d^2*x)]*Sinh[b/d] - 3*a*d*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*Si
nh[b/d] - 3*(b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/
d] + Sinh[b/d]) - 6*c*d*Sinh[(a + b*x)/(c + d*x)] - 6*d^2*x*Sinh[(a + b*x)/
(c + d*x)] + 2*c*d*Sinh[(3*(a + b*x))/(c + d*x)] + 2*d^2*x*Sinh[(3*(a + b*x
))/(c + d*x)] - 3*b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))]
+ 3*a*d*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - 3*b*c*Sinh[b
/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + 3*a*d*Sinh[b/d]*SinhIntegr
al[(-(b*c) + a*d)/(d*(c + d*x))] + 6*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*(-(b
*c) + a*d))/(d*(c + d*x))] - 6*a*d*Sinh[(3*b)/d]*SinhIntegral[(3*(-(b*c) +
a*d))/(d*(c + d*x))] - 3*b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*
x)] + 3*a*d*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*b*c*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - 3*a*d*Sinh[b/d]*SinhIntegral
[(b*c - a*d)/(c*d + d^2*x)])/(8*d^2)
```

fricas [B] time = 1.33, size = 717, normalized size = 3.70

$$-\frac{6\,(bc-ad)\mathrm{Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2\cosh\left(\frac{3\,b}{d}\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2-3\,(bc-ad)\mathrm{Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{3\,b}{d}\right)\sinh\left(\frac{bx+a}{dx+c}\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x +
c))^2*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^2 - 3*(b*c - a*d)*Ei(-3*(b*c -
a*d)/(d^2*x + c*d))*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^4 - 2*(d^2*x + c*
d)*sinh((b*x + a)/(d*x + c))^3 - 3*((b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x +
c*d))*cosh((b*x + a)/(d*x + c))^4 + (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c
*d)))*cosh(3*b/d) + 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a
*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) + 6*(d^2*x - (d^2*x + c*d)*co
sh((b*x + a)/(d*x + c))^2 + c*d)*sinh((b*x + a)/(d*x + c)) - 3*((b*c - a*d)
```

```
*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^4 - 2*(b*c - a*
d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2*sinh((b*x +
 a)/(d*x + c))^2 + (b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x +
 a)/(d*x + c))^4 - (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c*d)))*sinh(3*b/d)
 - 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh((b*x + a)/(d*x + c))^4 - 2*d^2*cosh
((b*x + a)/(d*x + c))^2*sinh((b*x + a)/(d*x + c))^2 + d^2*sinh((b*x + a)/(d
*x + c))^4)
```

giac [B] time = 24.02, size = 1383, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*c^2*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 6*a*b^2*c*d
*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 3*(b*x + a)*b^2*c^2*d*Ei(
-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) + 3*a^2*b*d^2*Ei(-3*(
b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) + 6*(b*x + a)*a*b*c*d^2*Ei(-3*(b -
(b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*(b*x + a)*a^2*d^3*Ei(-3*(
b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*b^3*c^2*Ei(-(b - (b*x
 + a)*d/(d*x + c))/d)*e^(b/d) + 6*a*b^2*c*d*Ei(-(b - (b*x + a)*d/(d*x + c))
/d)*e^(b/d) + 3*(b*x + a)*b^2*c^2*d*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b
/d)/(d*x + c) - 3*a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 6*
(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) +
3*(b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) -
3*b^3*c^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 6*a*b^2*c*d*Ei((b -
(b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 3*(b*x + a)*b^2*c^2*d*Ei((b - (b*x + a
)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - 3*a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x
 + c))/d)*e^(-b/d) - 6*(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d
)*e^(-b/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d
)*e^(-b/d)/(d*x + c) + 3*b^3*c^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*
b/d) - 6*a*b^2*c*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d) - 3*(b*x
+ a)*b^2*c^2*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) + 3
*a^2*b*d^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d) + 6*(b*x + a)*a*b
*c*d^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) - 3*(b*x +
a)*a^2*d^3*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) + b^2*c
^2*d*e^(3*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(3*(b*x + a)/(d*x + c)) + a^
2*d^3*e^(3*(b*x + a)/(d*x + c)) - 3*b^2*c^2*d*e^((b*x + a)/(d*x + c)) + 6*a
*b*c*d^2*e^((b*x + a)/(d*x + c)) - 3*a^2*d^3*e^((b*x + a)/(d*x + c)) + 3*b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) - 6*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) + 3
*a^2*d^3*e^(-(b*x + a)/(d*x + c)) - b^2*c^2*d*e^(-3*(b*x + a)/(d*x + c)) +
2*a*b*c*d^2*e^(-3*(b*x + a)/(d*x + c)) - a^2*d^3*e^(-3*(b*x + a)/(d*x + c))
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [B] time = 0.18, size = 700, normalized size = 3.61

$$-\frac{e^{-\frac{3(bx+a)}{dx+c}} a}{8\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{3(bx+a)}{dx+c}} cb}{8d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{3\,e^{-\frac{3b}{d}} \operatorname{Ei}\left(1,\frac{3da-3cb}{d(dx+c)}\right) a}{8d}-\frac{3\,e^{-\frac{3b}{d}} \operatorname{Ei}\left(1,\frac{3da-3cb}{d(dx+c)}\right) cb}{8d^{2}}+\frac{3\,e^{-\frac{bx+a}{dx+c}} a}{8\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{3\,e^{-\frac{bx+}{dx+}}}{8d\left(\frac{da}{dx+c}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c))^3,x)
```

```
[Out] -1/8*exp(-3*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/8/d*exp(-3*(b*x+
a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*c*b+3/8/d*exp(-3*b/d)*Ei(1,3*(a*d-b*c
)/d/(d*x+c))*a-3/8/d^2*exp(-3*b/d)*Ei(1,3*(a*d-b*c)/d/(d*x+c))*c*b+3/8*exp(
-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-3/8/d*exp(-(b*x+a)/(d*x+c))/(
d*a/(d*x+c)-b*c/(d*x+c))*c*b-3/8/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*a+3/
8/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*c*b+1/8*d*exp(3*(b*x+a)/(d*x+c))/
(a*d-b*c)*x*a-1/8*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*x*c*b+1/8*exp(3*(b*x+a)/
(d*x+c))/(a*d-b*c)*c*a-1/8/d*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+3/8/d*e
xp(3*b/d)*Ei(1,-3*(a*d-b*c)/d/(d*x+c))*a-3/8/d^2*exp(3*b/d)*Ei(1,-3*(a*d-b*
c)/d/(d*x+c))*c*b-3/8*d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*a+3/8*exp((b*x+a)/
(d*x+c))/(a*d-b*c)*x*c*b-3/8*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c*a+3/8/d*exp((
b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b-3/8/d*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a
+3/8/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(\frac{bx+a}{dx+c}\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)/(c + d*x))^3,x)
```

```
[Out] int(sinh((a + b*x)/(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.298 $\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal. Leaf size=121

$$\frac{f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

```
[Out] (-a*d+b*c)*f*Chi((-a*d+b*c)*f/d/(d*x+c))*cosh(e+b*f/d)/d^2-(-a*d+b*c)*f*Shi
((-a*d+b*c)*f/d/(d*x+c))*sinh(e+b*f/d)/d^2+(d*x+c)*sinh((b*f*x+d*e*x+a*f+c*
e)/(d*x+c))/d
```

Rubi [A] time = 0.26, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5609, 5607, 3297, 3303, 3298, 3301}

$$\frac{f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)])/d - ((b*c
- a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))])/d^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5609

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)dx = \int \sinh\left(\frac{ce+af+(de+bf)x}{c+dx}\right)dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{((bc-ad)f)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{de+bf}{d}-\frac{(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+d\ldots}\right.}{d^2}$$
$$= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{\left((bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)fx}{d}\right)}{x}\ldots\right.}{d^2}$$
$$= \frac{(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-a\ldots}{\ldots}$$

Mathematica [B] time = 1.48, size = 449, normalized size = 3.71

$$\frac{f(bc-ad)\left(\cosh\left(\frac{bf}{d}+e\right)-\sinh\left(\frac{bf}{d}+e\right)\right)\operatorname{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)+f(bc-ad)\left(\sinh\left(\frac{bf}{d}+e\right)+\cosh\left(\frac{bf}{d}+e\right)\right)\operatorname{Chi}\left(\frac{adf-b}{d(c+d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*f*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))]*(Cosh[e + (b*f)/
d] - Sinh[e + (b*f)/d]) + (b*c - a*d)*f*CoshIntegral[(-(b*c*f) + a*d*f)/(d*
(c + d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) + 2*c*d*Sinh[(c*e + a*f
 + d*e*x + b*f*x)/(c + d*x)] + 2*d^2*x*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c
+ d*x)] + b*c*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x)
)] - a*d*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] -
b*c*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + a*d*f
*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + b*c*f*Cosh
[e + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - a*d*f*Cosh[e
 + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + b*c*f*Sinh[e +
 (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - a*d*f*Sinh[e + (
b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))])/(2*d^2)
```

fricas [A] time = 1.61, size = 202, normalized size = 1.67

$$\frac{\left((bc-ad)f\operatorname{Ei}\left(\frac{(bc-ad)f}{d^2x+cd}\right)+(bc-ad)f\operatorname{Ei}\left(-\frac{(bc-ad)f}{d^2x+cd}\right)\right)\cosh\left(\frac{de+bf}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{ce+af+\left(de+bf\right)x}{dx+c}\right)-\left((bc-ad)\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-(b*
c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 2*(d^2*x + c*d)*sinh((c*e
+ a*f + (d*e + b*f)*x)/(d*x + c)) - ((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x
+ c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(d^2*x + c*d)))*sinh((d*e + b*f)/
d))/d^2
```

giac [B] time = 34.35, size = 1736, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^2*f^3*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/
d)*e^((b*f + d*e)/d) - 2*a*b^2*c*d*f^3*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*
f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d) + a^2*b*d^2*f^3*Ei(-(b*f + d*e -
 (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d) + b^2*c^2*d*
f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f +
d*e)/d + 1) - 2*a*b*c*d^2*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*
d/(d*x + c))/d)*e^((b*f + d*e)/d + 1) + a^2*d^3*f^2*Ei(-(b*f + d*e - (b*f*x
 + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d + 1) - (b*f*x + d*x*
e + a*f + c*e)*b^2*c^2*d*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d
/(d*x + c))/d)*e^((b*f + d*e)/d)/(d*x + c) + 2*(b*f*x + d*x*e + a*f + c*e)*
a*b*c*d^2*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*
e^((b*f + d*e)/d)/(d*x + c) - (b*f*x + d*x*e + a*f + c*e)*a^2*d^3*f^2*Ei(-(
b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d)/(
d*x + c) + b^2*c^2*d*f^2*e^((b*f*x + d*x*e + a*f + c*e)/(d*x + c)) - 2*a*b*
c*d^2*f^2*e^((b*f*x + d*x*e + a*f + c*e)/(d*x + c)) + a^2*d^3*f^2*e^((b*f*x
 + d*x*e + a*f + c*e)/(d*x + c)))*((b*f + d*e)*c/(b*c*f - a*d*f)^2 - (a*f +
 c*e)*d/(b*c*f - a*d*f)^2)/(b*d^2*f + d^3*e - (b*f*x + d*x*e + a*f + c*e)*d
^3/(d*x + c)) + 1/2*(b^3*c^2*f^3*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e
)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d) - 2*a*b^2*c*d*f^3*Ei((b*f + d*e - (b*f
*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d) + a^2*b*d^2*f^3*
Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)
/d) + b^2*c^2*d*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c)
)/d)*e^(-(b*f + d*e)/d + 1) - 2*a*b*c*d^2*f^2*Ei((b*f + d*e - (b*f*x + d*x*
e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d + 1) + a^2*d^3*f^2*Ei((b*f
 + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d + 1)
 - (b*f*x + d*x*e + a*f + c*e)*b^2*c^2*d*f^2*Ei((b*f + d*e - (b*f*x + d*x*e
 + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d)/(d*x + c) + 2*(b*f*x + d*x
*e + a*f + c*e)*a*b*c*d^2*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d
/(d*x + c))/d)*e^(-(b*f + d*e)/d)/(d*x + c) - (b*f*x + d*x*e + a*f + c*e)*a
^2*d^3*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-
(b*f + d*e)/d)/(d*x + c) - b^2*c^2*d*f^2*e^(-(b*f*x + d*x*e + a*f + c*e)/(d
*x + c)) + 2*a*b*c*d^2*f^2*e^(-(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) - a^2
*d^3*f^2*e^(-(b*f*x + d*x*e + a*f + c*e)/(d*x + c)))*((b*f + d*e)*c/(b*c*f
- a*d*f)^2 - (a*f + c*e)*d/(b*c*f - a*d*f)^2)/(b*d^2*f + d^3*e - (b*f*x + d
*x*e + a*f + c*e)*d^3/(d*x + c))
```

maple [B] time = 0.08, size = 459, normalized size = 3.79

$$-\frac{f\,\mathrm{e}^{-\frac{bfx+dex+af+ce}{dx+c}}a}{2\left(\frac{daf}{dx+c}-\frac{fcb}{dx+c}\right)}+\frac{f\,\mathrm{e}^{-\frac{bfx+dex+af+ce}{dx+c}}cb}{2d\left(\frac{daf}{dx+c}-\frac{fcb}{dx+c}\right)}+\frac{f\,\mathrm{e}^{-\frac{bf+de}{d}}\,\mathrm{Ei}\left(1,\frac{(da-cb)f}{d(dx+c)}\right)a}{2d}-\frac{f\,\mathrm{e}^{-\frac{bf+de}{d}}\,\mathrm{Ei}\left(1,\frac{(da-cb)f}{d(dx+c)}\right)cb}{2d^{2}}+\frac{f\,\mathrm{e}^{\frac{bfx+dex+af+ce}{dx+c}}a}{2d\left(\frac{af}{dx+c}-\frac{fcb}{d(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c)),x)
```

```
[Out] -1/2*f*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*a+
1/2/d*f*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*c
*b+1/2/d*f*exp(-(b*f+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*a-1/2/d^2*f*exp(-(
b*f+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*c*b+1/2/d*f*exp((b*f*x+d*e*x+a*f+c*
e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*a-1/2/d^2*f*exp((b*f*x+d*e*x+a*
f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b+1/2/d*f*exp((b*f+d*e)/d
)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*a-1/2/d^2*f*exp((b*
f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(e + \frac{(bx+a)f}{dx+c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e + \frac{f\,(a+b\,x)}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x)),x)
```

```
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(e + \frac{f(a+bx)}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(sinh(e + f*(a + b*x)/(c + d*x)), x)
```

3.299 $\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal. Leaf size=129

$$\frac{f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{af+bfx\cdots}{c\cdots}\right.}{d}$$

```
[Out] -(-a*d+b*c)*f*cosh(2*e+2*b*f/d)*Shi(2*(-a*d+b*c)*f/d/(d*x+c))/d^2+(-a*d+b*c
)*f*Chi(2*(-a*d+b*c)*f/d/(d*x+c))*sinh(2*e+2*b*f/d)/d^2+(d*x+c)*sinh((b*f*x
+d*e*x+a*f+c*e)/(d*x+c))^2/d
```

Rubi [A] time = 0.28, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5609, 5607, 3313, 12, 3303, 3298, 3301}

$$\frac{f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{af+bfx\cdots}{c\cdots}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*f*CoshIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))]*Sinh[2*(e + (b
*f)/d)])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)]^2)/d
- ((b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c
 + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5609

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)dx = \int \sinh^2\left(\frac{ce+af+(de+bf)x}{c+dx}\right)dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(2i(bc-ad)f)\,\text{Subst}\left(\int \frac{i\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)fx}{d}\right)}{2x}d\right.}{d^2}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{((bc-ad)f)\,\text{Subst}\left(\int \frac{\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)fx}{d}\right)}{x}dx,\right.}{d^2}$$
$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{\left((bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(b}{}\right.}{x}\right.}{d^2}$$
$$= \frac{(bc-ad)f\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(b}{}$$

Mathematica [A] time = 2.10, size = 136, normalized size = 1.05

$$\frac{2f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(adf-bcf)}{d(c+dx)}\right)+2f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(adf-bcf)}{d(c+dx)}\right)+d\left((c+dx)\cosh\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)]) + 2
*(b*c - a*d)*f*CoshIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))]*Sinh[2*(e
 + (b*f)/d)] + 2*(b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(-(b*c
*f) + a*d*f))/(d*(c + d*x))])/(2*d^2)
```

fricas [B] time = 1.65, size = 477, normalized size = 3.70

$$-\frac{d^2x-\left(d^2x+cd\right)\cosh\left(\frac{ce+af+\left(de+bf\right)x}{dx+c}\right)^2+\left((bc-ad)f\text{Ei}\left(-\frac{2\left(bc-ad\right)f}{d^2x+cd}\right)\cosh\left(\frac{2\left(de+bf\right)}{d}\right)-d^2x-cd\right)\sinh\left(\frac{ce+af+(}{dx-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 +
 ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh(2*(d*e + b*f)/d) -
d^2*x - c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 - ((b*c - a*d)*f
*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x +
 c))^2 - (b*c - a*d)*f*Ei(2*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(2*(d*e + b*f
)/d) - ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f +
(d*e + b*f)*x)/(d*x + c))^2 - (b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*
d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*Ei(2*(b*c
 - a*d)*f/(d^2*x + c*d)))*sinh(2*(d*e + b*f)/d))/(d^2*cosh((c*e + a*f + (d*
e + b*f)*x)/(d*x + c))^2 - d^2*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^
2)
```

giac [B] time = 154.69, size = 1703, normalized size = 13.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*c^2*f^3*Ei(-2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x +
c))/d)*e^(2*(b*f + d*e)/d) - 4*a*b^2*c*d*f^3*Ei(-2*(b*f + d*e - (b*f*x + d*
x*e + a*f + c*e)*d/(d*x + c))/d)*e^(2*(b*f + d*e)/d) + 2*a^2*b*d^2*f^3*Ei(-
2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(2*(b*f + d*e)
/d) - 2*b^3*c^2*f^3*Ei(2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x +
c))/d)*e^(-2*(b*f + d*e)/d) + 4*a*b^2*c*d*f^3*Ei(2*(b*f + d*e - (b*f*x + d*
x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-2*(b*f + d*e)/d) - 2*a^2*b*d^2*f^3*Ei(
2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-2*(b*f + d*e
)/d) - 2*(b*f*x + d*x*e + a*f + c*e)*b^2*c^2*d*f^2*Ei(-2*(b*f + d*e - (b*f*
x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(2*(b*f + d*e)/d)/(d*x + c) + 4*(b
*f*x + d*x*e + a*f + c*e)*a*b*c*d^2*f^2*Ei(-2*(b*f + d*e - (b*f*x + d*x*e +
 a*f + c*e)*d/(d*x + c))/d)*e^(2*(b*f + d*e)/d)/(d*x + c) - 2*(b*f*x + d*x*
e + a*f + c*e)*a^2*d^3*f^2*Ei(-2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d
/(d*x + c))/d)*e^(2*(b*f + d*e)/d)/(d*x + c) + 2*b^2*c^2*d*f^2*Ei(-2*(b*f +
 d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(2*(b*f + d*e)/d + 1)
- 4*a*b*c*d^2*f^2*Ei(-2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c
))/d)*e^(2*(b*f + d*e)/d + 1) + 2*a^2*d^3*f^2*Ei(-2*(b*f + d*e - (b*f*x + d
*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(2*(b*f + d*e)/d + 1) - 2*b^2*c^2*d*f^2
*Ei(2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-2*(b*f +
 d*e)/d + 1) + 4*a*b*c*d^2*f^2*Ei(2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e
)*d/(d*x + c))/d)*e^(-2*(b*f + d*e)/d + 1) - 2*a^2*d^3*f^2*Ei(2*(b*f + d*e
- (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-2*(b*f + d*e)/d + 1) + 2*
```

```
(b*f*x + d*x*e + a*f + c*e)*b^2*c^2*d*f^2*Ei(2*(b*f + d*e - (b*f*x + d*x*e
+ a*f + c*e)*d/(d*x + c))/d)*e^(-2*(b*f + d*e)/d)/(d*x + c) - 4*(b*f*x + d*
x*e + a*f + c*e)*a*b*c*d^2*f^2*Ei(2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e
)*d/(d*x + c))/d)*e^(-2*(b*f + d*e)/d)/(d*x + c) + 2*(b*f*x + d*x*e + a*f +
 c*e)*a^2*d^3*f^2*Ei(2*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c)
)/d)*e^(-2*(b*f + d*e)/d)/(d*x + c) + b^2*c^2*d*f^2*e^(2*(b*f*x + d*x*e + a
*f + c*e)/(d*x + c)) - 2*a*b*c*d^2*f^2*e^(2*(b*f*x + d*x*e + a*f + c*e)/(d*
x + c)) + a^2*d^3*f^2*e^(2*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) + b^2*c^2
*d*f^2*e^(-2*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) - 2*a*b*c*d^2*f^2*e^(-2
*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) + a^2*d^3*f^2*e^(-2*(b*f*x + d*x*e
+ a*f + c*e)/(d*x + c)) - 2*b^2*c^2*d*f^2 + 4*a*b*c*d^2*f^2 - 2*a^2*d^3*f^2
)*((b*f + d*e)*c/(b*c*f - a*d*f)^2 - (a*f + c*e)*d/(b*c*f - a*d*f)^2)/(b*d^
2*f + d^3*e - (b*f*x + d*x*e + a*f + c*e)*d^3/(d*x + c))
```

maple [B] time = 0.24, size = 468, normalized size = 3.63

$$-\frac{x}{2}+\frac{f\,\mathrm{e}^{-\frac{2\left(bfx+dex+af+ce\right)}{dx+c}}a}{\frac{4daf}{dx+c}-\frac{4fcb}{dx+c}}-\frac{f\,\mathrm{e}^{-\frac{2\left(bfx+dex+af+ce\right)}{dx+c}}cb}{4d\left(\frac{daf}{dx+c}-\frac{fcb}{dx+c}\right)}-\frac{f\,\mathrm{e}^{-\frac{2\left(bf+de\right)}{d}}\,\mathrm{Ei}\left(1,\frac{2(da-cb)f}{d(dx+c)}\right)a}{2d}+\frac{f\,\mathrm{e}^{-\frac{2\left(bf+de\right)}{d}}\,\mathrm{Ei}\left(1,\frac{2(da-cb)f}{d(dx+c)}\right)cb}{2d^2}+\frac{f\,\mathrm{e}^{\frac{2b}{}}}{4d\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^2,x)
```

```
[Out] -1/2*x+1/4*f*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)
*c*b)*a-1/4/d*f*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x
+c)*c*b)*c*b-1/2/d*f*exp(-2*(b*f+d*e)/d)*Ei(1,2*(a*d-b*c)*f/d/(d*x+c))*a+1/
2/d^2*f*exp(-2*(b*f+d*e)/d)*Ei(1,2*(a*d-b*c)*f/d/(d*x+c))*c*b+1/4/d*f*exp(2
*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*a-1/4/d^2*f
*exp(2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b+1
/2/d*f*exp(2*(b*f+d*e)/d)*Ei(1,-2*(a*d-b*c)*f/d/(d*x+c)-2*(b*f+d*e)/d-2*(-b
*f-d*e)/d)*a-1/2/d^2*f*exp(2*(b*f+d*e)/d)*Ei(1,-2*(a*d-b*c)*f/d/(d*x+c)-2*(
b*f+d*e)/d-2*(-b*f-d*e)/d)*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,x+\frac{1}{4}\,\int e^{\left(\frac{2\,bcf}{d^2x+cd}-2\,e-\frac{2\,af}{dx+c}-\frac{2\,bf}{d}\right)}\,dx+\frac{1}{4}\,\int e^{\left(-\frac{2\,bcf}{d^2x+cd}+2\,e+\frac{2\,af}{dx+c}+\frac{2\,bf}{d}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*integrate(e^(2*b*c*f/(d^2*x + c*d) - 2*e - 2*a*f/(d*x + c) - 2
*b*f/d), x) + 1/4*integrate(e^(-2*b*c*f/(d^2*x + c*d) + 2*e + 2*a*f/(d*x +
c) + 2*b*f/d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+\frac{f\,\left(a+b\,x\right)}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x))^2,x)
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**2,x)
[Out] Timed out
```

3.300 $\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal. Leaf size=226

$$-\frac{3f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\mathrm{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\mathrm{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\sinh\left(\frac{bf}{d}\right.}{4d^2}$$

```
[Out] -3/4*(-a*d+b*c)*f*Chi((-a*d+b*c)*f/d/(d*x+c))*cosh(e+b*f/d)/d^2+3/4*(-a*d+b
*c)*f*Chi(3*(-a*d+b*c)*f/d/(d*x+c))*cosh(3*e+3*b*f/d)/d^2+3/4*(-a*d+b*c)*f*
Shi((-a*d+b*c)*f/d/(d*x+c))*sinh(e+b*f/d)/d^2-3/4*(-a*d+b*c)*f*Shi(3*(-a*d+
b*c)*f/d/(d*x+c))*sinh(3*e+3*b*f/d)/d^2+(d*x+c)*sinh((b*f*x+d*e*x+a*f+c*e)/
(d*x+c))^3/d
```

Rubi [A] time = 0.48, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5609, 5607, 3313, 3303, 3298, 3301}

$$-\frac{3f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\mathrm{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\mathrm{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\sinh\left(\frac{bf}{d}\right.}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

```
[Out] (-3*(b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*
x))])/(4*d^2) + (3*(b*c - a*d)*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(b*c
 - a*d)*f)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b
*f*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b
*c - a*d)*f)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*f*Sinh[3*(e + (b*f)/d
)]*SinhIntegral[(3*(b*c - a*d)*f)/(d*(c + d*x))])/(4*d^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5607

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> -Dist[d^(-1), Subst[Int[Sinh[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5609

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)dx=\int \sinh^3\left(\frac{ce+af+(de+bf)x}{c+dx}\right)dx$$
$$=-\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}dx,x,\frac{1}{c+dx}\right)}{d}$$
$$=\frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{(3(bc-ad)f)\,\text{Subst}\left(\int\left(-\frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc-ad)fx}{d}\right)}{4x}\right.\right.}{d^2}$$
$$=\frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}+\frac{(3(bc-ad)f)\,\text{Subst}\left(\int \frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc-ad)fx}{d}\right)}{x}dx\right.}{4d^2}$$
$$=\frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{\left(3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)}{d}\right.}{x}\right.}{4d^2}$$
$$=-\frac{3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Chi}\left(\right.}{4d^2}$$

Mathematica [B] time = 6.00, size = 671, normalized size = 2.97

$$\frac{-6adf\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{3(adf-bcf)}{d(c+dx)}\right)+6bcf\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{3(adf-bcf)}{d(c+dx)}\right)+3f(bc-ad)\left(\sinh\left(\frac{bf}{d}+e\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

```
[Out] (6*b*c*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c +
d*x))] - 6*a*d*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(-(b*c*f) + a*d*f))/
(d*(c + d*x))] + 3*(b*c - a*d)*f*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
]*(-Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) - 3*(b*c - a*d)*f*CoshIntegral[(
-(b*c*f) + a*d*f)/(d*(c + d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) -
6*c*d*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)] - 6*d^2*x*Sinh[(c*e + a*f
 + d*e*x + b*f*x)/(c + d*x)] + 2*c*d*Sinh[(3*(c*e + a*f + d*e*x + b*f*x))/(
c + d*x)] + 2*d^2*x*Sinh[(3*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)] - 3*b*c
*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + 3*a*d*f*
Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + 3*b*c*f*Sin
h[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] - 3*a*d*f*Sinh[e
```

```
 + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] - 3*b*c*f*Cosh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 3*a*d*f*Cosh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - 3*b*c*f*Sinh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 3*a*d*f*Sinh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 6*b*c*f*Sinh[3*(e
 + (b*f)/d)]*SinhIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))] - 6*a*d*f*S
inh[3*(e + (b*f)/d)]*SinhIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))])/(8
*d^2)
```

fricas [B] time = 1.95, size = 942, normalized size = 4.17

$$-\frac{6\,(bc-ad)f\,\mathrm{Ei}\left(-\frac{3\,(bc-ad)f}{d^2x+cd}\right)\cosh\left(\frac{ce+af+\left(de+bf\right)x}{dx+c}\right)^2\cosh\left(\frac{3\left(de+bf\right)}{d}\right)\sinh\left(\frac{ce+af+\left(de+bf\right)x}{dx+c}\right)^2-3\,(bc-ad)f\,\mathrm{Ei}\left(-\frac{3\,(bc-}{d^2x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f +
(d*e + b*f)*x)/(d*x + c))^2*cosh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e +
b*f)*x)/(d*x + c))^2 - 3*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*c
osh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*(d^2
*x + c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^3 - 3*((b*c - a*d)*f*
Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x +
c))^4 + (b*c - a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(3*(d*e + b*f)
/d) + 3*((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-
(b*c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 6*(d^2*x - (d^2*x + c*d
)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + c*d)*sinh((c*e + a*f + (d
*e + b*f)*x)/(d*x + c)) - 3*((b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d
))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*(b*c - a*d)*f*Ei(-3*(b
*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2*si
nh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*Ei(-3*(b*c - a*
d)*f/(d^2*x + c*d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - (b*c -
a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*sinh(3*(d*e + b*f)/d) - 3*((b*c -
 a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(
d^2*x + c*d)))*sinh((d*e + b*f)/d))/(d^2*cosh((c*e + a*f + (d*e + b*f)*x)/(
d*x + c))^4 - 2*d^2*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2*sinh((c*e
 + a*f + (d*e + b*f)*x)/(d*x + c))^2 + d^2*sinh((c*e + a*f + (d*e + b*f)*x)
/(d*x + c))^4)
```

giac [B] time = 178.86, size = 3230, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="giac")

[Out] 1/8*(3*b^3*c^2*f^3*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d) - 6*a*b^2*c*d*f^3*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d) + 3*a^2*b*d^2*f^3*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d) - 3*b^3*c^2*f^3*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d) + 6*a*b^2*c*d*f^3*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d) - 3*a^2*b*d^2*f^3*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d) - 3*b^3*c^2*f^3*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d) + 6*a*b^2*c*d*f^3*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d) - 3*a^2*b*d^2*f^3*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d) + 3*b^3*c^2*f^3*Ei(3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d) - 6*a*b^2*c*d*f^3*Ei(3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d) + 3*a^2*b*d^2*f^3*Ei(3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d) - 3*(b*f*x + d*x*e + a*f + c*e)*b^2*c^2*d*f^2*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d)/(d*x + c) + 6*(b*f*x + d*x*e + a*f + c*e)*a*b*c*d^2*f^2*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d)/(d*x + c) - 3*(b*f*x + d*x*e + a*f + c*e)*a^2*d^3*f^2*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d)/(d*x + c) + 3*b^2*c^2*d*f^2*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d + 1) - 6*a*b*c*d^2*f^2*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d + 1) + 3*a^2*d^3*f^2*Ei(-3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(3*(b*f + d*e)/d + 1) - 3*b^2*c^2*d*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d + 1) + 6*a*b*c*d^2*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d + 1) - 3*a^2*d^3*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d + 1) + 3*(b*f*x + d*x*e + a*f + c*e)*b^2*c^2*d*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d)/(d*x + c) - 6*(b*f*x + d*x*e + a*f + c*e)*a*b*c*d^2*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d)/(d*x + c) + 3*(b*f*x + d*x*e + a*f + c*e)*a^2*d^3*f^2*Ei(-(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^((b*f + d*e)/d)/(d*x + c) - 3*b^2*c^2*d*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d + 1) + 6*a*b*c*d^2*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d + 1) - 3*a^2*d^3*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d + 1) + 3*b^2*c^2*d*f^2*Ei(3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d + 1) - 6*a*b*c*d^2*f^2*Ei(3*(b*f + d*e - (b*f*x + d*x*e + a

```
*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d + 1) + 3*a^2*d^3*f^2*Ei(3*(b*
f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d +
 1) + 3*(b*f*x + d*x*e + a*f + c*e)*b^2*c^2*d*f^2*Ei((b*f + d*e - (b*f*x +
d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d)/(d*x + c) - 6*(b*f*x
+ d*x*e + a*f + c*e)*a*b*c*d^2*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c
*e)*d/(d*x + c))/d)*e^(-(b*f + d*e)/d)/(d*x + c) + 3*(b*f*x + d*x*e + a*f +
 c*e)*a^2*d^3*f^2*Ei((b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/
d)*e^(-(b*f + d*e)/d)/(d*x + c) - 3*(b*f*x + d*x*e + a*f + c*e)*b^2*c^2*d*f
^2*Ei(3*(b*f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f
 + d*e)/d)/(d*x + c) + 6*(b*f*x + d*x*e + a*f + c*e)*a*b*c*d^2*f^2*Ei(3*(b*
f + d*e - (b*f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d)/
(d*x + c) - 3*(b*f*x + d*x*e + a*f + c*e)*a^2*d^3*f^2*Ei(3*(b*f + d*e - (b*
f*x + d*x*e + a*f + c*e)*d/(d*x + c))/d)*e^(-3*(b*f + d*e)/d)/(d*x + c) + b
^2*c^2*d*f^2*e^(3*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) - 2*a*b*c*d^2*f^2*
e^(3*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) + a^2*d^3*f^2*e^(3*(b*f*x + d*x
*e + a*f + c*e)/(d*x + c)) - 3*b^2*c^2*d*f^2*e^((b*f*x + d*x*e + a*f + c*e)
/(d*x + c)) + 6*a*b*c*d^2*f^2*e^((b*f*x + d*x*e + a*f + c*e)/(d*x + c)) - 3
*a^2*d^3*f^2*e^((b*f*x + d*x*e + a*f + c*e)/(d*x + c)) + 3*b^2*c^2*d*f^2*e^
(-(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) - 6*a*b*c*d^2*f^2*e^(-(b*f*x + d*x
*e + a*f + c*e)/(d*x + c)) + 3*a^2*d^3*f^2*e^(-(b*f*x + d*x*e + a*f + c*e)/
(d*x + c)) - b^2*c^2*d*f^2*e^(-3*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) + 2
*a*b*c*d^2*f^2*e^(-3*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)) - a^2*d^3*f^2*e
^(-3*(b*f*x + d*x*e + a*f + c*e)/(d*x + c)))*((b*f + d*e)*c/(b*c*f - a*d*f)
^2 - (a*f + c*e)*d/(b*c*f - a*d*f)^2)/(b*d^2*f + d^3*e - (b*f*x + d*x*e + a
*f + c*e)*d^3/(d*x + c))
```

maple [B] time = 0.22, size = 922, normalized size = 4.08

$$-\frac{f\,\mathrm{e}^{-\frac{3(bfx+dex+af+ce)}{dx+c}}a}{8\left(\frac{daf}{dx+c}-\frac{fcb}{dx+c}\right)}+\frac{f\,\mathrm{e}^{-\frac{3(bfx+dex+af+ce)}{dx+c}}cb}{8d\left(\frac{daf}{dx+c}-\frac{fcb}{dx+c}\right)}+\frac{3f\,\mathrm{e}^{-\frac{3(bf+de)}{d}}\,\mathrm{Ei}\left(1,\frac{3(da-cb)f}{d(dx+c)}\right)a}{8d}-\frac{3f\,\mathrm{e}^{-\frac{3(bf+de)}{d}}\,\mathrm{Ei}\left(1,\frac{3(da-cb)f}{d(dx+c)}\right)cb}{8d^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^3,x)
```

```
[Out] -1/8*f*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*
a+1/8/d*f*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*
b)*c*b+3/8/d*f*exp(-3*(b*f+d*e)/d)*Ei(1,3*(a*d-b*c)*f/d/(d*x+c))*a-3/8/d^2*
f*exp(-3*(b*f+d*e)/d)*Ei(1,3*(a*d-b*c)*f/d/(d*x+c))*c*b+3/8*f*exp(-(b*f*x+d
*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*a-3/8/d*f*exp(-(b*f*x+
d*e*x+a*f+c*e)/(d*x+c))/(d/(d*x+c)*a*f-f/(d*x+c)*c*b)*c*b-3/8/d*f*exp(-(b*f
+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*a+3/8/d^2*f*exp(-(b*f+d*e)/d)*Ei(1,(a*
d-b*c)*f/d/(d*x+c))*c*b+1/8/d*f*exp(3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*
x+c)*a*f-f/d/(d*x+c)*c*b)*a-1/8/d^2*f*exp(3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/
```

```
(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b+3/8/d*f*exp(3*(b*f+d*e)/d)*Ei(1,-3*(a*d
-b*c)*f/d/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)/d)*a-3/8/d^2*f*exp(3*(b*f+d*e)
/d)*Ei(1,-3*(a*d-b*c)*f/d/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)/d)*c*b-3/8/d*f
*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*a+3/8/d
^2*f*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(1/(d*x+c)*a*f-f/d/(d*x+c)*c*b)*c*b
-3/8/d*f*exp((b*f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e
)/d)*a+3/8/d^2*f*exp((b*f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(
-b*f-d*e)/d)*c*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(e + \frac{(bx+a)f}{dx+c}\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(e + \frac{f\,(a+b\,x)}{c+d\,x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x))^3,x)
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**3,x)
[Out] Timed out
```

3.301 $\int e^{a+bx} \sinh^4(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -1/48*exp(-3*b*x-3*a)/b+1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/12*exp(3*b*x+3
*a)/b+1/80*exp(5*b*x+5*a)/b
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

```
[Out] -E^(-3*a - 3*b*x)/(48*b) + E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) - E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{16x^4}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^4}\,dx, x, e^{a+bx}\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)dx, x, e^{a+bx}\right)}{16b}$$
$$= -\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.75

$$\frac{e^{-3(a+bx)}\left(60e^{2(a+bx)}+90e^{4(a+bx)}-20e^{6(a+bx)}+3e^{8(a+bx)}-5\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

```
[Out] (-5 + 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) - 20*E^(6*(a + b*x)) + 3*E^(8
*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

fricas [A] time = 0.77, size = 113, normalized size = 1.36

$$-\frac{\cosh(bx+a)^4-16\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-10\right)\sinh(bx+a)^2-}{120\,(b\cosh(bx+a)-b\sinh(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b*x + a)^2 - 20*cosh(b*x + a)^2 - 16*(c
osh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) - b*
sinh(b*x + a))
```

giac [A] time = 0.13, size = 60, normalized size = 0.72

$$\frac{5\left(12\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+3\,e^{(5\,bx+5\,a)}-20\,e^{(3\,bx+3\,a)}+90\,e^{(bx+a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/240*(5*(12*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) - 20
*e^(3*b*x + 3*a) + 90*e^(b*x + a))/b
```

maple [A] time = 0.02, size = 45, normalized size = 0.54

$$\frac{\left(\frac{8}{15}+\frac{\left(\sinh^4(bx+a)\right)}{5}-\frac{4\left(\sinh^2(bx+a)\right)}{15}\right)\cosh(bx+a)+\frac{\left(\sinh^5(bx+a)\right)}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^4,x)
```

```
[Out] 1/b*((8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)+1/5*sinh(b*x+a
)^5)
```

maxima [A] time = 0.40, size = 68, normalized size = 0.82

$$\frac{e^{(5\,bx+5\,a)}}{80\,b}-\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{3\,e^{(bx+a)}}{8\,b}+\frac{e^{(-bx-a)}}{4\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/80*e^(5*b*x + 5*a)/b - 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b + 1/4*e
^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

mupad [B] time = 0.53, size = 58, normalized size = 0.70

$$\frac{90\,\mathrm{e}^{a+b\,x}+60\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}-20\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(a + b*x)^4,x)
```

```
[Out] (90*exp(a + b*x) + 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) - 20*exp(3*a +
3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

sympy [A] time = 59.12, size = 139, normalized size = 1.67

$$\begin{cases}\frac{e^{a}e^{bx}\sinh^4(a+bx)}{5b}+\frac{4e^{a}e^{bx}\sinh^3(a+bx)\cosh(a+bx)}{5b}-\frac{4e^{a}e^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}-\frac{8e^{a}e^{bx}\sinh(a+bx)\cosh^3(a+bx)}{15b}+\frac{8e^{a}e^{bx}\cosh^4(a+}{15b} \\ xe^{a}\sinh^4(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) + 4*exp(a)*exp(b*x)*sinh(
a + b*x)**3*cosh(a + b*x)/(5*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(5*b) - 8*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b)
+ 8*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**
4, True))
```

3.302 $\int e^{a+bx} \sinh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[Out] 1/16*exp(-2*b*x-2*a)/b-3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 266, 43}

$$\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-2*a - 2*b*x)/(16*b) - (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32*
b) + (3*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2a+2bx}\right)}{16b}\\
&= \frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.79

$$\frac{e^{-2(a+bx)}-3e^{2(a+bx)}+\frac{1}{2}e^{4(a+bx)}+6bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^3,x]
[Out] (E^(-2*(a + b*x)) - 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

fricas [B] time = 0.65, size = 95, normalized size = 1.67

$$\frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3+6\,(2\,bx-1)\cosh(bx+a)-3\left(4\,bx+\cosh\right.}{32\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^3
 + 6*(2*b*x - 1)*cosh(b*x + a) - 3*(4*b*x + cosh(b*x + a)^2 + 2)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 57, normalized size = 1.00

$$\frac{12\,bx - 2\left(3\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 12\,a + e^{(4\,bx+4\,a)} - 6\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*x
 + 4*a) - 6*e^(2*b*x + 2*a))/b
```

maple [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{\left(\frac{\left(\sinh^{3}(bx+a)\right)}{4} - \frac{3\sinh(bx+a)}{8}\right)\cosh\left(bx+a\right) + \frac{3bx}{8} + \frac{3a}{8} + \frac{\left(\sinh^{4}(bx+a)\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*((1/4*sinh(b*x+a)^3-3/8*sinh(b*x+a))*cosh(b*x+a)+3/8*b*x+3/8*a+1/4*sinh
(b*x+a)^4)
```

maxima [A] time = 0.35, size = 53, normalized size = 0.93

$$\frac{3\,(bx+a)}{8\,b} + \frac{e^{(4\,bx+4\,a)}}{32\,b} - \frac{3\,e^{(2\,bx+2\,a)}}{16\,b} + \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b - 3/16*e^(2*b*x + 2*a)/b + 1/16*e^
(-2*b*x - 2*a)/b
```

mupad [B] time = 0.80, size = 42, normalized size = 0.74

$$\frac{3\,x}{8} + \frac{\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16} - \frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16} + \frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (3*x)/8 + (exp(- 2*a - 2*b*x)/16 - (3*exp(2*a + 2*b*x))/16 + exp(4*a + 4*b*
x)/32)/b
```

sympy [A] time = 17.82, size = 182, normalized size = 3.19

$$\begin{cases} \frac{3xe^a e^{bx} \sinh^3(a+bx)}{8} - \frac{3xe^a e^{bx} \sinh^2(a+bx)\cosh(a+bx)}{8} - \frac{3xe^a e^{bx} \sinh(a+bx)\cosh^2(a+bx)}{8} + \frac{3xe^a e^{bx} \cosh^3(a+bx)}{8} - \frac{e^a e^{bx} \sinh^3(a+bx)}{8b} + \\ xe^a \sinh^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*sin
h(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a +
b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - exp(a)*exp(b*x)*sinh(a
 + b*x)**3/(8*b) + 3*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*b) -
 3*exp(a)*exp(b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3,
 True))
```

3.303 $\int e^{a+bx} \sinh^2(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

[Out] -1/4*exp(-b*x-a)/b-1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 270}

$$-\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b*x)*Sinh[a + b*x]^2,x]

[Out] -E^(-a - b*x)/(4*b) - E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{4x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, e^{a+bx}\right)}{4b}$$
$$= -\frac{e^{-a-bx}}{4b} - \frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{12b}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{e^{-a-bx}\left(-6e^{2(a+bx)}+e^{4(a+bx)}-3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^2,x]
```

```
[Out] (E^(-a - b*x)*(-3 - 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

fricas [A] time = 0.80, size = 54, normalized size = 1.10

$$-\frac{\cosh(bx+a)^2 - 4\cosh(bx+a)\sinh(bx+a) + \sinh(bx+a)^2 + 3}{6\,(b\cosh(bx+a) - b\sinh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

giac [A] time = 0.14, size = 34, normalized size = 0.69

$$\frac{e^{(3\,bx+3\,a)} - 6\,e^{(bx+a)} - 3\,e^{(-bx-a)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

[Out] 1/12*(e^(3*b*x + 3*a) - 6*e^(b*x + a) - 3*e^(-b*x - a))/b

maple [A] time = 0.02, size = 35, normalized size = 0.71

$$\frac{\left(-\frac{2}{3}+\frac{\left(\sinh^2(bx+a)\right)}{3}\right)\cosh\left(bx+a\right)+\frac{\left(\sinh^3(bx+a)\right)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b*x+a)*sinh(b*x+a)^2,x)

[Out] 1/b*((-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+1/3*sinh(b*x+a)^3)

maxima [A] time = 0.38, size = 40, normalized size = 0.82

$$\frac{e^{(3\,bx+3\,a)}}{12\,b}-\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")

[Out] 1/12*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b

mupad [B] time = 0.61, size = 36, normalized size = 0.73

$$-\frac{6\,e^{a+b\,x}+3\,e^{-a-b\,x}-e^{3\,a+3\,b\,x}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(a + b*x)*sinh(a + b*x)^2,x)

[Out] -(6*exp(a + b*x) + 3*exp(- a - b*x) - exp(3*a + 3*b*x))/(12*b)

sympy [A] time = 5.15, size = 78, normalized size = 1.59

$$\begin{cases}\frac{e^a e^{bx}\sinh^2(a+bx)}{3b}+\frac{2e^a e^{bx}\sinh(a+bx)\cosh(a+bx)}{3b}-\frac{2e^a e^{bx}\cosh^2(a+bx)}{3b} & \text{for } b\neq 0\\ xe^a\sinh^2(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*sinh(b*x+a)**2,x)

[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*sinh(
a + b*x)*cosh(a + b*x)/(3*b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne
(b, 0)), (x*exp(a)*sinh(a)**2, True))

3.304 $\int e^{a+bx} \sinh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{2a+2bx}}{4b}-\frac{x}{2}$$

```
[Out] 1/4*exp(2*b*x+2*a)/b-1/2*x
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 14}

$$\frac{e^{2a+2bx}}{4b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\sinh(a+bx)\,dx = \frac{\mathrm{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\mathrm{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{\mathrm{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

fricas [B] time = 4.00, size = 50, normalized size = 2.17

$$-\frac{(2\,bx-1)\cosh\left(bx+a\right)-(2\,bx+1)\sinh\left(bx+a\right)}{4\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*x - 1)*cosh(b*x + a) - (2*b*x + 1)*sinh(b*x + a))/(b*cosh(b*x +
a) - b*sinh(b*x + a))
```

giac [A] time = 0.11, size = 24, normalized size = 1.04

$$-\frac{2\,bx+2\,a-e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x + 2*a - e^(2*b*x + 2*a))/b
```

maple [A] time = 0.01, size = 37, normalized size = 1.61

$$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{\left(\cosh^2(bx+a)\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a),x)
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a+1/2*cosh(b*x+a)^2)
```

maxima [A] time = 0.36, size = 24, normalized size = 1.04

$$-\frac{1}{2}x-\frac{a}{2b}+\frac{e^{(2bx+2a)}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

mupad [B] time = 0.07, size = 18, normalized size = 0.78

$$\frac{e^{2a+2bx}}{4b}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(a + b*x),x)
[Out] exp(2*a + 2*b*x)/(4*b) - x/2
```

sympy [A] time = 1.07, size = 80, normalized size = 3.48

$$\begin{cases} \frac{xe^a e^{bx}\sinh(a+bx)}{2}-\frac{xe^a e^{bx}\cosh(a+bx)}{2}-\frac{e^a e^{bx}\sinh(a+bx)}{2b}+\frac{e^a e^{bx}\cosh(a+bx)}{b} & \text{for } b\neq 0\\ xe^a\sinh(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)/2 - x*exp(a)*exp(b*x)*cosh(a + b
*x)/2 - exp(a)*exp(b*x)*sinh(a + b*x)/(2*b) + exp(a)*exp(b*x)*cosh(a + b*x)
/b, Ne(b, 0)), (x*exp(a)*sinh(a), True))
```

3.305 $\int e^{a+bx}\mathbf{csch}(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 12, 260}

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x],x]
```

```
[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\text{csch}(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x],x]
[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

fricas [A] time = 2.03, size = 30, normalized size = 1.58

$$\frac{\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

giac [A] time = 0.12, size = 24, normalized size = 1.26

$$\frac{\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] (log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.02, size = 19, normalized size = 1.00

$$x+\frac{\ln\left(\sinh\left(bx+a\right)\right)}{b}+\frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a),x)
[Out] x+1/b*ln(sinh(b*x+a))+a/b
```

maxima [A] time = 0.30, size = 27, normalized size = 1.42

$$\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

mupad [B] time = 0.07, size = 16, normalized size = 0.84

$$\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x),x)
[Out] log(exp(2*a + 2*b*x) - 1)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x), x)
```

3.306 $\int e^{a+bx}\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 288, 206}

$$\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^2,x]
```

```
[Out] (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^2}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{4\operatorname{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 0.90

$$\frac{2\left(-\frac{e^{a+bx}}{e^{2(a+bx)}-1}-\tanh^{-1}\left(e^{a+bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^2,x]
[Out] (2*(-(E^(a + b*x)/(-1 + E^(2*(a + b*x)))) - ArcTanh[E^(a + b*x)]))/b
```

fricas [B] time = 0.85, size = 157, normalized size = 3.74

$$-\frac{\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2-1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-}{b\cosh\left(bx+a\right)^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
```

```
 + 2*cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

giac [A] time = 0.13, size = 48, normalized size = 1.14

$$-\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) + log(e^(b*x + a) + 1) - log(abs(e^(b
*x + a) - 1)))/b
```

maple [A] time = 0.02, size = 25, normalized size = 0.60

$$\frac{-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{1}{\sinh(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^2,x)
```

```
[Out] 1/b*(-2*arctanh(exp(b*x+a))-1/sinh(b*x+a))
```

maxima [A] time = 0.38, size = 52, normalized size = 1.24

$$-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x + a)/(b*(e^(2*b
*x + 2*a) - 1))
```

mupad [B] time = 0.62, size = 52, normalized size = 1.24

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^2,x)
```

```
[Out] - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x)
)/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}^{2}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**2, x)
```

3.307 $\int e^{a+bx}\mathbf{csch}^3(a+bx)\,dx$

Optimal. Leaf size=31

$$-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[Out] -2*exp(4*b*x+4*a)/b/(1-exp(2*b*x+2*a))^2
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 12, 264}

$$-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(4*a + 4*b*x))/(b*(1 - E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{8\operatorname{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$

$$= -\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.94

$$-\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(4*a + 4*b*x))/(b*(-1 + E^(2*a + 2*b*x))^2)
```

fricas [B] time = 0.54, size = 88, normalized size = 2.84

$$-\frac{2\left(\cosh\left(bx+a\right)+3\,\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3-b\cosh\left(bx+a\right)+3\left(b\cosh\left(bx+a\right)^2-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)
^2 - b)*sinh(b*x + a))
```

giac [A] time = 0.13, size = 31, normalized size = 1.00

$$-\frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b\left(e^{(2\,bx+2\,a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] -2*(2*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

maple [A] time = 0.03, size = 24, normalized size = 0.77

$$\frac{-\coth(bx+a) - \frac{1}{2\sinh(bx+a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^3,x)
[Out] 1/b*(-coth(b*x+a)-1/2/sinh(b*x+a)^2)
```

maxima [B] time = 0.30, size = 68, normalized size = 2.19

$$-\frac{4\,e^{(2\,bx+2\,a)}}{b\left(e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 1\right)} + \frac{2}{b\left(e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1)) + 2/(b*(e^
(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

mupad [B] time = 0.58, size = 31, normalized size = 1.00

$$-\frac{2\left(2\,e^{2\,a+2\,b\,x}-1\right)}{b\left(e^{2\,a+2\,b\,x}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^3,x)
[Out] -(2*(2*exp(2*a + 2*b*x) - 1))/(b*(exp(2*a + 2*b*x) - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**3,x)
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**3, x)
```

3.308 $\int e^{a+bx}\mathbf{csch}^4(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3}+\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 8/3*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))
^2+exp(b*x+a)/b/(1-exp(2*b*x+2*a))+arctanh(exp(b*x+a))/b
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2282, 12, 288, 199, 206}

$$\frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3}+\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^4,x]
```

```
[Out] (8*E^(3*a + 3*b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1 -
 E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 - E^(2*a + 2*b*x))) + ArcTanh[E^(a
 + b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{16x^4}{\left(1-x^2\right)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{16\operatorname{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{8\operatorname{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.74

$$\frac{3e^{a+bx}-8e^{3(a+bx)}-3e^{5(a+bx)}+3\left(e^{2(a+bx)}-1\right)^3\tanh^{-1}\left(e^{a+bx}\right)}{3b\left(e^{2(a+bx)}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^4,x]
```

```
[Out] (3*E^(a + b*x) - 8*E^(3*(a + b*x)) - 3*E^(5*(a + b*x)) + 3*(-1 + E^(2*(a +
b*x)))^3*ArcTanh[E^(a + b*x)])/(3*b*(-1 + E^(2*(a + b*x)))^3)
```

fricas [B] time = 2.05, size = 705, normalized size = 6.98

$$-\frac{6\cosh(bx+a)^5+30\cosh(bx+a)\sinh(bx+a)^4+6\sinh(bx+a)^5+4\left(15\cosh(bx+a)^2+4\right)\sinh(bx+a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^3 + 16*cosh(b*x + a)^3 + 12*
(5*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3
*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*
x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 -
 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) +
sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 8*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a) - 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b
)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x +
a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 +
 b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b
*x + a))*sinh(b*x + a) - b)
```

giac [A] time = 0.14, size = 75, normalized size = 0.74

$$-\frac{\frac{2\left(3e^{(5bx+5a)}+8e^{(3bx+3a)}-3e^{(bx+a)}\right)}{\left(e^{(2bx+2a)}-1\right)^3}-3\log\left(e^{(bx+a)}+1\right)+3\log\left(\left|e^{(bx+a)}-1\right|\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*(3*e^(5*b*x + 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x +
 2*a) - 1)^3 - 3*log(e^(b*x + a) + 1) + 3*log(abs(e^(b*x + a) - 1)))/b
```

maple [A] time = 0.03, size = 37, normalized size = 0.37

$$\frac{-\frac{\operatorname{csch}(bx+a)\coth(bx+a)}{2} + \operatorname{arctanh}\left(e^{bx+a}\right) - \frac{1}{3\sinh(bx+a)^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/2*csch(b*x+a)*coth(b*x+a)+arctanh(exp(b*x+a))-1/3/sinh(b*x+a)^3)
```

maxima [A] time = 0.30, size = 100, normalized size = 0.99

$$\frac{\log\left(e^{(bx+a)}+1\right)}{2b} - \frac{\log\left(e^{(bx+a)}-1\right)}{2b} - \frac{3e^{(5bx+5a)} + 8e^{(3bx+3a)} - 3e^{(bx+a)}}{3b\left(e^{(6bx+6a)} - 3e^{(4bx+4a)} + 3e^{(2bx+2a)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/2*log(e^(b*x + a) + 1)/b - 1/2*log(e^(b*x + a) - 1)/b - 1/3*(3*e^(5*b*x +
 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(b*(e^(6*b*x + 6*a) - 3*e^(4*b*x
 + 4*a) + 3*e^(2*b*x + 2*a) - 1))
```

mupad [B] time = 0.60, size = 135, normalized size = 1.34

$$\frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+bx}}{b\,\left(e^{4a+4bx} - 2\,e^{2a+2bx} + 1\right)} - \frac{8\,e^{3a+3bx}}{3\,b\,\left(3\,e^{2a+2bx} - 3\,e^{4a+4bx} + e^{6a+6bx} - 1\right)} - \frac{e^{a+bx}}{b\,\left(e^{2a+2bx} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^4,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(3*
exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(a + b*
x)/(b*(exp(2*a + 2*b*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}^{4}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**4, x)
```

3.309 $\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}$$

```
[Out] -4/b/(1-exp(2*b*x+2*a))^4+32/3/b/(1-exp(2*b*x+2*a))^3-8/b/(1-exp(2*b*x+2*a)
)^2
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 12, 266, 43}

$$-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^5,x]
```

```
[Out] -4/(b*(1 - E^(2*a + 2*b*x))^4) + 32/(3*b*(1 - E^(2*a + 2*b*x))^3) - 8/(b*(1
 - E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{32x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{32\operatorname{Subst}\left(\int \frac{x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{16\operatorname{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2a+2bx}\right)}{b}\\ &= \frac{16\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, e^{2a+2bx}\right)}{b}\\ &= -\frac{4}{b\left(1-e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.67

$$-\frac{4\left(-4e^{2(a+bx)}+6e^{4(a+bx)}+1\right)}{3b\left(e^{2(a+bx)}-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^5,x]
```

```
[Out] (-4*(1 - 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(-1 + E^(2*(a + b*x))
)^4)
```

fricas [B] time = 2.73, size = 233, normalized size = 3.53

$$-\frac{}{3\left(b\cosh(bx+a)^6+6\,b\cosh(bx+a)\sinh(bx+a)^5+b\sinh(bx+a)^6-4\,b\cosh(bx+a)^4+\left(15\,b\cosh(bx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="fricas")

[Out] -4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)^2 - 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 4*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*cosh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 - 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 - 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x + a))*sinh(b*x + a) - 4*b)

giac [A] time = 0.13, size = 42, normalized size = 0.64

$$-\frac{4\left(6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)}{3\,b\left(e^{(2\,bx+2\,a)}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="giac")

[Out] -4/3*(6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^4)

maple [A] time = 0.12, size = 35, normalized size = 0.53

$$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^{2}}{3}\right)\coth\left(bx+a\right)-\frac{1}{4\sinh(bx+a)^{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(b*x+a)*csch(b*x+a)^5,x)

[Out] 1/b*((2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)-1/4/sinh(b*x+a)^4)

maxima [B] time = 0.36, size = 172, normalized size = 2.61

$$-\frac{8\,e^{(4\,bx+4\,a)}}{b\left(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)}+\frac{16\,e^{(2\,bx+2\,a)}}{3\,b\left(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="maxima")

[Out] -8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)) + 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a) -

```
4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b*(e
^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a)
+ 1))
```

mupad [B] time = 0.60, size = 42, normalized size = 0.64

$$-\frac{4\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{3\,b\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^5,x)
```

```
[Out] -(4*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) -
 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^a\int e^{bx}\operatorname{csch}^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**5,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**5, x)
```

3.310 $\int e^x \sinh^2(2x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

```
[Out] -1/12/exp(3*x)-1/2*exp(x)+1/20*exp(5*x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 270}

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[2*x]^2,x]
```

```
[Out] -1/(12*E^(3*x)) - E^x/2 + E^(5*x)/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \sinh^2(2x)\,dx = \text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{4x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1-x^4\right)^2}{x^4}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+x^4\right)dx, x, e^x\right)$$
$$= -\frac{1}{12}e^{-3x} - \frac{e^x}{2} + \frac{e^{5x}}{20}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{1}{12}e^{-3x} - \frac{e^x}{2} + \frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[2*x]^2,x]
[Out] -1/12*1/E^(3*x) - E^x/2 + E^(5*x)/20
```

fricas [B] time = 0.65, size = 47, normalized size = 1.81

$$-\frac{\cosh(x)^4 - 16\,\cosh(x)^3\sinh(x) + 6\,\cosh(x)^2\sinh(x)^2 - 16\,\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 15}{30\,(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="fricas")
[Out] -1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4 + 15)/(cosh(x) - sinh(x))
```

giac [A] time = 0.11, size = 17, normalized size = 0.65

$$\frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

maple [A] time = 0.04, size = 34, normalized size = 1.31

$$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}-\frac{\cosh(x)}{2}-\frac{\cosh(3x)}{12}+\frac{\cosh(5x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(2*x)^2,x)
```

```
[Out] -1/2*sinh(x)+1/12*sinh(3*x)+1/20*sinh(5*x)-1/2*cosh(x)-1/12*cosh(3*x)+1/20*
cosh(5*x)
```

maxima [A] time = 0.31, size = 17, normalized size = 0.65

$$\frac{1}{20}\,e^{(5\,x)}-\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

mupad [B] time = 0.09, size = 17, normalized size = 0.65

$$\frac{e^{5\,x}}{20}-\frac{e^{-3\,x}}{12}-\frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)^2*exp(x),x)
```

```
[Out] exp(5*x)/20 - exp(-3*x)/12 - exp(x)/2
```

sympy [B] time = 0.66, size = 42, normalized size = 1.62

$$\frac{7e^{x}\sinh^{2}(2x)}{15}+\frac{4e^{x}\sinh(2x)\cosh(2x)}{15}-\frac{8e^{x}\cosh^{2}(2x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)**2,x)
```

```
[Out] 7*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 - 8*exp(x)*cosh(
2*x)**2/15
```

3.311 $\int e^x \sinh(2x)\, dx$

Optimal. Leaf size=19

$$\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

[Out] 1/2/exp(x)+1/6*exp(3*x)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

Antiderivative was successfully verified.

[In] Int[E^x*Sinh[2*x],x]

[Out] 1/(2*E^x) + E^(3*x)/6

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh(2x)\,dx &= \text{Subst}\left(\int \frac{-1+x^4}{2x^2}\,dx,x,e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+x^4}{x^2}\,dx,x,e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{x^2}+x^2\right)\,dx,x,e^x\right)\\ &= \frac{e^{-x}}{2}+\frac{e^{3x}}{6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{6}e^{-x}\left(e^{4x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[2*x],x]
[Out] (3 + E^(4*x))/(6*E^x)
```

fricas [A] time = 0.59, size = 26, normalized size = 1.37

$$\frac{2\left(\cosh(x)^2-\cosh(x)\sinh(x)+\sinh(x)^2\right)}{3\,(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="fricas")
[Out] 2/3*(cosh(x)^2 - cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

giac [A] time = 0.13, size = 13, normalized size = 0.68

$$\frac{1}{6}\,e^{(3\,x)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="giac")
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

maple [A] time = 0.02, size = 22, normalized size = 1.16

$$-\frac{\sinh(x)}{2}+\frac{\sinh\left(3x\right)}{6}+\frac{\cosh(x)}{2}+\frac{\cosh\left(3x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(2*x),x)
[Out] -1/2*sinh(x)+1/6*sinh(3*x)+1/2*cosh(x)+1/6*cosh(3*x)
```

maxima [A] time = 0.32, size = 13, normalized size = 0.68

$$\frac{1}{6}\,e^{(3\,x)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="maxima")
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

mupad [B] time = 0.56, size = 12, normalized size = 0.63

$$\frac{\mathrm{e}^{-x}\,\left(\mathrm{e}^{4\,x}+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)*exp(x),x)
[Out] (exp(-x)*(exp(4*x) + 3))/6
```

sympy [A] time = 0.27, size = 20, normalized size = 1.05

$$-\frac{e^{x}\sinh\left(2x\right)}{3}+\frac{2e^{x}\cosh\left(2x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x)
[Out] -exp(x)*sinh(2*x)/3 + 2*exp(x)*cosh(2*x)/3
```

3.312 $\int e^x \mathbf{csch}(2x)\,dx$

Optimal. Leaf size=11

$$\tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

```
[Out] arctan(exp(x))-arctanh(exp(x))
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2282, 12, 298, 203, 206}

$$\tan^{-1}(e^x) - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[2*x],x]
```

```
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x\operatorname{csch}(2x)\,dx &= \operatorname{Subst}\left(\int \frac{2x^2}{-1+x^4}\,dx,x,e^x\right)\\ &= 2\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,e^x\right)\\ &= -\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^x\right)+\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^x\right)\\ &= \tan^{-1}\left(e^x\right)-\tanh^{-1}\left(e^x\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\tan^{-1}\left(e^x\right)-\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[2*x],x]
```

```
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

fricas [B] time = 0.76, size = 25, normalized size = 2.27

$$\arctan\left(\cosh(x)+\sinh(x)\right)-\frac{1}{2}\,\log\left(\cosh(x)+\sinh(x)+1\right)+\frac{1}{2}\,\log\left(\cosh(x)+\sinh(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="fricas")
```

```
[Out] arctan(cosh(x) + sinh(x)) - 1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x
) + sinh(x) - 1)
```

giac [B] time = 0.13, size = 19, normalized size = 1.73

$$\arctan\left(e^x\right)-\frac{1}{2}\,\log\left(e^x+1\right)+\frac{1}{2}\,\log\left(|e^x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="giac")
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

maple [C] time = 0.08, size = 34, normalized size = 3.09

$$-\frac{\ln\left(e^{x}+1\right)}{2}+\frac{\ln\left(e^{x}-1\right)}{2}+\frac{i\ln\left(e^{x}+i\right)}{2}-\frac{i\ln\left(e^{x}-i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(2*x),x)
[Out] -1/2*ln(exp(x)+1)+1/2*ln(exp(x)-1)+1/2*I*ln(exp(x)+I)-1/2*I*ln(exp(x)-I)
```

maxima [A] time = 0.41, size = 18, normalized size = 1.64

$$\arctan\left(e^{x}\right)-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="maxima")
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

mupad [B] time = 0.17, size = 26, normalized size = 2.36

$$\frac{\ln\left(4\,e^{x}-4\right)}{2}-\frac{\ln\left(-4\,e^{x}-4\right)}{2}-\mathrm{atan}\left(e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(2*x),x)
[Out] log(4*exp(x) - 4)/2 - log(- 4*exp(x) - 4)/2 - atan(exp(-x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}\left(2x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x)
[Out] Integral(exp(x)*csch(2*x), x)
```

3.313 $\int e^x \mathrm{csch}^2(2x)\,dx$

Optimal. Leaf size=32

$$\frac{e^x}{1-e^{4x}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)$$

```
[Out] exp(x)/(1-exp(4*x))-1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2282, 12, 288, 212, 206, 203}

$$\frac{e^x}{1-e^{4x}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[2*x]^2,x]
```

```
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x \operatorname{csch}^2(2x)\,dx &= \operatorname{Subst}\left(\int \frac{4x^4}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= 4\operatorname{Subst}\left(\int \frac{x^4}{\left(1-x^4\right)^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}\left(e^x\right) - \frac{1}{2}\tanh^{-1}\left(e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 1.00

$$\frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}\left(e^x\right) - \frac{1}{2}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[2*x]^2,x]
```

```
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

fricas [B] time = 2.30, size = 182, normalized size = 5.69

$$-\frac{2\left(\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)^2+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4-1\right)\arctan\left(\cosh(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*cos
h(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 -
1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x)
- 1) + 4*cosh(x) + 4*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)
```

giac [A] time = 0.11, size = 33, normalized size = 1.03

$$-\frac{e^x}{e^{(4\,x)}-1}-\frac{1}{2}\,\arctan\left(e^x\right)-\frac{1}{4}\,\log\left(e^x+1\right)+\frac{1}{4}\,\log\left(|e^x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e^x -
 1))
```

maple [C] time = 0.09, size = 46, normalized size = 1.44

$$-\frac{e^x}{e^{4x}-1}-\frac{\ln\left(e^x+1\right)}{4}+\frac{i\ln\left(e^x-i\right)}{4}-\frac{i\ln\left(e^x+i\right)}{4}+\frac{\ln\left(e^x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(2*x)^2,x)
```

```
[Out] -exp(x)/(exp(4*x)-1)-1/4*ln(exp(x)+1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)
+1/4*ln(exp(x)-1)
```

maxima [A] time = 0.40, size = 32, normalized size = 1.00

$$-\frac{e^x}{e^{(4\,x)}-1}-\frac{1}{2}\,\arctan\left(e^x\right)-\frac{1}{4}\,\log\left(e^x+1\right)+\frac{1}{4}\,\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x - 1)
```

mupad [B] time = 0.72, size = 36, normalized size = 1.12

$$\frac{\ln\left(1-e^x\right)}{4}-\frac{\ln\left(-e^x-1\right)}{4}-\frac{\operatorname{atan}\left(e^x\right)}{2}-\frac{e^x}{e^{4x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(2*x)^2,x)
```

```
[Out] log(1 - exp(x))/4 - log(- exp(x) - 1)/4 - atan(exp(x))/2 - exp(x)/(exp(4*x)
 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \operatorname{csch}^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(2*x)**2, x)
```

3.314 $\int e^x \sinh^2(3x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

```
[Out] -1/20/exp(5*x)-1/2*exp(x)+1/28*exp(7*x)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 270}

$$-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[3*x]^2,x]
```

```
[Out] -1/(20*E^(5*x)) - E^x/2 + E^(7*x)/28
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \sinh^2(3x)\,dx = \text{Subst}\left(\int \frac{(1-x^6)^2}{4x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x^6)^2}{x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^6}+x^6\right)dx, x, e^x\right)$$
$$= -\frac{1}{20}e^{-5x} - \frac{e^x}{2} + \frac{e^{7x}}{28}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$-\frac{1}{20}e^{-5x} - \frac{e^x}{2} + \frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[3*x]^2,x]
```

```
[Out] -1/20*1/E^(5*x) - E^x/2 + E^(7*x)/28
```

fricas [B] time = 3.42, size = 67, normalized size = 2.58

$$-\frac{\cosh(x)^6 - 36\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 - 120\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 - 36}{70(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)^6
 + 35)/(cosh(x) - sinh(x))
```

giac [A] time = 0.11, size = 17, normalized size = 0.65

$$\frac{1}{28}e^{(7x)} - \frac{1}{20}e^{(-5x)} - \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

maple [A] time = 0.03, size = 34, normalized size = 1.31

$$-\frac{\sinh(x)}{2}+\frac{\sinh(5x)}{20}+\frac{\sinh(7x)}{28}-\frac{\cosh(x)}{2}-\frac{\cosh(5x)}{20}+\frac{\cosh(7x)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(3*x)^2,x)
```

```
[Out] -1/2*sinh(x)+1/20*sinh(5*x)+1/28*sinh(7*x)-1/2*cosh(x)-1/20*cosh(5*x)+1/28*
cosh(7*x)
```

maxima [A] time = 0.30, size = 17, normalized size = 0.65

$$\frac{1}{28}\,e^{(7\,x)}-\frac{1}{20}\,e^{(-5\,x)}-\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

mupad [B] time = 0.58, size = 17, normalized size = 0.65

$$\frac{e^{7\,x}}{28}-\frac{e^{-5\,x}}{20}-\frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(3*x)^2*exp(x),x)
```

```
[Out] exp(7*x)/28 - exp(-5*x)/20 - exp(x)/2
```

sympy [B] time = 0.66, size = 42, normalized size = 1.62

$$\frac{17e^{x}\sinh^{2}(3x)}{35}+\frac{6e^{x}\sinh(3x)\cosh(3x)}{35}-\frac{18e^{x}\cosh^{2}(3x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)**2,x)
```

```
[Out] 17*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 - 18*exp(x)*cos
h(3*x)**2/35
```

3.315 $\int e^x \sinh(3x)\, dx$

Optimal. Leaf size=19

$$\frac{e^{-2x}}{4}+\frac{e^{4x}}{8}$$

[Out] 1/4/exp(2*x)+1/8*exp(4*x)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{-2x}}{4}+\frac{e^{4x}}{8}$$

Antiderivative was successfully verified.

[In] Int[E^x*Sinh[3*x],x]

[Out] 1/(4*E^(2*x)) + E^(4*x)/8

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x\sinh(3x)\,dx &= \text{Subst}\left(\int \frac{-1+x^6}{2x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+x^6}{x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{x^3}+x^3\right)dx, x, e^x\right)\\ &= \frac{e^{-2x}}{4}+\frac{e^{4x}}{8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{8}e^{-2x}\left(e^{6x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[3*x],x]
```

```
[Out] (2 + E^(6*x))/(8*E^(2*x))
```

fricas [B] time = 1.43, size = 40, normalized size = 2.11

$$\frac{3\,\cosh(x)^3-3\,\cosh(x)^2\sinh(x)+9\,\cosh(x)\sinh(x)^2-\sinh(x)^3}{8\left(\cosh(x)-\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*cosh(x)^3 - 3*cosh(x)^2*sinh(x) + 9*cosh(x)*sinh(x)^2 - sinh(x)^3)/(
cosh(x) - sinh(x))
```

giac [A] time = 0.11, size = 13, normalized size = 0.68

$$\frac{1}{8}\,e^{(4\,x)}+\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="giac")
```

```
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

maple [A] time = 0.04, size = 26, normalized size = 1.37

$$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}+\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(3*x),x)
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)+1/4*cosh(2*x)+1/8*cosh(4*x)
```

maxima [A] time = 0.30, size = 13, normalized size = 0.68

$$\frac{1}{8}e^{(4x)}+\frac{1}{4}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="maxima")
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

mupad [B] time = 0.05, size = 12, normalized size = 0.63

$$\frac{e^{-2x}\left(e^{6x}+2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(3*x)*exp(x),x)
[Out] (exp(-2*x)*(exp(6*x) + 2))/8
```

sympy [A] time = 0.26, size = 20, normalized size = 1.05

$$-\frac{e^{x}\sinh(3x)}{8}+\frac{3e^{x}\cosh(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x)
[Out] -exp(x)*sinh(3*x)/8 + 3*exp(x)*cosh(3*x)/8
```

3.316 $\int e^x \mathbf{csch}(3x)\, dx$

Optimal. Leaf size=54

$$\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(e^{2x}+e^{4x}+1\right)+\frac{\tan^{-1}\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*ln(1-exp(2*x))-1/6*ln(1+exp(2*x)+exp(4*x))+1/3*arctan(1/3*(1+2*exp(2*x)
)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2282, 12, 275, 292, 31, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(e^{2x}+e^{4x}+1\right)+\frac{\tan^{-1}\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[3*x],x]
```

```
[Out] ArcTan[(1 + 2*E^(2*x))/Sqrt[3]]/Sqrt[3] + Log[1 - E^(2*x)]/3 - Log[1 + E^(2
*x) + E^(4*x)]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x\operatorname{csch}(3x)\,dx &= \text{Subst}\left(\int \frac{2x^3}{-1+x^6}\,dx,x,e^x\right)\\ &= 2\,\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx,x,e^x\right)\\ &= \text{Subst}\left(\int \frac{x}{-1+x^3}\,dx,x,e^{2x}\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,e^{2x}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx,x,e^{2x}\right)\\ &= \frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,e^{2x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,e^{2x}\right)\\ &= \frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2e^{2x}\right)\\ &= \frac{\tan^{-1}\left(\frac{1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.41

$$-\frac{1}{2}e^{4x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[3*x],x]
[Out] -1/2*(E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, E^(6*x)])
```

fricas [A] time = 1.09, size = 83, normalized size = 1.54

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{3\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3\left(\cosh(x)-\sinh(x)\right)}\right)-\frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)+\frac{1}{3}\log\left(\frac{2}{\cosh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - s
inh(x))) - 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

giac [A] time = 0.13, size = 43, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right)-\frac{1}{6}\log\left(e^{(4x)}+e^{(2x)}+1\right)+\frac{1}{3}\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1)) - 1/6*log(e^(4*x) + e^(2*x)
 + 1) + 1/3*log(abs(e^(2*x) - 1))
```

maple [C] time = 0.08, size = 79, normalized size = 1.46

$$-\frac{\ln\left(e^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(e^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln\left(e^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(e^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(e^{2x}-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(3*x),x)
```

```
[Out] -1/6*ln(exp(2*x)+1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)+1/2+1/2*I*3^(1/2))*3^
(1/2)-1/6*ln(exp(2*x)+1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*x)+1/2-1/2*I*3^(1/2
))*3^(1/2)+1/3*ln(exp(2*x)-1)
```

maxima [A] time = 0.41, size = 73, normalized size = 1.35

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}+1)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^{x}-1)\right)-\frac{1}{6}\log\left(e^{(2x)}+e^{x}+1\right)-\frac{1}{6}\log\left(e^{(2x)}-e^{x}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) +
 1/3*log(e^x + 1) + 1/3*log(e^x - 1)
```

mupad [B] time = 0.18, size = 65, normalized size = 1.20

$$\frac{\ln\left(8e^{2x}-8\right)}{3}+\ln\left(24e^{2x}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(-24e^{2x}\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(3*x),x)
```

```
[Out] log(8*exp(2*x) - 8)/3 + log(24*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) - 8)*((3^(1/
2)*1i)/6 - 1/6) - log(- 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6) - 8)*((3^(1/2)*1
i)/6 + 1/6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x)
```

```
[Out] Integral(exp(x)*csch(3*x), x)
```

3.317 $\int e^x \operatorname{csch}^2(3x)\,dx$

Optimal. Leaf size=105

$$\frac{2e^x}{3\left(1-e^{6x}\right)}+\frac{1}{18}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{18}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}\left(e^x\right)$$

```
[Out] 2/3*exp(x)/(1-exp(6*x))-2/9*arctanh(exp(x))+1/18*ln(1-exp(x)+exp(2*x))-1/18
*ln(1+exp(x)+exp(2*x))+1/9*arctan(1/3*(1-2*exp(x))*3^(1/2))*3^(1/2)-1/9*arc
tan(1/3*(1+2*exp(x))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2282, 12, 288, 210, 634, 618, 204, 628, 206}

$$\frac{2e^x}{3\left(1-e^{6x}\right)}+\frac{1}{18}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{18}\log\left(e^x+e^{2x}+1\right)+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[3*x]^2,x]
```

```
[Out] (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - ArcTa
n[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x + E^(
2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x\text{csch}^2(3x)\,dx &= \text{Subst}\left(\int \frac{4x^6}{\left(1-x^6\right)^2}\,dx,x,e^x\right)\\
&= 4\,\text{Subst}\left(\int \frac{x^6}{\left(1-x^6\right)^2}\,dx,x,e^x\right)\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^6}\,dx,x,e^x\right)\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)}-\frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^x\right)-\frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,e^x\right)-\frac{2}{9}\text{Subst}\left(\int\right.\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)}-\frac{2}{9}\tanh^{-1}\left(e^x\right)+\frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,e^x\right)-\frac{1}{18}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\right.\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)}-\frac{2}{9}\tanh^{-1}\left(e^x\right)+\frac{1}{18}\log\left(1-e^x+e^{2x}\right)-\frac{1}{18}\log\left(1+e^x+e^{2x}\right)+\frac{1}{3}\text{Subst}\left(\int -\right.\\
&= \frac{2e^x}{3\left(1-e^{6x}\right)}+\frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}\left(e^x\right)+\frac{1}{18}\log\left(1-e^x+e^{2x}\right)-\frac{1}{18}\text{lo}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 34, normalized size = 0.32

$$\frac{2}{3}e^x\left(\frac{1}{1-e^{6x}}-{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[3*x]^2,x]
[Out] (2*E^x*((1 - E^(6*x))^(-1) - Hypergeometric2F1[1/6, 1, 7/6, E^(6*x)]))/3
```

fricas [B] time = 0.85, size = 560, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh
(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sin
h(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(
2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)) + 2*(sqrt(3)*cosh(
x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2 + 20*sq
rt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(3)*cosh
(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(2/3*sqrt(3)*cosh(x) + 2
/3*sqrt(3)*sinh(x) - 1/3*sqrt(3)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) -
 (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*s
inh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*lo
g((2*cosh(x) - 1)/(cosh(x) - sinh(x))) + 2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x)
 + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4
 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cosh(x) + sinh(x) + 1) - 2*(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cos
h(x) + sinh(x) - 1) + 12*cosh(x) + 12*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)
```

giac [A] time = 0.13, size = 86, normalized size = 0.82

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}-1\right)\right)-\frac{2e^{x}}{3\left(e^{(6x)}-1\right)}-\frac{1}{18}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{18}\log\left(e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(abs(e^x - 1))
```

maple [C] time = 0.10, size = 148, normalized size = 1.41

$$-\frac{2\,\mathrm{e}^{x}}{3\left(\mathrm{e}^{6x}-1\right)}+\frac{\ln\left(\mathrm{e}^{x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{18}+\frac{i\ln\left(\mathrm{e}^{x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{18}+\frac{\ln\left(\mathrm{e}^{x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{18}-\frac{i\ln\left(\mathrm{e}^{x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{18}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(3*x)^2,x)
```

```
[Out] -2/3*exp(x)/(exp(6*x)-1)+1/18*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/18*I*ln(exp(x)
-1/2-1/2*I*3^(1/2))*3^(1/2)+1/18*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/18*I*ln(exp
```

```
(x)-1/2+1/2*I*3^(1/2))*3^(1/2)-1/9*ln(exp(x)+1)-1/18*ln(exp(x)+1/2-1/2*I*3^
(1/2))+1/18*I*ln(exp(x)+1/2-1/2*I*3^(1/2))*3^(1/2)-1/18*ln(exp(x)+1/2+1/2*I
*3^(1/2))-1/18*I*ln(exp(x)+1/2+1/2*I*3^(1/2))*3^(1/2)+1/9*ln(exp(x)-1)
```

maxima [A] time = 0.40, size = 85, normalized size = 0.81

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}-1\right)\right)-\frac{2e^{x}}{3\left(e^{(6x)}-1\right)}-\frac{1}{18}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{18}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(e^x - 1)
```

mupad [B] time = 0.40, size = 91, normalized size = 0.87

$$\frac{\ln\left(\frac{2}{3}-\frac{2e^{x}}{3}\right)}{9}-\frac{\ln\left(-\frac{2e^{x}}{3}-\frac{2}{3}\right)}{9}+\frac{\ln\left(\left(\frac{2e^{x}}{3}-\frac{1}{3}\right)^{2}+\frac{1}{3}\right)}{18}-\frac{\ln\left(\left(\frac{2e^{x}}{3}+\frac{1}{3}\right)^{2}+\frac{1}{3}\right)}{18}-\frac{2e^{x}}{3\left(e^{6x}-1\right)}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2e^{x}}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(3*x)^2,x)
```

```
[Out] log(2/3 - (2*exp(x))/3)/9 - log(- (2*exp(x))/3 - 2/3)/9 + log(((2*exp(x))/3
 - 1/3)^2 + 1/3)/18 - log(((2*exp(x))/3 + 1/3)^2 + 1/3)/18 - (2*exp(x))/(3*
(exp(6*x) - 1)) - (3^(1/2)*atan(3^(1/2)*((2*exp(x))/3 - 1/3)))/9 - (3^(1/2)
*atan(3^(1/2)*((2*exp(x))/3 + 1/3)))/9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}^{2}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(3*x)**2, x)
```

3.318 $\int e^x \sinh^2(4x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

[Out] -1/28/exp(7*x)-1/2*exp(x)+1/36*exp(9*x)

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2282, 12, 270}

$$-\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

[In] Int[E^x*Sinh[4*x]^2,x]

[Out] -1/(28*E^(7*x)) - E^x/2 + E^(9*x)/36

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \sinh^2(4x)\,dx = \text{Subst}\left(\int \frac{\left(1-x^8\right)^2}{4x^8}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{\left(1-x^8\right)^2}{x^8}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^8}+x^8\right)dx, x, e^x\right)$$
$$= -\frac{1}{28}e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$-\frac{1}{28}e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[4*x]^2,x]
```

```
[Out] -1/28*1/E^(7*x) - E^x/2 + E^(9*x)/36
```

fricas [B] time = 1.40, size = 87, normalized size = 3.35

$$-\frac{\cosh(x)^8 - 64\,\cosh(x)^7\sinh(x) + 28\,\cosh(x)^6\sinh(x)^2 - 448\,\cosh(x)^5\sinh(x)^3 + 70\,\cosh(x)^4\sinh(x)^4 - 44}{126\,(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*co
sh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 + 63)/(cosh(x) - sinh(
x))
```

giac [A] time = 0.13, size = 17, normalized size = 0.65

$$\frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} - \frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

maple [A] time = 0.03, size = 34, normalized size = 1.31

$$-\frac{\sinh(x)}{2}+\frac{\sinh(7x)}{28}+\frac{\sinh(9x)}{36}-\frac{\cosh(x)}{2}-\frac{\cosh(7x)}{28}+\frac{\cosh(9x)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(4*x)^2,x)
```

```
[Out] -1/2*sinh(x)+1/28*sinh(7*x)+1/36*sinh(9*x)-1/2*cosh(x)-1/28*cosh(7*x)+1/36*
cosh(9*x)
```

maxima [A] time = 0.30, size = 17, normalized size = 0.65

$$\frac{1}{36}\,e^{(9\,x)}-\frac{1}{28}\,e^{(-7\,x)}-\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

mupad [B] time = 0.08, size = 17, normalized size = 0.65

$$\frac{e^{9\,x}}{36}-\frac{e^{-7\,x}}{28}-\frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(4*x)^2*exp(x),x)
```

```
[Out] exp(9*x)/36 - exp(-7*x)/28 - exp(x)/2
```

sympy [B] time = 0.65, size = 42, normalized size = 1.62

$$\frac{31e^{x}\sinh^{2}(4x)}{63}+\frac{8e^{x}\sinh(4x)\cosh(4x)}{63}-\frac{32e^{x}\cosh^{2}(4x)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)**2,x)
```

```
[Out] 31*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 - 32*exp(x)*cos
h(4*x)**2/63
```

3.319 $\int e^x \sinh(4x)\,dx$

Optimal. Leaf size=19

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

[Out] 1/6/exp(3*x)+1/10*exp(5*x)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2282, 12, 14}

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

Antiderivative was successfully verified.

[In] Int[E^x*Sinh[4*x],x]

[Out] 1/(6*E^(3*x)) + E^(5*x)/10

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh(4x)\,dx &= \text{Subst}\left(\int \frac{-1+x^8}{2x^4}\,dx, x, e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+x^8}{x^4}\,dx, x, e^x\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{x^4}+x^4\right)dx, x, e^x\right)\\ &= \frac{e^{-3x}}{6}+\frac{e^{5x}}{10}\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 19, normalized size = 1.00

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[4*x],x]
[Out] 1/(6*E^(3*x)) + E^(5*x)/10
```

fricas [**B**] time = 1.37, size = 46, normalized size = 2.42

$$\frac{4\left(\cosh(x)^4-\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2-\cosh(x)\sinh(x)^3+\sinh(x)^4\right)}{15\left(\cosh(x)-\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="fricas")
[Out] 4/15*(cosh(x)^4 - cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - cosh(x)*sinh(
x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

giac [**A**] time = 0.13, size = 13, normalized size = 0.68

$$\frac{1}{10}\,e^{(5\,x)}+\frac{1}{6}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="giac")
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

maple [A] time = 0.02, size = 26, normalized size = 1.37

$$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}+\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(4*x),x)
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)+1/6*cosh(3*x)+1/10*cosh(5*x)
```

maxima [A] time = 0.30, size = 13, normalized size = 0.68

$$\frac{1}{10}\,e^{(5\,x)}+\frac{1}{6}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="maxima")
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.68

$$\frac{\mathrm{e}^{-3\,x}}{6}+\frac{\mathrm{e}^{5\,x}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(4*x)*exp(x),x)
[Out] exp(-3*x)/6 + exp(5*x)/10
```

sympy [A] time = 0.26, size = 20, normalized size = 1.05

$$-\frac{e^{x}\sinh(4x)}{15}+\frac{4e^{x}\cosh(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x)
[Out] -exp(x)*sinh(4*x)/15 + 4*exp(x)*cosh(4*x)/15
```

3.320 $\int e^x \mathbf{csch}(4x)\,dx$

Optimal. Leaf size=113

$$-\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,e^x+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}(e$$

```
[Out] -1/2*arctan(exp(x))-1/2*arctanh(exp(x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/
2)+1/4*arctan(1+exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2
^(1/2)+1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {2282, 12, 301, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$-\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,e^x+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}(e$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[4*x],x]
```

```
[Out] -ArcTan[E^x]/2 - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*E
^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2
]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x \operatorname{csch}(4x)\,dx &= \text{Subst}\left(\int \frac{2x^4}{-1+x^8}\,dx, x, e^x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^4}{-1+x^8}\,dx, x, e^x\right)\\
&= -\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)+\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e\right.\\
&= -\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x}\right.\\
&= -\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{1}{2}\tanh^{-1}\left(e^x\right)-\frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-}\right.}{}\\
&= -\frac{1}{2}\tan^{-1}\left(e^x\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(e^x\right)-\frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right.}{4\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.19

$$-\frac{2}{5}e^{5x}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[4*x],x]
```

```
[Out] (-2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, E^(8*x)])/5
```

fricas [A] time = 1.33, size = 132, normalized size = 1.17

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4e^{(2x)}+4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1)
- 1) - 1/2*sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x +
4*e^(2*x) + 4) + 1) + 1/8*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 1/8*
sqrt(2)*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(e^x - 1)
```

giac [A] time = 0.13, size = 96, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(abs(e^x - 1))
```

maple [C] time = 0.09, size = 56, normalized size = 0.50

$$\frac{\ln(e^{x}-1)}{4}+2\left(\sum_{\_R=\mathrm{RootOf}\left(4096\_Z^{4}+1\right)}\_R\ln(e^{x}+8\_R)\right)-\frac{\ln(e^{x}+1)}{4}+\frac{i\ln(e^{x}-i)}{4}-\frac{i\ln(e^{x}+i)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(4*x),x)
```

```
[Out] 1/4*ln(exp(x)-1)+2*sum(_R*ln(exp(x)+8*_R),_R=RootOf(4096*_Z^4+1))-1/4*ln(ex
p(x)+1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)
```

maxima [A] time = 0.41, size = 95, normalized size = 0.84

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(e^x - 1)
```

mupad [B] time = 0.37, size = 106, normalized size = 0.94

$$\frac{\ln\left(128-128\,\mathrm{e}^{x}\right)}{4}-\frac{\ln\left(-128\,\mathrm{e}^{x}-128\right)}{4}-\frac{\mathrm{atan}\left(\mathrm{e}^{x}\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(128\,\mathrm{e}^{x}-64\,\sqrt{2}\right)}{128}\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(128\,\mathrm{e}^{x}+64\,\sqrt{2}\right)}{128}\right)}{4}-\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(4*x),x)
```

```
[Out] log(128 - 128*exp(x))/4 - log(- 128*exp(x) - 128)/4 - atan(exp(x))/2 + (2^(
1/2)*atan((2^(1/2)*(128*exp(x) - 64*2^(1/2)))/128))/4 + (2^(1/2)*atan((2^(1
/2)*(128*exp(x) + 64*2^(1/2)))/128))/4 - (2^(1/2)*log((128*exp(x) - 64*2^(1
/2))^2 + 8192))/8 + (2^(1/2)*log((128*exp(x) + 64*2^(1/2))^2 + 8192))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}\left(4x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x)
```

```
[Out] Integral(exp(x)*csch(4*x), x)
```

3.321 $\int e^x \mathbf{csch}^2(4x)\,dx$

Optimal. Leaf size=131

$$\frac{e^x}{2\left(1-e^{8x}\right)}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{1}{8}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}-$$

```
[Out] 1/2*exp(x)/(1-exp(8*x))-1/8*arctan(exp(x))-1/8*arctanh(exp(x))-1/16*arctan(
-1+exp(x)*2^(1/2))*2^(1/2)-1/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)+1/32*ln(1+
exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {2282, 12, 288, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{e^x}{2\left(1-e^{8x}\right)}+\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{1}{8}\tan^{-1}\left(e^x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}}-$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[4*x]^2,x]
```

```
[Out] E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sqrt[2])
 - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqrt[2]*E
^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \text{csch}^2(4x)\,dx = \text{Subst}\left(\int \frac{4x^8}{\left(1-x^8\right)^2}\,dx, x, e^x\right)$$
$$= 4\,\text{Subst}\left(\int \frac{x^8}{\left(1-x^8\right)^2}\,dx, x, e^x\right)$$
$$= \frac{e^x}{2\left(1-e^{8x}\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)$$
$$= \frac{e^x}{2\left(1-e^{8x}\right)} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= \frac{e^x}{2\left(1-e^{8x}\right)} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1-}{1+}\right.$$
$$= \frac{e^x}{2\left(1-e^{8x}\right)} - \frac{1}{8}\tan^{-1}\left(e^x\right) - \frac{1}{8}\tanh^{-1}\left(e^x\right) - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) - \frac{1}{16}\text{Subs}$$
$$= \frac{e^x}{2\left(1-e^{8x}\right)} - \frac{1}{8}\tan^{-1}\left(e^x\right) - \frac{1}{8}\tanh^{-1}\left(e^x\right) + \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{16\sqrt{2}} -$$
$$= \frac{e^x}{2\left(1-e^{8x}\right)} - \frac{1}{8}\tan^{-1}\left(e^x\right) + \frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{8\sqrt{2}} - \frac{1}{8}\tanh^{-1}\left(e^x\right) + \frac{\log\left(1\right.}{}$$

Mathematica [C] time = 0.03, size = 34, normalized size = 0.26

$$\frac{1}{2}e^x\left(\frac{1}{1-e^{8x}} - {}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{8x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[4*x]^2,x]
[Out] (E^x*((1 - E^(8*x))^(-1) - Hypergeometric2F1[1/8, 1, 9/8, E^(8*x)]))/2
```

fricas [B] time = 2.98, size = 207, normalized size = 1.58

$$\frac{4\left(\sqrt{2}\,e^{(8\,x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}\,e^x+\sqrt{2}\,\sqrt{\sqrt{2}\,e^x+e^{(2\,x)}+1}-1\right)+4\left(\sqrt{2}\,e^{(8\,x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}\,e^x+\frac{1}{2}\,\sqrt{2}\,\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - 4*(e^(8*x) -
 1)*arctan(e^x) - (sqrt(2)*e^(8*x) - sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x)
 + 4) + (sqrt(2)*e^(8*x) - sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 2
*(e^(8*x) - 1)*log(e^x + 1) + 2*(e^(8*x) - 1)*log(e^x - 1) - 16*e^x)/(e^(8*
x) - 1)
```

giac [A] time = 0.13, size = 108, normalized size = 0.82

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

maple [C] time = 0.10, size = 68, normalized size = 0.52

$$-\frac{e^{x}}{2\left(e^{8x}-1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^{4}+1\right)}\_R\ln\left(e^{x}-64\_R\right)\right)+\frac{i\ln\left(e^{x}-i\right)}{16}-\frac{i\ln\left(e^{x}+i\right)}{16}+\frac{\ln\left(e^{x}-1\right)}{16}-\frac{\ln\left(e^{x}+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(4*x)^2,x)
```

```
[Out] -1/2*exp(x)/(exp(8*x)-1)+4*sum(_R*ln(exp(x)-64*_R),_R=RootOf(16777216*_Z^4+
1))+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp(x)+I)+1/16*ln(exp(x)-1)-1/16*ln(exp(x
)+1)
```

maxima [A] time = 0.41, size = 107, normalized size = 0.82

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(e^x - 1)
```

mupad [B] time = 0.92, size = 120, normalized size = 0.92

$$\frac{\ln\left(\frac{1}{2}-\frac{e^x}{2}\right)}{16}-\frac{\ln\left(-\frac{e^x}{2}-\frac{1}{2}\right)}{16}-\frac{\operatorname{atan}\left(e^x\right)}{8}-\frac{e^x}{2\left(e^{8x}-1\right)}-\frac{\sqrt{2}\operatorname{atan}\left(2\sqrt{2}\left(\frac{e^x}{2}-\frac{\sqrt{2}}{4}\right)\right)}{16}-\frac{\sqrt{2}\operatorname{atan}\left(2\sqrt{2}\left(\frac{e^x}{2}+\frac{\sqrt{2}}{4}\right)\right)}{16}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(4*x)^2,x)
```

```
[Out] log(1/2 - exp(x)/2)/16 - log(- exp(x)/2 - 1/2)/16 - atan(exp(x))/8 - exp(x)
/(2*(exp(8*x) - 1)) - (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 - 2^(1/2)/4)))/16 -
 (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 + 2^(1/2)/4)))/16 + (2^(1/2)*log((exp(x)
/2 - 2^(1/2)/4)^2 + 1/8))/32 - (2^(1/2)*log((exp(x)/2 + 2^(1/2)/4)^2 + 1/8)
)/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}^{2}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(4*x)**2, x)
```

3.322 $\int F^{c(a+bx)} \sinh^3(d+ex)\,dx$

Optimal. Leaf size=202

$$-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6bce^2\log(F)\sinh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4}$$

```
[Out] -6*e^3*F^(c*(b*x+a))*cosh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F
)^4)+6*b*c*e^2*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^
2+b^4*c^4*ln(F)^4)+3*e*F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2/(9*e^2-b^2*c
^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)^3/(9*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5476, 5474}

$$-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6bce^2\log(F)\sinh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{6e^3\cosh(d+ex)F^{c(a+bx)}}{-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cosh[d + e*x])/(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b
^4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(9*e^4
- 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d
 + e*x]*Sinh[d + e*x]^2)/(9*e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a + b*x))*
Log[F]*Sinh[d + e*x]^3)/(9*e^2 - b^2*c^2*Log[F]^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = \frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^3(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{\left(6e^2\right)\int F}{9e}$$
$$= -\frac{6e^3F^{c(a+bx)}\cosh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sinh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} +$$

Mathematica [A] time = 0.69, size = 157, normalized size = 0.78

$$\frac{F^{c(a+bx)}\left(3\cosh(3(d+ex))\left(e^3-b^2c^2e\log^2(F)\right)+3\cosh(d+ex)\left(b^2c^2e\log^2(F)-9e^3\right)+2bc\log(F)\sinh(d+ex)\left(\right.}{4\left(b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(3*Cosh[3*(d + e*x)]*(e^3 - b^2*c^2*e*Log[F]^2) + 3*Cosh[d
 + e*x]*(-9*e^3 + b^2*c^2*e*Log[F]^2) + 2*b*c*Log[F]*(13*e^2 - b^2*c^2*Log[
F]^2 + Cosh[2*(d + e*x)]*(-e^2 + b^2*c^2*Log[F]^2))*Sinh[d + e*x]))/(4*(9*e
^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

fricas [B] time = 1.01, size = 2228, normalized size = 11.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c^3
*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*cosh
(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e*x +
 d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*l
og(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*
b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*c^3*cos
h(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2 - b^3*
c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) + (5*b^3*c
^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^2*e*cos
h(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)
^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3 - 3*(b^2*c^2
*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x + d)^2
```

```
+ b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*cosh(e*x + d)^2
+ (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*log(F)^
3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cosh(e*x + d)^2 - b
^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e^2*cosh(e*x + d)^
2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 - 27*b*c*
e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e^2)*log(F) + 6*(3*e
^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) + (b^3*c^
3*cosh(e*x + d)^5 - 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d))*log(
F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 - 2*b^2*c^2*e*cosh(e*x + d)^3 - b^2*c^2
*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 - 18*b*c*e^2*cosh(e*x
 + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*cosh((b*c*x + a*c
)*log(F)) + (3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(
F)^3 - 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(
b^3*c^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e
^2*cosh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cos
h(e*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3
*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^
2 - (5*b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2
 - b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) + (
5*b^3*c^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^
2*e*cosh(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e
*x + d)^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3 - 3*(
b^2*c^2*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x
+ d)^2 + b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*cosh(e*x
+ d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*
log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cosh(e*x + d
)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e^2*cosh(e*
x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 -
27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e^2)*log(F) +
 6*(3*e^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) +
(b^3*c^3*cosh(e*x + d)^5 - 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d
))*log(F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 - 2*b^2*c^2*e*cosh(e*x + d)^3 -
b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 - 18*b*c*e^2*c
osh(e*x + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*sinh((b*c*
x + a*c)*log(F)))/(b^4*c^4*cosh(e*x + d)^3*log(F)^4 - 10*b^2*c^2*e^2*cosh(e
*x + d)^3*log(F)^2 + 9*e^4*cosh(e*x + d)^3 + (b^4*c^4*log(F)^4 - 10*b^2*c^2
*e^2*log(F)^2 + 9*e^4)*sinh(e*x + d)^3 + 3*(b^4*c^4*cosh(e*x + d)*log(F)^4
- 10*b^2*c^2*e^2*cosh(e*x + d)*log(F)^2 + 9*e^4*cosh(e*x + d))*sinh(e*x + d
)^2 + 3*(b^4*c^4*cosh(e*x + d)^2*log(F)^4 - 10*b^2*c^2*e^2*cosh(e*x + d)^2*
log(F)^2 + 9*e^4*cosh(e*x + d)^2)*sinh(e*x + d))
```

giac [C] time = 0.27, size = 1239, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="giac")

[Out] 1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 3/4*(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/2*I*(6*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) - 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/2*I*(-6*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e) + 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/4*(2*(b*c*log(abs(F)) - 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d) - 1/2*I*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d)

maple [A] time = 0.13, size = 326, normalized size = 1.61

$$\left(\ln(F)^3 b^3 c^3 \mathrm{e}^{6ex+6d} - 3\ln(F)^3 b^3 c^3 \mathrm{e}^{4ex+4d} - 3\ln(F)^2 b^2 c^2 e\, \mathrm{e}^{6ex+6d} + 3\ln(F)^3 b^3 c^3 \mathrm{e}^{2ex+2d} + 3\ln(F)^2 b^2 c^2 e\, \mathrm{e}^{4ex+4d} - \mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^3,x)
```

```
[Out] 1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)-3*ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F
)^2*b^2*c^2*e*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)+3*ln(F)^2*b^2
*c^2*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)-ln(F)^3*b^3*c^3+3*ln(F)^
2*b^2*c^2*e*exp(2*e*x+2*d)+27*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+
6*d)-3*ln(F)^2*b^2*c^2*e-27*ln(F)*b*c*e^2*exp(2*e*x+2*d)-27*e^3*exp(4*e*x+4
*d)+ln(F)*b*c*e^2-27*e^3*exp(2*e*x+2*d)+3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d
)/(b*c*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

maxima [A] time = 0.33, size = 134, normalized size = 0.66

$$\frac{F^{ac}e^{\left(bcx\log(F)+3\,ex+3\,d\right)}}{8\left(bc\log(F)+3\,e\right)} - \frac{3\,F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{8\left(bc\log(F)+e\right)} + \frac{3\,F^{ac}e^{\left(bcx\log(F)-ex\right)}}{8\left(bce^{d}\log(F)-ee^{d}\right)} - \frac{F^{ac}e^{\left(bcx\log(F)-3\,ex\right)}}{8\left(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 3/8*F^(a*c)
*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F)
- e*x)/(b*c*e^d*log(F) - e*e^d) - 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(b*c
*e^(3*d)*log(F) - 3*e*e^(3*d))
```

mupad [B] time = 1.56, size = 166, normalized size = 0.82

$$-\frac{F^{a\,c+b\,c\,x}\,\left(-b^3\,c^3\,\sinh\left(d+e\,x\right)^3\,\ln(F)^3+3\,b^2\,c^2\,e\,\cosh\left(d+e\,x\right)\,\sinh\left(d+e\,x\right)^2\,\ln(F)^2-6\,b\,c\,e^2\,\cosh\left(d+e\,x\right)^2\,\mathrm{s}\right.}{b^4\,c^4\,\ln(F)^4-10\,b^2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x)^3,x)
```

```
[Out] -(F^(a*c + b*c*x)*(6*e^3*cosh(d + e*x)^3 - 9*e^3*cosh(d + e*x)*sinh(d + e*x
)^2 - b^3*c^3*sinh(d + e*x)^3*log(F)^3 + 7*b*c*e^2*sinh(d + e*x)^3*log(F) +
 3*b^2*c^2*e*cosh(d + e*x)*sinh(d + e*x)^2*log(F)^2 - 6*b*c*e^2*cosh(d + e*
x)^2*sinh(d + e*x)*log(F)))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(
F)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**3,x)
[Out] Timed out
```

3.323 $\int F^{c(a+bx)} \sinh^2(d+ex)\,dx$

Optimal. Leaf size=132

$$-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

```
[Out] -2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*co
sh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sinh(
e*x+d)^2/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5476, 2194}

$$-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b*c
*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5476

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x]^n)/(e^2*n^2 - b^2*c^
2*Log[F]^2), x] + (-Dist[(n*(n - 1)*e^2)/(e^2*n^2 - b^2*c^2*Log[F]^2), Int[
F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[(e*n*F^(c*(a + b*x))*C
osh[d + e*x]*Sinh[d + e*x]^(n - 1))/(e^2*n^2 - b^2*c^2*Log[F]^2), x]) /; Fr
eeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n,
1]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)} - \frac{(2e^2)\int F^{c}}{4e^2-b^2c^2}$$
$$= -\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}}{4e^2}$$

Mathematica [A] time = 0.21, size = 86, normalized size = 0.65

$$\frac{F^{c(a+bx)}\left(b^2c^2\log^2(F)\cosh(2(d+ex)) - b^2c^2\log^2(F) - 2bce\log(F)\sinh(2(d+ex)) + 4e^2\right)}{2b^3c^3\log^3(F) - 8bce^2\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 - b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log[
F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*Lo
g[F]^3)
```

fricas [B] time = 0.52, size = 703, normalized size = 5.33

$$\frac{\left(\left(b^2c^2\log(F)^2 - 2\,bce\log(F)\right)\sinh\left(ex+d\right)^4 + 8\,e^2\cosh\left(ex+d\right)^2 + 4\left(b^2c^2\cosh\left(ex+d\right)\log(F)^2 - 2\,bce\cosh\left(ex\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 + 8*e^2*cosh(e*x
+ d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*
sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*b^2*c^2*cosh(e*x + d)^2 + b^
2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x +
 d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)
^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) - 4*e^2*cosh(e*x + d
) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x +
d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(
e*x + d)^4 + 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*
b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*
b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*lo
g(F) - (3*b^2*c^2*cosh(e*x + d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d
)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3
*log(F) - 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x
```

```
 + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/(b^3*c^3*cosh(e
*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^3*c^3*log(F)^3 -
 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x + d)*log(F)^3 - 4*
b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

giac [C] time = 0.22, size = 904, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
 (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c
)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(2*I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c
*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
 2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi
*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b
*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x +
2*d) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c
*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*c
*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sg
n(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F
)) - 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) - 1/2*I
*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) + 2
*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I
*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d)
```

maple [A] time = 0.07, size = 143, normalized size = 1.08

$$\frac{\left(\ln(F)^2 b^2 c^2 e^{4ex+4d} - 2\ln(F)^2 b^2 c^2 e^{2ex+2d} - 2\ln(F) bce\, e^{4ex+4d} + b^2 c^2 \ln(F)^2 + 2\ln(F) bce + 8e^2 e^{2ex+2d}\right) e^{-2ex-2d} F^{c(}}{4\ln(F) bc\,(bc\ln(F) - 2e)\,(bc\ln(F) + 2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^2,x)
```

```
[Out] 1/4*(ln(F)^2*b^2*c^2*exp(4*e*x+4*d)-2*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)-2*ln(F
)*b*c*e*exp(4*e*x+4*d)+b^2*c^2*ln(F)^2+2*ln(F)*b*c*e+8*e^2*exp(2*e*x+2*d))/
ln(F)/b/c/(b*c*ln(F)-2*e)*exp(-2*e*x-2*d)/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

maxima [A] time = 0.32, size = 94, normalized size = 0.71

$$\frac{F^{ac}e^{\left(bcx\log(F)+2\,ex+2\,d\right)}}{4\left(bc\log(F)+2\,e\right)}+\frac{F^{ac}e^{\left(bcx\log(F)-2\,ex\right)}}{4\left(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)}\right)}-\frac{F^{bcx+ac}}{2\,bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)
*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/2*F^(b*c*x
 + a*c)/(b*c*log(F))
```

mupad [B] time = 0.92, size = 97, normalized size = 0.73

$$-\frac{F^{a\,c+b\,c\,x}\,\left(2\,e^2-\frac{b^2\,c^2\,{\ln(F)}^2}{2}+\frac{b^2\,c^2\,{\ln(F)}^2\,\cosh(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln(F)\,\mathrm{sinh}\left(2\,d+2\,e\,x\right)\right)}{b\,c\,\ln(F)\,\left(4\,e^2-b^2\,c^2\,{\ln(F)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x)^2,x)
```

```
[Out] -(F^(a*c + b*c*x)*(2*e^2 - (b^2*c^2*log(F)^2)/2 + (b^2*c^2*log(F)^2*cosh(2*
d + 2*e*x))/2 - b*c*e*log(F)*sinh(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 - b^2*c
^2*log(F)^2))
```

sympy [A] time = 34.10, size = 604, normalized size = 4.58

$$\begin{cases} \frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ \tilde{\infty}e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex) + \tilde{\infty}e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\sinh(d+ex)\cosh(d+ex) + \tilde{\infty}e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{b} \\ \tilde{\infty}e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex) + \tilde{\infty}e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\sinh(d+ex)\cosh(d+ex) + \tilde{\infty}e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\cosh \\ F^{ac}\left(\frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e}\right) \\ \frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ \frac{F^{ac}F^{bcx}b^2c^2\log(F)^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac}F^{bcx}bce\log(F)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac}F^{bcx}e^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} + \frac{2F^{ac}F^{bcx}e^2\cosh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh
(d + e*x)/(2*e), Eq(F, 1)), (zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c)
)**(b*c*x)*sinh(d + e*x)**2 + zoo*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c
))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x) + zoo*e**2*exp(-2*e/(b*c))**(a*c)*e
xp(-2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2, Eq(F, exp(-2*e/(b*c)))), (zoo*e**
2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2 + zoo*e**2
*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)*cosh(d + e*x)
+ zoo*e**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2,
Eq(F, exp(2*e/(b*c)))), (F**(a*c)*(x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**
2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (x*sinh(d + e*x)**2/2
- x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**
(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3
- 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sinh(d + e*x)*cos
h(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x
)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a
*c)*F**(b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(
F)), True))
```

3.324 $\int F^{c(a+bx)} \sinh(d+ex)\,dx$

Optimal. Leaf size=75

$$\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

```
[Out] e*F^(c*(b*x+a))*cosh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*s
inh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5474}

$$\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a +
 b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx=\frac{eF^{c(a+bx)}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

Mathematica [A] time = 0.11, size = 50, normalized size = 0.67

$$\frac{F^{c(a+bx)}(e\cosh(d+ex)-bc\log(F)\sinh(d+ex))}{(e-bc\log(F))(bc\log(F)+e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(e*Cosh[d + e*x] - b*c*Log[F]*Sinh[d + e*x]))/((e - b*c*Lo
g[F])*(e + b*c*Log[F]))
```

fricas [B] time = 0.53, size = 244, normalized size = 3.25

$$-\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log(F)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2-bc\right)\log(F)-2\left(bc\cosh\left(ex+d\right)\log(F)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x
 + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh
(e*x + d) + e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*log(F
) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e
*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) + e)*sinh((b*c*x + a*c)*log
(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*log(F)
^2 - e^2)*sinh(e*x + d))
```

giac [C] time = 0.20, size = 612, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] (2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) - 1/4*I*(-
2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I
*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) + 2*I*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)
/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)
) + (b*c*log(abs(F)) + e)*x + d) - (2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c
*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F)
- pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/
2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c
*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c
*log(abs(F)) - e)*x - d) - 1/4*I*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b
*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*
```

```
b*c*log(abs(F)) - 2*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1
/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log
(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

maple [A] time = 0.02, size = 77, normalized size = 1.03

$$\frac{\left(\ln(F)bc\,\mathrm{e}^{2ex+2d}-bc\ln(F)-e\,\mathrm{e}^{2ex+2d}-e\right)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2\,(bc\ln(F)-e)\,(e+bc\ln(F))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d),x)
```

```
[Out] 1/2*(ln(F)*b*c*exp(2*e*x+2*d)-b*c*ln(F)-e*exp(2*e*x+2*d)-e)/(b*c*ln(F)-e)*e
xp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

maxima [A] time = 0.32, size = 63, normalized size = 0.84

$$\frac{F^{ac}e^{\left(bcx\log(F)+ex+d\right)}}{2\left(bc\log(F)+e\right)}-\frac{F^{ac}e^{\left(bcx\log(F)-ex\right)}}{2\left(bce^{d}\log(F)-ee^{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/2*F^(a*c)*e^(b*
c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

mupad [B] time = 0.66, size = 73, normalized size = 0.97

$$\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e+e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln(F)-b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln(F)\right)}{2\,\left(e^2-b^2\,c^2\ln(F)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x),x)
```

```
[Out] (F^(a*c + b*c*x)*exp(- d - e*x)*(e + e*exp(2*d + 2*e*x) + b*c*log(F) - b*c*
exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

sympy [A] time = 7.85, size = 342, normalized size = 4.56

$$\begin{cases} \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\sinh(d+ex)}{2} - \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\cosh(d+ex)}{2} - \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\sinh(d+ex)}{2e} + \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\cosh(d+ex)}{e} & \text{for } F = -1 \wedge b \\ x\sinh(d) & \text{for } F = 1 \wedge e = \\ \tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F = e^{-\frac{e}{bc}} \\ \tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\sinh(d+ex) + \tilde{\infty}e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\cosh(d+ex) & \text{for } F = e^{\frac{e}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} - \frac{F^{ac}F^{bcx}e\cosh(d+ex)}{b^2c^2\log(F)^2-e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d),x)
```

```
[Out] Piecewise(((-1)**(a*c)*(-1)**(-I*e*x/pi)*x*sinh(d + e*x)/2 - (-1)**(a*c)*(-
1)**(-I*e*x/pi)*x*cosh(d + e*x)/2 - (-1)**(a*c)*(-1)**(-I*e*x/pi)*sinh(d +
e*x)/(2*e) + (-1)**(a*c)*(-1)**(-I*e*x/pi)*cosh(d + e*x)/e, Eq(F, -1) & Eq(
b, -I*e/(pi*c))), (x*sinh(d), Eq(F, 1) & Eq(e, 0)), (zoo*e*exp(-e/(b*c))**(
a*c)*exp(-e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e*exp(-e/(b*c))**(a*c)*exp(
-e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(-e/(b*c)))), (zoo*e*exp(e/(b*c)
)**(a*c)*exp(e/(b*c))**(b*c*x)*sinh(d + e*x) + zoo*e*exp(e/(b*c))**(a*c)*ex
p(e/(b*c))**(b*c*x)*cosh(d + e*x), Eq(F, exp(e/(b*c)))), (F**(a*c)*F**(b*c*
x)*b*c*log(F)*sinh(d + e*x)/(b**2*c**2*log(F)**2 - e**2) - F**(a*c)*F**(b*c
*x)*e*cosh(d + e*x)/(b**2*c**2*log(F)**2 - e**2), True))
```

3.325 $\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx$

Optimal. Leaf size=66

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{bc\log(F)+e}$$

```
[Out] -2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c
*ln(F)/e],exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5493}

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x],x]
```

```
[Out] (-2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx=-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Mathematica [A] time = 5.61, size = 93, normalized size = 1.41

$$\frac{F^{c(a+bx)}\left({}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;-\cosh(d+ex)-\sinh(d+ex)\right)-{}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;\cosh(d+ex)+\right.}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
-Cosh[d + e*x] - Sinh[d + e*x]] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 +
(b*c*Log[F])/e, Cosh[d + e*x] + Sinh[d + e*x]]))/(b*c*Log[F])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\operatorname{csch}\left(ex+d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\operatorname{csch}\left(ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4\,F^{ac}e\int\frac{e^{\left(bcx\log(F)+ex+d\right)}}{bc\log(F)+\left(bce^{(4\,d)}\log(F)-ee^{(4\,d)}\right)e^{(4\,ex)}-2\left(bce^{(2\,d)}\log(F)-ee^{(2\,d)}\right)e^{(2\,ex)}-e}\,dx-\frac{2\,F^{ac}e^{(bc}}{bc\log(F)-\left(bce^{(2\,d)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="maxima")
```

```
[Out] 4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*d)
*log(F) - e*e^(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2*e*
x) - e), x) - 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) - (b*c*e^(2*
d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x),x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{csch}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x), x)
```

3.326 $\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx$

Optimal. Leaf size=68

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

```
[Out] 4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c*
ln(F)/e],exp(2*e*x+2*d))/(b*c*ln(F)+2*e)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5493}

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Mathematica [A] time = 2.82, size = 87, normalized size = 1.28

$$-\frac{2F^{c(a+bx)}\left(\left(e^{2d}-1\right)\,{}_2F_1\left(1,\frac{bc\log(F)}{2e};\frac{bc\log(F)}{2e}+1;e^{2(d+ex)}\right)+\sinh(d)\operatorname{csch}(d+ex)(\cosh(ex)-\sinh(ex))\right)}{\left(e^{2d}-1\right)e}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

```
[Out] (-2*F^(c*(a + b*x))*((-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e)
, 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))] + Csch[d + e*x]*Sinh[d]*(Cosh[e*
x] - Sinh[e*x])))/(e*(-1 + E^(2*d)))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{csch}\,(ex+d)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{csch}\,(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$16\,F^{ac}bce\int -\frac{}{b^2c^2\log(F)^2-6\,bce\log(F)+8\,e^2-\left(b^2c^2e^{(6\,d)}\log(F)^2-6\,bcee^{(6\,d)}\log(F)+8\,e^2e^{(6\,d)}\right)e^{(6\,ex)}+3\left(b^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 16*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
8*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d))
*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2*e
^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F) +
8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + 4*(4*F^(a*c)*e + (F^(a*c)*b*c*e^(2*d
)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*
b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) +
 8*e^2*e^(4*d))*e^(4*e*x) - 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*l
og(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^2,x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**2, x)
```

3.327 $\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx$

Optimal. Leaf size=122

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{e^2}-\frac{bc\log(F)\text{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\text{coth}(d+e}{}$$

```
[Out] -1/2*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)/e-1/2*b*c*F^(c*(b*x+a))*csch(e*x
+d)*ln(F)/e^2+exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[
3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2
```

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5491, 5493}

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{e^2}-\frac{bc\log(F)\text{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\text{coth}(d+e}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

```
[Out] -(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x])/(2*e) - (b*c*F^(c*(a + b*x))
*Csch[d + e*x]*Log[F])/(2*e^2) + (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometri
c2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*(e
 - b*c*Log[F]))/e^2
```

Rule 5491

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csch[d + e*x]^(n - 2))/(e^2*(n - 1
)*(n - 2)), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a +
b*x))*Csch[d + e*x]^(n - 1)*Cosh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a,
 b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &
& NeQ[n, 2]
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\text{csch}(d+ex)\log(F)}{2e^2} - \frac{1}{2}\left(1-\frac{b^2c^2\ldots\right.$$

$$= -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\text{csch}(d+ex)\log(F)}{2e^2} + \frac{e^{d+ex}F^{c(a+bx}\ldots}{\ldots}$$

Mathematica [B] time = 20.20, size = 299, normalized size = 2.45

$$\frac{F^{c(a+bx)}\left(\frac{4\left(e^2-b^2c^2\log^2(F)\right)\left((\sinh(d)+\cosh(d)-1)\,{}_2F_1\left(1,\frac{bc\log(F)}{e};\frac{bc\log(F)}{e}+1;\cosh(d+ex)+\sinh(d+ex)\right)+1\right)}{bc\log(F)(\sinh(d)+\cosh(d)-1)} + \frac{4\left(e^2-b^2c^2\log^2(F)\right)\left(1-(\sinh(d)+\text{co}\ldots\right.}{b\ldots}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csch[(d + e*x)/2]^2) - 4*b*c*Csch[d]*Log[F] + Csch[d]
*((-4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) - e*Sech[(d + e*x)/2]^2 + (4*(e^2 -
 b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F
])/e, Cosh[d + e*x] + Sinh[d + e*x]]*(-1 + Cosh[d] + Sinh[d])))/(b*c*Log[F]
*(-1 + Cosh[d] + Sinh[d])) + (4*(e^2 - b^2*c^2*Log[F]^2)*(1 - Hypergeometri
c2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, -Cosh[d + e*x] - Sinh[d + e*x]]
*(1 + Cosh[d] + Sinh[d])))/(b*c*Log[F]*(1 + Cosh[d] + Sinh[d])) + 2*b*c*Csc
h[d/2]*Csch[(d + e*x)/2]*Log[F]*Sinh[(e*x)/2] + 2*b*c*Log[F]*Sech[d/2]*Sech
[(d + e*x)/2]*Sinh[(e*x)/2]))/(8*e^2)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{csch}\left(ex+d\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{csch}\left(ex+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^3, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$48\left(F^{ac}bcee^{d}\log(F)+F^{ac}e^{2}e^{d}\right)\int\frac{}{b^{2}c^{2}\log(F)^{2}-8\,bce\log(F)+15\,e^{2}+\left(b^{2}c^{2}e^{(8\,d)}\log(F)^{2}-8\,bcee^{(8\,d)}\log(F)+15\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F) +
 e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(F)
^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) - 4*(b^2*c^2*e^(6*d
)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^2*c^
2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x) - 4
*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*
e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) + (F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^
(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) +
15*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d
))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^
2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F)
 + 15*e^2*e^(2*d))*e^(2*e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{F^{c\,(a+b\,x)}}{\sinh(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^3,x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)} \operatorname{csch}^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**3,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**3, x)
```

3.328 $\int F^{c(a+bx)}\mathbf{csch}^4(d+ex)\,dx$

Optimal. Leaf size=131

$$-\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\text{csch}^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\coth(}{}$$

```
[Out] -1/3*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2/e-1/6*b*c*F^(c*(b*x+a))*csch(e
*x+d)^2*ln(F)/e^2-2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*
ln(F)/e],[2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5491, 5493}

$$-\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\text{csch}^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\coth(}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] -(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2)/(3*e) - (b*c*F^(c*(a + b*x
))*Csch[d + e*x]^2*Log[F])/(6*e^2) - (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hyp
ergeometric2F1[2, 1 + (b*c*Log[F])/(2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d +
 e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
```

Rule 5491

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csch[d + e*x]^(n - 2))/(e^2*(n - 1
)*(n - 2)), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a +
b*x))*Csch[d + e*x]^(n - 1)*Cosh[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a,
 b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &
& NeQ[n, 2]
```

Rule 5493

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[((-2)^n*E^(n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n
/2 + (b*c*Log[F])/(2*e), 1 + n/2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(e
*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{csch}^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\text{csch}^2(d+ex)\log(F)}{6e^2} - \frac{1}{6}\left(4 - \frac{b^2}{}\right.$$

$$= -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\text{csch}^2(d+ex)\log(F)}{6e^2} - \frac{2e^{2(d+ex)}F}{}$$

Mathematica [A] time = 7.06, size = 202, normalized size = 1.54

$$\frac{F^{c(a+bx)}\left(4e^2-b^2c^2\log^2(F)\right)\left(2\,{}_2F_1\left(1,\frac{bc\log(F)}{2e};\frac{bc\log(F)}{2e}+1;\cosh(2(d+ex))+\sinh(2(d+ex))\right)+\coth(d)-1\right)}{6e^3}-$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(-1 + Coth[d] + 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e),
 1 + (b*c*Log[F])/(2*e), Cosh[2*(d + e*x)] + Sinh[2*(d + e*x)]])*(4*e^2 - b
^2*c^2*Log[F]^2))/(6*e^3) - (F^(a*c + b*c*x)*Csch[d]*Csch[d + e*x]^2*(2*e*C
osh[d] + b*c*Log[F]*Sinh[d]))/(6*e^2) + (F^(a*c + b*c*x)*Csch[d]*Csch[d + e
*x]^3*Sinh[e*x])/(3*e) - (F^(a*c + b*c*x)*Csch[d]*Csch[d + e*x]*(4*e^2 - b^
2*c^2*Log[F]^2)*Sinh[e*x])/(6*e^3)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(F^{bcx+ac}\,\text{csch}\,(ex+d)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{(bx+a)c}\,\text{csch}\,(ex+d)^4\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^4, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\mathrm{csch}\,(ex+d)^4\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(-F^(b
*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 192*
e^3 - (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*b*c
*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d)*lo
g(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e
^3*e^(8*d))*e^(8*e*x) - 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)
*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 10*(b
^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*
d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) - 5*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b
^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e
^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F^(a*c)*b^2*
c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(a*c)*e^2*e^(
4*d))*e^(4*e*x) + 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*e^2*e^(2*d))*
e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^
2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F
)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*
e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(
F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*
e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x
) - 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e
^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e^(2*e*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,\left(a+b\,x\right)}}{{\mathrm{sinh}\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^4,x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)} \operatorname{csch}^4(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**4,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**4, x)
```

3.329 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx$

Optimal. Leaf size=250

$$\frac{e^{-4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\sinh^2(ac+b}{32bc}$$

```
[Out] 1/128*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(4*c*(b*x+a))-5/64*c
sch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+5/32*exp(2*c*
(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/128*exp(4*c*(b*x+a
))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/192*exp(6*c*(b*x+a))*csc
h(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/16*x*csch(b*c*x+a*c)*(sinh(b*c
*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\sinh^2(ac+b}{32bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128*b*c*E^(4*c*(a + b*x))) -
 (5*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(64*b*c*E^(2*c*(a + b*x)))
 + (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*
c) - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128
*b*c) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(19
2*b*c) - (5*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx$$

$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{\left(-1+x^2\right)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{\left(-1+x^2\right)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc}$$

$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{(-1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc}$$

$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\text{Subst}\left(\int \left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+x^2\right) d\right.}{64bc}$$

$$= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc} - \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh}}{64bc}$$

Mathematica [A] time = 0.11, size = 106, normalized size = 0.42

$$\frac{\left(\frac{1}{2}e^{-4c(a+bx)}-5e^{-2c(a+bx)}+10e^{2c(a+bx)}-\frac{5}{2}e^{4c(a+bx)}+\frac{1}{3}e^{6c(a+bx)}-20bcx\right)\sinh^2(c(a+bx))^{5/2}\text{csch}^5(c(a+bx))}{64bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) - (5
*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 - 20*b*c*x)*Csch[c*(a + b*x)]^5
*(Sinh[c*(a + b*x)]^2)^(5/2))/(64*b*c)
```

fricas [A] time = 0.67, size = 218, normalized size = 0.87

$$\frac{5\cosh(bcx+ac)^5+25\cosh(bcx+ac)\sinh(bcx+ac)^4-\sinh(bcx+ac)^5-5\left(2\cosh(bcx+ac)^2-3\right)\sinh(bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.19, size = 269, normalized size = 1.08

$$-\frac{120\,bcx\text{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-3\left(30\,e^{(4\,bcx+4\,ac)}\text{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-10\,e^{(2\,bcx+2\,ac)}\text{sgn}\left(e^{(bcx+ac)}-e^{(-bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*e^(4*b*c*
x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x + 2*a*c)
*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))*e^(-4*b*c*x - 4*a*c) - (2*e^(6*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)) + 60*e^(2*b*c*x + 14*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c
)))*e^(-12*a*c))/(b*c)
```

maple [A] time = 0.23, size = 184, normalized size = 0.74

$$\frac{8\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\ \cosh\left(c\left(bx+a\right)\right)\left(\sinh^{5}\left(c\left(bx+a\right)\right)\right)+8\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\ \left(\sinh^{6}\left(c\left(bx+a\right)\right)\right)-10\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] 1/48*(8*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+a))*sinh(c*(b*x+a))^5+8*(sinh
(c*(b*x+a))^2)^(1/2)*sinh(c*(b*x+a))^6-10*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*
(b*x+a))*sinh(c*(b*x+a))^3+15*cosh(c*(b*x+a))*(sinh(c*(b*x+a))^2)^(1/2)*sin
h(c*(b*x+a))-15*ln(cosh(c*(b*x+a))+(sinh(c*(b*x+a))^2)^(1/2))*sinh(c*(b*x+a
))+8*(sinh(c*(b*x+a))^2)^(1/2))/sinh(c*(b*x+a))/c/b
```

maxima [A] time = 0.41, size = 90, normalized size = 0.36

$$\frac{\left(2\,e^{(10\,bcx+10\,ac)}-15\,e^{(8\,bcx+8\,ac)}+60\,e^{(6\,bcx+6\,ac)}-30\,e^{(2\,bcx+2\,ac)}+3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc}-\frac{5\,(bcx+ac)}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{e}^{c\,(a+b\,x)}\,\left(\sinh\left(a\,c+b\,c\,x\right)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.330 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{16bc}-\frac{3e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}}{32bc}$$

```
[Out] 1/16*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))-3/16*ex
p(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/32*exp(4*c*(
b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+3/8*x*csch(b*c*x+a*c)
*(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{16bc}-\frac{3e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}}{32bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c*E^(2*c*(a + b*x))) -
(3*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c)
+ (E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*c)
+ (3*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx &= \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)} \sinh^3(ac+bcx)\,dx \\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc} \\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{2c(a+bx)}\right)}{16bc} \\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, e^{2c(a+bx)}\right)}{16bc} \\
&= \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc} - \frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2}}{16bc}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.47

$$\frac{\left(e^{-2c(a+bx)}-3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\sinh^2(c(a+bx))^{3/2}\operatorname{csch}^3(c(a+bx))}{16bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)
*Csch[c*(a + b*x)]^3*(Sinh[c*(a + b*x)]^2)^(3/2))/(16*b*c)
```

fricas [A] time = 0.80, size = 126, normalized size = 0.78

$$\frac{3 \cosh (bcx+ac)^3+9 \cosh (bcx+ac)\sinh (bcx+ac)^2-\sinh (bcx+ac)^3+6(2bcx-1)\cosh (bcx+ac)-3\left(4b\right.}{32(bc\cosh (bcx+ac)-bc\sinh (bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sin
h(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

giac [A] time = 0.15, size = 195, normalized size = 1.20

$$\frac{12\,bcx\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-2\,bcx-2\,ac}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2*b*c*x +
2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e^(-
b*c*x - a*c)))*e^(-2*b*c*x - 2*a*c) + (e^(4*b*c*x + 8*a*c)*sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b
*c*x - a*c)))*e^(-4*a*c))/(b*c)
```

maple [A] time = 0.18, size = 152, normalized size = 0.94

$$\frac{2\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\;\cosh (c(bx+a))\left(\sinh^3 (c(bx+a))\right)+2\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\;\left(\sinh^4 (c(bx+a))\right)-3\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] 1/8*(2*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+a))*sinh(c*(b*x+a))^3+2*(sinh(
c*(b*x+a))^2)^(1/2)*sinh(c*(b*x+a))^4-3*cosh(c*(b*x+a))*(sinh(c*(b*x+a))^2)
```

```
^(1/2)*sinh(c*(b*x+a))+3*ln(cosh(c*(b*x+a))+(sinh(c*(b*x+a))^2)^(1/2))*sinh
(c*(b*x+a))-2*(sinh(c*(b*x+a))^2)^(1/2))/sinh(c*(b*x+a))/c/b
```

maxima [A] time = 0.42, size = 62, normalized size = 0.38

$$\frac{\left(e^{(6\,bcx+6\,ac)} - 6\,e^{(4\,bcx+4\,ac)} + 2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc} + \frac{3\,(bcx + ac)}{8\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)
/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{c\,(a+b\,x)}\,\left(\sinh\left(a\,c+b\,c\,x\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.331 $\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\mathrm{csch}(ac+bcx)}{4bc}-\frac{1}{2}x\sqrt{\sinh^2(ac+bcx)}\,\mathrm{csch}(ac+bcx)$$

```
[Out] 1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-1/2*x*cs
ch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 14}

$$\frac{e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\mathrm{csch}(ac+bcx)}{4bc}-\frac{1}{2}x\sqrt{\sinh^2(ac+bcx)}\,\mathrm{csch}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(4*b*c) - (
x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx &= \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{-1+x^2}{2x}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{-1+x^2}{x}\,dx,x,e^{c(a+bx)}\right)}{2bc}\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx,x,e^{c(a+bx)}\right)}{2bc}\\
&= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{4bc}-\frac{1}{2}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bc
\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}-2bcx\right)\sqrt{\sinh^2(c(a+bx))}\operatorname{csch}(c(a+bx))}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)]*Sqrt[Sinh[c*(a + b*x)]^2])
/(4*b*c)
```

fricas [A] time = 0.69, size = 66, normalized size = 0.89

$$-\frac{(2\,bcx-1)\cosh(bcx+ac)-(2\,bcx+1)\sinh(bcx+ac)}{4\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

giac [A] time = 0.12, size = 71, normalized size = 0.96

$$-\frac{1}{2}\, x \operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) + \frac{e^{(2\,bcx+2\,ac)} \operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

maple [A] time = 0.20, size = 100, normalized size = 1.35

$$\frac{\cosh\left(c\,(bx+a)\right)\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}}{2cb} - \frac{\ln\left(\cosh\left(c\,(bx+a)\right)+\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\right)}{2cb} + \frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\ (\cos}{2cb\sinh\left(c\,(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x)
```

```
[Out] 1/2/c/b*cosh(c*(b*x+a))*(sinh(c*(b*x+a))^2)^(1/2)-1/2/c/b*ln(cosh(c*(b*x+a)
)+(sinh(c*(b*x+a))^2)^(1/2))+1/2/c/b*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+
a))^2/sinh(c*(b*x+a))
```

maxima [A] time = 0.41, size = 36, normalized size = 0.49

$$-\frac{bcx + ac}{2\,bc} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

mupad [B] time = 0.63, size = 77, normalized size = 1.04

$$-\frac{\left(x\,\mathrm{e}^{a\,c+b\,c\,x}-\frac{\mathrm{e}^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] -((x*exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2
- exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) - 1)
```

sympy [A] time = 17.30, size = 175, normalized size = 2.36

$$\begin{cases} x\sqrt{\sinh^2(ac)}\,e^{ac} & \text{for } b = 0 \\ 0 & \text{for } a = \frac{\log\left(-e^{-bcx}\right)}{c} \vee a = \\ \frac{x\sqrt{\sinh^2(ac+bcx)}\,e^{ac}e^{bcx}}{2} - \frac{x\sqrt{\sinh^2(ac+bcx)}\,e^{ac}e^{bcx}\cosh(ac+bcx)}{2\sinh(ac+bcx)} + \frac{\sqrt{\sinh^2(ac+bcx)}\,e^{ac}e^{bcx}\cosh(ac+bcx)}{2bc\sinh(ac+bcx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(sinh(a*c)**2)*exp(a*c), Eq(b, 0)), (0, Eq(c, 0) | Eq(a, l
og(exp(-b*c*x))/c) | Eq(a, log(-exp(-b*c*x))/c)), (x*sqrt(sinh(a*c + b*c*x)
**2)*exp(a*c)*exp(b*c*x)/2 - x*sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*
x)*cosh(a*c + b*c*x)/(2*sinh(a*c + b*c*x)) + sqrt(sinh(a*c + b*c*x)**2)*exp
(a*c)*exp(b*c*x)*cosh(a*c + b*c*x)/(2*b*c*sinh(a*c + b*c*x)), True))
```

3.332 $\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)/b/c/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b*c*x]^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\sinh(ac+bcx)\,\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(2\sinh(ac+bcx))\,\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.96

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b*c*Sqrt[Sinh[c*(a + b*x)]^
2])
```

fricas [A] time = 0.47, size = 42, normalized size = 0.91

$$\frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

giac [A] time = 0.14, size = 85, normalized size = 1.85

$$\frac{\log\left(e^{(bcx)} + e^{(-ac)}\right)\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) + \log\left(\left|e^{(bcx)} - e^{(-ac)}\right|\right)\operatorname{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] (log(e^(b*c*x) + e^(-a*c))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + log(ab
s(e^(b*c*x) - e^(-a*c)))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{2\,e^{c(bx+a)}}{\sqrt{-2+2\cosh\left(2bcx+2ac\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x)
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x)
```

maxima [A] time = 0.41, size = 39, normalized size = 0.85

$$\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{c\,(a+b\,x)}}{\sqrt{{\sinh\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\sqrt{\sinh^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sinh(a*c + b*c*x)**2), x)
```

3.333 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] -2*exp(4*c*(b*x+a))*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x+
a*c)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6720, 2282, 12, 264}

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqr
t[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^3(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\sinh(ac+bcx)\operatorname{Subst}\left(\int \frac{8x^3}{\left(-1+x^2\right)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(8\sinh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^3}{\left(-1+x^2\right)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 46, normalized size = 0.79

$$-\frac{4e^{5c(a+bx)}\sqrt{\sinh^2(c(a+bx))}}{bc\left(e^{2c(a+bx)}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-4*E^(5*c*(a + b*x))*Sqrt[Sinh[c*(a + b*x)]^2])/(b*c*(-1 + E^(2*c*(a + b*x
)))^3)
```

fricas [B] time = 0.81, size = 121, normalized size = 2.09

$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\,\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3-bc\cosh\left(bcx+ac\right)+3\left(bc\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

giac [**A**] time = 0.19, size = 87, normalized size = 1.50

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)}{bc\left(e^{(2\,bcx+2\,ac)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{8\,\mathrm{e}^{c(bx+a)}}{\left(-2+2\cosh\left(2bcx+2ac\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x)
```

maxima [**A**] time = 0.41, size = 84, normalized size = 1.45

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{2}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [**B**] time = 0.59, size = 76, normalized size = 1.31

$$-\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*((exp(a*c + b*c*x)/2 - ex
p(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int \frac{e^{bcx}}{\left(\sinh^{2}\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(sinh(a*c + b*c*x)**2)**(3/2), x)
```

3.334 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}+\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] -4*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+32/
3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)-8*si
nh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$-\frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}+\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Sinh[a*c + b*c*x
]^2]) + (32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3*Sqrt[Sinh[a
*c + b*c*x]^2]) - (8*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt
[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^5(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{\sinh(ac+bcx)\operatorname{Subst}\left(\int \frac{32x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(32\sinh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^5}{\left(-1+x^2\right)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(16\sinh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(16\sinh(ac+bcx))\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= -\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}} + \frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}} - \frac{}{bc}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 0.49

$$-\frac{4\left(-4e^{2c(a+bx)}+6e^{4c(a+bx)}+1\right)\sinh(c(a+bx))}{3bc\left(e^{2c(a+bx)}-1\right)^4\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sinh[c*(a + b*x)])/(3*b
*c*(-1 + E^(2*c*(a + b*x)))^4*Sqrt[Sinh[c*(a + b*x)]^2])
```

fricas [B] time = 0.68, size = 315, normalized size = 2.14

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^6+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^5+bc\sinh\left(bcx+ac\right)^6-4\,bc\cosh\left(bcx+ac\right)^4+\left(15\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

giac [A] time = 0.20, size = 122, normalized size = 0.83

$$-\frac{4\left(6\,e^{\left(4\,bcx+4\,ac\right)}\mathrm{sgn}\left(e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}\right)-4\,e^{\left(2\,bcx+2\,ac\right)}\mathrm{sgn}\left(e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}\right)+\mathrm{sgn}\left(e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}\right)\right)}{3\,bc\left(e^{\left(2\,bcx+2\,ac\right)}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 4*e^(
2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^4)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{32\,e^{c(bx+a)}}{(-2+2\cosh(2bcx+2ac))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x)
```

maxima [A] time = 0.42, size = 209, normalized size = 1.42

$$-\frac{8\,e^{(4\,bcx+4\,ac)}}{bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc\left(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 0.62, size = 89, normalized size = 0.61

$$-\frac{8\,\mathrm{e}^{a\,c+b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}\,\left(6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}-4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*
(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*a*c +
2*b*c*x) - 1)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.335 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx$

Optimal. Leaf size=199

$$\frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] -32/3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^6/(sinh(b*c*x+a*c)^2)^(1/2)+
192/5*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^5/(sinh(b*c*x+a*c)^2)^(1/2)-
48*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+64/
3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6720, 2282, 12, 266, 43}

$$\frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^6*Sqrt[Sinh[a*c + b*
c*x]^2]) + (192*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^(2*c*(a + b*x)))^5*Sqrt[Si
nh[a*c + b*c*x]^2]) - (48*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4
*Sqrt[Sinh[a*c + b*c*x]^2]) + (64*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a
+ b*x)))^3*Sqrt[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^7(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{\sinh(ac+bcx)\,\text{Subst}\left(\int \frac{128x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(128\sinh(ac+bcx))\,\text{Subst}\left(\int \frac{x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(64\sinh(ac+bcx))\,\text{Subst}\left(\int \frac{x^3}{(-1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(64\sinh(ac+bcx))\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^7}+\frac{3}{(-1+x)^6}+\frac{3}{(-1+x)^5}+\frac{1}{(-1+x)^4}\right) dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= -\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}} + \frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}} - \frac{}{bc\,(}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 0.42

$$-\frac{16\left(6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}-1\right)\sinh(c(a+bx))}{15bc\left(e^{2c(a+bx)}-1\right)^6\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b*x)))^6*Sqrt[Sinh[c*(a + b
*x)]^2])
```

fricas [B] time = 0.57, size = 592, normalized size = 2.97

$$-\frac{}{15\left(bc\cosh(bcx+ac)^9+9\,bc\cosh(bcx+ac)\sinh(bcx+ac)^8+bc\sinh(bcx+ac)^9-6\,bc\cosh(bcx+ac)^7+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

giac [A] time = 0.21, size = 161, normalized size = 0.81

$$-\frac{16\left(20\,e^{(6\,bcx+6\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-15\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+6\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}\right.\right.}{15\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 15
*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 6*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{128\,\mathrm{e}^{c(bx+a)}}{\left(-2+2\cosh\left(2bcx+2ac\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x)
```

```
[Out] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x)
```

maxima [B] time = 0.42, size = 386, normalized size = 1.94

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc\left(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{\cdots}{bc\left(e^{(12\,bcx+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

mupad [B] time = 0.62, size = 353, normalized size = 1.77

$$\frac{128\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}-\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}+\frac{96\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}-\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^4}+\frac{384\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{\cdots}}{5\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}-\mathrm{e}^{3\,a\,c\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(
1/2))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x
) - 1)^3) + (96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c
*x)/2)^2)^(1/2))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c
+ 2*b*c*x) - 1)^4) + (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(-
 a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))
*(exp(2*a*c + 2*b*c*x) - 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x
)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

3.336 $\int e^x \sinh(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{e^x\sinh(a+bx)}{1-b^2}-\frac{be^x\cosh(a+bx)}{1-b^2}$$

```
[Out] -b*exp(x)*cosh(b*x+a)/(-b^2+1)+exp(x)*sinh(b*x+a)/(-b^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5474}

$$\frac{e^x\sinh(a+bx)}{1-b^2}-\frac{be^x\cosh(a+bx)}{1-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + b*x],x]
```

```
[Out] -((b*E^x*Cosh[a + b*x])/(1 - b^2)) + (E^x*Sinh[a + b*x])/(1 - b^2)
```

Rule 5474

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
, x] + Simp[(e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2), x]
/; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sinh(a+bx)\,dx=-\frac{be^x\cosh(a+bx)}{1-b^2}+\frac{e^x\sinh(a+bx)}{1-b^2}$$

Mathematica [A] time = 0.05, size = 28, normalized size = 0.68

$$\frac{e^x(b\cosh(a+bx)-\sinh(a+bx))}{b^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + b*x],x]
```

```
[Out] (E^x*(b*Cosh[a + b*x] - Sinh[a + b*x]))/(-1 + b^2)
```

fricas [A] time = 0.89, size = 42, normalized size = 1.02

$$\frac{b\cosh\left(bx+a\right)\cosh(x)+b\cosh\left(bx+a\right)\sinh(x)-\left(\cosh(x)+\sinh(x)\right)\sinh\left(bx+a\right)}{b^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)*cosh(x) + b*cosh(b*x + a)*sinh(x) - (cosh(x) + sinh(x))*si
nh(b*x + a))/(b^2 - 1)
```

giac [A] time = 0.11, size = 32, normalized size = 0.78

$$\frac{e^{(bx+a+x)}}{2\left(b+1\right)}+\frac{e^{(-bx-a+x)}}{2\left(b-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a + x)/(b + 1) + 1/2*e^(-b*x - a + x)/(b - 1)
```

maple [A] time = 0.04, size = 62, normalized size = 1.51

$$-\frac{\sinh\left(\left(b-1\right)x+a\right)}{2\left(b-1\right)}+\frac{\sinh\left(\left(1+b\right)x+a\right)}{2+2b}+\frac{\cosh\left(\left(b-1\right)x+a\right)}{2b-2}+\frac{\cosh\left(\left(1+b\right)x+a\right)}{2+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(b*x+a),x)
```

```
[Out] -1/2/(b-1)*sinh((b-1)*x+a)+1/2/(1+b)*sinh((1+b)*x+a)+1/2*cosh((b-1)*x+a)/(b
-1)+1/2*cosh((1+b)*x+a)/(1+b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-b>0)', see `assume?` for more deta
ils)Is -b equal to -1?
```

mupad [B] time = 0.09, size = 45, normalized size = 1.10

$$\frac{e^{x-a-b\,x}\,\left(b-e^{2\,a+2\,b\,x}+b\,e^{2\,a+2\,b\,x}+1\right)}{2\,\left(b^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(a + b*x),x)
```

```
[Out] (exp(x - a - b*x)*(b - exp(2*a + 2*b*x) + b*exp(2*a + 2*b*x) + 1))/(2*(b^2
- 1))
```

sympy [A] time = 0.76, size = 99, normalized size = 2.41

$$\begin{cases} \frac{xe^{x}\sinh{(a-x)}}{2}+\frac{xe^{x}\cosh{(a-x)}}{2}+\frac{e^{x}\sinh{(a-x)}}{2} & \text{for } b=-1 \\ \frac{xe^{x}\sinh{(a+x)}}{2}-\frac{xe^{x}\cosh{(a+x)}}{2}+\frac{e^{x}\sinh{(a+x)}}{2} & \text{for } b=1 \\ \frac{be^{x}\cosh{(a+bx)}}{b^{2}-1}-\frac{e^{x}\sinh{(a+bx)}}{b^{2}-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 + exp(x)*sinh(a
- x)/2, Eq(b, -1)), (x*exp(x)*sinh(a + x)/2 - x*exp(x)*cosh(a + x)/2 + exp(
x)*sinh(a + x)/2, Eq(b, 1)), (b*exp(x)*cosh(a + b*x)/(b**2 - 1) - exp(x)*si
nh(a + b*x)/(b**2 - 1), True))
```

3.337 $\int e^x \sinh\left(a + cx^2\right) dx$

Optimal. Leaf size=85

$$\frac{\sqrt{\pi}\, e^{\frac{1}{4c}-a}\operatorname{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\, e^{a-\frac{1}{4c}}\operatorname{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*exp(-a+1/4/c)*erf(1/2*(-2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*exp(a-1/
4/c)*erfi(1/2*(2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5512, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\, e^{\frac{1}{4c}-a}\operatorname{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\, e^{a-\frac{1}{4c}}\operatorname{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + c*x^2],x]
```

```
[Out] (E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^(
a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh\left(a+cx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-a+x-cx^2}+\frac{1}{2}e^{a+x+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-a+x-cx^2}\,dx\right)+\frac{1}{2}\int e^{a+x+cx^2}\,dx\\ &= \frac{1}{2}e^{a-\frac{1}{4c}}\int e^{\frac{(1+2cx)^2}{4c}}\,dx-\frac{1}{2}e^{-a+\frac{1}{4c}}\int e^{-\frac{(1-2cx)^2}{4c}}\,dx\\ &= \frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.94

$$\frac{\sqrt{\pi}\,e^{-\frac{1}{4}/c}\left((\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)-e^{\frac{1}{2}/c}(\cosh(a)-\sinh(a))\mathrm{erf}\left(\frac{2cx-1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]))
 + Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c
)))
```

fricas [A] time = 0.88, size = 103, normalized size = 1.21

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{2\,cx-1}{2\,\sqrt{c}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c))
*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) + sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)/c
) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

giac [A] time = 0.12, size = 73, normalized size = 0.86

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c}\left(2\,x+\frac{1}{c}\right)\right)e^{\left(\frac{4\,ac-1}{4\,c}\right)}}{4\,\sqrt{-c}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c}\left(2\,x-\frac{1}{c}\right)\right)e^{\left(-\frac{4\,ac-1}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c)
 + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt(c
)
```

maple [A] time = 0.15, size = 72, normalized size = 0.85

$$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(c*x^2+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*
Pi^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

maxima [A] time = 0.31, size = 65, normalized size = 0.76

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\,\sqrt{-c}}\right)e^{\left(a-\frac{1}{4\,c}\right)}}{4\,\sqrt{-c}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\,\sqrt{c}}\right)e^{\left(-a+\frac{1}{4\,c}\right)}}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) - 1/4*sq
rt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^x\sinh\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(a + c*x^2),x)
[Out] int(exp(x)*sinh(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \sinh\left(a + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x**2+a),x)
[Out] Integral(exp(x)*sinh(a + c*x**2), x)
```

3.338 $\int e^x \sinh\left(a+bx+cx^2\right)\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt{\pi}\,e^{\frac{(1-b)^2}{4c}-a}\operatorname{erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,e^{a-\frac{(b+1)^2}{4c}}\operatorname{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*exp(-a+1/4*(1-b)^2/c)*erf(1/2*(-2*c*x-b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/
4*exp(a-1/4*(1+b)^2/c)*erfi(1/2*(2*c*x+b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5512, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{\frac{(1-b)^2}{4c}-a}\operatorname{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,e^{a-\frac{(b+1)^2}{4c}}\operatorname{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sqr
t[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]
)/(4*Sqrt[c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh\left(a+bx+cx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(1+b)x+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-a+(1-b)x-cx^2}\,dx\right)+\frac{1}{2}\int e^{a+(1+b)x+cx^2}\,dx\\ &= -\left(\frac{1}{2}e^{-a+\frac{(1-b)^2}{4c}}\int e^{-\frac{(1-b-2cx)^2}{4c}}\,dx\right)+\frac{1}{2}e^{a-\frac{(1+b)^2}{4c}}\int e^{\frac{(1+b+2cx)^2}{4c}}\,dx\\ &= \frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 92, normalized size = 0.91

$$\frac{\sqrt{\pi}\,e^{-\frac{(b+1)^2}{4c}}\left((\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)-e^{\frac{b^2+1}{2c}}(\cosh(a)-\sinh(a))\mathrm{erf}\left(\frac{b+2cx-1}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a]
 - Sinh[a])) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*S
qrt[c]*E^((1 + b)^2/(4*c)))
```

fricas [A] time = 0.65, size = 129, normalized size = 1.28

$$-\frac{\sqrt{\pi}\,\sqrt{-c}\left(\cosh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)+\sinh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\,\sqrt{c}\left(\cosh\left(-\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)-\sinh\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*(
b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) + sqrt(pi)*s
```

```
qrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c - 2*b
 + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

giac [A] time = 0.15, size = 91, normalized size = 0.90

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4ac+2b+1}{4c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4ac-2b+1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c + 2
*b + 1)/c)/sqrt(-c) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^(1
/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

maple [A] time = 0.23, size = 97, normalized size = 0.96

$$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2+2b-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1-b}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2-2b-1}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{1+b}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(c*x^2+b*x+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)
/c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^(1
/2)*x+1/2*(1+b)/(-c)^(1/2))
```

maxima [A] time = 0.32, size = 81, normalized size = 0.80

$$\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{-c}\,x-\frac{b+1}{2\sqrt{-c}}\right)e^{\left(a-\frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{c}\,x+\frac{b-1}{2\sqrt{c}}\right)e^{\left(-a+\frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c)
/sqrt(-c) - 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4*(
b - 1)^2/c)/sqrt(c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh \left(c\,x^2+b\,x+a\right)\,\mathrm{e}^x \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x + c*x^2)*exp(x),x)
```

```
[Out] int(sinh(a + b*x + c*x^2)*exp(x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x} \sinh \left(a + bx + cx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*sinh(a + b*x + c*x**2), x)
```

3.339 $\int e^{x^2} \sinh(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}\sqrt{\pi}\,e^{a-\frac{b^2}{4}}\operatorname{erfi}\left(\frac{1}{2}(b+2x)\right)-\frac{1}{4}\sqrt{\pi}\,e^{-a-\frac{b^2}{4}}\operatorname{erfi}\left(\frac{1}{2}(2x-b)\right)$$

```
[Out] 1/4*exp(-a-1/4*b^2)*erfi(1/2*b-x)*Pi^(1/2)+1/4*exp(a-1/4*b^2)*erfi(1/2*b+x)
*Pi^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5512, 2234, 2204}

$$\frac{1}{4}\sqrt{\pi}\,e^{a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(b+2x)\right)-\frac{1}{4}\sqrt{\pi}\,e^{-a-\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-b)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + b*x],x]
```

```
[Out] -(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*E
rfi[(b + 2*x)/2])/4
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\sinh(a+bx)\,dx &= \int \left(-\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right)dx \\
&= -\left(\frac{1}{2}\int e^{-a-bx+x^2}\,dx\right)+\frac{1}{2}\int e^{a+bx+x^2}\,dx \\
&= -\left(\frac{1}{2}e^{-a-\frac{b^2}{4}}\int e^{\frac{1}{4}(-b+2x)^2}\,dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4}}\int e^{\frac{1}{4}(b+2x)^2}\,dx \\
&= -\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(b+2x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 51, normalized size = 0.78

$$\frac{1}{4}\sqrt{\pi}\,e^{-\frac{b^2}{4}}\left((\cosh(a)-\sinh(a))\mathrm{erfi}\left(\frac{b}{2}-x\right)+(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{b}{2}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + b*x],x]
```

```
[Out] (Sqrt[Pi]*(Erfi[b/2 - x]*(Cosh[a] - Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Sin
h[a])))/(4*E^(b^2/4))
```

fricas [A] time = 0.76, size = 45, normalized size = 0.69

$$\frac{1}{4}\sqrt{\pi}\left(\mathrm{erfi}\left(\frac{1}{2}\,b+x\right)e^{\left(\frac{1}{4}\,b^2+a\right)}-\mathrm{erfi}\left(-\frac{1}{2}\,b+x\right)e^{\left(\frac{1}{4}\,b^2-a\right)}\right)e^{\left(-\frac{1}{2}\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(erfi(1/2*b + x)*e^(1/4*b^2 + a) - erfi(-1/2*b + x)*e^(1/4*b^2
 - a))*e^(-1/2*b^2)
```

giac [C] time = 0.12, size = 45, normalized size = 0.69

$$\frac{1}{4}i\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}-\frac{1}{4}i\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(1/
2*I*b - I*x)*e^(-1/4*b^2 - a)
```

maple [C] time = 0.12, size = 52, normalized size = 0.80

$$-\frac{i\sqrt{\pi}\,\mathrm{e}^{-a-\frac{b^2}{4}}\,\mathrm{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4}}\,\mathrm{erf}\left(ix+\frac{1}{2}ib\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(b*x+a),x)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*
b^2)*erf(I*x+1/2*I*b)
```

maxima [C] time = 0.31, size = 45, normalized size = 0.69

$$-\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(-1
/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{x^2}\,\sinh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(a + b*x),x)
```

```
[Out] int(exp(x^2)*sinh(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^{2}} \sinh{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + b*x), x)
```

3.340 $\int e^{x^2} \sinh\left(a + cx^2\right) dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\, e^a \operatorname{erfi}\left(\sqrt{c+1}\, x\right)}{4\sqrt{c+1}} - \frac{\sqrt{\pi}\, e^{-a} \operatorname{erfi}\left(\sqrt{1-c}\, x\right)}{4\sqrt{1-c}}$$

```
[Out] -1/4*erfi(x*(1-c)^(1/2))*Pi^(1/2)/exp(a)/(1-c)^(1/2)+1/4*exp(a)*erfi(x*(1+c
)^(1/2))*Pi^(1/2)/(1+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5512, 2204}

$$\frac{\sqrt{\pi}\, e^a \operatorname{Erfi}\left(\sqrt{c+1}\, x\right)}{4\sqrt{c+1}} - \frac{\sqrt{\pi}\, e^{-a} \operatorname{Erfi}\left(\sqrt{1-c}\, x\right)}{4\sqrt{1-c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + c*x^2],x]
```

```
[Out] -(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sq
rt[1 + c]*x])/(4*Sqrt[1 + c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=\int\left(-\frac{1}{2}e^{-a+(1-c)x^2}+\frac{1}{2}e^{a+(1+c)x^2}\right)dx$$
$$=-\left(\frac{1}{2}\int e^{-a+(1-c)x^2}\,dx\right)+\frac{1}{2}\int e^{a+(1+c)x^2}\,dx$$
$$=-\frac{e^{-a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

Mathematica [**A**] time = 0.11, size = 72, normalized size = 1.11

$$\frac{\sqrt{\pi}\left((c-1)\sqrt{c+1}\,(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\sqrt{c+1}\,x\right)-\sqrt{c-1}\,(c+1)(\cosh(a)-\sinh(a))\mathrm{erf}\left(\sqrt{c-1}\,x\right)\right)}{4\left(c^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]))
+ (-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c
^2))
```

fricas [**A**] time = 0.96, size = 75, normalized size = 1.15

$$-\frac{\sqrt{\pi}\,((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\ \mathrm{erf}\left(\sqrt{c-1}\,x\right)+\sqrt{\pi}\,((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{-c-1}\ \mathrm{er}}{4\left(c^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
 1)*x) + sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqrt
(-c - 1)*x))/(c^2 - 1)
```

giac [**A**] time = 0.14, size = 49, normalized size = 0.75

$$\frac{\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{c-1}\,x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{-c-1}\,x\right)e^{a}}{4\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sqr
t(-c - 1)*x)*e^a/sqrt(-c - 1)
```

maple [A] time = 0.35, size = 48, normalized size = 0.74

$$-\frac{\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erf}\left(\sqrt{c-1}\;x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{e}^{a}\,\mathrm{erf}\left(\sqrt{-1-c}\;x\right)}{4\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(c*x^2+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-
1-c)^(1/2)*erf((-1-c)^(1/2)*x)
```

maxima [A] time = 0.31, size = 47, normalized size = 0.72

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}\,x\right)e^{(-a)}}{4\,\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}\,x\right)e^{a}}{4\,\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt
(-c - 1)*x)*e^a/sqrt(-c - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{x^2}\sinh\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(a + c*x^2),x)
```

```
[Out] int(exp(x^2)*sinh(a + c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^{2}}\sinh\left(a+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + c*x**2), x)
```

3.341 $\int e^{x^2} \sinh\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\pi}\, e^{-a-\frac{b^2}{4(1-c)}} \operatorname{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}} + \frac{\sqrt{\pi}\, e^{a-\frac{b^2}{4(c+1)}} \operatorname{erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}$$

```
[Out] 1/4*exp(-a-1/4*b^2/(1-c))*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))*Pi^(1/2)/(1-c
)^(1/2)+1/4*exp(a-1/4*b^2/(1+c))*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2))*Pi^(1/
2)/(1+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5512, 2234, 2204}

$$\frac{\sqrt{\pi}\, e^{-a-\frac{b^2}{4(1-c)}} \operatorname{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}} + \frac{\sqrt{\pi}\, e^{a-\frac{b^2}{4(c+1)}} \operatorname{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a - b^2/(4*(1 - c))))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])])
/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c))))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x)
/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\sinh\left(a+bx+cx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-a-bx+(1-c)x^2}+\frac{1}{2}e^{a+bx+(1+c)x^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-a-bx+(1-c)x^2}\,dx\right)+\frac{1}{2}\int e^{a+bx+(1+c)x^2}\,dx\\
&= -\left(\frac{1}{2}e^{-a-\frac{b^2}{4(1-c)}}\int e^{\frac{(-b+2(1-c)x)^2}{4(1-c)}}\,dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4(1+c)}}\int e^{\frac{(b+2(1+c)x)^2}{4(1+c)}}\,dx\\
&= \frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}
\end{aligned}
$$

Mathematica [A] time = 0.40, size = 123, normalized size = 1.07

$$
\frac{\sqrt{\pi}\,e^{-\frac{b^2}{4c+4}}\left((c-1)\sqrt{c+1}\,(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)-\sqrt{c-1}\,(c+1)e^{\frac{b^2c}{2(c^2-1)}}(\cosh(a)-\sinh(a))\mathrm{erf}\left(\frac{b+2(c-1)x}{2\sqrt{c-1}}\right)\right)}{4\left(c^2-1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1
 + c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a])) + (-1 + c)*Sqrt[1 + c]*Erfi
[(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(
b^2/(4 + 4*c)))
```

fricas [A] time = 0.48, size = 164, normalized size = 1.43

$$
-\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\,\mathrm{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)+\sqrt{\pi}\left((c-1)\cosh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)\cdots\right.}{4\left(c^2-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*si
nh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b)
/sqrt(c - 1)) + sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)) +
```

```
(c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*(c
+ 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

giac [A] time = 0.15, size = 101, normalized size = 0.88

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c-1}\left(2\,x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c-1}\left(2\,x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*c
 - 4*a)/(c + 1))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x + b/
(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

maple [A] time = 0.39, size = 105, normalized size = 0.91

$$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2-4a}{4(c-1)}}\,\operatorname{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2+4a}{4+4c}}\,\operatorname{erf}\left(-\sqrt{-1-c}\,x+\frac{b}{2\sqrt{-1-c}}\right)}{4\sqrt{-1-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(c*x^2+b*x+a),x)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x
+1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-1-c)^(1/2
)*erf(-(-1-c)^(1/2)*x+1/2*b/(-1-c)^(1/2))
```

maxima [A] time = 0.32, size = 89, normalized size = 0.77

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c-1}\,x-\frac{b}{2\,\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\,\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c + 1
))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(-a
 + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x + c*x^2)*exp(x^2),x)
```

```
[Out] int(sinh(a + b*x + c*x^2)*exp(x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + b*x + c*x**2), x)
```

3.342 $\int f^{a+bx} \sinh\left(d + fx^2\right) dx$

Optimal. Leaf size=110

$$\frac{1}{4}\sqrt{\pi}\, f^{a-\frac{1}{2}} e^{d-\frac{b^2\log^2(f)}{4f}} \operatorname{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt{\pi}\, f^{a-\frac{1}{2}} e^{\frac{b^2\log^2(f)}{4f}-d} \operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] -1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*
Pi^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f
^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}\, f^{a-\frac{1}{2}} e^{d-\frac{b^2\log^2(f)}{4f}} \operatorname{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt{\pi}\, f^{a-\frac{1}{2}} e^{\frac{b^2\log^2(f)}{4f}-d} \operatorname{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] -(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f]
)/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
i[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sinh\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx-\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,dx\\
&= -\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2fx+b
\end{aligned}$$

Mathematica [A] time = 0.14, size = 103, normalized size = 0.94

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left((\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)-e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi]*(-(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Co
sh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

fricas [B] time = 0.45, size = 213, normalized size = 1.94

$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)\mathrm{erf}\left(\frac{\left(2fx+b\log(f)\right)\sqrt{-f}}{2f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)\mathrm{erf}\left(-\frac{2fx
}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*e
rf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) - sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(
f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) - sqrt
(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqr
t(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

giac [A] time = 0.14, size = 106, normalized size = 0.96

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{f}\left(2\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{4\,\sqrt{f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-f}\left(2\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*
a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*lo
g(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

maple [A] time = 0.15, size = 100, normalized size = 0.91

$$-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2-4df}{4f}}\operatorname{erf}\left(-\sqrt{-f}\,x+\frac{\ln(f)b}{2\sqrt{-f}}\right)}{4\sqrt{-f}}+\frac{\sqrt{\pi}\,f^a\mathrm{e}^{\frac{\ln(f)^2b^2-4df}{4f}}\operatorname{erf}\left(-\sqrt{f}\,x+\frac{\ln(f)b}{2\sqrt{f}}\right)}{4\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(-f)^(1/2)*erf(-(-f)^(1/2
)*x+1/2*ln(f)*b/(-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)
/f^(1/2)*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))
```

maxima [A] time = 0.32, size = 90, normalized size = 0.82

$$-\frac{1}{4}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}\,x-\frac{b\log(f)}{2\,\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4\,f}-d\right)}+\frac{\sqrt{\pi}\,f^a\operatorname{erf}\left(\sqrt{-f}\,x-\frac{b\log(f)}{2\,\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,f}+d\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))*
e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\sinh\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2), x)
```

3.343 $\int f^{a+bx} \sinh^2\left(d+fx^2\right) dx$

Optimal. Leaf size=148

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}\, f^{a-\frac{1}{2}} e^{\frac{b^2\log^2(f)}{8f}-2d} \operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\, f^{a-\frac{1}{2}} e^{2d-\frac{b^2\log^2(f)}{8f}} \operatorname{erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] -1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/4*
(4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*b^2*ln(f
)^2/f)*f^(-1/2+a)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2
)
```

Rubi [A] time = 0.19, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5512, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}\, f^{a-\frac{1}{2}} e^{\frac{b^2\log^2(f)}{8f}-2d} \operatorname{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}\, f^{a-\frac{1}{2}} e^{2d-\frac{b^2\log^2(f)}{8f}} \operatorname{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 - f^(a + b*x)/(
2*b*Log[f])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sinh^2\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx}\,dx-\frac{1}{2}\int f^{a+bx}\,dx\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2d-2fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2d+2fx^2+a\log(f)+bx\log(f)}\,dx\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{\frac{(4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{(-4fx+
\end{aligned}$$

$$= \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{4fx}{2}\right.$$

Mathematica [A] time = 0.77, size = 149, normalized size = 1.01

$$\frac{1}{16}f^a\left(\frac{\sqrt{2\pi}\,e^{\frac{b^2\log^2(f)}{8f}}(\cosh(2d)-\sinh(2d))\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)}{\sqrt{f}}+\frac{\sqrt{2\pi}\,e^{-\frac{b^2\log^2(f)}{8f}}(\sinh(2d)+\cosh(2d))\mathrm{erfi}\left(\frac{b\log(f)+4f}{2\sqrt{2}\sqrt{f}}\right.}{\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*((-8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4
*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (S
qrt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[2*
d]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

fricas [B] time = 0.71, size = 278, normalized size = 1.88

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,b\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}\left(4\,fx+b\log(f)\right)\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\,\sqrt{\pi}\,b\sqrt{f}\,\cosh\left(\frac{b^2\log(f)^2+8\,a}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2)*
sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*erf(-
1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)
*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f*log(f)
 - 16*d*f)/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh((b*x + a)*log(f)))/(b
*f*log(f))
```

giac [C] time = 0.18, size = 356, normalized size = 2.41

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{f}\left(4\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8\,af\log(f)-16\,df}{8\,f}\right)}}{16\,\sqrt{f}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{-f}\left(4\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log}{}\right.}}{16\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8*
(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)*e
rf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*
log(f) - 16*d*f)/f)/sqrt(-f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1
/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f)
- pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2
*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^
(b*x*log(abs(f)) + a*log(abs(f))) - 1/2*I*(2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2
*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4
```

```
*b*log(abs(f))) - 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*s
gn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x
*log(abs(f)) + a*log(abs(f)))
```

maple [A] time = 0.22, size = 126, normalized size = 0.85

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2} b^{2}-16 d f}{8 f}} \sqrt{2}\, \operatorname{erf}\left(-\sqrt{2}\, \sqrt{f}\, x+\frac{\ln(f) b \sqrt{2}}{4 \sqrt{f}}\right)}{16 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-16 d f}{8 f}} \operatorname{erf}\left(-\sqrt{-2 f}\, x+\frac{\ln(f) b}{2 \sqrt{-2 f}}\right)}{8 \sqrt{-2 f}}-\frac{f^{a} f^{b x}}{2 \ln(f) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-16*d*f)/f)*2^(1/2)/f^(1/2)*erf(-2^(
1/2)*f^(1/2)*x+1/4*ln(f)*b*2^(1/2)/f^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/8*(ln(f
)^2*b^2-16*d*f)/f)/(-2*f)^(1/2)*erf(-(-2*f)^(1/2)*x+1/2*ln(f)*b/(-2*f)^(1/2
))-1/2*f^a/ln(f)/b*f^(b*x)
```

maxima [A] time = 0.41, size = 127, normalized size = 0.86

$$\frac{\sqrt{2}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\,\sqrt{f}\, x-\frac{\sqrt{2}\, b \log(f)}{4\,\sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{8\, f}-2\, d\right)}}{16\,\sqrt{f}}+\frac{\sqrt{2}\,\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\,\sqrt{-f}\, x-\frac{\sqrt{2}\, b \log(f)}{4\,\sqrt{-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{8\, f}+2\, d\right)}}{16\,\sqrt{-f}}-\frac{f^{bx+}}{2\, b \log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqrt
(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf(s
qrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/f +
 2*d)/sqrt(-f) - 1/2*f^(b*x + a)/(b*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\sinh\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx} \sinh^{2}\left(d+fx^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**2, x)
```

3.344 $\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=239

$$\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\operatorname{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\operatorname{e}$$

```
[Out] -1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/6*(6*f*x-b*ln(f))*3^(1/
2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*er
fi(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/16*exp(-d+1/4*b^
2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*exp(
d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\operatorname{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\operatorname{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\operatorname{E}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 - (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sinh^3\left(d+fx^2\right)dx &= \int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}-\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx}\,dx-\frac{3}{8}\int\\
&= -\left(\frac{1}{8}\int e^{-3d-3fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{-d}\\
&= -\left(\frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx\right)+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{\frac{(6fx+b\log(f))^2}{12f}}\,dx-\\
&= \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{6f
\end{aligned}$$

Mathematica [A] time = 0.43, size = 287, normalized size = 1.20

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{-\frac{b^2\log^2(f)}{4f}}\left(3\sqrt{3}\,e^{\frac{b^2\log^2(f)}{2f}}(\cosh(d)-\sinh(d))\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-e^{\frac{b^2\log^2(f)}{3f}}(\cosh(3d)-\sinh(3d))\mathrm{er}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqr
t[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqrt
```

```
[3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d]) - 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt
[f])]*Sinh[d] - E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*
Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f*x +
b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f)))
```

fricas [B] time = 0.49, size = 445, normalized size = 1.86

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}\left(6\,fx+b\log(f)\right)\sqrt{-f}}{6\,f}\right)-\sqrt{3}\,\sqrt{\pi}\,\sqrt{f}\,\cosh\left(\frac{b^2\log(f)^2+12\,af\log(f)-36}{12\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) - sqrt(3)*sqrt(pi
)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*sqr
t(3)*(6*f*x - b*log(f))/sqrt(f)) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3
)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*
d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt
(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) - 9*sqrt(pi)*s
qrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(2*f*x +
b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + 4*a*f*l
og(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)
*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4*a*f*log(f)
 - 4*d*f)/f) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f)*s
inh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

giac [A] time = 0.17, size = 223, normalized size = 0.93

$$\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{f}\left(6\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{f}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{-f}\left(6\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2}{}\right.}}{48\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12*
(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi)*
erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a
*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*l
og(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) + 3/16*sq
```

```
rt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*
log(f) - 4*d*f)/f)/sqrt(-f)
```

maple [A] time = 0.27, size = 207, normalized size = 0.87

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-36 d f}{12 f}} \operatorname{erf}\left(-\sqrt{-3 f}\, x+\frac{\ln(f) b}{2 \sqrt{-3 f}}\right)}{16 \sqrt{-3 f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2} b^{2}-36 d f}{12 f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\, \sqrt{f}\, x+\frac{\ln(f) b \sqrt{3}}{6 \sqrt{f}}\right)}{48 \sqrt{f}}-\frac{3 \sqrt{\pi}\, f^{a} \mathrm{e}^{\ln(f}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/12*(ln(f)^2*b^2-36*d*f)/f)/(-3*f)^(1/2)*erf(-(-3*
f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(1/2))+1/48*Pi^(1/2)*f^a*exp(1/12*(ln(f)^2*b^
2-36*d*f)/f)*3^(1/2)/f^(1/2)*erf(-3^(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(
1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(ln(f)^2*b^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)
*x+1/2*ln(f)*b/f^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*f)/f)/(
-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

maxima [A] time = 0.42, size = 200, normalized size = 0.84

$$\frac{3}{16} \sqrt{\pi} f^{a-\frac{1}{2}} \operatorname{erf}\left(\sqrt{f} x-\frac{b \log(f)}{2 \sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{4 f}-d\right)}-\frac{\sqrt{3} \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{3} \sqrt{f} x-\frac{\sqrt{3} b \log(f)}{6 \sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{12 f}-3 d\right)}}{48 \sqrt{f}}+\frac{\sqrt{3} \sqrt{\pi} f^{a} e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqr
t(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt(3)
*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e^(-1
/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2
*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,\sinh\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**3, x)
```

3.345 $\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=115

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\operatorname{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

```
[Out] -1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/
2))*Pi^(1/2)+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*
ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*
Log[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\sinh\left(d+ex+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx}\right)dx \\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx}\,dx \\
&= -\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}\,dx \\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx\right)+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\int e^{\frac{(e+2fx+b\log(f))^2}{4f}} \\
&= -\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{e}
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 124, normalized size = 1.08

$$
\frac{1}{4}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+e^2}{4f}}\left((\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-(\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+e^2}{2f}}\mathrm{erf}\left(\frac{-b\log
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e
+ 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(e + 2*f*x + b
*Log[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f
)))
```

fricas [B] time = 0.56, size = 253, normalized size = 2.20

$$
-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+e^2-4df+2\left(be-2af\right)\log(f)}{4f}\right)\mathrm{erf}\left(\frac{\left(2fx+b\log(f)+e\right)\sqrt{-f}}{2f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+e^2-4df-2\left(be-2a\right.}{4f}\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a
*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) - sqrt(pi)*sqrt(f
)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(-1/
2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*lo
g(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a*f
)*log(f))/f) - sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f))*si
nh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

giac [A] time = 0.14, size = 134, normalized size = 1.17

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{f}\left(2\,x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-2\,be\log(f)-4\,df+e^2}{4\,f}\right)}}{4\,\sqrt{f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-f}\left(2\,x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-\cdots}{\cdots}\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^
2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(f) - 1/4*sqrt(pi)*er
f(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log
(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

maple [A] time = 0.13, size = 126, normalized size = 1.10

$$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{\ln(f)^2b^2+2\ln(f)be-4df+e^2}{4f}}\operatorname{erf}\left(-\sqrt{-f}\,x+\frac{e+b\ln(f)}{2\sqrt{-f}}\right)}{4\sqrt{-f}}+\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{\ln(f)^2b^2-2\ln(f)be-4df+e^2}{4f}}\operatorname{erf}\left(-\sqrt{f}\,x+\frac{b\ln(f)-e}{2\sqrt{f}}\right)}{4\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*f+e^2)/f)/(-f)^(1/2
)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(1/4*(l
n(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*ln(f)-e)
/f^(1/2))
```

maxima [A] time = 0.33, size = 102, normalized size = 0.89

$$-\frac{1}{4}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}\,x-\frac{b\log(f)-e}{2\,\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4\,f}\right)}+\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-f}\,x-\frac{b\log(f)+e}{2\,\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4\,f}\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
 + 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f
) + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2), x)
```

3.346 $\int f^{a+bx} \sinh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=161

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\operatorname{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\operatorname{erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f^{a}}{2b\log}$$

```
[Out] -1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(
1/4*(2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*
(2*e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2))
*2^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5512, 2194, 2287, 2234, 2205, 2204}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\operatorname{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\operatorname{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)-\frac{f}{2b\log}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 - f^(a + b*x)/(2*b*Log[f])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx &= \int\left(-\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx}\,dx-\frac{1}{2}\int f^{a+bx}\,dx\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int\exp\left(-2d-2fx^2+a\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^a\right)\int e^{-\frac{(-2e-4fx+b\log(f))^2}{8f}}dx+\frac{1}{4}\left(e^{2d-\frac{(2e+b\log(f))^2}{8f}}\right.\\
&= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\end{aligned}$$

Mathematica [A] time = 0.64, size = 220, normalized size = 1.37

$$\frac{f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+4e^2}{8f}}\left(\sqrt{\pi}\,b\log(f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2\log^2(f)+4e^2}{4f}}\operatorname{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)-4\sqrt{2}f^{b\left(\frac{e}{2f}+x\right)+\frac{1}{2}}e^{\frac{b^2\log^2(f)\cdots}{8f}}\right.}{8\sqrt{2}\,b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(-4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/
2 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*e
 + 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d]) +
b*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[
2*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f])
```

fricas [B] time = 0.43, size = 334, normalized size = 2.07

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,b\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2+4\,e^2-16\,df+4\left(be-2\,af\right)\log(f)}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}\left(4\,fx+b\log(f)+2\,e\right)\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\,\sqrt{\pi}\,b\sqrt{f}\,\cosh\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqrt(
-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 -
 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f) + 2
*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f*x +
b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*sqrt(2)*
(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f - 4*(b*e - 2*a*f)*log(f))/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh(
(b*x + a)*log(f)))/(b*f*log(f))
```

giac [C] time = 0.20, size = 390, normalized size = 2.42

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{f}\left(4\,x-\frac{b\log(f)-2\,e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8\,af\log(f)-4\,be\log(f)-16\,df+4\,e^2}{8\,f}\right)}}{16\,\sqrt{f}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{-f}\left(4\,x+\frac{b\log\cdots}{\cdots}\right)\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(f) - 2*e)/f))
*e^(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/f)/sq
rt(f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) +
2*e)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*log(f) + 4*b*e*log(f) - 16*d*f + 4*e
^2)/f)/sqrt(-f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f
) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) -
(pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f)
+ 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(
```

```
f)) + a*log(abs(f))) - 1/2*I*(2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1
/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f)
)) - 2*I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I
*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f))
+ a*log(abs(f)))
```

maple [A] time = 0.21, size = 158, normalized size = 0.98

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2} b^{2}-4\ln(f) b e-16 d f+4 e^{2}}{8 f}} \sqrt{2}\ \mathrm{erf}\left(-\sqrt{2}\ \sqrt{f}\ x+\frac{\left(b\ln(f)-2 e\right)\sqrt{2}}{4\sqrt{f}}\right)}{16\sqrt{f}}-\frac{\sqrt{\pi}\ f^{a}\mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4\ln(f) b e-16 d f+4 e^{2}}{8 f}}\ \mathrm{erf}\left(-\sqrt{-2 f}\ x+\right.}{8\sqrt{-2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(ln(f)^2*b^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)*2^(1/2
)/f^(1/2)*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))-1/8*Pi^
(1/2)*f^a*exp(-1/8*(ln(f)^2*b^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)/(-2*f)^(1/2)*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))-1/2*f^a/ln(f)/b*f^(b*x)
```

maxima [A] time = 0.43, size = 143, normalized size = 0.89

$$\frac{\sqrt{2}\,\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{-f}\,x-\frac{\sqrt{2}\left(b\log(f)+2\,e\right)}{4\,\sqrt{-f}}\right)e^{\left(2\,d-\frac{\left(b\log(f)+2\,e\right)^{2}}{8\,f}\right)}}{16\,\sqrt{-f}}+\frac{\sqrt{2}\,\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{f}\,x-\frac{\sqrt{2}\left(b\log(f)-2\,e\right)}{4\,\sqrt{f}}\right)e^{\left(-2\,d+\frac{(b\log(\right.}{8}\right.}}{16\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)*s
qrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f))*e
^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) - 1/2*f^(b*x + a)/(b*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\mathrm{sinh}\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**2, x)
```

3.347 $\int f^{a+bx} \sinh^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=257

$$\frac{3}{16}\sqrt{\pi} f^{a-\frac{1}{2}} e^{\frac{(e-b\log(f))^2}{4f}-d} \operatorname{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right) - \frac{1}{16}\sqrt{\frac{\pi}{3}} f^{a-\frac{1}{2}} e^{\frac{(3e-b\log(f))^2}{12f}-3d} \operatorname{erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right) - \frac{3}{16}\sqrt{\pi}$$

```
[Out] -1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/6*(3*e+6*f*x-b*ln(f
))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*(3*e+b*ln(f))^2/f)*f
^(-1/2+a)*erfi(1/6*(3*e+6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/
16*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/2)
)*Pi^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*l
n(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{3}{16}\sqrt{\pi} f^{a-\frac{1}{2}} e^{\frac{(e-b\log(f))^2}{4f}-d} \operatorname{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right) - \frac{1}{16}\sqrt{\frac{\pi}{3}} f^{a-\frac{1}{2}} e^{\frac{(3e-b\log(f))^2}{12f}-3d} \operatorname{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right) - \frac{3}{16}\sqrt{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 - (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)dx = \int \left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+\cdots}\right.$$

$$= -\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx}\,dx\right)+\frac{1}{8}\int \exp\left(6d+6ex+6fx^2-3\left(d+ex+f\cdots\right.\right.$$

$$= -\left(\frac{1}{8}\int \exp\left(-3d-3fx^2+a\log(f)-x(3e-b\log(f))\right)dx\right)+\frac{1}{8}\int \exp\left(3d\cdots\right.$$

$$= \frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}dx-\frac{1}{8}\left(e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^a\right)\int e^{-\frac{(-3e-\cdots}{}}$$

$$= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+\cdots}$$

Mathematica [A] time = 0.78, size = 354, normalized size = 1.38

$$\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{be+f}{2f}}e^{-\frac{b^2\log^2(f)+3e^2}{4f}}\left(3\sqrt{3}(\cosh(d)-\sinh(d))e^{\frac{b^2\log^2(f)+2e^2}{2f}}\mathrm{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)-(\cosh(3d)-\sinh(\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(-3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[
(e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Er
fi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 + b^
2*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[
d]) - 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*Sinh
[d] - E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sq
rt[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(3*
e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2 + b^2*
Log[f]^2)/(4*f)))
```

fricas [B] time = 0.52, size = 541, normalized size = 2.11

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\sqrt{-f}\,\cosh\left(\frac{b^2\log(f)^2+9\,e^2-36\,df+6\,(be-2\,af)\log(f)}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}\,(6\,fx+b\log(f)+3\,e)\sqrt{-f}}{6\,f}\right)-\sqrt{3}\,\sqrt{\pi}\,\sqrt{f}\,\cosh\left(\frac{b^2\log(f)^2+9\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f +
 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt(-
f)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f -
 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/sqrt(
f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sq
rt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*log(f)
)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*log(f))/
f) - 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sqrt(pi)*sqr
t(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
 b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt
(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

giac [A] time = 0.19, size = 285, normalized size = 1.11

$$\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{f}\left(6\,x-\frac{b\log(f)-3\,e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-6\,be\log(f)-36\,df+9\,e^2}{12\,f}\right)}}{48\,\sqrt{f}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{-f}\left(6\,x+\frac{b\log(\cdots}{\cdots}\right)\right)\cdots}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*log(f) - 3*e)/f))*
e^(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/f)/s
qrt(f) - 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) +
 3*e)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a*f*log(f) + 6*b*e*log(f) - 36*d*f +
9*e^2)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f
))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 2*b*e*log(f) - 4*d*f + e^2)/f)/sqr
t(f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b
^2*log(f)^2 - 4*a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/f)/sqrt(-f)
```

maple [A] time = 0.26, size = 265, normalized size = 1.03

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6\ln(f) b e-36 d f+9 e^{2}}{12 f}} \mathrm{erf}\left(-\sqrt{-3 f}\, x+\frac{3 e+b\ln(f)}{2\sqrt{-3 f}}\right)}{16\sqrt{-3 f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{\ln(f)^{2} b^{2}-6\ln(f) b e-36 d f+9 e^{2}}{12 f}} \sqrt{3}\, \mathrm{erf}\left(-\sqrt{3}\, \sqrt{f}\, x+\frac{(b}{}\right.}{48\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/12*(ln(f)^2*b^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)/(-3*
f)^(1/2)*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))+1/48*Pi^(1/2)*
f^a*exp(1/12*(ln(f)^2*b^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)*3^(1/2)/f^(1/2)*erf(
-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))-3/16*Pi^(1/2)*f^a*exp
(1/4*(ln(f)^2*b^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*l
n(f)-e)/f^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*f+
e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))
```

maxima [A] time = 0.43, size = 228, normalized size = 0.89

$$\frac{\sqrt{3}\,\sqrt{\pi}\, f^{a}\, \mathrm{erf}\left(\sqrt{3}\,\sqrt{-f}\, x-\frac{\sqrt{3}\,(b\log(f)+3\,e)}{6\,\sqrt{-f}}\right) e^{\left(3\,d-\frac{(b\log(f)+3\,e)^{2}}{12\,f}\right)}}{48\,\sqrt{-f}}+\frac{3}{16}\,\sqrt{\pi}\, f^{a-\frac{1}{2}}\, \mathrm{erf}\left(\sqrt{f}\, x-\frac{b\log(f)-e}{2\,\sqrt{f}}\right) e^{\left(-d+\frac{(b\log(f)-e}{4\,f}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(pi)
*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*log
(f) - e)^2/f) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqrt(
3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt(f) -
 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d - 1/4
*(b*log(f) + e)^2/f)/sqrt(-f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b x} \sinh\left(f x^2 + e x + d\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**3,x)
[Out] Timed out
```

3.348 $\int f^{a+cx^2} \sinh(d+ex)\, dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{4c\log(f)}-d} \operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4c\log(f)}} \operatorname{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5512, 2287, 2234, 2204}

$$\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{4c\log(f)}-d} \operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4c\log(f)}} \operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*Sq
rt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh(d+ex)\,dx &= \int\left(-\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+cx^2}\,dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\ &= -\left(\frac{1}{2}\left(e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx\right)+\frac{1}{2}\left(e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,dx\\ &= \frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 104, normalized size = 0.78

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}}\left((\sinh(d)-\cosh(d))\mathrm{erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[d]
 + Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] +
Sinh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.51, size = 217, normalized size = 1.63

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)\right)\mathrm{erf}\left(\frac{\left(2cx\log(f)+e\right)\sqrt{-c\log(f)}}{2c\log(f)}\right)-}{4c\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e
^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(
c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-
c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))*
erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

giac [A] time = 0.15, size = 132, normalized size = 0.99

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f}{}\right.}}{4\,\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*
log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*
d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.14, size = 117, normalized size = 0.88

$$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{4d\ln(f)c-e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{4d\ln(f)c+e^2}{4\ln(f)c}}\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-
(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*l
n(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f
))^(1/2))
```

maxima [A] time = 0.32, size = 105, normalized size = 0.79

$$\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}\,x-\frac{e}{2\,\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-c\log(f)}\,x+\frac{e}{2\,\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*
e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\mathrm{sinh}\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x), x)
```

3.349 $\int f^{a+cx^2}\sinh^2(d+ex)\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}-2d}\operatorname{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a\operatorname{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-e^2/c/ln(f))*f^a*erfi((-e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(
1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*erfi((e+c*x*ln(f))/c^
(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*f^a*erfi(x*c^(1/2)*ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5512, 2204, 2287, 2234}

$$-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}-2d}\operatorname{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)}}\operatorname{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a\operatorname{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^
(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[Lo
g[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*
Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}\right)dx$$

$$= \frac{1}{4}\int e^{-2d-2ex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d-2ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2d+2ex+a\log(f)}$$

$$= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\left(e^{2d-\frac{e^2}{c\log(f)}}f\right.$$

$$= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{e}}{8\sqrt{c}\,\sqrt{\text{lo}}}$$

Mathematica [A] time = 0.27, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}}\left((\cosh(2d)-\sinh(2d))\text{erfi}\left(\frac{cx\log(f)-e}{\sqrt{c}\,\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\text{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)-2e^{\frac{e^2}{c\log(f)}}\text{erfi}\left(\sqrt{c}\right.\right.}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-
e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e +
 c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]*E
^(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

fricas [A] time = 0.52, size = 244, normalized size = 1.52

$$\frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(a\log(f)\right)+\sqrt{\pi}\sinh\left(a\log(f)\right)\right)\operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)-\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{ac\log(f)^2}{c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))*
erf(sqrt(-c*log(f))*x) - sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2*c
*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(f) -
 e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(
-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))) +
 sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf((c*x*lo
g(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

giac [A] time = 0.16, size = 150, normalized size = 0.93

$$\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}\,x\right)}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x\right.\right.}{8\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e^2
)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e/(c
*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f
))
```

maple [A] time = 0.17, size = 139, normalized size = 0.86

$$\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{2d\ln(f)c+e^2}{\ln(f)c}}\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{\frac{2d\ln(f)c-e^2}{\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}-\frac{f^a\sqrt{\pi}\operatorname{erf}\left(\sqrt{\right.}{4\sqrt{-c\,\mathrm{l}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*l
n(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln
```

(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/4*f^a
*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)

maxima [A] time = 0.32, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{e}{\sqrt{-c \log(f)}}\right) e^{\left(2 d - \frac{e^{2}}{c \log(f)}\right)}}{8 \sqrt{-c \log(f)}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x + \frac{e}{\sqrt{-c \log(f)}}\right) e^{\left(-2 d - \frac{e^{2}}{c \log(f)}\right)}}{8 \sqrt{-c \log(f)}} - \frac{\sqrt{\pi} f^{a} \operatorname{erf}}{4 \sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(c
*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sqrt
(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*e
rf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c x^{2}+a} \sinh(d+e x)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^{2}} \sinh^{2}(d+ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**2, x)
```

3.350 $\int f^{a+cx^2}\sinh^3(d+ex)\,dx$

Optimal. Leaf size=271

$$-\frac{3\sqrt{\pi}\,f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+$$

```
[Out] 3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*erfi(1
/2*(-3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/1
6*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*
Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*erfi(1/2*(3*
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5512, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}\,f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) - (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = \int\left(-\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}-\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx$$
$$= -\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+cx^2}\,dx-\frac{3}{8}\int e^{d+ex}$$
$$= -\left(\frac{1}{8}\int e^{-3d-3ex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{8}\int e^{3d+3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int e^{-d-ex+}$$
$$= -\left(\frac{1}{8}\left(e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3e+2cx\log(f))^2}{4c\log(f)}}\,dx\right)+\frac{1}{8}\left(e^{3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}$$
$$= -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3e^{d-\frac{}{4c}}}{}$$

Mathematica [A] time = 0.46, size = 214, normalized size = 0.79

$$\frac{\sqrt{\pi}f^a e^{-\frac{9e^2}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e^2}{c\log(f)}}\mathrm{erfi}\left(\frac{2cx\log(f)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\mathrm{erfi}\right.\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c
*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e
```

```
 + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Erfi[
(-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[3*d] + Sinh[3*d])))/
(16*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.51, size = 427, normalized size = 1.58

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{\left(2\,cx\log(f)+3\,e\right)\sqrt{-c\log(f)}}{2\,c\log(f)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) - 9
*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f)))
 - 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(c
*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(
-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(
f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))
*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(
sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))) + sq
rt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))))*erf(1
/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

giac [A] time = 0.16, size = 264, normalized size = 0.97

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x+\frac{3\,e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\cdots}{}\right)}}{16\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*a
*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqr
t(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2
 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.21, size = 234, normalized size = 0.86

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3d\ln(f)c-\frac{9e^2}{4}}{c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4d\ln(f)c+3e^2\right)}{4\ln(f)c}} \operatorname{erf}\left(\sqrt{-c\ln(f)}\, x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}+\frac{3\sqrt{\pi}\, f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-3/4*(4
*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+3/2*e/(-
c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*l
n(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^
a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)
*x+1/2*e/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.33, size = 211, normalized size = 0.78

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{3\, e}{2\, \sqrt{-c\log(f)}}\right) e^{\left(3\, d-\frac{9\, e^2}{4\, c\log(f)}\right)}}{16\, \sqrt{-c\log(f)}}-\frac{3\, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{e}{2\, \sqrt{-c\log(f)}}\right) e^{\left(d-\frac{e^2}{4\, c\log(f)}\right)}}{16\, \sqrt{-c\log(f)}}+\frac{3\, \sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d - 9
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*
x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/1
6*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e
^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,{\mathrm{sinh}\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \sinh^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**3, x)
```

3.351 $\int f^{a+cx^2} \sinh\left(d+fx^2\right)\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\pi}\,e^d f^a \operatorname{erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,e^{-d} f^a \operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}$$

```
[Out] -1/4*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/4*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5512, 2287, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^d f^a \operatorname{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,e^{-d} f^a \operatorname{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] -(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d
*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\sinh\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d+a\log(f)-x^2(f-c\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+a\log(f)+x^2(f+c\log(f))}\,dx\\
&= -\frac{e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}
\end{aligned}
$$

Mathematica [A] time = 0.35, size = 76, normalized size = 0.94

$$
\frac{1}{4}\sqrt{\pi}f^a\left(\frac{(\sinh(d)+\cosh(d))\mathrm{erfi}\left(x\sqrt{c\log(f)+f}\right)}{\sqrt{c\log(f)+f}}-\frac{(\cosh(d)-\sinh(d))\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{\sqrt{f-c\log(f)}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c
*Log[f]]) + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log
[f]]))/4
```

fricas [B] time = 0.43, size = 146, normalized size = 1.80

$$
\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(a\log(f)-d\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(a\log(f)-d\right)\right)\sqrt{-c\log(f)+f}\ \mathrm{erf}\left(\sqrt{-c\log(f}\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)*
sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) - (sqrt(
pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*log(
```

```
f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2 - f^
2)
```

giac [A] time = 0.15, size = 75, normalized size = 0.93

$$-\frac{\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{-c\log(f)-f}\,x\right)e^{\left(a\log(f)+d\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{-c\log(f)+f}\,x\right)e^{\left(a\log(f)-d\right)}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) -
 f) + 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log
(f) + f)
```

maple [A] time = 0.12, size = 70, normalized size = 0.86

$$\frac{\sqrt{\pi}\ f^{a}\mathrm{e}^{d}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-f}\ x\right)}{4\sqrt{-c\ln(f)-f}}-\frac{\sqrt{\pi}\ f^{a}\mathrm{e}^{-d}\,\mathrm{erf}\left(x\sqrt{f-c\ln(f)}\right)}{4\sqrt{f-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)-1/4*Pi
^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))
```

maxima [A] time = 0.32, size = 69, normalized size = 0.85

$$-\frac{\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{-c\log(f)+f}\,x\right)e^{(-d)}}{4\sqrt{-c\log(f)+f}}+\frac{\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{-c\log(f)-f}\,x\right)e^{d}}{4\sqrt{-c\log(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1
/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2), x)
```

3.352 $\int f^{a+cx^2} \sinh^2\left(d+fx^2\right) dx$

Optimal. Leaf size=128

$$\frac{\sqrt{\pi}\, e^{-2d} f^a \operatorname{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, e^{2d} f^a \operatorname{erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}\, f^a \operatorname{erfi}\left(\sqrt{c}\, x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] -1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*e
rf(x*(2*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d
)*f^a*erfi(x*(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5512, 2204, 2287, 2205}

$$\frac{\sqrt{\pi}\, e^{-2d} f^a \operatorname{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, e^{2d} f^a \operatorname{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}\, f^a \operatorname{Erfi}\left(\sqrt{c}\, x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^
a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) +
(E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f]]
)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx=\int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}\right)dx$$
$$=\frac{1}{4}\int e^{-2d-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx$$
$$=-\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d+a\log(f)-x^2(2f-c\log(f))}\,dx+\frac{1}{4}\int e^{2d+a\log(f}$$
$$=-\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\,\mathrm{erfi}}{8\sqrt{2f}}$$

Mathematica [A] time = 0.54, size = 179, normalized size = 1.40

$$\frac{\sqrt{\pi}\,f^a\left(\left(8f^2-2c^2\log^2(f)\right)\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)+\sqrt{c}\,\sqrt{\log(f)}\,\left(\sqrt{2f-c\log(f)}\,(c\log(f)+2f)(\sinh(2d)-\cosh\right.\right.}{8\sqrt{c}\,\sqrt{\log(f)}\,\left(c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(8*f^2 - 2*c^2*Log[f]^2) + Sqrt
[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c
*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f - c*L
og[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[
f]]*(-4*f^2 + c^2*Log[f]^2))
```

fricas [B] time = 0.42, size = 254, normalized size = 1.98

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\cosh\left(a\log(f)-2\,d\right)+\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\sinh\left(a\log(f)-2\,d\right)\right)\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(p
i)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2*f
)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(f) +
 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) - 2*(sqrt(pi)*(c^
2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(a
*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*c*f^2*l
og(f))
```

giac [A] time = 0.14, size = 107, normalized size = 0.84

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(-\sqrt{-c \log\left(f\right)} x\right)}{4 \sqrt{-c \log\left(f\right)}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log\left(f\right) - 2 f} x\right) e^{\left(a \log\left(f\right) + 2 d\right)}}{8 \sqrt{-c \log\left(f\right) - 2 f}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log\left(f\right) + 2 f} x\right) e^{\left(a \log\left(f\right) - \cdots\right.}}{8 \sqrt{-c \log\left(f\right) + 2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8*s
qrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f) + 2
*f)
```

maple [A] time = 0.15, size = 101, normalized size = 0.79

$$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-2 d} \operatorname{erf}\left(x \sqrt{2 f - c \ln\left(f\right)}\right)}{8 \sqrt{2 f - c \ln\left(f\right)}} + \frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{2 d} \operatorname{erf}\left(\sqrt{-c \ln\left(f\right) - 2 f}\, x\right)}{8 \sqrt{-c \ln\left(f\right) - 2 f}} - \frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln\left(f\right)}\, x\right)}{4 \sqrt{-c \ln\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*d)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+1
/8*Pi^(1/2)*f^a*exp(2*d)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)-1
/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

maxima [A] time = 0.32, size = 100, normalized size = 0.78

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 2 f} x\right) e^{\left(2 d\right)}}{8 \sqrt{-c \log\left(f\right) - 2 f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 2 f} x\right) e^{\left(-2 d\right)}}{8 \sqrt{-c \log\left(f\right) + 2 f}} - \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right)} x\right)}{4 \sqrt{-c \log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f)
 + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f) +
2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2)**2, x)
```

3.353 $\int f^{a+cx^2} \sinh^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=171

$$\frac{3\sqrt{\pi}\,e^{-d}f^a\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}\,e^{-3d}f^a\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\,e^{d}f^a\text{erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}\,e^{3d}f^a\text{e}}{16}$$

```
[Out] 3/16*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)-1/16*f^
a*erf(x*(3*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*d)/(3*f-c*ln(f))^(1/2)-3/16*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d)*
f^a*erfi(x*(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5512, 2287, 2205, 2204}

$$\frac{3\sqrt{\pi}\,e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}\,e^{-3d}f^a\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\,e^{d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}\,e^{3d}f^a}{16}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) - (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
- (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)dx=\int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)dx$$
$$=-\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+cx^2}dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+cx^2}dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+cx^2}dx-\frac{3}{8}$$
$$=-\left(\frac{1}{8}\int e^{-3d+a\log(f)-x^2(3f-c\log(f))}dx\right)+\frac{1}{8}\int e^{3d+a\log(f)+x^2(3f+c\log(f))}dx+\frac{3}{8}\int e^{-}$$
$$=\frac{3e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^df^a\sqrt{\pi}}{16}$$

Mathematica [A] time = 1.23, size = 272, normalized size = 1.59

$$\frac{\sqrt{\pi}f^a\left(3\sqrt{f-c\log(f)}\left(-c^3\log^3(f)-c^2f\log^2(f)+9cf^2\log(f)+9f^3\right)(\cosh(d)-\sinh(d))\mathrm{erf}\left(x\sqrt{f-c\log(f}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*
f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*Lo
g[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[
f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x*Sqr
t[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) -
Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d]
 + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

fricas [B] time = 0.74, size = 492, normalized size = 2.88

$$\frac{\left(\sqrt{\pi}\left(c^3\log(f)^3+3c^2f\log(f)^2-cf^2\log(f)-3f^3\right)\cosh\left(a\log(f)-3d\right)+\sqrt{\pi}\left(c^3\log(f)^3+3c^2f\log(f)^2-c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(
f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(f)
+ 3*f)*x) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9
*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^
2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f
) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9
*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^
2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f
) - f)*x) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f
^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^
2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*erf(sqrt(-c*l
og(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

giac [A] time = 0.16, size = 155, normalized size = 0.91

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)-3\,f}\,x\right)e^{\left(a\log(f)+3\,d\right)}}{16\,\sqrt{-c\log(f)-3\,f}}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)-f}\,x\right)e^{\left(a\log(f)+d\right)}}{16\,\sqrt{-c\log(f)-f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)}\right.}{16\,\sqrt{-c\log(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*log
(f) - 3*f) + 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqr
t(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) -
d)/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^(a*l
og(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

maple [A] time = 0.21, size = 144, normalized size = 0.84

$$\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{3d}\operatorname{erf}\left(\sqrt{-c\ln(f)-3f}\,x\right)}{16\sqrt{-c\ln(f)-3f}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-3d}\operatorname{erf}\left(x\sqrt{3f-c\ln(f)}\right)}{16\sqrt{3f-c\ln(f)}}+\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}^{-d}\operatorname{erf}\left(x\sqrt{f-c\ln(f)}\right)}{16\sqrt{f-c\ln(f)}}-\frac{3\sqrt{\pi}\,f^{a}\mathrm{e}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(3*d)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x)
-1/16*Pi^(1/2)*f^a*exp(-3*d)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))
+3/16*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))-3/16*
Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

maxima [A] time = 0.33, size = 143, normalized size = 0.84

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) - 3f}\,x\right) e^{(3d)}}{16\sqrt{-c\log(f) - 3f}} + \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) + f}\,x\right) e^{(-d)}}{16\sqrt{-c\log(f) + f}} - \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) + 3f}\,x\right) e^{(-3}}{16\sqrt{-c\log(f) + 3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*f
) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f)
 - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f) +
 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a} \sinh\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \sinh^3\left(d + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2)**3, x)
```

3.354 $\int f^{a+cx^2} \sinh\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=140

$$\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \text{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

```
[Out] -1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi
(1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \text{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))
/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)dx = \int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+cx^2}\right)dx$$

$$= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+cx^2}\,dx$$

$$= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,dx$$

$$= -\left(\frac{1}{2}\left(e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right)+\frac{1}{2}\left(e^{d-\frac{}{4(f+c}}\right.$$

$$= -\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2x(f+c\log}{2\sqrt{f+c\log}}\right.}{4\sqrt{f+c\log(f)}}$$

Mathematica [A] time = 0.74, size = 166, normalized size = 1.19

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4(c\log(f)+f)}}\left(\sqrt{f-c\log(f)}(\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{2cx\log(f)+e+2fx}{2\sqrt{c\log(f)+f}}\right)-\sqrt{c\log(f)+f}(\cosh(d)-\sinh(d))e\right.}{4\sqrt{f-c\log(f)}\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*
x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d])) +
 Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]
```

```
*(Cosh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sqr
t[f + c*Log[f]])
```

fricas [B] time = 0.48, size = 322, normalized size = 2.30

$$\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\left(cd+af\right)\log(f)}{4\left(c\log(f)-f\right)}\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\left(cd+af\right)\log(}{4\left(c\log(f)-f\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c
*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a*c
*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*lo
g(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f)
- f)) - (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f + 4
*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1/4*(4*
a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c
*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(
f) + f)))/(c^2*log(f)^2 - f^2)
```

giac [A] time = 0.14, size = 172, normalized size = 1.23

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)+4\,df-e^2}{4\left(c\log(f)+f\right)}\right)}}{4\,\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)+f}\left(2\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4
*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f
))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e
/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*
d*f - e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

maple [A] time = 0.16, size = 147, normalized size = 1.05

$$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{4d\ln(f)c+4df-e^2}{4c\ln(f)+4f}}\,\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e}{2\sqrt{-c\ln(f)-f}}\right)}{4\sqrt{-c\ln(f)-f}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{4d\ln(f)c-4df+e^2}{4\left(-f+c\ln(f)\right)}}\,\operatorname{erf}\left(x\sqrt{f-c\ln(f)}+\frac{e}{2\sqrt{f-c\ln(}}\right.}{4\sqrt{f-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)
^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))-1/4*Pi^(1/2)*f^a
*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(x*(f-
c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))
```

maxima [A] time = 0.32, size = 127, normalized size = 0.91

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log(f)-f}}-\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\right.}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(
d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*f^a*erf(sqrt
(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) -
f))/sqrt(-c*log(f) + f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2), x)
```

3.355 $\int f^{a+cx^2} \sinh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=183

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{2f-c\log(f)}-2d} \operatorname{erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)+2f}} \operatorname{erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi} f^a \operatorname{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-
2*d+e^2/(2*f-c*ln(f)))*f^a*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^
(1/2)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*erfi((e+x*(2*f
+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5512, 2204, 2287, 2234, 2205}

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{2f-c\log(f)}-2d} \operatorname{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)+2f}} \operatorname{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^
(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sqr
t[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log[f
]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*Sq
rt[2*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)dx=\int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$=-\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2d-2ex+a\log(f)-x^2(2f-c$$

$$=-\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+2x(-2f}{4(-2f+c}$$

$$=-\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2}}{}$$

Mathematica [A] time = 1.46, size = 258, normalized size = 1.41

$$\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{2f-c\log(f)}}\left(2\left(4f^2-c^2\log^2(f)\right)e^{\frac{e^2}{c\log(f)-2f}}\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)-\sqrt{c}\,\sqrt{\log(f)}\left((2f-c\log(f))\sqrt{c\log(f)+2}\right.\right.}{8\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(2*E^(e^2/(-2*f + c*Log[f]))*Erfi[Sq
rt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[(e
 + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c*
Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))*Erf
i[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt[2*f
+ c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^2 + c^
2*Log[f]^2))
```

fricas [B] time = 0.66, size = 422, normalized size = 2.31

$$\frac{2\left(\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\cosh\left(a\log(f)\right)+\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\sinh\left(a\log(f)\right)\right)\sqrt{-c\log(f)}\ \mathrm{erf}\left(\sqrt{-c\log(f)}\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(
f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) - (sqr
t(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c
*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f)
)*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f) - 2*f)
))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + 2*f)
/(c*log(f) - 2*f)) - (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((a*c*log(
f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^
2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)
*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f) + 2*f*x +
 e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*c*f^2*log(f)
)
```

giac [A] time = 0.18, size = 198, normalized size = 1.08

$$\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-c\log(f)}\,x\right)}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log(f)-2f}\left(x+\frac{e}{c\log(f)+2f}\right)\right)e^{\left(\frac{ac\log(f)^2+2cd\log(f)+2af\log(f)+4df-e^2}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}}-\frac{\sqrt{\pi}\,\mathrm{e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*d*
log(f) + 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f
) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e^((a
```

```
*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - 2*f))/
sqrt(-c*log(f) + 2*f)
```

maple [A] time = 0.18, size = 177, normalized size = 0.97

$$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2d\ln(f)c-4df+e^{2}}{-2f+c\ln(f)}} \operatorname{erf}\left(x\sqrt{2f-c\ln(f)}+\frac{e}{\sqrt{2f-c\ln(f)}}\right)}{8\sqrt{2f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2d\ln(f)c+4df-e^{2}}{2f+c\ln(f)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\, x+\frac{e}{\sqrt{-c\ln(f}}\right.}{8\sqrt{-c\ln(f)-2f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))
^(1/2)*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f
)-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))-1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x)
```

maxima [A] time = 0.42, size = 161, normalized size = 0.88

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)-2\,f}\,x-\frac{e}{\sqrt{-c\log(f)-2\,f}}\right) e^{\left(2\,d-\frac{e^{2}}{c\log(f)+2\,f}\right)}}{8\,\sqrt{-c\log(f)-2\,f}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)+2\,f}\,x+\frac{e}{\sqrt{-c\log(f)+2\,f}}\right) e}{8\,\sqrt{-c\log(f)+2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^(
2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(
-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^{2}+a}\sinh\left(f\,x^{2}+e\,x+d\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

3.356 $\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=300

$$\frac{3\sqrt{\pi}\,f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{2x(c\ldots}{2\sqrt{\ldots}}\right)}{16\sqrt{c\log(f)+f}}$$

```
[Out] 3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))*f
^a*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(
f))^(1/2)-3/16*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+2*x*(f+c*ln(f)))/
(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9/4*e^2/(3*f+c*l
n(f)))*f^a*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(
3*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{3\sqrt{\pi}\,f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\text{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\text{Erfi}\left(\frac{2\ldots}{2\ldots}\right)}{16\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) - (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)dx = \int\left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right)f^{a+}\right.$$

$$= -\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+ex+f\right.\right.$$

$$= -\left(\frac{1}{8}\int\exp\left(-3d-3ex+a\log(f)-x^2(3f-c\log(f))\right)dx\right)+\frac{1}{8}\int\exp\left(3d\right.$$

$$= \frac{1}{8}\left(3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx-\frac{1}{8}\left(e^{-3d+\frac{9e}{12f-4c}}\right.$$

$$= \frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3e+2x(3f-}{2\sqrt{3f-c}}\right.}{16\sqrt{3f-c\log(f)}}$$

Mathematica [A] time = 5.94, size = 480, normalized size = 1.60

$$\sqrt{\pi}f^a\exp\left(-\frac{1}{4}e^2\left(\frac{9}{c\log(f)+3f}+\frac{1}{c\log(f)+f}\right)\right)\left(3\sqrt{f-c\log(f)}\left(-c^3\log^3(f)-c^2f\log^2(f)+9cf^2\log(f)+9f^3\right)(\cos\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3*
f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*S
qrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*
(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f + c*
Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f])/(2*
Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log
[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(3*f + c
*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f +
 c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) - E^(e^2/(4*(f + c*Log[f]))
)*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f]
)*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*((f + c*Log[
f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f
]^4))
```

fricas [B] time = 0.61, size = 848, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(f)
- 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)
*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(
f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e)*sqrt
(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log
(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*
(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)
^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c*
d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2*c*x*log(f)
 - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f)
^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^
2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3
- c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 +
 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2
*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)
*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(1/4*(4*a*c*l
og(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(
pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(1/4*(4*a*
c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)))*sqr
```

```
t(-c*log(f) - 3*f)*erf(1/2*(2*c*x*log(f) + 6*f*x + 3*e)*sqrt(-c*log(f) - 3*
f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

giac [A] time = 0.18, size = 352, normalized size = 1.17

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{3e}{c\log(f)+3f}\right)\right)e^{\left(\frac{4ac\log(f)^2+12cd\log(f)+12af\log(f)+36df-9e^2}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f)))
*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) + 36*d*f - 9*e^2)/(
c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) +
4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4*a*c*l
og(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) + 4*d*f - e^2)/(c*log(f) - f))/sqrt(-
c*log(f) + f) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x - 3*e/(c*
log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f) + 36
*d*f - 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

maple [A] time = 0.28, size = 302, normalized size = 1.01

$$-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{3d\ln(f)c+9df-\frac{9e^2}{4}}{3f+c\ln(f)}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)-3f}\;x+\frac{3e}{2\sqrt{-c\ln(f)-3f}}\right)}{16\sqrt{-c\ln(f)-3f}}-\frac{\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{3(4d\ln(f)c-12df+3e^2)}{4(-3f+c\ln(f))}}\,\mathrm{erf}\left(x\sqrt{3f-c\ln(f)}+\right.}{16\sqrt{3f-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))/(-c*ln
(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))-1/16
*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(-3*f+c*ln(f)))/(3*f-c*ln
(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))+3/16*Pi^(1/
2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf
(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(1/4*(4*
d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2
)*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

maxima [A] time = 0.35, size = 263, normalized size = 0.88

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) - 3f}\,x - \frac{3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(3d - \frac{9e^2}{4\left(c\log(f)+3f\right)}\right)}}{16\sqrt{-c\log(f) - 3f}} - \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f) - f}\,x - \frac{e}{2\sqrt{-c\log(f)-f}}\right.}{16\sqrt{-c\log(f) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f)
)*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*
f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^2/(
c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) +
 f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqrt(-c*
log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/sqrt(-c
*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

3.357 $\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{\pi}\,f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(f
))^2/c/ln(f))*f^a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^
(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5512, 2287, 2234, 2204}

$$\frac{\sqrt{\pi}\,f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
(e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sinh(d+ex)\,dx &= \int\left(-\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+bx+cx^2}\,dx\\ &= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)dx\right)+\frac{1}{2}\int \exp\left(d+\right.\\ &= -\left(\frac{1}{2}\left(e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\right)+\frac{1}{2}\left(e^{d-\frac{(e+b\log}{4c\log}}\right.\\ &= \frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2c}{2\sqrt{c}\sqrt{\log}}\right.}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.34, size = 135, normalized size = 0.88

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(2b\log(f)+e)}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{\log(f)(b+2cx)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)-e^{\frac{be}{c}}(\cosh(d)-\sinh(d))\mathrm{erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d])) + Erfi[(e + (b + 2*c*x)*Log[f
])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*
b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.68, size = 263, normalized size = 1.72

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{\left(b^2-4ac\right)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{\left(b^2-4ac\right)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)\right)\mathrm{erf}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(p
i)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log
(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log
(f))
```

giac [A] time = 0.15, size = 169, normalized size = 1.10

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\,l}{c}\right.\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e^
(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(
c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (
b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*l
og(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.14, size = 156, normalized size = 1.02

$$-\frac{\sqrt{\pi}\, f^{a}\mathrm{e}^{-\frac{\ln(f)^2b^2+2\ln(f)be-4d\ln(f)c+e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}+\frac{\sqrt{\pi}\, f^{a}\mathrm{e}^{-\frac{\ln(f)^2b^2-2\ln(f)be+4d\ln(f)c+e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\right.}{4\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1/2
))+1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c+e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1
/2))
```

maxima [A] time = 0.34, size = 129, normalized size = 0.84

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f)+e}{2 \sqrt{-c \log(f)}}\right) e^{\left(d - \frac{(b \log(f)+e)^{2}}{4 c \log(f)}\right)}}{4 \sqrt{-c \log(f)}} - \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f)-e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-d - \frac{(b \log(f)-e)^{2}}{4 c \log(f)}\right)}}{4 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f))
)*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(
b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c x^2+b x+a} \sinh(d+e x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2} \sinh(d+ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x), x)
```

3.358 $\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$

Optimal. Leaf size=219

$$-\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*e+b*ln(f)+2*c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-1/4*(2
*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)
*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5512, 2234, 2204, 2287}

$$-\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8
*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqr
t[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2ex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$=\frac{1}{4}\int\exp\left(-2d+a\log(f)+cx^2\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int\exp\left(2d+\right.$$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+b\log(f)}{4c\log}\right.$$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] time = 0.63, size = 183, normalized size = 0.84

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{e(b\log(f)+e)}{c\log(f)}}\left(e^{\frac{2be}{c}}(\cosh(2d)-\sinh(2d))\text{erfi}\left(\frac{\log(f)(b+2cx)-2e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)+\cosh(2d))\text{erfi}\left(\frac{\log(f)(b+2cx)+2}{2\sqrt{c}\sqrt{\log(f)}}\right.\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b
+ 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*x
)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e + (
```

```
b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*S
qrt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.59, size = 343, normalized size = 1.57

$$2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)-\sqrt{-c\log(f)}\left(\sqrt{\pi}\ c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(p
i)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c
) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 4
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*lo
g(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1
/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log(f))) +
sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f)
)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))/(c*log(f)
)))/(c*log(f))
```

giac [A] time = 0.16, size = 225, normalized size = 1.03

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(\right.}{}\right.}}{8\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*
a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
 + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8
*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-1/4*(
b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/(c*log
(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.22, size = 211, normalized size = 0.96

$$-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2-4\ln(f)be+8d\ln(f)c+4e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-2e}{2\sqrt{-c\ln(f)}}\right)}{8\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2+4\ln(f)be-8d\ln(f)c+4e^2}{4\ln(f)c}}\operatorname{erf}\left(-\sqrt{-c\ln(f\right.}{8\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c+4*e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f)*b*e-8*d*ln(f)*c+4*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*l
n(f))^(1/2))+1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.34, size = 185, normalized size = 0.84

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f)+2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(2 d - \frac{(b \log(f)+2 e)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x - \frac{b \log(f)-2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-2 d - \frac{(b \log(f)-2 e)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(f
)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(
pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^(-2
*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1
/4*b^2/c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sinh\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^{2}} \sinh^{2}(d+ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**2, x)
```

3.359 $\int f^{a+bx+cx^2} \sinh^3(d+ex)\,dx$

Optimal. Leaf size=315

$$-\frac{3\sqrt{\pi}\, f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}\, f^a e^{d-\frac{(b\log(f}{4c\log}}}{16}$$

```
[Out] 3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-1/4*(3*e-
b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(
1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f))*f^
a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln
(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(3*e+b*ln(
f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5512, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}\, f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}\, f^a e^{d-\frac{(b\log(}{4c\log}}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = \int\left(-\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}-\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+bx+}\right.$$

$$= -\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+bx+cx^2}\,d$$

$$= -\left(\frac{1}{8}\int \exp\left(-3d+a\log(f)+cx^2\log(f)-x(3e-b\log(f))\right)dx\right)+\frac{1}{8}\int \exp\left(3\right.$$

$$= \frac{1}{8}\left(3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx-\frac{1}{8}\left(e^{-3d-\frac{(3e-b\log}{4c\log}}\right.$$

$$= -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-b\log}{2}\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] time = 1.02, size = 263, normalized size = 0.83

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{-\frac{3e(2b\log(f)+3e)}{4c\log(f)}}\left((\sinh(d)+\cosh(d))\left(3(\cosh(2d)-\sinh(2d))e^{\frac{2e(b\log(f)+e)}{c\log(f)}}\mathrm{erfi}\left(\frac{\log(f)(b+2cx)-e}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sinh(2d)\right.\right.}{16\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((e*(2*e + b*Log[f])
)/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E
^((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) - E^((3*b*e)/c)*Erfi[(-3
*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d]))
)/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

fricas [B] time = 0.65, size = 527, normalized size = 1.67

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2+9\,e^2-6\,(2\,cd-be)\log(f)}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2+9\,e^2-6\,(2\,cd-be)\log(f)}{4\,c\log(f)}\right)\right)\mathrm{erf}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
- 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*l
og(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)
*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) - 3*sqrt(-c*log(f))*(sqrt(pi)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f)))
 + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f
))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*log(f))
) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt
(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c
*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/
(c*log(f))
```

giac [A] time = 0.18, size = 343, normalized size = 1.09

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x+\frac{b\log(f)-3\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+12\,cd\log(f)-6\,be\log(f)+9\,e^2}{4\,c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f))))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) + 9*
```

```
e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(
2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^
2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f))
)/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f)
 + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f)
 + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

maple [A] time = 0.24, size = 316, normalized size = 1.00

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6\ln(f) b e-12 d \ln(f) c+9 e^{2}}{4\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{3 e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{16\sqrt{-c\ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6\ln(f) b e+12 d \ln(f) c+9 e^{2}}{4\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c\ln\right.}{16\sqrt{-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*ln(f)*b*e-12*d*ln(f)*c+9*e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)
)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*ln(f)*b*e+12*d*ln(f)*c+9
*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln
(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e
)/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d
*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln
(f))/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.37, size = 263, normalized size = 0.83

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{b\log(f)+3 e}{2\sqrt{-c\log(f)}}\right) e^{\left(3 d-\frac{(b\log(f)+3 e)^{2}}{4 c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right) e^{\left(d-\frac{(b\log(f)+e)^{2}}{4 c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log(
f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^(d
- 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(
sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b*log(
f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f
```

```
))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f) - 3*e)
^2/(c*log(f)))/sqrt(-c*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sinh\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**3,x)
[Out] Timed out
```

3.360 $\int f^{a+bx+cx^2} \sinh\left(d+fx^2\right) dx$

Optimal. Leaf size=154

$$\frac{\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

```
[Out] 1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(f)
))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/
(f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(
1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d -
 (b^2*Log[f]^2)/(4*(f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int\exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)dx\right)+\frac{1}{2}\int\exp\left(d+\right.\\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right)+\frac{1}{2}\left(e^{d-\cdots}\right.\\
&= \frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x\cdots}{2\sqrt{f+\cdots}}\right)}{4\sqrt{f+c\log(f)}}
\end{aligned}
$$

Mathematica [A] time = 0.81, size = 179, normalized size = 1.16

$$
\frac{\sqrt{\pi}\,f^a e^{-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\left(\sqrt{f-c\log(f)}\,(\sinh(d)+\cosh(d))\mathrm{erfi}\left(\frac{\log(f)(b+2cx)+2fx}{2\sqrt{c\log(f)+f}}\right)-\sqrt{c\log(f)+f}\,(\cosh(d)-\sinh(d))e\cdots\right.}{4\sqrt{f-c\log(f)}\sqrt{c\log(f)+f}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] -
Sinh[d])) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[
f - c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
)*Sqrt[f - c*Log[f]]*Sqrt[f + c*Log[f]])
```

fricas [B] time = 0.49, size = 325, normalized size = 2.11

$$\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,df+4\left(cd+af\right)\log(f)}{4\left(c\log(f)-f\right)}\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,df+4\left(cd\right.}{4\left(c\log(f)-f\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4
*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*((
b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt
(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(
c*log(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*
sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f)
 + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*lo
g(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

giac [A] time = 0.15, size = 181, normalized size = 1.18

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\left(c\log(f)+f\right)}\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d
*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log
(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
 + f)
```

maple [A] time = 0.18, size = 160, normalized size = 1.04

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-4 d \ln(f) c-4 d f}{4\left(f+c \ln(f)\right)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-f}\, x+\frac{\ln(f) b}{2 \sqrt{-c \ln(f)-f}}\right)}{4 \sqrt{-c \ln(f)-f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+4 d \ln(f) c-4 d f}{4\left(-f+c \ln(f)\right)}} \operatorname{erf}\left(-x \sqrt{f-c \ln(f)}\right.}{4 \sqrt{f-c \ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))/(-c
*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))+1
/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)/(-f+c*ln(f)))/(f-c
*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))
```

maxima [A] time = 0.33, size = 139, normalized size = 0.90

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4\left(c \log(f)+f\right)}+d\right)}}{4 \sqrt{-c \log(f)-f}}-\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-\frac{b}{4(}\right.}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) - 1/4*sqrt
(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e^(-
1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c x^{2}+b x+a} \sinh\left(f x^{2}+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^{2}} \sinh\left(d+f x^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d),x)
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2), x)
```

3.361 $\int f^{a+bx+cx^2} \sinh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=225

$$-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log\ldots}}{\ldots}\right)}{4\sqrt{c}\sqrt{\log(\ldots}}$$

```
[Out] -1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(
f)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1/2)+1/8
*exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(2*f+c*ln(f
)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5512, 2234, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{l\ldots}}{\ldots}\right)}{4\sqrt{c}\sqrt{\log\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f^
a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])])
/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*f^
a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]])]
)/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)dx = \int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$=\frac{1}{4}\int\exp\left(-2d+a\log(f)+bx\log(f)-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int\exp\left(2a\right.$$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x}{4(-2f}\right.$$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2f-c\log(f)}{2\sqrt{2f-c\log(f)}}\right.}{8\sqrt{2f-c\log(f)}}$$

Mathematica [A] time = 2.27, size = 257, normalized size = 1.14

$$\frac{1}{8}\sqrt{\pi}f^a\left(-\frac{e^{-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\left(\sqrt{2f-c\log(f)}\,(c\log(f)+2f)(\cosh(2d)-\sinh(2d))e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\text{erf}\left(\frac{4fx-\log(f)(b+2cx)}{2\sqrt{2f-c\log(f)}}\right)+\right.}{c^2\log^2(f)-4f^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((-2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*E
rf[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Log[
f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)*Lo
g[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh
[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Lo
g[f]^2))))/8
```

fricas [B] time = 0.56, size = 466, normalized size = 2.07

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-16\,df+8\left(cd+af\right)\log(f)}{4\left(c\log(f)-2\,f\right)}\right)+\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log
(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*d +
 a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2
*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*
log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*
d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))/(c*log(
f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f))*sqrt
(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*co
sh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4
*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

giac [A] time = 0.17, size = 239, normalized size = 1.06

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-2\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)-8\,af\log(f)-16\,df}{4\left(c\log(f)+2\,f\right)}\right)}}{8\sqrt{-c\log(f)-2\,f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2*
f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f) -
 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sq
rt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) - 2*f)
)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c)
)*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

maple [A] time = 0.22, size = 217, normalized size = 0.96

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+8 d \ln(f) c-16 d f}{4\left(-2 f+c \ln(f)\right)}} \operatorname{erf}\left(-x \sqrt{2 f-c \ln(f)}+\frac{\ln(f) b}{2 \sqrt{2 f-c \ln(f)}}\right)}{8 \sqrt{2 f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-8 d \ln(f) c-16 d f}{4\left(2 f+c \ln(f)\right)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-\right.}{8 \sqrt{-c \ln(f)-2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*d*ln(f)*c-16*d*f)/(-2*f+c*ln(f)))
/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*d*ln(f)*c-16*d*f)/(2*f+c*ln(
f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)
-2*f)^(1/2))+1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

maxima [A] time = 0.33, size = 199, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-2 f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)-2 f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4 \left(c \log(f)+2 f\right)}+2 d\right)}}{8 \sqrt{-c \log(f)-2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+2 f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)+}}\right.}{8 \sqrt{-c \log(f)+2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
- 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f)
+ 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) + 2*f
) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/
(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2)**2, x)
```

3.362 $\int f^{a+bx+cx^2} \sinh^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=323

$$-\frac{3\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\operatorname{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\, f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)}}}{16}$$

```
[Out] -3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(
f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+b^2*ln(f)^
2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(
1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)-3/16*exp(d-1/4*b^2*ln(f)^2/(f+c*ln(f)))*
f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(
f))^(1/2)+1/16*exp(3*d-1/4*b^2*ln(f)^2/(3*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)
+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5512, 2287, 2234, 2205, 2204}

$$-\frac{3\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\operatorname{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\, f^a e^{d-\frac{b^2\log^2(}{4(c\log(f)}}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) -
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)dx=\int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3fx^2}f$$

$$=-\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx-}$$

$$=-\left(\frac{1}{8}\int\exp\left(-3d+a\log(f)+bx\log(f)-x^2(3f-c\log(f))\right)dx\right)+\frac{1}{8}\int\exp$$

$$=\frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx-\frac{1}{8}\left(e^{-3d+\frac{}{12}}\right.$$

$$=-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log}{}\right.}{16\sqrt{3f-c\log}}$$

Mathematica [B] time = 6.52, size = 2511, normalized size = 7.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(27*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^3*Cosh[d]*Erf[(2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]] +
 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f^2*Cosh[d]*Erf[(2*f*x - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]] - 3*c^
2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*f*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]] - 3*c^3*E^
((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Cosh[d]*Erf[(2*f*x - b*Log[f] - 2*c*x*L
og[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]] - 3*E^((b^2*Log[
f]^2)/(4*(3*f - c*Log[f])))*f^3*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log
[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]] - c*E^((b^2*Log[f]^2)/(
4*(3*f - c*Log[f])))*f^2*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]] + 3*c^2*E^((b^2*Log[f]^
2)/(4*(3*f - c*Log[f])))*f*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]] + c^3*E^((b^2*Log[f
]^2)/(4*(3*f - c*Log[f])))*Cosh[3*d]*Erf[(6*f*x - b*Log[f] - 2*c*x*Log[f])/
(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]] - (27*f^3*Cosh[d]*E
rfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Lo
g[f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (27*c*f^2*Cosh[d]*Erfi[(2*f*
x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Cosh[d]*Erfi[(2*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f
]])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Cosh[d]*Erfi[(2*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]])
/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*f^3*Cosh[3*d]*Erfi[(6*f*x + b*L
og[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((b
^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erfi[(6*f*x + b*
Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[
f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(6*f*x +
 b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*L
og[f]])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - 27*E^((b^2*Log[f]^2)/(4*(
f - c*Log[f])))*f^3*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Sqrt[f - c*Log[f]]*Sinh[d] - 27*c*E^((b^2*Log[f]^2)/(4*(f - c*Log[f]
)))*f^2*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f
]*Sqrt[f - c*Log[f]]*Sinh[d] + 3*c^2*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*
f*Erf[(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sq
rt[f - c*Log[f]]*Sinh[d] + 3*c^3*E^((b^2*Log[f]^2)/(4*(f - c*Log[f])))*Erf[
(2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f -
 c*Log[f]]*Sinh[d] - (27*f^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Lo
g[f]))) + (27*c*f^2*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Lo
g[f]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[
```

```
f])) + (3*c^2*f*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f
]))) - (3*c^3*Erfi[(2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])
]*Log[f]^3*Sqrt[f + c*Log[f]]*Sinh[d])/E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
) + 3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^3*Erf[(6*f*x - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] + c*E
^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f^2*Erf[(6*f*x - b*Log[f] - 2*c*x*Lo
g[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d] - 3*c
^2*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*f*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]]*Sinh[3*d] -
 c^3*E^((b^2*Log[f]^2)/(4*(3*f - c*Log[f])))*Erf[(6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d] +
 (3*f^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sq
rt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]
*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) -
(3*c^2*f*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*L
og[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Log[f
]))) + (c^3*Erfi[(6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])
]*Log[f]^3*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((b^2*Log[f]^2)/(4*(3*f + c*Lo
g[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3*f + c*Log[f
]))
```

fricas [B] time = 1.72, size = 852, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f)
 - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3
)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*lo
g(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f))*s
qrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*
f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*l
```

```
og(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log(f) - f)/(c*log(
f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^
3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/(c*l
og(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) +
3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/
(c*log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f
))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(
f)^2 + 9*f^4)
```

giac [A] time = 0.19, size = 369, normalized size = 1.14

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) + 3
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f
) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/
2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) + f))/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + b*lo
g(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f)
 + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/16*sqrt(pi
)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f)))*e^(-1/4
*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 36*d*f)/(
c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

maple [A] time = 0.29, size = 326, normalized size = 1.01

$$-\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2-12d\ln(f)c-36df}{4(3f+c\ln(f))}}\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-3f}}\right)}{16\sqrt{-c\ln(f)-3f}}+\frac{\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{\ln(f)^2b^2+12d\ln(f)c-36df}{4(-3f+c\ln(f))}}\operatorname{erf}\left(-x\sqrt{3f-}\right.}{16\sqrt{3f-c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*d*ln(f)*c-36*d*f)/(3*f+c*ln(f))
)/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*
f)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*d*ln(f)*c-36*d*f)/(-3*
f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f
```

```
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*d*ln(f)*c-4*d*f)
/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*
ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*d*ln(f)*c-4*d*f)/(f
+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-f)^(1/2))
```

maxima [A] time = 0.34, size = 287, normalized size = 0.89

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - 3\, f}\, x - \frac{b \log(f)}{2\, \sqrt{-c \log(f) - 3\, f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4\, \left(c \log(f) + 3\, f\right)} + 3\, d\right)}}{16\, \sqrt{-c \log(f) - 3\, f}} - \frac{3\, \sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - f}\, x - \frac{b \log(f)}{2\, \sqrt{-c \log(f) - f}}\right.}{16\, \sqrt{-c \log(f) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*f)
 - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 3/16
*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))
*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(p
i)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3*f))*e^
(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.363 $\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=161

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

```
[Out] -1/4*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*(e+b*ln
(f))^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1
/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5512, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[
f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) +
(E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f
] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx &= \int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx \\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx+cx^2}\,dx \\
&= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx\right)+\frac{1}{2} \\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right) \\
&= -\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\right.}{4\sqrt{f+c}}
\end{aligned}$$

Mathematica [A] time = 1.56, size = 252, normalized size = 1.57

$$\frac{\sqrt{\pi}\,e^{-\frac{b^2\log^2(f)+e^2}{4(c\log(f)+f)}}f^{a+\frac{bef}{c^2\log^2(f)-f^2}}\left((f-c\log(f))\sqrt{c\log(f)+f}\,(\sinh(d)+\cosh(d))f^{\frac{be}{2f-2c\log(f)}}\,\mathrm{erfi}\left(\frac{\log(f)(b+2cx)+e+2fx}{2\sqrt{c\log(f)+f}}\right)\right.}{4\left(f^2-c^2\log\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(-(E^((f*(e^2 + b^2*Log[f]^
2))/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[
f])*(Cosh[d] - Sinh[d])) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x + (
b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]
]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2
 - c^2*Log[f]^2))
```

fricas [B] time = 0.98, size = 363, normalized size = 2.25

$$\frac{\left(\sqrt{\pi}\left(c\log(f)+f\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2+e^2-4\,df+2\left(2\,cd-be+2\,af\right)\log(f)}{4\left(c\log(f)-f\right)}\right)+\sqrt{\pi}\left(c\log(f)+f\right)\sinh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2+\cdots}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d
*f + 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) +
 f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*
f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x +
b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) - (sqrt(pi)*(c*log(f) -
f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f
)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2 - 4*a*c
)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + f)))
*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqrt(-c*log(f
) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

giac [A] time = 0.22, size = 209, normalized size = 1.30

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)-f}\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)+2\,be\log(f)-4\,df+e^2}{4\left(c\log(f)+f\right)}\right)}}{4\sqrt{-c\log(f)-f}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
+ f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f)
 + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sq
rt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f)))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 2*b*
e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

maple [A] time = 0.17, size = 186, normalized size = 1.16

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^2 b^2+2\ln(f) b e-4 d\ln(f) c-4 d f+e^2}{4\left(f+c\ln(f)\right)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\; x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)-f}}\right)}{4\sqrt{-c\ln(f)-f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^2 b^2-2\ln(f) b e+4 d\ln(f) c-4 d f+e^2}{4\left(-f+c\ln(f)\right)}} \operatorname{erf}}{4\sqrt{f-c\ln}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*f+e^2)/
(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(
-c*ln(f)-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*l
n(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+
1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))
```

maxima [A] time = 0.36, size = 151, normalized size = 0.94

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)-f}\; x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right) e^{\left(-\frac{\left(b\log(f)+e\right)^2}{4\left(c\log(f)+f\right)}+d\right)}}{4\sqrt{-c\log(f)-f}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)+f}\; x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right) e^{\left(-\frac{(b}{4(c}\right.}}{4\sqrt{-c\log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*log
(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f)
- 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-c*l
og(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2), x)
```

3.364 $\int f^{a+bx+cx^2} \sinh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=239

$$-\frac{\sqrt{\pi}\, f^{a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a \exp\left(2d-\frac{(b}{4}\right.}{}$$

```
[Out] -1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(2
*e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(
1/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(2*e+b*ln(f)
+2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5512, 2234, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi}\, f^{a-\frac{b^2}{4c}} \operatorname{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\, f^a \exp\left(2d-\frac{(}{4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*
Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f + 4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sq
rt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)dx &= \int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+ \\
&= \frac{1}{4}\int \exp\left(-2d+a\log(f)-x(2e-b\log(f))-x^2(2f-c\log(f))\right)dx+\frac{1}{4} \\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\right)\int \\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2e-}{}\right.}{8\sqrt{2f-c\log(f)}}
\end{aligned}$$

Mathematica [A] time = 6.23, size = 339, normalized size = 1.42

$$-\frac{\sqrt{\pi}\,e^{-\frac{b^2\log^2(f)+4e^2}{4c\log(f)+8f}}f^{a+\frac{4bef}{c^2\log^2(f)-4f^2}}\left(\sqrt{2f-c\log(f)}\,(c\log(f)+2f)(\cosh(2d)-\sinh(2d))f^{\frac{be}{c\log(f)+2f}}\exp\left(\frac{f\left(b^2\log^2(f)\right.}{4f^2-c^2\log\right.}\right.}{8\left(c\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqr
t[Pi]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f
+ c*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f
]])]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((b*
e)/(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2*f
+ c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d])
))/(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[f]^2)
)
```

fricas [B] time = 0.50, size = 516, normalized size = 2.16

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(f)\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2+4\,e^2-16\,df+4\left(2\,cd-be+2\,af\right)\log(f)}{4\left(c\log(f)-2\,f\right)}\right)+\sqrt{\pi}\left(c^2\log(f)^2+2\,cf\log(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)) +
 sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-
c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*log(f)
+ 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/
4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f)
)/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f) +
2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 -
4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt
(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

giac [A] time = 0.20, size = 273, normalized size = 1.14

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-2\,f}\left(2\,x+\frac{b\log(f)+2\,e}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)-8\,af\log(f)+4\,be\log(f)-16\,df+4\,e^2}{4\left(c\log(f)+2\,f\right)}\right)}}{8\sqrt{-c\log(f)-2\,f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*log
(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*
log(f) + 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2*e)/
(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) +
8*a*f*log(f) - 4*b*e*log(f) - 16*d*f + 4*e^2)/(c*log(f) - 2*f))/sqrt(-c*log
(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^
2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

maple [A] time = 0.22, size = 249, normalized size = 1.04

$$-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{\ln(f)^{2} b^{2}-4\ln(f) b e+8 d\ln(f) c-16 d f+4 e^{2}}{4\left(-2 f+c\ln(f)\right)}} \operatorname{erf}\left(-x\sqrt{2 f-c\ln(f)}+\frac{b\ln(f)-2 e}{2\sqrt{2 f-c\ln(f)}}\right)}{8\sqrt{2 f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{\ln(f)^{2} b^{2}+4\ln(f) b e-8 d\ln(f) c-16 d f+4 e^{2}}{4\left(2 f+c\ln(f)\right)}}}{8\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*ln(f)*b*e+8*d*ln(f)*c-16*d*f+4*e^
2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln
(f)-2*e)/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*ln(f
)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-
c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f)-2*f)^(1/2))+1/4*Pi^(1/2)*f
^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*
ln(f))^(1/2))
```

maxima [A] time = 0.36, size = 215, normalized size = 0.90

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)-2 f}\, x-\frac{b\log(f)+2 e}{2\sqrt{-c\log(f)-2 f}}\right) e^{\left(-\frac{\left(b\log(f)+2 e\right)^{2}}{4\left(c\log(f)+2 f\right)}+2 d\right)}}{8\sqrt{-c\log(f)-2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)+2 f}\, x-\frac{b\log(f)-2}{2\sqrt{-c\log(f)+}}\right)}{8\sqrt{-c\log(f)+2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-c
*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*
log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*log(f
) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) - 2*f)
 - 2*d)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/
2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\mathrm{sinh}\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**2,x)
[Out] Timed out
```

3.365 $\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=344

$$-\frac{\sqrt{\pi} f^a \exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\operatorname{erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+$$

```
[Out] 3/16*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+(3*e-b*
ln(f))^2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*ln(f)))/(3*f
-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/
(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi
^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/(3*f+c*ln(f)))*f^
a*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3
*f+c*ln(f))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5512, 2287, 2234, 2205, 2204}

$$-\frac{\sqrt{\pi} f^a \exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\operatorname{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
- (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) - (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)dx = \int\left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right.\right.\right.$$

$$= -\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+e\right.\right.$$

$$= -\left(\frac{1}{8}\int\exp\left(-3d+a\log(f)-x(3e-b\log(f))-x^2(3f-c\log(f))\right)dx\right)$$

$$= -\left(\frac{1}{8}\left(\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-3e+b\log(f)+2x(-3}{4(-3f+c\log}\right.\right.$$

$$= \frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right.}{16}$$

Mathematica [B] time = 6.65, size = 2991, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log
[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e^2 +
 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*Erf[(e
 + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f
 - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) -
 (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)
/(4*(f - c*Log[f]))) - (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x
*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e
*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c*f^2*Cosh[3*d]*Erf[(3*e +
 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f
 - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f])
)) + (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt
[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f]
 - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c^3*Cosh[3*d]*Erf[(3*e + 6*f*x -
b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Lo
g[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (2
7*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[
f]])]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*
Log[f]))) + (27*c*f^2*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2
*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b
^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Cosh[d]*Erfi[(e + 2*f*x + b*Log
[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]])/E^
((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Cosh[d]*E
rfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*
Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]
))) + (3*f^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log
[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cos
h[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]]
)]*Log[f]^2*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/
(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*Log[f]])/E^((9*e^2
 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (27*f^3*Erf[(e + 2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*S
inh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*c
*f^2*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[
```

```
f]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(
f - c*Log[f]))) + (3*c^2*f*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqr
t[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log
[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^3*Erf[(e + 2*f*x - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]]*Sinh[d]
)/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*f^3*Erf
i[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*
Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f])))
 + (27*c*f^2*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]
^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f
])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2
*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Erfi[(e + 2*f*x +
b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]
]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*
f^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*S
qrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4
*(3*f - c*Log[f]))) + (c*f^2*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 +
 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (3*c^2*f*Erf[(3*e + 6
*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f
 - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f -
 c*Log[f]))) - (c^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f
 - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*
Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*f^3*Erfi[(3*e + 6*f*x + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]]*Sinh
[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*
Log[f]*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f
]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*
Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/
E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Erfi[
(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*
Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4
*(3*f + c*Log[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3
*f + c*Log[f]))
```

fricas [B] time = 0.70, size = 940, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
```

```
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*f)*
log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f
^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(
2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-
1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*
f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*f)*l
og(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*
log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d
- b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x
 - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f)
 + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*
sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*l
og(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*lo
g(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3
*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(
pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f) +
3*e)*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*lo
g(f)^2 + 9*f^4)
```

giac [A] time = 0.20, size = 431, normalized size = 1.25

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)+3e}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)+6be\log(f)-36df+9e^2}{4\left(c\log(f)+3f\right)}\right)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}\,\mathrm{er}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*lo
g(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a
*f*log(f) + 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) + 3*f))/sqrt(-c*log(f)
 - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/
(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*
a*f*log(f) + 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) + f))/sqrt(-c*log(f) - f
) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log
(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*lo
g(f) - 2*b*e*log(f) - 4*d*f + e^2)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/
16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e)/(c*log(f
```

```
) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*
log(f) - 6*b*e*log(f) - 36*d*f + 9*e^2)/(c*log(f) - 3*f))/sqrt(-c*log(f) +
3*f)
```

maple [A] time = 0.30, size = 384, normalized size = 1.12

$$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}+6\ln(f) b e-12 d\ln(f) c-36 d f+9 e^{2}}{4\left(3 f+c\ln(f)\right)}} \operatorname{erf}\left(-\sqrt{-c\ln(f)-3 f}\; x+\frac{3 e+b\ln(f)}{2\sqrt{-c\ln(f)-3 f}}\right)}{16\sqrt{-c\ln(f)-3 f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{\ln(f)^{2} b^{2}-6\ln(f) b e+12 d\ln(f) c-36 d f+}{4\left(-3 f+c\ln(f)\right)}}}{16\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*ln(f)*b*e-12*d*ln(f)*c-36*d*f+9*
e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3
*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6
*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*e^2)/(-3*f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*e
rf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*e)/(3*f-c*ln(f))^(1/2))-3/16*Pi^(1
/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f
)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^
(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*
f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*l
n(f))/(-c*ln(f)-f)^(1/2))
```

maxima [A] time = 0.41, size = 315, normalized size = 0.92

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - 3 f}\, x - \frac{b \log(f) + 3 e}{2 \sqrt{-c \log(f) - 3 f}}\right) e^{\left(-\frac{\left(b \log(f) + 3 e\right)^{2}}{4 \left(c \log(f) + 3 f\right)} + 3 d\right)}}{16 \sqrt{-c \log(f) - 3 f}} - \frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f) - f}\, x - \frac{b \log(f) + e}{2 \sqrt{-c \log(f) - f}}\right.}{16 \sqrt{-c \log(f) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(-
c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c
*log(f) - 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f
) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sq
rt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*lo
g(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)
/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*
(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/(c*log(f
) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\mathrm{sinh}\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**3,x)
[Out] Timed out
```

3.366 $\int (x+\sinh(x))^2\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}-\frac{x}{2}-2\sinh(x)+2x\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] -1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2637, 2635, 8}

$$\frac{x^3}{3}-\frac{x}{2}-2\sinh(x)+2x\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sinh[x])^2,x]
```

```
[Out] -x/2 + x^3/3 + 2*x*Cosh[x] - 2*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int (x+\sinh(x))^2\,dx &= \int \left(x^2+2x\sinh(x)+\sinh^2(x)\right)\,dx \\
&= \frac{x^3}{3}+2\int x\sinh(x)\,dx+\int \sinh^2(x)\,dx \\
&= \frac{x^3}{3}+2x\cosh(x)+\frac{1}{2}\cosh(x)\sinh(x)-\frac{\int 1\,dx}{2}-2\int \cosh(x)\,dx \\
&= -\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 1.00

$$\frac{1}{6}x\left(2x^2-3\right)-2\sinh(x)+\frac{1}{4}\sinh(2x)+2x\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sinh[x])^2,x]
[Out] (x*(-3 + 2*x^2))/6 + 2*x*Cosh[x] - 2*Sinh[x] + Sinh[2*x]/4
```

fricas [A] time = 0.55, size = 22, normalized size = 0.73

$$\frac{1}{3}\,x^3+2\,x\cosh(x)+\frac{1}{2}\,(\cosh(x)-4)\sinh(x)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 2*x*cosh(x) + 1/2*(cosh(x) - 4)*sinh(x) - 1/2*x
```

giac [A] time = 0.13, size = 35, normalized size = 1.17

$$\frac{1}{3}\,x^3+(x+1)e^{(-x)}+(x-1)e^x-\frac{1}{2}\,x+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

maple [A] time = 0.02, size = 25, normalized size = 0.83

$$-\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sinh(x))^2,x)
[Out] -1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

maxima [A] time = 0.32, size = 35, normalized size = 1.17

$$\frac{1}{3}x^3+(x+1)e^{(-x)}+(x-1)e^x-\frac{1}{2}x+\frac{1}{8}e^{(2x)}-\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

mupad [B] time = 0.60, size = 24, normalized size = 0.80

$$\frac{\cosh(x)\sinh(x)}{2}-2\sinh(x)-\frac{x}{2}+2x\cosh(x)+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sinh(x))^2,x)
[Out] (cosh(x)*sinh(x))/2 - 2*sinh(x) - x/2 + 2*x*cosh(x) + x^3/3
```

sympy [A] time = 0.18, size = 41, normalized size = 1.37

$$\frac{x^3}{3}+\frac{x\sinh^2(x)}{2}-\frac{x\cosh^2(x)}{2}+2x\cosh(x)+\frac{\sinh(x)\cosh(x)}{2}-2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))**2,x)
[Out] x**3/3 + x*sinh(x)**2/2 - x*cosh(x)**2/2 + 2*x*cosh(x) + sinh(x)*cosh(x)/2
- 2*sinh(x)
```

3.367 $\int (x+\sinh(x))^3\,dx$

Optimal. Leaf size=56

$$\frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh(x)-\frac{3\sinh^2(x)}{4}-6x\sinh(x)+\frac{\cosh^3(x)}{3}+5\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

```
[Out] -3/4*x^2+1/4*x^4+5*cosh(x)+3*x^2*cosh(x)+1/3*cosh(x)^3-6*x*sinh(x)+3/2*x*co
sh(x)*sinh(x)-3/4*sinh(x)^2
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6742, 3296, 2638, 3310, 30, 2633}

$$\frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh(x)-\frac{3\sinh^2(x)}{4}-6x\sinh(x)+\frac{\cosh^3(x)}{3}+5\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sinh[x])^3,x]
```

```
[Out] (-3*x^2)/4 + x^4/4 + 5*Cosh[x] + 3*x^2*Cosh[x] + Cosh[x]^3/3 - 6*x*Sinh[x]
+ (3*x*Cosh[x]*Sinh[x])/2 - (3*Sinh[x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (x+\sinh(x))^3\,dx &= \int \left(x^3+3x^2\sinh(x)+3x\sinh^2(x)+\sinh^3(x)\right)\,dx\\
&= \frac{x^4}{4}+3\int x^2\sinh(x)\,dx+3\int x\sinh^2(x)\,dx+\int \sinh^3(x)\,dx\\
&= \frac{x^4}{4}+3x^2\cosh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}-\frac{3\int x\,dx}{2}-6\int x\cosh(x)\,dx-\text{Sub}\\
&= -\frac{3x^2}{4}+\frac{x^4}{4}-\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\,\text{si}}{}\\
&= -\frac{3x^2}{4}+\frac{x^4}{4}+5\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3}{}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 48, normalized size = 0.86

$$\frac{1}{24}\left(6x\left(x^3-3x-24\sinh(x)+3\sinh(2x)\right)+18\left(4x^2+7\right)\cosh(x)-9\cosh(2x)+2\cosh(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sinh[x])^3,x]
```

```
[Out] (18*(7 + 4*x^2)*Cosh[x] - 9*Cosh[2*x] + 2*Cosh[3*x] + 6*x*(-3*x + x^3 - 24*
Sinh[x] + 3*Sinh[2*x]))/24
```

fricas [A] time = 0.46, size = 58, normalized size = 1.04

$$\frac{1}{4}x^4+\frac{1}{12}\cosh(x)^3+\frac{1}{8}(2\cosh(x)-3)\sinh(x)^2-\frac{3}{4}x^2+\frac{3}{4}\left(4x^2+7\right)\cosh(x)-\frac{3}{8}\cosh(x)^2+\frac{3}{2}(x\cosh(x)-4x)\sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/12*cosh(x)^3 + 1/8*(2*cosh(x) - 3)*sinh(x)^2 - 3/4*x^2 + 3/4*(4
*x^2 + 7)*cosh(x) - 3/8*cosh(x)^2 + 3/2*(x*cosh(x) - 4*x)*sinh(x)
```

giac [A] time = 0.13, size = 75, normalized size = 1.34

$$\frac{1}{4}x^4-\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}+\frac{3}{8}\left(4x^2+8x+7\right)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{8}\left(4x^2-8x+7\right)e^{x}+\frac{1}{24}e^{(3x)}+\frac{1}{24}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/8*(4*x^2 + 8*x + 7)*e^(-x) -
 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 7)*e^x + 1/24*e^(3*x) + 1/24*
e^(-3*x)
```

maple [A] time = 0.02, size = 52, normalized size = 0.93

$$\left(-\frac{2}{3}+\frac{\left(\sinh^2(x)\right)}{3}\right)\cosh(x)+\frac{3x\cosh(x)\sinh(x)}{2}-\frac{3x^2}{4}-\frac{3\left(\cosh^2(x)\right)}{4}+3x^2\cosh(x)-6x\sinh(x)+6\cosh(x)+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sinh(x))^3,x)
```

```
[Out] (-2/3+1/3*sinh(x)^2)*cosh(x)+3/2*x*cosh(x)*sinh(x)-3/4*x^2-3/4*cosh(x)^2+3*
x^2*cosh(x)-6*x*sinh(x)+6*cosh(x)+1/4*x^4
```

maxima [A] time = 0.35, size = 81, normalized size = 1.45

$$\frac{1}{4}x^4-\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}+\frac{3}{2}\left(x^2+2x+2\right)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{2}\left(x^2-2x+2\right)e^{x}+\frac{1}{24}e^{(3x)}-\frac{3}{8}e^{(-x)}+\frac{1}{24}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/2*(x^2 + 2*x + 2)*e^(-x) - 3
/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(-x
) + 1/24*e^(-3*x) - 3/8*e^x
```

mupad [B] time = 0.08, size = 46, normalized size = 0.82

$$5\cosh(x)+3x^2\cosh(x)-\frac{3\cosh(x)^2}{4}+\frac{\cosh(x)^3}{3}-6x\sinh(x)-\frac{3x^2}{4}+\frac{x^4}{4}+\frac{3x\cosh(x)\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sinh(x))^3,x)
```

```
[Out] 5*cosh(x) + 3*x^2*cosh(x) - (3*cosh(x)^2)/4 + cosh(x)^3/3 - 6*x*sinh(x) - (
3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2
```

sympy [A] time = 0.32, size = 85, normalized size = 1.52

$$\frac{x^4}{4}+\frac{3x^2\sinh^2(x)}{4}-\frac{3x^2\cosh^2(x)}{4}+3x^2\cosh(x)+\frac{3x\sinh(x)\cosh(x)}{2}-6x\sinh(x)+\sinh^2(x)\cosh(x)-\frac{3\sinh^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sinh(x)**2/4 - 3*x**2*cosh(x)**2/4 + 3*x**2*cosh(x) + 3*x*s
inh(x)*cosh(x)/2 - 6*x*sinh(x) + sinh(x)**2*cosh(x) - 3*sinh(x)**2/4 - 2*co
sh(x)**3/3 + 6*cosh(x)
```

3.368 $\int \frac{\sinh(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\ldots$$

```
[Out] 1/2*cosh(a+b*(-c)^(1/2)/d^(1/2))*Shi(b*x-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d
^(1/2)-1/2*cosh(a-b*(-c)^(1/2)/d^(1/2))*Shi(b*x+b*(-c)^(1/2)/d^(1/2))/(-c)^
(1/2)/d^(1/2)-1/2*Chi(b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a-b*(-c)^(1/2)/d^(1/2)
)/(-c)^(1/2)/d^(1/2)+1/2*Chi(-b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a+b*(-c)^(1/2)
/d^(1/2))/(-c)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5280, 3303, 3298, 3301}

$$-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] -(CoshIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sinh[a - (b*Sqrt[-c])/Sqrt[d]])/
(2*Sqrt[-c]*Sqrt[d]) + (CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sinh[a + (
b*Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a + (b*Sqrt[-c])/Sqrt[d]
]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a
- (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-
c]*Sqrt[d])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\,\sinh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\sinh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}+\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\sinh\left(a-\frac{b\sqrt{-}}{\sqrt{d}}\right.}{}$$

$$= -\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-}}$$

Mathematica [C] time = 0.33, size = 180, normalized size = 0.85

$$\frac{i\left(\sinh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(ibx-\frac{b\sqrt{c}}{\sqrt{d}}\right)-\sinh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(ixb+\frac{\sqrt{c}\,b}{\sqrt{d}}\right)+i\left(\cosh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)+\cosh\left(a\right.\right.\right.}{2\sqrt{c}\,\sqrt{d}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[-((b*Sqrt[c])/Sqrt[d]) + I*b*x]*Sinh[a - (I*b*Sqrt[c])/
Sqrt[d]] - CosIntegral[(b*Sqrt[c])/Sqrt[d] + I*b*x]*Sinh[a + (I*b*Sqrt[c])/
Sqrt[d]] + I*(Cosh[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
```

```
d] - I*b*x] + Cosh[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] + I*b*x])))/(Sqrt[c]*Sqrt[d])
```

fricas [B] time = 0.46, size = 316, normalized size = 1.48

$$-\frac{\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2*c/
d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-b^2*c/
d)))/(b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

maple [A] time = 0.07, size = 212, normalized size = 1.00

$$\frac{e^{-\frac{b\sqrt{-cd}+da}{d}}\operatorname{Ei}\left(1,-\frac{b\sqrt{-cd}-(bx+a)d+da}{d}\right)}{4\sqrt{-cd}}-\frac{e^{-\frac{-b\sqrt{-cd}+da}{d}}\operatorname{Ei}\left(1,\frac{b\sqrt{-cd}+(bx+a)d-da}{d}\right)}{4\sqrt{-cd}}-\frac{e^{\frac{b\sqrt{-cd}+da}{d}}\operatorname{Ei}\left(1,\frac{b\sqrt{-cd}-(bx+a)d+da}{d}\right)}{4\sqrt{-cd}}+\frac{e^{\frac{-b}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x^2+c),x)
```

```
[Out] 1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(1/2)-(b*x+a)
*d+d*a)/d)-1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1
/2)+(b*x+a)*d-d*a)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*
(-c*d)^(1/2)-(b*x+a)*d+d*a)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+d*a)/d
)*Ei(1,-(b*(-c*d)^(1/2)+(b*x+a)*d-d*a)/d)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx+a\right)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(a+b\,x\right)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+bx\right)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x**2), x)
```

3.369 $\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\cosh\left(a-\right.$$

```
[Out] cosh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*Shi(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))
/e)/(-4*c*e+d^2)^(1/2)-cosh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Shi(b*x+1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Chi(b*x+1/2*b*(d-(-4*c*e+d^2)
^(1/2))/e)*sinh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Chi(b*
x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sinh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-
4*c*e+d^2)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {6728, 3303, 3298, 3301}

$$\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\cosh\left(a-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CoshIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d - Sqr
t[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CoshIntegral[(b*(d + Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[
d^2 - 4*c*e] + (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$= \frac{(2e)\int \frac{\sinh(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\sinh(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sinh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sinh\left(\cdots\right.}{d+\cdots}}{\sqrt{d^2-4ce}}$$

$$= \frac{\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d+\sqrt{d\cdots}\right.}{2\cdots}\right.}{\sqrt{d^2-4ce}}$$

Mathematica [C] time = 0.54, size = 248, normalized size = 0.92

$$\frac{\sinh\left(a+\frac{b\left(\sqrt{d^2-4ce}-d\right)}{2e}\right)\mathrm{Ci}\left(\frac{ib\left(d+2ex-\sqrt{d^2-4ce}\right)}{2e}\right) - \sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\mathrm{Ci}\left(\frac{ib\left(d+2ex+\sqrt{d^2-4ce}\right)}{2e}\right) - \cosh\left(a-\frac{b\left(\sqrt{d^2-4c\cdots}\right.}{2e}\right.}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[a + b*x]/(c + d*x + e*x^2),x]

[Out] (CosIntegral[((I/2)*b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e]*Sinh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)] - CosIntegral[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)] - Cosh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] + I*Cosh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[((I/2)*b*(-d + Sqrt[d^2 - 4*c*e]))/e - I*b*x])/Sqrt[d^2 - 4*c*e]

fricas [B] time = 0.54, size = 671, normalized size = 2.48

$$-\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)-e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")

[Out] -1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d - 2*a*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx+a)}{ex^2+dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")

[Out] integrate(sinh(b*x + a)/(e*x^2 + d*x + c), x)

maple [A] time = 0.07, size = 376, normalized size = 1.39

$$-\frac{b\,\mathrm{e}^{-\frac{2ea-bd-\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{Ei}\left(1,\frac{2e(bx+a)-2ea+bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}+\frac{b\,\mathrm{e}^{-\frac{2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{Ei}\left(1,-\frac{-2e(bx+a)+2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(e*x^2+d*x+c),x)
```

```
[Out] -1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2
)^(1/2)))*Ei(1,1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/
2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(
1/2)))*Ei(1,-1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2
*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/
2)))*Ei(1,1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/2*b/
(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)^(1/2))
)*Ei(1,-1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c*e-d^2>0)', see `assume?` for mo
re details)Is 4*c*e-d^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(a+b\,x\right)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x + e*x^2),x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+bx\right)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x + e*x**2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```