Computer algebra independent integration tests

5-Inverse-trig-functions/5.5-Inverse-secant/5.5.2-Inverse-secant-functions

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July 22, 2021 Compiled on July 22, 2021 at 5:26am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [50]. This is test number [157].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (50)	% 0.00 (0)
Mathematica	% 98.00 (49)	% 2.00 (1)
Maple	% 74.00 (37)	% 26.00 (13)
Maxima	% 36.00 (18)	% 64.00 (32)
Fricas	% 56.00 (28)	% 44.00 (22)
Sympy	% 26.00 (13)	% 74.00 (37)
Giac	% 54.00 (27)	% 46.00 (23)
Mupad	% 20.00 (10)	% 80.00 (40)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

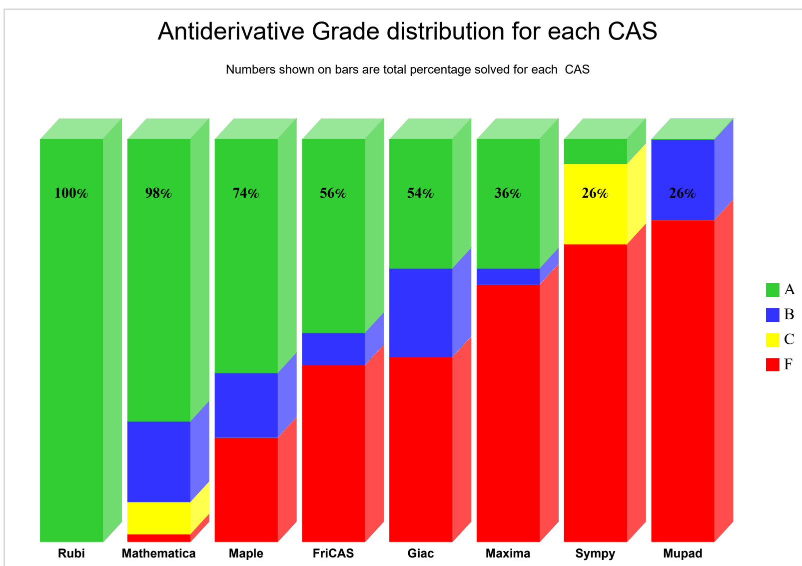
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

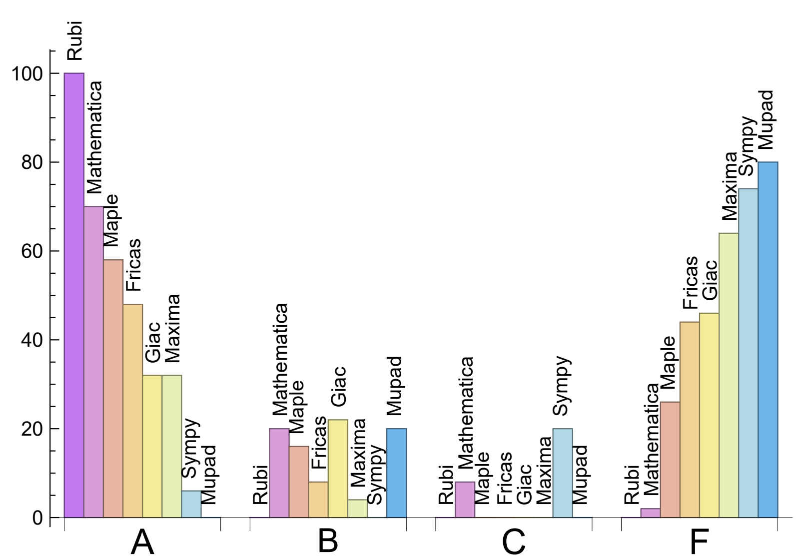
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.00	20.00	8.00	2.00
Maple	58.00	16.00	0.00	26.00
Maxima	32.00	4.00	0.00	64.00
Fricas	48.00	8.00	0.00	44.00
Sympy	6.00	0.00	20.00	74.00
Giac	32.00	22.00	0.00	46.00
Mupad	0.00	20.00	0.00	80.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	0.00 %	100.00 %	0.00 %
Maple	13	100.00 %	0.00 %	0.00 %
Maxima	32	93.75 %	6.25 %	0.00 %
Fricas	22	90.91 %	0.00 %	9.09 %
Sympy	37	89.19 %	10.81 %	0.00 %
Giac	23	95.65 %	0.00 %	4.35 %
Mupad	40	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

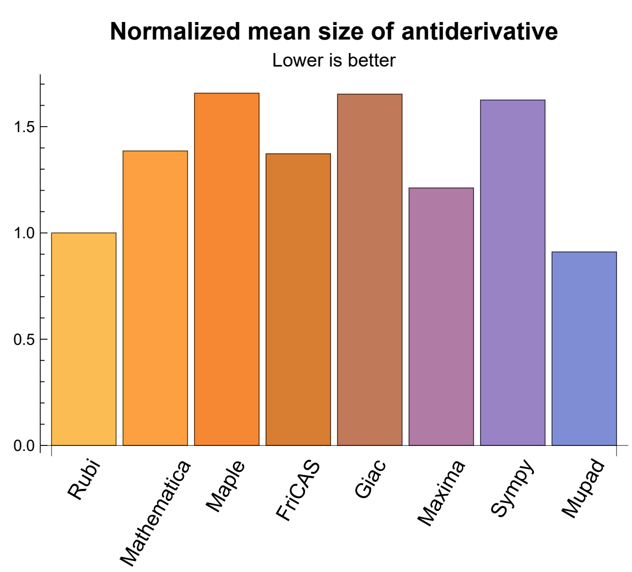
1.3 Performance

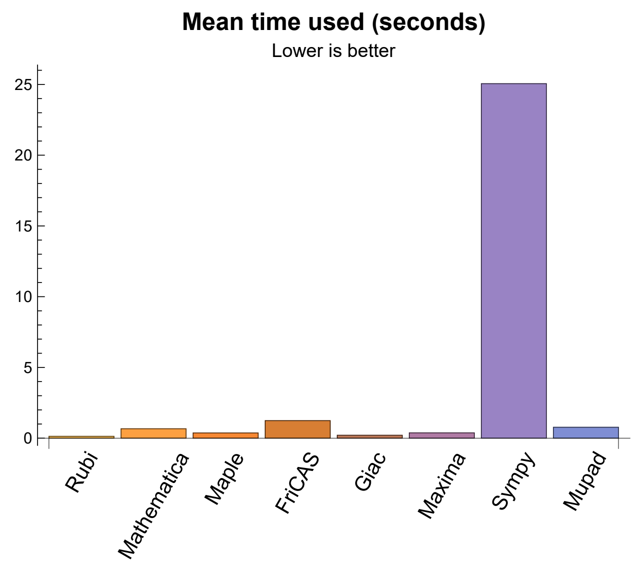
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	120.76	1.00	69.00	1.00
Mathematica	0.67	172.67	1.39	107.00	1.00
Maple	0.37	208.46	1.66	93.00	1.42
Maxima	0.38	56.67	1.21	54.50	1.20
Fricas	1.24	102.61	1.37	56.50	0.92
Sympy	25.05	76.31	1.63	58.00	1.61
Giac	0.20	120.96	1.65	82.00	1.72
Mupad	0.77	37.50	0.91	36.50	0.90

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {13, 23, 27, 28, 30, 31, 35, 36, 43, 44, 45, 46}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

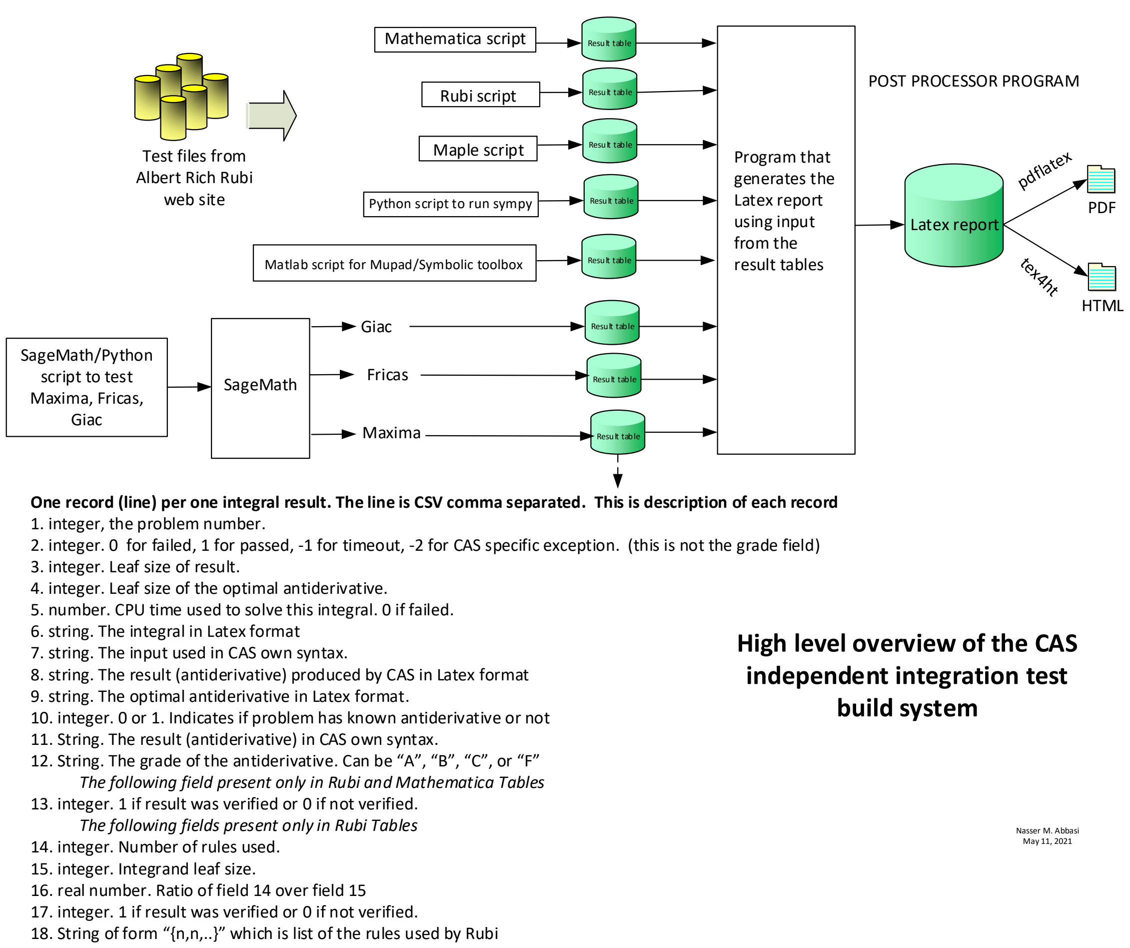
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 20, 21, 23, 27, 28, 29, 30, 33, 34, 35, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade: { 14, 22, 31, 32, 36, 38, 39, 40, 41, 42 }

C grade: { 17, 24, 25, 26 }

F grade: { 37 }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 21, 22, 23, 27, 28, 29, 30, 32, 33, 34, 38, 39, 40, 42, 50 }

B grade: { 11, 12, 18, 19, 20, 24, 25, 26 }

C grade: { }

F grade: { 1, 31, 35, 36, 37, 41, 43, 44, 45, 46, 47, 48, 49 }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 7, 10, 11, 12, 14, 15, 16, 22, 38, 39, 40, 41 }

B grade: { 8, 9 }

C grade: { }

F grade: { 1, 6, 13, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15, 18, 19, 20, 21, 25, 26, 38, 39, 40, 41, 47, 48, 49 }

B grade: { 14, 16, 22, 24 }

C grade: { }

F grade: { 1, 6, 13, 17, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 50 }

2.1.6 Sympy

A grade: { 10, 11, 12 }

B grade: { }

C grade: { 2, 3, 4, 5, 7, 8, 9, 14, 15, 16 }

F grade: { 1, 6, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

2.1.7 Giac

A grade: { 7, 8, 9, 10, 11, 12, 15, 16, 21, 24, 38, 39, 40, 41, 47, 50 }

B grade: { 2, 3, 4, 5, 14, 18, 19, 20, 22, 25, 26 }

C grade: { }

F grade: { 1, 6, 13, 17, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 48, 49 }

2.1.8 Mupad

A grade: { }

B grade: { 5, 7, 11, 12, 14, 22, 38, 39, 40, 41 }

C grade: { }

F grade: { 1, 2, 3, 4, 6, 8, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.038	0.180	0.000	2.017	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	43	66	32	119	152	-1
normalized size	1	1.00	0.69	0.74	1.14	0.55	2.05	2.62	-0.02
time (sec)	N/A	0.020	0.028	0.052	0.332	0.488	73.945	0.134	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	38	52	27	90	116	-1
normalized size	1	1.00	0.74	0.81	1.11	0.57	1.91	2.47	-0.02
time (sec)	N/A	0.018	0.022	0.049	0.333	0.772	25.692	0.138	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	31	38	20	58	80	-1
normalized size	1	1.00	0.78	0.86	1.06	0.56	1.61	2.22	-0.03
time (sec)	N/A	0.011	0.018	0.048	0.328	0.891	8.944	0.150	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	21	14	29	41	21
normalized size	1	1.00	1.00	1.39	1.17	0.78	1.61	2.28	1.17
time (sec)	N/A	0.004	0.006	0.048	0.332	0.932	4.423	0.127	1.149

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	63	0	0	0	0	-1
normalized size	1	1.00	0.96	1.12	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.079	0.029	0.115	0.000	0.605	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	46	51	19	76	30	28
normalized size	1	1.00	0.84	1.21	1.34	0.50	2.00	0.79	0.74
time (sec)	N/A	0.016	0.022	0.051	0.419	1.586	20.047	0.128	0.684

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	57	80	29	144	44	-1
normalized size	1	1.00	1.02	1.06	1.48	0.54	2.67	0.81	-0.02
time (sec)	N/A	0.019	0.034	0.049	0.437	1.691	53.237	0.142	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	45	67	106	35	180	58	-1
normalized size	1	1.00	0.66	0.99	1.56	0.51	2.65	0.85	-0.01
time (sec)	N/A	0.024	0.055	0.050	0.428	0.537	132.622	0.148	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	42	66	54	39	51	47	-1
normalized size	1	1.00	0.75	1.18	0.96	0.70	0.91	0.84	-0.02
time (sec)	N/A	0.042	0.035	0.081	0.433	1.089	0.488	0.136	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	93	46	38	41	39	38
normalized size	1	1.00	0.94	1.98	0.98	0.81	0.87	0.83	0.81
time (sec)	N/A	0.020	0.025	0.067	0.426	0.877	0.221	0.122	0.628

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	51	24	27	22	28	24
normalized size	1	1.00	1.00	1.96	0.92	1.04	0.85	1.08	0.92
time (sec)	N/A	0.009	0.012	0.060	0.340	0.909	0.146	0.127	0.109

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	76	0	0	0	0	-1
normalized size	1	1.00	1.00	1.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.024	0.118	0.000	0.916	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	93	41	52	107	29	61	29
normalized size	1	1.00	3.00	1.32	1.68	3.45	0.94	1.97	0.94
time (sec)	N/A	0.030	0.142	0.044	0.340	2.117	2.173	0.141	0.605

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	54	32	39	53	61	-1
normalized size	1	1.00	0.95	1.42	0.84	1.03	1.39	1.61	-0.03
time (sec)	N/A	0.021	0.022	0.056	0.422	0.646	1.250	0.176	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	98	64	142	100	80	-1
normalized size	1	1.00	1.15	1.63	1.07	2.37	1.67	1.33	-0.02
time (sec)	N/A	0.042	0.044	0.056	0.420	0.943	2.515	0.125	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	98	0	0	0	0	-1
normalized size	1	1.00	0.87	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.093	0.161	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	173	509	0	152	0	409	-1
normalized size	1	1.00	0.88	2.58	0.00	0.77	0.00	2.08	-0.01
time (sec)	N/A	0.232	0.186	0.088	0.000	0.751	0.000	0.180	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	150	359	0	130	0	299	-1
normalized size	1	1.00	0.97	2.32	0.00	0.84	0.00	1.93	-0.01
time (sec)	N/A	0.139	0.287	0.057	0.000	2.065	0.000	0.187	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	131	273	0	117	0	204	-1
normalized size	1	1.00	1.13	2.35	0.00	1.01	0.00	1.76	-0.01
time (sec)	N/A	0.090	0.178	0.057	0.000	2.182	0.000	0.170	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	110	126	0	104	0	133	-1
normalized size	1	1.00	1.41	1.62	0.00	1.33	0.00	1.71	-0.01
time (sec)	N/A	0.053	0.123	0.053	0.000	1.443	0.000	0.156	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	115	51	55	73	0	82	35
normalized size	1	1.00	3.11	1.38	1.49	1.97	0.00	2.22	0.95
time (sec)	N/A	0.023	0.144	0.049	0.333	1.160	0.000	0.171	0.862

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	284	374	0	0	0	0	-1
normalized size	1	1.00	1.42	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.323	0.836	0.000	0.974	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	112	154	0	281	0	94	-1
normalized size	1	1.00	1.60	2.20	0.00	4.01	0.00	1.34	-0.01
time (sec)	N/A	0.097	0.314	0.061	0.000	1.013	0.000	0.193	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	198	452	0	427	0	216	-1
normalized size	1	1.00	1.58	3.62	0.00	3.42	0.00	1.73	-0.01
time (sec)	N/A	0.192	1.052	0.064	0.000	2.712	0.000	0.221	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	241	760	0	548	0	451	-1
normalized size	1	1.00	1.33	4.20	0.00	3.03	0.00	2.49	-0.01
time (sec)	N/A	0.288	0.431	0.069	0.000	1.928	0.000	0.223	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	667	734	0	0	0	0	-1
normalized size	1	1.00	1.75	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	9.418	4.002	0.000	2.013	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	473	540	0	0	0	0	-1
normalized size	1	1.00	1.64	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	6.582	1.686	0.000	1.030	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	142	264	0	0	0	0	-1
normalized size	1	1.00	0.92	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.147	0.330	0.000	1.685	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	111	179	0	0	0	0	-1
normalized size	1	1.00	1.18	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.113	0.282	0.000	1.348	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	813	0	0	0	0	0	-1
normalized size	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	2.378	1.450	0.000	3.071	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	686	341	0	0	0	0	-1
normalized size	1	1.00	2.81	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	2.293	0.844	0.000	2.409	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	442	770	0	0	0	0	-1
normalized size	1	1.00	0.89	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	0.525	1.789	0.000	3.721	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	248	429	0	0	0	0	-1
normalized size	1	1.00	0.89	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	0.438	1.532	0.000	1.189	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	160	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.099	0.620	0.000	0.768	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	1058	0	0	0	0	0	-1
normalized size	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	4.055	1.563	0.000	1.760	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	180.002	1.441	0.000	2.226	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	148	81	71	96	0	100	52
normalized size	1	1.00	2.55	1.40	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.073	0.237	0.103	0.347	0.600	0.000	0.377	1.031

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	148	81	71	96	0	100	52
normalized size	1	1.00	2.55	1.40	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.076	0.155	0.101	0.360	0.798	0.000	0.401	0.774

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	137	81	71	96	0	100	52
normalized size	1	1.00	2.36	1.40	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.078	0.383	0.101	0.375	1.213	0.000	0.382	0.777

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	130	0	66	92	0	75	44
normalized size	1	1.00	2.65	0.00	1.35	1.88	0.00	1.53	0.90
time (sec)	N/A	0.072	0.340	0.156	0.359	1.010	0.000	0.279	1.084

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	280	116	0	0	0	0	-1
normalized size	1	1.00	3.29	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.855	0.168	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	95	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.312	0.054	0.000	1.339	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	107	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.197	0.051	0.000	0.955	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	54	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.060	0.049	0.000	0.973	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	79	0	0	0	0	0	-1
normalized size	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.054	0.050	0.000	0.991	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	0	0	23	0	51	-1
normalized size	1	1.00	0.87	0.00	0.00	0.59	0.00	1.31	-0.03
time (sec)	N/A	0.030	0.043	0.051	0.000	1.420	0.000	0.205	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	0	0	30	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.045	0.053	0.000	1.053	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	54	0	0	40	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.139	0.051	0.000	1.840	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	92	0	0	0	115	-1
normalized size	1	1.00	0.86	1.33	0.00	0.00	0.00	1.67	-0.01
time (sec)	N/A	0.091	0.050	0.351	0.000	0.527	0.000	0.591	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [34] had the largest ratio of [1.200]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	10	0.600
2	A	4	3	1.00	10	0.300
3	A	4	3	1.00	10	0.300
4	A	4	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	7	6	1.00	10	0.600
7	A	5	5	1.00	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	6	5	1.00	10	0.500
9	A	7	5	1.00	10	0.500
10	A	5	4	1.00	10	0.400
11	A	4	4	1.00	8	0.500
12	A	3	3	1.00	6	0.500
13	A	6	6	1.00	10	0.600
14	A	5	5	1.00	10	0.500
15	A	3	3	1.00	10	0.300
16	A	6	6	1.00	10	0.600
17	A	7	6	1.00	10	0.600
18	A	9	8	1.00	10	0.800
19	A	8	7	1.00	10	0.700
20	A	7	6	1.00	10	0.600
21	A	6	6	1.00	8	0.750
22	A	5	5	1.00	6	0.833
23	A	14	8	1.00	10	0.800
24	A	5	5	1.00	10	0.500
25	A	7	7	1.00	10	0.700
26	A	8	8	1.00	10	0.800
27	A	20	9	1.00	12	0.750
28	A	17	9	1.00	12	0.750
29	A	11	8	1.00	10	0.800
30	A	8	6	1.00	8	0.750
31	A	17	9	1.00	12	0.750
32	A	12	8	1.00	12	0.667
33	A	25	14	1.00	12	1.167
34	A	16	12	1.00	10	1.200
35	A	10	7	1.00	8	0.875
36	A	20	10	1.00	12	0.833
37	A	14	9	1.00	12	0.750
38	A	7	6	1.00	14	0.429
39	A	7	6	1.00	16	0.375
40	A	7	6	1.00	16	0.375
41	A	6	6	1.00	14	0.429
42	A	7	7	1.00	10	0.700
43	A	6	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	6	4	1.00	8	0.500
45	A	5	3	1.00	6	0.500
46	A	6	5	1.00	10	0.500
47	A	3	3	1.00	10	0.300
48	A	5	4	1.00	10	0.400
49	A	6	4	1.00	10	0.400
50	A	8	8	1.00	19	0.421

Chapter 3

Listing of integrals

3.1 $\int \frac{\sec^{-1}(ax^5)}{x} \, dx$

Optimal. Leaf size=62

$$\frac{1}{10} i \text{Li}_2\left(-e^{2i\sec^{-1}(ax^5)}\right) + \frac{1}{10} i \sec^{-1}\left(ax^5\right)^2 - \frac{1}{5} \sec^{-1}\left(ax^5\right) \log\left(1 + e^{2i\sec^{-1}(ax^5)}\right)$$

```
[Out] 1/10*I*arcsec(a*x^5)^2-1/5*arcsec(a*x^5)*ln(1+(1/a/x^5+I*(1-1/a^2/x^10)^(1/
2))^2)+1/10*I*polylog(2,-(1/a/x^5+I*(1-1/a^2/x^10)^(1/2))^2)
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5218, 4626, 3719, 2190, 2279, 2391}

$$\frac{1}{10} i \text{PolyLog}\left(2, -e^{2i\sec^{-1}(ax^5)}\right) + \frac{1}{10} i \sec^{-1}\left(ax^5\right)^2 - \frac{1}{5} \sec^{-1}\left(ax^5\right) \log\left(1 + e^{2i\sec^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a*x^5]/x,x]
```

```
[Out] (I/10)*ArcSec[a*x^5]^2 - (ArcSec[a*x^5]*Log[1 + E^((2*I)*ArcSec[a*x^5])])/5
 + (I/10)*PolyLog[2, -E^((2*I)*ArcSec[a*x^5])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5218

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{\sec^{-1}(ax)}{x}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{5}\operatorname{Subst}\left(\int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, \frac{1}{x^5}\right)\right)\\
&= \frac{1}{5}\operatorname{Subst}\left(\int x\tan(x)\,dx, x, \sec^{-1}\left(ax^5\right)\right)\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{2}{5}i\operatorname{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \sec^{-1}\left(ax^5\right)\right)\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right) + \frac{1}{5}\operatorname{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \sec
\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right) - \frac{1}{10}i\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\,\mathrm{se}
\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right) + \frac{1}{10}i\operatorname{Li}_2\left(-e^{2i\sec^{-1}\left(ax^5\right)}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.90

$$\frac{1}{10}i\left(\operatorname{Li}_2\left(-e^{2i\sec^{-1}\left(ax^5\right)}\right) + \sec^{-1}\left(ax^5\right)\left(\sec^{-1}\left(ax^5\right) + 2i\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a*x^5]/x,x]
```

```
[Out] (I/10)*(ArcSec[a*x^5]*(ArcSec[a*x^5] + (2*I)*Log[1 + E^((2*I)*ArcSec[a*x^5]
)]) + PolyLog[2, -E^((2*I)*ArcSec[a*x^5])])
```

fricas [F] time = 2.02, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arcsec}\left(ax^5\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(a*x^5)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arcsec(a*x^5)/x, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a*x^5)/x,x)
[Out] int(arcsec(a*x^5)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-5\,a^2 \int \frac{\sqrt{ax^5+1}\,\sqrt{ax^5-1}\,\log(x)}{a^4x^{11}-a^2x}\,dx - 5i\,a^2 \int \frac{\log(x)}{a^4x^{11}-a^2x}\,dx + \arctan\left(\sqrt{ax^5+1}\,\sqrt{ax^5-1}\right)\log(x) - \frac{1}{2}i\,\log\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^5)/x,x, algorithm="maxima")
[Out] -5*a^2*integrate(sqrt(a*x^5 + 1)*sqrt(a*x^5 - 1)*log(x)/(a^4*x^11 - a^2*x),
 x) - 5*I*a^2*integrate(log(x)/(a^4*x^11 - a^2*x), x) + arctan(sqrt(a*x^5 +
 1)*sqrt(a*x^5 - 1))*log(x) - 1/2*I*log(a^2*x^10)*log(x) + 1/2*I*log(a*x^5
+ 1)*log(x) + 1/2*I*log(-a*x^5 + 1)*log(x) + I*log(a)*log(x) + 5/2*I*log(x)
^2 + 1/10*I*dilog(a*x^5) + 1/10*I*dilog(-a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{1}{a\,x^5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a*x^5))/x,x)
[Out] int(acos(1/(a*x^5))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a*x**5)/x,x)
[Out] Integral(asec(a*x**5)/x, x)
```

3.2 $\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=58

$$\frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{28}(x-1)^{7/2} - \frac{3}{20}(x-1)^{5/2} - \frac{1}{4}(x-1)^{3/2} - \frac{\sqrt{x-1}}{4}$$

```
[Out] -1/4*(-1+x)^(3/2)-3/20*(-1+x)^(5/2)-1/28*(-1+x)^(7/2)+1/4*x^4*arcsec(x^(1/2
))-1/4*(-1+x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5270, 12, 43}

$$\frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{28}(x-1)^{7/2} - \frac{3}{20}(x-1)^{5/2} - \frac{1}{4}(x-1)^{3/2} - \frac{\sqrt{x-1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSec[Sqrt[x]],x]
```

```
[Out] -Sqrt[-1 + x]/4 - (-1 + x)^(3/2)/4 - (3*(-1 + x)^(5/2))/20 - (-1 + x)^(7/2)
/28 + (x^4*ArcSec[Sqrt[x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5270

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSec[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(m
+ 1)*Sqrt[u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[
u^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{x^3}{2\sqrt{-1+x}} dx \\
&= \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{8}\int \frac{x^3}{\sqrt{-1+x}} dx \\
&= \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{8}\int \left(\frac{1}{\sqrt{-1+x}} + 3\sqrt{-1+x} + 3(-1+x)^{3/2} + (-1+x)^{5/2}\right) dx \\
&= -\frac{1}{4}\sqrt{-1+x} - \frac{1}{4}(-1+x)^{3/2} - \frac{3}{20}(-1+x)^{5/2} - \frac{1}{28}(-1+x)^{7/2} + \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.69

$$\frac{1}{4}x^4\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{140}\sqrt{x-1}\left(5x^3+6x^2+8x+16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSec[Sqrt[x]],x]
[Out] -1/140*(Sqrt[-1 + x]*(16 + 8*x + 6*x^2 + 5*x^3)) + (x^4*ArcSec[Sqrt[x]])/4
```

fricas [A] time = 0.49, size = 32, normalized size = 0.55

$$\frac{1}{4}x^4\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{140}\left(5x^3+6x^2+8x+16\right)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x^(1/2)),x, algorithm="fricas")
[Out] 1/4*x^4*arcsec(sqrt(x)) - 1/140*(5*x^3 + 6*x^2 + 8*x + 16)*sqrt(x - 1)
```

giac [B] time = 0.13, size = 152, normalized size = 2.62

$$-\frac{1}{3584}x^{\frac{7}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^7-\frac{7}{2560}x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5+\frac{1}{4}x^4\arccos\left(\frac{1}{\sqrt{x}}\right)-\frac{7}{512}x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3-\frac{35}{512}\sqrt{x}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x^(1/2)),x, algorithm="giac")
[Out] -1/3584*x^(7/2)*(sqrt(-1/x + 1) - 1)^7 - 7/2560*x^(5/2)*(sqrt(-1/x + 1) - 1
)^5 + 1/4*x^4*arccos(1/sqrt(x)) - 7/512*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 - 35
/512*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/17920*(1225*x^3*(sqrt(-1/x + 1) - 1)^
6 + 245*x^2*(sqrt(-1/x + 1) - 1)^4 + 49*x*(sqrt(-1/x + 1) - 1)^2 + 5)/(x^(7
/2)*(sqrt(-1/x + 1) - 1)^7)
```

maple [A] time = 0.05, size = 43, normalized size = 0.74

$$\frac{x^4\operatorname{arcsec}\left(\sqrt{x}\right)}{4}-\frac{(x-1)\left(5x^3+6x^2+8x+16\right)}{140\sqrt{\frac{x-1}{x}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(x^(1/2)),x)
[Out] 1/4*x^4*arcsec(x^(1/2))-1/140*(x-1)*(5*x^3+6*x^2+8*x+16)/((x-1)/x)^(1/2)/x^
(1/2)
```

maxima [A] time = 0.33, size = 66, normalized size = 1.14

$$-\frac{1}{28}x^{\frac{7}{2}}\left(-\frac{1}{x}+1\right)^{\frac{7}{2}}-\frac{3}{20}x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}}+\frac{1}{4}x^4\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{4}x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x^(1/2)),x, algorithm="maxima")
[Out] -1/28*x^(7/2)*(-1/x + 1)^(7/2) - 3/20*x^(5/2)*(-1/x + 1)^(5/2) + 1/4*x^4*ar
csec(sqrt(x)) - 1/4*x^(3/2)*(-1/x + 1)^(3/2) - 1/4*sqrt(x)*sqrt(-1/x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(1/x^(1/2)),x)
```

```
[Out] int(x^3*acos(1/x^(1/2)), x)
```

sympy [C] time = 73.95, size = 119, normalized size = 2.05

$$\frac{x^4 \operatorname{asec}\left(\sqrt{x}\right)}{4} - \frac{\begin{cases} \frac{2x^3\sqrt{x-1}}{7} + \frac{12x^2\sqrt{x-1}}{35} + \frac{16x\sqrt{x-1}}{35} + \frac{32\sqrt{x-1}}{35} & \text{for } |x| > 1 \\ \frac{2ix^3\sqrt{1-x}}{7} + \frac{12ix^2\sqrt{1-x}}{35} + \frac{16ix\sqrt{1-x}}{35} + \frac{32i\sqrt{1-x}}{35} & \text{otherwise} \end{cases}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(x**(1/2)),x)
```

```
[Out] x**4*asec(sqrt(x))/4 - Piecewise((2*x**3*sqrt(x - 1)/7 + 12*x**2*sqrt(x - 1
)/35 + 16*x*sqrt(x - 1)/35 + 32*sqrt(x - 1)/35, Abs(x) > 1), (2*I*x**3*sqrt
(1 - x)/7 + 12*I*x**2*sqrt(1 - x)/35 + 16*I*x*sqrt(1 - x)/35 + 32*I*sqrt(1
- x)/35, True))/8
```

3.3 $\int x^2 \sec^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=47

$$\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{15}(x-1)^{5/2}-\frac{2}{9}(x-1)^{3/2}-\frac{\sqrt{x-1}}{3}$$

```
[Out] -2/9*(-1+x)^(3/2)-1/15*(-1+x)^(5/2)+1/3*x^3*arcsec(x^(1/2))-1/3*(-1+x)^(1/2
)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5270, 12, 43}

$$\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{15}(x-1)^{5/2}-\frac{2}{9}(x-1)^{3/2}-\frac{\sqrt{x-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[Sqrt[x]],x]
```

```
[Out] -Sqrt[-1 + x]/3 - (2*(-1 + x)^(3/2))/9 - (-1 + x)^(5/2)/15 + (x^3*ArcSec[Sq
rt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5270

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSec[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(m
+ 1)*Sqrt[u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[
u^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sec^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\frac{x^2}{2\sqrt{-1+x}}\,dx\\ &= \frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\frac{x^2}{\sqrt{-1+x}}\,dx\\ &= \frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\left(\frac{1}{\sqrt{-1+x}}+2\sqrt{-1+x}+(-1+x)^{3/2}\right)dx\\ &= -\frac{1}{3}\sqrt{-1+x}-\frac{2}{9}(-1+x)^{3/2}-\frac{1}{15}(-1+x)^{5/2}+\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.74

$$\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{45}\sqrt{x-1}\left(3x^2+4x+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[Sqrt[x]],x]
[Out] -1/45*(Sqrt[-1 + x]*(8 + 4*x + 3*x^2)) + (x^3*ArcSec[Sqrt[x]])/3
```

fricas [A] time = 0.77, size = 27, normalized size = 0.57

$$\frac{1}{3}x^3\,\mathrm{arcsec}\left(\sqrt{x}\right)-\frac{1}{45}\left(3\,x^2+4\,x+8\right)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x^(1/2)),x, algorithm="fricas")
[Out] 1/3*x^3*arcsec(sqrt(x)) - 1/45*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

giac [B] time = 0.14, size = 116, normalized size = 2.47

$$-\frac{1}{480}\,x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5-\frac{5}{288}\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3+\frac{1}{3}\,x^3\arccos\left(\frac{1}{\sqrt{x}}\right)-\frac{5}{48}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)+\frac{150\,x^2\left(\sqrt{-\frac{1}{x}+\right.}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x^(1/2)),x, algorithm="giac")
[Out] -1/480*x^(5/2)*(sqrt(-1/x + 1) - 1)^5 - 5/288*x^(3/2)*(sqrt(-1/x + 1) - 1)^
3 + 1/3*x^3*arccos(1/sqrt(x)) - 5/48*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/1440*
(150*x^2*(sqrt(-1/x + 1) - 1)^4 + 25*x*(sqrt(-1/x + 1) - 1)^2 + 3)/(x^(5/2)
*(sqrt(-1/x + 1) - 1)^5)
```

maple [A] time = 0.05, size = 38, normalized size = 0.81

$$\frac{x^3\mathrm{arcsec}\left(\sqrt{x}\right)}{3}-\frac{(x-1)\left(3x^2+4x+8\right)}{45\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(x^(1/2)),x)
[Out] 1/3*x^3*arcsec(x^(1/2))-1/45*(x-1)*(3*x^2+4*x+8)/((x-1)/x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.33, size = 52, normalized size = 1.11

$$-\frac{1}{15}\,x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}}+\frac{1}{3}\,x^3\,\mathrm{arcsec}\left(\sqrt{x}\right)-\frac{2}{9}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}\,\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x^(1/2)),x, algorithm="maxima")
[Out] -1/15*x^(5/2)*(-1/x + 1)^(5/2) + 1/3*x^3*arcsec(sqrt(x)) - 2/9*x^(3/2)*(-1/
x + 1)^(3/2) - 1/3*sqrt(x)*sqrt(-1/x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/x^(1/2)),x)
```

```
[Out] int(x^2*acos(1/x^(1/2)), x)
```

sympy [C] time = 25.69, size = 90, normalized size = 1.91

$$\frac{x^3 \operatorname{asec}\left(\sqrt{x}\right)}{3} - \frac{\begin{cases} \frac{2x^2\sqrt{x-1}}{5} + \frac{8x\sqrt{x-1}}{15} + \frac{16\sqrt{x-1}}{15} & \text{for } |x| > 1 \\ \frac{2ix^2\sqrt{1-x}}{5} + \frac{8ix\sqrt{1-x}}{15} + \frac{16i\sqrt{1-x}}{15} & \text{otherwise} \end{cases}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(x**(1/2)),x)
```

```
[Out] x**3*asec(sqrt(x))/3 - Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/15
 + 16*sqrt(x - 1)/15, Abs(x) > 1), (2*I*x**2*sqrt(1 - x)/5 + 8*I*x*sqrt(1 -
 x)/15 + 16*I*sqrt(1 - x)/15, True))/6
```

3.4 $\int x \sec^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}(x-1)^{3/2}-\frac{\sqrt{x-1}}{2}$$

```
[Out] -1/6*(-1+x)^(3/2)+1/2*x^2*arcsec(x^(1/2))-1/2*(-1+x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5270, 12, 43}

$$\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}(x-1)^{3/2}-\frac{\sqrt{x-1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[Sqrt[x]],x]
```

```
[Out] -Sqrt[-1 + x]/2 - (-1 + x)^(3/2)/6 + (x^2*ArcSec[Sqrt[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5270

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSec[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(m
+ 1)*Sqrt[u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[
u^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int x\sec^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{x}{2\sqrt{-1+x}}\,dx\\
&= \frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x}{\sqrt{-1+x}}\,dx\\
&= \frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\left(\frac{1}{\sqrt{-1+x}}+\sqrt{-1+x}\right)\,dx\\
&= -\frac{1}{2}\sqrt{-1+x}-\frac{1}{6}(-1+x)^{3/2}+\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.78

$$\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\sqrt{x-1}\,(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[Sqrt[x]],x]
```

```
[Out] -1/6*(Sqrt[-1 + x]*(2 + x)) + (x^2*ArcSec[Sqrt[x]])/2
```

fricas [A] time = 0.89, size = 20, normalized size = 0.56

$$\frac{1}{2}\,x^2\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{6}\,(x+2)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arcsec(sqrt(x)) - 1/6*(x + 2)*sqrt(x - 1)
```

giac [B] time = 0.15, size = 80, normalized size = 2.22

$$-\frac{1}{48}\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3+\frac{1}{2}\,x^2\arccos\left(\frac{1}{\sqrt{x}}\right)-\frac{3}{16}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)+\frac{9\,x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2+1}{48\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x^(1/2)),x, algorithm="giac")
```

```
[Out] -1/48*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 + 1/2*x^2*arccos(1/sqrt(x)) - 3/16*sqr
t(x)*(sqrt(-1/x + 1) - 1) + 1/48*(9*x*(sqrt(-1/x + 1) - 1)^2 + 1)/(x^(3/2)*
(sqrt(-1/x + 1) - 1)^3)
```

maple [A] time = 0.05, size = 31, normalized size = 0.86

$$\frac{x^2\operatorname{arcsec}\left(\sqrt{x}\right)}{2}-\frac{(x-1)\,(x+2)}{6\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(x^(1/2)),x)
```

```
[Out] 1/2*x^2*arcsec(x^(1/2))-1/6*(x-1)*(x+2)/((x-1)/x)^(1/2)/x^(1/2)
```

maxima [A] time = 0.33, size = 38, normalized size = 1.06

$$-\frac{1}{6}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}+\frac{1}{2}\,x^2\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/6*x^(3/2)*(-1/x + 1)^(3/2) + 1/2*x^2*arcsec(sqrt(x)) - 1/2*sqrt(x)*sqrt(
-1/x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\mathrm{acos}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/x^(1/2)),x)
```

```
[Out] int(x*acos(1/x^(1/2)), x)
```

sympy [C] time = 8.94, size = 58, normalized size = 1.61

$$\frac{x^2\operatorname{asec}\left(\sqrt{x}\right)}{2}-\frac{\begin{cases}\frac{x\sqrt{x-1}}{3}+\frac{2\sqrt{x-1}}{3} & \text{for } |x|>1\\ \frac{ix\sqrt{1-x}}{3}+\frac{2i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(x**(1/2)),x)
```

```
[Out] x**2*asec(sqrt(x))/2 - Piecewise((x*sqrt(x - 1)/3 + 2*sqrt(x - 1)/3, Abs(x)
 > 1), (I*x*sqrt(1 - x)/3 + 2*I*sqrt(1 - x)/3, True))/2
```

3.5 $\int \sec^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=18

$$x\sec^{-1}\left(\sqrt{x}\right)-\sqrt{x-1}$$

```
[Out] x*arcsec(x^(1/2))-(-1+x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5268, 12, 32}

$$x\sec^{-1}\left(\sqrt{x}\right)-\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]],x]
```

```
[Out] -Sqrt[-1 + x] + x*ArcSec[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5268

```
Int[ArcSec[u_], x_Symbol] :> Simp[x*ArcSec[u], x] - Dist[u/Sqrt[u^2], Int[S
implifyIntegrand[(x*D[u, x])/(u*Sqrt[u^2 - 1]), x], x], x] /; InverseFuncti
onFreeQ[u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{-1}\left(\sqrt{x}\right) dx &= x\sec^{-1}\left(\sqrt{x}\right)-\int \frac{1}{2\sqrt{-1+x}}\,dx\\ &= x\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{1}{\sqrt{-1+x}}\,dx\\ &= -\sqrt{-1+x}+x\sec^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$x\sec^{-1}\left(\sqrt{x}\right)-\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]],x]
```

```
[Out] -Sqrt[-1 + x] + x*ArcSec[Sqrt[x]]
```

fricas [A] time = 0.93, size = 14, normalized size = 0.78

$$x\,\mathrm{arcsec}\left(\sqrt{x}\right)-\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2)),x, algorithm="fricas")
```

```
[Out] x*arcsec(sqrt(x)) - sqrt(x - 1)
```

giac [B] time = 0.13, size = 41, normalized size = 2.28

$$x \arccos\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{2}\sqrt{x}\left(\sqrt{-\frac{1}{x}+1} - 1\right) + \frac{1}{2\sqrt{x}\left(\sqrt{-\frac{1}{x}+1} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arccos(1/sqrt(x)) - 1/2*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/2/(sqrt(x)*(sqrt
(-1/x + 1) - 1))
```

maple [A] time = 0.05, size = 25, normalized size = 1.39

$$x \operatorname{arcsec}\left(\sqrt{x}\right) - \frac{x-1}{\sqrt{\frac{x-1}{x}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2)),x)
```

```
[Out] x*arcsec(x^(1/2))-1/((x-1)/x)^(1/2)/x^(1/2)*(x-1)
```

maxima [A] time = 0.33, size = 21, normalized size = 1.17

$$x \operatorname{arcsec}\left(\sqrt{x}\right) - \sqrt{x}\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arcsec(sqrt(x)) - sqrt(x)*sqrt(-1/x + 1)
```

mupad [B] time = 1.15, size = 21, normalized size = 1.17

$$x \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) - \sqrt{x}\sqrt{1-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2)),x)
```

```
[Out] x*acos(1/x^(1/2)) - x^(1/2)*(1 - 1/x)^(1/2)
```

sympy [C] time = 4.42, size = 29, normalized size = 1.61

$$x \operatorname{asec}\left(\sqrt{x}\right) - \frac{\begin{cases} 2\sqrt{x-1} & \text{for}\ |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2)),x)
```

```
[Out] x*asec(sqrt(x)) - Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x),
True))/2
```

3.6 $\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=56

$$i\text{Li}_2\left(-e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)+i\sec^{-1}\left(\sqrt{x}\right)^2-2\sec^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] I*arcsec(x^(1/2))^2-2*arcsec(x^(1/2))*ln(1+(1/x^(1/2)+I*(1-1/x)^(1/2))^2)+I
*polylog(2,-(1/x^(1/2)+I*(1-1/x)^(1/2))^2)
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5218, 4626, 3719, 2190, 2279, 2391}

$$i\text{PolyLog}\left(2,-e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)+i\sec^{-1}\left(\sqrt{x}\right)^2-2\sec^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x,x]
```

```
[Out] I*ArcSec[Sqrt[x]]^2 - 2*ArcSec[Sqrt[x]]*Log[1 + E^((2*I)*ArcSec[Sqrt[x]])]
+ I*PolyLog[2, -E^((2*I)*ArcSec[Sqrt[x]])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5218

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\sec^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= 2\,\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 4i\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + 2\,\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \cos^{-}\right.\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) - i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}\left(\right.}\right.\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.96

$$i\left(\text{Li}_2\left(-e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right) + \sec^{-1}\left(\sqrt{x}\right)\left(\sec^{-1}\left(\sqrt{x}\right) + 2i\log\left(1+e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x,x]
```

```
[Out] I*(ArcSec[Sqrt[x]]*(ArcSec[Sqrt[x]] + (2*I)*Log[1 + E^((2*I)*ArcSec[Sqrt[x]
])]) + PolyLog[2, -E^((2*I)*ArcSec[Sqrt[x]])])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcsec}\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(sqrt(x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsec}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(sqrt(x))/x, x)
```

maple [A] time = 0.12, size = 63, normalized size = 1.12

$$i\text{arcsec}\left(\sqrt{x}\right)^2 - 2\,\text{arcsec}\left(\sqrt{x}\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+i\sqrt{1-\frac{1}{x}}\right)^2\right) + i\,\text{polylog}\left(2, -\left(\frac{1}{\sqrt{x}}+i\sqrt{1-\frac{1}{x}}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x,x)
```

```
[Out] I*arcsec(x^(1/2))^2-2*arcsec(x^(1/2))*ln(1+(1/x^(1/2)+I*(1-1/x)^(1/2))^2)+I
*polylog(2,-(1/x^(1/2)+I*(1-1/x)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsec(sqrt(x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x,x)
```

```
[Out] int(acos(1/x^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x,x)
```

```
[Out] Integral(asec(sqrt(x))/x, x)
```

3.7 $\int \frac{\sec^{-1}(\sqrt{x})}{x^2} \, dx$

Optimal. Leaf size=38

$$\frac{\sqrt{x-1}}{2x}+\frac{1}{2}\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arcsec(x^(1/2))/x+1/2*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5270, 12, 51, 63, 203}

$$\frac{\sqrt{x-1}}{2x}+\frac{1}{2}\tan^{-1}\left(\sqrt{x-1}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x^2,x]
```

```
[Out] Sqrt[-1 + x]/(2*x) - ArcSec[Sqrt[x]]/x + ArcTan[Sqrt[-1 + x]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5270

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSec[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(m
+ 1)*Sqrt[u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[
u^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\int \frac{1}{2\sqrt{-1+x}\,x^2}\,dx\\
&= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx\\
&= \frac{\sqrt{-1+x}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,x}\,dx\\
&= \frac{\sqrt{-1+x}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= \frac{\sqrt{-1+x}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\tan^{-1}\left(\sqrt{-1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.84

$$
\frac{\sqrt{x-1}-x\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)-2\sec^{-1}\left(\sqrt{x}\right)}{2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x^2,x]
[Out] (Sqrt[-1 + x] - 2*ArcSec[Sqrt[x]] - x*ArcSin[1/Sqrt[x]])/(2*x)
```

fricas [A] time = 1.59, size = 19, normalized size = 0.50

$$
\frac{(x-2)\operatorname{arcsec}\left(\sqrt{x}\right)+\sqrt{x-1}}{2\,x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^2,x, algorithm="fricas")
[Out] 1/2*((x - 2)*arcsec(sqrt(x)) + sqrt(x - 1))/x
```

giac [A] time = 0.13, size = 30, normalized size = 0.79

$$
\frac{\sqrt{-\frac{1}{x}+1}}{2\sqrt{x}}-\frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{x}+\frac{1}{2}\arccos\left(\frac{1}{\sqrt{x}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^2,x, algorithm="giac")
[Out] 1/2*sqrt(-1/x + 1)/sqrt(x) - arccos(1/sqrt(x))/x + 1/2*arccos(1/sqrt(x))
```

maple [A] time = 0.05, size = 46, normalized size = 1.21

$$
-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x-1}\left(\arctan\left(\frac{1}{\sqrt{x-1}}\right)x-\sqrt{x-1}\right)}{2\sqrt{\frac{x-1}{x}}\,x^{\frac{3}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x^2,x)
```

```
[Out] -arcsec(x^(1/2))/x-1/2*(x-1)^(1/2)*(arctan(1/(x-1)^(1/2))*x-(x-1)^(1/2))/((
x-1)/x)^(1/2)/x^(3/2)
```

maxima [A] time = 0.42, size = 51, normalized size = 1.34

$$-\frac{\sqrt{x}\sqrt{-\frac{1}{x}+1}}{2\left(x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\arctan\left(\sqrt{x}\sqrt{-\frac{1}{x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x)*sqrt(-1/x + 1)/(x*(1/x - 1) - 1) - arcsec(sqrt(x))/x + 1/2*arc
tan(sqrt(x)*sqrt(-1/x + 1))
```

mupad [B] time = 0.68, size = 28, normalized size = 0.74

$$\frac{\sqrt{1-\frac{1}{x}}}{2\sqrt{x}}-\frac{\operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)\left(\frac{2}{x}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x^2,x)
```

```
[Out] (1 - 1/x)^(1/2)/(2*x^(1/2)) - (acos(1/x^(1/2))*(2/x - 1))/2
```

sympy [C] time = 20.05, size = 76, normalized size = 2.00

$$\frac{\begin{cases} i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)+\frac{i\sqrt{-1+\frac{1}{x}}}{\sqrt{x}} & \text{for}\ \frac{1}{|x|}>1 \\ -\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)+\frac{1}{\sqrt{x}\sqrt{1-\frac{1}{x}}}-\frac{1}{x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}}{2}-\frac{\operatorname{asec}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x**2,x)
```

```
[Out] Piecewise((I*acosh(1/sqrt(x)) + I*sqrt(-1 + 1/x)/sqrt(x), 1/Abs(x) > 1), (-
asin(1/sqrt(x)) + 1/(sqrt(x)*sqrt(1 - 1/x)) - 1/(x**(3/2)*sqrt(1 - 1/x)), T
rue))/2 - asec(sqrt(x))/x
```

3.8 $\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{x-1}}{8x^2}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{x-1}}{16x}+\frac{3}{16}\tan^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] -1/2*arcsec(x^(1/2))/x^2+3/16*arctan((-1+x)^(1/2))+1/8*(-1+x)^(1/2)/x^2+3/1
6*(-1+x)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5270, 12, 51, 63, 203}

$$\frac{\sqrt{x-1}}{8x^2}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{x-1}}{16x}+\frac{3}{16}\tan^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x^3,x]
```

```
[Out] Sqrt[-1 + x]/(8*x^2) + (3*Sqrt[-1 + x])/(16*x) - ArcSec[Sqrt[x]]/(2*x^2) +
(3*ArcTan[Sqrt[-1 + x]])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5270

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSec[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(m
+ 1)*Sqrt[u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[
u^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !Function
```

```
OfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\int \frac{1}{2\sqrt{-1+x}\,x^3}\,dx\\
&= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx\\
&= \frac{\sqrt{-1+x}}{8x^2}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{16}\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx\\
&= \frac{\sqrt{-1+x}}{8x^2}+\frac{3\sqrt{-1+x}}{16x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{32}\int \frac{1}{\sqrt{-1+x}\,x}\,dx\\
&= \frac{\sqrt{-1+x}}{8x^2}+\frac{3\sqrt{-1+x}}{16x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{16}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= \frac{\sqrt{-1+x}}{8x^2}+\frac{3\sqrt{-1+x}}{16x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{16}\tan^{-1}\left(\sqrt{-1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.02

$$\sqrt{\frac{x-1}{x}}\left(\frac{1}{8x^{3/2}}+\frac{3}{16\sqrt{x}}\right)-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{3}{16}\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x^3,x]
[Out] (1/(8*x^(3/2)) + 3/(16*Sqrt[x]))*Sqrt[(-1 + x)/x] - ArcSec[Sqrt[x]]/(2*x^2)
 - (3*ArcSin[1/Sqrt[x]])/16
```

fricas [A] time = 1.69, size = 29, normalized size = 0.54

$$\frac{\left(3\,x^2-8\right)\mathrm{arcsec}\left(\sqrt{x}\right)+(3\,x+2)\sqrt{x-1}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/16*((3*x^2 - 8)*arcsec(sqrt(x)) + (3*x + 2)*sqrt(x - 1))/x^2
```

giac [A] time = 0.14, size = 44, normalized size = 0.81

$$\frac{3\sqrt{-\frac{1}{x}+1}}{16\sqrt{x}}+\frac{\sqrt{-\frac{1}{x}+1}}{8\,x^{\frac{3}{2}}}-\frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{2\,x^2}+\frac{3}{16}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^3,x, algorithm="giac")
[Out] 3/16*sqrt(-1/x + 1)/sqrt(x) + 1/8*sqrt(-1/x + 1)/x^(3/2) - 1/2*arccos(1/sqr
t(x))/x^2 + 3/16*arccos(1/sqrt(x))
```

maple [A] time = 0.05, size = 57, normalized size = 1.06

$$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{x-1}\,\left(-3\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{2}+3x\sqrt{x-1}\,+2\sqrt{x-1}\right)}{16\sqrt{\frac{x-1}{x}}\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arcsec(x^(1/2))/x^2+1/16*(x-1)^(1/2)*(-3*arctan(1/(x-1)^(1/2))*x^2+3*x
*(x-1)^(1/2)+2*(x-1)^(1/2))/((x-1)/x)^(1/2)/x^(5/2)
```

maxima [B] time = 0.44, size = 80, normalized size = 1.48

$$\frac{3\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}+5\,\sqrt{x}\,\sqrt{-\frac{1}{x}+1}}{16\left(x^{2}\left(\frac{1}{x}-1\right)^{2}-2\,x\left(\frac{1}{x}-1\right)+1\right)}-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2\,x^{2}}+\frac{3}{16}\,\arctan\left(\sqrt{x}\,\sqrt{-\frac{1}{x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*x^(3/2)*(-1/x + 1)^(3/2) + 5*sqrt(x)*sqrt(-1/x + 1))/(x^2*(1/x - 1)
^2 - 2*x*(1/x - 1) + 1) - 1/2*arcsec(sqrt(x))/x^2 + 3/16*arctan(sqrt(x)*sqr
t(-1/x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x^3,x)
```

```
[Out] int(acos(1/x^(1/2))/x^3, x)
```

sympy [C] time = 53.24, size = 144, normalized size = 2.67

$$\frac{\begin{cases}\frac{3i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4}-\frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}}+\frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}}+\frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for}\ \frac{1}{|x|}>1\\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4}+\frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}}-\frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}}-\frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise}\end{cases}}{4}-\frac{\operatorname{asec}\left(\sqrt{x}\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x**3,x)
```

```
[Out] Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*x
**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-3
*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1 - 1
/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))/4 - asec(sqrt(x))/(2*x**2)
```

3.9 $\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{x-1}}{18x^3}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5\sqrt{x-1}}{72x^2}+\frac{5\sqrt{x-1}}{48x}+\frac{5}{48}\tan^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] -1/3*arcsec(x^(1/2))/x^3+5/48*arctan((-1+x)^(1/2))+1/18*(-1+x)^(1/2)/x^3+5/
72*(-1+x)^(1/2)/x^2+5/48*(-1+x)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5270, 12, 51, 63, 203}

$$\frac{5\sqrt{x-1}}{72x^2}+\frac{\sqrt{x-1}}{18x^3}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5\sqrt{x-1}}{48x}+\frac{5}{48}\tan^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x^4,x]
```

```
[Out] Sqrt[-1 + x]/(18*x^3) + (5*Sqrt[-1 + x])/(72*x^2) + (5*Sqrt[-1 + x])/(48*x)
 - ArcSec[Sqrt[x]]/(3*x^3) + (5*ArcTan[Sqrt[-1 + x]])/48
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5270

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSec[u]))/(d*(m + 1)), x] - Dist[(b*u)/(d*(m
+ 1)*Sqrt[u^2]), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(u*Sqrt[
u^2 - 1]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
```

```
OfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{3}\int \frac{1}{2\sqrt{-1+x}\,x^4}\,dx\\
&= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{6}\int \frac{1}{\sqrt{-1+x}\,x^4}\,dx\\
&= \frac{\sqrt{-1+x}}{18x^3}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{36}\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx\\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx\\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{96}\int \frac{1}{\sqrt{-1+x}\,x}\,dx\\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\tan^{-1}\left(\sqrt{-1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.66

$$\frac{-15x^3\sin^{-1}\left(\frac{1}{\sqrt{x}}\right)+\sqrt{x-1}\left(15x^2+10x+8\right)-48\sec^{-1}\left(\sqrt{x}\right)}{144x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[-1 + x]*(8 + 10*x + 15*x^2) - 48*ArcSec[Sqrt[x]] - 15*x^3*ArcSin[1/Sq
rt[x]])/(144*x^3)
```

fricas [A] time = 0.54, size = 35, normalized size = 0.51

$$\frac{3\left(5x^3-16\right)\operatorname{arcsec}\left(\sqrt{x}\right)+\left(15x^2+10x+8\right)\sqrt{x-1}}{144x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/144*(3*(5*x^3 - 16)*arcsec(sqrt(x)) + (15*x^2 + 10*x + 8)*sqrt(x - 1))/x^
3
```

giac [A] time = 0.15, size = 58, normalized size = 0.85

$$\frac{5\sqrt{-\frac{1}{x}+1}}{48\sqrt{x}}+\frac{5\sqrt{-\frac{1}{x}+1}}{72x^{\frac{3}{2}}}+\frac{\sqrt{-\frac{1}{x}+1}}{18x^{\frac{5}{2}}}-\frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{3x^3}+\frac{5}{48}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 5/48*sqrt(-1/x + 1)/sqrt(x) + 5/72*sqrt(-1/x + 1)/x^(3/2) + 1/18*sqrt(-1/x
+ 1)/x^(5/2) - 1/3*arccos(1/sqrt(x))/x^3 + 5/48*arccos(1/sqrt(x))
```

maple [A] time = 0.05, size = 67, normalized size = 0.99

$$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3x^3}-\frac{\sqrt{x-1}\,\left(15\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^3-15x^2\sqrt{x-1}-10x\sqrt{x-1}-8\sqrt{x-1}\right)}{144\sqrt{\frac{x-1}{x}}\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x^4,x)
```

```
[Out] -1/3*arcsec(x^(1/2))/x^3-1/144*(x-1)^(1/2)*(15*arctan(1/(x-1)^(1/2))*x^3-15
*x^2*(x-1)^(1/2)-10*x*(x-1)^(1/2)-8*(x-1)^(1/2))/((x-1)/x)^(1/2)/x^(7/2)
```

maxima [B] time = 0.43, size = 106, normalized size = 1.56

$$-\frac{15\,x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}}+40\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}+33\,\sqrt{x}\,\sqrt{-\frac{1}{x}+1}}{144\left(x^3\left(\frac{1}{x}-1\right)^3-3\,x^2\left(\frac{1}{x}-1\right)^2+3\,x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3\,x^3}+\frac{5}{48}\arctan\left(\sqrt{x}\,\sqrt{-\frac{1}{x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/144*(15*x^(5/2)*(-1/x + 1)^(5/2) + 40*x^(3/2)*(-1/x + 1)^(3/2) + 33*sqrt
(x)*sqrt(-1/x + 1))/(x^3*(1/x - 1)^3 - 3*x^2*(1/x - 1)^2 + 3*x*(1/x - 1) -
1) - 1/3*arcsec(sqrt(x))/x^3 + 5/48*arctan(sqrt(x)*sqrt(-1/x + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x^4,x)
```

```
[Out] int(acos(1/x^(1/2))/x^4, x)
```

sympy [C] time = 132.62, size = 180, normalized size = 2.65

$$\frac{\begin{cases}\frac{5i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{8}-\frac{5i}{8\sqrt{x}\sqrt{-1+\frac{1}{x}}}+\frac{5i}{24x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}}+\frac{i}{12x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}}+\frac{i}{3x^{\frac{7}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for}\ \frac{1}{|x|}>1\\ -\frac{5\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{8}+\frac{5}{8\sqrt{x}\sqrt{1-\frac{1}{x}}}-\frac{5}{24x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}}-\frac{1}{12x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}}-\frac{1}{3x^{\frac{7}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise}\end{cases}}{6}-\frac{\operatorname{asec}\left(\sqrt{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x**4,x)
```

```
[Out] Piecewise((5*I*acosh(1/sqrt(x))/8 - 5*I/(8*sqrt(x)*sqrt(-1 + 1/x)) + 5*I/(2
4*x**(3/2)*sqrt(-1 + 1/x)) + I/(12*x**(5/2)*sqrt(-1 + 1/x)) + I/(3*x**(7/2)
*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-5*asin(1/sqrt(x))/8 + 5/(8*sqrt(x)*sqrt(
1 - 1/x)) - 5/(24*x**(3/2)*sqrt(1 - 1/x)) - 1/(12*x**(5/2)*sqrt(1 - 1/x)) -
 1/(3*x**(7/2)*sqrt(1 - 1/x)), True))/6 - asec(sqrt(x))/(3*x**3)
```

3.10 $\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=56

$$\frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2}-\frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{3}x^3\cos^{-1}\left(\frac{x}{a}\right)$$

```
[Out] 1/9*a^3*(1-x^2/a^2)^(3/2)+1/3*x^3*arccos(x/a)-1/3*a^3*(1-x^2/a^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5264, 4628, 266, 43}

$$\frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2}-\frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{3}x^3\cos^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a/x],x]
```

```
[Out] -(a^3*Sqrt[1 - x^2/a^2])/3 + (a^3*(1 - x^2/a^2)^(3/2))/9 + (x^3*ArcCos[x/a]
)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5264

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx &= \int x^2 \cos^{-1}\left(\frac{x}{a}\right) dx \\
&= \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\int \frac{x^3}{\sqrt{1-\frac{x^2}{a^2}}} dx}{3a} \\
&= \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x}{a^2}}} dx, x, x^2\right)}{6a} \\
&= \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\text{Subst}\left(\int \left(\frac{a^2}{\sqrt{1-\frac{x}{a^2}}} - a^2\sqrt{1-\frac{x}{a^2}}\right) dx, x, x^2\right)}{6a} \\
&= -\frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2} + \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.75

$$\frac{1}{3}x^3 \sec^{-1}\left(\frac{a}{x}\right) - \frac{1}{9}a\left(2a^2+x^2\right)\sqrt{1-\frac{x^2}{a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[a/x],x]
[Out] -1/9*(a*(2*a^2 + x^2)*Sqrt[1 - x^2/a^2]) + (x^3*ArcSec[a/x])/3
```

fricas [A] time = 1.09, size = 39, normalized size = 0.70

$$\frac{1}{3}\,x^3\,\mathrm{arcsec}\left(\frac{a}{x}\right) - \frac{1}{9}\left(2\,a^2x + x^3\right)\sqrt{\frac{a^2-x^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(a/x),x, algorithm="fricas")
[Out] 1/3*x^3*arcsec(a/x) - 1/9*(2*a^2*x + x^3)*sqrt((a^2 - x^2)/x^2)
```

giac [A] time = 0.14, size = 47, normalized size = 0.84

$$\frac{1}{3}\,x^3\arccos\left(\frac{x}{a}\right) - \frac{2}{9}\,a^3\sqrt{-\frac{x^2}{a^2}+1} - \frac{1}{9}\,ax^2\sqrt{-\frac{x^2}{a^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(a/x),x, algorithm="giac")
[Out] 1/3*x^3*arccos(x/a) - 2/9*a^3*sqrt(-x^2/a^2 + 1) - 1/9*a*x^2*sqrt(-x^2/a^2
+ 1)
```

maple [A] time = 0.08, size = 66, normalized size = 1.18

$$-a^3\left(-\frac{x^3\mathrm{arcsec}\left(\frac{a}{x}\right)}{3a^3} + \frac{\left(-1+\frac{a^2}{x^2}\right)\left(\frac{2a^2}{x^2}+1\right)x^4}{9\sqrt{\frac{\left(-1+\frac{a^2}{x^2}\right)x^2}{a^2}}\;a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(a/x),x)
```

```
[Out] -a^3*(-1/3/a^3*x^3*arcsec(a/x)+1/9*(-1+a^2/x^2)*(2*a^2/x^2+1)/((-1+a^2/x^2)
*x^2/a^2)^(1/2)/a^4*x^4)
```

maxima [A] time = 0.43, size = 54, normalized size = 0.96

$$\frac{1}{3}\,x^3\,\mathrm{arcsec}\left(\frac{a}{x}\right)-\frac{2\,a^4\sqrt{-\frac{x^2}{a^2}+1}+a^2x^2\sqrt{-\frac{x^2}{a^2}+1}}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsec(a/x) - 1/9*(2*a^4*sqrt(-x^2/a^2 + 1) + a^2*x^2*sqrt(-x^2/a^2
 + 1))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{x^3\,\mathrm{acos}\left(\frac{x}{a}\right)}{3}-\frac{\sqrt{a^2-x^2}\,\left(2\,a^2+x^2\right)}{9} & \text{if } 0<a \\ \int x^2\,\mathrm{acos}\left(\frac{x}{a}\right)\,dx & \text{if } \neg 0<a \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(x/a),x)
```

```
[Out] piecewise(0 < a, (x^3*acos(x/a))/3 - ((a^2 - x^2)^(1/2)*(2*a^2 + x^2))/9, ~
0 < a, int(x^2*acos(x/a), x))
```

sympy [A] time = 0.49, size = 51, normalized size = 0.91

$$\begin{cases} -\frac{2a^3\sqrt{1-\frac{x^2}{a^2}}}{9}-\frac{ax^2\sqrt{1-\frac{x^2}{a^2}}}{9}+\frac{x^3\,\mathrm{asec}\left(\frac{a}{x}\right)}{3} & \text{for } a\neq 0 \\ \tilde{\infty}x^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(a/x),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(1 - x**2/a**2)/9 - a*x**2*sqrt(1 - x**2/a**2)/9 + x
**3*asec(a/x)/3, Ne(a, 0)), (zoo*x**3, True))
```

3.11 $\int x \sec^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=47

$$-\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{4}a^2\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}x^2\cos^{-1}\left(\frac{x}{a}\right)$$

```
[Out] 1/2*x^2*arccos(x/a)+1/4*a^2*arcsin(x/a)-1/4*a*x*(1-x^2/a^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5264, 4628, 321, 216}

$$-\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{4}a^2\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}x^2\cos^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a/x],x]
```

```
[Out] -(a*x*Sqrt[1 - x^2/a^2])/4 + (x^2*ArcCos[x/a])/2 + (a^2*ArcSin[x/a])/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5264

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\int x \sec^{-1}\left(\frac{a}{x}\right) dx = \int x \cos^{-1}\left(\frac{x}{a}\right) dx$$

$$= \frac{1}{2}x^2 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}} dx}{2a}$$

$$= -\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{2}x^2 \cos^{-1}\left(\frac{x}{a}\right) + \frac{1}{4}a \int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} dx$$

$$= -\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{2}x^2 \cos^{-1}\left(\frac{x}{a}\right) + \frac{1}{4}a^2 \sin^{-1}\left(\frac{x}{a}\right)$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.94

$$\frac{1}{4}\left(-ax\sqrt{1-\frac{x^2}{a^2}} + a^2 \sin^{-1}\left(\frac{x}{a}\right) + 2x^2 \sec^{-1}\left(\frac{a}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a/x],x]
[Out] (-(a*x*Sqrt[1 - x^2/a^2]) + 2*x^2*ArcSec[a/x] + a^2*ArcSin[x/a])/4
```

fricas [A] time = 0.88, size = 38, normalized size = 0.81

$$-\frac{1}{4}x^2\sqrt{\frac{a^2-x^2}{x^2}} - \frac{1}{4}\left(a^2-2x^2\right)\operatorname{arcsec}\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(a/x),x, algorithm="fricas")
[Out] -1/4*x^2*sqrt((a^2 - x^2)/x^2) - 1/4*(a^2 - 2*x^2)*arcsec(a/x)
```

giac [A] time = 0.12, size = 39, normalized size = 0.83

$$-\frac{1}{4}a^2 \arccos\left(\frac{x}{a}\right) + \frac{1}{2}x^2 \arccos\left(\frac{x}{a}\right) - \frac{1}{4}ax\sqrt{-\frac{x^2}{a^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(a/x),x, algorithm="giac")
[Out] -1/4*a^2*arccos(x/a) + 1/2*x^2*arccos(x/a) - 1/4*a*x*sqrt(-x^2/a^2 + 1)
```

maple [B] time = 0.07, size = 93, normalized size = 1.98

$$\frac{x^2 \operatorname{arcsec}\left(\frac{a}{x}\right)}{2} + \frac{a\sqrt{-1+\frac{a^2}{x^2}}\, x \arctan\left(\frac{1}{\sqrt{-1+\frac{a^2}{x^2}}}\right)}{4\sqrt{\frac{\left(-1+\frac{a^2}{x^2}\right)x^2}{a^2}}} - \frac{\left(-1+\frac{a^2}{x^2}\right)x^3}{4a\sqrt{\frac{\left(-1+\frac{a^2}{x^2}\right)x^2}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(a/x),x)
```

```
[Out] 1/2*x^2*arcsec(a/x)+1/4*a*(-1+a^2/x^2)^(1/2)/((-1+a^2/x^2)*x^2/a^2)^(1/2)*x
*arctan(1/(-1+a^2/x^2)^(1/2))-1/4/a*(-1+a^2/x^2)/((-1+a^2/x^2)*x^2/a^2)^(1/
2)*x^3
```

maxima [A] time = 0.43, size = 46, normalized size = 0.98

$$\frac{1}{2}\,x^2\,\mathrm{arcsec}\left(\frac{a}{x}\right) + \frac{a^3\arcsin\left(\frac{x}{a}\right) - a^2x\sqrt{-\frac{x^2}{a^2}+1}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsec(a/x) + 1/4*(a^3*arcsin(x/a) - a^2*x*sqrt(-x^2/a^2 + 1))/a
```

mupad [B] time = 0.63, size = 38, normalized size = 0.81

$$\frac{a^2\,\mathrm{acos}\left(\frac{x}{a}\right)\left(\frac{2\,x^2}{a^2}-1\right)}{4} - \frac{a\,x\,\sqrt{1-\frac{x^2}{a^2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(x/a),x)
```

```
[Out] (a^2*acos(x/a)*((2*x^2)/a^2 - 1))/4 - (a*x*(1 - x^2/a^2)^(1/2))/4
```

sympy [A] time = 0.22, size = 41, normalized size = 0.87

$$\begin{cases} -\frac{a^2\,\mathrm{asec}\left(\frac{a}{x}\right)}{4} - \frac{ax\sqrt{1-\frac{x^2}{a^2}}}{4} + \frac{x^2\,\mathrm{asec}\left(\frac{a}{x}\right)}{2} & \text{for } a \neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(a/x),x)
```

```
[Out] Piecewise((-a**2*asec(a/x)/4 - a*x*sqrt(1 - x**2/a**2)/4 + x**2*asec(a/x)/2
, Ne(a, 0)), (zoo*x**2, True))
```

3.12 $\int \sec^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=26

$$x\cos^{-1}\left(\frac{x}{a}\right) - a\sqrt{1-\frac{x^2}{a^2}}$$

```
[Out] x*arccos(x/a)-a*(1-x^2/a^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5264, 4620, 261}

$$x\cos^{-1}\left(\frac{x}{a}\right) - a\sqrt{1-\frac{x^2}{a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x],x]
```

```
[Out] -(a*Sqrt[1 - x^2/a^2]) + x*ArcCos[x/a]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5264

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^{-1}\left(\frac{a}{x}\right) dx &= \int \cos^{-1}\left(\frac{x}{a}\right) dx \\
&= x\cos^{-1}\left(\frac{x}{a}\right) + \frac{\int \frac{x}{\sqrt{1-\frac{x^2}{a^2}}} dx}{a} \\
&= -a\sqrt{1-\frac{x^2}{a^2}} + x\cos^{-1}\left(\frac{x}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$x\sec^{-1}\left(\frac{a}{x}\right) - a\sqrt{1-\frac{x^2}{a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x],x]
```

```
[Out] -(a*Sqrt[1 - x^2/a^2]) + x*ArcSec[a/x]
```

fricas [A]   time = 0.91, size = 27, normalized size = 1.04

$$x \operatorname{arcsec}\left(\frac{a}{x}\right) - x\sqrt{\frac{a^2 - x^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x),x, algorithm="fricas")
[Out] x*arcsec(a/x) - x*sqrt((a^2 - x^2)/x^2)
```

giac [A]   time = 0.13, size = 28, normalized size = 1.08

$$a\left(\frac{x \arccos\left(\frac{x}{a}\right)}{a} - \sqrt{-\frac{x^2}{a^2} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x),x, algorithm="giac")
[Out] a*(x*arccos(x/a)/a - sqrt(-x^2/a^2 + 1))
```

maple [B]   time = 0.06, size = 51, normalized size = 1.96

$$-a\left(-\frac{x \operatorname{arcsec}\left(\frac{a}{x}\right)}{a} + \frac{x^2\left(-1 + \frac{a^2}{x^2}\right)}{\sqrt{\frac{\left(-1+\frac{a^2}{x^2}\right)x^2}{a^2}}\, a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x),x)
[Out] -a*(-1/a*x*arcsec(a/x)+1/((-1+a^2/x^2)*x^2/a^2)^(1/2)/a^2*x^2*(-1+a^2/x^2))
```

maxima [A]   time = 0.34, size = 24, normalized size = 0.92

$$x \operatorname{arcsec}\left(\frac{a}{x}\right) - a\sqrt{-\frac{x^2}{a^2} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x),x, algorithm="maxima")
[Out] x*arcsec(a/x) - a*sqrt(-x^2/a^2 + 1)
```

mupad [B]   time = 0.11, size = 24, normalized size = 0.92

$$x \operatorname{acos}\left(\frac{x}{a}\right) - a\sqrt{1 - \frac{x^2}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a),x)
[Out] x*acos(x/a) - a*(1 - x^2/a^2)^(1/2)
```

sympy [A]   time = 0.15, size = 22, normalized size = 0.85

$$\begin{cases} -a\sqrt{1 - \frac{x^2}{a^2}} + x \operatorname{asec}\left(\frac{a}{x}\right) & \text{for } a \neq 0 \\ \tilde{\infty} x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x),x)
[Out] Piecewise((-a*sqrt(1 - x**2/a**2) + x*asec(a/x), Ne(a, 0)), (zoo*x, True))
```

3.13 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2+\cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right)$$

```
[Out] -1/2*I*arccos(x/a)^2+arccos(x/a)*ln(1+(x/a+I*(1-x^2/a^2)^(1/2))^2)-1/2*I*po
lylog(2,-(x/a+I*(1-x^2/a^2)^(1/2))^2)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5264, 4626, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2+\cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x,x]
```

```
[Out] (-I/2)*ArcCos[x/a]^2 + ArcCos[x/a]*Log[1 + E^((2*I)*ArcCos[x/a])] - (I/2)*P
olyLog[2, -E^((2*I)*ArcCos[x/a])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5264

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}\,dx$$
$$= -\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{x}{a}\right)\right)$$
$$= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + 2i\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{x}{a}\right)\right)$$
$$= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + \cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right) - \text{Subst}\left(\int \log\left(1+e^{2ix}\right)dx, x, \cos^{-1}\left(\frac{x}{a}\right)\right)$$
$$= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + \cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right)$$
$$= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + \cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right) - \frac{1}{2}i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right)$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.00

$$-\frac{1}{2}i\text{Li}_2\left(-e^{2i\sec^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}i\sec^{-1}\left(\frac{a}{x}\right)^2 + \sec^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\sec^{-1}\left(\frac{a}{x}\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a/x]/x,x]
```

```
[Out] (-1/2*I)*ArcSec[a/x]^2 + ArcSec[a/x]*Log[1 + E^((2*I)*ArcSec[a/x])] - (I/2)
*PolyLog[2, -E^((2*I)*ArcSec[a/x])]
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcsec}\left(\frac{a}{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(a/x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsec}\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(a/x)/x, x)
```

maple [A] time = 0.12, size = 76, normalized size = 1.29

$$-\frac{i\text{arcsec}\left(\frac{a}{x}\right)^2}{2} + \text{arcsec}\left(\frac{a}{x}\right)\ln\left(1+\left(\frac{x}{a}+i\sqrt{1-\frac{x^2}{a^2}}\right)^2\right) - \frac{i\,\text{polylog}\left(2, -\left(\frac{x}{a}+i\sqrt{1-\frac{x^2}{a^2}}\right)^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x,x)
```

```
[Out] -1/2*I*arcsec(a/x)^2+arcsec(a/x)*ln(1+(x/a+I*(1-x^2/a^2)^(1/2))^2)-1/2*I*po
lylog(2,-(x/a+I*(1-x^2/a^2)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsec(a/x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x,x)
```

```
[Out] int(acos(x/a)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(\frac{a}{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x,x)
```

```
[Out] Integral(asec(a/x)/x, x)
```

3.14 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2} dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}-\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}$$

```
[Out] -arccos(x/a)/x+arctanh((1-x^2/a^2)^(1/2))/a
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5264, 4628, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}-\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x^2,x]
```

```
[Out] -(ArcCos[x/a]/x) + ArcTanh[Sqrt[1 - x^2/a^2]]/a
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5264

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x^2}\,dx$$
$$= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} - \frac{\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx}{a}$$
$$= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} + a\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{x^2}{a^2}}\right)$$
$$= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}$$

Mathematica [B] time = 0.14, size = 93, normalized size = 3.00

$$\frac{x\sqrt{\frac{a^2}{x^2}-1}\left(\log\left(\frac{a}{x\sqrt{\frac{a^2}{x^2}-1}}+1\right)-\log\left(1-\frac{a}{x\sqrt{\frac{a^2}{x^2}-1}}\right)\right)}{2a^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x]/x^2,x]
```

```
[Out] -(ArcSec[a/x]/x) + (Sqrt[-1 + a^2/x^2]*x*(-Log[1 - a/(Sqrt[-1 + a^2/x^2]*x)
] + Log[1 + a/(Sqrt[-1 + a^2/x^2]*x)]))/(2*a^2*Sqrt[1 - x^2/a^2])
```

fricas [B] time = 2.12, size = 107, normalized size = 3.45

$$-\frac{2\,ax\arctan\left(-\frac{x^2\sqrt{\frac{a^2-x^2}{x^2}}}{a^2-x^2}\right)-2\,(ax-a)\,\mathrm{arcsec}\left(\frac{a}{x}\right)-x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}+a\right)+x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}-a\right)}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*arctan(-x^2*sqrt((a^2 - x^2)/x^2)/(a^2 - x^2)) - 2*(a*x - a)*ar
csec(a/x) - x*log(x*sqrt((a^2 - x^2)/x^2) + a) + x*log(x*sqrt((a^2 - x^2)/x
^2) - a))/(a*x)
```

giac [B] time = 0.14, size = 61, normalized size = 1.97

$$\frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a}-\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a}\right)}{2\,|a|} - \frac{\arccos\left(\frac{x}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a*(log(abs(a + sqrt(a^2 - x^2)))/a - log(abs(-a + sqrt(a^2 - x^2)))/a)/
abs(a) - arccos(x/a)/x
```

maple [A] time = 0.04, size = 41, normalized size = 1.32

$$-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}+\frac{\ln\left(\frac{a}{x}+\frac{a\sqrt{1-\frac{x^2}{a^2}}}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x^2,x)
```

```
[Out] -arcsec(a/x)/x+1/a*ln(a/x+1/x*a*(1-x^2/a^2)^(1/2))
```

maxima [A] time = 0.34, size = 52, normalized size = 1.68

$$-\frac{\frac{2\,a\operatorname{arcsec}\left(\frac{a}{x}\right)}{x}-\log\left(\sqrt{-\frac{x^2}{a^2}+1}+1\right)+\log\left(-\sqrt{-\frac{x^2}{a^2}+1}+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*arcsec(a/x)/x - log(sqrt(-x^2/a^2 + 1) + 1) + log(-sqrt(-x^2/a^2
+ 1) + 1))/a
```

mupad [B] time = 0.60, size = 29, normalized size = 0.94

$$\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{a}-\frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x^2,x)
```

```
[Out] atanh(1/(1 - x^2/a^2)^(1/2))/a - acos(x/a)/x
```

sympy [C] time = 2.17, size = 29, normalized size = 0.94

$$-\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{x}-\frac{\begin{cases}-\operatorname{acosh}\left(\frac{a}{x}\right) & \text{for}\ \left|\frac{a^2}{x^2}\right|>1\\ i\operatorname{asin}\left(\frac{a}{x}\right) & \text{otherwise}\end{cases}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x**2,x)
```

```
[Out] -asec(a/x)/x - Piecewise((-acosh(a/x), Abs(a**2/x**2) > 1), (I*asin(a/x), T
rue))/a
```

3.15 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\cos^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] -1/2*arccos(x/a)/x^2+1/2*(1-x^2/a^2)^(1/2)/a/x
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5264, 4628, 264}

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\cos^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x^3,x]
```

```
[Out] Sqrt[1 - x^2/a^2]/(2*a*x) - ArcCos[x/a]/(2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5264

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx &= \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx\\ &= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{2x^2}-\frac{\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx}{2a}\\ &= \frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\cos^{-1}\left(\frac{x}{a}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.95

$$\frac{x\sqrt{1-\frac{x^2}{a^2}}-a\sec^{-1}\left(\frac{a}{x}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x]/x^3,x]
```

```
[Out] (x*Sqrt[1 - x^2/a^2] - a*ArcSec[a/x])/(2*a*x^2)
```

fricas [A] time = 0.65, size = 39, normalized size = 1.03

$$-\frac{a^2 \operatorname{arcsec}\left(\frac{a}{x}\right) - x^2 \sqrt{\frac{a^2-x^2}{x^2}}}{2\,a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*arcsec(a/x) - x^2*sqrt((a^2 - x^2)/x^2))/(a^2*x^2)
```

giac [A] time = 0.18, size = 61, normalized size = 1.61

$$\frac{a\left(\frac{a+\sqrt{a^2-x^2}}{a^2 x} - \frac{x}{\left(a+\sqrt{a^2-x^2}\right)a^2}\right)}{4\,|a|} - \frac{\arccos\left(\frac{x}{a}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*a*((a + sqrt(a^2 - x^2))/(a^2*x) - x/((a + sqrt(a^2 - x^2))*a^2))/abs(a
) - 1/2*arccos(x/a)/x^2
```

maple [A] time = 0.06, size = 54, normalized size = 1.42

$$-\frac{\frac{a^2 \operatorname{arcsec}\left(\frac{a}{x}\right)}{2x^2} - \frac{x\left(-1+\frac{a^2}{x^2}\right)}{2\sqrt{\frac{\left(-1+\frac{a^2}{x^2}\right)x^2}{a^2}}\;a}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x^3,x)
```

```
[Out] -1/a^2*(1/2*a^2/x^2*arcsec(a/x)-1/2/((-1+a^2/x^2)*x^2/a^2)^(1/2)/a*x*(-1+a^
2/x^2))
```

maxima [A] time = 0.42, size = 32, normalized size = 0.84

$$-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{2\,x^2} + \frac{\sqrt{-\frac{x^2}{a^2}+1}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arcsec(a/x)/x^2 + 1/2*sqrt(-x^2/a^2 + 1)/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x^3,x)
```

```
[Out] int(acos(x/a)/x^3, x)
```

sympy [C] time = 1.25, size = 53, normalized size = 1.39

$$-\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{2x^2}-\frac{\begin{cases}-\frac{\sqrt{\frac{a^2}{x^2}-1}}{a} & \text{for } \left|\frac{a^2}{x^2}\right|>1\\ -\frac{i\sqrt{-\frac{a^2}{x^2}+1}}{a} & \text{otherwise}\end{cases}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x**3,x)
```

```
[Out] -asec(a/x)/(2*x**2) - Piecewise((-sqrt(a**2/x**2 - 1)/a, Abs(a**2/x**2) > 1
), (-I*sqrt(-a**2/x**2 + 1)/a, True))/(2*a)
```

3.16 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4} dx$

Optimal. Leaf size=60

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}-\frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[Out] -1/3*arccos(x/a)/x^3+1/6*arctanh((1-x^2/a^2)^(1/2))/a^3+1/6*(1-x^2/a^2)^(1/
2)/a/x^2
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5264, 4628, 266, 51, 63, 208}

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}-\frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x^4,x]
```

```
[Out] Sqrt[1 - x^2/a^2]/(6*a*x^2) - ArcCos[x/a]/(3*x^3) + ArcTanh[Sqrt[1 - x^2/a^
2]]/(6*a^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
```

```
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5264

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx &= \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx\\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx}{3a}\\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{6a}\\
&= \frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{12a^3}\\
&= \frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{x^2}{a^2}}\right)}{6a}\\
&= \frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 1.15

$$\frac{-2a^3\sec^{-1}\left(\frac{a}{x}\right) + a^2x\sqrt{1-\frac{x^2}{a^2}} + x^3\log\left(\sqrt{1-\frac{x^2}{a^2}}+1\right) - x^3\log(x)}{6a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x]/x^4,x]
```

```
[Out] (a^2*x*Sqrt[1 - x^2/a^2] - 2*a^3*ArcSec[a/x] - x^3*Log[x] + x^3*Log[1 + Sqr
t[1 - x^2/a^2]])/(6*a^3*x^3)
```

fricas [B] time = 0.94, size = 142, normalized size = 2.37

$$-\frac{4\,a^3x^3\arctan\left(-\frac{x^2\sqrt{\frac{a^2-x^2}{x^2}}}{a^2-x^2}\right) - x^3\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}+a\right) + x^3\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}-a\right) - 2\,ax^2\sqrt{\frac{a^2-x^2}{x^2}} - 4\left(a^3x^3-a^3\right)\text{arcse}}{12\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^3*x^3*arctan(-x^2*sqrt((a^2 - x^2)/x^2)/(a^2 - x^2)) - x^3*log(x
*sqrt((a^2 - x^2)/x^2) + a) + x^3*log(x*sqrt((a^2 - x^2)/x^2) - a) - 2*a*x^
2*sqrt((a^2 - x^2)/x^2) - 4*(a^3*x^3 - a^3)*arcsec(a/x))/(a^3*x^3)
```

giac [A] time = 0.12, size = 80, normalized size = 1.33

$$\frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a^3}-\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a^3}+\frac{2\sqrt{a^2-x^2}}{a^2x^2}\right)}{12\,|a|}-\frac{\arccos\left(\frac{x}{a}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*a*(log(abs(a + sqrt(a^2 - x^2)))/a^3 - log(abs(-a + sqrt(a^2 - x^2)))/
a^3 + 2*sqrt(a^2 - x^2)/(a^2*x^2))/abs(a) - 1/3*arccos(x/a)/x^3
```

maple [A] time = 0.06, size = 98, normalized size = 1.63

$$-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{3x^3}+\frac{-1+\frac{a^2}{x^2}}{6a^3\sqrt{\frac{\left(-1+\frac{a^2}{x^2}\right)x^2}{a^2}}}+\frac{\sqrt{-1+\frac{a^2}{x^2}}\,x\ln\left(\frac{a}{x}+\sqrt{-1+\frac{a^2}{x^2}}\right)}{6a^4\sqrt{\frac{\left(-1+\frac{a^2}{x^2}\right)x^2}{a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x^4,x)
```

```
[Out] -1/3*arcsec(a/x)/x^3+1/6/a^3*(-1+a^2/x^2)/((-1+a^2/x^2)*x^2/a^2)^(1/2)+1/6/
a^4*(-1+a^2/x^2)^(1/2)/((-1+a^2/x^2)*x^2/a^2)^(1/2)*x*ln(a/x+(-1+a^2/x^2)^(
1/2))
```

maxima [A] time = 0.42, size = 64, normalized size = 1.07

$$\frac{\frac{\log\left(\frac{2\sqrt{-\frac{x^2}{a^2}+1}}{|x|}+\frac{2}{|x|}\right)}{a^2}+\frac{\sqrt{-\frac{x^2}{a^2}+1}}{x^2}}{6\,a}-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(log(2*sqrt(-x^2/a^2 + 1)/abs(x) + 2/abs(x))/a^2 + sqrt(-x^2/a^2 + 1)/x
^2)/a - 1/3*arcsec(a/x)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x^4,x)
```

```
[Out] int(acos(x/a)/x^4, x)
```

sympy [C] time = 2.52, size = 100, normalized size = 1.67

$$-\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{3x^3}-\frac{\begin{cases}-\frac{\sqrt{\frac{a^2}{x^2}-1}}{2ax}-\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^2} & \text{for } \left|\frac{a^2}{x^2}\right|>1\\ \frac{ia}{2x^3\sqrt{-\frac{a^2}{x^2}+1}}-\frac{i}{2ax\sqrt{-\frac{a^2}{x^2}+1}}+\frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{2a^2} & \text{otherwise}\end{cases}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x**4,x)
```

```
[Out] -asec(a/x)/(3*x**3) - Piecewise((-sqrt(a**2/x**2 - 1)/(2*a*x) - acosh(a/x)/
(2*a**2), Abs(a**2/x**2) > 1), (I*a/(2*x**3*sqrt(-a**2/x**2 + 1)) - I/(2*a*
x*sqrt(-a**2/x**2 + 1)) + I*asin(a/x)/(2*a**2), True))/(3*a)
```

3.17 $\int \frac{\sec^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=69

$$\frac{i\text{Li}_2\left(-e^{2i\sec^{-1}(ax^n)}\right)}{2n}+\frac{i\sec^{-1}\left(ax^n\right)^2}{2n}-\frac{\sec^{-1}\left(ax^n\right)\log\left(1+e^{2i\sec^{-1}(ax^n)}\right)}{n}$$

```
[Out] 1/2*I*arcsec(a*x^n)^2/n-arcsec(a*x^n)*ln(1+(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(
1/2))^2)/n+1/2*I*polylog(2,-(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(1/2))^2)/n
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5218, 4626, 3719, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(ax^n)}\right)}{2n}+\frac{i\sec^{-1}\left(ax^n\right)^2}{2n}-\frac{\sec^{-1}\left(ax^n\right)\log\left(1+e^{2i\sec^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a*x^n]/x,x]
```

```
[Out] ((I/2)*ArcSec[a*x^n]^2)/n - (ArcSec[a*x^n]*Log[1 + E^((2*I)*ArcSec[a*x^n])]
)/n + ((I/2)*PolyLog[2, -E^((2*I)*ArcSec[a*x^n])])/n
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5218

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}(ax^n)}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sec^{-1}(ax)}{x}\,dx, x, x^n\right)}{n} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, x^{-n}\right)}{n} \\
&= \frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cos^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right)dx, x, \cos^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n} \\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cos^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} - \frac{i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n} \\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cos^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n}
\end{aligned}
$$

Mathematica [C] time = 0.09, size = 60, normalized size = 0.87

$$
\frac{x^{-n}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{x^{-2n}}{a^2}\right)}{an} + \log(x)\left(\sin^{-1}\left(\frac{x^{-n}}{a}\right) + \sec^{-1}(ax^n)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a*x^n]/x,x]
```

```
[Out] HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, 1/(a^2*x^(2*n))]/(a*n*x^n) +
 (ArcSec[a*x^n] + ArcSin[1/(a*x^n)])*Log[x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\text{arcsec}(ax^n)}{x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(a*x^n)/x, x)
```

maple [A] time = 0.16, size = 98, normalized size = 1.42

$$\frac{i\mathrm{arcsec}\left(a\,x^{n}\right)^{2}}{2n}-\frac{\mathrm{arcsec}\left(a\,x^{n}\right)\ln\left(1+\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^{2}}}\right)^{2}\right)}{n}+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^{2}}}\right)^{2}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a*x^n)/x,x)
```

```
[Out] 1/2*I*arcsec(a*x^n)^2/n-arcsec(a*x^n)*ln(1+(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(
1/2))^2)/n+1/2*I*polylog(2,-(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(1/2))^2)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^{2}n\int\frac{\sqrt{ax^{n}+1}\,\sqrt{ax^{n}-1}\,\log(x)}{a^{4}xx^{2\,n}-a^{2}x}\,dx+\arctan\left(\sqrt{ax^{n}+1}\,\sqrt{ax^{n}-1}\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a^2*n*integrate(sqrt(a*x^n + 1)*sqrt(a*x^n - 1)*log(x)/(a^4*x*x^(2*n) - a^
2*x), x) + arctan(sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{acos}\left(\frac{1}{a\,x^{n}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a*x^n))/x,x)
```

```
[Out] int(acos(1/(a*x^n))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{asec}\left(ax^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a*x**n)/x,x)
```

```
[Out] Integral(asec(a*x**n)/x, x)
```

3.18 $\int x^4 \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=197

$$\frac{a^5 \sec^{-1}(a+bx)}{5b^5}+\frac{\left(53a^2+20\right)a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5}-\frac{\left(58a^2+9\right)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5}-\frac{\left(40a^4+40a^2+3\right)\tanh}{40b^5}$$

```
[Out] 1/5*a^5*arcsec(b*x+a)/b^5+1/5*x^5*arcsec(b*x+a)-1/40*(40*a^4+40*a^2+3)*arct
anh((1-1/(b*x+a)^2)^(1/2))/b^5+1/30*a*(53*a^2+20)*(b*x+a)*(1-1/(b*x+a)^2)^(
1/2)/b^5+11/60*a*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/20*x^3*(b*x+a)*(1-
1/(b*x+a)^2)^(1/2)/b^2-1/120*(58*a^2+9)*(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^5
```

Rubi [A] time = 0.23, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5258, 4426, 3782, 4056, 4048, 3770, 3767, 8}

$$\frac{\left(53a^2+20\right)a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5}-\frac{\left(58a^2+9\right)(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5}+\frac{a^5 \sec^{-1}(a+bx)}{5b^5}-\frac{\left(40a^4+40a^2+3\right)\tanh}{40b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSec[a + b*x],x]
```

```
[Out] (a*(20 + 53*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(30*b^5) + (11*a*x^2*(
a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(60*b^3) - (x^3*(a + b*x)*Sqrt[1 - (a +
b*x)^(-2)])/(20*b^2) - ((9 + 58*a^2)*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)])/
(120*b^5) + (a^5*ArcSec[a + b*x])/(5*b^5) + (x^5*ArcSec[a + b*x])/5 - ((3 +
 40*a^2 + 40*a^4)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(40*b^5)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^4 \sec^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))^4\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^5} \\
&= \frac{1}{5}x^5\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^5\,dx, x, \sec^{-1}(a+bx)\right)}{5b^5} \\
&= -\frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{1}{5}x^5\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^2\left(-4a^3+3\left(1\right.\right.\right.}{} \\
&= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{1}{5}x^5\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\right.}{} \\
&= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} - \frac{\left(9+58a^2\right)(a+bx)^2\sqrt{1-\frac{}{(a+}}}{120b^5} \\
&= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} - \frac{\left(9+58a^2\right)(a+bx)^2\sqrt{1-\frac{}{(a+}}}{120b^5} \\
&= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} - \frac{\left(9+58a^2\right)(a+bx)^2\sqrt{1-\frac{}{(a+}}}{120b^5} \\
&= \frac{a\left(20+53a^2\right)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5} + \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{}{(}}}{20b^2}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 173, normalized size = 0.88

$$\frac{-24a^5\sin^{-1}\left(\frac{1}{a+bx}\right) + \sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}\left(-9\left(4a^2+1\right)b^2x^2 + 2a\left(48a^2+31\right)bx + a^2\left(154a^2+71\right) + 16ab^3x^3 - 6\right.}{120b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSec[a + b*x],x]
```

```
[Out] (Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^2*(71 + 154*a^2) + 2*a
*(31 + 48*a^2)*b*x - 9*(1 + 4*a^2)*b^2*x^2 + 16*a*b^3*x^3 - 6*b^4*x^4) + 24
*b^5*x^5*ArcSec[a + b*x] - 24*a^5*ArcSin[(a + b*x)^(-1)] - 3*(3 + 40*a^2 +
40*a^4)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]
)])/(120*b^5)
```

fricas [A] time = 0.75, size = 152, normalized size = 0.77

$$\frac{24 b^5x^5 \operatorname{arcsec}(bx+a) + 48 a^5 \arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right) + 3\left(40a^4+40a^2+3\right)\log\left(-bx-a+\right.}{120 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^5*x^5*arcsec(b*x + a) + 48*a^5*arctan(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)) + 3*(40*a^4 + 40*a^2 + 3)*log(-b*x - a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1)) - (6*b^3*x^3 - 22*a*b^2*x^2 - 154*a^3 + (58*a^2 + 9)*
b*x - 71*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^5
```

giac [B] time = 0.18, size = 409, normalized size = 2.08

$$-\frac{1}{960}b\left(\frac{192(bx+a)^5\left(\frac{5a}{bx+a}-\frac{10a^2}{(bx+a)^2}+\frac{10a^3}{(bx+a)^3}-\frac{5a^4}{(bx+a)^4}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^6}-\frac{3(bx+a)^4\left(\sqrt{-\frac{1}{(bx+a)^2}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/960*b*(192*(b*x + a)^5*(5*a/(b*x + a) - 10*a^2/(b*x + a)^2 + 10*a^3/(b*x
 + a)^3 - 5*a^4/(b*x + a)^4 - 1)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a
))/b^6 - (3*(b*x + a)^4*(sqrt(-1/(b*x + a)^2 + 1) - 1)^4 + 40*(b*x + a)^3*a
*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 240*(b*x + a)^2*a^2*(sqrt(-1/(b*x + a)^
2 + 1) - 1)^2 + 960*(b*x + a)*a^3*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 24*(b*x
+ a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 360*(b*x + a)*a*(sqrt(-1/(b*x + a
)^2 + 1) - 1) + 24*(40*a^4 + 40*a^2 + 3)*log(-(sqrt(-1/(b*x + a)^2 + 1) - 1
)*abs(b*x + a)) - (120*(8*a^3 + 3*a)*(b*x + a)^3*(sqrt(-1/(b*x + a)^2 + 1)
- 1)^3 + 24*(10*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 40*
(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 3)/((b*x + a)^4*(sqrt(-1/(b*x
+ a)^2 + 1) - 1)^4))/b^6)
```

maple [B] time = 0.09, size = 509, normalized size = 2.58

$$\frac{11\left(-1+(bx+a)^2\right)x^2a}{60b^3\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)}-\frac{29\left(-1+(bx+a)^2\right)x\,a^2}{60b^4\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)}-\frac{3\sqrt{-1+(bx+a)^2}\,\ln\left(bx+a+\sqrt{-1+(bx+a)^2}\right)}{40b^5\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)}+\frac{7}{120b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsec(b*x+a),x)
```

```
[Out] 11/60/b^3*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*x^2*a-29/
60/b^4*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*x*a^2-3/40/b
```

```
^5*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*ln(b*x+a+(
-1+(b*x+a)^2)^(1/2))+71/120/b^5*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(
1/2)/(b*x+a)*a+77/60/b^5*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b
*x+a)*a^3+1/5*x^5*arcsec(b*x+a)-1/20/b^2*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*
x+a)^2)^(1/2)/(b*x+a)*x^3-1/5/b^5*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x
+a)^2)^(1/2)/(b*x+a)*a^5*arctan(1/(-1+(b*x+a)^2)^(1/2))-1/b^5*(-1+(b*x+a)^2
)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^4*ln(b*x+a+(-1+(b*x+a)^2
)^(1/2))-3/40/b^4*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*x
-1/b^5*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^2*ln
(b*x+a+(-1+(b*x+a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}\,x^5\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)-\int\frac{\left(b^2x^6+abx^5\right)e^{\left(\frac{1}{2}\,\log(bx+a+1)+\frac{1}{2}\,\log(bx+a-1)\right)}}{5\left(b^2x^2+2\,abx+a^2+\left(b^2x^2+2\,abx+a^2-1\right)e^{\left(\log(bx+a+1)+\log(bx+a-1)\right)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/5*(b^2*x^
6 + a*b*x^5)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acos(1/(a + b*x)),x)
```

```
[Out] int(x^4*acos(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\,\mathrm{asec}\,(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asec(b*x+a),x)
```

```
[Out] Integral(x**4*asec(a + b*x), x)
```

3.19 $\int x^3 \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{a^4\sec^{-1}(a+bx)}{4b^4}-\frac{\left(17a^2+2\right)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4}+\frac{\left(2a^2+1\right)a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4}+\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}-$$

```
[Out] -1/4*a^4*arcsec(b*x+a)/b^4+1/4*x^4*arcsec(b*x+a)+1/2*a*(2*a^2+1)*arctanh((1
-1/(b*x+a)^2)^(1/2))/b^4-1/12*(17*a^2+2)*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4-
1/12*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2+1/3*a*(b*x+a)^2*(1-1/(b*x+a)^2)^
(1/2)/b^4
```

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5258, 4426, 3782, 4048, 3770, 3767, 8}

$$-\frac{\left(17a^2+2\right)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4}-\frac{a^4\sec^{-1}(a+bx)}{4b^4}+\frac{\left(2a^2+1\right)a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4}-\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSec[a + b*x],x]
```

```
[Out] -((2 + 17*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(12*b^4) - (x^2*(a + b*x
)*Sqrt[1 - (a + b*x)^(-2)])/(12*b^2) + (a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-
2)])/(3*b^4) - (a^4*ArcSec[a + b*x])/(4*b^4) + (x^4*ArcSec[a + b*x])/4 + (a
*(1 + 2*a^2)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(2*b^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sec^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))^3\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^4}$$

$$= \frac{1}{4}x^4\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^4\,dx, x, \sec^{-1}(a+bx)\right)}{4b^4}$$

$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{1}{4}x^4\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))\left(-3a^3+\left(2+\right.\right.\right.}{}$$

$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} + \frac{1}{4}x^4\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} - \frac{a^4\sec^{-1}(a+bx)}{4b^4} + \frac{1}{4}x^4\sec^{-1}(a$$

$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} - \frac{a^4\sec^{-1}(a+bx)}{4b^4} + \frac{1}{4}x^4\sec^{-1}(a$$

$$= -\frac{\left(2+17a^2\right)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4} - \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}$$

Mathematica [A] time = 0.29, size = 150, normalized size = 0.97

$$\frac{3a^4\sin^{-1}\left(\frac{1}{a+bx}\right)+6\left(2a^2+1\right)a\log\left((a+bx)\left(\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}+1\right)\right)-\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}\left(13a^3+9a^2bx-3ab^2\right.}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSec[a + b*x],x]
```

```
[Out] (-(Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(2*a + 13*a^3 + 2*b*x +
 9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)) + 3*b^4*x^4*ArcSec[a + b*x] + 3*a^4*Ar
cSin[(a + b*x)^(-1)] + 6*a*(1 + 2*a^2)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 +
2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(12*b^4)
```

fricas [A] time = 2.07, size = 130, normalized size = 0.84

$$\frac{3\,b^4x^4\,\text{arcsec}\,(bx+a)-6\,a^4\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-6\left(2\,a^3+a\right)\log\left(-bx-a+\sqrt{b^2x^2-\right.}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4*arcsec(b*x + a) - 6*a^4*arctan(-b*x - a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 - 1)) - 6*(2*a^3 + a)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a
^2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b^2*x^2 - 4*a*b*x + 13*a^2 +
2))/b^4
```

giac [B] time = 0.19, size = 299, normalized size = 1.93

$$-\frac{1}{96}b\left(\frac{24\,(bx+a)^4\left(\frac{4\,a}{bx+a}-\frac{6\,a^2}{(bx+a)^2}+\frac{4\,a^3}{(bx+a)^3}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^5}+\frac{(bx+a)^3\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3+12\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/96*b*(24*(b*x + a)^4*(4*a/(b*x + a) - 6*a^2/(b*x + a)^2 + 4*a^3/(b*x + a
)^3 - 1)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^5 + ((b*x + a)^3*(s
qrt(-1/(b*x + a)^2 + 1) - 1)^3 + 12*(b*x + a)^2*a*(sqrt(-1/(b*x + a)^2 + 1)
 - 1)^2 + 72*(b*x + a)*a^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 9*(b*x + a)*(sq
rt(-1/(b*x + a)^2 + 1) - 1) + 48*(2*a^3 + a)*log(-(sqrt(-1/(b*x + a)^2 + 1)
 - 1)*abs(b*x + a)) - (9*(8*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1)
- 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)/((b*x + a)^3*(s
qrt(-1/(b*x + a)^2 + 1) - 1)^3))/b^5)
```

maple [B] time = 0.06, size = 359, normalized size = 2.32

$$\frac{x^4\mathrm{arcsec}\,(bx+a)}{4}-\frac{\left(-1+(bx+a)^2\right)x^2}{12b^2\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)}+\frac{\sqrt{-1+(bx+a)^2}\;a^4\arctan\left(\frac{1}{\sqrt{-1+(bx+a)^2}}\right)}{4b^4\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)}+\frac{\left(-1+(bx+a)^2\right)xa}{3b^3\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(b*x+a),x)
```

```
[Out] 1/4*x^4*arcsec(b*x+a)-1/12/b^2*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1
/2)/(b*x+a)*x^2+1/4/b^4*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/
2)/(b*x+a)*a^4*arctan(1/(-1+(b*x+a)^2)^(1/2))+1/3/b^3*(-1+(b*x+a)^2)/((-1+(
b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*x*a+1/b^4*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x
+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^3*ln(b*x+a+(-1+(b*x+a)^2)^(1/2))-13/12/b^
4*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^2+1/2/b^4*(-1+(
b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a*ln(b*x+a+(-1+(b*
x+a)^2)^(1/2))-1/6/b^4*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x
+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,x^4\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)-\int\frac{\left(b^2x^5+abx^4\right)e^{\left(\frac{1}{2}\,\log(bx+a+1)+\frac{1}{2}\,\log(bx+a-1)\right)}}{4\left(b^2x^2+2\,abx+a^2+\left(b^2x^2+2\,abx+a^2-1\right)e^{\left(\log(bx+a+1)+\log(bx+a-1)\right)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/4*(b^2*x^
5 + a*b*x^4)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(1/(a + b*x)),x)
```

```
[Out] int(x^3*acos(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asec}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(b*x+a),x)
```

```
[Out] Integral(x**3*asec(a + b*x), x)
```

3.20 $\int x^2 \sec^{-1}(a + bx)\, dx$

Optimal. Leaf size=116

$$\frac{a^3 \sec^{-1}(a+bx)}{3b^3}-\frac{\left(6a^2+1\right)\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}+\frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3}-\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2}+\frac{1}{3}x^3\sec^{-1}($$

```
[Out] 1/3*a^3*arcsec(b*x+a)/b^3+1/3*x^3*arcsec(b*x+a)-1/6*(6*a^2+1)*arctanh((1-1/
(b*x+a)^2)^(1/2))/b^3+5/6*a*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/6*x*(b*x+a)
*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5258, 4426, 3782, 3770, 3767, 8}

$$\frac{a^3 \sec^{-1}(a+bx)}{3b^3}-\frac{\left(6a^2+1\right)\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}+\frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3}-\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2}+\frac{1}{3}x^3\sec^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a + b*x],x]
```

```
[Out] (5*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*b^3) - (x*(a + b*x)*Sqrt[1 - (a
 + b*x)^(-2)])/(6*b^2) + (a^3*ArcSec[a + b*x])/(3*b^3) + (x^3*ArcSec[a + b*
x])/3 - ((1 + 6*a^2)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(6*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \sec^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))^2\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{1}{3}x^3\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^3\,dx, x, \sec^{-1}(a+bx)\right)}{3b^3}$$
$$= -\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{1}{3}x^3\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int\left(-2a^3+\left(1+6a^2\right)\sec(x)-\right.\right.}{6b^3}$$
$$= -\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\sec^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+bx) + \frac{(5a)\,\text{Subst}\left(\int \sec\right.}{}$$
$$= -\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\sec^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+bx) - \frac{\left(1+6a^2\right)\tanh^{-1}\left(\right.}{6b^3}$$
$$= \frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3} - \frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\sec^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+$$

Mathematica [A] time = 0.18, size = 131, normalized size = 1.13

$$\frac{-2a^3\sin^{-1}\left(\frac{1}{a+bx}\right)+\left(5a^2+4abx-b^2x^2\right)\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}-\left(6a^2+1\right)\log\left((a+bx)\left(\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}+1\right)\right)+}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[a + b*x],x]
```

```
[Out] ((5*a^2 + 4*a*b*x - b^2*x^2)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^
2] + 2*b^3*x^3*ArcSec[a + b*x] - 2*a^3*ArcSin[(a + b*x)^(-1)] - (1 + 6*a^2)
*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(6*
b^3)
```

fricas [A] time = 2.18, size = 117, normalized size = 1.01

$$\frac{2\,b^3x^3\,\text{arcsec}\,(bx+a)+4\,a^3\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+\left(6\,a^2+1\right)\log\left(-bx-a+\sqrt{b^2x^2+\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arcsec(b*x + a) + 4*a^3*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 - 1)) + (6*a^2 + 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2
- 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 5*a))/b^3
```

giac [B] time = 0.17, size = 204, normalized size = 1.76

$$-\frac{1}{24}b\left(\frac{8\,(bx+a)^3\left(\frac{3\,a}{bx+a}-\frac{3\,a^2}{(bx+a)^2}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^4}-\frac{(bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+12\,(bx+a)a\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/24*b*(8*(b*x + a)^3*(3*a/(b*x + a) - 3*a^2/(b*x + a)^2 - 1)*arccos(-1/((
b*x + a)*(a/(b*x + a) - 1) - a))/b^4 - ((b*x + a)^2*(sqrt(-1/(b*x + a)^2 +
1) - 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*(6*a^2 + 1)*l
og(-(sqrt(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - (12*(b*x + a)*a*(sqrt(-1
/(b*x + a)^2 + 1) - 1) + 1)/((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2))
/b^4)
```

maple [B] time = 0.06, size = 273, normalized size = 2.35

$$\frac{x^3\mathrm{arcsec}\,(bx+a)}{3}-\frac{\sqrt{-1+(bx+a)^2}\;a^3\arctan\left(\frac{1}{\sqrt{-1+(bx+a)^2}}\right)}{3b^3\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\;(bx+a)}-\frac{\left(-1+(bx+a)^2\right)x}{6b^2\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\;(bx+a)}-\frac{\sqrt{-1+(bx+a)^2}\;a^2\ln\left(\right.}{b^3\sqrt{\frac{-1+(b}{(bx+}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(b*x+a),x)
```

```
[Out] 1/3*x^3*arcsec(b*x+a)-1/3/b^3*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^
2)^(1/2)/(b*x+a)*a^3*arctan(1/(-1+(b*x+a)^2)^(1/2))-1/6/b^2*(-1+(b*x+a)^2)/
((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*x-1/b^3*(-1+(b*x+a)^2)^(1/2)/((-1+
(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^2*ln(b*x+a+(-1+(b*x+a)^2)^(1/2))+5/6/
b^3*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a-1/6/b^3*(-1+(
b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*ln(b*x+a+(-1+(b*x+
a)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)-\int\frac{\left(b^2x^4+abx^3\right)e^{\left(\frac{1}{2}\,\log(bx+a+1)+\frac{1}{2}\,\log(bx+a-1)\right)}}{3\left(b^2x^2+2\,abx+a^2+\left(b^2x^2+2\,abx+a^2-1\right)e^{\left(\log(bx+a+1)+\log(bx+a-1)\right)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/3*(b^2*x^
4 + a*b*x^3)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/(a + b*x)),x)
[Out] int(x^2*acos(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asec}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(b*x+a),x)
[Out] Integral(x**2*asec(a + b*x), x)
```

3.21 $\int x \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=78

$$-\frac{a^2\sec^{-1}(a+bx)}{2b^2}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2}+\frac{a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)$$

```
[Out] -1/2*a^2*arcsec(b*x+a)/b^2+1/2*x^2*arcsec(b*x+a)+a*arctanh((1-1/(b*x+a)^2)^
(1/2))/b^2-1/2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5258, 4426, 3773, 3770, 3767, 8}

$$-\frac{a^2\sec^{-1}(a+bx)}{2b^2}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2}+\frac{a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a + b*x],x]
```

```
[Out] -((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(2*b^2) - (a^2*ArcSec[a + b*x])/(2*b^
2) + (x^2*ArcSec[a + b*x])/2 + (a*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x\sec^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^2\,dx, x, \sec^{-1}(a+bx)\right)}{2b^2}\\
&= -\frac{a^2\sec^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int \sec^2(x)\,dx, x, \sec^{-1}(a+bx)\right)}{2b^2} + \frac{a\,\text{S}}{}\\
&= -\frac{a^2\sec^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx) + \frac{a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2} + \frac{\text{Subst}\left(\int 1\,dx, x,\right.}{}\\
&= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2} - \frac{a^2\sec^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx) + \frac{a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+b}}\right.}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 110, normalized size = 1.41

$$
\frac{-(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}} + 2a\log\left((a+bx)\left(\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}+1\right)\right) + a^2\sin^{-1}\left(\frac{1}{a+bx}\right) + b^2x^2\sec^{-1}(a+bx)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a + b*x],x]
```

```
[Out] (-((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]) + b^2*x^2*Ar
cSec[a + b*x] + a^2*ArcSin[(a + b*x)^(-1)] + 2*a*Log[(a + b*x)*(1 + Sqrt[(-
1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(2*b^2)
```

fricas [A] time = 1.44, size = 104, normalized size = 1.33

$$
\frac{b^2x^2\,\text{arcsec}\,(bx+a) - 2\,a^2\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right) - 2\,a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a}\right.}{2\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2*arcsec(b*x + a) - 2*a^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1)) - 2*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^2
```

giac [A] time = 0.16, size = 133, normalized size = 1.71

$$
-\frac{1}{4}b\left(\frac{2\,(bx+a)^2\left(\frac{2\,a}{bx+a}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^3} + \frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right) + 4\,a\log\left(-\left(\sqrt{-\frac{1}{(bx+a)^2}}\right.\right.}{b^3}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*(b*x + a)^2*(2*a/(b*x + a) - 1)*arccos(-1/((b*x + a)*(a/(b*x + a)
 - 1) - a))/b^3 + ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*a*log(-(sqr
```

```
t(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - 1/((b*x + a)*(sqrt(-1/(b*x + a)^
2 + 1) - 1)))/b^3)
```

maple [A] time = 0.05, size = 126, normalized size = 1.62

$$\frac{x^2 \operatorname{arcsec}(bx+a)}{2} - \frac{a^2 \operatorname{arcsec}(bx+a)}{2b^2} + \frac{\sqrt{-1+(bx+a)^2}\, a \ln\left(bx+a+\sqrt{-1+(bx+a)^2}\right)}{b^2 \sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)} - \frac{-1+(bx+a)^2}{2b^2 \sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(b*x+a),x)
```

```
[Out] 1/2*x^2*arcsec(b*x+a)-1/2*a^2*arcsec(b*x+a)/b^2+1/b^2/((-1+(b*x+a)^2)/(b*x+
a)^2)^(1/2)/(b*x+a)*(-1+(b*x+a)^2)^(1/2)*a*ln(b*x+a+(-1+(b*x+a)^2)^(1/2))-1
/2/b^2/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*(-1+(b*x+a)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2 \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right) - \int \frac{\left(b^2x^3+abx^2\right)e^{\left(\frac{1}{2}\log(bx+a+1)+\frac{1}{2}\log(bx+a-1)\right)}}{2\left(b^2x^2+2\,abx+a^2+\left(b^2x^2+2\,abx+a^2-1\right)e^{\left(\log(bx+a+1)+\log(bx+a-1)\right)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/2*(b^2*x^
3 + a*b*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acos}\left(\frac{1}{a+b\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/(a + b*x)),x)
```

```
[Out] int(x*acos(1/(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asec}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(b*x+a),x)
```

```
[Out] Integral(x*asec(a + b*x), x)
```

3.22 $\int \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

```
[Out] (b*x+a)*arcsec(b*x+a)/b-arctanh((1-1/(b*x+a)^2)^(1/2))/b
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5250, 372, 266, 63, 206}

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x])/b - ArcTanh[Sqrt[1 - (a + b*x)^(-2)]]/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5250

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSec[c + d*x]
)/d, x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{-1}(a+bx)\,dx = \frac{(a+bx)\sec^{-1}(a+bx)}{b} - \int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\,dx$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{(a+bx)^2}\right)}{2b}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

Mathematica [B] time = 0.14, size = 115, normalized size = 3.11

$$x\sec^{-1}(a+bx) - \frac{(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}\left(\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2+2abx+b^2x^2-1}}\right) - a\tan^{-1}\left(\sqrt{(a+bx)^2-1}\right)\right)}{b\sqrt{a^2+2abx+b^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x],x]
```

```
[Out] x*ArcSec[a + b*x] - ((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x
)^2]*(-(a*ArcTan[Sqrt[-1 + (a + b*x)^2]]) + ArcTanh[(a + b*x)/Sqrt[-1 + a^2
 + 2*a*b*x + b^2*x^2]]))/(b*Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2])
```

fricas [B] time = 1.16, size = 73, normalized size = 1.97

$$\frac{bx\,\text{arcsec}\,(bx+a) + 2\,a\arctan\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 - 1}\right) + \log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 - 1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*arcsec(b*x + a) + 2*a*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)) + log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

giac [B] time = 0.17, size = 82, normalized size = 2.22

$$\frac{1}{2}\,b\left(\frac{2\,(bx+a)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^2} - \frac{\log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(b*x + a)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^2 - (log(
sqrt(-1/(b*x + a)^2 + 1) + 1) - log(-sqrt(-1/(b*x + a)^2 + 1) + 1))/b^2)
```

maple [A] time = 0.05, size = 51, normalized size = 1.38

$$x \operatorname{arcsec}(bx+a) + \frac{\operatorname{arcsec}(bx+a)\,a}{b} - \frac{\ln\left(bx+a+(bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a),x)
```

```
[Out] x*arcsec(b*x+a)+1/b*arcsec(b*x+a)*a-1/b*ln(b*x+a+(b*x+a)*(1-1/(b*x+a)^2)^(1
/2))
```

maxima [A] time = 0.33, size = 55, normalized size = 1.49

$$\frac{2(bx+a)\operatorname{arcsec}(bx+a) - \log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right) + \log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arcsec(b*x + a) - log(sqrt(-1/(b*x + a)^2 + 1) + 1) + log(
-sqrt(-1/(b*x + a)^2 + 1) + 1))/b
```

mupad [B] time = 0.86, size = 35, normalized size = 0.95

$$-\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x)^2}}}\right) - \operatorname{acos}\left(\frac{1}{a+b\,x}\right)(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x)),x)
```

```
[Out] -(atanh(1/(1 - 1/(a + b*x)^2)^(1/2)) - acos(1/(a + b*x))*(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a),x)
```

```
[Out] Integral(asec(a + b*x), x)
```

3.23 $\int \frac{\sec^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=200

$$-i\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-i\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+b}}{\sqrt{1-a^2}+}\right.$$

```
[Out] -arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)*ln
(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b*x+a)*
ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+1/2*I*polylo
g(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-I*polylog(2,a*(1/(b*x+a)+I*(1-1
/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-I*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] time = 0.31, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5258, 4551, 4530, 3719, 2190, 2279, 2391, 4520}

$$-i\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-i\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)+\sec^{-1}(a+bx)\log\left(1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x,x]
```

```
[Out] ArcSec[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + Ar
cSec[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - ArcS
ec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] - I*PolyLog[2, (a*E^(I*ArcSe
c[a + b*x]))/(1 - Sqrt[1 - a^2])] - I*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/
(1 + Sqrt[1 - a^2])] + (I/2)*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
 Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4530

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tan[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sin[c + d*x]*Tan[c + d*x]^(n -
1))/(a + b*Cos[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4551

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[((e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Cos[c + d*x])
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}(a+bx)}{x}\,dx &= \text{Subst}\left(\int \frac{x\sec(x)\tan(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)\\
&= \text{Subst}\left(\int \frac{x\tan(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)\\
&= a\,\text{Subst}\left(\int \frac{x\sin(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right) + \text{Subst}\left(\int x\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)\\
&= -\left(2i\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)\right) - (ia)\,\text{Subst}\left(\int \frac{e^{ix}x}{1-\sqrt{1-a^2}-ae^{ix}}\,dx, x,\right.\\
&= \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a\\
&= \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a\\
&= \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a
\end{aligned}
$$

Mathematica [A] time = 0.32, size = 284, normalized size = 1.42

$$
-i\left(\text{Li}_2\left(-\frac{\left(\sqrt{1-a^2}-1\right)e^{i\sec^{-1}(a+bx)}}{a}\right)+\text{Li}_2\left(\frac{\left(\sqrt{1-a^2}+1\right)e^{i\sec^{-1}(a+bx)}}{a}\right)\right)+\log\left(1+\frac{\left(\sqrt{1-a^2}-1\right)e^{i\sec^{-1}(a+bx)}}{a}\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a + b*x]/x,x]
```

```
[Out] (-4*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*ArcTan[((1 + a)*Tan[ArcSec[a + b*x]
/2])/Sqrt[1 - a^2]] + (ArcSec[a + b*x] - 2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]
)*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + (ArcSec[a + b*x
] + 2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]])*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*A
rcSec[a + b*x]))/a] - ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] -
I*(PolyLog[2, -(((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a)] + PolyLog[
2, ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a]) + (I/2)*PolyLog[2, -E^((
2*I)*ArcSec[a + b*x])]
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcsec}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)/x, x)
```

maple [A] time = 0.84, size = 374, normalized size = 1.87

$$-\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x,x)
```

```
[Out] -arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-arcsec(b*x+a)*ln
(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+arcsec(b*x+a)*ln((-a*(1/(b*x+a)+I
*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+arcsec(b*x+a)
*ln((a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(
1/2)))-I*dilog((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)/(1
+(-a^2+1)^(1/2)))-I*dilog((a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(
1/2)-1)/(-1+(-a^2+1)^(1/2)))+I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)
))+I*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsec(b*x + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x,x)
```

```
[Out] int(acos(1/(a + b*x))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x,x)
```

```
[Out] Integral(asec(a + b*x)/x, x)
```

3.24 $\int \frac{\sec^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=70

$$\frac{2b\tan^{-1}\left(\frac{\sqrt{a+1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}-\frac{b\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{x}$$

```
[Out] -b*arcsec(b*x+a)/a-arcsec(b*x+a)/x+2*b*arctan((1+a)^(1/2)*tan(1/2*arcsec(b*
x+a))/(1-a)^(1/2))/a/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5258, 4426, 3783, 2659, 205}

$$\frac{2b\tan^{-1}\left(\frac{\sqrt{a+1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}-\frac{b\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x^2,x]
```

```
[Out] -((b*ArcSec[a + b*x])/a) - ArcSec[a + b*x]/x + (2*b*ArcTan[(Sqrt[1 + a]*Tan
[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}(a+bx)}{x^2}\,dx &= b\,\mathrm{Subst}\left(\int \frac{x\sec(x)\tan(x)}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)\\
&= -\frac{\sec^{-1}(a+bx)}{x} + b\,\mathrm{Subst}\left(\int \frac{1}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)\\
&= -\frac{b\sec^{-1}(a+bx)}{a} - \frac{\sec^{-1}(a+bx)}{x} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)}{a}\\
&= -\frac{b\sec^{-1}(a+bx)}{a} - \frac{\sec^{-1}(a+bx)}{x} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1-a+(1+a)x^2}\,dx, x, \tan\left(\frac{1}{2}\sec^{-1}(a+bx\right.\right.}{a}\\
&= -\frac{b\sec^{-1}(a+bx)}{a} - \frac{\sec^{-1}(a+bx)}{x} + \frac{2b\tan^{-1}\left(\frac{\sqrt{1+a}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}
\end{aligned}
$$

Mathematica [C] time = 0.31, size = 112, normalized size = 1.60

$$
-\frac{\sec^{-1}(a+bx)}{x}+\frac{b\left(\sin^{-1}\left(\frac{1}{a+bx}\right)-\frac{i\log\left(\frac{2\left(a\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}(a+bx)+\frac{i\left(a^2+abx-1\right)}{\sqrt{1-a^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/x^2,x]
```

```
[Out] -(ArcSec[a + b*x]/x) + (b*(ArcSin[(a + b*x)^(-1)] - (I*Log[(2*((I*a*(-1 + a
^2 + a*b*x))/Sqrt[1 - a^2] + a*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2
)/(a + b*x)^2]))/(b*x)])/Sqrt[1 - a^2]))/a
```

fricas [B] time = 1.01, size = 281, normalized size = 4.01

$$
\left[-\frac{2\left(a^2-1\right)bx\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{a^2-1}\,bx\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}\,a\right.}{x}\right.}{\left(a^3-a\right)x}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [-(2*(a^2 - 1)*b*x*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - s
qrt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^
2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + (a^3 -
 a)*arcsec(b*x + a))/((a^3 - a)*x), -(2*(a^2 - 1)*b*x*arctan(-b*x - a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-a^2 +
 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + (a
^3 - a)*arcsec(b*x + a))/((a^3 - a)*x)]
```

giac [A] time = 0.19, size = 94, normalized size = 1.34

$$b\left(\frac{2\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}\,a}+\frac{\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{a\left(\frac{a}{bx+a}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] b*(2*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + a)/sqrt(-a^2 + 1))/
(sqrt(-a^2 + 1)*a) + arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a*(a/(b*
x + a) - 1)))
```

maple [B] time = 0.06, size = 154, normalized size = 2.20

$$-\frac{\operatorname{arcsec}(bx+a)}{x}+\frac{b\sqrt{-1+(bx+a)^2}\,\arctan\left(\frac{1}{\sqrt{-1+(bx+a)^2}}\right)}{\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)\,a}-\frac{b\sqrt{-1+(bx+a)^2}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{-1+(bx+a)^2}+2a(bx+a)-2}{bx}\right)}{\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,(bx+a)\,a\sqrt{a^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x^2,x)
```

```
[Out] -arcsec(b*x+a)/x+b*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b
*x+a)/a*arctan(1/(-1+(b*x+a)^2)^(1/2))-b*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^
2)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2-1)^(1/2)*ln(2*((a^2-1)^(1/2)*(-1+(b*x+a)
^2)^(1/2)+a*(b*x+a)-1)/b/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx\int\frac{1}{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,bx^2+\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,ax}\,dx-\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] (x*integrate((b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/
(b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x)*e^
(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt(b*x + a + 1)*sqrt(
b*x + a - 1)))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x^2,x)
```

```
[Out] int(acos(1/(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asec}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x**2,x)
```

```
[Out] Integral(asec(a + b*x)/x**2, x)
```

3.25 $\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=125

$$\frac{b^2\sec^{-1}(a+bx)}{2a^2}-\frac{\left(1-2a^2\right)b^2\tan^{-1}\left(\frac{\sqrt{a+1}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x}-\frac{\sec^{-1}(a+bx)}{2x^2}$$

```
[Out] 1/2*b^2*arcsec(b*x+a)/a^2-1/2*arcsec(b*x+a)/x^2-(-2*a^2+1)*b^2*arctan((1+a)
^(1/2)*tan(1/2*arcsec(b*x+a))/(1-a)^(1/2))/a^2/(-a^2+1)^(3/2)+1/2*b*(b*x+a)
*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x
```

Rubi [A] time = 0.19, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5258, 4426, 3785, 3919, 3831, 2659, 205}

$$\frac{b^2\sec^{-1}(a+bx)}{2a^2}-\frac{\left(1-2a^2\right)b^2\tan^{-1}\left(\frac{\sqrt{a+1}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x}-\frac{\sec^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x^3,x]
```

```
[Out] (b*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(2*a*(1 - a^2)*x) + (b^2*ArcSec[a +
b*x])/(2*a^2) - ArcSec[a + b*x]/(2*x^2) - ((1 - 2*a^2)*b^2*ArcTan[(Sqrt[1 +
 a]*Tan[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(a^2*(1 - a^2)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx = b^2\,\text{Subst}\left(\int \frac{x\sec(x)\tan(x)}{(-a+\sec(x))^3}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= -\frac{\sec^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1-a^2-a\sec(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)}{2a\left(1-a^2\right)}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{}{-a\ldots}\right.}{2a^2\,(\ldots}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{}{1-\ldots}\right.}{2a^2\,(\ldots}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{\left(\left(1-2a^2\right)b^2\right)\text{Subst}\left(\int \frac{}{1-\ldots}\right.}{2a^2\,(\ldots}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{\left(1-2a^2\right)b^2\tan^{-1}\left(\frac{\sqrt{1+a}\,\text{ta}\ldots}{\ldots}\right.}{a^2\left(1-a^2\right)^{3/\ldots}}$$

Mathematica [C] time = 1.05, size = 198, normalized size = 1.58

$$-\frac{\frac{bx(a+bx)\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}}{a\left(a^2-1\right)} + \frac{i\left(2a^2-1\right)b^2x^2\log\left(\frac{4(a-1)a^2(a+1)\left(-\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}\,(a+bx)-\frac{i\left(a^2+abx-1\right)}{\sqrt{1-a^2}}\right)}{\left(2a^2-1\right)b^2x}\right)}{a^2\left(1-a^2\right)^{3/2}} + \frac{b^2x^2\sin^{-1}\left(\frac{1}{a+bx}\right)}{a^2} + \sec^{-1}(a+b\ldots}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/x^3,x]
```

```
[Out] -1/2*((b*x*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])/(a*(
-1 + a^2)) + ArcSec[a + b*x] + (b^2*x^2*ArcSin[(a + b*x)^(-1)])/a^2 + (I*(-
1 + 2*a^2)*b^2*x^2*Log[(4*(-1 + a)*a^2*(1 + a)*(((-I)*(-1 + a^2 + a*b*x))/S
qrt[1 - a^2] - (a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))
/((-1 + 2*a^2)*b^2*x)])/(a^2*(1 - a^2)^(3/2)))/x^2
```

fricas [A] time = 2.71, size = 427, normalized size = 3.42

$$\left[\frac{\left(2\,a^{2}-1\right)\sqrt{a^{2}-1}\,b^{2}x^{2}\log\left(\frac{a^{2}bx+a^{3}+\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\left(a^{2}+\sqrt{a^{2}-1}\,a-1\right)+\left(abx+a^{2}-1\right)\sqrt{a^{2}-1}-a}{x}\right)+2\left(a^{4}-2\,a^{2}+1\right)b^{2}x^{2}\,\mathrm{arcta}\cdots}{2\left(a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*((2*a^2 - 1)*sqrt(a^2 - 1)*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(a^
2 - 1) - a)/x) + 2*(a^4 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 - 1)) - (a^3 - a)*b^2*x^2 - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)*(a^3 - a)*b*x - (a^6 - 2*a^4 + a^2)*arcsec(b*x + a))/((a^6 - 2*a^4 + a^
2)*x^2), -1/2*(2*(2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*arctan(-(sqrt(-a^2 + 1)
*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) - 2*(a^
4 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))
 + (a^3 - a)*b^2*x^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^3 - a)*b*x + (a
^6 - 2*a^4 + a^2)*arcsec(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2)]
```

giac [B] time = 0.22, size = 216, normalized size = 1.73

$$-\frac{1}{2}\,b\left(\frac{2\left(2\,a^{2}b-b\right)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^{2}}+1}-1\right)+a}{\sqrt{-a^{2}+1}}\right)}{\left(a^{4}-a^{2}\right)\sqrt{-a^{2}+1}}+\frac{2\left((bx+a)ab\left(\sqrt{-\frac{1}{(bx+a)^{2}}+1}-1\right)+b\right)}{(bx+a)^{2}\left(\sqrt{-\frac{1}{(bx+a)^{2}}+1}-1\right)^{2}+2\,(bx+a)a\left(\sqrt{-\frac{1}{(bx+a)^{2}}+\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*(2*(2*a^2*b - b)*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) +
a)/sqrt(-a^2 + 1))/((a^4 - a^2)*sqrt(-a^2 + 1)) + 2*((b*x + a)*a*b*(sqrt(-1
/(b*x + a)^2 + 1) - 1) + b)/(((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2
+ 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)*(a^3 - a)) + (2*a*b/(b*
x + a) - b)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a^2*(a/(b*x + a)
- 1)^2))
```

maple [B] time = 0.06, size = 452, normalized size = 3.62

$$-\frac{\mathrm{arcsec}\left(bx+a\right)}{2x^{2}}-\frac{b^{2}\sqrt{-1+(bx+a)^{2}}\,\arctan\left(\frac{1}{\sqrt{-1+(bx+a)^{2}}}\right)}{2\sqrt{\frac{-1+(bx+a)^{2}}{(bx+a)^{2}}}\,(bx+a)\left(a^{2}-1\right)}+\frac{b^{2}\sqrt{-1+(bx+a)^{2}}\,a^{2}\ln\left(\frac{2\sqrt{a^{2}-1}\,\sqrt{-1+(bx+a)^{2}}+2a(bx+\cdots}{bx}\right.}{\sqrt{\frac{-1+(bx+a)^{2}}{(bx+a)^{2}}}\,(bx+a)\left(a^{2}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x^3,x)
```

```
[Out] -1/2*arcsec(b*x+a)/x^2-1/2*b^2*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)
^2)^(1/2)/(b*x+a)/(a^2-1)*arctan(1/(-1+(b*x+a)^2)^(1/2))+b^2*(-1+(b*x+a)^2)
^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^2/(a^2-1)^(5/2)*ln(2*((a^
2-1)^(1/2)*(-1+(b*x+a)^2)^(1/2)+a*(b*x+a)-1)/b/x)+1/2*b^2*(-1+(b*x+a)^2)^(1
/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2-1)*arctan(1/(-1+(b*x+
a)^2)^(1/2))-1/2*b*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/
a/(a^2-1)/x-3/2*b^2*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(
b*x+a)/(a^2-1)^(5/2)*ln(2*((a^2-1)^(1/2)*(-1+(b*x+a)^2)^(1/2)+a*(b*x+a)-1)/
b/x)+1/2*b^2*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/
a^2/(a^2-1)^(5/2)*ln(2*((a^2-1)^(1/2)*(-1+(b*x+a)^2)^(1/2)+a*(b*x+a)-1)/b/x
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2 \int \frac{\left(b^2 x + ab\right) e^{\left(\frac{1}{2} \log(bx+a+1) + \frac{1}{2} \log(bx+a-1)\right)}}{b^2 x^4 + 2\,abx^3 + \left(b^2 x^4 + 2\,abx^3 + \left(a^2-1\right)x^2\right)(bx+a+1)(bx+a-1) + \left(a^2-1\right)x^2}\,dx - \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*integrate(1/2*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*
x + a - 1))/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (
a^2 - 1)*x^2)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt(b*
x + a + 1)*sqrt(b*x + a - 1)))/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x^3,x)
```

```
[Out] int(acos(1/(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(a + bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x**3,x)
```

```
[Out] Integral(asec(a + b*x)/x**3, x)
```

3.26 $\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=181

$$-\frac{b^3\sec^{-1}(a+bx)}{3a^3}-\frac{\left(2-5a^2\right)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\left(1-a^2\right)^2x}+\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\left(1-a^2\right)x^2}+\frac{\left(6a^4-5a^2+2\right)b^3\tan^{-1}\left(\frac{\sqrt{a+1}\tan(\cdots}{\cdots}\right.}{3a^3\left(1-a^2\right)^{5/2}}$$

```
[Out] -1/3*b^3*arcsec(b*x+a)/a^3-1/3*arcsec(b*x+a)/x^3+1/3*(6*a^4-5*a^2+2)*b^3*ar
ctan((1+a)^(1/2)*tan(1/2*arcsec(b*x+a))/(1-a)^(1/2))/a^3/(-a^2+1)^(5/2)+1/6
*b*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x^2-1/6*(-5*a^2+2)*b^2*(b*x+a)*
(1-1/(b*x+a)^2)^(1/2)/a^2/(-a^2+1)^2/x
```

Rubi [A] time = 0.29, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5258, 4426, 3785, 4060, 3919, 3831, 2659, 205}

$$-\frac{\left(2-5a^2\right)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\left(1-a^2\right)^2x}-\frac{b^3\sec^{-1}(a+bx)}{3a^3}+\frac{\left(6a^4-5a^2+2\right)b^3\tan^{-1}\left(\frac{\sqrt{a+1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3\left(1-a^2\right)^{5/2}}+\frac{b(a+bx\cdots}{6a(1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x^4,x]
```

```
[Out] (b*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*a*(1 - a^2)*x^2) - ((2 - 5*a^2)*b
^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*a^2*(1 - a^2)^2*x) - (b^3*ArcSec[
a + b*x])/(3*a^3) - ArcSec[a + b*x]/(3*x^3) + ((2 - 5*a^2 + 6*a^4)*b^3*ArcT
an[(Sqrt[1 + a]*Tan[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(3*a^3*(1 - a^2)^(5/2
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx = b^3\,\text{Subst}\left(\int \frac{x\sec(x)\tan(x)}{(-a+\sec(x))^4}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= -\frac{\sec^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3\,\text{Subst}\left(\int \frac{1}{(-a+\sec(x))^3}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{\sec^{-1}(a+bx)}{3x^3} - \frac{b^3\,\text{Subst}\left(\int \frac{2(1-a^2)-2a\sec(x)-\sec^2(x)}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a-\right.}{6a(1-a^2)}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{\sec^{-1}(a+bx)}{3x^3} + \frac{b^3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\sec^{-1}(a+bx)}{3a^3} - \frac{\sec^{-1}(a+}{3x^3}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\sec^{-1}(a+bx)}{3a^3} - \frac{\sec^{-1}(a+}{3x^3}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\sec^{-1}(a+bx)}{3a^3} - \frac{\sec^{-1}(a+}{3x^3}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x} - \frac{b^3\sec^{-1}(a+bx)}{3a^3} - \frac{\sec^{-1}(a+}{3x^3}$$

Mathematica [C] time = 0.43, size = 241, normalized size = 1.33

$$\frac{1}{6}\left(\frac{2b^3\sin^{-1}\left(\frac{1}{a+bx}\right)}{a^3} - \frac{b\sqrt{\frac{a^2+2abx+b^2x^2-1}{(a+bx)^2}}\left(a^4-4a^3bx-a^2\left(5b^2x^2+1\right)+abx+2b^2x^2\right)}{a^2\left(a^2-1\right)^2x^2} - \frac{i\left(6a^4-5a^2+2\right)b^3\log\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/x^4,x]
```

```
[Out] (-((b*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^4 + a*b*x - 4*a^3
*b*x + 2*b^2*x^2 - a^2*(1 + 5*b^2*x^2)))/(a^2*(-1 + a^2)^2*x^2)) - (2*ArcSe
c[a + b*x])/x^3 + (2*b^3*ArcSin[(a + b*x)^(-1)])/a^3 - (I*(2 - 5*a^2 + 6*a^
4)*b^3*Log[(12*a^3*(-1 + a^2)^2*((I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2] + (a
+ b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))/((2 - 5*a^2 + 6*a
^4)*b^3*x)])/(a^3*(1 - a^2)^(5/2)))/6
```

fricas [A] time = 1.93, size = 548, normalized size = 3.03

$$\left[\frac{\left(6\,a^4-5\,a^2+2\right)\sqrt{a^2-1}\,b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}\,a-1\right)-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-4\left(a^6-3\,a^4+3\,a^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((6*a^4 - 5*a^2 + 2)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(b
^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)
*sqrt(a^2 - 1) - a)/x) - 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x -
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (5*a^5 - 7*a^3 + 2*a)*b^3*x^3 - 2*
(a^9 - 3*a^7 + 3*a^5 - a^3)*arcsec(b*x + a) + ((5*a^5 - 7*a^3 + 2*a)*b^2*x^
2 - (a^6 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9 - 3*a
^7 + 3*a^5 - a^3)*x^3), 1/6*(2*(6*a^4 - 5*a^2 + 2)*sqrt(-a^2 + 1)*b^3*x^3*a
rctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 +
1))/(a^2 - 1)) - 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x - a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (5*a^5 - 7*a^3 + 2*a)*b^3*x^3 - 2*(a^9 - 3
*a^7 + 3*a^5 - a^3)*arcsec(b*x + a) + ((5*a^5 - 7*a^3 + 2*a)*b^2*x^2 - (a^6
 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9 - 3*a^7 + 3*a
^5 - a^3)*x^3)]
```

giac [B] time = 0.22, size = 451, normalized size = 2.49

$$\frac{1}{3}b\left(\frac{\left(6a^4b^2-5a^2b^2+2b^2\right)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{\left(a^7-2a^5+a^3\right)\sqrt{-a^2+1}}+\frac{4(bx+a)^3a^3b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3+10(bx\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b*((6*a^4*b^2 - 5*a^2*b^2 + 2*b^2)*arctan(((b*x + a)*(sqrt(-1/(b*x + a)
^2 + 1) - 1) + a)/sqrt(-a^2 + 1))/((a^7 - 2*a^5 + a^3)*sqrt(-a^2 + 1)) + (4
*(b*x + a)^3*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 10*(b*x + a)^2*a^4*
b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 - (b*x + a)^3*a*b^2*(sqrt(-1/(b*x + a)
^2 + 1) - 1)^3 + (b*x + a)^2*a^2*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 16*
(b*x + a)*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 2*(b*x + a)^2*b^2*(sqrt(
-1/(b*x + a)^2 + 1) - 1)^2 - 7*(b*x + a)*a*b^2*(sqrt(-1/(b*x + a)^2 + 1) -
1) + 5*a^2*b^2 - 2*b^2)/((a^6 - 2*a^4 + a^2)*((b*x + a)^2*(sqrt(-1/(b*x + a
)^2 + 1) - 1)^2 + 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)^2) - (3
*a*b^2/(b*x + a) - 3*a^2*b^2/(b*x + a)^2 - b^2)*arccos(-1/((b*x + a)*(a/(b*
x + a) - 1) - a))/(a^3*(a/(b*x + a) - 1)^3))
```

maple [B] time = 0.07, size = 760, normalized size = 4.20

$$-\frac{\operatorname{arcsec}(bx+a)}{3x^3}+\frac{b^3\sqrt{-1+(bx+a)^2}\;a\arctan\left(\frac{1}{\sqrt{-1+(bx+a)^2}}\right)}{3\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\;(bx+a)\left(a^2-1\right)^2}-\frac{b^3\sqrt{-1+(bx+a)^2}\;a^3\ln\left(\frac{2\sqrt{a^2-1}\;\sqrt{-1+(bx+a)^2}+2a\cdots}{bx}\right)}{\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\;(bx+a)\left(a^2-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x^4,x)
```

```
[Out] -1/3*arcsec(b*x+a)/x^3+1/3*b^3*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)
^2)^(1/2)/(b*x+a)*a/(a^2-1)^2*arctan(1/(-1+(b*x+a)^2)^(1/2))-b^3*(-1+(b*x+a
)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)*a^3/(a^2-1)^(7/2)*ln(2*
((a^2-1)^(1/2)*(-1+(b*x+a)^2)^(1/2)+a*(b*x+a)-1)/b/x)-2/3*b^3*(-1+(b*x+a)^2
)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2-1)^2*arctan(1/(-1+(
b*x+a)^2)^(1/2))+5/6*b^2*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b
```

```
*x+a)/(a^2-1)^2/x+11/6*b^3*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^
(1/2)/(b*x+a)*a/(a^2-1)^(7/2)*ln(2*((a^2-1)^(1/2)*(-1+(b*x+a)^2)^(1/2)+a*(b
*x+a)-1)/b/x)-1/6*b*(-1+(b*x+a)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)
*a/(a^2-1)^2/x^2+1/3*b^3*(-1+(b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1
/2)/(b*x+a)/a^3/(a^2-1)^2*arctan(1/(-1+(b*x+a)^2)^(1/2))-1/3*b^2*(-1+(b*x+a
)^2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2-1)^2/x-7/6*b^3*(-1+(
b*x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2-1)^(7/2)*ln
(2*((a^2-1)^(1/2)*(-1+(b*x+a)^2)^(1/2)+a*(b*x+a)-1)/b/x)+1/6*b*(-1+(b*x+a)^
2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2-1)^2/x^2+1/3*b^3*(-1+(b*
x+a)^2)^(1/2)/((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)/(b*x+a)/a^3/(a^2-1)^(7/2)*ln
(2*((a^2-1)^(1/2)*(-1+(b*x+a)^2)^(1/2)+a*(b*x+a)-1)/b/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3 \int \frac{\left(b^2 x + ab\right) e^{\left(\frac{1}{2} \log(bx+a+1) + \frac{1}{2} \log(bx+a-1)\right)}}{b^2 x^5 + 2\,abx^4 + \left(a^2-1\right)x^3 + \left(b^2 x^5 + 2\,abx^4 + \left(a^2-1\right)x^3\right)(bx+a+1)(bx+a-1)}\,dx - \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^3*integrate(1/3*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*
x + a - 1))/(b^2*x^5 + 2*a*b*x^4 + (a^2 - 1)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (
a^2 - 1)*x^3)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt(b*
x + a + 1)*sqrt(b*x + a - 1)))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x^4,x)
```

```
[Out] int(acos(1/(a + b*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}{\left(a + bx \right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x**4,x)
```

```
[Out] Integral(asec(a + b*x)/x**4, x)
```

3.27 $\int x^3 \sec^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=381

$$-\frac{a^4 \sec^{-1}(a+bx)^2}{4b^4}+\frac{2ia^3\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^4}-\frac{2ia^3\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^4}-\frac{4ia^3\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b^4}+$$

```
[Out] -a*x/b^3+1/12*(b*x+a)^2/b^4-1/4*a^4*arcsec(b*x+a)^2/b^4+1/4*x^4*arcsec(b*x+
a)^2-4*I*a^3*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^4-2*
I*a*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^4+1/3*ln(b*x+
a)/b^4+3*a^2*ln(b*x+a)/b^4-2*I*a^3*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2)))/b^4+2*I*a^3*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4-
I*a*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4+I*a*polylog(2,-I*(
1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4-1/3*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x
+a)^2)^(1/2)/b^4-3*a^2*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4+a*(b
*x+a)^2*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4-1/6*(b*x+a)^3*arcsec(b*x+a)
*(1-1/(b*x+a)^2)^(1/2)/b^4
```

Rubi [A] time = 0.30, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5258, 4426, 4190, 4181, 2279, 2391, 4184, 3475, 4185}

$$\frac{2ia^3\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^4}-\frac{2ia^3\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^4}+\frac{ia\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^4}-\frac{ia\text{PolyLog}\left(2\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSec[a + b*x]^2,x]
```

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) - ((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*A
rcSec[a + b*x])/(3*b^4) - (3*a^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[
a + b*x])/b^4 + (a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/b^
4 - ((a + b*x)^3*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/(6*b^4) - (a^4*A
rcSec[a + b*x]^2)/(4*b^4) + (x^4*ArcSec[a + b*x]^2)/4 - ((2*I)*a*ArcSec[a +
 b*x]*ArcTan[E^(I*ArcSec[a + b*x])])/b^4 - ((4*I)*a^3*ArcSec[a + b*x]*ArcTa
n[E^(I*ArcSec[a + b*x])])/b^4 + Log[a + b*x]/(3*b^4) + (3*a^2*Log[a + b*x])
/b^4 + (I*a*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])])/b^4 + ((2*I)*a^3*PolyLo
g[2, (-I)*E^(I*ArcSec[a + b*x])])/b^4 - (I*a*PolyLog[2, I*E^(I*ArcSec[a + b
*x])])/b^4 - ((2*I)*a^3*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^4
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sec^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int x^2 \sec(x)(-a+\sec(x))^3 \tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^4}$$

$$= \frac{1}{4}x^4 \sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x(-a+\sec(x))^4\,dx, x, \sec^{-1}(a+bx)\right)}{2b^4}$$

$$= \frac{1}{4}x^4 \sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \left(a^4x - 4a^3x\sec(x) + 6a^2x\sec^2(x) - 4ax\sec^3(x) + x\right.\right.}{2b^4}$$

$$= -\frac{a^4\sec^{-1}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x\sec^4(x)\,dx, x, \sec^{-1}(a+bx)\right)}{2b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^4} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^4} - \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\text{se}}{b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^4} - \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\text{se}}{b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^4} - \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\text{se}}{b^4}$$

Mathematica [A] time = 9.42, size = 667, normalized size = 1.75

$$\frac{\left(1-\frac{a}{a+bx}\right)^3\left(-24a\left(2a^2+1\right)\left(2i\left(\text{Li}_2\left(-ie^{-i\sec^{-1}(a+bx)}\right)-\text{Li}_2\left(ie^{-i\sec^{-1}(a+bx)}\right)\right)+\left(\pi-2\sec^{-1}(a+bx)\right)\left(\log\left(1-i\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcSec[a + b*x]^2,x]
```

```
[Out] ((1 - a/(a + b*x))^3*(24*a*(2 + (1 + 2*a^2)*ArcSec[a + b*x]^2) + (2 + (-2 +
 24*a)*ArcSec[a + b*x] + 3*(1 - 4*a + 12*a^2)*ArcSec[a + b*x]^2)/(-1 + Sqrt
[1 - (a + b*x)^(-2)]) + 16*(1 + 9*a^2)*Log[(a + b*x)^(-1)] - 24*a*(1 + 2*a^
2)*((Pi - 2*ArcSec[a + b*x])*(Log[1 - I/E^(I*ArcSec[a + b*x])] - Log[1 + I/
E^(I*ArcSec[a + b*x])]) - Pi*Log[Cot[(Pi + 2*ArcSec[a + b*x])/4]] + (2*I)*(
PolyLog[2, (-I)/E^(I*ArcSec[a + b*x])] - PolyLog[2, I/E^(I*ArcSec[a + b*x])
])) - (3*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a + b*x]/2
])^4 + (4*ArcSec[a + b*x]*(1 + 6*a*ArcSec[a + b*x])*Sin[ArcSec[a + b*x]/2])
/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a + b*x]/2])^3 + (8*(2*ArcSec[a + b*x
] + 18*a^2*ArcSec[a + b*x] + 6*a^3*ArcSec[a + b*x]^2 + 3*a*(2 + ArcSec[a +
b*x]^2))*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a + b
*x]/2]) - (3*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*
x]/2])^4 + (4*ArcSec[a + b*x]*(1 - 6*a*ArcSec[a + b*x])*Sin[ArcSec[a + b*x]
/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])^3 - (2 + (2 - 24*a)*
ArcSec[a + b*x] + 3*(1 - 4*a + 12*a^2)*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b
*x]/2] + Sin[ArcSec[a + b*x]/2])^2 - (8*(-2*ArcSec[a + b*x] - 18*a^2*ArcSec
[a + b*x] + 6*a^3*ArcSec[a + b*x]^2 + 3*a*(2 + ArcSec[a + b*x]^2))*Sin[ArcS
ec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])))/(48*b^4
*(-1 + a/(a + b*x))^3)
```

fricas [F] time = 2.01, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3 \operatorname{arcsec}(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsec(b*x + a)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(a+b*x)]Simplific
ation assuming t_nostep near 0Simplification assuming t_nostep near 0Evalua
tion time: 0.59sym2poly/r2sym(const gen & e,const index_m & i,const vecteur
 & l) Error: Bad Argument Value
```

maple [A] time = 4.00, size = 734, normalized size = 1.93

$$\frac{x^4 \operatorname{arcsec}(bx+a)^2}{4} - \frac{a^4 \operatorname{arcsec}(bx+a)^2}{4b^4} - \frac{5ax}{6b^3} - \frac{\ln\left(1+\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{3b^4} + \frac{2\ln\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{3b^4} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(b*x+a)^2,x)
```

```
[Out] 1/4*x^4*arcsec(b*x+a)^2-1/4*a^4*arcsec(b*x+a)^2/b^4-5/6*a*x/b^3+6/b^4*a^2*l
n(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))-3/b^4*a^2*ln(1+(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2))^2)-1/3*I/b^4*arcsec(b*x+a)-3/2/b^3*((-1+(b*x+a)^2)/(b*x+a)^2)^
(1/2)*arcsec(b*x+a)*x*a^2-11/12/b^4*a^2+1/12/b^2*x^2-1/3/b^4*ln(1+(1/(b*x+a
)+I*(1-1/(b*x+a)^2)^(1/2))^2)+2/3/b^4*ln(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))
+1/b^4*a*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-2/b^4*a^
3*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2/b^4*a^3*arcse
c(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-1/b^4*a*arcsec(b*x+a)*
ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-13/6/b^4*((-1+(b*x+a)^2)/(b*x+a
)^2)^(1/2)*arcsec(b*x+a)*a^3+I/b^4*a*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2)))-1/3/b^4*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*a-1/6/b*((-
1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*x^3-1/3/b^3*((-1+(b*x+a)^2)/(b*
x+a)^2)^(1/2)*arcsec(b*x+a)*x-3*I/b^4*arcsec(b*x+a)*a^2+2*I/b^4*a^3*dilog(1
+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-2*I/b^4*a^3*dilog(1-I*(1/(b*x+a)+I*
(1-1/(b*x+a)^2)^(1/2)))-I/b^4*a*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
)))+1/2/b^2*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*x^2*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^4 \arctan\left(\sqrt{bx+a+1}\sqrt{bx+a-1}\right)^2 - \frac{1}{16}x^4 \log\left(b^2x^2+2abx+a^2\right)^2 - \int \frac{2\sqrt{bx+a+1}\sqrt{bx+a-1}\,bx^4 \operatorname{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/16*x^4*log(b^2*x^
2 + 2*a*b*x + a^2)^2 - integrate(1/4*(2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)
*b*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 4*(b^3*x^6 + 3*a*b^2*x
^5 + (3*a^2 - 1)*b*x^4 + (a^3 - a)*x^3)*log(b*x + a)^2 - (b^3*x^6 + 2*a*b^2
```

```
*x^5 + (a^2 - 1)*b*x^4 + 4*(b^3*x^6 + 3*a*b^2*x^5 + (3*a^2 - 1)*b*x^4 + (a^
3 - a)*x^3)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*
x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(1/(a + b*x))^2,x)
```

```
[Out] int(x^3*acos(1/(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asec}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(b*x+a)**2,x)
```

```
[Out] Integral(x**3*asec(a + b*x)**2, x)
```

3.28 $\int x^2 \sec^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=288

$$\frac{a^3 \sec^{-1}(a+bx)^2}{3b^3}-\frac{2ia^2\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^3}+\frac{2ia^2\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^3}+\frac{4ia^2\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b^3}-\frac{i\text{Li}_2(\ldots}{}$$

```
[Out] 1/3*x/b^2+1/3*a^3*arcsec(b*x+a)^2/b^3+1/3*x^3*arcsec(b*x+a)^2+2/3*I*arcsec(
b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3+4*I*a^2*arcsec(b*x+a)*
arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3-2*a*ln(b*x+a)/b^3-1/3*I*polyl
og(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-2*I*a^2*polylog(2,-I*(1/(b
*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+1/3*I*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*
x+a)^2)^(1/2)))/b^3+2*I*a^2*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))
)/b^3+2*a*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/3*(b*x+a)^2*arc
sec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.23, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5258, 4426, 4190, 4181, 2279, 2391, 4184, 3475, 4185}

$$-\frac{2ia^2\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^3}+\frac{2ia^2\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^3}-\frac{i\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{3b^3}+\frac{i\text{PolyLog}\left(2,ie^{i\ldots}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a + b*x]^2,x]
```

```
[Out] x/(3*b^2) + (2*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/b^3 -
((a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/(3*b^3) + (a^3*ArcSe
c[a + b*x]^2)/(3*b^3) + (x^3*ArcSec[a + b*x]^2)/3 + (((2*I)/3)*ArcSec[a + b
*x]*ArcTan[E^(I*ArcSec[a + b*x])])/b^3 + ((4*I)*a^2*ArcSec[a + b*x]*ArcTan[
E^(I*ArcSec[a + b*x])])/b^3 - (2*a*Log[a + b*x])/b^3 - ((I/3)*PolyLog[2, (-
I)*E^(I*ArcSec[a + b*x])])/b^3 - ((2*I)*a^2*PolyLog[2, (-I)*E^(I*ArcSec[a +
 b*x])])/b^3 + ((I/3)*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^3 + ((2*I)*a^2
*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \sec^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int x^2 \sec(x)(-a+\sec(x))^2 \tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{1}{3}x^3 \sec^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int x(-a+\sec(x))^3\,dx, x, \sec^{-1}(a+bx)\right)}{3b^3}$$
$$= \frac{1}{3}x^3 \sec^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int \left(-a^3x+3a^2x\sec(x)-3ax\sec^2(x)+x\sec^3(x)\right)\,dx, x, \sec\right.}{3b^3}$$
$$= \frac{a^3 \sec^{-1}(a+bx)^2}{3b^3} + \frac{1}{3}x^3 \sec^{-1}(a+bx)^2 - \frac{2\,\text{Subst}\left(\int x\sec^3(x)\,dx, x, \sec^{-1}(a+bx)\right)}{3b^3} + \frac{(}{}$$
$$= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^3} + \frac{a}{}$$
$$= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^3} + \frac{a}{}$$
$$= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^3} + \frac{a}{}$$
$$= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^3} + \frac{a}{}$$

Mathematica [A] time = 6.58, size = 473, normalized size = 1.64

$$\frac{2\left(-6a^2-1\right)\left(2i\left(\text{Li}_2\left(-ie^{-i\sec^{-1}(a+bx)}\right)-\text{Li}_2\left(ie^{-i\sec^{-1}(a+bx)}\right)\right)+\left(\pi-2\sec^{-1}(a+bx)\right)\left(\log\left(1-ie^{-i\sec^{-1}(a+bx)}\right)-\text{lo}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSec[a + b*x]^2,x]
```

```
[Out] (4 + 2*(1 + 6*a^2)*ArcSec[a + b*x]^2 + (ArcSec[a + b*x]*(2 + (-1 + 6*a)*Arc
Sec[a + b*x]))/(-1 + Sqrt[1 - (a + b*x)^(-2)]) + 24*a*Log[(a + b*x)^(-1)] +
 2*(-1 - 6*a^2)*((Pi - 2*ArcSec[a + b*x])*(Log[1 - I/E^(I*ArcSec[a + b*x])]
 - Log[1 + I/E^(I*ArcSec[a + b*x])]) - Pi*Log[Cot[(Pi + 2*ArcSec[a + b*x])/
4]] + (2*I)*(PolyLog[2, (-I)/E^(I*ArcSec[a + b*x])] - PolyLog[2, I/E^(I*Arc
Sec[a + b*x])])) + (2*ArcSec[a + b*x]^2*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec
[a + b*x]/2] - Sin[ArcSec[a + b*x]/2])^3 + (2*(2 + 12*a*ArcSec[a + b*x] + (
1 + 6*a^2)*ArcSec[a + b*x]^2)*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/
2] - Sin[ArcSec[a + b*x]/2]) - (2*ArcSec[a + b*x]^2*Sin[ArcSec[a + b*x]/2])
/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])^3 + (ArcSec[a + b*x]*(2
+ (1 - 6*a)*ArcSec[a + b*x]))/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]
/2])^2 - (2*(2 - 12*a*ArcSec[a + b*x] + (1 + 6*a^2)*ArcSec[a + b*x]^2)*Sin[
ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2]))/(12*
b^3)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2 \operatorname{arcsec}\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsec(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsec}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsec(b*x + a)^2, x)
```

maple [A] time = 1.69, size = 540, normalized size = 1.88

$$-\frac{\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,\operatorname{arcsec}(bx+a)\,x^2}{3b}+\frac{4\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,\operatorname{arcsec}(bx+a)\,xa}{3b^2}+\frac{x^3\operatorname{arcsec}(bx+a)^2}{3}+\frac{5\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,\operatorname{arcsec}}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(b*x+a)^2,x)
```

```
[Out] -1/3/b*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*x^2+4/3/b^2*((-1+(b*x
+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*x*a+1/3*x^3*arcsec(b*x+a)^2+5/3/b^3*(
(-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*a^2-2*I/b^3*a^2*dilog(1+I*(1/
(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2/b^3*a^2*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)
+I*(1-1/(b*x+a)^2)^(1/2)))-2/b^3*a^2*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1
/(b*x+a)^2)^(1/2)))+2*I/b^3*a*arcsec(b*x+a)+1/3*I/b^3*dilog(1-I*(1/(b*x+a)+
I*(1-1/(b*x+a)^2)^(1/2)))+2*I/b^3*a^2*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2)))-1/3/b^3*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))
+1/3/b^3*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+1/3*a^3*
arcsec(b*x+a)^2/b^3-1/3*I/b^3*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))
)+2/b^3*a*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-4/b^3*ln(1/(b*x+a)+I*
(1-1/(b*x+a)^2)^(1/2))*a+1/3/b^3*a+1/3*x/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}x^3\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)^2-\frac{1}{12}x^3\log\left(b^2x^2+2\,abx+a^2\right)^2-\int\frac{2\,\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,bx^3\operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/12*x^3*log(b^2*x^
2 + 2*a*b*x + a^2)^2 - integrate(1/3*(2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)
*b*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*(b^3*x^5 + 3*a*b^2*x
^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 - (b^3*x^5 + 2*a*b^2
*x^4 + (a^2 - 1)*b*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^
3 - a)*x^2)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*
x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/(a + b*x))^2,x)
```

```
[Out] int(x^2*acos(1/(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asec}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(b*x+a)**2,x)
```

```
[Out] Integral(x**2*asec(a + b*x)**2, x)
```

3.29 $\int x \sec^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=154

$$-\frac{a^2 \sec^{-1}(a+bx)^2}{2b^2}+\frac{2ia\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{2ia\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{\log(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a$$

```
[Out] -1/2*a^2*arcsec(b*x+a)^2/b^2+1/2*x^2*arcsec(b*x+a)^2-4*I*a*arcsec(b*x+a)*ar
ctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^2+ln(b*x+a)/b^2+2*I*a*polylog(2,-
I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2-2*I*a*polylog(2,I*(1/(b*x+a)+I*(
1-1/(b*x+a)^2)^(1/2)))/b^2-(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5258, 4426, 4190, 4181, 2279, 2391, 4184, 3475}

$$\frac{2ia\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{2ia\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{a^2\sec^{-1}(a+bx)^2}{2b^2}+\frac{\log(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a + b*x]^2,x]
```

```
[Out] -(((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/b^2) - (a^2*ArcSec[a
 + b*x]^2)/(2*b^2) + (x^2*ArcSec[a + b*x]^2)/2 - ((4*I)*a*ArcSec[a + b*x]*A
rcTan[E^(I*ArcSec[a + b*x])])/b^2 + Log[a + b*x]/b^2 + ((2*I)*a*PolyLog[2,
(-I)*E^(I*ArcSec[a + b*x])])/b^2 - ((2*I)*a*PolyLog[2, I*E^(I*ArcSec[a + b*
x])])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x\sec^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int x^2\sec(x)(-a+\sec(x))\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x(-a+\sec(x))^2\,dx, x, \sec^{-1}(a+bx)\right)}{b^2}\\
&= \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \left(a^2x-2ax\sec(x)+x\sec^2(x)\right)\,dx, x, \sec^{-1}(a+bx)\right)}{b^2}\\
&= -\frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x\sec^2(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^2} + \frac{(2
\\
&= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2} - \frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{4ia\sec
\\
&= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2} - \frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{4ia\sec
\\
&= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2} - \frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{4ia\sec
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 142, normalized size = 0.92

$$
\frac{-\frac{1}{2}a^2\sec^{-1}(a+bx)^2 + \frac{1}{2}b^2x^2\sec^{-1}(a+bx)^2 + 2ia\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right) - 2ia\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right) + \log(a+bx) - (a+}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a + b*x]^2,x]
```

```
[Out] (-((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]) - (a^2*ArcSec[a + b*
x]^2)/2 + (b^2*x^2*ArcSec[a + b*x]^2)/2 - (4*I)*a*ArcSec[a + b*x]*ArcTan[E^
(I*ArcSec[a + b*x])] + Log[a + b*x] + (2*I)*a*PolyLog[2, (-I)*E^(I*ArcSec[a
 + b*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^2
```

fricas [F] time = 1.68, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{arcsec}\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arcsec(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arcsec}(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsec(b*x + a)^2, x)
```

maple [A] time = 0.33, size = 264, normalized size = 1.71

$$-\frac{a^2 \operatorname{arcsec}(bx+a)^2}{2b^2} - \frac{2a \operatorname{arcsec}(bx+a) \ln\left(1 + i\left(\frac{1}{bx+a} + i\sqrt{1 - \frac{1}{(bx+a)^2}}\right)\right)}{b^2} + \frac{2a \operatorname{arcsec}(bx+a) \ln\left(1 - i\left(\frac{1}{bx+a} +\right.\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(b*x+a)^2,x)
```

```
[Out] -1/2*a^2*arcsec(b*x+a)^2/b^2-2/b^2*a*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1
/(b*x+a)^2)^(1/2)))+2/b^2*a*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^
2)^(1/2)))+2*I/b^2*a*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-2*I/b^2
*a*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+1/2*x^2*arcsec(b*x+a)^2-1
/b*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*x-1/b^2*((-1+(b*x+a)^2)/(
b*x+a)^2)^(1/2)*arcsec(b*x+a)*a-1/b^2*ln(1/(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} x^2 \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)^2 - \frac{1}{8} x^2 \log\left(b^2x^2 + 2\,abx + a^2\right)^2 - \int \frac{2\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,bx^2 \operatorname{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/8*x^2*log(b^2*x^2
 + 2*a*b*x + a^2)^2 - integrate(1/2*(2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*
b*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 2*(b^3*x^4 + 3*a*b^2*x^
3 + (3*a^2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a)^2 - (b^3*x^4 + 2*a*b^2*x^
3 + (a^2 - 1)*b*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2 - 1)*b*x^2 + (a^3 -
 a)*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2 - 1)*b*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/(a + b*x))^2,x)
```

```
[Out] int(x*acos(1/(a + b*x))^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asec}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(b*x+a)**2,x)
```

```
[Out] Integral(x*asec(a + b*x)**2, x)
```

3.30 $\int \sec^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=94

$$-\frac{2i\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{2i\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sec^{-1}(a+bx)^2}{b}+\frac{4i\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b}$$

```
[Out] (b*x+a)*arcsec(b*x+a)^2/b+4*I*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)
^2)^(1/2))/b-2*I*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b+2*I*po
lylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5252, 5216, 3757, 4181, 2279, 2391}

$$-\frac{2i\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{2i\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sec^{-1}(a+bx)^2}{b}+\frac{4i\sec^{-1}(a+bx)\tan^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x]^2)/b + ((4*I)*ArcSec[a + b*x]*ArcTan[E^(I*ArcSec
[a + b*x])])/b - ((2*I)*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])])/b + ((2*I)*
PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5216

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/c, Subst[I
nt[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n}
, x] && IGtQ[n, 0]
```

Rule 5252

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcSec[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\int \sec^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \sec^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int x^2\sec(x)\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} - \frac{2\,\text{Subst}\left(\int x\sec(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} + \frac{4i\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} + \frac{2\,\text{Subst}\left(\int \log\left(1-ie^{ix}\right)\right.}{b}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} + \frac{4i\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{(2i)\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx\right.}{b}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} + \frac{4i\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{2i\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b} +$$

Mathematica [A] time = 0.11, size = 111, normalized size = 1.18

$$\frac{-2i\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right) + 2i\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right) + \sec^{-1}(a+bx)\left((a+bx)\sec^{-1}(a+bx) - 2\log\left(1-ie^{i\sec^{-1}(a+bx)}\right) + \right.}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a + b*x]^2,x]
```

```
[Out] (ArcSec[a + b*x]*((a + b*x)*ArcSec[a + b*x] - 2*Log[1 - I*E^(I*ArcSec[a + b
*x])] + 2*Log[1 + I*E^(I*ArcSec[a + b*x])]) - (2*I)*PolyLog[2, (-I)*E^(I*Ar
cSec[a + b*x])] + (2*I)*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b
```

fricas [F] time = 1.35, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{arcsec}\,(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{arcsec}\,(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)^2, x)
```

maple [A] time = 0.28, size = 179, normalized size = 1.90

$$x\,\text{arcsec}\,(bx+a)^2 + \frac{\text{arcsec}\,(bx+a)^2\,a}{b} + \frac{2\,\text{arcsec}\,(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)}{b} - \frac{2\,\text{arcsec}\,(bx+a)\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^2,x)
```

```
[Out] x*arcsec(b*x+a)^2+1/b*arcsec(b*x+a)^2*a+2/b*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)
+I*(1-1/(b*x+a)^2)^(1/2)))-2/b*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2)))+2*I/b*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-2*I/b*di
log(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)^2-\frac{1}{4}\,x\log\left(b^2x^2+2\,abx+a^2\right)^2-\int\frac{2\,\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,bx\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/4*x*log(b^2*x^2 + 2*a*b
*x + a^2)^2 - integrate((2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*b*x*arctan(s
qrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2
 - 1)*b*x - a)*log(b*x + a)^2 - (b^3*x^3 + 2*a*b^2*x^2 + (a^2 - 1)*b*x + (b
^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a))*log(b^2*x^2
 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acos}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^2,x)
```

```
[Out] int(acos(1/(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**2,x)
```

```
[Out] Integral(asec(a + b*x)**2, x)
```

3.31 $\int \frac{\sec^{-1}(a+bx)^2}{x} \, dx$

Optimal. Leaf size=310

$$-2i\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2i\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+2\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+2\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)$$

```
[Out] -arcsec(b*x+a)^2*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)^
2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b*x
+a)^2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+I*arcs
ec(b*x+a)*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-2*I*arcsec(b*x+
a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-2*I*
arcsec(b*x+a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(
1/2)))-1/2*polylog(3,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+2*polylog(3,a*
(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+2*polylog(3,a*(1/(b
*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A] time = 0.49, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5258, 4551, 4530, 3719, 2190, 2531, 2282, 6589, 4520}

$$-2i\sec^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2i\sec^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+2\text{PolyLog}\left(3,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^2/x,x]
```

```
[Out] ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] +
ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] -
ArcSec[a + b*x]^2*Log[1 + E^((2*I)*ArcSec[a + b*x])] - (2*I)*ArcSec[a + b*x
]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] - (2*I)*ArcSec[
a + b*x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + I*ArcS
ec[a + b*x]*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + 2*PolyLog[3, (a*E^(I*A
rcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + 2*PolyLog[3, (a*E^(I*ArcSec[a + b*x
]))/(1 + Sqrt[1 - a^2])] - PolyLog[3, -E^((2*I)*ArcSec[a + b*x])]/2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
 Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4530

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tan[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sin[c + d*x]*Tan[c + d*x]^(n -
1))/(a + b*Cos[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4551

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[((e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Cos[c + d*x])
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx = \text{Subst}\left(\int \frac{x^2\sec(x)\tan(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= \text{Subst}\left(\int \frac{x^2\tan(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= a\,\text{Subst}\left(\int \frac{x^2\sin(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right) + \text{Subst}\left(\int x^2\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\left(2i\,\text{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)\right) - (ia)\,\text{Subst}\left(\int \frac{e^{ix}x^2}{1-\sqrt{1-a^2}-ae^{ix}}\,dx, x, \text{s}\right.$$
$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$
$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$
$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$
$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$

Mathematica [B] time = 2.38, size = 813, normalized size = 2.62

$$\log\left(\frac{e^{i\sec^{-1}(a+bx)}a}{\sqrt{1-a^2}-1}+1\right)\sec^{-1}(a+bx)^2+\log\left(\frac{e^{i\sec^{-1}(a+bx)}\left(\sqrt{1-a^2}-1\right)}{a}+1\right)\sec^{-1}(a+bx)^2+\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a + b*x]^2/x,x]
```

```
[Out] ArcSec[a + b*x]^2*Log[1 + (a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2])] +
 ArcSec[a + b*x]^2*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a]
- 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqrt[1
- a^2])*E^(I*ArcSec[a + b*x]))/a] + ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSe
c[a + b*x]))/(1 + Sqrt[1 - a^2])] + ArcSec[a + b*x]^2*Log[1 - ((1 + Sqrt[1
- a^2])*E^(I*ArcSec[a + b*x]))/a] + 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/
a]/Sqrt[2]]*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] - 2*ArcS
ec[a + b*x]^2*Log[1 + E^((2*I)*ArcSec[a + b*x])] + ArcSec[a + b*x]^2*Log[(2
*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(a + b*x)] - ArcSec[a + b*x
]^2*Log[1 + ((-1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-
2)]))/a] + 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1
+ Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] - ArcSec
[a + b*x]^2*Log[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a +
b*x)^(-2)]))/a] - 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1
- ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] -
(2*I)*ArcSec[a + b*x]*PolyLog[2, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 -
 a^2]))] - (2*I)*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 +
Sqrt[1 - a^2])] + I*ArcSec[a + b*x]*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])]
+ 2*PolyLog[3, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2]))] + 2*PolyL
og[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - PolyLog[3, -E^((2*I)
*ArcSec[a + b*x])]/2
```

fricas [F] time = 3.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcsec}\,(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x,x, algorithm="fricas")
[Out] integral(arcsec(b*x + a)^2/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x,x, algorithm="giac")
[Out] integrate(arcsec(b*x + a)^2/x, x)
```

maple [F] time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^2/x,x)
[Out] int(arcsec(b*x+a)^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x,x, algorithm="maxima")
[Out] integrate(arcsec(b*x + a)^2/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^2/x,x)
[Out] int(acos(1/(a + b*x))^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**2/x,x)
[Out] Integral(asec(a + b*x)**2/x, x)
```

3.32 $\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=244

$$-\frac{2b\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2b\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i}}{\sqrt{}}\right.}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsec(b*x+a)^2/a-arcsec(b*x+a)^2/x-2*I*b*arcsec(b*x+a)*ln(1-a*(1/(b*x+a
)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*I*b*arcse
c(b*x+a)*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(
-a^2+1)^(1/2)-2*b*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+
1)^(1/2)))/a/(-a^2+1)^(1/2)+2*b*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1
/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5258, 4426, 4191, 3321, 2264, 2190, 2279, 2391}

$$-\frac{2b\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2b\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2ib\sec^{-1}(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^2/x^2,x]
```

```
[Out] -((b*ArcSec[a + b*x]^2)/a) - ArcSec[a + b*x]^2/x - ((2*I)*b*ArcSec[a + b*x]
*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])
+ ((2*I)*b*ArcSec[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 -
a^2])])/(a*Sqrt[1 - a^2]) - (2*b*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 -
Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (2*b*PolyLog[2, (a*E^(I*ArcSec[a + b*x
]))/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx = b\,\text{Subst}\left(\int \frac{x^2\sec(x)\tan(x)}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^2}{x} + (2b)\,\text{Subst}\left(\int \frac{x}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^2}{x} + (2b)\,\text{Subst}\left(\int \left(-\frac{x}{a} + \frac{x}{a(1-a\cos(x))}\right)dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} + \frac{(2b)\,\text{Subst}\left(\int \frac{x}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} + \frac{(4b)\,\text{Subst}\left(\int \frac{e^{ix}x}{-a+2e^{ix}-ae^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{(4b)\,\text{Subst}\left(\int \frac{e^{ix}x}{2-2\sqrt{1-a^2}-2ae^{ix}}\,dx, x, \sec^{-1}(a+bx)\right)}{\sqrt{1-a^2}} +$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2ib\sec^{-1}}{}$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2ib\sec^{-1}}{}$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{2ib\sec^{-1}}{}$$

Mathematica [B] time = 2.29, size = 686, normalized size = 2.81

$$-\frac{\frac{(a+bx)\sec^{-1}(a+bx)^2}{x} + \frac{2b\left(i\left(\text{Li}_2\left(\frac{\left(i\sqrt{a^2-1}+1\right)\left(-a+\sqrt{a^2-1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)+1\right)}{a\left(a+\sqrt{a^2-1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)-1\right)}\right)-\text{Li}_2\left(\frac{\left(1-i\sqrt{a^2-1}\right)\left(-a+\sqrt{a^2-1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)+1\right)}{a\left(a+\sqrt{a^2-1}\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)-1\right)}\right)\right)+2\sec^{-1}(a+bx)\tan}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]^2/x^2,x]
```

```
[Out] -((((a + b*x)*ArcSec[a + b*x]^2)/x + (2*b*(2*ArcSec[a + b*x]*ArcTanh[((-1 +
 a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] - 2*ArcCos[a^(-1)]*ArcTanh[((1
+ a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] - (2*I)*ArcT
anh[((-1 + a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] + (2*I)*ArcTanh[((1 +
 a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sq
rt[a]*E^((I/2)*ArcSec[a + b*x])*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)
] + (2*I)*(ArcTanh[((-1 + a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] - ArcT
anh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*
E^((I/2)*ArcSec[a + b*x]))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (A
rcCos[a^(-1)] - (2*I)*ArcTanh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^
2]])*Log[((-1 + a)*(I + I*a + Sqrt[-1 + a^2])*(-I + Tan[ArcSec[a + b*x]/2])
)/(a*(-1 + a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))] - (ArcCos[a^(-1)] +
 (2*I)*ArcTanh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[((-1 +
 a)*(-I - I*a + Sqrt[-1 + a^2])*(I + Tan[ArcSec[a + b*x]/2]))/(a*(-1 + a +
Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))] + I*(-PolyLog[2, ((1 - I*Sqrt[-1 +
 a^2])*(1 - a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))/(a*(-1 + a + Sqrt[-
1 + a^2]*Tan[ArcSec[a + b*x]/2]))] + PolyLog[2, ((1 + I*Sqrt[-1 + a^2])*(1
```

```
- a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))/(a*(-1 + a + Sqrt[-1 + a^2]*T
an[ArcSec[a + b*x]/2]))]))))/Sqrt[-1 + a^2])/a)
```

fricas [F] time = 2.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcsec}(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(arcsec(b*x + a)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(arcsec(b*x + a)^2/x^2, x)
```

maple [A] time = 0.84, size = 341, normalized size = 1.40

$$-\frac{b\operatorname{arcsec}(bx+a)^2}{a}-\frac{\operatorname{arcsec}(bx+a)^2}{x}-\frac{2ib\sqrt{-a^2+1}\,\operatorname{arcsec}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a\left(a^2-1\right)}+\frac{2ib\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^2/x^2,x)
[Out] -b*arcsec(b*x+a)^2/a-arcsec(b*x+a)^2/x-2*I*b*(-a^2+1)^(1/2)/a/(a^2-1)*arcse
c(b*x+a)*ln((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-
a^2+1)^(1/2)))+2*I*b*(-a^2+1)^(1/2)/a/(a^2-1)*arcsec(b*x+a)*ln((a*(1/(b*x+a
)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-2*b*(-a^2
+1)^(1/2)/a/(a^2-1)*dilog((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^
(1/2)+1)/(1+(-a^2+1)^(1/2)))+2*b*(-a^2+1)^(1/2)/a/(a^2-1)*dilog((a*(1/(b*x+
a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^2/x^2,x)
```

```
[Out] int(acos(1/(a + b*x))^2/x^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**2/x**2,x)
```

```
[Out] Integral(asec(a + b*x)**2/x**2, x)
```

3.33 $\int x^2 \sec^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=494

$$\frac{a^3 \sec^{-1}(a+bx)^3}{3b^3} - \frac{6ia^2 \sec^{-1}(a+bx) \text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2 \sec^{-1}(a+bx) \text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2 \text{Li}_3\left(-ie^{i\sec}\right.}{b^3}$$

```
[Out] (b*x+a)*arcsec(b*x+a)/b^3-3*I*a*arcsec(b*x+a)^2/b^3+1/3*a^3*arcsec(b*x+a)^3
/b^3+1/3*x^3*arcsec(b*x+a)^3+6*I*a^2*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-
1/(b*x+a)^2)^(1/2))/b^3+I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2)))/b^3-arctanh((1-1/(b*x+a)^2)^(1/2))/b^3+6*a*arcsec(b*x+a)*ln(1
+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^3+I*arcsec(b*x+a)^2*arctan(1/(b*x
+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3+6*I*a^2*arcsec(b*x+a)*polylog(2,I*(1/(b*x+
a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-3*I*a*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)
^2)^(1/2))^2)/b^3-6*I*a^2*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2)))/b^3-I*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2)))/b^3+polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+6*a^2*p
olylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-polylog(3,I*(1/(b*x+a)
+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-6*a^2*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^
2)^(1/2)))/b^3+3*a*(b*x+a)*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^3-1/2*(b
*x+a)^2*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^3
```

Rubi [A] time = 0.40, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {5258, 4426, 4190, 4181, 2531, 2282, 6589, 4184, 3719, 2190, 2279, 2391, 4186, 3770}

$$-\frac{6ia^2 \sec^{-1}(a+bx) \text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{6ia^2 \sec^{-1}(a+bx) \text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^3} + \frac{6a^2 \text{PolyLog}\left(3\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x])/b^3 - ((3*I)*a*ArcSec[a + b*x]^2)/b^3 + (3*a*(a
 + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2)/b^3 - ((a + b*x)^2*Sqrt
[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2)/(2*b^3) + (a^3*ArcSec[a + b*x]^3)/(
3*b^3) + (x^3*ArcSec[a + b*x]^3)/3 + (I*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSe
c[a + b*x])])/b^3 + ((6*I)*a^2*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x
])])/b^3 - ArcTanh[Sqrt[1 - (a + b*x)^(-2)]]/b^3 + (6*a*ArcSec[a + b*x]*Log
[1 + E^((2*I)*ArcSec[a + b*x])])/b^3 - (I*ArcSec[a + b*x]*PolyLog[2, (-I)*E
^(I*ArcSec[a + b*x])])/b^3 - ((6*I)*a^2*ArcSec[a + b*x]*PolyLog[2, (-I)*E^(
I*ArcSec[a + b*x])])/b^3 + (I*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a +
b*x])])/b^3 + ((6*I)*a^2*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])
])/b^3 - ((3*I)*a*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])])/b^3 + PolyLog[3,
(-I)*E^(I*ArcSec[a + b*x])]/b^3 + (6*a^2*PolyLog[3, (-I)*E^(I*ArcSec[a + b*
x])])/b^3 - PolyLog[3, I*E^(I*ArcSec[a + b*x])]/b^3 - (6*a^2*PolyLog[3, I*E
^(I*ArcSec[a + b*x])])/b^3
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
```

```
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \sec^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int x^3 \sec(x)(-a+\sec(x))^2 \tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3 \sec^{-1}(a+bx)^3 - \frac{\text{Subst}\left(\int x^2(-a+\sec(x))^3\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3 \sec^{-1}(a+bx)^3 - \frac{\text{Subst}\left(\int \left(-a^3x^2+3a^2x^2\sec(x)-3ax^2\sec^2(x)+x^2\sec^3(x)\right) \ldots}{b^3}$$

$$= \frac{a^3\sec^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3 \sec^{-1}(a+bx)^3 - \frac{\text{Subst}\left(\int x^2\sec^3(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3} \ldots$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)^2}{b^3} - \frac{(a+bx)^2\sqrt{1-\ldots}}{\ldots}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx) \ldots}{b^3}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx) \ldots}{b^3}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx) \ldots}{b^3}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx) \ldots}{b^3}$$

Mathematica [A] time = 0.53, size = 442, normalized size = 0.89

$$\frac{1}{3}a^3\sec^{-1}(a+bx)^3+6a^2\left(\text{Li}_3\left(-ie^{i\sec^{-1}(a+bx)}\right)-i\sec^{-1}(a+bx)\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)\right)+6ia^2\left(\sec^{-1}(a+bx)\text{Li}_2\left(ie^{i\text{se}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x] + 3*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[
a + b*x]^2 - ((a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2)/2 + (
a^3*ArcSec[a + b*x]^3)/3 + (b^3*x^3*ArcSec[a + b*x]^3)/3 + I*ArcSec[a + b*x
]^2*ArcTan[E^(I*ArcSec[a + b*x])] + (6*I)*a^2*ArcSec[a + b*x]^2*ArcTan[E^(I
*ArcSec[a + b*x])] - ArcTanh[Sqrt[1 - (a + b*x)^(-2)]] - I*ArcSec[a + b*x]*
PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] + I*ArcSec[a + b*x]*PolyLog[2, I*E^(
I*ArcSec[a + b*x])] - (3*I)*a*(ArcSec[a + b*x]*(ArcSec[a + b*x] + (2*I)*Log
[1 + E^((2*I)*ArcSec[a + b*x])]) + PolyLog[2, -E^((2*I)*ArcSec[a + b*x])])
+ PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])] + 6*a^2*((-I)*ArcSec[a + b*x]*Poly
Log[2, (-I)*E^(I*ArcSec[a + b*x])] + PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])]
) + (6*I)*a^2*(ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])] + I*Poly
Log[3, I*E^(I*ArcSec[a + b*x])]) - PolyLog[3, I*E^(I*ArcSec[a + b*x])])/b^3
```

fricas [F] time = 3.72, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\,\text{arcsec}\,(bx+a)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsec(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{arcsec}\,(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsec(b*x + a)^3, x)
```

maple [A] time = 1.79, size = 770, normalized size = 1.56

$$\frac{x^3\text{arcsec}\,(bx+a)^3}{3}-\frac{\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,\text{arcsec}\,(bx+a)^2\,x^2}{2b}+\frac{2\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\,\text{arcsec}\,(bx+a)^2\,xa}{b^2}+\frac{a^3\text{arcsec}\,(bx+a)^3}{3b^3}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(b*x+a)^3,x)
```

```
[Out] 1/3*x^3*arcsec(b*x+a)^3-1/2/b*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a
)^2*x^2+2/b^2*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)^2*x*a+1/3*a^3*
arcsec(b*x+a)^3/b^3+5/2/b^3*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)^
2*a^2-3*I*a*arcsec(b*x+a)^2/b^3+1/b^2*arcsec(b*x+a)*x+1/b^3*arcsec(b*x+a)*a
-6*I*a^2*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^
3-3/b^3*a^2*arcsec(b*x+a)^2*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-3*I
*a*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^3-6*a^2*polylog(3,I*
(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+6*I*a^2*arcsec(b*x+a)*polylog(2,I*
(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+3/b^3*a^2*arcsec(b*x+a)^2*ln(1+I*(
```

```
1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+6*a^2*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b
*x+a)^2)^(1/2)))/b^3+6*a*arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1
/2))^2)/b^3+I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))
)/b^3+2*I/b^3*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))-1/2/b^3*arcsec(b*x+
a)^2*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-I*arcsec(b*x+a)*polylog(2,
-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-polylog(3,I*(1/(b*x+a)+I*(1-1/(
b*x+a)^2)^(1/2)))/b^3+1/2/b^3*arcsec(b*x+a)^2*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2)))+polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)^3-\frac{1}{4}\,x^3\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)\log\left(b^2x^2+2\,abx+a^2\right)^2-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 1/4*x^3*arctan(sqrt
(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate
(1/4*((4*b*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^3*log(b^
2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*(3*(b^3*x
^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 - (b^3
*x^5 + 2*a*b^2*x^4 + (a^2 - 1)*b*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 -
1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arcta
n(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a
^2 - 1)*b*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/(a + b*x))^3,x)
```

```
[Out] int(x^2*acos(1/(a + b*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{asec}^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(b*x+a)**3,x)
```

```
[Out] Integral(x**2*asec(a + b*x)**3, x)
```

3.34 $\int x \sec^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=278

$$-\frac{a^2 \sec^{-1}(a+bx)^3}{2b^2}+\frac{6ia \sec^{-1}(a+bx)\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{6ia \sec^{-1}(a+bx)\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{3i\text{Li}_2\left(-e^{2i\sec^{-1}(a-}\right.}{2b^2}$$

```
[Out] 3/2*I*arcsec(b*x+a)^2/b^2-1/2*a^2*arcsec(b*x+a)^3/b^2+1/2*x^2*arcsec(b*x+a)
^3-6*I*a*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^2-3*ar
csec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2+6*I*a*arcsec(b*
x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2-6*I*a*arcsec(b*x
+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2+3/2*I*polylog(2,-(
1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2-6*a*polylog(3,-I*(1/(b*x+a)+I*(1-
1/(b*x+a)^2)^(1/2)))/b^2+6*a*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)
))/b^2-3/2*(b*x+a)*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A] time = 0.26, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {5258, 4426, 4190, 4181, 2531, 2282, 6589, 4184, 3719, 2190, 2279, 2391}

$$\frac{6ia \sec^{-1}(a+bx)\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{6ia \sec^{-1}(a+bx)\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{3i\text{PolyLog}\left(2,-e^{2i\sec}\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*ArcSec[a + b*x]^2)/b^2 - (3*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*A
rcSec[a + b*x]^2)/(2*b^2) - (a^2*ArcSec[a + b*x]^3)/(2*b^2) + (x^2*ArcSec[a
 + b*x]^3)/2 - ((6*I)*a*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])])/b^
2 - (3*ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])])/b^2 + ((6*I)*a*A
rcSec[a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])])/b^2 - ((6*I)*a*ArcSe
c[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^2 + (((3*I)/2)*PolyLog[2,
 -E^((2*I)*ArcSec[a + b*x])])/b^2 - (6*a*PolyLog[3, (-I)*E^(I*ArcSec[a + b*
x])])/b^2 + (6*a*PolyLog[3, I*E^(I*ArcSec[a + b*x])])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\sec^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int x^3\sec(x)(-a+\sec(x))\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^2}$$
$$= \frac{1}{2}x^2\sec^{-1}(a+bx)^3 - \frac{3\,\text{Subst}\left(\int x^2(-a+\sec(x))^2\,dx, x, \sec^{-1}(a+bx)\right)}{2b^2}$$
$$= \frac{1}{2}x^2\sec^{-1}(a+bx)^3 - \frac{3\,\text{Subst}\left(\int \left(a^2x^2-2ax^2\sec(x)+x^2\sec^2(x)\right)dx, x, \sec^{-1}(a+bx)\right)}{2b^2}$$
$$= -\frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^3 - \frac{3\,\text{Subst}\left(\int x^2\sec^2(x)\,dx, x, \sec^{-1}(a+bx)\right)}{2b^2} +$$
$$= -\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^3 - \frac{6ia\,s}{}$$
$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec$$
$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec$$
$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec$$
$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec$$

Mathematica [A] time = 0.44, size = 248, normalized size = 0.89

$$\frac{1}{2}\left(x^2\sec^{-1}(a+bx)^3 - \frac{3\left(\frac{1}{3}a^2\sec^{-1}(a+bx)^3 + 4ia\sec^{-1}(a+bx)\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right) - i\text{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right) + 4a\left(\text{Li}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a + b*x]^3,x]
```

```
[Out] (x^2*ArcSec[a + b*x]^3 - (3*((-I)*ArcSec[a + b*x]^2 + (a + b*x)*Sqrt[1 - (a
 + b*x)^(-2)]*ArcSec[a + b*x]^2 + (a^2*ArcSec[a + b*x]^3)/3 + (4*I)*a*ArcSe
c[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] + 2*ArcSec[a + b*x]*Log[1 + E^((
2*I)*ArcSec[a + b*x])] + (4*I)*a*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a
 + b*x])] - I*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + 4*a*((-I)*ArcSec[a +
 b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] + PolyLog[3, (-I)*E^(I*ArcSec[
a + b*x])]) - 4*a*PolyLog[3, I*E^(I*ArcSec[a + b*x])]))/b^2)/2
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(x\,\text{arcsec}\left(bx+a\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arcsec(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arcsec}(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsec(b*x + a)^3, x)
```

maple [A] time = 1.53, size = 429, normalized size = 1.54

$$\frac{x^2 \operatorname{arcsec}(bx+a)^3}{2} - \frac{3\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\operatorname{arcsec}(bx+a)^2 x}{2b} - \frac{a^2 \operatorname{arcsec}(bx+a)^3}{2b^2} - \frac{3\sqrt{\frac{-1+(bx+a)^2}{(bx+a)^2}}\operatorname{arcsec}(bx+a)^2 a}{2b^2} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(b*x+a)^3,x)
```

```
[Out] 1/2*x^2*arcsec(b*x+a)^3-3/2/b*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*arcsec(b*x+a
)^2*x-1/2*a^2*arcsec(b*x+a)^3/b^2-3/2/b^2*((-1+(b*x+a)^2)/(b*x+a)^2)^(1/2)*
arcsec(b*x+a)^2*a+3/2*I*arcsec(b*x+a)^2/b^2+3/b^2*a*arcsec(b*x+a)^2*ln(1-I*
(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-6*I*a*arcsec(b*x+a)*polylog(2,I*(1/(b*
x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2+6*a*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a
)^2)^(1/2)))/b^2+6*I*a*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)
^2)^(1/2)))/b^2-3/b^2*a*arcsec(b*x+a)^2*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2)))-6*a*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2-3*arcsec
(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2+3/2*I*polylog(2,-(1
/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2 \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)^3 - \frac{3}{8}\,x^2 \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)\log\left(b^2x^2+2\,abx+a^2\right)^2 - \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/8*x^2*arctan(sqrt
(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate
(3/8*((4*b*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^2*log(b^
2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*(2*(b^3*x
^4 + 3*a*b^2*x^3 + (3*a^2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a)^2 - (b^3*x
^4 + 2*a*b^2*x^3 + (a^2 - 1)*b*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2 - 1)
*b*x^2 + (a^3 - a)*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sq
rt(b*x + a + 1)*sqrt(b*x + a - 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 -
 1)*b*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/(a + b*x))^3,x)
```

```
[Out] int(x*acos(1/(a + b*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asec}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(b*x+a)**3,x)
```

```
[Out] Integral(x*asec(a + b*x)**3, x)
```

3.35 $\int \sec^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=154

$$-\frac{6i\sec^{-1}(a+bx)\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{6i\sec^{-1}(a+bx)\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{6\text{Li}_3\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b}-\frac{6\text{Li}_3\left(ie^{i\sec^{-1}(\cdots}\right.}{b}$$

```
[Out] (b*x+a)*arcsec(b*x+a)^3/b+6*I*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2))/b-6*I*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2)))/b+6*I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))
)/b+6*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b-6*polylog(3,I*(1/
(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
```

Rubi [A] time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5252, 5216, 3757, 4181, 2531, 2282, 6589}

$$-\frac{6i\sec^{-1}(a+bx)\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{6i\sec^{-1}(a+bx)\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{6\text{PolyLog}\left(3,-ie^{i\,\text{se}\cdots}\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x]^3)/b + ((6*I)*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcS
ec[a + b*x])])/b - ((6*I)*ArcSec[a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b
*x])])/b + ((6*I)*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b +
(6*PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])])/b - (6*PolyLog[3, I*E^(I*ArcSec[
a + b*x])])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5216

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/c, Subst[I
nt[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n}
, x] && IGtQ[n, 0]
```

Rule 5252

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcSec[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \sec^{-1}(a+bx)^3\,dx &= \frac{\text{Subst}\left(\int \sec^{-1}(x)^3\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int x^3\sec(x)\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}\\
&= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} - \frac{3\,\text{Subst}\left(\int x^2\sec(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}\\
&= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} + \frac{6\,\text{Subst}\left(\int x\log\left(1-ie\right.\right.}{}\\
&= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6i\sec^{-1}(a+bx)\text{Li}_2\left(-ie\right.}{b}\\
&= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6i\sec^{-1}(a+bx)\text{Li}_2\left(-ie\right.}{b}\\
&= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6i\sec^{-1}(a+bx)\text{Li}_2\left(-ie\right.}{b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 160, normalized size = 1.04

$$\frac{-6i\sec^{-1}(a+bx)\left(\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)-\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)\right)+6\left(\text{Li}_3\left(-ie^{i\sec^{-1}(a+bx)}\right)-\text{Li}_3\left(ie^{i\sec^{-1}(a+bx)}\right)\right)+(a+b}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x]^3 - 3*ArcSec[a + b*x]^2*(Log[1 - I*E^(I*ArcSec[a
 + b*x])] - Log[1 + I*E^(I*ArcSec[a + b*x])]) - (6*I)*ArcSec[a + b*x]*(Poly
Log[2, (-I)*E^(I*ArcSec[a + b*x])] - PolyLog[2, I*E^(I*ArcSec[a + b*x])]) +
 6*(PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])] - PolyLog[3, I*E^(I*ArcSec[a + b
*x])]))/b
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\text{arcsec}\left(bx+a\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3,x, algorithm="fricas")
[Out] integral(arcsec(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsec}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3,x, algorithm="giac")
[Out] integrate(arcsec(b*x + a)^3, x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \operatorname{arcsec}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^3,x)
[Out] int(arcsec(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)^3-\frac{3}{4}x\arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)\log\left(b^2x^2+2\,abx+a^2\right)^2-\int \frac{3((\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3,x, algorithm="maxima")
[Out] x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/4*x*arctan(sqrt(b*x + a
 + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate(3/4*((4
*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x^2 + 2*a*
b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*((b^3*x^3 + 3*a*b^2*x
^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a)^2 - (b^3*x^3 + 2*a*b^2*x^2 + (
a^2 - 1)*b*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x
+ a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a -
 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^3,x)
[Out] int(acos(1/(a + b*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**3,x)
[Out] Integral(asec(a + b*x)**3, x)
```

3.36 $\int \frac{\sec^{-1}(a+bx)^3}{x} dx$

Optimal. Leaf size=430

$$-3i\sec^{-1}(a+bx)^2\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-3i\sec^{-1}(a+bx)^2\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+6\sec^{-1}(a+bx)\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+6\sec$$

```
[Out] -arcsec(b*x+a)^3*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)^
3*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b*x
+a)^3*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+3/2*I*
arcsec(b*x+a)^2*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-3*I*arcse
c(b*x+a)^2*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2
)))-3*I*arcsec(b*x+a)^2*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+
(-a^2+1)^(1/2)))-3/2*arcsec(b*x+a)*polylog(3,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2))^2)+6*arcsec(b*x+a)*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(
1-(-a^2+1)^(1/2)))+6*arcsec(b*x+a)*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2))/(1+(-a^2+1)^(1/2)))-3/4*I*polylog(4,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(
1/2))^2)+6*I*polylog(4,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1
/2)))+6*I*polylog(4,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)
))
```

Rubi [A] time = 0.53, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5258, 4551, 4530, 3719, 2190, 2531, 6609, 2282, 6589, 4520}

$$-3i\sec^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-3i\sec^{-1}(a+bx)^2\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+6\sec^{-1}(a+bx)\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^3/x,x]
```

```
[Out] ArcSec[a + b*x]^3*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] +
ArcSec[a + b*x]^3*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] -
ArcSec[a + b*x]^3*Log[1 + E^((2*I)*ArcSec[a + b*x])] - (3*I)*ArcSec[a + b*x
]^2*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] - (3*I)*ArcSe
c[a + b*x]^2*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + ((
3*I)/2)*ArcSec[a + b*x]^2*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + 6*ArcSec
[a + b*x]*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + 6*Arc
Sec[a + b*x]*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - (3
*ArcSec[a + b*x]*PolyLog[3, -E^((2*I)*ArcSec[a + b*x])])/2 + (6*I)*PolyLog[
4, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + (6*I)*PolyLog[4, (a*E^(
I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - ((3*I)/4)*PolyLog[4, -E^((2*I)*A
rcSec[a + b*x])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4520

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1))
, x] + (-Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x))), x], x] - Dist[I, Int[((e + f*x)^m*E^(I*(c + d*x)))/(a +
 Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4530

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tan[c +
d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sin[c + d*x]*Tan[c + d*x]^(n -
1))/(a + b*Cos[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4551

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[((e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*G[c + d*x]^p)/(b + a*Cos[c + d*x])
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
```

```
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = \text{Subst}\left(\int \frac{x^3\sec(x)\tan(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= \text{Subst}\left(\int \frac{x^3\tan(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= a\,\text{Subst}\left(\int \frac{x^3\sin(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right) + \text{Subst}\left(\int x^3\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= -\left(2i\,\text{Subst}\left(\int \frac{e^{2ix}x^3}{1+e^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)\right) - (ia)\,\text{Subst}\left(\int \frac{e^{ix}x^3}{1-\sqrt{1-a^2}-ae^{ix}}\,dx, x, \text{s}\right.$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \sec^{-1}(a$$

Mathematica [B] time = 4.06, size = 1058, normalized size = 2.46

$$2\log\left(\frac{e^{i\sec^{-1}(a+bx)}a}{\sqrt{1-a^2}-1}+1\right)\sec^{-1}(a+bx)^3+\log\left(\frac{e^{i\sec^{-1}(a+bx)}\left(\sqrt{1-a^2}-1\right)}{a}+1\right)\sec^{-1}(a+bx)^3+2\log\left(1-\frac{ae^{i\sec^{-1}(a+b}}{\sqrt{1-a^2}+}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a + b*x]^3/x,x]
```

```
[Out] 2*ArcSec[a + b*x]^3*Log[1 + (a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2])]
 + ArcSec[a + b*x]^3*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a
] - 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqr
t[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + 2*ArcSec[a + b*x]^3*Log[1 - (a*E^(I
*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + ArcSec[a + b*x]^3*Log[1 - ((1 + S
qrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(
-1 + a)/a]/Sqrt[2]]*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a]
- 3*ArcSec[a + b*x]^3*Log[1 + E^((2*I)*ArcSec[a + b*x])] + 2*ArcSec[a + b*x
]^3*Log[(2*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(a + b*x)] - ArcS
ec[a + b*x]^3*Log[1 + (a*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(-1
 + Sqrt[1 - a^2])] - ArcSec[a + b*x]^3*Log[1 + ((-1 + Sqrt[1 - a^2])*((a +
b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] + 6*ArcSec[a + b*x]^2*ArcSin[Sq
rt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*S
qrt[1 - (a + b*x)^(-2)]))/a] - ArcSec[a + b*x]^3*Log[1 - (a*((a + b*x)^(-1)
```

```
+ I*Sqrt[1 - (a + b*x)^(-2)]))/(1 + Sqrt[1 - a^2])] - ArcSec[a + b*x]^3*Lo
g[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a
] - 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 - ((1 + Sqrt
[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] - (3*I)*ArcSec
[a + b*x]^2*PolyLog[2, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2]))] -
(3*I)*ArcSec[a + b*x]^2*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 -
a^2])] + ((3*I)/2)*ArcSec[a + b*x]^2*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])
] + 6*ArcSec[a + b*x]*PolyLog[3, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 -
a^2]))] + 6*ArcSec[a + b*x]*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt
[1 - a^2])] - (3*ArcSec[a + b*x]*PolyLog[3, -E^((2*I)*ArcSec[a + b*x])])/2
+ (6*I)*PolyLog[4, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2]))] + (6*
I)*PolyLog[4, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - ((3*I)/4)*Po
lyLog[4, -E^((2*I)*ArcSec[a + b*x])]
```

fricas [F] time = 1.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcsec}\left(bx+a\right)^{3}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x,x, algorithm="fricas")
[Out] integral(arcsec(b*x + a)^3/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsec}\left(bx+a\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x,x, algorithm="giac")
[Out] integrate(arcsec(b*x + a)^3/x, x)
```

maple [F] time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsec}\left(bx+a\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^3/x,x)
[Out] int(arcsec(b*x+a)^3/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsec}\left(bx+a\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x,x, algorithm="maxima")
[Out] integrate(arcsec(b*x + a)^3/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^3/x,x)
```

```
[Out] int(acos(1/(a + b*x))^3/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**3/x,x)
```

```
[Out] Integral(asec(a + b*x)**3/x, x)
```

3.37 $\int \frac{\sec^{-1}(a+bx)^3}{x^2} dx$

Optimal. Leaf size=362

$$-\frac{6b\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6b\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{6ib\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6ib\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+}}{\sqrt{1-a^2}+}\right.}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsec(b*x+a)^3/a-arcsec(b*x+a)^3/x-3*I*b*arcsec(b*x+a)^2*ln(1-a*(1/(b*x
+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+3*I*b*arc
sec(b*x+a)^2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
/a/(-a^2+1)^(1/2)-6*b*arcsec(b*x+a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*arcsec(b*x+a)*polylog(2,a
*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)-6
*I*b*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/
(-a^2+1)^(1/2)+6*I*b*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a
^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5258, 4426, 4191, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{6b\sec^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6b\sec^{-1}(a+bx)\text{PolyLog}\left(2,\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{6ib\text{PolyLog}\left(3,\frac{ae^{i\sec^{-1}}}{1-\sqrt{1}}\right.}{a\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^3/x^2,x]
```

```
[Out] -((b*ArcSec[a + b*x]^3)/a) - ArcSec[a + b*x]^3/x - ((3*I)*b*ArcSec[a + b*x]
^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]
) + ((3*I)*b*ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[
1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*Arc
Sec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (6*b*ArcSec[a + b*
x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a
^2]) - ((6*I)*b*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/
(a*Sqrt[1 - a^2]) + ((6*I)*b*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt
[1 - a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4426

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((e + f
*x)^m*(a + b*Sec[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n
+ 1)), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5258

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx = b\,\text{Subst}\left(\int \frac{x^3\sec(x)\tan(x)}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^3}{x} + (3b)\,\text{Subst}\left(\int \frac{x^2}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^3}{x} + (3b)\,\text{Subst}\left(\int \left(-\frac{x^2}{a}+\frac{x^2}{a(1-a\cos(x))}\right)dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} + \frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} + \frac{(6b)\,\text{Subst}\left(\int \frac{e^{ix}x^2}{-a+2e^{ix}-ae^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{(6b)\,\text{Subst}\left(\int \frac{e^{ix}x^2}{2-2\sqrt{1-a^2}-2ae^{ix}}\,dx, x, \sec^{-1}(a+bx\right.}{\sqrt{1-a^2}}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{3ib\,\text{se}}{}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{3ib\,\text{se}}{}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{3ib\,\text{se}}{}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} + \frac{3ib\,\text{se}}{}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[ArcSec[a + b*x]^3/x^2,x]
[Out] $Aborted
```

fricas [F] time = 2.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcsec}\,(bx+a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x^2,x, algorithm="fricas")
[Out] integral(arcsec(b*x + a)^3/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcsec}\,(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)^3/x^2, x)
```

maple [F] time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^3/x^2,x)
```

```
[Out] int(arcsec(b*x+a)^3/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4 \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)^3 - 3 \arctan\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\right)\log\left(b^2x^2+2\,abx+a^2\right)^2 - 3\,x\int \frac{(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3*arctan(sqrt(b*x +
 a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - 4*x*integrate(3
/4*((4*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x^2
+ 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 4*((b^3*x^3 + 3*a
*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a)^2 + (b^3*x^3 + 2*a*b^2*x
^2 + (a^2 - 1)*b*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*lo
g(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sqrt(b*x + a + 1)*sqrt(b*x
 + a - 1)))/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2), x)
)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^3/x^2,x)
```

```
[Out] int(acos(1/(a + b*x))^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^3(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**3/x**2,x)
```

```
[Out] Integral(asec(a + b*x)**3/x**2, x)
```

3.38 $\int x \left(a + b \sec^{-1}\left(c + dx^2\right)\right) dx$

Optimal. Leaf size=58

$$\frac{ax^2}{2} + \frac{b\left(c + dx^2\right)\sec^{-1}\left(c + dx^2\right)}{2d} - \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{\left(c+dx^2\right)^2}}\right)}{2d}$$

```
[Out] 1/2*a*x^2+1/2*b*(d*x^2+c)*arcsec(d*x^2+c)/d-1/2*b*arctanh((1-1/(d*x^2+c)^2)
^(1/2))/d
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6715, 5250, 372, 266, 63, 206}

$$\frac{ax^2}{2} + \frac{b\left(c + dx^2\right)\sec^{-1}\left(c + dx^2\right)}{2d} - \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{\left(c+dx^2\right)^2}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSec[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*(c + d*x^2)*ArcSec[c + d*x^2])/(2*d) - (b*ArcTanh[Sqrt[1 - (
c + d*x^2)^(-2)]])/(2*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5250

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSec[c + d*x]
)/d, x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)dx &= \frac{1}{2}\text{Subst}\left(\int\left(a+b\sec^{-1}(c+dx)\right)dx,x,x^2\right)\\
&= \frac{ax^2}{2}+\frac{1}{2}b\,\text{Subst}\left(\int\sec^{-1}(c+dx)\,dx,x,x^2\right)\\
&= \frac{ax^2}{2}+\frac{b\left(c+dx^2\right)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{1}{2}b\,\text{Subst}\left(\int\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}dx,x,x^2\right)\\
&= \frac{ax^2}{2}+\frac{b\left(c+dx^2\right)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{b\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}dx,x,c+dx^2\right)}{2d}\\
&= \frac{ax^2}{2}+\frac{b\left(c+dx^2\right)\sec^{-1}\left(c+dx^2\right)}{2d}+\frac{b\,\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x}dx,x,\frac{1}{\left(c+dx^2\right)^2}\right)}{4d}\\
&= \frac{ax^2}{2}+\frac{b\left(c+dx^2\right)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{b\,\text{Subst}\left(\int\frac{1}{1-x^2}dx,x,\sqrt{1-\frac{1}{\left(c+dx^2\right)^2}}\right)}{2d}\\
&= \frac{ax^2}{2}+\frac{b\left(c+dx^2\right)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{\left(c+dx^2\right)^2}}\right)}{2d}
\end{aligned}
$$

Mathematica [B] time = 0.24, size = 148, normalized size = 2.55

$$
\frac{ax^2}{2}-\frac{b\left(c+dx^2\right)\sqrt{\frac{c^2+2cdx^2+d^2x^4-1}{\left(c+dx^2\right)^2}}\left(\tanh^{-1}\left(\frac{c+dx^2}{\sqrt{c^2+2cdx^2+d^2x^4-1}}\right)-c\tan^{-1}\left(\sqrt{\left(c+dx^2\right)^2-1}\right)\right)}{2d\sqrt{c^2+2cdx^2+d^2x^4-1}}+\frac{1}{2}bx^2\sec^{-1}\left(c+dx^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSec[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcSec[c + d*x^2])/2 - (b*(c + d*x^2)*Sqrt[(-1 + c^2 + 2
*c*d*x^2 + d^2*x^4)/(c + d*x^2)^2]*(-(c*ArcTan[Sqrt[-1 + (c + d*x^2)^2]]) +
 ArcTanh[(c + d*x^2)/Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]]))/(2*d*Sqrt[-1 +
 c^2 + 2*c*d*x^2 + d^2*x^4])
```

fricas [A] time = 0.60, size = 96, normalized size = 1.66

$$
\frac{bdx^2\,\text{arcsec}\left(dx^2+c\right)+adx^2+2\,bc\arctan\left(-dx^2-c+\sqrt{d^2x^4+2\,cdx^2+c^2-1}\right)+b\log\left(-dx^2-c+\sqrt{d^2x^4+2\,cdx^2+c^2-1}\right)}{2\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x^2*arcsec(d*x^2 + c) + a*d*x^2 + 2*b*c*arctan(-d*x^2 - c + sqrt(d
^2*x^4 + 2*c*d*x^2 + c^2 - 1)) + b*log(-d*x^2 - c + sqrt(d^2*x^4 + 2*c*d*x^
2 + c^2 - 1)))/d
```

giac [A] time = 0.38, size = 100, normalized size = 1.72

$$\frac{1}{2}\,ax^2+\frac{1}{4}\,bd\left(\frac{2\left(dx^2+c\right)\arccos\left(-\frac{1}{\left(dx^2+c\right)\left(\frac{c}{dx^2+c}-1\right)-c}\right)}{d^2}-\frac{\log\left(\sqrt{-\frac{1}{\left(dx^2+c\right)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{\left(dx^2+c\right)^2}+1}+1\right)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/4*b*d*(2*(d*x^2 + c)*arccos(-1/((d*x^2 + c)*(c/(d*x^2 + c) -
1) - c))/d^2 - (log(sqrt(-1/(d*x^2 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^2 +
 c)^2 + 1) + 1))/d^2)
```

maple [A] time = 0.10, size = 81, normalized size = 1.40

$$\frac{\operatorname{arcsec}\left(d\,x^2+c\right)x^2b}{2}+\frac{a\,x^2}{2}+\frac{\operatorname{arcsec}\left(d\,x^2+c\right)bc}{2d}-\frac{\ln\left(d\,x^2+c+\left(d\,x^2+c\right)\sqrt{1-\frac{1}{\left(d\,x^2+c\right)^2}}\right)b}{2d}+\frac{ac}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsec(d*x^2+c)),x)
```

```
[Out] 1/2*arcsec(d*x^2+c)*x^2*b+1/2*a*x^2+1/2/d*arcsec(d*x^2+c)*b*c-1/2/d*ln(d*x^
2+c+(d*x^2+c)*(1-1/(d*x^2+c)^2)^(1/2))*b+1/2/d*a*c
```

maxima [A] time = 0.35, size = 71, normalized size = 1.22

$$\frac{1}{2}\,ax^2+\frac{\left(2\left(dx^2+c\right)\operatorname{arcsec}\left(dx^2+c\right)-\log\left(\sqrt{-\frac{1}{\left(dx^2+c\right)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{\left(dx^2+c\right)^2}+1}+1\right)\right)b}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*(d*x^2 + c)*arcsec(d*x^2 + c) - log(sqrt(-1/(d*x^2 + c)^
2 + 1) + 1) + log(-sqrt(-1/(d*x^2 + c)^2 + 1) + 1))*b/d
```

mupad [B] time = 1.03, size = 52, normalized size = 0.90

$$\frac{a\,x^2}{2}-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{\left(d\,x^2+c\right)^2}}}\right)}{2\,d}+\frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^2+c}\right)\left(d\,x^2+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acos(1/(c + d*x^2))),x)
```

```
[Out] (a*x^2)/2 - (b*atanh(1/(1 - 1/(c + d*x^2)^2)^(1/2)))/(2*d) + (b*acos(1/(c +
 d*x^2))*(c + d*x^2))/(2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asec(d*x**2+c)),x)
```

```
[Out] Timed out
```

3.39 $\int x^2 \left(a + b\sec^{-1}\left(c + dx^3\right)\right) dx$

Optimal. Leaf size=58

$$\frac{ax^3}{3} + \frac{b\left(c + dx^3\right)\sec^{-1}\left(c + dx^3\right)}{3d} - \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{\left(c+dx^3\right)^2}}\right)}{3d}$$

```
[Out] 1/3*a*x^3+1/3*b*(d*x^3+c)*arcsec(d*x^3+c)/d-1/3*b*arctanh((1-1/(d*x^3+c)^2)
^(1/2))/d
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6715, 5250, 372, 266, 63, 206}

$$\frac{ax^3}{3} + \frac{b\left(c + dx^3\right)\sec^{-1}\left(c + dx^3\right)}{3d} - \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{\left(c+dx^3\right)^2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSec[c + d*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*(c + d*x^3)*ArcSec[c + d*x^3])/(3*d) - (b*ArcTanh[Sqrt[1 - (
c + d*x^3)^(-2)]])/(3*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5250

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSec[c + d*x]
)/d, x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\sec^{-1}\left(c+dx^3\right)\right)dx &= \frac{1}{3}\text{Subst}\left(\int\left(a+b\sec^{-1}(c+dx)\right)dx,x,x^3\right)\\
&= \frac{ax^3}{3}+\frac{1}{3}b\,\text{Subst}\left(\int\sec^{-1}(c+dx)\,dx,x,x^3\right)\\
&= \frac{ax^3}{3}+\frac{b\left(c+dx^3\right)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{1}{3}b\,\text{Subst}\left(\int\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}dx,\right.\\
&= \frac{ax^3}{3}+\frac{b\left(c+dx^3\right)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{b\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}dx,x,c+dx^3\right)}{3d}\\
&= \frac{ax^3}{3}+\frac{b\left(c+dx^3\right)\sec^{-1}\left(c+dx^3\right)}{3d}+\frac{b\,\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x}dx,x,\frac{1}{\left(c+dx^3\right)^2}\right)}{6d}\\
&= \frac{ax^3}{3}+\frac{b\left(c+dx^3\right)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{b\,\text{Subst}\left(\int\frac{1}{1-x^2}dx,x,\sqrt{1-\frac{1}{\left(c+dx^3\right)^2}}\right)}{3d}\\
&= \frac{ax^3}{3}+\frac{b\left(c+dx^3\right)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{\left(c+dx^3\right)^2}}\right)}{3d}
\end{aligned}
$$

Mathematica [B] time = 0.16, size = 148, normalized size = 2.55

$$
\frac{ax^3}{3}-\frac{b\left(c+dx^3\right)\sqrt{\frac{c^2+2cdx^3+d^2x^6-1}{\left(c+dx^3\right)^2}}\left(\tanh^{-1}\left(\frac{c+dx^3}{\sqrt{c^2+2cdx^3+d^2x^6-1}}\right)-c\tan^{-1}\left(\sqrt{\left(c+dx^3\right)^2-1}\right)\right)}{3d\sqrt{c^2+2cdx^3+d^2x^6-1}}+\frac{1}{3}bx^3\sec^{-1}\left(c+\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcSec[c + d*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcSec[c + d*x^3])/3 - (b*(c + d*x^3)*Sqrt[(-1 + c^2 + 2
*c*d*x^3 + d^2*x^6)/(c + d*x^3)^2]*(-(c*ArcTan[Sqrt[-1 + (c + d*x^3)^2]]) +
 ArcTanh[(c + d*x^3)/Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]]))/(3*d*Sqrt[-1 +
 c^2 + 2*c*d*x^3 + d^2*x^6])
```

fricas [A] time = 0.80, size = 96, normalized size = 1.66

$$
\frac{bdx^3\,\text{arcsec}\left(dx^3+c\right)+adx^3+2\,bc\arctan\left(-dx^3-c+\sqrt{d^2x^6+2\,cdx^3+c^2-1}\right)+b\log\left(-dx^3-c+\sqrt{d^2x^6-}\right.}{3\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d*x^3*arcsec(d*x^3 + c) + a*d*x^3 + 2*b*c*arctan(-d*x^3 - c + sqrt(d
^2*x^6 + 2*c*d*x^3 + c^2 - 1)) + b*log(-d*x^3 - c + sqrt(d^2*x^6 + 2*c*d*x^
3 + c^2 - 1)))/d
```

giac [A] time = 0.40, size = 100, normalized size = 1.72

$$\frac{1}{3}\,ax^3+\frac{1}{6}\,bd\left(\frac{2\left(dx^3+c\right)\arccos\left(-\frac{1}{\left(dx^3+c\right)\left(\frac{c}{dx^3+c}-1\right)-c}\right)}{d^2}-\frac{\log\left(\sqrt{-\frac{1}{\left(dx^3+c\right)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{\left(dx^3+c\right)^2}+1}+1\right)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/6*b*d*(2*(d*x^3 + c)*arccos(-1/((d*x^3 + c)*(c/(d*x^3 + c) -
1) - c))/d^2 - (log(sqrt(-1/(d*x^3 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^3 +
 c)^2 + 1) + 1))/d^2)
```

maple [A] time = 0.10, size = 81, normalized size = 1.40

$$\frac{\mathrm{arcsec}\left(d\,x^3+c\right)x^3b}{3}+\frac{x^3a}{3}+\frac{\mathrm{arcsec}\left(d\,x^3+c\right)bc}{3d}-\frac{\ln\left(d\,x^3+c+\left(d\,x^3+c\right)\sqrt{1-\frac{1}{\left(d\,x^3+c\right)^2}}\right)b}{3d}+\frac{ac}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsec(d*x^3+c)),x)
```

```
[Out] 1/3*arcsec(d*x^3+c)*x^3*b+1/3*x^3*a+1/3/d*arcsec(d*x^3+c)*b*c-1/3/d*ln(d*x^
3+c+(d*x^3+c)*(1-1/(d*x^3+c)^2)^(1/2))*b+1/3/d*a*c
```

maxima [A] time = 0.36, size = 71, normalized size = 1.22

$$\frac{1}{3}\,ax^3+\frac{\left(2\left(dx^3+c\right)\mathrm{arcsec}\left(dx^3+c\right)-\log\left(\sqrt{-\frac{1}{\left(dx^3+c\right)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{\left(dx^3+c\right)^2}+1}+1\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*(d*x^3 + c)*arcsec(d*x^3 + c) - log(sqrt(-1/(d*x^3 + c)^
2 + 1) + 1) + log(-sqrt(-1/(d*x^3 + c)^2 + 1) + 1))*b/d
```

mupad [B] time = 0.77, size = 52, normalized size = 0.90

$$\frac{a\,x^3}{3}-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{{\left(d\,x^3+c\right)}^2}}}\right)}{3\,d}+\frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^3+c}\right)\,\left(d\,x^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acos(1/(c + d*x^3))),x)
```

```
[Out] (a*x^3)/3 - (b*atanh(1/(1 - 1/(c + d*x^3)^2)^(1/2)))/(3*d) + (b*acos(1/(c +
 d*x^3))*(c + d*x^3))/(3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asec(d*x**3+c)),x)
```

```
[Out] Timed out
```

3.40 $\int x^3 \left(a + b\sec^{-1}\left(c + dx^4\right)\right) dx$

Optimal. Leaf size=58

$$\frac{ax^4}{4} + \frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d} - \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{\left(c+dx^4\right)^2}}\right)}{4d}$$

```
[Out] 1/4*a*x^4+1/4*b*(d*x^4+c)*arcsec(d*x^4+c)/d-1/4*b*arctanh((1-1/(d*x^4+c)^2)
^(1/2))/d
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6715, 5250, 372, 266, 63, 206}

$$\frac{ax^4}{4} + \frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d} - \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{\left(c+dx^4\right)^2}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcSec[c + d*x^4]),x]
```

```
[Out] (a*x^4)/4 + (b*(c + d*x^4)*ArcSec[c + d*x^4])/(4*d) - (b*ArcTanh[Sqrt[1 - (
c + d*x^4)^(-2)]])/(4*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5250

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSec[c + d*x]
)/d, x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+b\sec^{-1}\left(c+dx^4\right)\right) dx &= \frac{1}{4}\text{Subst}\left(\int \left(a+b\sec^{-1}(c+dx)\right) dx, x, x^4\right) \\
&= \frac{ax^4}{4}+\frac{1}{4}b\,\text{Subst}\left(\int \sec^{-1}(c+dx)\,dx, x, x^4\right) \\
&= \frac{ax^4}{4}+\frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\,dx, x, x\right. \\
&= \frac{ax^4}{4}+\frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx, x, c+dx^4\right)}{4d} \\
&= \frac{ax^4}{4}+\frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{\left(c+dx^4\right)^2}\right)}{8d} \\
&= \frac{ax^4}{4}+\frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{\left(c+dx^4\right)^2}}\right)}{4d} \\
&= \frac{ax^4}{4}+\frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{\left(c+dx^4\right)^2}}\right)}{4d}
\end{aligned}$$

Mathematica [B] time = 0.38, size = 137, normalized size = 2.36

$$\frac{ax^4}{4}-\frac{b\sqrt{\left(c+dx^4\right)^2-1}\left(\log\left(\frac{c+dx^4}{\sqrt{\left(c+dx^4\right)^2-1}}+1\right)-\log\left(1-\frac{c+dx^4}{\sqrt{\left(c+dx^4\right)^2-1}}\right)\right)}{8d\left(c+dx^4\right)\sqrt{1-\frac{1}{\left(c+dx^4\right)^2}}}+\frac{b\left(c+dx^4\right)\sec^{-1}\left(c+dx^4\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcSec[c + d*x^4]),x]
```

```
[Out] (a*x^4)/4 + (b*(c + d*x^4)*ArcSec[c + d*x^4])/(4*d) - (b*Sqrt[-1 + (c + d*x
^4)^2]*(-Log[1 - (c + d*x^4)/Sqrt[-1 + (c + d*x^4)^2]] + Log[1 + (c + d*x^4
)/Sqrt[-1 + (c + d*x^4)^2]]))/(8*d*(c + d*x^4)*Sqrt[1 - (c + d*x^4)^(-2)])
```

fricas [A] time = 1.21, size = 96, normalized size = 1.66

$$\frac{bdx^4\operatorname{arcsec}\left(dx^4+c\right)+adx^4+2\,bc\arctan\left(-dx^4-c+\sqrt{d^2x^8+2\,cdx^4+c^2-1}\right)+b\log\left(-dx^4-c+\sqrt{d^2x^8+2\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*d*x^4*arcsec(d*x^4 + c) + a*d*x^4 + 2*b*c*arctan(-d*x^4 - c + sqrt(d
^2*x^8 + 2*c*d*x^4 + c^2 - 1)) + b*log(-d*x^4 - c + sqrt(d^2*x^8 + 2*c*d*x^
4 + c^2 - 1)))/d
```

giac [A] time = 0.38, size = 100, normalized size = 1.72

$$\frac{1}{4}\,ax^4+\frac{1}{8}\,bd\left(\frac{2\left(dx^4+c\right)\arccos\left(-\frac{1}{\left(dx^4+c\right)\left(\frac{c}{dx^4+c}-1\right)-c}\right)}{d^2}-\frac{\log\left(\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}+1\right)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 1/8*b*d*(2*(d*x^4 + c)*arccos(-1/((d*x^4 + c)*(c/(d*x^4 + c) -
1) - c))/d^2 - (log(sqrt(-1/(d*x^4 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^4 +
 c)^2 + 1) + 1))/d^2)
```

maple [A] time = 0.10, size = 81, normalized size = 1.40

$$\frac{\operatorname{arcsec}\left(d\,x^4+c\right)x^4b}{4}+\frac{x^4a}{4}+\frac{\operatorname{arcsec}\left(d\,x^4+c\right)bc}{4d}-\frac{\ln\left(d\,x^4+c+\left(d\,x^4+c\right)\sqrt{1-\frac{1}{\left(d\,x^4+c\right)^2}}\right)b}{4d}+\frac{ac}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsec(d*x^4+c)),x)
```

```
[Out] 1/4*arcsec(d*x^4+c)*x^4*b+1/4*x^4*a+1/4/d*arcsec(d*x^4+c)*b*c-1/4/d*ln(d*x^
4+c+(d*x^4+c)*(1-1/(d*x^4+c)^2)^(1/2))*b+1/4/d*a*c
```

maxima [A] time = 0.37, size = 71, normalized size = 1.22

$$\frac{1}{4}\,ax^4+\frac{\left(2\left(dx^4+c\right)\operatorname{arcsec}\left(dx^4+c\right)-\log\left(\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}+1\right)\right)b}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*(d*x^4 + c)*arcsec(d*x^4 + c) - log(sqrt(-1/(d*x^4 + c)^
2 + 1) + 1) + log(-sqrt(-1/(d*x^4 + c)^2 + 1) + 1))*b/d
```

mupad [B] time = 0.78, size = 52, normalized size = 0.90

$$\frac{a\,x^4}{4}-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{\left(d\,x^4+c\right)^2}}}\right)}{4\,d}+\frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^4+c}\right)\left(d\,x^4+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acos(1/(c + d*x^4))),x)
```

```
[Out] (a*x^4)/4 - (b*atanh(1/(1 - 1/(c + d*x^4)^2)^(1/2)))/(4*d) + (b*acos(1/(c +
 d*x^4))*(c + d*x^4))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asec(d*x**4+c)),x)
```

```
[Out] Timed out
```

3.41 $\int x^{-1+n} \sec^{-1}(a + bx^n)\, dx$

Optimal. Leaf size=49

$$\frac{(a + bx^n)\sec^{-1}(a + bx^n)}{bn} - \frac{\tanh^{-1}\left(\sqrt{1 - \frac{1}{(a+bx^n)^2}}\right)}{bn}$$

[Out] (a+b*x^n)*arcsec(a+b*x^n)/b/n-arctanh((1-1/(a+b*x^n)^2)^(1/2))/b/n

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6715, 5250, 372, 266, 63, 206}

$$\frac{(a + bx^n)\sec^{-1}(a + bx^n)}{bn} - \frac{\tanh^{-1}\left(\sqrt{1 - \frac{1}{(a+bx^n)^2}}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcSec[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSec[a + b*x^n])/(b*n) - ArcTanh[Sqrt[1 - (a + b*x^n)^(-2)]]
/(b*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5250

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSec[c + d*x]
)/d, x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1+n}\sec^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \sec^{-1}(a+bx)\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx, x, a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{(a+bx^n)^2}\right)}{2bn}\\
&= \frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}\\
&= \frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}
\end{aligned}
$$

Mathematica [B] time = 0.34, size = 130, normalized size = 2.65

$$
\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn} - \frac{\sqrt{(a+bx^n)^2-1}\left(\log\left(\frac{a+bx^n}{\sqrt{(a+bx^n)^2-1}}+1\right)-\log\left(1-\frac{a+bx^n}{\sqrt{(a+bx^n)^2-1}}\right)\right)}{2bn(a+bx^n)\sqrt{1-\frac{1}{(a+bx^n)^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcSec[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSec[a + b*x^n])/(b*n) - (Sqrt[-1 + (a + b*x^n)^2]*(-Log[1 -
 (a + b*x^n)/Sqrt[-1 + (a + b*x^n)^2]] + Log[1 + (a + b*x^n)/Sqrt[-1 + (a +
b*x^n)^2]]))/(2*b*n*(a + b*x^n)*Sqrt[1 - (a + b*x^n)^(-2)])
```

fricas [A] time = 1.01, size = 92, normalized size = 1.88

$$
\frac{bx^n\,\text{arcsec}(bx^n+a) + 2a\arctan\left(-bx^n - a + \sqrt{b^2x^{2n} + 2abx^n + a^2 - 1}\right) + \log\left(-bx^n - a + \sqrt{b^2x^{2n} + 2abx^n}\right.}{bn}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*x^n*arcsec(b*x^n + a) + 2*a*arctan(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*b
*x^n + a^2 - 1)) + log(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)
))/(b*n)
```

giac [A] time = 0.28, size = 75, normalized size = 1.53

$$\frac{b\left(\frac{2\,(bx^n+a)\arccos\left(\frac{1}{bx^n+a}\right)}{b^2}-\frac{\log\left(\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)}{b^2}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(b*x^n + a)*arccos(1/(b*x^n + a))/b^2 - (log(sqrt(-1/(b*x^n + a)^2
 + 1) + 1) - log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/b^2)/n
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x^{n-1}\mathrm{arcsec}\left(a+b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arcsec(a+b*x^n),x)
```

```
[Out] int(x^(n-1)*arcsec(a+b*x^n),x)
```

maxima [A] time = 0.36, size = 66, normalized size = 1.35

$$\frac{2\,(bx^n+a)\,\mathrm{arcsec}\,(bx^n+a)-\log\left(\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arcsec(b*x^n + a) - log(sqrt(-1/(b*x^n + a)^2 + 1) + 1)
+ log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/(b*n)
```

mupad [B] time = 1.08, size = 44, normalized size = 0.90

$$-\frac{\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x^n)^2}}}\right)-\mathrm{acos}\left(\frac{1}{a+b\,x^n}\right)\,(a+b\,x^n)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acos(1/(a + b*x^n)),x)
```

```
[Out] -(atanh(1/(1 - 1/(a + b*x^n)^2)^(1/2)) - acos(1/(a + b*x^n))*(a + b*x^n))/(
b*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*asec(a+b*x**n),x)
```

```
[Out] Timed out
```

3.42 $\int \sec^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=85

$$\frac{i\text{Li}_2\left(-e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{i\sec^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\sec^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

```
[Out] 1/2*I*arcsec(c*exp(b*x+a))^2/b-arcsec(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+I*
(1-1/c^2/exp(b*x+a)^2)^(1/2))^2)/b+1/2*I*polylog(2,-(1/c/exp(b*x+a)+I*(1-1/
c^2/exp(b*x+a)^2)^(1/2))^2)/b
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2282, 5218, 4626, 3719, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{i\sec^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\sec^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[c*E^(a + b*x)],x]
```

```
[Out] ((I/2)*ArcSec[c*E^(a + b*x)]^2)/b - (ArcSec[c*E^(a + b*x)]*Log[1 + E^((2*I)
*ArcSec[c*E^(a + b*x)])])/b + ((I/2)*PolyLog[2, -E^((2*I)*ArcSec[c*E^(a + b
*x)])])/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5218

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule steps

$$\int \sec^{-1}\left(ce^{a+bx}\right) dx = \frac{\text{Subst}\left(\int \frac{\sec^{-1}(cx)}{x} dx, x, e^{a+bx}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\cos^{-1}\left(\frac{x}{c}\right)}{x} dx, x, e^{-a-bx}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int x\tan(x)\, dx, x, \cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$

$$= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}} dx, x, \cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b}$$

$$= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right) dx, x, c\right.}{b}$$

$$= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} - \frac{i\,\text{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{2i\cos^{-}}\right.}{2b}$$

$$= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{2b}$$

Mathematica [B] time = 0.86, size = 280, normalized size = 3.29

$$x\sec^{-1}\left(ce^{a+bx}\right) - \frac{e^{-a-bx}\left(-4\sqrt{1-c^2e^{2(a+bx)}}\,\text{Li}_2\left(\frac{1}{2}\left(1-\sqrt{1-c^2e^{2(a+bx)}}\right)\right) + \sqrt{1-c^2e^{2(a+bx)}}\left(\log^2\left(c^2e^{2(a+bx)}\right) + 2\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[c*E^(a + b*x)],x]
```

```
[Out] x*ArcSec[c*E^(a + b*x)] - (E^(-a - b*x)*(4*Sqrt[-1 + c^2*E^(2*(a + b*x))]*A
rcTan[Sqrt[-1 + c^2*E^(2*(a + b*x))]]*(2*b*x - Log[c^2*E^(2*(a + b*x))]) +
Sqrt[1 - c^2*E^(2*(a + b*x))]*(Log[c^2*E^(2*(a + b*x))]^2 - 4*Log[c^2*E^(2*
(a + b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] + 2*Log[(1 + Sqrt[1
- c^2*E^(2*(a + b*x))])/2]^2) - 4*Sqrt[1 - c^2*E^(2*(a + b*x))]*PolyLog[2,
(1 - Sqrt[1 - c^2*E^(2*(a + b*x))])/2]))/(8*b*c*Sqrt[1 - 1/(c^2*E^(2*(a + b
*x)))])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcsec}\left(ce^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsec(c*e^(b*x + a)), x)
```

maple [A] time = 0.17, size = 116, normalized size = 1.36

$$\frac{i\operatorname{arcsec}\left(c\,e^{bx+a}\right)^2}{2b} - \frac{\operatorname{arcsec}\left(c\,e^{bx+a}\right)\ln\left(1+\left(\frac{e^{-bx-a}}{c}+i\sqrt{1-\frac{e^{-2bx-2a}}{c^2}}\right)^2\right)}{b} + \frac{i\operatorname{polylog}\left(2,-\left(\frac{e^{-bx-a}}{c}+i\sqrt{1-\frac{e^{-2bx-2a}}{c^2}}\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(c*exp(b*x+a)),x)
```

```
[Out] 1/2*I*arcsec(c*exp(b*x+a))^2/b-arcsec(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+I*
(1-1/c^2/exp(b*x+a)^2)^(1/2))^2)/b+1/2*I*polylog(2,-(1/c/exp(b*x+a)+I*(1-1/
c^2/exp(b*x+a)^2)^(1/2))^2)/b
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acos}\left(\frac{e^{-a-b\,x}}{c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(exp(- a - b*x)/c),x)
```

```
[Out] int(acos(exp(- a - b*x)/c), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}\left(ce^{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(c*exp(b*x+a)),x)
```

```
[Out] Integral(asec(c*exp(a + b*x)), x)
```

3.43 $\int e^{\sec^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=99

$$\frac{\left(\frac{24}{5}+\frac{8i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{3}{2}-\frac{i}{2},4;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3}-\frac{\left(\frac{12}{5}+\frac{4i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{3}{2}-\frac{i}{2},3;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right.}{a^3}$$

```
[Out] (-12/5-4/5*I)*exp((1+3*I)*arcsec(a*x))*hypergeom([3, 3/2-1/2*I],[5/2-1/2*I]
,-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^3+(24/5+8/5*I)*exp((1+3*I)*arcsec(a*x)
)*hypergeom([4, 3/2-1/2*I],[5/2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^
3
```

Rubi [A] time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5266, 12, 4471, 2251}

$$\frac{\left(\frac{24}{5}+\frac{8i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{3}{2}-\frac{i}{2},4;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3}-\frac{\left(\frac{12}{5}+\frac{4i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{3}{2}-\frac{i}{2},3;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right.}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]*x^2,x]
```

```
[Out] ((-12/5 - (4*I)/5)*E^((1 + 3*I)*ArcSec[a*x])*Hypergeometric2F1[3/2 - I/2, 3
, 5/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a^3 + ((24/5 + (8*I)/5)*E^((1 + 3*I)*
ArcSec[a*x])*Hypergeometric2F1[3/2 - I/2, 4, 5/2 - I/2, -E^((2*I)*ArcSec[a*
x])])/a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4471

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5266

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -(a/b) + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x
, ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sec^{-1}(ax)}x^2\,dx = \frac{\text{Subst}\left(\int \frac{e^x\sec^3(x)\tan(x)}{a^2}\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int e^x\sec^3(x)\tan(x)\,dx, x, \sec^{-1}(ax)\right)}{a^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{16ie^{(1+3i)x}}{(1+e^{2ix})^4} - \frac{8ie^{(1+3i)x}}{(1+e^{2ix})^3}\right)dx, x, \sec^{-1}(ax)\right)}{a^3}$$
$$= -\frac{(8i)\,\text{Subst}\left(\int \frac{e^{(1+3i)x}}{(1+e^{2ix})^3}\,dx, x, \sec^{-1}(ax)\right)}{a^3} + \frac{(16i)\,\text{Subst}\left(\int \frac{e^{(1+3i)x}}{(1+e^{2ix})^4}\,dx, x, \sec^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\left(\frac{12}{5}+\frac{4i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{3}{2}-\frac{i}{2},3;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3} + \frac{\left(\frac{24}{5}+\frac{8i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{3}{2}\right.}{a^3}$$

Mathematica [A] time = 0.31, size = 95, normalized size = 0.96

$$\frac{e^{\sec^{-1}(ax)}\left(a^4x^4\left(\cos\left(2\sec^{-1}(ax)\right)-\sin\left(2\sec^{-1}(ax)\right)+5\right)-(4+4i)\left(ax\sqrt{1-\frac{1}{a^2x^2}}-i\right){}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e\right.\right.}{12a^4x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSec[a*x]*x^2,x]
```

```
[Out] (E^ArcSec[a*x]*((-4 - 4*I)*(-I + a*Sqrt[1 - 1/(a^2*x^2)]*x)*Hypergeometric2
F1[1/2 - I/2, 1, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])] + a^4*x^4*(5 + Cos[2*Ar
cSec[a*x]] - Sin[2*ArcSec[a*x]])))/(12*a^4*x)
```

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2e^{(\text{arcsec}(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(arcsec(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2e^{(\text{arcsec}(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(arcsec(a*x)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{\text{arcsec}(ax)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))*x^2,x)
```

```
[Out] int(exp(arcsec(a*x))*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{(\mathrm{arcsec}(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*e^(arcsec(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(acos(1/(a*x))),x)
[Out] int(x^2*exp(acos(1/(a*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\mathrm{asec}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))*x**2,x)
[Out] Integral(x**2*exp(asec(a*x)), x)
```

3.44 $\int e^{\sec^{-1}(ax)}x\,dx$

Optimal. Leaf size=91

$$\frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},3;2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^2}-\frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},2;2-\frac{i}{2};-e^{2i\sec^{-1}(a\ldots}\right.}{a^2}$$

```
[Out] (-8/5-4/5*I)*exp((1+2*I)*arcsec(a*x))*hypergeom([2, 1-1/2*I],[2-1/2*I],-(1/
a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^2+(16/5+8/5*I)*exp((1+2*I)*arcsec(a*x))*hyp
ergeom([3, 1-1/2*I],[2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^2
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5266, 12, 4471, 2251}

$$\frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},3;2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^2}-\frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},2;2-\frac{i}{2};-e^{2i\sec^{-1}(a\ldots}\right.}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]*x,x]
```

```
[Out] ((-8/5 - (4*I)/5)*E^((1 + 2*I)*ArcSec[a*x])*Hypergeometric2F1[1 - I/2, 2, 2
 - I/2, -E^((2*I)*ArcSec[a*x])])/a^2 + ((16/5 + (8*I)/5)*E^((1 + 2*I)*ArcSe
c[a*x])*Hypergeometric2F1[1 - I/2, 3, 2 - I/2, -E^((2*I)*ArcSec[a*x])])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4471

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5266

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -(a/b) + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x
, ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sec^{-1}(ax)}x\,dx = \frac{\text{Subst}\left(\int \frac{e^x\sec^2(x)\tan(x)}{a}\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int e^x\sec^2(x)\tan(x)\,dx, x, \sec^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{8ie^{(1+2i)x}}{(1+e^{2ix})^3} - \frac{4ie^{(1+2i)x}}{(1+e^{2ix})^2}\right)dx, x, \sec^{-1}(ax)\right)}{a^2}$$
$$= -\frac{(4i)\,\text{Subst}\left(\int \frac{e^{(1+2i)x}}{(1+e^{2ix})^2}\,dx, x, \sec^{-1}(ax)\right)}{a^2} + \frac{(8i)\,\text{Subst}\left(\int \frac{e^{(1+2i)x}}{(1+e^{2ix})^3}\,dx, x, \sec^{-1}(ax)\right)}{a^2}$$
$$= -\frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},2;2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^2} + \frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},\right.}{a^2}$$

Mathematica [A] time = 0.20, size = 107, normalized size = 1.18

$$\frac{\left(\frac{1}{5}+\frac{i}{10}\right)e^{\sec^{-1}(ax)}\left((-2+i)ax\left(\sqrt{1-\frac{1}{a^2x^2}}-ax\right)+(1+2i)\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)-e^{2i\sec^{-1}(ax)}\,{}_2F_1\left(1,1-\frac{i}{2}\right.\right.}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSec[a*x]*x,x]
[Out] ((1/5 + I/10)*E^ArcSec[a*x]*((-2 + I)*a*x*(Sqrt[1 - 1/(a^2*x^2)] - a*x) + (
1 + 2*I)*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcSec[a*x])] - E^
((2*I)*ArcSec[a*x])*Hypergeometric2F1[1, 1 - I/2, 2 - I/2, -E^((2*I)*ArcSec
[a*x])]))/a^2
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(xe^{(\text{arcsec}(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x,x, algorithm="fricas")
[Out] integral(x*e^(arcsec(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{(\text{arcsec}(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x,x, algorithm="giac")
[Out] integrate(x*e^(arcsec(a*x)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{\text{arcsec}(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))*x,x)
```

```
[Out] int(exp(arcsec(a*x))*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{(\operatorname{arcsec}(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*e^(arcsec(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(acos(1/(a*x))),x)
[Out] int(x*exp(acos(1/(a*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\operatorname{asec}{(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))*x,x)
[Out] Integral(x*exp(asec(a*x)), x)
```

3.45 $\int e^{\sec^{-1}(ax)}\,dx$

Optimal. Leaf size=91

$$\frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}-\frac{(1+i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}$$

```
[Out] (-1-I)*exp((1+I)*arcsec(a*x))*hypergeom([1, 1/2-1/2*I],[3/2-1/2*I],-(1/a/x+
I*(1-1/a^2/x^2)^(1/2))^2)/a+(2+2*I)*exp((1+I)*arcsec(a*x))*hypergeom([2, 1/
2-1/2*I],[3/2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5266, 4471, 2251}

$$\frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}-\frac{(1+i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x],x]
```

```
[Out] ((-1 - I)*E^((1 + I)*ArcSec[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2
, -E^((2*I)*ArcSec[a*x])])/a + ((2 + 2*I)*E^((1 + I)*ArcSec[a*x])*Hypergeom
etric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4471

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5266

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -(a/b) + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x
, ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sec^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int e^x \sec(x)\tan(x)\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{4ie^{(1+i)x}}{(1+e^{2ix})^2} - \frac{2ie^{(1+i)x}}{1+e^{2ix}}\right)dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= -\frac{(2i)\,\text{Subst}\left(\int \frac{e^{(1+i)x}}{1+e^{2ix}}\,dx, x, \sec^{-1}(ax)\right)}{a} + \frac{(4i)\,\text{Subst}\left(\int \frac{e^{(1+i)x}}{(1+e^{2ix})^2}\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= -\frac{(1+i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a} + \frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}\right.}{a}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.59

$$xe^{\sec^{-1}(ax)} - \frac{(1-i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSec[a*x],x]
[Out] E^ArcSec[a*x]*x - ((1 - I)*E^((1 + I)*ArcSec[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(e^{(\text{arcsec}(ax))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x)),x, algorithm="fricas")
[Out] integral(e^(arcsec(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(\text{arcsec}(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x)),x, algorithm="giac")
[Out] integrate(e^(arcsec(a*x)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int e^{\text{arcsec}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x)),x)
[Out] int(exp(arcsec(a*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(\text{arcsec}(ax))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsec(a*x)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x))),x)
```

```
[Out] int(exp(acos(1/(a*x))), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{asec}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x)),x)
```

```
[Out] Integral(exp(asec(a*x)), x)
```

3.46 $\int \frac{e^{\sec^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=45

$$2ie^{\sec^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)-ie^{\sec^{-1}(ax)}$$

```
[Out] -I*exp(arcsec(a*x))+2*I*exp(arcsec(a*x))*hypergeom([1, -1/2*I],[1-1/2*I],-(
1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5266, 12, 4442, 2194, 2251}

$$2ie^{\sec^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)-ie^{\sec^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x,x]
```

```
[Out] (-I)*E^ArcSec[a*x] + (2*I)*E^ArcSec[a*x]*Hypergeometric2F1[-I/2, 1, 1 - I/2
, -E^((2*I)*ArcSec[a*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 5266

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -(a/b) + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x
, ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{\sec^{-1}(ax)}}{x}\,dx = \frac{\operatorname{Subst}\left(\int ae^x \tan(x)\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= \operatorname{Subst}\left(\int e^x \tan(x)\,dx, x, \sec^{-1}(ax)\right)$$
$$= i\operatorname{Subst}\left(\int \left(-e^x + \frac{2e^x}{1+e^{2ix}}\right)dx, x, \sec^{-1}(ax)\right)$$
$$= -\left(i\operatorname{Subst}\left(\int e^x\,dx, x, \sec^{-1}(ax)\right)\right) + 2i\operatorname{Subst}\left(\int \frac{e^x}{1+e^{2ix}}\,dx, x, \sec^{-1}(ax)\right)$$
$$= -ie^{\sec^{-1}(ax)} + 2ie^{\sec^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)$$

Mathematica [A] time = 0.05, size = 79, normalized size = 1.76

$$-i\left(\left(\frac{1}{5}-\frac{2i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1,1-\frac{i}{2};2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)-e^{\sec^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcSec[a*x]/x,x]
```

```
[Out] (-I)*(-(E^ArcSec[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcSe
c[a*x])]) + (1/5 - (2*I)/5)*E^((1 + 2*I)*ArcSec[a*x])*Hypergeometric2F1[1,
1 - I/2, 2 - I/2, -E^((2*I)*ArcSec[a*x])])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{e^{(\operatorname{arcsec}(ax))}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arcsec(a*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\operatorname{arcsec}(ax))}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(e^(arcsec(a*x))/x, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{arcsec}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x,x)
```

```
[Out] int(exp(arcsec(a*x))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\mathrm{arcsec}(ax))}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsec(a*x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x,x)
```

```
[Out] int(exp(acos(1/(a*x)))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\mathrm{asec}\,(ax)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x,x)
```

```
[Out] Integral(exp(asec(a*x))/x, x)
```

3.47 $\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\,e^{\sec^{-1}(ax)}-\frac{e^{\sec^{-1}(ax)}}{2x}$$

```
[Out] -1/2*exp(arcsec(a*x))/x+1/2*a*exp(arcsec(a*x))*(1-1/a^2/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5266, 12, 4432}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\,e^{\sec^{-1}(ax)}-\frac{e^{\sec^{-1}(ax)}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x^2,x]
```

```
[Out] (a*E^ArcSec[a*x]*Sqrt[1 - 1/(a^2*x^2)])/2 - E^ArcSec[a*x]/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 5266

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -(a/b) + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x
, ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx &= \frac{\text{Subst}\left(\int a^2 e^x \sin(x)\,dx, x, \sec^{-1}(ax)\right)}{a}\\ &= a\,\text{Subst}\left(\int e^x \sin(x)\,dx, x, \sec^{-1}(ax)\right)\\ &= \frac{1}{2}ae^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{e^{\sec^{-1}(ax)}}{2x}\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.87

$$\frac{1}{2}a\left(\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{ax}\right)e^{\sec^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]/x^2,x]
```

```
[Out] (a*E^ArcSec[a*x]*(Sqrt[1 - 1/(a^2*x^2)] - 1/(a*x)))/2
```

fricas [A] time = 1.42, size = 23, normalized size = 0.59

$$\frac{\left(\sqrt{a^2x^2-1}-1\right)e^{(\mathrm{arcsec}(ax))}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 - 1) - 1)*e^(arcsec(a*x))/x
```

giac [A] time = 0.20, size = 51, normalized size = 1.31

$$\frac{1}{2}\,a\sqrt{-\frac{1}{a^2x^2}+1}\,e^{\left(\frac{1}{2}\,\pi-\arcsin\left(\frac{1}{ax}\right)\right)}-\frac{e^{\left(\frac{1}{2}\,\pi-\arcsin\left(\frac{1}{ax}\right)\right)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a*sqrt(-1/(a^2*x^2) + 1)*e^(1/2*pi - arcsin(1/(a*x))) - 1/2*e^(1/2*pi -
 arcsin(1/(a*x)))/x
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\mathrm{arcsec}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x^2,x)
```

```
[Out] int(exp(arcsec(a*x))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\mathrm{arcsec}(ax))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsec(a*x))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x^2,x)
```

```
[Out] int(exp(acos(1/(a*x)))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asec}{(ax)}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x**2,x)
```

```
[Out] Integral(exp(asec(a*x))/x**2, x)
```

3.48 $\int \frac{e^{\sec^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=41

$$\frac{1}{10}a^2e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)-\frac{1}{5}a^2e^{\sec^{-1}(ax)}\cos\left(2\sec^{-1}(ax)\right)$$

```
[Out] -1/5*a^2*exp(arcsec(a*x))*cos(2*arcsec(a*x))+1/10*a^2*exp(arcsec(a*x))*sin(
2*arcsec(a*x))
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5266, 12, 4469, 4432}

$$\frac{1}{10}a^2e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)-\frac{1}{5}a^2e^{\sec^{-1}(ax)}\cos\left(2\sec^{-1}(ax)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x^3,x]
```

```
[Out] -(a^2*E^ArcSec[a*x]*Cos[2*ArcSec[a*x]])/5 + (a^2*E^ArcSec[a*x]*Sin[2*ArcSec
[a*x]])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5266

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -(a/b) + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x
, ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{\sec^{-1}(ax)}}{x^3}\,dx = \frac{\text{Subst}\left(\int a^3 e^x \cos(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= a^2\,\text{Subst}\left(\int e^x \cos(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)$$
$$= a^2\,\text{Subst}\left(\int \frac{1}{2}e^x \sin(2x)\,dx, x, \sec^{-1}(ax)\right)$$
$$= \frac{1}{2}a^2\,\text{Subst}\left(\int e^x \sin(2x)\,dx, x, \sec^{-1}(ax)\right)$$
$$= -\frac{1}{5}a^2 e^{\sec^{-1}(ax)}\cos\left(2\sec^{-1}(ax)\right) + \frac{1}{10}a^2 e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)$$

Mathematica [**A**] time = 0.04, size = 30, normalized size = 0.73

$$\frac{1}{10}a^2 e^{\sec^{-1}(ax)}\left(\sin\left(2\sec^{-1}(ax)\right) - 2\cos\left(2\sec^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]/x^3,x]
[Out] (a^2*E^ArcSec[a*x]*(-2*Cos[2*ArcSec[a*x]] + Sin[2*ArcSec[a*x]]))/10
```

fricas [**A**] time = 1.05, size = 30, normalized size = 0.73

$$\frac{\left(a^2x^2 + \sqrt{a^2x^2-1} - 2\right)e^{(\text{arcsec}(ax))}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^3,x, algorithm="fricas")
[Out] 1/5*(a^2*x^2 + sqrt(a^2*x^2 - 1) - 2)*e^(arcsec(a*x))/x^2
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\text{arcsec}(ax))}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^3,x, algorithm="giac")
[Out] integrate(e^(arcsec(a*x))/x^3, x)
```

maple [**F**] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\text{arcsec}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x^3,x)
[Out] int(exp(arcsec(a*x))/x^3,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\text{arcsec}(ax))}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsec(a*x))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x^3,x)
```

```
[Out] int(exp(acos(1/(a*x)))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asec}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x**3,x)
```

```
[Out] Integral(exp(asec(a*x))/x**3, x)
```

3.49 $\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{3}{40}a^3e^{\sec^{-1}(ax)}\cos\left(3\sec^{-1}(ax)\right)+\frac{1}{40}a^3e^{\sec^{-1}(ax)}\sin\left(3\sec^{-1}(ax)\right)-\frac{a^2e^{\sec^{-1}(ax)}}{8x}+\frac{1}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\,e^{\sec^{-1}(ax)}$$

```
[Out] -1/8*a^2*exp(arcsec(a*x))/x-3/40*a^3*exp(arcsec(a*x))*cos(3*arcsec(a*x))+1/
40*a^3*exp(arcsec(a*x))*sin(3*arcsec(a*x))+1/8*a^3*exp(arcsec(a*x))*(1-1/a^
2/x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5266, 12, 4469, 4432}

$$\frac{1}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\,e^{\sec^{-1}(ax)}-\frac{a^2e^{\sec^{-1}(ax)}}{8x}-\frac{3}{40}a^3e^{\sec^{-1}(ax)}\cos\left(3\sec^{-1}(ax)\right)+\frac{1}{40}a^3e^{\sec^{-1}(ax)}\sin\left(3\sec^{-1}(ax)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x^4,x]
```

```
[Out] (a^3*E^ArcSec[a*x]*Sqrt[1 - 1/(a^2*x^2)])/8 - (a^2*E^ArcSec[a*x])/(8*x) - (
3*a^3*E^ArcSec[a*x]*Cos[3*ArcSec[a*x]])/40 + (a^3*E^ArcSec[a*x]*Sin[3*ArcSe
c[a*x]])/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5266

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -(a/b) + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x
, ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{\sec^{-1}(ax)}}{x^4} dx = \frac{\text{Subst}\left(\int a^4 e^x \cos^2(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= a^3 \text{Subst}\left(\int e^x \cos^2(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)$$
$$= a^3 \text{Subst}\left(\int \left(\frac{1}{4}e^x \sin(x) + \frac{1}{4}e^x \sin(3x)\right) dx, x, \sec^{-1}(ax)\right)$$
$$= \frac{1}{4}a^3 \text{Subst}\left(\int e^x \sin(x)\,dx, x, \sec^{-1}(ax)\right) + \frac{1}{4}a^3 \text{Subst}\left(\int e^x \sin(3x)\,dx, x, \sec^{-1}(ax)\right)$$
$$= \frac{1}{8}a^3 e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^2 e^{\sec^{-1}(ax)}}{8x} - \frac{3}{40}a^3 e^{\sec^{-1}(ax)} \cos\left(3\sec^{-1}(ax)\right) + \frac{1}{40}a^3 e^{\sec^{-1}(ax)} \sin\left(3\,\mathrm{se}\right.$$

Mathematica [A] time = 0.14, size = 54, normalized size = 0.64

$$\frac{1}{40}a^3 e^{\sec^{-1}(ax)}\left(5\sqrt{1-\frac{1}{a^2x^2}} - \frac{5}{ax} - 3\cos\left(3\sec^{-1}(ax)\right) + \sin\left(3\sec^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]/x^4,x]
[Out] (a^3*E^ArcSec[a*x]*(5*Sqrt[1 - 1/(a^2*x^2)] - 5/(a*x) - 3*Cos[3*ArcSec[a*x]
] + Sin[3*ArcSec[a*x]]))/40
```

fricas [A] time = 1.84, size = 40, normalized size = 0.48

$$\frac{\left(a^2x^2 + \left(a^2x^2+1\right)\sqrt{a^2x^2-1} - 3\right)e^{(\mathrm{arcsec}(ax))}}{10\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^4,x, algorithm="fricas")
[Out] 1/10*(a^2*x^2 + (a^2*x^2 + 1)*sqrt(a^2*x^2 - 1) - 3)*e^(arcsec(a*x))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\mathrm{arcsec}(ax))}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^4,x, algorithm="giac")
[Out] integrate(e^(arcsec(a*x))/x^4, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\mathrm{arcsec}(ax)}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x^4,x)
[Out] int(exp(arcsec(a*x))/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{(\mathrm{arcsec}(ax))}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^4,x, algorithm="maxima")
[Out] integrate(e^(arcsec(a*x))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x^4,x)
[Out] int(exp(acos(1/(a*x)))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asec}{\left(a x \right)}}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x**4,x)
[Out] Integral(exp(asec(a*x))/x**4, x)
```

3.50 $\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=69

$$\frac{i\text{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right)}{2d}+\frac{i\sec^{-1}(a+bx)^2}{2d}-\frac{\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{d}$$

```
[Out] 1/2*I*arcsec(b*x+a)^2/d-arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/
2))^2)/d+1/2*I*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/d
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5256, 12, 5218, 4626, 3719, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)}{2d}+\frac{i\sec^{-1}(a+bx)^2}{2d}-\frac{\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*ArcSec[a + b*x]^2)/d - (ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a +
b*x])])/d + ((I/2)*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4626

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n/Cot[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
```

```
]
```

Rule 5218

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 5256

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcSec[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{b\sec^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\sec^{-1}(x)}{x}\,dx, x, a+bx\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x}\,dx, x, \frac{1}{a+bx}\right)}{d} \\
&= \frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{1}{a+bx}\right)\right)}{d} \\
&= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{(2i)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{1}{a+bx}\right)\right)}{d} \\
&= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cos^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, c\right.}{d} \\
&= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cos^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} - \frac{i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-}}\right.}{2d} \\
&= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cos^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.86

$$
\frac{i\left(\text{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right)+\sec^{-1}(a+bx)\left(\sec^{-1}(a+bx)+2i\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*(ArcSec[a + b*x]*(ArcSec[a + b*x] + (2*I)*Log[1 + E^((2*I)*ArcSec[a
+ b*x])]) + PolyLog[2, -E^((2*I)*ArcSec[a + b*x])]))/d
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{b\,\text{arcsec}\,(bx+a)}{bdx+ad}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsec(b*x + a)/(b*d*x + a*d), x)
```

giac [A] time = 0.59, size = 115, normalized size = 1.67

$$-\frac{1}{4}b^2\left(\frac{2\,(bx+a)^2\arccos\left(\frac{1}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)\left(\frac{a}{bx+a}-1\right)+a}\right)}{b^3d}-\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)-\frac{1}{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)}}{b^3d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(2*(b*x + a)^2*arccos(1/(((b*x + a)*(a/(b*x + a) - 1) - a)*(a/(b*x
 + a) - 1) + a))/(b^3*d) - ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 1/((
b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1)))/(b^3*d))
```

maple [A] time = 0.35, size = 92, normalized size = 1.33

$$\frac{i\mathrm{arcsec}\,(bx+a)^2}{2d}-\frac{\mathrm{arcsec}\,(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{d}+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] 1/2*I*arcsec(b*x+a)^2/d-arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/
2))^2)/d+1/2*I*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{-i\left(b\left(\frac{\log(bx+a+1)\log(bx+a)-\log(bx+a)^2+\log(bx+a)\log(bx+a-1)}{b^2d}-\frac{\log(bx+a+1)\log(bx+a)+\mathrm{Li}_2(-bx-a)}{b^2d}-\frac{\log(bx+a)\log(-bx-a+1)+\mathrm{Li}_2(bx+}{b^2d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*integrate(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*log(b*x + a)/(b^3
*d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^3 - a)*d), x) + 2*I*b*d*int
egrate(log(b*x + a)/(b^3*d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^3 -
 a)*d), x) - 2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))*log(b*x + a) + I
*log(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) - I*log(b*x + a + 1)*log(b*x + a
) - I*log(b*x + a)^2 - I*log(b*x + a)*log(-b*x - a + 1) - I*dilog(b*x + a)
- I*dilog(-b*x - a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/(d*x + (a*d)/b),x)
```

```
[Out] int(acos(1/(a + b*x))/(d*x + (a*d)/b), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{\operatorname{asec}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/(a*d/b+d*x),x)
[Out] b*Integral(asec(a + b*x)/(a + b*x), x)/d
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```